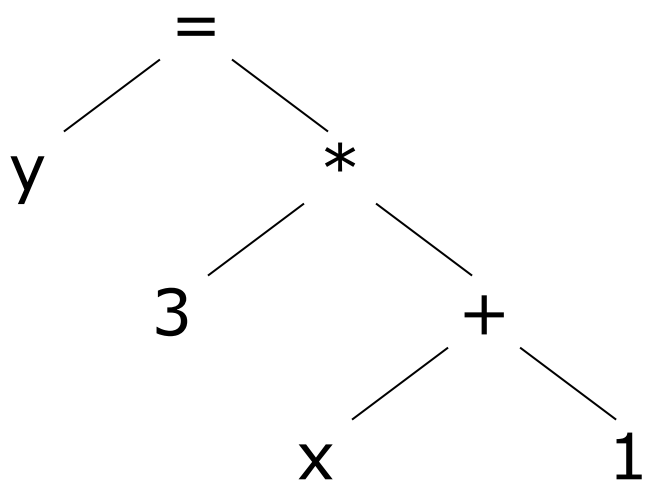
Expression Tree

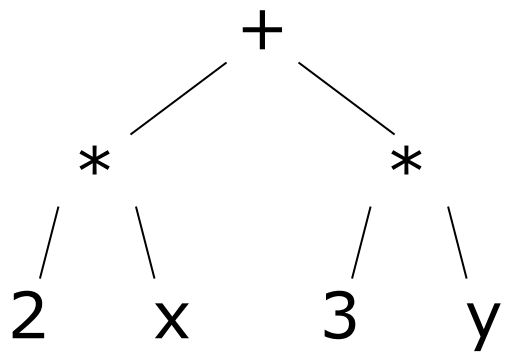
An expression tree is a tree that represents an expression.



y = 3 * (x + 1)

Expression Tree

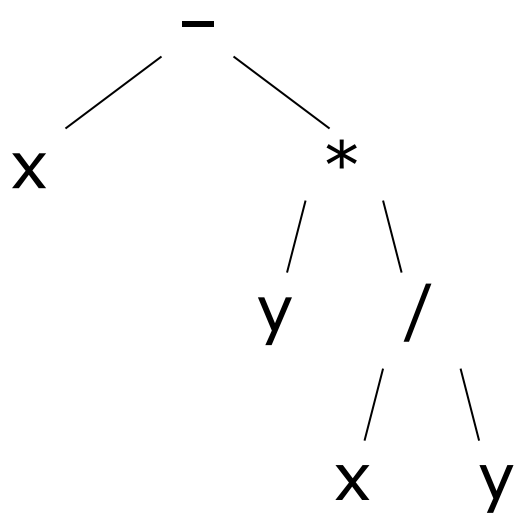
An expression tree is a tree that represents an expression.



2*x + 3*y

Expression Tree

An expression tree is a tree that represents an expression.



x – (y * (x / y))

Expression Tree

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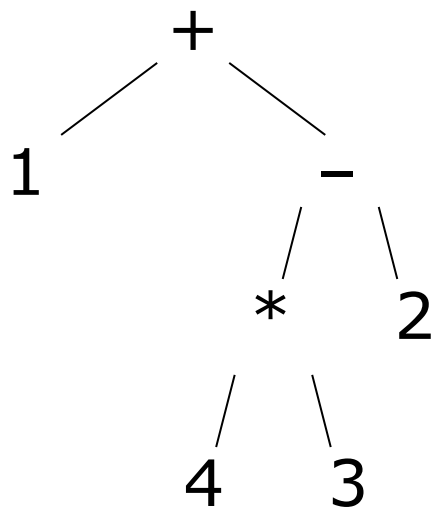
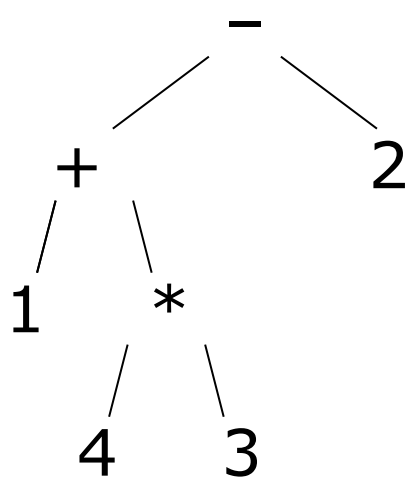


1 + (4 * 3 – 2)

Expression Tree

An expression tree is a tree that represents an expression.



(1 + 4 * 3) – 2

Expression Tree

An expression tree is a tree that represents an expression.

5

5

Expression Tree

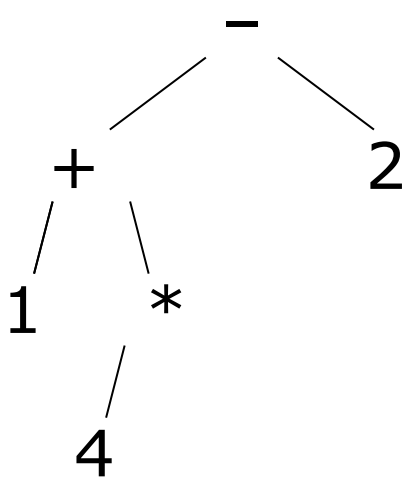
An expression tree is a tree that represents an expression.

x

x

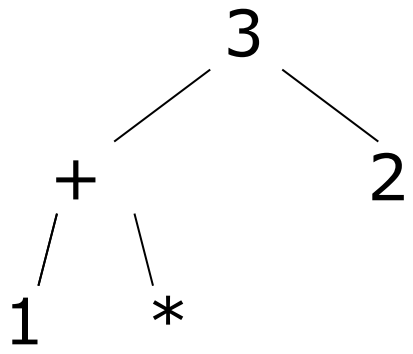
Expression Tree

This is NOT an expression tree.

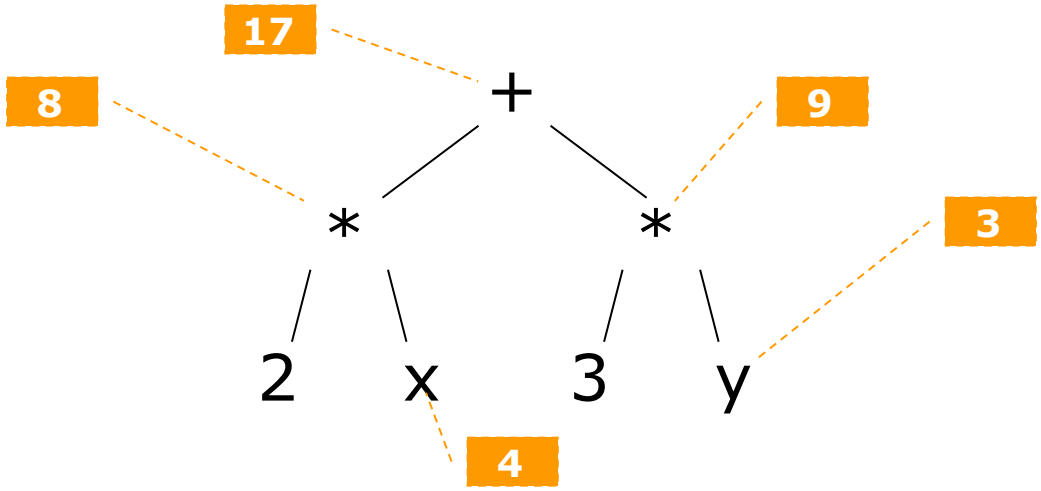


Expression Tree

This is NOT an expression tree.



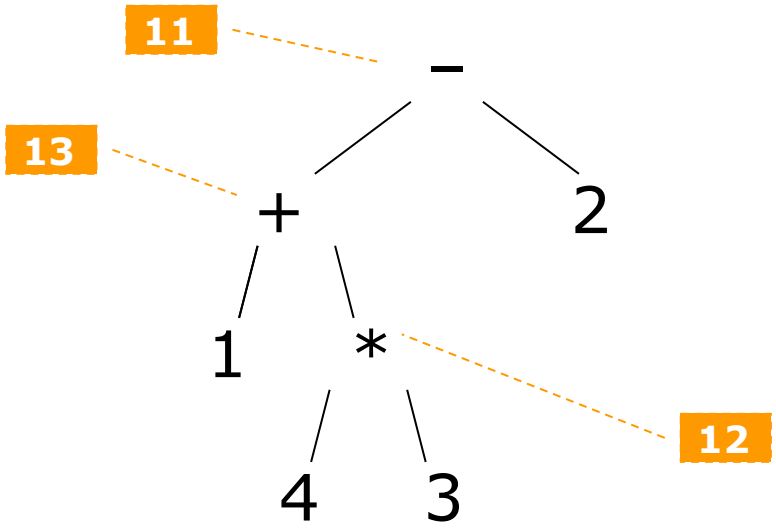
Evaluating an Expression



{x = 4, y = 3}

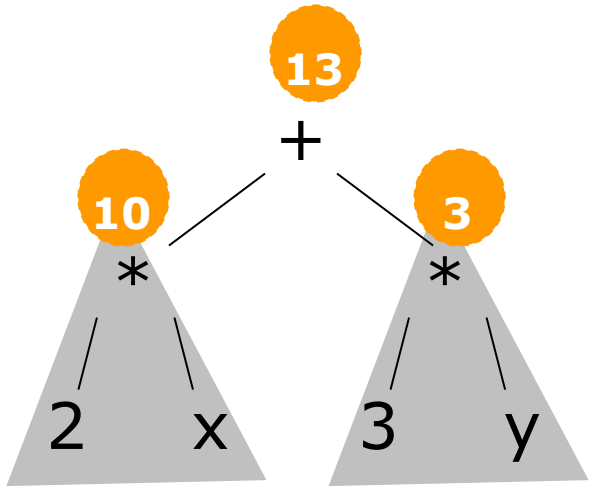
2*x + 3*y

Evaluating an Expression



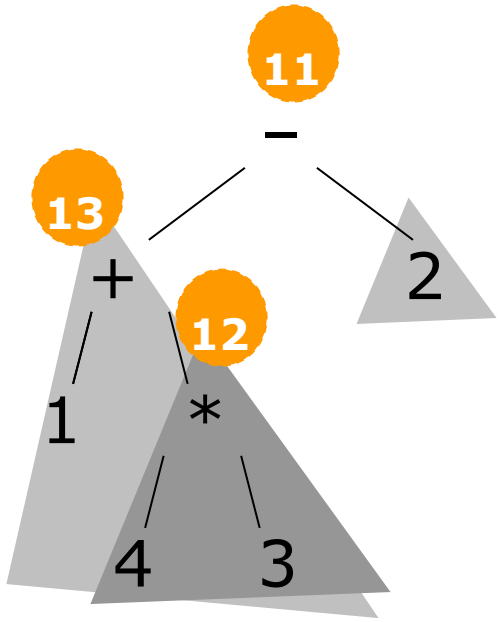
1 + 4 * 3 – 2

Evaluating an Expression: Another Viewpoint



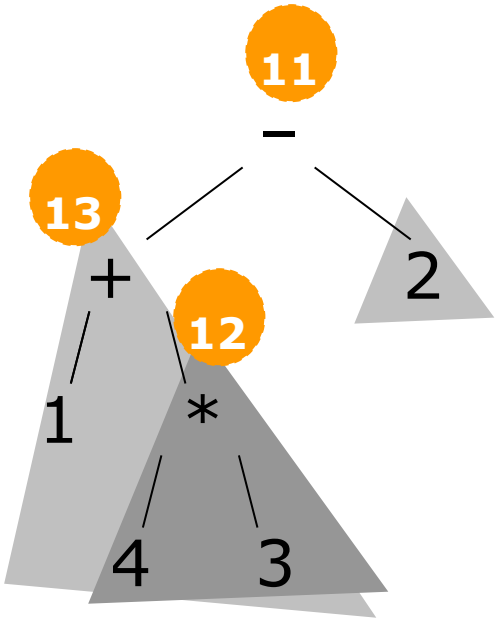
{x = 5, y = 1}

Evaluating an Expression: Another Viewpoint



1 + 4 * 3 – 2

Evaluating an Expression: Another Viewpoint



1 + 4 * 3 – 2

```
ExprTreeADT t1 = …;
…
int Result = Eval(t1);
```

```
/* In the implementation file of ExprTreeADT
*/
int Eval(ExprTreeADT t) {
    …
    …
    LHS = Eval(LeftExprSubtree(t));
    RHS = Eval(RightExprSubtree(t));

    op = ExprTreeRoot(t);
    return (LHS op RHS);
}
```

Evaluating an Expression: Another Viewpoint

5

```
/* In the implementation file of ExprTreeADT
*/
int Eval(ExprTreeADT t) {
    ...
    if (Height(t) == 1) return ExprTreeRoot(t);
    ...
    LHS = Eval(LeftExprSubtree(t));
    RHS = Eval(RightExprSubtree(t));

    op = ExprTreeRoot(t);
    return (LHS op RHS);
}
```

5

Evaluating an Expression: Another Viewpoint

What should **ExprTreeRoot(t)** return?

- an integer?
- an operator? (*But do you know how to return an operator, after all!*)

```
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*/
int Eval(ExprTreeADT t) {
   ...
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}
```

Evaluating an Expression: Another Viewpoint

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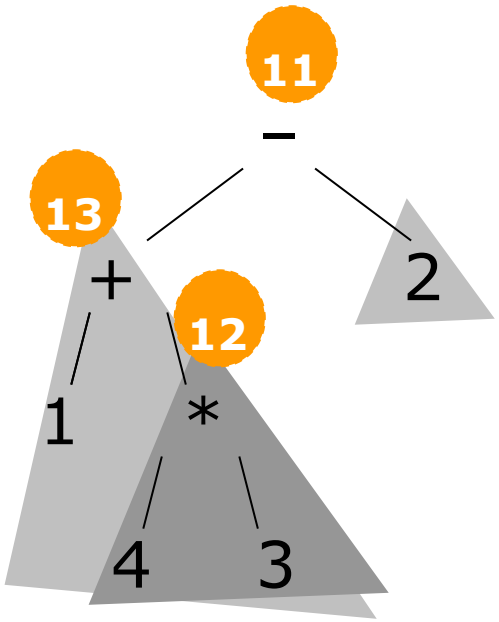
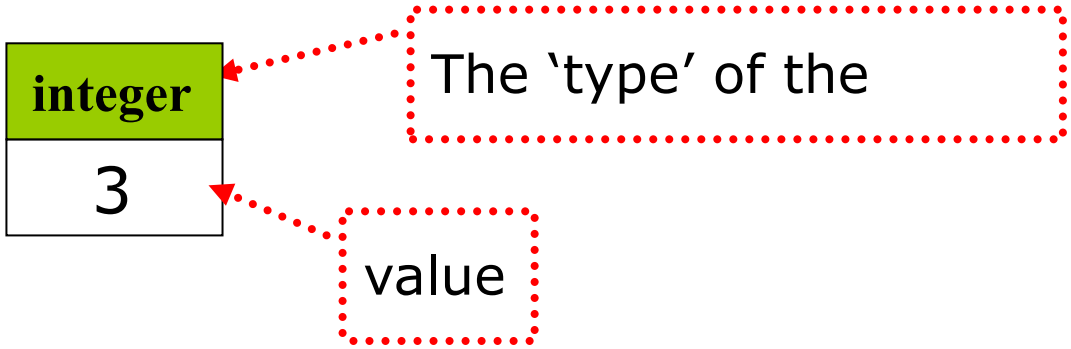


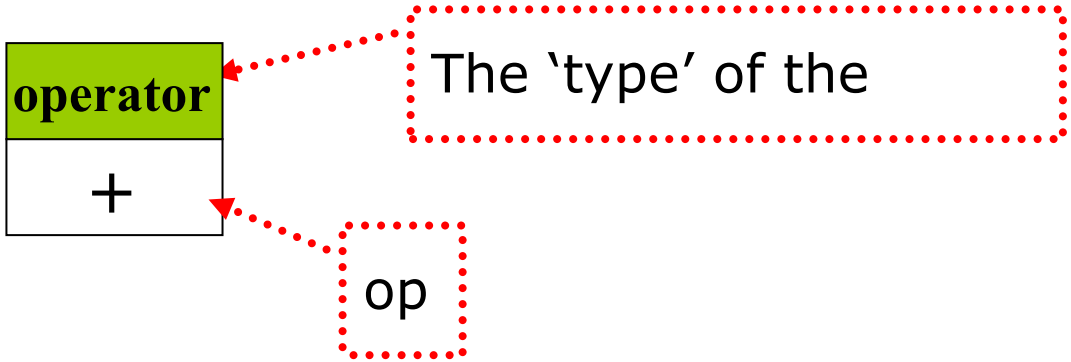
We see that a tree node can be an integer or an operator (For the time being, let's forget that a node can also be a variable.)

A Node of an Expression Tree
(Type is **ExprTreeNodeADT**)

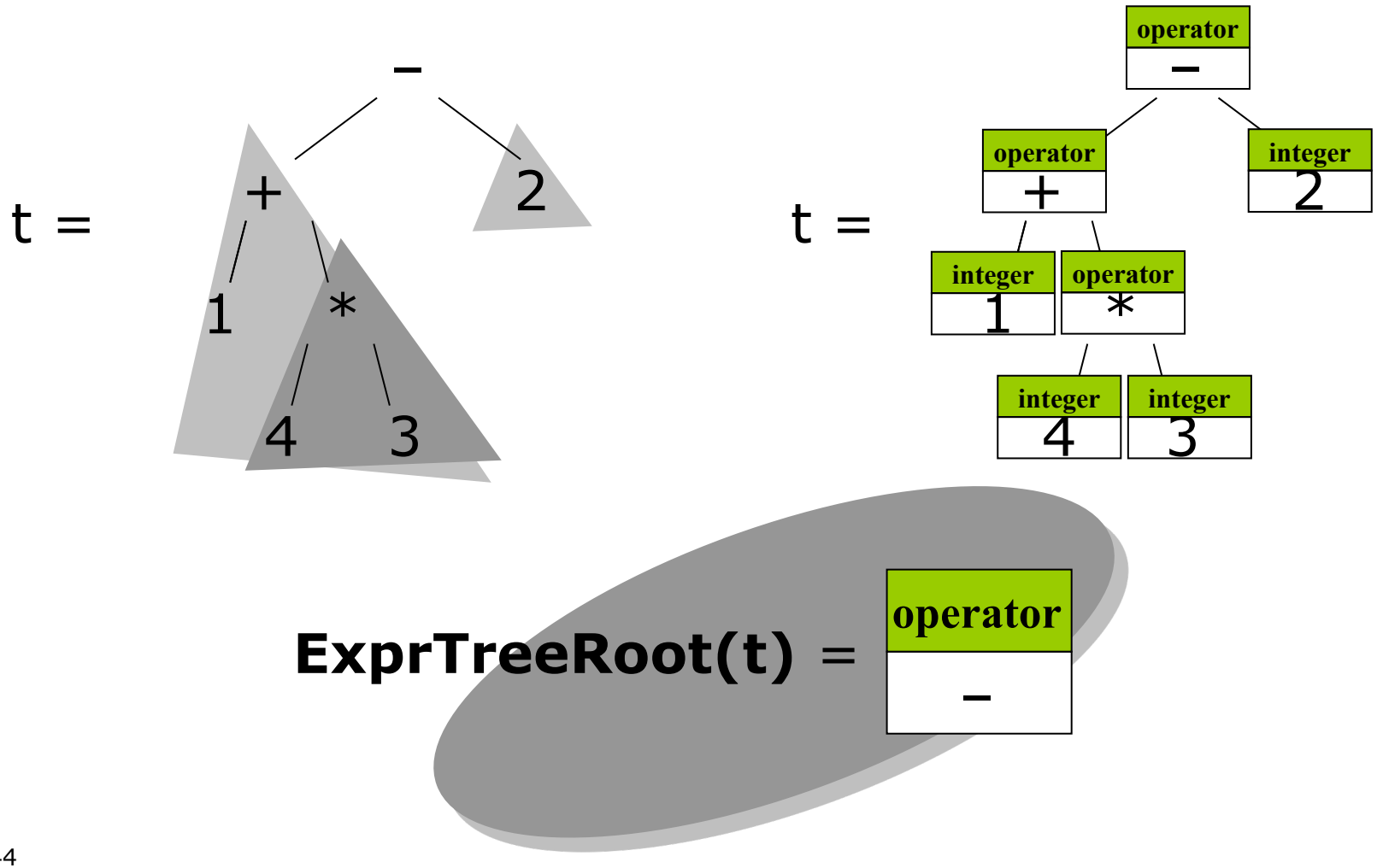
- If it is an integer:



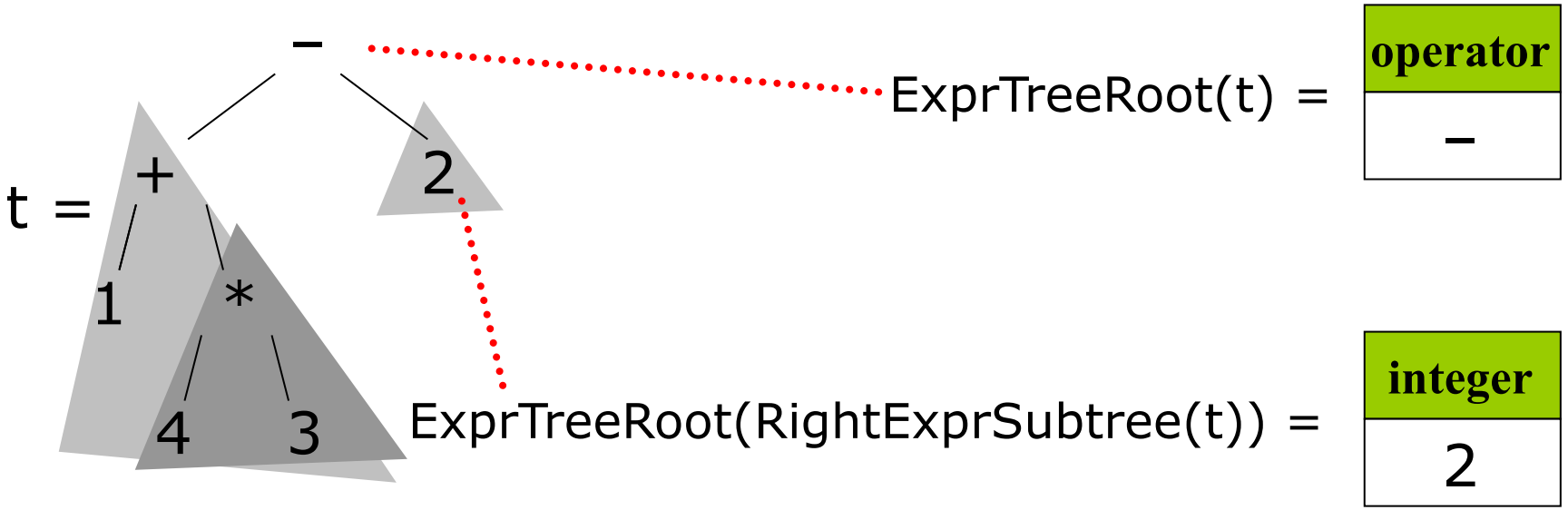
- If it is an operator:



Evaluating an Expression: Another Viewpoint

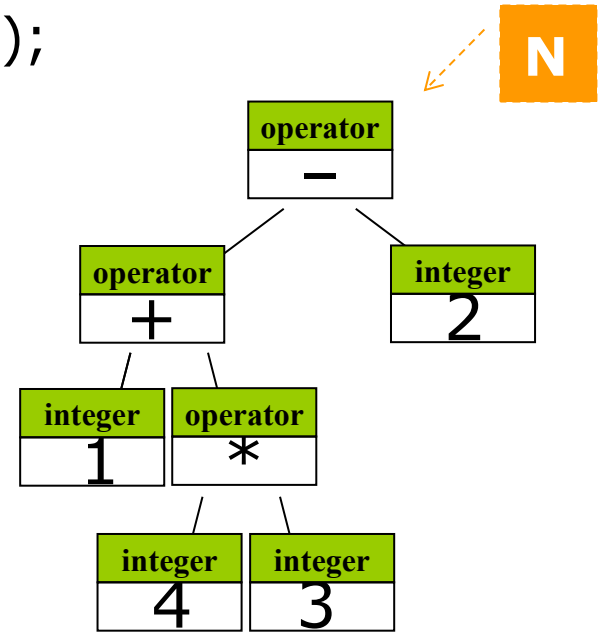


We introduce three more operations **NodeType**, **Node-Value** and **NodeOp**:



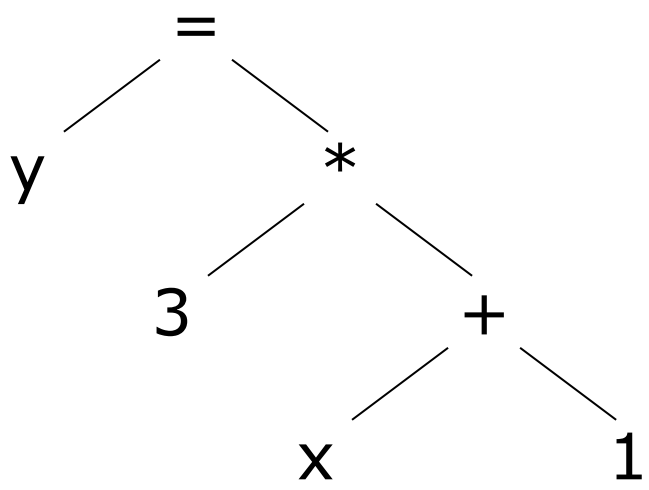
- **NodeType**(ExprTreeRoot(t)) returns **operator**
- **NodeOp**(ExprTreeRoot(t)) returns **'-'**
- **NodeType**(ExprTreeRoot(RightExprSubtree(t))) returns **integer**
- **NodeValue**(ExprTreeRoot(RightExprSubtree(t))) returns **2**

```
int Eval(ExprTreeADT t) {
   ExprTreeNodeADT N = ExprTreeRoot(t);
   switch (NodeType(N)) {
      case integer : return NodeValue(N);
      case operator : {
         int lhs = Eval(LeftExprSubtree(t));
         int rhs = Eval(RightExprSubtree(t));
         switch (NodeOp(N)) {
            case '+' : return lhs + rhs;
            case '–' : return lhs – rhs;
            case '*' : return lhs * rhs;
            case '/' : return lhs / rhs;
         }
   }
}
```



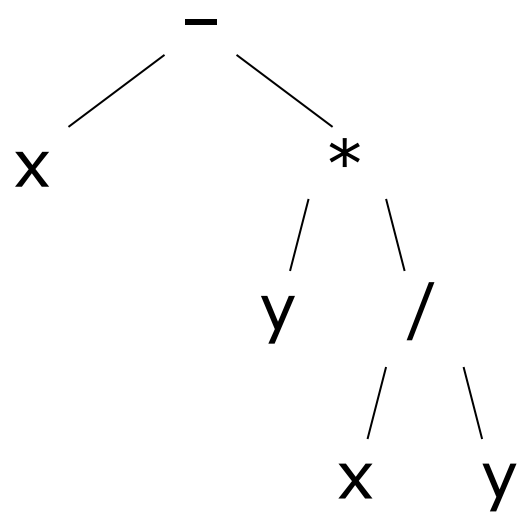
Traversing an Expression Tree

Now let’s consider how to reconstruct an expression from an expression tree.



Traversing an Expression Tree

Now let’s consider this expression tree.



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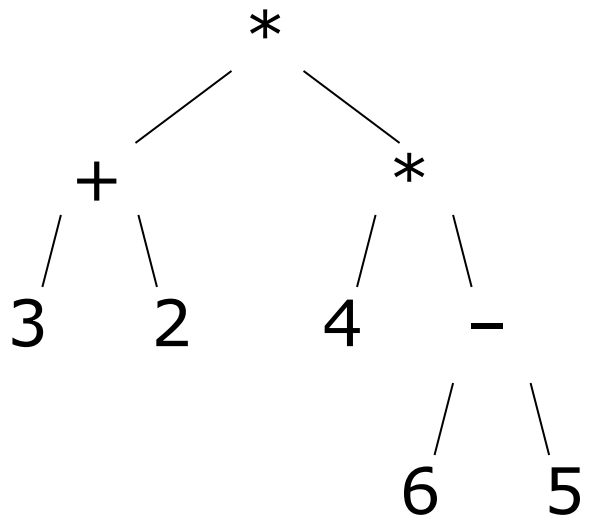

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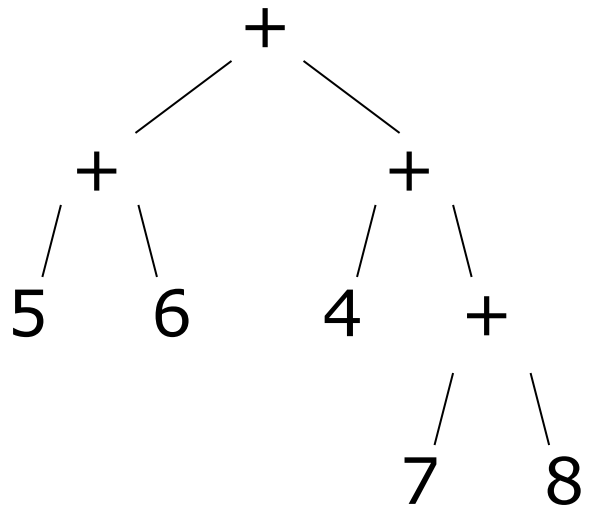

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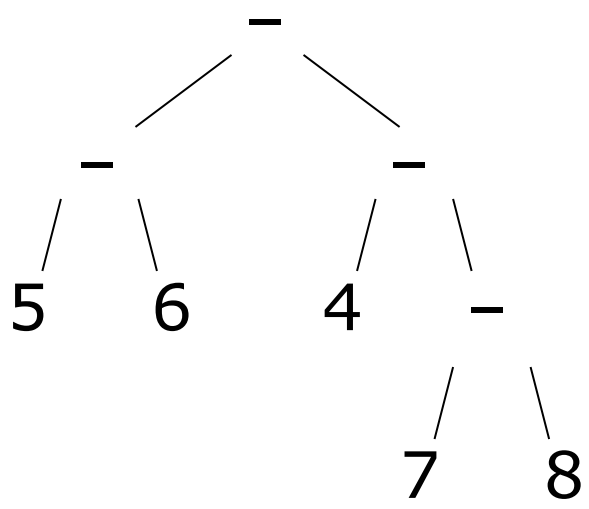
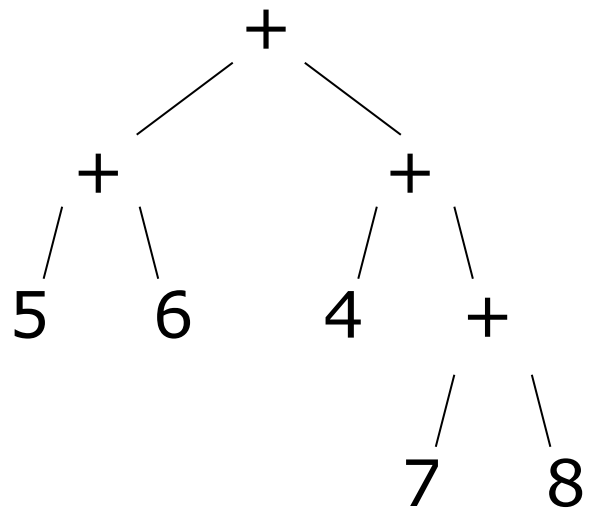
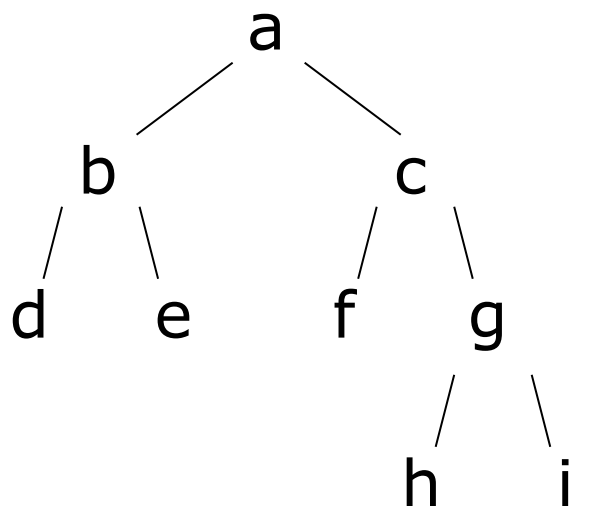


Traversing an Expression Tree

What we are doing is called an **'in-order' traversal of a tree**.



5+6+4+7+8

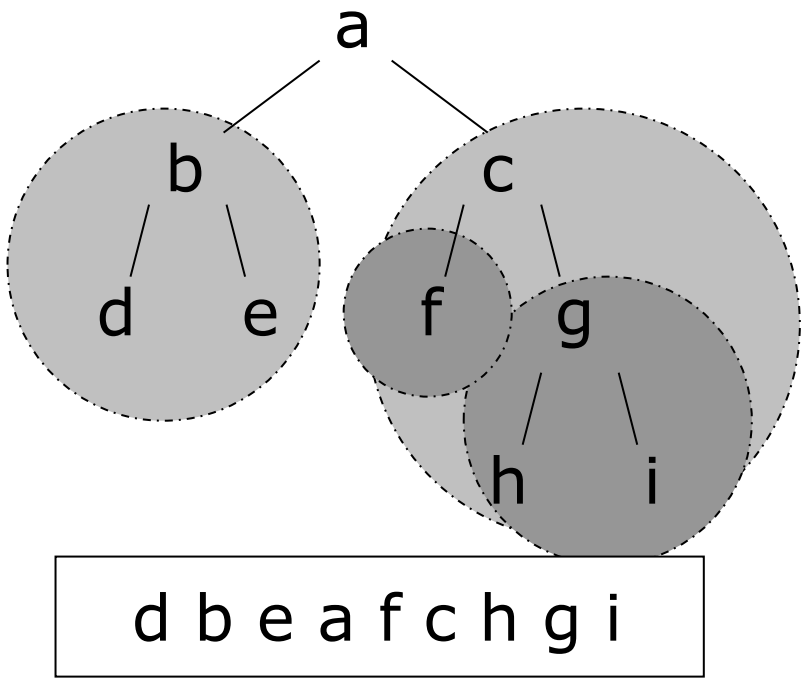


d b e a f c h g i

In-Order Traversal of a Tree

In an in-order traversal of a tree, we 'visit' the tree nodes in this order:

- We first visit the nodes in the left subtree, using in-order traversal.
- We then visit the root.
- We finally visit the nodes in the right subtree, using in-order traversal.



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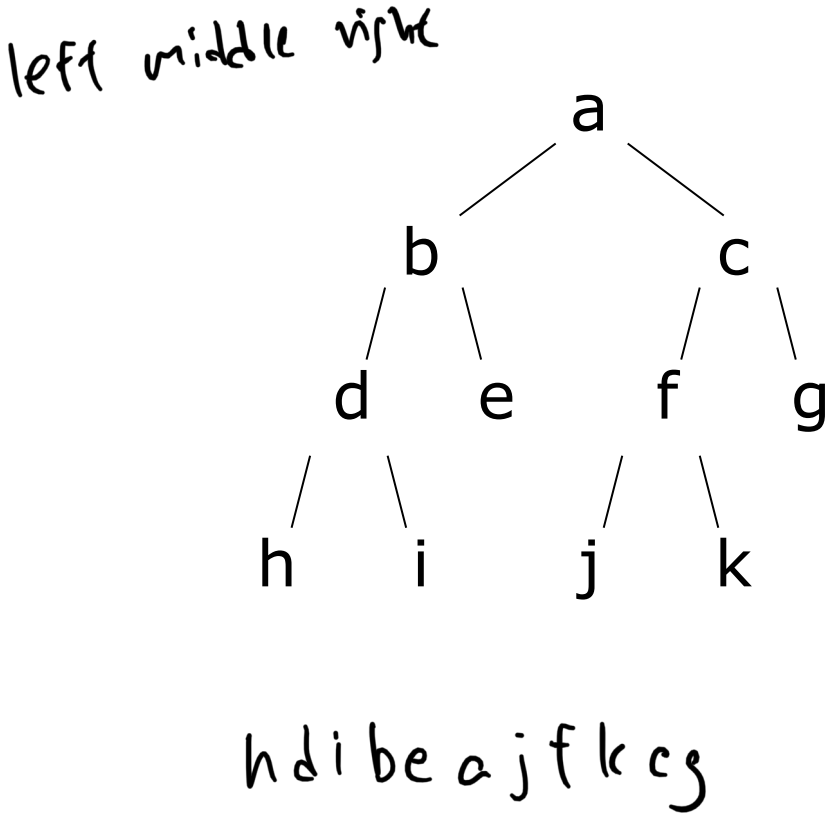

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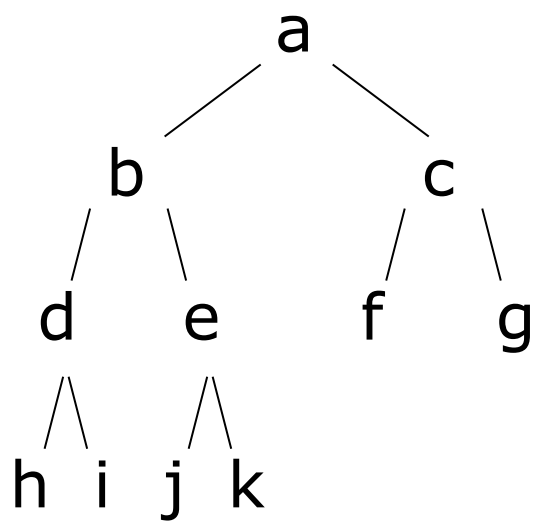

hdibjekafcg

In-Order Traversal of a Tree

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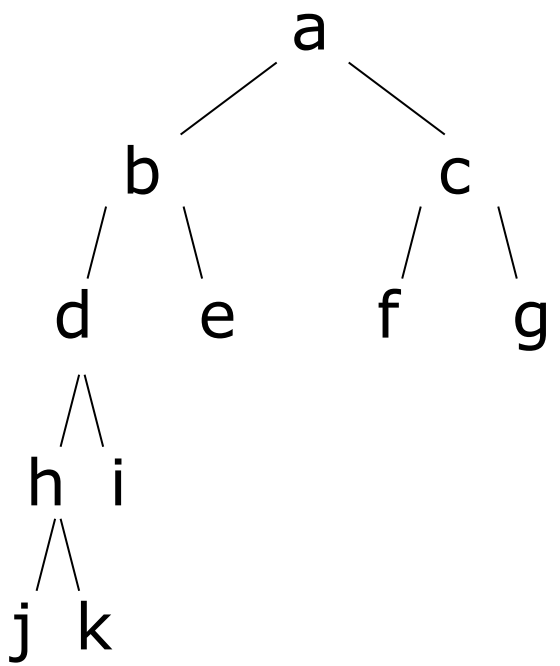
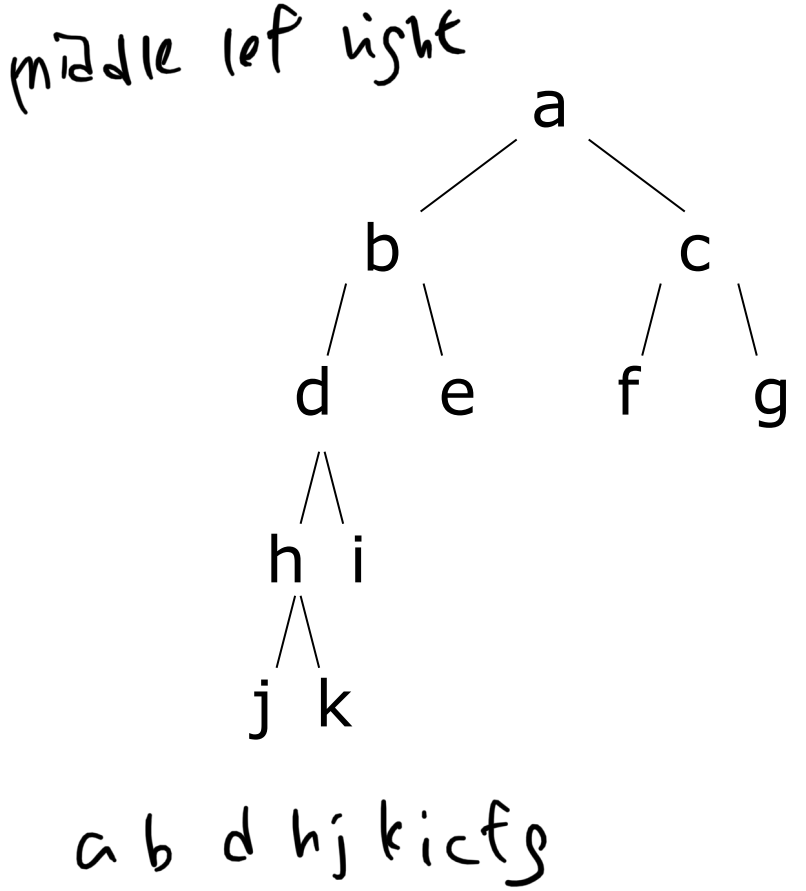


jhkdibeafcg

Pre-Order Traversal of a Tree

Similarly, we have the **'pre-order' traversal of a tree**:

- We first visit the root.
- We then visit the nodes in the left subtree, using pre-order traversal.
- We finally visit the nodes in the right subtree, using pre-order traversal.



Pre-Order Traversal of a Tree

Consider a pre-order traversal of this tree:

- We first visit the root.
- We then visit the nodes in the left subtree, using pre-order traversal.
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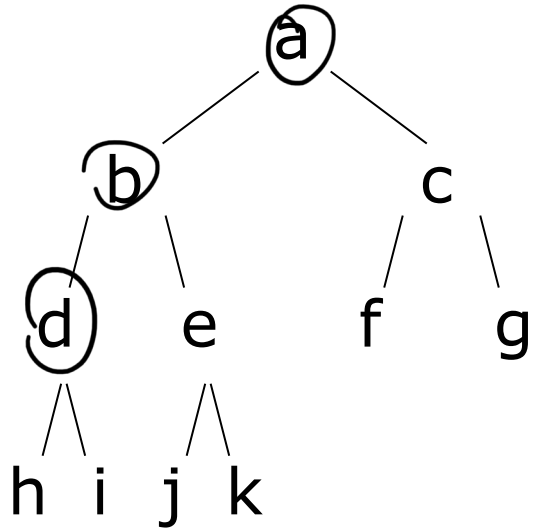
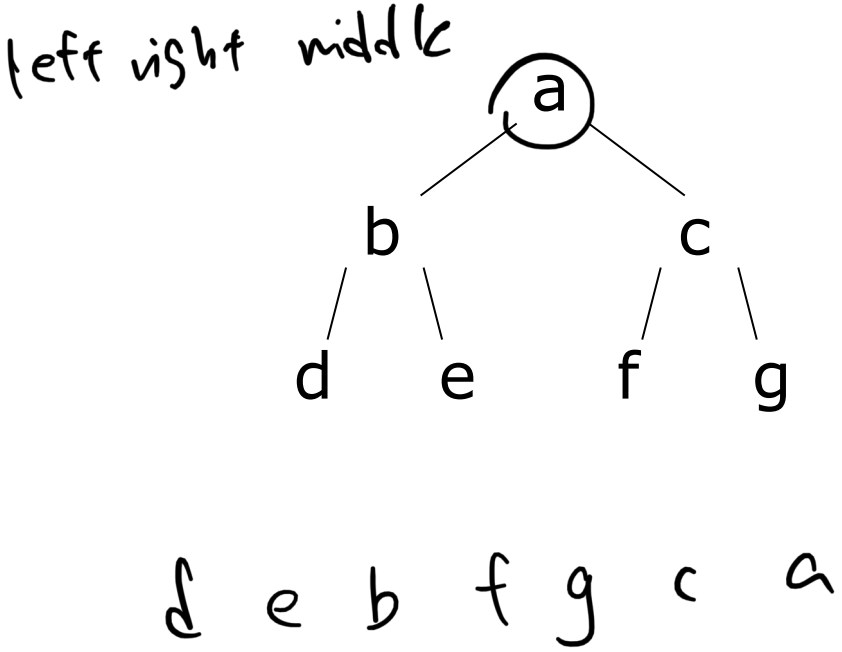


a b d h i e j k c f g

Post-Order Traversal of a Tree

Finally, we have the **'post-order' traversal of a tree**:

- We first visit the nodes in the left subtree, using post-order traversal.
- We then visit the nodes in the right subtree, using post-order traversal.



Post-Order Traversal of a Tree

Consider a post-order traversal of this tree:

- We first visit the nodes in the left subtree, using post-order traversal.
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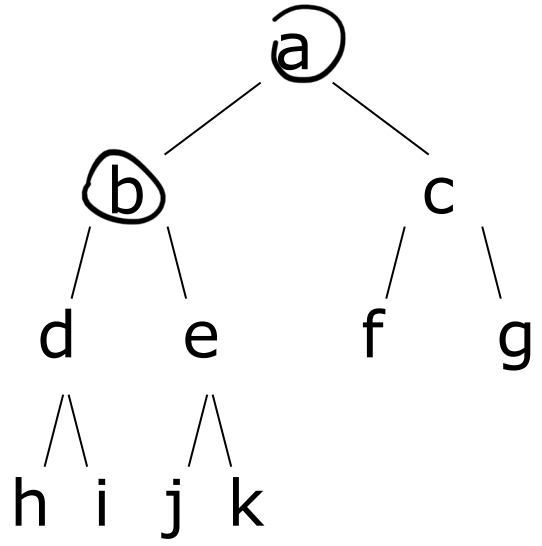

h i d j k e b f g c a

Post-Order Traversal of a Tree

Consider a post-order traversal of this tree:

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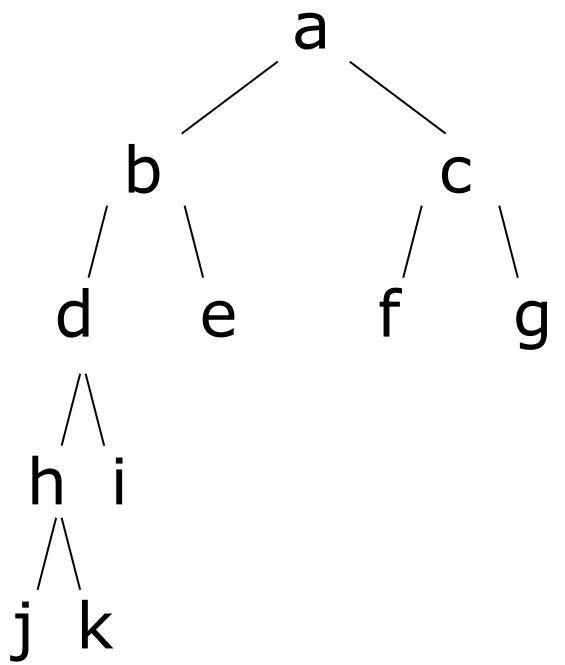


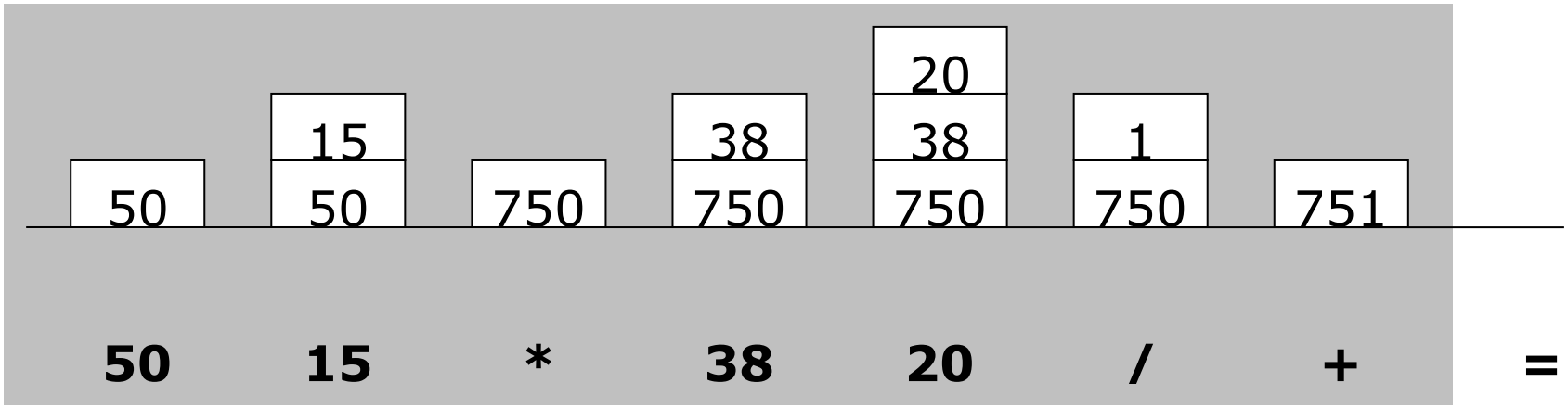
j k h i d e b f g c a

The Reverse Polish Notation RPN (Long Time Ago …)

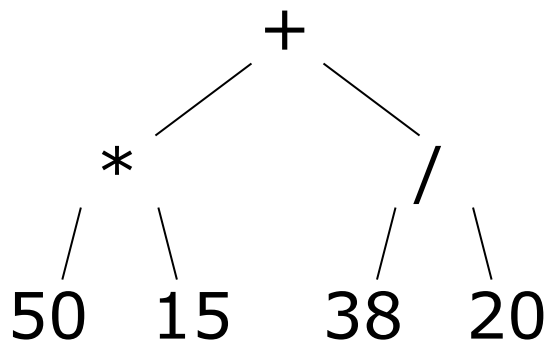
50 * 15 + 38 / 20

50 15 * 38 20 / + =

50↵
15↵
*↵
38↵
20↵
/↵
+↵
=↵



50 * 15 + 38 / 20



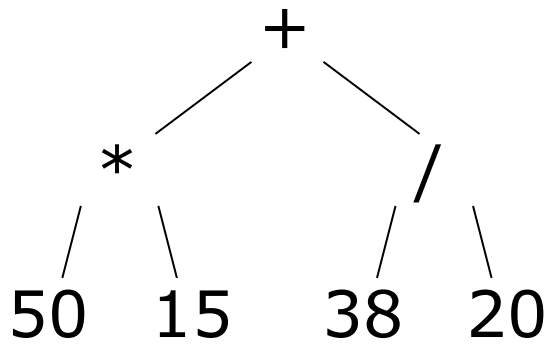
Post order (left right middle)

postfix form: **50 15 * 38 20 / +**

(operator put in the end) => operand operand operator

(left middle right) operand operator operand

In-order : 50 * 15 + 38 / 20 ← in-fix form



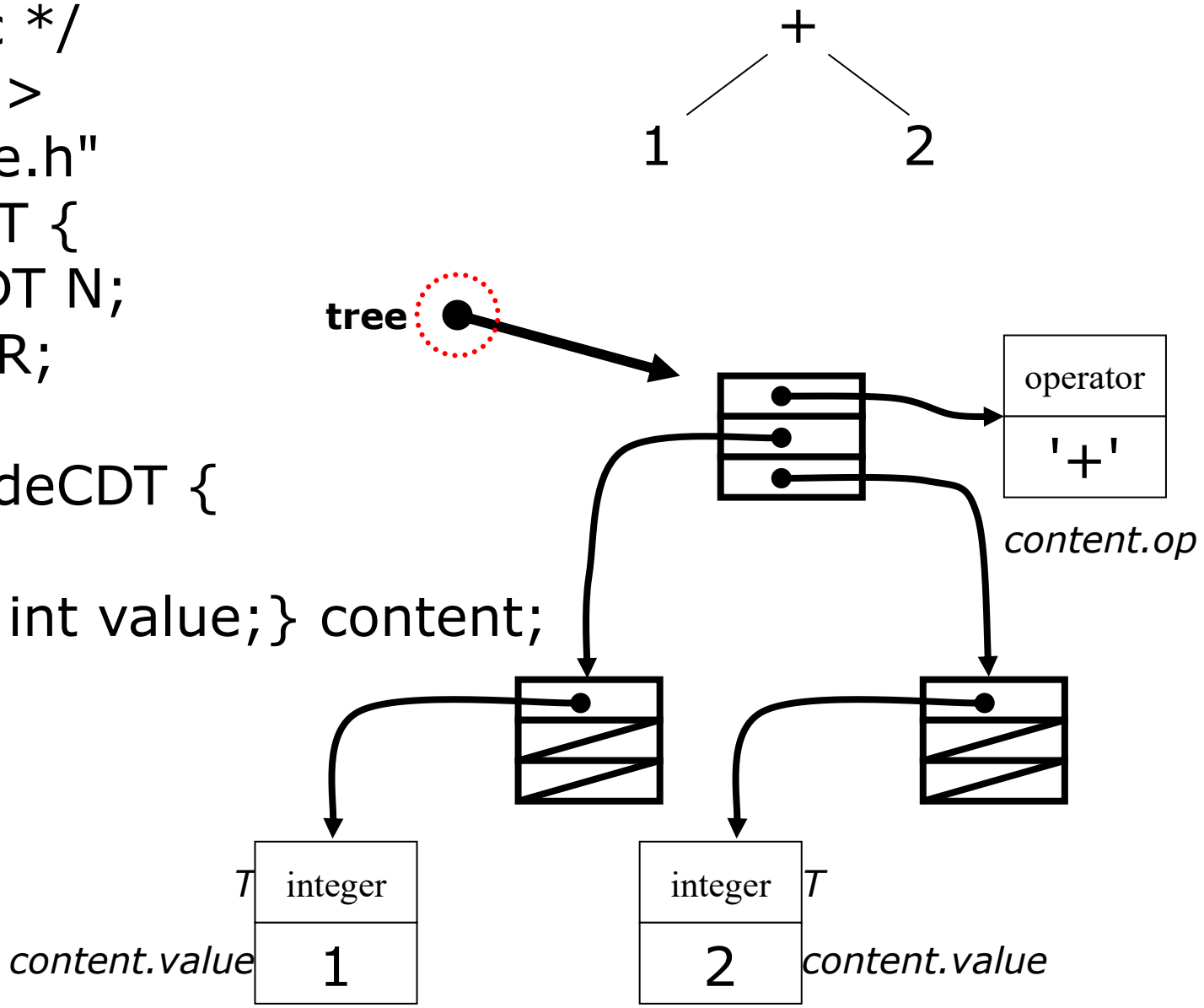
50 15 * 38 20 / +

Conclusion: the RPN of an expression can be obtained using a post-order traversal of the expression tree.

The Implementation

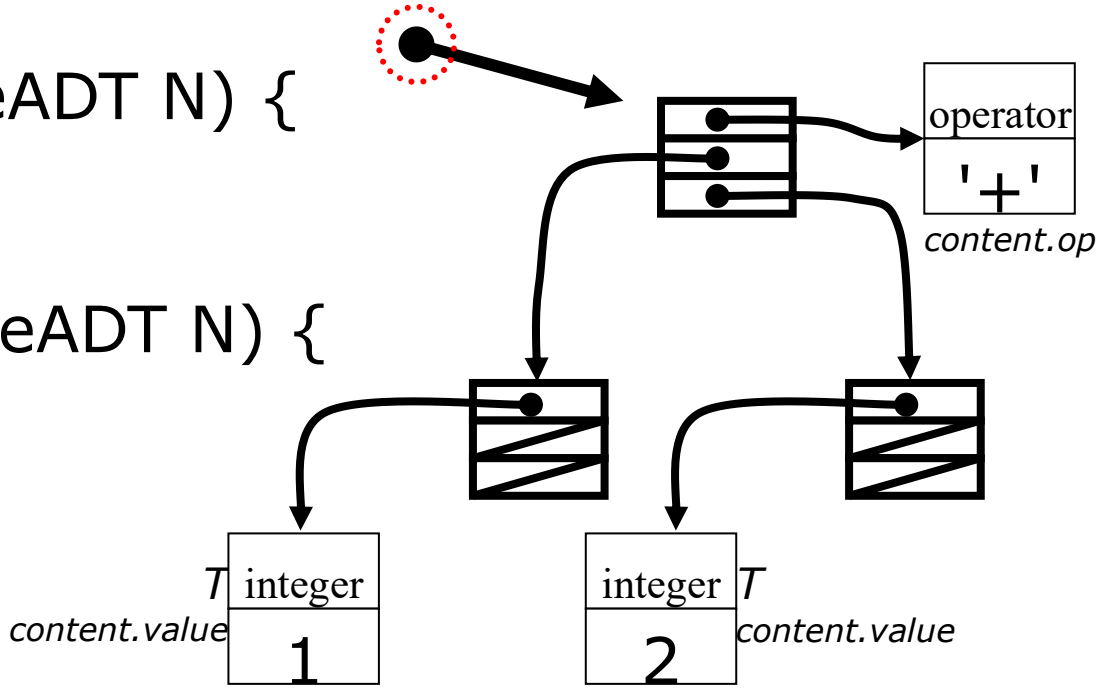
```
#include <stdbool.h>
typedef struct ExprTreeCDT * ExprTreeADT;
typedef struct ExprTreeNodeCDT *ExprTreeNodeADT;
enum nodetype {operator, integer};
typedef enum nodetype NodeTypeT;

NodeTypeT NodeType(ExprTreeNodeADT);
char NodeOp(ExprTreeNodeADT);
int NodeValue(ExprTreeNodeADT);
ExprTreeNodeADT ExprTreeRoot(ExprTreeADT);
ExprTreeADT EmptyExprTree(void);
ExprTreeADT NonemptyExprTree(ExprTreeNodeADT, ExprTreeADT, ExprTreeADT);
bool ExprTreeIsEmpty(ExprTreeADT);
ExprTreeADT LeftExprSubtree(ExprTreeADT);
ExprTreeADT RightExprSubtree(ExprTreeADT);
```

```
/* File: ExprTree.c */
#include <stdlib.h>
#include "ExprTree.h"
struct ExprTreeCDT {
   ExprTreeNodeADT N;
   ExprTreeADT L, R;
};
struct ExprTreeNodeCDT {
   NodeTypeT T;
   union {char op; int value;} content;
};
```



```
struct ExprTreeNodeCDT {
   NodeTypeT T;
   union {char op; int value;} content;
};
```

```
NodeTypeT NodeType(ExprTreeNodeADT N) {
   return N->T;
}
char NodeOp(ExprTreeNodeADT N) {
   return N->content.op;
}
int NodeValue(ExprTreeNodeADT N) {
   return N->content.value;
}
```



```
ExprTreeADT EmptyExprTree() {
  return (ExprTreeADT) NULL;
}

ExprTreeADT NonemptyExprSubtree(ExprTreeNode N,
        ExprTreeADT L, ExprTreeADT R) {
  ExprTreeADT t = (ExprTreeADT) malloc(sizeof(*t));
  t->N = N; t->L = L; t->R = R;
  return t;
}

bool ExprTreeIsEmpty(ExprTreeADT t) {
  return t == (ExprTreeADT) NULL;
}
```

```
ExprTreeNodeADT ExprTreeRoot(ExprTreeADT t) {
  return t->N;
}

ExprTreeADT LeftExprSubtree(ExprTreeADT t) {
  return t->L;
}

ExprTreeADT RightExprSubtree(ExprTreeADT t) {
  return t->R;
}
```