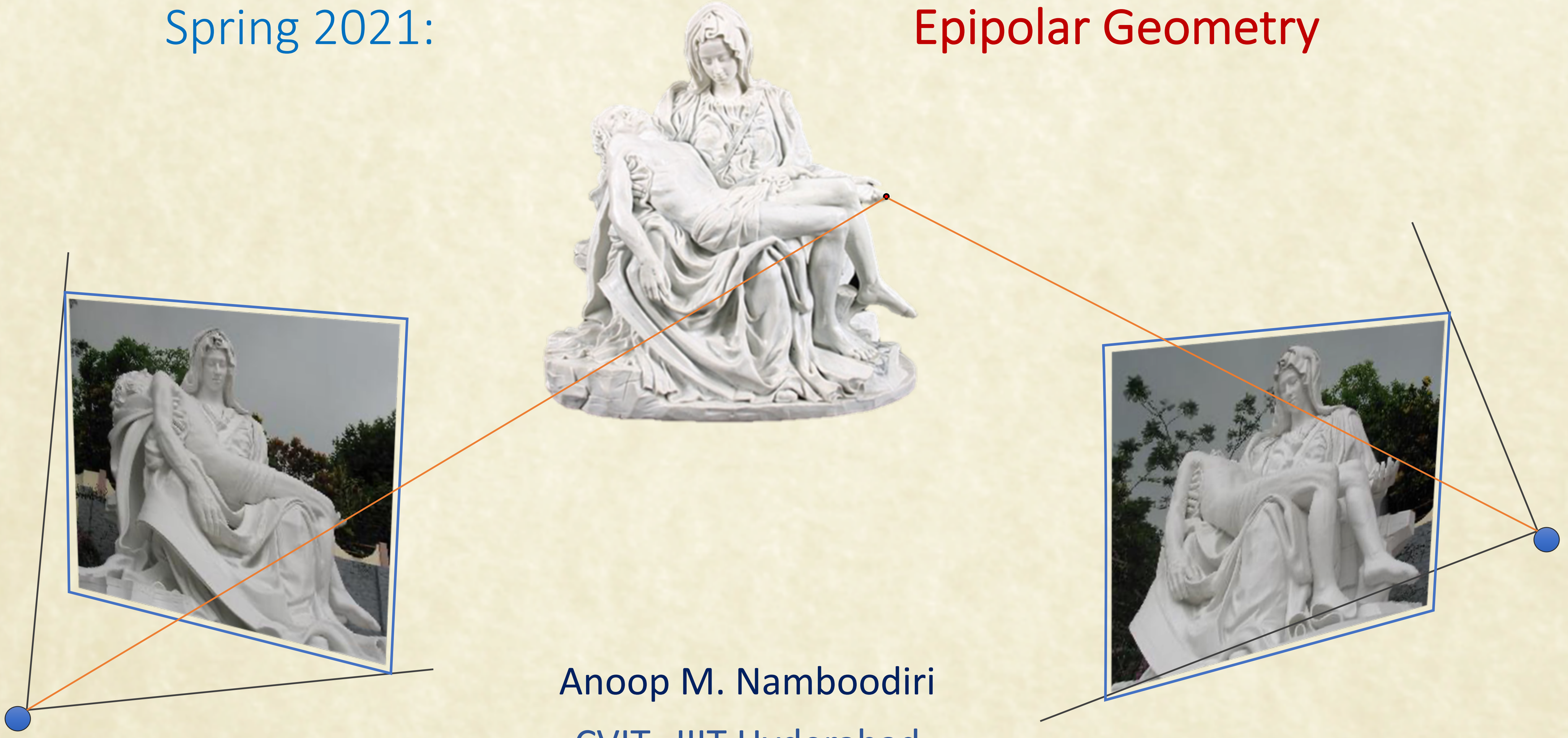
CSE 578: Computer Vision

Spring 2021: Epipolar Geometry



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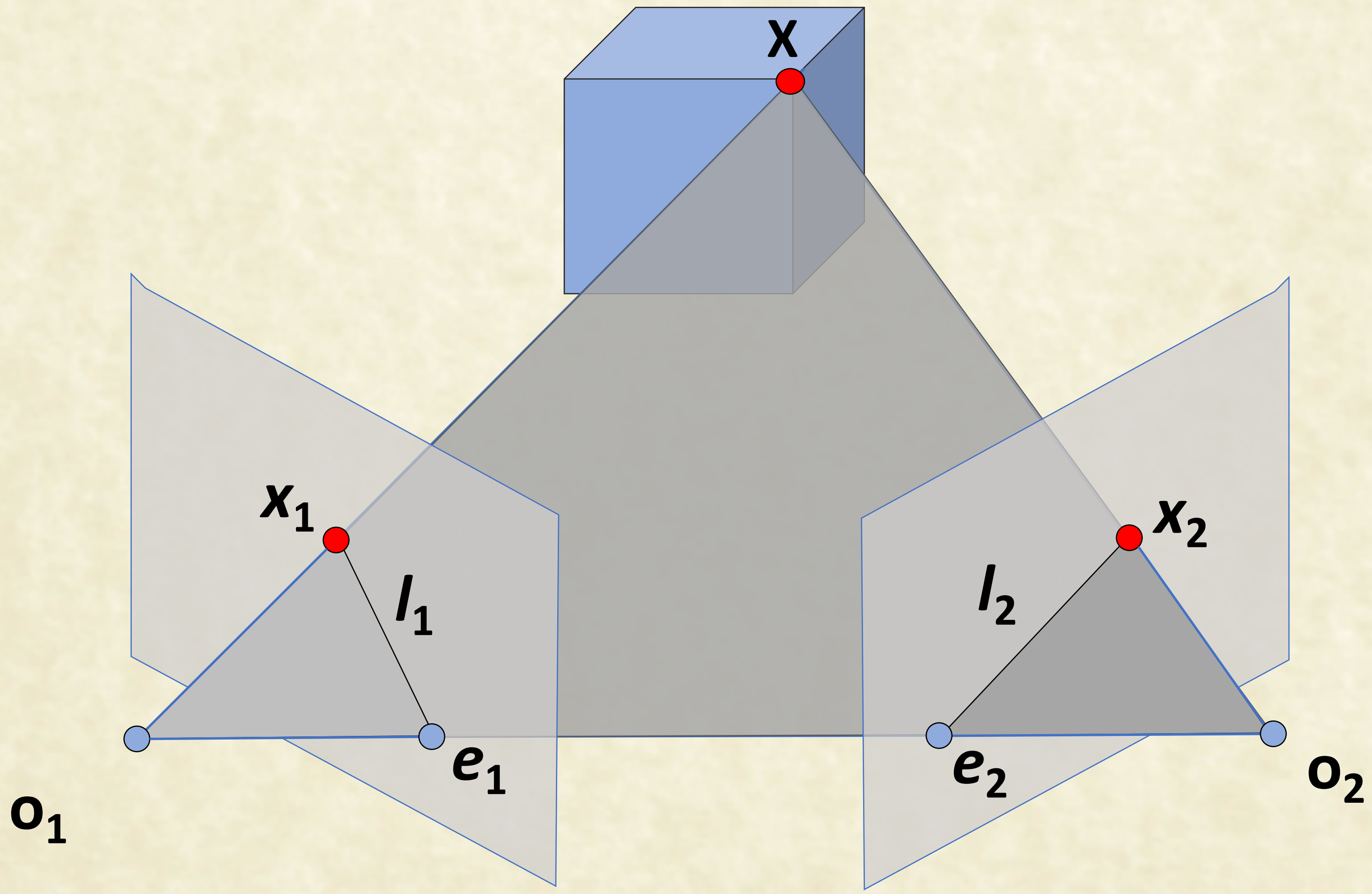
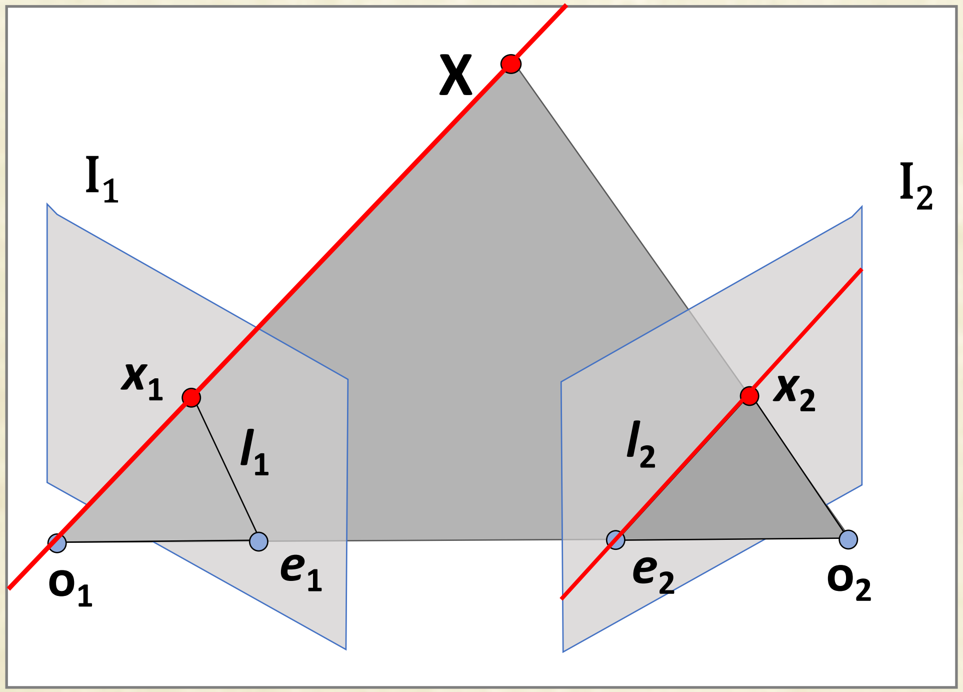
Cross Product: A Recap

- Consider $\mathbf{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$; and Let $\widehat{\mathbf{A}} = \begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{bmatrix}$
- $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{bmatrix} A_y B_z - B_y A_z \\ B_x A_z - A_x B_z \\ A_x B_y - B_x A_y \end{bmatrix}$
- $\widehat{\mathbf{A}}\mathbf{B} = \begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} A_y B_z - B_y A_z \\ B_x A_z - A_x B_z \\ A_x B_y - B_x A_y \end{bmatrix}$
- Note: The cross product, AxB or ÂB is a vector perpendicular to both A and B

Case 3: Generic World and Cameras



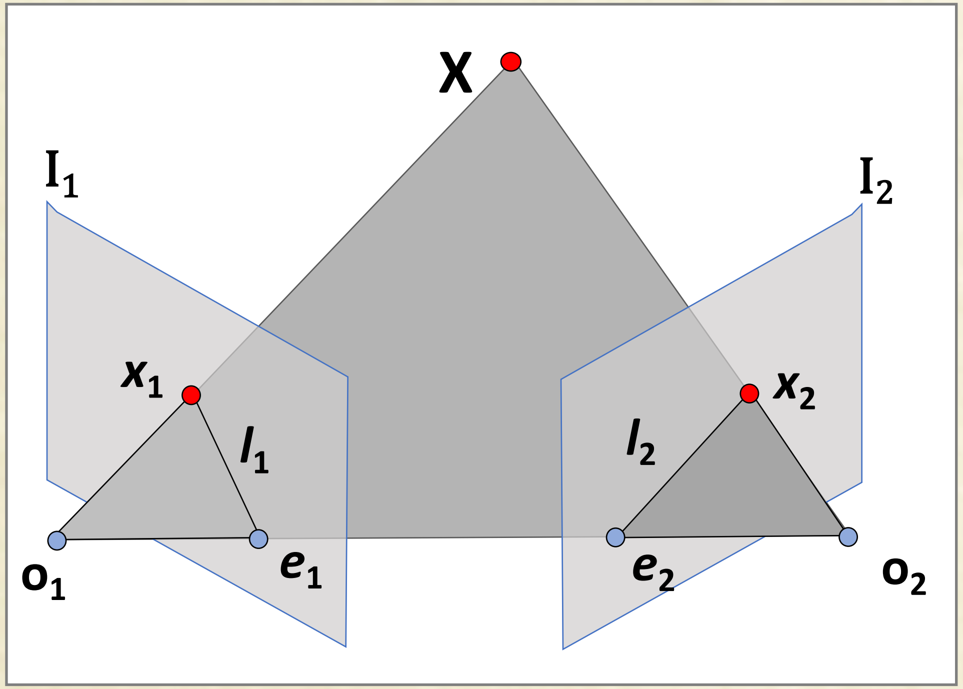
Epipolar Geometry



- All world points that map to $\boldsymbol{x}_1$ in $\mathbf{I}_1$ (pre-image of $\boldsymbol{x}_1$) map to a line $\boldsymbol{l}_2$ in $\mathbf{I}_2$, called an epipolar line. (so is $\boldsymbol{l}_1$)
- The image of $\boldsymbol{o}_1$ in $\mathbf{I}_2$ ($\boldsymbol{e}_2$) is an epipole. (so is $\boldsymbol{e}_1$)

- The plane containing these is called the epipolar plane.
- These result in a set of constraints, which are referred to as the *epipolar constraints* and the resulting geometry is called the *epipolar geometry.*

Epipolar Constraint: Essential Matrix



- Consider $\mathbf{X}$ in camera 1's coordinates:
  - $\lambda_1 \boldsymbol{x}_1 = \mathbf{X}$
- Now, viewing it in camera 2's coordinates:
- $\lambda_2 \boldsymbol{x}_2 = \mathbf{RX} + \mathbf{T}$

$$= \mathbf{R}(\lambda_1 \boldsymbol{x}_1) + \mathbf{T}$$

Pre-multiplying by $\hat{\mathbf{T}}$, and then by $\boldsymbol{x}_2{}^T$, we get

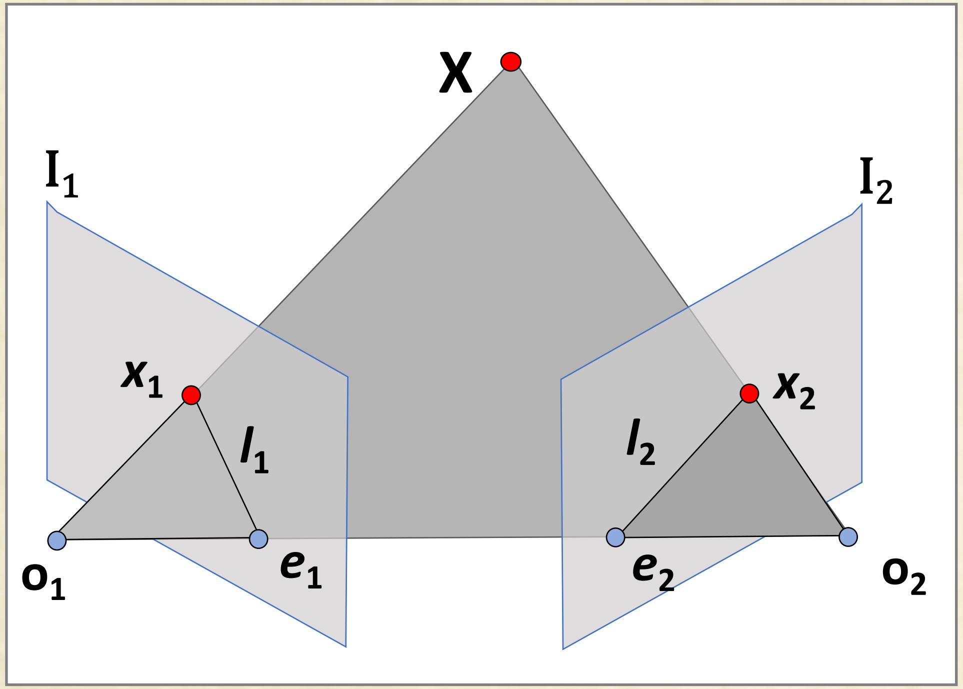
$$\hat{\mathbf{T}}\lambda_2 \boldsymbol{x}_2 = \hat{\mathbf{T}}\mathbf{R}\lambda_1 \boldsymbol{x}_1 + \cancel{\hat{\mathbf{T}}\mathbf{T}}$$

$$\lambda_2 \cancel{\boldsymbol{x}_2{}^T \hat{\mathbf{T}} \boldsymbol{x}_2} = \lambda_1 \boldsymbol{x}_2{}^T \hat{\mathbf{T}}\mathbf{R}\boldsymbol{x}_1 + 0$$

$$\boldsymbol{x}_2{}^T \hat{\mathbf{T}}\mathbf{R}\boldsymbol{x}_1 = 0$$

$$\boldsymbol{x}_2{}^T \mathbf{E}\boldsymbol{x}_1 = 0 \quad \text{or} \quad \boldsymbol{x}_1{}^T \mathbf{E}\boldsymbol{x}_2 = 0$$

Epipolar Constraint: Fundamental Matrix



$$\mathbf{x}_1 = \mathbf{K}_1\mathbf{X}$$

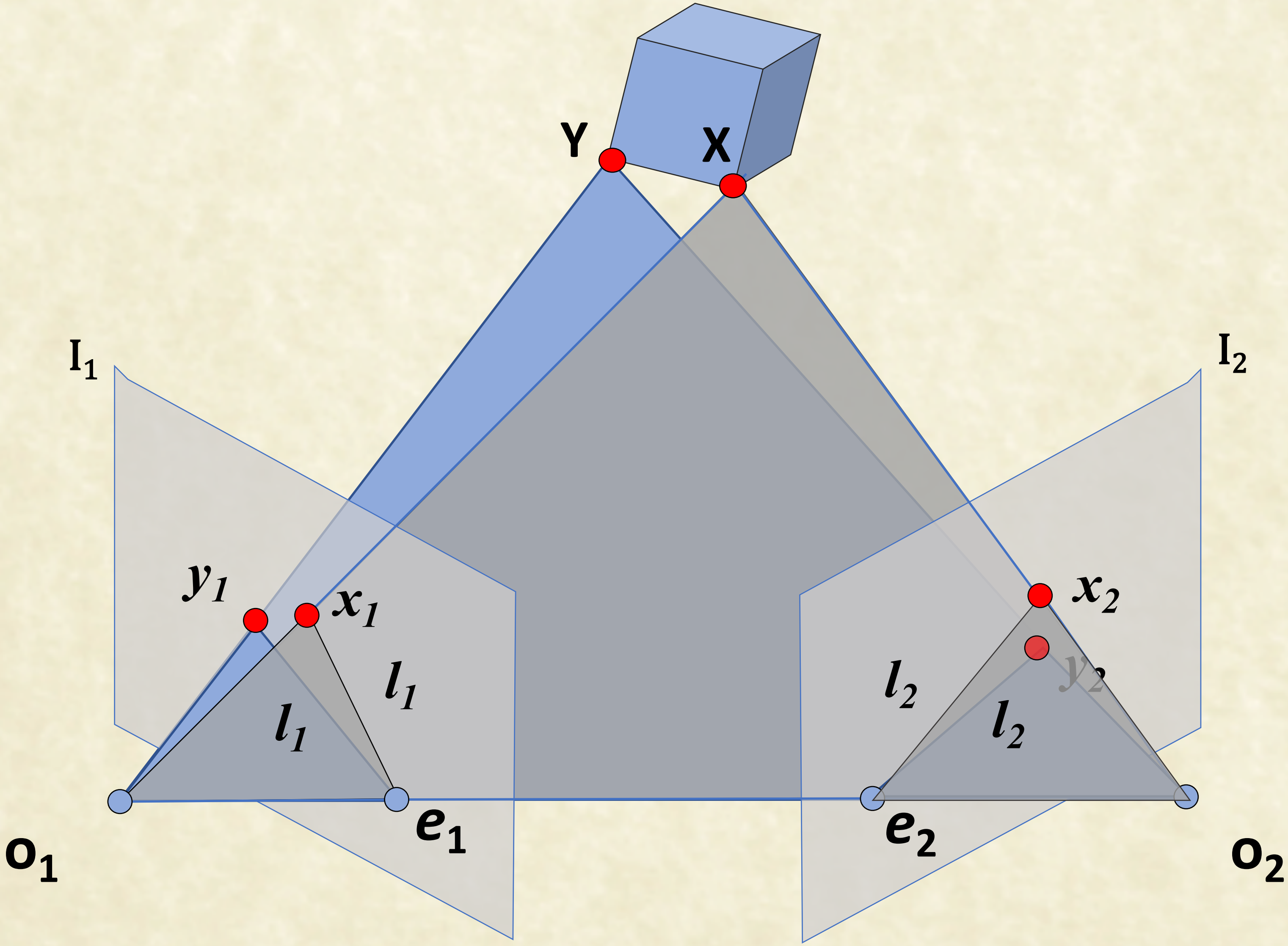
$$\mathbf{x}_2 = \mathbf{K}_2\mathbf{X}$$

$$\mathbf{x}_2^T\mathbf{K}_2^{-T}\hat{\mathbf{T}}\mathbf{R}\mathbf{K}_1^{-1}\mathbf{x}_1 = 0$$

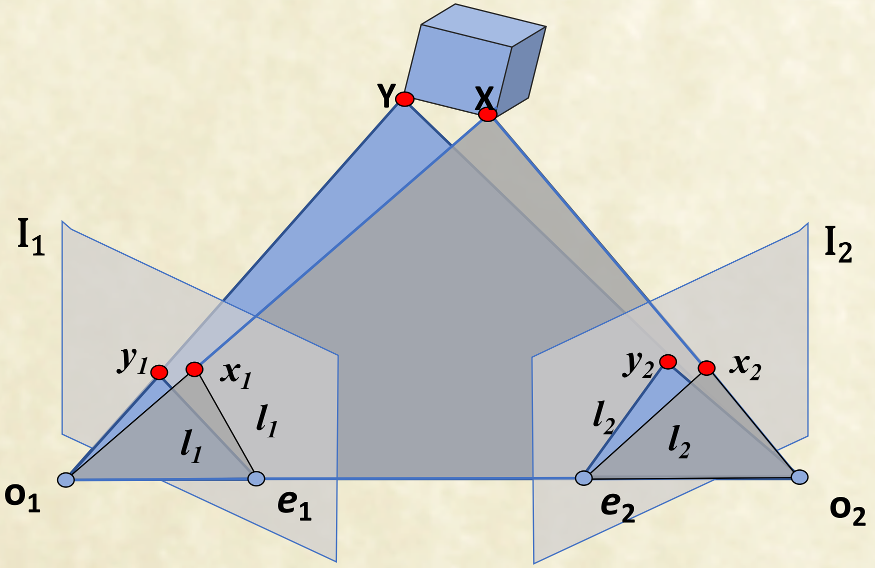
$$\mathbf{x}_1^T F\mathbf{x}_2 = 0$$

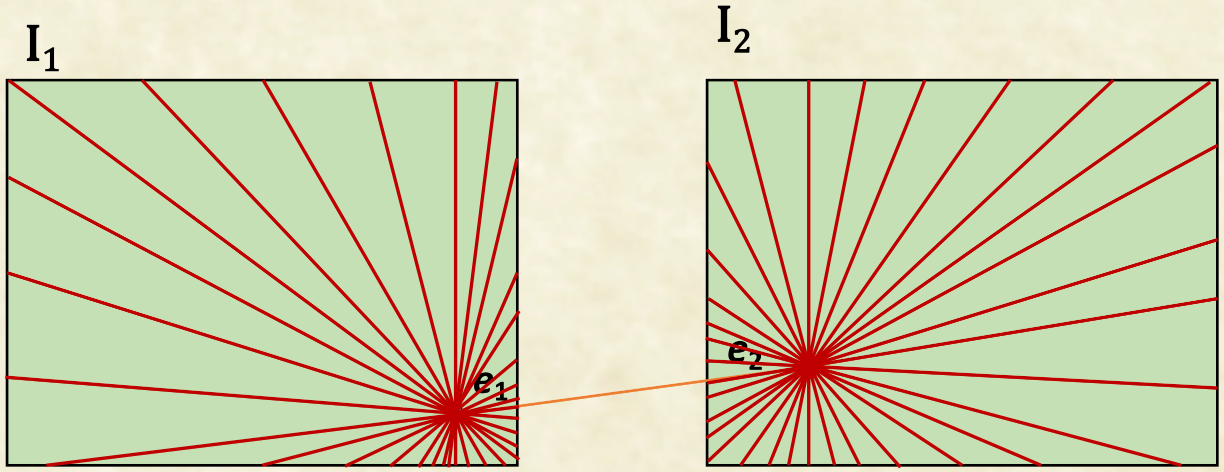
Both Essential and Fundamental matrices are 3x3 and are independent of the world point.

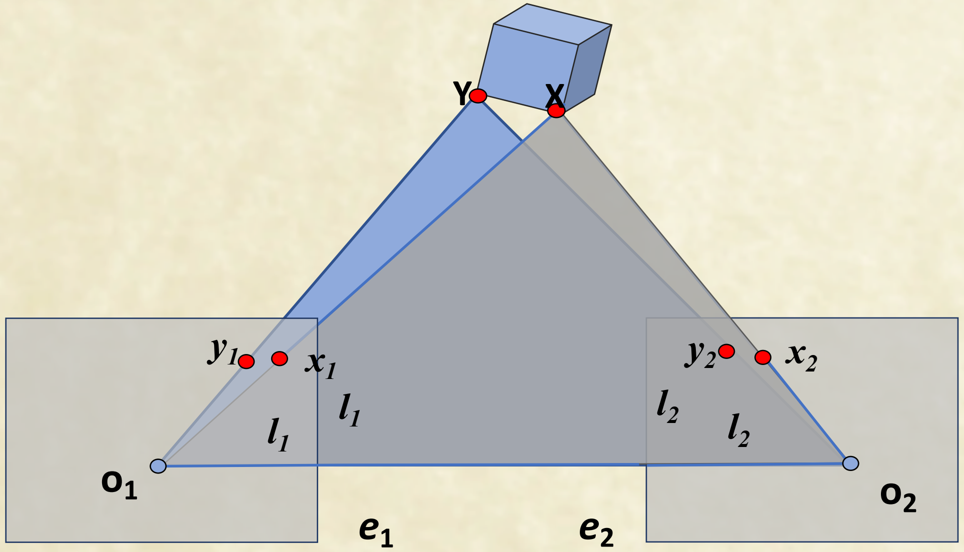
Epipolar Lines

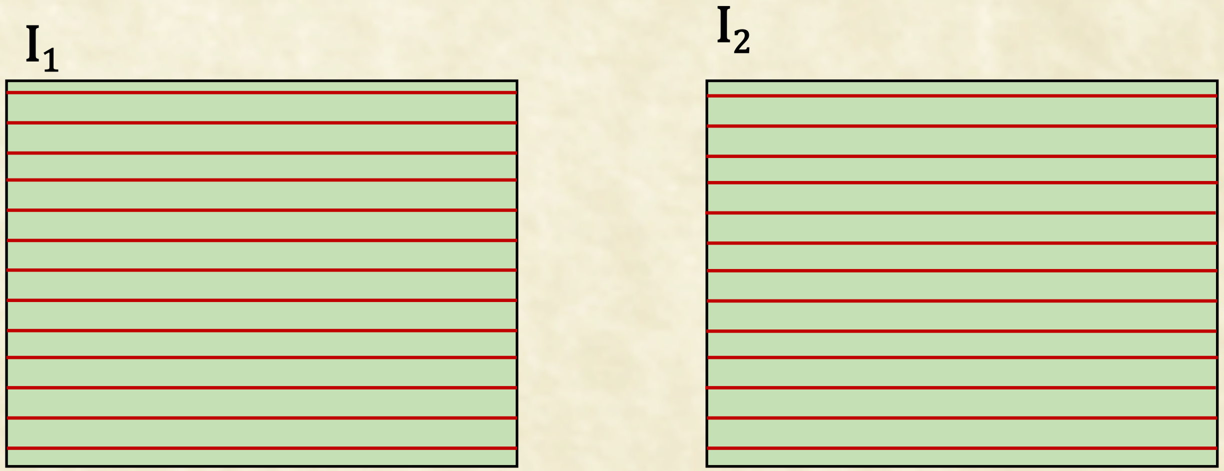


Epipolar Lines









Summary

1. Two views of a planar world are related by a homography. (H: non-singular 3x3 matrix)
2. Two views or an arbitrary world from the same camera position are related by a homography.
3. Two views of an arbitrary world are constrained by epipolar geometry
   - Strong Calib.: Essential Matrix: $\boldsymbol{E}$; $x_1{}^T E x_2 = 0$
   - Weak Calib.: Fundamental Matrix: $\boldsymbol{F}$; $x_1{}^T F x_2 = 0$
4. All epipolar lines pass through the epipole in the image
   - The epipolar lines become parallel when the two image planes are coplanar

Thank You