

Probability and Statistics: Lecture-4

Monsoon-2020

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<https://tinyurl.com/y2g93ofb>

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- * Since the candies to be distributed are identical, **ordering doesn't matter**

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- * Since all the candies are same, **repetitions are allowed**
- * Since the candies to be distributed are identical, **ordering doesn't matter**
- * This is a case of **combinations with repetitions**

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» Distributing Candies Among Kids: Solution Finally!

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Analyzing the problem, and fitting it into the known case

- * Number of combinations = size of combination
- * Number of options = Number of kids
- * The answer is $\binom{15 + (7 - 1)}{(7 - 1)} = \binom{21}{3} = 54264$

» Fair Distributions..

A problem with previous distribution...

No kid will like not having even a single candy. Infact, a kid would like to have it all!
How can we change the problem?

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» Fair Distributions..

A problem with previous distribution...

We want that each kid receives *atleast* one candy!



Let us change our previous problem so that each kid is ensured *atleast one* candy!

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Question

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- * Does our previous approach work here? What should you do?

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Quiz

- * Does our previous approach work here? What should you do?
- * Can we reduce this problem to previous problem? If yes, how?

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- * Give kids atleast one candy. Then we are left with $15-7=8$ candies...

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 - * #combinations = 8; #options = 7

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- * Now we can distribute these 8 candies as in previous problem!
- * It becomes a problem of **Combinations with Repetitions**
 - * #combinations = 8; #options = 7
- * Answer = $\binom{8 + (7 - 1)}{(7 - 1)} = \binom{14}{6} = 3003$

» **How many kids can have no candies?**

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Question

Assume that 15 identical candies are distributed among 7 kids. How many ways are there that will leave some kids without any candy?

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Hint

Use the answer to previous two problems!

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- * But what next? How many options for the 2nd digit?

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Problem

How many non-negative integer numbers are there below 10 000 such that their sum of digits is equal to 9?

Understanding the problem...

- * It is clear that we have nine options for the 1st digit
- * But what next? How many options for the 2nd digit?
- * It is already tricky...

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- * This belongs to the category: “unordered with repetitions”

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- * This belongs to the category: “unordered with repetitions”
- * #combinations = 9; #options = 4

Problem

How many non-negative integer numbers are there below 10 000 such that their sum of digits is equal to 9?

- * Idea: How many ways we can distribute 9 ones in 4 places?
- * Multiple ones will go to at least one of the four places, and get added
- * This belongs to the category: “unordered with repetitions”
- * #combinations = 9; #options = 4
- * Hence, answer is $\binom{9 + (4 - 1)}{(4 - 1)} = \binom{12}{3} = 220$