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This assignment is due on Friday, Sept. 3 to Gradescope by 6PM. There are 7 questions on this homework. You are expected to write or type up your solutions neatly. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own.

**Important:** Make sure to clearly write your full name and your student ID number at the top of your assignment. You may **neatly** type your solutions in LaTeX for extra credit on the assignment. Make sure that your images/scans are clear or you will lose points/possibly be given a 0. Additionally, please be sure to match the problems from the Gradescope outline to your uploaded images.

1. A supervisor believes that one of more of the hard-drives in a system is not working correctly.

The drives, X, Y, and Z are all able to report information about the status (good or bad) of the drives in the system.

The supervisor is unsure whether a drive is truly not working, or whether the problem is in the status reporting routines in one or more of the drives.

After polling each drive, the supervisor receives the following **status reports**:

**Drive X reports:** Drive Y is not working and drive Z is working.

**Drive Y reports:** Drive X is working if and only if Y is working.

**Drive Z reports:** At least one of the other two drives is not working.

- (a) Suppose: Write the **three status reports** in terms of  $X$ ,  $Y$ , and  $Z$  using the symbols of formal logic.

- (b) Complete the following truth table:

Suppose  $x$  : Drive  $X$  is working,

Suppose  $y$  : Drive  $Y$  is working,

Suppose  $z$  : Drive  $Z$  is working,

$x$	$y$	$z$	X's Report	Y's Report	Z's Report
T	T	T			
T	F	T			
F	T	T			
F	F	T			
T	T	F			
T	F	F			
F	T	F			
F	F	F			

- (c) Assuming that all of the reports are true, which drive(s) is/are working?

- (d) Assuming that all of the drives are working, which status report(s) is/are false?

**Solution:**

(a) X's report:  $\neg Y \wedge Z$ .

Y's report:  $X \leftrightarrow Y$ .

Z's report:  $\neg X \vee \neg Y$ .

(b)

<i>x</i>	<i>y</i>	<i>z</i>	X's Report	Y's Report	Z's Report
T	T	T	F	T	F
T	F	T	T	F	T
F	T	T	F	F	T
F	F	T	T	T	T
T	T	F	F	T	F
T	F	F	F	F	T
F	T	F	F	F	T
F	F	F	F	T	T

(c) Drive Z

(d) Status report x and status report z.

2. In mathematics/physics it is said that two ordered pairs  $(m, n)$  and  $(p, q)$  are considered equal when  $m = p$  and  $n = q$ .

Suppose  $a =$  'If  $(m, n) = (p, q)$ , then  $m = p$ .'

- (a) Write the converse of  $a$ . Is this converse true or false?
- (b) Write the inverse of  $a$ . Is the inverse true or false?
- (c) Write the contrapositive of  $a$ . Is this true or false?

**Solution:**

Suppose  $r$  is  $(m, n) = (p, q)$ , and  $k$  is  $m = p$

(a) Converse: 'If  $m = p$ ', then  $(m, n) = (p, q)$ . This is false when  $k$  is true and  $r$  is false. i.e. when  $n \neq q$ .

(b) Inverse: If  $(m, n) \neq (p, q)$ , then  $m \neq p$ . This is false when  $\neg r = T$  and  $\neg k = F$ , that is  $r = F$  and  $k = T$ . i.e. when  $m = p$  and  $n \neq q$ .

(c) Contrapositive: If  $m \neq p$ , then  $(m, n) \neq (p, q)$ . This is true when  $r = T$  and  $k = T$ .

The contrapositive has the same truth value as the original implication.

The contrapositive is False when  $\neg k = T$  and  $\neg r = F$ , i.e.  $k = F$  and  $r = T$ , or when  $m \neq p$  and  $(m, n) = (p, q)$ .

3. Suppose:

$p$  = 'You can drink alcohol.'

$q$  = 'You are under 21 years old.'

$r$  = 'You are operating machinery.'

(a) Translate the following statement into symbols of formal logic:

'You can't drink alcohol if you are under 21 years old or you are operating machinery.'

(b) Using the symbols of formal logic, what is the contrapositive of the statement in part 'a'?

(c) What is the contrapositive of the statement in part 'a' in English?

**Solution:**

(a)  $(q \vee r) \rightarrow \neg p$ .

(b)  $p \rightarrow (\neg q \wedge \neg r)$ .

(c) If you are allowed to drink, then it is because you are not under 21 years of age and you are not operating machinery.

4. Consider the following four propositions:

D: You have a CU diploma

P: You party every weekend.

C: You got a C or higher in every class.

T: You have unpaid parking fines at CU.

What is the translation (using propositional logic) of the statement:

You have a CU diploma only if you got a 'C' or higher in every class and  
you didn't party every weekend and you don't have unpaid parking fines at CU.

**Solution:**

(a)  $D \rightarrow (C \wedge (\neg P) \wedge (\neg T))$

5. Explain why the following compound proposition is or is not satisfiable.

(a)  $(p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee q)$

(b)  $(p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

**Solution:**

- (a) Upon constructing the truth table, when p and q are true, the given compound proposition is true and makes it satisfiable.
- (b) Upon constructing the truth table, all four combinations of truth values for p and q will lead the given compound proposition to false and makes it not satisfiable.

6. Consider the statement  $S = [\neg(p \rightarrow q)] \vee [\neg(p \vee q)]$ .

- (a) Construct a truth table for  $S$ .
- (b) What is a simpler expression that is logically equivalent to  $S$ ?

**Solution:**

(a)

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$p \vee q$	$\neg(p \vee q)$	$S = \neg(p \rightarrow q) \vee \neg(p \vee q)$	$\neg q$
T	T	T	F	T	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	T	F	F	F
F	F	T	F	F	T	T	T

(b)  $S$  is the same as  $\neg q$  as seen in the last column of the truth table seen in part 'a'.

7. Four students (A, B, C and D) at the University of Colorado Boulder want to cross the Broadway bridge near the UMC building. Everyone begins on the same side. You have 17 minutes to get all of them across to the other side. It's night time. The street lights are not working and there is only one flashlight with them. A maximum of two people can cross at one time. Any party who crosses, either one or two people, must have the flashlight with them. The flashlight must be walked back and forth, it cannot be thrown. Each person walks at a different speed. A pair must walk together at the rate of the slower person.

A: 1 minute to cross

B: 2 minutes to cross

C: 5 minutes to cross

D: 10 minutes to cross

For example, if A and D walk across first, 10 minutes have elapsed when they get to the other side of the bridge. If D returns with the flashlight, a total of 20 minutes have passed, and you have failed the mission.

Which combination would you choose so that all students reach the other side of the bridge within 17 minutes?

**Solution:**

A and B cross = 2 mins (total = 2)

A goes back = 1 mins (total = 3)

C and D cross = 10 mins (total = 13)

B goes back = 2 mins (total = 15)

A and B cross = 2 mins (total = 17)

or

A and B cross = 2 mins (total = 2)

B goes back = 2 mins (total = 4)

C and D cross = 10 minutes (total = 14)

A goes back = 1 min (total = 15)

A and B cross = 2 mins (total = 17)