14.09.2021

# Digital Image Processing (CSE/ECE 478)

## Lecture-7: Spatial Filtering

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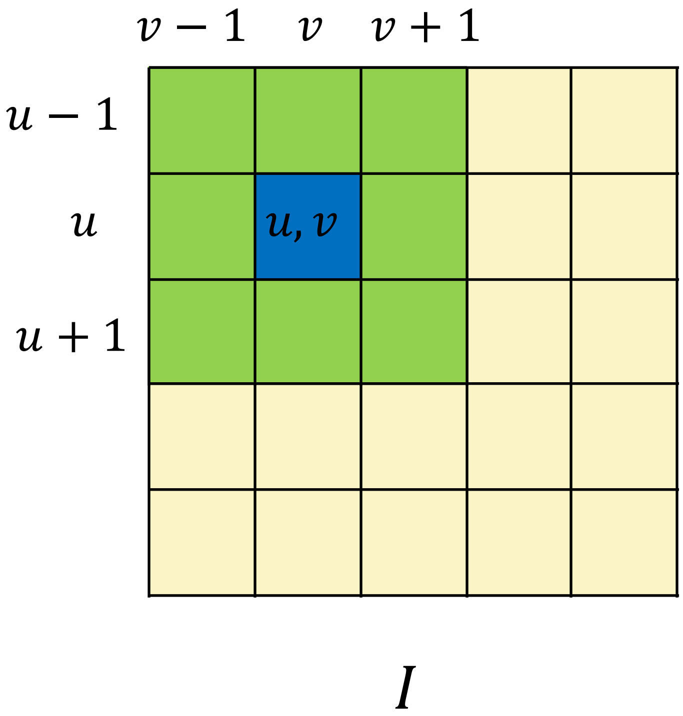
# Announcements

- Mini Quiz – 2 today

# Mean/Average Filter




Note: Coefficients sum to 1

H → 3 x 3 Weight Mask / Kernel / Filter

| | | |
|---|---|---|
| 1/9 | 1/9 | 1/9 |
| 1/9 | 1/9 | 1/9 |
| 1/9 | 1/9 | 1/9 |

$$I'(u,v) \leftarrow \frac{1}{9} \cdot \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i, v+j)$$

$$I'(u,v) \leftarrow \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i, v+j) \boxed{\cdot\ H(i,j)}$$

# Effect of Mask Size

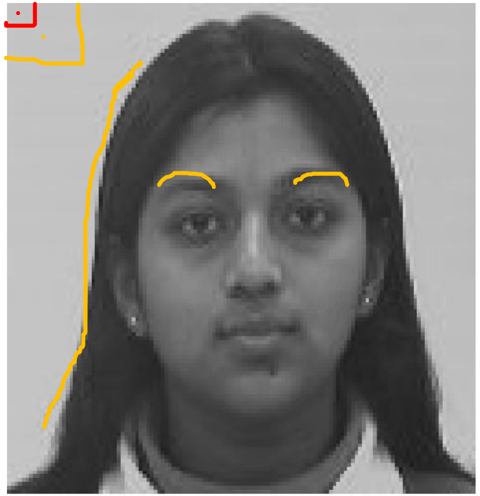
Original Image

[3x3]

[5x5]

[7x7]

# Repeated Averaging Using Same Filter



Before

After

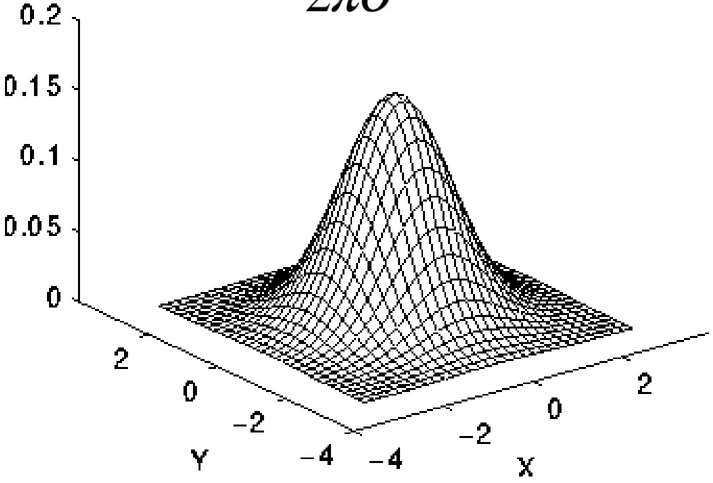
After repeated averaging

NOTE: Can get the <u>effect</u> of larger filters by smoothing repeatedly with smaller filters

# Gaussian Smoothing

- Mask weights are samples of a zero-mean 2-D Gaussian

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2 + y^2)/2\sigma^2\}$$

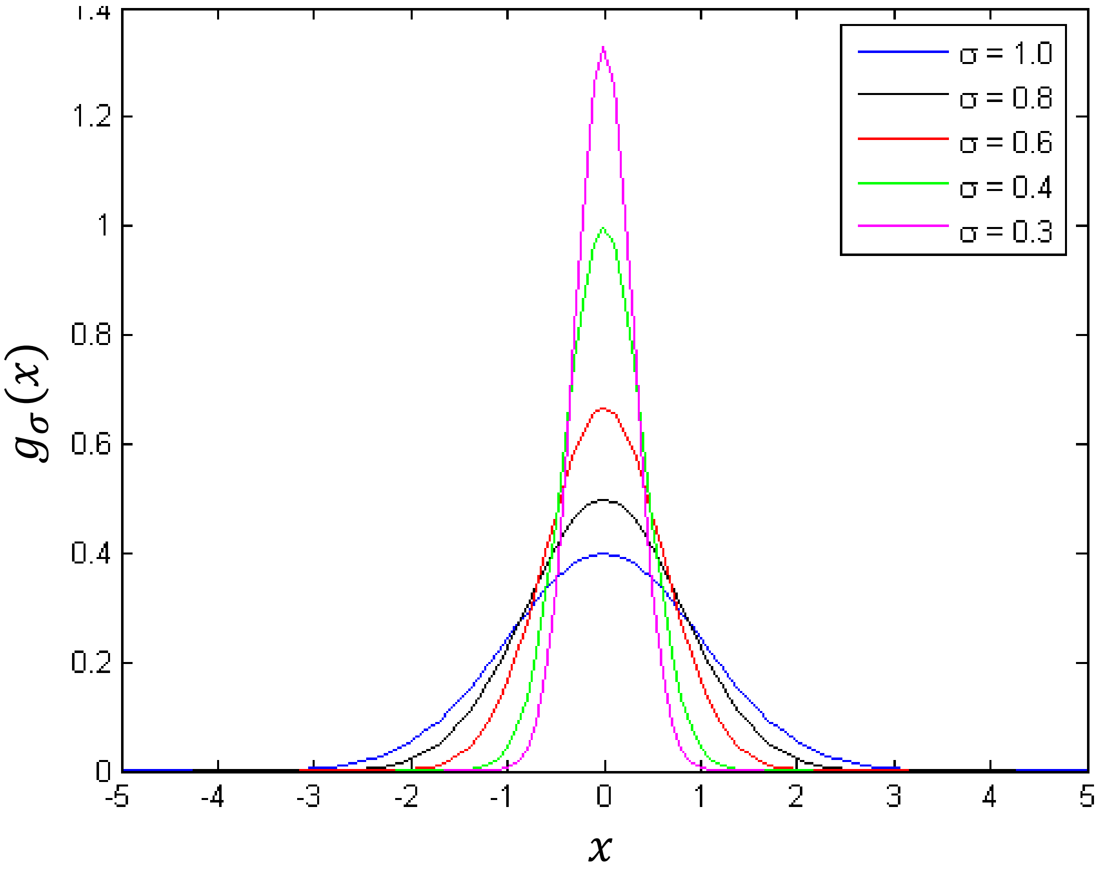



$\frac{1}{273}$

| | | | | |
|---|---|---|---|---|
| 1 | 4 | 7 | 4 | 1 |
| 4 | 16 | 26 | 16 | 4 |
| 7 | 26 | 41 | 26 | 7 |
| 4 | 16 | 26 | 16 | 4 |
| 1 | 4 | 7 | 4 | 1 |

5×5 Gaussian filter, $\sigma$=1

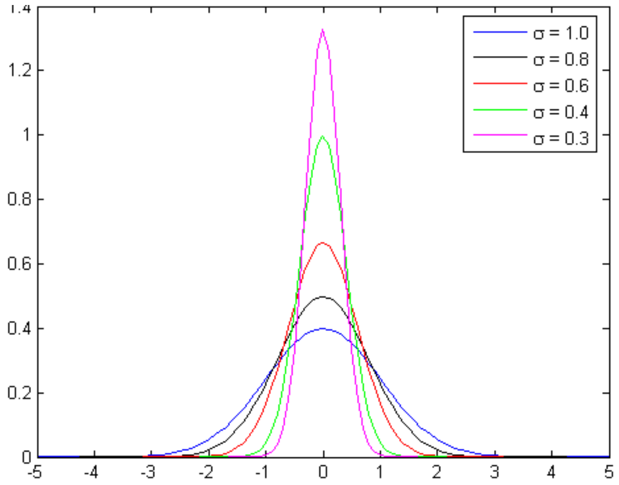
# How are Gaussian filter coefficients obtained ?



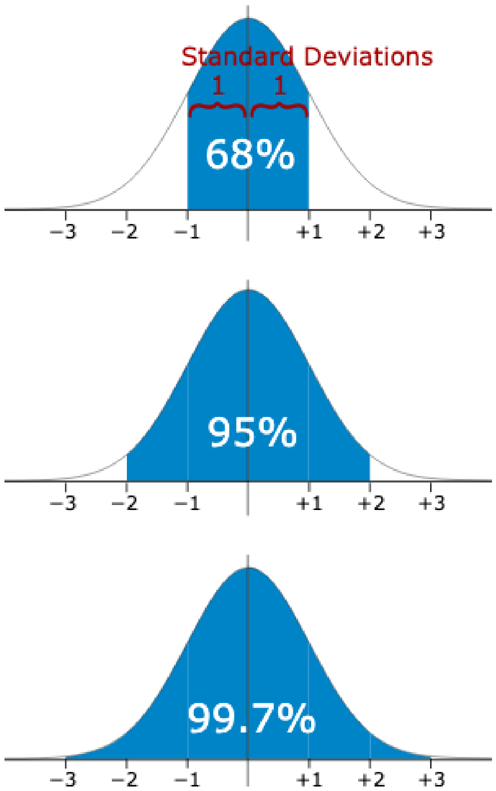

$$g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

Note: This is a PDF NOT Probability

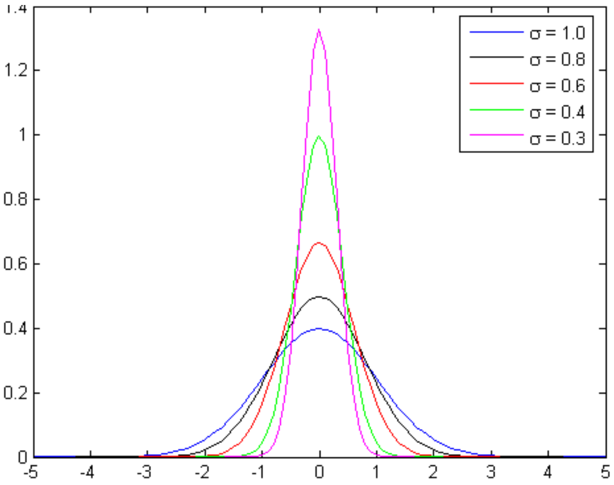
# How are Gaussian filter coefficients obtained ?




$$g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right)$$

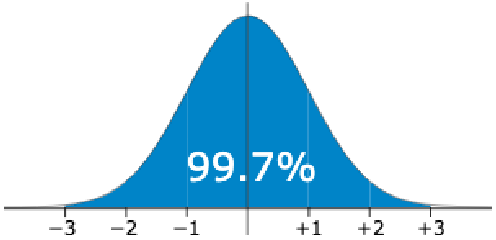


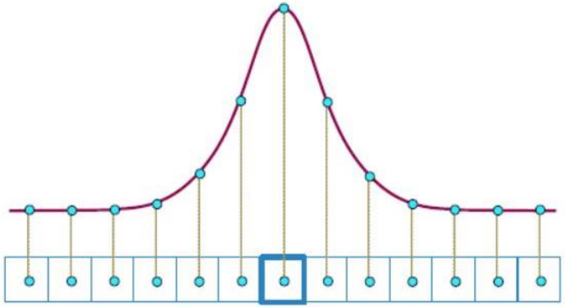
# How are Gaussian filter coefficients obtained ?




$$g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$P(X \leq x)$ $P(X = x)$



Let s be the size of the filter, i.e., sxs

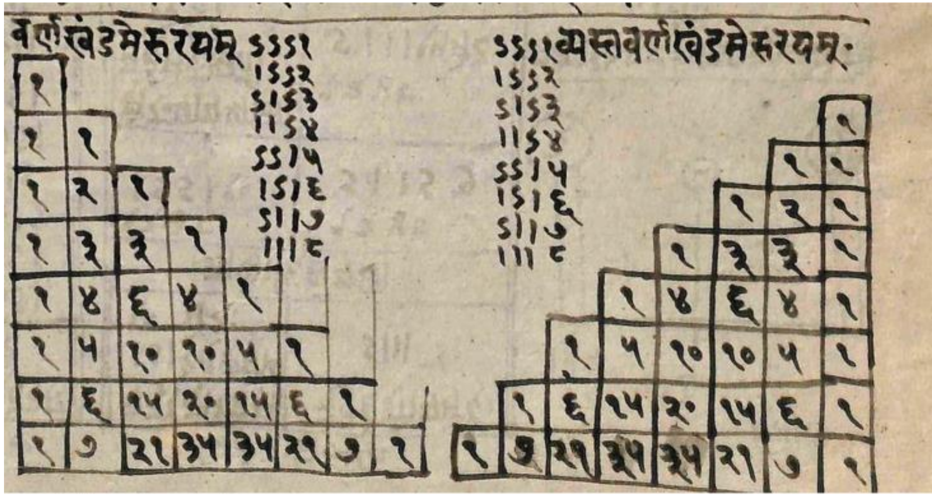
Heuristic:
s = round($\sigma$)

# How are Gaussian filter coefficients obtained ?

$$\sum_{k=0}^{N} {}^{N}C_k = 2^N$$

| Index N | Coefficients | Sum = $2^N$ |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 1 | 2 |
| 2 | 1 2 1 | 4 |
| 3 | 1 3 3 1 | 8 |
| 4 | 1 4 6 4 1 | 16 |
| 5 | 1 5 10 10 5 1 | 32 |
| 6 | 1 6 15 20 15 6 1 | 64 |
| 7 | 1 7 21 35 35 21 7 1 | 128 |
| 8 | 1 8 28 56 70 56 28 8 1 | 256 |
| 9 | 1 9 36 84 126 126 84 36 9 1 | 512 |
| 10 | 1 10 45 120 210 252 210 120 45 10 1 | 1024 |
| 11 | 1 11 55 165 330 462 462 330 165 55 11 1 | 2048 |
| 12 | 1 12 66 220 495 792 924 792 495 220 66 12 1 | 4096 |

*Meru Prastaara*, derived from Pingala's formulae (2 BCE), Manuscript from Raghunath Temple Library, Jammu



# How are Gaussian filter coefficients obtained ?

- Use the row *n* of Pascal's Triangle as a one-dimensional, n-point approximation of a Gaussian filter.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | 1 | | | | | | |
| | | | | | 1 | | 1 | | | | | |
| | | | | 1 | | 2 | | 1 | | | | |
| | | | 1 | | 3 | | 3 | | 1 | | | |
| | | 1 | | 4 | | 6 | | 4 | | 1 | | |
| | 1 | | 5 | | 10 | | 10 | | 5 | | 1 | |
| 1 | | 6 | | 15 | | 20 | | 15 | | 6 | | 1 |

Image Courtesy: http://ece.uprm.edu/~mtoledo/web/6088/F2018/Slides/6_Filtering.pdf

# How are Gaussian filter coefficients obtained ?

$$g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) \qquad \int_{-\infty}^{\infty} g_\sigma(x)dx = 1$$

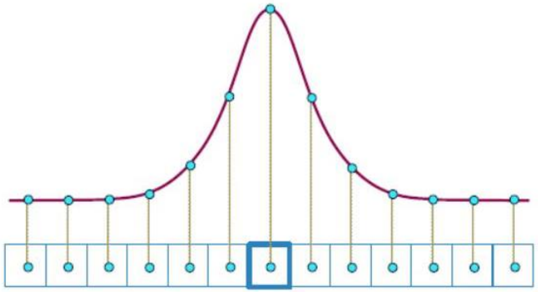
s = 7 x 7

$$\sum_{k=0}^{N} {}^{N}C_k = 2^N$$

| Index N | Coefficients | Sum = $2^N$ |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 1 | 2 |
| 2 | 1 2 1 | 4 |
| 3 | 1 3 3 1 | 8 |
| 4 | 1 4 6 4 1 | 16 |
| 5 | 1 5 10 10 5 1 | 32 |
| 6 | 1 6 15 20 15 6 1 | 64 |
| 7 | 1 7 21 35 35 21 7 1 | 128 |
| 8 | 1 8 28 56 70 56 28 8 1 | 256 |
| 9 | 1 9 36 84 126 126 84 36 9 1 | 512 |
| 10 | 1 10 45 120 210 252 210 120 45 10 1 | 1024 |
| 11 | 1 11 55 165 330 462 462 330 165 55 11 1 | 2048 |
| 12 | 1 12 66 220 495 792 924 792 495 220 66 12 1 | 4096 |

1/64 1 6 15 20 15 6 1

$$\frac{\sum_{k=0}^{N} {}^{N}C_k}{2^N} = 1$$



Points to note:

- N=s-1
- Require the kernel size such that it extends most of the Gaussian area
- Heuristic: For sigma=1, use 5xsigma to cover 98.76% of the area

# How are Gaussian filter coefficients obtained ?

$$g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right)$$

s = 7 x 7

| Index N | Coefficients | Sum = $2^N$ |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 1 | 2 |
| 2 | 1 2 1 | 4 |
| 3 | 1 3 3 1 | 8 |
| 4 | 1 4 6 4 1 | 16 |
| 5 | 1 5 10 10 5 1 | 32 |
| 6 | 1 6 15 20 15 6 1 | 64 |
| 7 | 1 7 21 35 35 21 7 1 | 128 |
| 8 | 1 8 28 56 70 56 28 8 1 | 256 |
| 9 | 1 9 36 84 126 126 84 36 9 1 | 512 |
| 10 | 1 10 45 120 210 252 210 120 45 10 1 | 1024 |
| 11 | 1 11 55 165 330 462 462 330 165 55 11 1 | 2048 |
| 12 | 1 12 66 220 495 792 924 792 495 220 66 12 1 | 4096 |

$$\begin{bmatrix} 1 & 6 & 15 & 20 & 15 & 6 & 1 \\ 6 & 36 & 90 & 120 & 90 & 36 & 6 \\ 15 & 90 & 225 & 300 & 225 & 90 & 15 \\ 20 & 120 & 300 & 400 & 300 & 120 & 20 \\ 15 & 90 & 225 & 300 & 225 & 90 & 15 \\ 6 & 36 & 90 & 120 & 90 & 36 & 6 \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{bmatrix} 1/4096$$

$$\frac{1}{\sqrt{2\pi}\sigma} = \frac{{}^{N}C_{N/2}}{2^N}$$

$$\frac{1}{64}\begin{bmatrix} 1 \\ 6 \\ 15 \\ 20 \\ 15 \\ 6 \\ 1 \end{bmatrix} \times \frac{1}{64}\begin{bmatrix} 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{bmatrix}$$

# What happens if other sampling values are used?

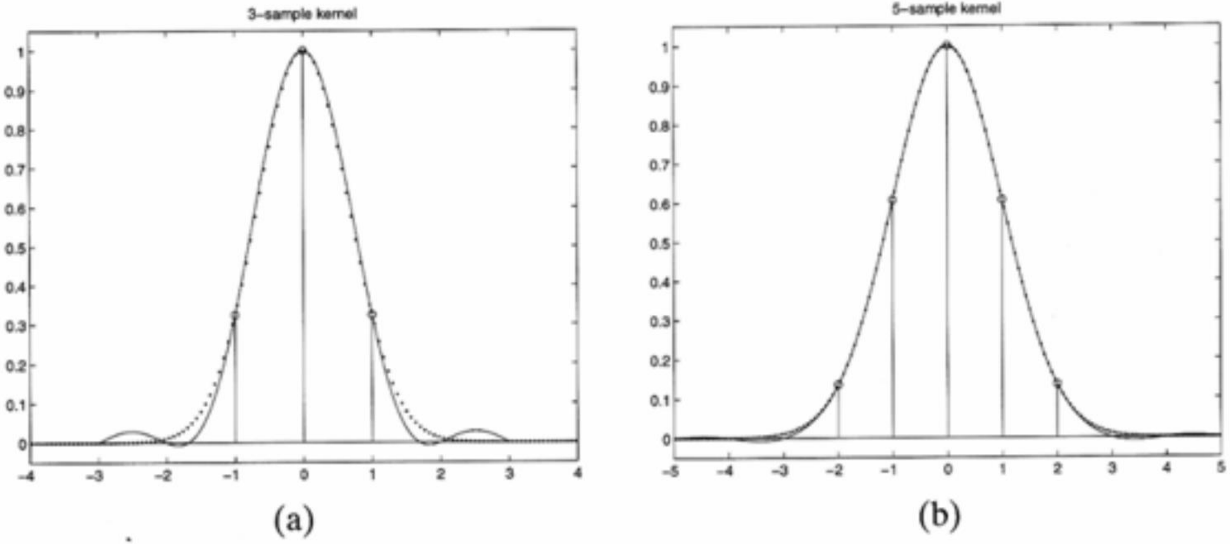
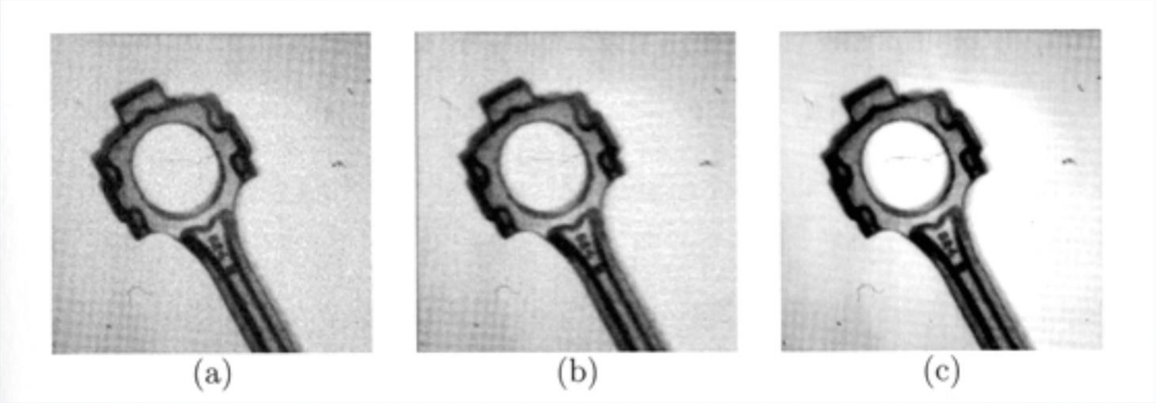



**Figure 3.6** Continuous Gaussian kernels (dotted), sampled real kernels, and continuous kernels reconstructed from samples (solid), for $\sigma = 0.6$ ($w = 3$) (a) and $\sigma = 1$ ($w = 5$) (b) respectively.

Image Courtesy: http://ece.uprm.edu/~mtoledo/web/6088/F2018/Slides/6_Filtering.pdf

# Example of Gaussian smoothing




Using the fifth row of Pascal's triangle as a Gaussian filter. (a) original; (b) After smoothing in the horizontal dir. (c) After smoothing in the vertical direction

Image Courtesy: http://ece.uprm.edu/~mtoledo/web/6088/F2018/Slides/6_Filtering.pdf

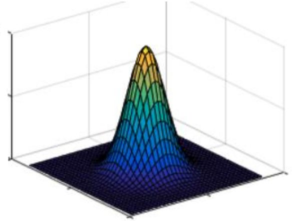
# Gaussian Smoothing – Effect of sigma

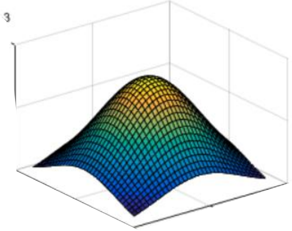
$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2 + y^2)/2\sigma^2\}$$

Original Image
(Sigma 0)

Gaussian Blur
(Sigma 0.7)

Gaussian Blur
(Sigma 2.8)





# Edge detection

Essentially what area V1 does in our visual cortex.

- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)

Source: D. Lowe

## First Derivative (Digital approximation)

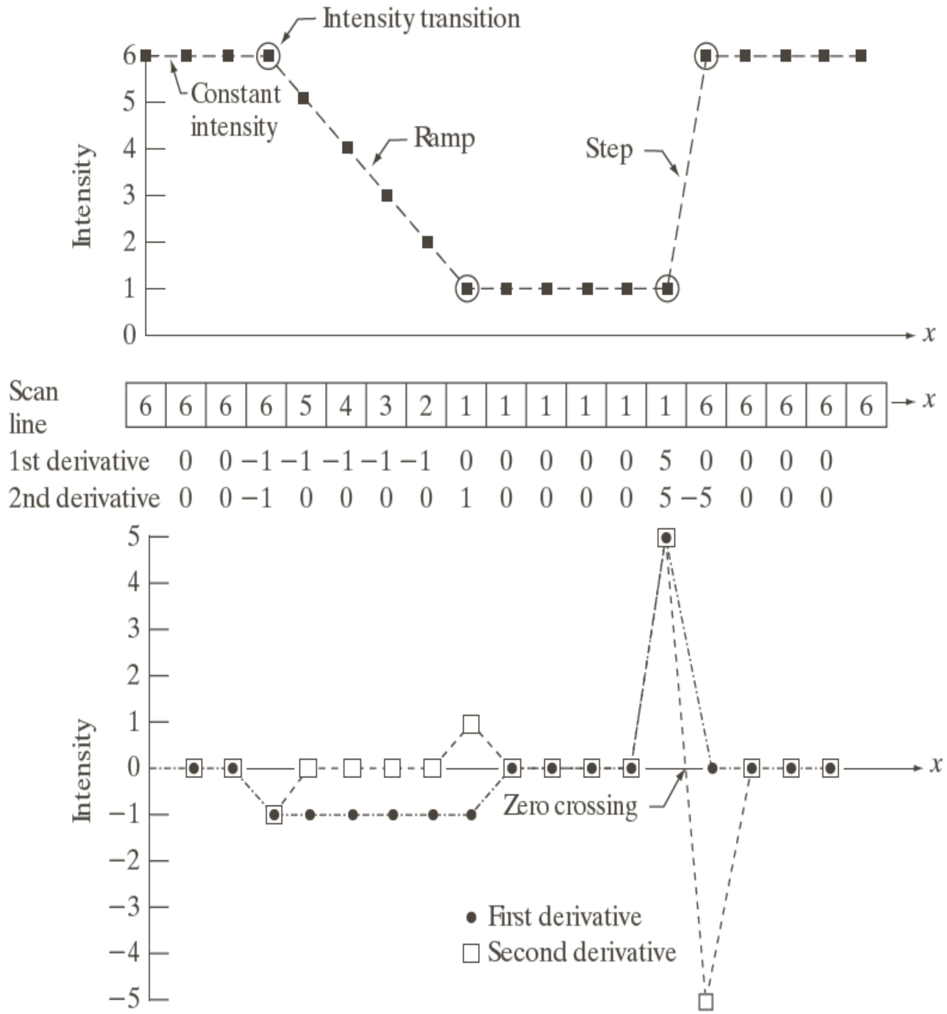
$$\frac{\partial f(x,y)}{\partial x} \sim f[x+1,y] - f[x,y]$$

| $-1$ | $1$ |
|---|---|
| $f[x,y]$ | $f[x+1,y]$ |

## Second Derivative (Digital Approximation)

$$\frac{\partial^2 f(x,y)}{\partial x^2} \sim (f[x+1,y] - f[x,y]) - (f[x,y] - f[x-1,y])$$

| $1$ | $-2$ | $1$ |
|---|---|---|
| $f[x-1,y]$ | $f[x,y]$ | $f[x+1,y]$ |



# Image Gradient and Edges

$$\frac{f(x+h,y)-f(x-h,y)}{2h} \Rightarrow \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

x-derivative

$$\frac{f(x,y+h)-f(x,y-h)}{2h} \Rightarrow \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

y-derivative

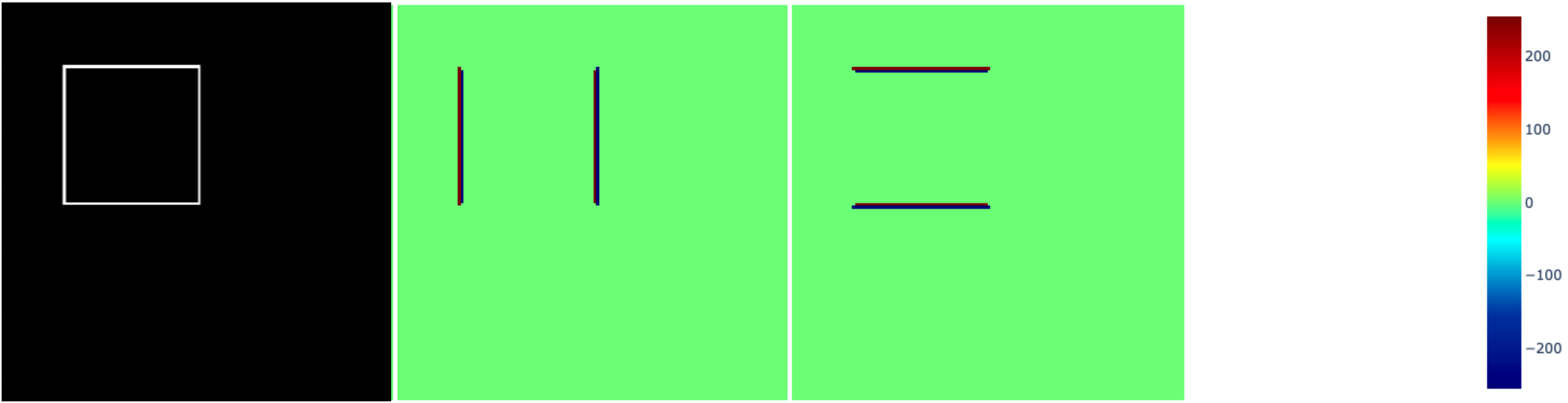



Image x-derivative y-derivative

# Prewitt Edge Filter

Dr. Prewitt

https://nihrecord.nih.gov/sites/recordNIH/files/pdf/1984/NIH-Record-1984-03-13.pdf

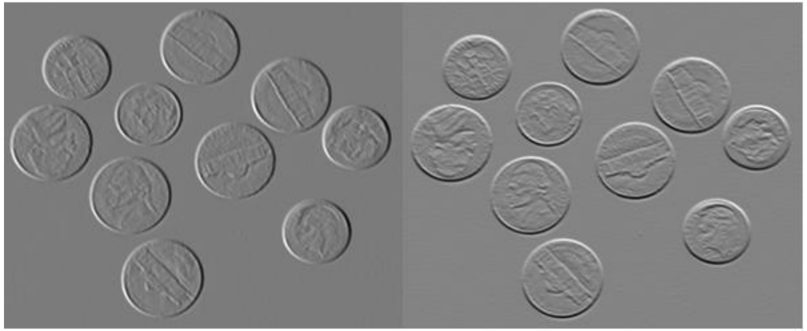
| | | |
|---|---|---|
| -1 | 0 | +1 |
| -1 | 0 | +1 |
| -1 | 0 | +1 |

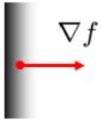
$G_x$

| | | |
|---|---|---|
| +1 | +1 | +1 |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

$G_y$

# Edge is perpendicular to gradient





$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$

$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$

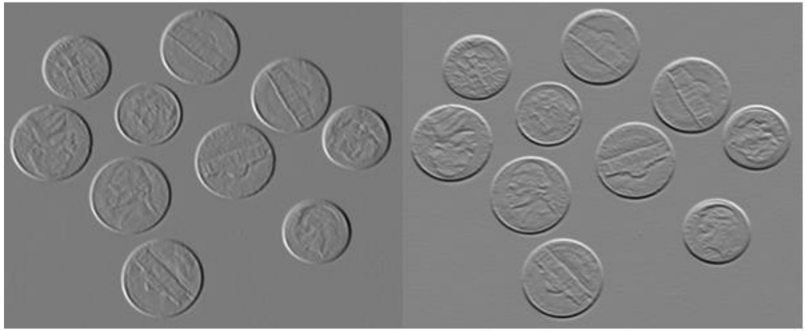
| | | |
|---|---|---|
| -1 | 0 | +1 |
| -1 | 0 | +1 |
| -1 | 0 | +1 |

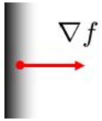
$G_x$

| | | |
|---|---|---|
| +1 | +1 | +1 |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

$G_y$

# Gradient Magnitude and Orientation





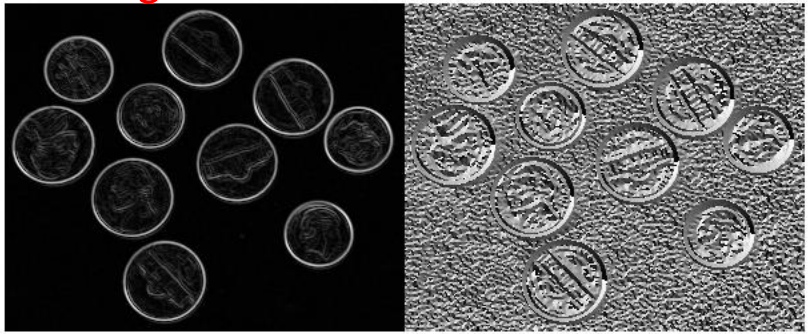
$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$

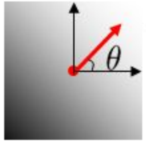
$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$

Magnitude

Orientation

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$






$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

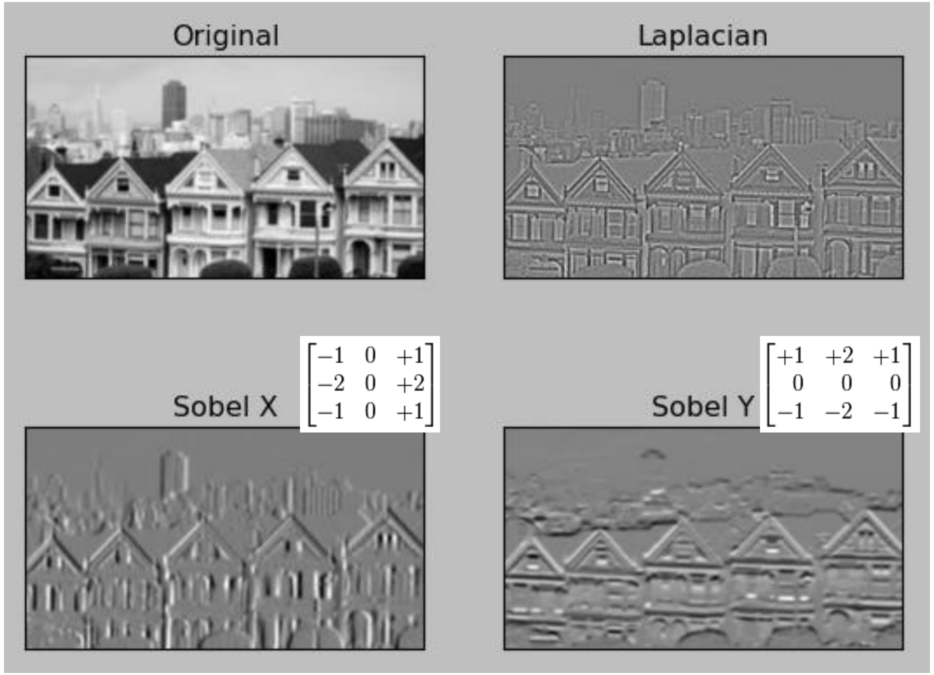
# 2-D Laplacian Filter

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | -4 | 1 |
| 0 | 1 | 0 |

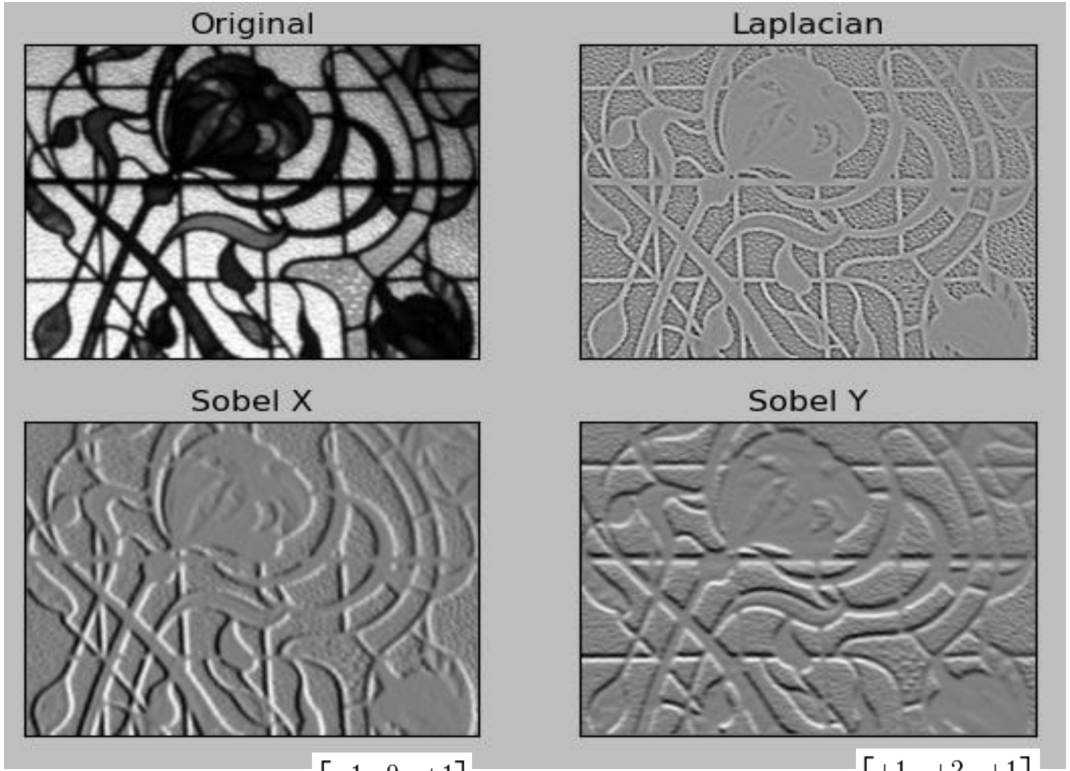
# Edge Masks – Sobel , Laplacian




| | | |
|---|---|---|
| 0 | −1 | 0 |
| −1 | 4 | −1 |
| 0 | −1 | 0 |

Note: Coefficients sum to 0

# Edge Masks – Sobel , Laplacian



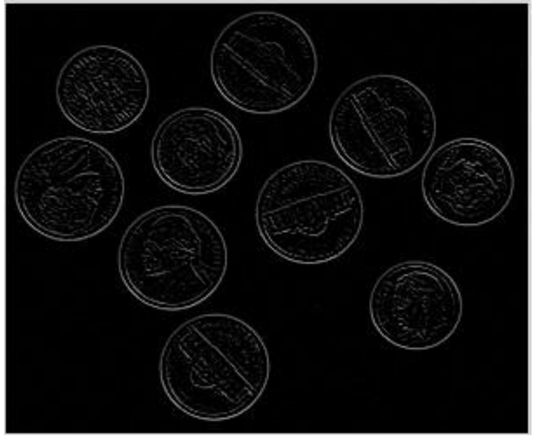

| | | |
|---|---|---|
| 0 | −1 | 0 |
| −1 | 4 | −1 |
| 0 | −1 | 0 |

$$\begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

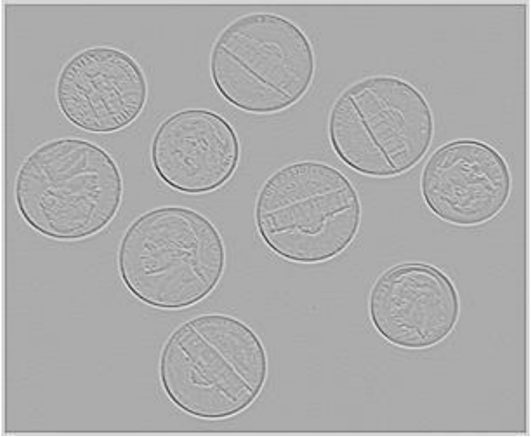
$$\begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

# Image Sharpening Using Laplacian

$I(u,v)$



$\nabla^2 I(u,v)$



$\nabla^2 I(u,v) + 128$

(For visualization)

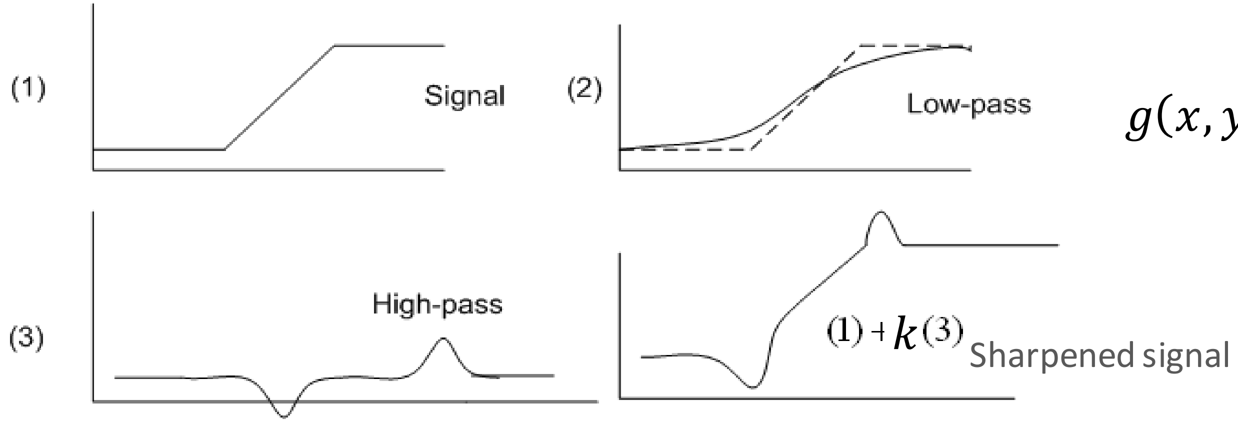
$I'(u,v) = I(u,v) + \nabla^2 I(u,v)$

# Image Sharpening (Unsharp Masking)

1. Blur the original image
2. Subtract the original image from the blurred image (outputs mask)
3. Add the mask back to the original image

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

$$g(x,y) = f(x,y) + k.\, g_{mask}(x,y)$$



# Unsharp masking

- What does blurring take away?


$-$


$=$


Difference between original and smoothed images

  - Let's add it back:


$+ \ k$


$=$


Add original and weighted detail images

# Unsharp Masking vs Highboost Filtering

$k \geq 0\ (k = 1)$

$k > 1$

# Unsharp Masking / Highboost Filtering as Spatial Filters

A=1

w = 9A-1

| | | |
|---|---|---|
| -1 | -1 | -1 |
| -1 | w | -1 |
| -1 | -1 | -1 |

A=2

w = 17

| | | |
|---|---|---|
| -1 | -1 | -1 |
| -1 | 17 | -1 |
| -1 | -1 | -1 |

- If **A=1**, we get unsharp masking. $I'(u,v) = I(u,v) + \nabla^2 I(u,v)$
- If **A>1**, original image is added back to detail image (highboost filtering).

# Corner cases, Padding

```
M = 3
For each valid location [x,y] in S
        a ← Average of intensities in a M x M neighborhood centered on [x,y]
        D[x,y] = round(a)
```

| | | | | |
|---|---|---|---|---|
| 120 | 190 | 140 | 150 | 200 |
| 17 | 21 | 30 | 8 | 27 |
| 89 | 123 | 150 | 73 | 56 |
| 10 | 178 | 140 | 150 | 18 |
| 190 | 14 | 76 | 69 | 87 |

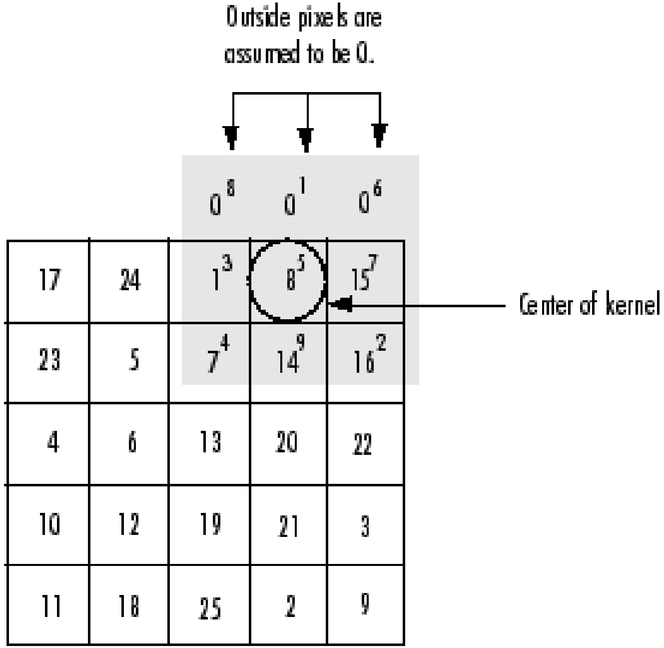
x

| | | |
|---|---|---|
| 1/9 | 1/9 | 1/9 |
| 1/9 | 1/9 | 1/9 |
| 1/9 | 1/9 | 1/9 |

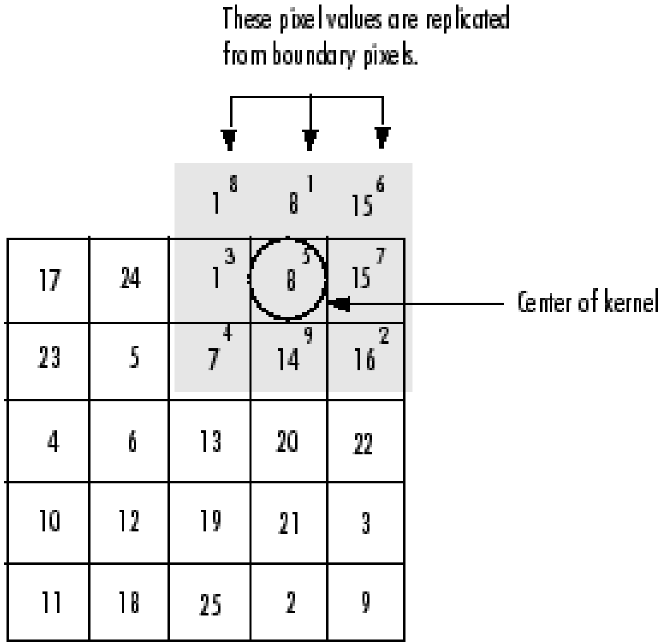
=

| | | | | |
|---|---|---|---|---|
| | | | | |
| | 98 | | | |
| | | | | |
| | | | | |
| | | | | |

# Image Padding




zero




replicate

# Edge Detection-Biometrics

(a) Lena (b)sobel (c)canny

(d)log (e)prewitt (g)paper method

Image Courtesy: Hua et al., “An edge detection method with boundary reserved based on non-subsampled contourlet transform for remote sensing imagery,” Optics and Photonics for Information Processing XI, SPIE, 2017

# Edge Detection-Biometrics

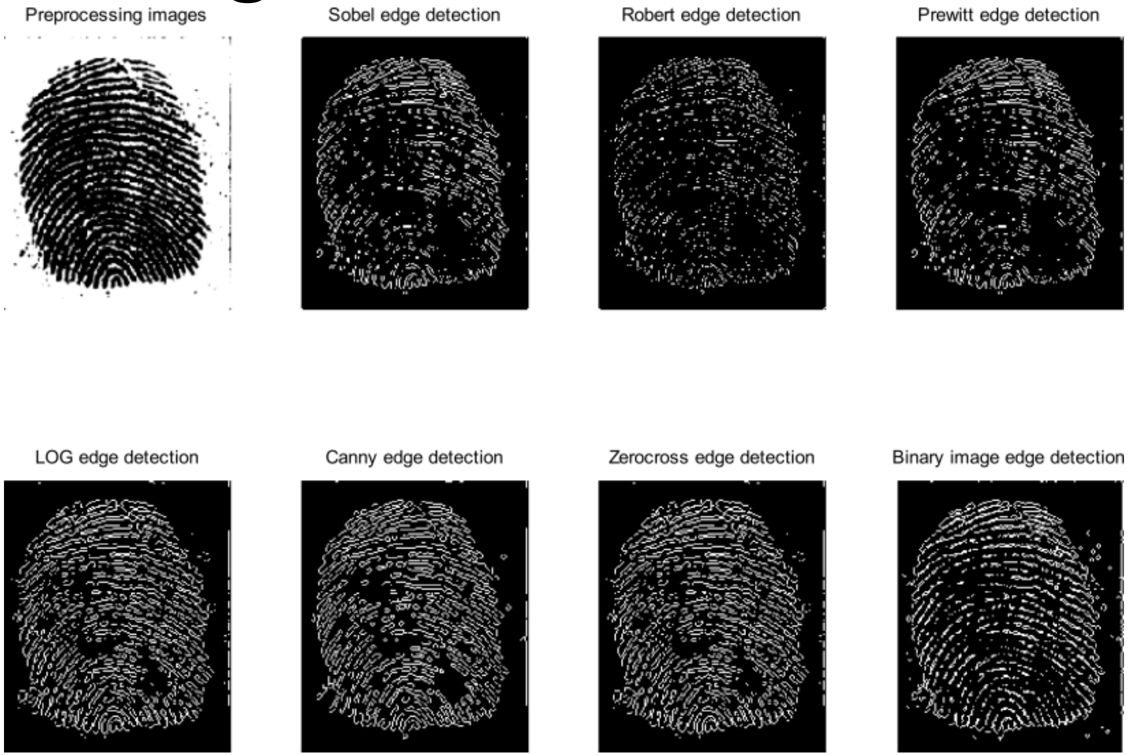



Image Courtesy: Cui et al., “The research of edge detection algorithm for fingerprint images,” World Automation Congress, 2008

# Edge Detection-Biometrics

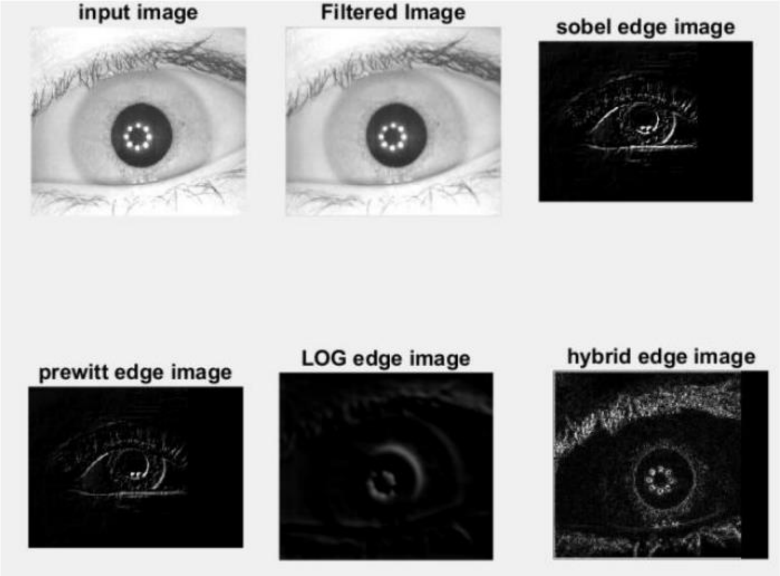



Image Courtesy: Kaur et al., “Comparison of edge detection techniques for iris recognition,” Intl. Journal of Computer Applications, 2016

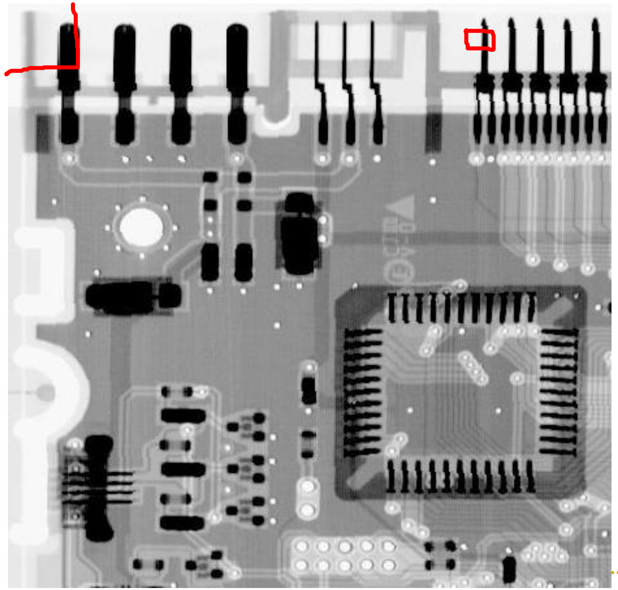
# References

- GW Chapter – 3.4.1, 3.5.1,3.6

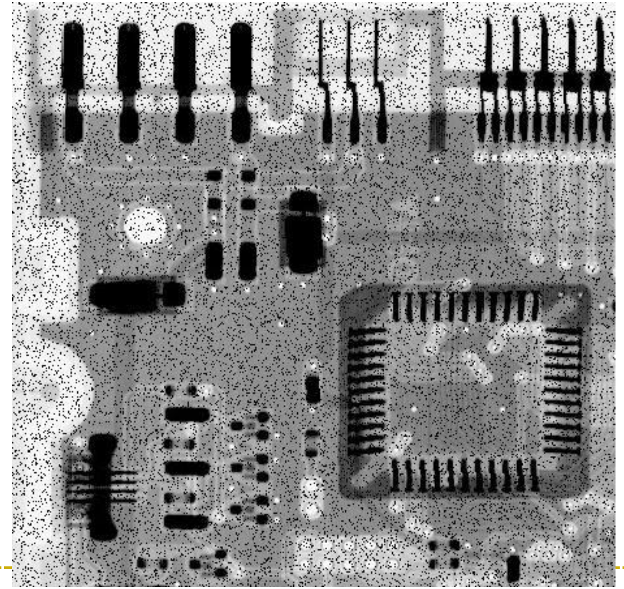
# Spatial Domain Filtering - Approaches

- Linear (Average, Gaussian, Prewitt, Sobel, Laplacian)
- Non-linear

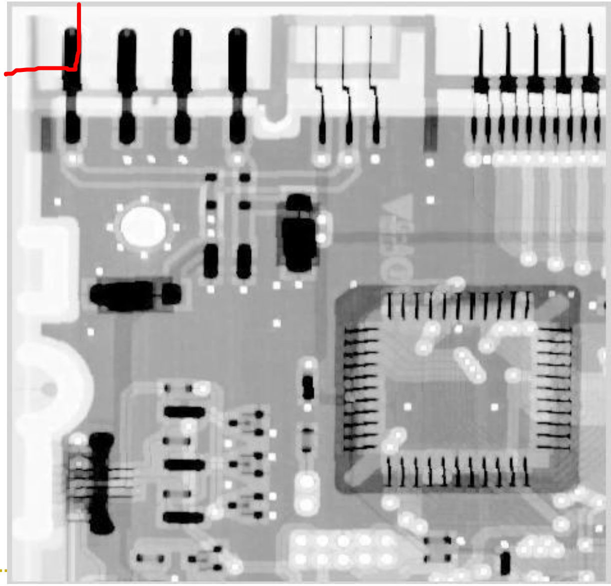
# Non-linear Spatial Filters (max)



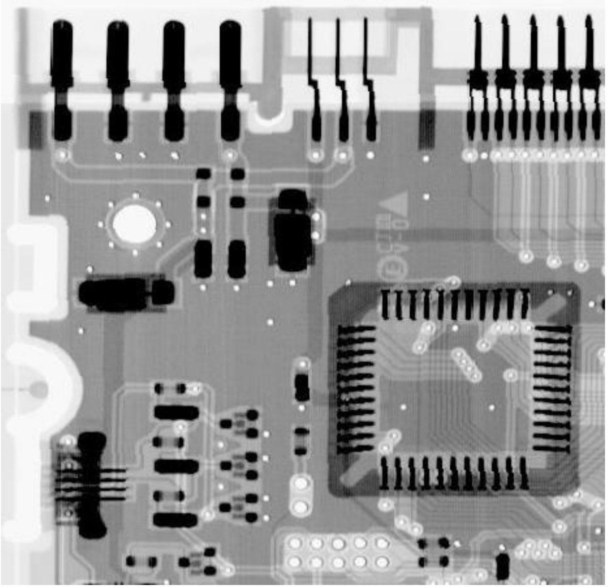
pepper noise



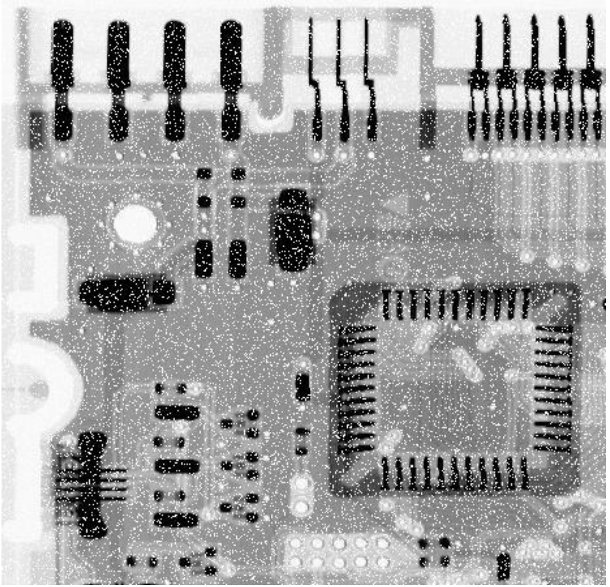
**After applying max filter**



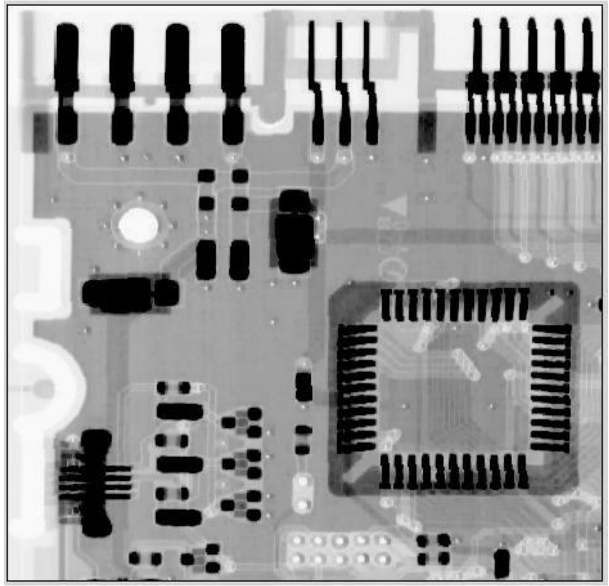
# Non-linear Spatial Filters (min)



salt noise



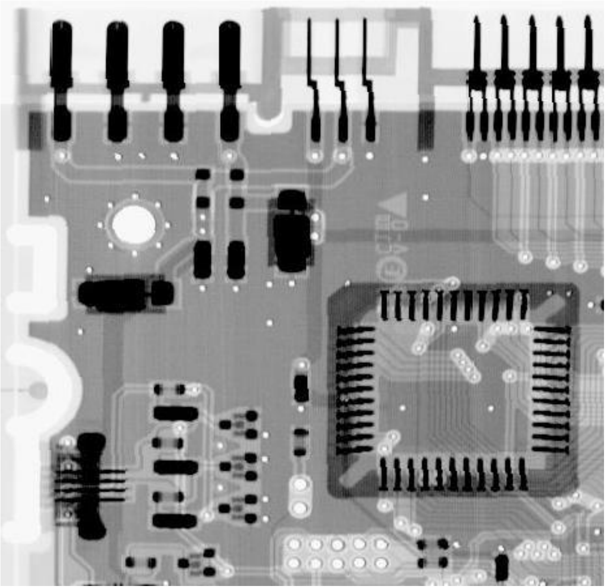
**After applying min filter**

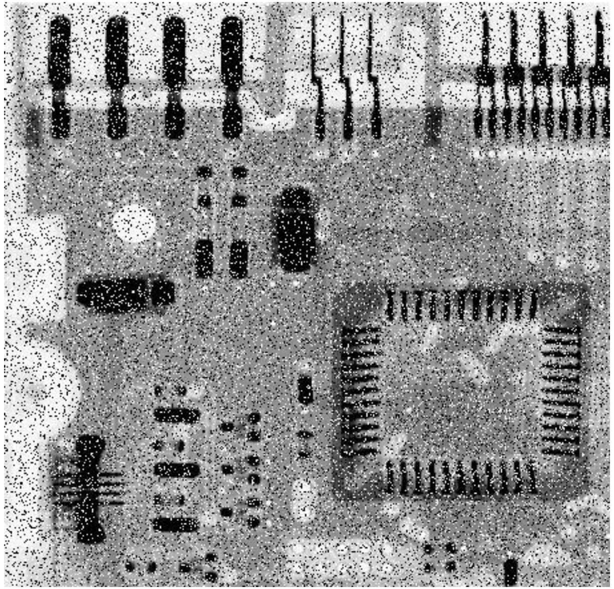


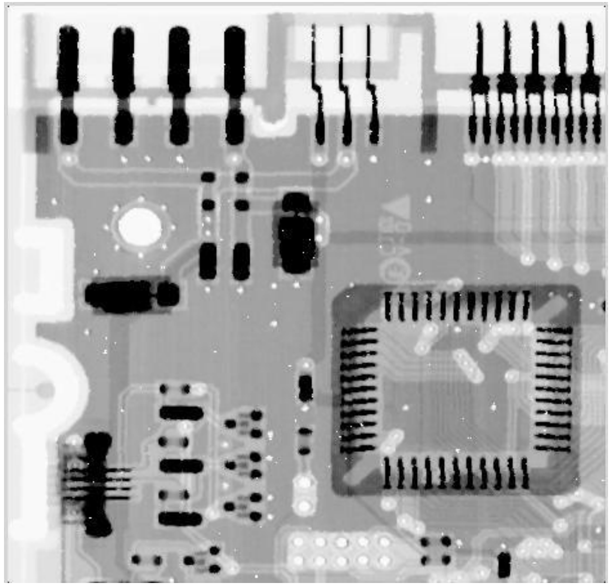
# Non-linear Spatial Filters (median)

salt & pepper noise

**After applying median filter**







max, min, median → also known as rank / order statistic filters

# Other Spatial Filters

- Geometric mean
- Harmonic mean
- Contra harmonic mean
- Mid Point filter
- Alpha trimmed mean filter
- …..

# Bilateral Filtering (Edge preserving smoothing)



Original image taken from cs.cityu.edu.hk

# References

- GW Chapter – 3.4