Rocky Erdenebat 109480099

1. $1 \cdot 2 + 2 \cdot 3 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Base step $f(1) = \frac{(1)(1+1)(1+2)}{3} = \frac{1 \cdot 2 \cdot 3}{3} = \frac{6}{3} = 2$

$1 \cdot 2 = 2 \quad 2 = 2$ true

Inductive step: $f(n) \Rightarrow f(n+1)$

$$f(n) = 1 \cdot 2 + 2 \cdot 3 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

$$f(n+1) = \underbrace{1 \cdot 2 + 2 \cdot 3 + \ldots + n(n+1)}_{f_n} + (n+1)(n+1+1)$$

We can rewrite $f(n+1)$ by replacing the same parts with $f(n)$

Therefore $f(n+1) = \frac{n(n+1)(n+2)}{3} + (n+1)(n+2)$

2. $f_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$ for $n > 0$

Basis step: $f_1 = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^1 - \left(\frac{1-\sqrt{5}}{2}\right)^1\right) = 1$

The first number in fib. sequence is 1

$1 = 1$ True

Induction step: Show $f_n \Rightarrow f_{n+1}$

$$f(n+1) = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right)$$

The fib. sequence is also defined as

$f_{n+1} = f_n + f_{n-1}$ which can be rewritten as

$$\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n} - \left(\frac{1-\sqrt{5}}{2}\right)^{n}\right) + \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n-1} - \left(\frac{1+\sqrt{5}}{2}\right)^{n-1}\right)$$

$$= \frac{1}{\sqrt{5}}\left(\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n} + \left(\frac{1+\sqrt{5}}{2}\right)^{n-1}\right) - \left(\left(\frac{1-\sqrt{5}}{2}\right)^{n} + \left(\frac{1-\sqrt{5}}{2}\right)^{n-1}\right)\right)$$

$$= \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right) = f_{n+1}$$

Therefore, f_{n+1} is true therefore f_n must also be true by strong induction

3. A $C_n = \sum_{i=1}^{n} 3^n$, $3^1 + 3^2 + 3^3 + \ldots + 3^n$

B. $C_n = \frac{3^{n+1} - 3}{2}$

By the definition of $C_n = \frac{3^{n+1}-3}{2}$ show that this is the same as $C_n = \sum_{i=1}^{n} 3^n$, as defined in part a.

Proof Using Mathematical Induction

$C_n = \frac{3^{n+1} - 3}{2}$ for $n > 0$

Assuming $C_n = \sum_{i=1}^{n} 3^n$ is true

Basis step: First kid receives 3 pieces.

$C_1 = \frac{3^{1+1}-3}{2} = \frac{9-3}{2} = 3$, $\underline{3 = 3 \quad \text{True}}$

Inductive step: Show $C_n \Rightarrow C_{n+1}$

$C_n = \frac{3^2 - 3}{2} + \frac{3^3 - 3}{2} + \ldots + \frac{3^{n+1} - 3}{2} = \frac{3^{n+1} - 3}{2}$

$C_{n+1} = \underbrace{\frac{3^2 - 3}{2} + \frac{3^3 - 3}{2} + \ldots + \frac{3^{n+1} - 3}{2}}_{C_n} + 3^{n+1}$

We can replace the part of C_{n+1} that is the same as C_n with the equivalent to P(k)

Now $C_{n+1} = \frac{3^{n+1} - 3}{2} + 3^{n+1} = \frac{3^{n+1} - 3 + (2 \cdot 3^{n+1})}{2}$

$$\frac{3^{n+1}-3+(2\cdot 3^{n+1})}{2} = \frac{3\cdot 3^{n+1}-3}{2}$$

$$= \frac{3^{(n+1)+1}-3}{2} = C_{n+1}$$

By definition of C_{n+1} above we know that C_n is true