1. $\sum\limits_{u=0}^{n} \binom{n}{u} 3^n \cdot 2^u$

Binomial Expansion

$(a+b)^n = \sum\limits_{u=0}^{n} \binom{n}{u} a^{n-u} \cdot b^u$

Set $a = 1$ and $b = 2$

$(1+2)^n = \sum\limits_{u=0}^{n} \binom{n}{u} (1)^{n-u} 2^u$

multiply each side by 3^n

$3^n \cdot 3^n = \sum\limits_{u=0}^{n} \binom{n}{u} 2^u \cdot 3^n$

$$\boxed{3^{2n} = \sum\limits_{u=0}^{n} \binom{n}{u} 3^n \cdot 2^u}$$

2. A. $\dfrac{13}{52} \cdot \dfrac{12}{51} = \dfrac{156}{2652} = \boxed{\dfrac{1}{17}}$

B. $\dfrac{13}{51} \cdot \dfrac{12}{50} = \dfrac{156}{2550} = \boxed{\dfrac{26}{425}}$

3. $x_1 + x_2 + x_3 + x_4 = 19$, $0 \leq x_i \leq 19$

$\quad {}_nC_r = \dfrac{n!}{r! \cdot (n-r)!}$ $\qquad n = 19 + 4 - 1$
$\qquad\qquad\qquad\qquad\qquad\qquad r = 4-1$

$\quad {}_{22}C_3 = \dfrac{22!}{3!(22-3)!} = \boxed{1540}$

4. A. 26 Possibilities, Length of 12

$\therefore \boxed{26^{12}}$

B. Length of scooby = 6

12 - 6 = 6 other Letters

Scooby can appear in 7 different spots.

$^l O ^l O ^l O ^l O ^l O ^l O ^l$

If O are letters and l is where scooby can be

$\therefore \boxed{26^6 \cdot 7}$

4.C. Daphne has same odds as Scooby of appearing here fore

$$26^{12} - 2(26^6 \cdot 7)$$

5. $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ where $k \leq n$

This is pascals identity

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(k-1)! \cdot (n-k)!} + \frac{(n-1)!}{k! \cdot (n-k-1)!}$$

$$= (n-1)! \left(\frac{k}{k! \cdot (n-k)!} + \frac{n-k}{k!(n-k)!} \right)$$

$$= (n-1)! \cdot \frac{n}{k!(n-k)!} = \frac{n!}{k!(n-k)!}$$

$$= \binom{n}{k}$$

6. 21 Consonants, 5 vowels, n = 5

A. Repetitions allowed $21^4 \cdot 5 \leftarrow$ num vowels

$\boxed{972,405}$ num combos of consonants

Not allowed
$21 \cdot 20 \cdot 19 \cdot 18 \cdot 5 = \boxed{718,200}$
num of consonants being able to used.

$\rightarrow$ ~~B. Repetitions allowed~~ ~~2 consonants max~~
~~$21^2 \cdot 5^3 \leftarrow$ combos of length 8 vowels only~~

~~Repetitions not allowed~~
~~$21 \cdot 20 \cdot 5 \cdot 4 \cdot 3$~~

C. Repetitions allowed, Start w x, y, or z ÷ 3
$3 \cdot 26^4 = \boxed{1,370,928}$

Repetitions not allowed
$3 \cdot 25 \cdot 24 \cdot 23 \cdot 22 = \boxed{910,800}$

6.

6. B. Repetitions

$21^2 \cdot 5^3$ 2 max consonants + 3 vowels

$21 \cdot 5^4$ 1 consonant + 4 vowels

$+ \quad 5^5$ all vowels
_____

$\boxed{71\,375}$

No repetitions

$21 \cdot 20 \cdot 5 \cdot 4 \cdot 3$

$21 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

$+ \quad 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or $5!$ all vowels
_____

$\boxed{27,840}$