Probability and Statistics: Lecture-16

Monsoon-2020

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» Example of Poisson Distribution...

$n >> 1$ Binomial (n, p)

Example

We want to send a bit string of length $n = 10^4$. The probability that a bit can be corrupted is $p = 10^{-6}$. What is the probability that the message will arrive uncorrupted?

$p << 1$

$$\begin{cases} n \to \infty \\ p \to \text{small} \end{cases}$$

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no corrupted bits

$$P(X = 0) = \frac{\lambda^0}{0!} e^{-\lambda}$$

$x=0$

$$= \frac{0.01^0}{0!} e^{-0.01}$$

$$\approx 0.9900498$$

99% probab.

10^4

* We could have solved this using Binomial $X \sim$ Binomial$(10^4, 10^{-6})$. This may have been harder to compute!

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$$
\begin{aligned}
P(X = 0) &= \frac{\lambda^i}{i!} e^{-\lambda} \\
&= \frac{0.01^0}{0!} e^{-0.01} \\
&\approx 0.9900498
\end{aligned}
$$

* We could have solved this using Binomial $X \sim \text{Binomial}(10^4, 10^{-6})$. This may have been harder to compute!
* When n large, and p small: can use Poisson!

» Another Example of Poisson Distribution...

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Example of Poisson Distribution

The Poisson distribution is often used to model the number of events that occur independently at any time in an interval of time or space, with a constant average rate.

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The Poisson distribution is often used to model the number of events that occur independently at any time in an interval of time or space, with a constant average rate. Earthquakes are a good example of this. Suppose there are an average of 2.8 major earthquakes in the world each year. What is the probability of getting more than one major earthquake next year?

$X \sim \text{Poisson}(2.8)$, $X = \#$ earthquake

$P(\boxed{X > 1})$

$= 1 - P(X=0) - P(X=1)$

$R_X = \{0, 1, 2, \ldots\}$

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* Let $X \sim \text{Poisson}(2.8)$ be the number of major earthquakes next year. We want $P(X > 1)$.
* We can use

$$P(X > 1) = 1 - P(X = 0) - P(X = 1).$$

Using PMF for Poisson

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Using PMF for Poisson

$$\begin{aligned} P(X > 1) &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - e^{-2.8}\frac{2.8^0}{0!} - e^{-2.8}\frac{2.8^1}{1!} \\ &= 1 - e^{-2.8} - 2.8e^{-2.8} \\ &\approx 1 - 0.06 - 0.17 = 0.77 \end{aligned}$$

77%

» Expectation of Poisson Distribution...

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Expectation of Poisson

Let $X \sim Poisson(\lambda)$, with PMF given by

$$P_X(k) = \begin{cases} \frac{e^{-\lambda}\lambda^k}{k!}, & k \in R_X \\ 0 & \text{otherwise} \end{cases}$$

$E[X] = \sum x\, p(x)$

Show that the expectation $E[X] = \lambda$.

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Show that the expectation $E[X] = \lambda$.

Proof

$$E[X] = \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!} = \sum_{x=1}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$$

$$= e^{-\lambda} \sum_{z=0}^{\infty} \frac{\lambda^{z+1}}{z!} = \lambda e^{-\lambda} \sum_{z=0}^{\infty} \frac{\lambda^z}{z!} = \lambda.$$

Recall Calculus / Real Analysis

If $f: \mathbb{R} \to \mathbb{R}$ is infinitely diff., then f can be expanded around $x = a$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \dots$$

$x = 0$

$$e^x : e^0 + \frac{e^0}{1} x + \frac{e^0 x^2}{2!} + \dots + \frac{x^n}{n!} + \dots$$

$x - 1 = z$

e^λ

Taylor's expansion

» Expectation of Binomial Distribution...

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Expectation of a Binomial Distribution

Show that $E[X] = np$, where $X \sim \text{Binomial}(n, p), 0 < p < 1$ with PMF given as follows

$$P_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{for } k = 0, 1, 2, \cdots, n \\ 0 & \text{otherwise,} \end{cases}$$

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Proof

0

$$E[X] = \sum_{x=0}^{n} x \underbrace{\binom{n}{x} p^x q^{n-x}}_{P_X \text{ PMF}} = \sum_{x=1}^{n} x \binom{n}{x} p^x q^{n-x}.$$

For $0 < x \leq n$, $x\binom{n}{x} = x\dfrac{n!}{(n-x)!x!} = \dfrac{n!}{(n-x)!(x-1)!} = n\binom{n-1}{x-1}$,

$$\implies E[X] = \sum_{x=1}^{n} n \binom{n-1}{x-1} p^x q^{n-x} = \sum_{z=0}^{n-1} n \binom{n-1}{z} p^{z+1} q^{n-1-z} = np.$$

$x - 1 = z$

take n & p common

» Variance

Motivation for Variance

* The expectation value $E(X)$ (also called the mean) of a discrete random variable is a rather coarse measure of how X is distributed. For example, consider the following three situations:

» Variance

Motivation for Variance

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  * You get Rs 1000

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* Let X be the discrete random variable corresponding to your winnings. In all three cases, $E(X) = Rs\ 1000$, but you will agree that your chances of actually getting any money are quite different in all three cases.

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* Let X be the discrete random variable corresponding to your winnings. In all three cases, $E(X) = Rs\ 1000$, but you will agree that your chances of actually getting any money are quite different in all three cases.
* The three cases differ in the spread of PMF. This can be measured by Variance.

» Variance and Standard Deviation...

» Variance and Standard Deviation...

Variance

Let $\mu = E[X]$. The variance $\mathrm{Var}(X)$ of X is defined by

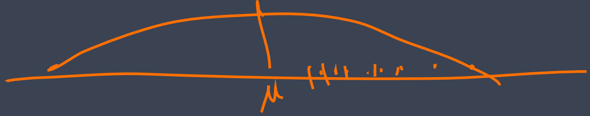
$E[g(x)] = \sum_x g(x) p_x(x)$

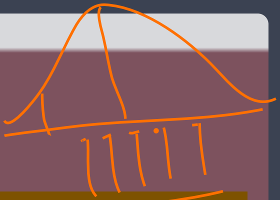
Variance

sometimes use p_x or f_X

Let $\mu = E[X]$. The **variance** $\mathrm{Var}(X)$ of X is defined by

$$\mathrm{Var}(X) = E(\underbrace{(X-\mu)^2}_{g(x)}) = \sum_x (x-\mu)^2 f_X(x)$$





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$$\sigma(X) = \sqrt{\sum_x (x-\mu)^2 f_X(x)} = \sqrt{\mathrm{Var}(X)}.$$

» Example of Computing Variance...

» Example of Computing Variance...

$R_X = \{1, 2, 3, 4, 5, 6\}$

x

Example

Let X be the value on one roll of a 6-sided die. Recall that $\underbrace{E[X]}_{\mu} = 7/2$. What is $Var(X)$?

$$Var(X) = E[(X-\mu)^2]$$

$$= \sum_x (x - 7/2)^2 \, p_X(x)$$

$$= \sum_x (x-7/2)^2 \cdot \frac{1}{6}$$

$$= (1-7/2)^2 \cdot \frac{1}{6} + (2-7/2)^2 \cdot \frac{1}{6} + \cdots + (6-7/2)^2 \cdot \frac{1}{6}$$

$$=$$

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Calculate Variance

- **First case:** There is only one outcome and it is the mean, variance is 0

» Variance

$\frac{1}{2}$ $\frac{1}{2}$

$R_X = \{0, 2000\}$

Motivation for Variance

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Calculate Variance

- First case: There is only one outcome and it is the mean, variance is 0
- Second case: $Var(X) = \frac{1}{2}(\underbrace{2000}_{x} - \underbrace{1000}_{\mu})^2 + \frac{1}{2}(\underbrace{0}_{x} - \underbrace{1000}_{\mu})^2 = 10^6$ ✓

$x = 0$

» Variance

Motivation for Variance

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  * A fair coin is tossed, and if it is head you get Rs 2000
  * A number is chosen from 1 to 1000 and if you can guess this, you get Rs 1 Million

Calculate Variance

* **First case:** There is only one outcome and it is the mean, variance is 0
* **Second case:** $\mathrm{Var}(X) = \frac{1}{2}(2000 - 1000)^2 + \frac{1}{2}(0 - 1000)^2 = 10^6$
* **Third case:** $\mathrm{Var}(X) = 10^{-3}(10^6 - 10^3)^2 + 999 \times 10^{-3}(0 - 10^3)^2 \approx 10^9$

» Remarks on Expectation, Variance, Standard Deviation...

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Example

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* If the standard deviation was only Rs 5,000, you would have a much better idea of what to expect for a starting salary at that company

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* Without calculating standard deviation, you can’t get a handle on whether the data are close to the average

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average Std dev

Same ~~Standard Deviation~~ Does Not Imply Same ~~Dataset~~

For example, the data sets 199, 200, 201 and 0, 200, 400 both have the same average (200) yet they have very different standard deviations.

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* Without calculating standard deviation, you can’t get a handle on whether the data are close to the average

Same Standard Deviation Does Not Imply Same Dataset

For example, the data sets 199, 200, 201 and 0, 200, 400 both have the same average (200) yet they have very different standard deviations. The first data set has a very small standard deviation (s=1) compared to the second data set (s=200).

» Another expression for the variance

Theorem (Another Expression for Variance)

If X is a discrete random variable with mean μ, then

$$\mathrm{Var}(X) = E[X^2] - \mu^2$$

Theorem (Another Expression for Variance)

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Proof

$$\begin{aligned}
\mathrm{Var}(X) &= \sum_x (x-\mu)^2 p_X(x) = \sum_x (x^2 - 2\mu x + \mu^2) p_X(x) \\
&= \sum_x x^2 p_X(x) - 2\mu \sum_x x\, p_X(x) + \mu^2 \sum_x p_X(x) \\
&= E[X^2] - 2\mu E[X] + \mu^2 = E[X^2] - \mu^2
\end{aligned}$$

1

μ

» Properties of Variance...

» Properties of Variance...

Theorem

Let X be a discrete random variable and α a constant. Then

$$\text{Var}(\alpha X) = \alpha^2 \text{Var}(X) \quad \text{and} \quad \text{Var}(X + \alpha) = \text{Var}(X)$$

$\boxed{E[X] = \mu}$

$E[\alpha X] = \alpha\mu$

$\text{Var}(\alpha X) = E[(\alpha X)^2] - \alpha^2\mu^2$

$= \alpha^2 E(X^2) - \alpha^2\mu^2$

$\text{Var}(\alpha X) = E[(\alpha X - \mu)^2]$

$= \alpha^2 [E(X^2) - \mu^2]$

$= \alpha^2 \text{Var}(X)$

$E[X+\alpha] = E[X] + E[\alpha]$

$= \mu + \alpha$

$\text{Var}(X+\alpha) = E[(X+\alpha)^2] - (\mu+\alpha)^2$

$= \text{Var}(X)$

» Computing Variance: Binomial

Binomial (n, p) $E[X] = np$

Variance of Binomial Distribution

Let $X \sim \text{Binomial}(n, p)$. Then the variance $\text{Var}(X) = np(1-p)$.

$$\text{Var}(X) = E(X^2) - \mu^2 = \underbrace{E[X(X-1)]} + \underbrace{E[X]} - \underbrace{E[X]}^2$$

$$E[X(X-1)] = \sum_{x=0}^{n} x\underbrace{(x-1)} \binom{n}{x} p^x (1-p)^{n-x}$$

$$\sum_{x=2}^{n} \frac{n!}{\underbrace{(x-2)!}\,(n-x)!} p^x (1-p)^{n-x}$$

» Variance of Binomial Distribution...

$$= \sum_{x=2}^{n} \frac{n!}{(x-2)!\,(n-x)!} p^{x} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$\begin{cases} m = n-2 \\ y = x-2 \end{cases}$$

$$= n(n-1)p^2 \underbrace{\sum \frac{m!}{y!\,(m-y)!} p^{y} (1-p)^{m-y}}$$

$$= \underline{n(n-1)p^2} \left(p + (1-p)\right)^m = n(n-1)p^2$$

$$= E[X(X-1)]$$

$$\mathrm{Var}(X) = E[X(X-1)] + E[X] - (E[X])^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$= np(1-p)$$

» Higher Order Moments...

$E[X]$

Define nth moment

The nth moment about the mean or nth central moment of a real valued random variable X is defined as follows

$$\mu_n = E[(X - E[X])^n],$$

where E is the expectation operator.

$E[X - \mu]$

$E[(X-\mu)^2]$

$E[(X-\mu)^3]$

» Higher Order Moments...

Define nth moment

The nth moment about the mean or nth central moment of a real valued random variable X is defined as follows

0

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where E is the expectation operator.

* The zeroth central moment μ_0 is 1

» Higher Order Moments...

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The *nth moment* about the mean or *nth central moment* of a real valued random variable X is defined as follows

$$\mu_n = E[(X - E[X])^n],$$

where E is the expectation operator.

$E[(X-\mu)]$
$= E[X] - \mu$
$= \mu - \mu = 0$

* The zeroth central moment μ_0 is 1
* The first central moment μ_1 is 0 (not the same as expected value!)

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    * First central moment about origin is indeed expectation!

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$E[(X-\mu)^2]$

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* The second central moment μ_2 is called variance, denoted by σ^2

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Generating Moments...

Is there a quick way to generate moments?

» Moment Generating Function...

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$e^{tX} = g(x)$

Moment Generating Function

The moment generating function $M_X(t)$ is the expectation value

$$M_X(t) = E[e^{tX}] = \sum_x e^{tx} p_X(x)$$

$\sum p_X(x) = 1$

$g(x)$ pmf f_X

Lemma

* $M_X(0) = 1$
* $E[X] = M'_X(0)$, where ' is the derivative w.r.t. t

$t \to 0$

derivative

$$\frac{d M_X(t)}{dt} = \sum x \cdot e^{tx} p_X(x)$$

$$= \sum x \, p_X(x) = E[X] = \mu$$