



Contents



GETTING STARTED

About this Course 28

Students Start Here 29

- Quick Start 29
- Using Desmos 31
- Additional Resources 34



UNIT 1 LINEAR EQUATIONS

Unit 1 Overview and Readiness 35

- Unit 1 Overview 35
- Solve Linear Equations: Mini-Lesson Review 35
- Classify Equations: Mini-Lesson Review 37
- Find Coordinates: Mini-Lesson Review 39
- Unit 1 Overview: Wrap Up 41

1.1

Exploring Expressions and Equations 43

- 1.1.0** Lesson Overview 43
- 1.1.1** Understanding Value 43
- 1.1.2** Creating Expressions to Estimate Cost, Part 1 43
- 1.1.3** Understanding Constraints 46
- 1.1.4** Creating Expressions to Estimate Cost, Part 2 48
- 1.1.5** Practice 49
- 1.1.6** Lesson Summary 51

1.2

Writing Equations to Model Relationships, Part 1 53

- 1.2.0** Lesson Overview 53
- 1.2.1** Finding the Percent of 200 53
- 1.2.2** Modeling with Equations to Find Edges in Platonic Solids 53
- 1.2.3** Writing Equations to Represent Relationships 56
- 1.2.4** Writing Equations to Represent Relationships with Percentages 58
- 1.2.5** Writing Equations to Represent T-Shirts 59
- 1.2.6** Practice 60
- 1.2.7** Lesson Summary 62

1.3

Writing Equations to Model Relationships, Part 2 63

- 1.3.0** Lesson Overview 63
- 1.3.1** Finding a Relationship between x and y 63
- 1.3.2** Describing Relationships Using Words and Equations 63
- 1.3.3** Identifying and Representing Relationships 66
- 1.3.4** Writing an Equation to Represent a Relationship 68
- 1.3.5** Practice 69
- 1.3.6** Lesson Summary 71

1.4 Equations and Their Solutions 73

- 1.4.0 Lesson Overview 73**
- 1.4.1 Finding the Solution to an Equation in One Variable, Part 1 73**
- 1.4.2 Writing Equations to Represent Constraints 73**
- 1.4.3 Finding the Solution to an Equation in Two Variables 75**
- 1.4.4 Finding the Solution to an Equation in One Variable, Part 2 76**
- 1.4.5 Practice 78**
- 1.4.6 Lesson Summary 80**

1.5 Equations and Their Graphs 81

- 1.5.0 Lesson Overview 81**
- 1.5.1 Analyzing Graphs of Linear Equations 81**
- 1.5.2 Graphing Linear Functions in Two Variables 82**
- 1.5.3 Examining an Equation in Two Variables and Its Graph, Part 1 83**
- 1.5.4 Writing, Graphing, and Solving a Linear Equation 86**
- 1.5.5 Examining an Equation in Two Variables and Its Graph, Part 2 89**
- 1.5.6 Practice 90**
- 1.5.7 Lesson Summary 92**

1.6 Equivalent Equations 93

- 1.6.0 Lesson Overview 93**
- 1.6.1 Exploring Equivalent Expressions 93**
- 1.6.2 Expressing Relationships as Equations 93**
- 1.6.3 Exploring Related Equations 95**
- 1.6.4 Recognizing Related Equations 97**
- 1.6.5 Practice 98**
- 1.6.6 Lesson Summary 100**

1.7 Explaining Steps for Rewriting Equations 101

- 1.7.0 Lesson Overview 101**
- 1.7.1 Determining if Zero Is a Solution 101**
- 1.7.2 Explaining Acceptable Moves to Solve an Equation 101**
- 1.7.3 Understanding Equations with No Solution or Infinitely Many 104**
- 1.7.4 Solving Equations by Dividing Values 107**
- 1.7.5 Practice 108**
- 1.7.6 Lesson Summary 109**

1.8 Choosing the Correct Variable to Solve For, Part 1 111

- 1.8.0 Lesson Overview 111**
- 1.8.1 Expressing Relationships between Two Quantities 111**
- 1.8.2 Identifying the Most Useful Form of an Equation 112**
- 1.8.3 Writing Equations and Isolating Variables 116**
- 1.8.4 Writing Equations Given Dimensions of Shapes 118**
- 1.8.5 Practice 119**
- 1.8.6 Lesson Summary 121**

1.9 Choosing the Correct Variable to Solve For, Part 2 123

- 1.9.0 Lesson Overview 123**

- 1.9.1** Rearranging an Equation in Three Variables 123
- 1.9.2** Writing an Equation to Represent a Constraint 124
- 1.9.3** Writing and Rearranging Equations in Two Variables 126
- 1.9.4** Solving for a Specified Variable 130
- 1.9.5** Practice 131
- 1.9.6** Lesson Summary 132

1.10 Connecting Equations to Graphs, Part 1 133

- 1.10.0** Lesson Overview 133
- 1.10.1** Interpreting Equations in Context 133
- 1.10.2** Graphing Equations in Context 134
- 1.10.3** Writing and Graphing Equations in Standard Form 140
- 1.10.4** Matching Equations and Graphs 141
- 1.10.5** Practice 143
- 1.10.6** Lesson Summary 145

1.11 Connecting Equations to Graphs, Part 2 147

- 1.11.0** Lesson Overview 147
- 1.11.1** Applying the Distributive Property to Rewrite Expressions 147
- 1.11.2** Relating Two-Variable Equations, Their Graphs, and Situations 147
- 1.11.3** Reasoning Symbolically and Abstractly about Linear Equations 151
- 1.11.4** Identifying the Features of a Graph 153
- 1.11.5** Practice 154
- 1.11.6** Lesson Summary 156

1.12 Writing the Equation of a Line 157

- 1.12.0** Lesson Overview 157
- 1.12.1** Using the Slope Formula 157
- 1.12.2** Writing an Equation Given the Slope and y -intercept 157
- 1.12.3** Writing an Equation Given the Slope and a Point 160
- 1.12.4** Writing Equations in Different Forms 162
- 1.12.5** Writing an Equation Given Two Points 164
- 1.12.6** Choosing a Method to Write an Equation 166
- 1.12.7** Practice 168
- 1.12.8** Lesson Summary 170

1.13 Lines from Tables and Graphs 171

- 1.13.0** Lesson Overview 171
- 1.13.1** Writing Linear Equations 171
- 1.13.2** Creating Tables from Verbal Descriptions 171
- 1.13.3** Matching Tables, Equations, and Graphs 174
- 1.13.4** Creating a Table from an Equation 176
- 1.13.5** Practice 177
- 1.13.6** Lesson Summary 179

1.14 Writing Equations of Parallel and Perpendicular Lines 181

- 1.14.0** Lesson Overview 181
- 1.14.1** Using Point-Slope Form to Write the Equation of a Line 181

1.14.2 Writing Equations for Point-Slope and Slope-Intercept	182
1.14.3 Writing an Equation of a Line Parallel to a Given Line	183
1.14.4 Writing an Equation of a Line Perpendicular to a Given Line	186
1.14.5 Writing an Equation of a Line Parallel or Perpendicular to an Axis	188
1.14.6 Drawing Parallel and Perpendicular Lines	192
1.14.7 Practice	193
1.14.8 Lesson Summary	195

1.15 Direct Variation 197

1.15.0 Lesson Overview	197
1.15.1 Solving Proportions	197
1.15.2 Modeling Equations Using Direct Variation	197
1.15.3 Using Direct Variation to Solve Application Problems, Part 1	199
1.15.4 Using Direct Variation to Solve Application Problems, Part 2	201
1.15.5 Practice	202
1.15.6 Lesson Summary	203

Project 1: Slopes and Intercepts 205

Project 1 Overview	205
Notice and Wonder Graph Features	205
Matching Graph and Equations	205

Unit 1 Wrap Up 208

UNIT 2 LINEAR INEQUALITIES AND SYSTEMS

Unit 2 Overview and Readiness 209

Unit 2 Overview	209
Determine the Ordered Pair: Mini-Lesson Review	209
Find the Slope and y-intercept: Mini-Lesson Review	212
Find the x- and y-intercepts: Mini-Lesson Review	214
Unit 2 Overview: Wrap Up	216

2.1 Writing and Graphing Systems of Equations 217

2.1.0 Lesson Overview	217
2.1.1 Understanding Constraints and Values	217
2.1.2 Writing and Graphing Equations	218
2.1.3 Meeting Constraints	222
2.1.4 Exploring a System of Equations	224
2.1.5 Practice	226
2.1.6 Lesson Summary	228

2.2 Writing Systems of Equations 229

2.2.0 Lesson Overview	229
2.2.1 Interpreting Graphs of Systems of Equations	229
2.2.2 Writing Systems of Equations from Tables	230
2.2.3 Graphs of Systems of Equations	232
2.2.4 Systems of Equations in the Real-World	237

- 2.2.5** Practice 238
- 2.2.6** Lesson Summary 241

2.3 Solving Systems by Substitution 243

- 2.3.0** Lesson Overview 243
- 2.3.1** Finding Connections between Graphs and Equations 243
- 2.3.2** Checking Solutions in Systems 244
- 2.3.3** Solving Equations Using Substitution 246
- 2.3.4** Solving More Equations 248
- 2.3.5** Practice 250
- 2.3.6** Lesson Summary 252

2.4 Solving Systems by Elimination, Part 1 253

- 2.4.0** Lesson Overview 253
- 2.4.1** Combining Two True Equations 253
- 2.4.2** Adding Equations 254
- 2.4.3** Adding and Subtracting Systems of Equations and Using Their Graphs 256
- 2.4.4** Determining the Best Method for Solving a System of Equations 260
- 2.4.5** Practice 261
- 2.4.6** Lesson Summary 263

2.5 Solving Systems by Elimination, Part 2 265

- 2.5.0** Lesson Overview 265
- 2.5.1** Writing and Evaluating New Equations 265
- 2.5.2** Adding Two Equations in a System 266
- 2.5.3** Solving Systems of Linear Equations in Two Variables 268
- 2.5.4** Solving Systems of Equations Using Real-World Examples 270
- 2.5.5** Practice 271
- 2.5.6** Lesson Summary 273

2.6 Solving Systems by Elimination, Part 3 275

- 2.6.0** Lesson Overview 275
- 2.6.1** Multiplying Equations by a Number 275
- 2.6.2** Writing a New System to Solve a Given System 276
- 2.6.3** Finding Solutions to Unordered Sets of Equivalent Systems 278
- 2.6.4** Building Equivalent Systems 280
- 2.6.5** Practice 281
- 2.6.6** Lesson Summary 283

2.7 Systems of Linear Equations and Their Solutions 285

- 2.7.0** Lesson Overview 285
- 2.7.1** Systems of Linear Equations with Infinitely Many Solutions 285
- 2.7.2** Systems of Linear Equations with No Solution 285
- 2.7.3** Sorting Systems of Equations Based on Number of Solutions 287
- 2.7.4** Writing Consistent and Inconsistent Systems of Equations 288
- 2.7.5** Practice 289
- 2.7.6** Lesson Summary 291

2.8 Representing Situations with Inequalities 293

- 2.8.0** Lesson Overview 293
- 2.8.1** Understanding Inequality Symbols 293
- 2.8.2** Identifying Constraints in Inequalities 293
- 2.8.3** Writing Inequalities to Represent Constraints 295
- 2.8.4** Writing More Inequalities 296
- 2.8.5** Practice 297
- 2.8.6** Lesson Summary 299

2.9 Solutions to Inequalities 301

- 2.9.0** Lesson Overview 301
- 2.9.1** Writing Solutions to Inequalities 301
- 2.9.2** Graphing Inequalities 301
- 2.9.3** Understanding the Meaning of an Inequality 304
- 2.9.4** Making Sense of Inequalities and Their Solutions 306
- 2.9.5** Comparing Equality and Inequality 306
- 2.9.6** Using Technology to Visualize Inequality Solutions 309
- 2.9.7** Identifying Inequality Solutions on a Graph 310
- 2.9.8** Practice 311
- 2.9.9** Lesson Summary 313

2.10 Writing and Solving Inequalities in One Variable 315

- 2.10.0** Lesson Overview 315
- 2.10.1** Writing an Inequality to Represent a Constraint 315
- 2.10.2** Using Inequalities to Solve a Problem 315
- 2.10.3** Different Ways of Solving an Inequality 317
- 2.10.4** Matching Inequalities and Solutions 319
- 2.10.5** Solving Inequalities 320
- 2.10.6** Writing Inequalities to Represent a Situation 323
- 2.10.7** Practice 325
- 2.10.8** Lesson Summary 327

2.11 Graphing Linear Inequalities in Two Variables 329

- 2.11.0** Lesson Overview 329
- 2.11.1** Using Computation and Reasoning 329
- 2.11.2** Finding Solutions to Inequalities on the Coordinate Plane 329
- 2.11.3** Sketching Solutions to Inequalities 332
- 2.11.4** Shading Boundary Line Regions 338
- 2.11.5** Practice 339
- 2.11.6** Lesson Summary 341

2.12 Using Linear Inequalities as Constraints 343

- 2.12.0** Lesson Overview 343
- 2.12.1** Sketching a Graph to Represent an Equation 343
- 2.12.2** Writing an Inequality to Represent a Constraint 344
- 2.12.3** Graphing Solutions and Interpreting Points 345
- 2.12.4** Recognizing Points Along Boundary Line 348
- 2.12.5** Recognizing Points for an Inequality 348

- 2.12.6** Practice 350
- 2.12.7** Lesson Summary 355

2.13 Solving Problems with Inequalities in Two Variables 357

- 2.13.0** Lesson Overview 357
- 2.13.1** Graphing Inequalities with Technology 357
- 2.13.2** Solving Problems with Inequalities in Two Variables 358
- 2.13.3** Matching Representations of Inequalities 362
- 2.13.4** Solving Inequality Problems Using Tables 364
- 2.13.5** Graphing Solutions to an Inequality 370
- 2.13.6** Practice 372
- 2.13.7** Lesson Summary 375

2.14 Solutions to Systems of Linear Inequalities in Two Variables 377

- 2.14.0** Lesson Overview 377
- 2.14.1** Modeling Equations to Solve a Riddle 377
- 2.14.2** Finding a Pair of Values that Satisfies Multiple Inequalities 377
- 2.14.3** Writing Systems of Inequalities that Represent Situations 380
- 2.14.4** Graphing Solutions of Systems of Inequalities 383
- 2.14.5** Writing Inequalities for Situations and Representing on a Graph 387
- 2.14.6** Practice 388
- 2.14.7** Lesson Summary 390

2.15 Solving Problems with Systems of Linear Inequalities in Two Variables 391

- 2.15.0** Lesson Overview 391
- 2.15.1** Analyzing Graphs that Represent Linear Equations and Inequalities 391
- 2.15.2** Determining if Points on Boundary Lines Are Solutions to a System 392
- 2.15.3** Solving Problems to Satisfy Multiple Constraints Simultaneously 396
- 2.15.4** Identifying Graph Regions Included in a System's Solution 399
- 2.15.5** Practice 400
- 2.15.6** Lesson Summary 403

Project 2: Modeling with Systems of Inequalities in Two Variables 405

- Project 2 Overview 405
- Determine a Solution to Inequalities 405
- Interpret a Set of Mathematical Models 405
- Perform Mathematical Modeling 406

Unit 2 Wrap Up 408



UNIT 3 TWO-VARIABLE STATISTICS

Unit 3 Overview and Readiness 409

- Unit 3 Overview 409
- Distinguish Patterns: Mini-Lesson Review 409
- Identify Positive and Negative Trends: Mini-Lesson Review 412
- Interpret Slope and y-intercept: Mini-Lesson Review 414
- Unit 3 Overview: Wrap Up 416

3.1 Linear Models 419

- 3.1.0** Lesson Overview 419
- 3.1.1** Exploring Scatter Plots 419
- 3.1.2** Creating a Scatter Plot Using Data 420
- 3.1.3** Interpreting the Slope and Vertical Intercept of a Linear Model 424
- 3.1.4** Interpreting the Slope and Vertical Intercept of a Scatter Plot 426
- 3.1.5** Using an Equation for a Fit Line 431
- 3.1.6** Practice 433
- 3.1.7** Lesson Summary 437

3.2 Fitting Lines 439

- 3.2.0** Lesson Overview 439
- 3.2.1** Selecting the Best Line 439
- 3.2.2** Determining the Best Line That Fits the Data 440
- 3.2.3** Writing Linear Models without Technology 443
- 3.2.4** Assessing the Fit of a Linear Model 446
- 3.2.5** Working with Fitted Lines 451
- 3.2.6** Practice 452
- 3.2.7** Lesson Summary 454

3.3 Residuals 455

- 3.3.0** Lesson Overview 455
- 3.3.1** Subtracting an Estimated Value from an Actual Value 455
- 3.3.2** Plotting and Analyzing Residuals 456
- 3.3.3** Matching Residuals Graphs to Scatter Plots 459
- 3.3.4** Identifying Best Fit Line for Residuals on a Graph 463
- 3.3.5** Practice 465
- 3.3.6** Lesson Summary 468

3.4 The Correlation Coefficient 469

- 3.4.0** Lesson Overview 469
- 3.4.1** Comparing Scatter Plots with Linear and Nonlinear Trends 469
- 3.4.2** Recognizing Differences in Scatter Plots 470
- 3.4.3** Finding the Correlation Coefficient with Technology 474
- 3.4.4** Matching Correlation Coefficients 477
- 3.4.5** Using Correlation Coefficient to Determine Best Fit 486
- 3.4.6** Practice 487
- 3.4.7** Lesson Summary 491

3.5 Using the Correlation Coefficient 493

- 3.5.0** Lesson Overview 493
- 3.5.1** Using Bivariate Data in Context 493
- 3.5.2** Finding and Using Correlation Coefficient to Interpret the Strength of Linear Relationships 495
- 3.5.3** Using Correlation Coefficient to Describe Relationships between Two Variables 499
- 3.5.4** Interpreting the Correlation Coefficient 501
- 3.5.5** Practice 502
- 3.5.6** Lesson Summary 504

3.6 Causal Relationships 505

- 3.6.0 Lesson Overview 505**
- 3.6.1 Describe the Relationship between Variables 505**
- 3.6.2 Describing How Two Variables Are Related 505**
- 3.6.3 Using the Term Causal Relationship 509**
- 3.6.4 Determining the Type of Relationship 511**
- 3.6.5 Practice 512**
- 3.6.6 Lesson Summary 513**

Project 3: Two-Variable Statistics 515

- Project 3 Overview 515
- Estimate Lengths 515
- Analyze Data 515

Unit 3 Wrap Up 516

UNIT 4 FUNCTIONS

Inquiry Project: Defining Functions 517

- Inquiry Project Overview 517
- Simulating Relationships 517
- What is a function? 518
- Different Kinds of Functions 520

Unit 4 Overview and Readiness 523

- Unit 4 Overview 523
- Find Slope: Mini-Lesson Review 523
- Describe Graphs: Mini-Lesson Review 527
- Extend a Pattern: Mini-Lesson Review 530
- Unit 4 Overview: Wrap Up 531

4.1 Describing and Graphing Situations 533

- 4.1.0 Lesson Overview 533**
- 4.1.1 Contrasting Two Relationships by Reasoning 533**
- 4.1.2 Reasoning Graphically about the Relationship between the Two Quantities 536**
- 4.1.3 Examining Relations and Functions 538**
- 4.1.4 Describing Functional Relationships 541**
- 4.1.5 Modeling Relationships between Two Variables 543**
- 4.1.6 Practice 544**
- 4.1.7 Lesson Summary 546**

4.2 Function Notation 547

- 4.2.0 Lesson Overview 547**
- 4.2.1 Interpreting Graphs to Answer Questions 547**
- 4.2.2 Interpreting Statements Written in Function Notation 549**
- 4.2.3 Finding a Unique Output for Each Input 551**
- 4.2.4 Understanding Function Notation Using Application Problems 554**

- 4.2.5** Practice 555
- 4.2.6** Lesson Summary 557

4.3 Interpreting Using Function Notation 559

- 4.3.0** Lesson Overview 559
- 4.3.1** Comparing Function Values 559
- 4.3.2** Writing Statements from Function Notation 560
- 4.3.3** Comparing Statements in Function Notation 562
- 4.3.4** Using Function Notation with a Real-World Example 563
- 4.3.5** Practice 565
- 4.3.6** Lesson Summary 567

4.4 Using Function Notation to Describe Rules, Part 1 569

- 4.4.0** Lesson Overview 569
- 4.4.1** Using Function Notation in Tables of Values 569
- 4.4.2** Defining Functions by a Rule 570
- 4.4.3** Using Tables to Find Function Rules 572
- 4.4.4** Writing a Rule Using Function Notation 575
- 4.4.5** Practice 576
- 4.4.6** Lesson Summary 578

4.5 Using Function Notation to Describe Rules, Part 2 579

- 4.5.0** Lesson Overview 579
- 4.5.1** Finding Solutions of Equations 579
- 4.5.2** Using the Vertical Line Test 579
- 4.5.3** Comparing Functions 583
- 4.5.4** Using Function Notation and Graphing Technology 584
- 4.5.5** Using Graphical and Algebraic Approaches for Finding Unknowns of Linear Functions 586
- 4.5.6** Practice 587
- 4.5.7** Lesson Summary 589

4.6 Features of Graphs 591

- 4.6.0** Lesson Overview 591
- 4.6.1** Interpreting Statements in Function Notation 591
- 4.6.2** Analyzing Graphs of Functions 592
- 4.6.3** Connecting Graphical and Verbal Representations of a Function 594
- 4.6.4** Key Features of Linear Functions 597
- 4.6.5** Interpreting Graphs 600
- 4.6.6** Practice 601
- 4.6.7** Lesson Summary 604

4.7 Finding Slope 605

- 4.7.0** Lesson Overview 605
- 4.7.1** Evaluating Fractions 605
- 4.7.2** Finding Slope From Tables, Graphs, and Points 605
- 4.7.3** Writing Linear Equations 610
- 4.7.4** Writing Equations From Two Points 612

4.7.5	Practice	614
4.7.6	Lesson Summary	615

4.8 Using Graphs to Find Average Rate of Change 617

4.8.0	Lesson Overview	617
4.8.1	Comparing Changes in Output Based on Input	617
4.8.2	Average Rate of Change and Slope	617
4.8.3	Finding and Interpreting Average Rates of Change	621
4.8.4	Rate of Change of Linear Functions	623
4.8.5	Interpreting Average Rates of Change with Real-World Examples	627
4.8.6	Practice	628
4.8.7	Lesson Summary	631

4.9 Interpreting and Creating Graphs 633

4.9.0	Lesson Overview	633
4.9.1	Analyzing and Comparing the Properties of Graphs	633
4.9.2	Interpreting Graphs Without Units	635
4.9.3	Sketching Graphs of Functions	640
4.9.4	Representing Quantities in a Situation	642
4.9.5	Using Verbal Descriptions to Create Graphs of Functions	643
4.9.6	Practice	645
4.9.7	Lesson Summary	647

4.10 Comparing Graphs 649

4.10.0	Lesson Overview	649
4.10.1	Analyzing Graphs and Statements in Function Notation	649
4.10.2	Interpreting Graphs and Statements in Terms of a Situation	650
4.10.3	Comparing Functions Represented in Separate Graphs	652
4.10.4	Comparing Graphs and Statements that Represent Functions without a Context	656
4.10.5	Analyzing Functions Using Real-World Context	658
4.10.6	Practice	659
4.10.7	Lesson Summary	662

4.11 Graphing a Function Using Transformations 663

4.11.0	Lesson Overview	663
4.11.1	Graphing Linear Functions	663
4.11.2	Vertical Shifts	664
4.11.3	Horizontal Shifts	665
4.11.4	Vertical Stretches and Compressions	670
4.11.5	Horizontal Stretches and Compressions	673
4.11.6	Graphing Using Transformations	680
4.11.7	Practice	681
4.11.8	Lesson Summary	684

4.12 Domain and Range, Part 1 685

4.12.0	Lesson Overview	685
4.12.1	Determining Reasonable Inputs and Outputs	685
4.12.2	Domain: The Input of a Function	686

- 4.12.3** Range: The Output of a Function 688
- 4.12.4** Using Graphs to Determine the Domain of a Function 692
- 4.12.5** Using Functions to Answer Real-World Questions 692
- 4.12.6** Practice 693
- 4.12.7** Lesson Summary 695

4.13 Domain and Range, Part 2 697

- 4.13.0** Lesson Overview 697
- 4.13.1** Different Features of a Graph 697
- 4.13.2** Graphs of Functions 698
- 4.13.3** Finding Domain and Range Using a Graph 702
- 4.13.4** Real-World Domain and Range 705
- 4.13.5** Graph of a Function and the Domain and Range Connection 708
- 4.13.6** Practice 709
- 4.13.7** Lesson Summary 712

4.14 Sequences 713

- 4.14.0** Lesson Overview 713
- 4.14.1** Describe the Pattern 713
- 4.14.2** What Is a Sequence? 713
- 4.14.3** The Connection between Sequences and Terms 714
- 4.14.4** Rules for a Sequence 716
- 4.14.5** Practice 717
- 4.14.6** Lesson Summary 718

4.15 Introducing Geometric Sequences 719

- 4.15.0** Lesson Overview 719
- 4.15.1** What Are the Patterns? 719
- 4.15.2** Using Tables and Graphs to Represent Geometric Sequences 719
- 4.15.3** Complete the Sequence 721
- 4.15.4** Connecting the Common Ratio and the Term in a Sequence 723
- 4.15.5** Practice 724
- 4.15.6** Lesson Summary 726

4.16 Introducing Arithmetic Sequences 727

- 4.16.0** Lesson Overview 727
- 4.16.1** Interpreting Function Notation 727
- 4.16.2** What Is an Arithmetic Sequence? 727
- 4.16.3** A Sequence Is a Type of Function 729
- 4.16.4** Identify and Generate Sequences 730
- 4.16.5** Practice 731
- 4.16.6** Lesson Summary 733

4.17 Representing Sequences 735

- 4.17.0** Lesson Overview 735
- 4.17.1** Define a Sequence 735
- 4.17.2** What Is a Recursive Definition? 735
- 4.17.3** Ways to Represent a Sequence 739

- 4.17.4** Geometric Recursive Definitions 741
- 4.17.5** Practice 742
- 4.17.6** Lesson Summary 744

4.18 The n^{th} Term of a Sequence 745

- 4.18.0** Lesson Overview 745
- 4.18.1** Repeated Operations 745
- 4.18.2** Identifying Domain for a Function 745
- 4.18.3** Define an Arithmetic Sequence by the n^{th} Term 746
- 4.18.4** Define a Geometric Sequence by the n^{th} Term 748
- 4.18.5** Interchanging between Formulas 750
- 4.18.6** Different Types of Equations 754
- 4.18.7** Practice 755
- 4.18.8** Lesson Summary 757

Project 4: Using Functions to Model Battery Power 759

- Project 4 Overview 759
- Estimate Time to Charge a Battery 759
- Analyze Percent Charge Over Time 759
- Write an Equation to Model Data 760

Unit 4 Wrap Up 762



UNIT 5 INTRODUCTION TO EXPONENTIAL FUNCTIONS

Unit 5 Overview and Readiness 763

- Unit 5 Overview 763
- Write Linear Functions from a Graph: Mini-Lesson Review 764
- Interpret Linear Functions: Mini-Lesson Review 766
- Evaluate Exponential Expressions: Mini-Lesson Review 767
- Unit 5 Overview: Wrap Up 769

5.1 Properties of Exponents 771

- 5.1.0** Lesson Overview 771
- 5.1.1** Understanding Exponents 771
- 5.1.2** Using Product and Quotient Properties for Exponents 771
- 5.1.3** Using Zero Exponent Property and Negative Exponents 775
- 5.1.4** Using Power Properties for Exponents 777
- 5.1.5** Simplifying Exponents 781
- 5.1.6** Practice 783
- 5.1.7** Lesson Summary 785

5.2 Rational Exponents 787

- 5.2.0** Lesson Overview 787
- 5.2.1** Perfect Squares Revisited 787
- 5.2.2** The n^{th} Root 787
- 5.2.3** Radicals and Rational Exponents 789
- 5.2.4** Properties of Exponents with Radicals 791

- 5.2.5** Practice 792
- 5.2.6** Lesson Summary 794

5.3 Patterns of Growth 795

- 5.3.0** Lesson Overview 795
- 5.3.1** Creating a Pictorial Representation of Exponential Growth 796
- 5.3.2** Exploring Different Growth Patterns 796
- 5.3.3** Compare and Contrast Two Patterns 799
- 5.3.4** Match Expressions and Tables with Situations 803
- 5.3.5** Describe Mathematically Changes in Growth 805
- 5.3.6** Practice 806
- 5.3.7** Lesson Summary 809

5.4 Representing Exponential Growth 811

- 5.4.0** Lesson Overview 811
- 5.4.1** Exponent Rules 811
- 5.4.2** Zero Exponent Rule 811
- 5.4.3** Exponential Change: The Growth Factor 813
- 5.4.4** Graphing Exponential Expressions 816
- 5.4.5** Interpret the Growth Factor of the Equation 820
- 5.4.6** Practice 821
- 5.4.7** Lesson Summary 824

5.5 Representing Exponential Decay 825

- 5.5.0** Lesson Overview 825
- 5.5.1** Identifying Linear and Exponential Quantities 825
- 5.5.2** Exponential Decay 825
- 5.5.3** Using Graphs to Represent Exponential Decay 827
- 5.5.4** Justifying Exponential Models 832
- 5.5.5** Writing Equations to Represent Exponential Decay 834
- 5.5.6** Practice 835
- 5.5.7** Lesson Summary 838

5.6 Negative Exponents and Scientific Notation 839

- 5.6.0** Lesson Overview 839
- 5.6.1** Checking Exponent Rules 839
- 5.6.2** Interpreting Negative Exponents in Exponential Growth 840
- 5.6.3** Interpreting Negative Exponents in Exponential Decay 842
- 5.6.4** Using Scientific Notation 846
- 5.6.5** Applying Scientific Notation 849
- 5.6.6** Interpreting Negative Exponents and Scientific Notation 852
- 5.6.7** Practice 853
- 5.6.8** Lesson Summary 855

5.7 Analyzing Graphs 857

- 5.7.0** Lesson Overview 857
- 5.7.1** Patterns in Real Numbers 857
- 5.7.2** Comparing Graphs 857

- 5.7.3** Describing Graphs 861
- 5.7.4** Quantities that Change Exponentially 866
- 5.7.5** Practice 867
- 5.7.6** Lesson Summary 870

5.8 Exponential Situations as Functions 871

- 5.8.0** Lesson Overview 871
- 5.8.1** Meaning of a Function Presented Graphically 871
- 5.8.2** Represent a Function as a Table of Values, Graph, and Equation 871
- 5.8.3** Using Function Language and Notation 873
- 5.8.4** Function Notation 875
- 5.8.5** Practice 876
- 5.8.6** Lesson Summary 878

5.9 Interpreting Exponential Functions 879

- 5.9.0** Lesson Overview 879
- 5.9.1** Evaluating Expressions for Different Values of x 879
- 5.9.2** Analyze Underlying Relationship in a Graph of a Function 879
- 5.9.3** Graph Equations to Solve Problems 881
- 5.9.4** Deciding on Graphing Window 883
- 5.9.5** Interpret Graphs to Find Approximate Values 884
- 5.9.6** Practice 886
- 5.9.7** Lesson Summary 887

5.10 Looking at Rates of Change 889

- 5.10.0** Lesson Overview 889
- 5.10.1** Calculate an Average Rate of Change from Two Points 889
- 5.10.2** Explore Average Rates of Change in an Exponential Growth Function 890
- 5.10.3** Explore Average Rates of Change in an Exponential Decay Context 893
- 5.10.4** Average Rate of Change 895
- 5.10.5** Practice 896
- 5.10.6** Lesson Summary 898

5.11 Modeling Exponential Behavior 899

- 5.11.0** Lesson Overview 899
- 5.11.1** Changing the Graphing Window 899
- 5.11.2** Choosing an Appropriate Model 900
- 5.11.3** Modeling with Exponential Functions 903
- 5.11.4** Examine Exponential Decay in Context 904
- 5.11.5** Using Relationship Models to Answer Questions 908
- 5.11.6** Practice 909
- 5.11.7** Lesson Summary 910

5.12 Reasoning about Exponential Graphs, Part 1 911

- 5.12.0** Lesson Overview 911
- 5.12.1** Modeling Exponential Decay 911
- 5.12.2** Equations and Their Graphs 911
- 5.12.3** Graphs Representing Exponential Decay 913

5.12.4 Possible Equation for a Function on a Graph	916
5.12.5 Practice	918
5.12.6 Lesson Summary	921

5.13 Reasoning about Exponential Graphs, Part 2 923

5.13.0 Lesson Overview	923
5.13.1 Comparing Functions	923
5.13.2 Creating Exponential Functions from Graphs	923
5.13.3 Comparing Exponential Functions	927
5.13.4 Interpreting Features of an Exponential Graph	929
5.13.5 Practice	930
5.13.6 Lesson Summary	932

5.14 Which One Changes Faster? 933

5.14.0 Lesson Overview	933
5.14.1 Distinguish Linear and Exponential Growth	933
5.14.2 Using Tables to Compare Linear and Exponential Growth Functions	934
5.14.3 Compare Linear Functions with Exponential Functions	937
5.14.4 Using Function Notation to Compare Linear and Exponential Functions	941
5.14.5 Practice	942
5.14.6 Lesson Summary	944

5.15 Changes Over Equal Intervals 945

5.15.0 Lesson Overview	945
5.15.1 Writing Equivalent Expressions	945
5.15.2 Linear Function Constant Rate of Change	945
5.15.3 Growth Rate of an Exponential Function	947
5.15.4 Comparing Linear and Exponential Growth	950
5.15.5 Practice	952
5.15.6 Lesson Summary	954

Project 5: Introduction to Exponential Functions 955

Project 5 Overview	955
Analyze Data	955
Write Equations to Model Populations	955
Open-ended Modeling with Data Investigation	956

Unit 5 Wrap Up 958

UNIT 6 WORKING WITH POLYNOMIALS

Unit 6 Overview and Readiness 959

Unit 6 Overview	959
Simplify Expressions Using Properties of Exponents: Mini-Lesson Review	959
Simplify Expressions Using the Distributive Property: Mini-Lesson Review	962
Find the Greatest Common Factor: Mini-Lesson Review	963
Unit 6 Overview: Wrap Up	965

Unit 6 Inquiry Project: Area Model Multiplication 967

- Inquiry Project Overview 967
- Discovering an Area Model 967
- Understanding an Area Model 968
- Using an Area Model 969
- Applying an Area Model to Polynomials 970

6.1 Add and Subtract Polynomials 973

- 6.1.0** Lesson Overview 973
- 6.1.1** Understanding Polynomials 973
- 6.1.2** Adding and Subtracting Polynomials 974
- 6.1.3** Evaluating a Polynomial Function for a Given Value 977
- 6.1.4** Adding and Subtracting Polynomial Functions 979
- 6.1.5** Reviewing Polynomial Addition and Subtraction 982
- 6.1.6** Practice 983
- 6.1.7** Lesson Summary 984

6.2 Multiplying Polynomials 985

- 6.2.0** Lesson Overview 985
- 6.2.1** Multiplying Monomials 985
- 6.2.2** Multiplying Binomials 986
- 6.2.3** Multiplying a Polynomial by a Polynomial 991
- 6.2.4** Multiplying Special Products 993
- 6.2.5** Multiplying Polynomial Functions 996
- 6.2.6** Practice 997
- 6.2.7** Lesson Summary 999

6.3 Dividing Polynomials 1001

- 6.3.0** Lesson Overview 1001
- 6.3.1** Dividing by Monomials 1001
- 6.3.2** Dividing Polynomials Using Long Division 1002
- 6.3.3** Dividing Polynomials Using Synthetic Division 1007
- 6.3.4** Dividing Polynomial Functions and the Remainder Theorem 1010
- 6.3.5** Using the Factor Theorem 1013
- 6.3.6** Practice 1015
- 6.3.7** Lesson Summary 1017

6.4 Greatest Common Factor and Factor by Grouping 1019

- 6.4.0** Lesson Overview 1019
- 6.4.1** Finding Factors 1019
- 6.4.2** Finding the GCF of Two or More Expressions 1019
- 6.4.3** Factoring the GCF from Polynomials 1021
- 6.4.4** Factoring Polynomials by Grouping 1024
- 6.4.5** Can You Factor the Polynomial? 1026
- 6.4.6** Practice 1027
- 6.4.7** Lesson Summary 1028

6.5 Factor Trinomials 1029

- 6.5.0 Lesson Overview 1029**
- 6.5.1 Undoing FOIL 1029**
- 6.5.2 Factoring Trinomials with Leading Coefficients of 1 1030**
- 6.5.3 Factoring Trinomials Using Trial and Error 1035**
- 6.5.4 Factoring Trinomials Using the “ac” Method and Substitution 1039**
- 6.5.5 Choosing Your Own Method to Factor Trinomials 1042**
- 6.5.6 Practice 1043**
- 6.5.7 Lesson Summary 1045**

6.6 Factor Special Products 1047

- 6.6.0 Lesson Overview 1047**
- 6.6.1 Identifying Perfect Square Trinomials 1047**
- 6.6.2 Factoring Perfect Square Trinomials 1047**
- 6.6.3 Factoring the Difference of Squares 1052**
- 6.6.4 Sorting Polynomials by Factoring Methods 1055**
- 6.6.5 Practice 1057**
- 6.6.6 Lesson Summary 1059**

6.7 General Strategy for Factoring Polynomials 1061

- 6.7.0 Lesson Overview 1061**
- 6.7.1 Factoring Strategies for Polynomials 1061**
- 6.7.2 Reviewing General Strategies to Factor a Polynomial 1061**
- 6.7.3 Implementing General Strategies for Factoring 1064**
- 6.7.4 Is the Polynomial Completely Factored? 1067**
- 6.7.5 Practice 1068**
- 6.7.6 Lesson Summary 1070**

Project 6: Polynomials and Rectangles 1071

- Project 6 Overview 1071
- Areas of Rectangles 1071
- Dimensions and Areas of Rectangles 1072
- Create Your Own Rectangles 1072

Unit 6 Wrap Up 1074

UNIT 7 INTRODUCTION TO QUADRATIC FUNCTIONS

Unit 7 Overview and Readiness 1075

- Unit 7 Overview 1075
- Multiply Binomials: Mini-Lesson Review 1075
- Factor Trinomials: Mini-Lesson Review 1077
- Compare Linear and Exponential Functions: Mini-Lesson Review 1079
- Unit 7 Overview: Wrap Up 1083

7.1 Patterns of Change 1085

- 7.1.0 Lesson Overview 1085**
- 7.1.1 Noticing New Patterns of Change 1086**

7.1.2	The Relationship between Length and Area	1087
7.1.3	Plotting Measurements	1088
7.1.4	The Input and Output of a Function	1090
7.1.5	Practice	1091
7.1.6	Lesson Summary	1093

7.2 Introduction to Quadratic Relationships 1095

7.2.0	Lesson Overview	1095
7.2.1	Squares in a Geometric Pattern	1095
7.2.2	Constant and Exponential Change	1096
7.2.3	Quadratic Expressions	1099
7.2.4	Quadratic Relationships	1101
7.2.5	Practice	1103
7.2.6	Lesson Summary	1105

7.3 Determining if a Function is Quadratic 1107

7.3.0	Lesson Overview	1107
7.3.1	Quadratic Expressions and Area	1107
7.3.2	Writing Equations for Patterns with Squares	1108
7.3.3	Quadratic Sequences	1110
7.3.4	Is the Pattern a Quadratic Function?	1112
7.3.5	Practice	1113
7.3.6	Lesson Summary	1115

7.4 Comparing Quadratic and Exponential Functions 1117

7.4.0	Lesson Overview	1117
7.4.1	Comparing Expressions	1117
7.4.2	Exploring Linear, Exponential, and Quadratic Growth	1117
7.4.3	Comparing Exponential and Quadratic Functions	1120
7.4.4	Comparing Exponential and Quadratic Expressions	1122
7.4.5	Practice	1123
7.4.6	Lesson Summary	1125

7.5 Building Quadratic Functions to Describe Situations, Part 1 1127

7.5.0	Lesson Overview	1127
7.5.1	Numerical Patterns	1127
7.5.2	A Time-Distance Quadratic Model	1127
7.5.3	Distance as a Quadratic Function of Elapsed Time	1129
7.5.4	A Quadratic Expression that Represents the Relationship between Distance and Time	1131
7.5.5	Practice	1132
7.5.6	Lesson Summary	1134

7.6 Building Quadratic Functions to Describe Situations, Part 2 1135

7.6.0	Lesson Overview	1135
7.6.1	Using Linear Functions to Describe Constant Speed	1135
7.6.2	The Force of Gravity Change in Quadratic Functions	1136
7.6.3	Using Quadratic Functions to Describe Height	1137
7.6.4	Interpreting Graphs of Quadratic Functions	1139

7.6.5	Practice	1140
7.6.6	Lesson Summary	1142

7.7 Domain, Range, Vertex, and Zeros of Quadratic Functions 1143

7.7.0	Lesson Overview	1143
7.7.1	Comparing Graphs of Functions	1144
7.7.2	Modeling Real-World Data with Quadratic Functions	1144
7.7.3	The Domain, Vertex, and Zero of Quadratic Functions	1147
7.7.4	Relate the Domain of a Function to Its Graph	1151
7.7.5	Practice	1152
7.7.6	Lesson Summary	1155

7.8 Equivalent Quadratic Expressions 1157

7.8.0	Lesson Overview	1157
7.8.1	Area Diagrams	1157
7.8.2	Using the Distributive Property to Write Equivalent Expressions	1157
7.8.3	Using Diagrams to Find Equivalent Quadratic Expressions	1159
7.8.4	Writing Equivalent Expressions	1161
7.8.5	Practice	1163
7.8.6	Lesson Summary	1165

7.9 Standard Form and Factored Form 1167

7.9.0	Lesson Overview	1167
7.9.1	Solving Equations Using Opposite Operations	1167
7.9.2	Finding Products of Differences	1167
7.9.3	Standard and Factored Forms of Quadratic Expressions	1169
7.9.4	Expressions in Standard Form	1170
7.9.5	Practice	1171
7.9.6	Lesson Summary	1172

7.10 Graphs of Functions in Standard and Factored Forms 1173

7.10.0	Lesson Overview	1173
7.10.1	Graphs of Linear Equations	1173
7.10.2	Quadratic Forms and Their Graphs	1174
7.10.3	The x - and y -intercepts of Quadratic Expressions	1176
7.10.4	Using the Standard Form of an Equation to Find the x - and y -intercepts	1180
7.10.5	Practice	1181
7.10.6	Lesson Summary	1182

7.11 Graphing from the Factored Form 1183

7.11.0	Lesson Overview	1183
7.11.1	Finding Coordinates	1183
7.11.2	The Factored Form of a Quadratic Expression and the x -intercepts of Its Graph	1184
7.11.3	Sketching a Graph of a Quadratic Function Using at Least Three Identifiable Points	1185
7.11.4	Using the Vertex and Axis of Symmetry of Quadratics	1187
7.11.5	Graphs of Quadratic Functions	1189
7.11.6	Practice	1190
7.11.7	Lesson Summary	1192

7.12 Graphing the Standard Form, Part 1 1193

- 7.12.0** Lesson Overview 1193
- 7.12.1** Using Coefficients and Constant Terms to Identify Graphs of Linear Equations 1193
- 7.12.2** Transformations with Quadratic Functions 1194
- 7.12.3** Understanding the Behaviors of a Graph in Relation to Its Quadratic Expression 1198
- 7.12.4** Representations of Quadratic Functions 1205
- 7.12.5** Identifying Graphs of Quadratic Equations 1207
- 7.12.6** Practice 1208
- 7.12.7** Lesson Summary 1212

7.13 Graphing the Standard Form, Part 2 1213

- 7.13.0** Lesson Overview 1213
- 7.13.1** Equivalent Expressions 1213
- 7.13.2** The Linear Term in a Quadratic Expression 1214
- 7.13.3** Writing Equations that Represent a Graph 1216
- 7.13.4** Writing Quadratic Equations from Real Solutions 1218
- 7.13.5** Graphing Quadratic Equations 1219
- 7.13.6** Practice 1221
- 7.13.7** Lesson Summary 1225

7.14 Graphs That Represent Situations 1227

- 7.14.0** Lesson Overview 1227
- 7.14.1** Evaluating Quadratic Functions in the Real World 1227
- 7.14.2** Interpreting a Functional Relationship between Two Quantities 1228
- 7.14.3** Analyzing Functions Using Different Representations 1230
- 7.14.4** Using Quadratic Functions to Solve Problems 1231
- 7.14.5** Identifying Key Features of Graphed Functions 1232
- 7.14.6** Practice 1233
- 7.14.7** Lesson Summary 1235

7.15 Vertex Form 1237

- 7.15.0** Lesson Overview 1237
- 7.15.1** Analyzing Two Sets of Equations 1237
- 7.15.2** The Vertex Form 1237
- 7.15.3** The Coordinates of the Vertex 1240
- 7.15.4** The Graph of a Function in Vertex Form 1242
- 7.15.5** Practice 1243
- 7.15.6** Lesson Summary 1245

7.16 Graphing from the Vertex Form 1247

- 7.16.0** Lesson Overview 1247
- 7.16.1** Three Forms of Quadratic Expressions 1247
- 7.16.2** Graph Functions in Vertex Form 1247
- 7.16.3** Quadratic Equations and Graphs 1250
- 7.16.4** Sketching A Graph of an Equation 1254
- 7.16.5** Practice 1255
- 7.16.6** Lesson Summary 1257

7.17 Changing the Vertex 1259

- 7.17.0 Lesson Overview 1259
- 7.17.1 Quadratic Expressions in Different Forms 1259
- 7.17.2 Modify Expressions to Translate Graphs 1260
- 7.17.3 Changing the Parameters of a Quadratic Expression 1264
- 7.17.4 Transforming the Parent Function 1266
- 7.17.5 Equations of Graph Translations 1267
- 7.17.6 Practice 1268
- 7.17.7 Lesson Summary 1271

Project 7: Design a Fountain 1273

- Project 7 Overview 1273
- Introduction to Fountain Design 1273
- Design a Fountain 1273

Unit 7 Wrap Up 1276

UNIT 8 QUADRATIC EQUATIONS

Unit 8 Overview and Readiness 1277

- Unit 8 Overview 1277
- Write Equivalent Expressions: Mini-Lesson Review 1277
- Find Intercepts: Mini-Lesson Review 1278
- Match Quadratic Graphs and Their Equations: Mini-Lesson Review 1280
- Unit 8 Overview: Wrap Up 1282

8.1 Finding Unknown Inputs 1283

- 8.1.0 Lesson Overview 1283
- 8.1.1 Understanding a Situation with Quadratic Equations 1283
- 8.1.2 Modeling a Quadratic Problem 1284
- 8.1.3 Formulating a Quadratic Equation to Represent the Model 1285
- 8.1.4 Interpreting a Solution 1288
- 8.1.5 Practice 1289
- 8.1.6 Lesson Summary 1291

8.2 When and Why Do We Write Quadratic Equations? 1293

- 8.2.0 Lesson Overview 1293
- 8.2.1 Interpreting Equations in Context 1293
- 8.2.2 Exploring Non-Graphing Strategies to Solve Quadratic Equations 1294
- 8.2.3 Solving a Quadratic Equation Set Equal to Zero 1295
- 8.2.4 Solving a Real-World Problem Using Quadratic Equations 1297
- 8.2.5 Practice 1298
- 8.2.6 Lesson Summary 1300

8.3 Solving Quadratic Equations by Reasoning 1301

- 8.3.0 Lesson Overview 1301
- 8.3.1 Determining the Number of Solutions 1301
- 8.3.2 Recognizing Pairs of Solutions 1301

- 8.3.3** Solving More Complex Quadratic Equations 1304
- 8.3.4** Finding the Solutions 1306
- 8.3.5** Practice 1307
- 8.3.6** Lesson Summary 1309

8.4

Solving Quadratic Equations with the Zero Product Property 1311

- 8.4.0** Lesson Overview 1311
- 8.4.1** Introducing the Zero Product Property 1311
- 8.4.2** Solving Equations of Increasing Complexity Using Reasoning 1311
- 8.4.3** Applying the Zero Product Property to Solve a Real-World Projectile Problem 1313
- 8.4.4** Solving Equations Using the Zero Product Property and Reasoning 1315
- 8.4.5** Practice 1316
- 8.4.6** Lesson Summary 1318

8.5

How Many Solutions? 1319

- 8.5.0** Lesson Overview 1319
- 8.5.1** Reviewing Zero Product Property 1319
- 8.5.2** Interpreting Graphs to Solve Quadratic Equations 1319
- 8.5.3** Choosing an Effective Strategy to Solve Quadratic Equations 1323
- 8.5.4** Analyzing Errors When Solving Quadratic Equations 1325
- 8.5.5** Practice 1326
- 8.5.6** Lesson Summary 1328

8.6

Rewriting Quadratic Expressions in Factored Form, Part 1 1329

- 8.6.0** Lesson Overview 1329
- 8.6.1** Finding and Reasoning Unknown Factors 1329
- 8.6.2** Using Diagrams to Understand Equivalent Expressions 1330
- 8.6.3** Rewriting Quadratic Expressions in Standard Form 1334
- 8.6.4** Finding the Missing Numbers in Pairs of Equivalent Expressions 1336
- 8.6.5** Practice 1337
- 8.6.6** Lesson Summary 1339

8.7

Rewriting Quadratic Expressions in Factored Form, Part 2 1341

- 8.7.0** Lesson Overview 1341
- 8.7.1** Understanding Sums and Products of Integers 1341
- 8.7.2** Interpreting Negative Constant Terms When Factoring Quadratic Expressions 1341
- 8.7.3** Analyzing Factors of 100 and -100 1344
- 8.7.4** Finding the Missing Terms 1348
- 8.7.5** Practice 1349
- 8.7.6** Lesson Summary 1351

8.8

Rewriting Quadratic Expressions in Factored Form, Part 3 1353

- 8.8.0** Lesson Overview 1353
- 8.8.1** Evaluating Expressions Using Mental Math 1353
- 8.8.2** Recognizing the Expanded Product of the Difference of Two Squares 1353
- 8.8.3** Factoring Quadratic Equations without a Linear Term 1356
- 8.8.4** Determining if an Expression Can Be Rewritten in Factored Form 1358

- 8.8.5** Practice 1359
- 8.8.6** Lesson Summary 1361

8.9 Solving Quadratic Equations by Using Factored Form 1363

- 8.9.0** Lesson Overview 1363
- 8.9.1** Finding a Solution through Substitution 1363
- 8.9.2** Using Factored Form and the Zero Product Property to Solve Quadratic Equations 1363
- 8.9.3** Writing an Equation to Represent a Quadratic Function with Only One Solution 1365
- 8.9.4** Solving More Quadratic Equations 1367
- 8.9.5** Practice 1368
- 8.9.6** Lesson Summary 1370

8.10 Rewriting Quadratic Expressions in Factored Form, Part 4 1371

- 8.10.0** Lesson Overview 1371
- 8.10.1** Analyzing Various Quadratic Expressions 1371
- 8.10.2** Working with Quadratic Factored Form with Leading Coefficient Other Than One 1371
- 8.10.3** Using Technology to Find Rational Factors 1375
- 8.10.4** Finding the Factors of Quadratic Expressions in Standard Form 1377
- 8.10.5** Solving Quadratic Equations by Any Method 1380
- 8.10.6** Practice 1381
- 8.10.7** Lesson Summary 1383

8.11 Writing Quadratic Equations Given Real Solutions 1385

- 8.11.0** Lesson Overview 1385
- 8.11.1** Relating a Quadratic Function and Its Zeros 1385
- 8.11.2** Finding a Quadratic Function from Its Zeros 1385
- 8.11.3** Finding a Quadratic Function from Its Zeros and a Point 1388
- 8.11.4** Writing Quadratics When Given Any Format 1392
- 8.11.5** Practice 1393
- 8.11.6** Lesson Summary 1395

8.12 Using Technology to Find the Quadratic Regression 1397

- 8.12.0** Lesson Overview 1397
- 8.12.1** Graphing a Quadratic Data Set Using Technology 1397
- 8.12.2** Finding the Curve of Best Fit 1398
- 8.12.3** Making Predictions Using a Quadratic Model 1401
- 8.12.4** Finding the Missing Data in the Set 1403
- 8.12.5** Practice 1405
- 8.12.6** Lesson Summary 1408

Project 8: Modeling Rocket Flight 1409

- Project 8 Overview 1409
- Relate Zeros to the Factored Form of Quadratic Equations 1409
- Convert between Factored Form and Standard Form 1410
- Model a Rocket Flight Using a Quadratic Equation 1410

Unit 8 Wrap Up 1412



UNIT 9 MORE QUADRATIC EQUATIONS

Unit 9 Overview and Readiness 1413

- Unit 9 Overview 1413
- Simplifying Radicals: Mini-Lesson Review 1413
- Factoring Perfect Square Trinomials: Mini-Lesson Review 1415
- Transformations: Mini-Lesson Review 1417
- Unit 9 Overview: Wrap Up 1418

9.1

What Are Perfect Squares? 1419

- 9.1.0 Lesson Overview 1419
- 9.1.1 Equations with Quadratic Expressions on Both Sides of the Equal Sign 1419
- 9.1.2 Recognizing Structure in Perfect-Square Expressions 1419
- 9.1.3 Solving Multi Step Quadratic Equations Using Perfect Squares 1421
- 9.1.4 Explain Why an Expression Is a Perfect Square 1423
- 9.1.5 Practice 1424
- 9.1.6 Lesson Summary 1426

9.2

Completing the Square, Part 1 1427

- 9.2.0 Lesson Overview 1427
- 9.2.1 Different Forms of Perfect Squares 1427
- 9.2.2 Standard and Factored Forms of Perfect Squares 1427
- 9.2.3 Completing the Square 1429
- 9.2.4 Making a Perfect Square 1432
- 9.2.5 Practice 1433
- 9.2.6 Lesson Summary 1436

9.3

Completing the Square, Part 2 1437

- 9.3.0 Lesson Overview 1437
- 9.3.1 Solving Equations with Fractions 1437
- 9.3.2 Using Completing the Square to Solve Equations 1437
- 9.3.3 Solve by Completing the Square 1440
- 9.3.4 Completing the Square with Fractions 1442
- 9.3.5 Practice 1443
- 9.3.6 Lesson Summary 1446

9.4

Completing the Square, Part 3 1447

- 9.4.0 Lesson Overview 1447
- 9.4.1 Perfect Squares with Coefficients Other than 1 1447
- 9.4.2 Rewrite Squared Expressions 1447
- 9.4.3 Standard Form and Squared Factors 1449
- 9.4.4 Three Different Methods for Solving an Equation 1451
- 9.4.5 Perfect Square and Squared Factors 1452
- 9.4.6 Practice 1453
- 9.4.7 Lesson Summary 1455

9.5

Quadratic Equations with Irrational Solutions 1457

- 9.5.0 Lesson Overview 1457
- 9.5.1 Roots of Squares 1457

9.5.2	Solutions Written as Square Roots	1457
9.5.3	Finding Irrational Solutions by Completing the Square	1459
9.5.4	Finding Exact Solutions	1461
9.5.5	Practice	1462
9.5.6	Lesson Summary	1466

9.6 The Quadratic Formula 1467

9.6.0	Lesson Overview	1467
9.6.1	Evaluating Expressions	1467
9.6.2	Choosing the Best Method to Solve an Equation	1467
9.6.3	The Quadratic Formula	1469
9.6.4	Solving Equations Using the Quadratic Formula	1471
9.6.5	Practice	1472
9.6.6	Lesson Summary	1474

9.7 Applying the Quadratic Formula 1475

9.7.0	Lesson Overview	1475
9.7.1	Evaluating Variable Expressions	1475
9.7.2	Common Calculation Errors When Using the Quadratic Formula	1475
9.7.3	Different Methods of Checking Solutions of Quadratic Equations	1478
9.7.4	Practice Spotting Calculation Errors	1480
9.7.5	Practice	1481
9.7.6	Lesson Summary	1483

9.8 Deriving the Quadratic Formula 1485

9.8.0	Lesson Overview	1485
9.8.1	Deriving the Quadratic Formula, Part 1	1486
9.8.2	Deriving the Quadratic Formula, Part 2	1488
9.8.3	Understanding That the Quadratic Formula Is the Combined Steps of Completing the Square	1490
9.8.4	Practice	1492
9.8.5	Lesson Summary	1494

9.9 Writing Quadratics in Different Forms 1495

9.9.0	Lesson Overview	1495
9.9.1	Finding the Vertex in Vertex Form	1495
9.9.2	Different Forms of Quadratics	1495
9.9.3	Vertex Form Given a Vertex and Point	1497
9.9.4	Rewriting Quadratics	1498
9.9.5	Practice	1500
9.9.6	Lesson Summary	1501

9.10 Rewriting Quadratic Expressions in Vertex Form 1503

9.10.0	Lesson Overview	1503
9.10.1	The Vertex and Intercepts of a Function	1503
9.10.2	Expanding from Factored Form to Standard Form	1504
9.10.3	Converting from Standard Form to Vertex Form	1505
9.10.4	Rewriting Expressions in Vertex Form	1506

- 9.10.5** Different Forms of Quadratic Expressions 1507
- 9.10.6** Vertex Form and Coordinates of the Vertex 1508
- 9.10.7** Practice 1509
- 9.10.8** Lesson Summary 1512

9.11 Using Quadratic Expressions in Vertex Form to Solve Problems 1513

- 9.11.0** Lesson Overview 1513
- 9.11.1** Maximum and Minimum Value of a Function 1513
- 9.11.2** Does the Vertex Represent the Minimum or Maximum Value? 1514
- 9.11.3** Comparing Maximums between Quadratics 1517
- 9.11.4** Maximum, Minimum, and Analyzing the Vertex 1519
- 9.11.5** Practice 1520
- 9.11.6** Lesson Summary 1522

Project 9: Using Quadratic Equations to Model Situations and Solve Problems 1523

- Project 9 Overview 1523
- Equations of Two Lines and a Curve 1523
- Analyze a Dive 1523
- A Linear Function and a Quadratic Function 1523
- Profit from a River Cruise 1524

Unit 9 Wrap Up 1526

Index 1527

About this Course



Rice University
6100 Main Street
Houston, TX 77005

www.openstax.org/details/books/algebra-1

Digital PDF:

ISBN: 978-1-961584-47-1

Publish date:

June 4, 2025



About OpenStax Algebra 1

Algebra 1 by OpenStax is licensed under Creative Commons Attribution-NonCommercial-ShareAlike License v4.0. *Algebra 1* was authored by OpenStax and is based, in part, on OpenStax *Prealgebra*, *Elementary Algebra*, *Intermediate Algebra*, and *College Algebra* materials, and the IM 6–12 Math™ by Illustrative Mathematics®. Based on OpenStax's open (CC-BY-NC-SA) license, educators may remix and reuse the elements and content in *Algebra 1* according to their approach.

OpenStax *Algebra 1* is a complete curriculum and an open education resource (OER) that is standards-aligned, easy to implement, supportive of diverse learners, and engaging for students. *Algebra 1* is supported by instructional resources, including implementation guidance, pacing suggestions, and ancillary engagement materials.

Senior Contributing Authors:

Lynn Marecek, Santa Ana College

MaryAnne Anthony-Smith, Formerly of Santa Ana College

Andrea Honeycutt Mathis, Northeast Mississippi Community College

Jay Abramson, Arizona State University

Sharon North, St. Louis Community College

Amy Baldwin, University of Central Arkansas

Alyssa Howell, OpenStax, Rice University

Errata

All OpenStax textbooks undergo a rigorous review process. However, like any professional-grade textbook, errors sometimes occur. Since our books are web based, we can make updates periodically when deemed pedagogically necessary. If you have a correction to suggest, submit it through the link on your book page on openstax.org. Subject matter experts review all errata suggestions. OpenStax is committed to remaining transparent about all updates, so you will also find a list of past errata changes on your book page on openstax.org.

STUDENTS START HERE

Quick Start

Welcome to your *Algebra 1* module! We're genuinely excited for you to join our community and embark on a transformative learning journey!

This user-friendly guide is your roadmap to navigating the features and resources of our unique *Algebra 1* content. Let's raise the bar on your algebra skills—get started with confidence today!

I. Where to Start

Algebra 1 has nine units, each of which begins with a special lesson called Overview and Readiness. These lessons provide a miniature review of the knowledge you need for the unit ahead. Completing these lessons will help you feel confident in your ability to handle what comes next.

Units 4 and 6 also have inquiry projects, meant to revisit previous knowledge while introducing you to upcoming topics. We recommend completing these alongside their respective Overview and Readiness lessons.

II. Unit Structure

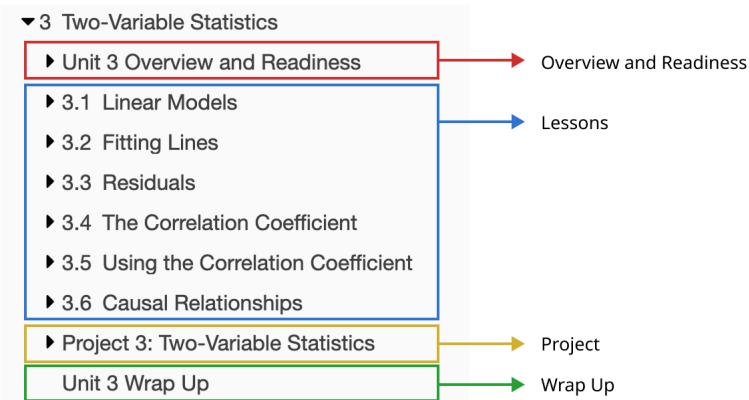


Image 1

After completing the **Overview and Readiness** lesson, you can progress through the regular **Lessons**.

Each Lesson has multiple pages to guide you through learning new skills and reinforcing old ones.

At the end of each unit, you will also find a **Project** and **Wrap Up** lesson covering the topics in the unit. Use these lessons to cement understanding of the skills you just learned.

III. Lesson Structure

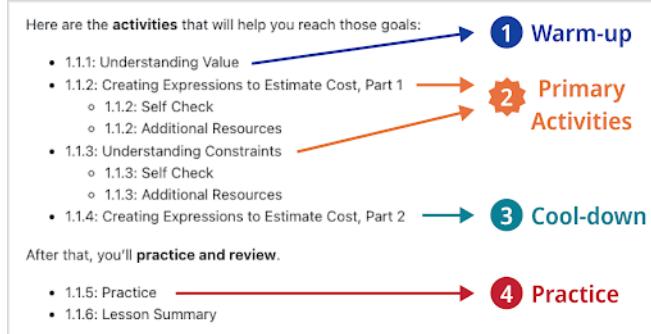


Image 2

Lesson Activities

The first page of a lesson is called "Lesson Overview." Here you will find the different types of activities included in the

lesson (Image 2).

- **Warm up:** This is the first activity in a lesson.
- **Primary activities:** These are unique and need extra focus. We'll take a close look at them in the next part of this guide.
- **Cool-down:** This is the last activity in a lesson.
- **Practice:** This is a practice page with problems on the topics you learned in the lesson.

IV. Primary Activities

Activity

Jada has time on the weekends to earn some money. A local bookstore is looking for someone to help sort books and will pay \$12.20 an hour. To get to and from the bookstore on a work day, however, Jada would have to spend \$7.15 on bus fare.

1. Write an equation that represents Jada's take-home earnings in dollars, E , if she works at the bookstore for h hours in one day.
[\[Show/Hide Solution\]](#)
2. One day, Jada takes home \$90.45 after working h hours and after paying the bus fare. Write an equation to represent this situation.
[\[Show/Hide Solution\]](#)
3. Is it a solution to the last equation you wrote?
 - If so, be prepared to explain how you know it is a solution.
 - If not, be prepared to explain why it is not a solution. Then, find the solution.[\[Show/Hide Solution\]](#)
4. Is it a solution to the last equation you wrote?
 - If so, be prepared to explain how you know it is a solution.
 - If not, be prepared to explain why it is not a solution. Then, find the solution.[\[Show/Hide Solution\]](#)
5. In this situation, what does the solution to the equation tell us?
[\[Show/Hide Solution\]](#)

Are you ready for more?

Extending Your Thinking

Jada has a second option to earn money. She could help some neighbors with errands and computer work for \$11 an hour. After re-considering her schedule, Jada realizes that she has about 9 hours available to work one day on the weekend.

What option should she choose, sorting books at the bookstore or helping her neighbors? Be prepared to show your reasoning.

[\[Show/Hide Solution\]](#)

SELF CHECK

Jada found a third job option and was offered a position working at a camp for \$10.50 per hour. The total bus fare to and from the camp is \$8.40. Write an equation that represents how much she will make, d , after working h hours.

a. $d = 8.40h + 10.50$
 b. $d = \frac{10.50}{8.40}h$
 c. $d = 10.50h + 8.40$
 d. $d = 10.50h - 8.40$

Additional Resources

Verify a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that make each side of the equation the same so that we end up with a true statement. Any value of the variable that makes the equation true is called a **solution of an equation**. It is the answer to the puzzle!

How to determine whether a number is a solution to an equation:

- Step 1 - Substitute the number in for the variable in the equation.
- Step 2 - Simplify the expressions on both sides of the equation.
- Step 3 - Determine whether the resulting equation is true (the left side is equal to the right side).

If it is true, the number is a solution.

If it is not true, the number is not a solution.

For example, the equation $2l + 2w = 72$ represents the relationship between the length, l , and the width, w , of a rectangle that has a perimeter of 72 units. If we know that the length is 15 units, what is the value of the width?

Step 1 - Substitute the number in for the variable in the equation.

$$2(15) + 2w = 72$$

This is an equation in one variable because w is the only quantity that we don't know. To solve this equation means to find a value of w that makes the equation true.

Step 2 - Simplify the expressions on both sides of the equation. In this case, 21 is the solution because substituting 21 for w in the equation results in a true statement.

$$2(15) + 2w = 72$$

$$2(15) + 2(21) = 72$$

$$30 + 42 = 72$$

Step 3 - Determine whether the resulting equation is true (the left side is equal to the right side).

If it is true, the number is a solution.

If it is not true, the number is not a solution.

$$72 = 72$$

TRY IT

Try It: Verify a Solution of an Equation

The equation $2l + 2w = 48$ represents the perimeter of a rectangle.

1. If the width of the rectangle is 10, what is the new equation?
[\[Show/Hide Solution\]](#)
2. What is the length of the rectangle that has a width of 10?
[\[Show/Hide Solution\]](#)
3. Verify the solution.
[\[Show/Hide Solution\]](#)

Image 3

Image 4

Primary Activities

Here's an example of one of these unique types of activities. Each lesson has 2-3 of these.

Some activities have a section called **Are you ready for more?** This section, meant to expand your thinking, is optional unless otherwise stated by your teacher (Image 3).

After you have finished the activity, you will do a multiple choice question called a **Self Check** (Image 3). If you feel confident in your skills, you may move on to the next lesson.

Additional Resources

If you want a refresher on the skills tested in the Self Check, read through the **Additional Resources** section, then complete the **Try It** section to test yourself on what you've learned. This section is optional unless otherwise stated by your teacher (Image 4).

Access for free at openstax.org

V. End of Lesson

In this lesson, you learned how to:

- Explain what it means for a value or pair of values to be a solution to an equation.
- Find solutions to equations by reasoning about a situation or by using algebra.

Here are the **activities** that helped you reach those goals:

- 1.4.1: Finding the Solution to an Equation in One Variable, Part 1
 - In this activity, you began to determine if a value was a solution to a given one-variable equation and identified what the solution could mean. For a value to be a solution to an equation, it must make the equation true.
- 1.4.2: Writing Equations to Represent Constraints
 - In this activity, you wrote equations from situations given the constraints of the problem.
 - 1.4.2: Self Check
 - 1.4.2: Additional Resources
- 1.4.3: Finding the Solution to an Equation in Two Variables
 - In this activity, you were able to find the solution to an equation with two variables, given the solution to one of the variables.
 - 1.4.3: Self Check
 - 1.4.3: Additional Resources
- 1.4.4: Finding the Solution to an Equation in One Variable, Part 2
 - In this activity, you worked to find multiple solutions to equations and identified what the solutions mean in this situation.

After these activities, you completed the following **practice**:

- 1.4.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

[\[Show/Hide Solution\]](#)

Image 5

Lesson Summary

Here you will review the topics covered and the activities you completed (Image 5). You will also have an opportunity to check-in on your progress with the learning goals from the lesson.

After you've finished the Unit, you can use the **Wrap Up** lesson to check in on the important skills from each lesson. If there are any skills you want to brush up on, use the Lesson Summary page to find the most relevant activities to revisit.

Useful Resources

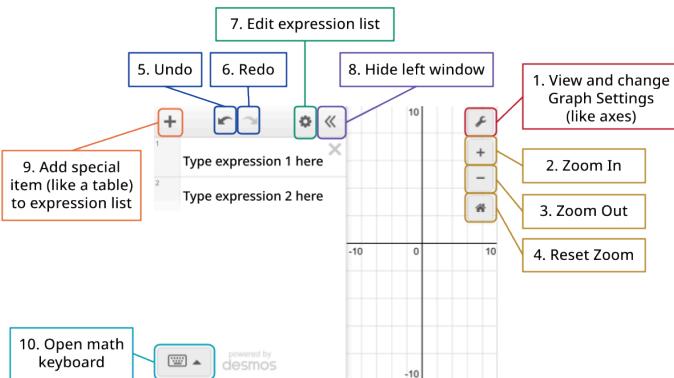
Want some study tips? Forgot the meaning of a word? Check out the sections **Students Start Here**, **Supporting All Learners**, and the **Index** for our curated resources.

Using Desmos

Throughout the *Algebra 1* curriculum, you will find Desmos activities and graphing calculators embedded in lessons. Desmos activities are interactive online problem sets located on the Desmos website. They can be assigned to your students and graded automatically if you have a Desmos account. Desmos graphing calculators are interactive online graphs embedded under graphing problems in the *Algebra 1* course. Note, the embedded calculator adheres to instructional requirements, not assessment requirements.

We have created a how-to guide with helpful screenshots and links to help you and your students use Desmos. If you need additional help, [view the official Desmos guide \(<https://openstax.org/r/desmos-graphing>\)](https://openstax.org/r/desmos-graphing).

I. Understanding Desmos Tools



1. View and change graph settings

- Change limits of the x -andy-axis and the scale for each
 - Toggle display color and font size
 - Toggle between radians and degrees
 - Turn on Braille mode
2. **Zoom in:** focus on one part of the graph and see less of the rest of it
3. **Zoom out:** see less of one part of the graph and more of the rest of it
4. **Reset zoom:** return to default window where $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.
5. **Undo:** undo your last action
6. **Redo:** redo your last action
7. **Edit expression list:**

- Quickly copy or delete expressions in the expression list
- Convert expressions to tables
- Select the colored dot to the left of the expression to change the line's color, thickness, opacity, and line type
- Quickly clear graph

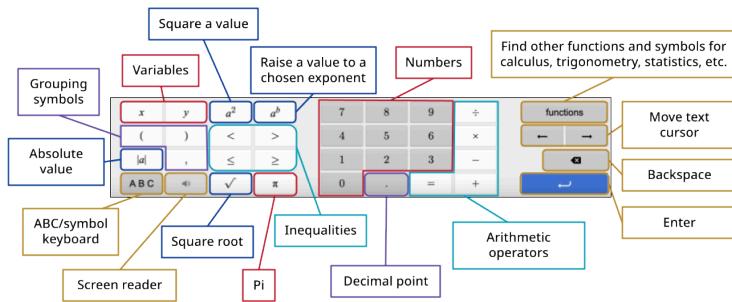
8. **Hide left window:** collapse or expand the window containing the expression list

9. **Add special item to expression list:** add an

- expression, an equation of a line;
- note, a text description or note about the graph (a good scaffold to support differentiation);
- table, a basic table where you can input x -andy-coordinates;
- image, a resizable image from your computer

10. **Open math keyboard:** open a digital keypad that allows you to quickly add special math functions and symbols, like \sin or \div , to your expression. View our guide to the Desmos math keyboard below.

- Tip: You can use your physical, non-Desmos keyboard to type math and interact with the graph. To learn more check out these guides [Math Notation Guide by PurpleMath](https://openstax.org/r/PurpleMath) (<https://openstax.org/r/PurpleMath>) and [Desmos Keyboard Shortcuts](https://help.desmos.com/hc/en-us/articles/4405966811021-Keyboard-Shortcuts) (<https://help.desmos.com/hc/en-us/articles/4405966811021-Keyboard-Shortcuts>)



II. Graphing Regressions Using the Desmos Graphing Calculator

You can use Desmos to graph a regression, regression line, regression curve, scatter plot, stat plot, and create a line of best fit. Check out our guide to using these features in Desmos, if you have additional questions, [view the official Desmos regressions guide](https://openstax.org/r/desmos-regressions) (<https://openstax.org/r/desmos-regressions>).

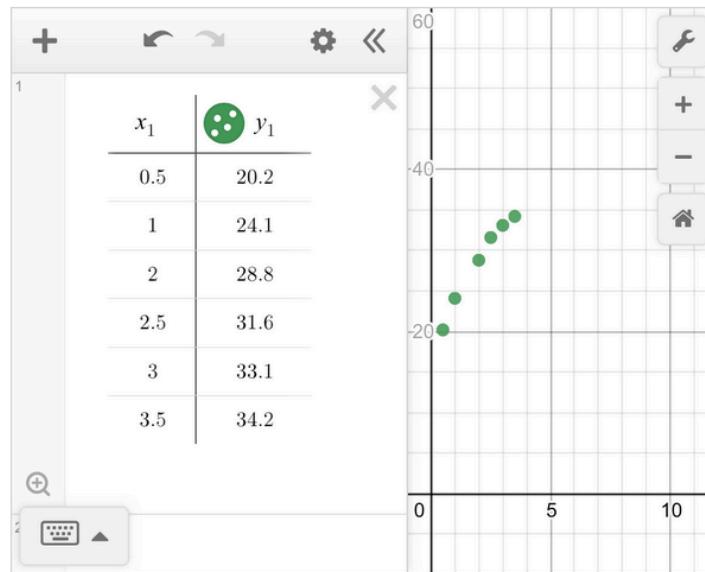


Image 1

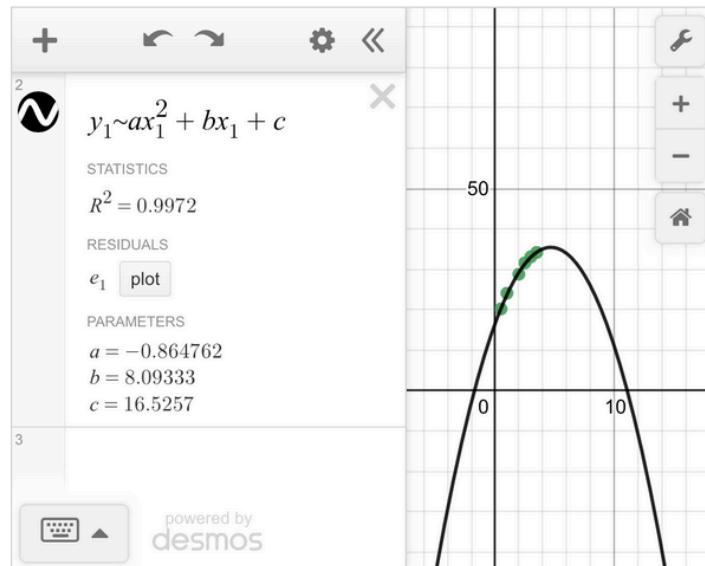


Image 2

Creating a Scatter Plot from a Table (Image 1)

1. On the Desmos graphing calculator, click the **+** button in the top left corner and add a table.
2. You can copy and paste an existing table in, or enter it manually. On the left side of the table, type the first x -coordinate from your reference table. On the right side, type the first y -coordinate. This will create a point on your graph.
3. Go to the next line of the table and repeat until all coordinates have been entered. Inputting all the coordinates will create a graph called a scatter plot, pictured above and left.
 - Tip: If you do not see your point graphed in the Desmos window, you may need to “Zoom” or change the graph settings for the window.

Creating a Line of Best Fit on a Scatter Plot (Image 2)

1. To create a line or curve that approximates the data trend, calculate a regression. Depending on the type of data (linear, exponential, or quadratic), the format of the regression equation will differ:
 - Linear: $y_1 \sim mx_1 + b$
 - Exponential: $y_1 \sim ax_1^b$
 - Quadratic: $y_1 \sim ax_1^2 + bx_1 + c$

2. Click the pointer below the table into entry box #2 to enter a new function. The cursor should start blinking. Enter the correct equation you found in step 4 using the following tips:
 - Notice that when you type the "1" after y or x , Desmos automatically places it as subscript.
 - Also notice that we are not using an equal sign. The symbol \sim means approximately and tells Desmos to approximate a line or curve of best fit. On your keyboard, you can type it with Shift+` (in the top left corner). If you are using the Desmos on-screen keyboard, it will be in the ABC/symbol keyboard.
 - To type an exponent, use the \wedge symbol on your physical keyboard. You can type it using Shift+6. You can also use the Desmos keyboard button a^2 or a^b .
3. Once you have entered the regression equation, you can press Enter. The graph of the line or curve of best fit will appear. Pictured above and right is a quadratic curve of best fit. Desmos will also report correlation statistics such as r and R^2 , as well as the a value, b value, and c value in the expression list below the quadratic equation you entered.
 - Tip: The window should be appropriately set to display the points in the table, but sometimes you have to adjust the window of the graph in order to properly view your data set. Click and drag the mouse to shift the viewing window. To zoom in or out, use the buttons in the top right or the scrolling wheel on the mouse. Or, edit the graph settings.
 - Tip: If you are having difficulty finding the correlation coefficient, r , type the command ' $corr(x_1, y_1)$ ' on another expression line. It may also be helpful to check the accuracy of the data entered into the x_1 and y_1 columns of the table.

Additional Resources

You may find it helpful to keep the following resources on hand as you progress through the course:

- [English-Spanish Glossary \(<https://openstax.org/r/english-spanish-glossary>\)](https://openstax.org/r/english-spanish-glossary)
- [Algebra 1 STAAR Reference Materials \(<https://openstax.org/r/STAAR-ref>\)](https://openstax.org/r/STAAR-ref)
- [Algebra 1 STAAR Graph Paper \(<https://openstax.org/r/STAAR-graph-paper>\)](https://openstax.org/r/STAAR-graph-paper)

UNIT 1 OVERVIEW AND READINESS

Unit 1 Overview

Why Should I Care?

Do you know what a budget is? Have you ever used one? Watch the video to learn a bit about how City Manager Arthur Noriega uses linear equations to manage the budget for the city of Miami.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-unit-1-overview>\)](http://openstax.org/books/algebra-1/pages/1-unit-1-overview)

In this unit, you will learn about using variables, expressions, and equations to model relationships. You will learn that you can represent a constraint, or limit on what is possible in a situation, using expressions, equations, and inequalities. This will be helpful whether you want to know how soon you'll be able to save for a new bicycle, or if you can afford to go to a concert with your friends in two weeks.

You will also learn to transform one equation into an equivalent equation. Sometimes you want to write an equation in a different form so you can solve for a certain variable or so you can write the equation in a form that makes it easier to identify information about the relationship. You will learn how the form and parts of linear equations are related to the features of its graph. A graph can be helpful to model a relationship and to make sense of the constraints.

Building Character: Social Intelligence

As a city manager, Arthur Noriega must work with others to develop budgets. Similarly, in this course, you will need to work with your classmates to solve problems. Working well with others is a skill that will be useful to you in this course, as well as outside of the classroom. Most jobs require the ability to work well, or collaborate, with other people.

Throughout this unit, you will work on developing your **social intelligence**. Social intelligence is the ability to connect with other people. Having this ability allows you to interact in a more positive manner with others. Being empathetic, or sensitive to what others are feeling, shows that you care about others. In fact, empathetic people are less likely to experience anxiety, depression, and addictions later in life. They are also more likely to be hired, promoted, earn more money, and have happier marriages and better-adjusted children.

Think about your current sense of social intelligence. How many of the following statements are true for you?

- I have a lot of relationships that are mutually beneficial, enjoyable, and supportive.
- Most of the time, I can tell how other people feel and have a good idea about how to respond appropriately.
- My relationships make me feel good about myself.
- The people in my life help me be my best.

Don't worry if none of these statements are true for you. Developing this trait takes time. Your first step starts today!

Am I Ready to Learn This?

Getting Ready for Unit 1

To be ready for this unit, you need to be able to:

- Solve linear equations using a general strategy.
- Classify equations.
- Identify points on a graph.

Next, you will answer some questions that will help you see how ready you are for this unit. If you need help, some mini-lessons will help you brush up on anything you need to work on.

Solve Linear Equations: Mini-Lesson Review

MINI LESSON QUESTION

Question #1: Solve Linear Equations

Solve: $2(m - 6) + 3 = m - 1$.

- a. $m = -10$
- b. $m = 14$

- c. $m = 8$
- d. $m = 2$

Solving Linear Equations Using a General Strategy

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation. For example, in $x + 2 = 3$, the value of the variable x is 1.

To find the solution to an equation in one variable, the goal is to isolate the variable on one side of the equation. For example,

$$3x + 1 = 9 - x$$

$$+x + x$$

$$4x + 1 = 9$$

$$-1 - 1$$

$$4x = 8$$

You can check the solution by substituting the value into the equation.

$$3(2) + 1 = 9 - 2$$

$$6 + 1 = 7$$

$$7 = 7$$

Equations may take several steps to solve, so it is helpful to have a clear and organized strategy. The following table shows a general strategy to solve any **linear equation** in one variable. You may not need to use every step.

General Strategy for Solving Linear Equations

Step 1

Simplify each side of the equation as much as possible.

Use the **Distributive Property** to remove any parentheses.

Combine like **terms**.

Step 2

Collect all the variable terms on one side of the equation.

Use the **Addition or Subtraction Property of Equality**.

Step 3

Collect all the constant terms on the other side of the equation.

Use the Addition or Subtraction Property of Equality.

Step 4

Make the **coefficient** of the variable term to equal to 1.

State the solution to the equation.

Step 5

Check the solution.

Substitute the solution into the original equation to make sure the result is a true statement.



Solving a Linear Equation Using a General Strategy

Solve the following using a general strategy:

Solve $2(3x - 8) = 19 - x$.

Check Your Understanding

Solve: $7(n - 3) - 8 = -15$.

- a. $n = -\frac{4}{7}$
 - b. $n = -2$
 - c. $n = 2$
 - d. $n = -3\frac{5}{7}$
-

Videos: Solving Linear Equations

Khan Academy: Equations with Parentheses

Watch this video to see how to use the Distributive Property to solve an equation with parentheses.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-solve-linear-equations-mini-lesson-review>\)](http://openstax.org/books/algebra-1/pages/1-solve-linear-equations-mini-lesson-review)

Khan Academy: Equations with Variables on Both Sides: Fractions

Watch this video to see how to solve an equation with fractional coefficients and variables on both sides.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-solve-linear-equations-mini-lesson-review>\)](http://openstax.org/books/algebra-1/pages/1-solve-linear-equations-mini-lesson-review)

Classify Equations: Mini-Lesson Review



MINI LESSON QUESTION

Question #2: Classify Equations

Which of the following correctly classifies the equation and the solution?

- $6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$
- a. contradiction; no solution
 - b. conditional equation; $1n = -\frac{1}{4}$
 - c. conditional equation; $n = 0$
 - d. identity; all real numbers

Use Solutions to Classify Equations

An equation that is true for one or more values of the variable and false for all other values of the variable is a **conditional equation**.

Consider the equation $7x + 8 = -13$.

Subtract 8 to get the constants on one side.	$7x = -21$
Divide by 7 to make the coefficient of the variable 1.	$x = -3$

The solution is $x = -3$. This means the equation $7x + 8 = -13$ is true when we replace the variable, x , with the value -3 , but it is not true when we replace x with any other value. Whether the equation $7x + 8 = -13$ is true depends on the value of the variable. The equation is a conditional equation.

An equation that is true for any value of the variable is called an **identity**. The solution of an identity is all real numbers.

Consider the equation $2y + 6 = 2(y + 3)$. Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for y .

Distribute.	$2y + 6 = 2y + 6$
Subtract $2y$ to get the variables on one side.	$6 = 6$

But $6 = 6$ is true. This means the equation $2y + 6 = 2(y + 3)$ is true for any value of y . The equation is an identity, and we say the solution is all real numbers.

An equation that is false for all values of the variable is called a **contradiction**. A contradiction has no solution.

Consider the equation $5z = 5z - 1$.

Subtract $5z$ to get the variables on one side.	$0 \neq -1$
---	-------------

The table summarizes the types of equations and solutions.

Type of Equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	True for any value of the variable	All real numbers
Contradiction	False for all values of the variable	No solution >



TRY IT

Use Solutions to Classify Equations

Classify the following: $5m + 3(9 + 3m) = 2(7m - 11)$.

Check Your Understanding

Which of the following correctly classifies the equation and the solution? $4 + 9(3x - 7) = -42x - 13 + 23(3x - 2)$

- identity; all real numbers
- conditional equation; $x = 0$
- conditional equation; $x = 2\frac{5}{27}$
- contradiction; no solution

Video: Solving Linear Equations

Khan Academy: Number of Solutions to Linear Equations

Watch the video to see how a linear equation may have one solution, no solution, or infinite solutions.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-classify-equations-mini-lesson-review>\)](http://openstax.org/books/algebra-1/pages/1-classify-equations-mini-lesson-review)

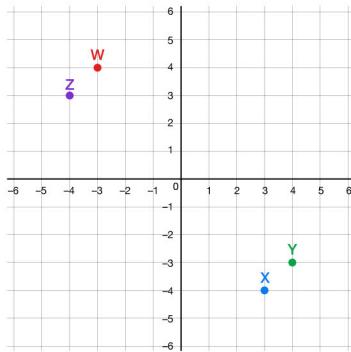
Find Coordinates: Mini-Lesson Review



MINI LESSON QUESTION

Question #3: Find Coordinates

Which point has coordinates $(3, -4)$?

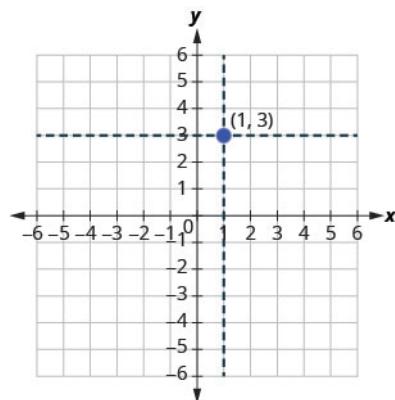


- Point Z
- Point Y
- Point W
- Point X

Identifying Coordinates of a Point on a Graph

In the rectangular coordinate system, every point is represented by an **ordered pair**. The first number in the ordered pair is the **x -coordinate** of the point, and the second number is the **y -coordinate** of the point. An ordered pair, (x, y) , gives the coordinates of a point in a rectangular coordinate system. The first number is the x -coordinate. The second number is the y -coordinate. The phrase "ordered pair" means the order is important. What is the ordered pair of the point where the axes cross? At that point, both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the origin.

We use the coordinates to locate a point on the xy -plane. Let's plot the point $(1, 3)$ as an example. First, locate 1 on the x -axis and lightly sketch a vertical line through $x = 1$. Then, locate 3 on the y -axis and sketch a horizontal line through $y = 3$. Now, find the point where these two lines meet—that is the point with coordinates $(1, 3)$.



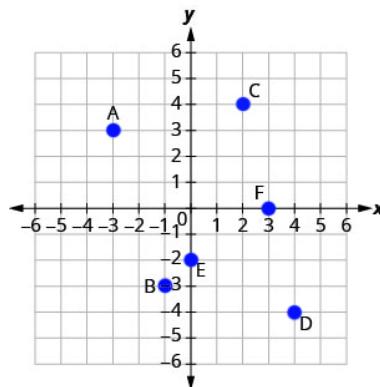
Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the x -coordinate of a point on a graph, read the number on the x -axis directly above or below the point. To identify the y -coordinate of a point, read the number on the y -axis directly to the left or right of the point. Remember, when you write the ordered pair, use the correct order, (x, y) .

> TRY IT

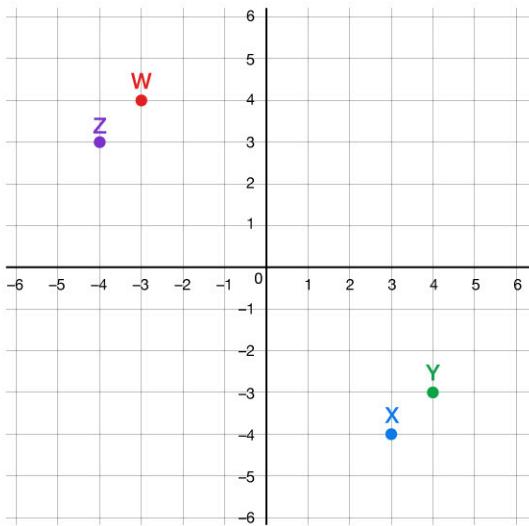
Identifying Coordinates of a Point on a Graph

Name the ordered pair of each point shown in the rectangular coordinate system.



Check Your Understanding

Which point has coordinates $(-4, 3)$?



- a. Point W
 - b. Point Z
 - c. Point X
 - d. Point Y
-

Video: Points on the Coordinate Plane

Khan Academy: Points on the Coordinate Plane

Watch this video to see how you can use a coordinate plane to plot points and to identify the coordinates of a point plotted on a coordinate plane.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-find-coordinates-mini-lesson-review>\)](http://openstax.org/books/algebra-1/pages/1-find-coordinates-mini-lesson-review)

Unit 1 Overview: Wrap Up

- Solve linear equations using a general strategy.
- Classify equations.
- Identify points on a graph.

If you are still struggling, be sure to reach out to your teacher for additional help. Nobody is good at problems like this on the first try. Keep at it, and it will get easier.

1.1 EXPLORING EXPRESSIONS AND EQUATIONS

1.1.0 Lesson Overview

In this lesson, you will experiment with expressions and equations to model a situation: planning a pizza party. For instance, what are the relevant quantities when planning for a pizza party? Are these quantities fixed or do they vary? How they might relate to one another? Are there any limitations or constraints on how much pizza should be ordered?

When you finish this lesson, you will be able to:

- Tell which quantities in a situation can vary and which ones cannot.
- Explain the meaning of the term “constraint.”
- Use letters and numbers to write expressions representing the quantities in a situation.

Here are the **activities** that will help you reach those goals:

- 1.1.1: Understanding Value
- 1.1.2: Creating Expressions to Estimate Cost, Part 1
 - 1.1.2: Self Check
 - 1.1.2: Additional Resources
- 1.1.3: Understanding Constraints
 - 1.1.3: Self Check
 - 1.1.3: Additional Resources
- 1.1.4: Creating Expressions to Estimate Cost, Part 2

After that, you'll **practice and review**.

- 1.1.5: Practice
- 1.1.6: Lesson Summary

1.1.1 Understanding Value

Warm Up

Here are some letters and what they represent. All costs are in dollars.

- m represents the cost of a main dish.
- n represents the number of side dishes.
- s represents the cost of a side dish.
- t represents the total cost of a meal.

For questions 1 – 4, discuss what each **equation** means with a partner. Then, describe the situation in words.

- | | |
|--|--|
| 1. $m = 7.50$ | 2. $m = s + 4.50$ |
| 3. $ns = 6$ | 4. $m + ns = t$ |
| 5. Write a new equation that could be true in this same situation. | 6. Describe what your new equation means in words. |

1.1.2 Creating Expressions to Estimate Cost, Part 1

Activity

Imagine your class is having a pizza party.

Work with your group to plan what to order and to estimate what the party would cost.



1. What is your favorite type of pizza to order?
2. How many toppings are on your favorite type of pizza?
3. What type of crust does your favorite pizza have?

For questions 4 – 9, use the following group instructions.

Work with a group to research the cost of ordering pizza from a local restaurant. Then, determine what to order for the class pizza party and determine how much it would cost. Answer the following questions regarding your group's plan.

4. What is the cost estimate for your group's planned class party?
5. Write down one **expression** that shows how your group's cost estimate was calculated.
6. In your expression, which quantities, if any, might change on the day of the party?
7. Rewrite your expression, replacing the quantities that might change with letters.
8. What do the letters represent?
9. How would you convince the class to go with your group's plan?

Are you ready for more?

Extending Your Thinking

A local pizzeria sells gourmet pizzas for the prices posted below. Remember the sizes are determined by the diameter of each pizza. Compare the cost per square inch of the sizes.

Pizza size	10" Small	12" Medium	14" Large	16" XLarge
Price	\$12.99	\$15.99	\$19.99	\$21.99

1. How did you determine the cost per square inch of each pizza?
2. Which size pizza is the better deal?
3. Which size pizza is the most expensive per square inch?
4. What is the difference in price per square inch between the most and least expensive sizes?

Self Check

A ninth-grade class is ordering pizza for lunch. There are n students in the class. Each slice of pizza costs \$3 and the delivery fee is \$8. Which expression represents the total cost if each student gets 2 slices of pizza?

- a. $3n + 8$
- b. $(2)3n + 8$
- c. $(2)3n$
- d. $3n$

Additional Resources

Translate an English Phrase to an Algebraic Expression

You can use the following operation symbols to translate English phrases into algebraic expressions.

Look closely at these phrases using the four operations:

The SUM of a and b $a + b$
The DIFFERENCE of a and b $a - b$
The PRODUCT of a and b $a \cdot b = ab = (a)(b)$
The QUOTIENT of a and b $\frac{a}{b} = a \div b$

 **CLASSROOM CONNECTION** Log into student.desmos.com (<https://student.desmos.com>) using the information provided by your teacher to complete the activity.

Identify other phrases that represent the same algebraic concepts by categorizing the following list according to the four operations.

b added to a	a divided by b	b subtracted from a
The product of a times b	a sets of b	The sum of a and b
a plus b	The ratio of a and b	a decreased by b
b less than a	a increased by b	b divided into a
The quotient of a and b	b more than a	The total of a and b
a times b	a minus b	The difference of a and b

Each phrase tells us to operate on two numbers. Look for the words "of" and "and" to find the numbers.

Later in this course, we'll apply our skills in algebra to solving applications. The first step will be to translate an English phrase to an algebraic expression.

EXAMPLE

The length of a rectangle is 14 less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Step 1 - Write a phrase about the length of the rectangle.

14 less than the width

Step 2 - Substitute w for "the width."

14 less than w

Step 3 - Rewrite **less than** as **subtracted from**.

14 subtracted from w

Step 4 - Translate the phrase into algebra.

$w - 14$

**TRY IT** **Translate an English Phrase to an Algebraic Expression**

Translate the following into an algebraic expression:

June has dimes and quarters in her purse. The number of dimes is seven less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

1.1.3 Understanding Constraints

Activity

A **constraint** is something that limits what is possible or reasonable in a situation.

For example, one constraint in a pizza party might be the number of slices of pizza each person could have, s . We can write $s < 4$ to say that each person gets fewer than 4 slices.

Look at the expressions you wrote when planning the pizza party earlier. Choose an expression that uses one or more letters.

1. What is the expression you are examining?
2. What does the first letter represent?
3. What values would be reasonable for this first letter? (For instance, could the value be greater than 50? Is it possible for the letter to be a non-whole number? A negative number?)
4. What does the second letter represent?
5. What values would be reasonable for this second letter? (For instance, could the value be greater than 50? Is it possible for the letter to be a non-whole number? A negative number?)
6. Write equations or inequalities that represent some constraints in your pizza party plan. If a quantity must be an exact value, use the = symbol. If it must be greater than or less than a certain value to be reasonable, use the > or < symbol.

Video: Writing an Inequality for the Constraint

Watch the following video to learn more about constraints.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-1-3-understanding-constraints>\)](http://openstax.org/books/algebra-1/pages/1-1-3-understanding-constraints)

Self Check

Marcus works s hours per week stocking shelves. He works m hours per week mowing lawns. Marcus works at least 8

hours per week stocking shelves. He works at most a total of 20 hours per week. Which of the following represent constraints for this situation?

- $s + m \leq 20; s \leq 8$
- $s + m \geq 20; s \leq 8$
- $s + m \leq 20; s \geq 8$
- $s + m \geq 20; s \geq 8$

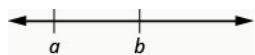
Additional Resources

Inequalities

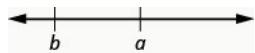
One way to represent a **constraint** is to use an inequality symbol. For example, you may decide that at most 3 toppings should be on each pizza. This could be represented by $t < 3$, if t represents the number of toppings.

On the number line, the numbers get larger as they go from left to right. The number line can be used to explain the symbols " $<$ " and " $>$ ".

$a < b$ is read " a is less than b ." a is to the left of b on the number line.



$a > b$ is read " a is greater than b ." a is to the right of b on the number line.



The expressions $a < b$ or $a > b$ can be read from left to right or right to left, though in English we usually read from left to right.

- For example, $7 < 11$ is equivalent to $11 > 7$.
- For example, $17 > 4$ is equivalent to $4 < 17$.

Inequality symbols	Words
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a \leq b$	a is less than or equal to b
$a > b$	a is greater than b
$a \geq b$	a is greater than or equal to b

Let's look at some specific examples. Translate the following statements into English phrases.

- $17 \leq 26$
- $12 > 27 \div 3$
- $y + 7 < 19$

TRY IT

Inequalities

Translate the following statements into English phrases.

- $14 \leq 27$
- $12 > 4 \div 2$
- $x - 7 < 1$

1.1.4 Creating Expressions to Estimate Cost, Part 2

Cool Down

At the end of the school year, all students in the ninth grade are invited to an ice cream party. Use the skills you learned from planning a pizza party to answer the following:

1. Identify and describe at least one letter that you will use to represent the cost of something you need to buy for the party.
2. Write an expression that could represent an estimated cost for the party using that letter. Note, you may use other letters, too, if needed.
3. Describe your expression in words.
4. Using the letter you identified in question 1, describe the values that would be reasonable for the quantity that the letter represents.

1.1.5 Practice

Complete the following questions to practice the skills you have learned in this lesson.

For questions 1 – 5, use the following scenario to identify the expression, equation, or inequality described.

To support a local senior citizens center, a student club sent a flier home to the n students in the school. The flier said, "Please bring in money to support the senior citizens center. Paper money and coins accepted!" Their goal is to raise T dollars.

Choose the expression, equation, or inequality, from the list, that describes the given quantity.

1. The dollar amount the club would have if they reached half of their goal.
 - a. $T - 0.5n$
 - b. $0.5n$
 - c. $0.25n$
 - d. $0.5T$
 - e. $T + 50$
2. The dollar amount the club would have if every student at the school donated 50 cents to the cause.
 - a. $T - 0.5n$
 - b. $0.5n$
 - c. $0.25n$
 - d. $0.5T$
 - e. $T + 50$
3. The dollar amount the club could donate if they made \$50 more than their goal.
 - a. $T - 0.5n$
 - b. $0.5n$
 - c. $0.25n$
 - d. $0.5T$
 - e. $T + 50$
4. The dollar amount the club would still need to raise to reach its goal after every student at the school donated 50 cents.
 - a. $T - 0.5n$
 - b. $0.5n$
 - c. $0.25n$
 - d. $0.5T$
 - e. $T + 50$
5. The dollar amount the club would have if half of the students at the school each gave 50 cents.
 - a. $T - 0.5n$
 - b. $0.5n$
 - c. $0.25n$
 - d. $0.5T$
 - e. $T + 50$
6. Each of the 10 students in the baking club made 2 chocolate cakes for a fundraiser. They all used the same recipe, using c cups of flour in total.

Write an expression that represents the amount of flour required for one cake.

- a. $\frac{2c}{10}$
- b. $c - 20$
- c. $\frac{c}{20}$
- d. $20c$

For questions 7 - 8, determine if each is an expression or an equation that represents the English phrase

7. $3(x - 7) = 27$

- a. equation
- b. expression

8. $5(4y - 2) - 7$

- a. equation
- b. expression

For questions 9 - 10, choose the algebraic expression that represents the English phrase

9. difference of $14x^2$ and 13

- a. $\frac{14x^2}{13}$
- b. $13 - 14x^2$
- c. $14x^2 - 13$
- d. $14x^2 + 13$

10. the quotient of $12x$ and 2

- a. $\frac{2}{12x}$
- b. $\frac{12x}{2}$
- c. $2 - 12x$
- d. $12x - 2$

1.1.6 Lesson Summary

In this lesson, you learned how to:

- Tell which quantities in a situation can vary and which ones cannot.
- Explain the meaning of the term “constraint.”
- Use letters and numbers to write expressions representing the quantities in a situation.

Here are the **activities** that helped you reach those goals:

- 1.1.1: Understanding Value
 - In this activity, you wrote equations to solve problems about a meal. The big thing you learned is that an equation is a statement that an expression has the same value as another expression. An equation can have letters, numbers, or a mix of letters and numbers.
- 1.1.2: Creating Expressions to Estimate Cost, Part 1
 - In this activity, you learned how expressions could represent the quantities in a situation like planning a pizza party. You also saw how expressions could change if quantities changed.
 - 1.1.2: Self Check
 - 1.1.2: Additional Resources
- 1.1.3: Understanding Constraints
 - In this activity, you continued working with the pizza party and learned that a constraint is something that limits what is possible or reasonable in a situation.
 - 1.1.3: Self Check
 - 1.1.3: Additional Resources
- 1.1.4: Creating Expressions to Estimate Cost, Part 2
 - In this activity, you wrote expressions and examined constraints for the cost of an ice cream party for the ninth-grade class.

After these activities, you completed the following **practice**:

- 1.1.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

1.2 WRITING EQUATIONS TO MODEL RELATIONSHIPS, PART 1

1.2.0 Lesson Overview

In this lesson, you will begin to learn how to write equations to model various situations. For instance, modeling scenarios with equations can help you figure out how much you need to pay for one pound of blueberries or how much tax you need to pay when buying a meal at a restaurant.

When you finish this lesson, you will be able to:

- Tell which quantities in a situation can vary and which ones cannot.
- Use letters and numbers to write equations representing the relationships in a situation.

Here are the **activities** that will help you reach those goals:

- 1.2.1: Finding the Percent of 200
- 1.2.2: Modeling with Equations to Find Edges in Platonic Solids
 - 1.2.2: Self Check
 - 1.2.2: Additional Resources
- 1.2.3: Writing Equations to Represent Relationships
 - 1.2.3: Self Check
 - 1.2.3: Additional Resources
- 1.2.4: Writing Equations to Represent Relationships with Percentages
 - 1.2.4: Self Check
 - 1.2.4: Additional Resources
- 1.2.5: Writing Equations to Represent T-Shirts

After that, you'll **practice and review**.

- 1.2.6: Practice
- 1.2.7: Lesson Summary

1.2.1 Finding the Percent of 200

Warm Up

You will be asked to evaluate the percentage of a number mentally. Sometimes, a visual is helpful to do this. If needed, use the GeoGebra activity to help you determine the requested values. Then, look for a pattern that can be used to describe how to find the percentage of any number.

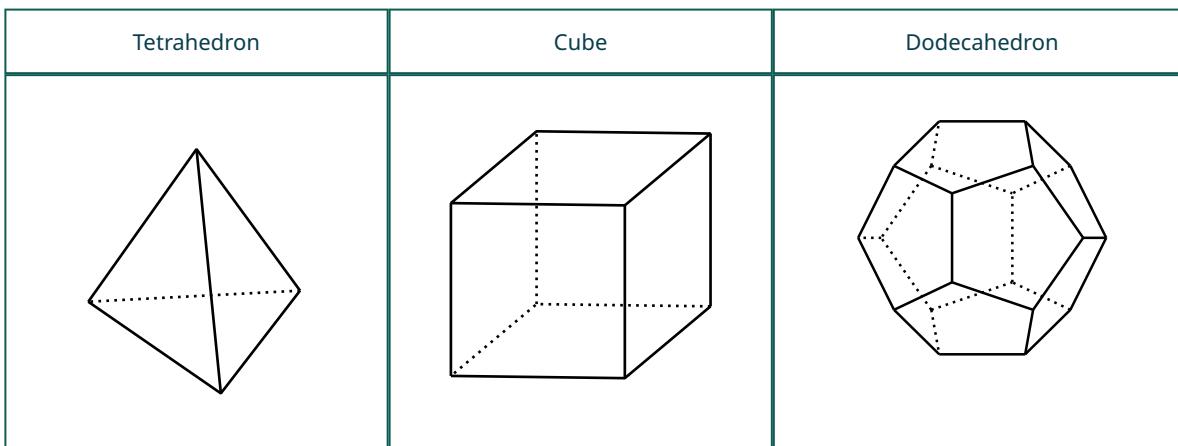
[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-2-1-finding-the-percent-of-200>\)](http://openstax.org/books/algebra-1/pages/1-2-1-finding-the-percent-of-200)

1. What is 25% of 200?
2. What is 12% of 200?
3. What is 8% of 200?
4. How would you determine $p\%$ of 200?

1.2.2 Modeling with Equations to Find Edges in Platonic Solids

Activity

These three figures are called Platonic solids.



	faces	vertices	edges
tetrahedron	4	4	6
cube			
dodecahedron	12	20	30

The table shows the number of faces, vertices, and edges for the tetrahedron and dodecahedron.

Determine the following values regarding the cube.

1. Number of faces.
2. Number of edges.
3. Number of vertices.

There are some interesting relationships between the number of faces (F), edges (E), and vertices (V) in all Platonic solids.

4. What do you notice about the relationships between the numbers of faces, edges, and vertices of the Platonic solids?
5. What do you wonder about the relationships between the numbers of faces, edges, and vertices of the Platonic solids?
6. There is a relationship that can be expressed with an equation using all three values: F , V , E . Write the equation that represents how the three values are related.

Are you ready for more?

Extending Your Thinking

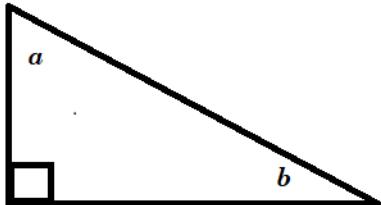
There are two more Platonic solids: an octahedron, which has 8 faces that are all triangles, and an icosahedron, which has 20 faces that are all triangles.

1. How many edges would each of these solids have? (Keep in mind that each edge is used in two faces.)
 - a. The octahedron has how many edges?
 - b. The icosahedron has how many edges?
2. Use your discoveries from the activity to determine how many vertices each of these solids would have.
 - a. The octahedron has how many vertices?
 - b. The icosahedron has how many vertices?

3. For all 5 Platonic solids, determine how many faces meet at each vertex.

Self Check

The sum of the measures of the angles of a triangle is 180° . The right triangle shown has angle measures a , b , and 90° . Which equation could model this relationship?



- a. $90ab = 180$
- b. $a + b + 90 = 180$
- c. $a + b - 90 = 180$
- d. $ab + 90 = 180$

Additonal Resources

Writing Equations Using Symbols

How to write statements using algebraic symbols

Step 1 - Read the problem.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Step 3 - Write a sentence using the relationship among the values.

Step 4 - Translate the sentence into an equation.

Remember that **variables** represent values in the problem that can change.

Let's examine two examples of how to write statements algebraically using symbols.

EXAMPLE 1

We want to express the following statement using symbolic language:

The sum of three consecutive integers is 372.

Step 1 - Read the problem.

The sum of three consecutive integers is 372.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Let x = the first integer. There are two more numbers that follow x .

Each number is 1 more than the number before it: $x + 1$ and $x + 2$

The sum of all three numbers is 372.



Step 3 - Write a sentence using the relationship among the values.

The sum of x , $x + 1$, and $x + 2$ is 372.

Step 4 - Translate the sentence into an equation.

$$x + (x + 1) + (x + 2) = 372$$

$$3x + 3 = 372$$

EXAMPLE 2

We want to express the following statement using symbolic language:

The sum of three consecutive odd integers is 93.

Step 1 - Read the problem.

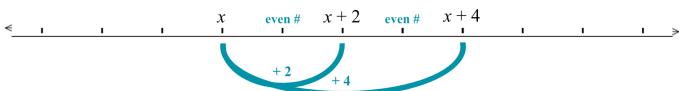
The sum of three consecutive odd integers is 93.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Let x = the first integer. There are two more ODD numbers that follow x .

Each number is 2 more than the number before it: $x + 2$ and $x + 4$

The sum of all three numbers is 93.



Step 3 - Write a sentence using the relationship among the values.

The sum of x , $x + 2$, and $x + 4$ is 93.

Step 4 - Translate the sentence into an equation.

$$x + (x + 2) + (x + 4) = 93$$

$$3x + 6 = 93$$



TRY IT

Writing Equations Using Symbols

Now it's your turn. Write the statement below algebraically using symbols.

The sum of three consecutive integers is 1,623.

1.2.3 Writing Equations to Represent Relationships

Activity

Write an **equation** to represent each situation.

1. Blueberries are \$4.99 a pound. Diego buys b pounds of blueberries and pays \$14.95.
2. Blueberries are \$4.99 a pound. Jada buys p pounds of blueberries and pays c dollars.
3. Blueberries are d dollars a pound. Lin buys q pounds of blueberries and pays t dollars.
4. Noah earned n dollars over the summer. Mai earned \$275, which is \$45 more than Noah did.
5. Noah earned v dollars over the summer. Mai earned m dollars, which is 45 dollars more than Noah did.
6. Noah earned w dollars over the summer. Mai earned x dollars, which is y dollars more than Noah did.

7. How are the equations you wrote for the blueberry purchases like the equations you wrote for Mai's and Noah's summer earnings? How are they different?

Self Check

Max and Jules are renting a car for their 3-day trip. The rental company charges \$50 per day and \$0.40 per mile driven. Which is a possible equation for their cost of the rental car?

- a. $C = 50(3) + 0.40m$
- b. $C = 50(3) + 40(3) + 0.40m$
- c. $C = 50m + 40(3)$
- d. $C = 50(3) + 40m$

Additional Resources

Writing an Equation to Represent a Real-World Problems

Here are the steps to writing an equation to represent a real-world scenario:

Step 1 - Read the problem.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Step 3 - Write a sentence using the relationship among the values.

Step 4 - Translate the sentence into an equation.

EXAMPLE

Write the following statement algebraically using symbols: Peanuts cost \$3.59 per pound. Max buys p pounds of peanuts and pays \$10.77.

Step 1 - Read the problem.

Peanuts cost \$3.59 per pound. Max buys p pounds of peanuts and pays \$10.77.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

\$3.59 = cost per pound of peanuts

\$10.77 = total cost of peanuts bought

p = number of pounds of peanuts bought

Step 3 - Write a sentence using the relationship among the values.

The cost per pound times the number of pounds bought equals the total cost of peanuts

Step 4 - Translate the sentence into an equation.

$$3.59(p) = 10.77$$

TRY IT

Writing an Equation to Represent a Real-World Problem

Translate the following scenario into an equation:

A married couple together earns \$110,000 a year. The wife earns \$16,000 less than twice what her husband earns. What does the husband earn?

1. After reading the problem, what are some of the known variables identified in the description?
2. Identify a variable and describe what it represents for this scenario.
3. What sentence can be used to describe the relationship among the values?
4. What equation can be used to represent the scenario?

Video: Writing an Equation to Represent a Real-World Problem

Watch the following video to learn more about how to write an equation to represent a real-world problem.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-2-3-writing-equations-to-represent-relationships>\)](http://openstax.org/books/algebra-1/pages/1-2-3-writing-equations-to-represent-relationships)

1.2.4 Writing Equations to Represent Relationships with Percentages

Activity

The tax on the sale of a car in Michigan is 6%. At a dealership in Ann Arbor, a car purchase also involves \$120 in miscellaneous charges.

There are several quantities in this situation: the original car price, sales tax, miscellaneous charges, and total price.

For questions 1 – 4, write an equation to describe the relationship between all the quantities when given specific information.

1. The original car price is \$9,500.
2. The original car price is \$14,699.
3. The total price is \$22,480.
4. The original price is p .
5. How would each equation you wrote change if the tax on car sales is $r\%$ and the miscellaneous charges are m dollars?

Self Check

Sweet's Candy Store charges \$5.00 to fill a bag of candy. There is also a sales tax of 7% per purchase. Mrs. Smith buys 4 bags of candy for her children for Christmas. What is an equation that can represent the cost of the candy that Mrs. Smith buys?

- a. $C = 0.07 + 4(5)$
- b. $C = 4(5)$
- c. $C = 0.07(4(5))$
- d. $C = 0.07(4(5)) + 4(5)$

Additional Resources

Writing an Equation to Represent a Real-World Problem with Percentages

The steps for writing an equation to represent a real-world scenario with percentages are the same as before. Just be sure to change the given percent to a decimal before you use it in the equation.

Writing an Equation

Step 1 - Read the problem.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Step 3 - Write a sentence using the relationship among the values.

Step 4 - Translate the sentence into an equation.

For example, what is an equation that can represent this scenario?

The label on Audrey's yogurt said that one serving provided 12 grams of protein, which is 24% of the recommended daily amount. What is the total recommended daily amount of protein?

Here's how to use the steps:

Step 1 - Read the problem.

The label on Audrey's yogurt said that one serving provided 12 grams of protein, which is 24% of the recommended daily amount. What is the total recommended daily amount of protein?

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

1 serving = 12 grams of protein

1 serving is 24% of recommended daily amount

Let a = total amount of protein (in grams).

Step 3 - Write a sentence using the relationship among the values.

12 grams is 24% of the total amount

Step 4 - Translate the sentence into an equation.

$$12 = 0.24a$$

TRY IT**Writing an Equation to Represent a Real-World Problem with Percentages**

Analyze the situation and then write an equation that can represent the scenario.

One serving of wheat square cereal has 7 grams of fiber, which is 28% of the recommended daily amount. What is the total recommended daily amount of fiber?

1. After reading the problem, what are some of the known variables identified in the description?
2. Identify a variable and describe what it represents for this scenario.
3. What sentence can be used to describe the relationship among the values?
4. What equation can be used to represent the scenario?

1.2.5 Writing Equations to Represent T-Shirts

Cool Down

A school choir needs to make T-shirts for its 75 members and has set aside some money in their budget to pay for them. The members of the choir decided to order from a printing company that charges \$3 per shirt, plus a \$50 fee for each color to be printed on the shirts.

1. Write an equation that represents the relationship between the number of T-shirts ordered, the number of colors on the shirts, and the total cost of the order. If you use a **variable**, specify what it represents.
2. In this situation, which quantities do you think can vary? Which might be fixed?

1.2.6 Practice

Complete the following questions to practice the skills you have learned in this lesson.

1. Translate to an equation: Which equation means nine more than x is equal to 52?
 - a. $\frac{52}{x} = 9$
 - b. $9 - x = 52$
 - c. $9x = 52$
 - d. $x + 9 = 52$
2. Translate to an equation: Which equation means ten less than m is -14 ?
 - a. $-14 - m = 10$
 - b. $m - 10 = -14$
 - c. $m - 10 = 14$
 - d. $10 - m = -14$
3. Translate to an equation: Which equation means the sum of y and -30 is 40 ?
 - a. $y + (-30) = 40$
 - b. $y + 40 = -30$
 - c. $y = 40 + (-30)$
 - d. $-30y = 40$
4. Translate to an equation: For a family birthday dinner, Celeste bought a turkey that weighed 5 pounds less than the one she bought for Thanksgiving. The birthday turkey weighed 16 pounds. Which equation represents how much the Thanksgiving turkey weighed?
 - a. $-x + 5 = 16$
 - b. $x + 5 = 16$
 - c. $x - 5 = 16$
 - d. $5 - x = 16$
5. Translate to an equation: Which equation represents 45% of 120?
 - a. $x = 45 + 120$
 - b. $x = 0.45(120)$
 - c. $x = 4.5(120)$
 - d. $x = 45(120)$
6. 250% of 65 can also be written as:
 - a. $x = 250(65)$
 - b. $x = 25(65)$
 - c. $x = 0.25(65)$
 - d. $x = 2.5(65)$
7. Translate to an equation: Geneva treated her parents to dinner at their favorite restaurant. The bill was \$74.25. Geneva wants to leave 16% of the total bill as a tip. How much should the tip be?
 - a. $t = 16 + (74.25)$
 - b. $t = 1.6(74.25)$
 - c. $t = 0.16(74.25)$
 - d. $t = 16(74.25)$
8. Large cheese pizzas cost \$5 each, and large one-topping pizzas cost \$6 each.
Choose the equation that represents the total cost, T , of c large cheese pizzas and d large one-topping pizzas.
 - a. $T = 6c - 5d$
 - b. $T = 5c - 6d$

- c. $T = 5c + 6d$
 - d. $T = 6c + 5d$
9. Jada plans to serve milk and healthy cookies for a book club meeting. She is preparing 12 ounces of milk and 4 cookies per person. Including herself, there are 15 people in the club. A package of cookies contains 24 cookies and costs \$4.50. A 1-gallon jug of milk contains 128 ounces and costs \$3. Let n represent the number of people in the club, m represent the ounces of milk, c represent the number of cookies, and b represent Jada's budget in dollars.
- Select **three** of the equations that could represent the quantities and constraints in this situation.
- a. $m = 12(15)$
 - b. $3m + 4.5c = b$
 - c. $4n = c$
 - d. $4(4.50) = c$
 - e. $b = 2(3) + 3(4.50)$

1.2.7 Lesson Summary

In this lesson, you learned how to:

- Tell which quantities in a situation can vary and which ones cannot.
- Use letters and numbers to write equations representing the relationships in a situation.

Here are the **activities** that helped you reach those goals:

- 1.2.1: Finding the Percent of 200
 - In this activity, you found different percentages of 200 by changing the percent to a decimal and writing an expression.
- 1.2.2: Modeling with Equations to Find Edges in Platonic Solids
 - In this activity, you modeled the relationship between the edges, vertices, and faces of Platonic solids with an equation.
 - 1.2.2: Self Check
 - 1.2.2: Additional Resources
- 1.2.3: Writing Equations to Represent Relationships
 - In this activity, you worked to write more equations that modeled different situations.
 - 1.2.3: Self Check
 - 1.2.3: Additional Resources
- 1.2.4: Writing Equations to Represent Relationships with Percentages
 - In this activity, you continued to write equations to model given situations but incorporated percentages.
 - 1.2.4: Self Check
 - 1.2.4: Additional Resources
- 1.2.5: Writing Equations to Represent T-Shirts
 - In this activity, you wrote an equation about the cost of T-shirts for a choir group. You also identified which quantities were fixed in the situation and which quantities could change.

After these activities, you completed the following **practice**:

- 1.2.6: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

1.3

WRITING EQUATIONS TO MODEL RELATIONSHIPS, PART 2

1.3.0 Lesson Overview

In this lesson, you will continue to develop your ability to identify, describe, and model relationships with mathematics.

When you finish this lesson, you will be able to:

- Use words and equations to describe the patterns in a table of values or in a set of calculations.
- Use representations, like diagrams and tables, to help make sense of a described situation and write equations for it.

Here are the **activities** that will help you reach those goals:

- 1.3.1: Finding a Relationship between x and y
- 1.3.2: Describing Relationships Using Words and Equations
 - 1.3.2: Self Check
 - 1.3.2: Additional Resources
- 1.3.3: Identifying and Representing Relationships
 - 1.3.3: Self Check
 - 1.3.3: Additional Resources
- 1.3.4: Writing an Equation to Represent a Relationship

After that, you'll **practice and review**.

- 1.3.5: Practice
- 1.3.6: Lesson Summary

1.3.1 Finding a Relationship between x and y

Warm Up

Here is a table of values. The two quantities, x and y , are related.

x	y
1	0
3	8
5	24
7	48

What are some strategies you could use to find a relationship between x and y ? Brainstorm as many ways as possible.

1.3.2 Describing Relationships Using Words and Equations

Activity

Create a math story that describes how the two quantities in each table are related.

1. Table A

Number of Laps, x	0	1	2.5	6	9
Meters Run, y	0	400	1,000	2,400	3,600

2. Table B

Meters From Home, x	0	75	128	319	396
Meters From School, y	400	325	272	81	4

3. Table C

Electricity Bills in Dollars, x	85	124	309	816
Total Expenses in Dollars, y	485	524	709	1,216

4. Table D

Monthly Salary in Dollars, x	872	998	1,015	2,110
Amount Deposited in Dollars, y	472	598	615	1,710

For questions 5 – 8, match each table from 1 - 4 to an equation that represents the relationship.

5. Equation 1: $400 + x = y$

6. Equation 2: $x - 400 = y$

7. Equation 3: $x + y = 400$

8. Equation 4: $400 \cdot x = y$

Are you ready for more?

Extending Your Thinking

Express every number between 1 and 20 at least one way using exactly four 4s and any operations. For example, 1 could be written as $\frac{4}{4} + 4 - 4$.

Video: Describing Relationships

Watch the following video to learn more about describing relationships using words and equations, specifically looking at the base and height of different rectangles.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-3-2-describing-relationships-using-words-and-equations>\)](http://openstax.org/books/algebra-1/pages/1-3-2-describing-relationships-using-words-and-equations)

Self Check

Miles from Home, x	0	5	10
Miles from School, y	30	25	20

Using the table above, what is an equation that shows the relationship between miles from home and miles from school?

- a. $y = 30x$
- b. $y = 30 - x$
- c. $y = 20 + x$
- d. $y = 30 + x$

Additional Resources

Modeling Linear Equations

Given a real-world problem, model a linear equation to fit it using these steps:

Step 1 - Read the problem.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Step 3 - Write a sentence or create a table using the relationship among the values.

Step 4 - Translate the sentence or table into an equation

Let's look at an example. Using the steps above, we can model a linear equation to solve a real-world application.

EXAMPLE

Cell phone Company A charges a monthly service fee of \$34 plus \$0.05/minute talk time.

Step 1 - Read the problem.

Cell phone Company A charges a monthly service fee of \$34 plus \$0.05/minute talk time.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

t : total cost (in dollars)

m : number of minutes

Step 3 - Write a sentence or create a table using the relationship among the values.

Minutes Talked in One Month	0	1	2	3	100
Total for One Month (Dollars)	34	34.05	34.10	34.15	39

Company A: The total cost, t , equals the fee, 34, plus 0.05 per minute, $0.05m$.

Step 4 - Translate the sentence or table into an equation.

$$t = 34 + 0.05m$$



TRY IT

Modeling Linear Equations

Cell phone Company B charges a monthly service fee of \$40 plus \$0.04/min talk-time. Write an equation to describe the relationship between the number of minutes and the total cost.

1.3.3 Identifying and Representing Relationships

Activity

1. The table represents the relationship between the base length and the height of some parallelograms. Both measurements are in inches.

1	48
2	24
3	16
4	12
6	8

2. Visitors to a carnival are invited to guess the number of beans in a jar. The person who guesses the correct number wins \$300. If multiple people guess correctly, the prize will be divided evenly among them. What is the relationship between the number of people who guess correctly and the amount of money each person will receive?

What is the relationship between the base length and the height of these parallelograms?

3. A $\frac{1}{2}$ -gallon jug of milk can fill 8 cups, while 32 fluid ounces of milk can fill 4 cups.

What is the relationship between the number of gallons and ounces? If you get stuck, try creating a table.

Building Character: Social Intelligence



Social intelligence is the ability to connect with other people. Think about your current sense of social intelligence. Are the following statements true for you??

- I have a lot of relationships that are mutually beneficial, enjoyable, and supportive.
- Most of the time, I can tell how other people feel and have a good idea about how to respond appropriately.

Don't worry if none of these statements are true for you. Developing this trait takes time. Your first step starts today!

Self Check

Number of guests, g	1	2	3	12
Number of cupcakes per guest, n	24	12	8	2

What is one way to write the relationship between the number of guests and the number of cupcakes per guest from the table above?

- a. $n = 24 - g$
- b. $n = \frac{g}{24}$
- c. $n = \frac{24}{g}$
- d. $n = 24g$

Additional Resources**Finding Relationships between Quantities**

There are times when the relationship between quantities may not be obvious. In some cases, the relationship between quantities might take a bit of work to figure out, by doing calculations several times or by looking for a pattern.

Here are two examples.

EXAMPLE 1

A plane departed from New Orleans and is heading to San Diego. The table shows its distance from New Orleans, x , and its distance from San Diego, y , at some points along the way.

miles from New Orleans	miles from San Diego
100	1,500
300	1,300
500	1,100
	1,020
900	700
1,450	
x	y

1. What is the relationship between the two distances?
2. Do you see any patterns in how each quantity is changing?
3. What is the value of x when $y = 1020$?
4. What is the value of y when $x = 1450$?

EXAMPLE 2

A company decides to donate \$50,000 to charity. It will select up to 20 charitable organizations, as nominated by its employees. Each selected organization will receive an equal donation amount.

What is the relationship between the number of students, s , and the dollar amount each student will receive, d ? To begin, let's examine some specific values to help uncover the pattern.

5. If 5 organizations are selected, how much will each charity receive?
6. If 10 organizations are selected, how much will each charity receive?
7. If 20 organizations are selected, how much will each charity receive?

Do you notice a pattern here? 10,000 is $\frac{50,000}{5}$, 5,000 is $\frac{50,000}{10}$, and 2,500 is $\frac{50,000}{20}$.

We can generalize that the amount each organization receives is 50,000 divided by the number of selected organizations, or $d = \frac{50,000}{n}$.

TRY IT

Finding Relationships between Quantities

A local business is going to hand out \$20,000 in scholarships to students at local high schools.

What is the relationship between the number of students, s , and the dollar amount each student will receive, d ?

1. If 2 students are selected, what is the amount of the scholarship they will receive?
2. If there are 5 students selected, what is the amount of the scholarship they will receive?
3. If there are 10 students selected, what is the amount of the scholarship they will receive?
4. If there are 20 students selected, what is the amount of the scholarship they will receive?
5. What equation can be used to model the relationship between the number of students, s , receiving scholarships and the dollar amount, d , they receive?

1.3.4 Writing an Equation to Represent a Relationship

Cool Down

Clare volunteers at a local library during the summer. Her work includes putting labels on 750 books.

1. How many minutes will she need to finish labeling all books if she takes no breaks and labels 10 books a minute?
2. How many minutes will she need to finish labeling all books if she takes no breaks and labels 15 books a minute?
3. Suppose Clare labels the books at a constant speed of s books per minute. Write an equation that represents the relationship between her labeling speed and the number of minutes it would take her to finish labeling.

1.3.5 Practice

Complete the following questions to practice the skills you have learned in this lesson.

For questions 1 – 2, use the following scenario to identify the equation described.

Two different telephone carriers offer the following plans that a person is considering. Company A has a monthly fee of \$20 and charges \$0.05/minute for calls. Company B has a monthly fee of \$5 and charges \$0.10/minute for calls.

1. Find the model for the entire cost, c , when Company A's plan is used for m minutes.
 - a. $c = 5 + 0.05m$
 - b. $c = 20 + 0.10m$
 - c. $c = 5 + 0.10m$
 - d. $c = 20 + 0.05m$
2. Find the model for the entire cost, c , when Company B's plan is used for m minutes.
 - a. $c = 5 + 0.05m$
 - b. $c = 20 + 0.10m$
 - c. $c = 5 + 0.10m$
 - d. $c = 20 + 0.05m$
3. If a carpenter sawed a 10-ft board into two sections and one section was n ft long, how long would the other section, S , be in terms of n ?
 - a. $S = \frac{10}{2}n$
 - b. $S = 10 - 2n$
 - c. $S = 10 - n$
 - d. $S = \frac{10}{n}$

Use the table below for problems 4 and 5:

Tyler needs to complete this table for his consumer science class. He knows that 1 tablespoon contains 3 teaspoons and that 1 cup contains 16 tablespoons.

Number of Teaspoons	Number of Tablespoons	Number of Cups
a. ____	b. ____	2
36	12	c. ____
d. ____	48	3

4. For each of the missing spaces in the table above, a-d, select the missing value that helps complete the table.
 - a. ____
 - a. 144
 - b. 96
 - c. 64
 - d. 32
 - e. $\frac{3}{4}$
 - b. ____
 - a. 144
 - b. 96

c. 64

d. 32

e. $\frac{3}{4}$

c. _____

a. 144

b. 96

c. 64

d. 32

e. $\frac{3}{4}$

d. _____

a. 144

b. 96

c. 64

d. 32

e. $\frac{3}{4}$

5. Choose the equation that represents the number of teaspoons, t , contained in a cup, C .

a. $c = \frac{t}{3}$

b. $c = t - 48$

c. $c = \frac{t}{48}$

d. $c = 48t$

6. The volume of dry goods, like apples or peaches, can be measured using bushels, pecks, and quarts. A bushel contains 4 pecks, and a peck contains 8 quarts.

What is the relationship between the number of bushels, b , and the number of quarts, q ?

a. $b = 32q$

b. $b = \frac{q}{64}$

c. $b = \frac{q}{12}$

d. $b = \frac{q}{32}$

1.3.6 Lesson Summary

In this lesson, you learned how to:

- Use words and equations to describe the patterns in a table of values or in a set of calculations.
- Use representations, like diagrams and tables, to help make sense of a described situation and write equations for it.

Here are the **activities** that helped you reach those goals:

- 1.3.1: Finding a Relationship between x and y
 - In this activity, you brainstormed ways to find the relationship between variables in a table.
- 1.3.2: Describing Relationships Using Words and Equations
 - In this activity, you matched tables with the equations that could model the relationships in each table.
 - 1.3.2: Self Check
 - 1.3.2: Additional Resources
- 1.3.3: Identifying and Representing Relationships
 - In this activity, you created tables from a situation and then wrote an equation to model the relationship.
 - 1.3.3: Self Check
 - 1.3.3: Additional Resources
- 1.3.4: Writing an Equation to Represent a Relationship
 - In this activity, you wrote an equation to model a relationship in a situation and also began to solve an equation given one of the quantities in the equation.

After these activities, you completed the following **practice**:

- 1.3.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

1.4 EQUATIONS AND THEIR SOLUTIONS

1.4.0 Lesson Overview

In middle school, you learned that a solution to an equation is a value or values that make the equation true. In this lesson, you will revisit what you learned about solutions to equations in one variable and two variables. You will also continue to practice modeling relationships with equations and to make sense of equations and their solutions in context.

When you finish this lesson, you will be able to:

- Explain what it means for a value or pair of values to be a solution to an equation.
- Find solutions to equations by reasoning about a situation or by using algebra.

Here are the **activities** that will help you reach those goals:

- 1.4.1: Finding the Solution to an Equation in One Variable, Part 1
- 1.4.2: Writing Equations to Represent Constraints
 - 1.4.2: Self Check
 - 1.4.2: Additional Resources
- 1.4.3 Finding the Solution to an Equation in Two Variables
 - 1.4.3: Self Check
 - 1.4.3: Additional Resources
- 1.4.4: Finding the Solution to an Equation in One Variable, Part 2

After that, you'll **practice and review**.

- 1.4.5: Practice
- 1.4.6: Lesson Summary

1.4.1 Finding the Solution to an Equation in One Variable, Part 1

Warm Up

A granola bite contains 27 calories. Most of the calories come from c grams of carbohydrates. The rest come from other ingredients. One gram of carbohydrate contains 4 calories.

The equation $4c + 5 = 27$ represents the relationship between these quantities.

1. What could the 5 represent in this situation?
2. Priya said that neither 8 nor 3 could be the solution to the equation. Is she correct?
3. Marcus said the solution is 5.5. Is he correct?
Explain.

1.4.2 Writing Equations to Represent Constraints

Activity

Jada has time on the weekends to earn some money. A local bookstore is looking for someone to help sort books and will pay \$12.20 an hour. To get to and from the bookstore on a work day, however, Jada would have to spend \$7.15 on bus fare.

1. Write an equation that represents Jada's take-home earnings in dollars, E , if she works at the bookstore for h hours in one day.
2. One day, Jada takes home \$90.45 after working h hours and after paying the bus fare. Write an equation to represent this situation.

3. Is 4 a solution to the last equation you wrote?
 - If so, be prepared to explain how you know it is a solution.
 - If not, be prepared to explain why it is not a solution. Then, find the solution.

4. Is 7 a solution to the last equation you wrote?
 - If so, be prepared to explain how you know it is a solution.
 - If not, be prepared to explain why it is not a solution. Then, find the solution.

5. In this situation, what does the solution to the equation tell us?

Are you ready for more?

Extending Your Thinking

Jada has a second option to earn money. She could help some neighbors with errands and computer work for \$11 an hour. After reconsidering her schedule, Jada realizes that she has about 9 hours available to work one day of the weekend.

Which option should she choose, sorting books at the bookstore or helping her neighbors? Be prepared to show your reasoning.

Self Check

Jada found a third job option and was offered a position working at a camp for \$10.50 per hour. The total bus fare to and from the camp is \$8.40. Write an equation that represents how much she will make in a day, d , after working h hours.

- a. $d = 8.40h + 10.50$
- b. $d = \frac{10.50h}{8.40}$
- c. $d = 10.50h + 8.40$
- d. $d = 10.50h - 8.40$

Additional Resources

Verify a Solution of an Equation

Solving an **equation** is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the **variable** that make each side of the equation the same so that we end up with a true statement. Any value of the variable that makes the equation true is called a **solution of an equation**. It is the answer to the puzzle!

How to determine whether a number is a solution to an equation:

Step 1 - Substitute the number in for the variable in the equation.

Step 2 - Simplify the expressions on both sides of the equation

Step 3 - Determine whether the resulting equation is true (the left side is equal to the right side).

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

For example, the equation $2l + 2w = 72$ represents the relationship between the length, l , and the width, w , of a rectangle that has a perimeter of 72 units. If we know that the length is 15 units, what is the value of the width?

Step 1 - Substitute the number in for the variable in the equation.

$$2(15) + 2w = 72$$

This is an equation in one variable because w is the only quantity that we don't know. To solve this equation means to find a value of w that makes the equation true.

Step 2 - Simplify the expressions on both sides of the equation. In this case, 21 is the solution because substituting 21 for

w in the equation results in a true statement.

$$2(15) + 2w = 72$$

$$2(15) + 2(21) = 72$$

$$30 + 42 = 72$$

Step 3 - Determine whether the resulting equation is true (the left side is equal to the right side).

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

$$72 = 72$$

TRY IT

Verify a Solution of an Equation

The equation $2l + 2w = 48$ represents the perimeter of a rectangle.

1. If the width of the rectangle is 10, what is the new equation?
2. What is the length of the rectangle that has a width of 10?
3. Verify the solution.

1.4.3 Finding the Solution to an Equation in Two Variables

Activity

Use the following scenario for questions 1 – 3:

One gram of protein contains 4 calories. One gram of fat contains 9 calories. A snack has 60 calories from p grams of protein and f grams of fat.

1. Determine if 5 grams of protein and 2 grams of fat could be the number of grams of protein and fat in the snack. Explain your reasoning.
2. Determine if 10.5 grams of protein and 2 grams of fat could be the number of grams of protein and fat in the snack. Explain your reasoning.
3. Determine if 8 grams of protein and 4 grams of fat could be the number of grams of protein and fat in the snack. Explain your reasoning.
4. If there are 6 grams of fat in the snack, how many grams of protein are there? Be prepared to show your reasoning.
5. In this situation, what does a solution to the equation $4p + 9f = 60$ tell us? Give an example of a solution.

Video: Working Through the Equation

Watch the following video to learn more about how to determine a solution to this particular equation: $4p + 9f = 60$.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-4-3-finding-the-solution-to-an-equation-in-two-variables>\)](http://openstax.org/books/algebra-1/pages/1-4-3-finding-the-solution-to-an-equation-in-two-variables)

Self Check

Which of the following is a solution (x, y) to $2x + 3y = 7$?

- a. $(-2, 4)$
- b. $(4, -1)$
- c. $(1, 2)$
- d. $(2, 1)$

Additional Resources

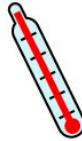
Solutions to Equations in Two Variables

An equation that contains two unknown quantities or two quantities that vary is called an equation in two variables.

Equations in two variables:

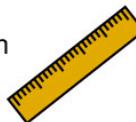
$$F = \frac{9}{5}C + 32$$

Converts temperature from degrees Fahrenheit (F) to degrees Celsius (C)



$$i = 2.54c$$

Converts a measurement from centimeters (c) to inches (i)



A solution to such an equation is a pair of numbers that makes the equation true.

EXAMPLE

Suppose Tyler spends \$45 on T-shirts and socks. A T-shirt costs \$10, and a pair of socks costs \$2.50. If t represents the number of T-shirts and p represents the number of pairs of socks that Tyler buys, we can represent this situation with the equation:

$$10t + 2.50p = 45$$

This is an equation in two variables. More than one pair of values for t and p make the equation true.

Which pair of values makes the equation $10t + 2.50p = 45$ true?

- 1. $t = 3$ and $p = 6$
- 2. $t = 4$ and $p = 2$
- 3. $t = 1$ and $p = 10$

In this situation, one **constraint** is that the combined cost of shirts and socks must equal \$45.

Solutions to the equations are pairs of t and p values that satisfy this constraint, such as in questions 1 – 2.

Combinations such as $t = 1$ and $p = 10$, as in question 3, are not solutions because they don't meet the constraint. When these pairs of values are substituted into the equation, they result in statements that are false.



TRY IT

Solutions to Equations in Two Variables

Is $a = 3$ and $b = 5$ a solution to $6a - 3b = 3$?

1.4.4 Finding the Solution to an Equation in One Variable, Part 2

Cool Down

An empty shipping box weighs 250 grams. The box is then filled with T-shirts. Each T-shirt weighs 132.5 grams.

The equation $W = 250 + 132.5T$ represents the relationship between the quantities in this situation, where W is the weight, in grams, of the filled box and T is the number of shirts in the box.

- 1. Name two possible solutions to the equation $W = 250 + 132.5T$.
- 2. What do the solutions mean in this situation?

3. Consider the equation $2,900 = 250 + 132.50T$. In this situation, what does the solution to this equation tell us?

1.4.5 Practice

Complete the following questions to practice the skills you have learned in this lesson.

1. Is $y = \frac{4}{3}$ a solution of $9y + 2 = 6y + 3$?
 - a. Yes
 - b. No
2. Is $y = \frac{7}{5}$ a solution of $5y + 3 = 10y - 4$?
 - a. Yes
 - b. No
3. Is $y = \frac{5}{3}$ a solution of $6y + 10 = 12y$?
 - a. Yes
 - b. No
4. Is $u = -\frac{1}{2}$ a solution of $8u - 1 = 6u$?
 - a. Yes
 - b. No
5. Tickets to the museum cost \$4.50 for children and \$9.00 for adults. The equation $63 = 4.5c + 9a$ represents the total cost for c children's tickets and a adult tickets. How many children's tickets were sold if 4 adult tickets were sold?

For questions 6 and 7, use the following scenario: T-shirts are on sale for \$12.50. Isla has a coupon for \$10.00 off.

6. Choose the equation that represents the total cost, c , if she buys n T-shirts.
 - a. $c = 12.5(10n)$
 - b. $c = 12.5(n - 10)$
 - c. $c = 10n - 12.5$
 - d. $c = 12.5n - 10$
7. How many T-shirts can she buy if she spends all of her \$40?
8. An artist is selling children's crafts. Necklaces cost \$2.25 each, and bracelets cost \$1.50 per each. Select **three** combinations of necklaces and bracelets that the artist could sell for exactly \$12.00
 - a. 5 necklaces and 1 bracelet
 - b. 2 necklaces and 5 bracelets
 - c. 3 necklaces and 3 bracelet
 - d. 4 necklaces and 2 bracelets
 - e. 3 necklaces and 5 bracelets
 - f. 6 necklaces and no bracelets
 - g. No necklaces and 8 bracelets
9. Volunteer drivers are needed to bring 80 students to the championship baseball game. Drivers either have cars, which can seat 4 students, or vans, which can seat 6 students. The equation $4c + 6v = 80$ describes the relationship between the number of cars, c , and number of vans, v , that can transport exactly 80 students. Select **four** statements that are true about the situation.
 - a. If 12 cars go, then 2 vans are needed.
 - b. $c = 14$ and $v = 4$ are a pair of solutions to the equation.
 - c. If 6 cars go and 11 vans go, there will be extra space.
 - d. 10 cars and 8 vans isn't enough to transport all the students.
 - e. If 20 cars go, no vans are needed.
 - f. 8 vans and 8 cars are numbers that meet the constraints in this situation.

Use the following scenario for questions 11 and 12:

The drama club is printing T-shirts for its members. The printing company charges a certain amount for each T-shirt plus a setup fee of \$40. There are 21 students in the drama club.

- 10.** If there are 21 students in the club and the T-shirt order costs a total of \$187, how much does each T-shirt cost?
- 11.** The equation $201.50 = f + 6.50(21)$ represents the cost of printing the T-shirts at a second printing company. Find the solution to the equation.

1.4.6 Lesson Summary

In this lesson, you learned how to:

- Explain what it means for a value or pair of values to be a solution to an equation.
- Find solutions to equations by reasoning about a situation or by using algebra.

Here are the **activities** that helped you reach those goals:

- 1.4.1: Finding the Solution to an Equation in One Variable, Part 1
 - In this activity, you began to determine if a value was a solution to a given one-variable equation and identified what the solution could mean. For a value to be a solution to an equation, it must make the equation true.
- 1.4.2: Writing Equations to Represent Constraints
 - In this activity, you wrote equations from situations given the constraints of the problem.
 - 1.4.2: Self Check
 - 1.4.2: Additional Resources
- 1.4.3: Finding the Solution to an Equation in Two Variables
 - In this activity, you were able to find the solution to an equation with two variables, given the solution to one of the variables.
 - 1.4.3: Self Check
 - 1.4.3: Additional Resources
- 1.4.4: Finding the Solution to an Equation in One Variable, Part 2
 - In this activity, you worked to find multiple solutions to equations and identified what the solutions mean in this situation.

After these activities, you completed the following **practice**:

- 1.4.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

1.5 EQUATIONS AND THEIR GRAPHS

1.5.0 Lesson Overview

In this lesson, you will analyze points on and off a graph and interpret them in context. In explaining correspondences between equations, verbal descriptions, and graphs, you will hone your skill at making sense of problems.

When you finish this lesson, you will be able to:

- Use graphing technology to graph linear equations and identify solutions to the equations.
- Explain how the coordinates of the points on the graph of a linear equation are related to the equation.
- Explain the meaning of points on a graph in terms of the situation it represents when given the graph of a linear equation.

Here are the **activities** that will help you reach those goals:

- 1.5.1: Analyzing Graphs of Linear Equations
- 1.5.2: Graphing Linear Functions in Two Variables
 - 1.5.2: Self Check
 - 1.5.2: Additional Resources
- 1.5.3: Examining an Equation in Two Variables and Its Graph, Part 1
 - 1.5.3: Self Check
 - 1.5.3: Additional Resources
- 1.5.4: Writing, Graphing, and Solving a Linear Equation
 - 1.5.4: Self Check
 - 1.5.4: Additional Resources
- 1.5.5: Examining an Equation in Two Variables and Its Graph, Part 2

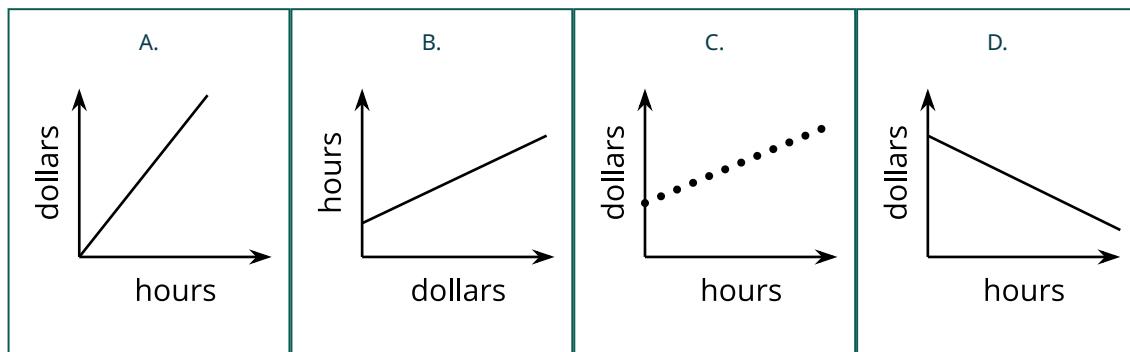
After that, you'll **practice and review**.

- 1.5.6: Practice
- 1.5.7: Lesson Summary

1.5.1 Analyzing Graphs of Linear Equations

Warm Up

Which one doesn't belong?



1.5.2 Graphing Linear Functions in Two Variables

Activity

For questions 1 – 7, use the equation $3x + 4y = 12$

1. Find the value of y when $x = 0$.
2. Find the value of x when $y = 0$.
3. Plot both of those points on a coordinate grid and use a ruler to draw a line.
4. What is the coordinate of the x -intercept?
5. What is the coordinate of the y -intercept?
6. What are two other names for the x -intercept?
7. What is the slope of the line?

Self Check

Graph the equation $5x - 2y = 10$. Which of the following describes the intercepts and slope of the graph?

- x -intercept: $x = -5$; y -intercept: $y = 2$; Slope: $\frac{5}{2}$
- x -intercept: $x = 2$; y -intercept: $y = -5$; Slope: $\frac{5}{2}$
- x -intercept: $x = -2$; y -intercept: $y = 5$; Slope: $-\frac{5}{2}$
- x -intercept: $x = 2$; y -intercept: $y = -5$; Slope: $\frac{5}{2}$

Additional Resources

Graphing a Line and Describing Characteristics

There are many ways to graph an equation of a line. One way is to use the x -intercepts and y -intercepts as two points on the line.

EXAMPLE

Graph $3x - 6y = 18$ using the intercepts, then identify key characteristics.

Step 1 - Find the x -intercept.

To find the x -intercept, let $y = 0$.

$$\begin{aligned} 3x - 6y &= 18 \\ 3x - 6(0) &= 18 \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

$(6, 0)$ is the x -intercept

The x -intercept is also called a solution or zero.

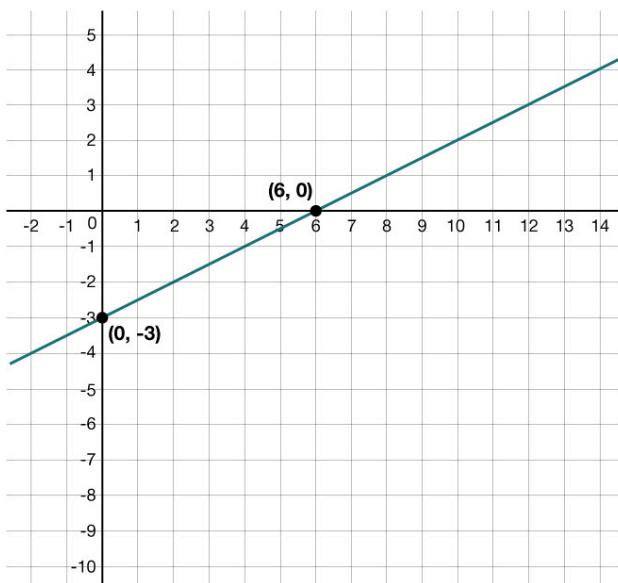
Step 2 -Find the y -intercept.

To find the y -intercept, let $x = 0$.

$$\begin{aligned} 3x - 6y &= 18 \\ 3(0) - 6y &= 18 \\ -6y &= 18 \\ y &= -3 \end{aligned}$$

$(0, -3)$ is the y -intercept

Step 3 -Graph the line by connecting the y -intercepts.



Step 4 - Find the slope of the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{3}{6} = \frac{1}{2}$$



Graphing a Line and Describing Characteristics

For questions 1 – 2, use the equation $7x - 3y = 21$.

1. Graph the equation using the intercepts.
2. What is the slope of the line?

1.5.3 Examining an Equation in Two Variables and Its Graph, Part 1

Activity

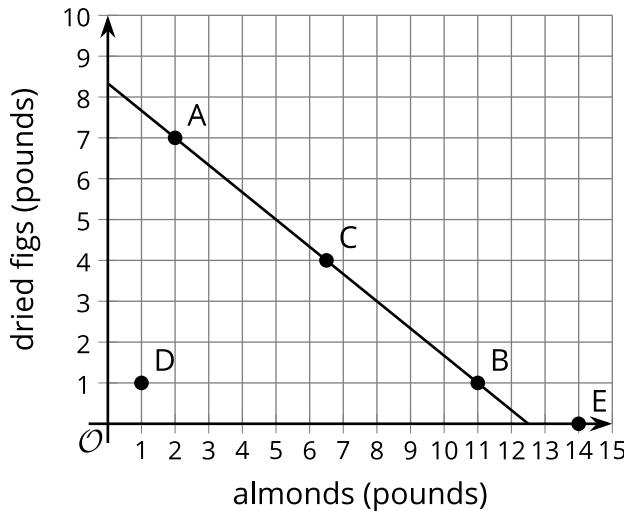
To get snacks for a class trip, Clare went to the “bulk” section of the grocery store, where she could buy any quantity of a product and the prices are usually good.

Clare purchased some salted almonds at \$6 a pound and some dried figs at \$9 per pound. She spent \$75 before tax.



- If she bought 2 pounds of almonds, how many pounds of figs did she buy?
- If she bought 1 pound of figs, how many pounds of almonds did she buy?
- Write an **equation** that describes the relationship between pounds of figs and pounds of almonds that Clare bought and the dollar amount that she paid. Be sure to specify what the **variables** represent.

For questions 4 and 5, use the graph that represents the quantities in the situation.



- Choose any point on the line, state its coordinates, and explain what it tells us.
- Choose any point that is not on the line, state its coordinates, and explain what it tells us.

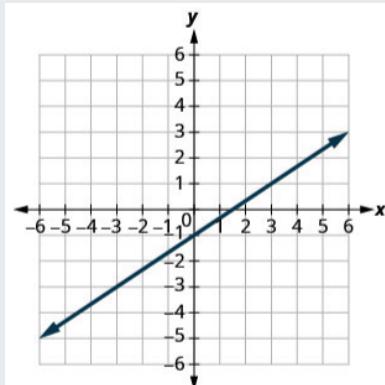
Video: Using a Graph to Solve an Equation

Watch the following video to further explore the above scenario.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-5-3-examining-an-equation-in-two-variables-and-its-graph-part-1>\)](http://openstax.org/books/algebra-1/pages/1-5-3-examining-an-equation-in-two-variables-and-its-graph-part-1)

Self Check

Which point is a solution to the line graphed below?



- (-3, 3)
- (1, 3)
- (0, 1)

d. (3, 1)

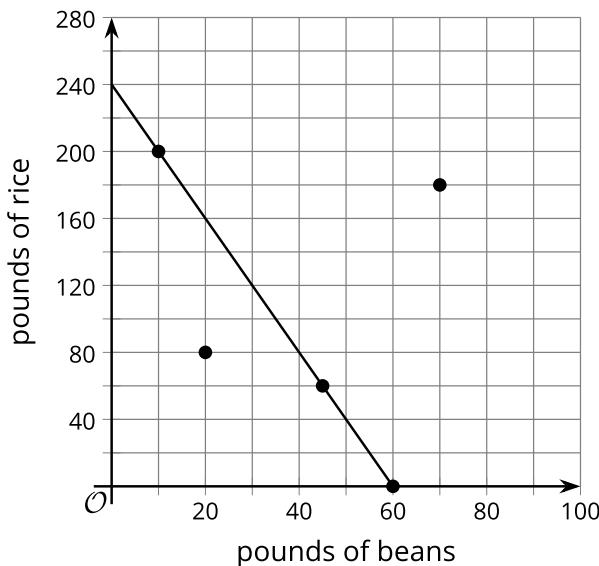
Additional Resources

Determining the Meaning of Solutions of a Graphed Line

Suppose we are buying beans and rice to feed a large gathering of people, and we plan to spend \$120 on the two ingredients. Beans cost \$2 a pound, and rice costs \$0.50 a pound.

If x represents pounds of beans and y pounds of rice, the equation $2x + 0.50y = 120$ can represent the constraints in this situation.

The graph of $2x + 0.50y = 120$ shows a straight line.



Each point on the line is a pair of x - and y -values that make the equation true and is thus a solution. It is also a pair of values that satisfy the constraints in the situation.

- The point (10, 200) is on the line. If we buy 10 pounds of beans and 200 pounds of rice, the cost will be $2(10) + 0.50(200)$, which equals 120.
- The points (60, 0) and (45, 60) are also on the line. If we buy only beans—60 pounds of them—and no rice, we will spend \$120. If we buy 45 pounds of beans and 60 pounds of rice, we will also spend \$120.

What about points that are *not* on the line? They are not solutions because they don't satisfy the constraints, but they still have meaning in the situation.

- The point (20, 80) is not on the line. Buying 20 pounds of beans and 80 pounds of rice costs $2(20) + 0.50(80)$ or 80, which does not equal 120. This combination costs less than what we intend to spend.
- The point (70, 180) means that we buy 70 pounds of beans and 180 pounds of rice. It will cost $2(70) + 0.50(180)$ or 230, which is over our budget of 120.

Let's look at an example.

EXAMPLE

1. Looking at the graph about purchasing beans and rice, is the point (30, 120) a solution?

Solution

Yes, because $2(30) + 0.50(120) = 60 + 60 = 120$.

2. What does the point (30, 120) mean in this situation?

Solution

Compare your answer:

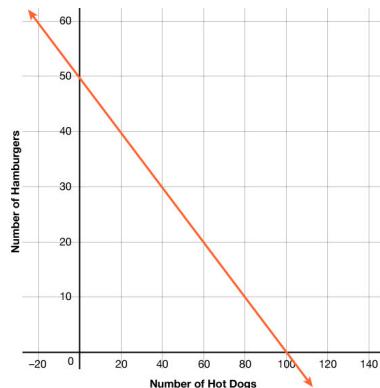
The point means that 30 pounds of beans and 120 pounds of rice were purchased. Since the point is on the line, the total spent was \$120.

 TRY IT

Determining the Meaning of Solutions of a Graphed Line

At a high school baseball game, the concession stand sold hot dogs and hamburgers. Hot dogs were \$1.50, and hamburgers cost \$3. The goal is to make \$150 for the game.

What is the meaning of the point (40, 30) on the graph?



1.5.4 Writing, Graphing, and Solving a Linear Equation

Activity



Access the [Desmos guide PDF](https://openstax.org/r/desmos-guide) (<https://openstax.org/r/desmos-guide>) for tips on solving problems with the Desmos graphing calculator.

Use the following scenario for questions 1 – 5:

A student has a savings account with \$475 in it. She deposits \$125 of her paycheck into the account every week. Her goal is to save \$7000 for college.

1. How much will be in the account after 3 weeks?
2. How many weeks will it take until she has \$1350?
3. Write an equation that represents the relationship between the dollar amount in her account and the number of weeks of saving.
4. Graph your equation using graphing technology. Mark the points on the graph that represent the amount after 3 weeks and the week she has \$1350. Write down the coordinates. Use the graphing tool or technology outside the course. Graph the equation that represents this scenario using the Desmos tool above.
5. What is the x -intercept?
6. What other information does the x -intercept identify in the function?
7. What is the y -intercept?

8. What is the slope?

Are you ready for more?

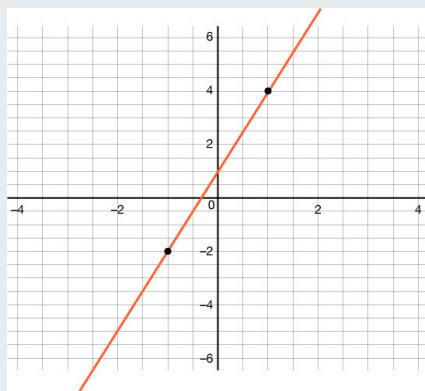
Extending Your Thinking

Use the following information to answer questions 1 – 5. A 450-gallon tank full of water is draining at a rate of 20 gallons per minute.

1. Write an equation that represents the relationship between the gallons of water in the tank and hours the tank has been draining.
2. Write an equation that represents the relationship between the gallons of water in the tank and seconds the tank has been draining.
3. Graph each of your new equations. Use the graphing tool or technology outside the course. Graph the equation that represents this scenario using the Desmos tool above.
4. In what way are all of the graphs the same? In what way are they all different?
5. How would these graphs change if we used quarts of water instead of gallons? What would stay the same?

Self Check

Which of the following points is a solution to the graph below?



- a. $(-2, -6)$
- b. $(7, 2)$
- c. $(0, 3)$
- d. $(2, 7)$

Additional Resources

Writing Equations Using Graphs in Situations

An equation that contains two unknown quantities or two quantities that vary is called an equation in two variables. A solution to such an equation is a pair of numbers that makes the equation true.

Suppose Tyler spends \$40 on T-shirts and socks. A T-shirt costs \$10 and a pair of socks costs \$2.50. If t represents the number of T-shirts and p represents the number of pairs of socks that Tyler buys, what is an equation that represents the equation?

EXAMPLE 1

Step 1 - Create a two-variable equation.

The cost is \$10 per t-shirt ($10t$) plus \$2.50 per pair of socks ($2.50p$) which equals \$40.

$$10t + 2.50p = 40$$

Now, we have to graph the equation. We will let $t = x$ and $p = y$.

Step 2 - Find the x -intercept.

To find the x -intercept, let $p = 0$.

$$10t + 2.50p = 40$$

$$10t + 2.50(0) = 40$$

$$10t + 0 = 40$$

$$t = 4$$

(4, 0) is the x -intercept.

The x -intercept is also called a solution or zero.

In this scenario, 4 represents the number of T-shirts Tyler can buy if he doesn't purchase any socks with \$40.

Step 3 - Find the y -intercept.

To find the y -intercept, let $t = 0$.

$$10t + 2.50p = 40$$

$$10(0) + 2.50p = 40$$

$$(0) + 2.50p = 40$$

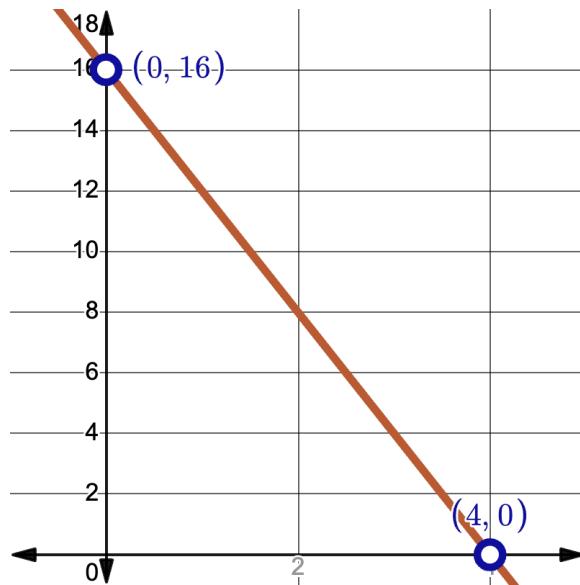
$$2.50p = 16$$

(0, 16) is the y -intercept.

In this scenario, 16 represents the number of socks Tyler can buy if he doesn't purchase any T-shirts with \$40.

Step 4 - Graph the line by connecting the intercepts.

Let's look at the graph of this equation:



Let's reflect about the graph and what it means.

EXAMPLE 2

What is the slope of the graph?

Solution

$$m = -4$$

EXAMPLE 3

What does the point $(4, 6)$ mean on this graph?

Solution

Compare your answer:

If Tyler bought 4 T-shirts and 6 pairs of socks, it would cost more than \$40.

TRY IT**Writing Equations Using Graphs in Situations**

Use Desmos or a graphing calculator to create a graph for $40x + 20y = 180$.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-5-4-writing-graphing-and-solving-a-linear-equation>\)](http://openstax.org/books/algebra-1/pages/1-5-4-writing-graphing-and-solving-a-linear-equation)

1. If x represents the number of pairs of shoes and y represents the number of pairs of jeans, what is one combination that is a solution?
2. What does the combination you identified mean on the graph?

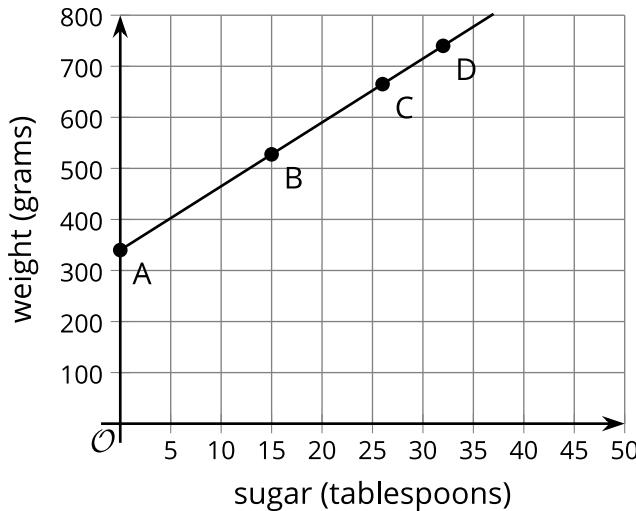
1.5.5 Examining an Equation in Two Variables and Its Graph, Part 2

Cool Down

A ceramic sugar bowl weighs 340 grams when empty. It is then filled with sugar. One tablespoon of sugar weighs 12.5 grams.

1. Write an equation to represent the relationship between the total weight of the bowl in grams, W , and the tablespoons of sugar, T .
2. When the sugar bowl is full, it weighs 740 grams. How many tablespoons of sugar can the bowl hold? Be prepared to show your reasoning.

The graph represents the relationship between the number of tablespoons of sugar in the bowl and the total weight of the bowl.

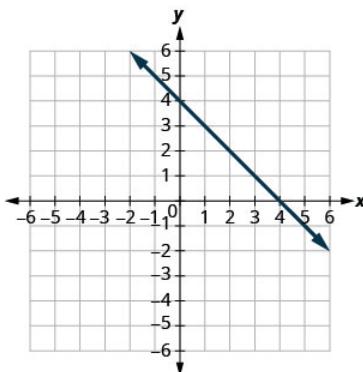


3. Which point on the graph could represent your answer to the previous question?
4. About how many tablespoons of sugar are in the bowl when the total weight is 600 grams?

1.5.6 Practice

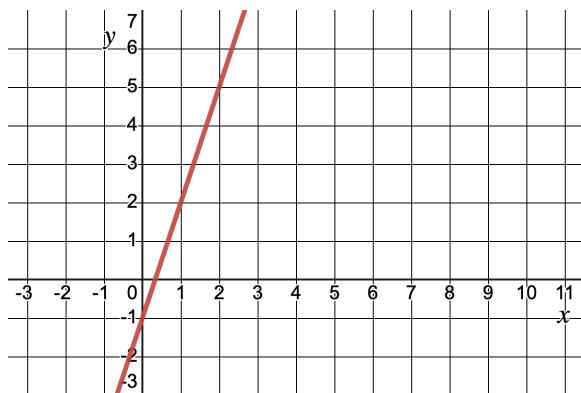
Complete the following questions to practice the skills you have learned in this lesson.

Use the graph for questions 1 - 2.



1. Which of the following ordered pairs are on the line? Select **three** answers.
 - a. (0, 4)
 - b. (2, 2)
 - c. (-2, 6)
 - d. (-1, 3)
 - e. (-4, 0)
2. Which of the following ordered pairs are solutions to the equation? Select **three** solutions.
 - a. (0, 4)
 - b. (2, 2)
 - c. (-2, 6)
 - d. (-1, 3)
 - e. (-4, 0)

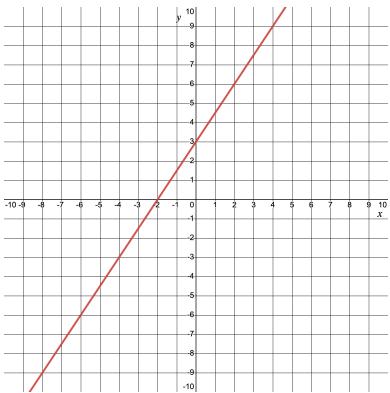
For questions 3 and 4, use the graph of $y = 3x - 1$ as shown.



3. Which of the following ordered pairs are on the line? Select **three** solutions.
 - a. (0, -1)
 - b. (1, 2)
 - c. (2, 5)
 - d. (1, 1)
 - e. (0, 1)
4. Which of the following ordered pairs are solutions to the equation? Select **three** solutions.
 - a. (0, -1)
 - b. (1, 2)

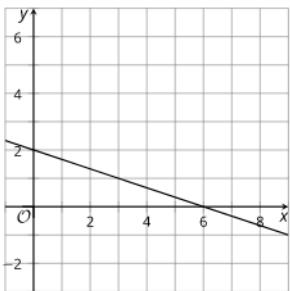
- c. (2, 5)
- d. (1, 1)
- e. (0, 1)

5. Select three points that are on the graph of the equation $4y - 6x = 12$.



- a. (-4, -3)
- b. (-1, 1.5)
- c. (0, 3)
- d. (0, -2)
- e. (6, 4)

6. Here is a graph of the equation $x + 3y = 6$.



Select three coordinate pairs that represent a solution to the equation.

- a. (0, 2)
- b. (6, 0)
- c. (2, 6)
- d. (3, 1)
- e. (8, -1)
- f. (-1, 2)

1.5.7 Lesson Summary

In this lesson, you learned how to:

- Use graphing technology to graph linear equations and identify solutions to the equations.
- Explain how the coordinates of the points on the graph of a linear equation are related to the equation.
- Explain the meaning of points on a graph in terms of the situation it represents when given the graph of a linear equation.

Here are the **activities** that helped you reach those goals:

- 1.5.1: Analyzing Graphs of Linear Equations
 - In this activity, you compared the characteristics of similar, yet different, graphs. In graphs, slopes, intercepts, axis labels, and where points are located can all create different meanings.
- 1.5.2: Graphing Linear Functions in Two Variables
 - In this activity, you reviewed how to graph points on a coordinate plane and then used your understanding to find and graph the x - and y -intercepts of an equation written in standard form. With these intercepts, you calculated the slope of the line.
 - 1.5.2: Self Check
 - 1.5.2: Additional Resources
- 1.5.3: Examining an Equation in Two Variables and Its Graph, Part 1
 - In this activity, you determined if a point was on the graph of a linear equation and what the meaning of that point was in a given situation
 - 1.5.3: Self Check
 - 1.5.3: Additional Resources
- 1.5.4: Writing, Graphing, and Solving a Linear Equation
 - In this activity, you wrote equations, graphed the equations, and used the graph to help solve the equations and find the meaning of solutions.
 - 1.5.4: Self Check
 - 1.5.4: Additional Resources
- 1.5.5: Examining an Equation in Two Variables and Its Graph, Part 2
 - In this activity, you wrote equations to express relationships. You also looked at graphs and determined the meaning of points on the graph to answer questions.

After these activities, you completed the following **practice**:

- 1.5.6: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

1.6 EQUIVALENT EQUATIONS

1.6.0 Lesson Overview

In this lesson, you will learn that equivalent equations are equations with identical solutions. Specifically, you will explore how applying properties of real numbers can create equivalent equations. Additionally, you will use context to interpret the solution to equivalent equations.

When you finish this lesson, you will be able to:

- Tell whether two expressions are equivalent and explain why or why not.
- Identify the moves that can be made to transform an equation into an equivalent one.
- Explain what it means for two equations to be equivalent and how equivalent equations can be used to describe the same situation in different ways.

Here are the **activities** that will help you reach those goals:

- 1.6.1: Exploring Equivalent Expressions
- 1.6.2: Expressing Relationships as Equations
 - 1.6.2: Self Check
 - 1.6.2: Additional Resources
- 1.6.3: Exploring Related Equations
 - 1.6.3: Self Check
 - 1.6.3: Additional Resources
- 1.6.4: Recognizing Related Equations

After that, you'll **practice and review**.

- 1.6.5: Practice
- 1.6.6: Lesson Summary

1.6.1 Exploring Equivalent Expressions

Warm Up

Your teacher will assign you one of these expressions:

$$\frac{n^2 - 9}{2(4 - 3)} \quad \text{or} \quad (n + 3) \cdot \frac{n - 3}{8 - 3 \cdot 2}$$

For questions 1 – 4, evaluate your expression when n is the given value.

- | | |
|--|-------------|
| 1. $n = 5$ | 2. $n = 7$ |
| 3. $n = 13$ | 4. $n = -1$ |
| 5. What did you discover when you compared your answers? Explain your rationale. | |

1.6.2 Expressing Relationships as Equations

Activity

In questions 1 – 3, write as many equations as possible that could represent the relationship between the ages of the two children in each family described. Be prepared to explain what each part of your equation represents.

1. In Family A, the youngest child is 7 years younger than the oldest, who is 18.
2. In Family B, the middle child is 5 years older than the youngest child.

3. Tyler thinks that the relationship between the ages of the children in Family B can be described with $2m - 2y = 10$, where m is the age of the middle child and y is the age of the youngest. Describe how Tyler came up with this equation.
4. Select three equations that are equivalent to $3a + 6 = 15$.
- $a + 3 = 12$
 - $3a = 9$
 - $\frac{1}{3}a = 1$
 - $a + 2 = 5$
5. Explain your reasoning for the equivalent equations in number 4.

Are you ready for more?

Extending Your Thinking

Here is a puzzle:

- $m + m = N$
- $N + N = p$
- $m + p = Q$
- $p + Q = ?$

Write two expressions that are equivalent to $p + Q$.

Self Check

Which equation is equivalent to $3x - 6 = 18$?

- $3x + 12 = 0$
- $x - 2 = 18$
- $3x = 12$
- $3x = 24$

Additional Resources

Finding Equivalent Equations

Equivalent equations are equations that have the same solutions. They are often found by using inverse operations or by multiplying each term in the equation by the same value.

EXAMPLE

Write three equivalent equations to $4x - 2 = 8$ using one of three strategies.

- Using an inverse operation
- Using division
- Using multiplication

Using an inverse operation:

An equivalent equation can be determined by adding 2 to both sides.

$$\begin{aligned}4x - 2 &= 8 \\4x - 2 + 2 &= 8 + 2 \\4x &= 10\end{aligned}$$

$(4x - 2 = 8)$ and $(4x = 10)$ are equivalent.

Using division:

An equivalent equation can be determined by dividing by 2 (or multiplying by $\frac{1}{2}$).

$$\begin{aligned}4x - 2 &= 8 \\(\frac{1}{2})(4x - 2) &= (\frac{1}{2})(8) \\2x - 1 &= 4\end{aligned}$$

$(4x - 2 = 8)$ and $(2x - 1 = 4)$ are equivalent.

Using multiplication:

An equivalent equation can be determined by multiplying the same number to both sides.

What is the equivalent equation if you multiply each term in the equation $4x - 2 = 8$ by 3?

TRY IT**Finding Equivalent Equations**

1. Determine the equivalent equation if you subtract 5 from each side of the equation,
 $3x + 5 = 15$.
2. Explain how $(3x = 10)$ is related to
 $(x = \frac{10}{3})$.
3. Multiply every term by the same number to determine an equivalent equation to
 $3x + 5 = 15$.
4. Is $(3x + 5 = 15)$ equivalent to
 $(6x + 15 = 60)$?
5. Explain your reasoning for your answer to number 4 and whether the equations $(3x + 5 = 15)$ and $(6x + 15 = 60)$ are equivalent.

1.6.3 Exploring Related Equations

Activity

With a partner, discuss the following situations and equations. For each, consider:

- What does the solution mean in the context of the situation?
- Are the given values solutions?

If operations are applied correctly, the solution to an equation is also the solution to all equations equivalent to it. If operations are incorrectly applied, the solution of each equation is different.

For questions 1 and 2, use the following situation and equation:

Noah is buying a pair of jeans and using a coupon for 10% off. The total price is \$56.70, which includes \$2.70 in sales tax. Noah's purchase can be modeled by the equation:

$$x - 0.1x + 2.70 = 56.70$$

1. What does the solution to the equation mean in this situation?
2. How can you verify that 70 is not a solution but 60 is the solution?
3. What was done to Noah's equation to make this one?
4. What is the interpretation of this new equation?
5. Is the solution the same?
6. What was done to Noah's equation to make this one?
7. What is the interpretation of this new equation?
8. Is the solution the same?
9. What was done to Noah's equation to make this one?
10. What is the interpretation of this new equation?
11. Is the solution the same?

For questions 12 – 14, use the equation $x - 0.1x = 56.70$.

12. What was done to Noah's equation to make this one?
13. What is the interpretation of this new equation?
14. Is the solution the same?

For questions 15 – 17, use the equation $x - 0.1x = 59.40$

15. What was done to Noah's equation to make this one?
16. What is the interpretation of this new equation?
17. Is the solution the same?

For questions 18–20, use the equation $2(x - 0.1x + 2.70) = 56.70$.

18. What was done to Noah's equation to make this one?
19. What is the interpretation of this new equation?
20. Is the solution the same?
21. Based on your work above, which of the six equations are equivalent to the original equation, $x - 0.1x + 2.70 = 56.70$? Select the **three** equations that are equivalent to the original.
- $100x - 10x + 270 = 5670$
 - $x - 0.1x = 54$
 - $0.9x + 2.70 = 56.70$
 - $x - 0.1x = 56.70$
 - $x - 0.1x = 59.40$
 - $2(x - 0.1x + 2.70) = 56.70$

Video: Looking at Equivalent Equations

Watch the following video to learn more about why these are equivalent equations.

[Access multimedia content \(<http://openstax.org/books/algebra-1/pages/1-6-3-exploring-related-equations>\)](http://openstax.org/books/algebra-1/pages/1-6-3-exploring-related-equations)

Self Check

Which of the following is a correct next step to create an equation equivalent to $x - 0.2x - 4.2 = 13.4$?

- $2(x - 0.2x) = 17.6$
- $x - 0.2x = 9.2$
- $10(x - 0.2x - 4.2) = 13.4$
- $0.8x - 4.2 = 13.4$

Additional Resources

Properties of Equality

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Subtraction Property of Equality For any real numbers a , b , and c , if $a = b$, then $a - c = b - c$.	Addition Property of Equality For any real numbers a , b , and c , if $a = b$, then $a + c = b + c$.
Division Property of Equality For any real numbers a , b , and c , and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.	Multiplication Property of Equality For any real numbers a , b , and c , if $a = b$, then $ac = bc$.

For equations to be equivalent, inverse operations are used and must be applied to both sides of an equation so it remains balanced.

Inverse operations are operations that “undo” other operations.

EXAMPLE 1

Solve $3x = 15$.

Since 3 is being multiplied by x , to solve for x , divide both sides by 3.

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

EXAMPLE 2

Solve $\frac{x}{2} + 4 = 12$.

Step 1 - Subtract 4 from both sides.

$$\frac{x}{2} + 4 - 4 = 12 - 4$$

Step 2 - Simplify.

$$\frac{x}{2} = 8$$

Step 3 - Multiply both sides by 2.

$$2 \times \frac{x}{2} = 8 \times 2$$

Step 4 - Simplify.

$$x = 16$$

**TRY IT****Properties of Equality**

For questions 1 – 2, use the scenario:

Denae bought 6 pounds of grapes for \$10.74.

1. Write an equation for the situation.
2. Solve the equation.

1.6.4 Recognizing Related Equations

Cool Down

A cardboard box, which weighs 0.6 pound when empty, is filled with 15 bags of beans and a 4-pound bag of rice. The total weight of the box and the contents inside it is 25.6 pounds. One way to represent this situation is with the equation $0.6 + 15b + 4 = 25.6$.

1. In this situation, what does the solution to the equation represent?
2. Identify the **three** equations that are equivalent to $0.6 + 15b + 4 = 25.6$.
 - a. $15b + 4 = 25.6$
 - b. $15b + 4 = 25$
 - c. $3(0.6 + 15b + 4) = 76.8$
 - d. $15b = 25.6$
 - e. $15b = 21$

1.6.5 Practice

Complete the following questions to practice the skills you have learned in this lesson.

1. Select three equations that have the same solution as the equation $3x - 12 = 24$.

- a. $15x - 60 = 120$
- b. $3x = 12$
- c. $3x = 36$
- d. $x - 4 = 8$
- e. $12x - 12 = 24$

2. Select three equations that have the same solution as $2x - 5 = 15$.

- a. $2x = 10$
- b. $2x = 20$
- c. $2(x - 5) = 15$
- d. $2x - 20 = 0$
- e. $4x - 10 = 30$
- f. $15 = 5 - 2x$

3. Select four of the equations that are equivalent to: $x + 0.4x - 4.2 = 12.8$.

- a. $1.4x - 4.2 = 12.8$
- b. $1.4x = 8.6$
- c. $1.4x = 17$
- d. $14x - 42 = 128$
- e. $10x + 4x - 42 = 12.8$
- f. $14x = 170$

4. Select two of the following that are equivalent to $3x - 6 = 12$.

- a. $3x = 6$
- b. $3(x - 2) = 12$
- c. $x - 2 = 4$
- d. $x - 2 = 12$
- e. $x - 6 = 4$

5. Identify which step the mistake is made in finding an equivalent equation to $2(x - 4) = 16$.

Step 1: $2x - 4 = 16$

Step 2: $2x = 20$

Step 3: $x = 10$

- a. Step 1
- b. Step 2
- c. Step 3

6. Identify which step the mistake is made in finding an equivalent equation to $2x + 3x - 4 = 20$.

Step 1: $5x - 4 = 20$

Step 2: $5x = 16$

Step 3: $x = 16/5$

- a. Step 1
- b. Step 2
- c. Step 3

7. Identify which step the mistake is made in finding an equivalent equation to $7x - 4x - 5 = 16$.

Step 1: $3x - 5 = 16$

Step 2: $3x = 21$

Step 3: $x = 62$

- a. Step 1
 - b. Step 2
 - c. Step 3
8. Fill in the blank to complete the equivalent equation to $3(x - 5) = 30$.
- $$3x - \underline{\hspace{2cm}} = 30$$
9. Fill in the blank to complete the equivalent equation to $\frac{x}{2} - 5 = 11$.
- $$x - \underline{\hspace{2cm}} = 22$$
10. Fill in the blank to complete the equivalent equation to $3(x - 9) = 24$.
- $$x - \underline{\hspace{2cm}} = 8$$

1.6.6 Lesson Summary

In this lesson, you learned how to:

- Tell whether two expressions are equivalent and explain why or why not.
- Identify the moves that can be made to transform an equation into an equivalent one.
- Explain what it means for two equations to be equivalent, and how equivalent equations can be used to describe the same situation in different ways.

Here are the **activities** that helped you reach those goals:

- 1.6.1 Exploring Equivalent Expressions
 - In this activity, you substituted values into equivalent expressions to show they are equivalent.
- 1.6.2 Expressing Relationships as Equations
 - In this activity, you wrote equivalent equations given different relationships.
 - 1.6.2 Self Check
 - 1.6.2 Additional Resources
- 1.6.3 Exploring Related Equations
 - In this activity, you identified changes that were made to create equivalent equations then determined if other equations were also equivalent after certain moves were made to the equation.
 - 1.6.3 Self Check
 - 1.6.3 Additional Resources
- 1.6.4 Recognizing Related Equations
 - In this activity, you determined which equations in a given group were all equivalent to each other.

After these activities, you completed the following **practice**:

- 1.6.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?