CSE251
Basics of Computer Graphics
Module: Graphics Pipeline

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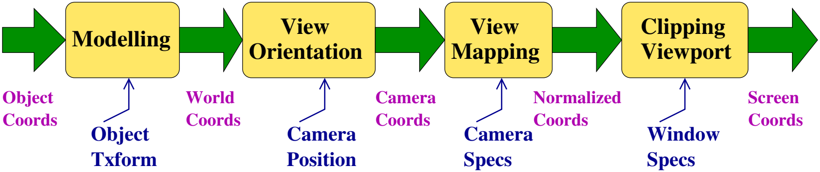
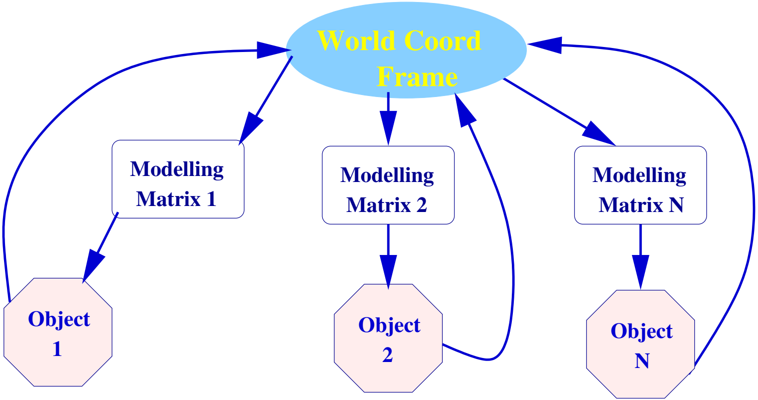
3D Graphics Pipeline

- Objects are specified in their own coordinate system and placed in the world coordinate frame.
- Camera is also placed in the world coordinate frame.
- Camera-to-world geometry is first projected to normalized coordinates and then to screen.



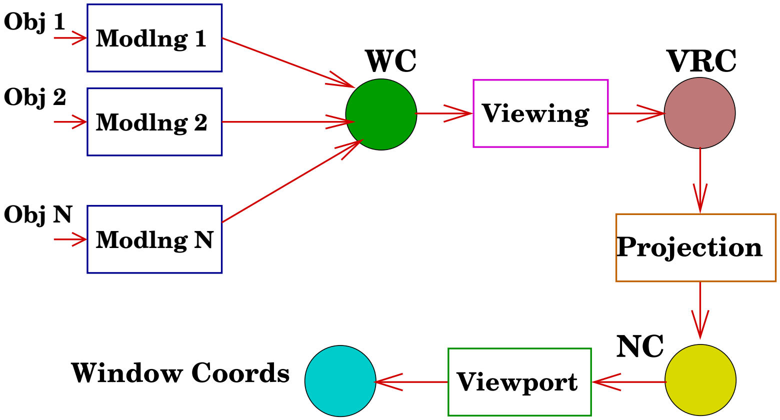
Modelling Different Objects



Modelling & Viewing: Summary

- Place objects in the world coordinate frame
- Place camera in the world coordinate frame
- Can compute object points in camera coordinate frame
- $\mathbf{P}_{\mathsf{VRC}} = \mathbf{V} \cdot \mathbf{M} \cdot \mathbf{P}_{\mathsf{ORC}}$

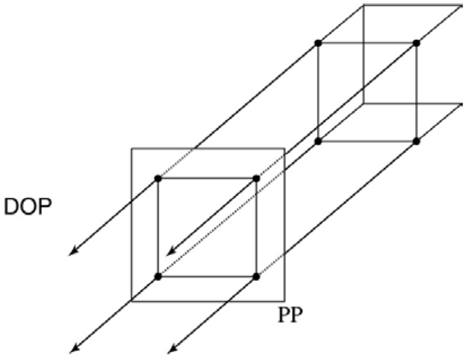
3D Graphics: Block Diagram



Projections

- Projection involves *projectors* starting from 3D points and hitting the 2D *projection plane*, forming the *image* of the point.
- Two types of projections.
- **Parallel projection**: Projectors are parallel to each other, all have the same *direction of projection* (**DOP**).

Projections (cont.)



Projections

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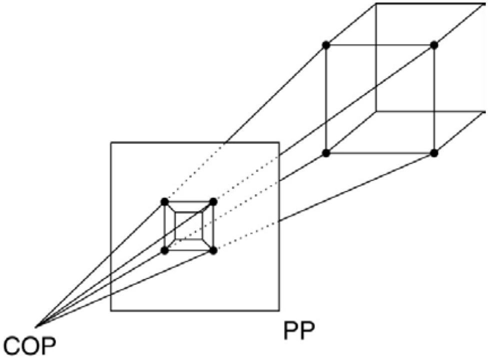
- Two types of projections.
- **Parallel projection**: Projectors are parallel to each other, all have the same *direction of projection* (**DOP**).
- **Perspective projection**: All projects pass through a point in space called the *centre of projection* (**COP**).

Projections (cont.)

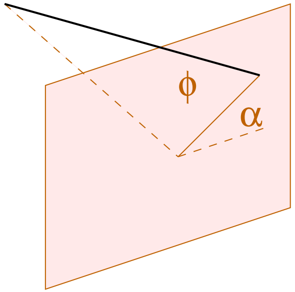


Parallel Projections

- **Orthographic:** Projection plane is perpendicular to the direction of projection.
  - If direction of projection parallel to the axes: *plan, elevation, side elevation*.
  - If DoP $(\pm 1, \pm 1, \pm 1)$: *isometric* projection.
- **Oblique:** Otherwise.
  - *Cavalier* when projectors make 45 degrees with the projection plane.
  - *Cabinet* when they make `arctan(2)` degrees with the projection plane.

Oblique Projections

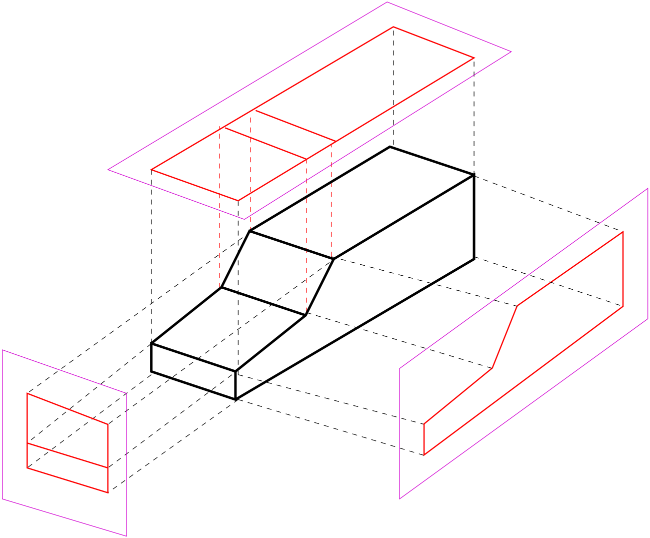
- Projector angle ϕ and angle with horizontal α.
- Cavalier: Length along the depth axis preserved.
- Cabinet: Length along depth axis halved. More realistic.
- α is between 30 and 45 degrees.
- Orthographic when $\phi = 90$.



Orthographic Projections

- Lengths parallel to the projection plane are preserved.
- Only direction of projection matters; distance from the point to the projection plane doesn't.
- Good approximation for a camera with a long focal length. (Orthographic with uniform scaling).
- Plan, elevation, side views etc.

Orthographic Projections (cont.)



Orthographic Projection Equation

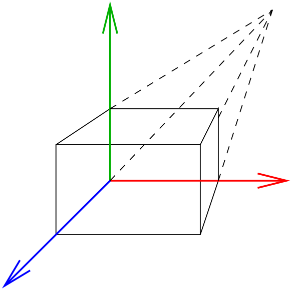
- Can be expressed as a matrix equation:

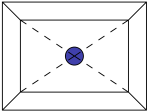
$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- If uniform scaling is involved, the top two 1's should be the scale factor.

Perspective Projections

- Can be characterized by the number of **vanishing points**. (projections of points at infinity).
- Depends on the number of axes the projection plane intersects.
- 1-point, 2-point, and 3-point perspective projections.

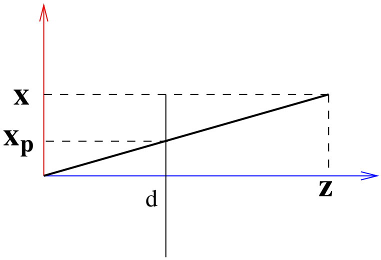




Geometry of Perspective Projection

- What is x_p, y_p, z_p?
- We know x, z, and d.
- Remember similar triangles?

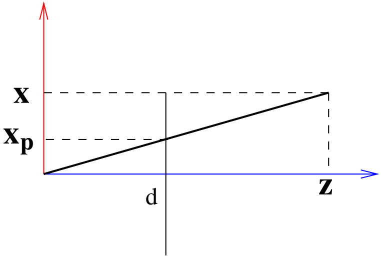
$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ w \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



Geometry of Perspective Projection

- $\frac{x_p}{d} = \frac{x}{z}, \quad \frac{y_p}{d} = \frac{y}{z}, \quad z = d.$
- In matrix form,

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

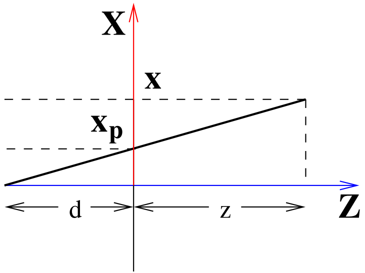


- Coordinates scaled down proportional to the depth or z values.

Another View

- Shift origin to lie on the projection plane, CoP at $(0, 0, -d)$, we get the matrix:

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



Another View

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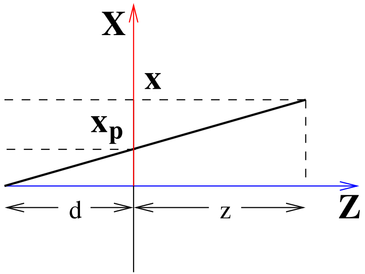
$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- Orthographic Projection matrix is a special case when $d \to \infty$.



$$x_p = \frac{xd}{z+d} = \frac{x}{1+\frac{z}{d}}, \quad z_p = 0$$

Projections: Summary

Perspective

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$d < \infty$$

Orthographic

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$d = \infty$$

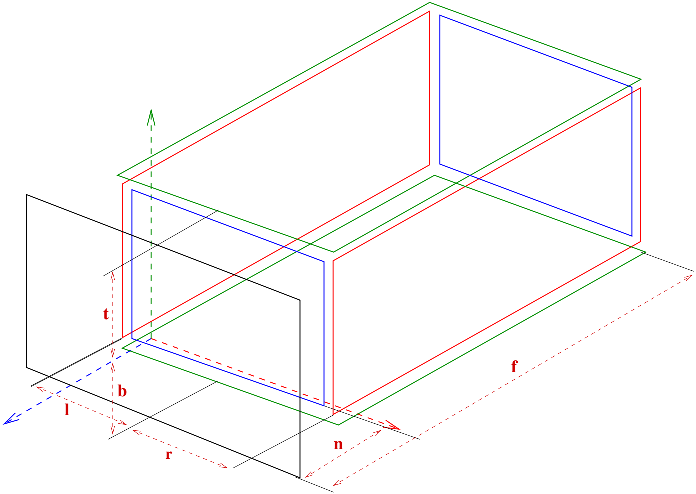
Volume of Visibility

- Cameras have finite fields of view in horizontal and vertical directions.
- What is the shape of its visible space?
- A cylinder for orthographic projections and a cone starting from the CoP for perspective may seem natural.
- Mathematics is difficult for cones; rectangular structures are easier!
- **View Volume:** The volume of potentially visible space.

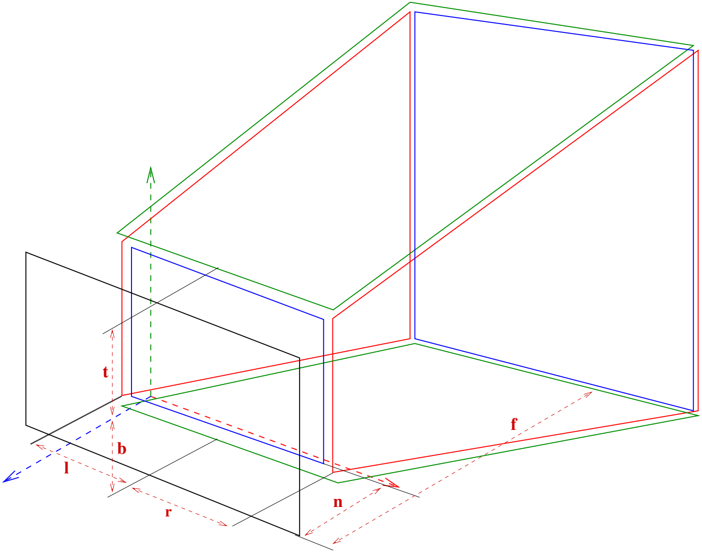
View Volume

- View volume is a cube for orthographic cameras and a (truncated) pyramid for perspective projections.
- 4 planes (left, right, top, bottom) define the view volume.
- Graphics cameras use 2 additional planes to limit visibility: **near & far!**
- Planes are specfied in VRC; they move with the camera

Orthographic View Volume

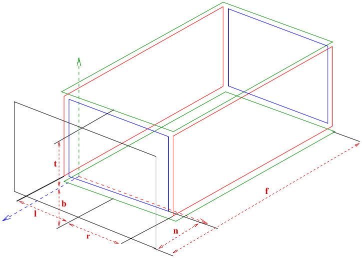


Perspective View Volume

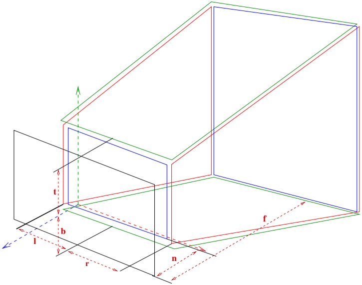


Orthographic & Perspective

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix}$$



What About the Focal Length?

- An ideal pin-hole camera has the whole world in focus.
- Finite focal-length lenses introduce the effect of focus in real cameras.
- Even for them, the depth of field (region in focus) increases as the f-stop increases or the aperture gets smaller.
- Computer Graphics simulates ideal pin-hole cameras.
- Depth of field can be simulated by intentional blurring.

View Volume Specification

- View volume is specified by 6 planes:
  left, right, top, bottom, near, far. All values in VRC

  `glm::frustum()` or `glm::ortho()`

- left, right, top, bottom: signed distances.
  near, far: positive distances to planes.

- Needn't be symmetric!

Alternate Specification

- Symmetric view volumes: horizontal and vertical fields of view $\theta_{\mathbf{h}}, \quad \theta_{\mathbf{v}}$
- For symmetric perspective view volumes:

  $\tan \frac{\theta_h}{2} = \ldots?$

  $\tan \frac{\theta_v}{2} = \ldots?$
- Aspect ratio: top / right.

  Using `glm::perspective()`

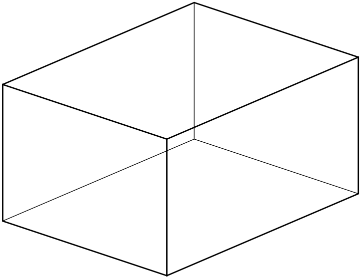
Canonical View Volume

- Projection is not performed right away; instead, map the view volume to a cube of fixed dimensions, called the **canonical view volume** or a standard view volume
- A **normalizing matrix** performs this transformation.
- Why?
  - Easier to eliminate objects outside the view volume.
  - Orthographic & perspective aren't different.
  - The z-coordinates not thrown away. (Used later!)

Canonical View Volume: Dimensions

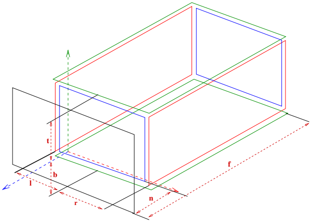
- OpenGL:

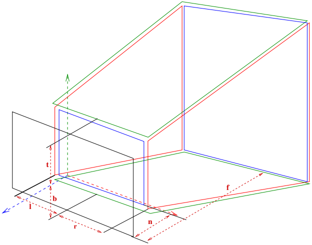
  $$-1 \leq x \leq 1$$
  $$-1 \leq y \leq 1$$
  $$-1 \leq z \leq 1$$



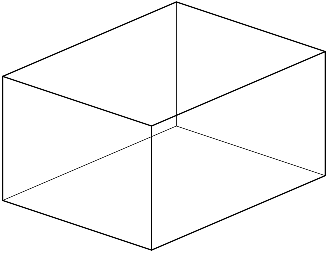
- Canonical view volume is the Orthographic View Volume, with appropriate scaling.

Orthographic & Perspective





To target view volume:



Orthographic Normalizing Matrix

- What are the side lengths on start?
- What are the side lengths at end?
- Whats the scale factor?
- Where is the origin at strat? At end?
- How do we achieve that?

Orthographic Normalizing Matrix

- Lengths (right - left), (top - bottom) and (far - near) scaled to 2.
- Shift origin so as to range from -1 to +1.
- Matrix??

Orthographic Normalizing Matrix (cont.)

- Matrix:

$$\begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & \frac{2}{far-near} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

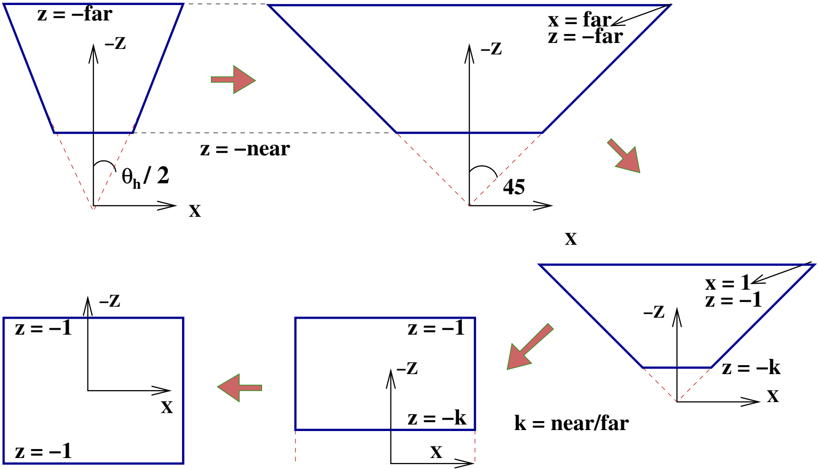
- $(0,0,-n)$ maps to $+1$ and $(0,0,-f)$ maps to -1.
- Drop z and use the (x,y) coordinates as the (normalized) window coordinates.

Symmetric Perspective Proj Matrix

- More complicated than orthographic case, as a frustum has to be mapped to a cube. Do it in steps.
- First, scale the horizontal and vertical extents so that the vertical and horizontal fields of view are 90 degres.
- A scaling transformation with $s_x = ??, s_y = ??, s_z = 1$
- View volume is almost right except for a uniform scale.
- Next, scale uniformly so that the far plane is at -1. We will also have $-1 \leq x, y \leq 1$ at the far plane after this.

Symmetric Perspective Proj Matrix (cont.)

- $s_x = s_y = s_z = ??$



Symmetric Perspective Proj Matrix (cont.)

- M_1 with $s_x = \cot\frac{\theta_h}{2}, \quad s_y = \cot\frac{\theta_v}{2}, \quad s_z = 1$
- M_2 with $s_x = s_y = s_z = \frac{1}{\mathbf{far}}$
- $M_2 M_1 = \begin{bmatrix} \frac{\cot\theta_h/2}{\mathbf{far}} & 0 & 0 & 0 \\ 0 & \frac{\cot\theta_v/2}{\mathbf{far}} & 0 & 0 \\ 0 & 0 & \frac{1}{\mathbf{far}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- The near plane is now at $\mathbf{k} = \frac{\mathbf{near}}{\mathbf{far}}$.

Symmetric Perspective Proj Matrix (cont.)

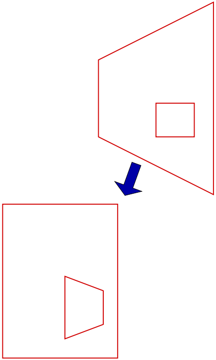
- View volume fits into the canonical view volume, but is still a frustum!
- Scale by to $-z$ to convert to a cube using a matrix with last row $\begin{bmatrix} 0 & 0 & -1 & 0 \end{bmatrix}$.
- Simultaneously send third component to range $[-1, 1]$. $z = -k$ maps to 1 and $z = -1$ maps to -1.
- Scale z by $\frac{1+k}{1-k}$ and translate by $\frac{2k}{1-k}$.
- Perspective division: convert from homogeneous to cartesian by dividing by the last component.
- Keep third component for later use. Relative ordering needs to be preserved. The values are not important.

Perspective Normalizing Txform

- Matrix M_3 for this step:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1+k}{1-k} & \frac{2k}{1-k} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

- $(x, y, -k)$ & $(x, y, -1)$ go to?
- Final matrix: $M = M_3 M_2 M_1$
- Frustum becomes a cube



Final 2D Coordinates

$$(u, v, d) \equiv \begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = M_3 M_2 M_1 \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{\text{VRC}}$$

- **Perspective division:** Divide x', y' coordinates by the w to get the normalized coordinates (u, v). (z' maintains ordering and can be used without division.)
- The normalized d component has non-linear precision. Higher around the *near* plane and lower around the *far* plane due to the division by z.

OpenGL Normalizing/Perspective Matrix

- The projection matrix in OpenGL is given by

$$P = \begin{bmatrix} \frac{2n}{r-l} & 0 & A & 0 \\ 0 & \frac{2n}{t-b} & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

where,

$$A = \frac{r+l}{r-l}, B = \frac{t+b}{t-b}, C = \frac{f+n}{f-n}, D = \frac{2nf}{f-n}$$

Actual Projection

- We have already performed the perspective division.
- Projection involves simply dropping the z coordinate and scaling x-y to the viewport.
- Why go through with the z-coordinates?
- The ordering is preserved along the depth dimension. z values can be used for visibility determination.

Where is the Film?

- Turns out: **It does not matter**.
- A final scaling is in the viewport transformations.
- As long as the film is in front of the camera, we will see an upright image.
- Can consider the near plane as the film plane.

Viewport Txformation: To Window

- Image of size -1 to +1 in X and Y is ready. The **viewport** transformation maps it to the actual window on screen.
- From $[1,1]$, map x and y to $[0,W]$ and $[0,H]$.
- First step: set sizes by scaling: $\mathbf{S}(\frac{\mathbf{W}}{\mathbf{2}}, \frac{\mathbf{H}}{\mathbf{2}})$.
  Next: Translate origin to South-West corner: $\mathbf{T}(\frac{\mathbf{W}}{\mathbf{2}}, \frac{\mathbf{H}}{\mathbf{2}})$
- Overall: $\mathbf{M} = \mathbf{T}(\frac{\mathbf{W}}{\mathbf{2}}, \frac{\mathbf{H}}{\mathbf{2}})\ \mathbf{S}(\frac{\mathbf{W}}{\mathbf{2}}, \frac{\mathbf{H}}{\mathbf{2}}) = \begin{bmatrix} \frac{W}{2} & 0 & \frac{W}{2} \\ 0 & \frac{H}{2} & \frac{H}{2} \\ 0 & 0 & 1 \end{bmatrix}$

General Viewport Txform

- General command: `glViewport(l, b, r, t)`.
- Translate so the range of x, y is $0 \cdots 2$.
- Scale so x varies from 0 to $(r-l)$ and y varies from 0 to $(t-b)$.
- Translate so x range is l to r and y range is b to t.
- Matrix for this? $\mathbf{T}(l,b)\,\mathbf{S}(\frac{r-l}{2},\frac{t-b}{2})\,\mathbf{T}(1,1)$

$$= \begin{bmatrix} \frac{r-l}{2} & 0 & \frac{r+l}{2} \\ 0 & \frac{t-b}{2} & \frac{t+b}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

General Viewport Txform (cont.)

- $[-1 \quad -1 \quad 1]^\intercal$ maps to $[l \quad b \quad 1]^\intercal$.
- $[1 \quad 1 \quad 1]^\intercal$ maps to $[r \quad t \quad 1]^\intercal$.

(Point) Pipeline in Action

- Points are transformed from Object to World to Canonical to Window coordinates.
- Each 3D point maps to a pixel (i, j) in the window space.
- Lines are made out of two points. Triangles and polygons are made out of 3 or more points.

Recapitulation

- 3D Graphics additionally involves projecting the 3D world to the 2D image plane of the camera.
- Compute the 3D world with respect to the camera. Or compute the relative geometry first.
- This involves a series of rigid transfomrations. For complex objects/environments, each object or its part is described in its own coordinate system.
- **Modelling** places these different objects in the world coordinate system. This could involve a hierarchy of transforms for objects made up of complex parts.

Recapitulation (cont.)

- **View Orientation** computes the world in the VRC.
- Camera can be perspective or parallel (orthographic, oblique). 6 planes give the view volume and defines the camera.
- **View Mapping** involves mapping the world to a canonical view volume, which is an orthographic view volume. This **Normalizing Transformation** has different forms for parallel and perspective cameras.
- **Projection** and **Clipping** are easy to perform in the canonical view volume. An image with dimensions from -1 to +1 results.

Recapitulation (cont.)

- **Viewport** tranformation is the final step, involving a 2D scaling and translation to map to window coordinates that can be used to address the frame buffer.
- Given a description of the 3D world primitives, project each point to 2D to get 2D primitives. These can be scan-converted using standard algorithms.