Tutorial 6

Probability and Statistics

Question 1: Binomial MGF using Bernoulli

Derive the MGF for Binomial distribution indirectly, i.e., using the MGF for Bernoulli.

Start by deriving the MGF of Bernoulli from definition.

Question 2: Poisson Distribution

A train bridge is constructed across a wide river. Trains arrive at the bridge according to a Poisson process of rate $\lambda = 3$ per day.

(a) If a train arrives on day 0, find the probability that there will be no trains on days 1, 2, and 3.

(b) Find the probability that the next train to arrive after the first train on day 0, takes more than 3 days to arrive.

(c) Find the probability that no trains arrive in the first 2 days, but 4 trains arrive on the 4th day.

(d) Find the probability that it takes more than 2 days for the 5th train to arrive at the bridge

Question 3: Exponential Distribution

Beginning at time $t = 0$, we begin using bulbs, one at a time, to illuminate a room. Bulbs are replaced immediately upon failure. Each new bulb is selected independently by an equally likely choice between a type-A bulb and a type-B bulb. The lifetime, X, of any particular bulb of a particular type is a random variable, independent of everything else, with the following PDF:

$$\text{for type-A Bulbs:} \quad f_X(x) = \begin{cases} e^{-x}, & x \geq 0, \\ 0, & \text{otherwise;} \end{cases}$$

$$\text{for type-B Bulbs:} \quad f_X(x) = \begin{cases} 3e^{-3x}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the expected time until the first failure.

(b) Find the probability that there are no bulb failures before time t.

(c) Given that there are no failures until time t, determine the conditional probability that the first bulb used is a type-A bulb.