Linear Algebra – MAT 2610

Section 2.1 (Matrix Operations)

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Theorem 1

Let A, B, C be $m \times n$ matrices, and let s and t be any scalars. Then

a) $A + B = B + A$ (commutative law)

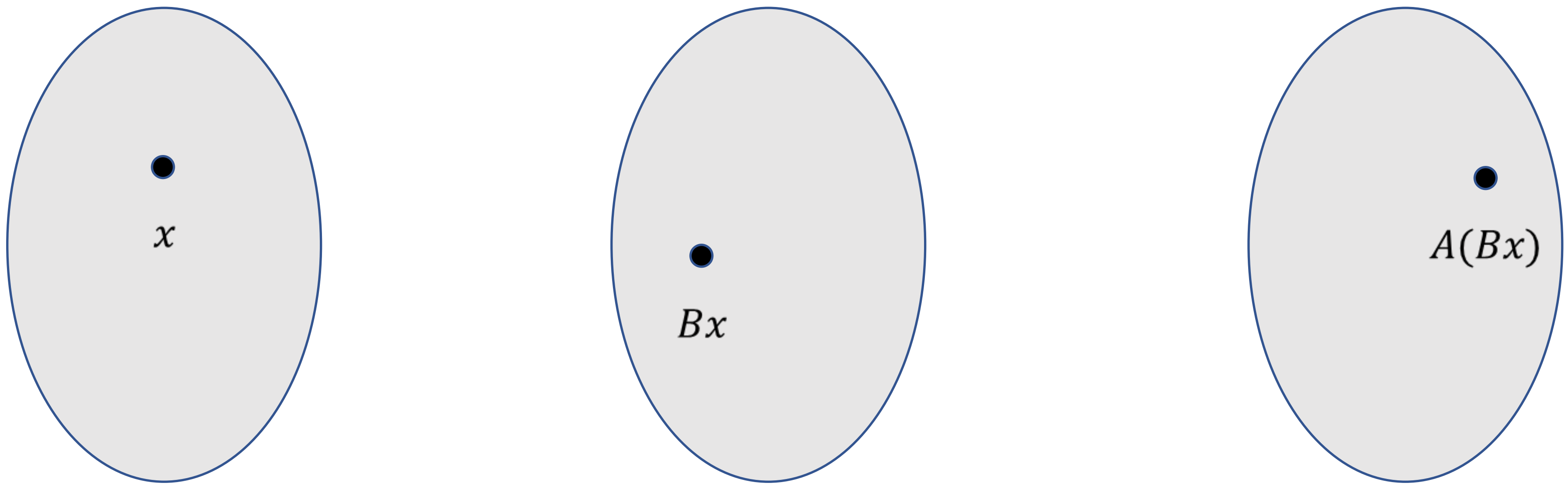
b) $(A + B) + C = A + (B + C)$ (associative law)

c) $A + 0 = A$

d) $s(A + B) = sA + sB$

e) $(s + t)A = sA + tA$

f) $(st)A = s(tA)$


x
Bx
$A(Bx)$

Definition: The product AB of the $m \times n$ matrix A and the $n \times p$ matrix B is defined as

$$AB = [Ab_1 \quad Ab_2 \quad \ldots \quad Ab_p]$$

where b_i is the i^{th} column of B

Example:

$$[2 \quad 3 \quad -2]\begin{bmatrix} r & -2 \\ s & t \\ 2 & -3 \end{bmatrix} =$$

Note: The product AB of the $m \times n$ matrix A and the $n \times p$ matrix B results in a $m \times p$ matrix.

Example: If A is a 4×3 matrix and B is a 3×5 matrix, then

- Its product AB will be a 4×5 matrix
- The entry in the $1st$ row and $1st$ column will be the product of the $1st$ row of A and the $1st$ column of B
- …
- The entry in the i^{th} row and j^{th}column will be the product of the i^{th} row of A and the j^{th} column of B

Row-Column rule for the (i, j)-entry of a Matrix Product:

To compute the (i, j)-entry of a Matrix Product AB, locate the i^{th} row of A and the j^{th} column of B as in the diagram. Moving across the the i^{th} row of A and the j^{th} column of B, multiply each entry of the row by the corresponding entry of the column. The sum of these products is the (i, j)-entry AB:

$$(AB)_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$$

Row j of B

Row i of A

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} a_{11} & \cdots & b_{1j} & \cdots & b_{1p} \\ b_{21} & \cdots & b_{2j} & \cdots & b_{2p} \\ \vdots & & \vdots & & \vdots \\ b_{n1} & \cdots & b_{nj} & \cdots & b_{np} \end{bmatrix}$$

Example: Let $A = \begin{bmatrix} -3 & 3 \\ 2 & -1 \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} 2 & x & 1 \\ y & -3 & z \end{bmatrix}$

1. $AB =$

2. $BA =$

Theorem 2

Let A and B be $k \times m$ matrices, C be an $m \times n$ matrix, and P and Q be $n \times p$ matrices. Then:

1. $(AC)P = A(CP)$
2. $C(P + Q) = CP + CQ$
3. $(A + B)C = AC + BC$
4. $s(AC) = (sA)C = A(sC)$ for any scalar s
5. $I_k A = A = AI_k$

Very Important Point I: Matrix Multiplication is not commutative. In other words, $AB \neq BA$ in general

Example: Let $A = \begin{bmatrix} -3 & 3 \\ 2 & -1 \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}$

1. $AB =$

2. $BA =$

Very Important Point II: Cancellation laws do not hold in general for matrix multiplication

Example: Let $A = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 5 \\ 0 & 5 \end{bmatrix}$

1. $AC = \begin{bmatrix} -12 & 0 \\ 8 & 0 \end{bmatrix}$

2. $BC = \begin{bmatrix} -12 & 0 \\ 8 & 0 \end{bmatrix}$

Very Important Point III: If the product of matrices is the zero matrix, you cannot conclude that either of your original matrices are zero

Example: Let $A = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}$, $\mathrm{B} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$

Then $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Definition: If A is an $n \times n$ matrix, then

$$AA = A^2$$
$$AAA = A^3$$
$$AAAA = A^4$$

and so on.

Also, $A^0 = I_n$

Definition: The (i, j)-entry of a matrix A is a **diagonal entry** if $i = j$. The set of diagonal entries is the **diagonal** of A. A square matrix A is a **diagonal matrix** if all non-diagonal entries are 0.

Theorem 3

Let A, B be $m \times n$ matrices, and let s be any scalar. Then

a) $(A^T)^T = A$

b) $(A + B)^T = A^T + B^T$

c) $(sA^T) = sA^T = s(A^T)$

d) $(AB)^T = B^T A^T$

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