Linear Algebra – MAT 2610

Section 2.3 (Characterizations of Invertible Matrices)

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Theorem 8 (Invertible Matrix Theorem)

If A is a square $n \times n$ matrix, then the following are equivalent:

a. The matrix A is invertible
b. The matrix A is row equivalent to I_n
c. There are n pivot positions in A
d. The equation $Ax = 0$ has only the trivial solution
e. The columns of A are linearly independent
f. The linear transformation $x \longmapsto Ax$ is one-to-one
g. The equation $Ax = b$ has at least one solution for every b in R^n
h. The columns of A span R^n
i. The linear transformation $x \longmapsto Ax$ maps R^n onto R^n
j. There is an $n \times n$ matrix C such that $AC = CA = I_n$
k. The matrix A^T is invertible

Ax

Theorem 9

Let $T: \mathrm{R}^{\mathrm{n}} \rightarrow \mathrm{R}^{\mathrm{n}}$ be a linear transformation and let A be the standard matrix for T. Then T is invertible if and only if A is an invertible matrix. In that case, the linear transformation S given by $S(x) = A^{-1}(x)$ is the unique function satisfying

- $S(T(X)) = x$ for all x in R^n; and
- $T(S(X)) = x$ for all x in R^n

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