Probability and Statistics: Lecture-16

Monsoon-2020

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on September 11, 2020

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1. Quiz

2. Special Distributions ← Discrete
   * Uniform Distribution
   * Bernoulli Distribution
   * Geometric Distribution
   * Binomial Distribution

PMF ← distribution

CDF

→ You may login to Skype for alternative chat

Attend the Quiz on Gradescope!

(1) Log in to gradescope with IIIT email

(2) Submit the Quiz: Online quiz-1.

(3) Time limit is 5 min.

Quiz starts at ~~9:33~~ 9:33 AM

» Motivation for Uniform Distribution: Distribution of a Die Roll...

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Example: Motivation for Uniform Distribution

Consider rolling a fair die. The possible outcomes are X $\{1, 2, 3, 4, 5, 6\}$. Then the PMF is given by

Uniform value

$$\frac{1}{6} + \frac{1}{6} + \cdots + \frac{1}{6} = 1$$

$$p(x) = \begin{cases} \frac{1}{6}, & x \in \{1, 2, 3, 4, 5, 6\} \\ 0, & otherwise \end{cases} \qquad \sum p($$

We note here that $\sum_{x \in \mathbb{Z}} p(x) = 1$. We note here that PMF takes uniform values for all values of $X = x$.

» Uniform Distribution...

Definition: Uniform Distribution

Motivated from the previous example, we now define uniform distribution on $\{1, 2, \ldots, n\}$ by

$$p(x) = \begin{cases} \frac{1}{n}, & x \in \{1, 2, \ldots, n\} \\ 0, & \textit{otherwise} \end{cases}$$

We verify here that $\sum_{x \in \mathbb{Z}} p(x) = 1$.

» Bernoulli Distribution

unfair coin p H

$1-p$ T

Bernoulli distribution

A random variable X is called a **Bernoulli random variable** with parameter p, denoted by $X \sim Bernoulli(p)$, if its **PMF** is given by

$$P_X(x) = \begin{cases} p & \text{for } x = 1, \\ 1-p & \text{for } x = 0, \\ 0 & \text{otherwise,} \end{cases}$$

H, Pass, Success

T, Fail, Failure

where $0 < p < 1$.

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* This models random experiments that have two possible outcomes
* Example: You take a pass-fail exam. You either pass or fail
* Example: A coin is tossed, the outcome is either heads or tails

» Geometric Distribution

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Definition of Geometric Distribution

A random variable X is called geometric random variable with parameter p, denoted by $X \sim Geometric(p)$, if its PMF is given by

$$P_X(k) = \begin{cases} p(1-p)^{k-1}, & \text{for } k = 1, 2, 3, \dots \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

first H at k^{th} toss

$(1-p)^{k-1}\, p$

$k-1$ $\quad$ k^{th}

an event that happen for the 1st time at k^{th} trial

$p \quad (1-p)$

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k

k−1 failure

Success at kth

where $0 < p < 1$.

* Example: Suppose we have an unfair coin with $P(H) = p$. We toss the coin until we obtain first heads. Let RV X be the total number of tosses. Then X have geometric distribution.
* Caution: Some books define geometric random variable X as total number of failures before observing first success. Then

$X = k$

k

k+1 success

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where $0 < p < 1$.

p^k

$(1-p)^{n-k}$

k heads in n-trials

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* Example: Consider an unfair coin with $P(H) = p$. Consider tossing the coin n times and let X be the total number of heads we observe. Then X is a Binomial with parameters n and p.

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$$Ber(p) \begin{cases} p, & x = 1 \\ 1-p, & x = 0 \end{cases}$$

1 coin toss

where $0 < p < 1$.

- Example: Consider an unfair coin with $P(H) = p$. Consider tossing the coin n times and let X be the total number of heads we observe. Then X is a Binomial with parameters n and p.
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* **Example:** Consider an unfair coin with $P(H) = p$. Consider tossing the coin n times and let X be the total number of heads we observe. Then X is a Binomial with parameters n and p.
* Binomial(n, p) is a sum of n independent Bernoulli(p) random variables.
* If $X_1, X_2, \ldots, X_n$ are independent Bernoulli(p) random variables, then $X = X_1 + \cdots + X_n$ has ~~Bernoulli~~(n, p) distribution.

  Binomial

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n k

$$P_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{for } k = 0, 1, 2, \cdots, n \\ 0 & \text{otherwise,} \end{cases}$$

$X_1 + X_2 \sim X \sim$ Binomial$(2, p)$

where $0 < p < 1$.

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* Binomial(n, p) is a sum of n independent Bernoulli(p) random variables.
* If $X_1, X_2, \ldots, X_n$ are independent Bernoulli(p) random variables, then $X = X_1 + \cdots + X_n$ has ~~Bernoulli~~ Binomial(n, p) distribution.
* We verify that $\sum_{x \in \mathbb{Z}} p(x) = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = 1$ check

» Example

n - ind. Bernoulli

m - ind Bernoulli

Example

Let $X \sim Binomial(n, p)$ and $Y \sim Binomial(m, p)$ be two independent random variables. We define a random variable $Z = X + Y$. What is the PMF of Z?

$\Rightarrow X = X_1 + X_2 + \dots + X_n$, where X_i's are Bernoulli

$\Rightarrow Y = Y_1 + Y_2 + \dots + Y_m$, where Y_i's are Bernoulli

$$Z = X + Y = \underbrace{X_1 + X_2 + X_3 + \dots + X_n + Y_1 + \dots + Y_m}_{\text{Sum of } m+n \text{ ind. Bernoulli r.v.}}$$

$= Binomial(m+n, p)$

» Scratch Space...

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» Motivation for Poisson Distribution...

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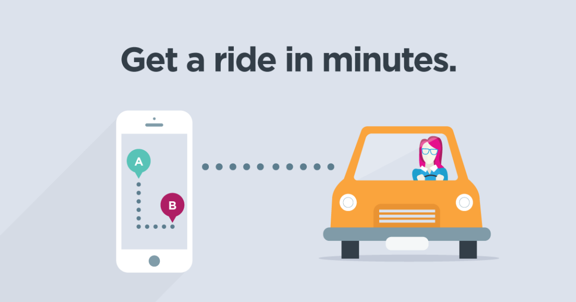

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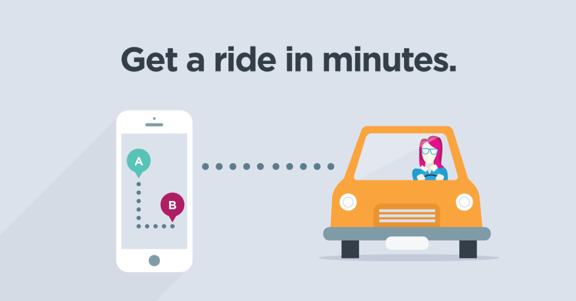

Example

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1st 2nd 3rd 5 60th

1 minute

0	0	1	0	1	...	...	0	1	0	1	1	0

60 second

0, or 1

in Binomial

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  * break a minute into 60 seconds;

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Binomial (k,

E(X)

  * break a minute into 60 seconds; each second is independent trial with request or no request
  * Let X = number of requests in a minute
  * $E[X] = \lambda = 5 = np$, where p is the probability of request

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0	0	1	0	1	...	...	0	1	0	1	1	0

60 bins

  - break a minute into 60 seconds; each second is independent trial with request or no request
  - Let X = number of requests in a minute
  - $E[X] = \lambda = 5 = np$, where p is the probability of request

5 hts

50

- We now identify this problem as $X \sim Binomial(n = 60, p = 5/60)$

$p = 5/n$

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n trials = 60

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- $P(X = k) = \binom{60}{k} \left(\frac{5}{60}\right)^k \left(1 - \frac{5}{60}\right)^{n-k}$

» Motivation for Possion Distribution...

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* What should we do next?

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* But there is still no guarantee that there will be only one request per millisecond
* What should we do next? Here comes calculus...

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Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
* $P(X = k) = \binom{60}{k}\binom{5}{60}^{k}\left(1 - \dfrac{5}{60}\right)^{n-k}$
* There is a problem! What is that? Time is continuous!
* Since time is continuous, no guarantee that there will be only one request per second
* What do we do now? We can make the interval smaller, take time intervals as milliseconds?
* But there is still no guarantee that there will be only one request per millisecond
* What should we do next? Here comes calculus... to go from discrete to continuous...

» Binomial in the Limit is Poisson Distribution...

» Binomial in the Limit is Poisson Distribution...

Derivation:

$$P(X=k) = \lim_{n \to \infty} \binom{n}{k} \left(\frac{\lambda}{n}\right)^k \left(1 - \frac{\lambda}{n}\right)^{n-k}$$

$$= \lim_{n \to \infty} \frac{n!}{k!\,(n-k)!} \frac{\lambda^k}{n^k} \frac{\left(1 - \lambda/n\right)^n}{\left(1 - \lambda/n\right)^k}$$

$\lim n \to \infty$

$$= \lim_{n \to \infty} \frac{n\,(n-)\cdots(n-k+1)}{n^k} \frac{(n-k)!}{(n-k)!} \frac{\lambda^k}{k!} \cdot \frac{e^{-\lambda}}{\left(1 - \frac{\lambda}{n}\right)^k}$$

$$= \lim_{n \to \infty} \frac{n^k}{n^k} \cdot \frac{\lambda^k}{k!} e^{-\lambda} = \frac{\lambda^k}{k!} e^{-\lambda}$$

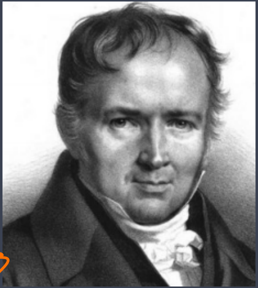
» Definition of Poisson Distribution...

» Definition of Poisson Distribution...

Definition of Poisson

A random variable X is said to be a Poisson random variable with parameter λ, shown as $X \sim Poisson(\lambda)$, if its range is $R_X = \{0, 1, 2, \ldots,\}$, and its PMF is given by

$$P_X(k) \begin{cases} \frac{e^{-\lambda}\lambda^k}{k!}, & k \in R_X \\ 0 & \text{otherwise} \end{cases}$$



* Simeon-Denis Poisson, was a French mathematician (1781-1840)
* He published his first paper at 18, professot at 21
* He published over 300 papers