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1. Given n be a 3 digit numbers n
Let x, y, z are the 3 digits of n
and $x + y + z = 9k$. Given that k is all natural numbers

I will use a direct proof

$n = 100x + 10y + z$

$9k = x + y + z$

$z = 9k - x - y$

$n = 100(x) + 10y + 9k - x - y$

$n = 99x + 9y + 9k$

$n = 9(11x + y + k)$

Anything multiplied by 9 is divisible by 9

$\therefore$ n is divisible by 9.

13. I will use a proof by contradiction
Given N is any natural numbers
$n^2 + n + 17$ is not always prime
Let $n = 20$

$20^2 + 20 + 17 = 437$

The factors of 437 is 1, 437, 19, & 23

437 has 4 factors

437 is not prime

$\therefore$ N can be 20 therefore $n^2 + n + 17$ is not always prime

2. Given that n is even if and only if $7n+4$ is even.

I will use direct proof

Suppose n is even

then $n = 2k$ for some integer k

$7(2k)+4 = 14k+4 = 2(7k+4)$

$\Rightarrow$ Any number multiplied by 2 is even

Suppose that $7n+4$ is even

Then n is odd and $n = 2k+1$ for some integer k

$7(2k+1)+4 = 14k+7+4 = 14k+11$

$14k+10+1 = 2(7k+5)+1$

$\Rightarrow$ Any number multiplied by two plus one is always odd

If n is even and $n = 2k$ for some integer k,

$7(2k)+4 = 14k+4 = 2(7k+4)$

$\Rightarrow$ Any number multiplied by 2 is even

$\therefore$ n is even if and only if $7n+4$ is even

2.B. n is odd if and only if $5n+6$ is odd

I will use a proof by cases

Case 1 n is odd, $n = 2k+1$ for some int. k

$5(2k+1)+6$

$10k+5+6 = 10k+10+1 = 2(2k+5)+1$

Any integer multiplied by 2 and plus one is odd

Case 2 n is even, $n = 2k$ for some int K

$5(2k)+6 = 10k+6 = 2(5k+3)$

Any integer multiplied by 2 is even

$\therefore$ n is odd if and only if $5n+6$ is odd

3. Proof by cases

Given that x and y are positive integers

$x^4 \leq 2401 = x^2 = 49 = x \leq 7$

$1 \leq x \leq 7$

Case 1: $x=1$

$1^4 + y^4 = 2401$ $y^4 = 2400$ $y = 6.99$

$6.99 \neq$ int

Case 2: $x = 2$

$2^4 + y^4 = 2401$, $y^4 = 2385$, $y = 6.98 \neq$ integer

Case 3: $x=3$

$3^4 + y^4 = 2401$, $y^4 = 2320$, $y = 6.94 \neq$ int

Case 4: $x=4$

$4^4 + y^4 = 2401$, $y^4 = 2145$, $y = 6.80 \neq$ int

Case 5: $x=5$

$5^5 + y^4 = 2401$, $y^4 = 1776$, $y = 6.49 \neq$ int

Case 6: $x=6$

$6^4+y^4=2401$, $y^4=1105$. $y=5.77 \nexists$ int

Case 7: $x=7$

$7^4+y^4=2401$, $y^4=0$, y is not a positive integer

$\therefore$ There are no solutions for $x^4+y^4=2401$, when x and y are both positive integers

4. A. I will use a proof by contrapositive

Given that $c=a+b$

Suppose that A and B do not have the same parity

$A=2k$, $B=2k+1$

$C=2k+2k+1 = 4k+1 = 2(2k)+1$

C is odd

The same thing can be said if A was odd and B was even

$\therefore$ C is even if and only if A and B have the same parity

4.B Proof by construction

buckets 11g 7g

0g $\xleftarrow{7g}$ 7g fill

7g $\xleftarrow{4g}$ 7g fill

Empty $\leftarrow$ 11g 3g

0g $\leftarrow$ 3g

3g $\leftarrow$ 7g fill

$\boxed{10g \quad 0g}$

5\. A. This seems like an attempt for a proof by cases but it only uses one case where n is 12. I would use a direct proof

Suppose $n = 2k$, $k \in \mathbb{Z}$

$n^2 = 2k^2 = 4k^2 = 4 \cdot k^2$

any number multiplied by 4 is divisible by 4.

B.

6\. A. Direct proof Given that $(P \times R) \cap (Q \times R)$ then

Suppose $x \in (P \times R) \cap (Q \times R)$

$\Leftrightarrow x \in (P \times R) \wedge x \in (Q \times R)$

~~$(x \in P \times x \in R) \wedge (x \in Q \times x \in R)$~~

$x \in R (x \in P \wedge x \in Q)$

$R \cdot (P \cap Q)$

4\. i For all x that are a real number, x^2 cannot ever equal -1. This is true.

II. There exists some x that is a real number in which $x^3 = -1$. This is true

III. For all xs that are an integer, $x-1$ is an integer. This is also true