Tutorial 3

Probability and Statistics

Problem 1

Generalized Monty Hall problem

There are N doors, with 1 car and N-1 goats. After your initial choice, the host opens p losing doors for you.

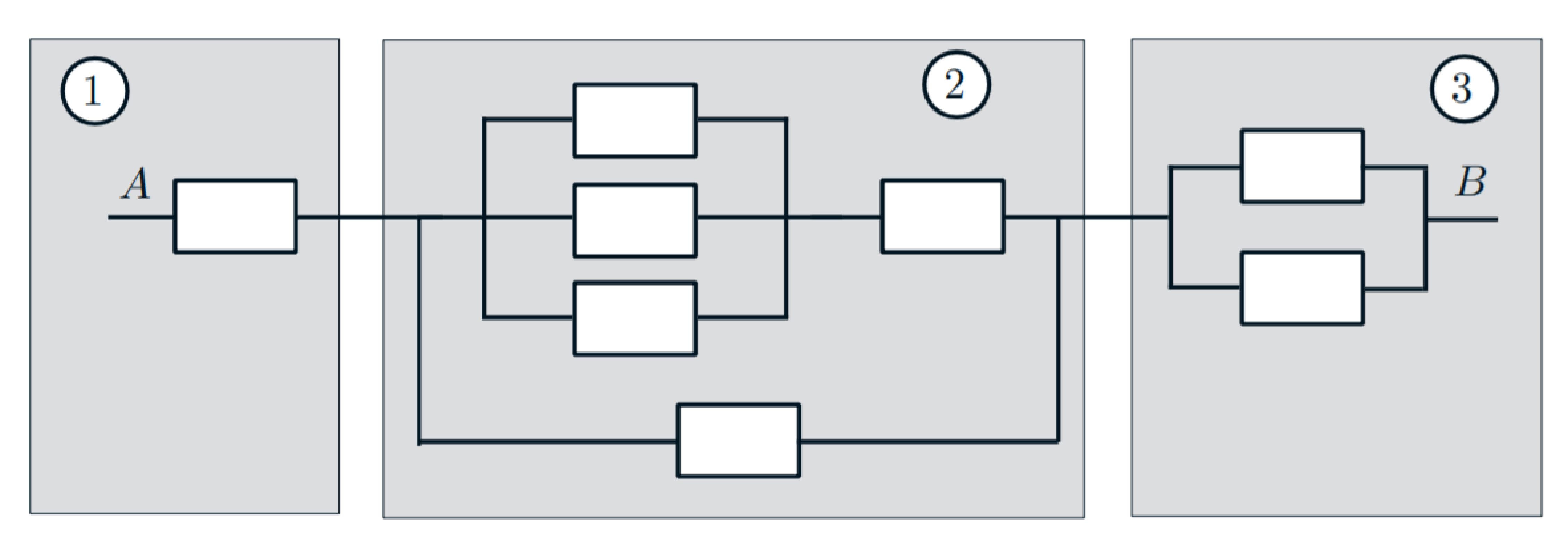
1. What is the probability of winning if you switch?

2. What is the probability of winning if you don't switch?

3. Should you always switch?

Problem 2

An electrical system consists of identical components that are operational with probability p independently of other components. The components are connected in three subsystems, as shown in the figure. The system is operational if there is a path that starts at point A, ends at point B, and consists of operational components. This is the same as requiring that all three subsystems are operational. What are the probabilities that the three subsystems, as well as the entire system, are operational?



Problem 3

The chess problem

This year Belmont chess championship is to be selected by the following procedure. B and C the leading challengers first play a two game match. If one of them wins both games, he gets to play a two-game second round with A, the current champion. A retains his championship unless a second round is required and the challenger beats A in both games. If A wins the initial game of the second round, no more games are played.

Furthermore we know the following:-

1. Probability that B will beat C in any particular game is 0.6
2. Probability that A will beat B in any particular game is 0.5
3. Probability that A will beat C in any particular game is 0.7

Assume no tie games are possible and all games are independent.

Problem 3 (Contd.)

(a) Determine the apriori probability that

1. A second round will be required.
2. B will win first round.
3. A will retain the championship this year.

(b) Given that a second round is required determine the conditional probability that

1. B is the surviving challenger.
2. A retains his championship.

(c) Given that the second round was required and that it comprised of only one game, what is the conditional probability that it was B who won the first round?

Problem 4

Most mornings, Victor checks the weather report before deciding whether to carry an umbrella. If the forecast is “rain,” the probability of actually having rain that day is 80%. On the other hand, if the forecast is “no rain,” the probability of it actually raining is equal to 10%. During fall and winter the forecast is “rain” 70% of the time and during summer and spring it is 20%.

1. One day, Victor missed the forecast and it rained. What is the probability that the forecast was “rain” if it was during the winter?
2. The probability of Victor missing the morning forecast is equal to 0.2 on any day in the year. If he misses the forecast, Victor will flip a fair coin to decide whether to carry an umbrella. On any day of a given season he sees the forecast, if it says “rain” he will always carry an umbrella, and if it says “no rain,” he will not carry an umbrella. Are the events “Victor is carrying an umbrella,” and “The forecast is no rain” independent? Does your answer depend on the season?
3. Victor is carrying an umbrella and it is not raining. What is the probability that he saw the forecast? Does it depend on the season?

Problem 5

Law of Large Numbers on Small Number of Samples

Let there be an urn that contains N balls out of which k *are* red and $N-k$ are blue ($k=0,1,\cdots,N$). The probability of any k is equal (I.e. all N+1 urns are equally likely to be chosen). We have chosen one such urn.

We pick balls from this urn. After the ball's color has been recorded, the ball is returned to the urn. Assume the red ball showed up s times and based on that observation predict the probability of the red ball showing up on the next trial.

Problem 6

11. Let[12] the probability p_n that a family has exactly n children be αp^n when $n \geq 1$, and $p_0 = 1 - \alpha p(1+p+p^2+\cdots)$. Suppose that all sex distributions of n children have the same probability. Show that for $k \geq 1$ the probability that a family has exactly k boys is $2\alpha p^k/(2-p)^{k+1}$.