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About this Course

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About Openstax Algebra 1

OpenStax *Algebra 1* is a complete curriculum and an open education resource (OER) that is standards-aligned, easy to implement, supportive of diverse learners, and engaging for students. *Algebra 1* is supported by instructional resources, including implementation guidance, pacing suggestions, and ancillary engagement materials.

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Errata

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STUDENTS START HERE

Quick Start

Welcome to your *Algebra 1* module! We're genuinely excited for you to join our community and embark on a transformative learning journey!

This user-friendly guide is your roadmap to navigating the features and resources of our unique *Algebra 1* content. Let's raise the bar on your algebra skills—get started with confidence today!

I. Where to Start

Algebra 1 has nine units, each of which begins with a special lesson called Overview and Readiness. These lessons provide a miniature review of the knowledge you need for the unit ahead. Completing these lessons will help you feel confident in your ability to handle what comes next.

Units 4 and 6 also have inquiry projects, meant to revisit previous knowledge while introducing you to upcoming topics. We recommend completing these alongside their respective Overview and Readiness lessons.

II. Unit Structure

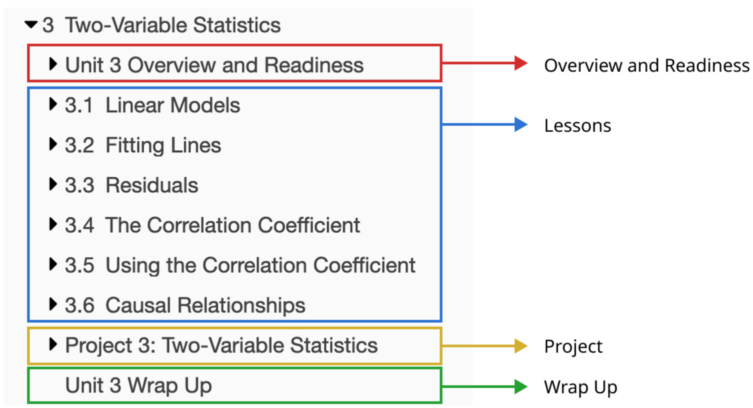


Image 1

After completing the **Overview and Readiness** lesson, you can progress through the regular **Lessons**.

Each Lesson has multiple pages to guide you through learning new skills and reinforcing old ones.

At the end of each unit, you will also find a **Project** and **Wrap Up** lesson covering the topics in the unit. Use these lessons to cement understanding of the skills you just learned.

III. Lesson Structure

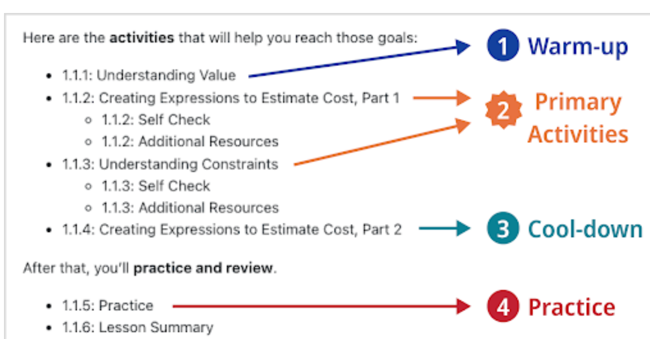


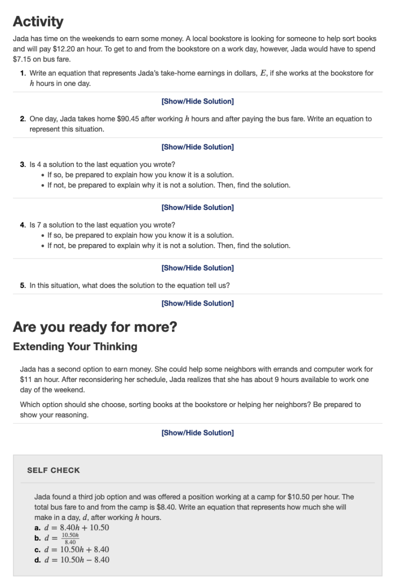
Image 2

Lesson Activities

The first page of a lesson is called "Lesson Overview." Here you will find the different types of activities included in the lesson (Image 2).

- **Warm up:** This is the first activity in a lesson.
- **Primary activities:** These are unique and need extra focus. We'll take a close look at them in the next part of this guide.
- **Cool-down:** This is the last activity in a lesson.
- **Practice:** This is a practice page with problems on the topics you learned in the lesson.

IV. Primary Activities



Activity

Jada has time on the weekends to earn some money. A local bookstore is looking for someone to help sort books and will pay $12.20 an hour. To get to and from the bookstore on a work day, however, Jada would have to spend $7.15 on bus fare.

1. Write an equation that represents Jada's take-home earnings in dollars, E, if she works at the bookstore for h hours in one day.

[Show/Hide Solution]

2. One day, Jada takes home $90.45 after working h hours and after paying the bus fare. Write an equation to represent this situation.

[Show/Hide Solution]

3. Is 4 a solution to the last equation you wrote?
   - If so, be prepared to explain how you know it is a solution.
   - If not, be prepared to explain why it is not a solution. Then, find the solution.

[Show/Hide Solution]

4. Is 7 a solution to the last equation you wrote?
   - If so, be prepared to explain how you know it is a solution.
   - If not, be prepared to explain why it is not a solution. Then, find the solution.

[Show/Hide Solution]

5. In this situation, what does the solution to the equation tell us?

[Show/Hide Solution]

Are you ready for more?

Extending Your Thinking

Jada has a second option to earn money. She could help some neighbors with errands and computer work for $11 an hour. After reconsidering her schedule, Jada realizes that she has about 9 hours available to work one day of the weekend.

Which option should she choose, sorting books at the bookstore or helping her neighbors? Be prepared to show your reasoning.

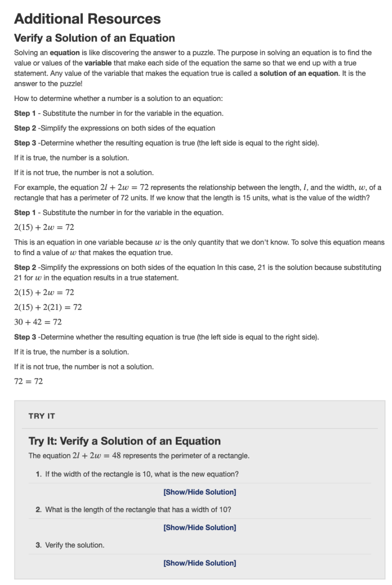
[Show/Hide Solution]

SELF CHECK

Jada found a third job option and was offered a position working at a camp for $10.50 per hour. The total bus fare to and from the camp is $8.40. Write an equation that represents how much she will make in a day, d, after working h hours.

a. $d = 8.40h + 10.50$
b. $d = \frac{10.50h}{8.40}$
c. $d = 10.50h + 8.40$
d. $d = 10.50h - 8.40$

Image 3



Additional Resources

Verify a Solution of an Equation

Solving an **equation** is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the **variable** that make each side of the equation the same so that we end up with a true statement. Any value of the variable that makes the equation true is called a **solution of an equation**. It is the answer to the puzzle!

How to determine whether a number is a solution to an equation:

Step 1 - Substitute the number in for the variable in the equation.

Step 2 -Simplify the expressions on both sides of the equation

Step 3 -Determine whether the resulting equation is true (the left side is equal to the right side).

If it is true, the number is a solution.

If it is not true, the number is not a solution.

For example, the equation $2l + 2w = 72$ represents the relationship between the length, l, and the width, w, of a rectangle that has a perimeter of 72 units. If we know that the length is 15 units, what is the value of the width?

Step 1 - Substitute the number in for the variable in the equation.

$2(15) + 2w = 72$

This is an equation in one variable because w is the only quantity that we don't know. To solve this equation means to find a value of w that makes the equation true.

Step 2 -Simplify the expressions on both sides of the equation In this case, 21 is the solution because substituting 21 for w in the equation results in a true statement.

$2(15) + 2w = 72$

$2(15) + 2(21) = 72$

$30 + 42 = 72$

Step 3 -Determine whether the resulting equation is true (the left side is equal to the right side).

If it is true, the number is a solution.

If it is not true, the number is not a solution.

$72 = 72$

TRY IT

Try It: Verify a Solution of an Equation

The equation $2l + 2w = 48$ represents the perimeter of a rectangle.

1. If the width of the rectangle is 10, what is the new equation?

[Show/Hide Solution]

2. What is the length of the rectangle that has a width of 10?

[Show/Hide Solution]

3. Verify the solution.

[Show/Hide Solution]

Image 4

Primary Activities

Here's an example of one of these unique types of activities. Each lesson has 2-3 of these.

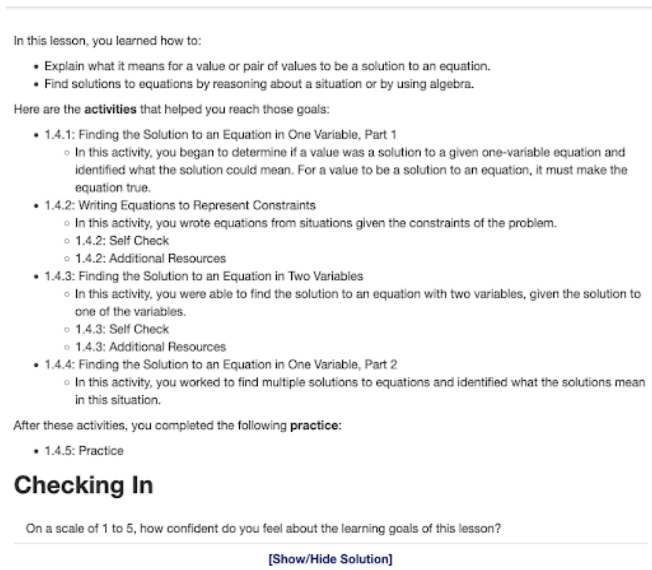
Some activities have a section called **Are you ready for more?** This section, meant to expand your thinking, is optional unless otherwise stated by your teacher (Image 3).

After you have finished the activity, you will do a multiple choice question called a **Self Check** (Image 3). If you feel confident in your skills, you may move on to the next lesson.

Additional Resources

If you want a refresher on the skills tested in the Self Check, read through the **Additional Resources** section, then complete the **Try It** section to test yourself on what you've learned. This section is optional unless otherwise stated by your teacher (Image 4).

V. End of Lesson



In this lesson, you learned how to:

- Explain what it means for a value or pair of values to be a solution to an equation.
- Find solutions to equations by reasoning about a situation or by using algebra.

Here are the **activities** that helped you reach those goals:

- 1.4.1: Finding the Solution to an Equation in One Variable, Part 1
  - In this activity, you began to determine if a value was a solution to a given one-variable equation and identified what the solution could mean. For a value to be a solution to an equation, it must make the equation true.
- 1.4.2: Writing Equations to Represent Constraints
  - In this activity, you wrote equations from situations given the constraints of the problem.
  - 1.4.2: Self Check
  - 1.4.2: Additional Resources
- 1.4.3: Finding the Solution to an Equation in Two Variables
  - In this activity, you were able to find the solution to an equation with two variables, given the solution to one of the variables.
  - 1.4.3: Self Check
  - 1.4.3: Additional Resources
- 1.4.4: Finding the Solution to an Equation in One Variable, Part 2
  - In this activity, you worked to find multiple solutions to equations and identified what the solutions mean in this situation.

After these activities, you completed the following **practice**:

- 1.4.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

[Show/Hide Solution]

Image 5

Lesson Summary

Here you will review the topics covered and the activities you completed (Image 5). You will also have an opportunity to check-in on your progress with the learning goals from the lesson.

After you've finished the Unit, you can use the **Wrap Up** lesson to check in on the important skills from each lesson. If there are any skills you want to brush up on, use the Lesson Summary page to find the most relevant activities to revisit.

Useful Resources

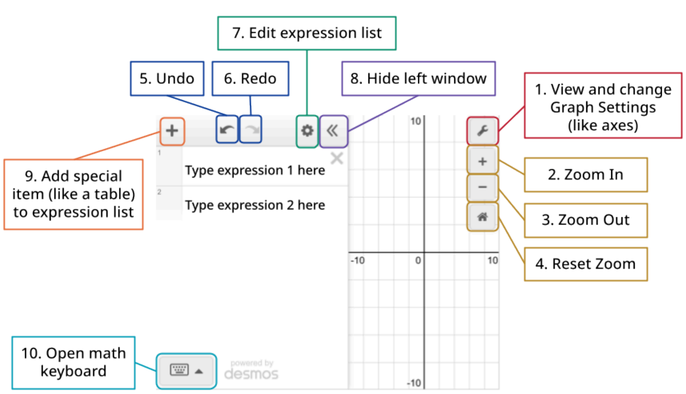
Want some study tips? Forgot the meaning of a word? Check out the sections **Students Start Here**, **Supporting All Learners**, and the **Index** for our curated resources.

Using Desmos

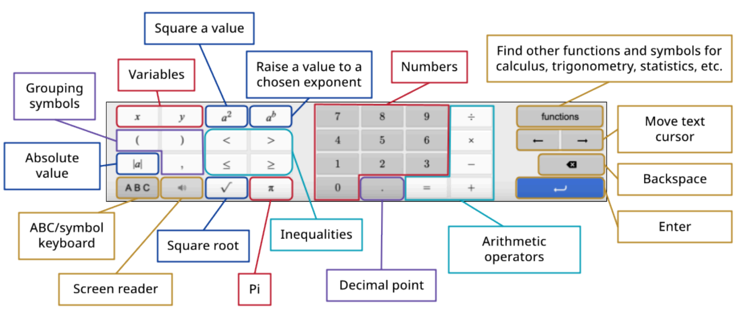
Throughout the *Algebra 1* curriculum, you will find Desmos activities and graphing calculators embedded in lessons. Desmos activities are interactive online problem sets located on the Desmos website. They can be assigned to your students and graded automatically if you have a Desmos account. Desmos graphing calculators are interactive online graphs embedded under graphing problems in the *Algebra 1* course. Note, the embedded calculator adheres to instructional requirements, not assessment requirements.

We have created a how-to guide with helpful screenshots and links to help you and your students use Desmos. If you need additional help, [view the official Desmos guide (https://openstax.org/r/desmos-graphing)](https://openstax.org/r/desmos-graphing).

I. Understanding Desmos Tools



1. **View and change graph settings**
    - Change limits of the x- and y-axis and the scale for each
    - Toggle display color and font size
    - Toggle between radians and degrees
    - Turn on Braille mode
2. **Zoom in:** focus on one part of the graph and see less of the rest of it
3. **Zoom out:** see less of one part of the graph and more of the rest of it
4. **Reset zoom:** return to default window where $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.
5. **Undo:** undo your last action
6. **Redo:** redo your last action
7. **Edit expression list:**
    - Quickly copy or delete expressions in the expression list
    - Convert expressions to tables
    - Select the colored dot to the left of the expression to change the line's color, thickness, opacity, and line type
    - Quickly clear graph
8. **Hide left window:** collapse or expand the window containing the expression list
9. **Add special item to expression list:** add an
    - expression, an equation of a line;
    - note, a text description or note about the graph (a good scaffold to support differentiation);
    - table, a basic table where you can input x- and y-coordinates;
    - image, a resizable image from your computer
10. **Open math keyboard:** open a digital keypad that allows you to quickly add special math functions and symbols, like sin or ÷, to your expression. View our guide to the Desmos math keyboard below.
    - Tip: You can use your physical, non-Desmos keyboard to type math and interact with the graph. To learn more check out these guides Math Notation Guide by PurpleMath (https://openstax.org/r/PurpleMath) and Desmos Keyboard Shortcuts (https://help.desmos.com/hc/en-us/articles/4405966811021-Keyboard-Shortcuts)



II. Graphing Regressions Using the Desmos Graphing Calculator

You can use Desmos to graph a regression, regression line, regression curve, scatter plot, stat plot, and create a line of best fit. Check out our guide to using these features in Desmos, if you have additional questions, view the official Desmos regressions guide (https://openstax.org/r/desmos-regressions).

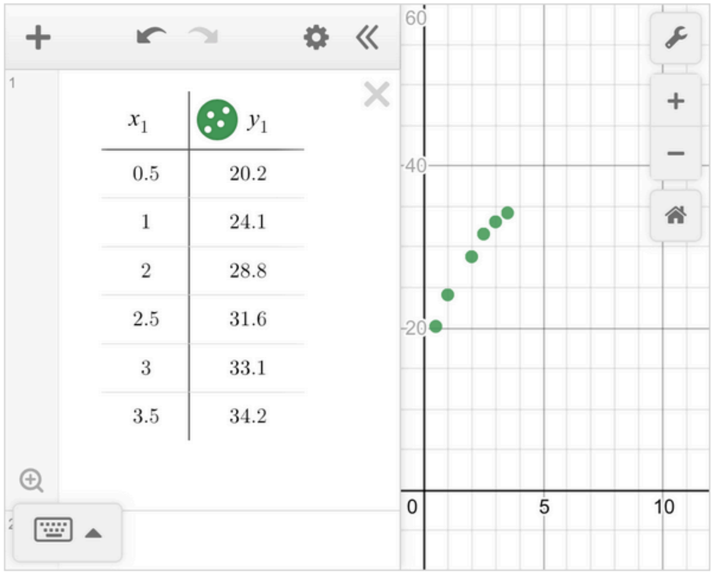


Image 1

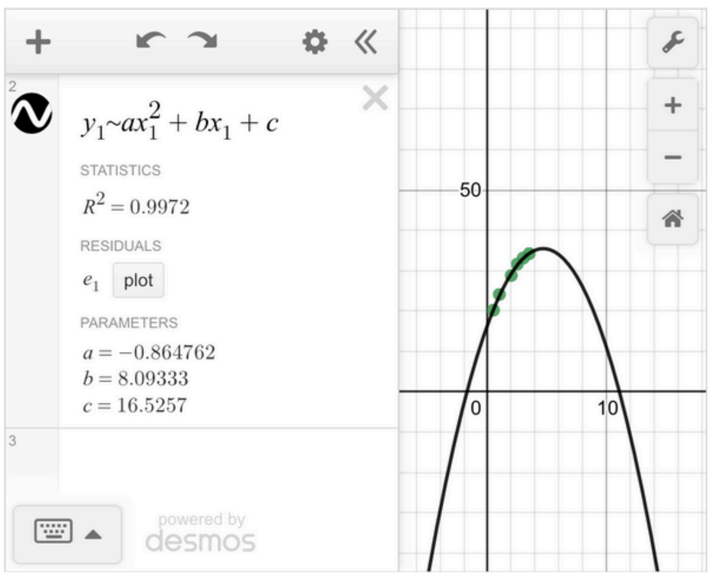


Image 2

Creating a Scatter Plot from a Table (Image 1)

1. On the Desmos graphing calculator, click the + button in the top left corner and add a table.
2. You can copy and paste an existing table in, or enter it manually. On the left side of the table, type the first x -coordinate from your reference table. On the right side, type the first y-coordinate. This will create a point on your graph.
3. Go to the next line of the table and repeat until all coordinates have been entered. Inputting all the coordinates will create a graph called a scatter plot, pictured above and left.
   - Tip: If you do not see your point graphed in the Desmos window, you may need to “Zoom” or change the graph settings for the window.

Creating a Line of Best Fit on a Scatter Plot (Image 2)

1. To create a line or curve that approximates the data trend, calculate a regression. Depending on the type of data (linear, exponential, or quadratic), the format of the regression equation will differ:
   - Linear: $y_1 \sim mx_1 + b$
   - Exponential: $y_1 \sim ax_1 b$
   - Quadratic: $y_1 \sim ax_1^2 + bx_1 + c$

2. Click the pointer below the table into entry box #2 to enter a new function. The cursor should start blinking. Enter the correct equation you found in step 4 using the following tips:
    - Notice that when you type the "1" after y or x, Desmos automatically places it as subscript.
    - Also notice that we are not using an equal sign. The symbol ~ means approximately and tells Desmos to approximate a line or curve of best fit. On your keyboard, you can type it with Shift+` (in the top left corner). If you are using the Desmos on-screen keyboard, it will be in the ABC/symbol keyboard.
    - To type an exponent, use the ^ symbol on your physical keyboard. You can type it using Shift+6. You can also use the Desmos keyboard button a^2 or a^b.
3. Once you have entered the regression equation, you can press Enter. The graph of the line or curve of best fit will appear. Pictured above and right is a quadratic curve of best fit. Desmos will also report correlation statistics such as r and R^2, as well as the a value, b value, and c value in the expression list below the quadratic equation you entered.
    - Tip: The window should be appropriately set to display the points in the table, but sometimes you have to adjust the window of the graph in order to properly view your data set. Click and drag the mouse to shift the viewing window. To zoom in or out, use the buttons in the top right or the scrolling wheel on the mouse. Or, edit the graph settings.
    - Tip: If you are having difficulty finding the correlation coefficient, r, type the command $‘corr(x_1, y_1)’$ on another expression line. It may also be helpful to check the accuracy of the data entered into the x_1 and y_1 columns of the table.

Additional Resources

You may find it helpful to keep the following resources on hand as you progress through the course:

- English-Spanish Glossary (https://openstax.org/r/english-spanish-glossary)
- Algebra 1 STAAR Reference Materials (https://openstax.org/r/STAAR-ref)
- Algebra 1 STAAR Graph Paper (https://openstax.org/r/STAAR-graph-paper)

UNIT 1 OVERVIEW AND READINESS

Unit 1 Overview

Why Should I Care?

Do you know what a budget is? Have you ever used one? Watch the video to learn a bit about how City Manager Arthur Noriega uses linear equations to manage the budget for the city of Miami.

Access multimedia content (http://openstax.org/books/algebra-1/pages/1-unit-1-overview)
In this unit, you will learn about using variables, expressions, and equations to model relationships. You will learn that you can represent a constraint, or limit on what is possible in a situation, using expressions, equations, and inequalities. This will be helpful whether you want to know how soon you'll be able to save for a new bicycle, or if you can afford to go to a concert with your friends in two weeks.

You will also learn to transform one equation into an equivalent equation. Sometimes you want to write an equation in a different form so you can solve for a certain variable or so you can write the equation in a form that makes it easier to identify information about the relationship. You will learn how the form and parts of linear equations are related to the features of its graph. A graph can be helpful to model a relationship and to make sense of the constraints.

Building Character: Social Intelligence

As a city manager, Arthur Noriega must work with others to develop budgets. Similarly, in this course, you will need to work with your classmates to solve problems. Working well with others is a skill that will be useful to you in this course, as well as outside of the classroom. Most jobs require the ability to work well, or collaborate, with other people.

Throughout this unit, you will work on developing your **social intelligence**. Social intelligence is the ability to connect with other people. Having this ability allows you to interact in a more positive manner with others. Being empathetic, or sensitive to what others are feeling, shows that you care about others. In fact, empathetic people are less likely to experience anxiety, depression, and addictions later in life. They are also more likely to be hired, promoted, earn more money, and have happier marriages and better-adjusted children.

Think about your current sense of social intelligence. How many of the following statements are true for you?

- I have a lot of relationships that are mutually beneficial, enjoyable, and supportive.
- Most of the time, I can tell how other people feel and have a good idea about how to respond appropriately.
- My relationships make me feel good about myself.
- The people in my life help me be my best.

Don't worry if none of these statements are true for you. Developing this trait takes time. Your first step starts today!

Am I Ready to Learn This?

Getting Ready for Unit 1

To be ready for this unit, you need to be able to:

- Solve linear equations using a general strategy.
- Classify equations.
- Identify points on a graph.

Next, you will answer some questions that will help you see how ready you are for this unit. If you need help, some mini-lessons will help you brush up on anything you need to work on.

Solve Linear Equations: Mini-Lesson Review

? **MINI LESSON QUESTION**

Question #1: Solve Linear Equations

Solve: $2(m-6)+3=m-1$.

a. $m=-10$
b. $m=14$

c. $m = 8$
d. $m = 2$

Solving Linear Equations Using a General Strategy

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation. For example, in $x + 2 = 3$, the value of the variable x is 1.

To find the solution to an equation in one variable, the goal is to isolate the variable on one side of the equation. For example,

$3x + 1 = 9 - x$

$+x + x$

$4x + 1 = 9$

$-1 - 1$

$4x = 8$

You can check the solution by substituting the value into the equation.

$3(2) + 1 = 9 - 2$

$6 + 1 = 7$

$7 = 7$

Equations may take several steps to solve, so it is helpful to have a clear and organized strategy. The following table shows a general strategy to solve any **linear equation** in one variable. You may not need to use every step.

General Strategy for Solving Linear Equations
Step 1 Simplify each side of the equation as much as possible. Use the **Distributive Property** to remove any parentheses. Combine like **terms** .
Step 2 Collect all the variable terms on one side of the equation. Use the **Addition or Subtraction Property of Equality**.
Step 3 Collect all the constant terms on the other side of the equation. Use the Addition or Subtraction Property of Equality.
Step 4 Make the **coefficient** of the variable term to equal to 1. State the solution to the equation.
Step 5

Check the solution.

Substitute the solution into the original equation to make sure the result is a true statement.

TRY IT **Solving a Linear Equation Using a General Strategy**

Solve the following using a general strategy:

Solve $2(3x - 8) = 19 - x$.

Check Your Understanding

Solve: $7(n - 3) - 8 = -15$.

a. $n = -\frac{4}{7}$

b. $n = -2$

c. $n = 2$

d. $n = -3\frac{5}{7}$

Videos: Solving Linear Equations

Khan Academy: Equations with Parentheses

Watch this video to see how to use the Distributive Property to solve an equation with parentheses.

Access multimedia content (http://openstax.org/books/algebra-1/pages/1-solve-linear-equations-mini-lesson-review)

Khan Academy: Equations with Variables on Both Sides: Fractions

Watch this video to see how to solve an equation with fractional coefficients and variables on both sides.

Access multimedia content (http://openstax.org/books/algebra-1/pages/1-solve-linear-equations-mini-lesson-review)

Classify Equations: Mini-Lesson Review

MINI LESSON QUESTION **Question #2: Classify Equations**

Which of the following correctly classifies the equation and the solution?

$6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$

a. contradiction; no solution
b. conditional equation; $1n = -\frac{1}{4}$
c. conditional equation; $n = 0$
d. identity; all real numbers

Use Solutions to Classify Equations

An equation that is true for one or more values of the variable and false for all other values of the variable is a **conditional equation**.

Consider the equation $7x + 8 = -13$.

Subtract 8 to get the constants on one side.	$7x = -21$
Divide by 7 to make the coefficient of the variable 1.	$x = -3$

The solution is $x = -3$. This means the equation $7x + 8 = -13$ is true when we replace the variable, x, with the value -3, but it is not true when we replace x with any other value. Whether the equation $7x + 8 = -13$ is true depends on the value of the variable. The equation is a conditional equation.

An equation that is true for any value of the variable is called an **identity**. The solution of an identity is all real numbers.

Consider the equation $2y + 6 = 2(y + 3)$. Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for y.

Distribute.	$2y + 6 = 2y + 6$
Subtract $2y$ to get the variables on one side.	$6 = 6$

But $6 = 6$ is true. This means the equation $2y + 6 = 2(y + 3)$ is true for any value of y. The equation is an identity, and we say the solution is all real numbers.

An equation that is false for all values of the variable is called a **contradiction**. A contradiction has no solution.

Consider the equation $5z = 5z - 1$.

Subtract $5z$ to get the variables on one side.	$0 \neq -1$

The table summarizes the types of equations and solutions.

Type of Equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution >

TRY IT **Use Solutions to Classify Equations**

Classify the following: $5m + 3(9 + 3m) = 2(7m - 11)$.

Check Your Understanding

Which of the following correctly classifies the equation and the solution?$4+9(3x-7)=-42x-13+23(3x-2)$

a. identity; all real numbers

b. conditional equation; $x=0$

c. conditional equation; $x=2\frac{5}{27}$

d. contradiction; no solution

Video: Solving Linear Equations

Khan Academy: Number of Solutions to Linear Equations

Watch the video to see how a linear equation may have one solution, no solution, or infinite solutions.

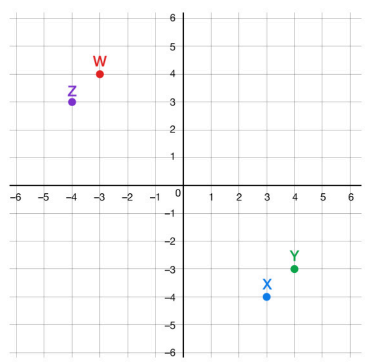
[Access multimedia content (http://openstax.org/books/algebra-1/pages/1-classify-equations-mini-lesson-review)](http://openstax.org/books/algebra-1/pages/1-classify-equations-mini-lesson-review)

Find Coordinates: Mini-Lesson Review

? **MINI LESSON QUESTION**

Question #3: Find Coordinates

Which point has coordinates $(3,-4)$?

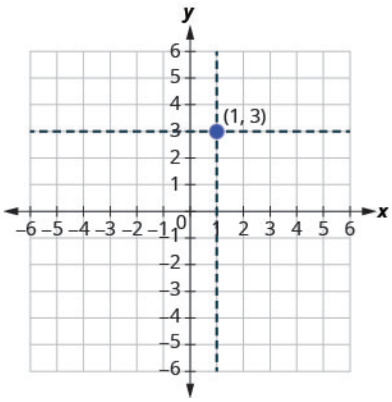


a. Point Z
b. Point Y
c. Point W
d. Point X

Identifying Coordinates of a Point on a Graph

In the rectangular coordinate system, every point is represented by an **ordered pair**. The first number in the ordered pair is the x**-coordinate** of the point, and the second number is the y**-coordinate** of the point. An ordered pair, (x, y), gives the coordinates of a point in a rectangular coordinate system. The first number is the x-coordinate. The second number is the y-coordinate. The phrase “ordered pair” means the order is important. What is the ordered pair of the point where the axes cross? At that point, both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the origin.

We use the coordinates to locate a point on the xy-plane. Let’s plot the point $(1, 3)$ as an example. First, locate 1 on the x-axis and lightly sketch a vertical line through $x=1$. Then, locate 3 on the y-axis and sketch a horizontal line through $y=3$. Now, find the point where these two lines meet—that is the point with coordinates $(1, 3)$.

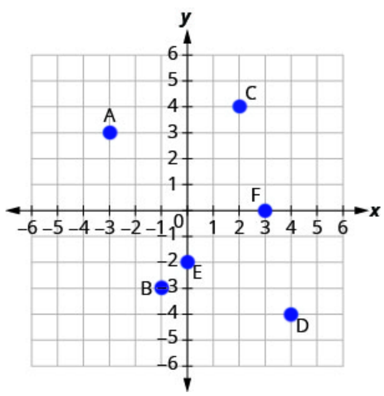


Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the x-coordinate of a point on a graph, read the number on the x-axis directly above or below the point. To identify the y-coordinate of a point, read the number on the y-axis directly to the left or right of the point. Remember, when you write the ordered pair, use the correct order, (x, y).

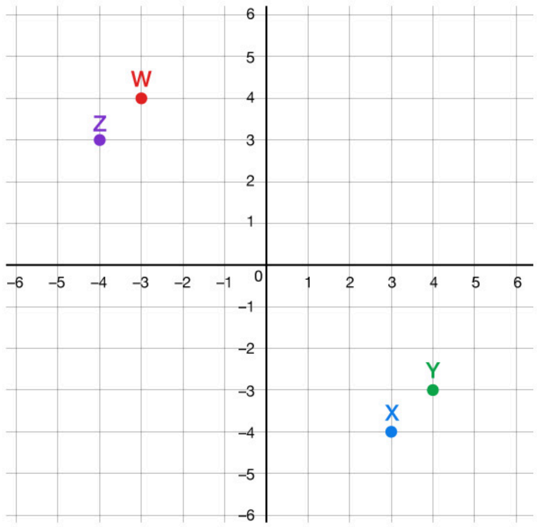
TRY IT **Identifying Coordinates of a Point on a Graph**

Name the ordered pair of each point shown in the rectangular coordinate system.



Check Your Understanding

Which point has coordinates $(-4, 3)$?



a. Point W

b. Point Z

c. Point X

d. Point Y

Video: Points on the Coordinate Plane

Khan Academy: Points on the Coordinate Plane

Watch this video to see how you can use a coordinate plane to plot points and to identify the coordinates of a point plotted on a coordinate plane.

Access multimedia content (http://openstax.org/books/algebra-1/pages/1-find-coordinates-mini-lesson-review)

Unit 1 Overview: Wrap Up

- Solve linear equations using a general strategy.
- Classify equations.
- Identify points on a graph.

If you are still struggling, be sure to reach out to your teacher for additional help. Nobody is good at problems like this on the first try. Keep at it, and it will get easier.

1.1 EXPLORING EXPRESSIONS AND EQUATIONS

1.1.0 Lesson Overview

In this lesson, you will experiment with expressions and equations to model a situation: planning a pizza party. For instance, what are the relevant quantities when planning for a pizza party? Are these quantities fixed or do they vary? How they might relate to one another? Are there any limitations or constraints on how much pizza should be ordered?

When you finish this lesson, you will be able to:

- Tell which quantities in a situation can vary and which ones cannot.
- Explain the meaning of the term “constraint.”
- Use letters and numbers to write expressions representing the quantities in a situation.

Here are the **activities** that will help you reach those goals:

- 1.1.1: Understanding Value
- 1.1.2: Creating Expressions to Estimate Cost, Part 1
  - 1.1.2: Self Check
  - 1.1.2: Additional Resources
- 1.1.3: Understanding Constraints
  - 1.1.3: Self Check
  - 1.1.3: Additional Resources
- 1.1.4: Creating Expressions to Estimate Cost, Part 2

After that, you’ll **practice and review**.

- 1.1.5: Practice
- 1.1.6: Lesson Summary

1.1.1 Understanding Value

Warm Up

Here are some letters and what they represent. All costs are in dollars.

- m represents the cost of a main dish.
- n represents the number of side dishes.
- s represents the cost of a side dish.
- t represents the total cost of a meal.

For questions 1 – 4, discuss what each **equation** means with a partner. Then, describe the situation in words.

1. $m = 7.50$

2. $m = s + 4.50$

3. $ns = 6$

4. $m + ns = t$

5. Write a new equation that could be true in this same situation.

6. Describe what your new equation means in words.

1.1.2 Creating Expressions to Estimate Cost, Part 1

Activity

Imagine your class is having a pizza party.

Work with your group to plan what to order and to estimate what the party would cost.

1. What is your favorite type of pizza to order?
2. How many toppings are on your favorite type of pizza?
3. What type of crust does your favorite pizza have?

For questions 4 – 9, use the following group instructions.

> Work with a group to research the cost of ordering pizza from a local restaurant. Then, determine what to order for the class pizza party and determine how much it would cost. Answer the following questions regarding your group's plan.

4. What is the cost estimate for your group's planned class party?
5. Write down one **expression** that show how your group's cost estimate was calculated.
6. In your expression, which quantities, if any, might change on the day of the party?
7. Rewrite your expression, replacing the quantities that might change with letters.
8. What do the letters represent?
9. How would you convince the class to go with your group's plan?

Are you ready for more?

Extending Your Thinking

A local pizzeria sells gourmet pizzas for the prices posted below. Remember the sizes are determined by the diameter of each pizza. Compare the cost per square inch of the sizes.

Pizza size	10” Small	12” Medium	14” Large	16” XLarge
Price	\$12.99	\$15.99	\$19.99	\$21.99

1. How did you determine the cost per square inch of each pizza?
2. Which size pizza is the better deal?
3. Which size pizza is the most expensive per square inch?
4. What is the difference in price per square inch between the most and least expensive sizes?

Self Check

A ninth-grade class is ordering pizza for lunch. There are n students in the class. Each slice of pizza costs \$3 and the delivery fee is \$8. Which expression represents the total cost if each student gets 2 slices of pizza?

a. $3n + 8$
b. $(2)3n + 8$
c. $(2)3n$
d. $3n$

Additional Resources

Translate an English Phrase to an Algebraic Expression

You can use the following operation symbols to translate English phrases into algebraic expressions.

Look closely at these phrases using the four operations:

The **SUM** of a and b $a + b$
The **DIFFERENCE** of a and b $a - b$
The **PRODUCT** of a and b $a \cdot b = ab = (a)(b)$
The **QUOTIENT** of a and b $\frac{a}{b} = a \div b$

CLASSROOM CONNECTION Log into [student.desmos.com (https://student.desmos.com)](https://student.desmos.com) using the information provided by your teacher to complete the activity.

Identify other phrases that represent the same algebraic concepts by categorizing the following list according to the four operations.

b added to a	a divided by b	b subtracted from a
The product of a times b	a sets of b	The sum of a and b
a plus b	The ratio of a and b	a decreased by b
b less than a	a increased by b	b divided into a
The quotient of a and b	b more than a	The total of a and b
a times b	a minus b	The difference of a and b

Each phrase tells us to operate on two numbers. Look for the words "of" and "and" to find the numbers.

Later in this course, we'll apply our skills in algebra to solving applications. The first step will be to translate an English phrase to an algebraic expression.

EXAMPLE

The length of a rectangle is 14 less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Step 1 - Write a phrase about the length of the rectangle.

14 less than the width

Step 2 - Substitute w for "the width."

14 less than w

Step 3 - Rewrite **less than** as **subtracted from**.

14 subtracted from w

Step 4 - Translate the phrase into algebra.

$w - 14$

TRY IT **Translate an English Phrase to an Algebraic Expression**

Translate the following into an algebraic expression:

June has dimes and quarters in her purse. The number of dimes is seven less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

1.1.3 Understanding Constraints

Activity

A **constraint** is something that limits what is possible or reasonable in a situation.

For example, one constraint in a pizza party might be the number of slices of pizza each person could have, s. We can write $s < 4$ to say that each person gets fewer than 4 slices.

Look at the expressions you wrote when planning the pizza party earlier. Choose an expression that uses one or more letters.

1. What is the expression you are examining?
2. What does the first letter represent?
3. What values would be reasonable for this first letter? (For instance, could the value be greater than 50? Is it possible for the letter to be a non-whole number? A negative number?)
4. What does the second letter represent?
5. What values would be reasonable for this second letter? (For instance, could the value be greater than 50? Is it possible for the letter to be a non-whole number? A negative number?)
6. Write equations or inequalities that represent some constraints in your pizza party plan. If a quantity must be an exact value, use the = symbol. If it must be greater than or less than a certain value to be reasonable, use the $>$ or $<$ symbol.

Video: Writing an Inequality for the Constraint

Watch the following video to learn more about constraints.

[Access multimedia content (http://openstax.org/books/algebra-1/pages/1-1-3-understanding-constraints)](http://openstax.org/books/algebra-1/pages/1-1-3-understanding-constraints)

Self Check

Marcus works s hours per week stocking shelves. He works m hours per week mowing lawns. Marcus works at least 8 hours per week stocking shelves. He works at most a total of 20 hours per week. Which of the following represent constraints for this situation?

a. $s + m \leq 20; s \leq 8$
b. $s + m \geq 20; s \leq 8$
c. $s + m \leq 20; s \geq 8$
d. $s + m \geq 20; s \geq 8$

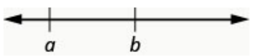
Additional Resources

Inequalities

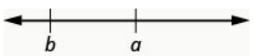
One way to represent a **constraint** is to use an inequality symbol. For example, you may decide that at most 3 toppings should be on each pizza. This could be represented by t < 3, if t represents the number of toppings.

On the number line, the numbers get larger as they go from left to right. The number line can be used to explain the symbols "<" and ">."

$a < b$ is read "a is less than b." a is to the left of b on the number line.



$a > b$ is read "a is greater than b." a is to the right of b on the number line.



The expressions $a < b$ or $a > b$ can be read from left to right or right to left, though in English we usually read from left to right.

- For example, $7 < 11$ is equivalent to $11 > 7$.
- For example, $17 > 4$ is equivalent to $4 < 17$.

Inequality symbols	Words
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a \leq b$	a is less than or equal to b
$a > b$	a is greater than b
$a \geq b$	a is greater than or equal to b

Let's look at some specific examples. Translate the following statements into English phrases.

1. $17 \leq 26$

2. $12 > 27 \div 3$

3. $y + 7 < 19$

TRY IT **Inequalities**

Translate the following statements into English phrases.

1. $14 \leq 27$

2. $12 > 4 \div 2$

3. $x - 7 < 1$

1.1.4 Creating Expressions to Estimate Cost, Part 2

Cool Down

At the end of the school year, all students in the ninth grade are invited to an ice cream party. Use the skills you learned from planning a pizza party to answer the following:

1. Identify and describe at least one letter that you will use to represent the cost of something you need to buy for the party.
2. Write an expression that could represent an estimated cost for the party using that letter. Note, you may use other letters, too, if needed.
3. Describe your expression in words.
4. Using the letter you identified in question 1, describe the values that would be reasonable for the quantity that the letter represents.

1.1.5 Practice

Complete the following questions to practice the skills you have learned in this lesson.

For questions 1 – 5, use the following scenario to identify the expression, equation, or inequality described.

> To support a local senior citizens center, a student club sent a flier home to the n students in the school. The flier said, "Please bring in money to support the senior citizens center. Paper money and coins accepted!" Their goal is to raise T dollars.

Choose the expression, equation, or inequality, from the list, that describes the given quantity.

1. The dollar amount the club would have if they reached half of their goal.

a. $T - 0.5n$
b. $0.5n$
c. $0.25n$
d. $0.5T$
e. $T + 50$

2. The dollar amount the club would have if every student at the school donated 50 cents to the cause.

a. $T - 0.5n$
b. $0.5n$
c. $0.25n$
d. $0.5T$
e. $T + 50$

3. The dollar amount the club could donate if they made $50 more than their goal.

a. $T - 0.5n$
b. $0.5n$
c. $0.25n$
d. $0.5T$
e. $T + 50$

4. The dollar amount the club would still need to raise to reach its goal after every student at the school donated 50 cents.

a. $T - 0.5n$
b. $0.5n$
c. $0.25n$
d. $0.5T$
e. $T + 50$

5. The dollar amount the club would have if half of the students at the school each gave 50 cents.

a. $T - 0.5n$
b. $0.5n$
c. $0.25n$
d. $0.5T$
e. $T + 50$

6. Each of the 10 students in the baking club made 2 chocolate cakes for a fundraiser. They all used the same recipe, using c cups of flour in total.

Write an expression that represents the amount of flour required for one cake.

a. $\frac{2c}{10}$
b. $c - 20$
c. $\frac{c}{20}$
d. $20c$

For questions 7 - 8, determine if each is an expression or an equation that represents the English phrase

7. $3(x - 7) = 27$
   a. equation
   b. expression
8. $5(4y - 2) - 7$
   a. equation
   b. expression

For questions 9 - 10, choose the algebraic expression that represents the English phrase

9. difference of $14x^2$ and 13
   a. $\frac{14x^2}{13}$
   b. $13 - 14x^2$
   c. $14x^2 - 13$
   d. $14x^2 + 13$
10. the quotient of $12x$ and 2
   a. $\frac{2}{12x}$
   b. $\frac{12x}{2}$
   c. $2 - 12x$
   d. $12x - 2$

1.1.6 Lesson Summary

In this lesson, you learned how to:

- Tell which quantities in a situation can vary and which ones cannot.
- Explain the meaning of the term "constraint."
- Use letters and numbers to write expressions representing the quantities in a situation.

Here are the **activities** that helped you reach those goals:

- 1.1.1: Understanding Value
  - In this activity, you wrote equations to solve problems about a meal. The big thing you learned is that an equation is a statement that an expression has the same value as another expression. An equation can have letters, numbers, or a mix of letters and numbers.
- 1.1.2: Creating Expressions to Estimate Cost, Part 1
  - In this activity, you learned how expressions could represent the quantities in a situation like planning a pizza party. You also saw how expressions could change if quantities changed.
  - 1.1.2: Self Check
  - 1.1.2: Additional Resources
- 1.1.3: Understanding Constraints
  - In this activity, you continued working with the pizza party and learned that a constraint is something that limits what is possible or reasonable in a situation.
  - 1.1.3: Self Check
  - 1.1.3: Additional Resources
- 1.1.4: Creating Expressions to Estimate Cost, Part 2
  - In this activity, you wrote expressions and examined constraints for the cost of an ice cream party for the ninth-grade class.

After these activities, you completed the following **practice:**

- 1.1.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

1.2 WRITING EQUATIONS TO MODEL RELATIONSHIPS, PART 1

1.2.0 Lesson Overview

In this lesson, you will begin to learn how to write equations to model various situations. For instance, modeling scenarios with equations can help you figure out how much you need to pay for one pound of blueberries or how much tax you need to pay when buying a meal at a restaurant.

When you finish this lesson, you will be able to:

- Tell which quantities in a situation can vary and which ones cannot.
- Use letters and numbers to write equations representing the relationships in a situation.

Here are the **activities** that will help you reach those goals:

- 1.2.1: Finding the Percent of 200
- 1.2.2: Modeling with Equations to Find Edges in Platonic Solids
    - 1.2.2: Self Check
    - 1.2.2: Additional Resources
- 1.2.3: Writing Equations to Represent Relationships
    - 1.2.3: Self Check
    - 1.2.3: Additional Resources
- 1.2.4: Writing Equations to Represent Relationships with Percentages
    - 1.2.4: Self Check
    - 1.2.4: Additional Resources
- 1.2.5: Writing Equations to Represent T-Shirts

After that, you'll **practice and review**.

- 1.2.6: Practice
- 1.2.7: Lesson Summary

1.2.1 Finding the Percent of 200

Warm Up

You will be asked to evaluate the percentage of a number mentally. Sometimes, a visual is helpful to do this. If needed, use the GeoGebra activity to help you determine the requested values. Then, look for a pattern that can be used to describe how to find the percentage of any number.

Access multimedia content (http://openstax.org/books/algebra-1/pages/1-2-1-finding-the-percent-of-200)

1. What is 25% of 200?

2. What is 12% of 200?

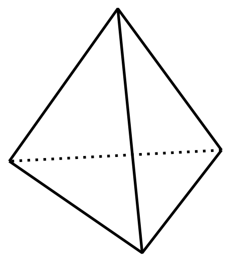
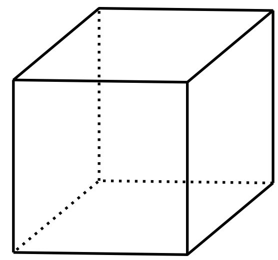
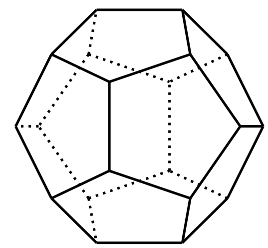
3. What is 8% of 200?

4. How would you determine $p\%$ of 200?

1.2.2 Modeling with Equations to Find Edges in Platonic Solids

Activity

These three figures are called Platonic solids.

Tetrahedron	Cube	Dodecahedron
		

	faces	vertices	edges
tetrahedron	4	4	6
cube			
dodecahedron	12	20	30

The table shows the number of faces, vertices, and edges for the tetrahedron and dodecahedron.

Determine the following values regarding the cube.

1. Number of faces.

2. Number of edges.

3. Number of vertices.

There are some interesting relationships between the number of faces (F), edges (E), and vertices (V) in all Platonic solids.

4. What do you notice about the relationships between the numbers of faces, edges, and vertices of the Platonic solids?

5. What do you wonder about the relationships between the numbers of faces, edges, and vertices of the Platonic solids?

6. There is a relationship that can be expressed with an equation using all three values: F, V, E. Write the equation that represents how the three values are related.

Are you ready for more?

Extending Your Thinking

There are two more Platonic solids: an octahedron, which has 8 faces that are all triangles, and an icosahedron, which has 20 faces that are all triangles.

1. How many edges would each of these solids have? (Keep in mind that each edge is used in two faces.)

a. The octahedron has how many edges?

b. The icosahedron has how many edges?

2. Use your discoveries from the activity to determine how many vertices each of these solids would have.

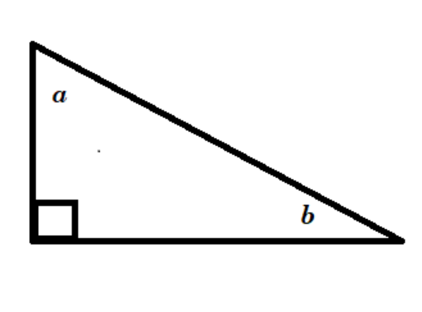
a. The octahedron has how many vertices?

b. The icosahedron has how many vertices?

3. For all 5 Platonic solids, determine how many faces meet at each vertex.

Self Check

The sum of the measures of the angles of a triangle is $180°$. The right triangle shown has angle measures a, b, and $90°$. Which equation could model this relationship?



a. $90ab = 180$
b. $a + b + 90 = 180$
c. $a + b - 90 = 180$
d. $ab + 90 = 180$

Additonal Resources

Writing Equations Using Symbols

How to write statements using algebraic symbols

Step 1 - Read the problem.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Step 3 - Write a sentence using the relationship among the values.

Step 4 - Translate the sentence into an equation.

Remember that **variables** represent values in the problem that can change.

Let's examine two examples of how to write statements algebraically using symbols.

EXAMPLE 1

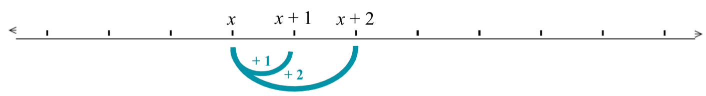
We want to express the following statement using symbolic language:

The sum of three consecutive integers is 372.

Step 1 - Read the problem.
The sum of three consecutive integers is 372.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Let x = the first integer. There are two more numbers that follow x.
Each number is 1 more than the number before it: $x + 1$ and $x + 2$
The sum of all three numbers is 372.



Step 3 - Write a sentence using the relationship among the values.
The sum of x, $x + 1$, and $x + 2$ is 372.

Step 4 - Translate the sentence into an equation.
$x + (x + 1) + (x + 2) = 372$
$3x + 3 = 372$

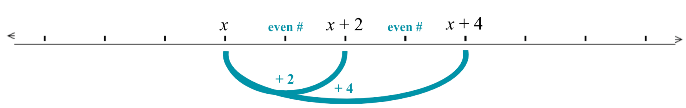
EXAMPLE 2

We want to express the following statement using symbolic language:
The sum of three consecutive odd integers is 93.

Step 1 - Read the problem.
The sum of three consecutive odd integers is 93.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Let x = the first integer. There are two more ODD numbers that follow x.
Each number is 2 more than the number before it: $x + 2$ and $x + 4$
The sum of all three numbers is 93.



Step 3 - Write a sentence using the relationship among the values.
The sum of x, $x + 2$, and $x + 4$ is 93.

Step 4 - Translate the sentence into an equation.
$x + (x + 2) + (x + 4) = 93$
$3x + 6 = 93$

TRY IT **Writing Equations Using Symbols**
Now it's your turn. Write the statement below algebraically using symbols.

The sum of three consecutive integers is 1,623.

1.2.3 Writing Equations to Represent Relationships

Activity

Write an **equation** to represent each situation.

1. Blueberries are \$4.99 a pound. Diego buys b pounds of blueberries and pays \$14.95.
2. Blueberries are \$4.99 a pound. Jada buys p pounds of blueberries and pays c dollars.
3. Blueberries are d dollars a pound. Lin buys q pounds of blueberries and pays t dollars.
4. Noah earned n dollars over the summer. Mai earned \$275, which is \$45 more than Noah did.
5. Noah earned v dollars over the summer. Mai earned m dollars, which is 45 dollars more than Noah did.
6. Noah earned w dollars over the summer. Mai earned x dollars, which is y dollars more than Noah did.

7. How are the equations you wrote for the blueberry purchases like the equations you wrote for Mai's and Noah's summer earnings? How are they different?

Self Check

Max and Jules are renting a car for their 3-day trip. The rental company charges $50 per day and $0.40 per mile driven. Which is a possible equation for their cost of the rental car?

a. $C = 50(3) + 0.40m$
b. $C = 50(3) + 40(3) + 0.40m$
c. $C = 50m + 40(3)$
d. $C = 50(3) + 40m$

Additional Resources

Writing an Equation to Represent a Real-World Problems

Here are the steps to writing an equation to represent a real-world scenario:

Step 1 - Read the problem.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Step 3 - Write a sentence using the relationship among the values.

Step 4 - Translate the sentence into an equation.

EXAMPLE

Write the following statement algebraically using symbols: Peanuts cost $3.59 per pound. Max buys p pounds of peanuts and pays $10.77.

Step 1 - Read the problem.

Peanuts cost $3.59 per pound. Max buys p pounds of peanuts and pays $10.77.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

$3.59 = cost per pound of peanuts

$10.77 = total cost of peanuts bought

p = number of pounds of peanuts bought

Step 3 - Write a sentence using the relationship among the values.

The cost per pound times the number of pounds bought equals the total cost of peanuts

Step 4 - Translate the sentence into an equation.

$3.59(p) = 10.77$

TRY IT **Writing an Equation to Represent a Real-World Problem**

Translate the following scenario into an equation:

A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

1. After reading the problem, what are some of the known variables identified in the description?
2. Identify a variable and describe what it represents for this scenario.
3. What sentence can be used to describe the relationship among the values?
4. What equation can be used to represent the scenario?

Video: Writing an Equation to Represent a Real-World Problem

Watch the following video to learn more about how to write an equation to represent a real-world problem.

[Access multimedia content (http://openstax.org/books/algebra-1/pages/1-2-3-writing-equations-to-represent-relationships)](http://openstax.org/books/algebra-1/pages/1-2-3-writing-equations-to-represent-relationships)

1.2.4 Writing Equations to Represent Relationships with Percentages

Activity

The tax on the sale of a car in Michigan is 6%. At a dealership in Ann Arbor, a car purchase also involves $120 in miscellaneous charges.

There are several quantities in this situation: the original car price, sales tax, miscellaneous charges, and total price.

For questions 1 – 4, write an equation to describe the relationship between all the quantities when given specific information.

1. The original car price is $9,500.
2. The original car price is $14,699.
3. The total price is $22,480.
4. The original price is p.
5. How would each equation you wrote change if the tax on car sales is r% and the miscellaneous charges are m dollars?

Self Check

Sweet's Candy Store charges $5.00 to fill a bag of candy. There is also a sales tax of 7% per purchase. Mrs. Smith buys 4 bags of candy for her children for Christmas. What is an equation that can represent the cost of the candy that Mrs. Smith buys?

a. $C = 0.07 + 4(5)$
b. $C = 4(5)$
c. $C = 0.07(4(5))$
d. $C = 0.07(4(5)) + 4(5)$

Additional Resources

Writing an Equation to Represent a Real-World Problem with Percentages

The steps for writing an equation to represent a real-world scenario with percentages are the same as before. Just be sure to change the given percent to a decimal before you use it in the equation.

Writing an Equation

Step 1 - Read the problem.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Step 3 - Write a sentence using the relationship among the values.

Step 4 - Translate the sentence into an equation.

For example, what is an equation that can represent this scenario?

The label on Audrey's yogurt said that one serving provided 12 grams of protein, which is 24% of the recommended daily amount. What is the total recommended daily amount of protein?

Here's how to use the steps:

Step 1 - Read the problem.

The label on Audrey's yogurt said that one serving provided 12 grams of protein, which is 24% of the recommended daily amount. What is the total recommended daily amount of protein?

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

1 serving = 12 grams of protein
1 serving is 24% of recommended daily amount
Let a = total amount of protein (in grams).

Step 3 - Write a sentence using the relationship among the values.

12 grams is 24% of the total amount

Step 4 - Translate the sentence into an equation.

$12 = 0.24a$

TRY IT **Writing an Equation to Represent a Real-World Problem with Percentages**

Analyze the situation and then write an equation that can represent the scenario.

One serving of wheat square cereal has 7 grams of fiber, which is 28% of the recommended daily amount. What is the total recommended daily amount of fiber?

1. After reading the problem, what are some of the known variables identified in the description?
2. Identify a variable and describe what it represents for this scenario.
3. What sentence can be used to describe the relationship among the values?
4. What equation can be used to represent the scenario?

1.2.5 Writing Equations to Represent T-Shirts

Cool Down

A school choir needs to make T-shirts for its 75 members and has set aside some money in their budget to pay for them. The members of the choir decided to order from a printing company that charges $3 per shirt, plus a $50 fee for each color to be printed on the shirts.

1. Write an equation that represents the relationship between the number of T-shirts ordered, the number of colors on the shirts, and the total cost of the order. If you use a **variable**, specify what it represents.
2. In this situation, which quantities do you think can vary? Which might be fixed?

1.2.6 Practice

Complete the following questions to practice the skills you have learned in this lesson.

1. Translate to an equation: Which equation means nine more than x is equal to 52?

a. $\frac{52}{x} = 9$
b. $9 - x = 52$
c. $9x = 52$
d. $x + 9 = 52$

2. Translate to an equation: Which equation means ten less than m is −14?

a. $-14 - m = 10$
b. $m - 10 = -14$
c. $m - 10 = 14$
d. $10 - m = -14$

3. Translate to an equation: Which equation means the sum of y and −30 is 40?

a. $y + (-30) = 40$
b. $y + 40 = -30$
c. $y = 40 + (-30)$
d. $-30y = 40$

4. Translate to an equation: For a family birthday dinner, Celeste bought a turkey that weighed 5 pounds less than the one she bought for Thanksgiving. The birthday turkey weighed 16 pounds. Which equation represents how much the Thanksgiving turkey weighed?

a. $-x + 5 = 16$
b. $x + 5 = 16$
c. $x - 5 = 16$
d. $5 - x = 16$

5. Translate to an equation: Which equation represents 45% of 120?

a. $x = 45 + 120$
b. $x = 0.45(120)$
c. $x = 4.5(120)$
d. $x = 45(120)$

6. 250% of 65 can also be written as:

a. $x = 250(65)$
b. $x = 25(65)$
c. $x = 0.25(65)$
d. $x = 2.5(65)$

7. Translate to an equation: Geneva treated her parents to dinner at their favorite restaurant. The bill was $74.25. Geneva wants to leave 16% of the total bill as a tip. How much should the tip be?

a. $t = 16 + (74.25)$
b. $t = 1.6(74.25)$
c. $t = 0.16(74.25)$
d. $t = 16(74.25)$

8. Large cheese pizzas cost $5 each, and large one-topping pizzas cost $6 each.
Choose the equation that represents the total cost, T, of c large cheese pizzas and d large one-topping pizzas.

a. $T = 6c - 5d$
b. $T = 5c - 6d$

c. $T = 5c + 6d$
d. $T = 6c + 5d$

9. Jada plans to serve milk and healthy cookies for a book club meeting. She is preparing 12 ounces of milk and 4 cookies per person. Including herself, there are 15 people in the club. A package of cookies contains 24 cookies and costs \$4.50. A 1-gallon jug of milk contains 128 ounces and costs \$3. Let n represent the number of people in the club, m represent the ounces of milk, c represent the number of cookies, and b represent Jada's budget in dollars.
Select **three** of the equations that could represent the quantities and constraints in this situation.

a. $m = 12(15)$
b. $3m + 4.5c = b$
c. $4n = c$
d. $4(4.50) = c$
e. $b = 2(3) + 3(4.50)$

1.2.7 Lesson Summary

In this lesson, you learned how to:

- Tell which quantities in a situation can vary and which ones cannot.
- Use letters and numbers to write equations representing the relationships in a situation.

Here are the **activities** that helped you reach those goals:

- 1.2.1: Finding the Percent of 200
  - In this activity, you found different percentages of 200 by changing the percent to a decimal and writing an expression.
- 1.2.2: Modeling with Equations to Find Edges in Platonic Solids
  - In this activity, you modeled the relationship between the edges, vertices, and faces of Platonic solids with an equation.
  - 1.2.2: Self Check
  - 1.2.2: Additional Resources
- 1.2.3: Writing Equations to Represent Relationships
  - In this activity, you worked to write more equations that modeled different situations.
  - 1.2.3: Self Check
  - 1.2.3: Additional Resources
- 1.2.4: Writing Equations to Represent Relationships with Percentages
  - In this activity, you continued to write equations to model given situations but incorporated percentages.
  - 1.2.4: Self Check
  - 1.2.4: Additional Resources
- 1.2.5: Writing Equations to Represent T-Shirts
  - In this activity, you wrote an equation about the cost of T-shirts for a choir group. You also identified which quantities were fixed in the situation and which quantities could change.

After these activities, you completed the following **practice**:

- 1.2.6: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

1.3 WRITING EQUATIONS TO MODEL RELATIONSHIPS, PART 2

1.3.0 Lesson Overview

In this lesson, you will continue to develop your ability to identify, describe, and model relationships with mathematics.

When you finish this lesson, you will be able to:

- Use words and equations to describe the patterns in a table of values or in a set of calculations.
- Use representations, like diagrams and tables, to help make sense of a described situation and write equations for it.

Here are the **activities** that will help you reach those goals:

- 1.3.1: Finding a Relationship between x and y
- 1.3.2: Describing Relationships Using Words and Equations
  - 1.3.2: Self Check
  - 1.3.2: Additional Resources
- 1.3.3: Identifying and Representing Relationships
  - 1.3.3: Self Check
  - 1.3.3: Additional Resources
- 1.3.4: Writing an Equation to Represent a Relationship

After that, you'll **practice and review**.

- 1.3.5: Practice
- 1.3.6: Lesson Summary

1.3.1 Finding a Relationship between x and y

Warm Up

Here is a table of values. The two quantities, x and y, are related.

x	y
1	0
3	8
5	24
7	48

What are some strategies you could use to find a relationship between x and y? Brainstorm as many ways as possible.

1.3.2 Describing Relationships Using Words and Equations

Activity

Create a math story that describes how the two quantities in each table are related.

1. **Table A**

Number of Laps, x	0	1	2.5	6	9
Meters Run, y	0	400	1,000	2,400	3,600

2. **Table B**

Meters From Home, x	0	75	128	319	396
Meters From School, y	400	325	272	81	4

3. **Table C**

Electricity Bills in Dollars, x	85	124	309	816
Total Expenses in Dollars, y	485	524	709	1,216

4. **Table D**

Monthly Salary in Dollars, x	872	998	1,015	2,110
Amount Deposited in Dollars, y	472	598	615	1,710

For questions 5 – 8, match each table from 1 - 4 to an equation that represents the relationship.

5. Equation 1: $400 + x = y$
6. Equation 2: $x - 400 = y$
7. Equation 3: $x + y = 400$
8. Equation 4: $400 \cdot x = y$

Are you ready for more?

Extending Your Thinking

Express every number between 1 and 20 at least one way using exactly four 4s and any operations. For example, 1 could be written as $\frac{4}{4} + 4 - 4$.

Video: Describing Relationships

Watch the following video to learn more about describing relationships using words and equations, specifically looking at the base and height of different rectangles.

[Access multimedia content (http://openstax.org/books/algebra-1/pages/1-3-2-describing-relationships-using-words-and-equations)](http://openstax.org/books/algebra-1/pages/1-3-2-describing-relationships-using-words-and-equations)

Self Check

Miles from Home, x	0	5	10
Miles from School, y	30	25	20

Using the table above, what is an equation that shows the relationship between miles from home and miles from school?

a. $y = 30x$
b. $y = 30 - x$
c. $y = 20 + x$
d. $y = 30 + x$

Additional Resources

Modeling Linear Equations

Given a real-world problem, model a linear equation to fit it using these steps:

Step 1 - Read the problem.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.

Step 3 - Write a sentence or create a table using the relationship among the values.

Step 4 - Translate the sentence or table into an equation

Let's look at an example. Using the steps above, we can model a linear equation to solve a real-world application.

EXAMPLE

Cell phone Company A charges a monthly service fee of $34 plus $0.05/minute talk time.

Step 1 - Read the problem.
Cell phone Company A charges a monthly service fee of $34 plus $0.05/minute talk time.

Step 2 - Identify the variables and known values. If needed, sketch a picture of the scenario.
t: total cost (in dollars)
m: number of minutes

Step 3 - Write a sentence or create a table using the relationship among the values.

Minutes Talked in One Month	0	1	2	3	100
Total for One Month (Dollars)	34	34.05	34.10	34.15	39

Company A: The total cost, t, equals the fee, 34, plus 0.05 per minute, $0.05m$.

Step 4 - Translate the sentence or table into an equation.
$t = 34 + 0.05m$

TRY IT **Modeling Linear Equations**

Cell phone Company B charges a monthly service fee of $40 plus $0.04/min talk-time. Write an equation to describe the relationship between the number of minutes and the total cost.

1.3.3 Identifying and Representing Relationships

Activity

1. The table represents the relationship between the base length and the height of some parallelograms. Both measurements are in inches.

1	48
2	24
3	16
4	12
6	8

What is the relationship between the base length and the height of these parallelograms?

2. Visitors to a carnival are invited to guess the number of beans in a jar. The person who guesses the correct number wins $300. If multiple people guess correctly, the prize will be divided evenly among them.
What is the relationship between the number of people who guess correctly and the amount of money each person will receive?

3. A $\frac{1}{2}$-gallon jug of milk can fill 8 cups, while 32 fluid ounces of milk can fill 4 cups.
What is the relationship between the number of gallons and ounces? If you get stuck, try creating a table.

Building Character: Social Intelligence

Social intelligence is the ability to connect with other people. Think about your current sense of social intelligence. Are the following statements true for you??

- I have a lot of relationships that are mutually beneficial, enjoyable, and supportive.
- Most of the time, I can tell how other people feel and have a good idea about how to respond appropriately.

Don’t worry if none of these statements are true for you. Developing this trait takes time. Your first step starts today!

Self Check

Number of guests, g	1	2	3	12
Number of cupcakes per guest, n	24	12	8	2

What is one way to write the relationship between the number of guests and the number of cupcakes per guest from the table above?

a. $n = 24 - g$
b. $n = \frac{g}{24}$
c. $n = \frac{24}{g}$
d. $n = 24g$

Additional Resources

Finding Relationships between Quantities

There are times when the relationship between quantities may not be obvious. In some cases, the relationship between quantities might take a bit of work to figure out, by doing calculations several times or by looking for a pattern.

Here are two examples.

EXAMPLE 1

A plane departed from New Orleans and is heading to San Diego. The table shows its distance from New Orleans, x, and its distance from San Diego, y, at some points along the way.

miles from New Orleans	miles from San Diego
100	1,500
300	1,300
500	1,100
	1,020
900	700
1,450	
x	y

1. What is the relationship between the two distances?
2. Do you see any patterns in how each quantity is changing?
3. What is the value of x when y = 1020?
4. What is the value of y when x = 1450?

EXAMPLE 2

A company decides to donate $50,000 to charity. It will select up to 20 charitable organizations, as nominated by its employees. Each selected organization will receive an equal donation amount.

What is the relationship between the number of students, s, and the dollar amount each student will receive, d? To begin, let's examine some specific values to help uncover the pattern.

5. If 5 organizations are selected, how much will each charity receive?
6. If 10 organizations are selected, how much will each charity receive?
7. If 20 organizations are selected, how much will each charity receive?

Do you notice a pattern here? 10,000 is $\frac{50{,}000}{5}$, 5,000 is $\frac{50{,}000}{10}$, and 2,500 is $\frac{50{,}000}{20}$.

We can generalize that the amount each organization receives is 50,000 divided by the number of selected organizations, or $d = \frac{50{,}000}{n}$.

TRY IT **Finding Relationships between Quantities**

A local business is going to hand out $20,000 in scholarships to students at local high schools.

What is the relationship between the number of students, s, and the dollar amount each student will receive, d?

1. If 2 students are selected, what is the amount of the scholarship they will receive?
2. If there are 5 students selected, what is the amount of the scholarship they will receive?
3. If there are 10 students selected, what is the amount of the scholarship they will receive?
4. If there are 20 students selected, what is the amount of the scholarship they will receive?
5. What equation can be used to model the relationship between the number of students, s, receiving scholarships and the dollar amount, d, they receive?

1.3.4 Writing an Equation to Represent a Relationship

Cool Down

Clare volunteers at a local library during the summer. Her work includes putting labels on 750 books.

1. How many minutes will she need to finish labeling all books if she takes no breaks and labels 10 books a minute?
2. How many minutes will she need to finish labeling all books if she takes no breaks and labels 15 books a minute?
3. Suppose Clare labels the books at a constant speed of s books per minute. Write an equation that represents the relationship between her labeling speed and the number of minutes it would take her to finish labeling.

1.3.5 Practice

Complete the following questions to practice the skills you have learned in this lesson.

For questions 1 – 2, use the following scenario to identify the equation described.

Two different telephone carriers offer the following plans that a person is considering. Company A has a monthly fee of $20 and charges $0.05/minute for calls. Company B has a monthly fee of $5 and charges $0.10/minute for calls.

1. Find the model for the entire cost, c, when Company A's plan is used for m minutes.

a. $c = 5 + 0.05m$
b. $c = 20 + 0.10m$
c. $c = 5 + 0.10m$
d. $c = 20 + 0.05m$

2. Find the model for the entire cost, c, when Company B's plan is used for m minutes.

a. $c = 5 + 0.05m$
b. $c = 20 + 0.10m$
c. $c = 5 + 0.10m$
d. $c = 20 + 0.05m$

3. If a carpenter sawed a 10-ft board into two sections and one section was n ft long, how long would the other section, S, be in terms of n?

a. $S = \frac{10}{2}n$
b. $S = 10 - 2n$
c. $S = 10 - n$
d. $S = \frac{10}{n}$

Use the table below for problems 4 and 5:

Tyler needs to complete this table for his consumer science class. He knows that 1 tablespoon contains 3 teaspoons and that 1 cup contains 16 tablespoons.

Number of Teaspoons	Number of Tablespoons	Number of Cups
a. ____	b. ____	2
36	12	c. ____
d. ____	48	3

4. For each of the missing spaces in the table above, a-d, select the missing value that helps complete the table.

a. ________

a. 144
b. 96
c. 64
d. 32
e. $\frac{3}{4}$

b. ________

a. 144
b. 96

c. 64
d. 32
e. $\frac{3}{4}$

c. ________

a. 144
b. 96
c. 64
d. 32
e. $\frac{3}{4}$

d. ________

a. 144
b. 96
c. 64
d. 32
e. $\frac{3}{4}$

5. Choose the equation that represents the number of teaspoons, t, contained in a cup, C.

a. $c = \frac{t}{3}$
b. $c = t - 48$
c. $c = \frac{t}{48}$
d. $c = 48t$

6. The volume of dry goods, like apples or peaches, can be measured using bushels, pecks, and quarts. A bushel contains 4 pecks, and a peck contains 8 quarts.
What is the relationship between the number of bushels, b, and the number of quarts, q?

a. $b = 32q$
b. $b = \frac{q}{64}$
c. $b = \frac{q}{12}$
d. $b = \frac{q}{32}$

1.3.6 Lesson Summary

In this lesson, you learned how to:

- Use words and equations to describe the patterns in a table of values or in a set of calculations.
- Use representations, like diagrams and tables, to help make sense of a described situation and write equations for it.

Here are the **activities** that helped you reach those goals:

- 1.3.1: Finding a Relationship between x and y
  - In this activity, you brainstormed ways to find the relationship between variables in a table.
- 1.3.2: Describing Relationships Using Words and Equations
  - In this activity, you matched tables with the equations that could model the relationships in each table.
  - 1.3.2: Self Check
  - 1.3.2: Additional Resources
- 1.3.3: Identifying and Representing Relationships
  - In this activity, you created tables from a situation and then wrote an equation to model the relationship.
  - 1.3.3: Self Check
  - 1.3.3: Additional Resources
- 1.3.4: Writing an Equation to Represent a Relationship
  - In this activity, you wrote an equation to model a relationship in a situation and also began to solve an equation given one of the quantities in the equation.

After these activities, you completed the following **practice** :

- 1.3.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

1.4 EQUATIONS AND THEIR SOLUTIONS

1.4.0 Lesson Overview

In middle school, you learned that a solution to an equation is a value or values that make the equation true. In this lesson, you will revisit what you learned about solutions to equations in one variable and two variables. You will also continue to practice modeling relationships with equations and to make sense of equations and their solutions in context.

When you finish this lesson, you will be able to:

- Explain what it means for a value or pair of values to be a solution to an equation.
- Find solutions to equations by reasoning about a situation or by using algebra.

Here are the **activities** that will help you reach those goals:

- 1.4.1: Finding the Solution to an Equation in One Variable, Part 1
- 1.4.2: Writing Equations to Represent Constraints
  - 1.4.2: Self Check
  - 1.4.2: Additional Resources
- 1.4.3 Finding the Solution to an Equation in Two Variables
  - 1.4.3: Self Check
  - 1.4.3: Additional Resources
- 1.4.4: Finding the Solution to an Equation in One Variable, Part 2

After that, you'll **practice and review**.

- 1.4.5: Practice
- 1.4.6: Lesson Summary

1.4.1 Finding the Solution to an Equation in One Variable, Part 1

Warm Up

A granola bite contains 27 calories. Most of the calories come from c grams of carbohydrates. The rest come from other ingredients. One gram of carbohydrate contains 4 calories.

The equation $4c + 5 = 27$ represents the relationship between these quantities.

1. What could the 5 represent in this situation?
2. Priya said that neither 8 nor 3 could be the solution to the equation. Is she correct?
3. Marcus said the solution is 5.5. Is he correct? Explain.

1.4.2 Writing Equations to Represent Constraints

Activity

Jada has time on the weekends to earn some money. A local bookstore is looking for someone to help sort books and will pay $12.20 an hour. To get to and from the bookstore on a work day, however, Jada would have to spend $7.15 on bus fare.

1. Write an equation that represents Jada's take-home earnings in dollars, E, if she works at the bookstore for h hours in one day.
2. One day, Jada takes home $90.45 after working h hours and after paying the bus fare. Write an equation to represent this situation.

3. Is 4 a solution to the last equation you wrote?
   - If so, be prepared to explain how you know it is a solution.
   - If not, be prepared to explain why it is not a solution. Then, find the solution.

4. Is 7 a solution to the last equation you wrote?
   - If so, be prepared to explain how you know it is a solution.
   - If not, be prepared to explain why it is not a solution. Then, find the solution.

5. In this situation, what does the solution to the equation tell us?

Are you ready for more?

Extending Your Thinking

Jada has a second option to earn money. She could help some neighbors with errands and computer work for $11 an hour. After reconsidering her schedule, Jada realizes that she has about 9 hours available to work one day of the weekend.

Which option should she choose, sorting books at the bookstore or helping her neighbors? Be prepared to show your reasoning.

Self Check

Jada found a third job option and was offered a position working at a camp for $10.50 per hour. The total bus fare to and from the camp is $8.40. Write an equation that represents how much she will make in a day, d, after working h hours.

a. $d = 8.40h + 10.50$
b. $d = \frac{10.50h}{8.40}$
c. $d = 10.50h + 8.40$
d. $d = 10.50h - 8.40$

Additional Resources

Verify a Solution of an Equation

Solving an **equation** is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the **variable** that make each side of the equation the same so that we end up with a true statement. Any value of the variable that makes the equation true is called a **solution of an equation**. It is the answer to the puzzle!

How to determine whether a number is a solution to an equation:

Step 1 - Substitute the number in for the variable in the equation.

Step 2 - Simplify the expressions on both sides of the equation

Step 3 - Determine whether the resulting equation is true (the left side is equal to the right side).

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

For example, the equation $2l + 2w = 72$ represents the relationship between the length, l, and the width, w, of a rectangle that has a perimeter of 72 units. If we know that the length is 15 units, what is the value of the width?

Step 1 - Substitute the number in for the variable in the equation.

$2(15) + 2w = 72$

This is an equation in one variable because w is the only quantity that we don't know. To solve this equation means to find a value of w that makes the equation true.

Step 2 - Simplify the expressions on both sides of the equation In this case, 21 is the solution because substituting 21 for

w in the equation results in a true statement.

$2(15) + 2w = 72$

$2(15) + 2(21) = 72$

$30 + 42 = 72$

Step 3 - Determine whether the resulting equation is true (the left side is equal to the right side).

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

$72 = 72$

TRY IT **Verify a Solution of an Equation**

The equation $2l + 2w = 48$ represents the perimeter of a rectangle.

1. If the width of the rectangle is 10, what is the new equation?
2. What is the length of the rectangle that has a width of 10?
3. Verify the solution.

1.4.3 Finding the Solution to an Equation in Two Variables

Activity

Use the following scenario for questions 1 – 3:

One gram of protein contains 4 calories. One gram of fat contains 9 calories. A snack has 60 calories from p grams of protein and f grams of fat.

1. Determine if 5 grams of protein and 2 grams of fat could be the number of grams of protein and fat in the snack. Explain your reasoning.
2. Determine if 10.5 grams of protein and 2 grams of fat could be the number of grams of protein and fat in the snack. Explain your reasoning.
3. Determine if 8 grams of protein and 4 grams of fat could be the number of grams of protein and fat in the snack. Explain your reasoning.
4. If there are 6 grams of fat in the snack, how many grams of protein are there? Be prepared to show your reasoning.
5. In this situation, what does a solution to the equation $4p + 9f = 60$ tell us? Give an example of a solution.

Video: Working Through the Equation

Watch the following video to learn more about how to determine a solution to this particular equation: $4p + 9f = 60$.

[Access multimedia content (http://openstax.org/books/algebra-1/pages/1-4-3-finding-the-solution-to-an-equation-in-two-variables)](http://openstax.org/books/algebra-1/pages/1-4-3-finding-the-solution-to-an-equation-in-two-variables)

Self Check

Which of the following is a solution (x, y) to $2x + 3y = 7$?

a. $(-2, 4)$
b. $(4, -1)$
c. $(1, 2)$
d. $(2, 1)$

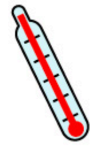
Additional Resources

Solutions to Equations in Two Variables

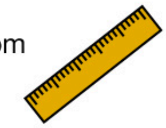
An equation that contains two unknown quantities or two quantities that vary is called an equation in two variables.

Equations in two variables:

$F = \frac{9}{5}C + 32$ Converts temperature from degrees Fahrenheit (F) to degrees Celsius (C)



$i = 2.54c$ Converts a measurement from centimeters (c) to inches (i)



A solution to such an equation is a pair of numbers that makes the equation true.

EXAMPLE

Suppose Tyler spends $45 on T-shirts and socks. A T-shirt costs $10, and a pair of socks costs $2.50. If t represents the number of T-shirts and p represents the number of pairs of socks that Tyler buys, we can represent this situation with the equation:

$10t + 2.50p = 45$

This is an equation in two variables. More than one pair of values for t and p make the equation true.

Which pair of values makes the equation $10t + 2.50p = 45$ true?

1. $t = 3$ and $p = 6$
2. $t = 4$ and $p = 2$
3. $t = 1$ and $p = 10$

In this situation, one **constraint** is that the combined cost of shirts and socks must equal $45.

Solutions to the equations are pairs of t and p values that satisfy this constraint, such as in questions 1 – 2.

Combinations such as $t = 1$ and $p = 10$, as in question 3, are not solutions because they don't meet the constraint. When these pairs of values are substituted into the equation, they result in statements that are false.

TRY IT **Solutions to Equations in Two Variables**

Is $a = 3$ and $b = 5$ a solution to $6a - 3b = 3$?

1.4.4 Finding the Solution to an Equation in One Variable, Part 2

Cool Down

An empty shipping box weighs 250 grams. The box is then filled with T-shirts. Each T-shirt weighs 132.5 grams.

The equation $W = 250 + 132.5T$ represents the relationship between the quantities in this situation, where W is the weight, in grams, of the filled box and T is the number of shirts in the box.

1. Name two possible solutions to the equation $W = 250 + 132.5T$.
2. What do the solutions mean in this situation?

3. Consider the equation $2{,}900 = 250 + 132.50T$. In this situation, what does the solution to this equation tell us?

1.4.5 Practice

Complete the following questions to practice the skills you have learned in this lesson.

1. Is $y = \frac{4}{3}$ a solution of $9y + 2 = 6y + 3$?

a. Yes
b. No

2. Is $y = \frac{7}{5}$ a solution of $5y + 3 = 10y - 4$?

a. Yes
b. No

3. Is $y = \frac{5}{3}$ a solution of $6y + 10 = 12y$?

a. Yes
b. No

4. Is $u = -\frac{1}{2}$ a solution of $8u - 1 = 6u$?

a. Yes
b. No

5. Tickets to the museum cost $4.50 for children and $9.00 for adults. The equation $63 = 4.5c + 9a$ represents the total cost for c children's tickets and a adult tickets. How many children's tickets were sold if 4 adult tickets were sold?

For questions 6 and 7, use the following scenario: T-shirts are on sale for $12.50. Isla has a coupon for $10.00 off.

6. Choose the equation that represents the total cost, c, if she buys n T-shirts.

a. $c = 12.5(10n)$
b. $c = 12.5(n - 10)$
c. $c = 10n - 12.5$
d. $c = 12.5n - 10$

7. How many T-shirts can she buy if she spends all of her $40?

8. An artist is selling children's crafts. Necklaces cost $2.25 each, and bracelets cost $1.50 per each. Select **three** combinations of necklaces and bracelets that the artist could sell for exactly $12.00

a. 5 necklaces and 1 bracelet
b. 2 necklaces and 5 bracelets
c. 3 necklaces and 3 bracelet
d. 4 necklaces and 2 bracelets
e. 3 necklaces and 5 bracelets
f. 6 necklaces and no bracelets
g. No necklaces and 8 bracelets

9. Volunteer drivers are needed to bring 80 students to the championship baseball game. Drivers either have cars, which can seat 4 students, or vans, which can seat 6 students. The equation $4c + 6v = 80$ describes the relationship between the number of cars, c, and number of vans, v, that can transport exactly 80 students. Select **four** statements that are true about the situation.

a. If 12 cars go, then 2 vans are needed.
b. $c = 14$ and $v = 4$ and are a pair of solutions to the equation.
c. If 6 cars go and 11 vans go, there will be extra space.
d. 10 cars and 8 vans isn't enough to transport all the students.
e. If 20 cars go, no vans are needed.
f. 8 vans and 8 cars are numbers that meet the constraints in this situation.

Use the following scenario for questions 11 and 12:

The drama club is printing T-shirts for its members. The printing company charges a certain amount for each T-shirt plus a setup fee of $40. There are 21 students in the drama club.

10. If there are 21 students in the club and the T-shirt order costs a total of $187, how much does each T-shirt cost?

11. The equation $201.50 = f + 6.50(21)$ represents the cost of printing the T-shirts at a second printing company. Find the solution to the equation.

1.4.6 Lesson Summary

In this lesson, you learned how to:

- Explain what it means for a value or pair of values to be a solution to an equation.
- Find solutions to equations by reasoning about a situation or by using algebra.

Here are the **activities** that helped you reach those goals:

- 1.4.1: Finding the Solution to an Equation in One Variable, Part 1
  - In this activity, you began to determine if a value was a solution to a given one-variable equation and identified what the solution could mean. For a value to be a solution to an equation, it must make the equation true.
- 1.4.2: Writing Equations to Represent Constraints
  - In this activity, you wrote equations from situations given the constraints of the problem.
  - 1.4.2: Self Check
  - 1.4.2: Additional Resources
- 1.4.3: Finding the Solution to an Equation in Two Variables
  - In this activity, you were able to find the solution to an equation with two variables, given the solution to one of the variables.
  - 1.4.3: Self Check
  - 1.4.3: Additional Resources
- 1.4.4: Finding the Solution to an Equation in One Variable, Part 2
  - In this activity, you worked to find multiple solutions to equations and identified what the solutions mean in this situation.

After these activities, you completed the following **practice**:

- 1.4.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

1.5 EQUATIONS AND THEIR GRAPHS

1.5.0 Lesson Overview

In this lesson, you will analyze points on and off a graph and interpret them in context. In explaining correspondences between equations, verbal descriptions, and graphs, you will hone your skill at making sense of problems.

When you finish this lesson, you will be able to:

- Use graphing technology to graph linear equations and identify solutions to the equations.
- Explain how the coordinates of the points on the graph of a linear equation are related to the equation.
- Explain the meaning of points on a graph in terms of the situation it represents when given the graph of a linear equation.

Here are the **activities** that will help you reach those goals:

- 1.5.1: Analyzing Graphs of Linear Equations
- 1.5.2: Graphing Linear Functions in Two Variables
    - 1.5.2: Self Check
    - 1.5.2: Additional Resources
- 1.5.3: Examining an Equation in Two Variables and Its Graph, Part 1
    - 1.5.3: Self Check
    - 1.5.3: Additional Resources
- 1.5.4: Writing, Graphing, and Solving a Linear Equation
    - 1.5.4: Self Check
    - 1.5.4: Additional Resources
- 1.5.5: Examining an Equation in Two Variables and Its Graph, Part 2

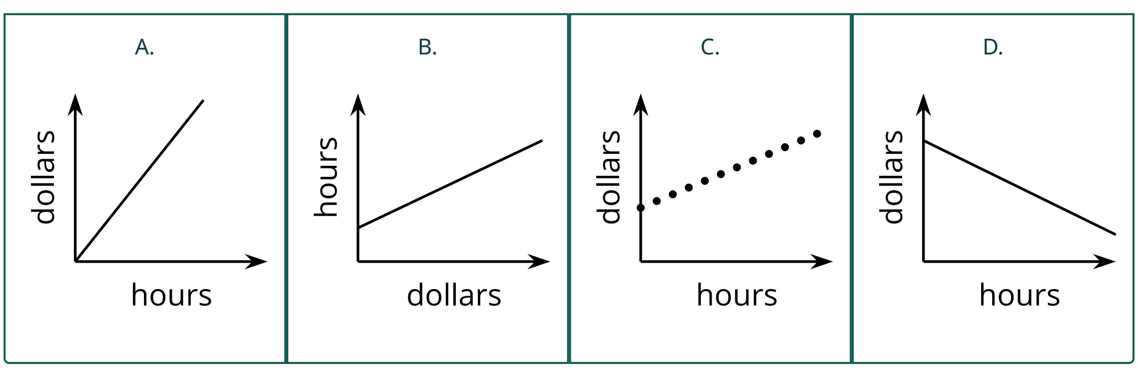
After that, you'll **practice and review**.

- 1.5.6: Practice
- 1.5.7: Lesson Summary

1.5.1 Analyzing Graphs of Linear Equations

Warm Up

Which one doesn't belong?



1.5.2 Graphing Linear Functions in Two Variables

Activity

For questions 1 – 7, use the equation $3x + 4y = 12$

1. Find the value of y when $x = 0$.
2. Find the value of x when $y = 0$.
3. Plot both of those points on a coordinate grid and use a ruler to draw a line.
4. What is the coordinate of the x-intercept?
5. What is the coordinate of the y-intercept?
6. What are two other names for the x-intercept?
7. What is the slope of the line?

Self Check

Graph the equation $5x - 2y = 10$. Which of the following describes the intercepts and slope of the graph?

a. x-intercept: $x = -5$; y-intercept: $y = 2$; Slope: $\frac{5}{2}$
b. x-intercept: $x = 2$; y-intercept: $y = -5$; Slope: $\frac{5}{2}$
c. x-intercept: $x = -2$; y-intercept: $y = 5$; Slope: $-\frac{5}{2}$
d. x-intercept: $x = 2$; y-intercept: $y = -5$; Slope: $\frac{5}{2}$

Additional Resources

Graphing a Line and Describing Characteristics

There are many ways to graph an equation of a line. One way is to use the x-intercepts and y-intercepts as two points on the line.

EXAMPLE

Graph $3x - 6y = 18$ using the intercepts, then identify key characteristics.

Step 1 - Find the x-intercept.
To find the x-intercept, let $y = 0$.

$$\begin{aligned} 3x - 6y &= 18 \\ 3x - 6(0) &= 18 \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

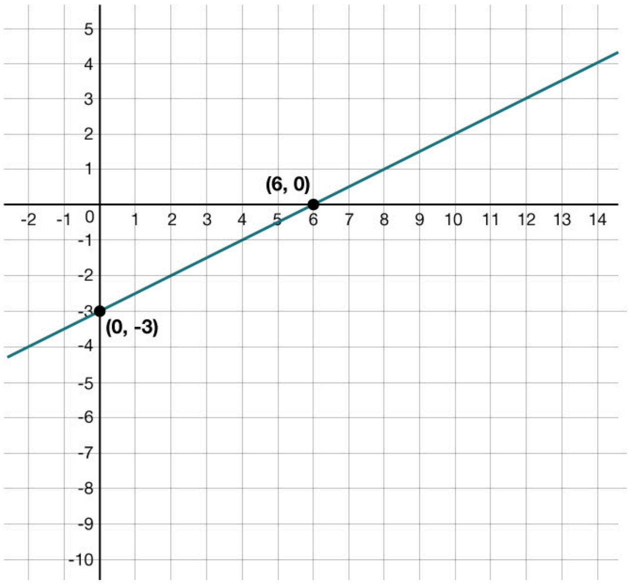
$(6, 0)$ is the x-intercept
The x-intercept is also called a solution or zero.

Step 2 -Find the y-intercept.
To find the y-intercept, let $x = 0$.

$$\begin{aligned} 3x - 6y &= 18 \\ 3(0) - 6y &= 18 \\ -6y &= 18 \\ y &= -3 \end{aligned}$$

$(0, -3)$ is the y-intercept

Step 3 -Graph the line by connecting the y-intercepts.



Step 4 - Find the slope of the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{3}{6} = \frac{1}{2}$$

TRY IT **Graphing a Line and Describing Characteristics**

For questions 1 – 2, use the equation $7x - 3y = 21$.

1. Graph the equation using the intercepts.
2. What is the slope of the line?

1.5.3 Examining an Equation in Two Variables and Its Graph, Part 1

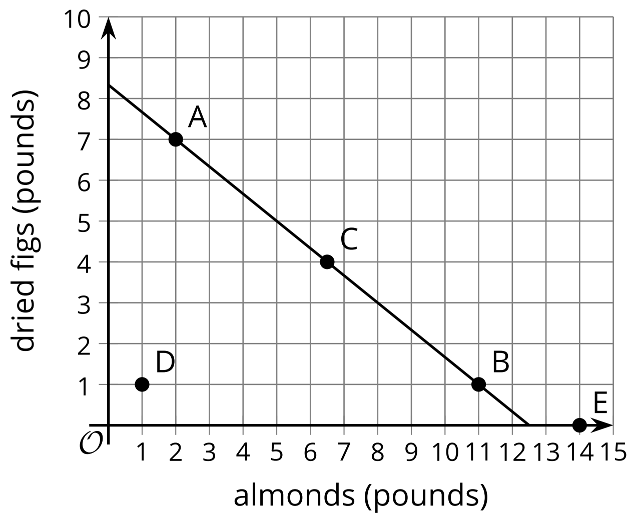
Activity

To get snacks for a class trip, Clare went to the “bulk” section of the grocery store, where she could buy any quantity of a product and the prices are usually good.

Clare purchased some salted almonds at $6 a pound and some dried figs at $9 per pound. She spent $75 before tax.

1. If she bought 2 pounds of almonds, how many pounds of figs did she buy?
2. If she bought 1 pound of figs, how many pounds of almonds did she buy?
3. Write an **equation** that describes the relationship between pounds of figs and pounds of almonds that Clare bought and the dollar amount that she paid. Be sure to specify what the **variables** represent.

For questions 4 and 5, use the graph that represents the quantities in the situation.



4. Choose any point on the line, state its coordinates, and explain what it tells us.
5. Choose any point that is not on the line, state its coordinates, and explain what it tells us.

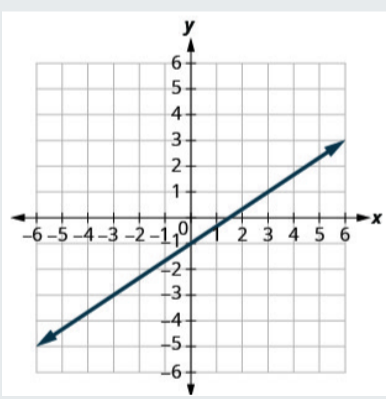
Video: Using a Graph to Solve an Equation

Watch the following video to further explore the above scenario.

[Access multimedia content (http://openstax.org/books/algebra-1/pages/1-5-3-examining-an-equation-in-two-variables-and-its-graph-part-1)](http://openstax.org/books/algebra-1/pages/1-5-3-examining-an-equation-in-two-variables-and-its-graph-part-1)

Self Check

Which point is a solution to the line graphed below?



a. $(-3, 3)$
b. $(1, 3)$
c. $(0, 1)$

d. $(3, 1)$

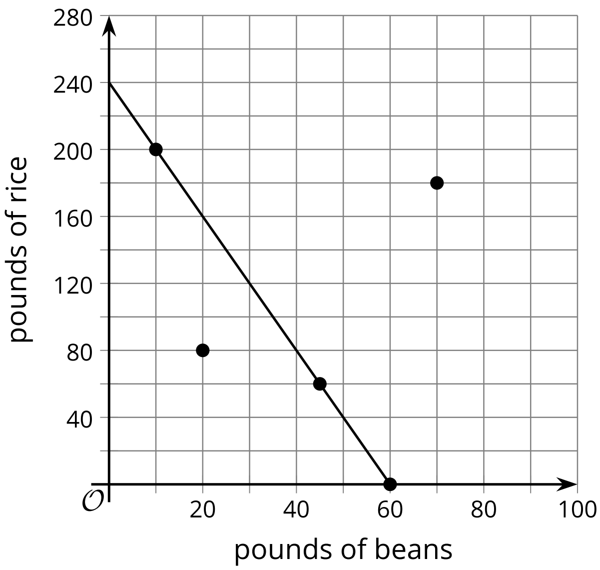
Additional Resources

Determining the Meaning of Solutions of a Graphed Line

Suppose we are buying beans and rice to feed a large gathering of people, and we plan to spend $120 on the two ingredients. Beans cost $2 a pound, and rice costs $0.50 a pound.

If x represents pounds of beans and y pounds of rice, the equation $2x + 0.50y = 120$ can represent the constraints in this situation.

The graph of $2x + 0.50y = 120$ shows a straight line.



Each point on the line is a pair of x- and y-values that make the equation true and is thus a solution. It is also a pair of values that satisfy the constraints in the situation.

- The point $(10, 200)$ is on the line. If we buy 10 pounds of beans and 200 pounds of rice, the cost will be $2(10) + 0.50(200)$, which equals 120.
- The points $(60, 0)$ and $(45, 60)$ are also on the line. If we buy only beans—60 pounds of them—and no rice, we will spend $120. If we buy 45 pounds of beans and 60 pounds of rice, we will also spend $120.

What about points that are *not* on the line? They are not solutions because they don't satisfy the constraints, but they still have meaning in the situation.

- The point $(20, 80)$ is not on the line. Buying 20 pounds of beans and 80 pounds of rice costs $2(20) + 0.50(80)$ or 80, which does not equal 120. This combination costs less than what we intend to spend.
- The point $(70, 180)$ means that we buy 70 pounds of beans and 180 pounds of rice. It will cost $2(70) + 0.50(180)$ or 230, which is over our budget of 120.

Let's look at an example.

EXAMPLE

1. Looking at the graph about purchasing beans and rice, is the point (30, 120) a solution?

Solution

Yes, because $2(3) + 0.50(12) = 60 + 60 = 120$.

2. What does the point (30, 120) mean in this situation?

Solution

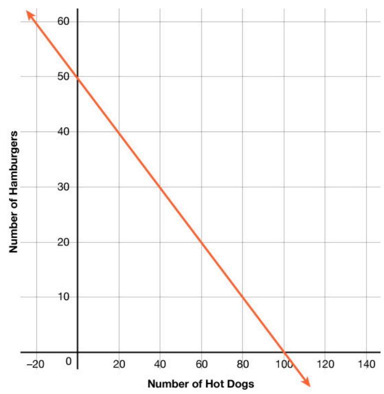
Compare your answer:

The point means that 30 pounds of beans and 120 pounds of rice were purchased. Since the point is on the line, the total spent was $120.

TRY IT **Determining the Meaning of Solutions of a Graphed Line**

At a high school baseball game, the concession stand sold hot dogs and hamburgers. Hot dogs were $1.50, and hamburgers cost $3. The goal is to make $150 for the game.

What is the meaning of the point $(40, 30)$ on the graph?



1.5.4 Writing, Graphing, and Solving a Linear Equation

Activity

Access the Desmos guide PDF (https://openstax.org/r/desmos-guide) for tips on solving problems with the Desmos graphing calculator.

Use the following scenario for questions 1 – 5:
A student has a savings account with $475 in it. She deposits $125 of her paycheck into the account every week. Her goal is to save $7000 for college.

1. How much will be in the account after 3 weeks?
2. How many weeks will it take until she has $1350?
3. Write an equation that represents the relationship between the dollar amount in her account and the number of weeks of saving.

Access multimedia content (http://openstax.org/books/algebra-1/pages/1-5-4-writing-graphing-and-solving-a-linear-equation)

4. Graph your equation using graphing technology. Mark the points on the graph that represent the amount after 3 weeks and the week she has $1350. Write down the coordinates. Use the graphing tool or technology outside the course. Graph the equation that represents this scenario using the Desmos tool above.
5. What is the x-intercept?
6. What other information does the x-intercept identify in the function?
7. What is the the y-intercept?

8. What is the slope?

Are you ready for more?

Extending Your Thinking

Use the following information to answer questions 1 – 5. A 450-gallon tank full of water is draining at a rate of 20 gallons per minute.

1. Write an equation that represents the relationship between the gallons of water in the tank and hours the tank has been draining.

2. Write an equation that represents the relationship between the gallons of water in the tank and seconds the tank has been draining.

Access multimedia content (http://openstax.org/books/algebra-1/pages/1-5-4-writing-graphing-and-solving-a-linear-equation)

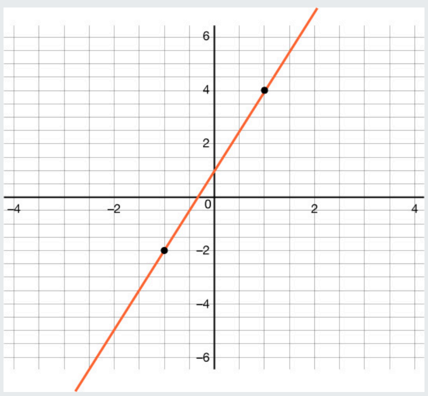
3. Graph each of your new equations. Use the graphing tool or technology outside the course. Graph the equation that represents this scenario using the Desmos tool above.

4. In what way are all of the graphs the same? In what way are they all different?

5. How would these graphs change if we used quarts of water instead of gallons? What would stay the same?

Self Check

Which of the following points is a solution to the graph below?



a. $(-2, -6)$
b. $(7, 2)$
c. $(0, 3)$
d. $(2, 7)$

Additional Resources

Writing Equations Using Graphs in Situations

An equation that contains two unknown quantities or two quantities that vary is called an equation in two variables. A solution to such an equation is a pair of numbers that makes the equation true.

Suppose Tyler spends \$40 on T-shirts and socks. A T-shirt costs \$10 and a pair of socks costs \$2.50. If t represents the number of T-shirts and p represents the number of pairs of socks that Tyler buys, what is an equation that represents the equation?

EXAMPLE 1

Step 1 - Create a two-variable equation.

The cost is $10 per t-shirt $(10t)$ plus $2.50 per pair of socks $(2.50p)$ which equals $40.
$10t + 2.50p = 40$

Now, we have to graph the equation. We will let $t = x$ and $p = y$.

Step 2 - Find the x-intercept.
To find the x-intercept, let $p = 0$.

$10t + 2.50p = 40$
$10t + 2.50(0) = 40$
$10t + 0 = 40$
$t = 4$

$(4, 0)$ is the x-intercept.
The x-intercept is also called a solution or zero.

In this scenario, 4 represents the number of T-shirts Tyler can buy if he doesn't purchase any socks with $40.

Step 3 - Find the y-intercept.
To find the y-intercept, let $t = 0$.

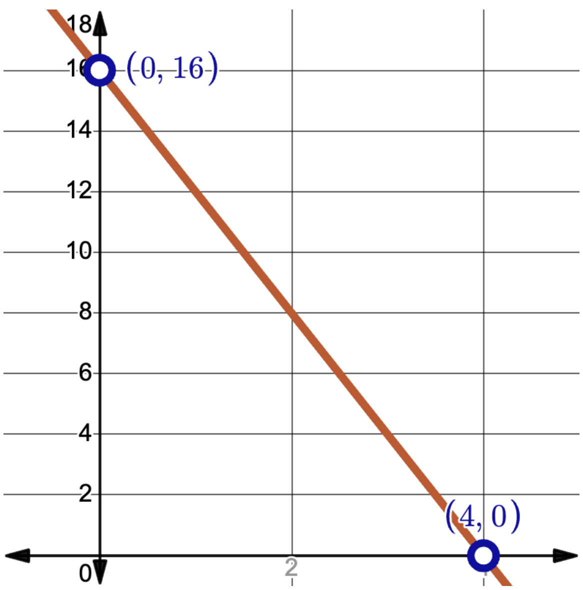
$10t + 2.50p = 40$
$10(0) + 2.50p = 40$
$(0) + 2.50p = 40$
$2.50p = 16$

$(0, 16)$ is the y-intercept.

In this scenario, 16 represents the number of socks Tyler can buy if he doesn't purchase any T-shirts with $40.

Step 4 - Graph the line by connecting the intercepts.

Let’s look at the graph of this equation:



Let’s reflect about the graph and what it means.

EXAMPLE 2

What is the slope of the graph?

Solution

$m = -4$

EXAMPLE 3

What does the point $(4, 6)$ mean on this graph?

Solution

Compare your answer:

If Tyler bought 4 T-shirts and 6 pairs of socks, it would cost more than $40.

TRY IT **Writing Equations Using Graphs in Situations**

Use Desmos or a graphing calculator to create a graph for $40x + 20y = 180$.

[Access multimedia content (http://openstax.org/books/algebra-1/pages/1-5-4-writing-graphing-and-solving-a-linear-equation)](http://openstax.org/books/algebra-1/pages/1-5-4-writing-graphing-and-solving-a-linear-equation)

1. If x represents the number of pairs of shoes and y represents the number of pairs of jeans, what is one combination that is a solution?
2. What does the combination you identified mean on the graph?

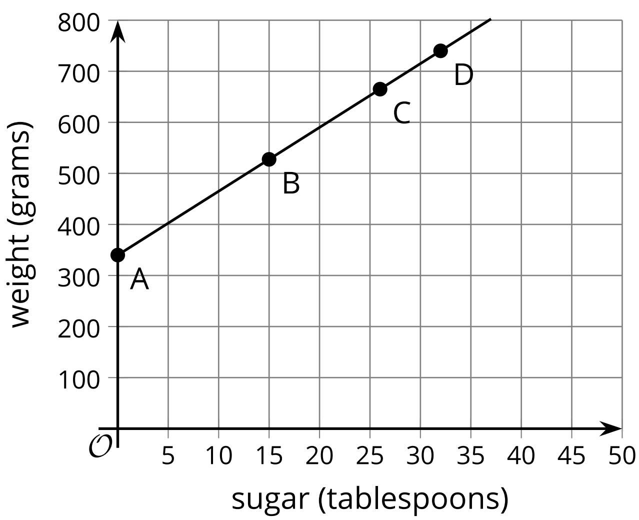
1.5.5 Examining an Equation in Two Variables and Its Graph, Part 2

Cool Down

A ceramic sugar bowl weighs 340 grams when empty. It is then filled with sugar. One tablespoon of sugar weighs 12.5 grams.

1. Write an equation to represent the relationship between the total weight of the bowl in grams, W, and the tablespoons of sugar, T.
2. When the sugar bowl is full, it weighs 740 grams. How many tablespoons of sugar can the bowl hold? Be prepared to show your reasoning.

The graph represents the relationship between the number of tablespoons of sugar in the bowl and the total weight of the bowl.

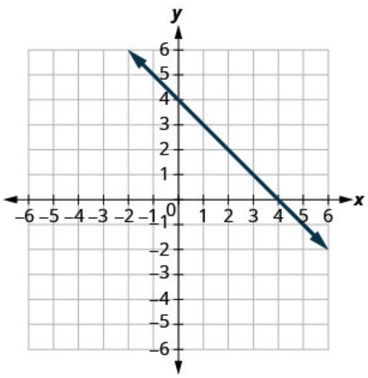


3. Which point on the graph could represent your answer to the previous question?
4. About how many tablespoons of sugar are in the bowl when the total weight is 600 grams?

1.5.6 Practice

Complete the following questions to practice the skills you have learned in this lesson.

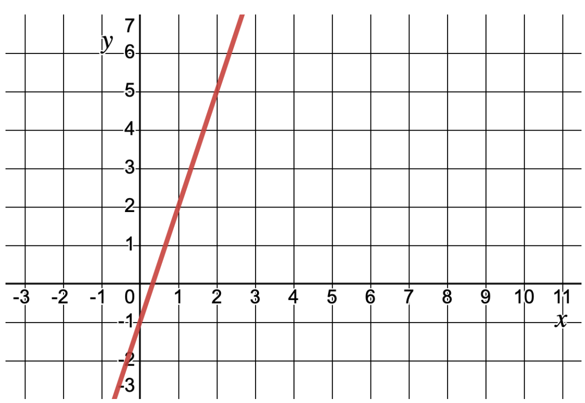
Use the graph for questions 1 - 2.



1. Which of the following ordered pairs are on the line? Select **three** answers.
   a. $(0, 4)$
   b. $(2, 2)$
   c. $(-2, 6)$
   d. $(-1, 3)$
   e. $(-4, 0)$

2. Which of the following ordered pairs are solutions to the equation? Select **three** solutions.
   a. $(0, 4)$
   b. $(2, 2)$
   c. $(-2, 6)$
   d. $(-1, 3)$
   e. $(-4, 0)$

For questions 3 and 4, use the graph of $y = 3x - 1$ as shown.

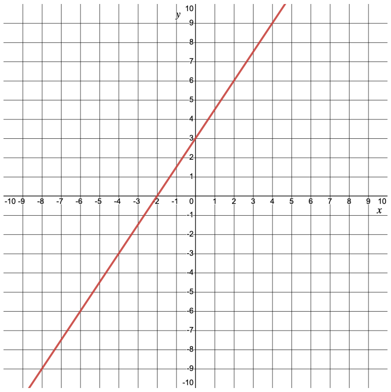


3. Which of the following ordered pairs are on the line? Select **three** solutions.
   a. $(0, -1)$
   b. $(1, 2)$
   c. $(2, 5)$
   d. $(1, 1)$
   e. $(0, 1)$

4. Which of the following ordered pairs are solutions to the equation? Select **three** solutions.
   a. $(0, -1)$
   b. $(1, 2)$

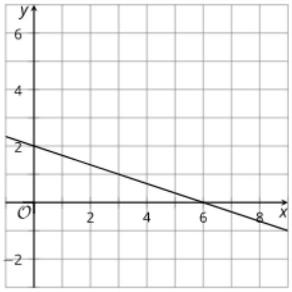
c. $(2, 5)$
d. $(1, 1)$
e. $(0, 1)$

5. Select three points that are on the graph of the equation $4y - 6x = 12$.



a. $(-4, -3)$
b. $(-1, 1.5)$
c. $(0, 3)$
d. $(0, -2)$
e. $(6, 4)$

6. Here is a graph of the equation $x + 3y = 6$.



Select three coordinate pairs that represent a solution to the equation.

a. $(0, 2)$
b. $(6, 0)$
c. $(2, 6)$
d. $(3, 1)$
e. $(8, -1)$
f. $(-1, 2)$

1.5.7 Lesson Summary

In this lesson, you learned how to:

- Use graphing technology to graph linear equations and identify solutions to the equations.
- Explain how the coordinates of the points on the graph of a linear equation are related to the equation.
- Explain the meaning of points on a graph in terms of the situation it represents when given the graph of a linear equation.

Here are the **activities** that helped you reach those goals:

- 1.5.1: Analyzing Graphs of Linear Equations
  - In this activity, you compared the characteristics of similar, yet different, graphs. In graphs, slopes, intercepts, axis labels, and where points are located can all create different meanings.
- 1.5.2: Graphing Linear Functions in Two Variables
  - In this activity, you reviewed how to graph points on a coordinate plane and then used your understanding to find and graph the x- and y-intercepts of an equation written in standard form. With these intercepts, you calculated the slope of the line.
  - 1.5.2: Self Check
  - 1.5.2: Additional Resources
- 1.5.3: Examining an Equation in Two Variables and Its Graph, Part 1
  - In this activity, you determined if a point was on the graph of a linear equation and what the meaning of that point was in a given situation
  - 1.5.3: Self Check
  - 1.5.3: Additional Resources
- 1.5.4: Writing, Graphing, and Solving a Linear Equation
  - In this activity, you wrote equations, graphed the equations, and used the graph to help solve the equations and find the meaning of solutions.
  - 1.5.4: Self Check
  - 1.5.4: Additional Resources
- 1.5.5: Examining an Equation in Two Variables and Its Graph, Part 2
  - In this activity, you wrote equations to express relationships. You also looked at graphs and determined the meaning of points on the graph to answer questions.

After these activities, you completed the following **practice**:

- 1.5.6: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?

1.6 EQUIVALENT EQUATIONS

1.6.0 Lesson Overview

In this lesson, you will learn that equivalent equations are equations with identical solutions. Specifically, you will explore how applying properties of real numbers can create equivalent equations. Additionally, you will use context to interpret the solution to equivalent equations.

When you finish this lesson, you will be able to:

- Tell whether two expressions are equivalent and explain why or why not.
- Identify the moves that can be made to transform an equation into an equivalent one.
- Explain what it means for two equations to be equivalent and how equivalent equations can be used to describe the same situation in different ways.

Here are the **activities** that will help you reach those goals:

- 1.6.1: Exploring Equivalent Expressions
- 1.6.2: Expressing Relationships as Equations
  - 1.6.2: Self Check
  - 1.6.2: Additional Resources
- 1.6.3: Exploring Related Equations
  - 1.6.3: Self Check
  - 1.6.3: Additional Resources
- 1.6.4: Recognizing Related Equations

After that, you'll **practice and review**.

- 1.6.5: Practice
- 1.6.6: Lesson Summary

1.6.1 Exploring Equivalent Expressions

Warm Up

Your teacher will assign you one of these expressions:

$$\frac{n^2-9}{2(4-3)} \qquad \text{or} \qquad (n+3)\cdot\frac{n-3}{8-3\cdot 2}$$

For questions 1 – 4, evaluate your expression when n is the given value.

1. $n=5$

2. $n=7$

3. $n=13$

4. $n=-1$

5. What did you discover when you compared your answers? Explain your rationale.

1.6.2 Expressing Relationships as Equations

Activity

In questions 1 – 3, write as many equations as possible that could represent the relationship between the ages of the two children in each family described. Be prepared to explain what each part of your equation represents.

1. In Family A, the youngest child is 7 years younger than the oldest, who is 18.

2. In Family B, the middle child is 5 years older than the youngest child.

3. Tyler thinks that the relationship between the ages of the children in Family B can be described with $2m - 2y = 10$, where m is the age of the middle child and y is the age of the youngest. Describe how Tyler came up with this equation.

4. **Select three** equations that are equivalent to $3a + 6 = 15$.
   a. $a + 3 = 12$
   b. $3a = 9$
   c. $\frac{1}{3}a = 1$
   d. $a + 2 = 5$

5. Explain your reasoning for the equivalent equations in number 4.

Are you ready for more?

Extending Your Thinking

Here is a puzzle:

- $m + m = N$
- $N + N = p$
- $m + p = Q$
- $p + Q = ?$

Write two expressions that are equivalent to $p + Q$.

Self Check

Which equation is equivalent to $3x - 6 = 18$?

a. $3x + 12 = 0$
b. $x - 2 = 18$
c. $3x = 12$
d. $3x = 24$

Additional Resources

Finding Equivalent Equations

Equivalent equations are equations that have the same solutions. They are often found by using inverse operations or by multiplying each term in the equation by the same value.

EXAMPLE

Write three equivalent equations to $4x - 2 = 8$ using one of three strategies.

- Using an inverse operation
- Using division
- Using multiplication

Using an inverse operation:
An equivalent equation can be determined by adding 2 to both sides.

$4x - 2 = 8$
$4x - 2 + 2 = 8 + 2$
$4x = 10$
$(4x - 2 = 8)$ and $(4x = 10)$ are equivalent.

Using division:
An equivalent equation can be determined by dividing by 2 (or multiplying by $\frac{1}{2}$).

$4x - 2 = 8$
$(\frac{1}{2})(4x - 2) = (\frac{1}{2})(8)$
$2x - 1 = 4$

$(4x - 2 = 8)$ and $(2x - 1 = 4)$ are equivalent.

Using multiplication:
An equivalent equation can be determined by multiplying the same number to both sides.

What is the equivalent equation if you multiply each term in the equation $4x - 2 = 8$ by 3?

TRY IT **Finding Equivalent Equations**

1. Determine the equivalent equation if you subtract 5 from each side of the equation, $3x + 5 = 15$.
2. Explain how $(3x = 10)$ is related to $(x = \frac{10}{3})$.
3. Multiply every term by the same number to determine an equivalent equation to $3x + 5 = 15$.
4. Is $(3x + 5 = 15)$ equivalent to $(6x + 15 = 60)$?
5. Explain your reasoning for your answer to number 4 and whether the equations $(3x + 5 = 15)$ and $(6x + 15 = 60)$ are equivalent.

1.6.3 Exploring Related Equations

Activity

With a partner, discuss the following situations and equations. For each, consider:

- What does the solution mean in the context of the situation?
- Are the given values solutions?

If operations are applied correctly, the solution to an equation is also the solution to all equations equivalent to it. If operations are incorrectly applied, the solution of each equation is different.

For questions 1 and 2, use the following situation and equation:

Noah is buying a pair of jeans and using a coupon for 10% off. The total price is $56.70, which includes $2.70 in sales tax. Noah's purchase can be modeled by the equation:

$x - 0.1x + 2.70 = 56.70$

1. What does the solution to the equation mean in this situation?
2. How can you verify that 70 is not a solution but 60 is the solution?
3. What was done to Noah's equation to make this one?
4. What is the interpretation of this new equation?
5. Is the solution the same?
6. What was done to Noah's equation to make this one?
7. What is the interpretation of this new equation?
8. Is the solution the same?
9. What was done to Noah's equation to make this one?
10. What is the interpretation of this new equation?
11. Is the solution the same?

For questions 12 – 14, use the equation $x - 0.1x = 56.70$.

12. What was done to Noah's equation to make this one?
13. What is the interpretation of this new equation?
14. Is the solution the same?

For questions 15 – 17, use the equation $x - 0.1x = 59.40$

15. What was done to Noah's equation to make this one?

16. What is the interpretation of this new equation?

17. Is the solution the same?

For questions 18–20, use the equation $2(x - 0.1x + 2.70) = 56.70$.

18. What was done to Noah's equation to make this one?

19. What is the interpretation of this new equation?

20. Is the solution the same?

21. Based on your work above, which of the six equations are equivalent to the original equation, $x - 0.1x + 2.70 = 56.70$? Select the **three** equations that are equivalent to the original.
    a. $100x - 10x + 270 = 5670$
    b. $x - 0.1x = 54$
    c. $0.9x + 2.70 = 56.70$
    d. $x - 0.1x = 56.70$
    e. $x - 0.1x = 59.40$
    f. $2(x - 0.1x + 2.70) = 56.70$

Video: Looking at Equivalent Equations

Watch the following video to learn more about why these are equivalent equations.

Access multimedia content (http://openstax.org/books/algebra-1/pages/1-6-3-exploring-related-equations)

Self Check

Which of the following is a correct next step to create an equation equivalent to $x - 0.2x - 4.2 = 13.4$?

a. $2(x - 0.2x) = 17.6$
b. $x - 0.2x = 9.2$
c. $10(x - 0.2x - 4.2) = 13.4$
d. $0.8x - 4.2 = 13.4$

Additional Resources

Properties of Equality

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Subtraction Property of Equality For any real numbers a, b, and c, if $a = b$, then $a - c = b - c$.	Addition Property of Equality For any real numbers a, b, and c, if $a = b$, then $a + c = b + c$.
Division Property of Equality For any real numbers a, b, and c, and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.	Multiplication Property of Equality For any real numbers a, b, and c, if $a = b$, then $ac = bc$.

For equations to be equivalent, inverse operations are used and must be applied to both sides of an equation so it remains balanced.

Inverse operations are operations that "undo" other operations.

EXAMPLE 1

Solve $3x = 15$.

Since 3 is being multiplied by x, to solve for x, divide both sides by 3.

$\frac{3x}{3} = \frac{15}{3}$

$x = 5$

EXAMPLE 2

Solve $\frac{x}{2} + 4 = 12$.

Step 1 - Subtract 4 from both sides.
$\frac{x}{2} + 4 - 4 = 12 - 4$

Step 2 - Simplify.
$\frac{x}{2} = 8$

Step 3 - Multiply both sides by 2.
$2 \times \frac{x}{2} = 8 \times 2$

Step 4 - Simplify.
$x = 16$

TRY IT **Properties of Equality**

For questions 1 – 2, use the scenario:

Denae bought 6 pounds of grapes for $10.74.

1. Write an equation for the situation.
2. Solve the equation.

1.6.4 Recognizing Related Equations

Cool Down

A cardboard box, which weighs 0.6 pound when empty, is filled with 15 bags of beans and a 4-pound bag of rice. The total weight of the box and the contents inside it is 25.6 pounds. One way to represent this situation is with the equation $0.6 + 15b + 4 = 25.6$.

1. In this situation, what does the solution to the equation represent?
2. Identify the **three** equations that are equivalent to 0.6+15b+4=25.6.
   a. $15b + 4 = 25.6$
   b. $15b + 4 = 25$
   c. $3(0.6 + 15b + 4) = 76.8$
   d. $15b = 25.6$
   e. $15b = 21$

1.6.5 Practice

Complete the following questions to practice the skills you have learned in this lesson.

1. **Select three** equations that have the same solution as the equation $3x - 12 = 24$.
   a. $15x - 60 = 120$
   b. $3x = 12$
   c. $3x = 36$
   d. $x - 4 = 8$
   e. $12x - 12 = 24$

2. **Select three** equations that have the same solution as $2x - 5 = 15$.
   a. $2x = 10$
   b. $2x = 20$
   c. $2(x - 5) = 15$
   d. $2x - 20 = 0$
   e. $4x - 10 = 30$
   f. $15 = 5 - 2x$

3. **Select four** of the equations that are equivalent to: $x + 0.4x - 4.2 = 12.8$.
   a. $1.4x - 4.2 = 12.8$
   b. $1.4x = 8.6$
   c. $1.4x = 17$
   d. $14x - 42 = 128$
   e. $10x + 4x - 42 = 12.8$
   f. $14x = 170$

4. **Select two** of the following that are equivalent to $3x - 6 = 12$.
   a. $3x = 6$
   b. $3(x - 2) = 12$
   c. $x - 2 = 4$
   d. $x - 2 = 12$
   e. $x - 6 = 4$

5. Identify which step the mistake is made in finding an equivalent equation to $2(x - 4) = 16$.

   Step 1: $2x - 4 = 16$
   Step 2: $2x = 20$
   Step 3: $x = 10$

   a. Step 1
   b. Step 2
   c. Step 3

6. Identify which step the mistake is made in finding an equivalent equation to $2x + 3x - 4 = 20$.

   Step 1: $5x - 4 = 20$
   Step 2: $5x = 16$
   Step 3: $x = 16/5$

   a. Step 1
   b. Step 2
   c. Step 3

7. Identify which step the mistake is made in finding an equivalent equation to $7x - 4x - 5 = 16$.

   Step 1: $3x - 5 = 16$
   Step 2: $3x = 21$
   Step 3: $x = 62$

a. Step 1
b. Step 2
c. Step 3

8. Fill in the blank to complete the equivalent equation to $3(x-5)=30$.

$3x-$ ____ $=30$

9. Fill in the blank to complete the equivalent equation to $\frac{x}{2}-5=11$.

$x-$ ______ $=22$

10. Fill in the blank to complete the equivalent equation to $3(x-9)=24$.

$x-$ ____ $=8$

1.6.6 Lesson Summary

In this lesson, you learned how to:

- Tell whether two expressions are equivalent and explain why or why not.
- Identify the moves that can be made to transform an equation into an equivalent one.
- Explain what it means for two equations to be equivalent, and how equivalent equations can be used to describe the same situation in different ways.

Here are the **activities** that helped you reach those goals:

- 1.6.1 Exploring Equivalent Expressions
  - In this activity, you substituted values into equivalent expressions to show they are equivalent.
- 1.6.2 Expressing Relationships as Equations
  - In this activity, you wrote equivalent equations given different relationships.
  - 1.6.2 Self Check
  - 1.6.2 Additional Resources
- 1.6.3 Exploring Related Equations
  - In this activity, you identified changes that were made to create equivalent equations then determined if other equations were also equivalent after certain moves were made to the equation.
  - 1.6.3 Self Check
  - 1.6.3 Additional Resources
- 1.6.4 Recognizing Related Equations
  - In this activity, you determined which equations in a given group were all equivalent to each other.

After these activities, you completed the following **practice**:

- 1.6.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?