$$\overbrace{n^2 \text{ is odd}}^{P} \leftrightarrow \overbrace{n \text{ is odd}}^{Q}, \quad n \in \mathbb{Z}$$

$P \to Q$ If n^2 is odd $\longrightarrow$ n is odd

I will use a proof by contraposition $\neg Q \to \neg P$

Suppose n is an Even integer, so $n = 2k$, for some integer k

So $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$

Because $2(2k^2)$ is the form of an even number, we have proven by contraposition that If n^2 is odd then n is odd

$Q \to P$ If n is odd then n^2 is odd

I will use a direct proof

Suppose that n is odd, so $n = 2k+1$, for some integer k

So $n^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$

Because we have shown that $n^2 = 2(2k^2+2k)+1$, that is the form of an odd number.

Therefore, we have shown by direct proof that if n is odd then so is n^2.

$$\overbrace{\text{If ab is even}}^{P} \text{ then } \overbrace{\text{a or b is even}}^{Q}$$

I will prove this by a proof by contrapositive

$\neg Q \rightarrow \neg P$

$\neg Q =$ If a and b are odd then $\neg P =$ ab is odd

Supposing that a and b are both odd then

$a = 2k+1$ and $b = 2k+1$, for some integer k.

Then $ab = (2k+1)(2k+1) = 4k^2+4k+1 = 2(2k^2+2k)+1$

$2(2k^2+2k)+1$ is the form of an odd number.

Hence, we have shown, by contraposition, that if ab is even then a or b is even.

There is no positive integer such that
$n^3 + n^2 = 100$

I will use a proof by Existence

Suppose $n = 4$

Then $n^3 + n^2 = 4^3 + 4^2 = 64 + 16 = 70$

$70 < 100$ so $n > 4$

Suppose $n = 5$

Then $n^3 + n^2 = 5^3 + 5^2 = 125 + 25 = 150$

$100 < 150$, so $n < 5$

Therefore, $4 < n < 5$

Hence, we have proven, by existence, that n is not a positive integer. This is because $4 < n < 5$.

Q.E.D