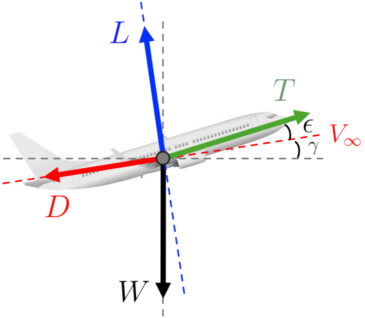
The fundamental equations that govern the aircraft's translational motion through air

Consider an aircraft in flight, **four physical forces** are acting on the aircraft:

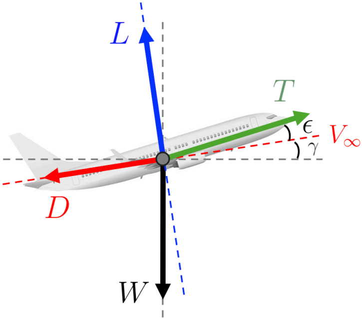
- **Lift (L):** perpendicular to the relative wind (V_∞)
- **Drag (D):** parallel to the relative wind (V_∞)
- **Weight (W):** acts vertically toward the center of the earth
- **Thrust (T):** 'approximately' parallel to the relative wind (V_∞)



The flight path: direction of motion of the aircraft, same as the relative wind V_∞

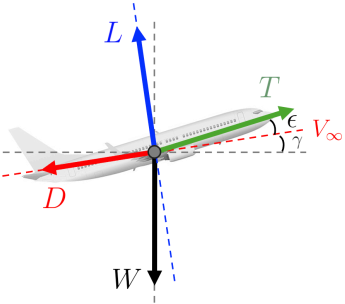
Two angles in a general flight:

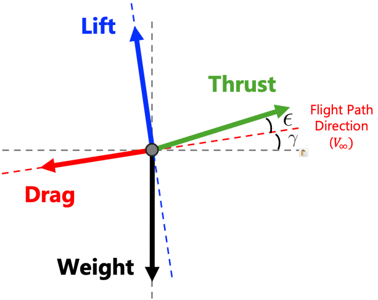
- **The flight path angle (γ):** the angle between the flight path and horizontal
  - It describes whether the aircraft is climbing or descending
- **The engine inclination angle (ϵ):** the angle between thrust and the flight path
  - The engine mounted at an angle relative to the aircraft's longitudinal axis
  - Can achieve enhanced aerodynamic efficiency and improved performance



The force diagram

The flight path can be **curved** (curvilinear) or **straight** (rectilinear)





Next, under the general **accelerated flight** and **curvilinear path**, we analyze forces along two mutually perpendicular axes:

1. Along the direction of the flight path
2. Normal to the flight path

The equations of motion

The equations of motion for an aircraft are statements of **Newton's Second Law** (Force $=$ mass $\times$ acceleration)

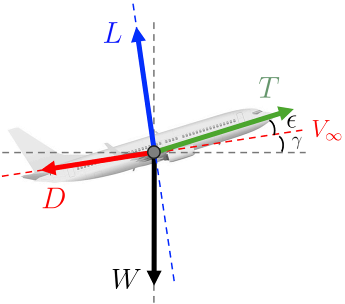
Then, we apply Newton's Second Law along both axes: the summation of all forces

- Parallel to the flight path

$$\sum F_{\parallel} = ma_{\parallel}$$

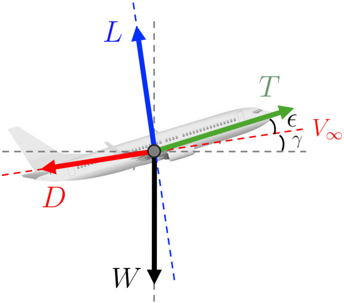
- Normal to the flight path

$$\sum F_{\perp} = ma_{\perp}$$



The equations of motion

Parallel to the flight path: $\sum F_{\parallel} = ma_{\parallel}$

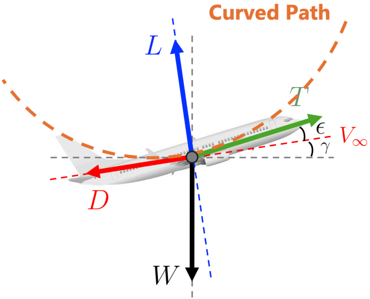


$$\sum F_{\parallel} = T\cos\epsilon - D - W\sin\gamma$$

$$ma_{\parallel} = m\frac{dV}{dt}$$

$$\boxed{T\cos\epsilon - D - W\sin\gamma = m\frac{dV}{dt}}$$

Normal to the flight path: $\sum F_\perp = ma_\perp$, centrifugal force on curved path (radius r_c)



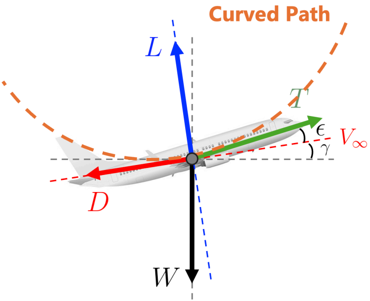
$$\sum F_\perp = L + T\sin\epsilon - W\cos\gamma$$

$$ma_\perp = m\frac{V^2}{r_c}$$

$$\boxed{L + T\sin\epsilon - W\cos\gamma = m\frac{V^2}{r_c}}$$

The equations of motion

In summary, the equations of motion for an aircraft in **translational flight**:



$$\boxed{\begin{aligned} T\cos\epsilon - D - W\sin\gamma &= m\frac{dV}{dt} \\ L + T\sin\epsilon - W\cos\gamma &= m\frac{V^2}{r_c} \end{aligned}}$$

Currently, the general 2-D translational motion does not include side-wise motion

The equations of motion

We are first interested in **static performance**, which is useful in the modeling of **many vital scenarios**, such as maximum velocity, maximum rate of climb, maximum range, etc.

In **steady and level flight**,

- Acceleration is zero: $a = 0$
- Flight path is along the horizontal: $\gamma = 0$

$$\boxed{\begin{aligned} T\cos\epsilon - D - 0 &= 0 \\ L + T\sin\epsilon - W &= 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} T\cos\epsilon &= D \\ L + T\sin\epsilon &= W \end{aligned}}$$

For most conventional aircraft, ϵ is small enough, such that $\cos\epsilon \approx 1$ and $\sin\epsilon \approx 0$

$$\boxed{\begin{aligned} T &= D \\ L &= W \end{aligned}}$$

This is an almost trivial but very useful result!