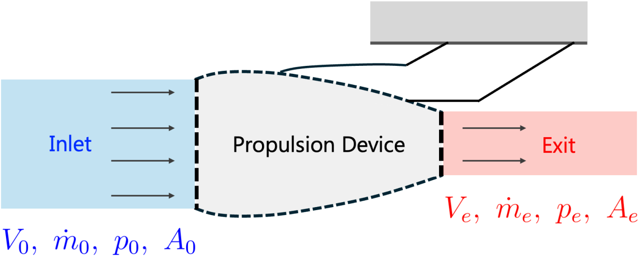
# The thrust equation

**How to calculate thrust? – The fundamental problem**

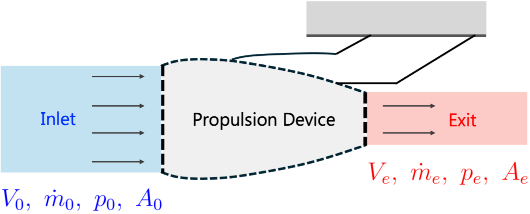



**Given:** flow conditions at the inlet and exit of a control volume (propulsion device)

- Inlet: station "$0$", free stream conditions, same as "$\infty$"
- Exit: station "$e$"

**Mass flow rate ($\dot{m} = \rho V A$):** the mass property to track for a moving fluid; the amount of mass moving through a given plane over some amount of time

**How to calculate thrust?**



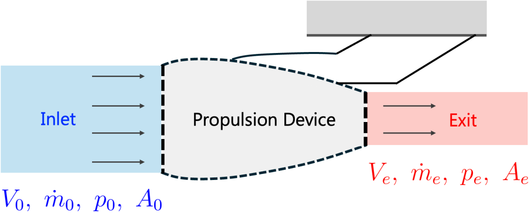

Overall, thrust generated by a propulsion system is **a result of two effects:**

1. **Change in momentum of the airflow:** thrust can be generated by the time rate of change of momentum (the product of the mass and velocity) of the airflow
   - **Newton's second law:** force is the rate of change of momentum w.r.t. time

$$F = \frac{dp}{dt} = \frac{d(mV)}{dt}$$

2. **Net change of pressure:** if the exit pressure is different from the inlet pressure

**1. Change in momentum of the airflow**




**Thrust** $=$ rate of change of momentum with respect to time

$$T = \frac{[mV]_{\text{Exit}} - [mV]_{\text{Inlet}}}{t_{\text{Exit}} - t_{\text{Inlet}}} = \frac{m_e V_e - m_0 V_0}{t_e - t_0}$$

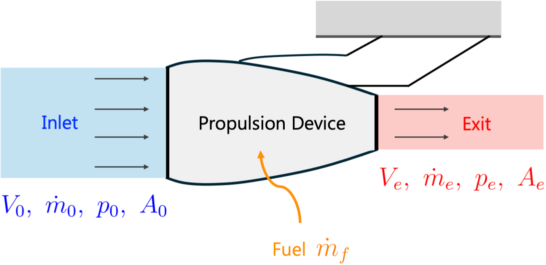
- With constant mass:

$$T = m\frac{V_e - V_0}{t_e - t_0} = ma$$

- With changing mass:

$$T = \frac{m_e}{t_e - t_0} V_e - \frac{m_0}{t_e - t_0} V_0 = \dot{m}_e V_e - \dot{m}_0 V_0$$

**1. Change in momentum of the airflow**



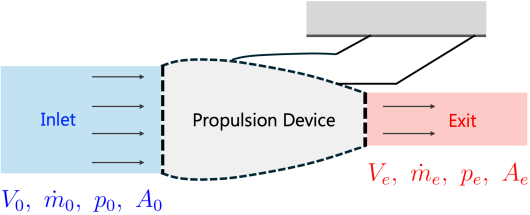

**The mass flow of air is changing!** Fuel is added and burned inside

- The mass flow of air entering the duct is $\dot{m}_0$
- The mass flow of gas leaving the duct is $\dot{m}_0 + \dot{m}_f$

Hence, thrust generated by change in momentum of the airflow is

$$T = \dot{m}_e V_e - \dot{m}_0 V_0 = (\dot{m}_0 + \dot{m}_f) V_e - \dot{m}_0 V_0$$

**2. Net change of pressure**




When the exit pressure is different from the inlet pressure, across the exit area there is an additional force term (although this effect is relatively small)

$$F = (p_e - p_0)A_e$$

**The general thrust equation** is then given by

$$\boxed{T = (\dot{m}_0 + \dot{m}_f)V_e - \dot{m}_0 V_0 + (p_e - p_0)A_e}$$

## The thrust equation

### Design approaches of aircraft engines

Some simplifications on the thrust equation:

- The mass of fuel added is usually small compared to the mass of air
- The change of pressure effect is relatively small

$$
\begin{aligned}
T &= (\dot{m}_0 + \underbrace{\cancel{\dot{m}_f}}_{\text{Small}})\,V_e - \dot{m}_0\,V_0 + \underbrace{\cancel{(p_e - p_0)A_e}}_{\text{Small}} \\
&= \boxed{\dot{m}_0(V_e - V_0)}
\end{aligned}
$$

**Two major ways** to produce **high thrust:**

1. **Make $\dot{m}_0$ as high as possible:** a large amount of air is processed each second, but the velocity is not changed very much
   - Examples: propeller aircraft, high-bypass turbofan engines
2. **Make $V_e$ very much greater than $V_0$:** a moderate amount of flow is accelerated to a high velocity in these engines
   - Examples: pure turbojets, turbojets with afterburners, and rockets