Thrust required

Consider an aircraft in **steady and level flight** at a given altitude and a given velocity, from **the equations of motion**, we have

$$T = D$$

$$L = W$$

The **thrust required (T_R)** to obtain steady flight is

$$T_R = D = q_\infty S C_D$$

Then, by adding

The drag polar $\qquad C_D = C_{D,0} + K C_L^2$

Lift coefficient $\qquad C_L = \frac{L}{q_\infty S} = \frac{W}{q_\infty S}$

We have

$$T_R = q_\infty S \left(C_{D,0} + K \frac{W^2}{q_\infty^2 S^2} \right) = q_\infty S C_{D,0} + \frac{K W^2}{q_\infty S}$$

Thrust required

From the last result

$$T_R = q_\infty S C_{D,0} + \frac{KW^2}{q_\infty S}$$

We can further add $q_\infty = (1/2)\rho_\infty V_\infty^2$ and get

$$\begin{aligned} T_R &= \frac{1}{2}\rho_\infty V_\infty^2 S C_{D,0} + \frac{2KW^2}{\rho_\infty V_\infty^2 S} \\ &= \left(\frac{1}{2}\rho_\infty S C_{D,0}\right) V_\infty^2 + \left(\frac{2KW^2}{\rho_\infty S}\right)\frac{1}{V_\infty^2} \end{aligned}$$

Similar to the total drag of the aircraft, T_R also consists of **two parts**:

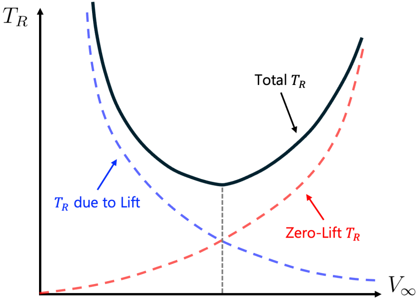
$$\boxed{T_R = \underbrace{\left(\frac{1}{2}\rho_\infty S C_{D,0}\right) V_\infty^2}_{\text{Zero-lift } T_R} + \underbrace{\left(\frac{2KW^2}{\rho_\infty S}\right)\frac{1}{V_\infty^2}}_{T_R \text{ due to lift}}}$$

The graphical representation of T_R

$$T_R = \underbrace{\left(\frac{1}{2}\rho_\infty S C_{D,0}\right) V_\infty^2}_{\text{Zero-lift } T_R} + \underbrace{\left(\frac{2KW^2}{\rho_\infty S}\right)\frac{1}{V_\infty^2}}_{T_R \text{ due to lift}}$$

We observe that

- Zero-lift T_R is proportional to V_∞^2
- T_R due to lift is proportional to $1/V_\infty^2$

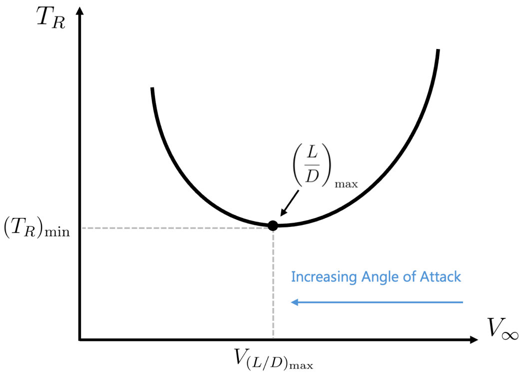


Thrust required

Thrust required is dictated by the **airframe** (aerodynamics and weight) of the aircraft

Overall, the **thrust required curve** is a **convex curve**!

- T_R varies inversely as L/D: minimum T_R corresponds to $(L/D)_{\max}$
- T_R varies with angle of attack: minimum T_R is reached at a specific α



Thrust available (T_A): the propulsive thrust provided by the engine

- **Turbojet engine:** T_A is reasonably constant with V_∞ for subsonic flight

$$T_A = \rho_\infty A_0 V_\infty (V_e - V_\infty)$$

As V_∞ is increased, the two effects tend to cancel (V_e staying about the same)

- **Propeller engine:** T_A decreases with V_∞