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Write clearly and in the box:

Name:	Student ID:
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**Exam Rules:**

**DO NOT TURN THIS PAGE OVER UNTIL THE EXAM BEGINS.**

- There are a total of 6 questions (some with parts) on this exam. Questions are on the front AND BACK of pages in this booklet - be sure to look on both sides of each page.
  - All cell phones must be stored in your backpack. If you have a cell phone anywhere on your body or at your desk during this exam you will receive a 0 on this quiz.
  - You are allowed a one-sided 8.5" x 11" crib sheet with hand-written (not typed) notes
  - You are allowed a non-graphing calculator.
  - No other outside resources including tablets, smartphones, smartwatches or any other electronic devices allowed.
  - No collaboration with other students is allowed during this exam.
  - Any violation of these rules will result in an F in the course and a report to the CU Honor Council.
  - **On EVERY problem: Show all steps justifying your answer(s) and simplify your answers!** Answers with no justification will receive no points.
  - You have **50 minutes** for this exam.
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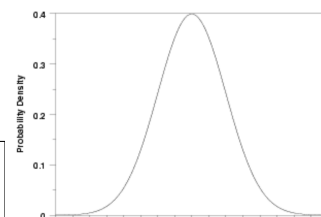
**Standard Normal Distribution:** Here  $\Phi(z)$  is the cumulative distribution function for the standard normal distribution evaluated at  $z$ . Its equivalent form in Python is  $\Phi(z) = \text{stats.norm.cdf}(z)$ .

$\Phi(4.75) \approx 1.000$     $\Phi(3.00) = 0.999$     $\Phi(2.58) = 0.995$     $\Phi(2.32) = 0.990$     $\Phi(2.00) = 0.977$     $\Phi(1.96) = 0.975$     $\Phi(1.88) = 0.970$   
 $\Phi(1.75) = 0.960$     $\Phi(1.64) = 0.950$     $\Phi(1.44) = 0.925$     $\Phi(1.28) = 0.900$     $\Phi(1.15) = 0.875$     $\Phi(1.04) = 0.850$     $\Phi(0.93) = 0.825$   
 $\Phi(0.84) = 0.800$     $\Phi(0.75) = 0.773$     $\Phi(0.67) = 0.749$     $\Phi(0.65) = 0.742$     $\Phi(0.60) = 0.725$     $\Phi(0.5) = 0.691$     $\Phi(0.45) = 0.675$   
 $\Phi(0.39) = 0.650$     $\Phi(0.32) = 0.625$     $\Phi(0.25) = 0.600$     $\Phi(0.19) = 0.575$     $\Phi(0.13) = 0.550$     $\Phi(0.06) = 0.525$     $\Phi(0.00) = 0.500$

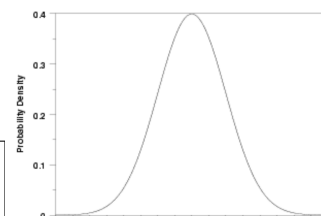
1. (15 pts) Let  $Z \sim N(0, 1)$ .

(a) What is  $P(Z \leq -0.65)$ ?

Write your answer as a number in the box AND then label/show what this number represents on the graph of the PDF.



(b) What is the critical value  $z_{0.15}$ ? Write your answer as a number in the box AND then label/show what this number represents on the graph of the PDF.



(c) What is  $E[Z]$ ? Write your answer as a number in the box.

2. (20 pts)

The grade point averages (GPAs) of a large population of college students are normally distributed with mean 2.4 and standard deviation 0.8.

(a) What percentage of students in this population possess a grade point average between 2.0 and 3.0?

(b) What GPA is the 40th percentile for this population of college students?

3. (18 pts) For each of the random variables described below:

- (i) Give the name of the distribution the random variable follows and give the value of any parameters, i.e. **your answer should be of the form:**  $RandomVariable \sim DistributionName(parameters)$ .
  - (ii) Give the distribution (i.e. either the probability mass function (pmf) or the probability density function (pdf)) for the random variable. Be sure to specify which values the random variable can take on (i.e. the domain of the pmf/pdf).
- (a) Suppose the amount of time a service technician needs to change the oil in a car is equally likely to be any real number between 10 and 21 minutes. Let  $Y$ =the time needed to change the oil in a car (in minutes).
- i). Name of Distribution:
  - ii). pmf/pdf and domain

- (b) Consider the following function related to a series of coin flips with a biased coin that lands on Heads with probability  $h$ . What distribution does the return value of the function belong to?

```
def flippy_flip(h=2/3):
    x = 1
    while np.random.choice([0,1], p=[1-h, h]) == 0:
        x += 1
    return x
```

- i). Name of Distribution:
- ii). pmf/pdf and domain

4. (10 pts) Suppose that a temperature is represented by the random variable  $X$ , whose units are in Kelvins, with **mean** 350 Kelvins and **standard deviation** 4 Kelvins.

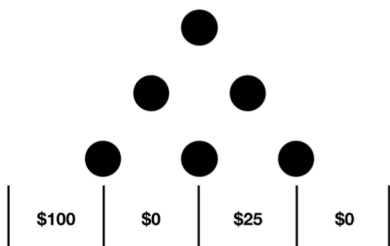
Let  $Y$  be the equivalent random variable with units converted to degrees Fahrenheit.

Recall that the formula for converting a temperature  $X$ , in Kelvin, to a temperature  $Y$ , in Fahrenheit is  $Y = \frac{9}{5}X - 460$ .

What is the **variance** of  $Y$ ? Fully simplify your answer.

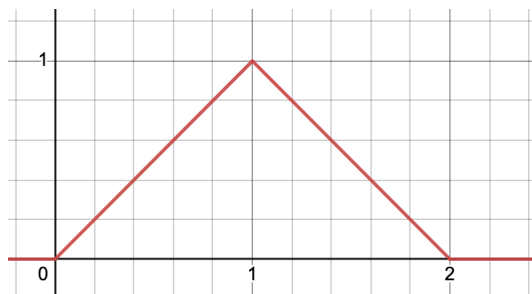
5. (15 pts)

A game of Plinko is to be played on the board shown below. You notice that the \$100 bin is on the left so you tilt the board such that the probability that the Plinko disk moves **left is three times as likely as moving right**. As usual, movements at all pegs are independent from each other, and there is no funny business except for your board tilting, which no one seems to have noticed. Furthermore, the disc can only be dropped from directly above the top-most peg. What is the **expected value of your winnings with a single disc**? Give your answer rounded to the nearest cent.



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6. (22 pts) The probability density function (pdf) for a continuous random variable  $X$  is shown here:



- (a) What is  $P(\frac{1}{2} < X < 1.4)$ ? Give your answer as a decimal rounded to the nearest thousandth.

- (b) What is the cumulative distribution function (cdf) of  $X$ ? Give your answer as a piecewise function, fully simplified.

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