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A calibration equation for carbon resistance thermometers

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Abstract A three-parameter equation is found which gives

temperature as a function of resistance for Speer 470 Ω , $\frac{1}{2}$ w carbon resistors with an accuracy to $\frac{1}{3}\%$ over the range 0.6–4.2°K; the parameters can be determined from data at three calibration points. If separate equations, each with three parameters, are used for the ranges 0.6–2.2°K and 2.2–4.2°K the accuracy is everywhere better than 2 mdegK

1 Introduction

Although carbon resistors have several properties (high temperature coefficient of resistance, small thermal capacity and simplicity of measurement) which make them very well suited for use as low temperature thermometers, they also suffer from two serious disadvantages: irreproducibility on cycling from liquid helium temperatures to room temperature and back, and the lack of a simple equation accurately relating resistance and temperature. In consequence, if temperatures are to be measured with a precision of 1 mdegK or less, it is necessary to recalibrate a thermometer each time it is cooled. This involves bringing the thermometer into thermal equilibrium with a standard thermometer (such as a helium vapour pressure bath) at a large number of different temperatures and measuring its resistance. Then a complicated equation must be found to interpolate between the calibration points. There is no difficulty about the calculation if a computer is available, but the laborious task of calibration can take as much time as the experiment in which the thermometer is to be used.

Many authors have devised equations to describe the temperature variation of resistance of various carbon thermometers (see, for example, Clement and Quinell 1952, Pearce, Markham and Dillinger 1956, Rayne 1956, Nicol and Soller 1957). While these equations are excellent general descriptions of the behaviour of the thermometers, we have found them to be too inaccurate for use in low temperature calorimetry. We have found a new equation which describes the behaviour of our thermometers (Speer 470 Ω , $\frac{1}{2}$ w) with sufficient accuracy (i.e. to within about 1 mdegK) over the temperature range 0.6–4.2°K. Further, we have found it possible to determine the constants of the equation using a much smaller number of calibration points than in previous procedures.

We have tested our equation using several series of calibration data obtained for two thermometers over a period of several months. For purposes of comparison we describe the previous interpolation procedure, as well as presenting the results obtained with the new equation.

2 Thermometer calibration

The calibration data were taken during the course of a series

of specific heat measurements on alloy samples over the temperature range 0.6–4.2°K. The apparatus used was a ^3He cryostat, designed, built and described by Howe (1967 DPhil Thesis, University of Sussex). The thermometers were mounted on sample holders, which, for calibration purposes, could be thermally connected, via exchange gas and a mechanical heat switch in parallel, to the helium vapour pressure baths. In the range 2.2–4.2°K the temperature was determined by measuring the vapour pressure over a liquid ^4He bath, using a 20 mm bore mercury manometer which could be read with a cathetometer with an accuracy to 0.02 mm. Below 2.2°K a liquid ^3He bath was used, the pressure being measured with a similar mercury manometer for temperatures down to 1.5°K, and with a manometer containing Apiezon B oil for lower temperatures. Cooling to 0.6°K was achieved by pumping on a separate ^3He bath. This was to avoid the possibility of errors due to thermal gradients across liquid–solid interfaces, which might occur if the vapour pressure were measured over the bath through which heat is extracted from the samples. It was not considered necessary to use separate ^4He baths, since the Kapitza boundary resistance is much smaller for ^4He than for ^3He . The lowest vapour pressures were corrected for thermomolecular pressure effects using the equation given by Roberts and Syordiak (1956), and temperatures were obtained using the 1958 ^4He and 1962 ^3He temperature scales.

The dates of the various calibrations, and the symbols used to represent the corresponding data in the diagrams, are given in table 1. There were a few occasions, between the dates

Table 1

Thermometer	Run	Date	Symbol
1	11	6 March	●
1	12	23 April	■
1	13	29 May	▲
2	21	6 March	●
2	22	23 April	■
2	23	7 May	▲
2	24	5 Sept.	▼
2	25	17 Sept.	◆

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given, when the thermometers were cycled over the temperature range 293 to 4.2°K, but no data were obtained owing to failures of various parts of the apparatus.

3 Reproducibility

In order to demonstrate the variation of the characteristics of the thermometers from run to run, equations have been devised representing the average behaviour of each, over a number of runs. (We defer discussing the details of these equations until a later section.) The deviations of the temperatures calculated from these equations from those measured are plotted as functions of measured temperature in figure 1. The deviations vary by about 10 mdegK at 0.6°K to about 45 mdegK at 4.2°K or, alternatively, the fractional variation is about 2% at 0.6°K and 1% at 4.2°K. There seems to be no obvious way in which the variations in the characteristics of

a thermometer can be correlated with the number of times it has been thermally cycled, so the necessity of some kind of recalibration during each experiment is well established.

4 Orthogonal polynomials

In previous specific heat work the calibration data have been fitted by an equation of the form:

$$T^{-1} = b_{-1}x^{-1} + b_0 + b_1x + b_2x^2 + b_3x^3 \quad (1)$$

where $x = \lg R$, T is the temperature (°K) and R is the resistance of the thermometer (Ω). If b_2 and b_3 are set equal to zero, equation (1) becomes identical with that of Clement and Quinnell (1952), but the extra terms have been found necessary in order to achieve the required accuracy with our thermometers. For the purpose of determining the coefficients b in equation (1) by a least-squares method, while avoiding the rounding errors often associated with the numerical solution of several simultaneous equations,† it is convenient to rewrite equation (1) in the following form:

$$xT^{-1} = s_0P_0 + s_1P_1 + s_2P_2 + s_3P_3 + s_4P_4 \quad (2)$$

where P_n is an n th order polynomial in x , defined by:

$$\begin{aligned} P_0 &= 1 \\ P_1 &= (x - k_1) \\ &\dots\dots\dots \\ P_n &= (x - k_n) P_{n-1} - l_n P_{n-2} \quad n \geq 2. \end{aligned} \quad (3)$$

The constants k_n , l_n and s_n are computed from the N pairs of calibration data (x_i , T_i) using the recurrence relations:

$$\left. \begin{aligned} k_n &= \frac{\sum_{i=1}^N x_i P_{n-1}^2(x_i)}{\sum_{i=1}^N P_{n-1}^2(x_i)} \\ l_n &= \frac{\sum_{i=1}^N P_{n-1}^2(x_i)}{\sum_{i=1}^N P_{n-2}^2(x_i)} \\ s_n &= \frac{\sum_{i=1}^N (x_i T_i^{-1}) P_n(x_i)}{\sum_{i=1}^N P_n^2(x_i)} \end{aligned} \right\} \quad (4)$$

In this way the terms in equation (2) can be computed successively, each being independent of those that follow, and the process continued until the equation fits the data with sufficient accuracy. For our data it was found that no improvement to the fit could be obtained by adding fifth or higher order polynomials, so the computation was continued only to the fourth order.

The constants computed from the calibration data are given in table 2. The coefficients b_n are sufficient to define the fitted equations, but, for the sake of illustration, the values of k_n , l_n and s_n computed for thermometer 1 (runs 11, 12 and 13) are also tabulated. The deviations of the computed equations from the measured temperatures are plotted in figures 2 and 3 (for thermometers 1 and 2).

On the whole the run to run variation of the parameters k , l and s is quite small, reflecting a maximum irreproducibility of about 2% in the thermometer characteristic. It is, however, very inconvenient to have to use twelve parameters to describe the behaviour of a thermometer, but, when the equations are rewritten in the form of (1) there seems to be no connection between them whatsoever; the coefficients b_n fluctuate enormously. This indicates that (1) is in fact rather a poor representation of the characteristic, but can be forced to fit the data by the powerful method of orthogonal polynomials. In theory it is possible to determine the five coefficients b_n from only five data points, but the rounding errors that would

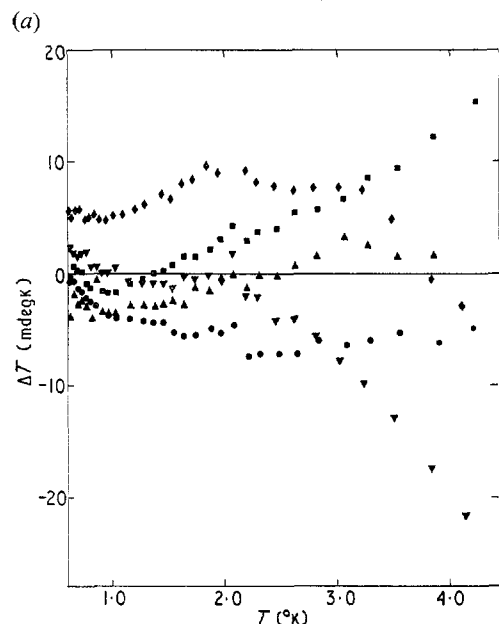
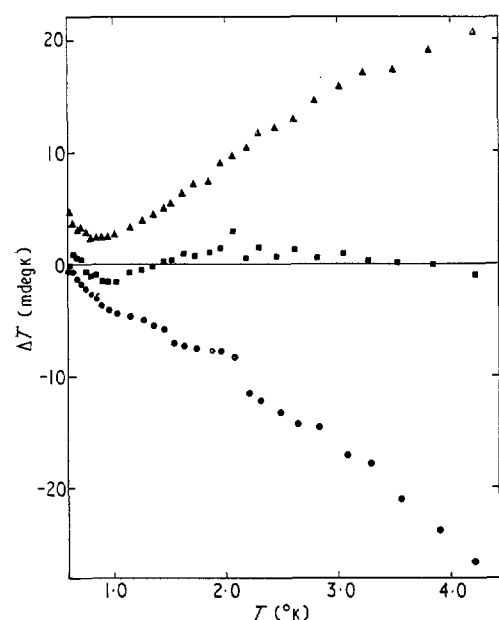


Figure 1 ΔT plotted against T for (a) thermometer 1 and (b) thermometer 2. $\Delta T = T_c - T_m$; T_m , measured temperature; T_c , temperature calculated from equation (6) with average parameters given in table 3

† For a complete discussion of the method, see Howe (1967 DPhil Thesis, University of Sussex).

Table 2 Constants used for polynomials

Run	n	k_n	l_n	s_n	
11	0	—	—	2.595918	
	1	3.205085	—	11.967456	
	2	3.258115	0.020040	13.938948	
	3	3.241460	0.014944	2.183418	
	4	3.258999	0.017061	0.733846	
12	0	—	—	2.601364	
	1	3.204776	—	11.965837	
	2	3.257162	0.020075	13.892616	
	3	3.239303	0.015122	2.258151	
	4	3.257073	0.017334	-0.122467	
13	0	—	—	2.613900	
	1	3.204771	—	12.038575	
	2	3.257505	0.020109	14.146652	
	3	3.240021	0.015324	3.102485	
	4	3.257162	0.017781	1.710332	
	b_{-1}	b_0	b_1	b_2	b_3
11	116.4106	-109.3256	39.1172	-7.3298	0.7338
12	19.4767	9.5654	-15.7193	-11.8742	-0.1225
13	194.3247	-214.2030	91.6701	-19.0625	1.7103
21	124.7661	-118.6370	42.6152	-7.9042	0.7609
22	31.9004	-4.1925	-10.2736	2.9586	-0.7591
23	-258.1984	365.4419	-186.8733	40.4556	-3.0614
24	-20.8828	59.3154	-38.8202	8.6336	-0.4963
25	-514.4775	674.8380	-326.3890	68.2843	-5.1312

occur in the solution of five simultaneous equations might be very large and, even if the coefficients could be determined exactly, there could be significant deviations from the data in between the calibration points. These difficulties are avoided by finding polynomials orthogonal to the data, provided that enough are available. While the method seems useful for fitting an equation to a complete set of data with a precision

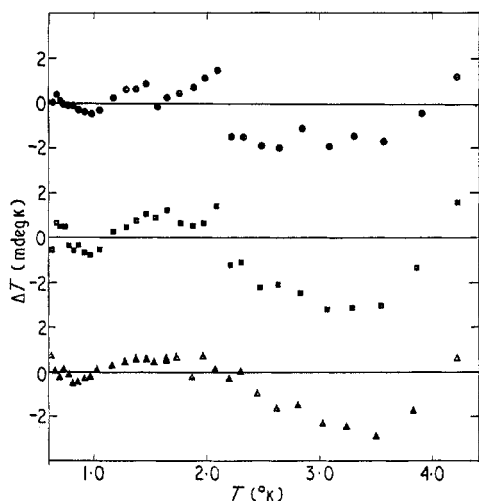


Figure 2 ΔT plotted against T for thermometer 1. $\Delta T = T_c - T_m$; T_c , temperature calculated from equation (1) with coefficients given in table 2

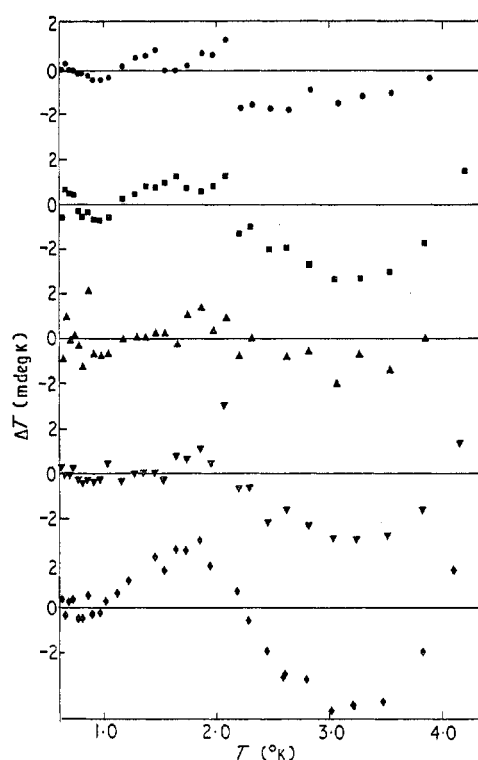


Figure 3 ΔT plotted against T for thermometer 2. $\Delta T = T_c - T_m$; T_c , temperature calculated from equation (1) with coefficients given in table 2

of about 0.1%, there is little hope that it could be adapted to calculating the variation of thermometer characteristics from run to run without taking a full set of data each time. Accordingly, with this object in view, we have attempted to devise a simpler equation which can be fitted more naturally to the data.

5 A three-parameter equation

Many authors have used simple two-parameter equations of the form suggested by Pearce *et al.* (1956):

$$T = A \lg R / (\lg R - B)^2. \quad (5)$$

This equation fits our data quite well, but the maximum deviation, about 10 mdegK, is not tolerable in accurate calorimetry. We have achieved better results with the equation:

$$T = A / (\lg R - B)^P \quad (6)$$

using three pairs of calibration data to determine the parameters A , B and P . Initially, the temperatures at which calibration data were taken were about 0.75, 1.6 and 3.0°K; the values of A , B and P found for each run are given in table 3, and the deviations of equation (6) from the rest of

Table 3 Parameters of equation (6) determined from data taken at 0.75, 1.6 and 3.0°K

Run	A	B	P
11	0.264671	2.80803	1.78927
12	0.264627	2.81034	1.77330
13	0.263868	2.80624	1.79282
Average	0.264389	2.80820	1.78513
21	0.266302	2.80920	1.77629
22	0.266074	2.81239	1.75762
23	0.267217	2.81152	1.75987
24	0.263951	2.81363	1.76053
25	0.262566	2.81259	1.76640
Average	0.265222	2.81187	1.76414

the data are plotted in figures 4 and 5 (thermometers 1 and 2). On the whole these deviations are not very much greater than those shown in figures 2 and 3, and, considering they result from three pairs of calibration rather than thirty, represent a worthwhile saving in time and labour. There are, however, two unsatisfactory aspects in these deviation graphs: they have a steep negative slope between 0.6 and 1.0°K and, in several cases, there is a discontinuity of a few millidegrees at about 2.2°K. The second is almost certainly due to some discrepancy in the use of helium vapour pressure measurements to determine the temperature; below 2.2°K ^3He was used, and at higher temperatures ^4He . The cause of this discrepancy has not yet been discovered, but it seems likely that the precautions taken to ensure that the helium liquid at the top of the baths was in thermal equilibrium with the thermometers were inadequate. There seems to be some qualitative difference between the deviations above 3°K for both thermometers for runs before and after the end of April, but we are not aware of any changes made in the experimental conditions at this time. The differences between the deviations are, however, much smaller than the differences between the corresponding calibration equations which can, therefore, be considered moderately successful.

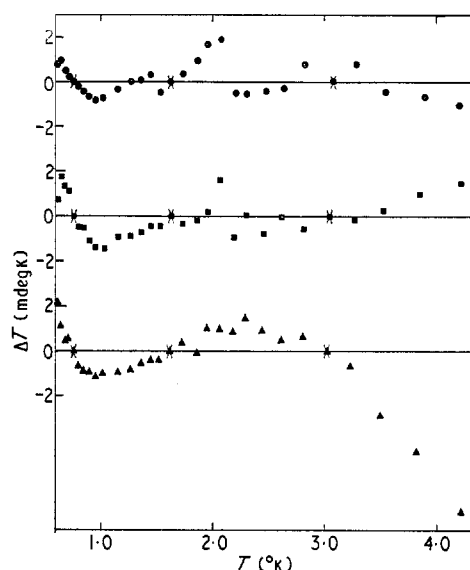


Figure 4 ΔT plotted against T for thermometer 1. $\Delta T = T_c - T_m$; T_c , temperature calculated from equation (6) with parameters given in table 3

Attempts to fit a smooth equation to discontinuous data can achieve only limited success (this also applies to the orthogonal polynomials, whose fit seems to have been affected by this discrepancy) so separate equations have been found to fit the data in the temperature ranges above and below 2.2°K. In the higher range, equation (6) was fitted to calibration data taken at about 2.3, 3.0 and 4.2°K, and in the lower range at about 0.75, 1.5 and 2.0°K. The last two temperatures

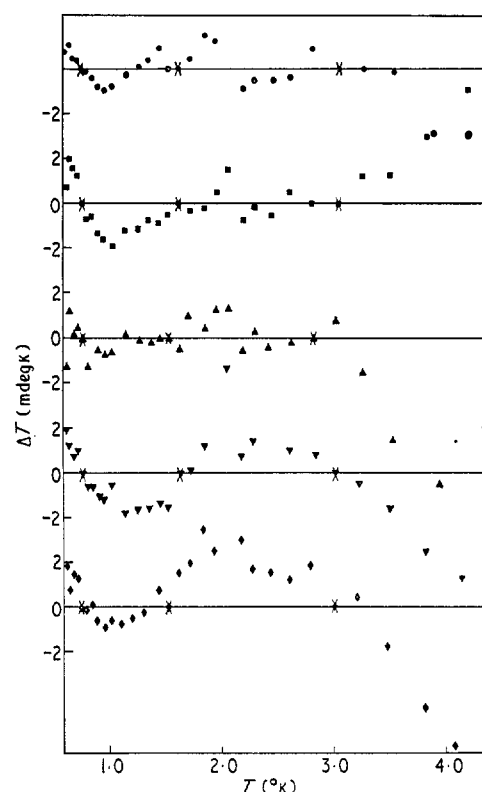


Figure 5 ΔT plotted against T for thermometer 2. $\Delta T = T_c - T_m$; T_c , temperature calculated from equation (6) with parameters given in table 3

Table 4

Run	Helium-3 range (fitted at 0.75, 1.5 and 2.0°K)			Helium-4 range (fitted at 2.3, 3.0 and 4.2°K)		
	A	B	P	A	B	P
11	0.265319	2.80586	1.79706	0.266407	2.81042	1.77229
12	0.265714	2.80544	1.79311	0.263774	2.80872	1.78398
13	0.264699	2.80250	1.80812	0.264328	2.81026	1.77097
21	0.266701	2.80838	1.77825	0.268515	2.81271	1.75248
22	0.267201	2.80692	1.78000	0.263997	2.80821	1.78462
23	0.267846	2.80758	1.77636	0.271429	2.81948	1.70882
24	0.266594	2.80170	1.80876	0.263769	2.81537	1.75219
25	0.263820	2.80383	1.80400	0.262485	2.81524	1.75319

were chosen as being likely to give the most accurate data, the ^3He vapour pressures being the highest measured with the oil and mercury manometers respectively. 0.75°K was chosen because there are considerable uncertainties in the temperature measurements below this point.

The values of A , B and P calculated for each range are given in table 4 and the corresponding deviations are plotted in figures 6 and 7 (thermometers 1 and 2). In every case the majority of the deviations are less than one millidegree, and all of them are less than two. The fit is therefore considerably better than that obtained by the method of orthogonal polynomials, and has the added advantage that it requires only six pairs of calibration data. There is still some evidence of systematic deviation below 1.0°K, which is not so apparent in the case of the orthogonal polynomials; presumably the latter equations, containing a higher number of terms, are better able to fit rapidly varying data. The low-temperature deviation could possibly be reduced by adding correction terms to the calibration equation, but it is doubtful whether the loss in simplicity would be compensated by the gain in accuracy.

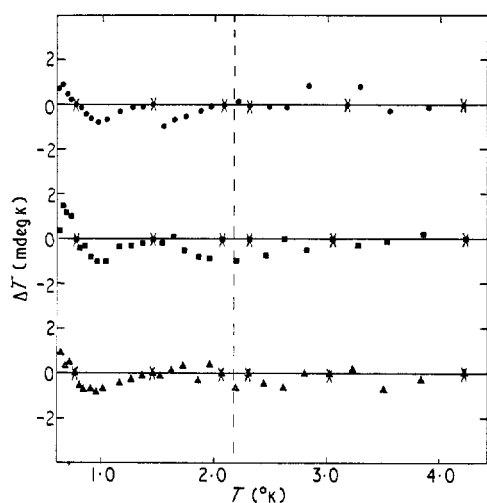


Figure 6 ΔT plotted against T for thermometer 1. $\Delta T = T_c - T_m$; T_c , temperature calculated from equation (6) with parameters given in table 4. The vertical line divides the ^3He and ^4He temperature ranges

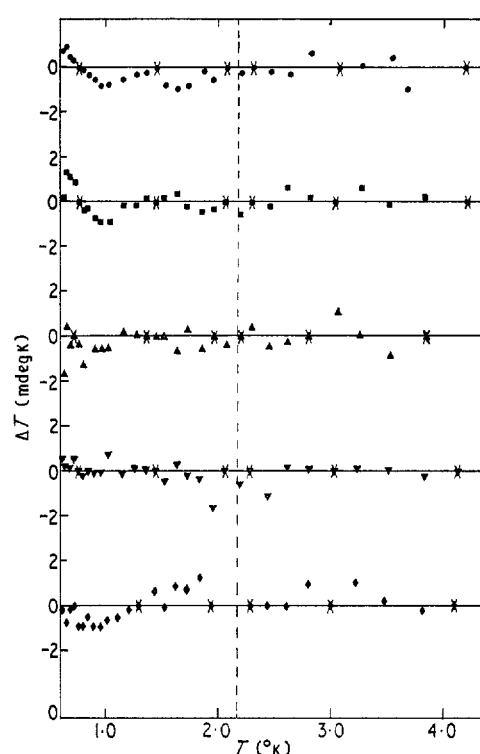


Figure 7 ΔT plotted against T for thermometer 2. $\Delta T = T_c - T_m$; T_c , temperature calculated from equation (6) with parameters given in table 4. The vertical line divides the ^3He and ^4He temperature ranges

An undesirable consequence of using different equations for different temperatures is that there is a discontinuity at the point common to both ranges. The magnitude of this discontinuity is given for each run in table 5. At present there is no satisfactory way of removing the discontinuity, since it is present in the original data.

6 Conclusions

In spite of the various difficulties discussed above, it is in fact possible to find three parameters for equation (6) which will give a fit to the characteristics of our 470 Ω Speer resistors to within $\frac{1}{2}\%$ over the temperature range 0.6–4.2°K. Further, to achieve this it is necessary to calibrate the thermometer at only three temperatures. Such a fit is comparable to that obtained by the method of orthogonal polynomials, which involves much more laborious calibration. A considerably

Table 5 Discontinuities in calculated temperatures at 2.2°K

Run	$T_4 - T_3$ (mdegk)
11	3.04
12	2.20
13	-0.04
21	2.75
22	1.98
23	2.37
24	4.24
25	2.44

T_4 , temperature calculated from data in ^4He range; T_3 , temperature calculated from data in ^3He range

better fit can be obtained by using two different equations for the ranges 0.6–2.2 and 2.2–4.2°K, using three calibration points in each. The systematic behaviour of the deviations of equation (6) from the data and the discontinuities themselves in the data at 2.2°K indicate that our temperature scale is somewhat suspect, and should be investigated. This would possibly lead to a better-fitting equation.

We are presently searching for a similar equation to describe the behaviour of 220 Ω Speer resistors, which we use in the temperature range 0.04–1.0°K, but so far our success has been limited by considerable scatter in the calibration data.

Acknowledgments

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