

# INSTRUCTMPC: A Human-LLM-in-the-Loop Framework for Context-Aware Power Grid Control

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## Abstract

The transition toward power grids with high renewable penetration demands context-aware decision making frameworks. Traditional operational paradigms, which rely on static optimization of history-based load forecasting, often fail to capture the complex nature of real-time operational conditions, such as operator-issued maintenance mandates, emergency topology changes, or event-driven load surges. To address this challenge, we introduce INSTRUCTMPC, a closed-loop framework that integrates Large Language Models (LLMs) to generate context-aware predictions, enabling the controller to optimize power system operation. Our method employs a Contextual Disturbances Predictor (CDP) module to translate contextual information into predictive disturbance trajectories, which are then incorporated into the Model Predictive Control (MPC) optimization. Unlike conventional open-loop forecasting frameworks, INSTRUCTMPC features an online tuning mechanism where the predictor's parameters are continuously updated based on the realized control cost with a theoretical guarantee, achieving a regret bound of  $O(\sqrt{T \log T})$  for linear dynamics when optimized via a tailored loss function, ensuring task-aware learning and adaption to non-stationary grid conditions.

## 1 Introduction

Efficient decision-making in power system operation is crucial to ensuring grid reliability and economic viability, particularly in the presence of uncertainty sourced from increasing renewable energy penetration and distributed energy resources [1, 2]. Traditional operation techniques, such as optimal power flow, rely on static optimization models or expert-provided heuristics to schedule the energy resource. However, the modern power operating environment is characterized by rapid fluctuations and dynamic uncertainties that demand timely adaptation [3, 4]. A critical limitation of existing approaches is their reliance on historical data patterns or heuristic expert rules for prediction, which lack *context awareness*. For instance, classic load forecasting may fail to capture behavior shifts driven by temporary social events, leading to control actions that are misaligned with the actual real-time environment.

To bridge the gap between scheduling and the need for dynamic real-time adaptation, advanced control strategies are required. Among these strategies, Model Predictive Control (MPC) is a versatile control strategy widely applied in domains such as energy management [5, 6], building control [7], and autonomous driving [8, 9]. These applications leverage MPC's ability to optimize the system's behavior over a predictive horizon, making it a cornerstone of modern control engineering.

In real-world scenarios, however, the efficacy of MPC hinges on the accuracy of its predictions, which cannot be derived only from historical data. Instead, accurate prediction often requires interpreting unstructured, semantic information regarding the current operational context. For example, in the operation of active distributed networks, MPC must anticipate and adapt to scenarios such as operator-issued mandates to prioritize voltage stability during emergencies, temporary topology changes due to unmapped maintenance schedules, and localized demand spikes caused by large-scale social events that are not captured in historical data. This context dependency underscores a critical question: “*How can we incorporate in-context instructions into MPC?*” Key challenges in addressing this issue revolve around adapting to evolving human instructions and ensuring generalizability in changing environments. The MPC system must be capable of dynamically adjusting its behavior in response to shifting human inputs and contexts, which demands a high level of flexibility and responsiveness to maintain effectiveness. Furthermore, the learning-based integration of these instructions must be designed to adapt to time-varying conditions, allowing the system to not only respond to immediate changes

but also to generalize its learning effectively, ensuring consistent performance over time. This combination of adaptability and robust generalization is essential for overcoming the complexities presented by dynamic human interactions and fluctuating environmental factors.

To tackle these challenges, we introduce INSTRUCTMPC, a novel framework that employs a human-LLM-in-the-loop approach to form a closed-loop system. INSTRUCTMPC takes in-context instructions from humans, processes them through a Large Language Model (LLM) together with an easily tunable Neural Network (NN) to generate context-aware predictions, and feeds these into the downstream MPC module. During online implementation, the model loss from MPC is propagated back to the tunable NN, which is fine-tuned to enhance adaptability and generalizability across scenarios.

Recent advancements in MPC have explored contextual and language-based enhancements to improve performance in power system operations, autonomous systems and personalized applications.

**Context-awareness in power system operation** In power system operations, context-aware control has focused on integrating exogenous variables (i.e., weather condition, market signals) into the decision-making process to mitigate uncertainty. For instance, data-driven MPC frameworks often utilize historical data to refine load and generation forecasts, thereby reducing conservatism in active distribution networks [10, 11]. Similarly, in building energy management, occupancy-based control algorithms adjust temperature setpoints based on detected human presence of schedules, effectively using occupancy state as context to optimize energy efficiency [12, 13]. More advanced approaches include scenario-based MPC, in which the controller switches between pre-defined control strategies triggered by events [14, 15]. Despite these advancements, existing methods treat context as numerical values or pre-defined scenarios, lacking the capability to utilize unstructured instructions from human operators. More critically, these approaches treat prediction and control as decoupled processes, making the predictor unable to refine itself based on the downstream control cost or real-time environmental disturbances.

**Context-aware MPC for autonomous systems** In autonomous systems, context-aware MPC leverages environmental and semantic information to enhance control performance. For instance, semantically informed MPC [16] employs deep neural networks to encode semantic labels (e.g., “find a couch”) into cost maps, guiding navigation in unknown environments with continuous control, achieving an improvement in success rates through mid-level visual cues. However, it focuses on discrete, predefined contexts without real-time human input. Similarly, [17] adapts MPC parameters to environmental contexts (e.g., rain) using contextual Bayesian optimization. While effective for model and objective tuning, it lacks mechanisms for incorporating real-time instructions from humans. Additionally, designed for crowded environments, the framework in [18] integrates contextual cues like human poses and activities into MPC. It prioritizes computational efficiency but does not generalize to in-context instructions beyond navigation tasks.

**Language-Based MPC personalization** Language-based approaches harness natural language to personalize MPC systems for improved usability and flexibility. For instance, the LanguageMPC in [19] utilizes LLMs as decision-makers in autonomous driving, combining LLM reasoning with low-level controllers to enhance safety, efficiency, and generalizability in complex scenarios, though it does not support real-time human interaction. Meanwhile, ChatMPC [20] uses BERT to classify natural language into a set of predefined circumstances, each linked to a predefined modification of MPC parameters; however, it is constrained by its predefined updating rule, and also lacks state-of-the-art human-LLM interaction and theoretical performance guarantees.

The INSTRUCTMPC framework integrates real-time human instructions through a closed-loop human-LLM system for dynamic adaptation and provides a theoretical performance guarantee for linear dynamics, ensuring features absent in existing context-aware and language-based approaches. Our contributions are two-fold:

1. **INSTRUCTMPC Framework:** INSTRUCTMPC, a closed-loop framework that dynamically tunes MPC by integrating real-time, human-provided in-context instructions through a Contextual Disturbances Predictor (CDP) and economically fine-tuning a small amount of CDP’s parameters in a closed-loop. The novelty lies in enabling the MPC controller to leverage unstructured, semantic information that is widely available but often ignored in power systems. By transforming high-level instructive information into predicted disturbance trajectories, INSTRUCTMPC enhances the adaptability classic control strategies used in power grids.
2. **Theoretical Guarantees:** For linear dynamics, we provide theoretical performance guarantees (Theorem 4.1) for closed-loop parameter tuning. Unlike traditional online optimization methods that directly

minimize regret, our approach decouples the CDP’s loss from the regret objective, which relies on MPC cost information and is challenging to compute online due to unknown future disturbances. In particular, Theorem 4.1 suggests a tailored loss function for tuning CDP, achieving a regret bound of  $O(\sqrt{T} \log T)$  as derived in Corollary 4.1. Furthermore, we demonstrate the robustness of INSTRUCTMPC in Theorem 4.2, proving that the system converges to a safe, unbiased baseline even when the upstream context provides no informative signal for grid state prediction.

## 2 Preliminaries and Problem Formulation

Denote  $[T] := \{0, 1, \dots, T - 1\}$ . We consider a finite horizon linear dynamical system

$$x_{t+1} = Ax_t + Bu_t + w_t, \quad t \in [T] \quad (1)$$

where  $x_t \in \mathbb{R}^n$ ,  $u_t \in \mathbb{R}^m$  denote the system state and control input at each time  $t \in [T]$ ;  $A \in \mathbb{R}^{n \times n}$  and  $B \in \mathbb{R}^{n \times m}$  are system matrices. The disturbance  $w_t \in \mathbb{R}^n$  is unknown to the controller at time  $t \in [T]$ . The controller’s goal is to minimize the following quadratic costs:

$$J^* := \min_{(u_t : t \in [T])} \sum_{t=0}^{T-1} (x_t^\top Q x_t + u_t^\top R u_t) + x_T^\top P x_T, \quad (2a)$$

$$\text{subject to (1),} \quad (2b)$$

where  $Q, R > 0$  are positive definite matrices, and  $P$  is the solution to the following discrete algebraic Riccati equation (DARE),<sup>1</sup>

$$P = Q + A^\top P A - A^\top P B (R + B^\top P B)^{-1} B^\top P A. \quad (3)$$

The disturbance  $w_t$  is bounded by a constant  $W > 0$ , i.e.  $w_t \in \mathcal{W} := \{w : \|w\| \leq W\}$  for all  $t \in [T]$ . Furthermore, we make a standard assumption on the system (c.f. [23]).

**Assumption 1.** *The system  $(A, B)$  is stabilizable.<sup>2</sup>*

### 2.1 Model Predictive Control

Fix a prediction horizon to be an integer  $k$  and define  $\mathcal{T} := \min\{t + k - 1, T - 1\}$ . We consider the setting in which, at each time  $t \in [T]$ ,  $k$ -step predictions  $\hat{w}_{t:\mathcal{T}|t} := (\hat{w}_{t|t}, \dots, \hat{w}_{\mathcal{T}|t}) \in \mathcal{W}^k$  corresponding to future disturbances  $(w_t, \dots, w_{\mathcal{T}})$  are available.

In this context, model predictive control naturally incorporates these predictions into its optimization framework. At each time  $t \in [T]$ , the controller solves the following optimization problem given the current state  $x_t$  and disturbance predictions in  $\hat{w}_{t:\mathcal{T}|t}$ :

$$\begin{aligned} u_{t:\mathcal{T}}^{\text{MPC}} &:= \arg \min_u \left( \sum_{\tau=t}^{\mathcal{T}} (x_\tau^\top Q x_\tau + u_\tau^\top R u_\tau) + x_{\mathcal{T}+1}^\top P x_{\mathcal{T}+1} \right) \\ \text{s.t. } x_{\tau+1} &= Ax_\tau + Bu_\tau + \hat{w}_{\tau|t}, \forall t \leq \tau < \mathcal{T}. \end{aligned} \quad (4)$$

Only the first element of  $u_{t:\mathcal{T}}^{\text{MPC}}$  is applied as a control input to the system, and the rest are discarded. However, in many real-world scenarios, system disturbances are hard to predict [24, 25]. In this paper, we propose a novel framework that leverages contextual information to predict future disturbances and incorporates these predictions into the decision-making process.

### 2.2 Contextual MPC with Model Fine-Tuning

Our goal is to improve the performance of the classic MPC defined in (4) by incorporating external contextual information at time  $t \in [T]$ , denoted by  $c_{t:\mathcal{T}|t} := (c_{t|t}, \dots, c_{\mathcal{T}|t})$  to predict future disturbances  $\hat{w}_{t:\mathcal{T}|t} \in \mathcal{W}^k$ .<sup>3</sup> Hence, the main challenge is to integrate a Neural Network (NN) to embed the contextual information into an

<sup>1</sup>To simplify the presentation of our main results, we fix the terminal cost in (2a) to be  $P$ . The arguments extend to more general terminal costs as well, since the overall cost only differs by an  $O(1)$  term [21, 22].

<sup>2</sup>In other words, there exists  $K \in \mathbb{R}^{m \times n}$  such that  $A - BK$  has a spectral radius  $\rho$  less than 1. Thus, the Gelfand’s formula implies that there exist  $C > 0$ ,  $\rho \in (0, 1)$  such that  $\|A - BK\|^t \leq C\rho^t$  for all  $t \geq 0$ .

<sup>3</sup>The predicted disturbance trajectory  $\hat{w}_{t:\mathcal{T}|t}$  is generated in real-time, operating on the same time scale as the MPC decision-making process.

embedding space  $\mathcal{D}$  and map these embeddings into numerical prediction that is useful for downstream MPC. In our framework, the mapping from embedding to prediction is modeled by another NN module  $g_\theta(\cdot) : \mathcal{D} \rightarrow \mathcal{W}$ , and the parameter  $\theta \in \Theta$  is refined as the MPC executes and true disturbances are revealed. Let  $\|\cdot\|$  denote the  $\ell_2$ -norm of a vector. The hypothesis set  $\Theta$  has bounded diameter so that  $\|\theta - \theta'\| \leq D$  for all  $\theta, \theta' \in \Theta$  and some constant  $D > 0$ . To facilitate a tractable analysis of the online updating process of parameter  $\theta$ , we impose the following assumption on the structure of  $g_\theta$ , which serve as approximations for widely used architectures, including transformers [26] and input convex neural networks [27].

**Assumption 2.** *The NN model  $g_\theta$  is differentiable and affine in  $\theta$  and its gradient satisfies  $\|\nabla_\theta g_\theta\| \leq L$  for some  $L > 0$ .*

A hyper-parameter  $\theta_t$  is updated at  $t \in [T]$ . We evaluate performance via the following *regret*, casting the problem as an online optimization over  $\theta_{1:T} := (\theta_t : t \in [T])$ :

$$J(\theta_{1:T}) - \min_{\theta \in \Theta} J(\theta), \quad (5)$$

where  $J(\theta_1, \dots, \theta_T)$  and  $J(\theta^*)$  are the quadratic costs defined in (2a) induced by the MPC actions in (4) corresponding to the hyper-parameters  $\theta_1, \dots, \theta_T$  and the optimal hyper-parameter  $\theta^*$ , respectively. In other words, the regret in Equation (5) measures the difference between the cumulative loss incurred by  $(\theta_t : t \in [T])$ , and the cumulative loss incurred by the optimal  $\theta^*$  in hindsight. Next, we introduce the INSTRUCTMPC framework for contextual MPC.

### 3 InstructMPC

In this section, we introduce INSTRUCTMPC, a framework that enables MPC controller to leverage external contextual information to effectively manage environmental disturbances. Fig. 1 presents the overall architecture of INSTRUCTMPC. In the following subsections, we explain the core component of INSTRUCTMPC, Contextual Disturbances Predictor (CDP) in details.

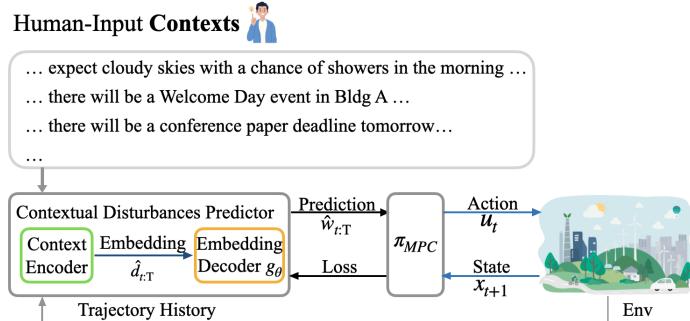


Figure 1: System framework of INSTRUCTMPC. The blue lines represent interactions between INSTRUCTMPC and the environment where INSTRUCTMPC receives the state  $x_t$ , and outputs the control input  $u_t$ . The black lines represents the *information loop*, within which external contextual information  $c_{t:\mathcal{T}|t}$  is passed to the CDP to produce predicted disturbances  $\hat{w}_{t:\mathcal{T}|t}$ . Then, the MPC controller  $\pi_{MPC}$  utilizes  $\hat{w}_{t:\mathcal{T}|t}$  and the current state  $x_t$  to determine a control input  $u_t^{MPC}$  via (4). After executing the MPC control input  $u_t^{MPC}$ , the environment reveals the true disturbance  $w_t$ . The discrepancy between  $w_t$  and  $\hat{w}_t$  is then sent back to the CDP module as a loss signal.

#### 3.1 Contextual Disturbances Predictor (CDP) Module

During the execution of INSTRUCTMPC, the CDP predicts future environmental disturbances  $\hat{w}_{t:\mathcal{T}|t}$  based on contextual information available at time  $t$ . After each control step, the decoder  $g_\theta$  refines its parameters  $\theta$  by comparing these predictions with the actual revealed disturbances.

At each time  $t$ , given contextual information  $c_{t:\mathcal{T}|t}$ , the encoder module first maps the input to a set of categorical probability distributions corresponding to environmental features (e.g., sunlight intensity, CPU load). For each feature and future time step  $\tau$ , the encoder predicts the probability of the feature belonging to

predefined discrete levels (e.g., {low, medium, high}). For each  $\tau \in \{t, \dots, T\}$ , the output embedding  $\hat{d}_{\tau|t}$  is then calculated by taking the weighted sum of the numerical values associated with each level according to their predicted probabilities. Finally, the complete embedding sequence  $\hat{d}_{t:T|t} := (\hat{d}_{t|t}, \dots, \hat{d}_{T|t}) \in \mathcal{D}^k$  is fed into the decoder model  $g_\theta$  to produce the predicted disturbance trajectory  $\hat{w}_{t:T|t}$ .

### 3.2 INSTRUCTMPC Framework

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**Algorithm 1:** INSTRUCTMPC

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for  $t = 0, \dots, T - 1$  do
     $\mathcal{T} \leftarrow \min(t + k - 1, T - 1)$ 
    Get  $d_{t:\mathcal{T}|t}$  from encoder with instruction  $c_{t:\mathcal{T}|t}$ :
     $(\hat{w}_{t|t}, \dots, \hat{w}_{\mathcal{T}|t}) \leftarrow (g_{\theta_t}(d_{t|t}), \dots, g_{\theta_t}(d_{\mathcal{T}|t}))$ 
    Generate an action  $u_t$  by solving MPC in (4)
    Update
         $x_{t+1} = Ax_t + Bu_t + w_t$ 
         $\theta_{t+1} = \Pi_\Theta(\theta_t - \eta_t \nabla_\theta L_{t-k+1}(\theta_{t-k+1}))$ 
end

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The decoder module  $g_\theta$  is updated at every time  $t$  through a loss function  $L_{t-k+1} : \Theta \times \mathcal{W}^{2k} \rightarrow \mathbb{R}$ , which also depends on real disturbances  $w_{t:\mathcal{T}}$  and predictions  $\hat{w}_{t:\mathcal{T}|t}$ . An example of such a loss function can be found in Corollary 4.1 in Section 4. At every time  $t \in [T]$ , the parameter  $\theta$  of the decoder module is updated according to:

$$\theta_{t+1} = \Pi_\Theta(\theta_t - \eta_t \nabla_\theta L_{t-k+1}(\theta_{t-k+1})), \quad (6)$$

where  $\eta_t \nabla_\theta L_{t-k+1}$  denotes the gradient of  $L_{t-k+1}$  with respect to  $\theta$ ,  $\Pi_\Theta$  denotes projection onto the hypothesis set  $\Theta$  and  $\eta_t$  is a time-varying learning rate. This last layer adaptation technique is widely used to efficiently fine-tune NN models (see [28–30]). The overall procedure of INSTRUCTMPC is summarized in Algorithm 1. Since  $\nabla_\theta L_t(\theta_t)$  is available only after  $(w_t, \dots, w_T)$  are revealed at time  $t + k$ , a delayed gradient is used in the update rule (6). For convenience, we define  $L_{t-k+1} = 0$  for  $t < k$ . In practice, when  $t < k$ , the NN model does not update due to insufficient observed data.

## 4 Theoretical Guarantee

Our main result in this section explores the online optimization of  $(\theta_t : t \in [T])$  in the online fine-tuning process. We make the following standard assumption on the loss function  $L_t$ . Throughout this section, we use  $d_\tau$  to denote  $d_{\tau|t}$ , which is the corresponding component of the sequence  $d_{\tau:\mathcal{T}|t}$ , omitting the subscript  $t$  whenever the time  $t$  is clear from the context.

**Assumption 3.** *The gradient  $\nabla_\theta L_t(\theta)$  is uniformly bounded, i.e., there exists  $G > 0$  such that  $\|\nabla_\theta L_t(\theta)\| \leq G$  for all  $\theta \in \Theta$ .*

A fundamental challenge is that the surrogate loss function used to train  $g_\theta$  does not match the true, regret-optimal objective (4), which is unknown at training time. This misalignment creates the primary difficulty, i.e., ensuring that training the upstream decoder module  $g_\theta$  actually improves the downstream MPC controller's performance. Our solution is to define and analyze the *loss discrepancy*, a term that quantifies the divergence between the gradient of the decoder module surrogate loss  $L$  and that of the true decision loss in Equation (4).

**Definition 1.** *The loss discrepancy between two loss functions  $L_1(\theta)$  and  $L_2(\theta)$ ,  $\text{LD}(L_1, L_2)$  is defined as*

$$\text{LD}(L_1, L_2) := \sup_{\theta \in \Theta} \|\nabla_\theta L_1(\theta) - \nabla_\theta L_2(\theta)\|.$$

Next, we will provide the theoretical guarantee of INSTRUCTMPC from both consistency and robustness aspect.

**Consistency of INSTRUCTMPC Framework** The theorem below provides a regret bound (see Equation (5)) for the proposed INSTRUCTMPC, which interacts with the dynamic system described via Equation (1).

**Theorem 4.1.** *Under Assumption 2,3, if the learning rate  $\eta_t$  is non-increasing, then*

$$\begin{aligned} J(\theta_{1:T}) - J(\theta^*) &\leq \frac{D^2}{2\eta_{T-1}} + \left(k - \frac{1}{2}\right) G^2 \sum_{t=0}^{T-1} \eta_t \\ &+ D \sum_{t=0}^{T-1} \text{LD}(L_t, \psi_t^\top H \psi_t) + (k-1)GD. \end{aligned} \quad (7)$$

Furthermore, if we choose  $\eta_t = \frac{D}{G\sqrt{2(2k-1)(t+1)}}$ ,

$$\begin{aligned} J(\theta_{1:T}) - J(\theta^*) &\leq 2GD\sqrt{(k-\frac{1}{2})T} \\ &+ D \sum_{t=0}^{T-1} \text{LD}(L_t, \psi_t^\top H \psi_t) + (k-1)GD, \end{aligned}$$

where  $H$  is defined in Lemma 1, and we define

$$\psi_t(\theta) := \sum_{\tau=t}^{T-1} (F^\top)^{\tau-t} P w_\tau - \sum_{\tau=t}^{\mathcal{T}} (F^\top)^{\tau-t} P g_{\theta_t}(d_\tau).$$

*Proof.* By Theorem 3.2 in [31], given predictions  $\hat{w}_{t:\mathcal{T}|t}$ , the MPC solution  $u_t^{\text{MPC}}$  to the problem defined in (4) is

$$u_t^{\text{MPC}} = -(R + B^\top PB)^{-1} B^\top \left( PAx_t + \sum_{\tau=t}^{\mathcal{T}} (F^\top)^{\tau-t} P \hat{w}_{\tau|t} \right),$$

where  $F := A - B(R + B^\top PB)^{-1}B^\top PA := A - BK$ . Thus, applying Lemma 1 twice, we obtain

$$J(\theta_{1:T}) - J(\theta^*) = \sum_{t=0}^{T-1} \psi_t(\theta_t)^\top H \psi_t(\theta_t) - \psi_t(\theta^*)^\top H \psi_t(\theta^*),$$

where  $\psi_t(\theta)$  is defined in Theorem 4.1. For notational simplicity, we define  $\psi_t := \psi_t(\theta_t)$  and  $\phi_t := \psi_t(\theta^*)$ . By our model assumption,  $Q, R > 0$  implies  $H \geq 0$ , using the convexity of  $x^\top H x$ ,

$$\begin{aligned} \frac{1}{2} \sum_{t=0}^{T-1} \psi_t^\top H \psi_t - \phi_t^\top H \phi_t &\leq \sum_{t=0}^{T-1} \psi_t^\top H (\psi_t - \phi_t) = \\ \sum_{t=0}^{T-1} \psi_t^\top H \left( \sum_{\tau=t}^{\mathcal{T}} (F^\top)^{\tau-t} P (g_{\theta^*}(d_\tau) - g_{\theta_t}(d_\tau)) \right). \end{aligned}$$

Applying Assumption 2, since  $g_\theta$  is affine in  $\theta$ , continuing from above,

$$\begin{aligned} J(\theta_{1:T}) - J(\theta^*) &\leq \\ &- 2 \sum_{t=0}^{T-1} \psi_t^\top H \left( \sum_{\tau=t}^{\mathcal{T}} (F^\top)^{\tau-t} P \nabla_\theta g_{\theta_t}(d_\tau)^\top \right) (\theta_t - \theta^*) \\ &= \sum_{t=0}^{T-1} \nabla_{\theta_t} (\psi_t^\top H \psi_t)^\top (\theta_t - \theta^*), \end{aligned} \quad (8)$$

where we denote  $\nabla_{\theta_t} (\psi_t^\top H \psi_t) := \nabla_{\theta_t} (\psi_t^\top H \psi_t)|_{\theta=\theta_t}$ , and have used Assumption 2. Denote the gradient of

the loss function as  $\nabla_\theta L_{t-k+1}(\theta_{t-k+1})$  as  $l_{t-k+1}$ . It follows that the RHS of (8) can be rewritten as

$$\begin{aligned} & \sum_{t=0}^{T-1} \nabla_{\theta_t} (\psi_t^\top H \psi_t)^\top (\theta_t - \theta^*) \\ &= \sum_{t=0}^{T-1} l_{t-k+1}^\top (\theta_t - \theta^*) + \sum_{t=0}^{T-1} (l_t - l_{t-k+1})^\top (\theta_t - \theta^*) \\ & \quad + \sum_{t=0}^{T-1} (\nabla_{\theta_t} (\psi_t^\top H \psi_t)^\top - l_t^\top) (\theta_t - \theta^*). \end{aligned} \quad (9)$$

The bound in (7) is obtained by bounding the terms in Equation (9) separately, and the details are relegated to Appendix A.  $\square$

Furthermore, for a wide range of loss functions ( $L_t : t \in [T]$ ), as exemplified in Corollary 4.1, they exhibit a bounded discrepancy from the MPC cost with the discrepancy decaying exponentially as  $k$  increases, i.e.,  $\text{LD}(L_t, \psi_t^\top H \psi_t) \leq C\rho^k$ .

**Corollary 4.1.** *Under Assumption 1, for the case when  $L_t(\theta) = \hat{\psi}_t(\theta)^\top H \hat{\psi}_t(\theta)$  with*

$$\hat{\psi}_t(\theta) := \sum_{\tau=t}^T (F^\top)^{\tau-t} P w_\tau - \sum_{\tau=t}^T (F^\top)^{\tau-t} P g_{\theta_t}(d_\tau),$$

*there exist constants  $C$  and  $0 < \rho < 1$  such that*

$$\text{LD}(L_t, \psi_t^\top H \psi_t) \leq C\rho^k, \text{ for all } t \in [T],$$

*where  $\psi_t(\theta)$  is defined in Theorem 4.1.*

For the particular choice of  $L_t$  in Corollary 4.1, the term  $D \sum_{t=0}^{T-1} \text{LD}(L_t, \psi_t^\top H \psi_t)$  in the bound (7) of Theorem 4.1 simplifies to  $CDT\rho^k$ . This provides an insightful guideline for selecting  $k$ . Specifically, by setting  $k = \log_{1/\rho} T$ , we can achieve a regret bound of  $O(\sqrt{T \log T})$ .

Given the sub-linear regret bound guarantees performance under general conditions, we also need to ensure the *robustness* to low-quality inputs in practical power system operations. In real-world operations, the upstream LLM or encoder might fail to extract meaningful information in provided context and yield a low quality embedding. Next, we provide the robustness guarantee of INSTRUCTMPC framework.

**Robustness of INSTRUCTMPC Framework** We will analyze the robustness result in a case, where environmental disturbance and context pairs are sampled independently and identically distributed from a fixed joint distribution. First, to ensure that the convergence result is theoretically tractable, we impose that the optimal solution lies within the interior of the hypothesis set. Second, since the context are random variables, the resulting feature embeddings are also stochastic, we further assume the feature embeddings are independent, which incurs no loss of generality, as we can always restrict our attention to a basis of the feature space. Third, to simplify the presentation, we consider an over-actuated system  $(A, B)$  with some full rank matrix  $B$ . It guarantees that the controller possess full authority to correct disturbances in any direction. Finally, existing literature [32] suggests that, when using gradient descent, the distance between the current parameter and the optimal one shrinks proportionally to the inverse of time (e.g.,  $O(1/t)$  or  $O(1/\sqrt{t})$ ). Therefore, we assume a sufficiently large horizon to ensure the online updates have adequate time to converge.

**Assumption 4.** *We need the following assumptions:*

1. *The unconstrained minimizer of the loss in Corollary 4.1 lies in the interior of the hypothesis set  $\Theta$ ;*
2. *The entries of embedding  $d$  are pair-wise independent;*
3. *The system matrix  $B \in \mathbb{R}^{n \times m}$  has full row rank;*
4. *The control task horizon  $T$  (see (1)) is large enough.*

To theoretically guarantee robustness, we consider a practical affine decoder  $g_{\theta_t}(d) = C_t d + b_t$ , where  $\theta_t = [C_t, b_t]^\top$ . It serves as a first-order approximation of the NN's behavior and allows us to derive explicit convergence limits.

**Theorem 4.2.** Under Assumption 3 and 4, consider the affine decoder defined above. If the embedded features  $d$  are statistically uncorrelated with the disturbances  $w$ , then for the loss function  $L_t(\theta)$  defined in Corollary 4.1, the sequence of parameters  $\{\theta_t\}_{t \in [T]}$  produced by Algorithm 1 satisfies  $\lim_{t \rightarrow \infty} C_t = 0$  and  $\lim_{t \rightarrow \infty} b_t = \mathbb{E}[w]$ , where  $\mathbb{E}[w]$  is the expected value of the disturbance.

This theorem provides a theoretical guarantee for the robustness of the INSTRUCTMPC framework, ensuring that even when the LLM or encoder outputs uncorrelated hallucinations, the framework can effectively ignore the noise and reduce to use safe and unbiased predictions.

## 5 Numerical Experiment

In Subsection 5.1 and Subsection 5.2, we demonstrate the effectiveness of INSTRUCTMPC framework in two scenarios. The first is a trajectory tracking task, where an agent is required to follow an unknown trajectory. In this setting, the target position is revealed only immediately before the control decision is made. Moreover, environmental disturbances, such as wind force, are described textually and passed through the CDP module to assist the MPC controller. We plot the resulting trajectory to directly demonstrate the superiority of the loss function designed in Corollary 4.1. The second is a battery management task in a real-world power system under uncertain future compute loads for which textual context is available. Power demand is predicted from compute job descriptions using different prediction models (LLM-based, metadata-based and classical baselines) and overall performance graphs are plotted, demonstrating the improved performance and robustness of our LLM-based CDP.

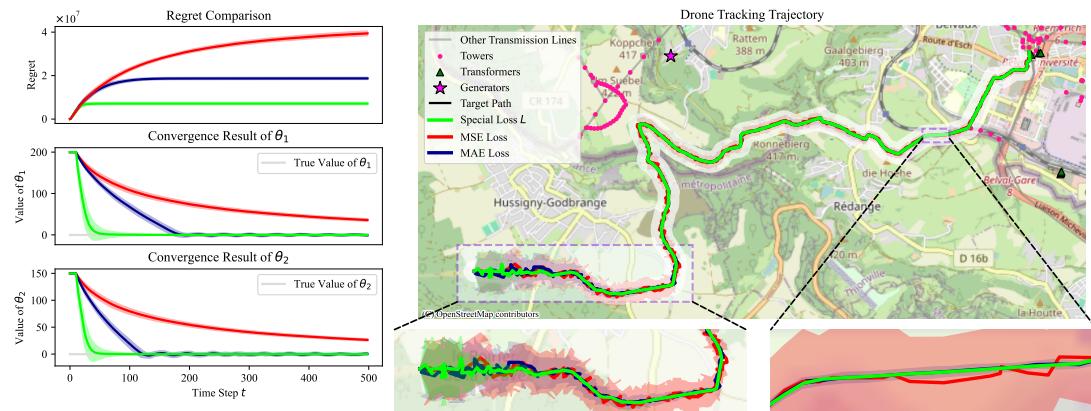


Figure 2: **Application 1: Power Infrastructure Inspection (Section 5.1).** Performance comparison of predictors tuned with three different loss functions: 1. MSE Loss in (10) (Red); 2. MAE Loss in (11) (Blue); 3. The proposed Special Loss tailored for the control task defined in Corollary 4.1 (Green). (**Left**) The evolution of cumulative regret (top) and the convergence of the predictor parameters,  $\theta_1$  (middle) and  $\theta_2$  (bottom), over time. (**Right**) The corresponding average drone tracking trajectories projected onto the real-world power infrastructure map [33]. For all plots, solid lines represent the mean over 50 independent experiments, while the shaded regions denote the standard deviation.

### 5.1 Application 1: Power Infrastructure Inspection

**Problem description.** We examine a real-world power infrastructure inspection scenario where a drone is used to track transmission lines for routine maintenance and fault detection [34]. The simulation is conducted on a real geographical map containing actual power system components [33], where the total length of the target inspection path is 16.32 kilometers, as visualized in Fig. 2. Unlike static patrol mission, this scenario requires the drone to adapt to dynamic instructions from the grid manager. For instance, while inspecting a primary transmission line, the drone may receive a temporary, urgent command to divert and inspect a secondary line or a transformer station. Consequently, the reference trajectory  $y_t$  is not fully known *a priori*; instead, the specific target position is revealed only immediately before the control decision is made. During the inspection, the parameters of the decoder  $g_\theta$  are updated online based on the cost incurred by the real-time control input.

Mathematically, let  $p_t \in \mathbb{R}^2$  and  $v_t \in \mathbb{R}^2$  denote the location and velocity of the drone at time  $t$ . The system follows  $p_{t+1} = p_t + 0.2v_t$ . The velocity is influenced by the control input  $u_t$  and external environmental disturbances  $Z_t$ , specifically variable wind forces encountered during flight. Let  $x_t := p_t - y_t$  represent the deviation between the drone's actual position and the target inspection point. The system dynamics are modeled as:

$$\begin{bmatrix} x_{t+1} \\ v_{t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ v_t \end{bmatrix} + Bu_t + w_t,$$

where

$$A := \begin{bmatrix} 1 & 0 & 0.2 & 0 \\ 0 & 1 & 0 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$w_t := Ay_t - y_{t+1} + Z_t, Z_t := \begin{bmatrix} 0 & 0 & -0.2Z_t^{(1)} & -0.2Z_t^{(2)} \end{bmatrix}^\top,$$

where  $Z_t$  is a random variable representing the random wind force, and  $Z_t^{(1)}, Z_t^{(2)} \sim \mathcal{U}(-20, 20)$ . To track the trajectory, the controller uses quadratic cost in Equation (2a) with

$$Q := \text{diag}(1, 1, 0, 0), R := \text{diag}(10^{-4}, 10^{-4}).$$

In this experiment, we use a affine function  $g_\theta$  to map the random wind force (encoded context) to predictions of disturbances:

$$\hat{w}_t = Ay_t - y_{t+1} + \hat{Z}_t, \hat{Z}_t = \begin{bmatrix} 0 & 0 & \theta_1 Z_t^{(1)} & \theta_2 Z_t^{(2)} \end{bmatrix}^\top.$$

**Experimental results.** In our first experiment, we plot the drone trajectory to demonstrate the impact of different loss function to tune the decoder  $g_\theta$ , in Fig. 2, zoom-in figures are used to demonstrate the exact performance of predictors tuned by different loss functions. We compare three settings:

1. *MSE Loss* (red line): Loss function

$$L_t = \frac{1}{\mathcal{T}-t} \sum_{\tau=t}^{\mathcal{T}} (w_\tau - g_{\theta_t}(d_{\tau|t}))^2 \quad (10)$$

is used in Algorithm 1 to tune the decoder model  $g_{\theta_t}$ .

2. *MAE Loss* (blue line): Loss function

$$L_t = \frac{1}{\mathcal{T}-t} \sum_{\tau=t}^{\mathcal{T}} |w_\tau - g_{\theta_t}(d_{\tau|t})| \quad (11)$$

is used in Algorithm 1 to tune the decoder model  $g_{\theta_t}$ .

3. *Special Loss* (green line): The task-aware loss derived in Corollary 4.1, which incorporate the specific structure of the downstream MPC control task to align the predictor update.

As shown in Fig. 2, the proposed *Special Loss* (green) outperforms the *MSE* (red) and *MAE* (blue) baselines. Quantitatively, the *Special Loss* (green) yields lower cumulative regret and faster parameter convergence, validating that control-aware gradients accelerate learning in dynamic environments. Qualitatively, this advantage is particularly evident in the left zoom-in region, where the trajectory produced by *Special Loss* converges to the target path significantly faster than the trajectories produced by baseline loss functions. These baseline loss functions suffer from prolonged oscillations due to the objective mismatch, our approach ensures rapid stabilization and tight adherence throughout the inspection mission.

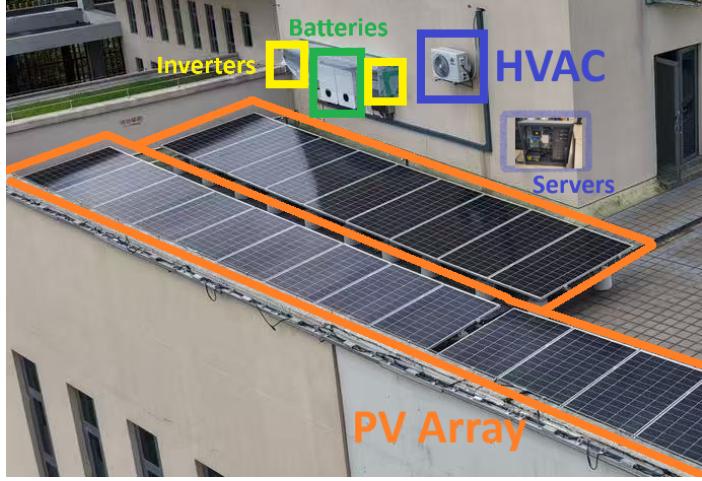


Figure 3: Application 2: OpenCEM on-campus installation (Section 5.2).

## 5.2 Application 2: Battery Management with SoC Target

**Problem description.** We examine the problem of battery management for an on-campus installed power system including a photovoltaic (PV) array of 26 solar panels (480W) for power generation, two 10kWh lithium-ion batteries for energy storage, two grid-connected inverters rated for 8kW, and as load two high-end workstations for CPU- and GPU-intensive workloads with total TDP>1kW and a 1.5HP HVAC system. The system serves as an academic test system to research algorithms for sustainable AI training and inference, named OpenCEM [35].

It processes various compute jobs using its two servers that are installed in an office next to the installation. The power demand on the system depends on number and details of the compute job as well as on the office usage patterns, which influence HVAC load. For this experiment we will only consider battery management for battery connected to the workstations.

Our goal for short-term battery management in this setup is two-fold. On one hand, we want to avoid low battery levels because it might require buying expensive energy during peak hours. On the other hand, with a full battery, we risk losing free energy when more solar power can be generated. Thus we aim to track a constant target SoC and penalize the squared deviation. We can buy or sell energy to or from the grid at any timestep and both incur a quadratic cost modeling transmission losses and hardware wear. Our system dynamics are given as  $x_{t+1} = x_t + u_t + w_t$ , where  $x_t$  is the deviation from the target SoC,  $u_t$  is the net energy bought from the grid (negative if energy is fed back to the grid in timestep  $t$ ), and  $w_t$  is the difference of energy generated from solar power and energy consumed by the workstations. The planning horizon is three days, divided into  $T = 2160$  two-minute steps, and the cost factors are  $P = 10^{-2}$ , and  $Q = 10^{-4}$  respectively. For prediction, we can use textual context about the compute jobs that are scheduled to be run on the system, see Table 1 for examples. In this experiment, we limit the context to shell commands as in the final row of the example table.

**Experimental results.** For  $g_\theta$  we compare three tuned prediction models with two baselines. For LLM-based prediction, the language model ChatGPT 5.1 is given the job description to be run and asked to classify the relative effort as low, medium, or high, which are mapped to 1, 2, 3 respectively, separately for the CPU-jobs on each workstation and GPU-jobs on one of the workstations. The classification is based on the LLM's pretrained world knowledge, for example, the final log entry shown in Table 1 contains the command to run the unit test suite of a large C++ library with several thousand unit tests, which is correctly classified as high effort. This mode of prompting yields for each timestep an embedding  $d_t \in \{0, 1, 2, 3\}^3$ , where 0 corresponds to no job context in the given timestep. Load is then predicted as  $\hat{w}_t = \theta_{t,0} + \sum_{i=1}^3 \theta_{t,i} d_{t,i}$ .

For a more classical prediction model, we use a linear regression model on job metadata, including for CPU-jobs the number of files to be processed, the number of CPU cores reserved, and for GPU-jobs the number of parameters in the model to be trained and the optimizer used for training. A final context-less model only trains the bias. As baselines we use a prediction that is fixed to the average of  $w_t$  and a constant zero prediction. The loss function with each model is the loss function given in Corollary 4.1.

The running cumulative cost is shown in Figure 4. We find that the LLM-based approach performs best outperforming the metadata-based regression, indicating that the LLM's effort estimation has better

Table 1: Context Examples (entries from internal OpenCEM users and system logs).

Start Date/Time	End Date/Time	Loc.	Description of Event	Source	Recorded Date/Time
2025-09-13 23:00	2025-09-13 23:20	2	The hardware configuration is: CPU: Intel XEON GOLD 6526Y GPU: NVIDIA RTX 2000 ...	team	2025-09-14 06:44
2025-07-28 18:00	2025-07-29 16:00	2	Tomorrow I will run a CPU-intensive, multi-core numeric robustness test for a day.	team	2025-07-27 18:00
2025-07-29 17:00	2025-07-31 15:00	2	In the evening, I will run a CPU-intensive, multi-core numeric robustness test for two days.	team	2025-07-28 06:00
2025-07-31 04:28	2025-07-31 08:00	2	Unexpected reboot.	log	2025-07-31 04:28
2025-07-28 18:11	2025-07-28 19:11	2	Shell command “cd dev/boost/libs”...	log	2025-07-28 18:11
2025-07-28 18:26	2025-07-28 19:26	2	Shell command “b2 -j8 cxxflags=’-O2’” in folder “.../geometry/test”	log	2025-07-28 18:26

directional correlation with a compute job’s power requirements than the available structured metadata. The final coefficients are  $\theta_T = [-245.61, -40.95, -44.18, -94.49]^\top$ , which aligns well with the summed idle consumption of roughly 240W, and the relative TDPs minus idle consumption of the CPU of the first workstation, the larger CPU of the second workstation, and the GPU of the first workstation.

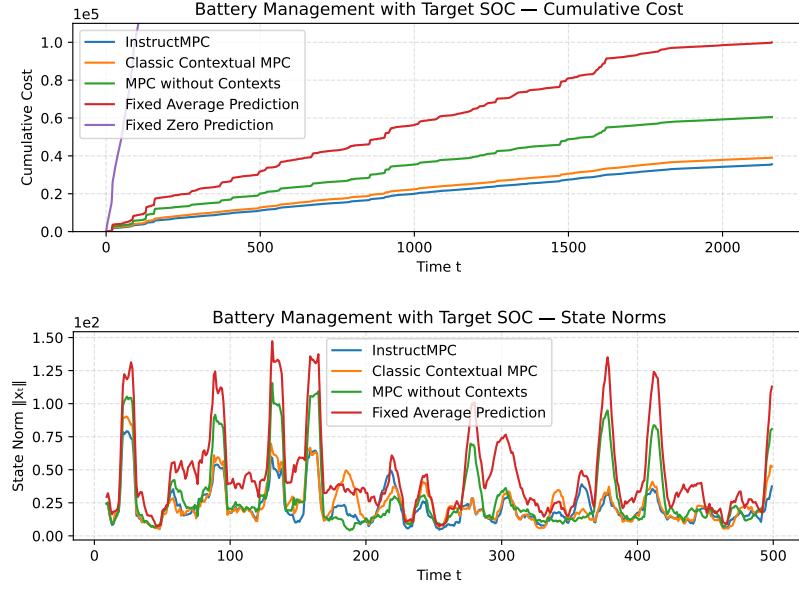


Figure 4: **Application 2: Battery Management with SoC Target (Section 5.2).** Performance comparison and state norm for 1. INSTRUCTMPC, 2. classic contextual MPC, 3. MPC without contexts, 4. fixed average prediction, and 5. fixed zero prediction (state omitted). State truncated to 500 time steps for readability.

## 6 Conclusion

We have presented the INSTRUCTMPC framework that integrates real-time human instructions into Model Predictive Control (MPC) via a human-LLM-in-the-loop approach. This work demonstrates how leveraging contextual information can significantly improve MPC’s predictive accuracy and performance in complex, real-world power system applications. By dynamically generating context-aware disturbance predictions

and refining them using a last layer adaptation technique, INSTRUCTMPC offers improved adaptability and performance across a wide range of applications. For linear dynamics, we established a robust performance guarantee, proving that INSTRUCTMPC achieves a regret bound of  $O(\sqrt{T \log T})$ . Future research will explore robust mechanisms to verify LLM outputs against physical grid constraints before they influence the control loop. Additionally, we aim to explore how multiple local controllers, each equipped with its own context-aware predictor, can coordinate to maintain system-wide stability while interpreting localized, heterogeneous human instructions.

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## A Proof of Theorem 4.1

The following lemma characterizes the quadratic cost gap.

**Lemma 1** (Lemma 13 in [21]). *For any  $\psi_t \in \mathbb{R}^n$ , if at each time  $t \in [T]$ , a controller  $\pi$  implements a control input  $u_t^\pi = -(R + B^\top PB)^{-1}B^\top(PAx_t + \sum_{\tau=t}^{T-1}(F^\top)^{\tau-t}Pw_\tau - \psi_t)$ , then the difference between the optimal cost  $J^*$  and the algorithm cost  $J(\pi)$  is given by  $J(\pi) - J^* = \sum_{t=0}^{T-1}\psi_t^\top H\psi_t$ , where  $H := B(R + B^\top PB)^{-1}B^\top$  and  $F := A - HPA$ .*

Now, we bound the terms in Equation (9) separately:

$$\begin{aligned} & \sum_{t=0}^{T-1} \nabla_{\theta_t} (\psi_t^\top H\psi_t)^\top (\theta_t - \theta^*) \\ &= \underbrace{\sum_{t=0}^{T-1} l_{t-k+1}^\top (\theta_t - \theta^*)}_{(a)} + \underbrace{\sum_{t=0}^{T-1} (l_t - l_{t-k+1})^\top (\theta_t - \theta^*)}_{(b)} \\ &\quad + \sum_{t=0}^{T-1} (\nabla_{\theta_t} (\psi_t^\top H\psi_t)^\top - l_t^\top)(\theta_t - \theta^*). \end{aligned}$$

First, based on the gradient update rule in (6) of INSTRUCTMPC,

$$\begin{aligned} & \|\theta_{t+1} - \theta^*\|^2 \\ &= \|\Pi_\Theta(\theta_t - \eta_t l_{t-k+1}) - \theta^*\|^2 \\ &\leq \|\theta_t - \eta_t l_{t-k+1} - \theta^*\|^2 \\ &= \|\theta_t - \theta^*\|^2 - 2\eta_t l_{t-k+1}^\top (\theta_t - \theta^*) + \eta_t^2 \|l_{t-k+1}\|^2. \end{aligned} \tag{12}$$

Then, rearranging the terms in Equation (13) and noting that the gradients are bounded by Assumption 3,

$$2l_{t-k+1}^\top (\theta_t - \theta^*) \leq \frac{\|\theta_t - \theta^*\|^2 - \|\theta_{t+1} - \theta^*\|^2}{\eta_t} + \eta_t G^2.$$

Summing above from  $t = 0$  to  $T - 1$ , for (a),

$$\begin{aligned} & 2 \sum_{t=0}^{T-1} l_{t-k+1}^\top (\theta_t - \theta^*) \\ &\leq \sum_{t=0}^{T-1} \frac{\|\theta_t - \theta^*\|^2 - \|\theta_{t+1} - \theta^*\|^2}{\eta_t} + \eta_t G^2 \\ &\leq \sum_{t=0}^{T-1} \|\theta_t - \theta^*\|^2 \left( \frac{1}{\eta_t} - \frac{1}{\eta_{t-1}} \right) + G^2 \sum_{t=0}^{T-1} \eta_t \\ &\leq \frac{D^2}{\eta_{T-1}} + G^2 \sum_{t=0}^{T-1} \eta_t. \end{aligned} \tag{14}$$

Furthermore, the second term in Equation (9) can be bounded from above as

$$\begin{aligned}
(b) &= \sum_{t=0}^{T-1} (l_t - l_{t-k+1})^\top (\theta_t - \theta^*) \\
&= \sum_{t=0}^{T-k} l_t^\top (\theta_t - \theta_{t+k-1}) + \sum_{t=T-k+1}^{T-1} l_t^\top (\theta_t - \theta^*) \\
&\leq \sum_{t=0}^{T-k} (G \|\theta_t - \theta_{t+k-1}\|) + (k-1)GD \\
&= (k-1)GD + G \sum_{t=0}^{T-k} \left\| \sum_{\tau=t}^{t+k-2} \eta_\tau l_{\tau-k+1} \right\| \\
&\leq (k-1)GD + G^2 \sum_{t=0}^{T-k} \sum_{\tau=t}^{t+k-2} \eta_\tau \\
&\leq (k-1)GD + (k-1)G^2 \sum_{t=0}^{T-1} \eta_t. \tag{15}
\end{aligned}$$

Recalling (8),

$$\begin{aligned}
&J(\theta_{1:T}) - J(\theta^*) \\
&\leq \frac{D^2}{2\eta_{T-1}} + \frac{G^2}{2} \sum_{t=0}^{T-1} \eta_t + (k-1)GD + (k-1)G^2 \sum_{t=0}^{T-1} \eta_t \\
&\quad + \sum_{t=0}^{T-1} \|\nabla_{\theta_t} (\psi_t^\top H \psi_t) - l_t\| \|\theta_t - \theta^*\| \tag{16} \\
&\leq \frac{D^2}{2\eta_{T-1}} + (k - \frac{1}{2})G^2 \sum_{t=0}^{T-1} \eta_t + (k-1)GD \\
&\quad + D \sum_{t=0}^{T-1} \text{LD}(L_t, \psi_t^\top H \psi_t), \tag{17}
\end{aligned}$$

where we have used (14) and (15) to derive (16); the last inequality (17) holds by the loss discrepancy in Definition 1. Finally, applying  $\eta_t = D / (G\sqrt{2(2k-1)(t+1)})$  to (17),

$$\begin{aligned}
&J(\theta_{1:T}) - J(\theta^*) \\
&= GD \sqrt{(k - \frac{1}{2})T} + \sqrt{k - \frac{1}{2}} \frac{GD}{2} \sum_{t=0}^{T-1} \frac{1}{\sqrt{t+1}} \\
&\quad + D \sum_{t=0}^{T-1} \text{LD}(L_t, \psi_t^\top H \psi_t) + (k-1)GD \\
&\leq 2GD \sqrt{(k - \frac{1}{2})T} + D \sum_{t=0}^{T-1} \text{LD}(L_t, \psi_t^\top H \psi_t) + (k-1)GD.
\end{aligned}$$

The inequality above in the second statement of Theorem 4.1 is obtained by using  $\sum_{t=1}^T \frac{1}{\sqrt{t}} \leq 2\sqrt{T}$ .

## B Proof of Corollary 4.1

Suppose  $L_t(\theta) = \hat{\psi}_t(\theta)^\top H \hat{\psi}_t(\theta)$  where

$$\hat{\psi}_t(\theta) := \sum_{\tau=t}^T (F^\top)^{\tau-t} P w_\tau - \sum_{\tau=t}^T (F^\top)^{\tau-t} P g_\theta^{(\tau-t+1)}(d_\tau).$$

By Definition 1,

$$\begin{aligned} LD(L_t, \psi_t^\top H \psi_t) &= \left\| \frac{\partial(\psi_t^\top H \psi_t)}{\partial \theta} - \frac{\partial(\hat{\psi}_t^\top H \hat{\psi}_t)}{\partial \theta} \right\| \\ &= \left\| 2(\psi_t - \hat{\psi}_t)^\top H \frac{\partial \psi_t}{\partial \theta} \right\| \end{aligned} \quad (18)$$

$$\leq 2\|H\| \|\psi_t - \hat{\psi}_t\| \left\| \frac{\partial \psi_t}{\partial \theta} \right\|. \quad (19)$$

We have used the fact that  $\frac{\partial \psi_t}{\partial \theta} = \frac{\partial \hat{\psi}_t}{\partial \theta}$  to get (18). Note that if  $\mathcal{T} = T - 1$ , then  $\psi_t = \hat{\psi}_t$ ,  $LD(L_t, \psi_t^\top H \psi_t) = 0$ . Thus, it suffices to restrict our analysis to the case when  $\mathcal{T} = t + k - 1$ . It follows that for some  $C > 0$ ,  $\rho < 1$

$$\begin{aligned} \|\psi_t - \hat{\psi}_t\| &= \left\| \sum_{\tau=t+k}^{T-1} (F^\top)^{\tau-t} P w_\tau \right\| \\ &\leq W \|P\| \sum_{\tau=t+k}^{T-1} \|F\|^{\tau-t} \end{aligned} \quad (20)$$

$$\leq W \|P\| \frac{C \rho^k}{1 - \rho}, \quad (21)$$

where (20) follows from the Assumption 1 so that  $\|w_t\| \leq W$  for all  $t \in [T]$ , and (21) follows from Gelfand's formula. Moreover, by Assumption 2,  $\|\nabla_\theta g_{\theta_t}^{(\tau-t+1)}(d_\tau)\| \leq L$ ,

$$\begin{aligned} \left\| \frac{\partial \psi_t}{\partial \theta} \right\| &= \left\| \sum_{\tau=t}^{t+k-1} (F^\top)^{\tau-t} P \nabla_\theta g_{\theta_t}^{(\tau-t+1)}(d_\tau) \right\| \\ &\leq L \|P\| \sum_{\tau=t}^{t+k-1} \|F\|^{\tau-t} \leq L \|P\| \frac{C}{1 - \rho}. \end{aligned}$$

Finally, combining (19), (21), and (22) we have

$$LD(L_t, \psi_t^\top H \psi_t) \leq 2 \frac{LW \|P\|^2 \|H\| C^2 \rho^k}{(1 - \rho)^2}. \quad (22)$$

## C Proof of Theorem 4.2

The following lemma characterizes the non-singularity of  $\sum_{i=0}^{k-1} (F^\top)^i P$

**Lemma 2.** *Under Assumption 1, for any finite horizon  $k \geq 0$ , the matrix sum  $\sum_{i=0}^{k-1} (F^\top)^i P$  is invertible.*

*Proof.* Let  $G_k := \sum_{i=0}^{k-1} (F^\top)^i$ , using the algebraic identity for finite geometric series, we have that  $(I - F^\top)G_k = I - (F^\top)^k$ . Since  $\rho(F) < 1$ , the matrix  $I - F^\top$  is invertible. Thus, we can write  $G_k = (I - F^\top)^{-1}(I - (F^\top)^k)$ . The eigenvalues of  $(F^\top)^k$  are  $\lambda^k$  for  $\lambda \in \text{spec}(F)$ . Since  $|\lambda| < 1$ , it follows that  $1 \notin \text{spec}((F^\top)^k)$ , which means  $I - (F^\top)^k$  is invertible. As the product of two invertible matrices,  $G_k$  is invertible. Finally, since  $Q, R > 0$ , we have  $P$  in (3) a positive definite matrix. Consequently, the matrix sum is invertible.  $\square$

Next, we start to formally prove Theorem 4.2. We immediately see that Assumption 2 holds, so both the sublinear regret bound established in Corollary 4.1 and Theorem 2 in [32] guarantee that the parameter sequence  $\{\theta_t\}_{t \in [T]}$  converges to the minimizer of the expected loss. Let  $\theta^* = [C^*, b^*]^\top$  denote the limit point. Apply Assumption 4(1) and denote the expected loss function as  $\mathcal{L}_t(\theta) = \mathbb{E}[L_t(\theta)]$  we have:  $\nabla_\theta \mathcal{L}_t(\theta^*) = 0$ . Now, we analyze the components  $b$  and  $C$  separately.

*Convergence of  $b$ :* We calculate the gradient of  $J$  with respect to  $b$  and set it to zero:

$$\frac{\partial \mathcal{L}_t}{\partial b} = \mathbb{E} \left[ \frac{\partial \hat{\psi}_t(\theta)^\top H \hat{\psi}_t(\theta)}{\partial b} \right] = \mathbb{E} \left[ 2 \frac{\partial \hat{\psi}_t(\theta)}{\partial b} H \hat{\psi}_t(\theta) \right] = 0,$$

where

$$\frac{\partial \hat{\psi}_t(\theta)}{\partial b} = - \sum_{\tau=t}^{\mathcal{T}} (F^\top)^{\tau-t} P \quad (23)$$

Since  $H$  is positive-definite by Assumption 4(3), and applying Lemma 2, the gradient in (23) is of full rank. Therefore,

$$\mathbb{E} [\hat{\psi}_t(\theta)] = \sum_{\tau=t}^{\mathcal{T}} (F^\top)^{\tau-t} P (\mathbb{E}[w] - C^* \mathbb{E}[d] - b^*) = 0.$$

Applying Lemma 2 again implies that:  $b^* = \mathbb{E}[w] - C^* \mathbb{E}[d]$ . *Convergence of  $C$ :* Denote the stacked centered disturbances vector  $\widetilde{\mathbf{W}}_t$  and the stacked centered embedding vector  $\widetilde{\mathbf{D}}_t$  as:

$$\widetilde{\mathbf{W}}_t := [\tilde{w}_t \ \dots \ \tilde{w}_{\mathcal{T}}]^\top, \quad \widetilde{\mathbf{D}}_t := [\tilde{d}_t \ \dots \ \tilde{d}_{\mathcal{T}}]^\top,$$

where  $\tilde{w}_t$  and  $\tilde{d}$  are centered variables defined as:

$$\tilde{w}_t := w_t - \mathbb{E}[w] \quad \text{and} \quad \tilde{d}_t := d_t - \mathbb{E}[d] \quad (24)$$

Let  $\mathbf{M} := [P \ F^\top P \ \dots \ (F^\top)^k P]$ , function  $\mathcal{L}_t$  becomes:  $\mathcal{L}_t = \mathbb{E}[(\mathbf{M}\widetilde{\mathbf{W}}_t - \mathbf{MC}\widetilde{\mathbf{D}}_t)^\top H(\mathbf{M}\widetilde{\mathbf{W}}_t - \mathbf{MC}\widetilde{\mathbf{D}}_t)]$ , where  $\mathbf{C} := I_{k \times k} \otimes C$  and  $\otimes$  denotes Kronecker product. Using the linearity of expectation,  $\mathcal{L}_t$  becomes:

$$\begin{aligned} & \mathbb{E} \left[ \underbrace{(\mathbf{MC}\widetilde{\mathbf{D}}_t)^\top H(\mathbf{MC}\widetilde{\mathbf{D}}_t)}_{\text{Quadratic term}} \right] - 2\mathbb{E} \left[ \underbrace{(\mathbf{M}\widetilde{\mathbf{W}}_t)^\top H(\mathbf{MC}\widetilde{\mathbf{D}}_t)}_{\text{Cross term}} \right] \\ & + \mathbb{E} \left[ \underbrace{(\mathbf{M}\widetilde{\mathbf{W}}_t)^\top H(\mathbf{M}\widetilde{\mathbf{W}}_t)}_{\text{Constant with respect to } \mathbf{C}} \right]. \end{aligned}$$

Since the Cross term is a scalar, the expectation of the Cross term is:  $\text{Trace}(\mathbf{M}^\top H \mathbf{MC} \mathbb{E}[\widetilde{\mathbf{D}}_t \widetilde{\mathbf{W}}_t^\top])$ , Recalling (24),  $\mathbb{E}[\widetilde{\mathbf{D}}_t \widetilde{\mathbf{W}}_t^\top]$  is the cross-covariance matrix of  $d$  and  $w$ , which is zero by assumption. Therefore, minimizing  $\mathcal{L}_t$  becomes equivalent to minimizing the Quadratic term. Similarly, the expectation of the quadratic term can be written as:  $\text{Trace}((\mathbf{MC})^\top H(\mathbf{MC}) \mathbb{E}[\widetilde{\mathbf{D}}_t \widetilde{\mathbf{D}}_t^\top])$ , where  $\mathbb{E}[\widetilde{\mathbf{D}}_t \widetilde{\mathbf{D}}_t^\top]$  is the variance of  $\widetilde{\mathbf{D}}_t$  since the expectation of  $\widetilde{\mathbf{D}}_t$  is zero by (24). Therefore, due to the positive-definiteness of  $H$ , minimizing the Quadratic term is equivalent to setting  $\mathbf{MC}$  to zero, whose necessary condition is:  $\sum_{\tau=t}^{\mathcal{T}} (F^\top)^{\tau-t} PC = 0$ , which directly leads to  $C = 0$  by using Lemma 2. Substituting  $C = 0$  to the convergence result of  $b$  yields  $b = \mathbb{E}[w]$ .