The drag polar of the complete aircraft

Describing the relationship between C_D and C_L, the drag polar includes **all necessary aerodynamic information** for the performance analysis of an aircraft.

Previously, we have learned that

$$C_D = C_{D,0} + \frac{C_L^2}{\pi e \mathsf{AR}}$$

where

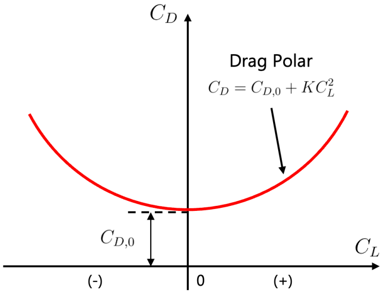
- C_D: the drag coefficient for the complete aircraft
- $C_{D,0}$: the parasite drag coefficient at zero lift ($\alpha = \alpha_{L=0}$)
- C_L: the total lift coefficient, including the small contributions from the tail and fuselage

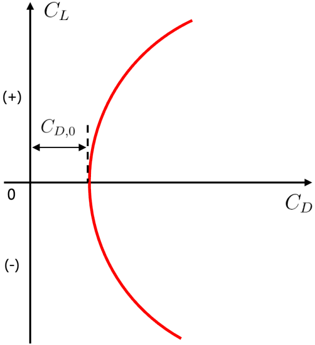
For simplification, let $K = 1/(\pi e \mathsf{AR})$ (essentially wing properties at this stage), we have

$$\boxed{C_D = C_{D,0} + K C_L^2}$$

The two identical representations

C_D vs. C_L, and C_L vs. C_D graphs

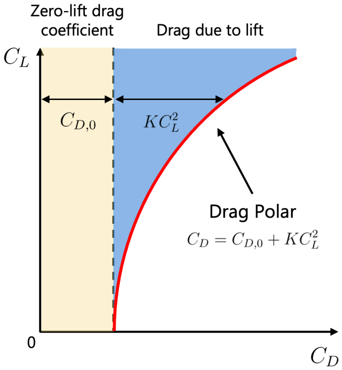




- The C_L vs. C_D representation is more common
- Negative C_L's do not occur frequently. Usually only the positive C_L portion is shown

The C_L vs. C_D graph

$$\underbrace{C_D}_{\text{Total drag}} = \underbrace{C_{D,0}}_{\text{Zero-lift drag}} + \underbrace{KC_L^2}_{\text{Drag due to lift}}$$



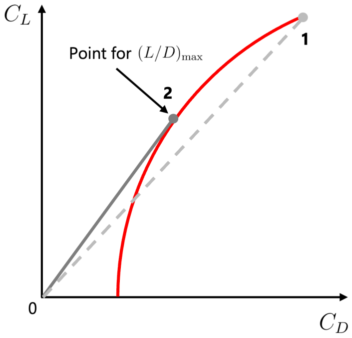
Each point on the drag polar corresponds to a <u>different angle of attack</u> for the aircraft

The maximum lift-to-drag ratio $(L/D)_{\max}$

Previously, we know that $(L/D)_{\max}$ **is essentially** $(C_L/C_D)_{\max}$

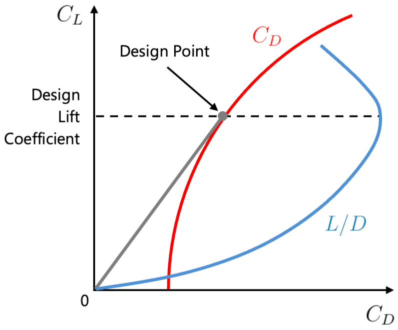
Construct a straight line **from the origin**,

- The slope of the line is equal to L/D
- The tangent line to the drag polar locates the point of $(L/D)_{\max}$



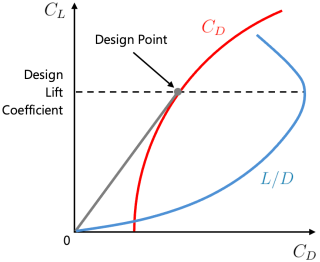
From the C_L vs. C_D graph, we can identify the following design features for the aircraft

- **The design point:** the tangent point corresponding to $(L/D)_{\max}$
- **Thr design lift coefficient:** the C_L corresponding to $(L/D)_{\max}$



As C_L increases, L/D first increases, then decreases after the design lift coefficient.
How to obtain the design lift coefficient?

Obtaining the design lift coefficient



Take derivative of C_L/C_D w.r.t. C_L:

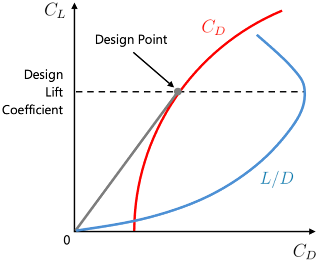
$$\frac{d}{dC_L}\left(\frac{C_L}{C_D}\right) = \frac{d}{dC_L}\left(\frac{C_L}{C_{D,0}+KC_L^2}\right) = 0$$

$$C_{D,0} + KC_L^2 - 2KC_L^2 = 0$$

$$C_{D,0} = KC_L^2$$

$$\boxed{C_L = \sqrt{\frac{C_{D,0}}{K}}}$$

Solution for the maximum lift-to-drag ratio

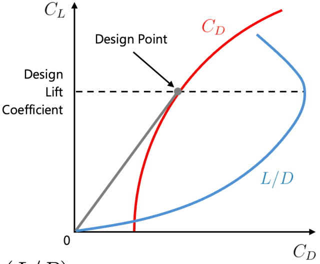


We can obtain the design lift coefficient when $C_{D,0} = KC_L^2$, i.e., when zero-lift drag and drag due to lift have **equal magnitudes**.

With the design lift coefficient $C_L = \sqrt{C_{D,0}/K}$, we have the **maximum lift-to-drag ratio**

$$\left(\frac{L}{D}\right)_{\max} = \frac{\sqrt{C_{D,0}/K}}{C_{D,0} + C_{D,0}} = \boxed{\sqrt{\frac{1}{4KC_{D,0}}}}$$

Solution for the corresponding airspeed



To obtain the airspeed to for $(L/D)_{\max}$, we use

- Steady and level flight condition:

$$L = W$$

- The lift equation

$$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$$

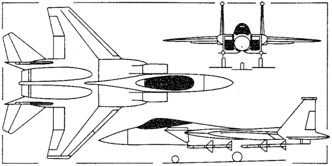
Hence,

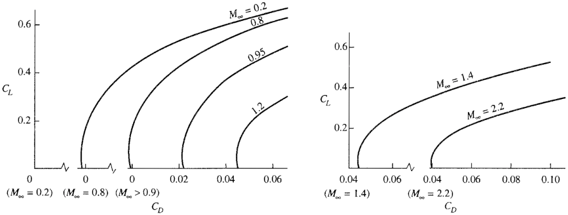
$$V_\infty = \sqrt{\frac{L}{\frac{1}{2}\rho_\infty S C_L}} = \sqrt{\frac{2}{\rho_\infty}\left(\frac{W}{S}\right)\frac{1}{C_L}} = \boxed{\sqrt{\frac{2}{\rho_\infty}\left(\frac{W}{S}\right)\sqrt{\frac{K}{C_{D,0}}}}}$$

Influence of Mach number

Because both C_L and C_D are functions of the Mach number, the same aircraft will have **different drag polars at different Mach numbers**

At **low subsonic Mach numbers**, the differences will be small and can be ignored





(a) Subsonic and Transonic

(b) Supersonic