

Answer:

7. (10 points) Let $A = \{1, 2, 3, 4\}$, and $R = \{(1, 4), (3, 1), (3, 2), (3, 3), (4, 2)\}$
- (a) Show the corresponding matrix M_R and draw the digraph of R ;
 - (b) Compute the matrix of R^2 ;
 - (c) Compute the matrix of the transitive closure of R .

Answer:

8. (10 points) Let $S = \{1, 2, 3, 4, 5\}$ and $A = S \times S$. Define the following relation R on A : $(a, b) R (a', b')$ if and only if $b = b'$.

- (a) Show that R is an equivalence relation
- (b) Compute A/R and $|A/R|$.

Answer:

9. (10 points) Let $A = \{1, 2, 3, 4, 5, 6\}$ and $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 5 & 4 & 6 \end{pmatrix}$ be a permutation of A , i.e. a

bijection from A to A : $p(1) = 3, p(2) = 1, p(3) = 2, \dots, p(6) = 6$

- (a) Compute p^{-1} and $p^2 = p \circ p$, and write them in the form above.
- (b) Is it possible that p^n is the identity function for some n ? If yes, why and what is such a possible integer? If not, why?

Answer:

10. (10 points) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two functions that are everywhere defined. Prove

- (a) If f and g are surjections, then $g \circ f$ is a surjection from A to C .
- (b) If $g \circ f$ is surjective (onto), then g is surjective.
- (c) Disprove that the following proposition: If $g \circ f$ is surjective, then f is surjective.

Answer: