27.08.2021

Digital Image Processing (CSE/ECE 478)

Lecture-3: Recap/Discussion

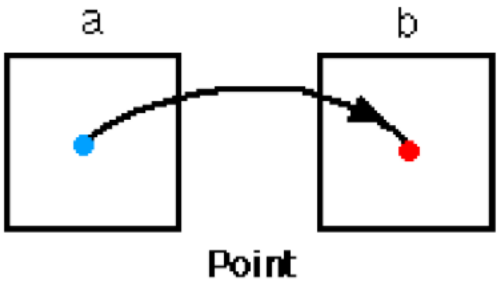
Center for Visual Information Technology (CVIT), IIIT Hyderabad

Ravi Kiran

Sudipta Banerjee

Spatial Domain Processing

- Manipulating Pixels Directly in Spatial Domain
- 3 approaches
- 1. Point to Point



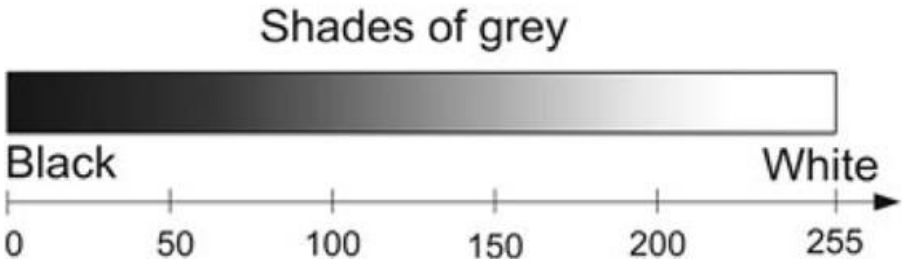
Linear Intensity Transforms

- $T(z) = z + K$

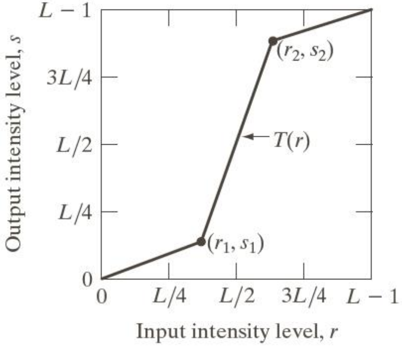
- $T(z) = z - K$

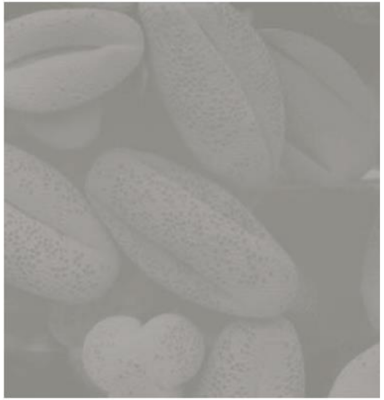
- $T(z) = Kz$

- $T(z) = K_1 z + K_2$

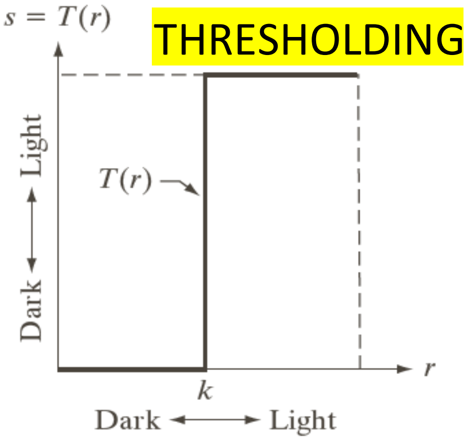


Piecewise-Linear Transformations



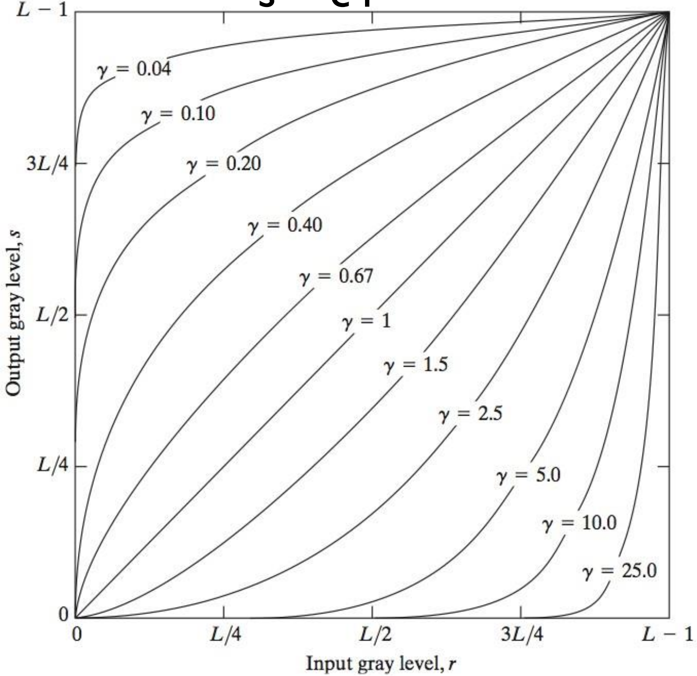


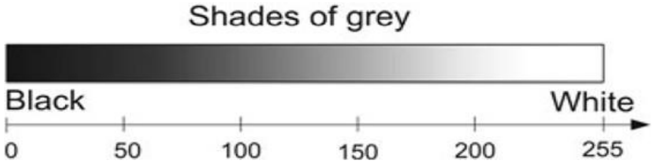
THRESHOLDING



Power-Law Transformations

$s = c\,r^{\gamma}$



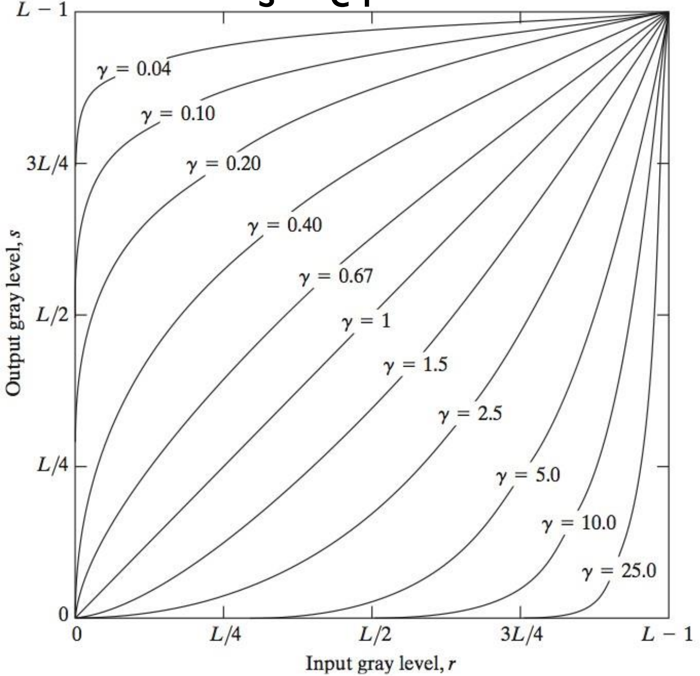


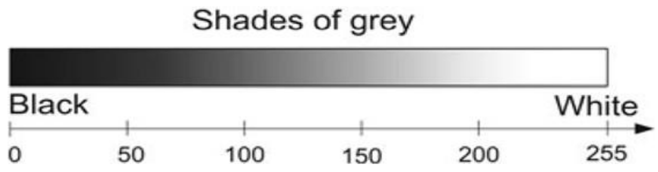
a b
c d

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

Power-Law Transformations

$s = c\, r^{\gamma}$



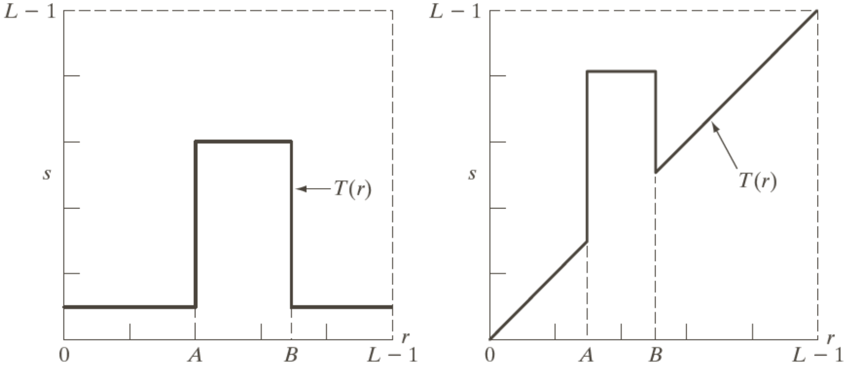


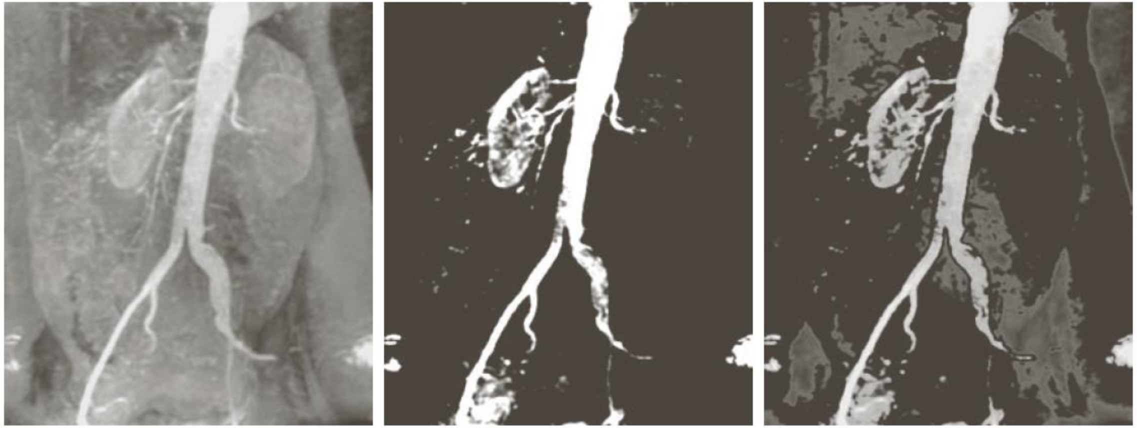
Demo:
https://colab.research.google.com/drive/11qIL0VKleZnONtPuxAryAf9WkUC7kEMI#scrollTo=aU5WQaqOpSCr&line=12&uniqifier=1

Intensity Slicing

a b

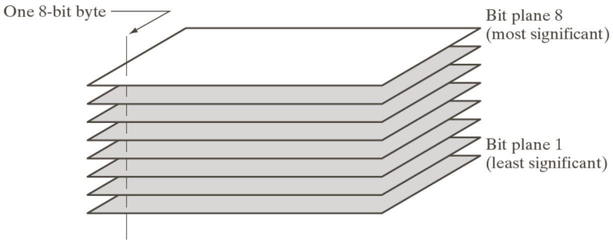
FIGURE 3.11 (a) This transformation highlights intensity range $[A, B]$ and reduces all other intensities to a lower level. (b) This transformation highlights range $[A, B]$ and preserves all other intensity levels.





a b c

Bit plane slicing



a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

27.08.2021

Digital Image Processing (CSE/ECE 478)

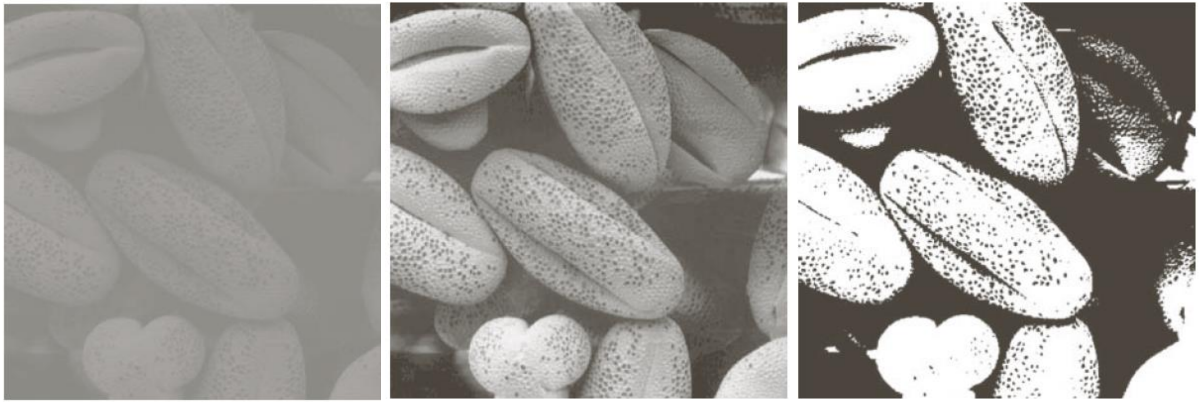
Lecture-4: Histogram Processing

Center for Visual Information Technology (CVIT), IIIT Hyderabad

Ravi Kiran

Sudipta Banerjee

Piecewise-Linear Transformations

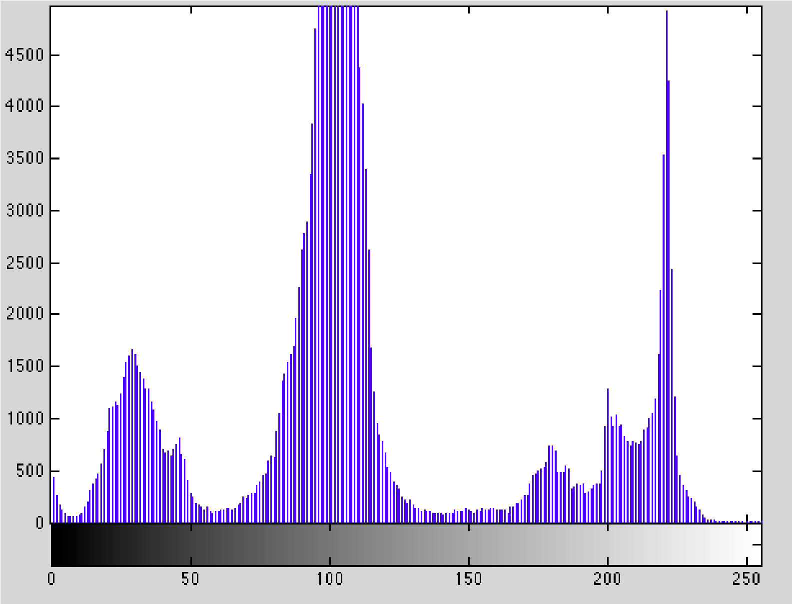


Histogram: An image representation + visualization

$$h_r(i) = n_i$$

i → intensity value, range [0, L-1]

n_i → number of pixels with intensity i

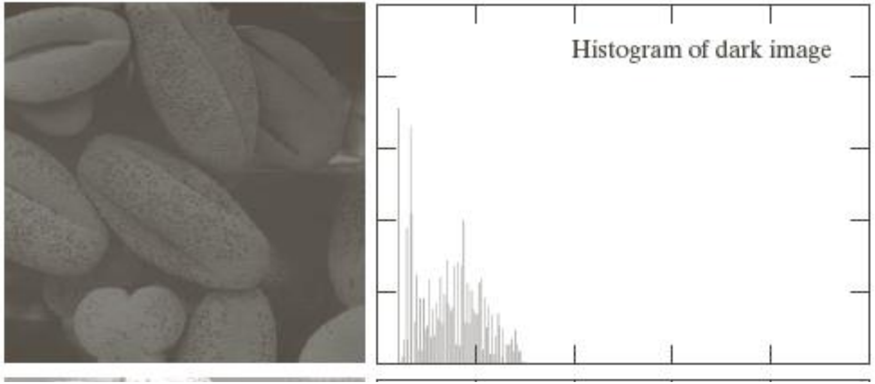


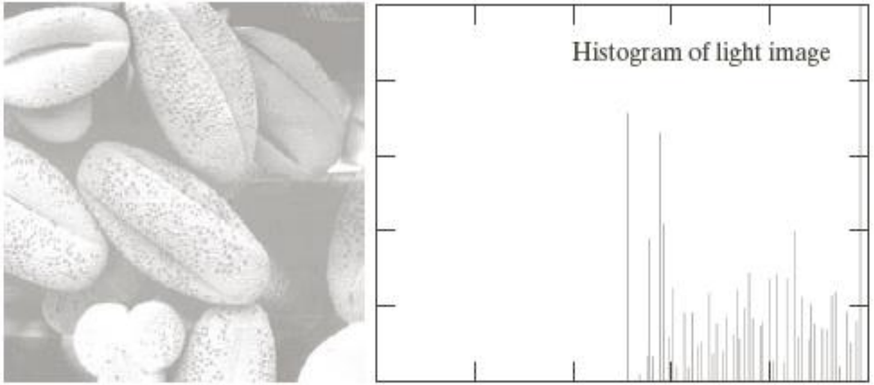
Histograms

- What can we infer from histograms?

Histogram viewing standard in most DSLR cameras

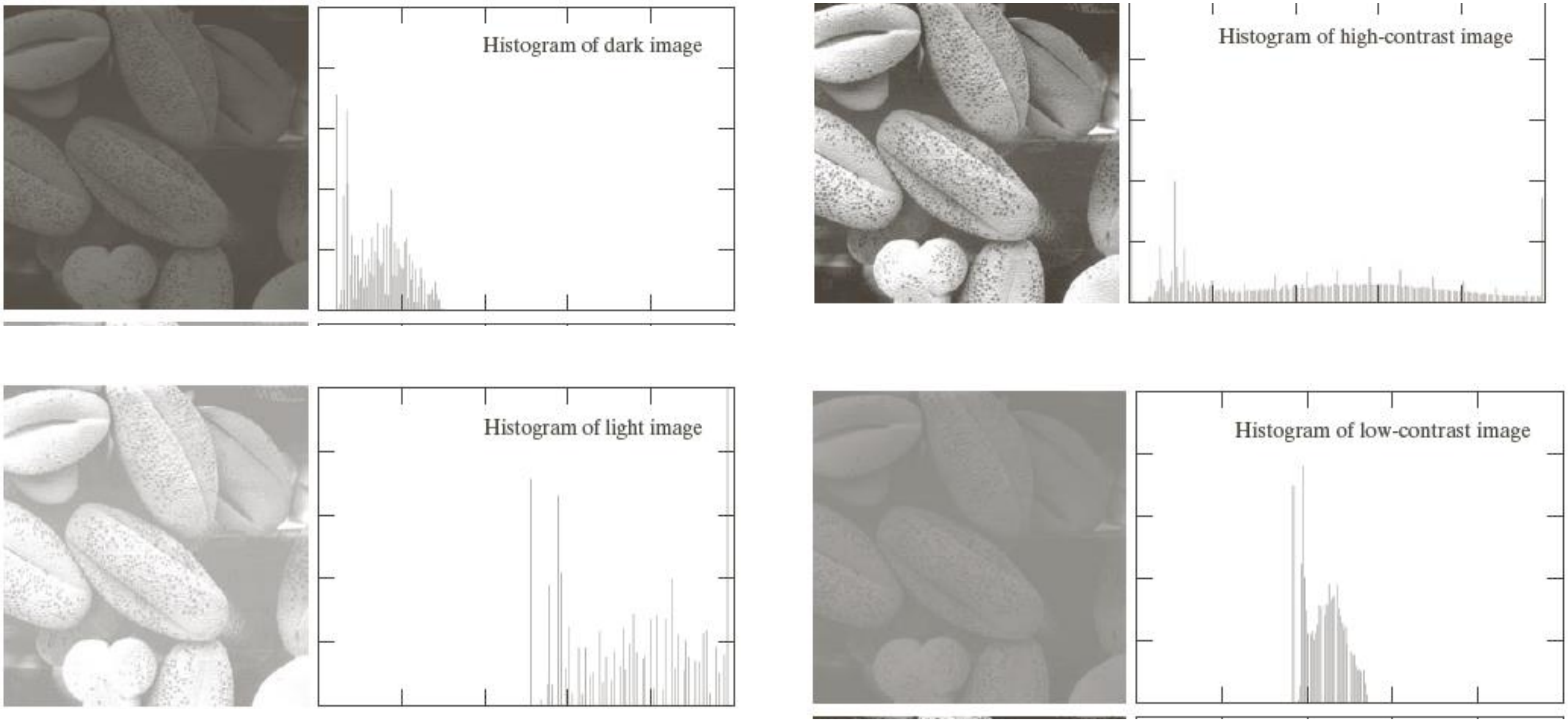
Histograms and Contrast



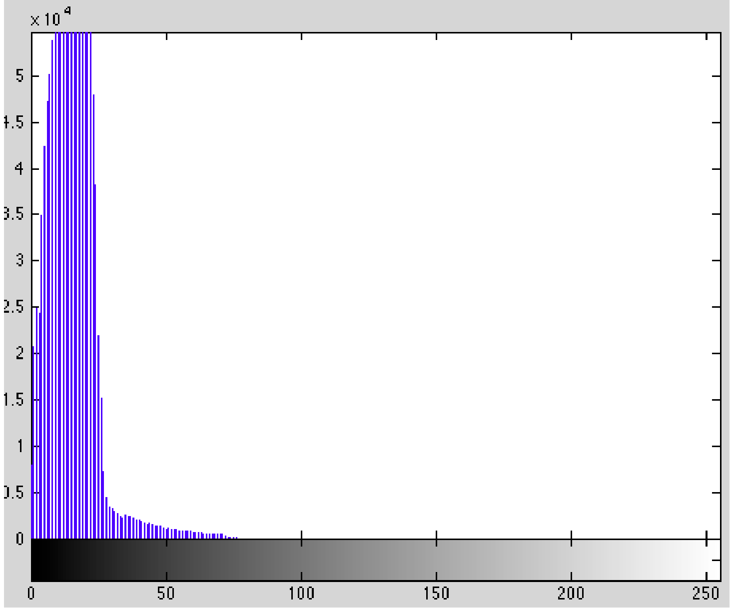


$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Histograms and Contrast

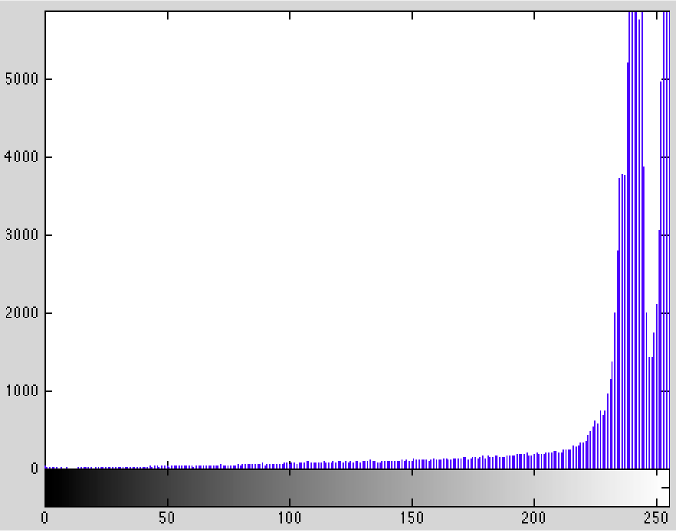


Histograms



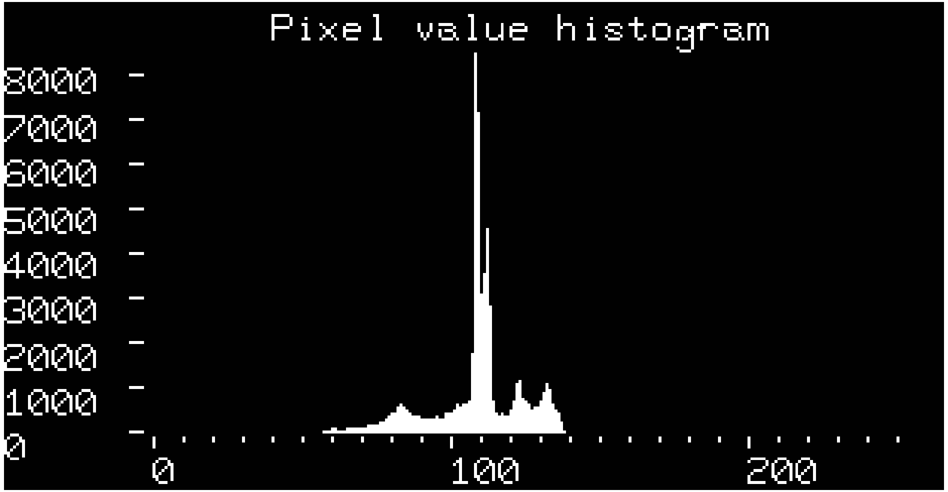
Under exposure

Histograms



Over exposure

A low-contrast image and its histogram



Contrast Stretching

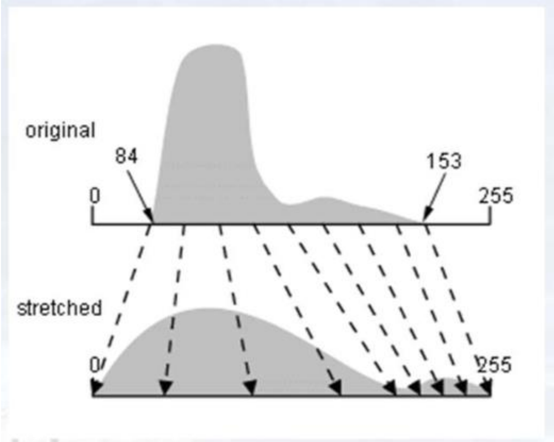
$$f_{\text{ac}}(a) = a_{\text{min}} + (a - a_{\text{low}}) \cdot \frac{a_{\text{max}} - a_{\text{min}}}{a_{\text{high}} - a_{\text{low}}}$$

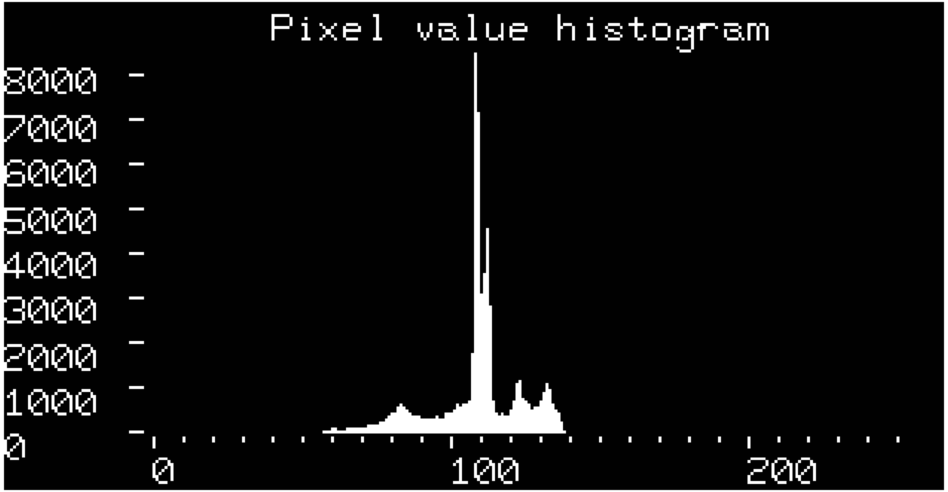
Contrast Stretching

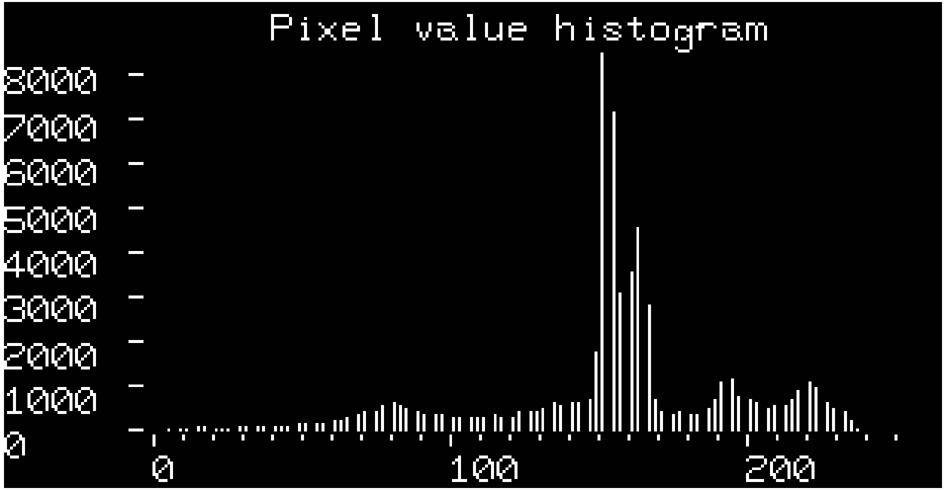
$$f_{\text{ac}}(a) = a_{\min} + (a - a_{\text{low}}) \cdot \frac{a_{\max} - a_{\min}}{a_{\text{high}} - a_{\text{low}}}$$

If a_{min} = 0 and a_{max} = 255

$$f_{\text{ac}}(a) = (a - a_{\text{low}}) \cdot \frac{255}{a_{\text{high}} - a_{\text{low}}}$$




Pixel value histogram
8000
7000
6000
5000
4000
3000
2000
1000
0
0
100
200


Pixel value histogram
8000
7000
6000
5000
4000
3000
2000
1000
0
0
100
200

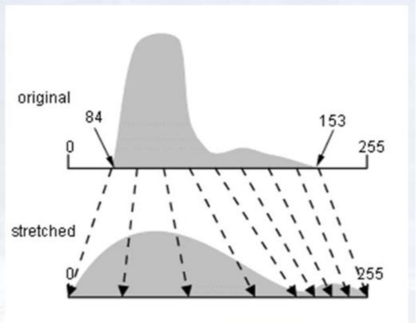
Contrast Stretching

Suppose we have a <u>single</u> pixel with intensity 255 in the original intensity range.
What happens ?

$$f_{\mathrm{ac}}(a) = a_{\mathrm{min}} + (a - a_{\mathrm{low}}) \cdot \frac{a_{\mathrm{max}} - a_{\mathrm{min}}}{a_{\mathrm{high}} - a_{\mathrm{low}}}$$

If a_{min} = 0 and a_{max} = 255

$$f_{\mathrm{ac}}(a) = (a - a_{\mathrm{low}}) \cdot \frac{255}{a_{\mathrm{high}} - a_{\mathrm{low}}}$$

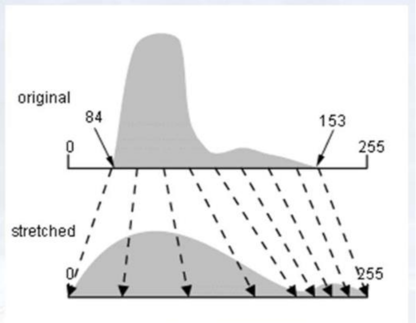


Contrast Stretching

$$f_{ac}(a) = a_{min} + (a - a_{low}) \cdot \frac{a_{max} - a_{min}}{a_{high} - a_{low}}$$

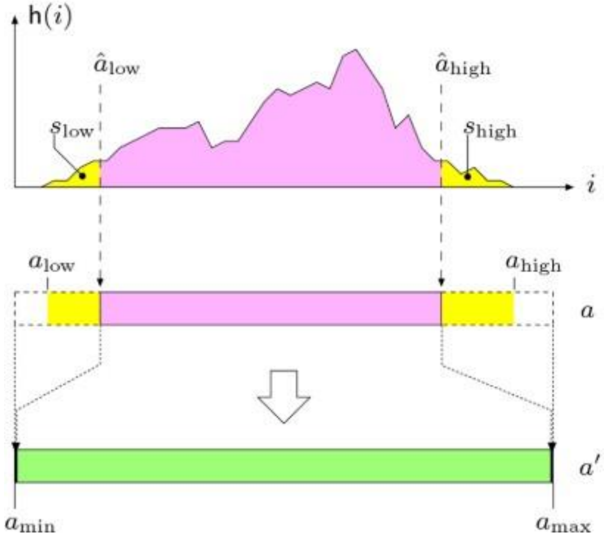
If a_{min} = 0 and a_{max} = 255

$$f_{ac}(a) = (a - a_{low}) \cdot \frac{255}{a_{high} - a_{low}}$$



Suppose we have a <u>single</u> pixel with intensity 0 in the original intensity range.
What happens ?

Contrast Stretching ver. 2

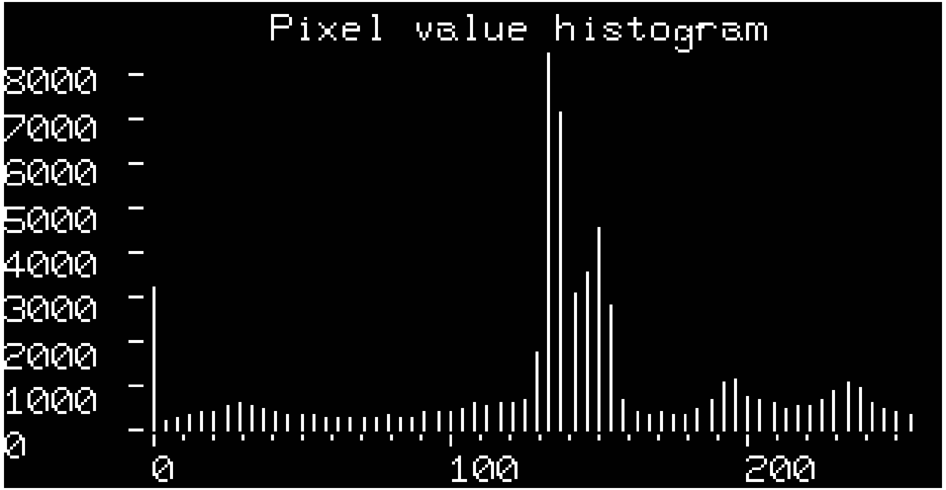


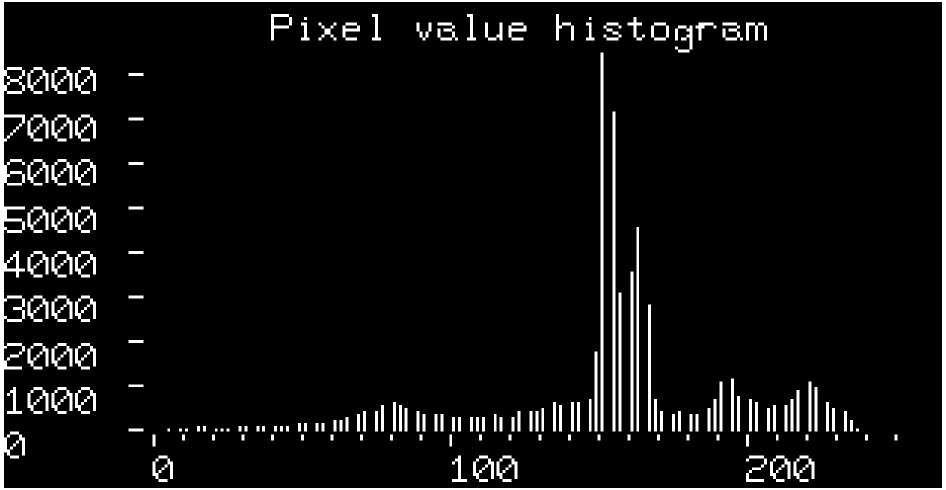
$$\hat{a}_{\text{low}} = \min\{\, i \mid \mathsf{H}(i) \geq M \cdot N \cdot s_{\text{low}} \}$$

$$\hat{a}_{\text{high}} = \max\{\, i \mid \mathsf{H}(i) \leq M \cdot N \cdot (1 - s_{\text{high}}) \}$$

$$f_{\text{mac}}(a) = \begin{cases} a_{\min} & \text{for } a \leq \hat{a}_{\text{low}} \\ a_{\min} + (a - \hat{a}_{\text{low}}) \cdot \dfrac{a_{\max} - a_{\min}}{\hat{a}_{\text{high}} - \hat{a}_{\text{low}}} & \text{for } \hat{a}_{\text{low}} < a < \hat{a}_{\text{high}} \\ a_{\max} & \text{for } a \geq \hat{a}_{\text{high}} \end{cases}$$

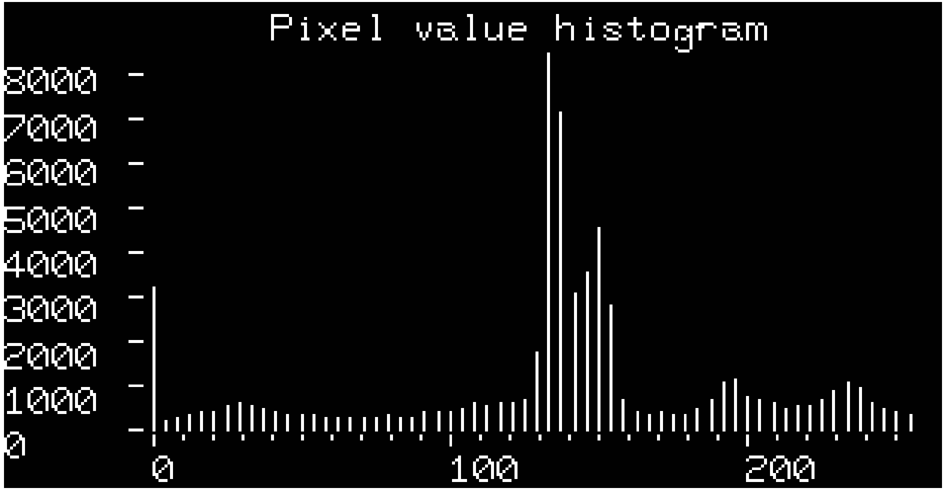
Ver. 2


Pixel value histogram
8000
7000
6000
5000
4000
3000
2000
1000
0
0
100
200


Pixel value histogram
8000
7000
6000
5000
4000
3000
2000
1000
0
0
100
200

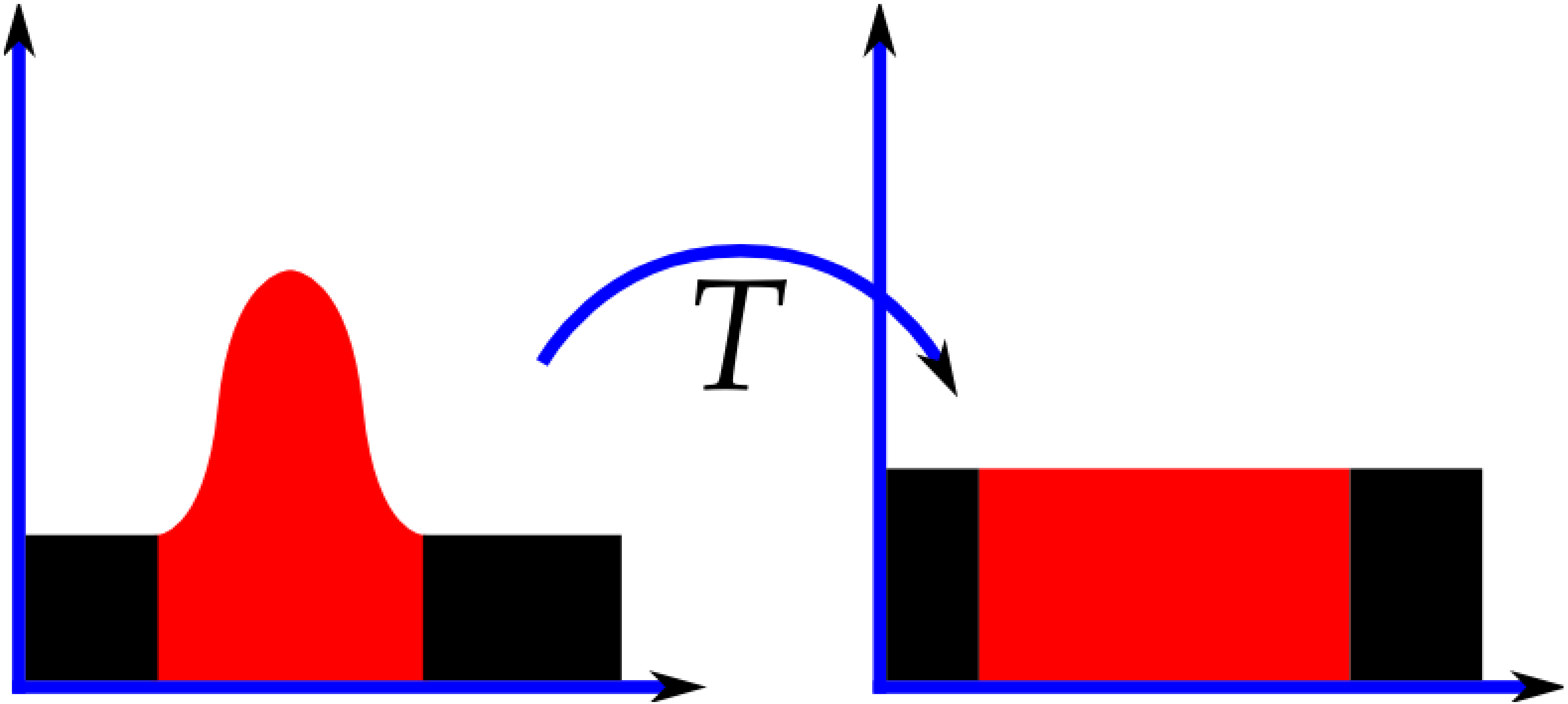
Are all intensities well represented ?

Ver. 2

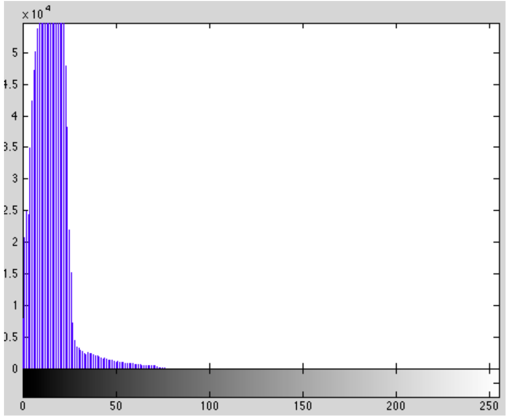


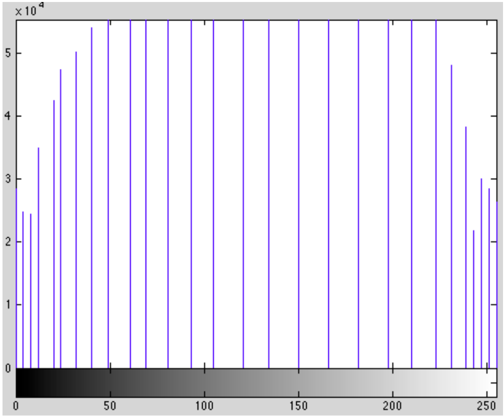
ALL INTENSITIES

Histogram Equalization



Histogram Equalization





The issue with contrast stretching

$$f_{\mathrm{ac}}(a) = a_{\mathrm{min}} + (a - a_{\mathrm{low}}) \cdot \frac{a_{\mathrm{max}} - a_{\mathrm{min}}}{a_{\mathrm{high}} - a_{\mathrm{low}}}$$

If a_{min} = 0 and a_{max} = 255

$$f_{\mathrm{ac}}(a) = (a - a_{\mathrm{low}}) \cdot \frac{255}{a_{\mathrm{high}} - a_{\mathrm{low}}}$$

Histogram Equalization

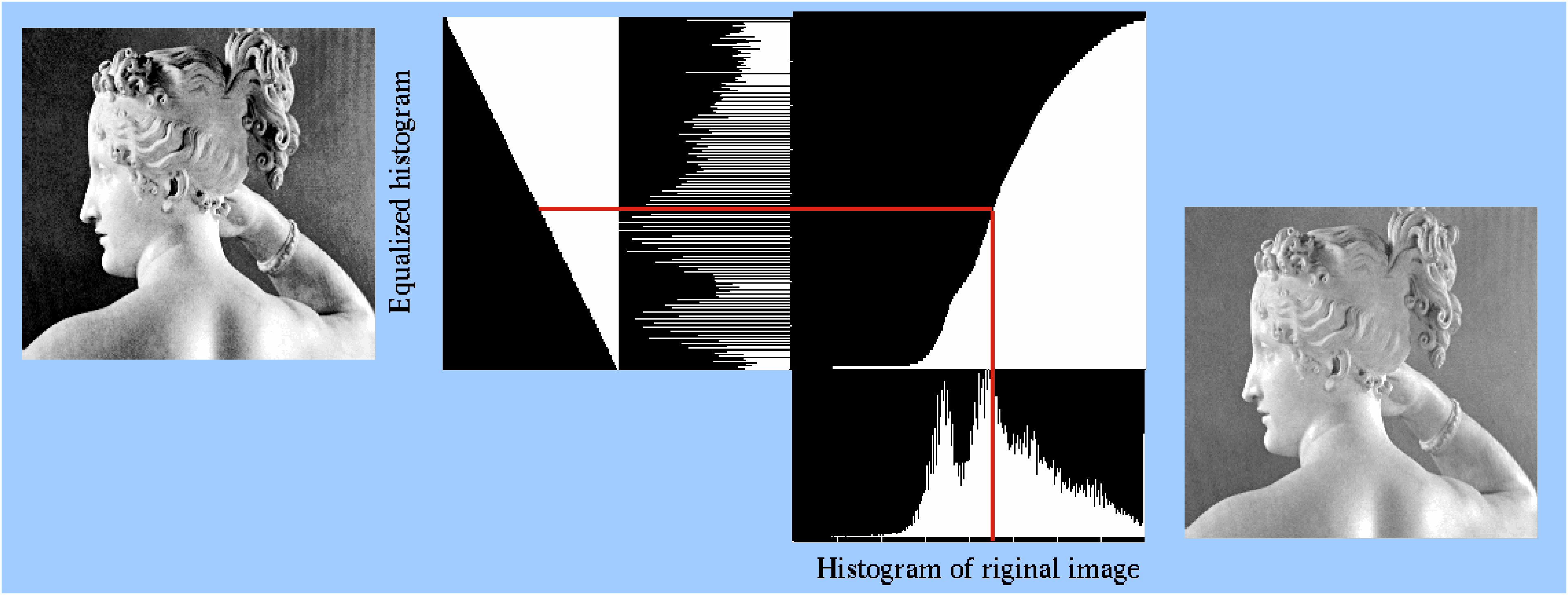
Histogram Equalization

Histogram Equalization

Histogram Equalization

Contrast Stretching

Histogram Equalization

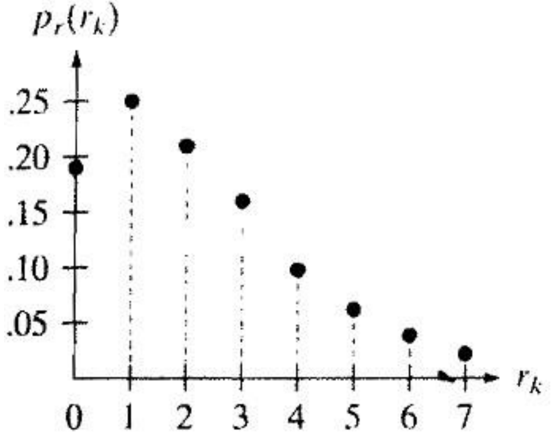

Equalized histogram
Histogram of riginal image

Histogram Equalization - Example

64 x 64 image

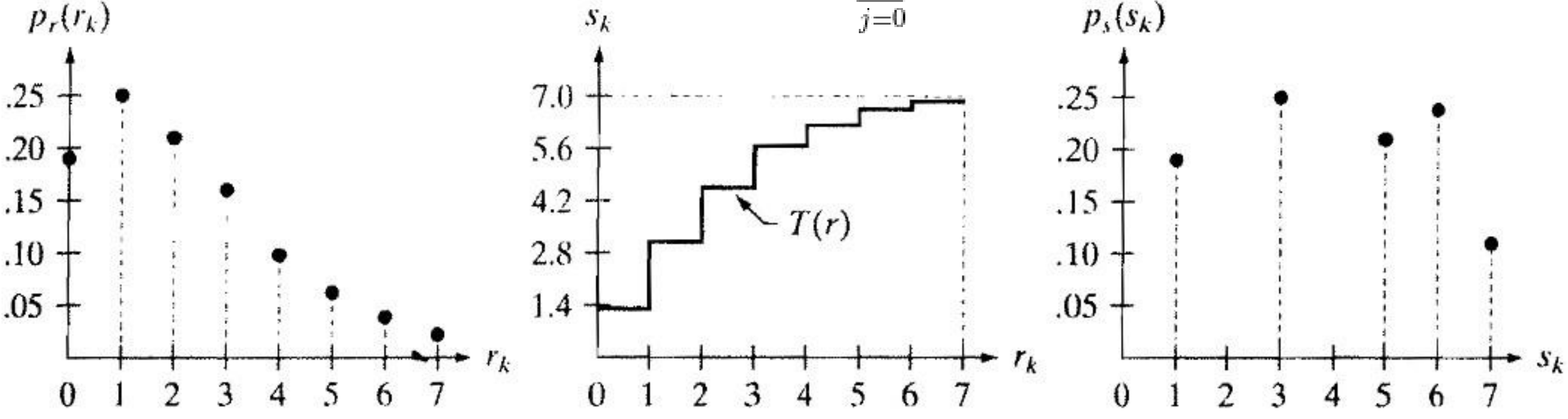
3-bits / pixel

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



Histogram Equalization - Example

$$s_k = \text{round}[(L-1)\sum_{j=0}^{k} p_r(r_k)]$$



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

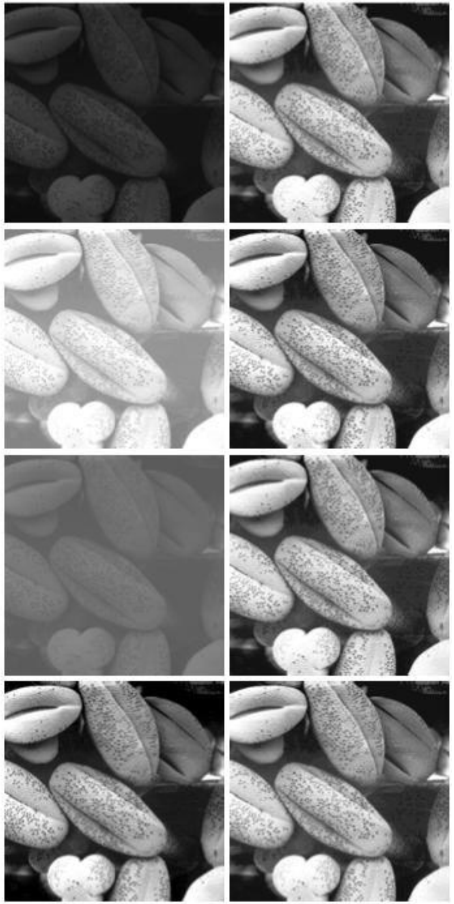
Histogram Equalization

1

2

3

4



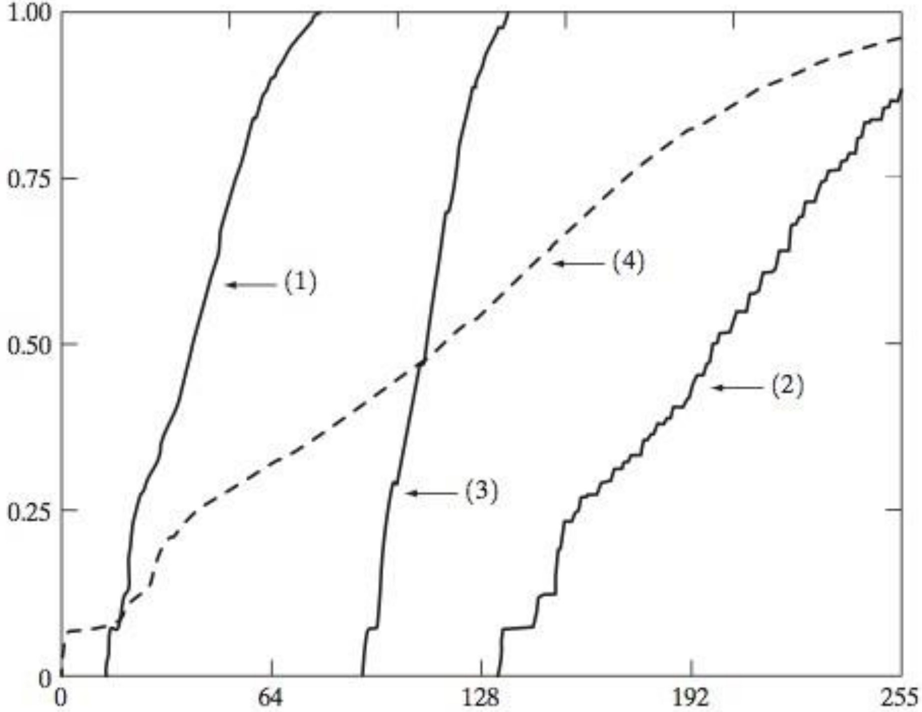


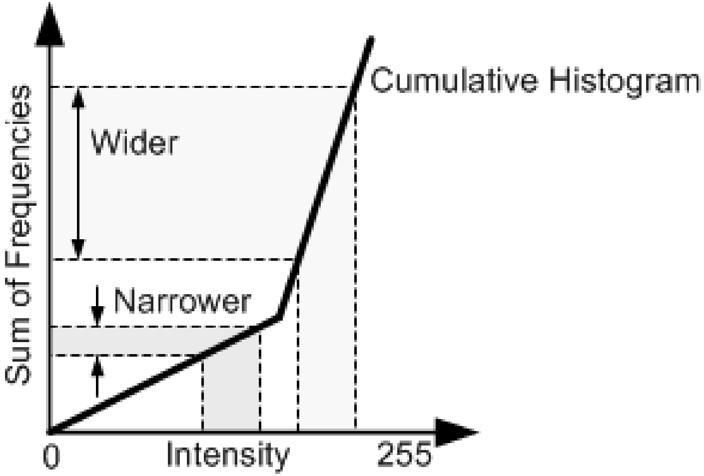
Image Courtesy: Gonzalez and Woods

Histogram Equalization v/s Contrast Enhancement

Contrast Enhancement

Histogram equalization

Histogram Equalization : A Visual Explanation

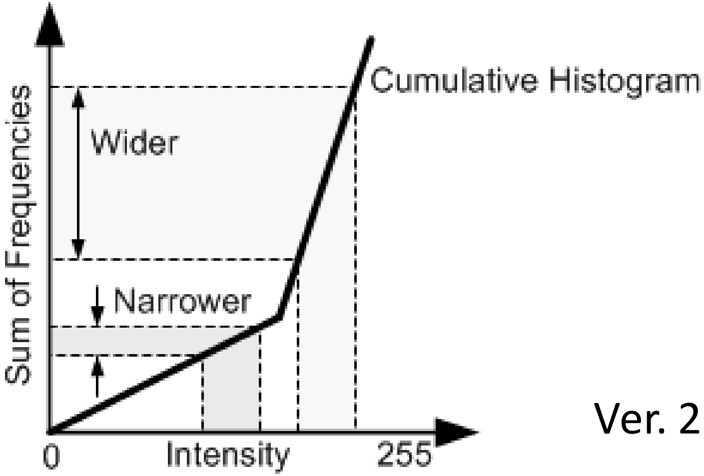


$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$s_k = T(r_k) = \mathtt{round}\left((L-1)\sum_{j=0}^{j=k} p_r(r_j)\right)$$

Histogram Equalization (ver. 2)

$$h[i] = \text{constant}, \qquad 0 \le i \le L-1$$



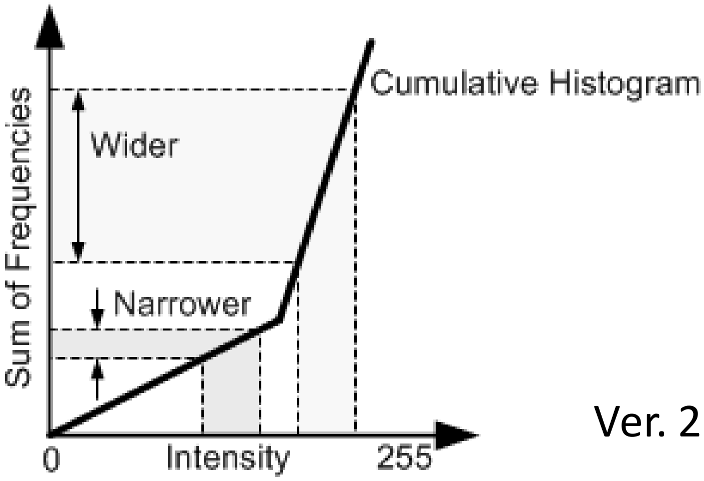
$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$s_k = T(r_k) = \texttt{round}\left((L-1)\underbrace{\sum_{j=0}^{j=k} p_r(r_j)}\right)$$

Ver. 2 $$s_k = T(r_k) = \texttt{round}\left((L-1) * \frac{cdf(r_k) - cdf_{min}}{1 - cdf_{min}}\right)$$

Histogram Equalization (ver. 2)

$$h[i] = \text{constant}, \qquad 0 \leq i \leq L-1$$



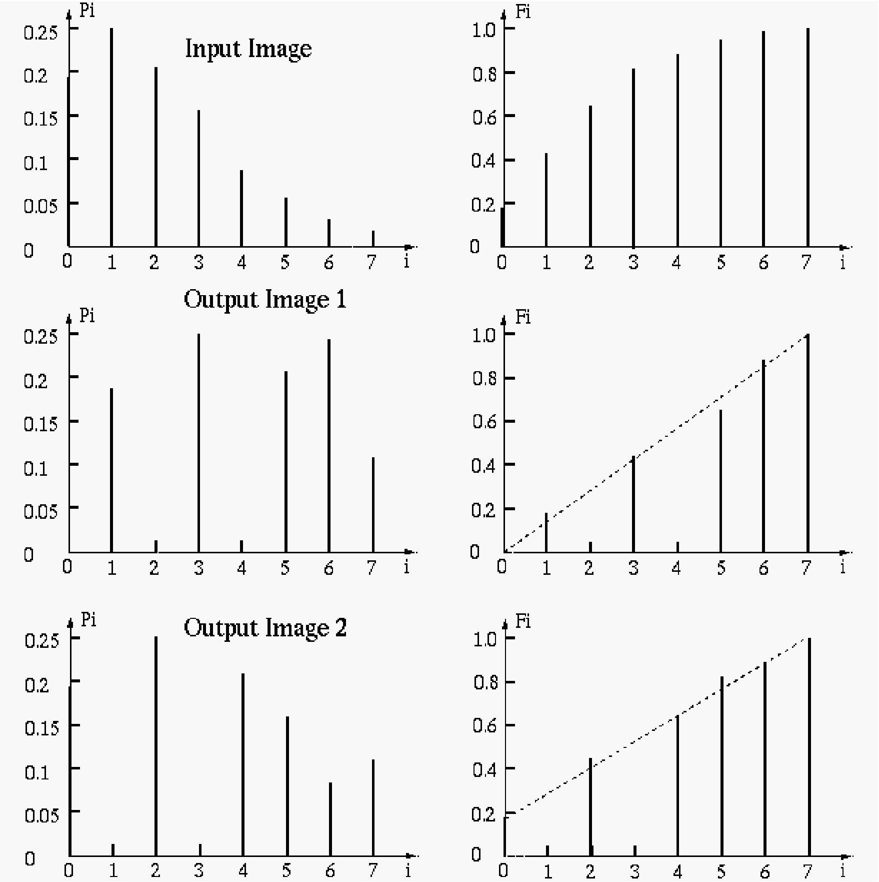
$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$s_k = T(r_k) = \texttt{round}\left((L-1)\underbrace{\sum_{j=0}^{j=k} p_r(r_j)}\right)$$

Ver. 2 $$s_k = T(r_k) = \texttt{round}\left((L-1) * \frac{cdf(r_k) - cdf_{min}}{1 - cdf_{min}}\right)$$

$$cdf_{min} = p_r(r_a) \text{ where } r_a = \min\{r_t | p_r(r_t) > 0\}; 0 \leqslant r_t \leqslant (L-1)$$

Histogram Equalization (default v/s ver. 2)



$$s_k = T(r_k) = \texttt{round}\left((L-1)\sum_{j=0}^{j=k} p_r(r_j)\right)$$

Ver. 2 $$s_k = T(r_k) = \texttt{round}\left((L-1) * \frac{cdf(r_k) - cdf_{min}}{1 - cdf_{min}}\right)$$

$$cdf_{min} = p_r(r_a) \text{ where } r_a = \min\{r_t | p_r(r_t) > 0\}; 0 \leqslant r_t \leqslant (L-1)$$

Histogram Equalization

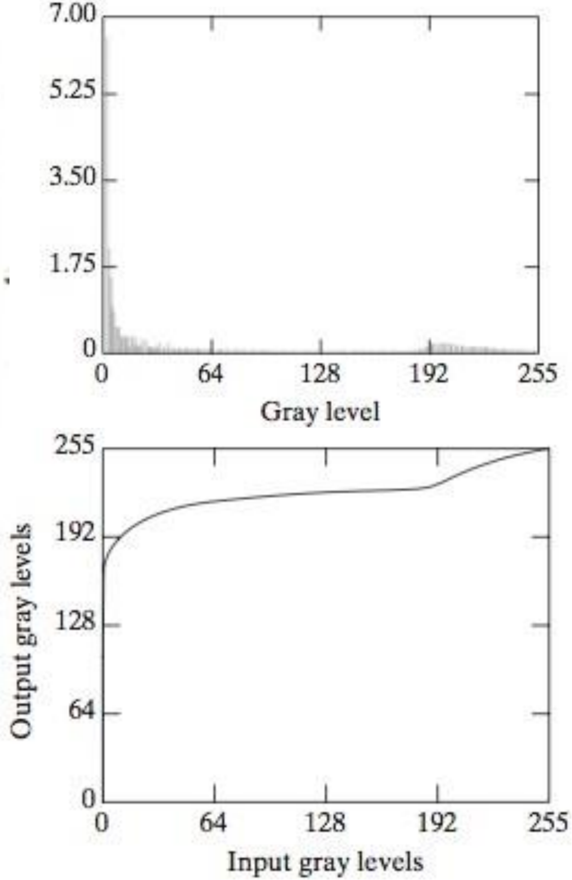
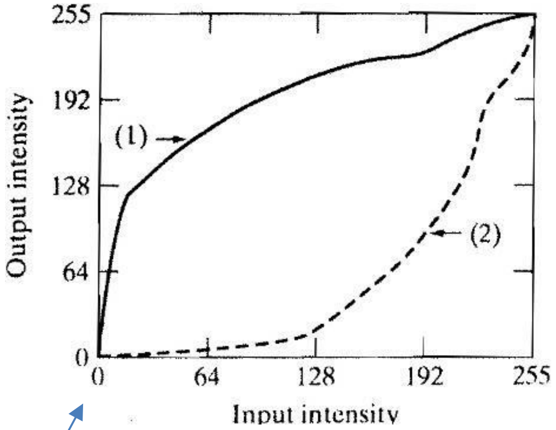


Image Courtesy: Gonzalez and Woods

Histogram specification

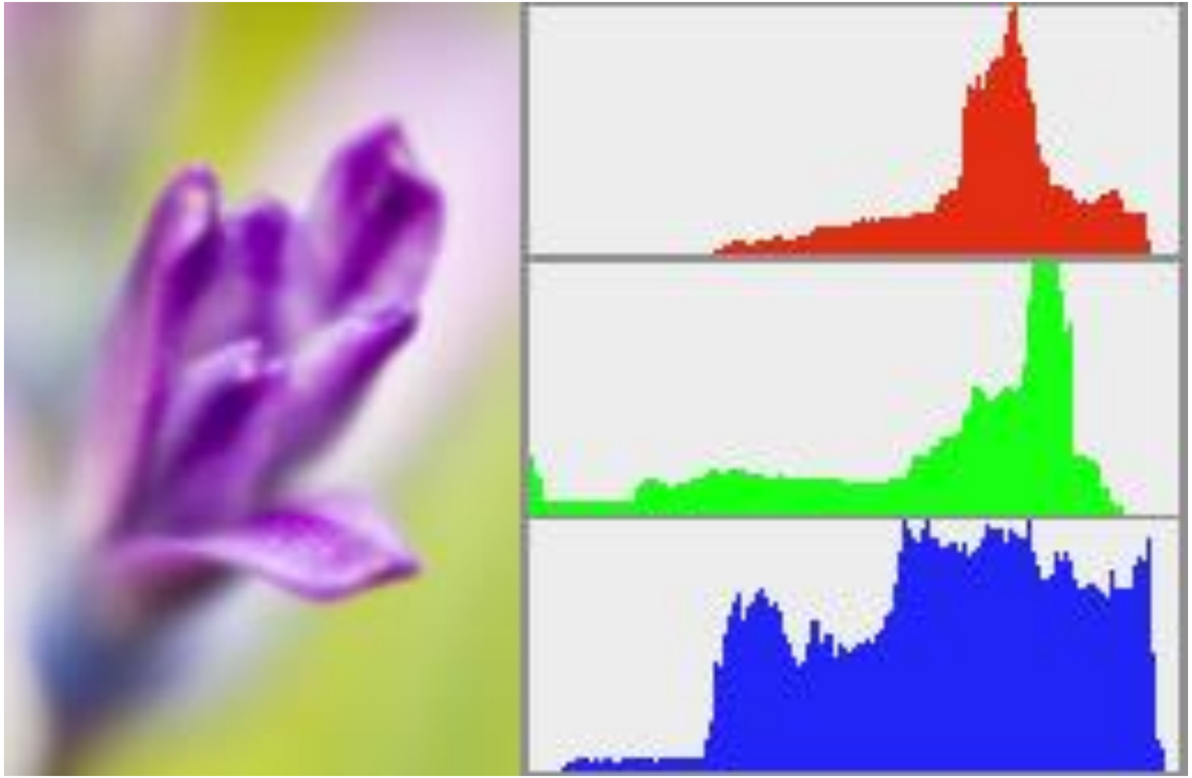
Histogram Specification / Matching [Section 3.3.2]

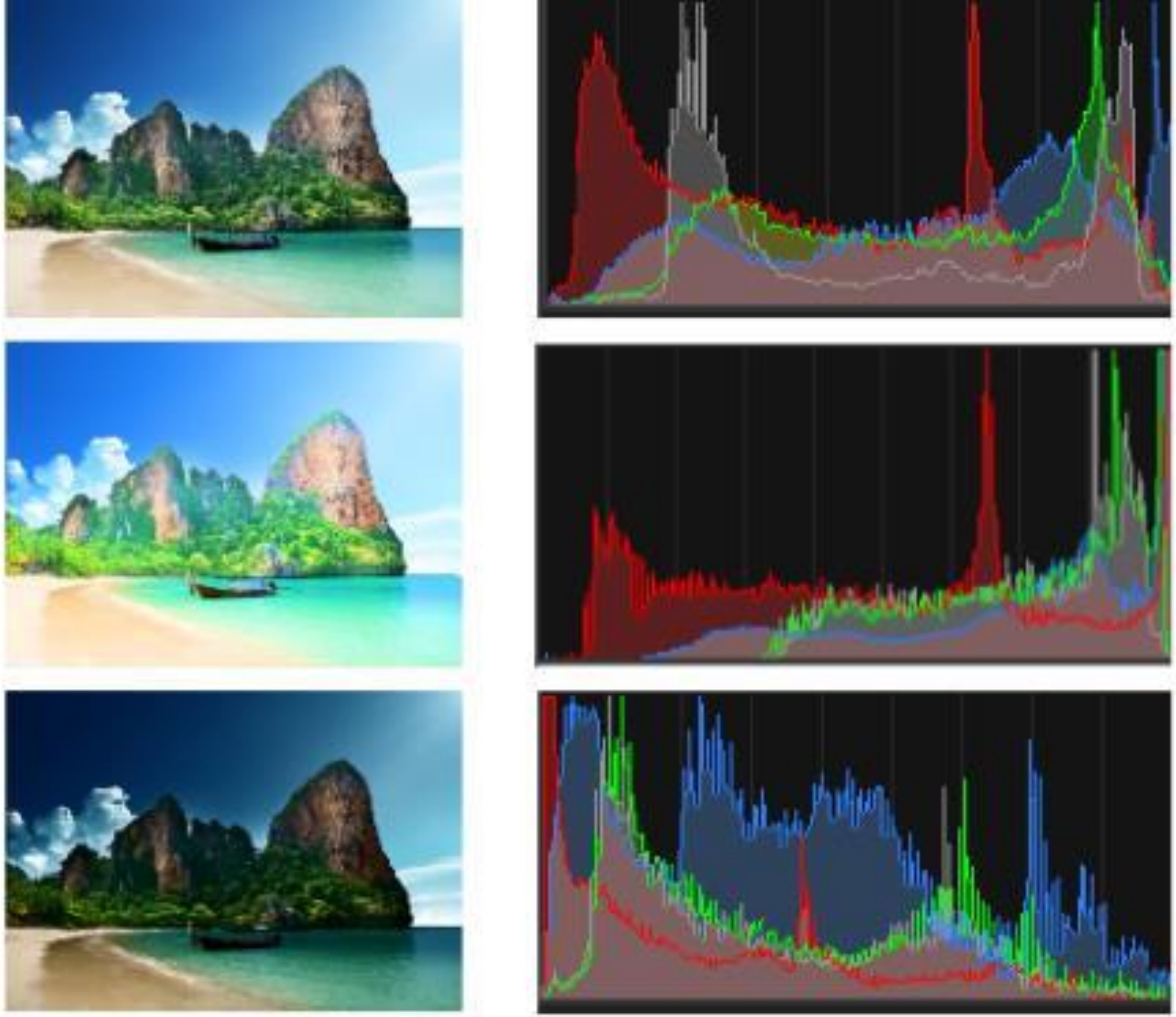


Compare with the curves we saw for contrast enhancement. What's the difference ?

Image Courtesy: Gonzalez and Woods

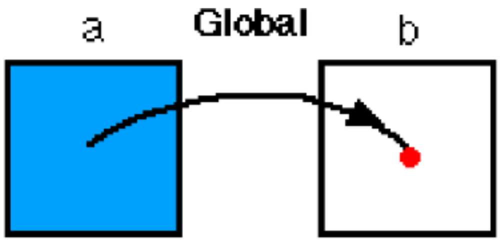
Histograms for RGB images





Histogram Processing

- Global to Point



Histogram : Discussion

- A visualization
- A useful statistical representation of image intensities
  - Not dependent on image size
- Drawbacks
  - No spatial information
  - Intensity-centric
  - Raw (unnormalized form): Image-size dependent
- Equalization:
  - An image ‘normalization’ approach
  - Improves global contrast, but can also boost noise

References

- GW Chapter – 3.3.1 to 3.3.3

- Transformations of Random Variables
  - http://www.randomservices.org/random/dist/Transformations.html
  - Section 1 of http://www.cs.cmu.edu/~minx/transform.pdf
  - Leibnitz Integration Rule : https://en.wikipedia.org/wiki/Leibniz_integral_rule#Alternative_derivation
  - Univariate transformation of a random variable