Assignment-3 Part-I

MDL Team 64

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~~S1 | S2 | S3 | S4 | S5 | S6~~

The roll no used is 2019101056.

$$x = 1 - \left(((1056) \% 30 + 1)/100\right) = 0.93$$
$$y = (56) \% 4 + 1 = 1.$$

Left

$A \in \{L, R\}$

Right

A : Action given $P(A) = x$ & $P(A') = 1 - x$

$\therefore \boxed{P(A) = 0.93}$

unsuccesfull action

As $y = 1$ $\therefore$ Table 1 will be used for observation probabilities.

$P(O=G|S=R) = 0.05$ ← $P(\text{observation} = \text{Red} | \text{State} = \text{Red}) = 0.95$

$P(O=R|S=G) = 0.2$ ← $P(\text{Observation} = \text{Green} | \text{State} = \text{Green}) = 0.8$

We will solve each part one by one.

① Agent took the action Right and observed green (Gre):-
This is the first action agent is taking and it is know that, it is equi-probable to find agent either in $S_1/S_3/S_6$ initially.

Page:
Date: / /

$\therefore$ Initial belief state: $b = [\overset{S1}{0.333}, \overset{S2}{0}, \overset{S3}{0.333}, \overset{S4}{0}, \overset{S5}{0}, \overset{S6}{0.333}]$.

From POMDP, we know that

$$b'(s') = \frac{O(s', a, o) \cdot \sum T(s, a, s') \cdot b(s)}{P(o \mid a, b)}$$

~~where s': fidertina~~

where s': new belief state — b: old belief state

s: old state — a: action

b': new belief state — o: observation.

Firstly, let's calculate unnormalized belief state and at last, we will normalize the new belief state. (by dividing the denominator).

$$\because \sum_s b'(s') = 1 = \frac{\text{Sum of un-normalized prob.}}{\text{denominator}}$$

$\therefore$ $\boxed{\text{denominator} = \text{Sum of un-normalized prob.}}$

Let's start with computation:-

S1	S2	S3	S4	S5	S6
R	G	R	G	G	R

R: Red
G: Green

(S1) $\therefore$ $b'(s_1)$ = Prob. of s_1 in new belief state b'

Numerator $= P(G \mid s_1, R) \times [P(s_1 \mid s_1, R) \times b(s_1) + P(s_1 \mid s_2, R) \times b(s_2) + P(s_1 \mid s_3, R) \cdot b(s_3) + P(s_1 \mid s_4, R) \cdot b(s_4) + P(s_1 \mid s_5, R) \cdot b(s_5) + P(s_1 \mid s_6, R) \cdot b(s_6)]$.

$= 0.05 \times [0.07 \times 0.333 + 0.07 \times 0 + 0 + 0 + 0 + 0]$

$= 0.001166$

Page:
Date: / /

(S2) $b'(s_2) =$ Prob. of s_2 in new belief state b'

Numerator $= P(Gr|s_2, R) \times [P(s_2|s_1, R) \times b(s_1)$
$+ P(s_2|s_2, R) \cdot b(s_2) + P(s_2|s_3, R) \cdot b(s_3)$
$+ P(s_2|s_4, R) \cdot b(s_4) + P(s_2|s_5, R) \cdot b(s_5) + P(s_2|s_6, R) \cdot b(s_6)]$

$= 0.8 \times [0.93 \times 0.333 + 0 + 0.07 \times 0.333 + 0 + 0 + 0]$
$=$ ~~0.264~~ 0.266666

(S3) $b'(s_3) =$ Prob. of s_3 in new belief state b'

Numerator $= P(Gr|s_3, R) \cdot [P(s_3|s_1, R) \cdot b(s_1)$
$+ P(s_3|s_2, R) \cdot b(s_2) + P(s_3|s_3, R) \cdot b(s_3) + P(s_3|s_4, R) \cdot b(s_4)$
$+ P(s_3|s_5, R) \cdot b(s_5) + P(s_3|s_6, R) \cdot b(s_6)]$.

$= 0.05 \times [0 + 0.93 \times 0 + 0 + 0.07 \times 0 + 0 + 0]$
$= 0.0$

(S4) $b'(s_4) =$ Prob. of s_4 in new belief state b'

Numerator $= P(Gr|s_4, R) \cdot [P(s_4|s_1, R) \cdot b(s_1) + P(s_4|s_2, R) \cdot b(s_2)$
$+ P(s_4|s_3, R) \cdot b(s_3) + P(s_4|s_4, R) \cdot b(s_4)$
$+ P(s_4|s_5, R) \cdot b(s_5) + P(s_4|s_6, R) \cdot b(s_6)]$.

$= 0.8 \times [0 + 0 + 0.93 \times 0.333 + 0 + 0.07 \times 0 + 0]$
$=$ ~~0.24552~~ 0.248

(S5) $b'(s_5) =$ Prob. of s_5 in new belief state b'

continued

Page:
Date: / /

Numerator $= P(Gr|S_5,R) \times [P(S_5|S_1,R)\cdot b(S_1) + P(S_5|S_2,R)\cdot b(S_2)$
$+ P(S_5|S_3,R)\cdot b(S_3) + P(S_5|S_4,R)\cdot b(S_4)$
$+ P(S_5|S_5,R)\cdot b(S_5) + P(S_5|S_6,R)\cdot b(S_6)]$

$= 0.8 \times [0 + 0 + 0 + 0.93 \times 0 + 0 + 0.07 \times 0.333]$
$=$ ~~0.01818~~ 0.018666

(56) $b'(S_6)$ = Prob. of S_6 in new belief state b'

Numerator $= P(Gr|S_6,R)\cdot[P(S_6|S_1,R)\cdot b(S_1) + P(S_6|S_2,R)\cdot b(S_2)$
$+ P(S_6|S_3,R)\cdot b(S_3) + P(S_6|S_4,R)\cdot b(S_4)$
$+ P(S_6|S_5,R)\cdot b(S_5) + P(S_6|S_6,R)\cdot b(S_6)]$

$= 0.05 \times [0+0+0+0+0.93\times 0 + 0.93 \times 0.3333]$
$=$ ~~0.015345~~ 0.0155

So now we have calculated all the un-normalized probabilities, so now find the denominator & normalize these prob.

~~$\sum_s b'$~~

~~∴ denominator = Sum of all un~~

∴ denominator = Sum of all un-normalized prob.

∴ denominator $= 0.00156 + 0.266 + 0 + 0.248 + 0.018666 + 0.0155$ ~~$+ 0.015345 = 0.5115$~~
$= 0.55$

∴ $b'(S_1) = 0.001155 / 0.55 =$ ~~0.02083~~ 0.002121
$b'(S_2) = 0.264 / 0.55 = 0.484848$
$b'(S_3) =$ ~~0/0.5115~~ $0/0.55 = 0.0$
$b'(S_4) = 0.24552 / 0.55 =$ ~~0.44627~~ 0.450909
$b'(S_5) = 0.01848 / 0.55 =$ ~~0.034~~ 0.033939
$b'(S_6) = 0.015345 / 0.55 =$ ~~0.02767~~ 0.028181

∴ New belief state:

$b' = [0.002121, 0.484848, 0.0, 0.450909, 0.033939, 0.028181]$
$\quad\quad\quad

Page:
Date: / /

(53) $b''(s_3)$ = Prob. of s_3 in new belief state b''

Numerator $= P(Re|s_3, L) \times [P(s_3|s_1, L) \cdot b'(s_1) + P(s_3|s_2, L) \cdot b'(s_2)$
$+ P(s_3|s_3, L) \cdot b'(s_3) + P(s_3|s_4, L) \cdot b'(s_4)$
$P(s_3|s_5, L) \cdot b'(s_5) + P(s_3|s_6, L) \cdot b'(s_6)]$

$= 0.95 \times [0 + 0.07 \times 0.4848 + 0 + 0.93 \times \overset{0.450909}{\cancel{0.4427}} + 0 + 0]$
$= 0.95 \times [0.033936 + 0.4 \cancel{1} 345]$
$= \cancel{0.42336465}\ 0.430620$

(54) $b''(s_4)$ = Prob. of s_4 in new belief state b''

Numerator $= P(Re|s_4, L) \times [P(s_4|s_1, L) \cdot b'(s_1) + P(s_4|s_2, L) \cdot b'(s_2)$
$+ P(s_4|s_3, L) \cdot b'(s_3) + P(s_4|s_4, L) \cdot b'(s_4)$
$+ P(s_4|s_5, L) \cdot b'(s_5) + P(s_4|s_6, L) \cdot b'(s_6)]$

$= 0.2 \times [0 + 0 + 0.07 \times 0 + 0 + 0.93 \times \overset{0.033939}{\cancel{0.034}} + 0]$
$= \cancel{0.06}\ \cancel{0.006324}\ 0.0063127$

(55) $b''(s_5)$ = Prob. of s_5 in new belief state b''

Numerator $= P(Re|s_5, L) \cdot [P(s_5|s_1, L) \cdot b'(s_1) + P(s_5|s_2, L) \cdot b'(s_2)$
$+ P(s_5|s_3, L) \cdot b'(s_3) + P(s_5|s_4, L) \cdot b'(s_4)$
$+ P(s_5|s_5, L) \cdot b'(s_5) + P(s_5|s_6, L) \cdot b'(s_6)]$

$= 0.2 \times [0 + 0 + 0 + 0.07 \times \overset{0.450909}{\cancel{0.4427}} + 0 + 0.93 \times \overset{0.028181}{\cancel{0.0267}}]$
$\cancel{= 0.2 \times [0.030989 + 0.0257331]}$
$\cancel{= 0.0113444 2}$
$= 0.2 \times [0.03156363 + 0.02620833]$
$= 0.0115545$

(S6) $b''(S_6)$ = Prob. of S_6 in new belief state b''

Numerator = $P(Re|S_6, L) \times [P(S_6|S_1, L) \cdot b'(S_1) + P(S_6|S_2, L) \cdot b'(S_2)$
$+ P(S_6|S_3, L) \cdot b'(S_3) + P(S_6|S_4, L) \cdot b'(S_4)$
$+ P(S_6|S_5, L) \cdot b'(S_5) + P(S_6|S_6, L) \cdot b'(S_6)]$

$= 0.95 \times [0 + 0 + 0 + 0 + 0.07 \times \cancel{0.034}\ 0.033939 + 0.07 \times \cancel{0.0275}\ 0.028181]$
$\cancel{= 0.95 \times E = 0.00410055} = 0.00413106$

So now we have calculated all the un-normalized prob. for 2nd step of agent. Let's normalize them by dividing by denominator and calculate the ~~updat~~ new belief state.

$\therefore$ denominator $= \cancel{0.446724105}\ 0.430237 + \cancel{0.00029162}\ 0.0000296 + \cancel{0.42336465}\ 0.430620$
$+ \cancel{0.006324}\ 0.0063127 + \cancel{0.01134442}\ 0.0115545 + \cancel{0.00410055}\ 0.00413106$
$= \cancel{0.89214985}\ 0.8828863$

$\therefore b''(S_1) = 0.446724105 / \cancel{0.89214985}$ denominator $= \cancel{0.50073}\ 0.487308$
$b''(S_2) = 0.00029162 / \cancel{0.89214985}$ denominator $= \cancel{0.000327}\ 0.000036$
$b''(S_3) = 0.42336465 / \cancel{0.89214985}$ denominator $= \cancel{0.4745}\ 0.487741$
$b''(S_4) = 0.006324 / \cancel{0.89214985}$ denominator $= \cancel{0.00709}\ 0.007150$
$b''(S_5) = 0.01134442 / \cancel{0.89214985}$ denominator $= \cancel{0.012716}\ 0.013087$
$b''(S_6) = 0.00410055 / \cancel{0.89214985}$ denominator $= \cancel{0.0046}\ 0.004679$

~~$\therefore$ New belief state:~~
~~$b'' = [0.50073, 0.000327, 0.4745, 0.00709, 0.012716, 0.0046]$~~
~~S1 S2 S3 S4 S5 S6~~

$\therefore$ New belief state:
$b'' = [0.487308, 0.000036, 0.487741, 0.007150, 0.013087, 0.004679]$
S1 S2 S3 S4 S5 S6

Page:
Date: / /

③ Agent took the action Left and observed Green:-
Each and every step will be same except that, ~~as~~ now we have to use belief state b'' as our updated belief state. (because this is the 3rd step of agent).

(S1) $b'''(s_1)$ = Prob. of s_1 in new belief state b'''

Numerator $= P(Gr|s_1, L) \times [P(s_1|s_1, L) \cdot b''(s_1) + P(s_1|s_2, L) \cdot b''(s_2)$
$+ P(s_1|s_3, L) \cdot b''(s_3) + P(s_1|s_4, L) \cdot b''(s_4)$
$+ P(s_1|s_5, L) \cdot b''(s_5) + P(s_1|s_6, L) \cdot b''(s_6)]$

$= 0.05 \times [0.93 \times \overset{0.487308}{\cancel{0.50073}} + 0.93 \times \overset{0.000036}{\cancel{0.000327}} + 0+0+0+0]$
$= 0.05 \times [\overset{0.45319644}{\cancel{0.4656789}} + \overset{0.00003348}{\cancel{0.00030411}}]$
$= \cancel{0.0233}\ \cancel{0.0233}\ 0.02266139$

(S2) $b'''(s_2)$ = Prob. of s_2 in new belief state b'''

Numerator $= P(Gr|s_2, L) \times [P(s_2|s_1, L) \cdot b''(s_1) + P(s_2|s_2, L) \cdot b''(s_2)$
$+ P(s_2|s_3, L) \cdot b''(s_3) + P(s_2|s_4, L) \cdot b''(s_4)$
$P(s_2|s_5, L) \cdot b''(s_5) + P(s_2|s_6, L) \cdot b''(s_6)]$

$= 0.8 \times [0.07 \times \overset{0.487308}{\cancel{0.50073}} + 0 + 0.93 \times \overset{0.487741}{\cancel{0.4745}} + 0+0+0]$
$\cancel{= 0.8 \times [0.0350511 + 0.441285]}$
$= \cancel{0.38107}\ 0.390169$

(S3) $b'''(s_3)$ = Prob. of s_3 in new belief state b'''

Numerator $= P(Gr|s_3, L) \times [P(s_3|s_1, L) \cdot b''(s_1) + P(s_3|s_2, L) \cdot b''(s_2)$
$+ P(s_3|s_3, L) \cdot b''(s_3) + P(s_3|s_4, L) \cdot b''(s_4)$
$+ P(s_3|s_5, L) \cdot b''(s_5) + P(s_3|s_6, L) \cdot b''(s_6)]$.

$$= 0.05 \times [\, 0 + 0.07 \times \overset{0.000036}{\cancel{0.000327}} + 0 + 0.93 \times \overset{0.007150}{\cancel{0.00709}} + 0 + 0 \,]$$

$$\cancel{= 0.05 \times [\, 0.00002289 + 0.0065937 \,]}$$

$$\cancel{= 0.000331} = 0.000332597$$

(54) $b'''(s_4)$ = Prob. of s_4 in new belief state b'''.

$$\text{Numerator} = P(Gr \mid s_4, L) \times [\, P(s_4 \mid s_1, L) \cdot b''(s_1) + P(s_4 \mid s_2, L) \cdot b''(s_2) + P(s_4 \mid s_3, L) \cdot b''(s_3) + P(s_4 \mid s_4, L) \cdot b''(s_4) \; P(s_4 \mid s_5, L) \cdot b''(s_5) + P(s_4 \mid s_6, L) \cdot b''(s_6) \,]$$

$$= 0.8 \times [\, 0 + 0 + 0.07 \times \overset{0.487741}{\cancel{0.4745}} + 0 + 0.93 \times \overset{0.013087}{\cancel{0.012716}} + 0 \,]$$

$$\cancel{= 0.8 \times [\, 0.033215 + 0.0118258 \,]}$$

$$= \cancel{0.036033} \; 0.037050448$$

(55) $b'''(s_5)$ = Prob. of s_5 in new belief state b'''

$$\text{Numerator} = P(Gr \mid s_5, L) \times [\, P(s_5 \mid s_1, L) \times b''(s_1) + P(s_5 \mid s_2, L) \times b''(s_2) + P(s_5 \mid s_3, L) \cdot b''(s_3) + P(s_5 \mid s_4, L) \cdot b''(s_4) + P(s_5 \mid s_5, L) \cdot b''(s_5) + P(s_5 \mid s_6, L) \cdot b''(s_6) \,]$$

$$= 0.8 \times [\, 0 + 0 + 0 + 0.07 \times \overset{0.007150}{\cancel{0.00709}} + 0 + 0.93 \times \overset{0.004679}{\cancel{0.0046}} \,]$$

$$\cancel{= 0.8 \times [\, 0.0004963 + 0.004278 \,]}$$

$$= \cancel{0.003382} \; 0.0038816$$

(56) $b'''(s_6)$ = Prob. of s_6 in new belief state b'''

$$\text{Numerator} = P(Gr \mid s_6, L) \times [\, P(s_6 \mid s_1, L) \cdot b''(s_1) + P(s_6 \mid s_2, L) \cdot b''(s_2) + P(s_6 \mid s_3, L) \cdot b''(s_3) + P(s_6 \mid s_4, L) \cdot b''(s_4) + P(s_6 \mid s_5, L) \cdot b''(s_5) + P(s_6 \mid s_6, L) \cdot b''(s_6) \,].$$

Page:
Date: / /

$= 0.05 \times [0+0+0+0+0.07 \times \overset{0.013087}{\cancel{0.012716}} + \overset{0.07}{\cancel{0.93}} \times \overset{0.004679}{\cancel{0.0046}}]$

$\cancel{= 0.05 \times [0.00089012 + 0.004278]}$

$= \cancel{0.0002584}\ 0.000062181978$

So now we have calculated all the un-normalized prob. for 3rd step of agent. Let's normalize them by dividing by denominator and calculate the new beleif state.

$\therefore$ denominator $= \overset{0.02266139}{\cancel{0.0233}} + \overset{0.390169}{\cancel{0.38107}} + \overset{0.000332597}{\cancel{0.000331}} + \overset{0.037050448}{\cancel{0.036033}}$
$+ \overset{0.0038816}{\cancel{0.00382}} + \overset{0.000062181978}{\cancel{0.0002584}}$

$= \cancel{0.4448124}\ 0.4541574$

$\therefore$ $b'''(s_1) = 0.0233 / \overset{\text{denominator}}{\cancel{0.4448124}} = \overset{0.049897}{\cancel{0.0524}}$

$b'''(s_2) = 0.38107 / \overset{\text{denominator}}{\cancel{0.4448124}} = \overset{0.859105}{\cancel{0.8567}}$

$b'''(s_3) = 0.000331 / \overset{\text{denominator}}{\cancel{0.4448124}} = \overset{0.000732}{\cancel{0.000744}}$

$b'''(s_4) = 0.036033 / \overset{\text{denominator}}{\cancel{0.4448124}} = 0.081580$

$b'''(s_5) = 0.00382 / \overset{\text{denominator}}{\cancel{0.4448124}} = \overset{0.008546}{\cancel{0.0086}}$

$b'''(s_6) = 0.0002584 / \overset{\text{denominator}}{\cancel{0.4448124}} = \overset{0.0001369}{\cancel{0.000581}}$

$\cancel{\therefore \text{ New beleif state: } b''' = [0.0524, 0.8567, 0.000744, 0.081, 0.0086, 0.000581]}$

$\therefore$ New beleif state:
$b''' = [0.049897, 0.859105, 0.000732, 0.081580, 0.008546, 0.0001369]$.