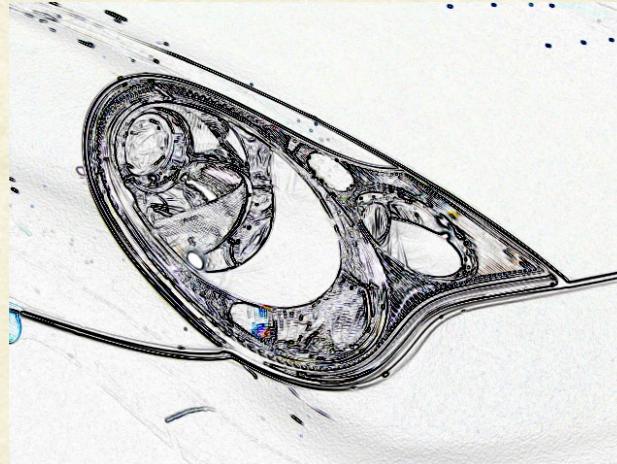




CS7.505: Computer Vision

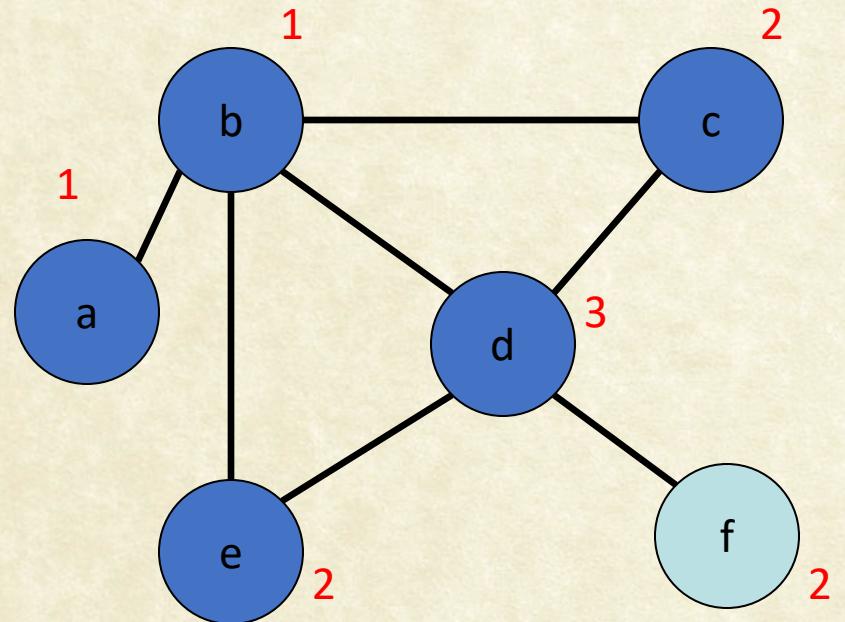
Spring 2022: Multi-Label MRF



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IIIT Hyderabad



The General Problem



Graph $G = (V, E)$

Discrete label set $L = \{1, 2, \dots, h\}$

Assign a label to each vertex
 $f: V \rightarrow L$

Cost of a labelling $Q(f)$

Unary Cost

Pairwise Cost

Find $f^* = \arg \min Q(f)$



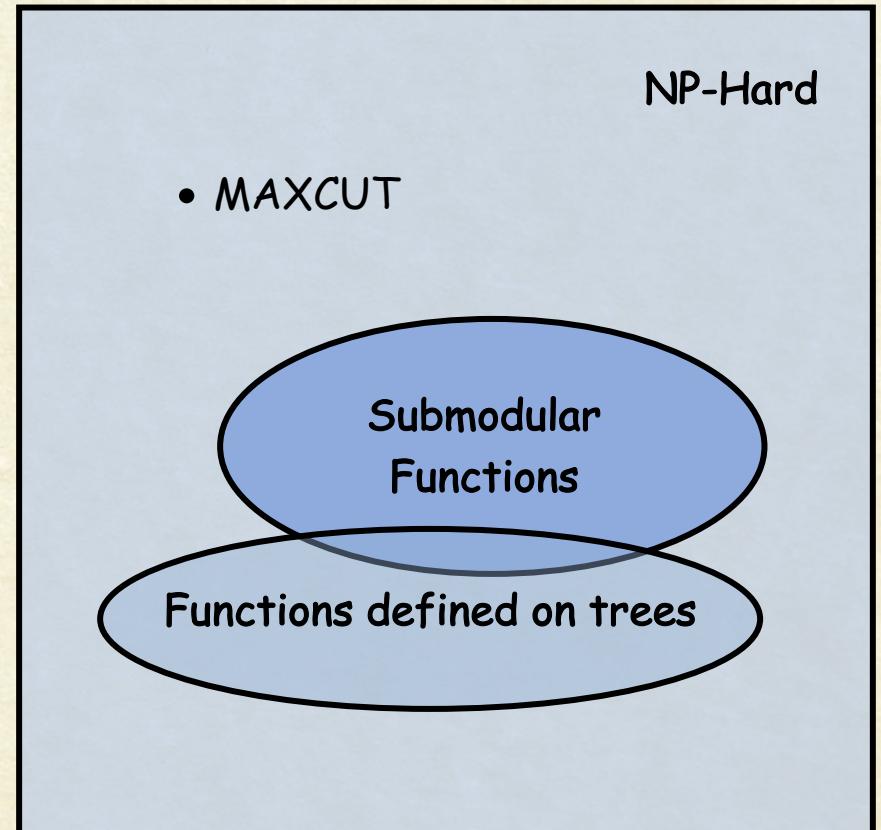
Minimizing Energy Functions

- **General Energy Functions**

- NP-hard to minimize
- Only approximate minimization possible

- **Easy energy functions**

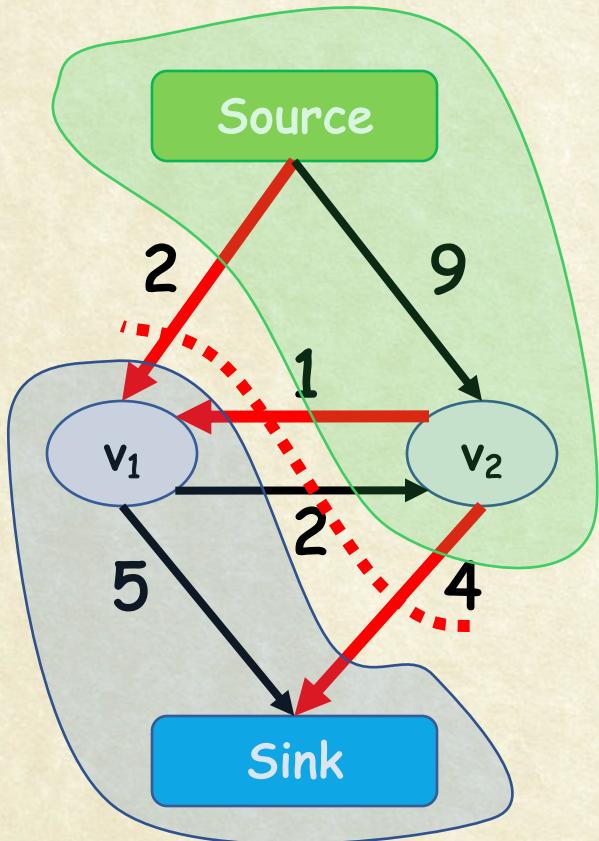
- Solvable in polynomial time
- Submodular $\sim O(n^6)$



Space of Function
Minimization Problems



The st-Mincut Problem



$$2 + 1 + 4 = 7$$

What is an st-cut?

An st-cut (S, T) divides the nodes between source and sink.

What is the cost of an st-cut?

Sum of cost of all edges going from S to T

What is the st-mincut?

st-cut with the minimum cost



Robust Interactions

- NP-hard problem (3 or more labels)
 - two labels can be solved via $s-t$ cuts (Greig et. al., 1989)
- α -expansion approximation algorithm
 - (Boykov, Veksler, Zabih 1998, 2001)
 - Guaranteed approximation quality (Veksler, thesis 2001)
 - within a factor of 2 from the global minima (Potts model)
 - Applies to a wide class of energies with robust interactions
 - Potts model (BVZ 1998)
 - “Metric” interactions (BVZ 2001)
 - Can be extended to arbitrary interactions with weaker guarantees
 - truncation (Kolmogorov et al. 2005)
 - QPBO (Boros and Hummer, 2002)
- Other “move” algorithms (e.g. $\alpha-\beta$ swap, jump-moves)



Metric and Semimetric Interactions

$$E(f) = \sum_{\{p,q\} \in \mathcal{N}} V_{p,q}(f_p, f_q) + \sum_{p \in \mathcal{P}} D_p(f_p)$$

$$V(\alpha, \beta) = 0 \iff \alpha = \beta,$$

$$V(\alpha, \beta) = V(\beta, \alpha) \geq 0,$$

$$V(\alpha, \beta) \leq V(\alpha, \gamma) + V(\gamma, \beta),$$

- where α, β , and γ are labels.
- Metric: If all three conditions are satisfied.
- Semi-metric: If only the first two are satisfied.
- Examples?



Multi-label Graphcut: Move Making

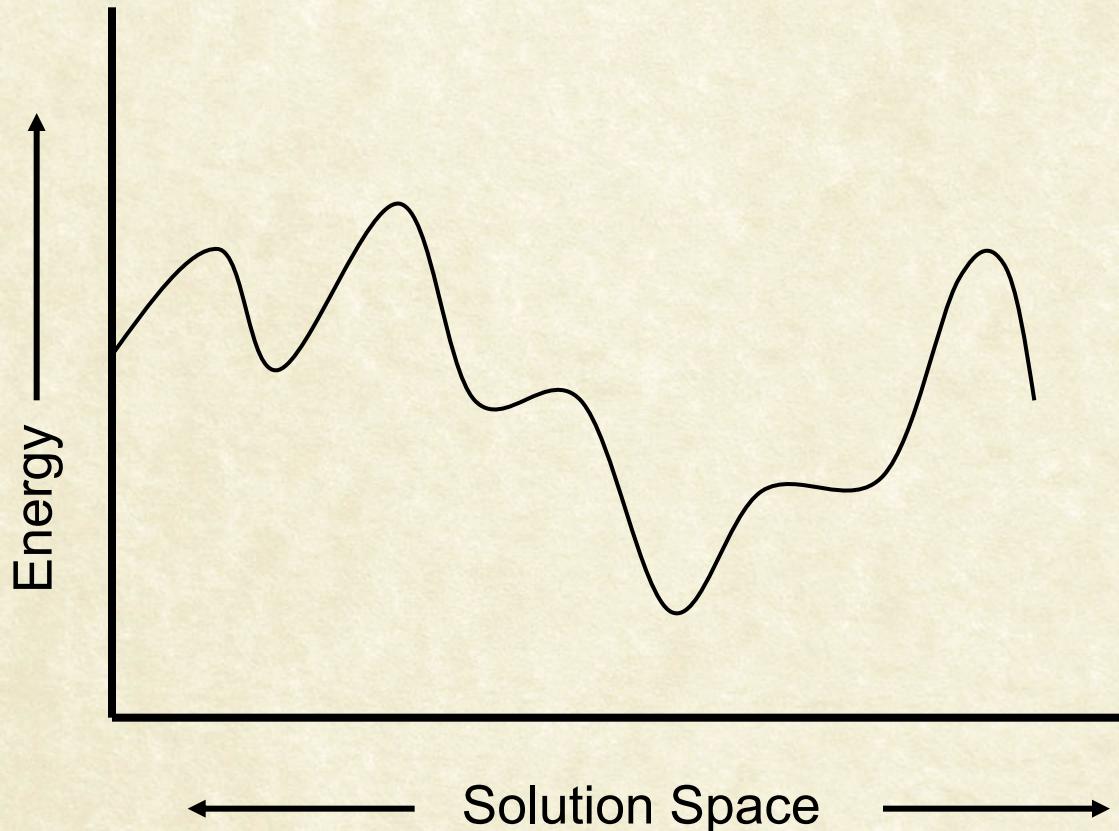
$$E(x) = \sum_i \theta_i(x_i) + \sum_{i,j} \theta_{ij}(x_i, x_j)$$

$$x \in Labels L = \{l_1, l_2, \dots, l_k\}$$

- Commonly used for solving **non-submodular** multi-label problems
- Extremely efficient and produce good solutions
- Not Exact: Produce local optima

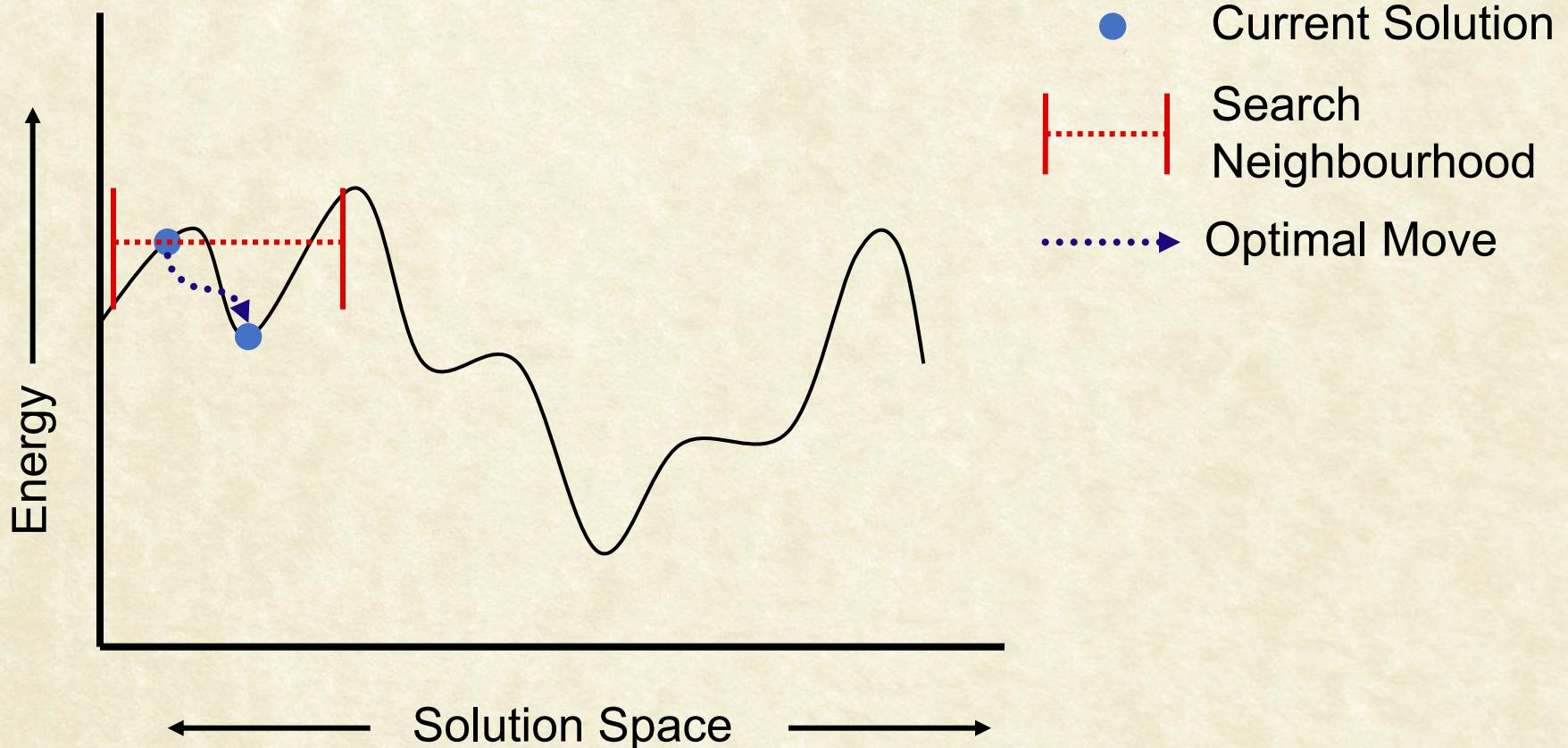


Move Making Algorithms



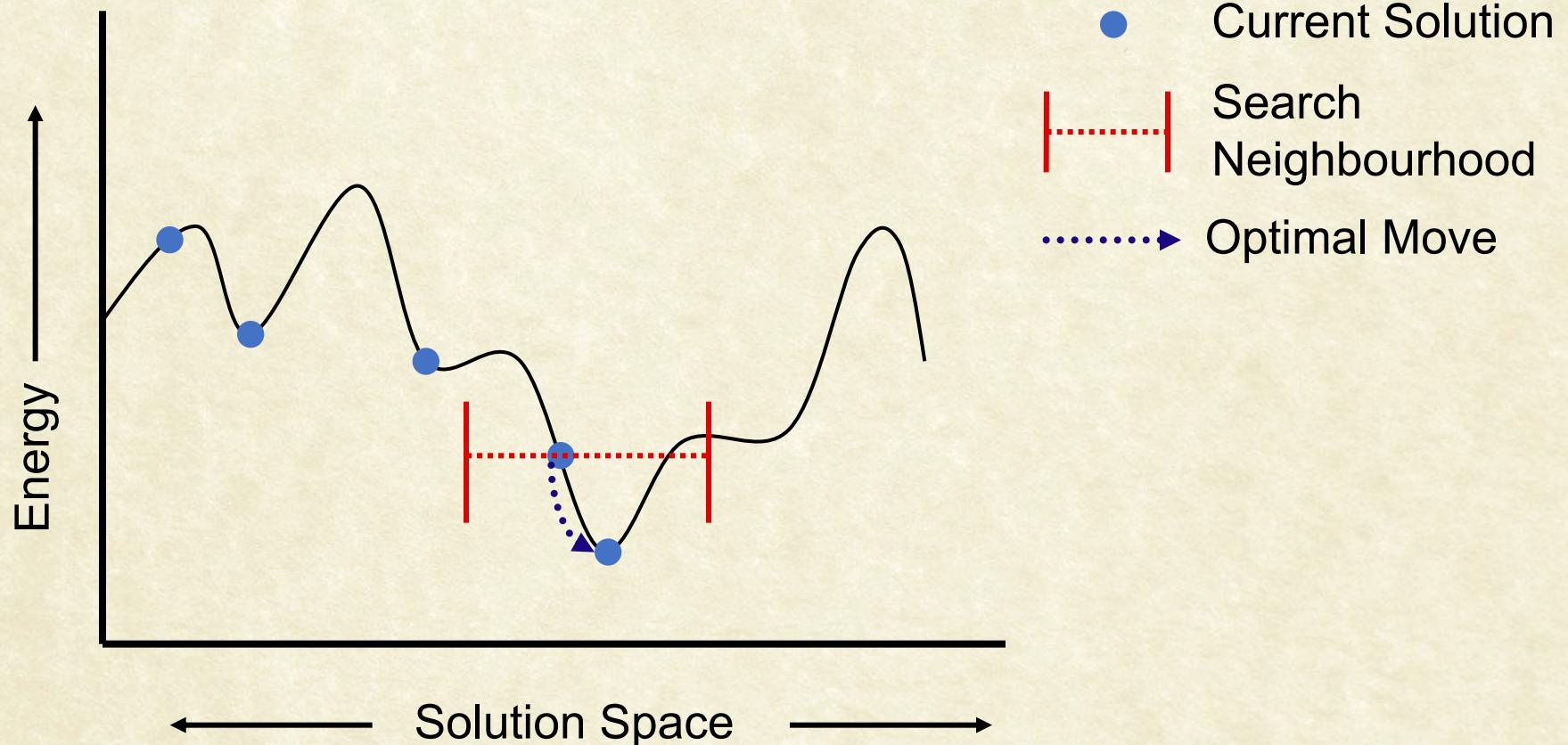


Move Making Algorithms



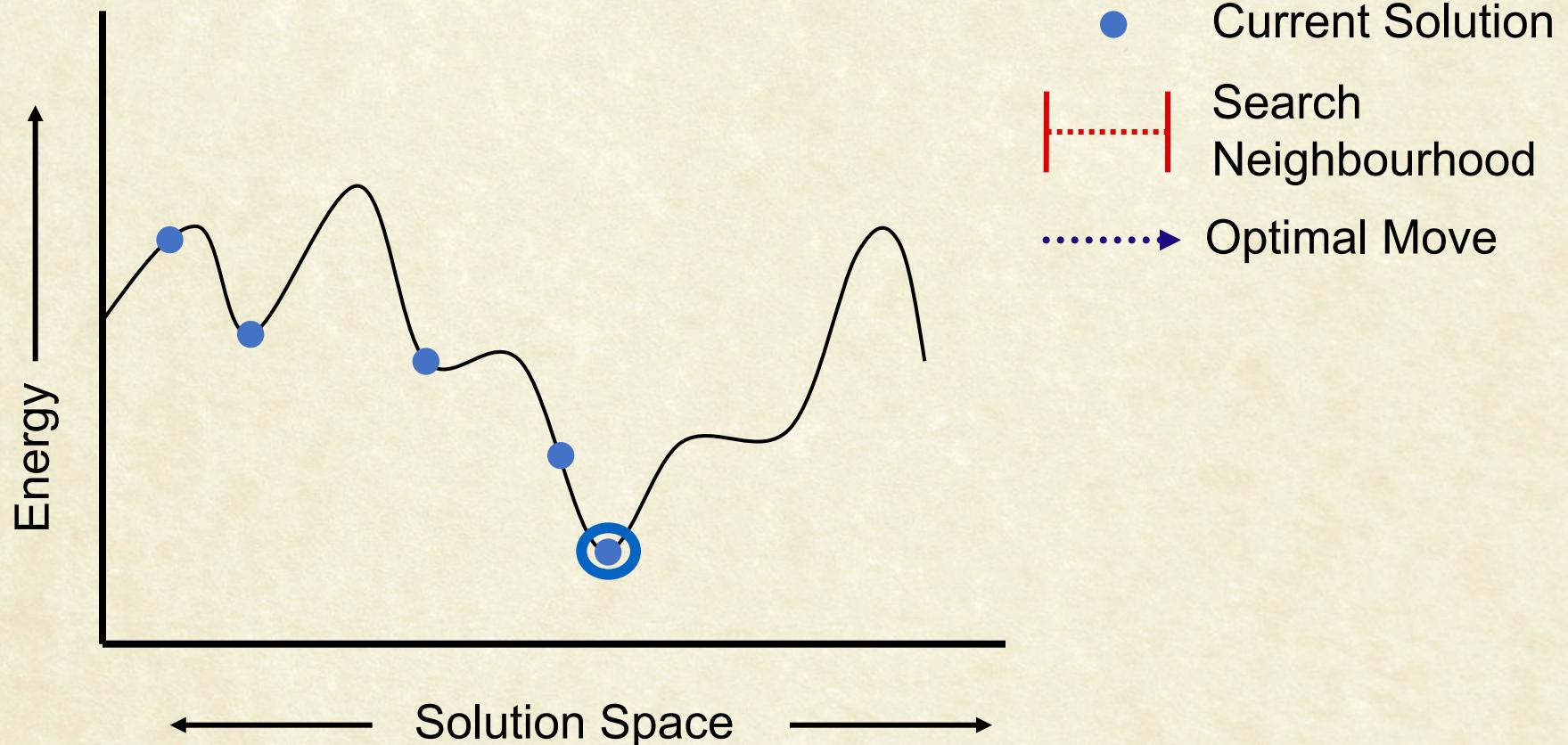


Move Making Algorithms



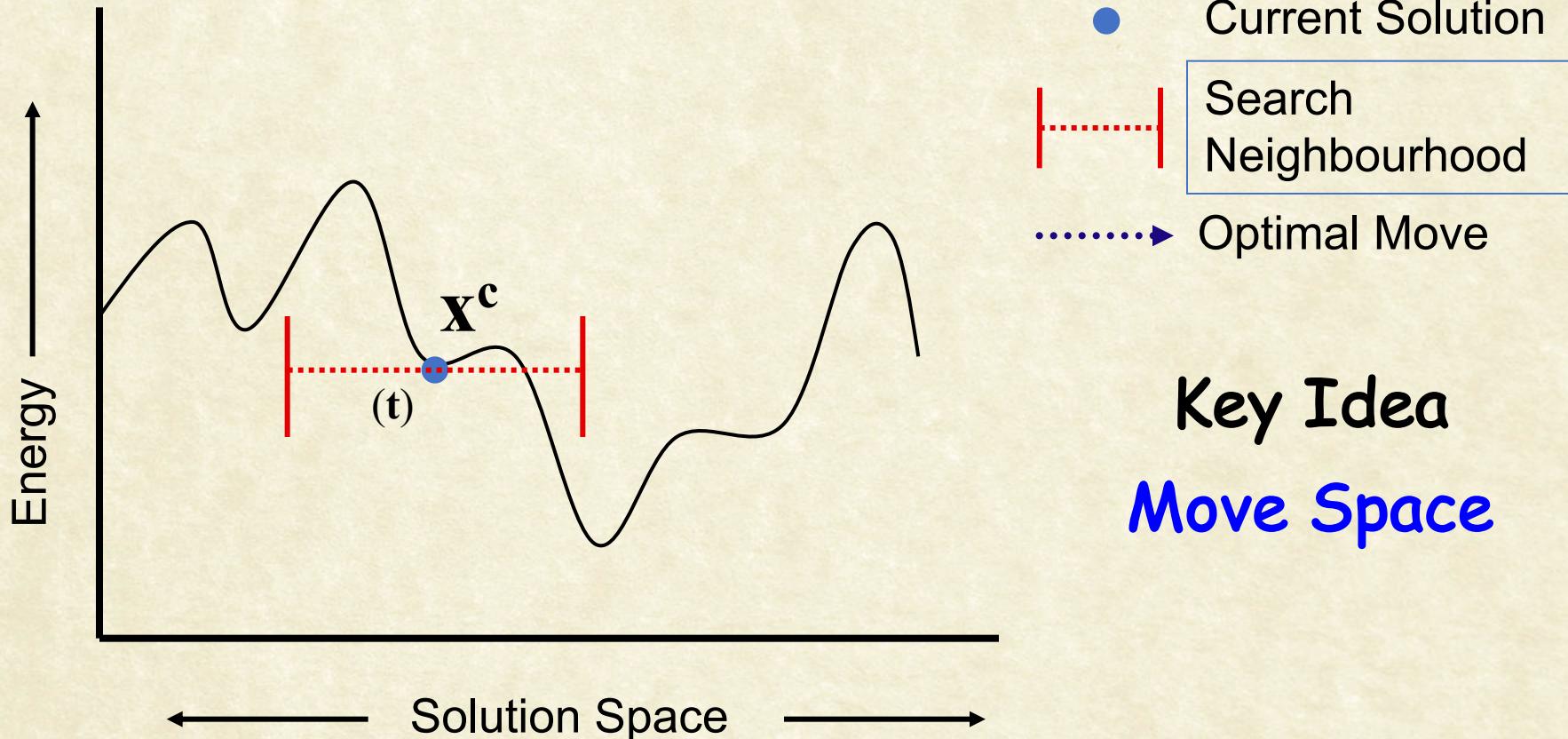


Move Making Algorithms

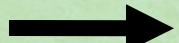




Computing the Optimal Move



Bigger move space



- Better solutions
- Finding the optimal move hard



What are we doing?

- Cast segmentation as a generic optimization prob. on a graph
- The graph optimization problem is generic:
 - Assign any one of k labels $L = \{l_1, l_2, \dots, l_k\}$ to each vertex of a graph
 - Cost of assigning labels in terms of unary and pair-wise costs
 - Derive algorithms that minimize the total cost
- The labels need not be object membership
 - Depth values, Class labels, Instance labels, Motion
- Unary potential need not be based on image pixel values
 - Stereo disparity

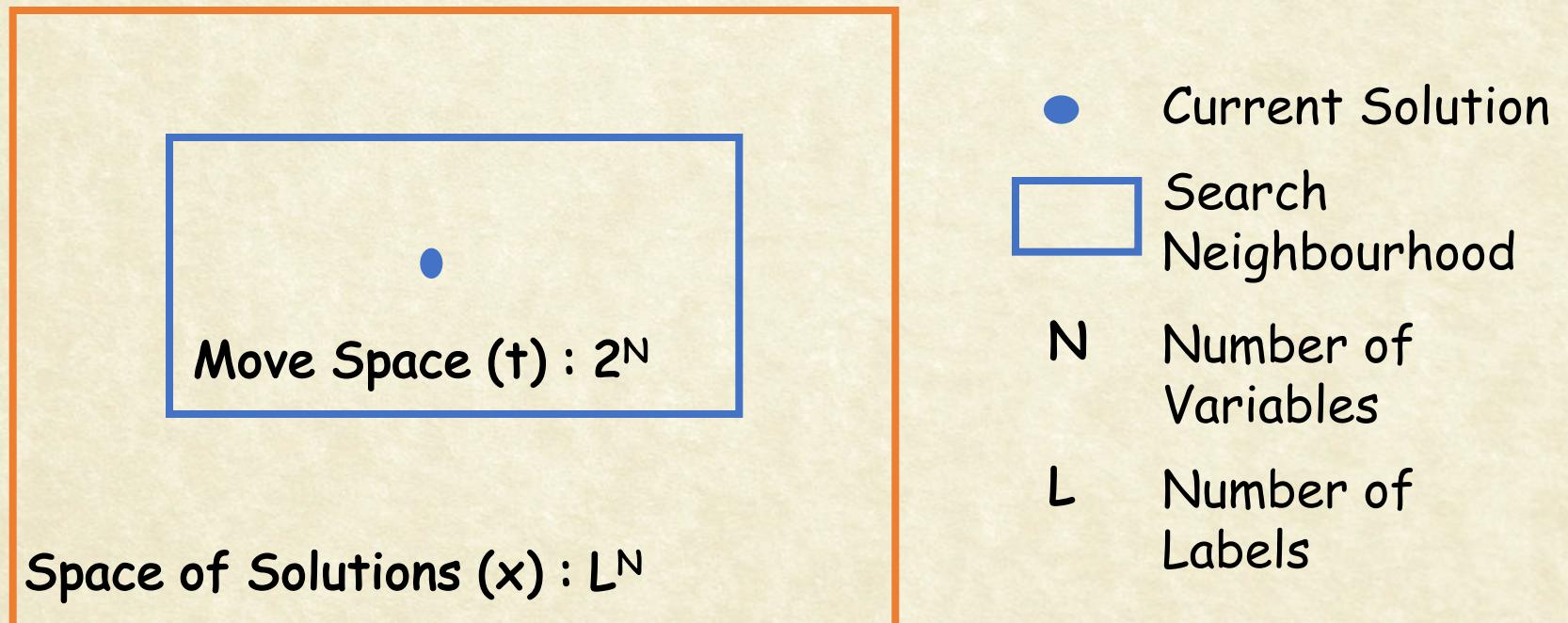


Moves using Graph Cuts

Expansion and Swap move algorithms

[Boykov, Veksler and Zabih, PAMI 2001]

- Makes a series of changes to the solution (moves)
- Each move results in a solution with smaller energy



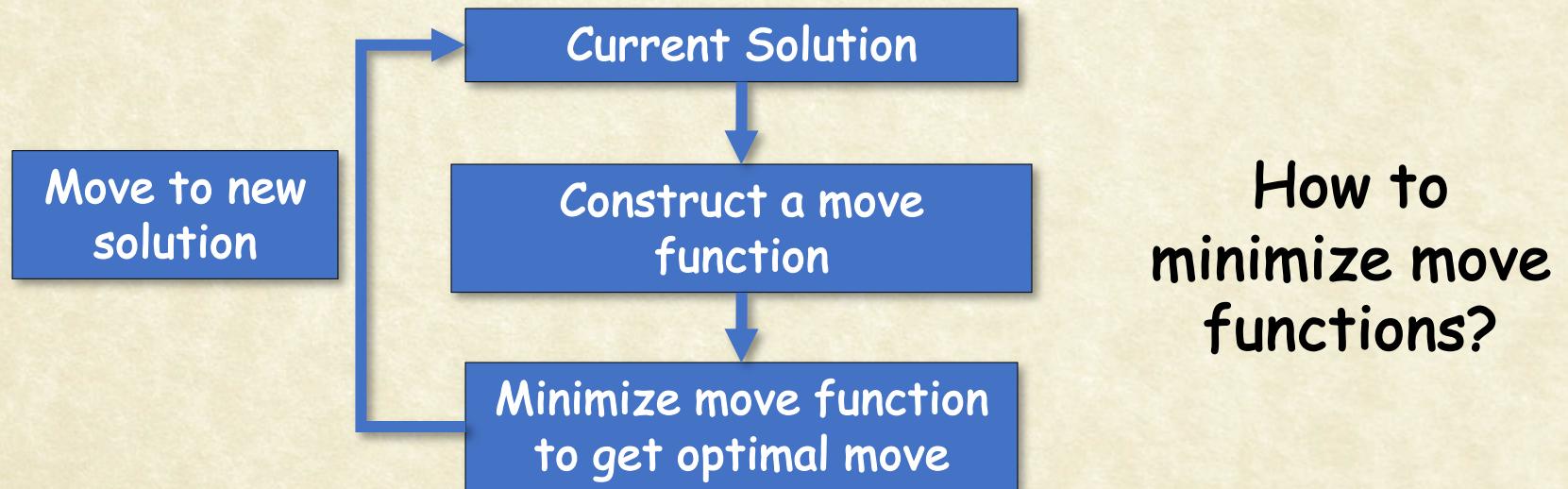


Moves using Graph Cuts

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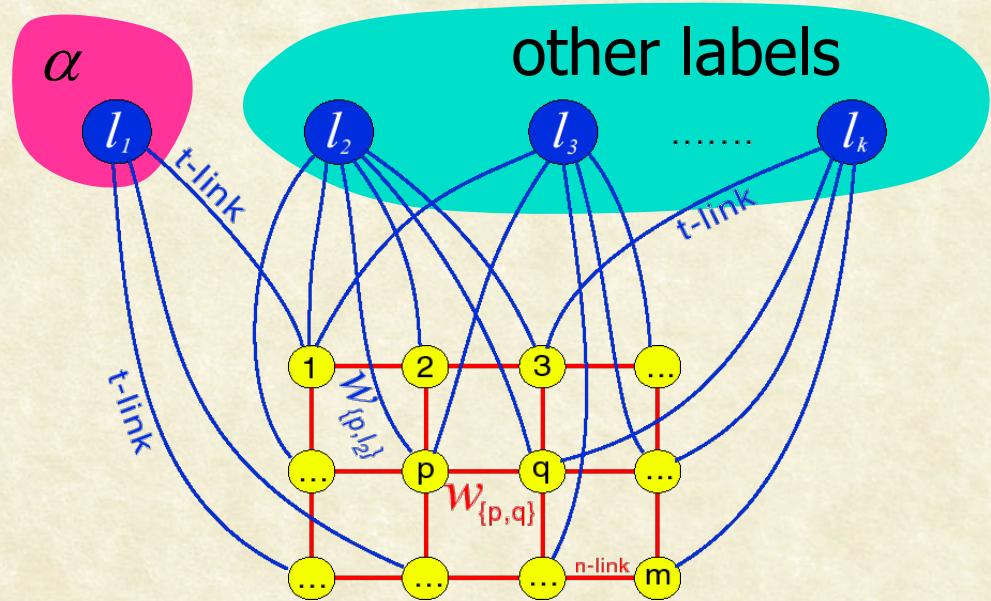
α -expansion algorithm

1. Start with any initial solution
2. For each label “ α ” in any (e.g. random) order
 1. *Compute optimal α -expansion move ($s-t$ graph cut)*
 2. *Decline the move if there is no energy decrease*
3. *Stop when no expansion move would decrease energy*



α -expansion move

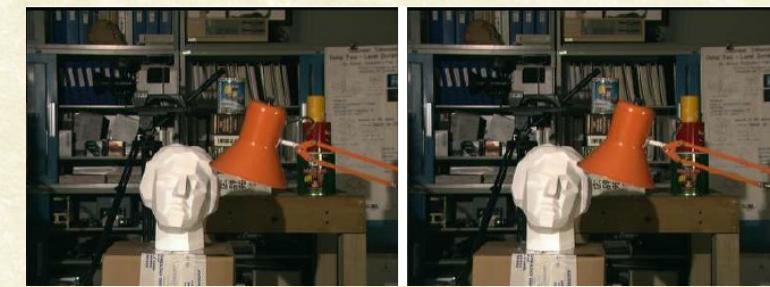
- Basic Idea: Break multi-way cut computation into a **sequence of binary $s-t$ cuts**





α -expansion move

In each α -expansion a given label “ α ” grabs space from other labels



initial solution

● -expansion

For each move we choose expansion that gives the largest decrease in the energy: **binary optimization problem**



Metric and Semimetric Interactions

$$E(f) = \sum_{\{p,q\} \in \mathcal{N}} V_{p,q}(f_p, f_q) + \sum_{p \in \mathcal{P}} D_p(f_p)$$

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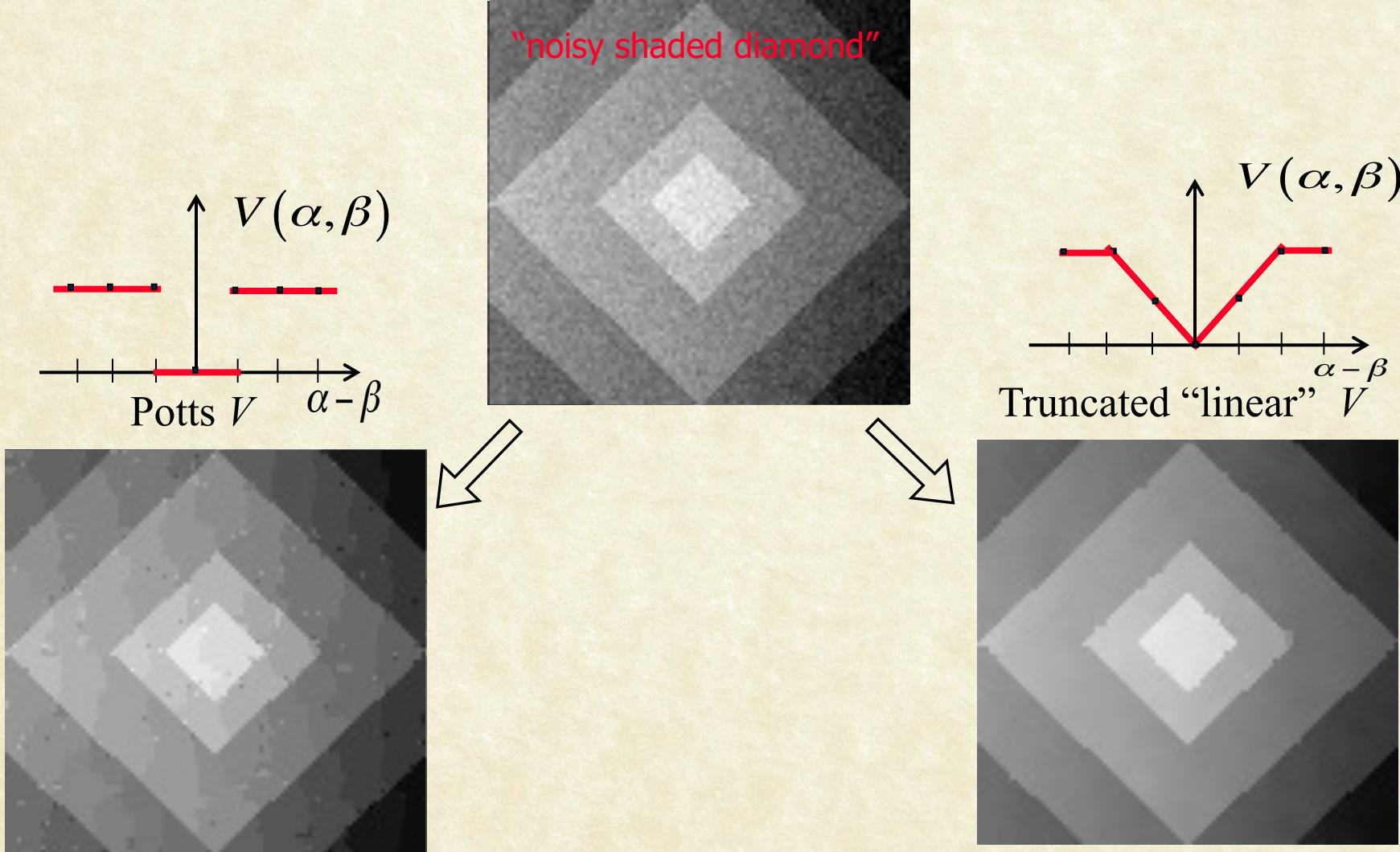
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$$V(\alpha, \beta) \leq V(\alpha, \gamma) + V(\gamma, \beta),$$

- where α, β , and γ are labels.
- Metric: If all three conditions are satisfied.
- Semimetric: If only the first two are satisfied.
- Examples?



α -expansions: Examples of *metric* interactions





Summary, Additional Reading

- Binary Labelling Problems: Exact Solution (s-t cut)
- Multi-label MRF: Efficient Approximate Solutions
 - α -expansion
 - Belief Propagation, Loopy BP, etc.
- Resources:
 - Vladimir Kolmogorov and Ramin Zabih, “What Energy Functions Can Be Minimized via Graph Cuts?”, TPAMI-2004
 - Yuri Boykov, Olga Veksler, Ramin Zabih, “Fast Approximate Energy Minimization via Graph Cuts”, ICCV 1999
 - Course on PGM: Daphne Koller, Stanford



Ink-Bleed Removal

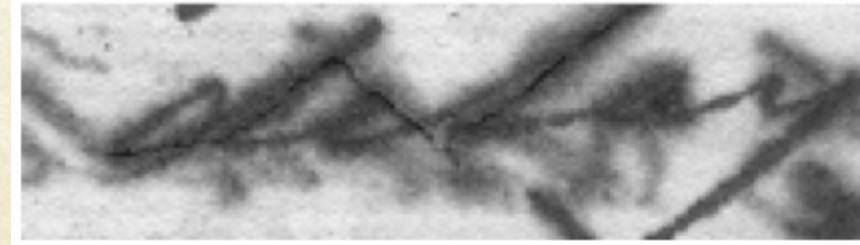
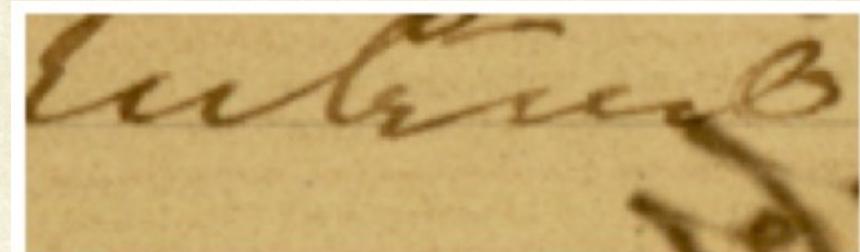
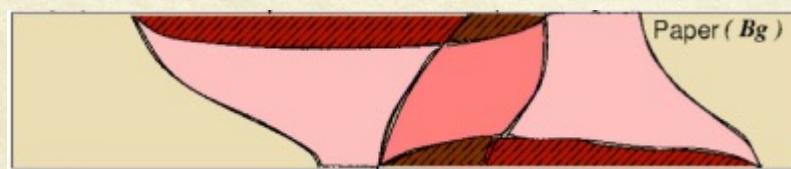
Yi Huang Michael S. Brown Dong Xu

“A Framework for Reducing Ink-Bleed in Old Documents”,
Proc. CVPR, June 2008, Anchorage, AK, USA.



What is Ink Bleed?

- Ink bloats through paper and appears on the reverse side



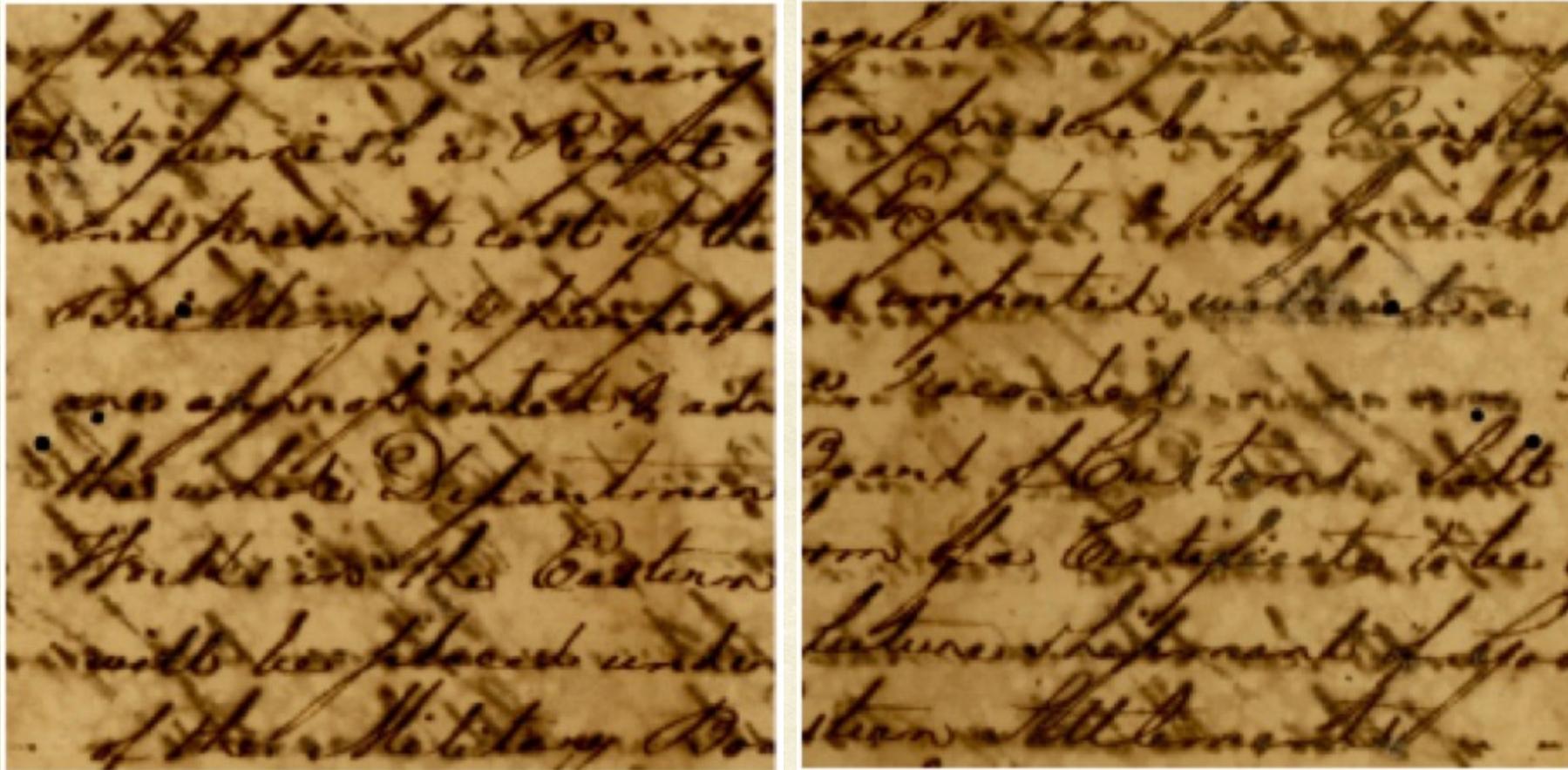


Recto and Verso Images

Recto (front) Side



Verso (back) Side

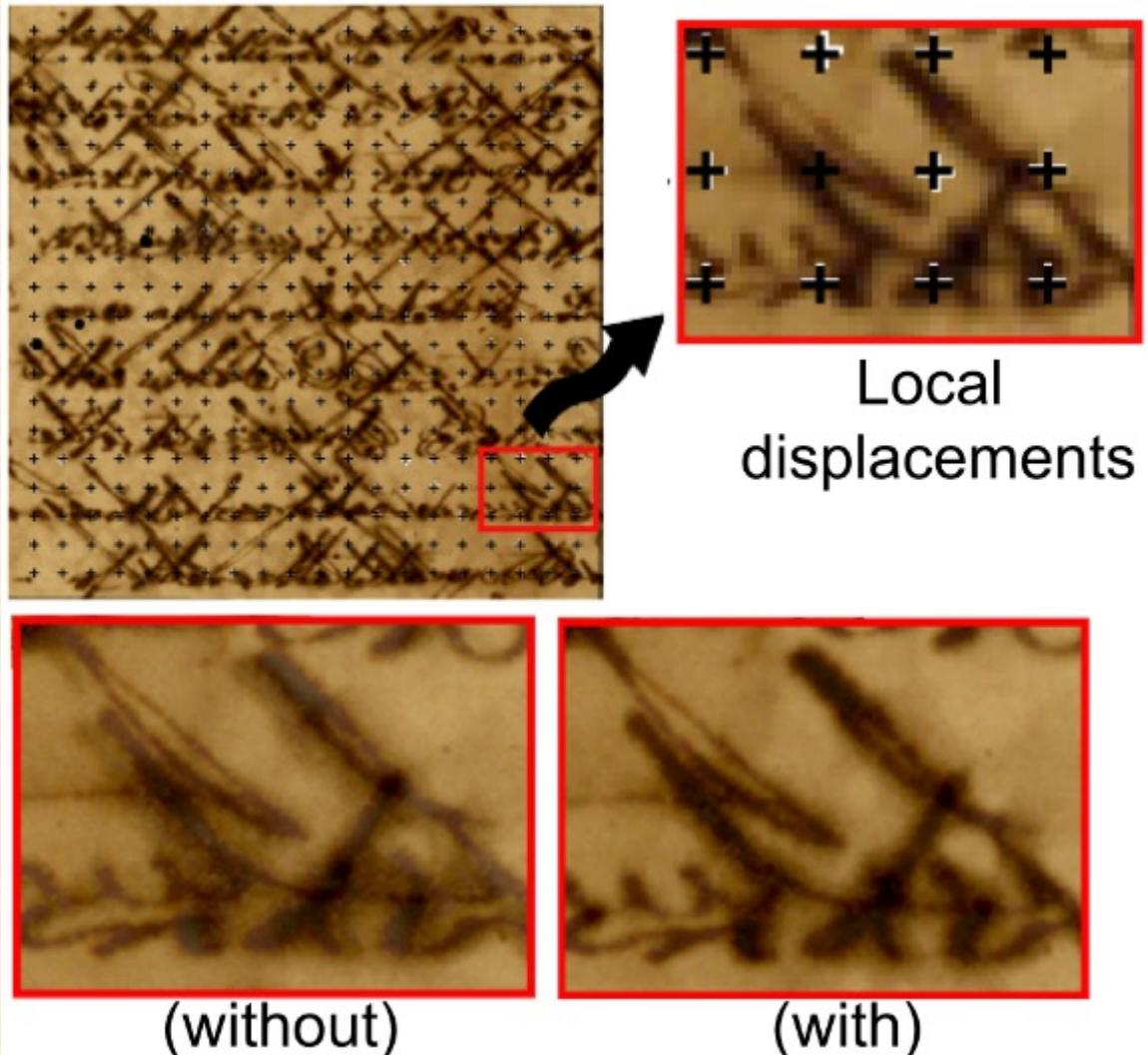


- Treat the two restoration problems together



Local Alignment

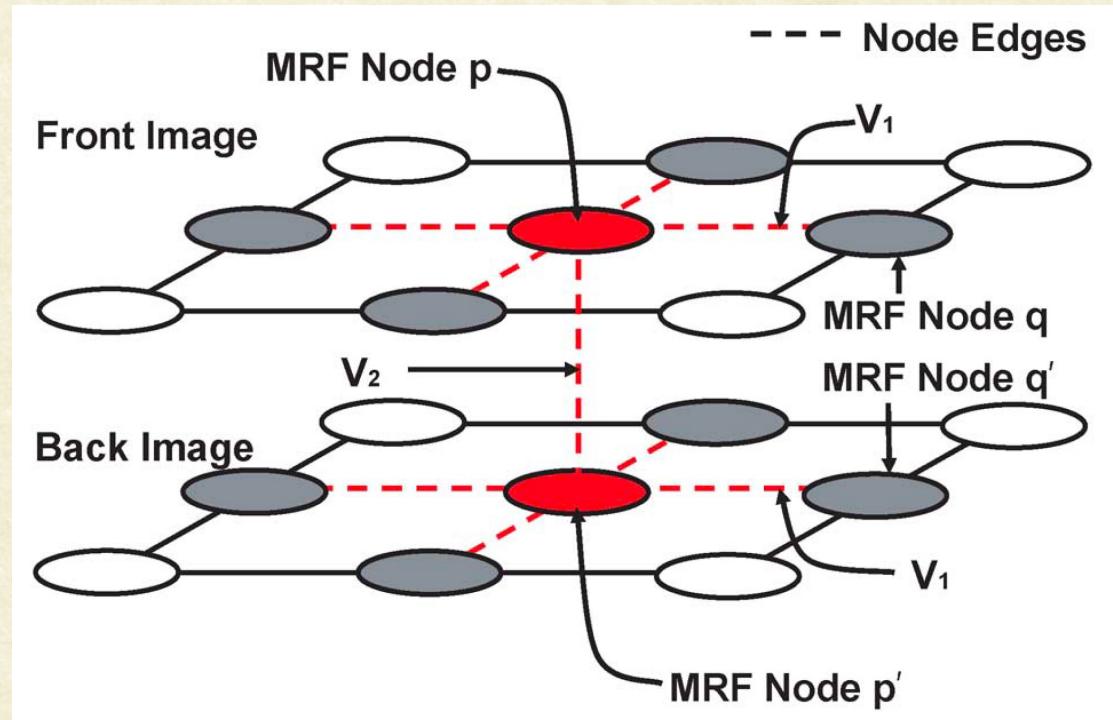
- Compute correlations at a set of grid points: 60x60 patch; [-10, 10] displacement
- Smoothen the local displacements (TPS)
- Warp the verso image





Dual Layer MRF

- Consider two sets of nodes, one for recto and one for verso
- The pixels/nodes of the images are aligned
- Connect corresponding pixels from either side.





Defining Potentials: Data Term

- Class Similarity:

$$S_{\mathcal{F}} = \sum_{m \in \pi^{\mathcal{F}}} \exp(-d_{pm}^2 / d_p^2)$$

$$S_{\mathcal{J}} = \sum_{m \in \pi^{\mathcal{J}}} \exp(-d_{pm}^2 / d_p^2)$$

$$S_{\mathcal{B}} = \sum_{m \in \pi^{\mathcal{B}}} \exp(-d_{pm}^2 / d_p^2),$$

where d_{pm} is the distance from pixel p to the m^{th} cluster center and d_p is the average distance to K nearest cluster centers in training data.

- Unary:

$$E_d(l_p = \mathcal{F}) = \frac{S_{\mathcal{J}} + S_{\mathcal{B}}}{2 \times (S_{\mathcal{F}} + S_{\mathcal{J}} + S_{\mathcal{B}})}$$

$$E_d(l_p = \mathcal{J}) = \frac{S_{\mathcal{F}} + S_{\mathcal{B}}}{2 \times (S_{\mathcal{F}} + S_{\mathcal{J}} + S_{\mathcal{B}})}$$

$$E_d(l_p = \mathcal{B}) = \frac{S_{\mathcal{F}} + S_{\mathcal{J}}}{2 \times (S_{\mathcal{F}} + S_{\mathcal{J}} + S_{\mathcal{B}})}$$



Defining Potentials: Smoothness Term

- Pairwise:

$$E_s = \sum_{p,q \in \mathcal{N}} V_1(l_p, l_q) + \sum_{p,p' \in \mathcal{M}} V_2(l_p, l_{p'})$$

ξ_{pq} :

- Intra-Layer:

$$V_1(l_p, l_q) = \frac{1}{1 + (\xi_{pq})^2}$$

l_p	l_q		
	Foreground	Ink-Bleed	Background
Foreground	∞	d_{pq}^ρ	d_{pq}^c
Ink-Bleed	d_{pq}^ρ	∞	d_{pq}^ρ
Background	d_{pq}^c	d_{pq}^ρ	∞

- Inter-Layer:

l_p	$l_{p'}$		
	Foreground	Ink-Bleed	Background
Foreground	0	0	0
Ink-Bleed	0	∞	∞
Background	0	∞	2ω



President Committee

Despatched with letter from the President Government
to Mr. B. H. Miller reporting the case.
Circular addressed to Robert Jeff Miller
Pahet, enjoining him to take measures
for apprehending certain persons men-
tioned therein.

Authorised to open the leading post office
Branches intended to be opened
in the first of August, & to sign a money
order or bill of exchange.

Directed to give a copy of the number
and an account of the Government
Buildings & Roads to which they
are appropriated, & that
the whole may be sent to the
British Consul at Calcutta.

1

2

3

Authorised to open the post office
Branches intended to be opened
in the first of August, & to sign a money
order or bill of exchange.

Despatched to give publicity to his instructions
to the Postmaster to neglect the Calcutta family
property tax.

162.16

Date	Subject	Amount
1st Dec.	President Committed, &c.	
2nd	Landed with letter from the Resident Com. N.W. to Hickey reporting the late treacherous capture of Lord Clive off Battala Pahar, & requested to take measures for apprehending certain persons con- cerned therein	
3rd	Authorised to give the Treasury of Letters & Papers - Medals to the rebels	
4th	The last of Rep'd. &c to sign a treaty with the King of Pagan	10.
5th	Directed to forward a Report of the manner which presents cost of the Government Buildings & Apartments to which they are appropriated & directed that the whole Department of Public Works in the Eastern Provinces will be placed under the direction of the Building Board in Bengal. 10.	
6th	To make inquiries into the state of the Bengal Cavalry Pahar	10.
7th	Appointed a jury to take statements under oath of all of Revenue and Charge of the Receipts & expenditure of Siraj-ud-din for 12834. A.D. 115	115
8th	Paid Expenses	
9th	Requested to give publicity to his Satisfaction 114.	
10th	Letter respecting the late Sultan's family & property 115	115



Results: Comparison

	Comparison 1	Comparison 2	Comparison 3
Original			
Adaptive [5]			
Wavelet [15]			
Single Layer MRF			
Dual Layer MRF			



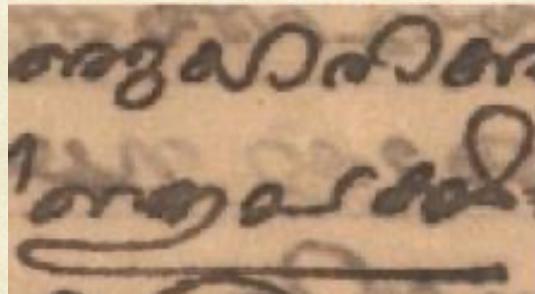
A Closer Look

Dear Madam
As you will see
Mr. J. V. F.
I venture to a

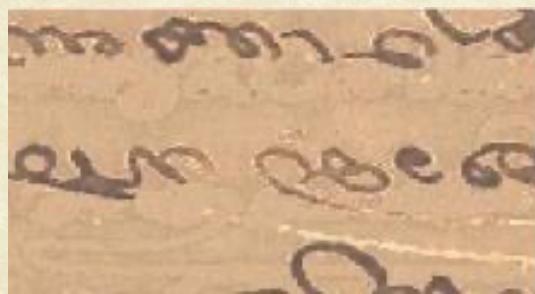


Going Beyond Segmentation

- Can we separate the layers or de-bleed the document?



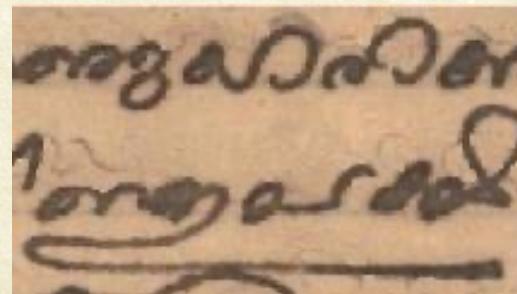
Input Image



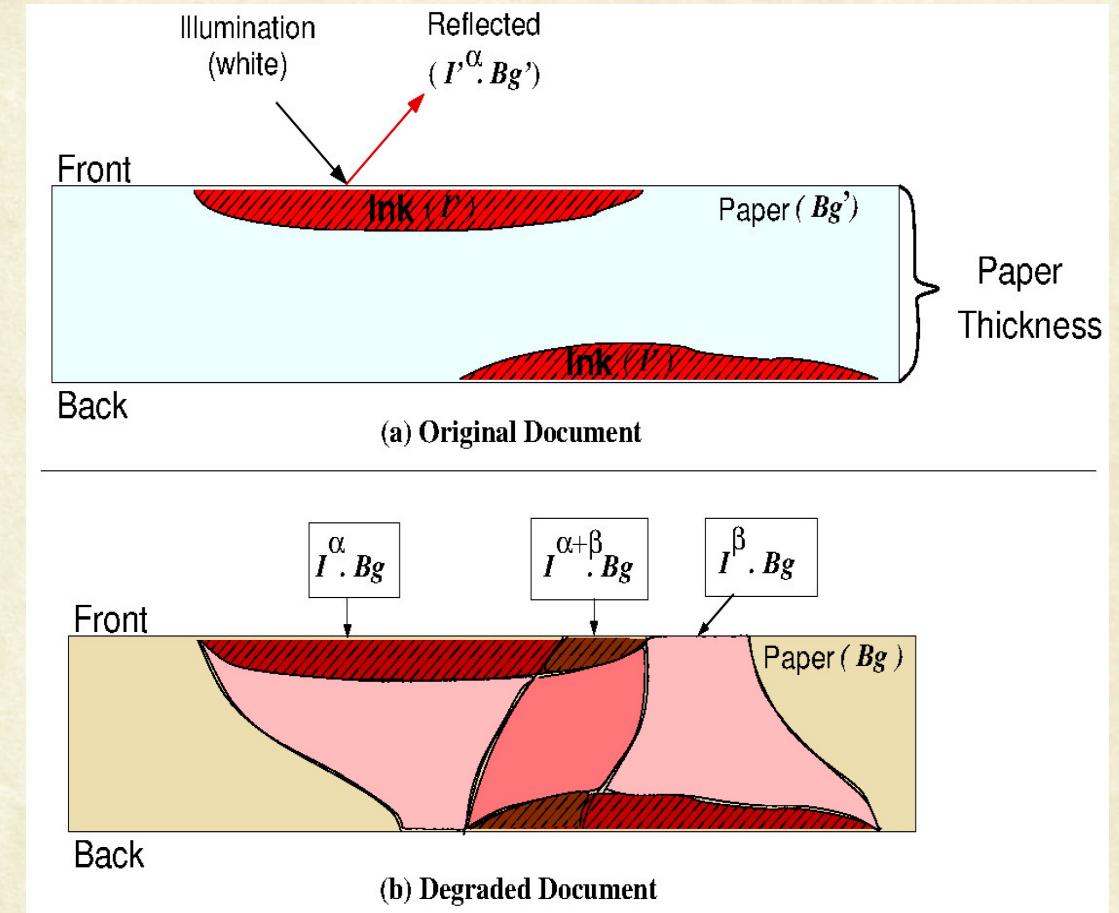
Bleed Layer



Background Layer



Output (Ink Layer)



Each pixel contain different levels of ink, bleed and background.



Multi-Label Classification Model



(a) $\alpha=1, \beta=.25$



(b) $\alpha=.8, \beta=.3$



(c) $\alpha=1, \beta=1$

$$Ink_p = I^{(\alpha_p + \beta_p)} B g_p$$

- Assume α and β are discrete $\{0, 0.25, 0.5, 0.75, 1\}$
- Reduces to 20 possible labels for a pixel.
- Define the energy terms and solve for labels.



Defining Potentials

- Total Energy:

$$E = E_d + \phi E_s$$

- Unary:

$$E_d(l_p = l^i) = \frac{\sum_j S^j - S^i}{19 \times \sum_j S^j}$$

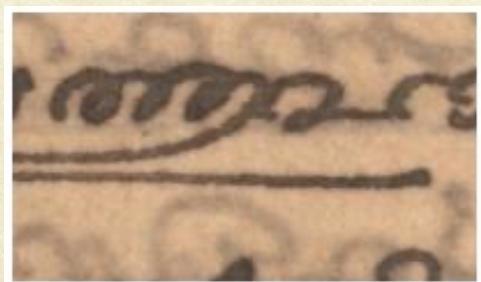
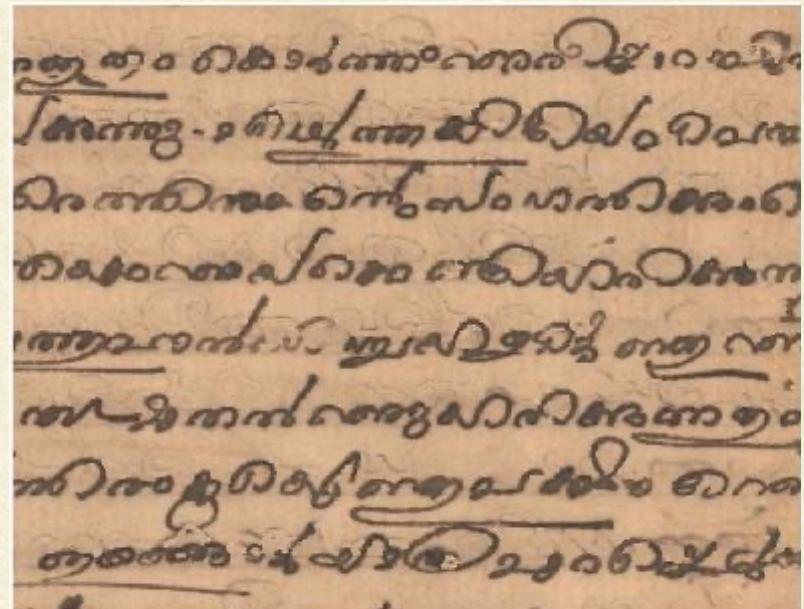
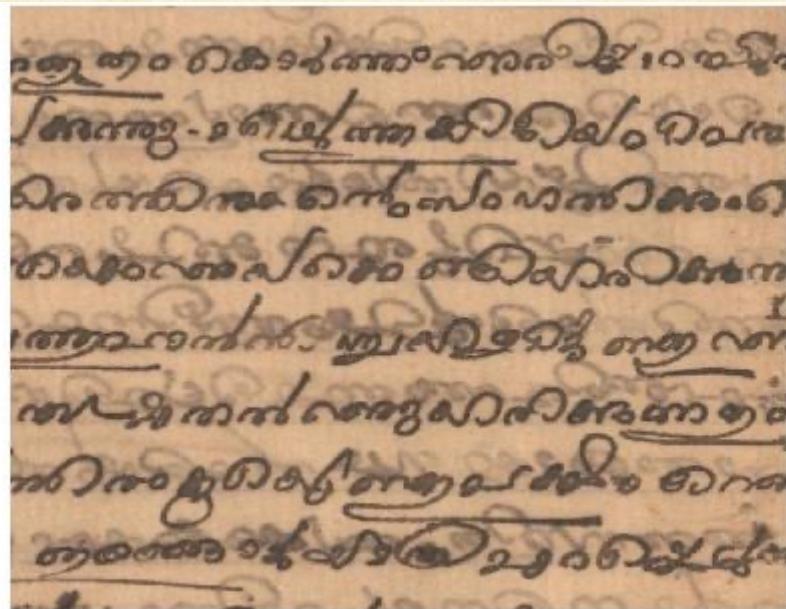
- Pairwise:

$$E_s = \sum_{p,q \in \mathcal{N}} V_1(l_p, l_q) + \sum_{p,p' \in \mathcal{M}} V_2(l_p, l_{p'})$$

- Solved using α -expansion move of graph cuts.



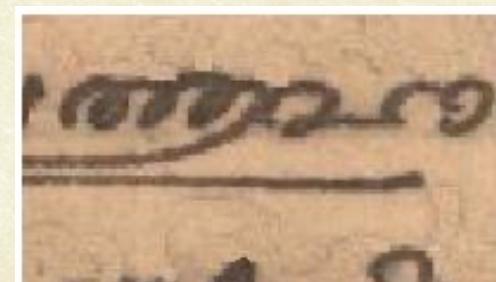
Results



Input

Binary DMRF

Multilabel DMRF





Questions?