

Probability and Statistics: Lecture-36

Monsoon-2020

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on November 6, 2020

» Solved Example

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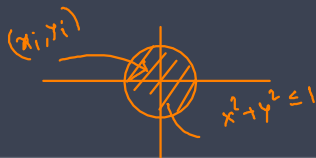
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The joint PDF of X and Y is given by

$$f_{XY}(x, y) = \begin{cases} c & (x, y) \in D \\ 0 & \text{otherwise} \end{cases}$$

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Are X and Y **uncorrelated**? $\rightarrow \text{Cov}(X, Y) = 0$?

$$\text{Cov}(X, Y) = \underline{\underline{E[XY]}} - \underline{\underline{E[X]}} \underline{\underline{E[Y]}}$$

» Answer to previous problem...

Earlier we found that X & Y are not independent &

$$X|Y \sim \text{Uniform}(-\sqrt{1-Y^2}, \sqrt{1-Y^2}) \Rightarrow f_{XY} = 0$$

$\Rightarrow X, Y$ uncorrelated.

We want

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y]$$

$$E[X] = E[E[X|Y]] \quad [L.O.T.E.]$$

$$= 0 \quad \text{Since } X|Y \sim \text{Uniform}(\dots)$$

$$E[XY] = E[E[XY|Y]] = E[Y E[X|Y]] = 0$$

$$\Rightarrow \text{Cov}(X, Y) = E[XY] - E[X]E[Y] = 0$$

» Bivariate Normal Distribution...

Example (Sum of Two Normal Distribution May Not be Normal)

Let RVs $X \sim N(0, 1)$ and $W \sim \text{Bernoulli}\left(\frac{1}{2}\right)$ be two **independent** RVs.

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Find the **PDF** of Y

» Answer to previous problem...

Due to symmetry, if $N(0,1)$ about zero, $-X$ is also a $N(0,1)$.

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &\stackrel{\uparrow}{=} P(Y \leq y | W=0) P(W=0) \\ &\quad + P(Y \leq y | W=1) P(W=1) \\ &= \frac{1}{2} P(Y \leq y | W=0) + \frac{1}{2} P(Y \leq y | W=1) \\ &= \frac{1}{2} P(X \leq y) + \frac{1}{2} (-X \leq y) \end{aligned}$$

$$= \frac{1}{2} \bar{\Phi}(y) + \frac{1}{2} \Phi(y)$$

$$= \Phi(y)$$

\uparrow CDF of Std. Normal.

$$\Rightarrow Y \sim N(0,1)$$

$\underline{\underline{=}}$

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5. If $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$ are **jointly normal**, then

$$\underline{X + Y} \sim N(\underbrace{\mu_X + \mu_Y}, \underbrace{\sigma_X^2 + \sigma_Y^2 + 2\rho(X, Y)\sigma_X\sigma_Y})$$

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6. Can we provide a simple way to generate jointly normal random variables?

» Bivariate Normal Distribution and Properties...

Handwritten notes:

- $N(0, 1)$ is circled.
- $X \sim N(\mu_X, \sigma_X^2)$ is written.
- $Z = \frac{X - \mu_X}{\sigma_X}$ is boxed.
- $X = \sigma_X^2 Z + \mu_X$ is written in a box.

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$$X + Y \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2 + 2\rho(X, Y)\sigma_X\sigma_Y)$$

6. Can we provide a simple way to generate jointly normal random variables?
7. We first introduce standard bivariate normal distribution

» Example of Bivariate Normal...

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Let Z_1 and Z_2 be two independent $N(0, 1)$ RVs.

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$$X = Z_1$$

$$Y = \rho Z_1 + \sqrt{1 - \rho^2} Z_2,$$

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where ρ is a real number in $(-1, 1)$.

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2. What is the **joint PDF** of X and Y ?

» Example of Bivariate Normal...

$$g: \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \end{pmatrix}$$

Example (Bivariate Normal Variable)

Let Z_1 and Z_2 be two **independent** $N(0, 1)$ RVs. We define

$$\begin{cases} X = Z_1 \\ Y = \rho Z_1 + \sqrt{1 - \rho^2} Z_2, \end{cases}$$

where ρ is a real number in $(-1, 1)$.

1. Is X and Y **bivariate normal**? ✓
2. What is the **joint PDF** of X and Y ?
3. Find $\rho(X, Y)$

» Answer to previous problem...

Note that z_1 and z_2 are normal and independent,

$$\begin{aligned}\underline{f_{z_1 z_2}(z_1, z_2)} &= f_{z_1}(z_1) f_{z_2}(z_2) \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{z_1^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{z_2^2}{2}} \\ &= \frac{1}{2\pi} e^{-\frac{1}{2}(z_1^2 + z_2^2)} \quad \leftarrow\end{aligned}$$

(a) Need to show that $ax + by$ is normal $\forall a, b \neq 0$

$$\begin{aligned}ax + by &= az_1 + b(\rho z_1 + (1-\rho^2)z_2) \\ &= \underbrace{(a + b\rho)}_{\text{linear combination of } z_1 \text{ and } z_2} z_1 + \underbrace{b(1-\rho^2)}_{\text{linear combination of } z_1 \text{ and } z_2} z_2 \\ &\text{which is a linear combination of } z_1 \text{ and } z_2 \Rightarrow \text{it is } \underline{\text{normal}}.\end{aligned}$$

» Answer to previous problem...

b) Using the method of transform.
to find the joint PDF of X & Y .

(i) Inverse:

$$z_1 = X \equiv h_1(x, y)$$

$$z_2 = \frac{-\rho}{\sqrt{1-\rho^2}} X + \frac{Y}{\sqrt{1-\rho^2}} \equiv h_2(x, y)$$

Recall:

$$f_{XY}(z_1, z_2) = f_{z_1 z_2}(h_1(x, y), h_2(x, y)) \bigg| \frac{1}{|J|} \Rightarrow$$

$\longrightarrow \textcircled{*}$

$$J = \det \begin{bmatrix} \frac{\partial h_1}{\partial x} & \frac{\partial h_1}{\partial y} \\ \frac{\partial h_2}{\partial x} & \frac{\partial h_2}{\partial y} \end{bmatrix}$$

$$= \det \begin{bmatrix} 1 & 0 \\ \frac{-\rho}{\sqrt{1-\rho^2}} & \frac{1}{\sqrt{1-\rho^2}} \end{bmatrix}$$

$$= \frac{1}{\sqrt{1-\rho^2}}$$

$$f_{XY}(x, y) = \frac{1}{2\pi} \exp \left\{ -\frac{1}{2} \left[x^2 + \frac{1}{1-\rho^2} (-\rho x + y)^2 \right] \right\} \cdot \frac{1}{\sqrt{1-\rho^2}}$$

» Standard Bivariate Normal Distribution...

$$f_{XY}(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}[x^2 - 2\rho xy + y^2]\right\}$$

If $\rho = 0 \Rightarrow f_{XY}(x,y) = \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2)}$

↗ This is a joint PDF of std. Normal

» Standard Bivariate Normal Distribution...

Definition of Standard Bivariate Normal Distribution

Two RVs X and Y are said to have the **standard bivariate normal distribution** with correlation coefficient ρ if their joint PDF is given by

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$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y]$$

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where $\rho \in (-1, 1)$. If $\rho = 0$, then we call X and Y to have standard ~~normal~~ bivariate normal distribution *with $\rho = 0$*

» Definition of Bivariate Normal Distribution...

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$$N(0,1) \rightarrow N(\mu_X, \sigma_X^2)$$

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Two random variables X and Y are said to have a **bivariate normal distribution** with parameters $\mu_X, \sigma_X^2, \mu_Y, \sigma_Y^2$, and ρ ,

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$$X \sim N(\mu_X, \sigma_X^2) \quad X = \sigma_X z_1 + \mu_X$$
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$$f_{XY} = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \cdot$$

$$\exp \left\{ -\frac{1}{2(1-\rho^2)} \left[\left(\frac{x-\mu_X}{\sigma_X} \right)^2 + \left(\frac{y-\mu_Y}{\sigma_Y} \right)^2 - 2\rho \frac{(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} \right] \right\},$$

$$z_1 = \frac{x-\mu_X}{\sigma_X}, \quad z_2 = \frac{y-\mu_Y}{\sigma_Y}$$

z_1, z_2 are $N(0,1)$.

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where $\mu_X, \mu_Y \in \mathbb{R}, \sigma_X, \sigma_Y > 0$ and $\rho \in (-1, 1)$ are all constant.

» Creating a Jointly Normal Distribution...

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Creation of Jointly Normal Random Variables

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If we want two **jointly normal** random variables X and Y such that

$$\underbrace{X \sim N(\mu_X, \sigma_X^2)}, \quad \underbrace{Y \sim N(\mu_Y, \sigma_Y^2)}, \quad \text{and} \quad \underbrace{\rho(X, Y) = \rho}$$

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we start with two **independent standard normal** RVs Z_1 and Z_2 and define

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we start with two independent standard normal RVs Z_1 and Z_2 and define

$$\begin{cases} X = \sigma_X Z_1 + \mu_X \\ Y = \sigma_Y(\rho Z_1 + \sqrt{1 - \rho^2} Z_2) + \mu_Y \end{cases}$$

and follow the above procedure: solve for Z_1, Z_2 , and apply method of transformation