Singular Value Decomposition (SVD)

Introduction to NLP, Spring ‘22

Tutorial - 3 [Part I]

Eigenvalues and Eigenvectors

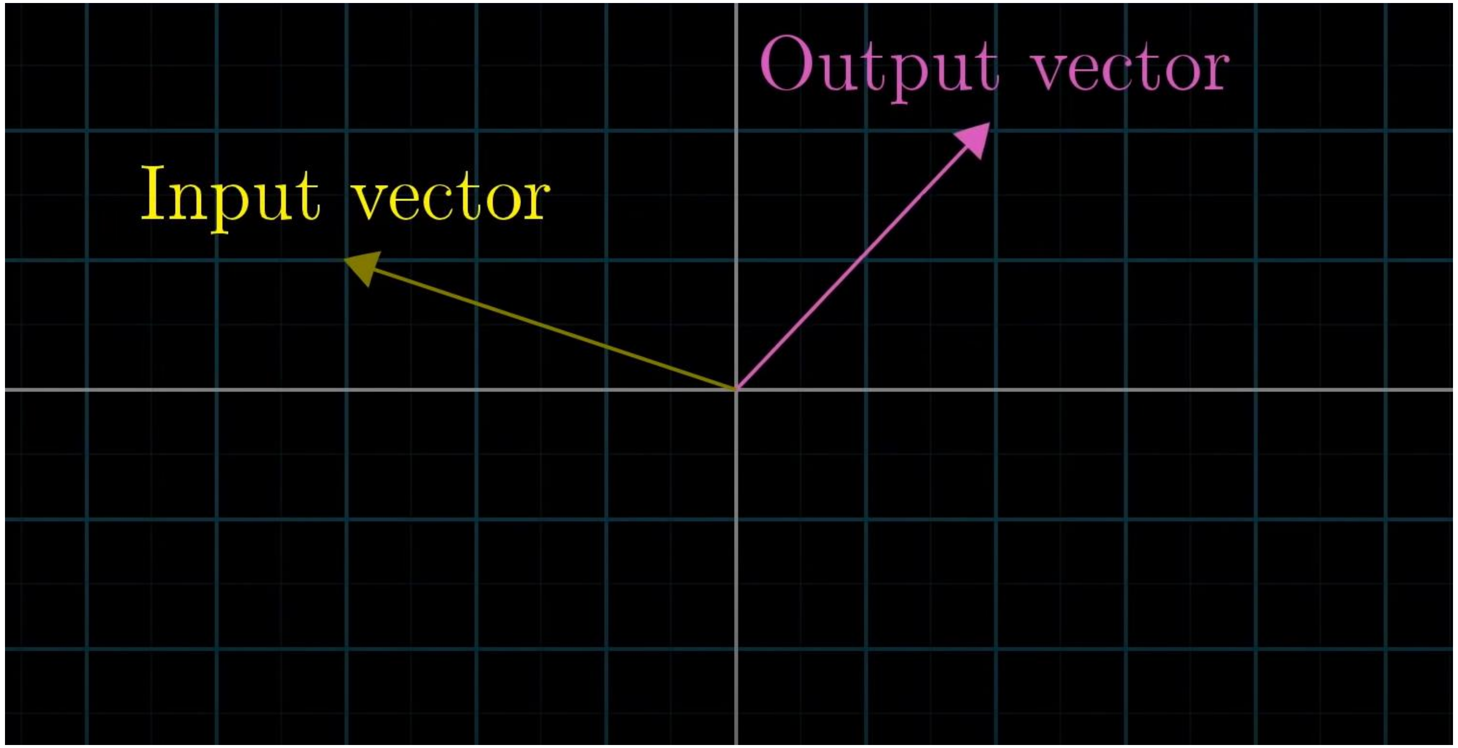
- Matrix Multiplications are nothing but [Linear Transformations]() (stretching or rotating)

"2x2 Matrix"

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{x \begin{bmatrix} a \\ c \end{bmatrix} + y \begin{bmatrix} b \\ d \end{bmatrix}}_{\text{Where all the intuition is}} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

Eigenvalues and Eigenvectors

- Transforming a vector (r, ϴ) to another vector (r’, ϴ’) requires changing its length and its angle.



Eigenvalues and Eigenvectors

- Consider a matrix X which spans the vector space V
- Let’s say we transform the basis vector X with matrix A
- Due to the transformation, all the vectors within the space must also be transformed (change in angle and length).

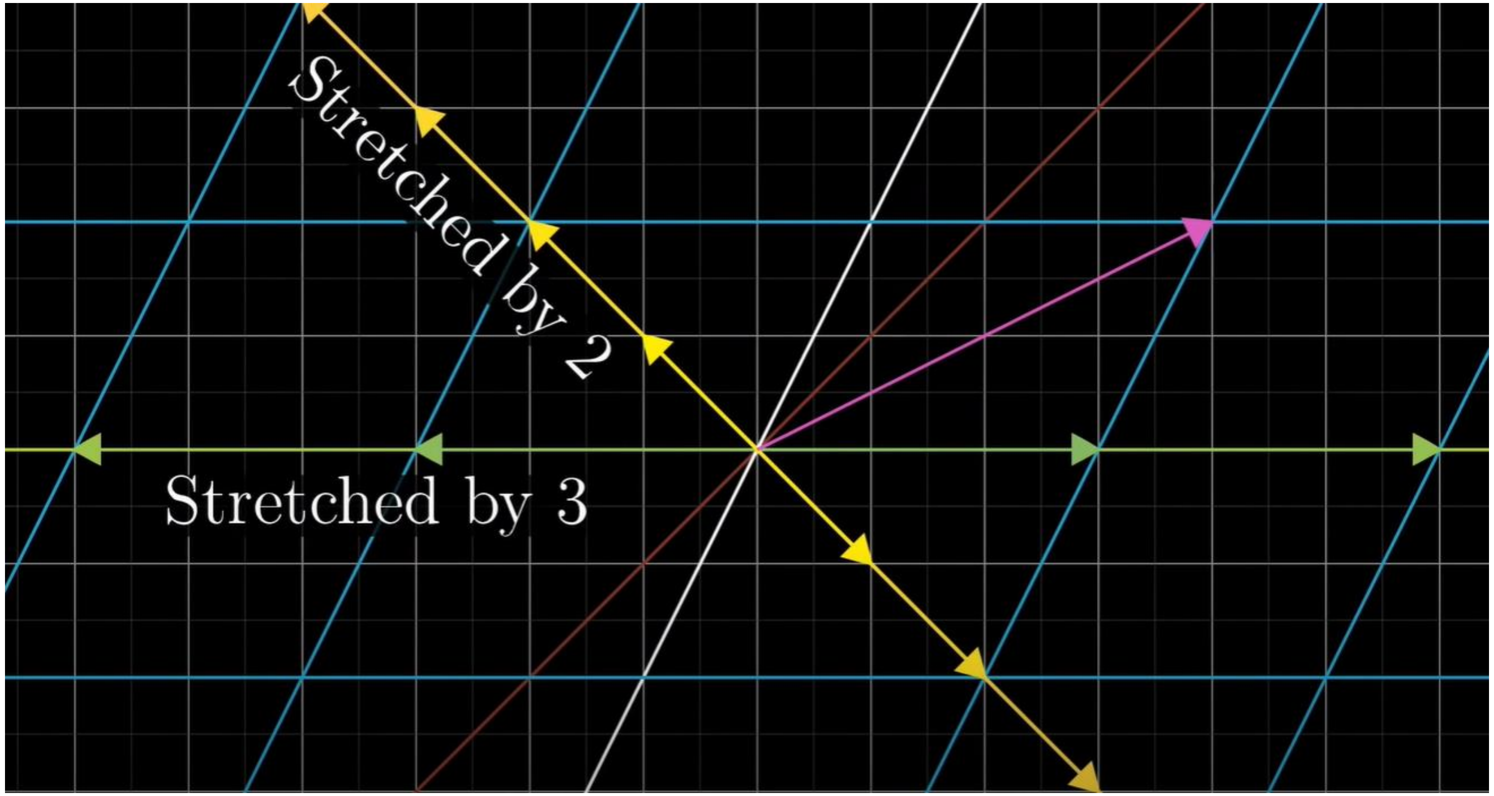
Eigenvalues and Eigenvectors

- Except a few, which just incur a change in length.
- The vectors which only change by a constant on transformation are called eigen vectors.
- And the constant it changes by is called eigen value.

$$Ax = \lambda \mathrm{x}$$

Eigenvalues and Eigenvectors

- Red represents non-eigen vector before transformation. Pink represents non-eigen vector after rotation. Yellow and Green are eigenvectors.



Orthogonal and Semi-Orthogonal Matrix

- Orthogonal matrices are square matrix for which $A^T = A^{-1}$. This entails that $A^T A = A^{-1} A = I$
- Semi-Orthogonal Matrix are non-square matrix which act like orthogonal matrix, and follow the property: $A^T A = I$

Diagonalisation

- Diagonal Matrix are square matrix with every element except diagonal ones equal to 0
- Diagonalisation is a transformation of a square matrix to a diagonal matrix

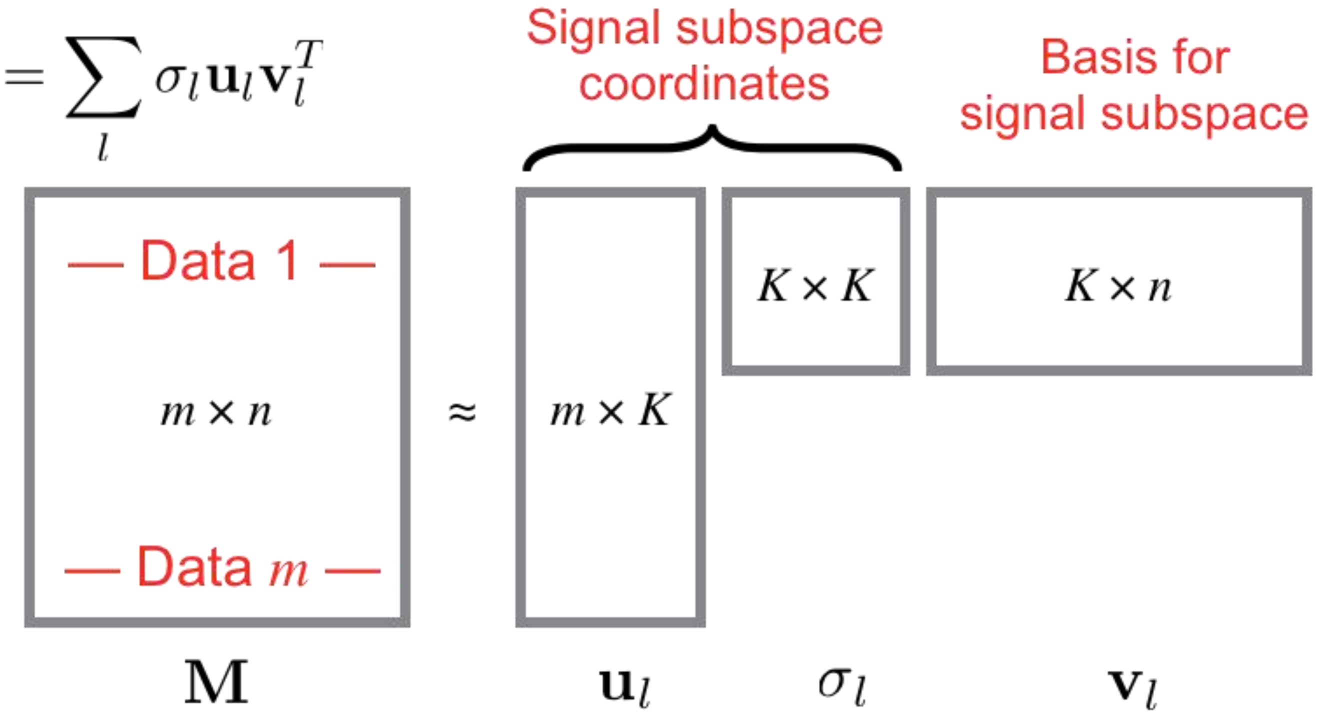
$$A = \begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & \cdots & 0 \\ 0 & 0 & a_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix}_{n \times n}$$

Eigen Decomposition

- $M = P\ D\ P^{-1}$
- M is original matrix you want to diagonalise.
- P is an orthogonal matrix composed of eigenvectors of M
- D is a diagonal matrix made up of eigenvalues of M
- There are some cases where M is not diagonalisable (M is not square, or not full rank)

Singular Value Decomposition

$$\mathbf{M} = \sum_l \sigma_l \mathbf{u}_l \mathbf{v}_l^T$$



Project data to signal subspace : $\{\mathbf{v}_1, \ldots, \mathbf{v}_K\}$

Coordinates in the signal subspace : $[\sigma_1 \mathbf{u}_1 \ \cdots \ \sigma_K \mathbf{u}_K]$

Singular Value Decomposition

$$M = U \Sigma V^T$$

$$M^T M = V \Sigma^T \underbrace{(U^T \cdot U)}_{I \text{ (orthogonal)}} \Sigma V^T$$

$$= V \underbrace{(\Sigma^T \Sigma)}_{\text{diag. matrix}} V^T$$

diagonalisation of $M^T M$

do the same for U by computing $M \cdot M^T$

Doubts