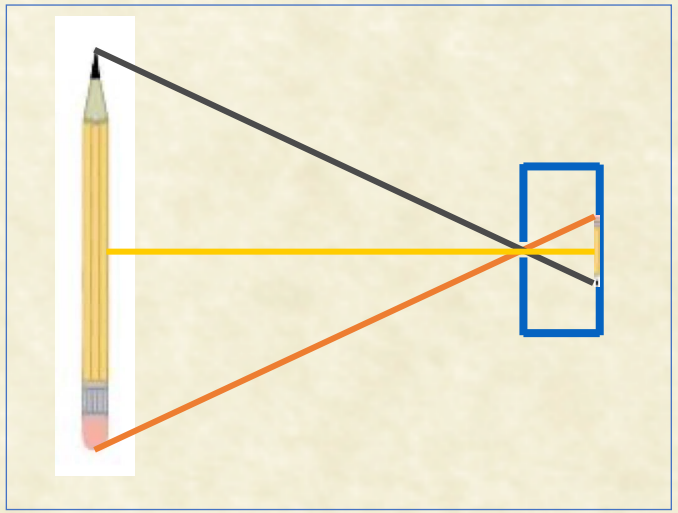
CSE 578: Computer Vision

Spring 2021: Pinhole Camera Model



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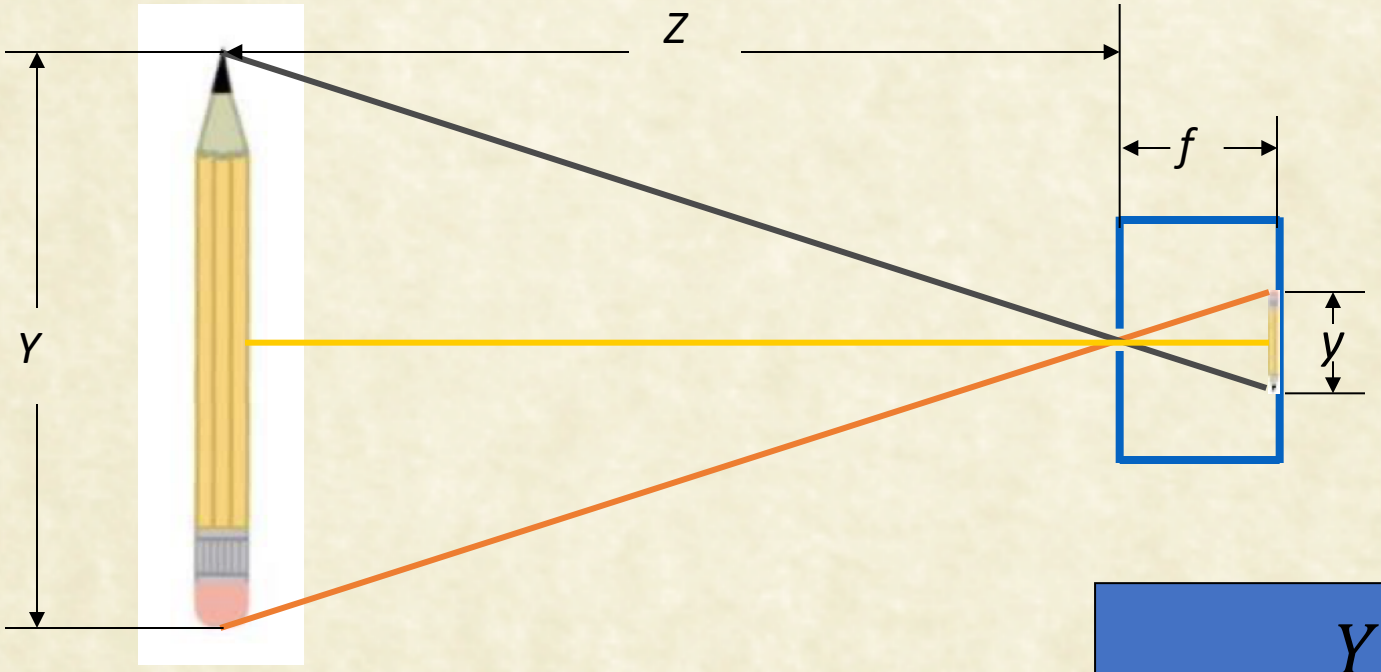
The Pinhole Camera

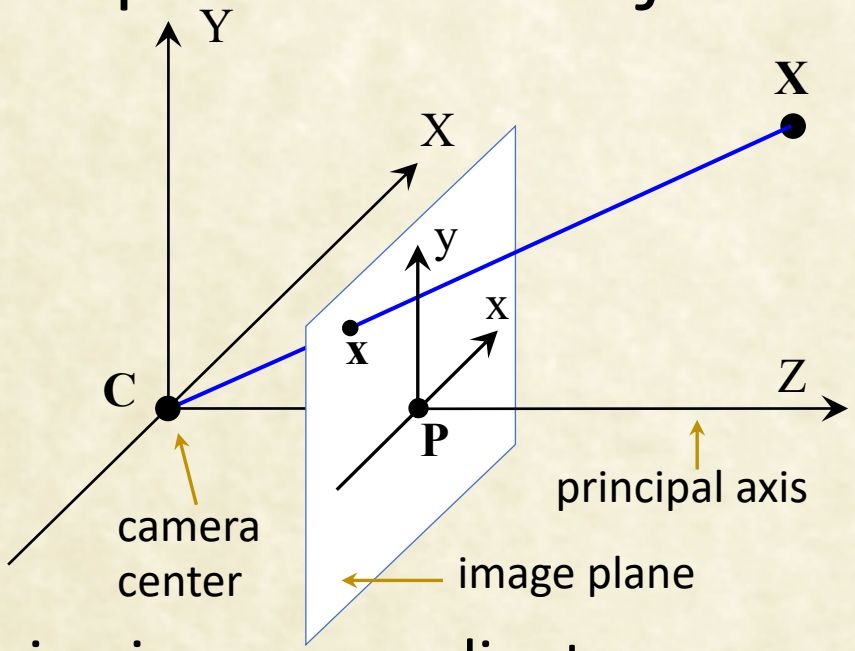


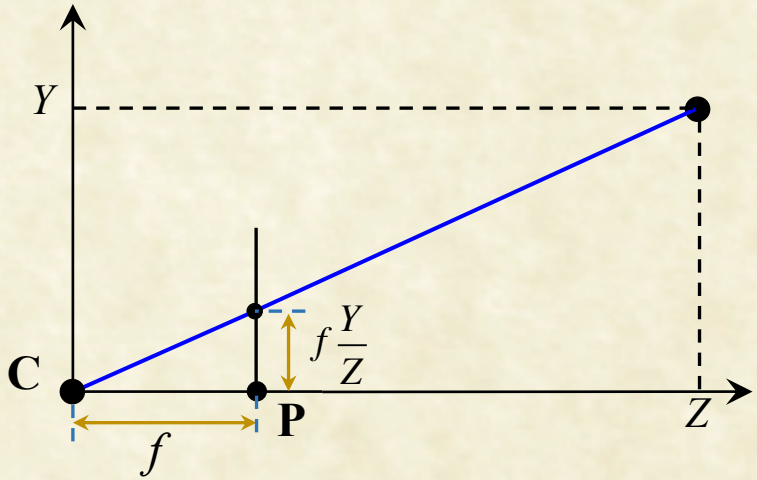
$$y = f\frac{Y}{Z}$$

Problem

- You have a person who is **1.75m** tall standing at a distance of **7m** from a camera. The pinhole camera has a focal length of **20mm**. The sensor is 1cm tall and has a resolution of **4000x3000**.
  - Find the height of the person in pixels in the image.
  - If the camera is raised by **1m**, how much does the person move in the sensor (in pixels)?
  - How much does the Sun move in the above case
    Note: Sun is **150 million kms** away (in pixels)?

Perspective Projection





- Cartesian image coordinates:

$$x = f\frac{X}{Z}, \qquad y = f\frac{Y}{Z}$$

- In matrix form (homogeneous):

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{PX}$$

Basic Camera Equation

A pinhole camera projects a 3D point $\mathbf{X_c}$ in camera coords to an image point **x** via the 3x4 camera matrix **P** as:

$$\mathbf{x} = \mathbf{P}\mathbf{X}_{\boldsymbol{c}} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{I}|\mathbf{0}]\mathbf{X}_{\boldsymbol{c}} = \mathbf{K}[\mathbf{I}|\mathbf{0}]\mathbf{X}_{\mathbf{c}},$$

where **K** is the internal camera calibration matrix.

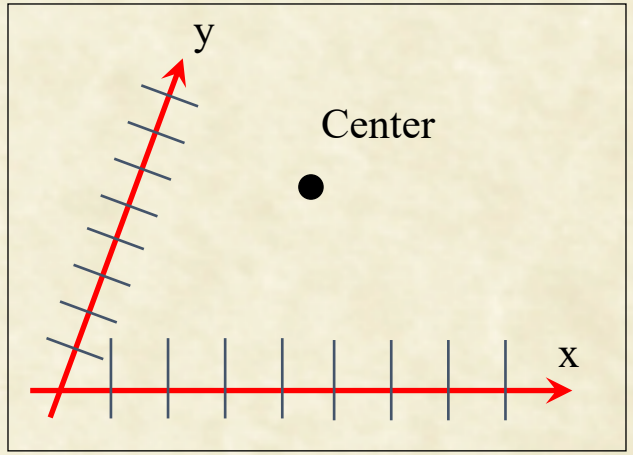
Note that:

- The camera is at the origin
- Z is the Camera or Optical axis
- Principal Point: Center of the image
- Focal length in pixel units
- Orthogonal image axes with uniform scale

A General Camera

Image center at (x_0,y_0), Non-orthogonal axes with skew s, and different scales for axes with focal lengths, α_x and α_y.

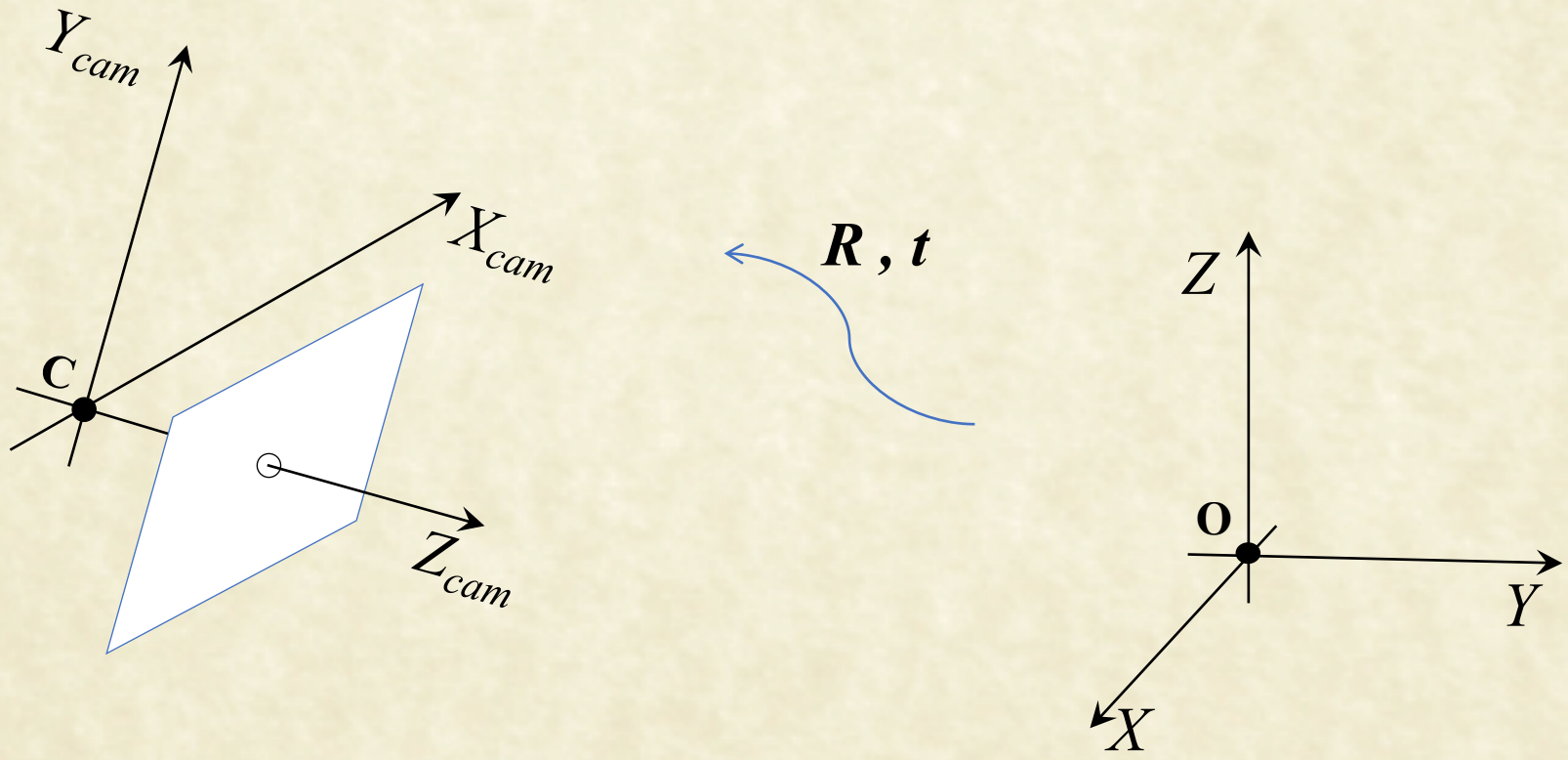
$$\mathbf{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$



$\mathbf{K}$ an upper diagonal matrix with 5 degrees of freedom.

Moving the Camera from Origin

- General Setting: Camera is not at origin and Z is not the optical axis.
- Camera is at a point C in world coordinates. The camera axes are also rotated by a matrix R.



General Camera Equation

- Camera and world are related by: $\mathbf{X_c} = \begin{bmatrix} \mathbf{R} & \mathbf{-RC} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \mathbf{X_w}$
- 2D projection $\mathbf{x}$ of a 3D point $\mathbf{X_w}$ given by:
  - $\mathbf{x = K[\ I \mid 0\ ]\ X_c = K\ [\ R \mid -RC\ ]\ X_w}$
- $\mathbf{x = PX_w}$; camera matrix $\mathbf{P = [KR \mid -KRC] = [M \mid p_4]}$
- Common K:

$$\begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

General K:

$$\begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Questions?

Motion of Object in Image