Ans. 1 (1) We are given a total of 10 shots so
$X = \{0, 1, 2, \ldots 10\}$.

PMF of X :-

$$P(X) \begin{cases} {}^{10}C_0 (0.2)^0 (0.8)^{10} & X = 0 \\ {}^{10}C_1 (0.2)^1 (0.8)^9 & X = 1 \\ {}^{10}C_2 (0.2)^2 (0.8)^8 & X = 2 \\ {}^{10}C_3 (0.2)^3 (0.8)^7 & X = 3 \\ {}^{10}C_4 (0.2)^4 (0.8)^6 & X = 4 \\ {}^{10}C_5 (0.2)^5 (0.8)^5 & X = 5 \\ {}^{10}C_6 (0.2)^6 (0.8)^4 & X = 6 \\ {}^{10}C_7 (0.2)^7 (0.8)^3 & X = 7 \\ {}^{10}C_8 (0.2)^8 (0.8)^2 & X = 8 \\ {}^{10}C_9 (0.2)^9 (0.8)^1 & X = 9 \\ {}^{10}C_{10} (0.2)^{10} (0.8)^0 & X = 10 \end{cases}$$

Where 0.2 = Probability of hitting the target
$1 - 0.2 = 0.8$ = Probability of <u>not</u> hitting the target.

(2) Expectation $= np = 10 \times 0.2 = 2$ hits
Variance $= np(1-p) = 10 \times 0.2 \times (1 - 0.2) = 1.6$

~~(3) Expectation y = Expecta~~ → Expectation(x)

(3) Expectation(y) $= 2 \times (\$2) - (\$3) = \$1$
Variance (Y) $= \cancel{1.6 \times (\$2) - (\$3) = \$0.2}$
~~Variance (x)~~
$=$ Variance$(2X - 3) =$ Variance$(2X)$
$= 4$ Variance$(X) = 4 \times 1.6 = 6.4$.

(4) By PMF $P(x) = {}^{10}C_x (0.2)^x (0.8)^{10-x}$

$\downarrow$

Tharum ~~shoot~~ did x hits

Profit : $g(x) = x^2$

Asked = $E[g(x)]$

$\therefore \cancel{\sum_{x=0}^{10} P(x) \cdot g(x)}$

$$\therefore E[g(x)] = \sum_{x=0}^{10} P(x) \cdot g(x)$$

$$\sum_{x=0}^{10} \underbrace{{}^{10}C_x (0.2)^x \cdot (0.8)^{10-x} * x^2}$$

$\downarrow$

$$\frac{10}{x} * x^2 \cdot {}^{9}C_{x-1} (0.2)^x \cdot (0.8)^{10-x}$$

$$= 10x \cdot {}^{9}C_{x-1} (0.2)^x \cdot (0.8)^{10-x}$$

$$= \cancel{10(x-1)} \; 10(x-1+1) \cdot {}^{9}C_{x-1} (0.2)^x \cdot (0.8)^{10-x}$$

$$= 10 \cdot \left[(x-1) \cdot {}^{9}C_{x-1} \cdot (0.2)^{x} \cdot (0.8)^{10-x} + 1 \cdot {}^{9}C_{x-1} \cdot (0.2)^{x} (0.8)^{10-x} \right]$$

$$\therefore = 10 \left[(x-1) \cdot \frac{9}{(x-1)} \cdot {}^{8}C_{x-2} \cdot (0.2)^{x} \cdot (0.8)^{10-x} + {}^{9}C_{x-1} (0.2)^{x} (0.8)^{10-x} \right]$$

$$= 10 \left[9 \cdot {}^{8}C_{x-2} (0.2)^{x} \cdot (0.8)^{10-x} + {}^{9}C_{x-1} (0.2)^{x} (0.8)^{10-x} \right]$$

$$\therefore \text{Asked} = 10 \times 9 \times \sum_{x=2}^{10} {}^{8}C_{x-2} (0.2)^{x} \cdot (0.8)^{10-x}$$

$$+ \quad 10 \cdot \sum_{x=1}^{10} {}^{9}C_{x-1} (0.2)^{x} (0.8)^{10-x}$$

~~Asked = 90 ($\sum_{x}$~~

$$= 1$$

$$\text{Asked} = 90 \times (0.2)^{2} \cdot \overbrace{\sum_{x=2}^{10} {}^{8}C_{x-2} (0.2)^{x-2} \cdot (0.8)^{10-x}}$$

$$+ 10 \cdot (0.2) \underbrace{\sum_{x=1}^{10} {}^{9}C_{x-1} (0.2)^{x-1} \cdot (0.8)^{10-x}}_{=1}$$

$$= 90 \times (0.2)^{2} \times 1 + 10 \times (0.2) \times 1 = 5.6$$

Ans. 2 Asked = P(BW or WB) → after rearrangement

→ Both are same thing

5W 3B	→ 2 balls →	3W 5B

2 Balls choosen can be WW, WB/BW, BB

$$\text{Asked} = \frac{{}^5C_2}{{}^8C_2}\left(\frac{{}^5C_1 \cdot {}^5C_1}{{}^{10}C_2}\right) + \frac{{}^5C_1 \cdot {}^3C_1}{{}^8C_2}\left(\frac{{}^4C_1 \cdot {}^6C_1}{{}^{10}C_2}\right)$$

$$+ \frac{{}^3C_2}{{}^8C_2}\left(\frac{{}^3C_1 \cdot {}^7C_1}{{}^{10}C_2}\right) = \frac{673}{1260}.$$

Ans.3 $D \geq 0$ to have atleast 1 real root

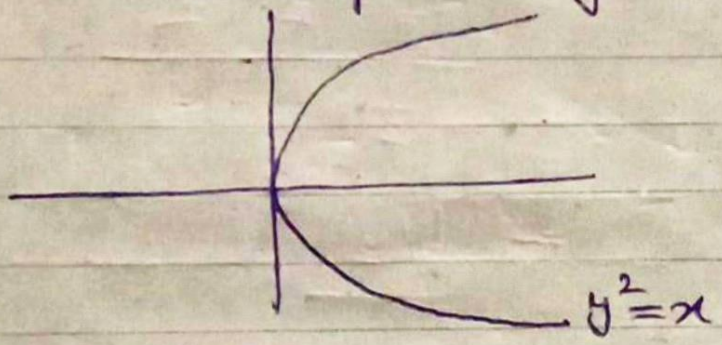
$\therefore 4k^2 - 4m \geq 0 \quad \therefore k^2 - m \geq 0.$

Let $y = k$ and $x = m$ where $x, y \in R$.

$\therefore y^2 - x \geq 0.$

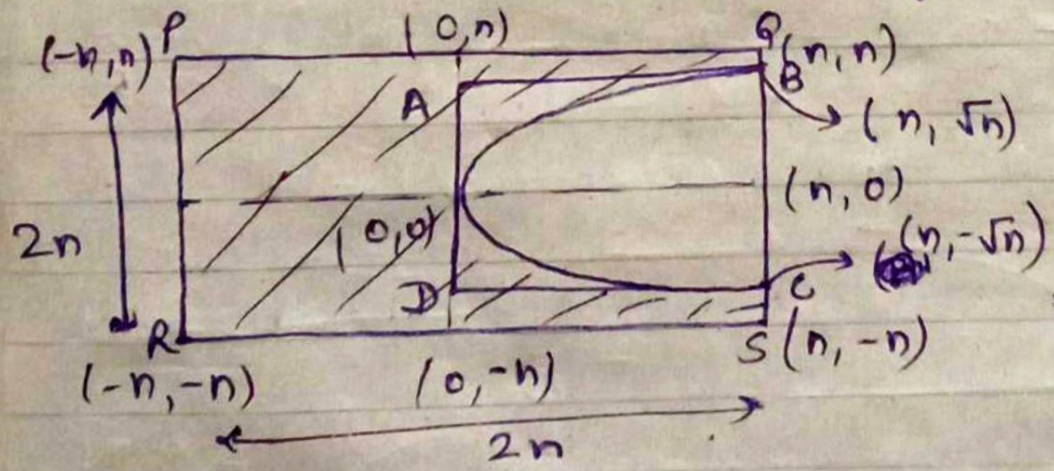
Assume x, y to be Cartesian Co-ordinates.

Let's plot $y^2 = x$.



Let's divide the whole region into $2n$ segments of x and $2n$ segments of y.

$\therefore x \in (-n, n)$ and $y \in (-n, n)$ where $n \to \infty$.



We should have $D \geq 0$

$\Rightarrow y^2 - x \geq 0$

$\Rightarrow$ Points outside the parabola.

Shaded Area = Favourable Area.

Area of Rectangle (here square) $= 2n \times 2n$.

We know that Area enclosed by parabola

$= \frac{2}{3} \times$ Area of Rectangle ABCD.

Area of Rectangle ABCD $= n \times 2\sqrt{n} = 2n\sqrt{n}$.

Area of Parabola $= \frac{2}{3} \times 2n\sqrt{n} = \frac{4n\sqrt{n}}{3}$.

~~∴ Ast~~

$$\therefore \text{prob.} = \frac{2n^2 + \left(2n^2 - \frac{4n\sqrt{n}}{3}\right)}{4n^2} = 1 - \frac{1}{3\sqrt{n}}$$

$$\therefore \lim_{n\to\infty} \text{Prob} = \lim_{n\to\infty}\left(1 - \frac{1}{3\sqrt{n}}\right) = 1.$$

Ans 4 There are 4 possibilities for the 2 Children :-

$\{GG, BB, BG, GB\}$

(1) P(Atsi is a girl / Atsi has a sister)

$$= \frac{GG}{GG + GB + BG} = \frac{1}{3}.$$

(2) P(Atsi is a boy / Atsi has a sister)

$$= \frac{BG + GB}{GG + GB + BG} = \frac{2}{3}.$$

skipped

Ans. 5 F L A S H

correct letter → F A S T → extra

skip

skip

insert

$$\therefore \text{Prob} = (0.8) \times (0.1) \times (0.8) \times (0.8) \times (0.1) \times (0.1) \times \frac{1}{26}$$

$$= \frac{1}{26} \times (0.8)^3 \times (0.1)^3$$

$$\approx 1.97 \times 10^{-5}$$

choose T from 26 letters

Ans 6 Asked = 1 − P (they all have diff. birth days)

$$= 1 - \frac{{}^{365}C_{30}}{(365)^{30}}.$$

365 : No. of days in a ~~normal~~ year.

Ans] Asked = $\dfrac{\left(\text{Total no° of paths from } (0,0) \text{ to } (x,y)\right) * \left(\text{Total no° of paths from } (x,y) \text{ to } (n-1, m-1)\right)}{\text{Total no° of paths from } (0,0) \text{ to } (n-1, m-1)}$

$$= \frac{\dfrac{(x+y)!}{x!\,y!} + \dfrac{(n-x+m-y)!}{(n-x)!\,(m-y)!}}{\dfrac{(n+m)!}{n!\,m!}}$$

Ans. 8 E and F are independent

$\therefore \quad P(E \cap F) = P(E) \cdot P(F)$

$= \frac{1}{6}$

$P(E' \cap F') = 1 - P(E \cup F) = 1/3$

$\boxed{P(E \cup F) = 2/3}$

$P(E \cup F) = \frac{2}{3} = P(E) + P(F) - P(E \cap F)$

$P(E) + P(F) = 5/6$ Let $P(E) = x$

$P(E) \cdot P(F) = 1/6$ $\therefore P(F) = 5/6 - x$

$\therefore \quad x \cdot \left(\frac{5}{6} - x\right) = \frac{1}{6}$

$\Rightarrow x = \frac{1}{2}$ or $\frac{1}{3}$.

Case-1) $P(E) = \frac{1}{2}$ $\therefore P(F) = \frac{1}{3}$

Given $(P(E) - P(F))\underbrace{(1 - P(F))}_{\text{always +ve}} > 0$

$\Rightarrow P(E) - P(F) > 0$

$\Rightarrow P(E) > P(F)$

$\Rightarrow$ Case-1 satisfies it ✓

Case-2 $P(E) = 1/3$ $\therefore P(F) = 1/2$

According to given condition

$P(E) > P(F)$

$\Rightarrow$ Case-2 voilates it.

✗

$\Rightarrow P(E) = \frac{1}{2}$ $P(F) = 1/3$

$\therefore \frac{P(E)}{P(F)} = \frac{1/2}{1/3} = \frac{3}{2}$

$\therefore \boxed{2P(E) = 3P(F)}$

Ans:9 Possible favourable cases are:-

$\{\underbrace{1, 1, - - - 1}_{n \text{ identical boxes}}$

$+$

$\{2, 1, - - - 1, 0$

$\{2, 2, 1, - - 1, 0\ 0$

$\vdots$

$\lfloor n/2 \rfloor$ times $\{2, 2, - - 0\ 0$

because if n is even then we have perfectly $n/2$ times 2 balls in $n/2$ boxes and 0 balls in remaining $n/2$ boxes.

- If n is odd then ~~also~~ ~~n+2~~ $\frac{n-1}{2}$ because at last we have $\left(\frac{n-1}{2}\right)$ boxes having 2 balls, 1 box having 1 ball and remaining $\left(\frac{n-1}{2}\right)$ boxes having 0 balls.

$\therefore$ Total favourable cases $= \lfloor n/2 \rfloor + 1$

Ans 10

B1 → 1, 2, 3

B2 → 1, 2, 3, 4, 5

B3 → 1, 2, 3, 4, 5, 6, 7

Total cases = ${}^3C_1 \cdot {}^5C_1 \cdot {}^7C_1$ = 105

Let d be the common difference of AP

d=0 → (1, 1, 1), (2, 2, 2), (3, 3, 3).

→ 3 possibilities

d=1 → (1, 2, 3), (2, 3, 4), (3, 4, 5)

All three no can be choosen from any of the boxes

∴ $(3 \times 2 \times 1)$

choosing numbers

4 can be choosen only from B_2 / B_3 and rest from any of the boxes

∴ $(2 \times 1) \times 2$

choosing 2 and 3 numbers.

choosing 4

4, 5 can't be selected from B_1, so 3 must be choosen from B_1. Both 4 & 5 can be choosen from $B2 / B3$

∴ (2×1)

$= (3 \times 2 \times 1) + (2 \times 1) \times 2 + (2 \times 1) = 6 + 4 + 2 = 12.$

d=2 (1, 3, 5), (2, 4, 6), (3, 5, 7)

2×2 1 1

$= 4 + 1 + 1 = 6.$

$\underline{\underline{d=3}}$ $(1, 4, 7)$ → only this possibility

$\downarrow$

$\underline{1}$

$$An = d=0 + d=1 + d=2 + d=3$$

$$= 3 + 12 + 6 + 1$$

$$= 22.$$

$\therefore$ Probability $= 22/105.$

Ans 11 Since C_i's form the partition of the sample space we can apply the law of total probability

$$P(A \cap B) = \sum_{i=1}^{M} P(A \cap B / C_i) \cdot P(C_i)$$

as A and B are conditionally independent

B is independent of all C_i's

$$= \sum_{i=1}^{M} P(A / C_i) \; P(B) \; P(C_i)$$

$$= P(B) \sum_{i=1}^{M} P(A / C_i) \; P(C_i)$$

by law of total probability

$P(A \cap B) = P(B)\,P(A)$

$\Rightarrow$ A and B are independent. H.P.

~~Ans 10~~ Ans 12 $P(\text{truth}) = 2/5$.

Asked = P(Koushiks says true) * P(Neeraj guess true)

+ P(Koushik says false) * P(Neeraj guess false)

$$= \frac{2}{5} \times \frac{1}{2} + \frac{3}{5} \times \frac{1}{2} = \frac{2+3}{10} = \frac{5}{10} = \frac{1}{2}.$$

Ans 13 A = he makes guess

B = he copies answer

C = he knows answer

D = Answer is correct

$P(A) = \frac{1}{3}$ $P(B) = \frac{1}{6}$

$P(A) + P(B) + P(C) = 1$

$P(C) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$

$P(D/B) = 1/8$

$\frac{P(D \cap B)}{P(B)} = \frac{1}{8} \Rightarrow P(D \cap B) = \frac{1}{6} \times \frac{1}{8}$

$P(D \cap C)$ = when he knows the answer, that his answer will be correct.

$= 1/2$.

when he make a guess, the chances of correct answer will be 1/4 [MCQ question]

$\Rightarrow P(D \cap A) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{3 \times 4}$

$$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$$

$$P(D) = \frac{1}{3 \times 4} + \frac{1}{6 \times 8} + \frac{1}{2}$$

$$P(D) = \frac{29}{48}.$$

$$P(C/D) = \frac{(1/2)}{(29/48)} = \frac{24}{29}.$$

Ans: 4 Asked = $1 -$ P(no dice shows pairs)

$$= 1 - \left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right)$$

$$= 1 - \left(\frac{5}{6}\right)^3 = \frac{6^3 - 5^3}{6^3} = \frac{91}{216}.$$

Ans. 15 $\quad P(H) = p \quad \therefore P(T) = 1 - p = q$ (let's say)

Asked $= P(\text{Head on } 2^{nd} \text{ toss}) + P(\text{Head on } 4^{th} \text{ toss}) + \cdots$

$$= qp + q^3 p + q^5 p + \cdots$$

$$= qp(1 + q^2 + q^4 + \cdots)$$

$$= qp \left(\frac{1}{1-q^2}\right) = \frac{qp}{1-q^2}$$

$$= \frac{(1-p)\,p}{1-(1-p)^2} = \frac{1-p}{2-p}.$$

Ans 16 Let the length of rod be L

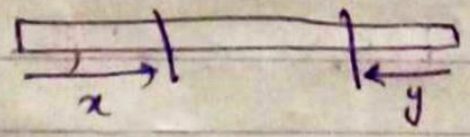
a) Avg of $\left(0, \frac{L}{2}\right) \Rightarrow \frac{L}{4}$

b) Avg of $\left(\frac{L}{2}, L\right) \Rightarrow \frac{3L}{4}$

$$\frac{\text{avg of } (0, L/2)}{\text{avg of } (L/2, L)} = 1:3$$

OR a) and b) parts can be calculated by integral but as it is linear $\Rightarrow$ we can take average.

c)



$$E[x] = \sum_{x} x\, p(x)$$

for continuous case

$$E(x) = \int_0^L \int_0^{L-x} x \, dx \, dy$$

$$= \int_0^L x \frac{dx}{L} \cdot \frac{dy}{L}$$

$$= \frac{1}{L^2} \int_0^L x(L-x)\,dx$$

$$= \frac{1}{L^2}\left(\frac{L^3}{2} - \frac{L^3}{3}\right) = \frac{L}{6}.$$

Ans. 17

G	S	G
G	S	S

$$\text{As Red} = P(GG / \text{get } G)$$

$$= \frac{P(GG)}{P(GG) + P(GS)}$$

$$= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{2}} = \frac{2}{3}.$$

Ans 18 Asked = P (confident) $\qquad$ $P(\text{hit}) = p$

= (She hits α times in α trials) +

(She hits α times in $(\alpha+1)$ trials) +

(She hits α time in $(\alpha+2)$ trials) +

⋮

(She hits α time in $(\alpha+\beta-1)$ trials)

this is because if trials are more than $(\alpha+\beta-1)$ then Anya will ~~return~~ have β misses earlier than α hits and she will return disappointed. Also in each $(\alpha+k)$ trials ($k \in [0, \beta-1]$) the last trial should be "hit" only.

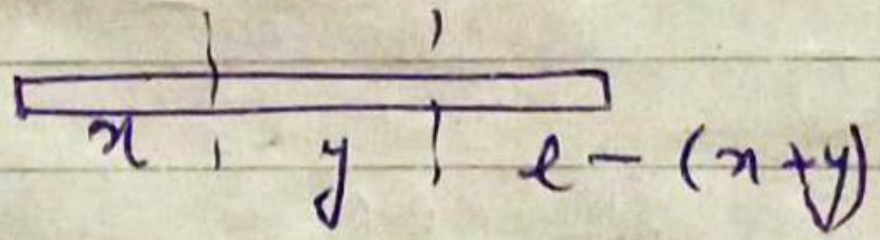
$$\therefore \text{Prob.} = p\left({}^{\alpha-1}C_0 (1-p)^0 p^{\alpha-1} + {}^{\alpha}C_1 (1-p)^1 (p)^{\alpha-1} + {}^{\alpha+1}C_2 (1-p)^2 p^{\alpha-1} + \cdots {}^{\alpha-1+(\beta-1)}C_{\beta-1} (1-p)^{\beta-1} (p)^{\alpha-1}\right)$$

for last term

$$= (p)^{\alpha}\left(\sum_{i=0}^{\beta-1} {}^{\alpha+i-1}C_i (1-p)^i\right).$$

Ans. 19 Let $l = 2\pi r$.

Wire



Constraints $0 < x < 3$ $0 < y < 3$ $0 < l - x + y < 3$

Δ inequality

$x + y > l - x + y \Rightarrow x + y > l/2$

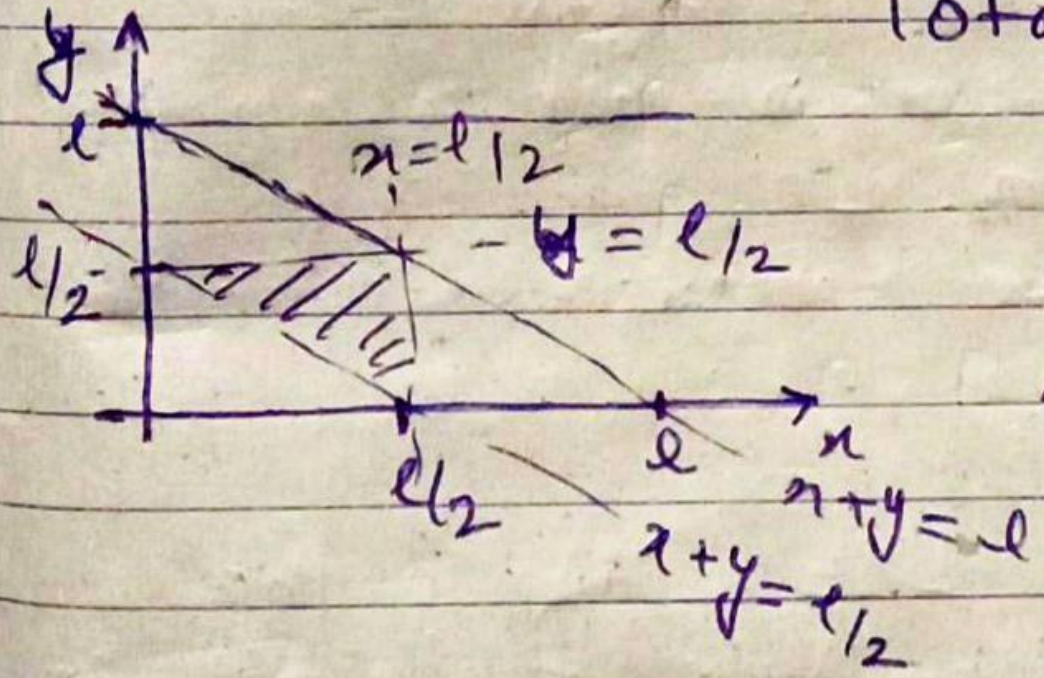
$x + l - (x+y) > y \Rightarrow y < l/2$

$y + l - (x+y) > x \Rightarrow x < l/2$

$x + y < l$

Let x, y be co-ordinates on 2-D plane.
So problem can be modeled as ~~favourable~~

$$\frac{\text{Favourable Area}}{\text{Total Area}}$$

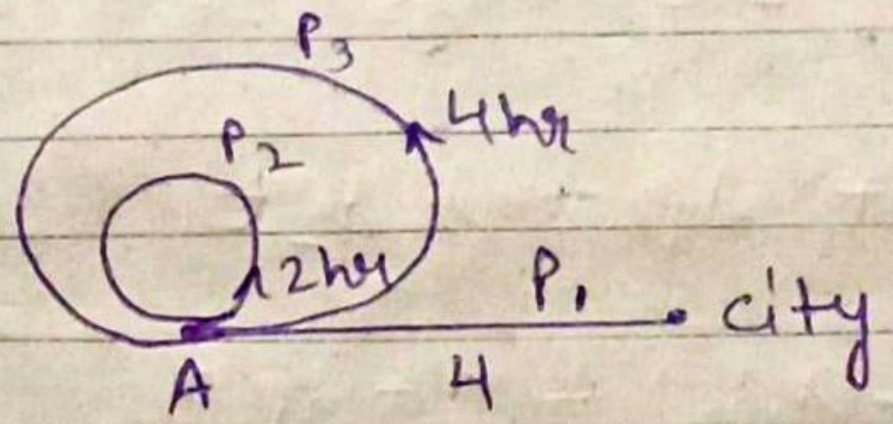


Shaded Area = Favourable Area

$$\therefore \text{Ans} = \frac{\frac{1}{2} \times \frac{l}{2} \times \frac{l}{2}}{\frac{1}{2} \times l \times l}$$

$$= \boxed{1/4}.$$

Ans. 20



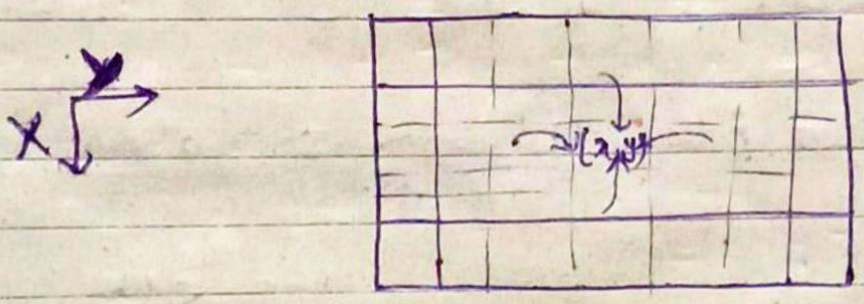
Let E: expected time

$$\therefore \quad E = \underbrace{\frac{1}{3}\times 3}_{\text{following } P_1 \text{ and directly to city}} + \underbrace{\frac{1}{3}\times (E+2)}_{P_2 \text{ and same point}} + \underbrace{\frac{1}{3}\times (E+4)}_{P_3 \text{ and same point}}$$

$$E = 1 + \frac{2}{3} + \frac{4}{3} + \frac{2E}{3}$$

$$\frac{E}{3} = 3 \quad \rightarrow \quad \boxed{E = 9 \text{ hrs}}$$

Ans 2) Suppose we are at (x,y) after k jumps, then just before the last jump we have to be either in the same row as of (x,y) (i.e. x) or in the same column as of (x,y) i.e. y. After reaching at either of these points we just have to make one more jump to reach (x,y).



Let $P(x,y,k)$: denotes the probability of reaching point (x,y) after exactly k moves.

$$\therefore P(x,y,k) = \frac{1}{2\times(N-1)}\left[P(1,y,k-1) + P(2,y,k-1) + \cdots P(x-1,y,k-1) + P(x+1,y,k-1) + \cdots P(N,y,k-1)\right]$$

$$+ \frac{1}{2\times(N-1)}\left[P(x,1,k-1) + P(x,2,k-1) + \cdots P(x,y-1,k-1) + P(x,y+1,k-1) + \cdots P(x,N,k-1)\right]$$

$\frac{1}{2}$ is probability of choosing a row jump or column jump

Probability of jumping to (x,y) from (x,y') or (x',y) $\rightarrow$ where $y' \neq y$ $y \in [1,N]$, $x' \neq x$ $x \in [1,N]$

Note:- In the recurrence relation we haven't included $P(x, y, k-1)$ in either of the expressions because we can't jump to the same point again. We just have $(N-1)$ choices in both rows and columns.

Base Case:- $P(x, y, 0)$ is 1 whereas

$$P(x', y', 0) = 0$$

for $x' \in [1, N] - \{x\}$

$y' \in [1, N] - \{y\}$

Also $P(x, y, k-1) = 0$ (Not included in recursion)

Answer to the problem will be $\boxed{P(x, y, k)}$

$$P(x, y, k) = \frac{1}{2 \times (N-1)} \sum_{\substack{i=1 \\ i \neq x}}^{N} P(i, y, k-1) + \frac{1}{2 \times (N-1)} \sum_{\substack{i=1 \\ i \neq y}}^{N} P(x, i, k-1)$$

Ans. 22 For the constraints $N > S$,

Prove:

$$P(\text{Mr. Nariyal is always ahead of Mr. Samso}) = \frac{N-S}{N+S}$$

when $N = S$ Probability $= 0$ as they always tied at end.

when $N > 0$ and $S = 0$, the probability is 1 as Mr Samso doesn't get any vote.

Let this result be true for $(N-1, S)$ and $(N, S-1)$.

The last vote is cast to Mr. N while counting with probability $= \frac{N}{N+S}$ and it

goes to Mr. Samso with probability $= \frac{S}{N+S}$.

Thus the probability that Mr N is always ahead of Mr. S

$$= \left(\frac{N}{N+S}\right)\left(\frac{(N-1)-S}{N+S-1}\right) + \left(\frac{S}{N+S}\right)\left(\frac{N-(S-1)}{N+S-1}\right)$$

$$= \frac{N-S}{N+S} \quad \text{H.P.}$$

(a) Probability that they are tied at a point is

$$1 - \left(\frac{N-S}{N+S}\right) = \frac{2S}{N+S}.$$

Thus, Probability that there exists a point while counting the votes one by one that they tied is $\frac{2S}{N+S}$.

(b) Note that the sequence of counting votes where Mr N is strictly ahead with values $(N+1, S)$ is equal to the number of sequences of counting votes where Mr S never goes ahead of Mr N, or where ties are allowed with values (N, S).

$\therefore$ Total no of accepted sequences =

$$\frac{N+1-S}{N+S+1} \cdot \binom{N+S+1}{S} = \frac{N+1-S}{\cancel{N+S+1}} \cdot \frac{\cancel{N+S+1}}{N+1} \binom{N+S}{S}$$

$$= \frac{N+1-S}{N+1} \binom{N+S}{S}$$

Total no of sequences = $\binom{N+S}{S}$

$$\therefore \boxed{\text{Prob.} = \frac{N-S+1}{N+1}}$$

Now that we need Prob. that Mr. Nariyal never gets ahead of Mr Samso,

$$\text{Prob} = \frac{S-N+1}{S+1}$$

$N=S$ is given $\Rightarrow$ Prob. $= 1/S+1$.

Thus the prob. that Mr N ~~mo~~ never gets ahead of Mr S while counting votes and when $N=S$ is

$$= 1/S+1 \quad \underline{\text{Ans.}}$$

Ans. 23 Say we have a permutation of any k days out of n possible:

$a_1, a_2, \ldots a_k$ ~~and~~

where a_1: birthday 1st person

a_2: birthday of 2nd person

so on.

Birthdays are unique

Let the set of days be $S_1 = \{a_1, a_2, \ldots a_k\}$

and let no of permutations of set of days S_1 be $Per(S_1)$.

$\therefore$ P (dist. birthdays from S_1) $= Per(S_1) \prod_{i=1}^{k} P(a_i)$

We need to find probability for all such S_1. $\therefore$ Such sets will be ${}^nC_k$.

$$\therefore P(\text{dist. birthdays}) = \sum_{i=1}^{{}^nC_k} \left(Per(S_i) \cdot \prod_{i=1}^{k} P_{a_i} \right)$$

where each S_i is different from each other

$$\cancel{\max(} \max P(\text{dist. bdays}) = \max \sum (Per \times \Pi)$$

$$= \sum \max (Per. \Pi)$$

$$Per(S_i) = k!$$

$$\therefore \max(P(\text{dist. bdays})) = k! \sum \max(\pi)$$

by A.M. $\geq$ G.M. inequality we have:-

$$\frac{\sum P_i}{n} \geq \sqrt[n]{\Pi P_i} \Rightarrow \frac{1}{n} \geq \sqrt[n]{\pi P_i}$$

Note that equality gives us the maxima of ΠP_i which is only achieved in case of $P_{a_1} = P_{a_2} = P_{a_3} = \cdots 1/n$.

Applying inequality for all ΠP_i we get

$$\boxed{P_1 = P_2 = P_3 = \cdots P_n = 1/n} \#$$

Ans 24 P(she comes out confident)

$\Rightarrow$ α consecutive hits before β consecutive misses $\Rightarrow P(c)$

$$P(c) = p^{\alpha}\Big(1 + \underbrace{\sum_{1\le k_1\le \beta-1}(1-p)^{k-1} + \sum_{\substack{1\le k_1\le \beta-1\\ 1\le k_2\le \alpha-1}}(1-p)^{k_1}p^{k_2}(1+\cdots)}_{\text{Let this be } t}\Big)$$

$$\Rightarrow \quad t = 1 + \lambda_1 + \lambda_1\lambda_2 t$$

$$t(1-\lambda_1\lambda_2) = 1+\lambda_1$$

$$t = \frac{1+\lambda_1}{1-\lambda_1\lambda_2}$$

$$\Rightarrow \quad \frac{1 + \frac{(1-p)}{p}\left(1-(1-p)^{\beta-1}\right)}{1 - \frac{(1-p)}{p}\left(1-(1-p)^{\beta-1}\right)\frac{p}{1-p}\left(1-p^{\alpha-1}\right)}$$

$$= \frac{\frac{p + 1-p + (1-p)(1-p)^{\beta-1}}{p}}{1-\left(1-(1-p)^{\beta-1}\right)\left(1-p^{\alpha-1}\right)}$$

$$P(c) = \frac{p^{\alpha-1}\left(1+(1-p)^{\beta}\right)}{1-\left(1-p^{\alpha-1}\right)\left(1-(1-p)^{\beta-1}\right)}$$

Ans. 25 (1) $X = 2$ $Y = 3$

(1) B (2) B (3) B (4) → Facing this side

Girls can't occupy 4th position. Either girls can occupy 2 of (1) or (2) or (3) position or they can both be a position (1) or (2).

Hence total no° of ways in which girls can ~~be~~ be seated is

$$= {}^3C_2 \times 2! \times 3! + {}^2C_1 \times 2! \times 3! = 60.$$

Number of ways in which 3B an 2G can be ~~arranged~~ seated $= 5!$

$$\therefore \text{Required Prob.} = \frac{60}{5!} = \frac{60}{120} = \frac{1}{2}.$$

Ans. 25(2) Let us first prove that ~~that~~ ~~at~~ the probability that at any point in the line the no of boys is strictly greater than the no of girls ~~at~~ when there are p boys and q girls

is $\frac{p-q}{p+q}$.

when $p=q$ Prob. $=0$. as no of boys & girls is equal at the end of the line.

when $q=0$ probability is 1 as no of girls can never match no of boys.

If possible the result is true for $(p-1, q)$ and $(p, q-1)$.

Then probability that the last child is a boy $= \frac{p}{p+q}$ and for girl is $\frac{q}{p+q}$

Prob. for $(p,q) \Rightarrow \frac{p}{p+q}\left(\frac{p-1-q}{p-1+q}\right) + \frac{q}{p+q}\frac{(p-(q-1))}{(p+(q-1))}$

$$= \frac{p(p-q-1) + q(p-q+1)}{(p+q)(p+q-1)}$$

$$= \frac{(p-q)(p+q-1)}{(p+q)(p+q-1)}$$

Hence Proved by induction

we observe that the no of line sequences, where at each point in the line the no of girls were never greater than the number of boys, ~~or were tie~~ or where ties are allowed for $p = x$, $q = y$ is the same as the number of line sequences where no of boys is strictly greater than no of girls at all points in the line for $p = x+1$, $q = y$.

$\therefore$ The no of sequences where ties are allowed is

$$\cancel{\left(\frac{x+1-y}{x+1+y}\right) \cdot \left(\frac{x+1+y}{x+1}\right) = \frac{x+1-y}{x+1+y} \cdot \frac{x+1+y}{x+1} \cdot \binom{x+y}{x}}$$

$$\left(\frac{x+1-y}{x+1+y}\right) {}^{x+1+y}C_{x+1} = \left(\frac{x+1-y}{\cancel{x+1+y}}\right) \cdot \left(\frac{\cancel{x+1+y}}{x+1}\right) \cdot {}^{x+y}C_x$$

$$= \left(\frac{x+1-y}{x+1}\right) \cdot {}^{x+y}C_x$$

$\therefore$ Total no of line sequences $= {}^{x+y}C_x$

$\therefore$ The probability $= \dfrac{x-y+1}{x+1}$

Now we observe no of boys in front of a girl is atleast 1 more than no of

girls in front of her, means that including that girl. the no of boys till that point in the line is greater than or equal to no of girls till that point in the line.

$\therefore$ Probability of that happening $= \frac{y-x+1}{y+1}$

for 1st part $y = 3$ $x = 2$

$\therefore$ Prob $= \frac{3-2+1}{3+1} = \frac{1}{2}$ (verified)

for 2nd part $x = y$

$\therefore$ Prob $= \frac{y-y+1}{y+1} = \frac{1}{y+1}$.

Turn: 1st 2nd 3rd 4th 5th

Ans 26 $P(\text{Hit}) = 1, 0, x, y, z, \ldots$

Probability of hitting target

$$\boxed{x = \frac{1}{1+1} = \frac{1}{2}}$$

According to constraints given in question.

$y = ?$

In the 3rd turn, it is either Hit / Miss.

e.g. For 3rd turn $P(\text{hit}) = 1/2$ $P(\text{Miss}) = 1/2$

So for first 3 turns there are 2 cases

H M H OR H M M

$$y = \underbrace{\frac{1}{2}}_{\substack{3^{rd}\text{ turns}\\ \text{was hit}}} \times \underbrace{\left(\frac{2}{3}\right)}_{\substack{2\text{ hit}\\ \text{and}\\ 3\text{ arrows}}} + \underbrace{\frac{1}{2}}_{\substack{3^{rd}\\ \text{turn}\\ \text{was}\\ \text{Miss}}} \times \underbrace{\left(\frac{1}{3}\right)}_{\substack{1\text{ hit}\\ \text{and}\\ 3\text{ arrows}}} = \frac{1}{2}$$

$$\Rightarrow \boxed{y = 1/2}$$

$z = ?$

Now for z there are 4 cases

HMHH OR HMHM or HMMH or HMMM

$$z = \underbrace{\frac{1}{2}}_{\substack{\text{For } 3^{rd}\\ \text{turn}\\ (H)}} \times \underbrace{\frac{1}{2}}_{\substack{\text{For}\\ 4^{th}\\ \text{turn}\\ (H)}}\left(\frac{3}{4}\right) + \frac{1}{2}\times\frac{1}{2}\times\left(\frac{2}{4}\right) + \frac{1}{2}\times\frac{1}{2}\times\left(\frac{2}{4}\right) + \frac{1}{2}\times\frac{1}{2}\times\left(\frac{1}{4}\right)$$

$$\Rightarrow \boxed{z = 1/2}$$

$\therefore$ By induction hypothesis we can conclude that

$$P(\text{Hit}) = 1, 0, 1/2, 1/2, 1/2, \ldots$$

~~We~~ There are total 100 targets. We want to hit 50 of them. In the first 2 turns 1st will definetely be a Hit and second a miss

$\Rightarrow$ choosing 49 hits from 98 remaining arrows

$$\therefore \text{Prob.} = {}^{98}C_{49} \cdot \underbrace{\left(\frac{1}{2}\right)^{49}}_{\text{hit}} \cdot \underbrace{\left(\frac{1}{2}\right)^{49}}_{\text{Miss}} = {}^{98}C_{49} \cdot \left(\frac{1}{2}\right)^{98}$$

Ans. 27 (1) Minimum number of locks needed = b.
Because in the worst case, assume all the y or more children have same sets of keys.
In that they can open exactly b locks and not more than that.

~~⇒ Almirah should have~~

⇒ Almirah can have min. of b locks.

(2) Min no of keys to the locks each student must carry = 1.
Because if almirah has a locks and each student is carrying b keys and suppose if a > b then to open almirah ~~atleast~~ each student must ~~have a key different from rest~~.
have atleast 1 key different from rest.

⇒ Min. = 1.

Ans. 28 (1) There are 3 possibilities for the 3 trials we have

1st/2nd/3rd trial :- B R R R B R R R B

$$\text{Prob.} = \frac{2}{3} \times \frac{1}{4} \times \frac{2}{5} + \frac{1}{3} \times \frac{2}{4} \times \frac{2}{5} + \frac{1}{3} \times \frac{2}{4} \times \frac{2}{5}$$

$$= \frac{4+4+4}{3 \times 4 \times 5} = \frac{4 \times 3}{3 \times 4 \times 5} = \frac{1}{5}.$$

(2) P(RRR / all balls of same color)

$$= \frac{P(RRR)}{P(RRR) + P(BBB)} = \frac{\frac{1}{3} \times \frac{2}{4} \times \frac{3}{5}}{\frac{1}{3} \times \frac{2}{4} \times \frac{3}{5} + \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}}$$

$$= \frac{1 \times 2 \times 3}{1 \times 2 \times 3 + 2 \times 3 \times 4} = \frac{6}{6+24} = \frac{6}{30} = \frac{1}{5}.$$

(3) 1 − P(No blue ball is drawn)

$$1 - P(RRR) = 1 - \frac{1}{3} \times \frac{2}{4} \times \frac{3}{5} = \frac{9}{10}.$$

(4) 1 − P(No red ball is drawn)

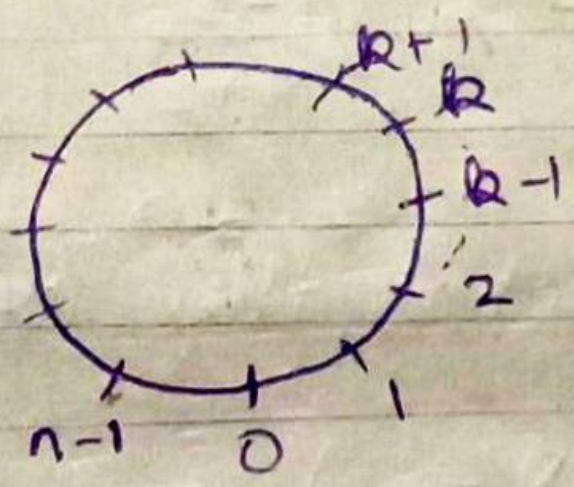
$$1 - P(BBB) = 1 - \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{3}{5}.$$

Ans. 29 Let $P(k)$ be the probability of visiting point k for the first time and already visiting rest of the points.

$\therefore$ Before k is visited one of $(k-1)$ and $(k+1)$ must be visited. Consider the time point when this happens for the first time. Because of symmetry, we can assume that it is $k-1$ that is visited. That k is the last point visited means that $(k+1)$ must be visited before k and this can only ~~happen~~ occur if the random walk passes clockwise from $(k-1)$ to $(k+1)$ before it visits k.

$P(0) = 0$, since robot starts from 0, so it is already visited.

Rest all points are equivalent.

so $\sum_{i=1}^{n-1} P(i) = 1$

$\Rightarrow \quad P(1) = P(2) \cdots = P(n-1) = \frac{1}{n-1}$.