

# Probability and Statistics: Lecture-9

Monsoon-2020

by Pawan Kumar (IIIT, Hyderabad)

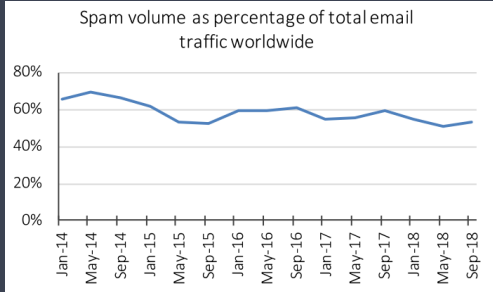
on August 28, 2020

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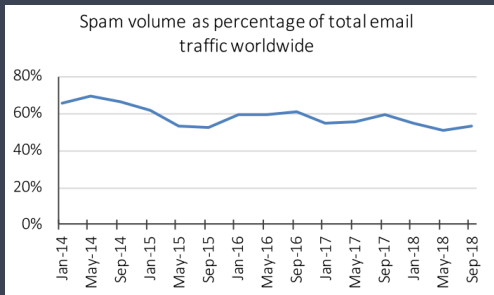
### 1. Conditional Probability, Bayes Theorem

### 2. The Monty Hall Problem

## » Bayes Theorem. Why?



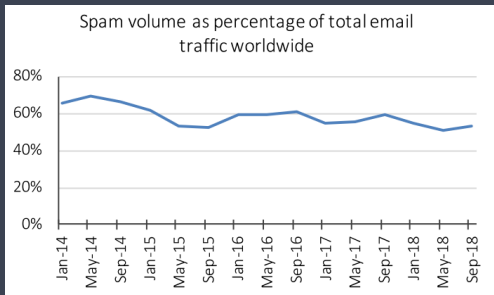
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$$P(E|F) = P(\text{Dear}|\text{Spam})$$

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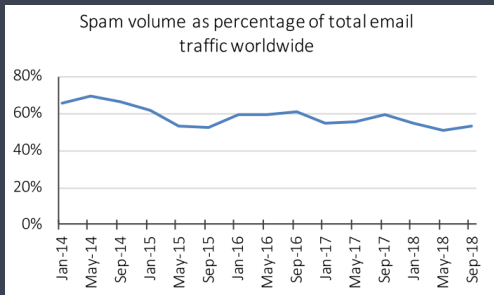
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## » Bayes Theorem

### Bayes Theorem

For any events  $E$  and  $F$  where  $P(E) > 0$  and  $P(F) > 0$ ,

$$P(F|E) = \frac{P(E|F)P(F)}{P(E)}$$

Proof of Bayes Theorem:



## » Bayes Theorem with Total Probability...

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### Bayes Theorem

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$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^c)P(F^c)}$$

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### Spam Email Example

Given the following:

- \* 60% of all email in 2016 is spam
- \* 20% of spam has the word “Dear”
- \* 1% of non-spam (aka ham) has the word “Dear”

You get an email with the word “Dear” in it. What is the probability that the email is spam?

Solution:



## » Application of Bayes Theorem...

### Example

A test is 98% effective at detecting a disease (“true positive”). However, the test has a “false positive” rate of 1%. The 0.5% of the US population has disease. What is the likelihood you have the disease, if you test positive?

Solution:





## » Conditional Probability and Game of Chance Movie...

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Movie Monty Hall Movie 21 Video Clip Here!

Another Monty Hall Youtube Movie Here!

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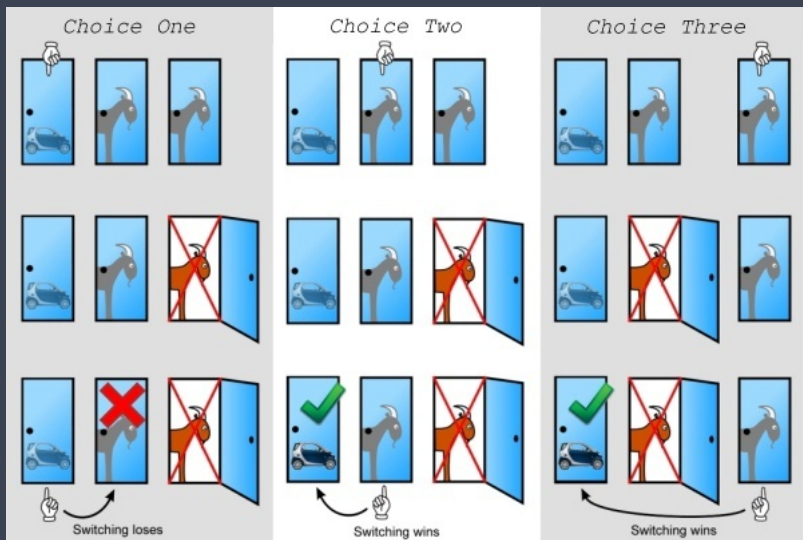
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- \* **Question:** if the host always opens goat door,

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  - \* then the host opens a door
- \* **Question:** if the host always opens goat door, is it wise to change your door?

## » Solution to Monty Hall Problem with Graphical Illustration

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Graphical illustration of Monty hall problem. Source: Google

## » Solution to Game Show: Choice Tree, Conditional Probability



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1	1	2/3	win	lose

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1	2	3	lose	win
1	3	2	lose	win
2	1	3	lose	win
2	2	$1/3$	win	lose
2	3	1	lose	win
3	1	2	lose	win
3	2	1	lose	win
3	3	$1/2$	win	lose

Exhaustive list of possibilities

### Conclusion

If you switch, the probability that you win a car is  $2/3$ ,

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Exhaustive list of possibilities

### Conclusion

If you switch, the probability that you win a car is  $2/3$ , and if you switch, the probability that you win goat is  $1/3$ .