

CSCI3170 Introduction to Database Systems

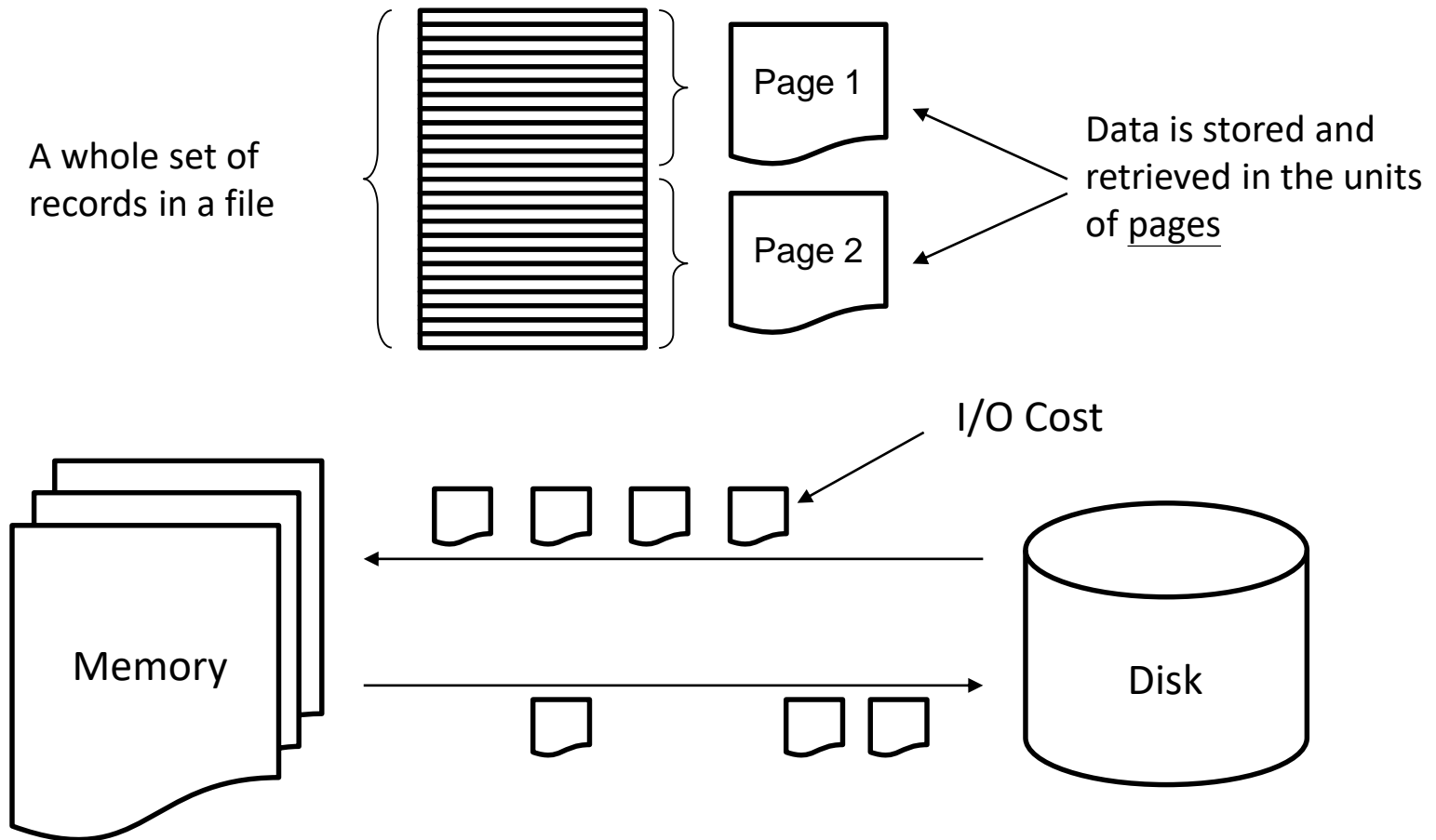
Tutorial 7 – Storage and Indexes

Outline

- Overview of Storage and Indexes
- Tree-structured Indexing
 - B+ Tree
- Hash-based Indexes
 - Static Hashing
 - Dynamic Hashing

Storage

- Files and pages



Record and Index

- Record
 - Record ID = $\langle \text{Page ID} \rangle + \langle \text{Offset} \rangle$
 - E.g. Record ID: $\langle 3 \rangle + \langle 10 \rangle$ = The 10th record in the 3rd page
- Index
 - Given a search key K, index can be used to speed up the selection of a set of particular pages.
 - An index file contains a collection of data entries.
 - The data entry of search key K is denoted as K^* .
 - $K^* = \langle K, \text{Record ID (rid)} \rangle$

Index Classification

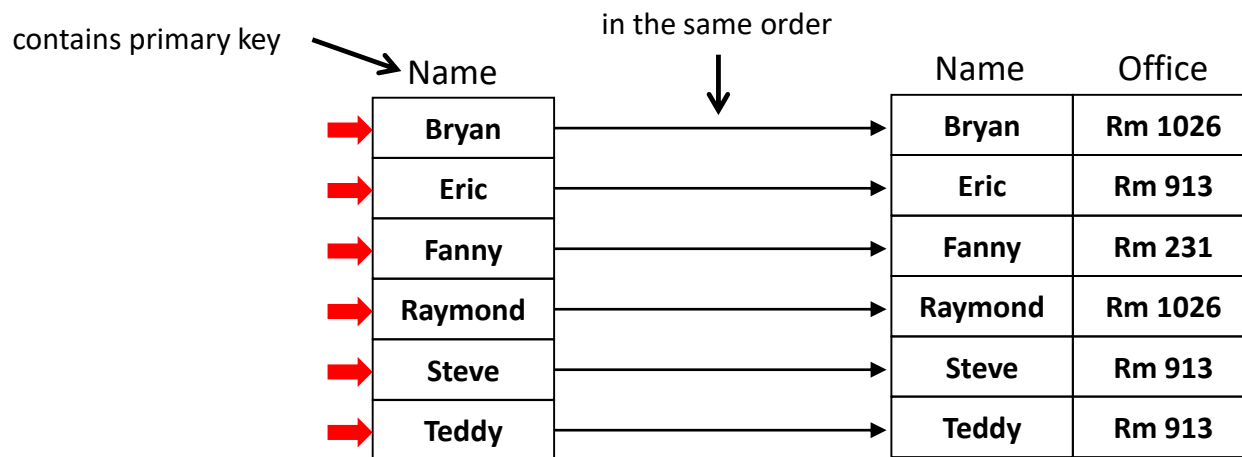
- Primary and Secondary
 - Primary
 - Search key contains primary key.
 - Secondary
 - Otherwise
- Dense and Sparse
 - Dense
 - K^* appear for ALL search key
 - Sparse
 - Otherwise

Index Classification (2)

- Clustered and Unclustered
 - Clustered
 - Data entries and records are sorted by K.
 - Order of records are equal/close to the order of data entries in index.
 - Support range search.
 - Unclustered
 - Otherwise

Index Classification: an Example

- Employee = { Name , Office }

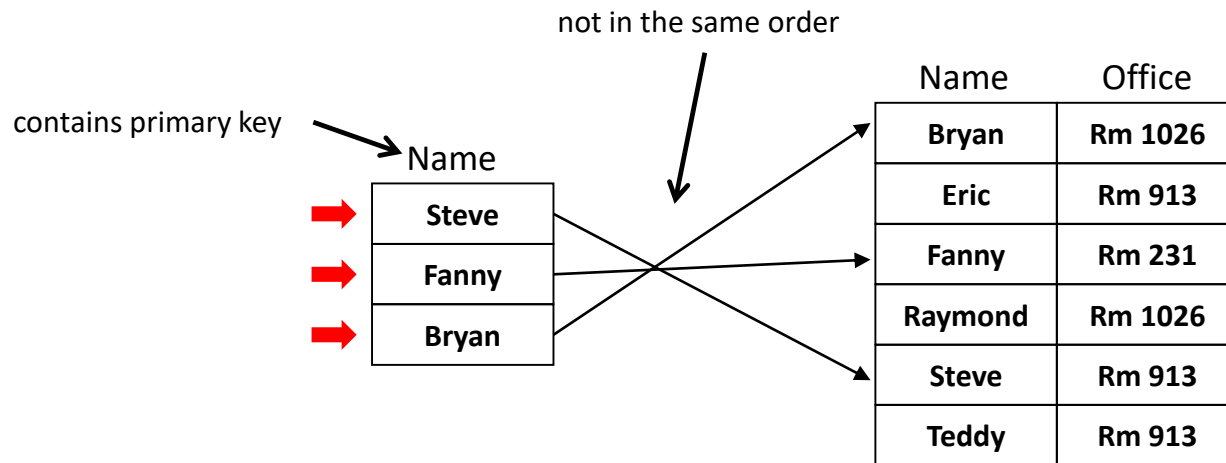


Primary	Secondary
Clustered	Unclusterd
Dense	Sparse

Name	
Bryan	✓
Eric	✓
Fanny	✓
Raymond	✓
Steve	✓
Teddy	✓

Index Classification: an Example

- Employee = { Name , Office }

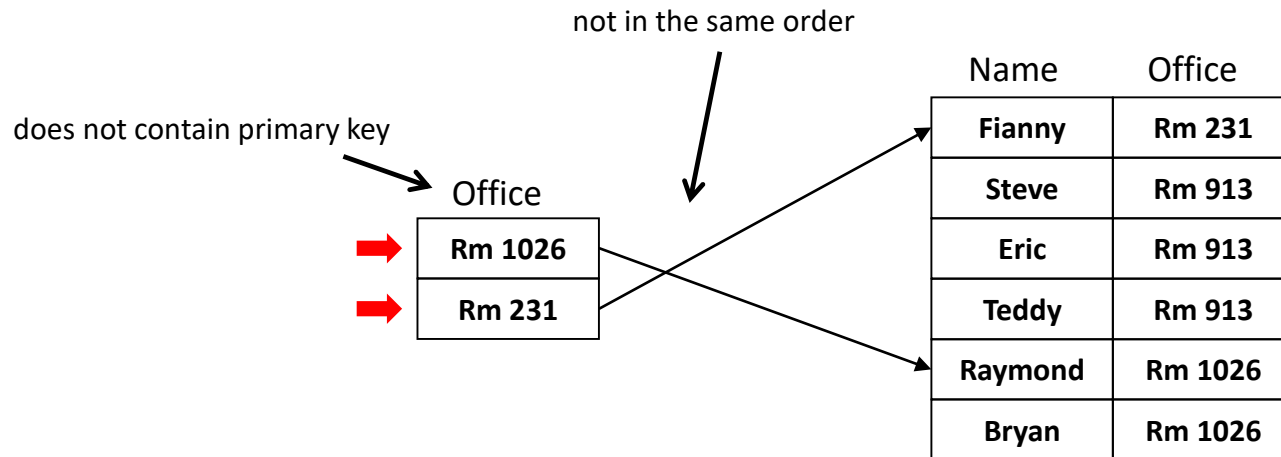


Primary	Secondary
Clustered	Unclusterd
Dense	Sparse

Name	
Bryan	✓
Eric	✗
Fanny	✓
Raymod	✗
Steve	✓
Teddy	✗

Index Classification: an Example

- Employee = { Name , Office }

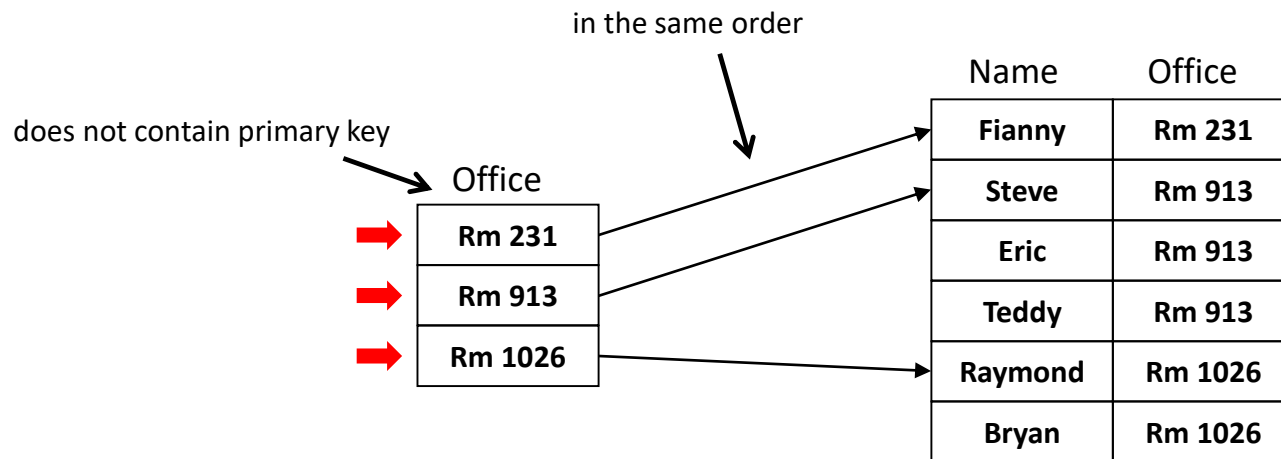


Primary	Secondary
Clustered	Unclusterd
Dense	Sparse

Name
→ Rm 231 ✓
Rm 913 ✗
Rm1026 ✓

Index Classification: an Example

- Employee = { Name , Office }

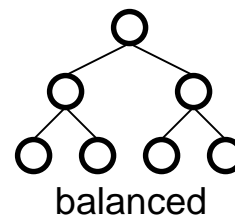
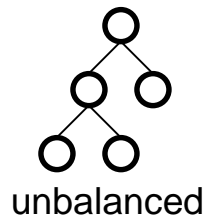


Primary	Secondary
Clustered	Unclusterd
Dense	Sparse

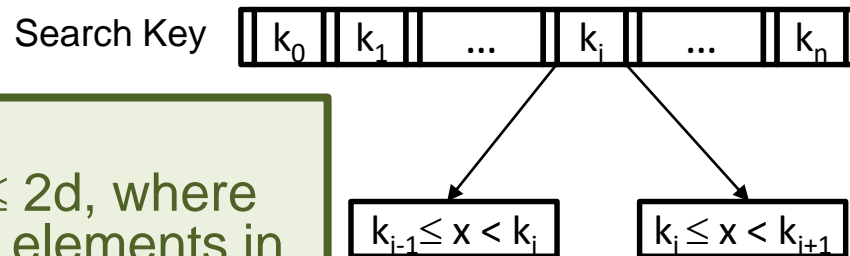
Name
Rm 231 ✓
Rm 913 ✓
Rm1026 ✓

Tree-structured Indexing

- B+ Tree Index Files
 - Balanced Tree – The length of every path from the root to a leaf is the same, maintaining efficient searching



- Support range search and equality selection

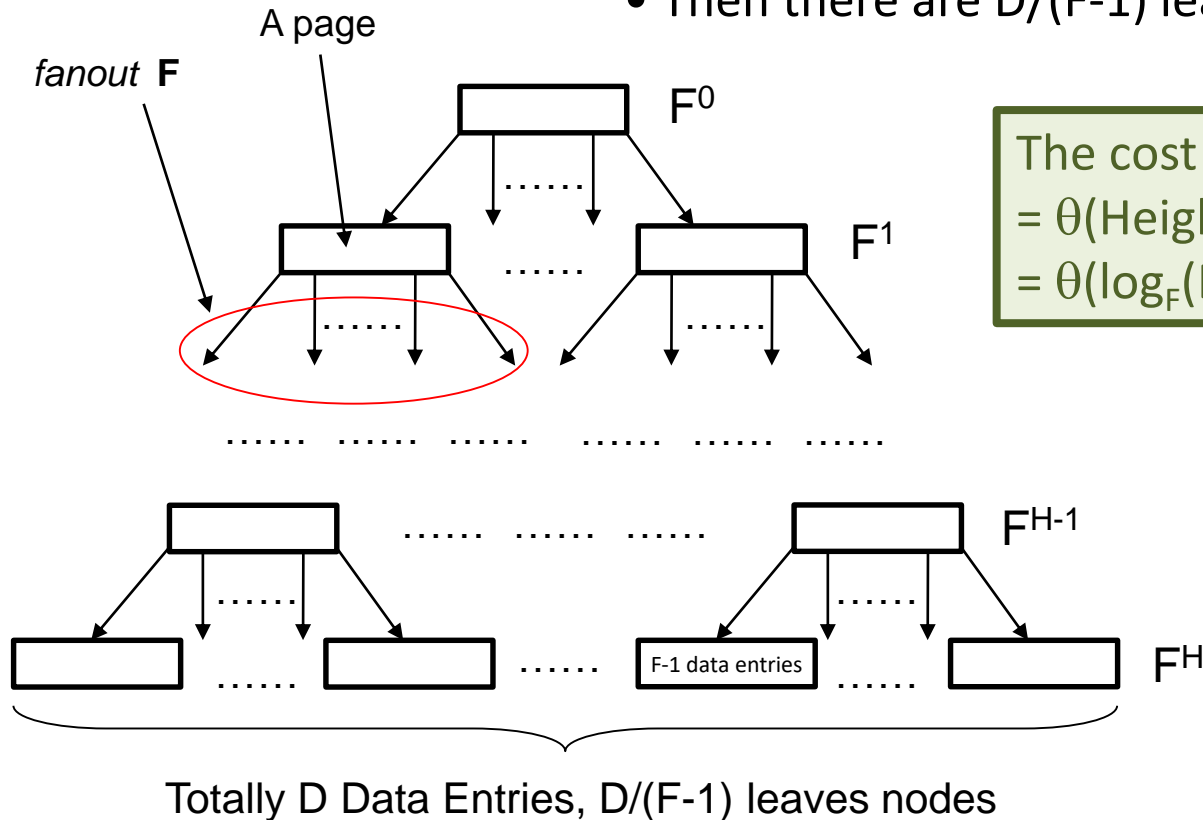


Order: d

- Root node: $1 \leq n_r \leq 2d$, where n_r is the number of elements in the root node
- Non-root nodes: $d \leq n_i \leq 2d$, where n_i is the number of elements in any non-root node

Search Cost of B+ Tree

- Let average number of pointers (*fanout*) is F
- Each leaf node can at most store $F-1$ data entries
- Suppose there are totally D data entries
- Then there are $D/(F-1)$ leaf nodes



The cost to reach a leave node:
 $= \theta(\text{Height of tree})$
 $= \theta(\log_F(D/(F-1)))$

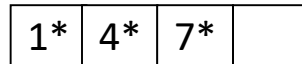
Insertion of B+ Tree

Order 2 B+ Tree

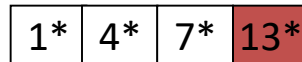
Root node: $1 \leq \# \text{ elements} \leq 4$

Non-root nodes: $2 \leq \# \text{ elements} \leq 4$

Original Tree

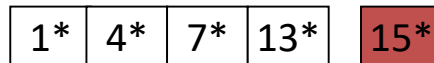


Insert 13



(No overflow)

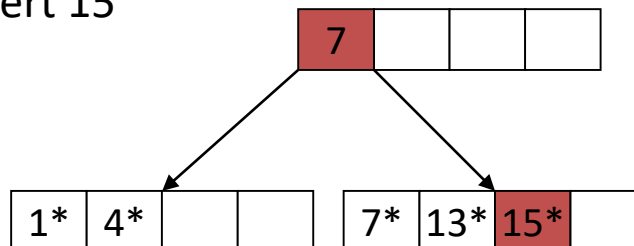
Insert 15



(Overflow)

Leaf node overflow: split leaf node, **copy** middle **key** to the parent.

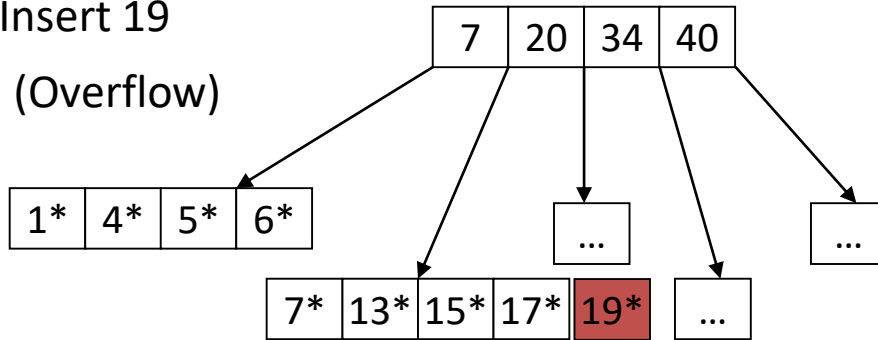
After Insert 15



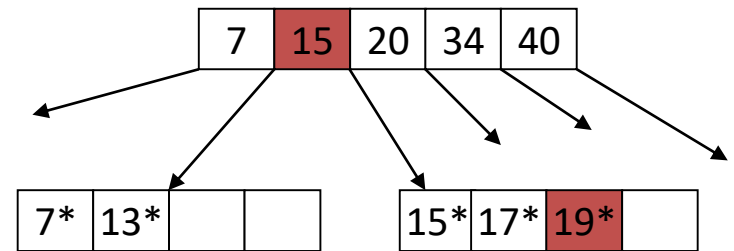
← Tree grows

Insertion of B+ Tree

Insert 19
(Overflow)

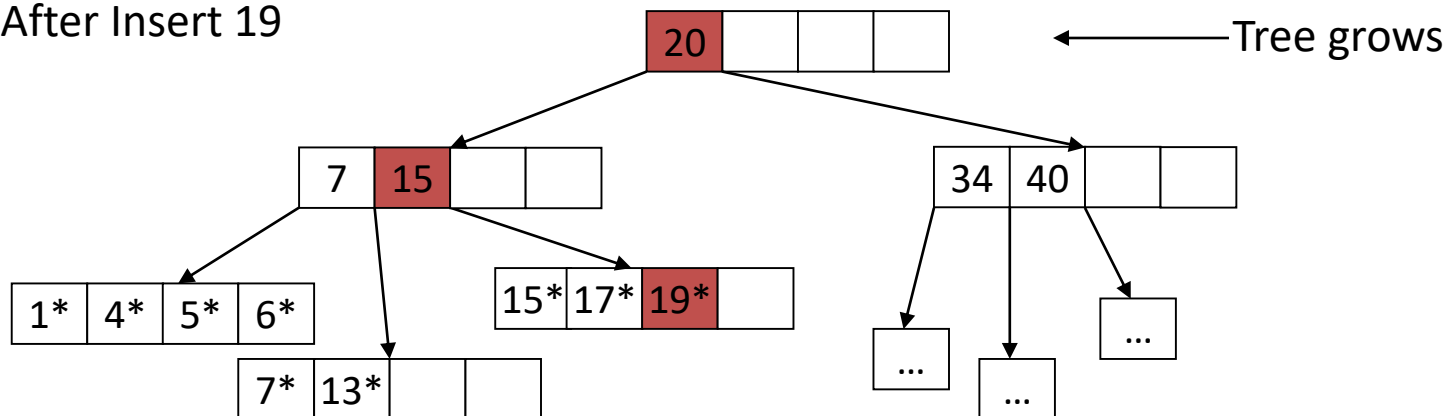


1. Leaf node overflow: split leaf node.



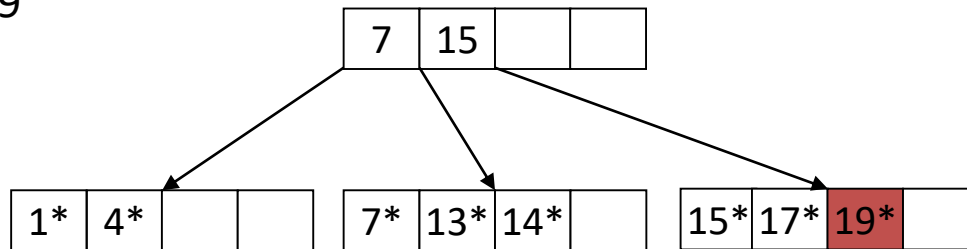
2. Non-leaf node overflow: split non-leaf node, **push** middle key up.

After Insert 19



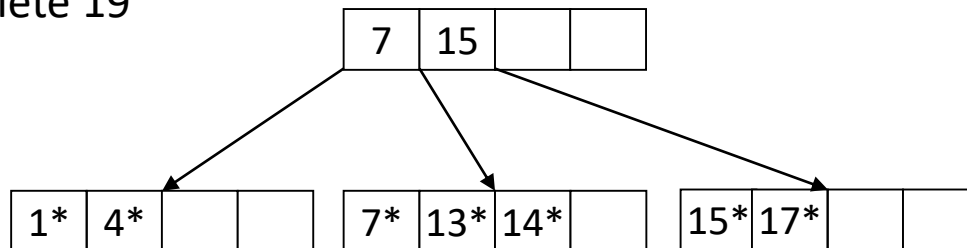
Deletion of B+ Tree

Delete 19



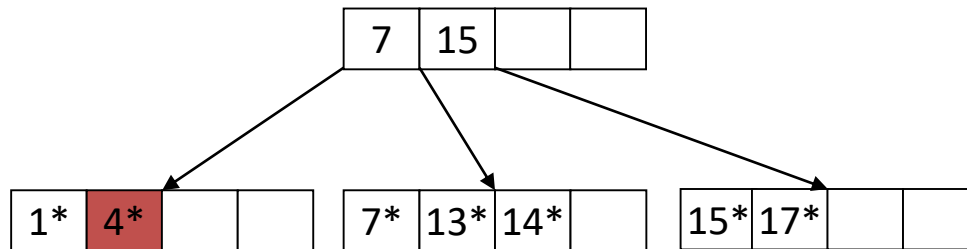
(No underflow)

After Delete 19



Deletion of B+ Tree

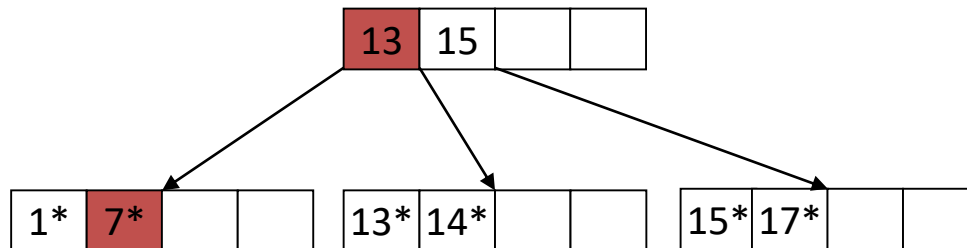
Delete 4



(Underflow)

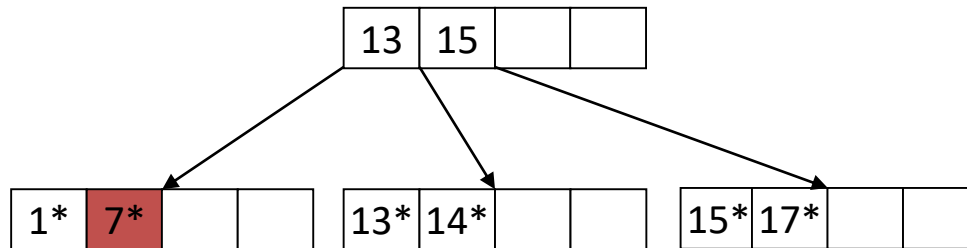
After delete 4, the page will underflow, need to borrow from sibling, update the parent.

After Delete 4



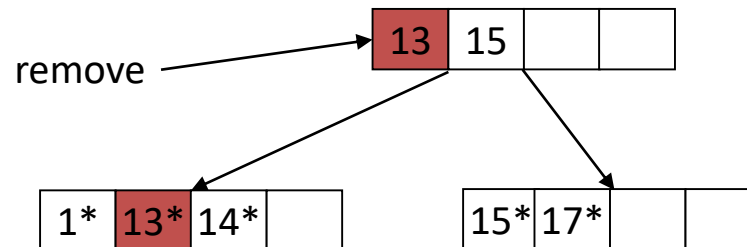
Deletion of B+ Tree

Delete 7

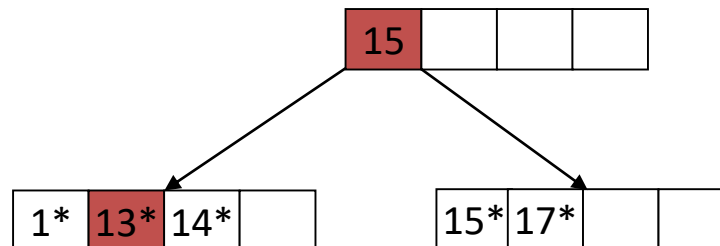


(Underflow)

After delete 7, the page will underflow, can't borrow from sibling, need to merge with sibling and update the parent.



After Delete 7

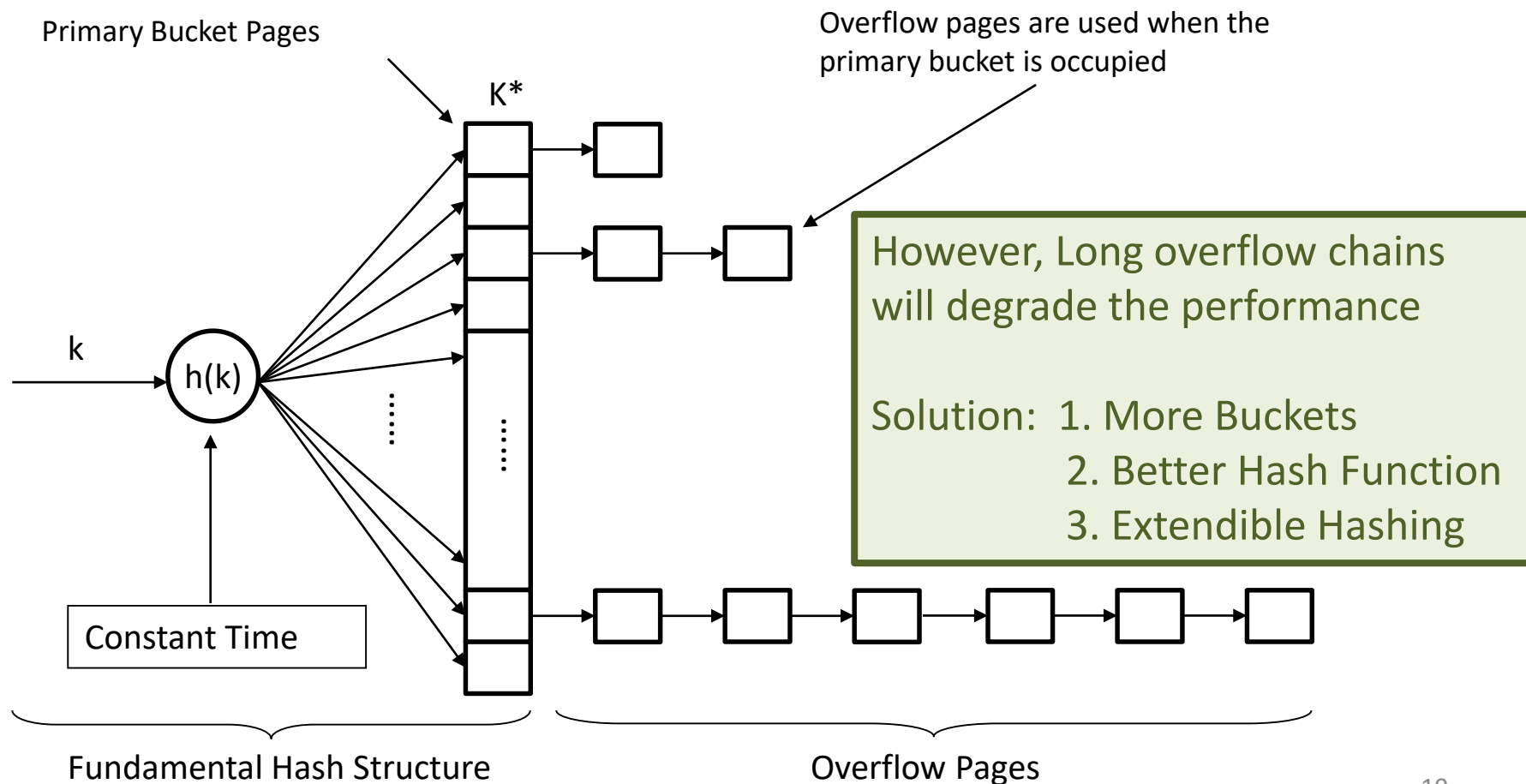


Hashing

- Properties
 - Data entries are kept in bucket.
 - Hashing function $h(k)$ = address of the bucket.
 - $h(k)$ should distribute K^* uniformly.
 - Best for equality selection.
 - Not support range search.
 - Constant time retrieval.

Static Hashing

No. of primary pages fixed, allocated sequentially, never de-allocated; overflow pages if needed.

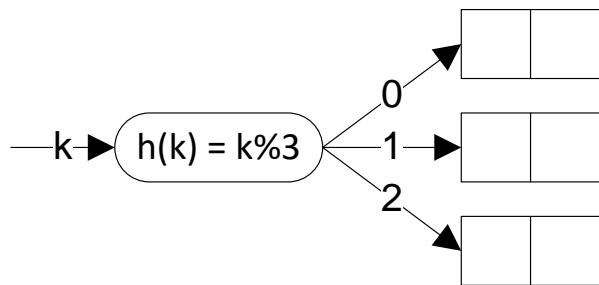


Static Hashing: an example

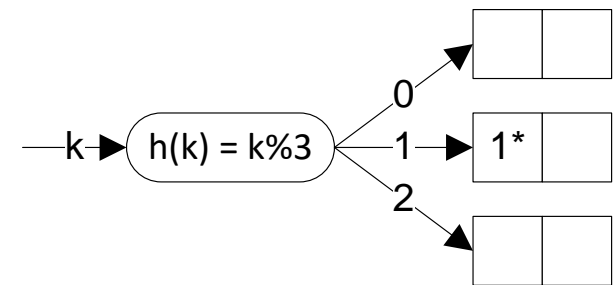
- Suppose the hash function is $h(x) = x \bmod 3$ and a bucket can hold at most 2 data entries. Below show a static hash structure after inserting 1, 4, 5, 7, 8, 2.

$$h(1) = 1$$

Insert 1



After insert 1

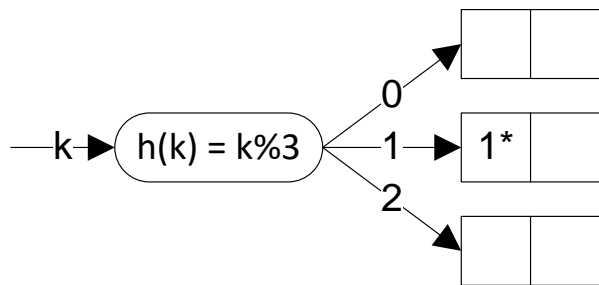


Static Hashing: an example

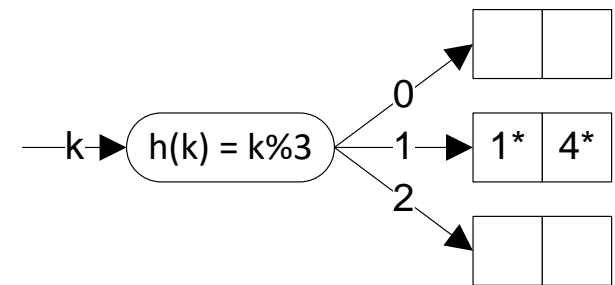
- Suppose the hash function is $h(x) = x \bmod 3$ and a bucket can hold at most 2 data entries. Below show a static hash structure after inserting 1, 4, 5, 7, 8, 2.

$$h(4) = 1$$

Insert 4



After insert 4

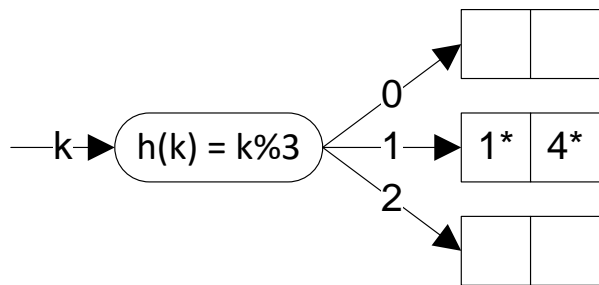


Static Hashing: an example

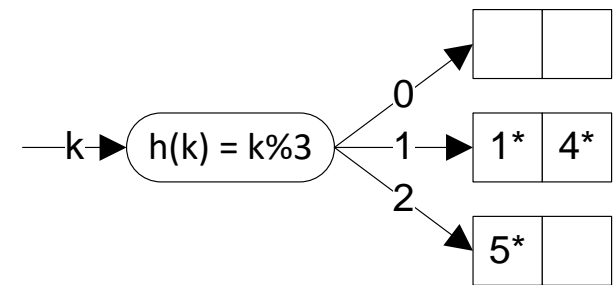
- Suppose the hash function is $h(x) = x \bmod 3$ and a bucket can hold at most 2 data entries. Below show a static hash structure after inserting 1, 4, 5, 7, 8, 2.

$$h(5) = 2$$

Insert 5



After insert 5

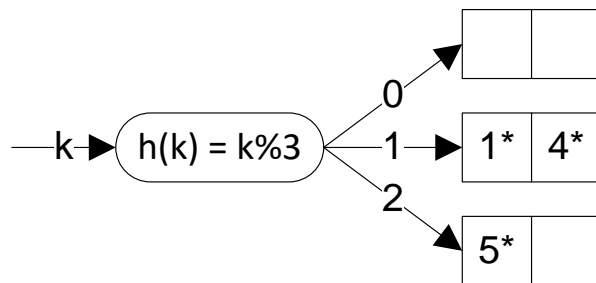


Static Hashing: an example

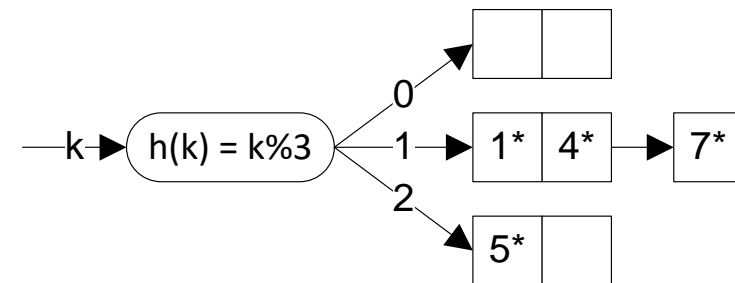
- Suppose the hash function is $h(x) = x \bmod 3$ and a bucket can hold at most 2 data entries. Below show a static hash structure after inserting 1, 4, 5, 7, 8, 2.

$$h(7) = 1$$

Insert 7



After insert 7

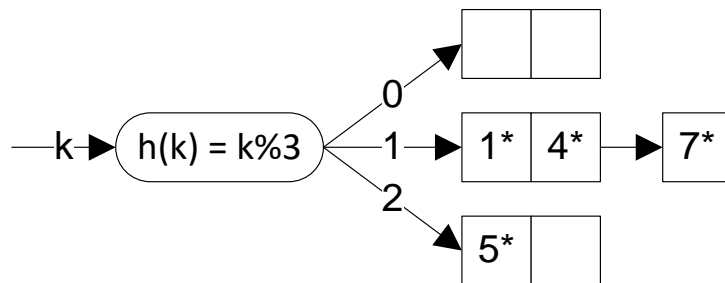


Static Hashing: an example

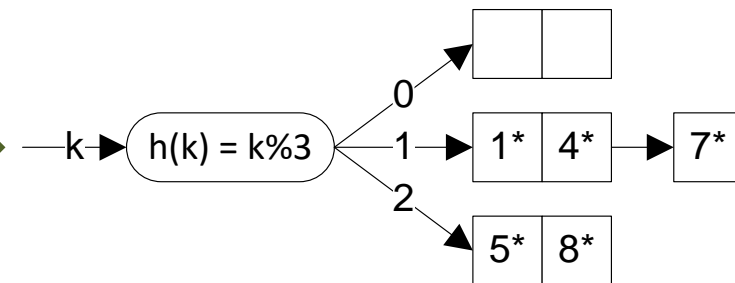
- Suppose the hash function is $h(x) = x \bmod 3$ and a bucket can hold at most 2 data entries. Below show a static hash structure after inserting 1, 4, 5, 7, 8, 2.

$$h(8) = 2$$

Insert 8



After insert 8

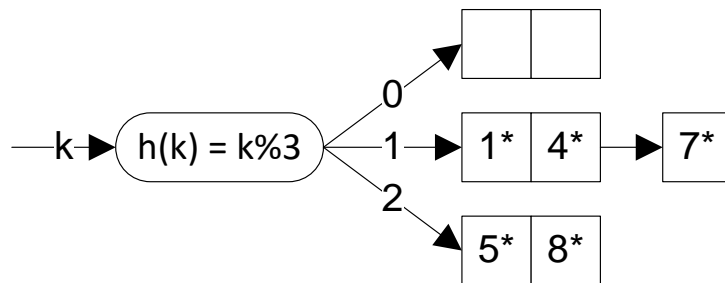


Static Hashing: an example

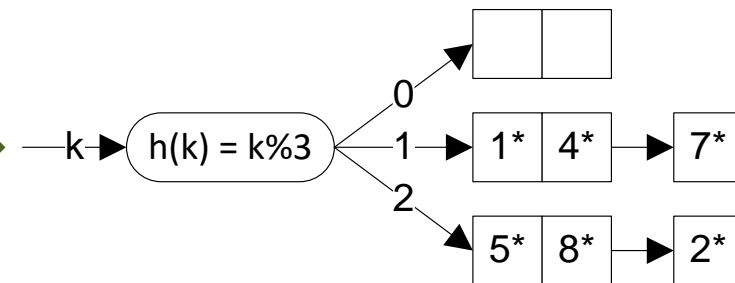
- Suppose the hash function is $h(x) = x \bmod 3$ and a bucket can hold at most 2 data entries. Below show a static hash structure after inserting 1, 4, 5, 7, 8, 2.

$$h(2) = 2$$

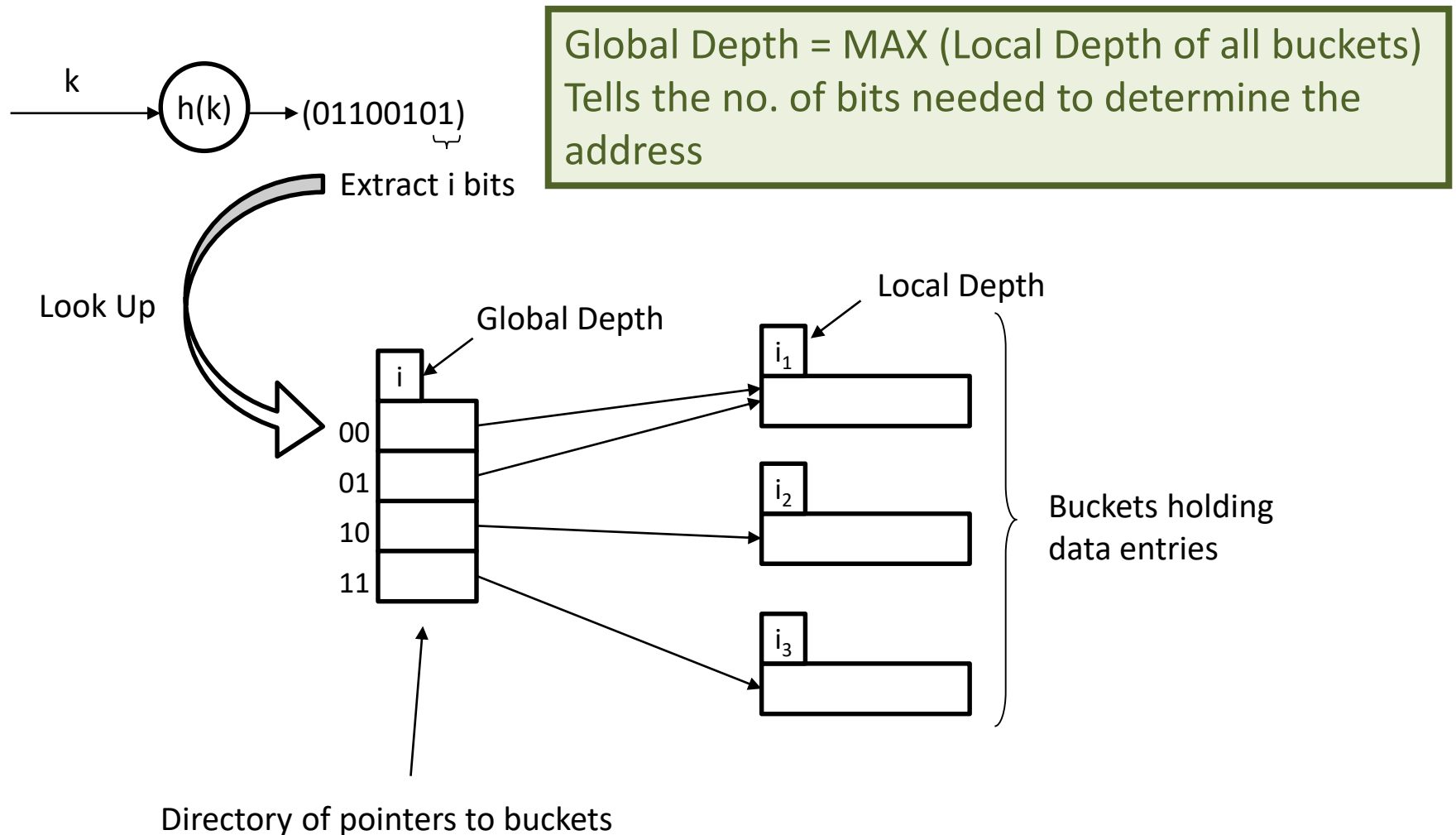
Insert 2



After insert 2



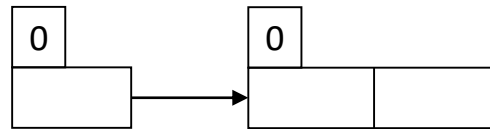
Extendible Hashing



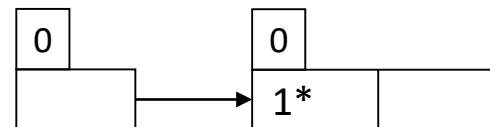
Insertion of Extendible Hashing

- Suppose the hash function is $h(x) = x \bmod 8$ and each bucket can hold at most 2 data entries. Below show an extendable hash structure after inserting 1, 4, 5, 7, 8, 2.

Initial:



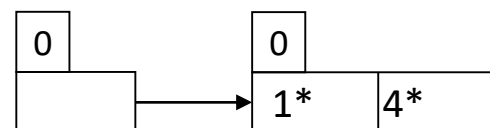
After insert 1:



(ok)

$$h(1) = 001$$

After insert 4:

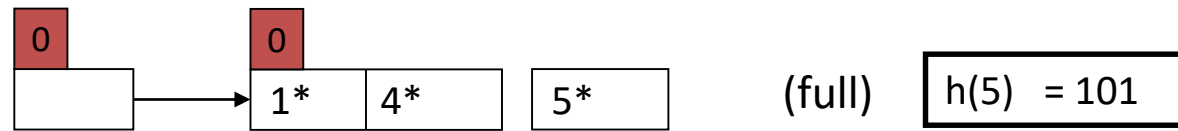


(ok)

$$\begin{aligned} h(1) &= 001 \\ h(4) &= 100 \end{aligned}$$

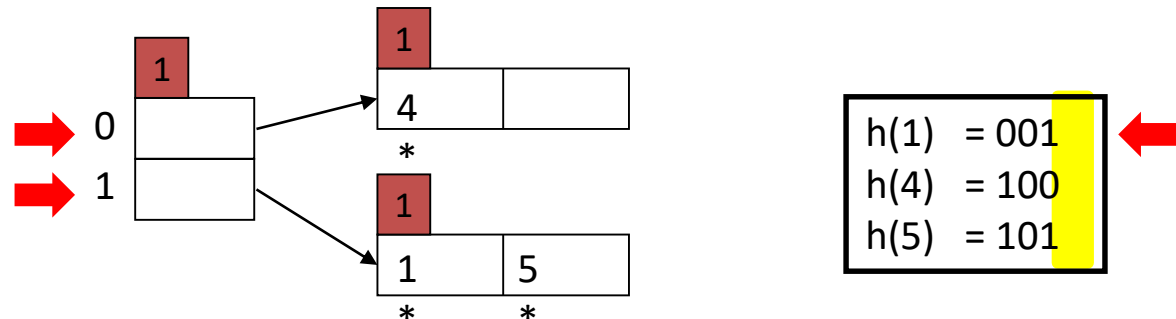
Insertion of Extendible Hashing

Insert 5:



If bucket with local depth = global depth, then double the directory and split the bucket.

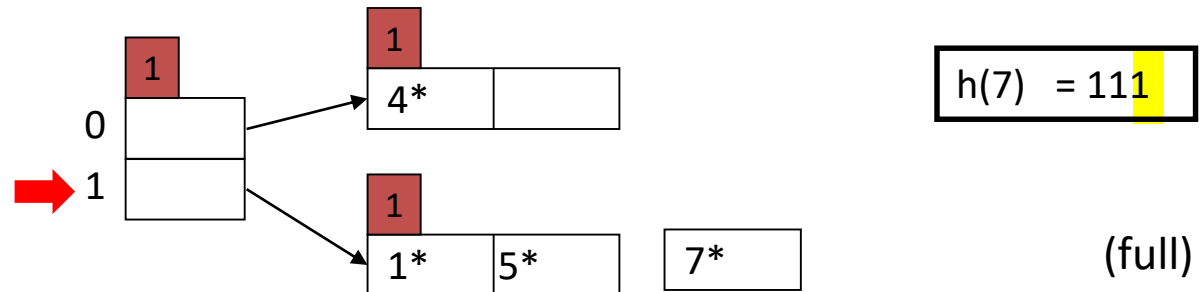
After insert 5:



Increase the global depth and local depth of the new buckets by 1.

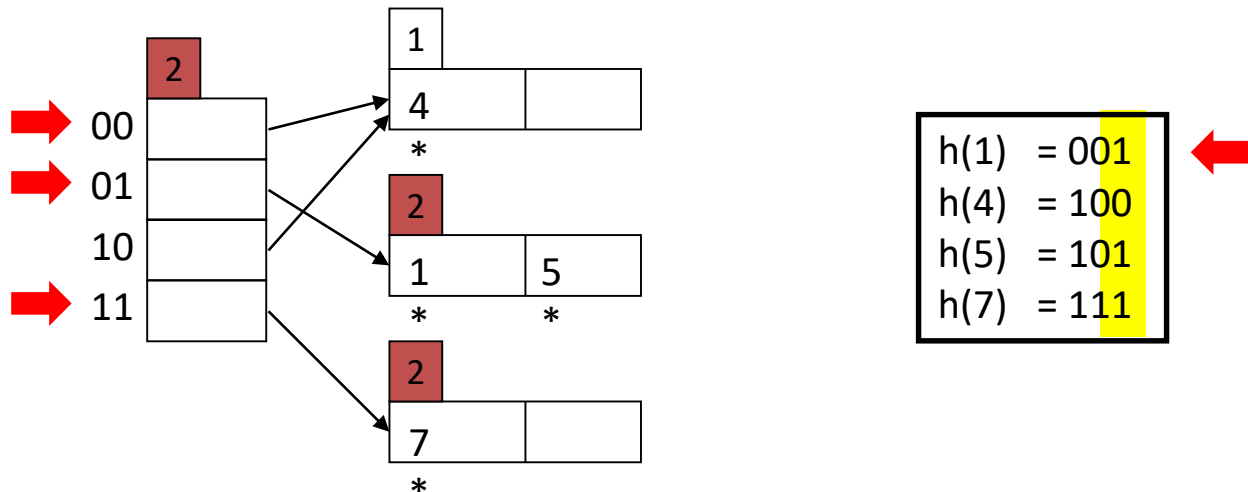
Insertion of Extendible Hashing

Insert 7:



If bucket with local depth = global depth, then double the directory and split the bucket.

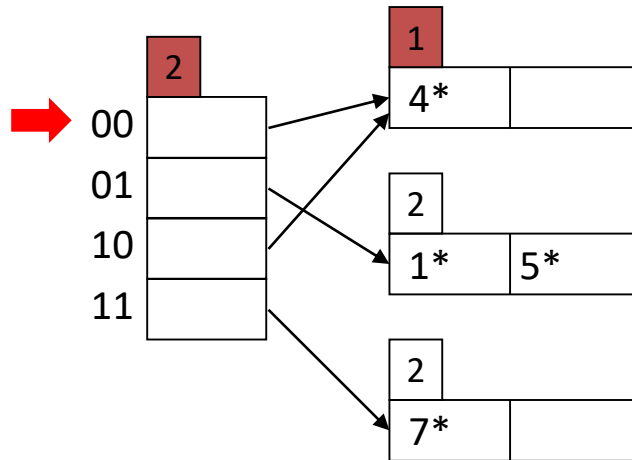
After insert 7:



Increase the global depth and local depth of the new buckets by 1.

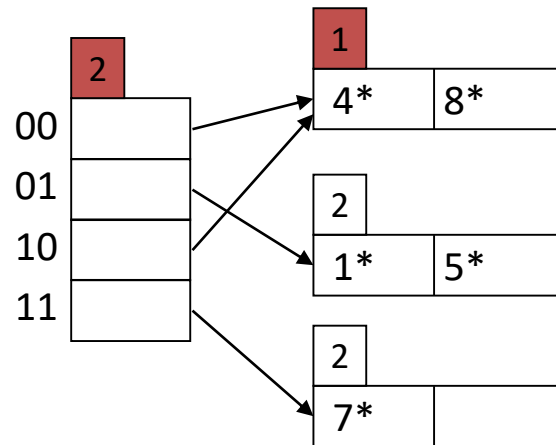
Insertion of Extendible Hashing

Insert 8:



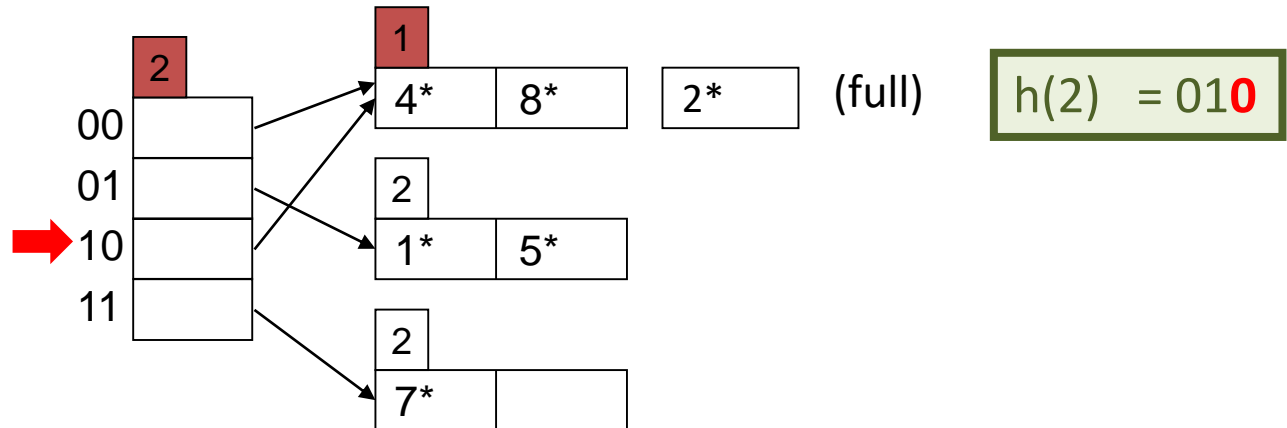
$$h(8) = 00\mathbf{0}$$

After insert 8:



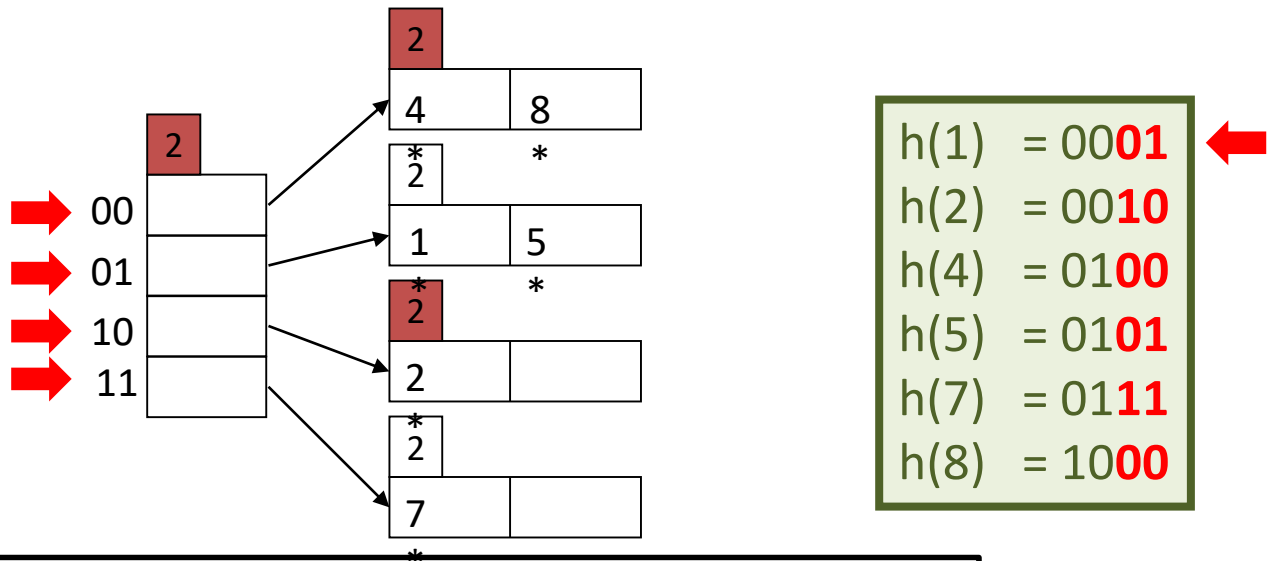
Insertion of Extendible Hashing

Insert 2:



If bucket with local depth < global depth, then split the bucket and update the pointers.

After insert 2:

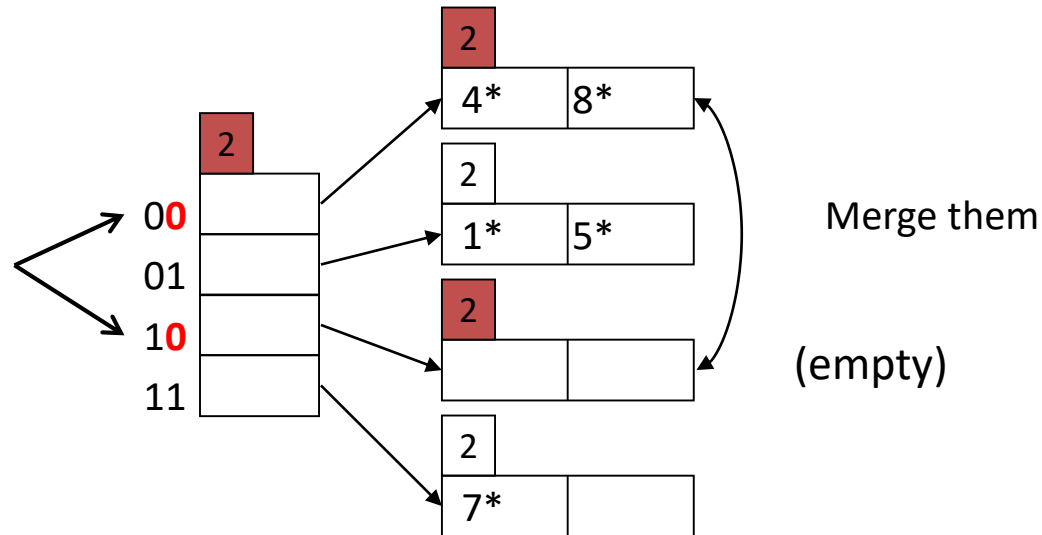


Increase the local depth of the new buckets by 1.

Deletion of Extendible Hashing

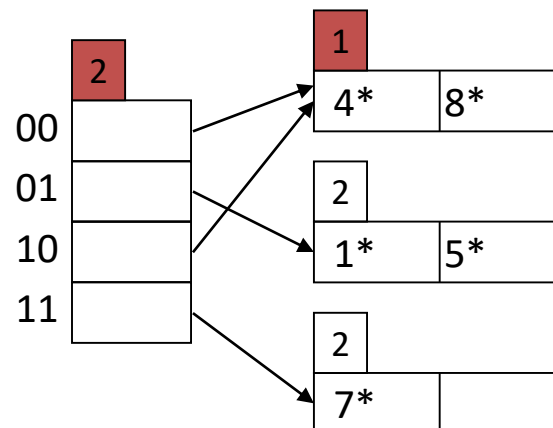
Delete 2:

Look at the last
(global depth-1) bits



If the removal makes the bucket empty, then remove the bucket and update the pointer.

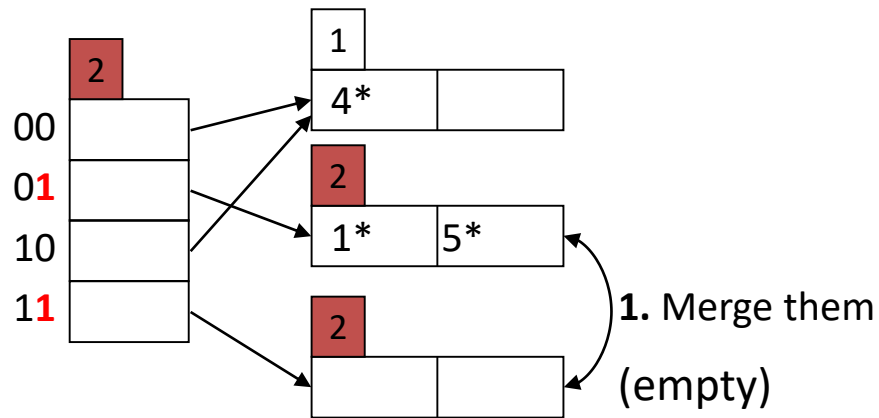
After delete 2:



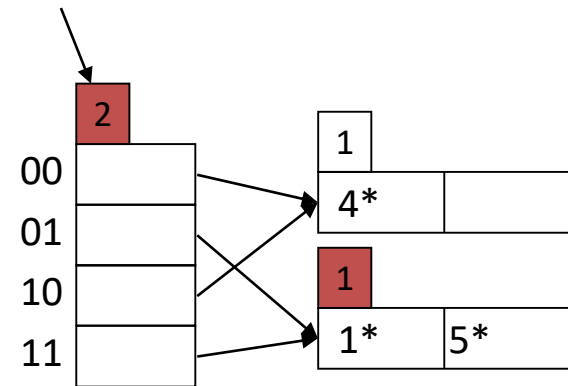
Decrease the local depth of the new bucket by 1.

Deletion of Extendible Hashing

Delete 7:

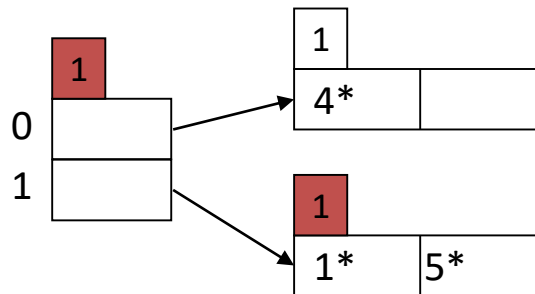


2. Halve the directory



If the merge makes $\max(\text{local depth of all buckets}) < \text{global depth}$, then halve the size of directory.

After delete 7:



Decrease the global depth of the new bucket by 1.