Let $P(n) = 1\cdot 1! + 2\cdot 2! + \dots + n\cdot n! = (n+1)! - 1$

Basis step $P(1) = (1+1)! - 1 = 1, \quad 1\cdot 1! = 1$

$1 = 1$ true, $P(1) =$ True

Inductive step $P(n) \Rightarrow P(n+1)$

$$P(n) = \underbrace{1\cdot 1! + 2\cdot 2! + \dots + n\cdot n!} = (n+1)! - 1$$

$$P(n+1) = \underbrace{1\cdot 1! + 2\cdot 2! + \dots + n\cdot n!}_{P(n)} + (n+1)\cdot(n+1)! = (n+2)! - 1$$

We can substitute the formula for $P(n)$ into $P(n+1)$

$$(n+1)! - 1 + (n+1)\cdot(n+1)! = (n+2)! - 1$$

$$(n+1+1)(n+1)! - 1 = (n+2)! - 1$$

$$(n+2)(n+1)! - 1 = (n+2)! - 1$$

$$(n+2)! - 1 = (n+2)! - 1 \quad \text{True}$$

Because $P(1)$ is true, and $P(n) \Rightarrow P(n+1)$, we have shown by weak mathematical induction that $1\cdot 1! + 2\cdot 2! + \dots + n\cdot n! = (n+1)! - 1$ is true for any positive integer $n$

Simplify first three terms

$(x^2-2)^{15}$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

$$(x^2-2)^{15} = ((2x)+(-2))^{15}$$

$$(x^2-2)^{15} = \sum_{k=0}^{15} \binom{15}{0} (x^2)^{n-k} (-2)^k$$

first 3 terms = $\underbrace{\binom{15}{0}(x^2)^{15}(-2)^0}_{1\text{ term}} + \underbrace{\binom{15}{1}(x^2)^{14}(-2)^1}_{2\text{nd term}} + \underbrace{\binom{15}{2}(x^2)^{13}(-2)^2}_{3\text{rd term}}$

$$\binom{15}{0} x^{30} + \binom{15}{1} -2x^{28} + \binom{15}{2} 4x^{26}$$

$${}_nC_r = \frac{n!}{(n-r)!(r!)} = \binom{n}{r}$$

$$\binom{15}{0} = \frac{15!}{15!\,0!} = 1 \cdot x^{30} = x^{30}$$

$$\binom{15}{1} = \frac{15!}{14!\,1!} = 15 \cdot -2x^{28} = -30x^{28}$$

$$\binom{15}{2} = \frac{15!}{13!\,2!} = 105 \cdot 4x^{26} = 420x^{26}$$

$$\boxed{x^{30} - 30x^{28} + 420x^{26}}$$

Find Recursive definition 2, 5, 8, 11, 14, 17

This is an arithmetic sequence
where last term is added by 3
for next term, with $a_0 = 2$

Therefore...

$$\boxed{a_n = a_{n-1} + 3}$$