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1.A. $\exists x\, Kx \land \exists x \neg Kx$

B. $\forall x \neg Kx$

2. T = Knights, truth, F = knaves, liars

A	B	C	$A \lor \neg B$	$A \land \neg C$	$C \cup B$
T	T	T	F	F	F
- T	T	F	T	T	T
T	F	F	F	T	F
F	F	F	T	F	F
F	F	T	T	F	T
F	T	T	F	F	F
T	F	T	F	F	T
F	T	F	T	F	T

Islanders A and B are Knights while C is a knave. This is because given the three statements and combining them into one propositional statement we get $(A \lor \neg B) \land (A \land \neg C) \land (C \cup B)$. This is only satisfiable when A & B are True and C is false

3.A.

P	Q	$(P \leftrightarrow Q)$	$(\neg P \leftrightarrow Q)$	$(P \leftrightarrow Q) \leftrightarrow (\neg P \leftrightarrow Q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	T	F	F

The symbol $\leftrightarrow$ is only true when both P and Q are the same boolean value. This equation is unsatisfiable because there remains both a P and $\neg P$, both in seperate statements consisting of $\leftrightarrow$.

B. i.

P	Q	r	$(P \to r)$	$(Q \to r)$	$(P \to r) \vee (Q \to r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	T
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	F	T
F	T	T	T	T	T

P	Q	R	$(P \wedge Q)$	$(P \wedge Q) \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	F	F	F	T
F	F	T	F	T
F	T	F	F	T
F	T	T	F	T

ii. $(P \to r) \vee (Q \to r) \equiv (P \wedge Q) \to R$

↳ $(\neg P \vee r) \vee (\neg Q \vee r)$ Distributive, table 6 red arrow

↳ $(\neg P \vee \neg Q) \vee r$ Distributive

↳ $\neg(P \wedge Q) \vee r$ De morgans

↳ $(P \wedge Q) \to r$ Distributive

This proposition is satisfiable because there is atleast 1 combination of boolean values for P, Q, R where they both are True. I.E when P, Q + R, are all true

4. A. $498 < 2^n$ Every student has to have atleast 1 unique trail/node. "n" is "number of unique trails. $498 < 512$
n is 9, because 2^9 is 512. Therefore 9 is the least number trails required for each student to have a unique combo of trails walked

B. There is an unlimited # of trails that can be walked where the criteria is still met. Given that each member walks atleast 1 trail this problem can be represented mathematically by $2^n - 1$.

5. A. Splinter: Any toppings
i. $L \land \neg P$
II. $\neg S$
III. $P \to \neg L$
IV. $L \leftrightarrow (S \lor G)$

B. Yes, they can order a two topping pizza with licorice and granola (L and G). This pizza would satisfy all four conditions given by the ninja turtles

6. Modus Tollens: · If P, then Q

· ¬Q

∴ ¬P

· $\overbrace{\text{If}}^{\text{If}}\ \overbrace{\text{Q is a rhombus,}}^{\text{P}}\ \overbrace{\text{then}}^{\text{then}}\ \overbrace{\text{Q is a Parallelogram}}^{\text{Q}}$

$\overbrace{\text{Q is } \overbrace{\text{not}}^{\text{Not}} \text{ a Parallelogram}}^{\text{Q}}$

¬P [∴ Therefore, we can say that Q is not a rhombus