

《数学基础 II》期中考试试卷(下午)

(考试形式： 闭卷 考试时间：2 小时)

姓名：

学号：

班级：

注意：答案一律写在本试题卷中。请把此卷与草稿纸一并交回

1. (10 points) Let A, B and C be sets, decide if the following statements are true. Mark the correct statements with \checkmark and false statements with \times .

- (a) $P(\emptyset) = \emptyset$
- (b) $\emptyset \in P(\emptyset)$ and $\emptyset \subseteq P(\emptyset)$
- (c) If $B \neq C$, then $A \cap B \neq A \cap C$
- (d) If $A - B = A - C$, then $B = C$.
- (e) If $|A \cap B| = 4$, $|A| = 10$, $|B| = 9$, then $|A \cup B| = 15$.

2. (10 points) Let $A = \{a, b, c, d\}$, $B = \{0, 1\}$.

- (a) How many relations there are from A to B?
- (b) Let $B^A = \{f \mid f: A \rightarrow B, f \text{ is everywhere defined}\}$. Compute $|B^A|$.
- (c) Is there a bijection $k: B^A \rightarrow 2^A$, where 2^A is the power set of A? If the answer is yes, please define such a function. If the answer is no, explain why.

Answers:

3. (10 points) Let $A = \{1, 2, 3, 4\}$. Define the following binary relations on A:

$$R_1 = \{(1, 1), (1, 2), (2, 2), (3, 3), (2, 1), (4, 4)\};$$

$$R_2 = \{(1, 2), (3, 3), (3, 4)\};$$

$$R_3 = \{(1, 2), (2, 3), (4, 4)\},$$

$$R_4 = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

For every relation above state if it is reflexive, symmetric, antisymmetric and transitive by filling table 1. Mark Y for yes and N for no.

Table 1

	reflexive	Symmetric	antisymmetric	transitive
R_1				
R_2				
R_3				
R_4				

4. (10 points) Suppose that following assumptions:

- (1) Logic is not difficult, or not many students like logic;
- (2) If mathematics is easy, then logic is not difficult.

By translating these assumptions into statements involving propositional variables and connectives, deciding whether each of the following is a valid conclusion of these assumptions:

- (a) That mathematics is not easy, if many students like logic;
- (b) That not many students like logic, if mathematics is not easy;
- (c) That logic is not difficult or mathematics is not easy.

Answer:

5. (10 points) Define the following propositions and answer the following questions by drawing their truth tables.

$$A. p \Rightarrow \sim p \quad B. (p \Rightarrow q) \vee (q \Rightarrow p)$$

$$C. ((p \Rightarrow q) \wedge \sim q) \Rightarrow p \quad D. (p \Rightarrow q) \wedge (r \Rightarrow q) \wedge (p \vee r) \Rightarrow q$$

- (a) Which of the propositions above are contingencies?
- (b) Which of the propositions above are tautologies?
- (c) Which of the propositions above are absurdities?

Answers:

6. (10 points) Let W be the set of propositions containing three proposition variables p_1, p_2 and p_3 . Define a relation R on W such that $p R q$ if and only if $p \leftrightarrow q$ is a tautology.

- (a) Prove that R is an equivalence relation on W.
- (b) Compute $|W/R|$.