Probability and Statistics: Lecture-35

Monsoon-2020

by Dr. Pawan Kumar (IIIT, Hyderabad)
on November 4, 2020

» Solved Example 2

» Solved Example 2

Example (Solved Example)

Let X, Y be jointly continuous RVs with joint PDF

» Solved Example 2

Example (Solved Example)

Let X, Y be jointly continuous RVs with joint PDF

$$f_{X,Y} = \begin{cases} 6e^{-(2x+3y)} & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

» Solved Example 2

Example (Solved Example)

Let X, Y be jointly continuous RVs with joint PDF

$$f_{X,Y} = \begin{cases} 6e^{-(2x+3y)} & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- Are X and Y independent?

» Solved Example 2

Example (Solved Example)

Let X, Y be jointly continuous RVs with joint PDF

$$f_{X,Y} = \begin{cases} 6e^{-(2x+3y)} & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- Are X and Y independent?
- Find $E[Y \mid X > 2]$

» Solved Example 2

Example (Solved Example)

Let X, Y be jointly continuous RVs with joint PDF

$$f_{X,Y} = \begin{cases} 6e^{-(2x+3y)} & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- Are X and Y independent?
- Find $E[Y \mid X > 2]$
- Find $P(X > Y)$

$\iint_D f_{X,Y}(x,y)$

» Answer to previous problem...

① $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dy$

$= \int_0^{\infty} 6 \cdot e^{-(2x+3y)}\, dy$

$= 6e^{-2x} \int_0^{\infty} e^{-3y}\, dy$

$= 6e^{-2x} \left[\frac{e^{-3y}}{-3} \right]_0^{\infty}$

$= -2e^{-2x} \left[e^{-3y} \right]_0^{\infty}$

$= -2e^{-2x}(-1) = 2e^{-2x}$

Similarly,

$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dx$

$= \int_0^{\infty} 6e^{-(2x+3y)}\, dx$

$= 6e^{-3y} \int_0^{\infty} e^{-2x}\, dx$

$= 6e^{-3y} \left[\frac{e^{-2x}}{-2} \right]_0^{\infty}$

$= -3e^{-3y} \left[e^{-2x} \right]_0^{\infty}$

$= 3e^{-3y}$

$\Rightarrow f_{XY}(x,y) = 6e^{-(2x+3y)} = f_X(x) \cdot f_Y(y)$

$\Rightarrow X, Y$ are independent

» Solved Example 3

(2) $E[Y \mid X>2]$

$= E[Y]$ Since X, Y are ind.

We observe that

$f_Y(y) = 3e^{-3y}$, $y \geq 0$

$\Rightarrow Y$ is $\exp(3)$.

$\Rightarrow E[Y] = \frac{1}{3}$. //

(3) Try.

» Solved Example 3

Example (Solved example)

Let X be a continuous RV with PDF

» Solved Example 3

Example (Solved example)

Let X be *a* continuous RV with PDF

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

» Solved Example 3

Example (Solved example)

Let X be a continuous RV with PDF

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

We are also know that given $X = x$, the RV Y is uniformly distributed on $[-x, x]$.

» Solved Example 3

$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)}$

$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)}$

Example (Solved example)

Let X be a continuous RV with PDF

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

We are also know that given $X = x$, the RV Y is uniformly distributed on $[-x, x]$.

- Find the joint PDF $f_{XY}(x,y)$
- Find $P_Y(y)$
- Find $P(|Y| < X^3)$

$P((x,y) \in A)$ $\iint_A f_{XY}(x,y)\,dx\,dy$

Exercise

» Answer to previous problem...

» Answer to previous problem...

» Solved Example 4

» Solved Example 4

Example (Solved example)

Let X, Y be two jointly continuous RVs with joint PDF

» Solved Example 4

Example (Solved example)

Let X, Y be two jointly continuous RVs with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \leq x \leq 1,\ 0 \leq y \leq \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

» Solved Example 4

Example (Solved example)

Let X, Y be two jointly continuous RVs with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \le x \le 1,\ 0 \le y \le \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

1. Plot R_{XY}

» Solved Example 4

Example (Solved example)

Let X, Y be two jointly continuous RVs with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \leq x \leq 1,\ 0 \leq y \leq \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

1. Plot R_{XY}
2. Find $f_X(x)$ and $f_Y(y)$

» Solved Example 4

Example (Solved example)

Let X, Y be two jointly continuous RVs with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \le x \le 1,\ 0 \le y \le \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

1. Plot R_{XY}
2. Find $f_X(x)$ and $f_Y(y)$
3. Are X and Y independent?

» Solved Example 4

Example (Solved example)

Let X, Y be two jointly continuous RVs with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \le x \le 1,\ 0 \le y \le \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

1. Plot R_{XY}
2. Find $f_X(x)$ and $f_Y(y)$
3. Are X and Y independent?
4. Find the conditional PDF of X given $Y = y$, $f_{X|Y}(x \mid y)$

» Solved Example 4

Example (Solved example)

Let X, Y be two jointly continuous RVs with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \le x \le 1,\ 0 \le y \le \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

1. Plot R_{XY}
2. Find $f_X(x)$ and $f_Y(y)$
3. Are X and Y independent?
4. Find the conditional PDF of X given $Y = y$, $f_{X|Y}(x \mid y)$
5. Find $E[X \mid Y = y], 0 \le y \le 1$

Example (Solved example)

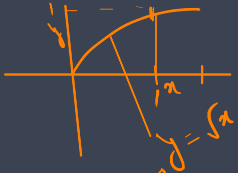
Let X, Y be two jointly continuous RVs with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \le x \le 1,\ 0 \le y \le \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

$\Rightarrow \boxed{y^2 \le x \le 1}$

1. Plot R_{XY}
2. Find $f_X(x)$ and $f_Y(y)$
3. Are X and Y independent?
4. Find the conditional PDF of X given $Y = y$, $f_{X|Y}(x \mid y)$
5. Find $E[X \mid Y = y], 0 \le y \le 1$
6. Find $\mathrm{Var}(X \mid Y = y)$ for $0 \le y \le 1$

» Answer to previous problem...

① 

② $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dy$

$= \int_0^{\sqrt{x}} 6xy\, dy = 6x \left[\frac{y^2}{2}\right]_0^{\sqrt{x}}$

$= 3x \cdot x = 3x^2$

$\Rightarrow f_X(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dx$

$= \int_{y^2}^{1} 6xy\, dx = 6y \left[\frac{x^2}{2}\right]_{y^2}^{1}$

$= 3y(1-y^4)$

$f_Y(y) = \begin{cases} 3y(1-y^4) & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$

③ No, $f_{XY}(x,y) \neq f_X(x) f_Y(y)$

④

» Answer to previous problem...

④ $$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

$$= \begin{cases} \dfrac{6xy^{2}}{3y(1-y^4)} & y^2 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

⑤ $$E[X|Y=y] = \int_{-\infty}^{\infty} x \, f_{X|Y}(x|y)\,dx$$

$$= \int_{y^2}^{1} x \cdot \frac{2x}{1-y^4}$$

$$\frac{2}{1-y^4} \int_{y^2}^{1} x^2 = \frac{2}{1-y^4}\left[\frac{x^3}{3}\right]_{y^2}^{1}$$

$$= \frac{2(1-y^6)}{3(1-y^4)}.$$

⑥ $$\mathrm{Var}(X|Y=y)$$

$$= \underbrace{E[X^2|Y=y]}_{\uparrow \text{ do this}} - \underbrace{(E[X|Y=y])^2}_{\uparrow \text{ found}}$$

$$=$$

» Definition of Covariance...

» Definition of Covariance...

Definition of Covariance

Let X and Y be two random variables.

» Definition of Covariance...

Definition of Covariance

Let X and Y be two random variables. The covariance between X and Y is defined as

Definition of Covariance

Let X and Y be two random variables. The covariance between X and Y is defined as

$$\mathrm{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - (E[X])(E[Y])$$

Derivation

$$E[XY - XE[Y] - E[X]Y + E[X]E[Y]]$$

$$= E[XY] - \cancel{E[Y]}E[X] - \cancel{E[X]}E[Y] + E[X]E[Y]$$

» Solved Example

» Solved Example

$\boxed{Y|X \sim \text{Exp}(X)}$ L.O.T.E

Example (Solved Example)

Suppose $X \sim \text{Uniform}(1, 2)$ and given $X = x$, Y is exponential with parameter $\lambda = x$. Find $\text{Cov}(X, Y)$.

$\text{Cov}(X,Y) = E[XY] - E[X]E[Y]$. We have $E[X] = \frac{3}{2}$.

We have

$$E[Y] = E[E[Y|X]] \quad \text{(by L.O.T.E)}$$

$$= E\left[\frac{1}{X}\right] = \int_1^2 \frac{1}{x} \cdot 1 = [\ln x]_1^2 = \ln 2$$

$$E[XY] = E[E[XY|X]] \quad \text{(L.O.T.E)}$$

$$= E[X\,E[Y|X]] = E\left[X \cdot \frac{1}{X}\right] = 1.$$

$\boxed{Y|X \sim \text{Exp}(X)}$

$$\boxed{\text{Cov}(X,Y) = 1 - \frac{3}{2} \cdot \ln 2}$$

Answer

» Properties of Covariance...

» Properties of Covariance...

$\text{Cov}(X,Y) = E[XY] - E[X]E[Y]$

If X, Y ind $\Rightarrow$ $E[XY] = E[X]E[Y]$

Properties of Covariance

1. $\text{Cov}(X,X) = \text{Var}(X)$

» Properties of Covariance...

Properties of Covariance

1. $\mathrm{Cov}(X, X) = \mathrm{Var}(X)$
2. If X and Y are independent, then $\mathrm{Cov}(X, Y) = 0$. [Note: converse is not true!]

Properties of Covariance

1. $\text{Cov}(X, X) = \text{Var}(X)$
2. If X and Y are independent, then $\text{Cov}(X, Y) = 0$. [Note: converse is not true!]
3. $\text{Cov}(X, Y) = \text{Cov}(Y, X)$

» Properties of Covariance...

Properties of Covariance

1. $\mathrm{Cov}(X, X) = \mathrm{Var}(X)$
2. If X and Y are independent, then $\mathrm{Cov}(X, Y) = 0$. [Note: converse is not true!]
3. $\mathrm{Cov}(X, Y) = \mathrm{Cov}(Y, X)$
4. $\mathrm{Cov}(aX, Y) = a\mathrm{Cov}(X, Y)$

» Properties of Covariance...

Properties of Covariance

1. $\mathrm{Cov}(X, X) = \mathrm{Var}(X)$
2. If X and Y are independent, then $\mathrm{Cov}(X, Y) = 0$. [Note: converse is not true!]
3. $\mathrm{Cov}(X, Y) = \mathrm{Cov}(Y, X)$
4. $\mathrm{Cov}(aX, Y) = a\mathrm{Cov}(X, Y)$
5. $\mathrm{Cov}(X + c, Y) = \mathrm{Cov}(X, Y)$

Properties of Covariance

1. $\mathrm{Cov}(X, X) = \mathrm{Var}(X)$
2. If X and Y are independent, then $\mathrm{Cov}(X, Y) = 0$. [Note: converse is not true!]
3. $\mathrm{Cov}(X, Y) = \mathrm{Cov}(Y, X)$
4. $\mathrm{Cov}(aX, Y) = a\,\mathrm{Cov}(X, Y)$
5. $\mathrm{Cov}(X + c, Y) = \mathrm{Cov}(X, Y)$
6. $\mathrm{Cov}(X + Y, Z) = \mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z)$

$$E[(X+Y)Z] - E[(X+Y)]E[Z]$$

$$= E[XZ] + E[YZ] - E[X]E[Z] - E[Y]E[Z]$$

$$\mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z).$$

Properties of Covariance

1. $\mathrm{Cov}(X, X) = \mathrm{Var}(X)$
2. If X and Y are independent, then $\mathrm{Cov}(X, Y) = 0$. [Note: converse is not true!]
3. $\mathrm{Cov}(X, Y) = \mathrm{Cov}(Y, X)$
4. $\mathrm{Cov}(aX, Y) = a\,\mathrm{Cov}(X, Y)$
5. $\mathrm{Cov}(X + c, Y) = \mathrm{Cov}(X, Y)$
6. $\mathrm{Cov}(X + Y, Z) = \mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z)$
7. More generally, we have

» Properties of Covariance...

Properties of Covariance

1. $\mathrm{Cov}(X, X) = \mathrm{Var}(X)$
2. If X and Y are independent, then $\mathrm{Cov}(X, Y) = 0$. [Note: converse is not true!]
3. $\mathrm{Cov}(X, Y) = \mathrm{Cov}(Y, X)$
4. $\mathrm{Cov}(aX, Y) = a\,\mathrm{Cov}(X, Y)$
5. $\mathrm{Cov}(X + c, Y) = \mathrm{Cov}(X, Y)$
6. $\mathrm{Cov}(X + Y, Z) = \mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z)$
7. More generally, we have

$$\mathrm{Cov}\left(\sum_{i=1}^{m} a_i X_i, \sum_{j=1}^{n} b_j Y_j\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_i b_j\,\mathrm{Cov}(X_i, Y_j)$$

Combining 4 & 6 inductively

» Solved Example...

Example (Solved example)

Let X and Y be two independent random variables following standard normal distribution, and

$$Z = 1 + X + XY^2$$
$$W = 1 + X$$

Find $\mathrm{Cov}(Z, W)$.

Solution

Prop 5.3

$\mathrm{Cov}(Z, W) = \mathrm{Cov}(1+X+XY^2, 1+X) = \mathrm{Cov}(X+XY^2, X)$

$= \mathrm{Cov}(X, X) + \mathrm{Cov}(XY^2, X)$ (Prop 6)

$= \mathrm{Var}(X) + E[X^2Y^2] - E[XY^2]E[X]$ (Since X, Y ind & $X \sim N(0,1)$)

$= 1 + E[X^2]E[Y^2] - E[X]^2E[Y^2]$

$= 1 + 1 \cdot 1 - 0 \cdot 1 = 1 + 1 = \underline{\underline{2}}$

» Variance of a Sum...

» Variance of a Sum...

Example (Variance of a Sum)

Let $Z = X + Y$, then

» Variance of a Sum...

Example (Variance of a Sum)

Let $Z = X + Y$, then

$$\mathrm{Var}(Z) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X, Y)$$

» Variance of a Sum...

Example (Variance of a Sum)

Let $Z = X + Y$, then

$$\mathsf{Var}(Z) = \mathsf{Var}(X) + \mathsf{Var}(Y) + 2\mathsf{Cov}(X, Y)$$

More generally, for $a, b \in \mathbb{R}$, and $Z = aX + bY$, we have

Example (Variance of a Sum)

Let $Z = X + Y$, then

$$\mathrm{Var}(Z) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X, Y)$$

More generally, for $a, b \in \mathbb{R}$, and $Z = aX + bY$, we have

$$\mathrm{Var}(Z) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y) + 2ab\,\mathrm{Cov}(X, Y)$$

Try this

Solution

$$\mathrm{Var}(Z) = \mathrm{Cov}(Z, Z) = \mathrm{Cov}(X+Y, X+Y)$$

$$= \mathrm{Cov}(X, X) + \mathrm{Cov}(X, Y) + \mathrm{Cov}(Y, X) + \mathrm{Cov}(Y, Y)$$

$$= \mathrm{Var}(X) + 2\,\mathrm{Cov}(X, Y) + \mathrm{Var}(Y)$$

» Correlation Coefficient...

» Correlation Coefficient...

Correlation coefficient

The correlation coefficient ρ_{XY} or $\rho(X, Y)$ is defined as follows

» Correlation Coefficient...

Correlation coefficient

The correlation coefficient ρ_{XY} or $\rho(X, Y)$ is defined as follows

$$\rho_{XY} = \rho(X, Y) = \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}} = \frac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

Correlation coefficient

The correlation coefficient ρ_{XY} or $\rho(X, Y)$ is defined as follows

$$\rho_{XY} = \rho(X, Y) = \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}} = \frac{\mathrm{Cov}(X, Y)}{\rho_X\,\rho_Y}$$

* Given two RVs X and Y, define standardized versions of these as follows

Correlation coefficient

The correlation coefficient ρ_{XY} or $\rho(X, Y)$ is defined as follows

$$\rho_{XY} = \rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\,\text{Var}(Y)}} = \frac{\text{Cov}(X, Y)}{\rho_X\,\rho_Y}$$

* Given two RVs X and Y, define standardized versions of these as follows

$$U = \frac{X - E[X]}{\rho_X}, \quad V = \frac{Y - E[Y]}{\rho_Y}$$

» Correlation Coefficient...

Correlation coefficient

The correlation coefficient ρ_{XY} or $\rho(X, Y)$ is defined as follows

$$\rho_{XY} = \rho(X, Y) = \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}} = \frac{\mathrm{Cov}(X, Y)}{\rho_X\, \rho_Y}$$

* Given two RVs X and Y, define standardized versions of these as follows

$$U = \frac{X - E[X]}{\rho_X}, \quad V = \frac{Y - E[Y]}{\rho_Y}$$

* With this by computing the covariance $\mathrm{Cov}(U, V)$, we have

Correlation coefficient

The correlation coefficient ρ_{XY} or $\rho(X, Y)$ is defined as follows

$$\rho_{XY} = \rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\,\text{Var}(Y)}} = \frac{\text{Cov}(X, Y)}{\rho_X\, \rho_Y}$$

* Given two RVs X and Y, define standardized versions of these as follows

$$U = \frac{X - E[X]}{\rho_X}, \quad V = \frac{Y - E[Y]}{\rho_Y}$$

$$\frac{X}{\rho_X} - \frac{E[X]}{\rho_X}$$

const

* With this by computing the covariance Cov(U, V), we have

$$\rho_{XY} = \text{Cov}(U, V) = \text{Cov}\left(\frac{X - E[X]}{\rho_X}, \frac{Y - E[Y]}{\rho_Y}\right)$$

$$= \text{Cov}\left(\frac{X}{\rho_X}, \frac{Y}{\rho_Y}\right) = \frac{\text{Cov}(X, Y)}{\rho_X \rho_Y} = \text{Correlation coeff.}$$

» Properties of Correlation Coefficient...

» Properties of Correlation Coefficient...

Properties of correlation coefficient

Let X, Y be two RVs.

» Properties of Correlation Coefficient...

Properties of correlation coefficient

Let X, Y be two RVs. These *are* some properties of correlation coefficient

» Properties of Correlation Coefficient...

Properties of correlation coefficient

Let X, Y be two RVs. These are some properties of correlation coefficient

1. $-1 \leq \rho(X, Y) \leq 1$

» Properties of Correlation Coefficient...

Properties of correlation coefficient

Let X, Y be two RVs. These *are* some properties of correlation coefficient

1. $-1 \leq \rho(X, Y) \leq 1$
2. If $\rho(X, Y) = 1$,

» Properties of Correlation Coefficient...

Properties of correlation coefficient

Let X, Y be two RVs. These are some properties of correlation coefficient

1. $-1 \leq \rho(X, Y) \leq 1$
2. If $\rho(X, Y) = 1$, then $Y = aX + b$, where $a > 0$

» Properties of Correlation Coefficient...

Properties of correlation coefficient

Let X, Y be two RVs. These are some properties of correlation coefficient

1. $-1 \leq \rho(X, Y) \leq 1$
2. If $\rho(X, Y) = 1$, then $Y = aX + b$, where $a > 0$
3. If $\rho(X, Y) = 1$,

» Properties of Correlation Coefficient...

Properties of correlation coefficient

Let X, Y be two RVs. These are some properties of correlation coefficient

1. $-1 \leq \rho(X, Y) \leq 1$
2. If $\rho(X, Y) = 1$, then $Y = aX + b$, where $a > 0$
3. If $\rho(X, Y) = 1$, then $Y = aX + b$, where $a < 0$

» Properties of Correlation Coefficient...

Properties of correlation coefficient

Let X, Y be two RVs. These are some properties of correlation coefficient

1. $-1 \leq \rho(X, Y) \leq 1$
2. If $\rho(X, Y) = 1$, then $Y = aX + b$, where $a > 0$
3. If $\rho(X, Y) = 1$, then $Y = aX + b$, where $a < 0$
4. $\rho(aX + b, cY + d) = \rho(X, Y)$ for $a, c > 0$

check

Exercise

» Answer to previous problem...

» Positive Correlation, Negative Correlation, Uncorrelation...

» Positive Correlation, Negative Correlation, Uncorrelation...

Definition of positive, negative correlation

Let X and Y be two RVs.

Definition of positive, negative correlation

Let X and Y be two RVs.

1. If $\rho(X, Y) = 0$, we say that X and Y are uncorrelated

» Positive Correlation, Negative Correlation, Uncorrelation...

Definition of positive, negative correlation

Let X and Y be two RVs.

1. If $\rho(X, Y) = 0$, we say that X and Y are uncorrelated
2. If $\rho(X, Y) > 0$, we say that X and Y are positively correlated

» Positive Correlation, Negative Correlation, Uncorrelation...

Definition of positive, negative correlation

Let X and Y be two RVs.

1. If $\rho(X, Y) = 0$, we say that X and Y are uncorrelated
2. If $\rho(X, Y) > 0$, we say that X and Y are positively correlated
3. If $\rho(X, Y) < 0$, we say that X and Y are negatively correlated

» Positive Correlation, Negative Correlation, Uncorrelation...

Definition of positive, negative correlation

Let X and Y be two RVs.

1. If $\rho(X, Y) = 0$, we say that X and Y are uncorrelated
2. If $\rho(X, Y) > 0$, we say that X and Y are positively correlated
3. If $\rho(X, Y) < 0$, we say that X and Y are negatively correlated

Pairwise uncorrelation and Variance

If X and Y are uncorrelated, then

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) \qquad + \cancel{\text{Cov}(X, Y)}^{\,0}$$

More generally, if $X_1, X_2, \ldots, X_n$, are pairwise uncorrelated, i.e., $\rho(X_i, X_j) = 0$ when $i \neq j$, then

Definition of positive, negative correlation

Let X and Y be two RVs.

1. If $\rho(X, Y) = 0$, we say that X and Y are uncorrelated
2. If $\rho(X, Y) > 0$, we say that X and Y are positively correlated
3. If $\rho(X, Y) < 0$, we say that X and Y are negatively correlated

Pairwise uncorrelation and Variance

If X and Y are uncorrelated, then

$$\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$$

More generally, if $X_1, X_2, \ldots, X_n$, are pairwise uncorrelated, i.e., $\rho(X_i, X_j) = 0$ when $i \neq j$, then

$$\mathrm{Var}(X_1 + X_2 + \cdots + X_n) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \cdots + \mathrm{Var}(X_n)$$

0

$+ \mathrm{Cov}(X_1, X_2) + \mathrm{Cov}(X_1, X_3) + \cdots$

0