(10 points) Let A, B and C be sets, decide if the following statements are true. Mark the correct statements with √ and false statements with ×.

(a) $P(\varnothing) = \varnothing$ ×

(b) $\varnothing \in P(\varnothing)$ and $\varnothing \subseteq P(\varnothing)$ √

(c) If $B \neq C$, then $A \cap B \neq A \cap C$ ×

(d) If $A - B = A - C$, then $B = C$. ×

(e) If $|A \cap B| = 4$, $|A| = 10$, $|B| = 9$, then $|A \cup B| = 15$. √

(10 points) Let A = { a , b , c ,d}, B={0,1}.

(a) How many relations there are from A to B?

(b) Let B^A={f | f : A -> B, f is everywhere defined}. Compute $|B^A|$.

(c) Is there a bijection k : B^A ->2^A, where 2^A is the power set of A? If the answer is yes, please define such a function. If the answer is no, explain why.

3. (10 points) Let A = {1,2,3,4}. Define the following binary relations on A:

R_1 = {(1,1),(1,2),(2,2),(3,3),(2,1),(4,4)};

R_2 = {(1,2),(3,3),(3,4)};

R_3 = {(1,2), (2,3),(4,4)},

R_4 = {(1,2), (2, 1), (3,4), (4,3)}

For every relation above state if it is reflexive, symmetric, antisymmetric and transitive by filling table 1. Mark Y for yes and N for no.

Table 1

	reflexive	Symmetric	antisymmetric	transitive
R_1	Y	Y	N	Y
R_2	N	N	Y	Y
R_3	N	N	Y	N
R_4	N	Y	N	N

4. (10 points) Suppose that following assumptions:

(1) Logic is not difficult, or not many students like logic;

(2) If mathematics is easy, then logic is not difficult.

By translating these assumptions into statements involving propositional variables and connectives, deciding whether each of the following is a valid conclusion of these assumptions:

(a) That mathematics is not easy, if many students like logic;

(b) That not many students like logic, if mathematics is not easy;

(c) That logic is not difficult or mathematics is not easy.

(c). valid.

5. (10 points) Define the following propositions and answer the following questions by drawing their truth tables.

   A. $p \Rightarrow \sim p$　　B. $(p \Rightarrow q) \vee (q \Rightarrow p)$

   C. $((p \Rightarrow q) \wedge \sim q) \Rightarrow p$　　D. $(p \Rightarrow q) \wedge (r \Rightarrow q) \wedge (p \vee r) \Rightarrow q$

   (a) Which of the propositions above are contingencies?

   (b) Which of the propositions above are tautologies?

   (c) Which of the propositions above are absurdities?

6. (10 points) Let W be the set of propositions containing three proposition variables p_1, p_2, and p_3. Define a relation R on W such that p R q if and only if $p \Leftrightarrow q$ is a tautology.

   (a) Prove that R is an equivalence relation on W.

   (b) Compute |W/R|.

(b).

$$|W/R| = 2^8 = 256.$$

7. **(10 points)** Let A={1, 2, 3, 4}, and R={(1, 4), (3, 1), (3, 2), (3, 3), (4, 2)}

   (a) Show the corresponding matrix M_R and draw the digraph of R;

   (b) Compute the matrix of R^2;

   (c) Compute the matrix of the transitive closure of R.

8. (10 points) Let S = {1, 2, 3, 4, 5} and A=S×S. Define the following relation R on A: (a, b) R (a', b') if and only if b=b'.

   (a) Show that R is an equivalence relation

   (b) Compute A/R and |A/R|.

9. (10 points) Let A={1, 2, 3, 4, 5, 6} and $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 5 & 4 & 6 \end{pmatrix}$ be a permutation of A, i.e. a bijection from A to A: p(1) = 3, p(2) = 1, p(3) = 2, ..., p(6) = 6

   (a) Compute p^{-1} and $p^2 = p \circ p$, and write them in the form above.

   (b) Is it possible that p^n is the identity function for some n? If yes, why and what is such a possible integer? If not, why?

10. (10 points) Let f: A->B and g: B ->C be two functions that are everywhere defined. Prove

   (a) If f and g are surjections, then $g \circ f$ is a surjection from A to C.

   (b) If $g \circ f$ is surjective(onto), then g is surjective.

   (c) Disprove that the following proposition: If $g \circ f$ is surjective, then f is surjective.