State Feedback Control of Unbalanced Seesaw

Tengis Tserendondog
Mongolian University of Science and Technology
Department of Computer Engineering
Ulaanbaatar, Mongolia
tengis@must.edu.mn

Byambajav Ragchaa
Mongolian University of Science and Technology
Department of Computer Engineering
Ulaanbaatar, Mongolia
byamjav.r@must.edu.mn

Luubaatar Badarch
Mongolian University of Science and Technology
Department of Computer Engineering
Ulaanbaatar, Mongolia
luubaatar@must.edu.mn

Batmunkh Amar
Mongolian University of Science and Technology
Department of Computer Engineering
Ulaanbaatar, Mongolia
abatmunkh@must.edu.mn

***Abstract* - Many articles have been written on unmanned flying vehicle with multiple rotors. Manipulation of rotor speed is the basic approach for controlling the object. This article considers state feedback approach for controlling rotor speed fixed on one end of a lever of which central point is fixed as pivot point and the other end of the lever has some passive counterweight. Our goal is to balance the seesaw system using state space method in MATLAB environment and to compare the simulation and experimental results.**

Keywords – full state feedback, counterweight, pole, BLDC motor

I. INTRODUCTION

During last few years development of small sized and high performance microcontrollers have accelerated many types of unmanned flying vehicles with multiple rotors. Among them the quad copter takes important role due to its optimality and wide range of applications. Many research papers address quad copter related problems. Several papers [1, 2, 3, 4] describe mathematical models of quad copter based on state space approach and must be noted that in each of the research works uses feedback with PID control method due to its simplicity.

In order for controlling a multi rotor system it is necessary to study dynamic behavior of a simple unstable system. Unbalanced seesaw could be considered as one of many examples of such systems.

Considering above information and results of experiments we set a goal to derive simplified linear dynamic equations for seesaw in state space form with state feedback and simulate the model in MATLAB programming environment. In the study, it was assumed that the reference input parameter is an angle between horizontal axis and inclined seesaw lever.

II. DERIVATION OF SEESAW DYNAMIC EQUATIONS

Figure 1 illustrates seesaw dynamics where *a* is a pivot point and gravitational forces, due to two masses on two ends of the seesaw, causing torque to rotate. The torque is caused a sum of these forces tangential components to the rotating multiplied with corresponding distances from the pivot point. An external force F created by random touch acts in a distance r_1 from the pivot.

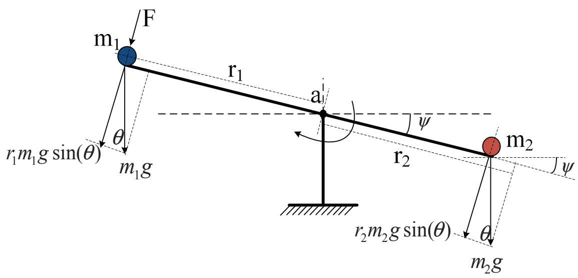

Fig. 1. Torques acting on seesaw.

The system's block diagram is drawn below and further is created in MATLAB Simulink library. Figure 2 shows the block diagram of the open loop system.

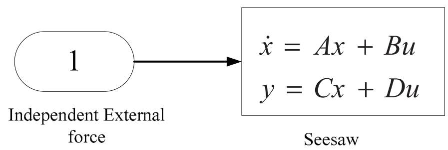

Fig. 2. Block diagram of open loop model for the seesaw system.

According to the Figure 1, assuming that ideal seesaw lever does not have a mass, following equation of motion is derived:

$$r_2 m_2 g \sin(\theta) - r_1 m_1 g \sin(\theta) = (m_1 r_1^2 + m_2 r_2^2)\ddot{\theta} + F r_2 \tag{1a}$$

Each component of the equation (1) has unit of torque [Joule]. From Figure 1, the angle between horizontal axis and the lever is $\psi = \pi/2 - \theta$. Since the lever is a rigid body, first and second derivatives of angles are $\dot{\psi} = -\dot{\theta}$ and $\ddot{\psi} = -\ddot{\theta}$, respectively. Consequently, we have

$$r_2 m_2 g \sin(\theta) - r_1 m_1 g \sin(\theta) = -(m_1 r_1^2 + m_2 r_2^2)\ddot{\theta} + F r_2 \tag{1b}$$

III. STATE SPACE REPRESENTATION OF A SEESAW

One of main advantages of state space method is modeling of multiple-input and multiple-output control system. As seen from the equation (1) the system under control is highly nonlinear, due to sine and cosine components [4], and it is necessary to apply linearization.

It is well known approximation that for small angles the function $Sin\ \theta\ \approx 1$ and $Cos\ \theta\ \approx 1$. Considering seesaw lever to be stable when $\theta = \pi/2$ so that θ angle swings around $\pi/2$ we can have approximate $Sin\ \theta$ to be $Sin\ \theta\ \approx 1+\psi$. In this case (1b) became (2).

$$(r_2m_2 - r_1m_1)g(1 \pm \psi) = -(m_1r_1^2 + m_2r_2^2)\ddot{\psi} + Fr_2 \qquad (2)$$

Denoting above parameters as $\psi = x_1$, $\dot{\psi} = \dot{x}_1 = x_2$, $\ddot{\psi} = \dot{x}_2$ to be state variables, one can reach following state space form (4).

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{(m_2r_2 - m_1r_1)}{(m_1r_1^2 + m_2r_2^2)}gx_1 + \frac{r_2}{(m_1r_1^2 + m_2r_2^2)}F - \frac{(m_2r_2 - m_1r_1)}{(m_1r_1^2 + m_2r_2^2)} \qquad (3)$$

Below shown the state space representation of the system in standard matrix form.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\dfrac{(m_2r_2 - m_2r_2)g}{(m_1r_1^2 + m_2r_2^2)} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} +$$
$$\begin{bmatrix}0 & 0\\ \frac{r_2}{(m_1r_1^2+m_2r_2^2)} & \frac{(m_2r_2-m_1r_1)}{(m_1r_1^2+m_2r_2^2)}\end{bmatrix}\begin{bmatrix}F\\ -1\end{bmatrix} \qquad (4)$$

Where matrices A and B can be written in the form of matrices for the open loop system:

$$A = \begin{bmatrix}0 & 1\\ -\frac{r_2m_2 - r_1m_1}{(m_1r_1^2+m_2r_2^2)}g & 0\end{bmatrix}$$
$$B = \begin{bmatrix}0 & 0\\ \frac{r_2}{(m_1r_1^2+m_2r_2^2)} & \frac{(m_2r_2-m_1r_1)}{(m_1r_1^2+m_2r_2^2)}\end{bmatrix} \qquad (5)$$

IV. FULL STATE FEEDBACK CONTROL

As it was mentioned before, control of systems, usually, carried out using PID control method. However, in this article we tried to consider usage of full state feedback control and its simulation in MATLAB.

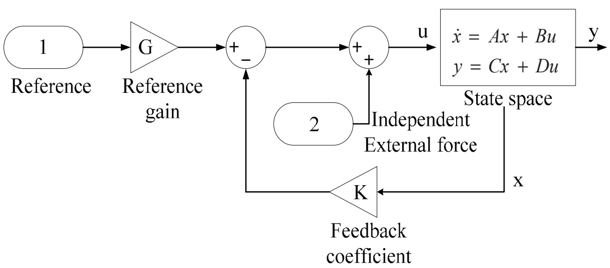

Fig. 3. Closed loop model block diagram of the system with state feedback control, reference and independent external force.

Full state feedback equations of the system in Figure 3 is written as (6) in time domain.

$$\dot{x} = A\dot{x} + B(G \cdot ref - Kx + Fr_3) \qquad (6)$$

where G is reference gain, K is feedback gain and F is external force. After some manipulation of (6) the updated matrix B becomes

$$B_{new} = B[G \quad 1] \qquad (7)$$

and size of which is *2x2*and

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = (A - BK)\begin{bmatrix}x_1\\ x_2\end{bmatrix} + B_{new}\begin{bmatrix}ref\\ F\end{bmatrix} \qquad (8)$$

The *ref* and *F* have size of $1 \times n$ and where *n* is number of simulation steps.

To derive state feedback law for a system with feedback its dynamic equations should be rewritten in Laplace domain. For the system shown in Figure 3 the state equations in *s*-domain could be written as (9).

$$sX(s) = AX(s) + B(G \cdot Ref(s) - K \cdot X(s) + F)$$
$$Y(s) = CX(s) \qquad (9)$$

From (9) the output of the system is (10).

$$Y(s) = \frac{CBG}{sI-(A-BK)}Ref(s) + \frac{CB}{sI-(A-BK)}F(s) \qquad (10)$$

As seen from (10) the system has two control inputs, 1-reference and 2-the independent external force (see Figure 3).

A. *Overshoot and settling time*

The procedure of pole placement set such that the rise time, overshoot and settling time are set in the same manner as in [7, 8, 9]. Using well known formulas (11), and under the given conditions

$$\xi = \frac{-\ln(OS/100)}{\sqrt{\pi^2 + (\ln(OS/100))^2}}$$
$$t_{settling} \approx \frac{4}{\xi\omega_n} \qquad (11)$$

B. *Pole placement*

Feedback gain matrix K should be defined after pole placement procedure of the characteristic equation of the system. Then using the pole dominant criterion we can choose, considering the requirements of overshoot and settling time, pole locations using *lsim()* function.

C. *Reference input gain*

For the system the state feedback control reference is the angle θ (see Figure 1). Assuming that all state variables are readable (measureable) that means that it is not necessary to use "observer" based feedback. Using state feedback law (10), and allowing the reference input, ref, we get the state feedback control system as shown in Figure 3.

To get full correspondence of the feedback states and the reference input there must be some relation, G (in some literature denoted as N_x, N_u), that is expressed as (12).

$$G = -(C(A - BK)^{-1}B)^{-1} \qquad (12)$$

V. SIMULATION AND RESULTS

The simulation is done in two steps. The first step is simulating open loop control and the second is simulating closed loop control with reference. The sample time step $dt = 0.005\ sec$ and duration of simulation time is within 10 seconds for continuous time model.

A. *Open loop simulation*

The simulation code for the open loop system shown in Figure 4 is follows:

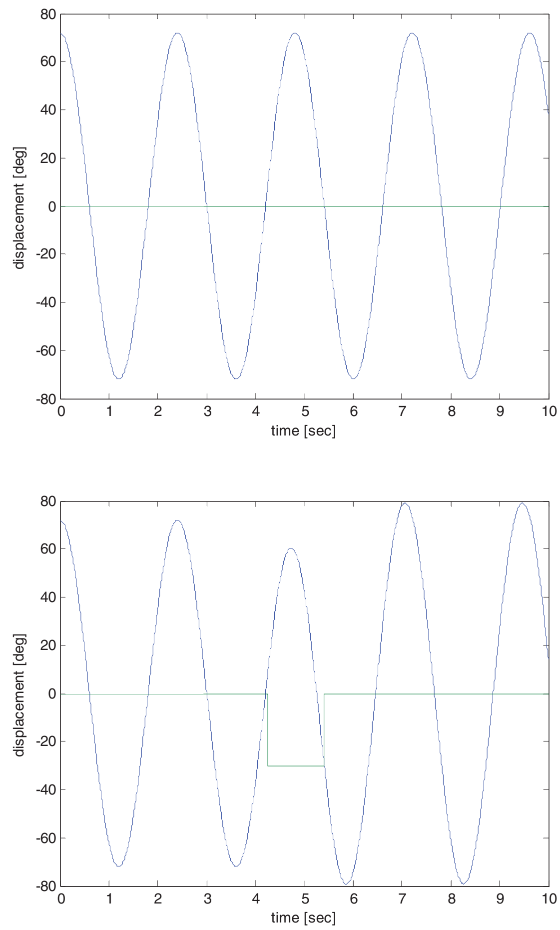

Fig 4. Open loop simulation result. Upper image shows open loop response without external force and lower show with negative external force respectively.

B. *Closed loop control and reference input*

We present, intuitively, the settling time for real model of the seesaw about 2 seconds and overshoot about no more than 4%. Using (11) calculated values for damping ratio $\xi = 0.72$ and natural frequency $\omega_n = 2.79Hz$.

Considering the above estimated values we have two poles $(-2 + 1.97j)$ and $(-2 - 1.96j)$.

Figure 5 and 6 correspondingly shows timing diagrams without and with positive independent external forces acting on seesaw at different time instances.

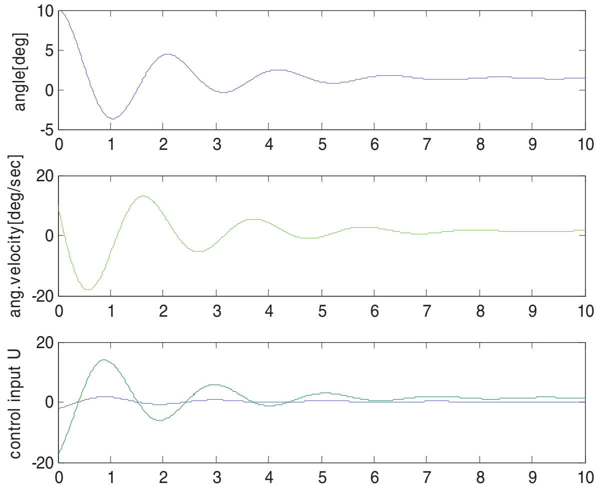

Fig 5. Closed loop simulation results without external force.

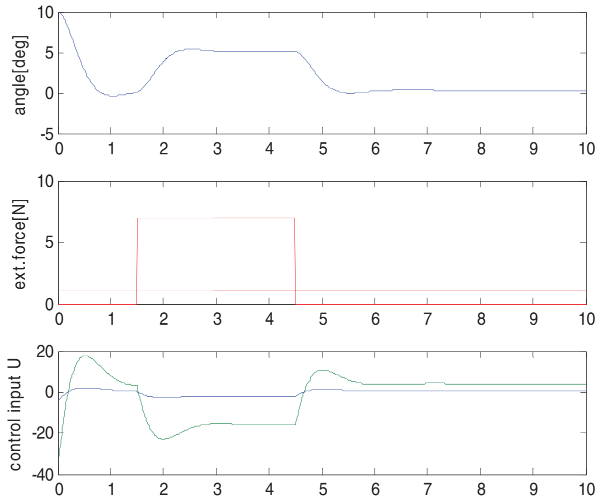

Fig 6. Closed loop simulation results with external force. Diagrams show positive force response.

VI. Experimental results of BLDC motor lifting force determination

Due to nonlinearity of BLDC and complexity of aerodynamics of propeller defining a precise mathematical model for our actuator system is not preferable.

In this case, we did some experimentations for determining response time of BLDC, applied voltage versus speed ratio and lifting force (Fig. 7, Fig. 8). The experiment is done on propeller Detrum BM2810CD-KV1080 BLDC and Detrum ESC 18A ESC.

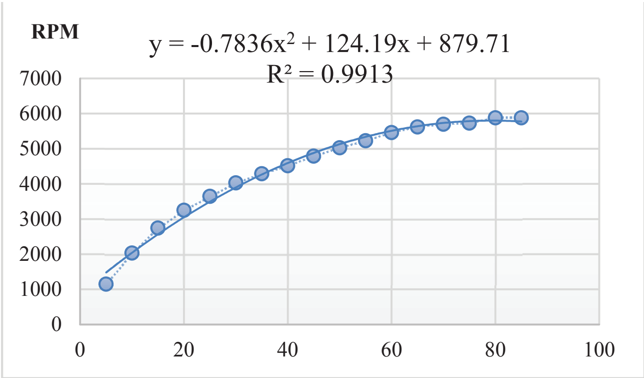

Fig. 7. *Applied voltage (by PWM duty cycle) and rotation speed*

Firstly, we changed the duty cycle of PWM signal to ESC and measured rotation speed of BLDC. The experimental result is provided in Figure 4.

We simply determined the response time by changing the voltage given to BLDC and measured the time the rotation speed reaches its estimated speed and that was about 1.53 seconds.

Although it is possible to determine lifting force by using some formula given as (13), in our seesaw actuator system we did some experimentation as shown in Figure 9.

$$F_D = \frac{1}{2} C_D \rho A_D v^2 \tag{13}$$

Here, C- is the lifting coefficient of propeller, ρ –air density, A- propeller rotation square, v- rotation speed of propeller.

In our lifting force determination experiments, we used various weights on the opposing side of the propeller on the seesaw and we incremented the duty cycle of PWM signal given to BLDC motor until it pulls the load up. The experimental results are shown in table 1.

TABLE I.

Duty cycle versus lifting force

№	Duty (%)	RPM	m (kg)	g	mg (N)	cos(30)* mg (N)
1	7	1670	0.03	9.81	0.29	0.25
2	12	2360	0.05	9.81	0.49	0.42
3	17	2820	0.07	9.81	0.68	0.59
4	23	3530	0.08	9.81	0.78	0.68
5	26	3750	0.10	9.81	0.98	0.85
6	30	4000	0.12	9.81	1.17	1.02
7	38	4390	0.14	9.81	1.37	1.19
8	43	4670	0.16	9.81	1.56	1.36
9	48	4950	0.18	9.81	1.76	1.53
10	51	5150	0.20	9.81	1.96	1.70
11	57	5446	0.22	9.81	2.15	1.87
12	62	5580	0.24	9.81	2.35	2.04

When seesaw is in its initial state, there is 30 degrees angle between ground and propeller side of the seesaw. Assuming the mass is m, the weight which is direct to earth is $W=mg$, and the actual force from propeller is perpendicular to the seesaw. In the other case, seesaw perpendicular component of W opposes propeller force. From our measurement the lifting force and voltage applied to BLDC has following relation as shown in Figure 8.

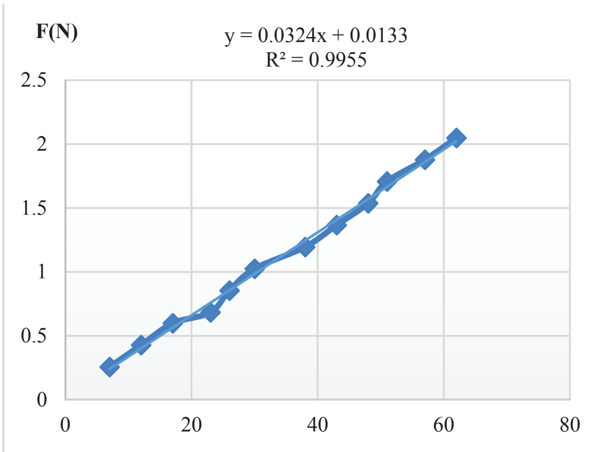

Fig. 8. *Lifting force and duty cycle of PWM relation.*

Actual experimental environment for seesaw lever is shown in Figure 9. While on the propeller opposite side there is stereo camera weighs 180gr, BLDC motor weighs 130gr.

Fig. 9. The experimental environment of the seesaw system.

The actual system's reaction to maintain the balancing is graphically depicted in Figure 10. The upper graph illustrates angular values with respect to time while lower part depicts the system's force to return seesaw into the stable state.

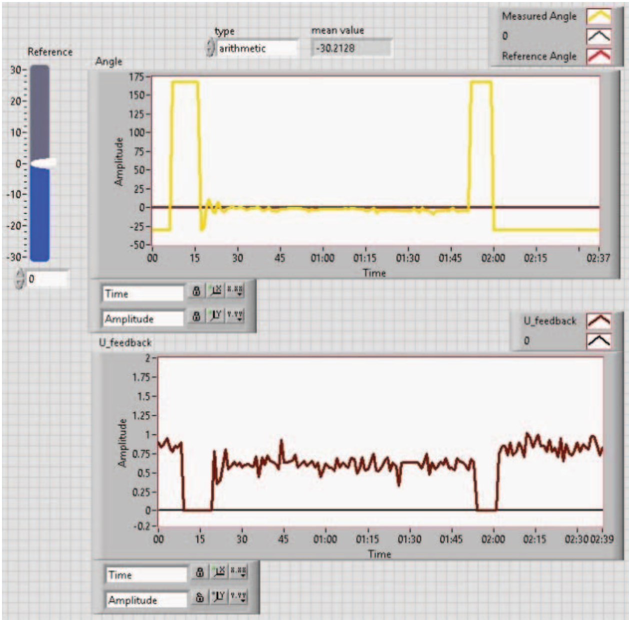

Fig. 10. Experimental results

CONCLUTION

In this article, we considered model of seesaw based on full state feedback control. The method uses simplified mathematical model considering several simplifications such as linearization, zeroing of some nonlinear functions. Although the model was highly simplified but the simulation result shows relatively real modeling.

The linearized model is simulated in open loop and closed loop modes. During the simulation there was chosen reference input control as angle. The results obtained from the simulation shown graphically in MATLAB environments. We hope that the model derived here will be useful to those who studies system balancing methods.

Experimental results prove that the model is applicable in a real environment. Filtering should be considered due to the control output distortion while the seesaw is stabilized. Our research is going to continue further by getting angular value from stereo camera and extended to be applied to a quad copter.

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