Linear Algebra – MAT 2610

Section 1.5 (Solution Sets of Linear Systems)

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Homogeneous

A system of linear equations is **homogeneous** if it can be written in the form $Ax = 0$ where A is an $m \times n$ matrix and 0 is the zero vector in R^m.

This system always has a solution $x = 0$ (the zero vector in R^n). This is called the **trivial solution**. Other solutions are **non-trivial solutions**.

The homogeneous equation $Ax = 0$ has a non-trivial solution if and only if the equation has at least one free variable.

Solution Sets

Below is an augmented matrix after row reduction representing a homogeneous system. Does it have a non-trivial solution? Describe the solution set.

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 2 & 0 \\ 0 & 1 & -5 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Parametric Vector Form

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 2 & 0 \\ 0 & 1 & -5 & 0 & -3 & 2 \\ 0 & 0 & 0 & 1 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Homogenous vs. Non-Homogenous

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 2 & 0 \\ 0 & 1 & -5 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ 5 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -2 \\ 3 \\ 0 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 2 & 0 \\ 0 & 1 & -5 & 0 & -3 & 2 \\ 0 & 0 & 0 & 1 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 4 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 5 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -2 \\ 3 \\ 0 \\ 4 \\ 1 \end{bmatrix}$$

Theorem 6

Suppose the equation $Ax = b$ is consistent for some given b, and let p be a solution. Then the solution set of $Ax = b$ is the set of all vectors of the form $w = p + v_h$ where v_h is any solution of the homogeneous equation $Ax = 0$.

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