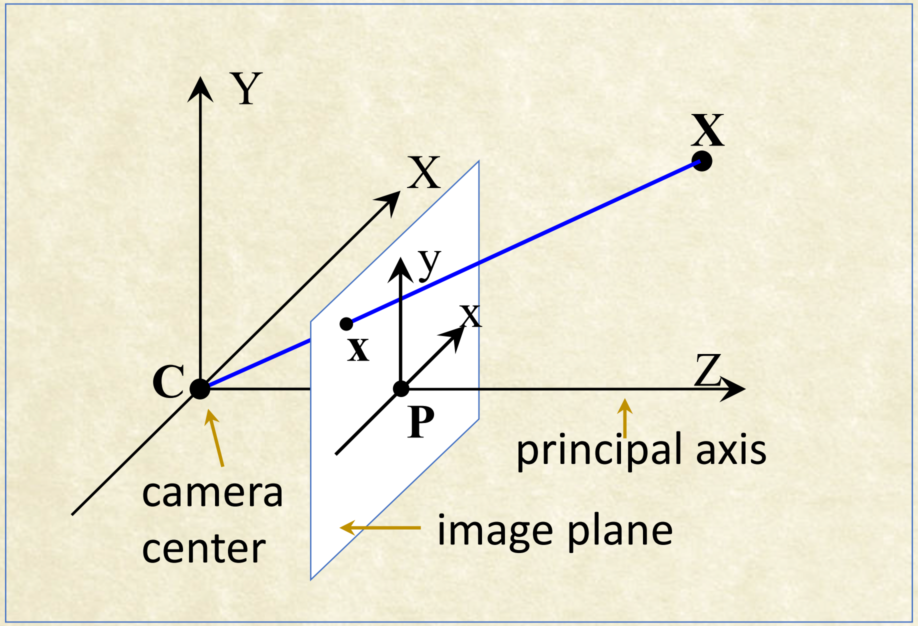
CS7.505: Computer Vision

Spring 2022: Camera Calibration



$$\mathbf{P} = \left[\left[\begin{array}{ccc:c} \alpha_x & s & x_0 & t_1 \\ 0 & \alpha_y & y_0 & t_2 \\ 0 & 0 & 1 & 0 \end{array}\right]\right]$$

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General Camera Equation

- Camera and world are related by: $\mathbf{X}_c = \begin{bmatrix} \mathbf{R} & \mathbf{-RC} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \mathbf{X}_w$
- 2D projection $\mathbf{x}$ of a 3D point $\mathbf{X_w}$ is given by: $\mathbf{x} = \mathbf{PX_w}$
- Camera matrix: $\mathbf{P} = [\mathbf{KR} \mid \mathbf{-KRC}] = [\mathbf{M} \mid \mathbf{p_4}]$
- Common K: General K:

$$\begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Any 3 × 4 matrix P with non-singular left submatrix represents a camera! It can be decomposed as:

- A non-singular upper diagonal K
- Orthonormal R and a vector C

The Camera Matrix: Summary

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathrm{P}^1 \\ \mathrm{P}^2 \\ \mathrm{P}^3 \end{bmatrix}$$

$$= [\mathrm{p}_1 \quad \mathrm{p}_2 \quad \mathrm{p}_3 \quad \mathrm{p}_4]$$

$$= \mathrm{K}\ [\mathrm{R} \mid -\mathrm{RC}]$$

Camera Calibration

$[R \mid -RC]$

P ?

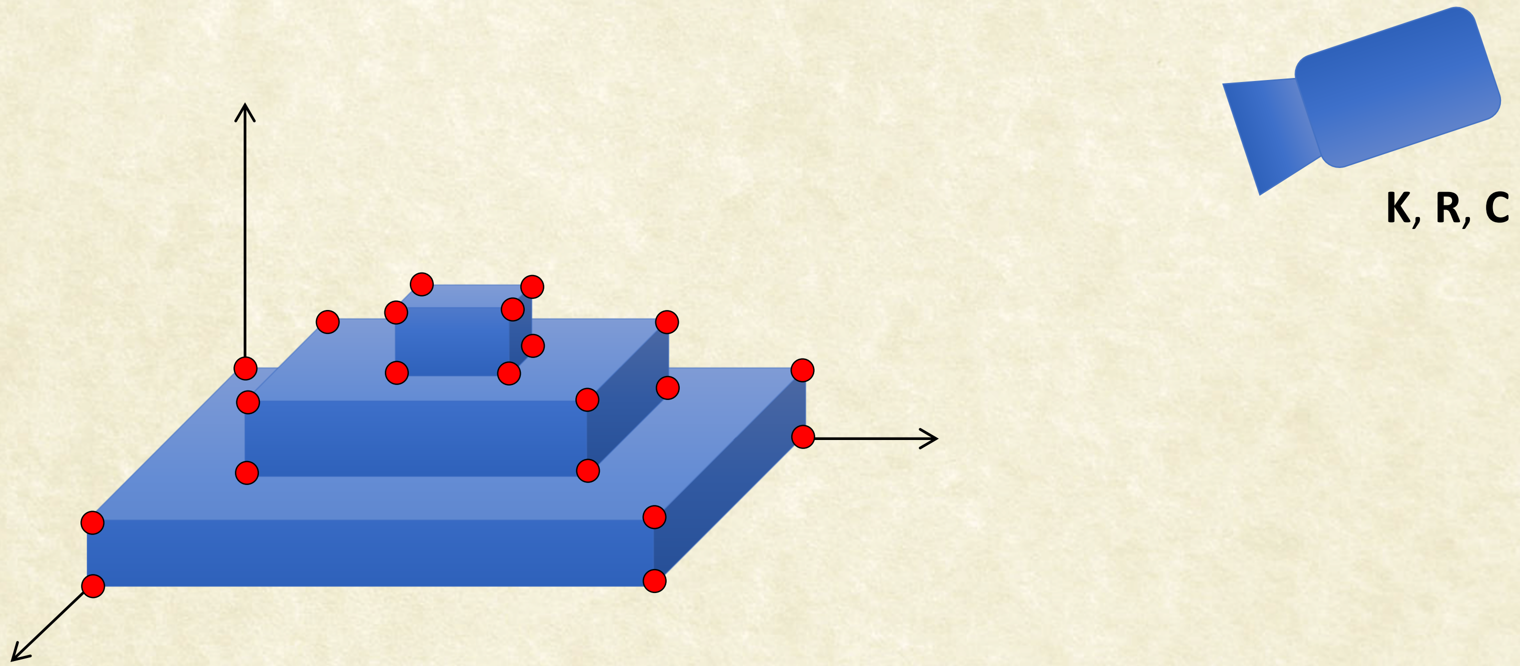
K

$$\begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} ?$$

O_W

3D Reference Object Based



Given a set of world points: (X_i,Y_i,Z_i) and their corresponding image coordinates: (x_i,y_i), we can write a set of linear equations in p_{mn}, the entries of the camera matrix.

3D Reference Object Based

$$\begin{bmatrix} x_i \\ y_i \\ w_i \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

But we know only: $(x_i / w_i \, , y_i / w_i)$, Call it $(u_i \, , v_i)$.

$$u_i = \frac{x_i}{w_i} = \frac{p_{11}X_i + p_{12}Y_i + p_{13}Z_i + p_{14}}{\boxed{p_{31}X_i + p_{32}Y_i + p_{33}Z_i + p_{34}}}$$

$$v_i = \frac{y_i}{w_i} = \frac{p_{21}X_i + p_{22}Y_i + p_{23}Z_i + p_{24}}{\boxed{p_{31}X_i + p_{32}Y_i + p_{33}Z_i + p_{34}}}$$

3D Reference Object Based

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_iX_i & -u_iY_i & -u_iZ_i & -u_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_iX_i & -v_iY_i & -v_iZ_i & -v_i \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ \vdots \\ p_{34} \end{bmatrix} = [\mathbf{0}]$$

- Stack equations for all points to get $\mathbf{Gp=0}$.
- Solving this over-determined linear system of equations, we can recover the camera matrix.
- The matrix $\mathbf{P}$ can then be decomposed into the external and internal parameters: $\mathbf{K}$, $\mathbf{R}$ and $\mathbf{t}$.

Decomposing $\mathbf{P}$

$$\mathbf{P} = \mathbf{K}\left[\mathbf{R} \mid \mathbf{t}\right]; \qquad \mathbf{K} = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Let $\mathbf{P}=[\mathbf{M}\ \mathbf{p}_4]$; where $\mathbf{M}=\mathbf{KR}$, and $\mathbf{p}_4=\mathbf{Kt}$.
- $\mathbf{MM}^T = \mathbf{KRR}^T\mathbf{K}^T = \mathbf{KK}^T$. We can solve for elements of $\mathbf{K}$.
- $\mathbf{R} = \mathbf{K}^{-1}\mathbf{M}$, and $\mathbf{t} = \mathbf{K}^{-1}\mathbf{p}_4$.

Solving for $\mathbf{K}$ (the naïve way)

$$\mathbf{KK}^{\mathrm{T}} = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 & 0 \\ \gamma & \beta & 0 \\ u_0 & v_0 & 1 \end{bmatrix} = \mathbf{MM}^{\mathrm{T}}$$

$$\begin{bmatrix} \boxed{\alpha^2+\gamma^2+{u_0}^2} & \beta\gamma+u_0v_0 & u_0 \\ \boxed{\beta\gamma+u_0v_0} & \boxed{\beta^2+{v_0}^2} & v_0 \\ u_0 & v_0 & 1 \end{bmatrix} = \begin{bmatrix} \boxed{mm_{11}} & mm_{12} & mm_{13} \\ \boxed{mm_{21}} & \boxed{mm_{22}} & mm_{23} \\ mm_{31} & mm_{32} & mm_{33} \end{bmatrix}$$

- We get u_0 and v_0 directly.
- Compute β
- Then γ; then α.

Solving for **K** (the Linear Algebra way)

- $\mathbf{M} = \mathbf{KR}$,
  - where K is upper triangular and R is orthonormal
- Use "QR factorization" of M to obtain K and R.
  - The RQ variant to be precise
- $\mathbf{t} = \mathbf{K}^{-1}\mathbf{p}_4$.

Now we have all the intrinsic parameters in **K** and all the extrinsic parameter, **R** and **t**.

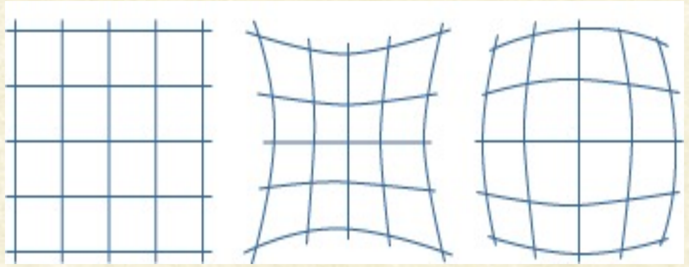
Refining P: Non-linear Optimization

- The distance metric used in the linear solution is not geometrically meaningful.
- We would like to minimize the distance between the points project by $\mathbf{P}$ and the observed points. i.e.,

$$\min_{p} \sum_{i} \|x_i - \phi(P, X_i)\|^2$$

- Can be solved by Levenberg-Marquardt algorithm.
- Use the linear solution as starting point.

Dealing with Radial Distortion



- Each pixel moves radially away from (barrel) or towards (pincushion) the image center (c).
- As a function of squared distance from c: ${r_c}^2 = {x_c}^2 + {y_c}^2$.
- The shift γ can be modelled as: $\gamma = 1 + k_1 {r_c}^2 + k_2 {r_c}^4$, where k_1 and k_2 are radial distortion parameters.
- The modified co-ordinates are:

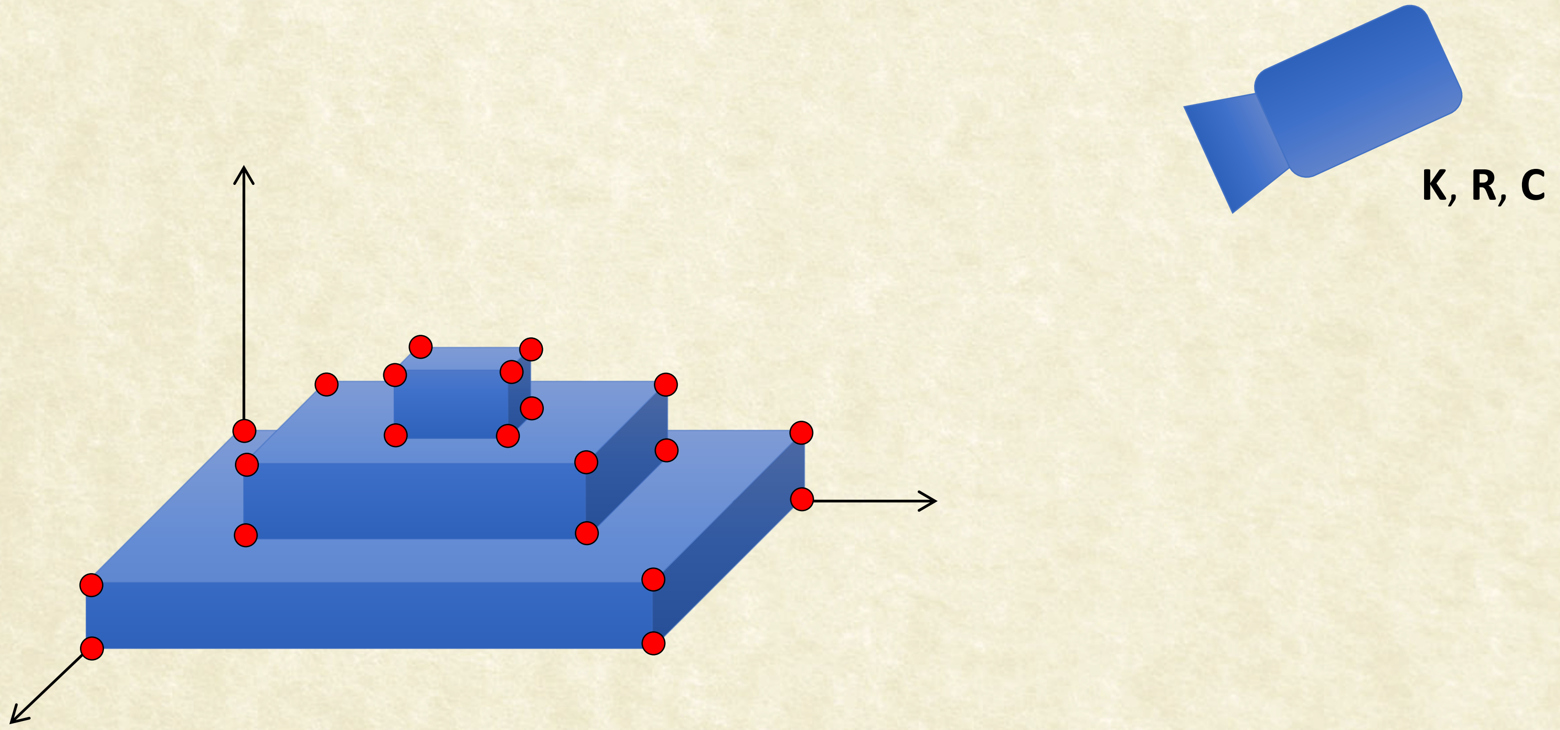
$$\hat{x}_c = \gamma x_c$$

$$\hat{y}_c = \gamma y_c$$

- This is applied before the focal-length multiplier and center shift are applied.
- Use radial-distortion-corrected image for computing the camera parameters
  - Can be done from knowledge of straight lines

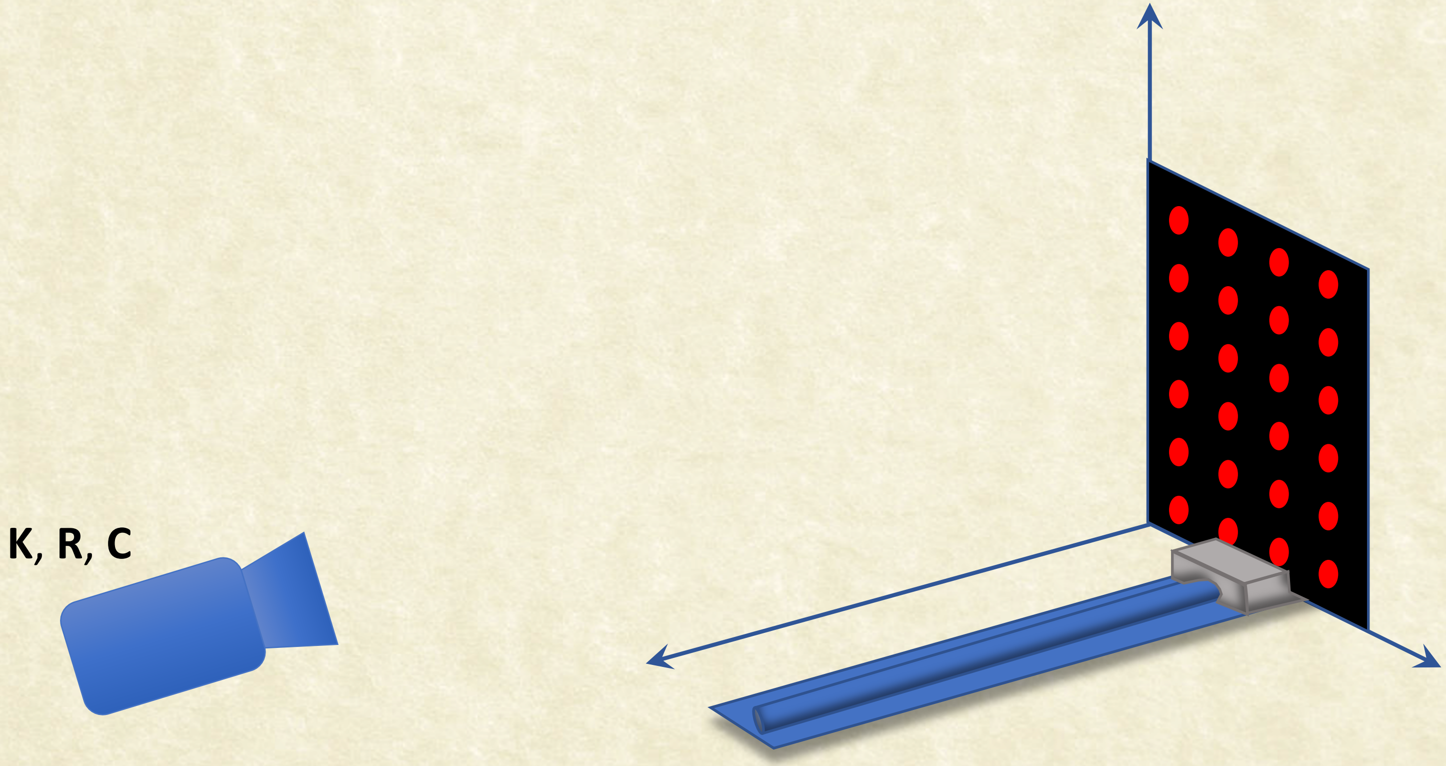
Calibration Methods

1. 3D Reference Object based calibration



Calibration Methods

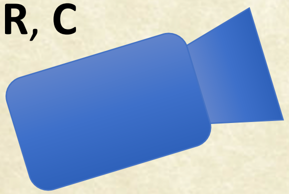
1. 3D Reference Object based calibration
2. Calibration from a precisely moving plane (R.Y. Tsai)

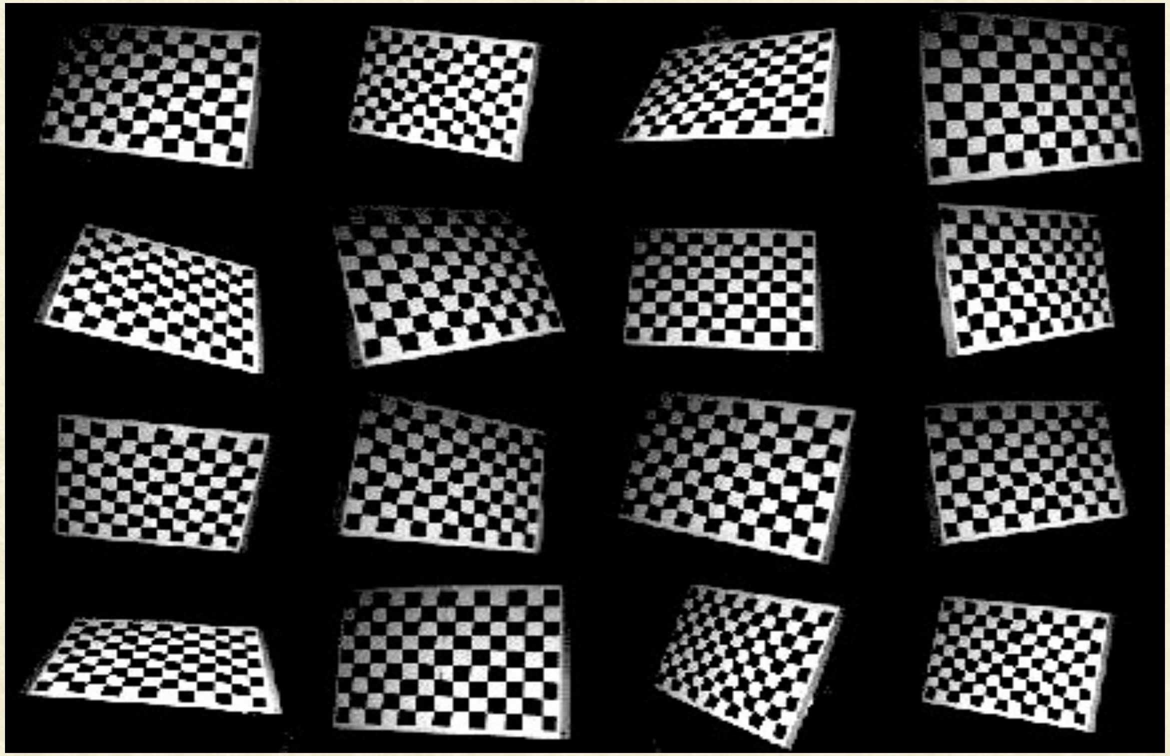


Calibration Methods

1. 3D Reference Object based calibration
2. Calibration from a precisely moving plane (R.Y. Tsai)
3. Calibration using a plane with unknown motion

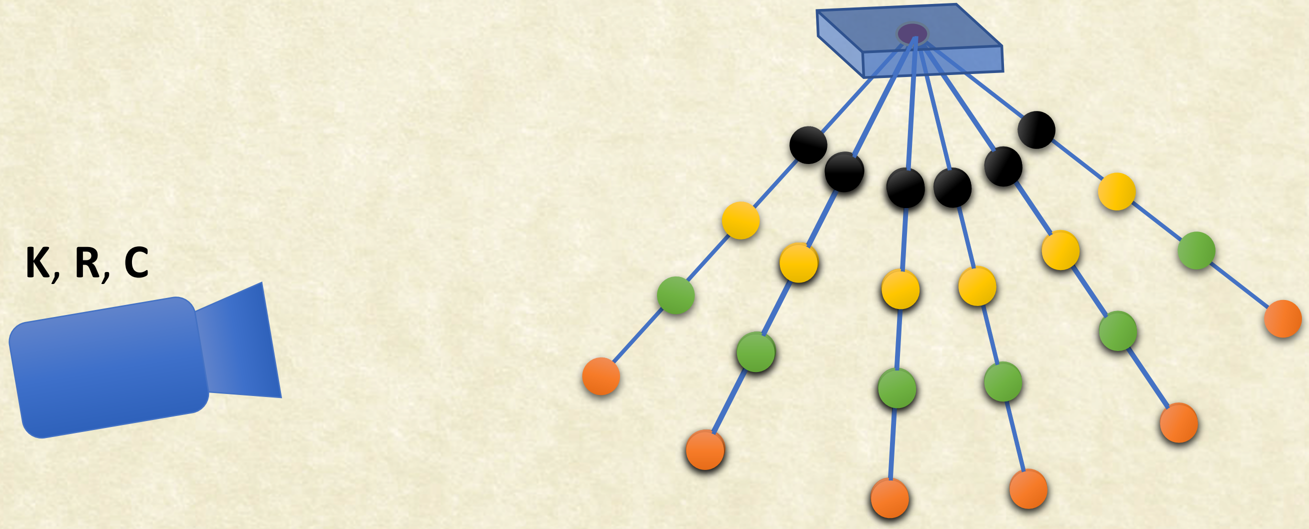
K, R, C





Calibration Methods

1. 3D Reference Object based calibration
2. Calibration from a precisely moving plane (R.Y. Tsai)
3. Calibration using a plane with unknown motion
4. Calibration from a set of collinear points that moves such that the lines passing through a fixed point



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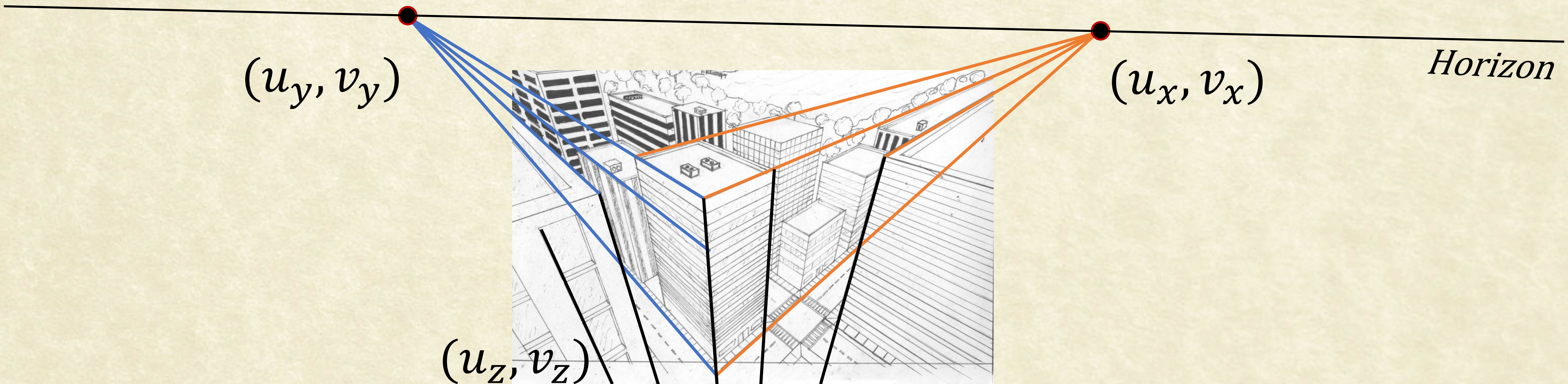
5. Calibration from Vanishing points in orthogonal directions



Calibration Methods

1. 3D Reference Object based calibration
2. Calibration from a precisely moving plane (R.Y. Tsai)
3. Calibration using a plane with unknown motion
4. Calibration from a set of collinear points that moves such that the lines passing through a fixed point
5. Calibration from Vanishing points in orthogonal directions
6. Self Calibration (rigid static world and point correspondences across images are available).

The calibration process requires fewer constraints as we move downwards, but results become less precise.

Questions?