Linear Algebra – MAT 2610

Section 1.3 (Vector Equations)

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A **row vector** is a matrix with one row, and a **column vector** is a matrix with one column. When we say **vector**, we will mean a column vector.

Given a vector u, the i^{th} entry in the vector is denoted u_i

$$u = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, u_2 = -1 \qquad v = \begin{bmatrix} 2 & a & b & -4 \end{bmatrix}, v_3 = b$$

In general, an upper-case letter is used to refer to a matrix, while a lower-case letter is used to refer to an individual element or a vector (although there are exceptions to this rule)

Vector Addition

Given two vectors u and v, their **sum** is the vector $u + v$ obtained by adding corresponding entries of u and v. Note that both vectors must be the same size.

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 + 1 \\ 4 - 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

A **scalar** is a real number (or a variable representing a real number)

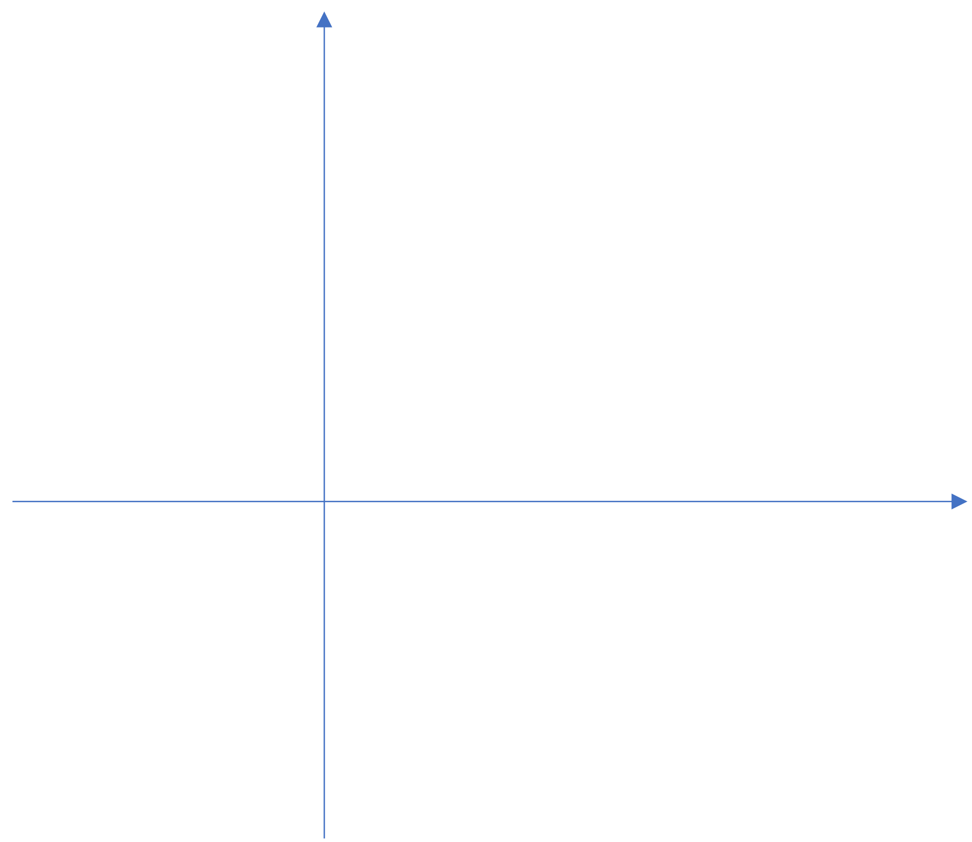
A scalar multiple of a vector u refers to the vector obtained by multiplying every entry in u by the same constant

$$2\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$-3\begin{bmatrix} -2 \\ \mathrm{b} - 2\mathrm{c} \end{bmatrix} = \begin{bmatrix} 6 \\ -3\mathrm{b} + 6\mathrm{c} \end{bmatrix}$$

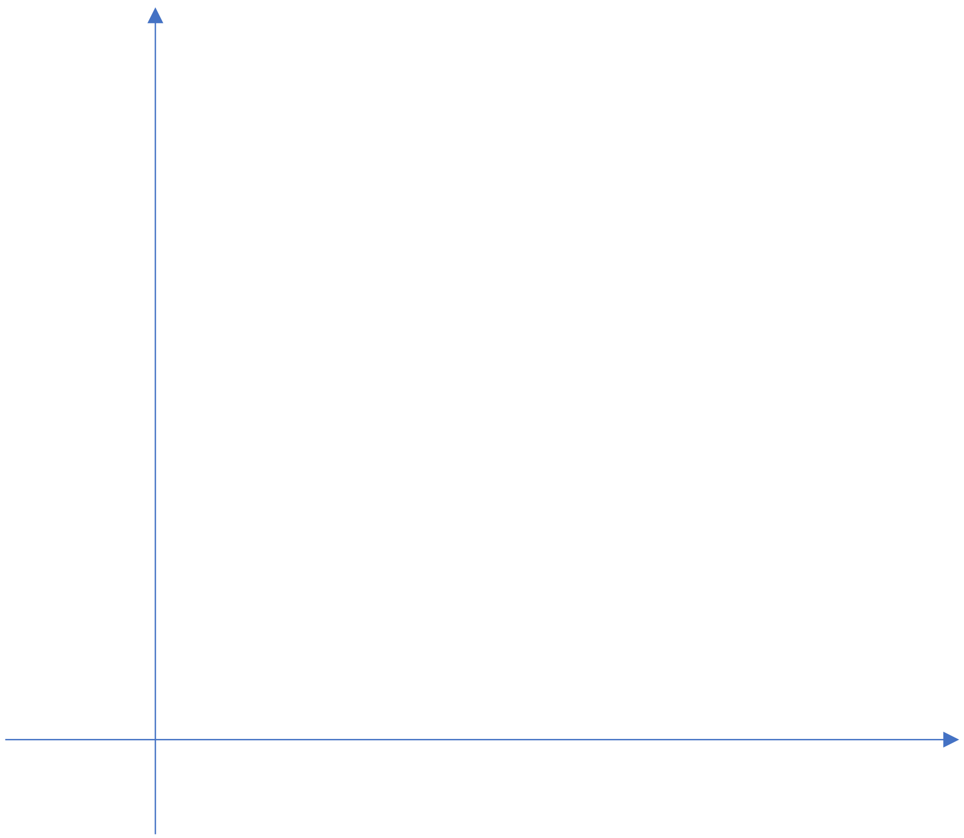
Geometric Descriptions of R^2

A vector in R^2 (such as $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ for example) can be represented by a geometric point on the



We can represent vector addition graphically using arrows by the *parallelogram law.* To add non-zero vectors u and v, first form a parallelogram with the adjacent sides u and v. Then the sum $u + v$ is the arrow along the diagonal of the parallelogram.

Example: Sketch the vectors $u = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$. Use the Parallelogram Law to sketch and calculate $u + v$.



Algebraic Properties of Vector Addition and Scalar Multiplication

Let u, v, w be vectors in R^n, and let s and t be any scalars. Then

a) $u + v = v + u$ (commutative law)

b) $(u + v) + w = u + (v + w)$ (associative law)

c) $u + 0 = u$

d) $u + (-u) = 0$

e) $s(u + v) = su + sv$

f) $(s + t)u = su + t\mathrm{u}$

g) $(st)u = s(tu)$

h) $1u = u$

Linear Combinations

A **linear combination of vectors** $u_1, u_2, \dots, u_n$ is any sum of the form $c_1u_1 + c_2u_2 + \cdots + c_nu_n$ where $c_1, c_2, \dots, c_n$ are scalars.

Examples:

If $u = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, then $3u - 2v = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$ is a linear combination of u and v.

$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ since $\begin{bmatrix} 1 \\ 3 \end{bmatrix} = -3\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Linear Combinations

Example:

If $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$, determine if $w = \begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix}$ is a linear combination of u and v.

Observation

A vector equation

$$c_1u_1 + c_2u_2 + \cdots + c_nu_n = b$$

Has the same solution set as the linear syste whose augmented matrix is

$$[u_1 \quad u_2 \quad \ldots \quad u_n \quad b]$$

In particular, b can be generated by a linear combination of $u_1, \ldots, u_n$ if and only if there exists a solution to the linear system corresponding to the matrix $[u_1 \quad u_2 \quad \ldots \quad u_n \quad b]$

Span

Definition: The **span** of a non-empty set of vectors $\{u_1, u_2, \ldots, u_k\}$ is the set of all possible linear combinations of those vectors. We denote this as $Span(\{u_1, u_2, \ldots, u_k\})$

Until now, we have been asking "can I build this specific vector from this other group of vectors". Now we are going to generalize this and ask "what are all the possible vectors that I can build from this group of vectors".

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