

Propulsive efficiency

$$\eta_p = \frac{\text{Useful power available}}{\text{Total power generated}}$$

Power available (P_A): power = force \times velocity

$$P_A = TV_0$$

Power wasted (P_W): producing wasted kinetic energy in the air jet behind the device

$$P_W = \frac{1}{2} \dot{m}_0 (V_e - V_0)^2$$

where $(V_e - V_0)$ is the relative velocity of the exhaust jet seen by the airplane.

Therefore, the **total power** generated by propulsive device is

$$P = TV_0 + \frac{1}{2} \dot{m}_0 (V_e - V_0)^2$$

Propulsive efficiency

$$\eta_p = \frac{TV_0}{TV_0 + \frac{1}{2}\dot{m}_0(V_e - V_0)^2}$$

With the general thrust equation

$$T = \dot{m}_0(V_e - V_0)$$

We have

$$\begin{aligned}\eta_p &= \frac{\dot{m}_0(V_e - V_0)V_0}{\dot{m}_0(V_e - V_0)V_0 + \frac{1}{2}\dot{m}_0(V_e - V_0)^2} \\ &= \frac{1}{1 + (1/2)(V_e - V_0)/V_0} \\ &= \boxed{\frac{2}{1 + V_e/V_0}}\end{aligned}$$

Eventually, the propulsive efficiency comes down to the ratio of V_e and V_0 !

There is a trade-off between thrust and efficiency!

- Propulsive efficiency

$$\eta_p = \frac{2}{1 + V_e/V_0}$$

- Thrust

$$T = \dot{m}_0 (V_e - V_0)$$

By examining both equations, we find out that

- Maximum (100%) propulsive efficiency is obtained when $V_e = V_0$. **No thrust!**
- In general, **more thrust means less efficiency**.
- This trade-off is the reason why different types of engine exist today.

“The choice of a proper engine depends on what you want the airplane to do”

Comparison among different types of engines

Different propulsion systems generate thrust in slightly different ways, according to the needs for thrust and efficiency. **And it is all about performance! Or excess thrust.**

