CS 458 Assignment 1:
Gaussian Discriminant Analysis(GDA)

Rocky Chen[1],
[1]Purdue University, College of Science, chen4514@purdue.edu

I. Introduction

Classification is a key task in machine learning, where the goal is to assign data points to the correct category based on their features. Linear Discriminant Analysis (LDA) and Quadratic Discriminant Analysis (QDA) are popular statistical methods that model class distributions using Gaussian assumptions. LDA creates linear decision boundaries by assuming shared covariance across classes. And QDA allows distinct covariances for each class, resulting in quadratic boundaries that capture more complex patterns.

II. Problem Formulation

We are given images $\mathbf{I} \in \mathbb{R}^{H\times W\times 3}$. For classification, we process each pixel individually, where each pixel is represented as $\mathbf{P} \in \mathbb{R}^3$. For the segmentation task, our goal is to assign a label to each pixel, classifying it into one of two categories:

$$h(x) = \begin{cases} 0 & \text{, if } x \text{ is part of the red barrel,} \\ 1 & \text{, if } x \text{ is not part of the red barrel.} \end{cases}$$

To create the classifer, we utilize gaussian discriminant analysis to perform LDA and QDA. The only difference is that LDA assume that different classes would share the same covariance matrix. To apply LDA or QDA, we must estimate the class parameters μ_k, Σ_k, and π_k for $k \in \{0, 1\}$. Here, μ_k is the class mean, Σ_k is the covariance matrix, and π_k is the prior probability of class k.
LDA:

$$\begin{aligned}\sigma_{\text{LDA}}(x) &= (\mu_0 - \mu_1)^\top \Sigma^{-1} x \\ &\quad - \frac{1}{2}\left(\mu_0^\top \Sigma^{-1}\mu_0 \;-\; \mu_1^\top \Sigma^{-1}\mu_1\right) \;+\; \ln\frac{\pi_0}{\pi_1} \quad (1)\end{aligned}$$

QDA:

$$\begin{aligned}\sigma_{\text{QDA}}(x) &= x^\top \left(\Sigma_-^{-1} - \Sigma_+^{-1}\right) x + 2\left(\Sigma_+^{-1}\mu_+ - \Sigma_-^{-1}\mu_-\right)^\top x \\ &\quad + (\mu_-^\top \Sigma_-^{-1}\mu_-) - (\mu_+^\top \Sigma_+^{-1}\mu_+) + \ln\frac{|\Sigma_-|}{|\Sigma_+|} + 2\ln\frac{\pi_+}{\pi_-} \quad (2)\end{aligned}$$

Using these techniques, we can then perform classification for each pixel:

$$h(x) = \begin{cases} 0 & \sigma(x) < 0, \\ 1 & \sigma(x) \geq 0. \end{cases}$$

III. Technical Approach

We begin by iterating through all pixels in the training images and computing the mean of each class. For class k, the mean can be calculated by

$$\mu_k = \frac{1}{N_k}\sum_{i=1}^{N_k} P_i,$$

Then prior for class k can be calculated by

$$\pi_k = \frac{N_k}{N_0 + N_1}.$$

Next, calculate the covariance matrix.

$$\Sigma_k = \frac{1}{N_k}\sum_{i=1}^{N_k}(P_i - \mu_k)(P_i - \mu_k)^\top.$$

Once these parameters are obtained, they are substituted into LDA or QDA to perform pixel-wise classification and construct the final segmentation map.

IV. Result

From the results, we observe that GDA with a common variance achieves high recall for the positive class but have extremely low precision, meaning it correctly identifies almost all positive pixels but also misclassifies many negative pixels as positive. On the other hand, GDA with variable variance significantly improves the balance, achieving both higher precision and strong recall for the positive class, and also maintaining high performance on the negative class.

TABLE I
GDA with Common Variance
(Label 0 = Positive, Label 1 = Negative)

	Precision	Recall
Label 0 (Positive)	9.96%	97.77%
Label 1 (Negative)	99.92%	75.19%

TABLE II
GDA with Variable Variance
(Label 0 = Positive, Label 1 = Negative)

	Precision	Recall
Label 0 (Positive)	31.48%	96.05%
Label 1 (Negative)	99.88%	94.13%

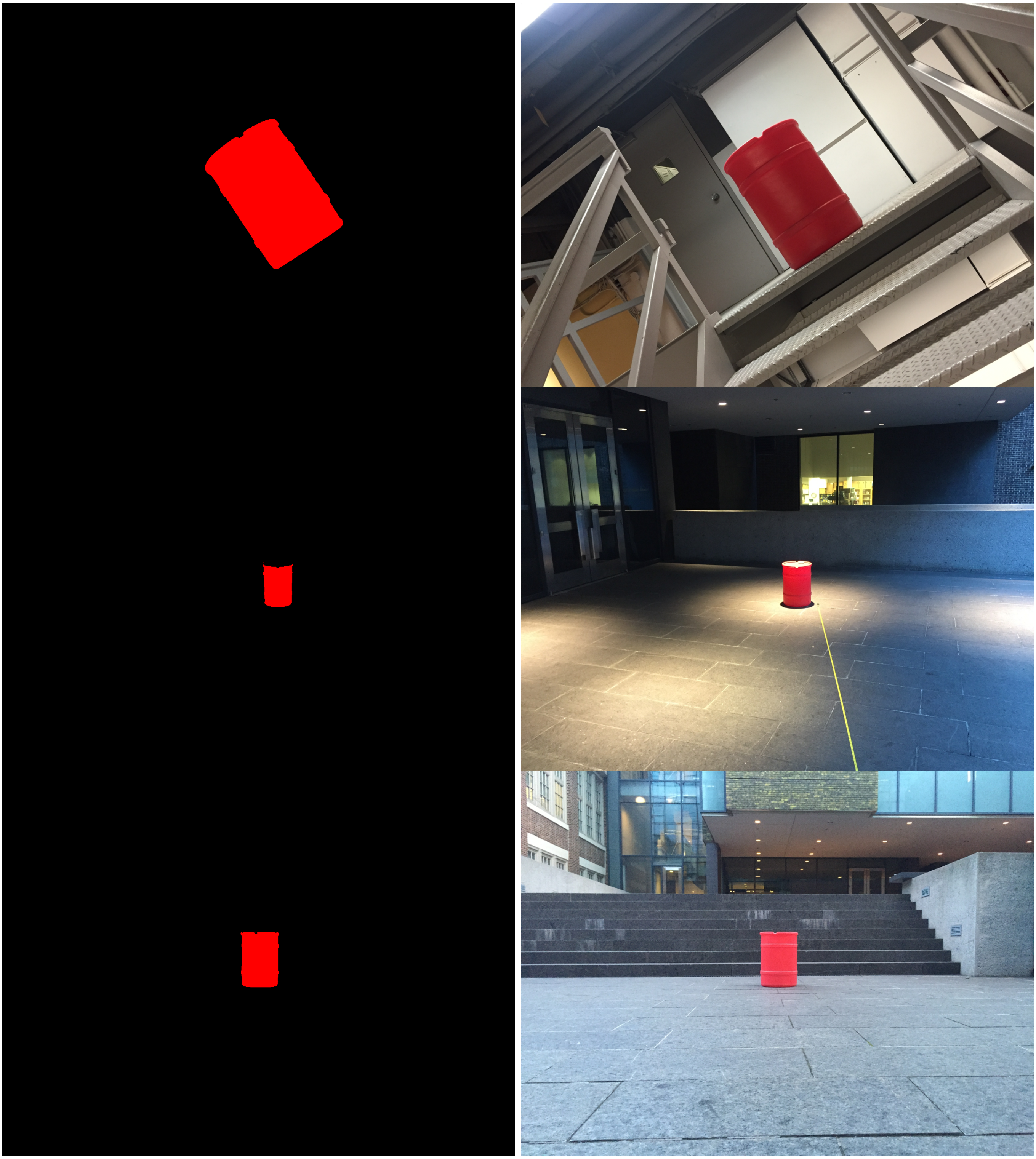

Fig. 1. Segmentation results with their corresponding original images.