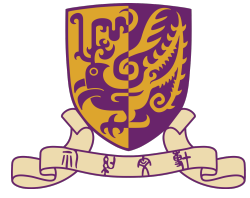
CSCI3230 (ESTR3108)
Fundamentals of Artificial Intelligence

Tutorial 2

Tao Huang

Email: thuang22@cse.cuhk.edu.hk
Office: Room 1026, 10/F, SHB

Dept. of Computer Science & Engineering
The Chinese University of Hong Kong



Outline

Part 1. Gradient Descent

Part 2. Play with gradient descent in Python

Part 1. Gradient Descent

Gradient descent algorithm

Review the algorithm of gradient descent:

Gradient descent

Ensure: $\alpha > 0$

Initialize $\Theta \leftarrow \Theta_0$ randomly

while not converge **do**

$\Theta \leftarrow \Theta - \alpha \nabla f(\Theta)$

end while

- An iterative method of finding the minimum.
- α is a small enough hyper-parameter called **learning rate**.

Before gradient descent

Specify these things:

- Learning rate α
- How to initialize Θ: usually sampled from a consistent distribution.
- Converging criterion (how to tell convergence):
  Usually when the distance between results in the last iteration and current iteration are less than a threshold.
- The objective function $f(\Theta)$ and its gradient $\nabla f(\Theta)$:
  In general, find the analytic form of $\nabla f(\Theta)$ so that we can retrieve the precise results of gradients.

Problem

Apply gradient descent to find the θ^* that minimizes $f(\theta) = \theta^2$. We set $\alpha = 1.0$, θ is initialized to 10, and regard the algorithm converges once $|f(\theta^*) - f(\theta)| \leq 0.01$. What is the number of iterations required at least to guarantee the convergence?

A 33 iterations

B 16 iterations

C 7 iterations

D not convergent

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Correct Answer: D

Solution to the Problem

- Objective function: $f(\theta) = \theta^2$
- Gradient: $\nabla f(\theta) = 2\theta$
- Learning rate: 1.0
- Converging criterion: $|f(\theta^*) - f(\theta)| \leq 0.01 \iff |f(\theta)| \leq 0.01$

- For the i-th iteration, $\theta_i = \theta_{i-1} - 1.0 \cdot 2 \cdot \theta_{i-1} = -\theta_i$
- $f(\theta)$ is symmetric: $f(\theta) = f(-\theta)$.
- $f(\theta_i) = f(\theta_{i-1})$, i.e., no descent!
- $\implies$ Never converge.

Part 2. Play with gradient descent in Python