MODULE 4

Problems:-

1. In an elementary school examination, the mean grade of 32 boys was 72 with a standard deviation of 8. While the mean grade of 36 girls was 75 with a standard deviation of 6. Test the hypothesis that the performance of girls is better than boys.

By data; $n_B = 32$; $\bar{x}_B = 72$ $\sigma_B = 8$

$n_G = 36$; $\bar{x}_G = 75$ $\sigma_G = 6$

We've;

Tests of significance of difference of mean

$$\boxed{Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}} = (\cancel{36} - \cancel{32})$$

$$= \frac{75 - 72}{\sqrt{\frac{\cancel{36}}{\cancel{36}} + \frac{\cancel{64}\,\cancel{8}\,2}{\cancel{32}\,4}}} = \boxed{\cancel{\frac{3}{\sqrt{1+\frac{8}{9}}}} \quad \cancel{\frac{3}{\frac{\sqrt{15}}{\sqrt{3}}}}}$$

$$= \frac{3}{\sqrt{1+2}} = \frac{3}{\sqrt{3}} = \sqrt{3} \;//$$

$$= 1.73$$

$Z = 1.732 \begin{cases} > Z_{0.05} = 1.645 \rightarrow \text{Reject (one tailed)} \\ < Z_{0.01} = 2.33 \rightarrow \text{accept (one tailed)} \end{cases}$ near to one tail

2. The difference of performance of girls and boys in the examination is significant at 5% level but not at 1% level.

[Z should be close or slightly greater than]

The hypothesis has been accepted at 1% level of significance [one-tailed test] and both 5% and 1% level of significance in 2-tailed test.

2. A sample of 100 bulbs produced by a company A showed a mean life of 1190 hours and a standard deviation of 90 hours. Also a sample of 75 bulbs produced by a company B showed a mean life of 1230 hours and a standard deviation of 120 hours. Is there a difference between mean lifetime of the bulbs manufactured by 2 companies. at (i) 5% level of significance (ii) 1% level of significance

$\sigma_A = 90 \quad \sigma_B = 120$

$\bar{x}_A = 1190 \quad \bar{x}_B = 1230$

$n_A = 100 \quad n_B = 75$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(1230 - 1190)}{\sqrt{\frac{(90)^2}{100} + \frac{(120)^2}{75}}}$$

$$= \frac{40}{\sqrt{81 + 192}} = \frac{40}{\sqrt{273}}$$

$$= \frac{40}{16.52}$$

$$Z = 2.42$$

(ii) The hypothesis is accepted at 1% level of significance in 2-tailed test. and rejected at 1% level in one-tailed test

(i) The hypothesis is rejected at 5% level of significance in both 1-tailed and 2 tailed test

There is a difference b/w the mean life of the bulbs at 1% level of significance (2 tailed test).

3. The mean of 2 large samples of 1000 and 2000 members are 168.75 cm and 170 cm respectively. Can the samples be regarded as drawn from the same population of standard deviation 6.25 cm?

$n_1 = 1000$; $n_2 = 2000$

$\bar{x}_1 = 168.75$; $\bar{x}_2 = 170$

$\sigma = 6.25$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{\overset{170-168.75}{(\cancel{2000-1000})}}{6.25\sqrt{\frac{1}{1000} + \frac{1}{2000}}} = \frac{1.250}{6.25\sqrt{0.001 + 0.0005}}$$

$$= \frac{\cancel{1000}\,1.25}{6.25 * 0.0387}$$

$$= 5.167$$

The hypothesis is rejected at both 1% and 5% level in both one and 2-tailed tests.

$\therefore$ We conclude that samples cannot be regarded as drawn from the same population.

4. The random sample of 1000 workers in a company has mean wage of ₹50/day and standard deviation of ₹15. Another sample of 1500 workers from another company has mean wage of ₹45/day and standard deviation of ₹20. Does the mean rate of wages varies b/w 2 companies?

$n_1 = 1000$; $\bar{x}_1 = 50$; $\sigma_1 = 15$; $n_2 = 1500$; $\bar{x}_2 = 45$; $\sigma_2 = 20$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(50-45)}{\sqrt{\frac{225}{1000} + \frac{400}{1500}}} = \frac{5}{\sqrt{0.225 + 0.266}} = 7.1355$$

Hypothesis is rejected at both 1% & 5% in one & 2-tailed. $\therefore$ wages doesn't varies //

Student's t distribution:

It is a test of distribution is to find the significance of a sample mean where the sample [small] is drawn from a normal population.
Let x_i $(i=1,2,3,4 \ldots\ldots n)$ be a sample of size n drawn from a normal population with mean μ and variance σ^2 then student's t distribution is defined as

$$\boxed{t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}} \Rightarrow \boxed{t = \frac{\bar{x}-\mu}{s} \cdot \sqrt{n}}$$

Here $\bar{x} = \frac{1}{n} \sum x_i$ is a sample mean.

$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$ is a sample variance

$\gamma = n-1 = v$ is Degree of freedom.

NOTE:- Degree of freedom:- The number of degrees of freedom usually denoted by $\gamma/(v)$ is the number of values in a set which may be assigned arbitrarily. It can be inpreted as a number of independent values generated by a sample of small size for estimating a population parameter.

1. A machine is expected to produce nails of length of 3 inches A random sample of 25 nails gave an average length 3.1 inch with standard deviation 0.3. Can it be said that the machine is producing nails as per specification? [$t_{0.05}$ for 24 d.f is 2.064]
degree of freedom

$n=25$ $s = \cancel{\sigma} = 0.3$ $\bar{x} = 3.1$

$\mu = 3$ ~~$s = \sigma^2 = 0.09$~~

$S = 0.3$

$t = \frac{\bar{x}-\mu}{s}\sqrt{n}$

$t = \frac{3.1-3}{0.3}\sqrt{25} = \frac{0.1}{0.3} \times 5 = 1.67$

$t_{0.05} = 2.064$ for 24 degreer of fredom

$\gamma = n - 1 = v$

$n = 25$

$\gamma = 25 - 1 = 24$

$t = 1.674$ is less than 2.064, thus the hypothesis that the machine is producing 25 nails as per specification is accepted at 5% level of significance.
$\hookrightarrow 0.05$

2. 10 individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches ($t_{0.05} = 2.262$ for 9 d.f)

$n = 10 \quad S = ? \quad \bar{x} = ?$

$\mu = 66$

$$\bar{x} = \frac{1}{n} \Sigma x_i$$

$$= \frac{63+63+66+67+68+69+70+70+71+71}{10}$$

$$\bar{x} = 67.8$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= \frac{1}{10-1} \left[(-4.8)^2 + (-4.8)^2 + (-1.8)^2 + (0.8)^2 + (0.2)^2 + (1.2)^2 + (2.2)^2 + (2.2)^2 + (3.2)^2 + (3.2)^2 \right]$$

$$= \frac{23.04 + 23.04 + 3.24 + 0.64 + 0.04 + 1.44 + 4.84 + 4.84 + 10.24 + 10.24}{9}$$

$S^2 = 9.069 \Rightarrow \boxed{S = 3.[illegible]}$

$\boxed{S = 3}$

$$t = \frac{\bar{x} - \mu}{S}\sqrt{n} = \frac{67.8 - 66}{3.01}\sqrt{10} = \frac{1.8 \times 3.16}{3.01} = 1.88 \Rightarrow \boxed{t = 1.88}$$

$t = 1.88$ is less than 2.262, thus the hypothesis that mean heigh is 66 at 5%.

3. Find the student's t for the following variable values in a sample of 8, −4, −2, −2, 0, 2, 2, 3, 3, taking the mean of universe to be 0.

$$\bar{x} = \frac{1}{n}\sum x \Rightarrow \bar{x} = \frac{\cancel{8}\,4-2-2+0+2+2+3+3}{8} = \frac{20}{8}$$

$$\bar{x} = 0.25$$

$$s^2 = \frac{1}{8-1}\Big[(-4-0.25)^2 + (-2-0.25)^2 + (-2-0.5)^2 + (0-0.25)^2 + (2-0.25)^2 + (2-0.25)^2 + (3-0.25)^2 + (3-0.25)^2\Big]$$

$$= \frac{1}{7}\Big[18.06 + 5.06 + 5.06 + 0.0625 + 3.06 + 3.06 + 7.56 + 7.56\Big]$$

$$s^2 = 7.07$$

$$\boxed{s = 2.65}$$

$$t = \frac{\bar{x} - \mu}{s}\cdot\sqrt{n} = \frac{0.25 - 0}{2.65}\sqrt{8}$$

$$\boxed{t = 0.266}$$ //

4. A group of boys and girls were given in an intelligence test. The mean score, standard deviation score and numbers in each group are as follows

	Boys	Girls
Mean	74	70
SD	8	10
n	12	10

Is a difference b/w the means of 2 groups significant at 5% level of significance ($t_{0.05} = 2.086$ for 20 d f).

We have students t distribution for significance of diff of sample mean is given by

$$\boxed{t = \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}}$$ where $$\boxed{S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$$

$n_1 = 12$; $n_2 = 10$; $S_1 = 8$; $S_2 = 10$; $x = 74$; $y = 70$

$$S^2 = \frac{(12)(8)^2 + (10)(10)^2}{12 + 10 - 2} = \frac{768 + 1000}{20}$$

$$S^2 = \frac{1768}{20} = 88.4$$

$$S = \sqrt{88.4} = 9.4$$

$$\boxed{S = 9.4}$$

$$t = \frac{74 - 70}{9.4\sqrt{\frac{1}{12} + \frac{1}{10}}} = \frac{4}{9.4\sqrt{0.08 + 0.1}}$$

$$= \frac{4}{9.4 * 0.42}$$

$$\boxed{t = 1.00}$$

$$t = 1 < 2.086$$

$\gamma = \overset{n-1}{12 - 1}$ $\quad \gamma = 10 - 1$

$= 11 \quad\quad = 9$

$\boxed{\text{total} = 20}$

Thus the hypothesis that there is a dif b/w the means of 2 groups is accepted at 5% level of significance.

5. A group of 10 boys fed on a diet A and another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weights.

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	10	1	2	8		

Test whether the diets A and B differ significantly regarding their effect on increase in weight.

$n_A = 10 \quad n_B = 8$

Diet A: $\bar{x} = \frac{5+6+8+1+12+4+3+9+6+10}{10} = 6.4$

Diet B: $\bar{y} = \frac{2+3+6+8+10+1+2+8}{8} = 5$

We have $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ for one group. but for 2 groups,

$$S^2 = \frac{1}{(n_1+n_2-2)}\left[\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2\right]$$

$$s^2 = \frac{1}{10+8-2}\Big[(-1.4)^2 + (-0.4)^2 + (1.6)^2 + (-5.4)^2 + (5.6)^2 + (-2.4)^2 + (-3.4)^2 + (+2.6)^2 + (-0.4)^2 + (3.6)^2 + (-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (5)^2 + (-4)^2 + (-3)^2 + (2)^2\Big]$$

$$= \frac{1}{16}\Big[1.96 + 0.16 + 2.56 + 29.16 + 31.36 + 5.76 + 11.56 + 6.76 + 0.16 + 12.96 + 9 + 4 + 1 + 9 + 25 + 16 + 9 + 4\Big]$$

$$s^2 = 11.21$$

$$\boxed{S = 3.39}$$

$$t = \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{6.4 - 5}{3.39\sqrt{\frac{1}{10} + \frac{1}{8}}} = \frac{1.4}{3.39 * 0.47}$$

$$\boxed{t = 0.87}$$

$t = 0.87 < 2.12$

Hypothesis is accepted thus doesn't differ.

Chi-square distribution: 'χ' → chi

Chi-square distribution provides a measure of correspondence b/w theoretical frequencies and observed frequencies.

If O_i ($i = 1, \ldots, n$) and E_i ($i = 1, \ldots, n$) are the observed and expected frequencies, the quantity Chi-square denoted by 'χ^2' and is defined as;

$$\boxed{\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}}$$

and degree of freedom;

$$\boxed{\gamma = n - 1}$$

→ Chi-square test as a test of goodness of fit

Chi-square test help us to test the goodness of fit of these distributions i.e., binomial, poisson, normal.

→ If the calculated value χ^2 is less than the table value of χ^2 at a specified value (level) of significance the hypothesis is accepted, otherwise, the hypothesis is rejected

1. A die is thrown 264 times and the number appearing on the face x follows the following frequency distribution.

x	1	2	3	4	5	6
f	40	32	28	58	54	60

Calculate the value of χ^2

The frequencies in the given table are the observed frequencies, now we need to find expected frequency as;

$$E_i = \frac{\overset{44}{\cancel{264}}}{6}$$ → 264 time, → No of faces

$$\boxed{E_i = 44}$$

We've;
$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(40-44)^2}{44} + \frac{(32-44)^2}{44} + \frac{(28-44)^2}{44} + \frac{(58-44)^2}{44} + \frac{(54-44)^2}{44} + \left(\frac{60-44}{44}\right)^2$$

$$= \frac{16}{44} + \frac{144}{44} + \frac{256}{44} + \frac{196}{44} + \frac{100}{44} + \frac{256}{44} = \frac{968}{44}$$

$$\boxed{\chi^2 = 22}$$ //

Q. 4 coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and test the goodness of fit ($\chi^2_{0.05} = 9.49$ for 4 d.f).

No of heads	0	1	2	3	4
Frequency	5	29	36	25	5

In the following data, observed frequencies are given, we need to find expected frequency by using binomial distribution.

Let 'x' denotes the no of heads & f is the corresponding frequencies.

$\therefore$ Mean $\mu = \dfrac{\Sigma f_i x_i}{\Sigma f} = \dfrac{0(5)+1(29)+2(36)+3(25)+4(5)}{5+29+36+25+5}$

$= \dfrac{29+72+75+20}{100}$

$\mu = 1.96$

$\boxed{\mu = 1.96}$

We've, $\mu = np$

$1.96 = 4p$

$p = 0.49 \quad q = 0.51$

We've, Binomial distribution,

$P(x) = nC_x p^x q^{n-x}$

$P(x) = 4C_x (0.49)^x (0.51)^{4-x}$

$x=0 \quad P(x) = 4C_0 (0.49)^0 (0.51)^4 = 0.067 \times 100 = 6.7$

$x=1 \quad P(1) = 4C_1 (0.49)^1 (0.51)^3 = 0.259 = 25.9$

$x=2 \quad P(2) = 0.374 = 37.4$

$x=3 \quad P(3) = 0.2400 = 24$

$x=4 \quad P(4) = 0.0576 = 5.76$

O_i	5	29	36	25	5
E_i	7	26	37	24	6

We've;

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(5-7)^2}{7} + \frac{(29-26)^2}{26} + \frac{(36-37)^2}{37} + \frac{(25-24)^2}{24} + \frac{(5-6)^2}{6}$$

$$= \frac{4}{7} + \frac{9}{26} + \frac{1}{37} + \frac{1}{24} + \frac{1}{6}$$

$$\chi^2 = 1.152 < 9.49$$

∴ Thus the hypothesis is that the fitness is good and can be accepted.

3. Fit a poisson distribution for the following data and test the goodness of fit, given that $\chi^2_{0.05} = 7.815$ for 3 d.f

x	0	1	2	3	4
f	122	60	15	2	1

We've, Poisson distribution; $P(x) = \frac{m^x e^{-m}}{x!}$

$$\mu = m = \frac{\Sigma x f}{\Sigma f} = \frac{60+30+6+4}{122+60+15+2+1}$$

$$= \frac{100}{200}$$

$$m = \mu = 0.5$$

$$P(x) = \frac{(0.5)^x e^{-0.5}}{x!}$$

$P(0) = 0.606$; $P(1) = 0.303$; $P(2) = 0.07$; $P(3) = 0.0126$
$\times 200$ $\times 200$ $\times 200$ $\times 200$
$P(4) = 0.00157$
$\times 200$

$P(0) = 121$; $P(1) = 60$; $P(2) = 15$; $P(3) = 3$; $P(4) =$

Q. 4 coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and test the goodness of fit ($\chi^2_{0.05} = 9.49$ for 4 d.f)

No of heads	0	1	2	3	4
Frequency	5	29	36	25	5

In the following data, observed frequencies are given, we need to find the expected frequency by using binomial distribution.

Let 'x' denotes the no of heads & f is the corresponding frequencies.

$$\therefore \text{Mean } \mu = \frac{\Sigma f_i x_i}{\Sigma f} = \frac{0(5)+1(29)+2(36)+3(25)+4(5)}{5+29+36+25+5}$$

$$= \frac{29+72+75+20}{100}$$

$$\mu = 1.96$$

$$\boxed{\mu = 1.96}$$

We've, $\mu = np$

$1.96 = 4p$

$p = 0.49 \quad q = 0.51$

We've, Binomial distribution.

$$P(x) = nC_x p^x q^{n-x}$$

$$P(x) = 4C_x (0.49)^x (0.51)^{4-x}$$

$x=0 \quad P(x) = 4C_0 (0.49)^0 (0.51)^4 = 0.067 \times 100 = 6.7$

$x=1 \quad P(1) = 4C_1 (0.49)^1 (0.51)^3 = 0.259 = 25.9$

$x=2 \quad P(2) = 0.374 = 37.4$

$x=3 \quad P(3) = 0.2400 = 24$

$x=4 \quad P(4) = 0.0576 = 5.76$

O_i	5	29	36	25	5
E_i	7	26	37	24	6

we,ve;

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(5-7)^2}{7} + \frac{(29-26)^2}{26} + \frac{(36-37)^2}{37} + \frac{(25-24)^2}{24} + \frac{(5-6)^2}{6}$$

$$= \frac{4}{7} + \frac{9}{26} + \frac{1}{37} + \frac{1}{24} + \frac{1}{6}$$

$$\chi^2 = 1.152 < 9.49$$

$\therefore$ Thus the hypothesis is that the fitness is good and can be accepted.

3. Fit a poisson distribution for the following data and test the goodness of fit, given that $\chi^2_{0.05} = 7.815$ for 3 d.f

x	0	1	2	3	4
f	122	60	15	2	1

We've, Poisson distribution; $P(x) = \frac{m^x e^{-m}}{x!}$

$$\mu = m = \frac{\Sigma x f}{\Sigma f} = \frac{60+30+6+4}{122+60+15+2+1}$$

$$= \frac{100}{200}$$

$$m = \mu = 0.5$$

$$P(x) = \frac{(0.5)^x e^{-0.5}}{x!}$$

$P(0) = 0.606$; $P(1) = 0.303$; $P(2) = 0.07$; $P(3) = 0.0126$
$\times 1200$ $\times 200$ $\times 200$ $\times 200$
$P(4) = 0.00157$
$\times 200$

$P(0) = 121$; $P(1) = 61$; $P(2) = 15$; $P(3) = 3$; $P(4) =$

O_i	122	60	15	$\underbrace{2+1}_{3}$	1
E_i	121	61	15	$\underbrace{3+0}_{3}$	0

Neglect last one and add it to previous because degree of freedom is 3 i.e., $\boxed{n=4}$

So, only 4 values to be taken.

We've; $\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$

$$= \frac{(122-121)^2}{121} + \frac{(60-61)^2}{61} + \frac{(15-15)^2}{1} + \frac{(3-3)^2}{3}$$

$$= \frac{1}{121} + \frac{1}{61} + \frac{0}{1} + 0$$

$$= 0.0246 < 7.815$$

The goodness of fit is accepted. //

Module 5

Random Process

1. **Random process:** The probabilistic model used for characterizing a random signal is called random process or stochastic process.

A stochastic process is defined as collection of random variables defined on a common probability space where the sample space is a probability measure and random variables are indexed by some set.

2. **Mean:** If $X(t)$ is a random variable associated with random process, the mean of $X(t)$ is denoted by $\mu_x(t)$ and it is defined by

$$\boxed{\mu_x(t) = E[X(t)]}$$

This is also called the time dependent mean or ensemble [collection] average of the stochastic process.

3. **Auto correlation:** (ACF) The Auto correlation of $X(t)$ is denoted by $R(t_1, t_2)$ and it is defined by

$$\boxed{R(t_1, t_2) = E[X(t_1) \cdot X(t_2)]}$$

4. **Covariance or Auto covariance:** The covariance or auto-covariance of $X(t)$ is denoted by $C(t_1, t_2)$ and it is defined by

$$\boxed{C(t_1, t_2) = R(t_1, t_2) - \mu_x(t_1) \cdot \mu_x(t_2)}$$

5. **Correlation coefficient:** The coefficient of correlation of $X(t)$ is denoted by $r(t_1, t_2)$ and it is defined by

$$r(t_1, t_2) = \frac{C(t_1, t_2)}{\sqrt{C(t_1, t_1) \cdot C(t_2, t_2)}}$$

Classification of Stochastic process

Let 'S' be a sample space of random variable (random experiment) and 'R' be the set of real no's. The random variable 'X' is defined as $X = f(s) \quad s \in S$

We define an index set $T \subset R$ indexed by the time parameter T. Stochastic process is a subset of random variables $X(t), t \in T$ defined on S with parameter T.

A stochastic process is said to be stationary, if its distribution function (or) certain expected values are invariant w.r.t time t.

Classification of Stationary process:-

1. Strict-Sense stationary (SSS): A random process $X(t)$ is said to be stationary in the strict sense, if all the distribution functions associated with the process are invariant under a translation of time i.e.,

$$\mu_x(t) = \text{constant} \quad \text{and}$$
$$R(t_1, t_2) = \text{constant} \quad \text{for SSS}$$

2. Wide-Sense stationary (WSS): A random process $X(t)$ is said to be wide-sense stationary process, if $E[X(t)] = \mu_x(t) = \text{constant}$ and

$$E[X(t) \cdot X(t+T)] = R(T) \quad \text{or} \quad R(t_1, t_2) \text{ is function of } |t_2 - t_1|$$

3. Ergodicity:- A stationary process is said to be Ergodic if its ensemble averages is equal to approximate time averages.

→ Ergodicity of the mean:

Consider a stochastic process, $X(t)$ with constant mean μ_x i.e.,

$$E[X(t)] = \mu_x$$

$$[\mu_x]_T = \frac{1}{T}\int_{-T/2}^{T} X(t)\, dt$$

$$E\left[[\mu_x]_T\right] = \mu_x$$

NOTE :- A stochastic process $X(t)$ is mean ergodic or ergodic w.r.t mean, if $[\mu_X]_T \to \mu_X$ as $T \to \infty$ and

$$E[(\mu_X(t))^2] - \sigma_T^2 = \mathrm{Var}\left[[\mu_X]_T\right] \to 0 \quad \text{as } T \to \infty$$

$$\Rightarrow \sigma_T^2 = \int_a^b \int_a^b C(t_1, t_2)\, dt_1\, dt_2 \quad \text{where}$$

$C(t_1, t_2)$ is an auto covariance.

A stochastic process $X(t)$ is mean ergodic if it auto covariance is such that $\frac{1}{T^2}\left[\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} C(t_1,t_2)\, dt_1\, dt_2\right] \to 0$ as $t \to \infty$

Problems:

1. Find the autocorrelation of $R[t_1, t_2]$ of stochastic process defined by $X(t) = A\cos(\omega t + \alpha)$ where the random variables A and α are independent and α is uniform in the interval $[-\pi, \pi]$.

By the defn of autocorrelation;

$$R(t_1, t_2) = E[X(t_1) \cdot X(t_2)] \longrightarrow (1)$$

$$X(t) = A\cos(\omega t + \alpha)$$

$$X(t_1) = A\cos(\omega t_1 + \alpha)$$

$$X(t_2) = A\cos(\omega t_2 + \alpha)$$

(1) may be rewritten as

$$R(t_1, t_2) = E[A\cos(\omega t_1 + \alpha) \cdot A\cos(\omega t_2 + \alpha)]$$

$$= E[A^2 \cos(\omega t_1 + \alpha) \cdot \cos(\omega t_2 + \alpha)]$$

$$= E(A^2) \cdot E[\cos(\omega t_1 + \alpha) \cdot \cos(\omega t_2 + \alpha)]$$

$$= EA^2 \cdot E\left[\frac{1}{2}\left[\cos(\omega(t_1 + t_2) + 2\alpha) + \cos(\omega(t_1 - t_2))\right]\right]$$

$$R(t_1, t_2) = EA^2\, \frac{1}{2} E\{[\cos(\omega(t_1+t_2) + 2\alpha] + \cos(\omega(t_1-$$

(I) → (2) (II)

$\lambda_k = (-\pi, \pi)$; $P(\lambda_k) = \frac{1}{2\pi}$ $\longrightarrow$ 2π's $\rightarrow \pi + \pi$ $\quad \pi - (-\pi)$

Consider ① ; $E\left[\cos(\omega(t_1+t_2)+2\alpha)\right]$

Partially Integrate w.r.t α.

$$\int_{-\pi}^{\pi} \frac{1}{2\pi}\left[\cos(\omega(t_1+t_2)+2\alpha)\right] d\alpha$$

$$= \frac{1}{2\pi}\left[\left(\frac{\sin(\omega(t_1+t_2)+2\alpha)}{2}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{4\pi}\left[\sin(2\pi+\omega(t_1+t_2)) - \underbrace{\sin(-2\pi+\omega(t_1+t_2))}_{\sin(-\theta) = -\sin\theta}\right]$$

$$= \frac{1}{4\pi}\left[\underbrace{\sin(2\pi+\omega(t_1+t_2))}_{\boxed{\sin(2\pi+\theta)}} + \underbrace{\sin(2\pi-\omega(t_1+t_2))}_{\boxed{\sin(2\pi-\theta)}}\right]$$

NOTE: $\sin(2\pi+\omega(t_1+t_2))$ w.r.t the angle $(2\pi+\theta)$

$$\sin(2\pi+\theta) = \sin\theta$$
$$\sin(2\pi-\theta) = -\sin\theta.$$

$$= \frac{1}{4\pi}\left[\sin(\omega(t_1+t_2)) - \sin(\omega(t_1+t_2))\right]$$

$$= \frac{1}{4\pi}[0] = 0$$

Consider (II): $E\left[\cos(\omega(t_1-t_2))\right]$

Partially Integrate w.r.t α

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \cos(\omega(t_1-t_2)).1\, d\alpha$$

$$= \frac{1}{2\pi}\left[\cos(\omega(t_1-t_2))\,\alpha\right]_{-\pi}^{\pi}$$

$$= \frac{\cos(\omega(t_1-t_2))}{2\cancel{\pi}}\left[\cancel{\pi}+\cancel{\pi}\right]$$

$$= \cos(\omega(t_1-t_2))$$

Substituting the above integral functions in (2)

$$R(t_1,t_2) = \frac{1}{2} EA^2 \cos\omega(t_1-t_2) \;//$$

2. Find autocorrelation function (ACF) of the stochastic process defined by $X(t) = A\cos(10t+\theta)$ where A is a random variable with mean 0 and variance 1 and θ is uniformly distributed in the interval $(-\pi,\pi)$.

$E(A^2) \rightarrow$ variance $= 1$

$$R(t_1,t_2) = E\left[x(t_1).\,x(t_2)\right] \longrightarrow (1)$$

$$X(t) = A\cos(10t+\theta)$$

$$X(t_1) = A\cos(10t_1+\theta)$$

$$X(t_2) = A\cos(10t_2+\theta)$$

$$(1) \Rightarrow R(t_1,t_2) = E\left[A\cos(10t_1+\theta) * A\cos(10t_2+\theta)\right]$$

$$= E\,A^2\left[\frac{1}{2}\left[\cos(10t_1+\theta+10t_2+\theta)+\cos(10t_1+\cancel{\theta}-10t_2-\cancel{\theta})\right]\right]$$

$$= E\,A^2\frac{1}{2}\left[\underbrace{\cos(10(t_1+t_2)+2\theta)}_{\text{I}}+\underbrace{\cos(10(t_1-t_2))}_{\text{II}}\right] \rightarrow (2)$$

$\lambda_K = (-\pi, \pi)$; $P(\lambda_k) = \frac{1}{2\pi}$

Integrate (I) $\rightarrow$ $E \cos(10(t_1+t_2)+2\theta)$ w.r.t θ

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(10(t_1+t_2)+2\theta)\, d\theta$$

$$= \frac{1}{2\pi} \left[\frac{\sin(10(t_1+t_2)+2\theta)}{2} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{4\pi} \left[\sin(10(t_1+t_2)+2\pi) - \sin(-2\pi + 10(t_1+t_2)) \right]$$

$$= \frac{1}{4\pi} \left[\underbrace{\sin(2\pi + 10(t_1+t_2))} + \sin(2\pi - 10(t_1+t_2)) \right]$$

$$= \frac{1}{4\pi} \left[\sin(10(t_1+t_2)) - \sin(10(t_1+t_2)) \right]$$

$$= 0_{//}$$

Integrate (II) $\rightarrow$ $\cos(10(t_1-t_2))$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \underbrace{\cos(10(t_1-t_2))}_{\text{constant}}\, d\theta$$

$$= \frac{1}{2\pi} ((\cos(10(t_1-t_2))) \left[\theta \right]_{-\pi}^{\pi}$$

$$= \frac{\cos(10(t_1-t_2)) \left[\cancel{2\pi} \right]}{\cancel{2\pi}}$$

$$= \cos(10(t_1-t_2))$$

$\therefore$ (2) $\Rightarrow$ $R(t_1, t_2) = \frac{1}{2} E(A^2) \cos(10(t_1-t_2)) \rightarrow$ (3)

Given that mean is 0 and variance $=1$

$E[(\mu_X(t))^2] = \sigma^2 = E[A^2] = 1$

(3) $\Rightarrow$ $R(t_1, t_2) = \frac{1}{2} \cos(10(t_1-t_2))_{//}$

(3) Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide sense stationary (WSS), if A & ω_0 are constants and θ is uniformly distributed RV in $(0, 2\pi)$

Random variable

$$\lambda_k = (0, 2\pi) \Rightarrow P(\lambda_k) = \frac{1}{2\pi}$$

$$X(t) = A\cos(\omega_0 t + \theta)$$

We've $E(X(t)) = E[A\cos(\omega_0 t + \theta)]$

we need to show; $R(t_1, t_2)$ is a function $t_1 - t_2$ | $t_2 - t_1$

$$R(t_1, t_2) = E[X(t_1) \cdot X(t_2)]$$

$$= E[A\cos(\omega_0 t_1 + \theta) \cdot A\cos(\omega_0 t_2 + \theta)]$$

$$= \frac{EA^2}{2}[\cos(\omega_0 t_1 + \theta + \omega_0 t_2 + \theta) + \cos(\omega_0 t_1 - \omega_0 t_2 + \cancel{\theta} - \cancel{\theta})]$$

$$= \frac{E(A^2)}{2}[\cos(\omega_0(t_1 + t_2) + 2\theta) + \cos(\omega_0(t_1 - t_2))]$$

$$= \frac{E(A^2)}{2}[\underbrace{\cos(\omega_0(t_1 + t_2) + 2\theta)}_{I} + \underbrace{\cos(\omega_0(t_1 - t_2))}_{II}]$$

$\to$ 1

Integrate I: $= \frac{1}{2\pi}\int_0^{2\pi}\cos(\omega_0(t_1 + t_2) + 2\theta)\, d\theta$

$$= \frac{1}{2\pi}\left[\frac{\sin(\omega_0(t_1 + t_2) + 2\theta)}{2}\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}[\sin(4\pi + \omega_0(t_1 + t_2)) + \sin(\cancel{4\pi} - \omega_0(t_1 + t_2))]$$

$\sin\theta \qquad - \sin\theta$

$= 0$

Integrate II: $= \frac{1}{2\pi}\int_0^{2\pi}\cos(\omega_0(t_1 - t_2))\, d\theta = \frac{1}{\cancel{2\pi}}\cos(\omega_0(t_1 - t_2))(\cancel{2\pi})$

$= \cos(\omega_0(t_1 - t_2))$

1 $\Rightarrow \frac{E(A^2)}{2}[\cos(\omega_0(t_1 - t_2))]$ //

(I) $\Rightarrow = \frac{1}{4\pi}\left[\sin\left(4\pi + w_0(t_1+t_2)\right) - \sin\left(w_0(t_1+t_2)\right)\right]$

$= \frac{1}{4\pi}\left[\sin w_0(t_1+t_2) - \sin\left(w_0(t_1+t_2)\right)\right]$

$= 0$

(II) $\Rightarrow \frac{1}{2\pi}\int_0^{2\pi} \cos\left(w_0(t_1-t_2)\right)d\theta$

$= \frac{1}{\cancel{2\pi}}\left[\cos\left(w_0(t_1-t_2)\right)\left[\cancel{2\pi - 0}\right]\right]$

$= \cos\left(w_0(t_1-t_2)\right)$

$R(t_1,t_2)$ (1) $\Rightarrow \frac{1}{2}E(A^2)\left[\cos\left(w_0(t_1-t_2)\right)\right]$

It is in terms of $t_1 - t_2$

It is WSS//

4. If $X(t)$ is a wide sense stationary process with auto correlation $R(\tau) = Ae^{-\alpha|t|}$. Determine the second order moment of RV $X(8) - X(5)$.

Soln:

We have, second order moment of RV $X(8)-X(5)$ is given by

$E[X(8)-X(5)]^2$

$= E[X^2(8)] + E[X^2(5)] - 2E[\underset{t_1}{X(8)}.\underset{t_2}{X(5)}]$ → (1)

Also we have, $\boxed{E[X^2(t)] = R(t_1,t) = A}$ → (2)

Given; $R(\tau) = A e^{-\alpha(t)}$ ⟶ t

W.K.T. $R(t_1, t_2) = A e^{-\alpha(t_1 - t_2)}$

By the given RV, $E[X(8)\,X(5)] = A \cdot e^{-\alpha(8-5)}$
$[X(8) = X(t_1);\ X(5) = X(t_2)]$ $\Rightarrow A e^{-3\alpha} \rightarrow (3)$

Using (2) $E[X^2(8)] = R(t_1, 8) = A \rightarrow (4)$
$E[X^2(5)] = R(t_1, 5) = A \rightarrow (5)$

(3), (4) & (5) in (1)

$= A + A - 2Ae^{-3\alpha}$

$= 2A(1 - e^{-3\alpha})$

5. Given that autocorrelation function for a stationary ergodic process with no periodic components is $R_{xx}(\tau) = 25 + \frac{4}{1+6\tau^2}$. Find the mean value and variance of the process $X(t)$.

By the property of autocorrelation function, we have, $\boxed{\mu_X^2 = \lim_{\tau \to \infty} R_{xx}(\tau)}$

Given; $R_{xx}(\tau) = 25 + \frac{4}{1+6\tau^2}$

$\therefore \mu_x^2 = \lim_{\tau \to \infty} \left[25 + \frac{4}{1+6\tau^2}\right]$

$\therefore 25 + \frac{4}{1+\infty} = 25 + 0 = 25$

$\mu_x^2 = 25$

$\boxed{\mu_x = 5} = E[X^2(t)]$

To find variance, we assume $R_{xx}(\tau) = E^2[x(t)] = 25$ (Property autocorrelation)

also we have variance;

$$\boxed{Var[x(t)] = E[x^2(t)] - E^2[x(t)]}$$

but $E[x^2(t)] = R_{xx}(0) = 25 + \frac{4}{1+0}$
$\tau = 0$

$= 29$

$Var(x(t)) = 29 - 25$

$= 4$

6. Find the power spectral density with autocorrelation function $R(\tau) = e^{-\alpha\tau^2}$

By the defn of Power spectral density

$$\boxed{S(\omega) = \int_{-\infty}^{\infty} R(\tau)\,[\cos\omega\tau - i\sin\omega\tau]\,d\tau}$$

Rewrite equn:

$$S(\omega) = \int_{-\infty}^{\infty} R(\tau)\,(e^{-i\omega\tau})\,d\tau$$

Given; $R(\tau) = e^{-\alpha\tau^2}$

$$S(\omega) = \int_{-\infty}^{\infty} e^{-\alpha\tau^2} e^{-i\omega\tau}\,d\tau$$

$$= \int_{-\infty}^{\infty} e^{-[\alpha\tau^2 + i\omega\tau]}\,d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\alpha\left[\tau^2 + \frac{i\omega\tau}{\alpha}\right]}\,d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\alpha\left[\left(\tau^2 + \frac{2i\omega\tau}{2\alpha} + \frac{i^2\omega^2}{4\alpha^2}\right) - \frac{i^2\omega^2}{4\alpha^2}\right]} d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\alpha \times -\frac{i^2\omega^2}{4\alpha^2} - \alpha\left(\tau^2 + \frac{2i\omega\tau}{2\alpha} + \frac{i^2\omega^2}{4\alpha^2}\right)} d\tau$$

$$= e^{-\frac{\omega^2}{4\alpha}} \left[\int_{-\infty}^{\infty} e^{-\alpha\left[\tau^2 + \frac{i^2\omega^2}{4\alpha^2} + \frac{2i\omega\tau}{2\alpha}\right]} d\tau\right]$$

$$x = \sqrt{\alpha}\left[\tau + \frac{i\omega}{2\alpha}\right]^2$$

Differentiate w.r.t τ $\quad dx = \sqrt{\alpha}\, d\tau$

when $\tau = -\infty \Rightarrow x = -\infty$

$\tau = \infty \Rightarrow x = \infty$

$$= e^{-\frac{\omega^2}{4\alpha}} \left[\int_{x=-\infty}^{\infty} e^{-x^2} \frac{1}{\sqrt{\alpha}}\, dx\right]$$

$$= \frac{e^{-\frac{\omega^2}{4\alpha}}}{\sqrt{\alpha}} \left[\int_{x=-\infty}^{\infty} e^{-x^2}\, dx\right]$$

By defn of gamma fn:-

$$S(\omega) = \frac{e^{-\frac{\omega^2}{4\alpha}}}{\sqrt{\alpha}} \cdot \sqrt{\pi}.$$

7. If the WSS process $X(t)$ is given by $X(t) = 10\cos(100t + \theta)$ where θ is uniformly distributed over $(-\pi, \pi)$. Prove that $X(t)$ is correlation ergodic.

We have correlation ergodic process, the stationary process $X(t)$ is said to be correlation ergodic if the process, $Y(t)$ is mean ergodic, where $Y(t)$

$$\boxed{Y(t) = X(t+\tau)\cdot X(t)}$$

Given that; $X(t) = 10\cos(100t+\theta)$

$E(X(t+\tau)\cdot X(t)]$ $t=\tau$, $\tau = 0$

$R(\tau) = E\left[10\cos(100t + (100\tau+\theta)\cdot 10\cos(100t+\theta)\right]$

By the definition of ergodic process, if $X(t)$ is said to be distribution ergodic, if $Y(t)$ is mean ergodic and is defined as

$$\{y(t)\} = \frac{1}{2T}\int_{-T}^{T} y(t)\,dt$$

$$\rightarrow E[\{y(t)\}] \text{ as } T\rightarrow\infty$$

$$Z = \{y(t)\} = \frac{1}{2T}\int_{-T}^{T}\left[10\underbrace{\cos(100t + 100\tau+\theta)}_{A}\cdot 10\underbrace{\cos(100t+\theta)}_{B}\right]dt$$

$$= \frac{100}{2T}\times\frac{1}{2}\int_{-T}^{T}\cos(100t+100\tau+\theta+100t+\theta) + \cos(100t+100\tau+\theta-100t-\theta)$$

$$= \frac{25}{T}\int_{-T}^{T}\cos(200t+100\tau+2\theta) + \cos(100\tau)\,dt$$

$$= \frac{25}{T}\left[\frac{\sin(200t+100\tau+2\theta)}{200} + \cos(100\tau)(t)\right]_{-T}^{T}$$

$\sin(200\pi+\theta) = \sin\theta$

$$= \frac{25}{T}\left|\frac{\sin(100\tau+2\theta)}{200}\cos(100\tau)(T) - \right.$$

$\sin(100\tau+2\theta) + \cos(10$

$= \frac{25}{\cancel{T}} 2\cos(100\tau)\cancel{T}$

$\bar{z} = 50\cos(100\tau)$

$\underset{T\to\infty}{Lt}\ \bar{z} = \underset{T\to\infty}{Lt}\{y(t)\} = R(\tau) = 50\cos(100\tau)$

$\therefore$ It is autocorrelation ergodic.

Thus $X(t)$ is a autocorrelation ergodic.

Power Spectral Density function:

The spectral density function of a real random function is an even function and it is defined as

$$\boxed{S(\omega) = \int_{-\infty}^{\infty} R(\tau)\, e^{-i\omega\tau}\, d\tau}$$

The power spectral density of autocorrelation is given by

$$\boxed{R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\, e^{i\tau\omega}\, d\omega}$$

8. If the Power spectral density of WSS process is given by

$$S(\omega) = \begin{cases} \frac{b}{a}(a - |\omega|) & \text{where } |\omega| \le a \\ 0 & |\omega| > a \end{cases}$$

. Find the autocorrelation function of the process.

We have; $R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\, e^{i\tau\omega}\, d\omega$

$= \frac{1}{2\pi}\int_{-a}^{a} \frac{b}{a}(a - |\omega|)\, e^{i\tau\omega}\, d\omega$

PSD is even & +ve.

$|\omega| \to \omega \to \omega > 0$
$-\omega \to \omega < 0$

$= \frac{1}{2\pi}\int_{-a}^{a} \frac{b}{a}(a-\omega)\left[\underbrace{\cos\omega\tau}_{\text{even}} + i\,\underbrace{\cancel{\sin\omega\tau}}_{\text{odd}}\right]$

$$= \frac{1}{2\pi}\left[\int_{-a}^{a} \frac{b}{a}(a-\omega)\cdot\cos\omega\tau \, d\omega\right]$$

$$= \frac{b}{2a\pi}\int_{-a}^{a} \underbrace{(a-\omega)}_{u}\underbrace{\cos\omega\tau}_{v}\, d\omega = \frac{b}{2a\pi}\times 2\int_{0}^{a}(a-\omega)\cos\omega\tau \, d\omega$$

$$= \frac{2b}{2a\pi}\left[(a-\omega)\frac{\sin\omega\tau}{\tau} - (-1)\frac{(-\cos\omega\tau)}{\tau^2}\right]_0^a$$

$$= \frac{\cancel{2}b}{\cancel{2}a\pi}\left[(a-\omega)\frac{\sin\omega\tau}{\tau} - \frac{\cos\omega\tau}{\tau^2}\right]_0^a$$

$$= \frac{b}{a\pi}\left(0 - \frac{\cos a\tau}{\tau^2} - \left[0 - \frac{\cos 0}{\tau^2}\right]\right)$$

$$= \frac{b}{a\pi}\left[-\frac{\cos a\tau}{\tau^2} + \frac{1}{\tau^2}\right]$$

$$= \frac{b}{a\pi\tau^2}\left[1-\cos a\tau\right]$$

$$= \frac{b}{a\pi\tau^2}\, 2\sin^2\left(\frac{a\tau}{2}\right)$$

$$= \frac{b}{a\pi\tau^2}\times\frac{a}{a}\times\frac{2}{2}\times 2\sin^2\left(\frac{a\tau}{2}\right)$$

$$= \frac{ab}{2\pi\left(\frac{a^2\tau^2}{4}\right)}\sin^2\left(\frac{a\tau}{2}\right) = \frac{ab}{2\pi}\,\frac{\sin^2\left(\frac{a\tau}{2}\right)}{\left(\frac{a\tau}{2}\right)^2}$$

$$= \frac{ab}{2\pi}\left(\frac{\sin\left(\frac{a\tau}{2}\right)}{\left(\frac{a\tau}{2}\right)}\right)^2 //$$

9. The power spectral density function of a zero mean WSS process $X(t)$ is given by $S(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & \text{elsewhere.} \end{cases}$ Find $R(\tau)$ and show that $X(t)$ & $X\left(t+\frac{\pi}{\omega_0}\right)$ are uncorrelated.

By the defn of PSD of autocorrelation;

$$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\, e^{i\tau\omega}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} 1\, e^{i\tau\omega}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{i\tau\omega}}{i\tau}\right]_{-\omega_0}^{\omega_0}$$

$$= \frac{1}{2\pi i\tau}\left[e^{i\tau\omega}\right]_{-\omega_0}^{\omega_0} =$$

$$= \frac{1}{2\pi i\tau}\left[\cancel{\cos\tau\omega + i\sin\tau\omega}\right]_{-\omega_0}^{\omega_0} e^{i\tau\omega} =$$

$$= \frac{1}{2\pi i\tau}\left[e^{i\omega_0\tau} - e^{-i\omega_0\tau}\right]$$

$$= \frac{1}{2\pi i\tau}\left(\cancel{\cos\omega_0\tau} + i\sin\omega_0\tau - \left(\cancel{\cos\omega_0\tau} + i\sin\omega_0\tau\right)\right)$$

$$\frac{1}{2\pi}\left[\cancel{e^{i\tau\omega}}\right.$$

$$\frac{1}{2\pi}\left[\int_{-\omega_0}^{\omega_0} \left(\cos\tau\omega + i\underbrace{\sin\tau\omega}_{\text{odd}}\right)\right]$$

$$\frac{1}{2\pi}\left[\frac{\sin\tau\omega}{\tau}\right]_{-\omega_0}^{\omega_0}$$

$$= \frac{1}{2\pi}\left(\frac{\sin\omega_0\tau}{\tau} + \frac{\sin\omega_0\tau}{\tau}\right)$$

Correct.

$$= \frac{\cancel{2}\cancel{i}\sin\omega_0\tau}{\cancel{2}\pi\cancel{i}\tau} = \frac{\sin\omega_0\tau}{\pi\tau}$$

$$\boxed{R(\tau) = \frac{\sin\omega_0\tau}{\pi\tau}}$$

To show $X(t)$ and $X\left(t+\frac{\pi}{\omega_0}\right)$ take the product.

$X_1(t) \cdot X_2(t)$ $\quad \boxed{\tau < \frac{\pi}{\omega_0}}$

$$E\left[X(t)\cdot X\left(t+\frac{\pi}{\omega_0}\right)\right] = R(\tau) = \frac{1}{\pi}\left[\frac{\sin\left(\cancel{\omega_0}\frac{\pi}{\cancel{\omega_0}}\right)}{\frac{\pi}{\omega_0}}\right]$$

$$= \frac{1}{\omega_0\pi^2}\sin\pi\, \omega_0 = 0$$

∴ Uncorrelated.

Poisson process:

we have the following definitions;

$$P[X(t)=k] = \frac{e^{-\lambda t}\cdot(\lambda t)^k}{k!}$$ [if probability is not given]

$$P[X(t)=k] = \frac{e^{-\lambda p t}(\lambda p t)^k}{k!}$$ [if probability given]

10. The radioactive source emits particles at a rate of 5 per [min] inaccordance with poisson's process, each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4-min period.

we have $P(X(t)=k) = \frac{e^{-\lambda p t}(\lambda p t)^k}{k!}$

$\lambda = 5$

$p = 0.6$

$k = 10$

$t = 4$

$$= \frac{e^{-5(0.6)(4)}\left(5(0.6)(4)\right)^{10}}{10!}$$

$$P(X(t)=k) = 0.1048$$