# Probability and Statistics: Lecture-8

## Monsoon-2020

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on August 26, 2020

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**Question**

Roll two 6-sided dice, yielding values $D_1$ and $D_2$. Let $E$ be the event $D_1 + D_2 = 4$. What is $P(E)$?

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$P(E|F) =$

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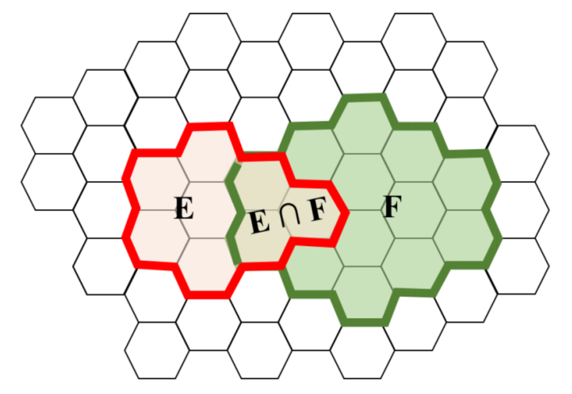
* Sample space is all possible outcomes consistent with $F$ (i.e., $S \cap F$)
* Event is all outcomes in $E$ consistent with $F$ (i.e., $E \cap F$)

With equally likely outcomes:

$$P(E|F) = \frac{|E \cap F|}{|S \cap F|} = \frac{|E \cap F|}{|F|}$$

## » Graphical Illustration/Example of Conditional Probability...




* **Question:** What is $P(E)$?

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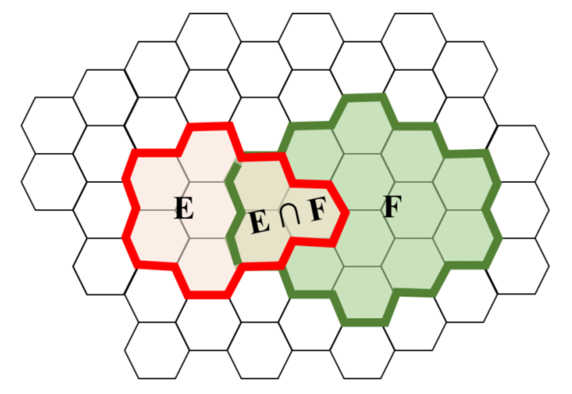


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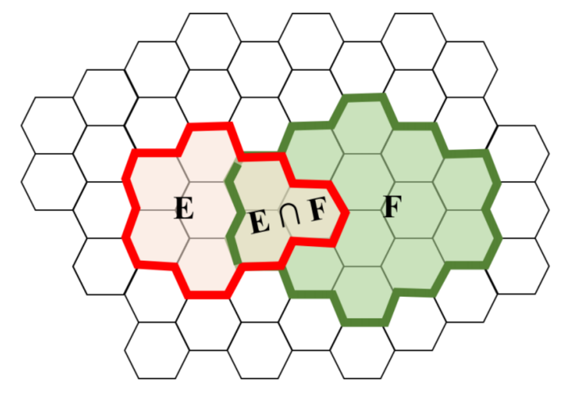


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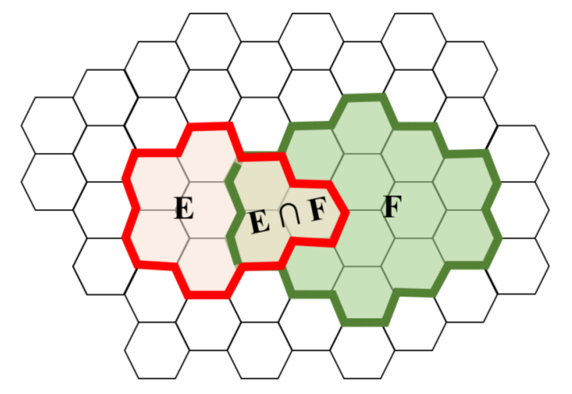


- Question: What is $P(E)$? Here $P(E) = \frac{8}{50} \approx 0.16$
- Question: What is $P(E \mid F)$? Here $P(E \mid F) = \frac{3}{14} \approx 0.21$

# » Probability of Receiving Spam Emails...

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**Email Spam Conditional Probability Problem**

24 emails are sent, 6 each to 4 users.

* 10 of the 24 emails are spam.
* All possible outcomes are equally likely.

**Question-1**

Let event $E$ = user 1 receives 3 spam emails. What is $P(E)$?

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* 10 of the 24 emails are spam.
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**Question-1**

Let event $E$ = user 1 receives 3 spam emails. What is $P(E)$?

**Question-2**

Let event $F$ = user 2 receives 6 spam emails. What is $P(E|F)$?

## » Law of Total Probability...

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## Conditional Probability Implies Chain Rule...

$$P(E|F) = \frac{P(E \cap F)}{P(F)} \implies P(E \cap F) = P(F)P(E|F)$$

These hold even when outcomes are not equally likely!

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$$P(E) = P(E \mid F)P(F) + P(E|F^c)P(F^c)$$

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### Law of Total Probability (Theorem)

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Proof:

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You win if you roll a 6. What is P(winning)?

Solution using probability tree:

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Solution using **total probability:**