Science-1: Problem Set on Special Theory of Relativity

Due on 16 Sept 2020 at 9pm; upload a single PDF document, can be scanned pages

1. Write the Lorentz Transform and the Inverse Lorentz Transform, with the setup of the inertial frames involved.
2. Consider the following scenarios:
   (a) Observer fixed, object moving away from the observer
   (b) Observer fixed, object moving toward the observer
   (c) Object fixed, Observer moving away from the object
   (d) Object fixed, observer moving towards the object

   Let the object be (a) rod of rest length L and (b) a clock with rest interval of τ. Make a table showing the respective measured properties in observer's frame.
3. A rocket has an antenna of length rest length L, sticking at an angle of θ with respect to the direction of motion in rocket's rest frame. What will be the angle as seen by earth, if the rocket is moving directly away from earth with speed v?
4. How fast does the spacecraft travel for one day on it to correspond to two days on earth?
5. It is known that in space, the plant grows at the rate of 1 inch per day, given that all other conditions are ideal. From a space station, rocket travels at a speed of $0.8c$ and returns after two weeks. By how much has the plant grown in height, assuming that the conditions were ideal on the trip?
6. It is known that Ursa Major (constellation) is moving away at the earth at the speed of 150,000 km/s. It is known that the due to the chemical composition of this constellation, a observer at rest with respect to the constellation should see high intensity at light wavelength of 550nm. Your astrophysicist friend is trying to detect Ursa Major using this light; what would you advice your friend about the wavelength of light received by his telescope on earth?
7. A spacecraft is launched from earth to travel to Ursa Major on a straight line path at a speed of $0.2c$.
   (a) What is the speed of constellation from the space craft?
   (b) If the distance to Ursa Major from earth is 330,000 light years, how many years will the rocket take for the round trip? (assume that the distance does not change)
   (c) It is estimated that the holographic data storage system located on the space craft can store the all data generated in 1 year in 1 micrometer cube of Unobtanium crystal. What is the size of the Unobtanium crystal required for the round trip to Ursa Major?
8. Find the equation of motion for the rocket in free space, if it spews exhaust gasses at a velocity of v_g (in rocket frame). Can you numerically solve by plugging in some rough numbers and plot velocity of rocket as a function of time?
9. The spacecraft sent to Ursa Major will send a FM signal (Frequency Modulation) with the carrier wave frequency of ν_0. While the trip could be on a straight line path, because of the requirement that spacecraft avoid or visit specific points, the path will deviate from the straight line path, but always at the same speed of $0.2c$. Given that earth also is not stationary, we need to calculate the formula for the carrier frequency for a simpler configuration: given that the angle made by earth and spacecraft velocities with respect to the line joining earth to spacecraft is given by θ_e and θ_s, what is the carrier frequency as seen by earth observatory?
10. A moving electron collides with electron at rest; as a result a electron-positron pair is created. If all four particles after the reaction have same velocity, the Kinetic energy is minimum (can you guess why?). Find this minimum KE value. Note that positron has same mass as electron, but has positive charge.
11. The spacecraft traveling to Ursa Major has a dilithium core: the dilithium core can be imagined to be a cubical volume within which matter-antimatter reaction can be contained without exploding and damaging the craft. The size of the core will regulate the amount of matter-antimatter reactants in it, and hence the power of the craft. It is known that dilithium core of 1 cm cube safely react 1 microgram of electron and positron releasing M MeV of energy.
    - Assuming that the setting of dilithium is such that, or every electron-positron reaction only photons are released. What is the minimum number of photons per electron-postion pair annihilation and why?
    - Assuming all generated energy goes into powering the craft, and space craft weighs 10,000 metric tonnes, and it expected to reach the terminal velocity in 60 seconds (this is the maximal amount of time a person can stand high g's reqired), what should be the size of dilithium core?