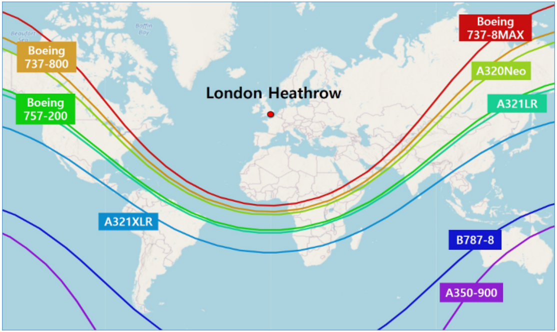
Range (R): the **total distance** (measured with respect to the ground) traversed by an aircraft on one tank of fuel

What aircraft design features and operating parameters are relevant to range?

Throughout the 20th century, range was one of the most important design features for both commercial and military aircraft



Aircraft weights (in kg)

- W_0**: gross weight** of the aircraft including everything – structure, fuel, payload, crew, etc.
- W_f**: weight of fuel**, an instantaneous value which changes during flight
- W_1**:** weight of the aircraft **without fuel**

At **any instant during the flight**, we have

$$W_0 = W_1 + W_f$$

W_f and W_0 are decreasing during flight. The **time rate of change of weight** is:

$$\frac{dW}{dt} = \frac{dW_f}{dt} = \dot{W}_f$$

The rate of fuel consumption $\dot{W}_f$ is an engine feature; and engine performance is important for aircraft range!

Specific fuel consumption (SFC): the **fuel mass flow rate** $\dot{m}_f$ required to produce a given engine output (thrust, power, etc.)

- SFC is one of the most important metrics in aviation to **measure fuel efficiency**
- SFC **reduces the effect of size** and facilitates comparison of engine performance

SFC is different for jet engines and piston engines:

- **Jet engine: Thrust-specific fuel consumption (TSFC)**

$$\boxed{\mathsf{TSFC} = \frac{\dot{m}_f}{T}} \quad \text{Unit: kg/hr}\cdot\text{N}$$

$$\dot{W}_f = -\mathsf{TSFC} \cdot T$$

- **Piston engine: Power-specific fuel consumption (PSFC)**

$$\boxed{\mathsf{PSFC} = \frac{\dot{m}_f}{P}} \quad \text{Unit: kg/hr}\cdot\text{hp}$$

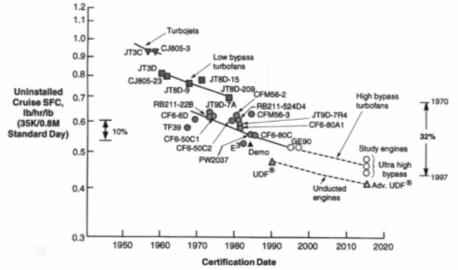
$$\dot{W}_f = -\mathsf{TSFC} \cdot P$$

Thrust-specific fuel consumption (TSFC)

$$\boxed{\begin{aligned}\text{TSFC} &= \frac{\dot{m}_f}{T} \\ \dot{W}_f &= -\text{TSFC} \cdot T\end{aligned}}$$

- Low TSFC = high efficiency
- Ways to minimize fuel consumption: (1) ↓ TSFC, (2) ↓ thrust required T_R

Engine TSFC has been decreasing quickly over the years



The Breguet range equation

Now, we already have the **two pieces of information** to derive the range equation

1. The time rate of change of weight

$$\frac{dW}{dt} = \dot{W}_f$$

2. The fuel consumption via TSFC

$$\dot{W}_f = -\mathsf{TSFC} \cdot T$$

We obtain the differential equation

$$\boxed{\frac{dW}{dt} = -\mathsf{TSFC} \cdot T}$$

Divide both sides by W:

$$\frac{dW}{W} = -\mathsf{TSFC} \cdot \frac{T}{W} dt$$

The Breguet range equation

In steady, level flight, $L = W$ and $T = D$. Hence

$$\frac{dW}{W} = -\mathsf{TSFC} \cdot \frac{T}{W} dt \qquad \longrightarrow \qquad \frac{dW}{W} = -\mathsf{TSFC} \cdot \frac{D}{L} dt$$

And we have

$$dt = -\frac{1}{\mathsf{TSFC}} \left(\frac{L}{D}\right) \frac{dW}{W}$$

Multiply both sides by V_∞:

$$V_\infty dt = -\frac{V_\infty}{\mathsf{TSFC}} \left(\frac{L}{D}\right) \frac{dW}{W} s$$

Then, integrate both sides from the <u>start of the mission (0)</u> to the <u>end of the mission (1)</u>

$$\int_{t_0}^{t_1} V_\infty dt = -\int_{W_0}^{W_1} \frac{V_\infty}{\mathsf{TSFC}} \left(\frac{L}{D}\right) \frac{dW}{W}$$

The Breguet range equation

$$\int_{t_0}^{t_1} V_\infty \, dt = -\int_{W_0}^{W_1} \frac{V_\infty}{\mathsf{TSFC}} \left(\frac{L}{D}\right) \frac{dW}{W}$$

$$V_\infty (t_1 - t_0) = \frac{V_\infty}{\mathsf{TSFC}} \left(\frac{L}{D}\right) (\ln W_0 - \ln W_1)$$

Finally, we have the Breguet range equation

$$\boxed{R = \frac{V_\infty}{\mathsf{TSFC}} \left(\frac{L}{D}\right) \ln\left(\frac{W_0}{W_1}\right)}$$

Observations:

- Parameters that influence R include TSFC, V_∞, L/D, and fuel fraction
- However, the parameters are not all independent of one another
- For a specific aircraft, to maximize range, the product $V_\infty(L/D)$ is maximized

Endurance (E): the **amount of time** an aircraft can stay in the air on one tank of fuel

For some specific designs or missions (e.g., air surveillance), the objective is to stay in the air for the longest possible time

Endurance equation:

From previous,

$$dt = -\frac{1}{\mathsf{TSFC}}\left(\frac{L}{D}\right)\frac{dW}{W}$$

Integrate both sides from the start of the mission (0) to the end of the mission (1)

$$\int_{t_0}^{t_1} dt = -\int_{W_0}^{W_1} \frac{1}{\mathsf{TSFC}}\left(\frac{L}{D}\right)\frac{dW}{W}$$

Finally, we have

$$\boxed{E = \frac{1}{\mathsf{TSFC}}\left(\frac{L}{D}\right)\ln\left(\frac{W_0}{W_1}\right)}$$

Graphical representation – cruise velocities for maximum range and endurance

Let's look at each optimization problem:

- **Maximize endurance:**

$$\min_{V_\infty} \frac{\text{Mass of fuel}}{\text{Time}} = \text{TSFC} \cdot T_R$$

- **Maximize range:**

$$\min_{V_\infty} \frac{\text{Mass of fuel}}{\text{Distance}} = \frac{\text{TSFC} \cdot T_R}{V_\infty}$$

Since **TSFC is an engine property**, these two problems are equivalent to:

- **Maximize endurance:**

$$\min_{V_\infty} \; T_R$$

- **Maximize range:**

$$\min_{V_\infty} \; \frac{T_R}{V_\infty}$$

Graphical representation – cruise velocities for maximum range and endurance

- **Maximize endurance:** $\min\limits_{V_\infty} \; T_R$
- **Maximize range:** $\min\limits_{V_\infty} \; \dfrac{T_R}{V_\infty}$