

Merkle - Damgård Hashing

Def:- It is a method of building collision resistant hash fns using one-way compression fns.

Proof of security:- There are 2 cases based on 2.

Case-1 ($L \neq L'$)

For last steps:

$$H^S(x) \Rightarrow Z_{B+1} = h^S(Z_B || L)$$

$$H^S(x') \Rightarrow Z'_{B'+1} = h^S(Z'_B || L')$$

$\therefore h^S(Z_B || L) = h^S(Z'_B || L')$ [$H^S(x) = H^S(x')$]
but $L \neq L' \therefore Z_B || L$ & $Z'_B || L'$ are different, hence there is a collision of 2 different strings.

Case-2 ($L = L'$)

$$\therefore B = B', \quad x_{B+1} = x'_{B+1}$$

$$\therefore x \neq x' \text{ \& } |x| = |x'|,$$

there exists at least one i such that $x_i \neq x'_i$

Let $i \leq B+1$ be the highest index s.t.

$$Z_{i-1} || x_i \neq Z'_{i-1} || x'_i$$

If $i = B+1$, then $Z_B || x_{B+1}$ and $Z'_B || x'_{B+1}$ are different.

$$\begin{aligned} \therefore h^S(Z_B || x_{B+1}) &= Z_{B+1} = H^S(x) = H^S(x') \\ &= Z'_{B+1} = h^S(Z'_B || x'_{B+1}) \end{aligned}$$

If $i \leq B$, then maximality of i implies $Z_i = Z'_i$

$\therefore Z_{i-1} || x_i$ & $Z'_{i-1} || x'_i$ are different & just a collision.