第9章 正弦稳态电路的分析

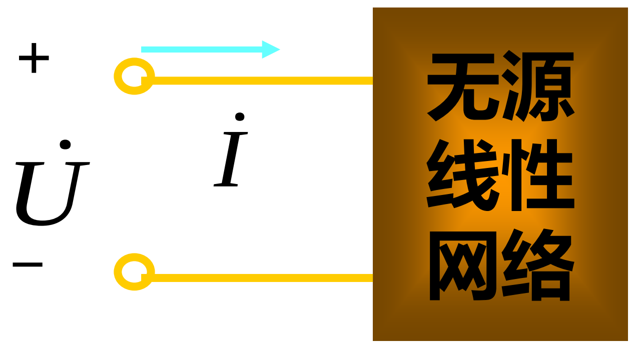
● 重点：

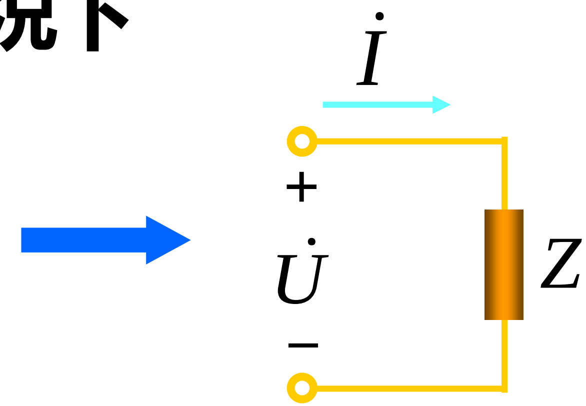
1. 阻抗和导纳；
2. 正弦稳态电路的分析；
3. 正弦稳态电路的功率分析；

9.1 阻抗和导纳

1. 阻抗

正弦稳态情况下





$$Z \overset{\text{def}}{=} \frac{\dot{U}}{\dot{I}} = |Z| \angle \varphi_z \, \Omega$$

欧姆定律的相量形式

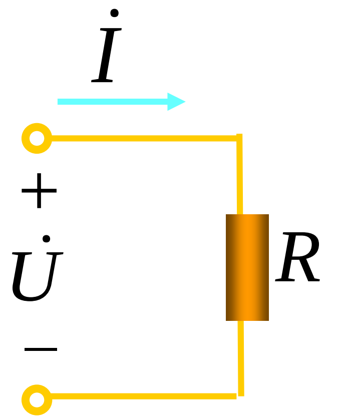
$$\begin{cases} |Z| = \dfrac{U}{I} \Omega \\ \varphi_z = \psi_u - \psi_i \end{cases}$$

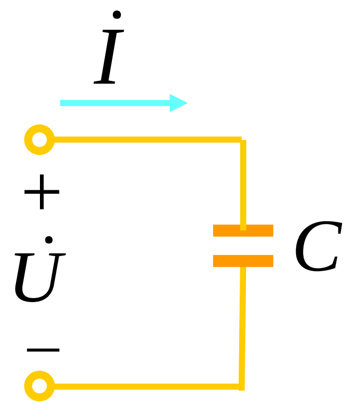
阻抗模

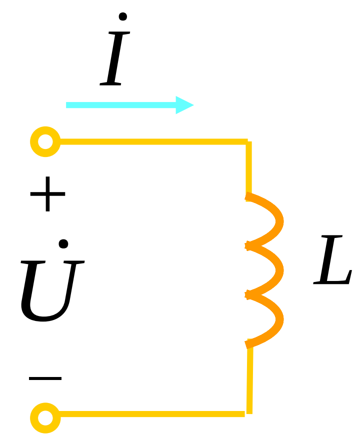
阻抗角

当无源网络内为单个元件时有：

$$Z \overset{\text{def}}{=} \frac{\dot{U}}{\dot{I}} = |Z|\angle\varphi_z \,\Omega$$







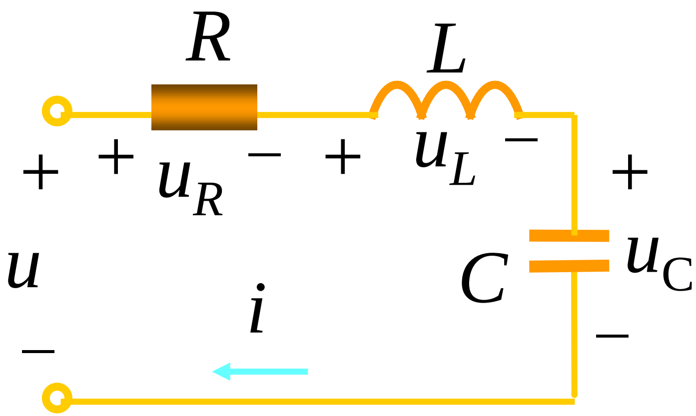
$$Z = \frac{\dot{U}}{\dot{I}} = R$$

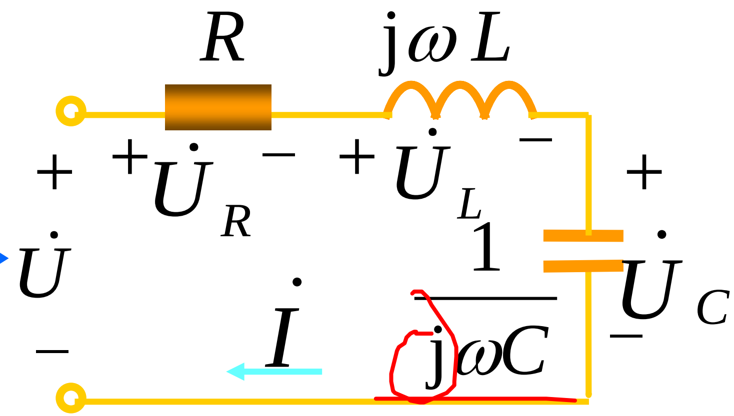
$$Z = \frac{\dot{U}}{\dot{I}} = -\mathrm{j}\frac{1}{\omega C} = \mathrm{j}X_C$$

$$Z = \frac{\dot{U}}{\dot{I}} = \mathrm{j}\omega L = \mathrm{j}X_L$$

表明　Z **可以是实数，也可以是虚数。**

2. *RLC*串联电路





KVL： $\dot{U} = \dot{U}_R + \dot{U}_L + \dot{U}_C = R\dot{I} + j\omega L\dot{I} - j\dfrac{1}{\omega C}\dot{I}$

$$= [R + j(\omega L - \frac{1}{\omega C})]\dot{I} = [R + j(X_L + X_C)]\dot{I} = (R + jX)\dot{I}$$

$$Z = \frac{\dot{U}}{\dot{I}} = R + j\omega L - j\frac{1}{\omega C} = R + jX = |Z|\angle\varphi_z$$

Z z 1 1

Z — **复阻抗；** $|Z|$ —**复阻抗的模；** φ_Z —**阻抗角；**

R —**电阻(阻抗的实部)；** X—**电抗(阻抗的虚部)。**

转换关系：

$$\begin{cases} |Z| = \sqrt{R^2 + X^2} \\ \varphi_z = \arctan \dfrac{X}{R} \end{cases}$$

$$Z = \frac{\dot{U}}{\dot{I}} = R + \mathrm{j}\omega L - \mathrm{j}\frac{1}{\omega C} = R + \mathrm{j}X = |Z|\angle\phi_Z$$

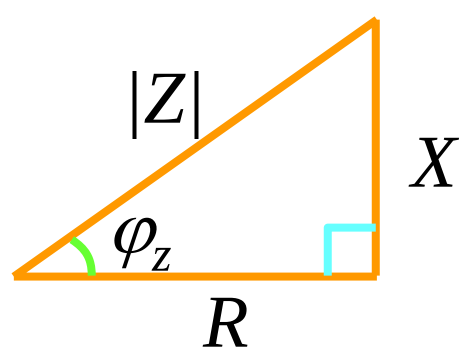
或

$$\begin{cases} R = |Z|\cos\varphi_Z \\ X = |Z|\sin\varphi_Z \end{cases}$$

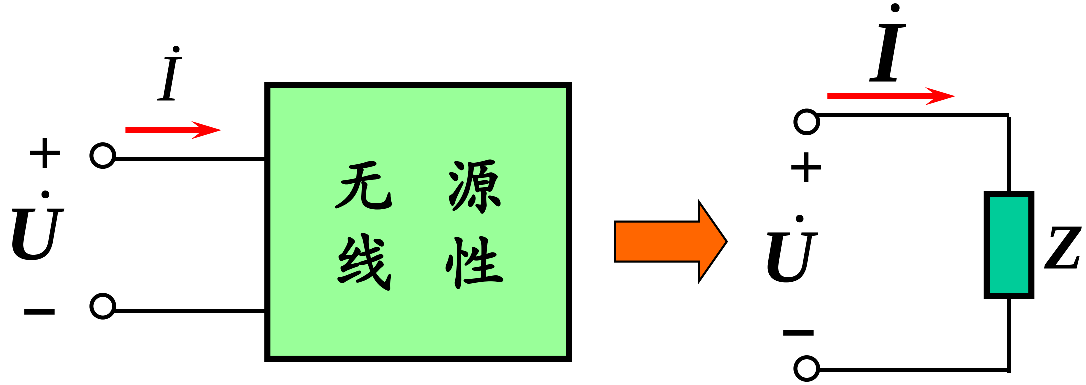
$$|Z| = \frac{U}{I}$$

$$\varphi_z = \psi_u - \psi_i$$

阻抗三角形



正弦激励下



复阻抗(impedance) $\quad Z = \dfrac{\dot{U}}{\dot{I}} = R + \mathrm{j}(\omega L - \dfrac{1}{\omega C}) = R + \mathrm{j}X$

电阻

电抗

特殊情况：

纯电阻 $\quad Z_R = R$

纯电感 $\quad Z_L = \mathrm{j}\omega L = \mathrm{j}X_L$

纯电容 $\quad Z_C = 1/\mathrm{j}\omega C = \mathrm{j}X_C$

$X_L = \omega L \qquad X_C = -1/\omega C$

感抗 $\qquad$ 容抗

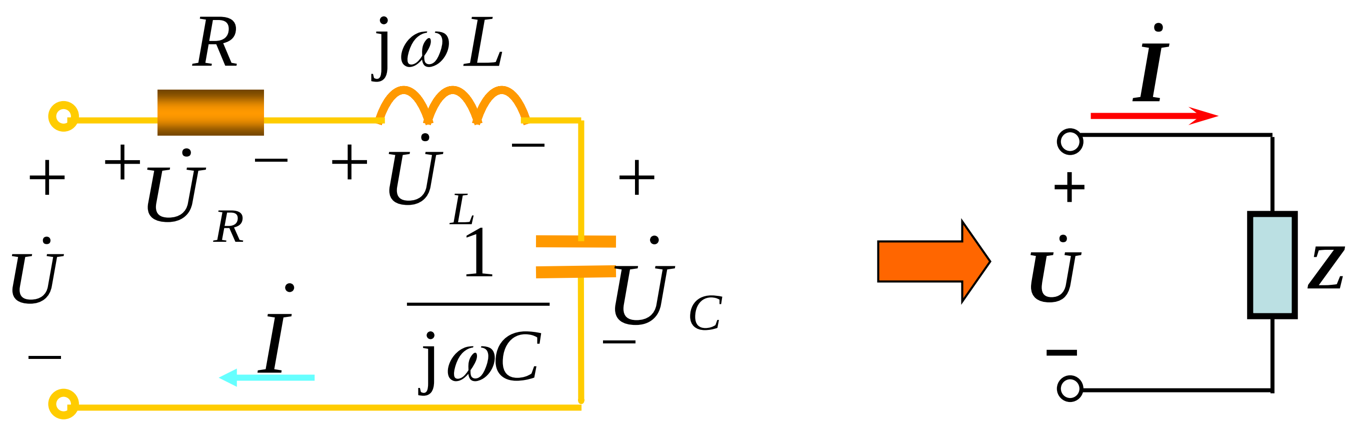
$$\dot{U} = R\dot{I}$$

$$\dot{U} = \mathrm{j}\omega L\dot{I}$$

$$\dot{U} = \frac{1}{\mathrm{j}\omega C}\dot{I}$$

分析 *R*、*L*、*C* 串联电路得出：

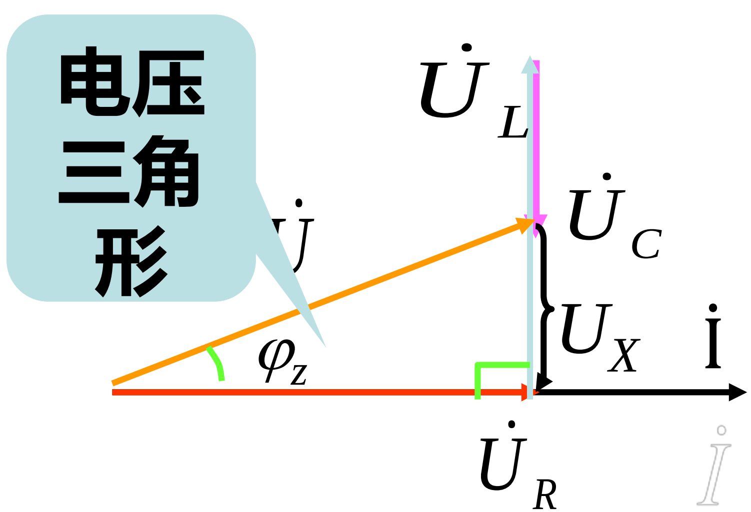
（1）$Z=R+\mathrm{j}(\omega L-1/\omega C)=|Z|\angle\varphi_Z$ **为复数，称复阻抗**

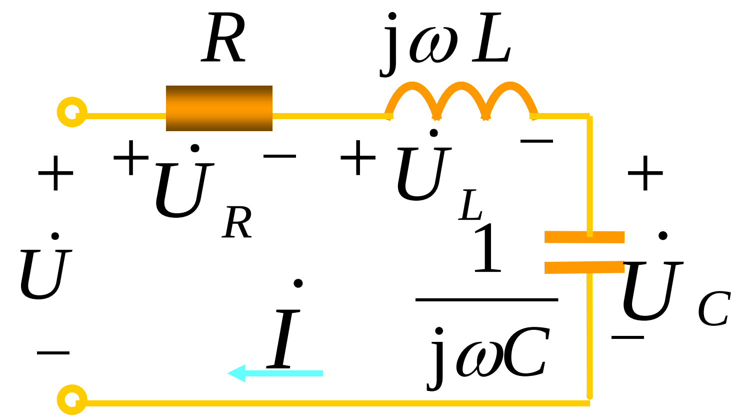


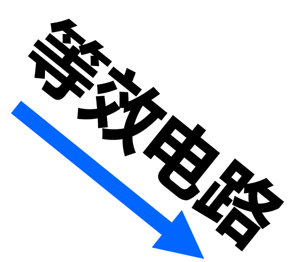
（2）$\omega L > 1/\omega C$， $X>0$， $\varphi_z>0$， $Z=R+\mathrm{j}(\omega L-1/\omega C)$

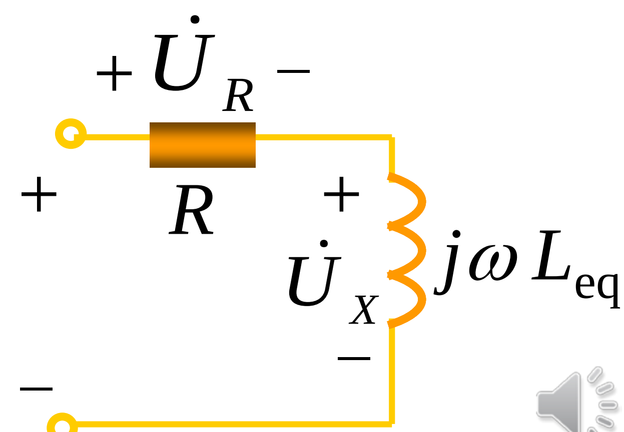
电路为感性， 电压超前电流。

相量图：一般选电流为参考向量， $\psi_i = 0$







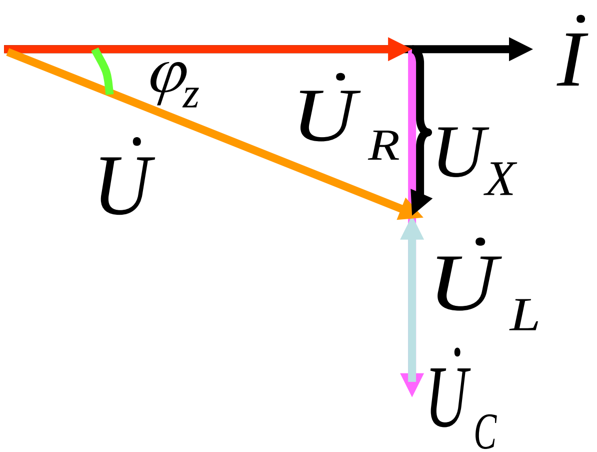


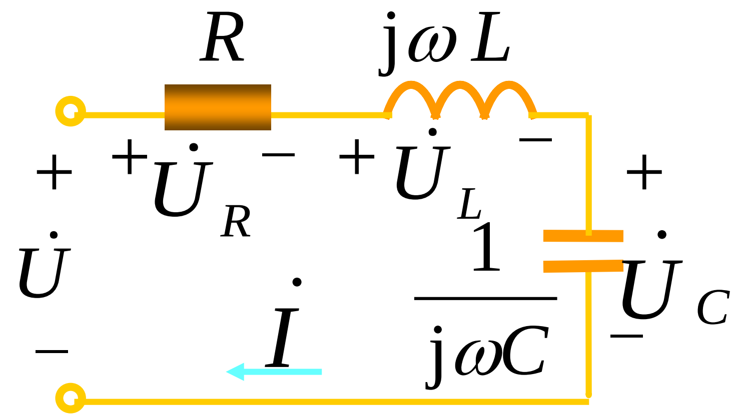
$$U=\sqrt{U_R^2+U_X^2}=\sqrt{U_R^2+(U_L-U_C)^2}$$

（3）$\omega L<1/\omega C$， $X<0$， $\varphi_z<0$， $Z=R+\mathrm{j}(\omega L-1/\omega C)$

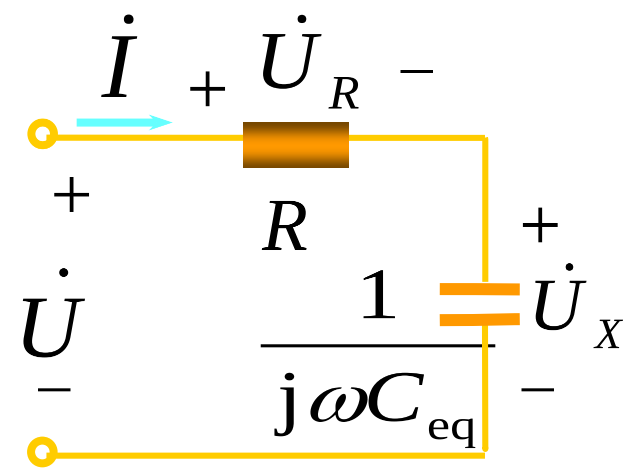
电路为容性， 电压落后电流。

相量图：一般选电流为参考向量





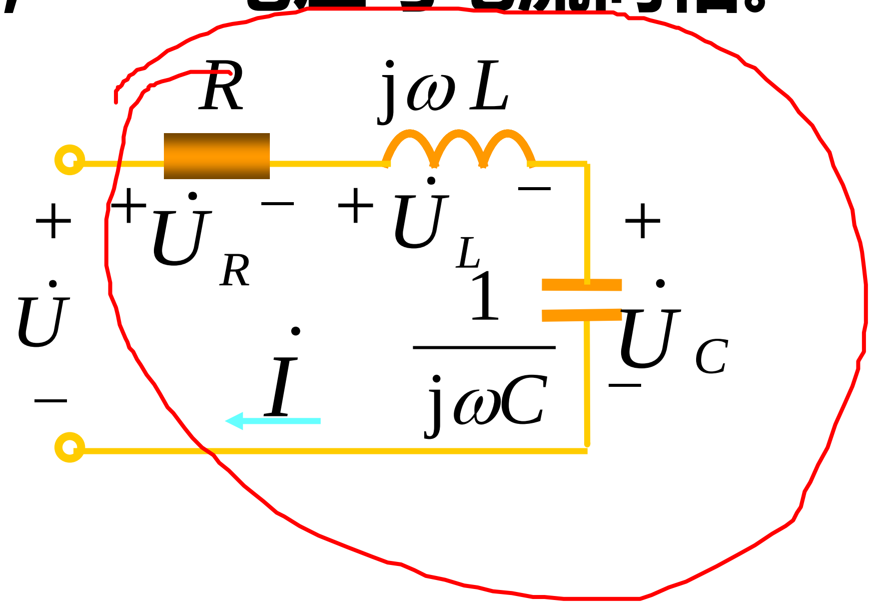
等效电路

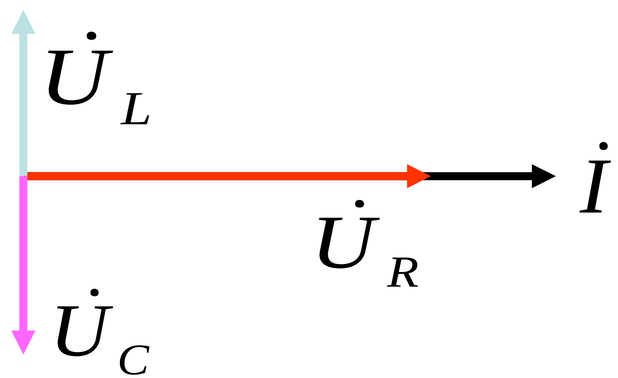


$$U=\sqrt{U_R^2+U_X^2}=\sqrt{U_R^2+(U_C-U_L)^2}$$

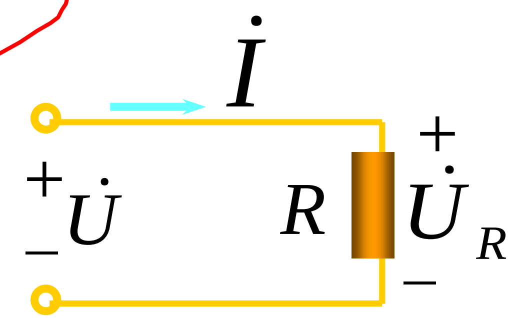
（4）$\omega L=1/\omega C$，$X=0$，$\varphi_z=0$，$Z=R+\mathrm{j}(\omega L-1/\omega C)$

电路为电阻性，　　电压与电流同相。





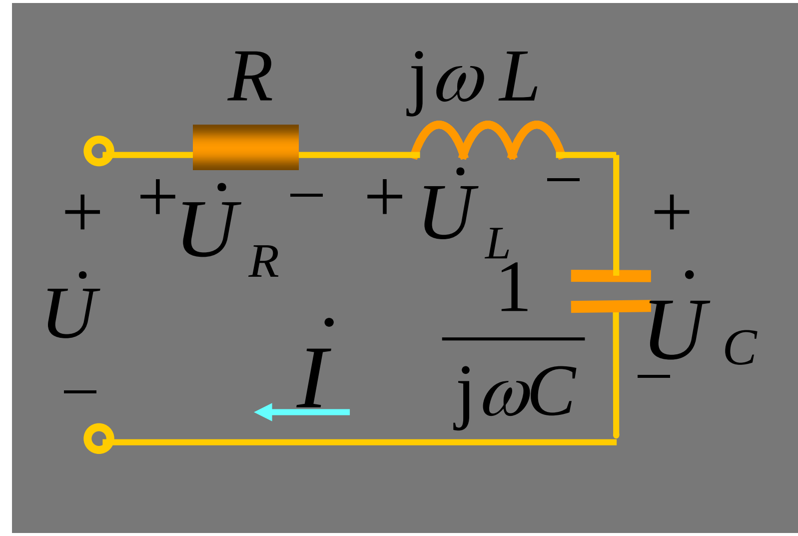
等效电路



例　**已知**：$R=15\Omega$, $L=0.3\text{mH}$, $C=0.2\mu\text{F}$,

$$u=5\sqrt{2}\cos(\omega t+60^\circ), f=3\times10^4\text{Hz}.$$

求 i, u_R , u_L , u_C .



解　**画出相量模型**

$$\dot{U}=5\angle60^\circ\ \text{V}$$

$$\text{j}\omega L=\text{j}2\pi\times3\times10^4\times0.3\times10^{-3}$$

$$=\text{j}56.5\Omega = 56.5\angle 90^\circ$$

$$-\text{j}\frac{1}{\omega C}=-\text{j}\frac{1}{2\pi\times3\times10^4\times0.2\times10^{-6}}=-\text{j}26.5\Omega = 26.5\angle-90^\circ$$

$$Z=R+\text{j}\omega L-\text{j}\frac{1}{\omega C}=15+\text{j}56.5-\text{j}26.5$$

$$=33.54\angle63.4^\circ\ \Omega$$

$$\dot{I}=\frac{\dot{U}}{Z}=\frac{5\angle 60^{\circ}}{33.54\angle 63.4^{\circ}}=0.149\angle -3.4^{\circ}\ \mathrm{A}$$

$$\dot{U}_R=R\dot{I}=15\times 0.149\angle -3.4^{\circ}=2.235\angle -3.4^{\circ}\ \mathrm{V}$$

$$\dot{U}_L=\mathrm{j}\omega L\dot{I}=56.5\angle 90^{\circ}\times 0.149\angle -3.4^{\circ}=8.42\angle 86.4^{\circ}\ \mathrm{V}$$

$$\dot{U}_C=-\mathrm{j}\frac{1}{\omega \mathrm{C}}\dot{I}=26.5\angle -90^{\circ}\times 0.149\angle -3.4^{\circ}=3.95\angle -93.4^{\circ}\ \mathrm{V}$$

则

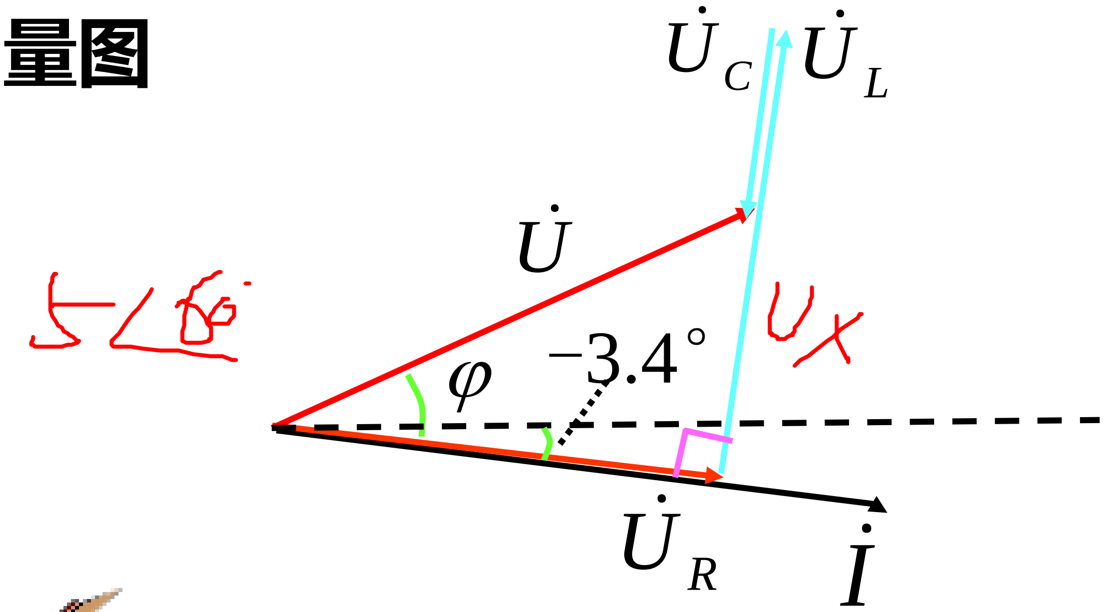
$$i=0.149\sqrt{2}\cos(\omega t-3.4^{\circ})\ \mathrm{A}$$

$$u_R=2.235\sqrt{2}\cos(\omega t-3.4^{\circ})\ \mathrm{V}$$

$$u_L=8.42\sqrt{2}\cos(\omega t+86.6^{\circ})\ \mathrm{V}$$

$$u_C=3.95\sqrt{2}\cos(\omega t-93.4^{\circ})\ \mathrm{V}$$

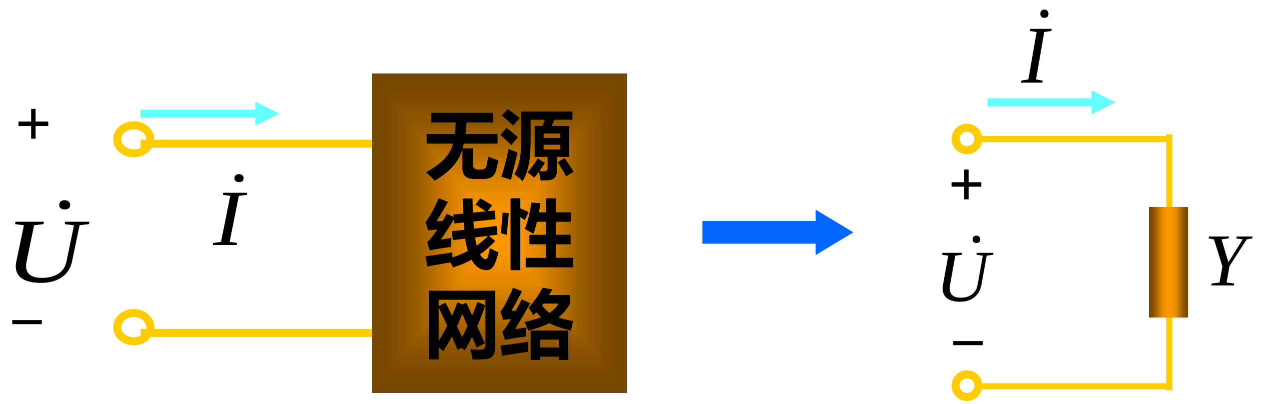
相量图



注意

U_L=8.42>U=5，**分电压大于总电压。**

3.导纳　　正弦稳态情况下



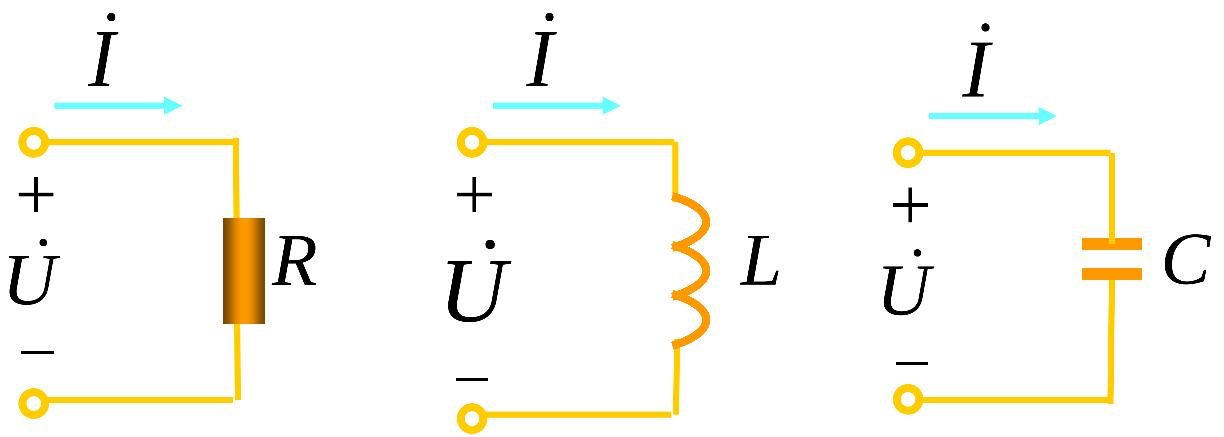
定义导纳 $Y = \dfrac{\dot{I}}{\dot{U}} = |Y| \angle \varphi_y$　S

$$\begin{cases} |Y| = \dfrac{I}{U} & \text{导纳模} \\ \varphi_y = \psi_i - \psi_u & \text{导纳角} \end{cases}$$

对同一二端网络:

$$Z=\frac{1}{Y}\ ,\ Y=\frac{1}{Z}$$

当无源网络内为单个元件时有:



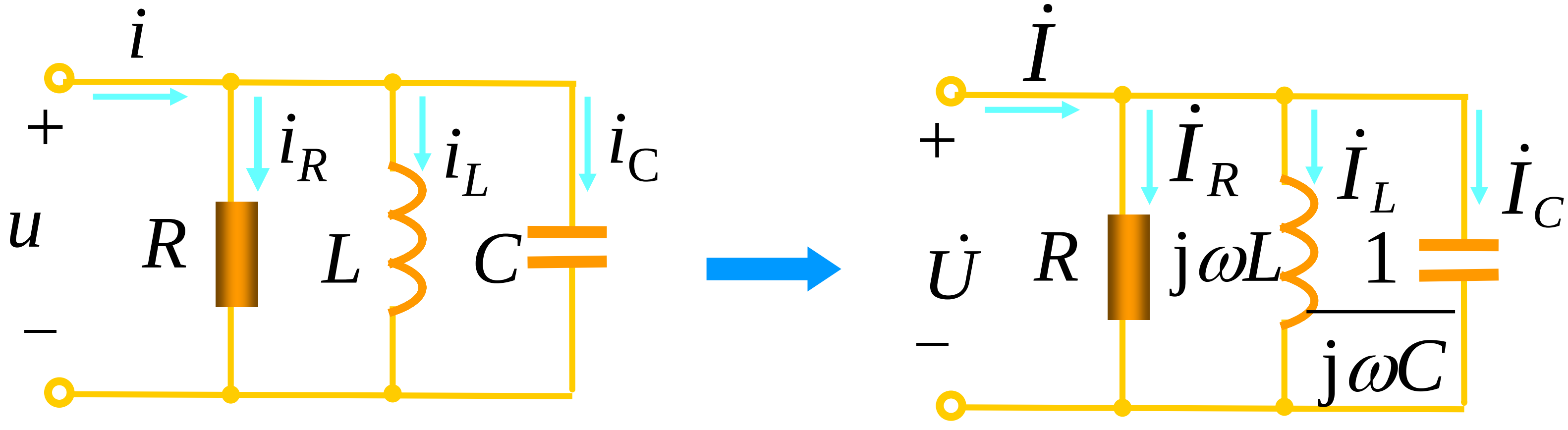
$$Y=\frac{\dot{I}}{\dot{U}}=\mathrm{j}\omega C=\mathrm{j}B_C$$

$$Y=\frac{\dot{I}}{\dot{U}}=\frac{1}{R}=G \qquad Y=\frac{\dot{I}}{\dot{U}}=\frac{1}{\mathrm{j}\omega L}=\mathrm{j}B_L$$

a+jb

表明 Y **可以是实数，也可以是虚数。**

4. *RLC*并联电路



由KCL：$\dot{I} = \dot{I}_R + \dot{I}_L + \dot{I}_C = G\dot{U} - \mathrm{j}\dfrac{1}{\omega L}\dot{U} + \mathrm{j}\omega C\dot{U}$

$$= (G - \mathrm{j}\frac{1}{\omega L} + \mathrm{j}\omega C)\dot{U} = [G + \mathrm{j}(B_L + B_C)]\dot{U} = (G + \mathrm{j}B)\dot{U}$$

$$Y = \frac{\dot{I}}{\dot{U}} = G + \mathrm{j}\omega C - \mathrm{j}\frac{1}{\omega L} = G + \mathrm{j}B = |Y|\angle\varphi_y$$

Y—复导纳；$|Y|$—复导纳的模；φ_y—导纳角； BL

G—电导(导纳的实部)；B—电纳(导纳的虚部)； BC

转换关系：
$$\begin{cases} |Y| = \sqrt{G^2 + B^2} \\ \varphi_y = \arctan \dfrac{B}{G} \end{cases}$$

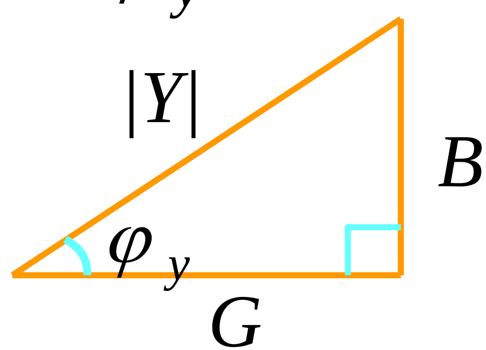
或
$$\begin{cases} G = |Y|\cos\varphi_y \\ B = |Y|\sin\varphi_y \end{cases}$$

$$\begin{cases} |Y| = \dfrac{I}{U} \\ \varphi_y = \psi_i - \psi_u \end{cases}$$

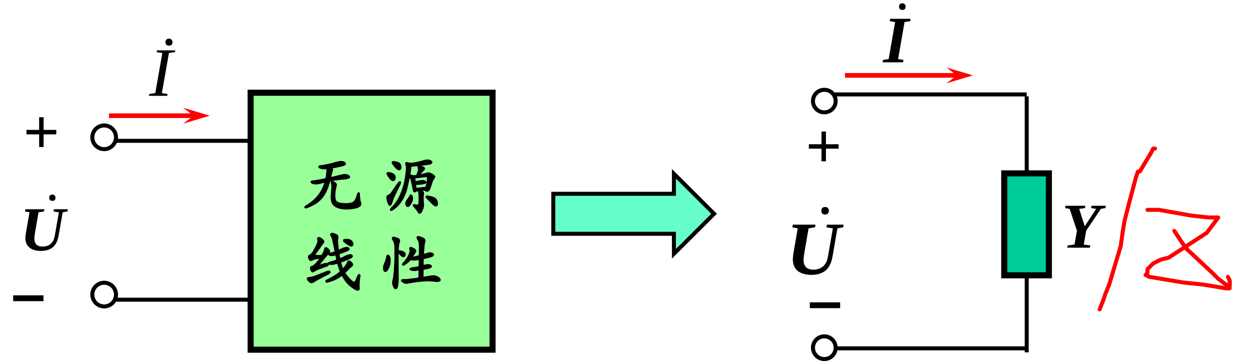
$|Z| = \dfrac{U}{I}$

$\varphi_z = \psi_u - \psi_i$

导纳三角形



正弦激励下



复导纳(admittance)

$$Y=\frac{\dot{I}}{\dot{U}}=G+\mathrm{j}(B_L+B_C)=G+\mathrm{j}B$$

电导　电纳

特殊情况：

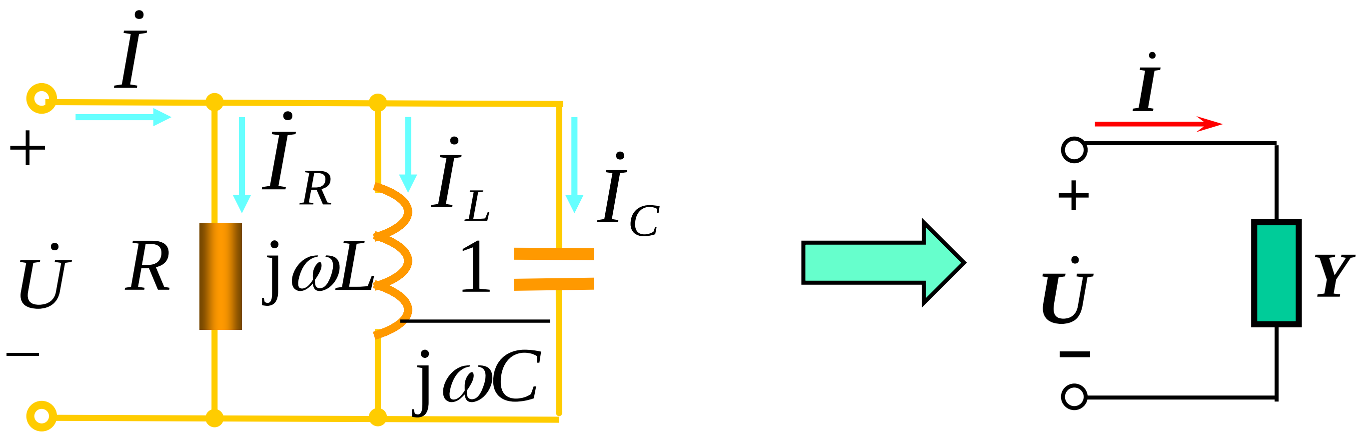
- 纯电阻　$Y_R=G$
- 纯电感　$Y_L=1/\mathrm{j}\omega L=\mathrm{j}B_L$
- 纯电容　$Y_C=\mathrm{j}\omega C=\mathrm{j}B_C$

$$B_L=-1/\omega L \qquad B_C=\omega C$$

感纳　　容纳

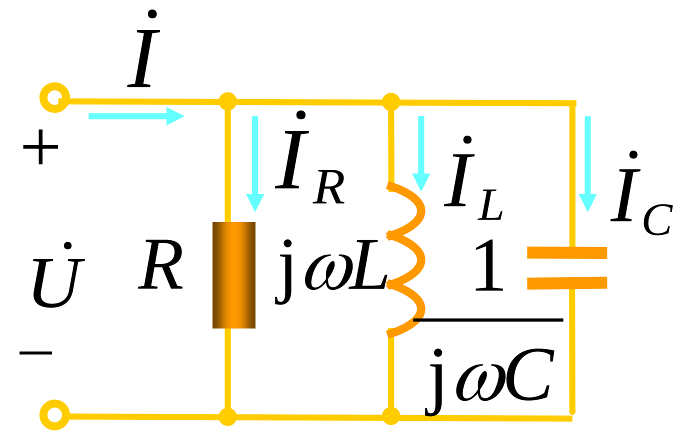
分析 R、L、C 并联电路得出：

（1）$Y=G+\mathrm{j}(\omega C-1/\omega L)=|Y|\angle\varphi_y$为复数，称复导纳；



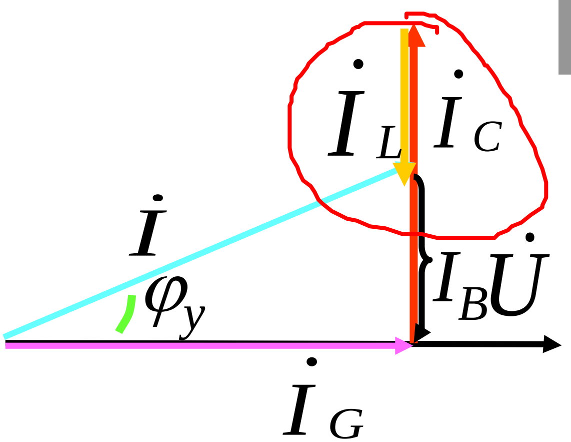
（2）$\omega C > 1/\omega L$， $B>0$， $\varphi_y>0$， $Y=G+\mathrm{j}(\omega C-1/\omega L)$

电路为容性， 电流超前电压。

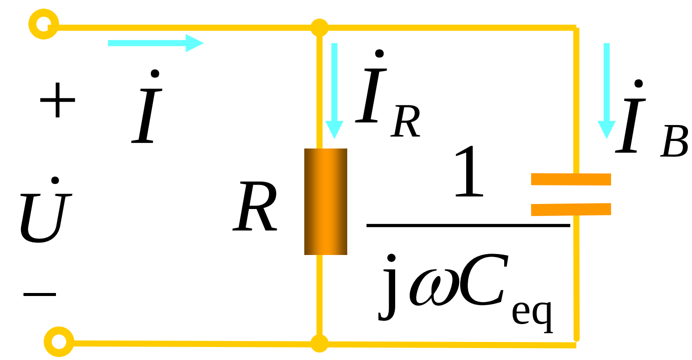


相量图：选电压为参考向量，

$$\psi_u = 0$$



等效电路



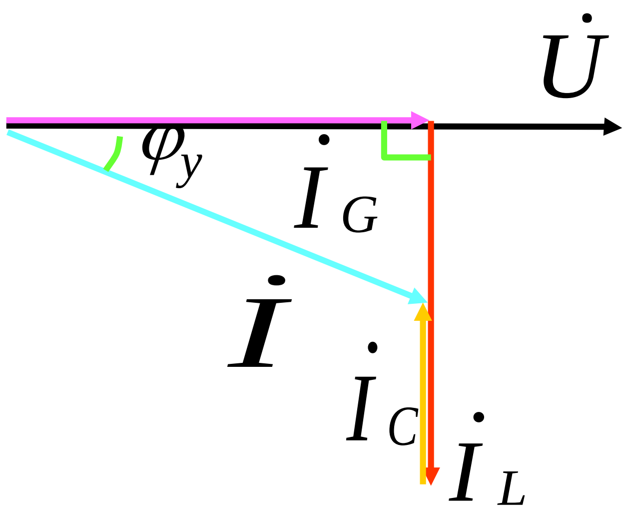
注意

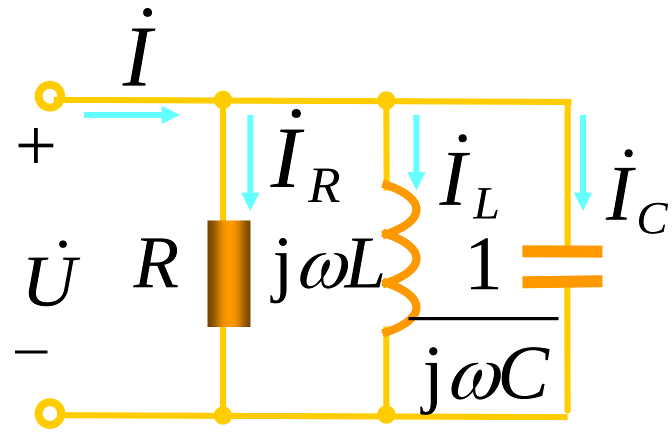
$$I=\sqrt{I_G^2+I_B^2}=\sqrt{I_G^2+(I_C-I_L)^2}$$

***RLC*并联电路会出现分电流大于总电流的现象**

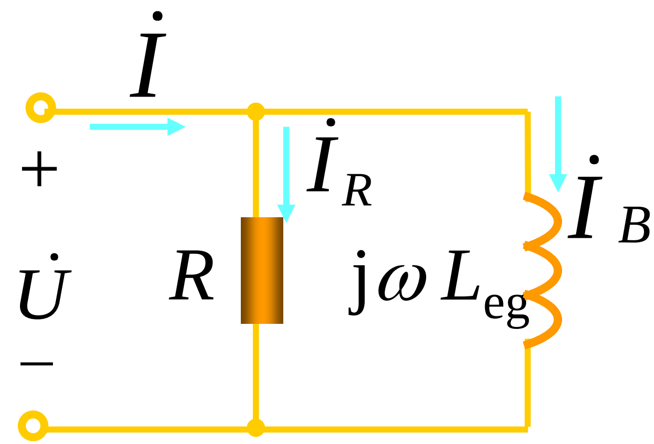
（3）$\omega C<1/\omega L$， $B<0$， $\varphi_y<0$， $Y=G+j(\omega C-1/\omega L)$

电路为感性， 电流落后电压；





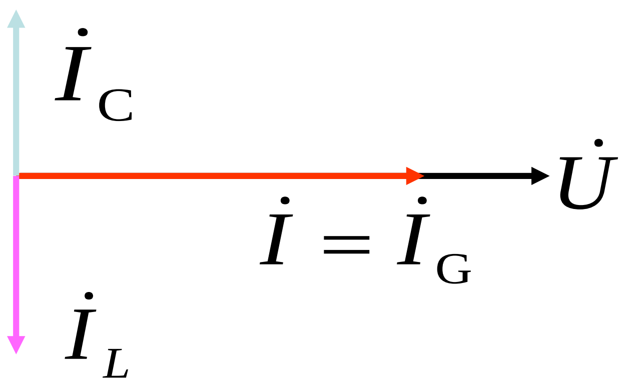
等效电路

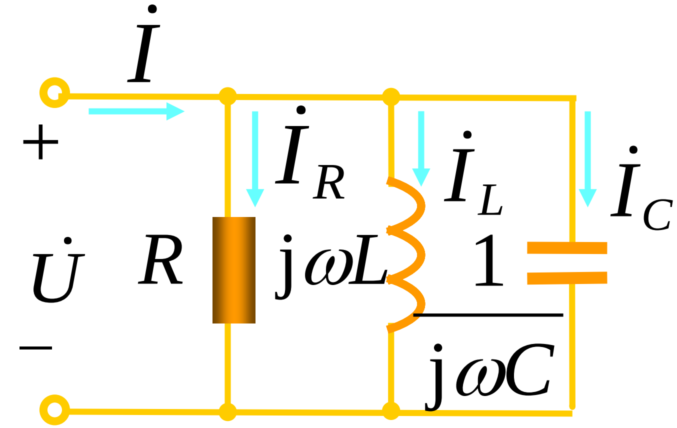


$$I=\sqrt{I_G^2+I_B^2}=\sqrt{I_G^2+(I_L-I_C)^2}$$

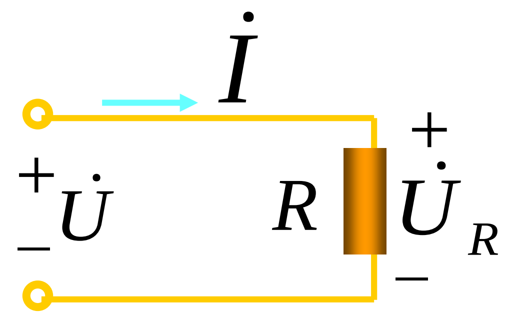
（4）$\omega C=1/\omega L$， $B=0$， $\varphi_y=0$　　$Y=G+\mathrm{j}(\omega C-1/\omega L)$

电路为电阻性， 电流与电压同相。

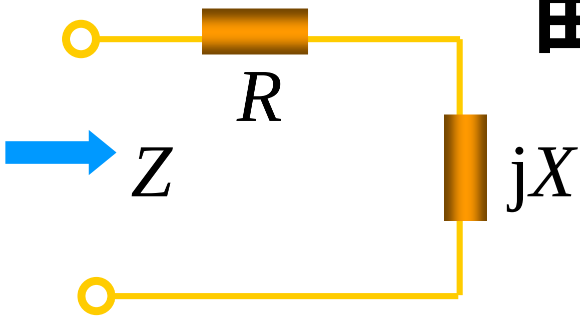




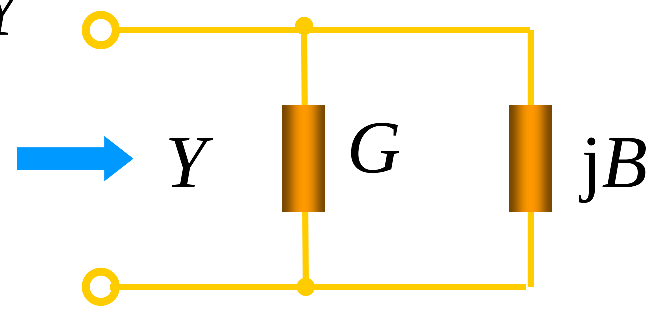
等效电路



5. 复阻抗和复导纳的等效互换



由Z变为Y



$$Z = R + \mathrm{j}X = |Z|\angle\varphi_z \quad \Leftrightarrow \quad Y = G + \mathrm{j}B = |Y|\angle\varphi_y$$

$$Y = \frac{1}{Z} = \frac{1}{R+\mathrm{j}X} = \frac{R-\mathrm{j}X}{R^2+X^2} = G + \mathrm{j}B$$

$$\therefore \quad G = \frac{R}{R^2+X^2}, \quad B = \frac{-X}{R^2+X^2} \quad |Y| = \frac{1}{|Z|}, \quad \varphi_y = -\varphi_z$$

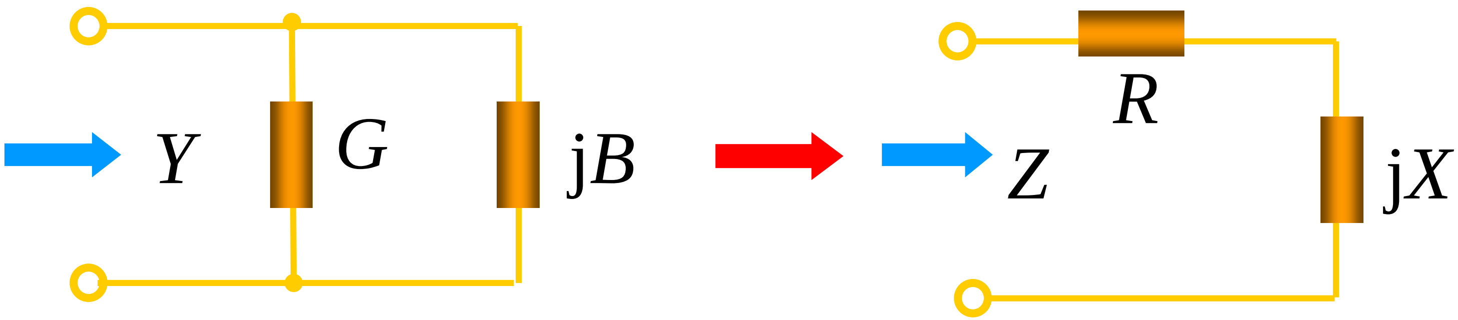
注意

一般情况$G \neq 1/R$，$B \neq 1/X$。

若Z为感性，$X>0$，**则** $B<0$，**即Y仍为感性。**

若Z为容性，$X<0$，**则** $B>0$，**即Y仍为容性。**

同样，若由Y变为Z，则有：



$$Y = G + \mathrm{j}B = |Y| \angle \varphi_y, \quad Z = R + \mathrm{j}X = |Z| \angle \varphi_z$$

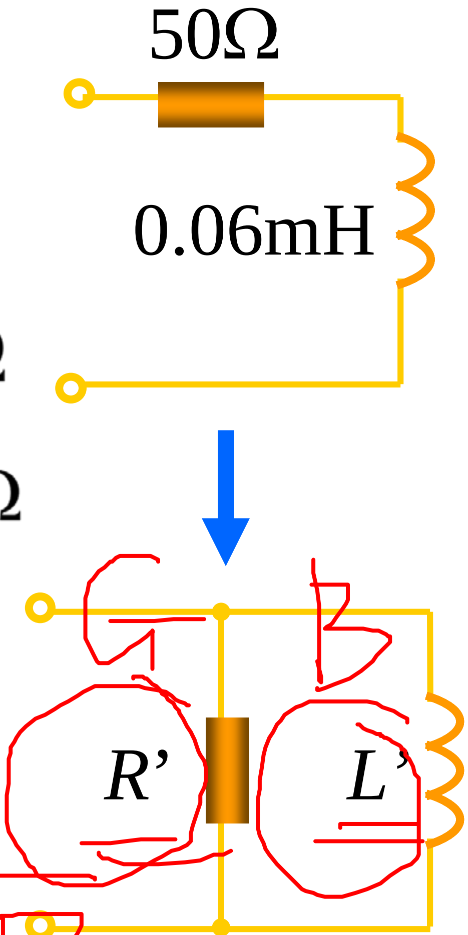
$$Z = \frac{1}{Y} = \frac{1}{G+\mathrm{j}B} = \frac{G-\mathrm{j}B}{G^2+B^2} = R + \mathrm{j}X$$

$$\therefore \quad R = \frac{G}{G^2+B^2}, \quad X = \frac{-B}{G^2+B^2}$$

$$|Y| = \frac{1}{|Z|}, \quad \varphi_z = -\varphi_y$$

例　RL串联电路如图，求在$\omega = 10^6 \mathrm{rad/s}$时的等效并联电路。

解　RL串联电路的阻抗为：



$$X_L = \omega L = 10^6 \times 0.06 \times 10^{-3} = 60\Omega$$

$$Z = R + \mathrm{j}X_L = 50 + \mathrm{j}60 = 78.1\angle 50.2^0 \Omega$$

$$Y = \frac{1}{Z} = \frac{1}{78.1\angle 50.2^0} = 0.0128\angle -50.2^0 \Omega$$

$$= 0.0082 - \mathrm{j}0.0098 \ \ \mathrm{S}$$

$G + \mathrm{j}B$

$$\begin{cases} R' = \dfrac{1}{G'} = \dfrac{1}{0.0082} = 122\Omega \\ L' = \dfrac{1}{0.0098\,\omega} = 0.102\,\mathrm{mH} \end{cases}$$

$L \Rightarrow \dfrac{1}{\omega B}$

$B = -\dfrac{1}{\omega L}$

注意

VCR KCL

①一端口N_0的阻抗或导纳是由其内部的参数、结构和正弦电源的频率决定的，在一般情况下，其每一部分都是频率的函数，随频率而变；

LW KVL

②一端口N_0中如不含受控源，则有

$$|\varphi_z| \le 90^\circ \quad \text{或} \quad |\varphi_y| \le 90^\circ$$

但有受控源时，可能会出现

$$|\varphi_z| \ge 90^\circ \quad \text{或} \quad |\varphi_y| \ge 90^\circ$$

其实部将为负值，其等效电路要设定受控源来表示实部；

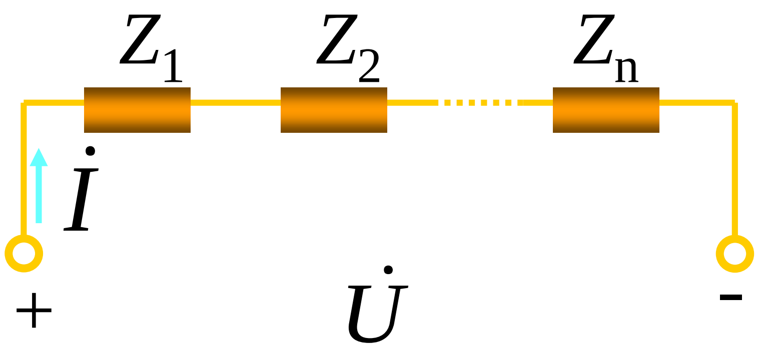
注意

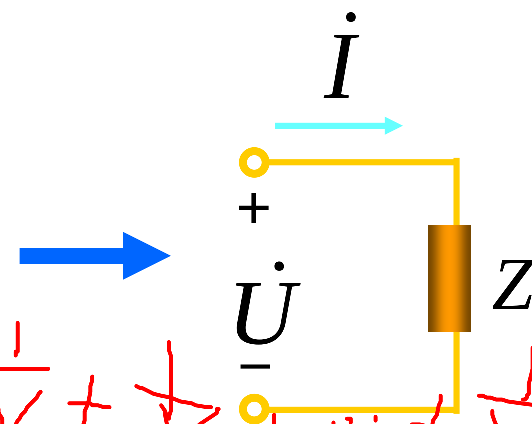
③一端口N_0的两种参数Z和Y具有同等效用，彼此可以等效互换，其极坐标形式表示的互换条件为

$$|Z||Y|=1 \qquad \varphi_z+\varphi_y=0$$

6. 阻抗（导纳）的串联和并联

①阻抗的串联





$\frac{1}{Y_1}+\frac{1}{Y_2}+\cdots+\frac{1}{Y_n} \quad 1/Y$

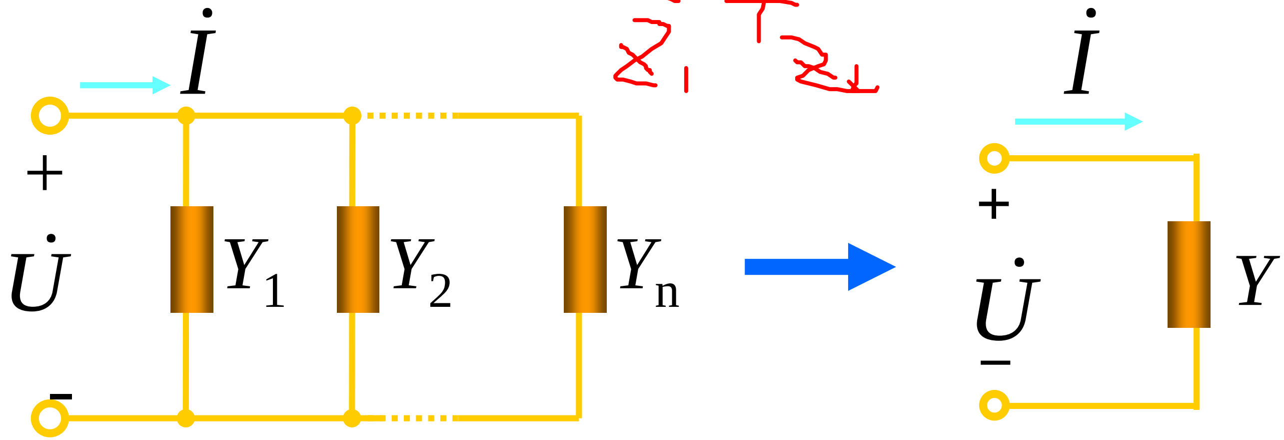
$$\dot{U}=\dot{U}_1+\dot{U}_2+\cdots+\dot{U}_n=\dot{I}(Z_1+Z_2+\cdots+Z_n)=\dot{I}Z$$

$$Z=\sum_{k=1}^{n}Z_k=\sum_{k=1}^{n}(R_k+jX_k)$$

分压公式 → $\dot{U}_i=\frac{Z_i}{Z}\dot{U}$

②导纳的并联

$\frac{1}{Z_1}+\frac{1}{Z_2}=\frac{1}{Z}$



$$\dot{I}=\dot{I}_1+\dot{I}_2+\cdots+\dot{I}_n=\dot{U}(Y_1+Y_2+\cdots+Y_n)=\dot{U}Y$$

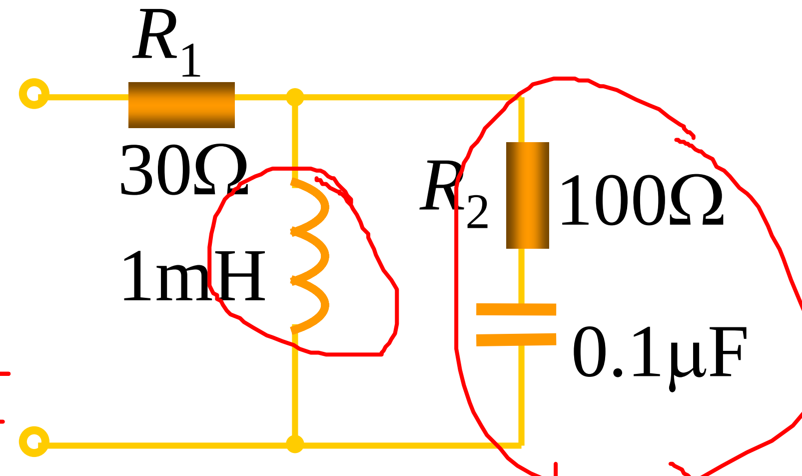
$$Y=\sum_{k=1}^{n}Y_k=\sum_{k=1}^{n}(G_k+\mathrm{j}B_k)$$

分流公式

$$\dot{I}_i=\frac{Y_i}{Y}\dot{I}$$

两个阻抗Z_1、Z_2**的并联等效阻抗为：** $Z=\frac{Z_1Z_2}{Z_1+Z_2}$

例1 **求图示电路的等效阻抗，** $\omega = 10^5 \text{rad/s}$ 。



解 **感抗和容抗为：**

100j

$$X_L = \omega L = 10^5 \times 1 \times 10^{-3} = 100\Omega$$

-100j

$$X_C = -\frac{1}{\omega C} = -\frac{1}{10^5 \times 0.1 \times 10^{-6}} = -100\Omega$$

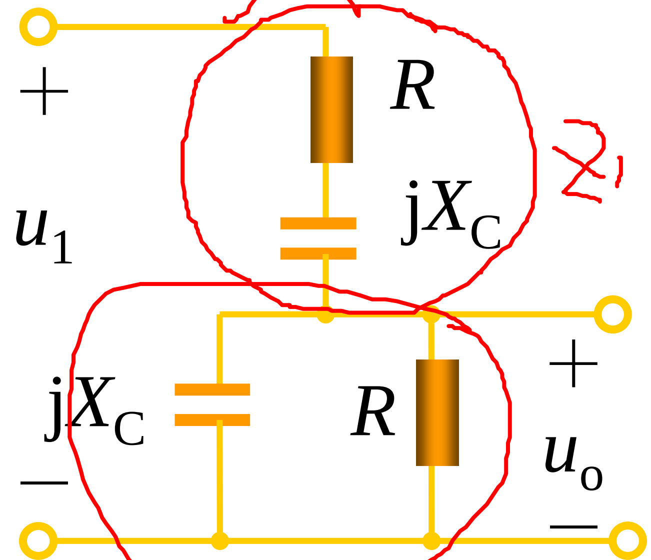
$$Z = R_1 + \frac{\mathrm{j}X_L(R_2 + \mathrm{j}X_C)}{\mathrm{j}X_L + R_2 + \mathrm{j}X_C} = 30 + \frac{\mathrm{j}100 \times (100 - \mathrm{j}100)}{100}$$

$$= 130 + \mathrm{j}100\Omega$$

例2 图为RC选频网络，求u_1和u_0同相位的条件及 $\dfrac{\dot U_1}{\dot U_0}=?$

$\psi = 0$

解 设：$Z_1=R+jX_C,\ Z_2=R//jX_C$



$$\dot U_o=\frac{\dot U_1 Z_2}{Z_1+Z_2}$$

$$\frac{R\cdot jX_c}{R+jX_c}$$

$$\frac{\dot U_1}{\dot U_o}=\frac{Z_1+Z_2}{Z_2}=1+\frac{Z_1}{Z_2}$$

$$\frac{Z_1}{Z_2}=\frac{R+jX_C}{jRX_C/(R+jX_C)}=\frac{(R+jX_C)^2}{jRX_C}$$

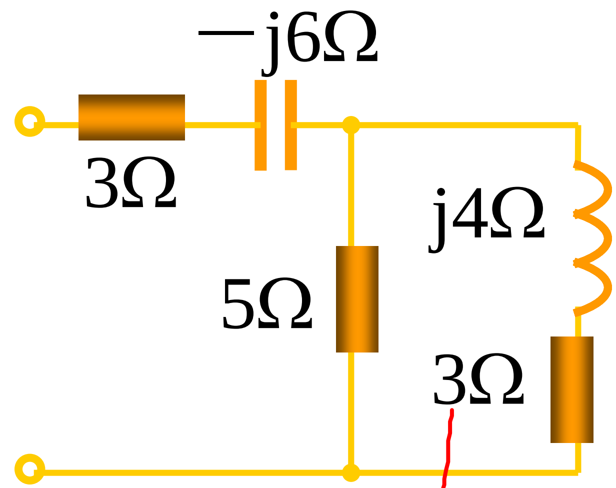
$$=\frac{R^2-X_C^2+j2RX_C}{jRX_C}=2-j\frac{R^2-X_C^2}{RX_C}=\text{实数}$$

$=0$

$$R=|X_C|$$

$$\frac{\dot U_1}{\dot U_o}=1+2=3$$

例3 **图示电路对外呈现感性还是容性?**



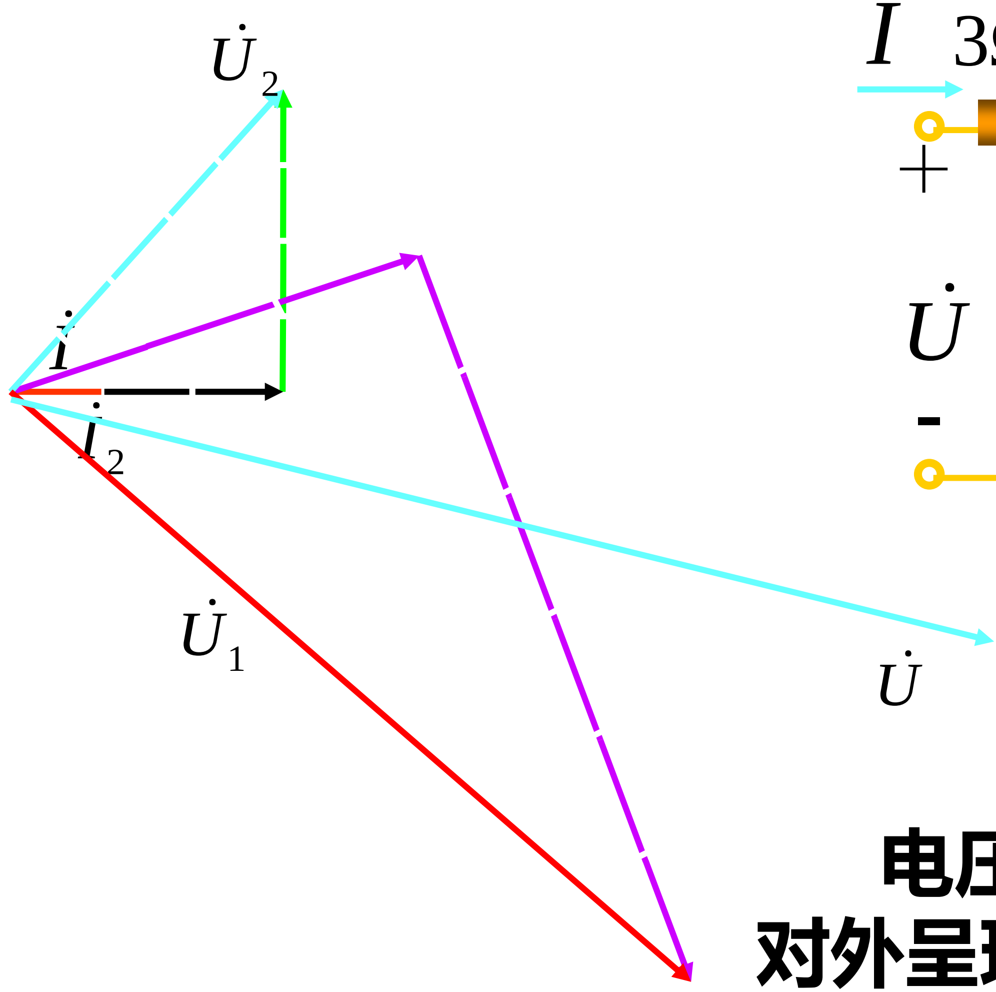
解1 **等效阻抗为:**

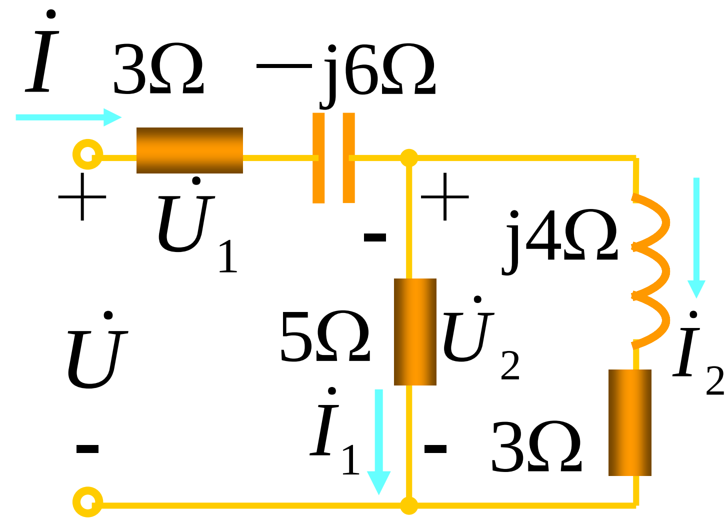
$$Z = 3 - j6 + \frac{5(3 + j4)}{5 + (3 + j4)}$$

$$= 3 - j6 + \frac{25\angle 53.1^{0}}{8 + j4} = 5.5 - j4.75\Omega$$

电路对外呈现容性

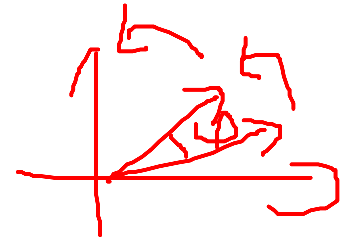
解2　　用相量图求解，取电感电流为参考相量：





电压滞后于电流，电路对外呈现容性。

相量图(phasor diagram)



流程：

参考电路并联（串联）部分的电压（电流）相量;

根据支路的VCR关系确定各并联（串联）支路的电流（电压）相量与电压（电流）相量之间的夹角;

根据结点上的KCL（KVL）方程，用相量平移求和法则，画出结点上各支路电流（电压）相量组成的多边形。

（a）同频率正弦量的相量，才能表示在同一张相量图中;

（b）逆时针旋转——正角度增加的方向;

（c）选定一个参考相量（设其初相位为零——水平线方向）。

9.3 正弦稳态电路的分析

电阻电路与正弦电流电路的分析比较：

电阻电路：

$$\begin{cases} \text{KCL}: \quad \sum i = 0 \\ \text{KVL}: \quad \sum u = 0 \\ \text{元件约束关系:} \\ u = Ri \quad \text{或} \quad i = Gu \end{cases}$$

正弦电路相量分析:

$$\begin{cases} \text{KCL}: \quad \sum \dot{I} = 0 \\ \text{KVL}: \quad \sum \dot{U} = 0 \\ \text{元件约束关系:} \\ \dot{U} = Z\dot{I} \quad \text{或} \quad \dot{I} = Y\dot{U} \end{cases}$$

结论

KCL KVL VCR

1.引入相量法，电阻电路和正弦电流电路依据的电路定律是相似的。

2.引入电路的相量模型，把列写时域微分方程转为直接列写相量形式的代数方程。

3.引入阻抗以后，可将电阻电路中讨论的所有网络定理和分析方法都推广应用于正弦稳态的相量分析中。直流 $(f=0)$是一个特例。

相量法求解正弦激励下动态电路的稳态响应

步骤：

① 画相量电路模型　$R, L, C \rightarrow$ 复阻抗

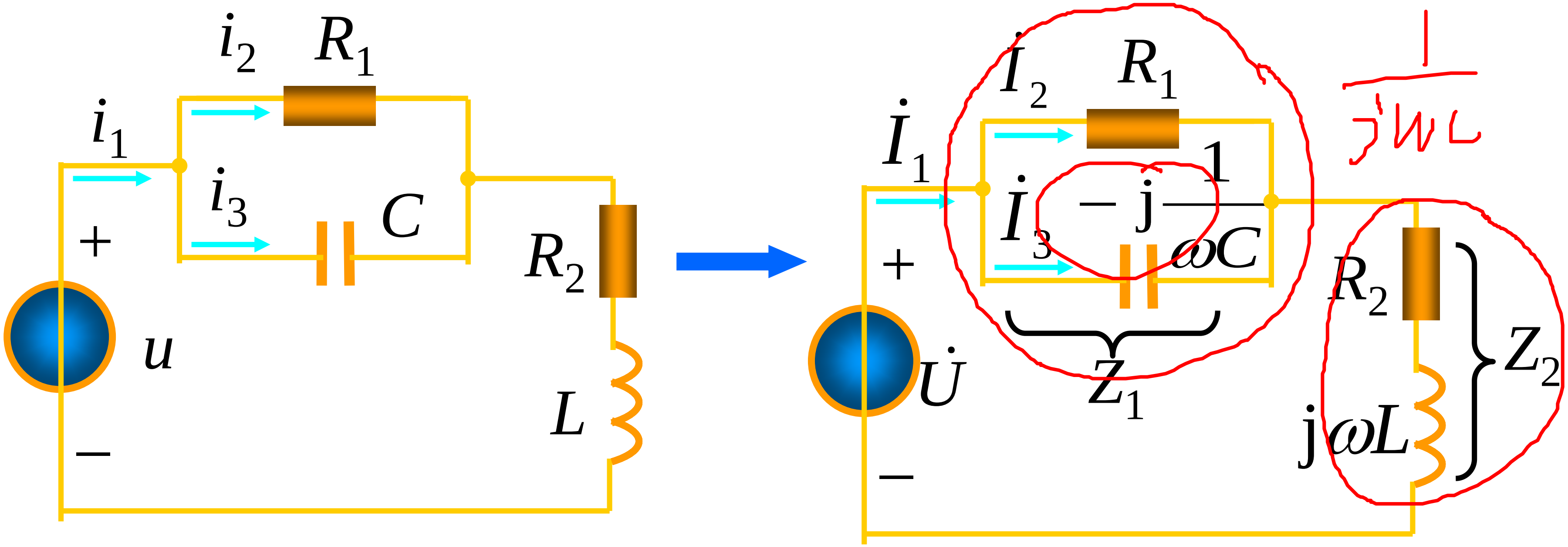
$$i, u \rightarrow \dot{U}, \dot{I}$$

② 使用电阻电路的各种方法进行分析或等效

③ 列写满足KVL、KCL的相量形式的代数方程

④ 求解相量形式的代数方程

例1 **已知:** $R_1=1000\Omega$, $R_2=10\Omega$, $L=500\text{mH}$, $C=10\mu\text{F}$, $U=100\text{V}$, $\omega=314\text{rad/s}$, **求:各支路电流。**



解 **画出电路的相量模型**

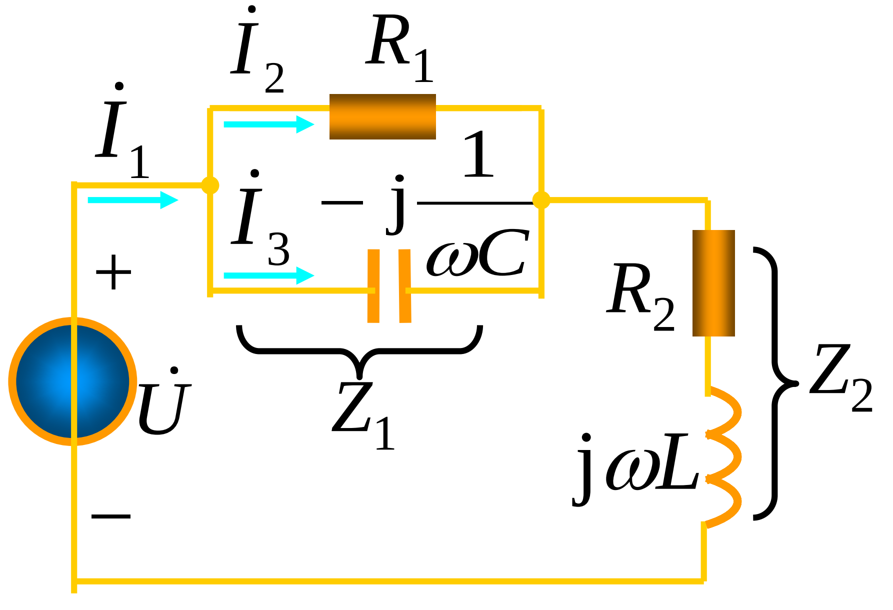
$$Z_1=\frac{R_1(-\text{j}\,1/\omega C)}{R_1-\text{j}\,1/\omega C}=\frac{1000\times(-\text{j}318.47)}{1000-\text{j}318.47}=\frac{318.47\times10^3\angle-90^\circ}{1049.5\angle-17.7^\circ}$$

$$Z_1 = 303.45\angle -72.3^\circ = 92.11 - j289.13\ \Omega$$

$$Z_2 = R_2 + \mathrm{j}\omega L = 10 + \mathrm{j}157\ \Omega$$

$$Z = Z_1 + Z_2 = 92.11 - \mathrm{j}289.13 + 10 + \mathrm{j}157$$

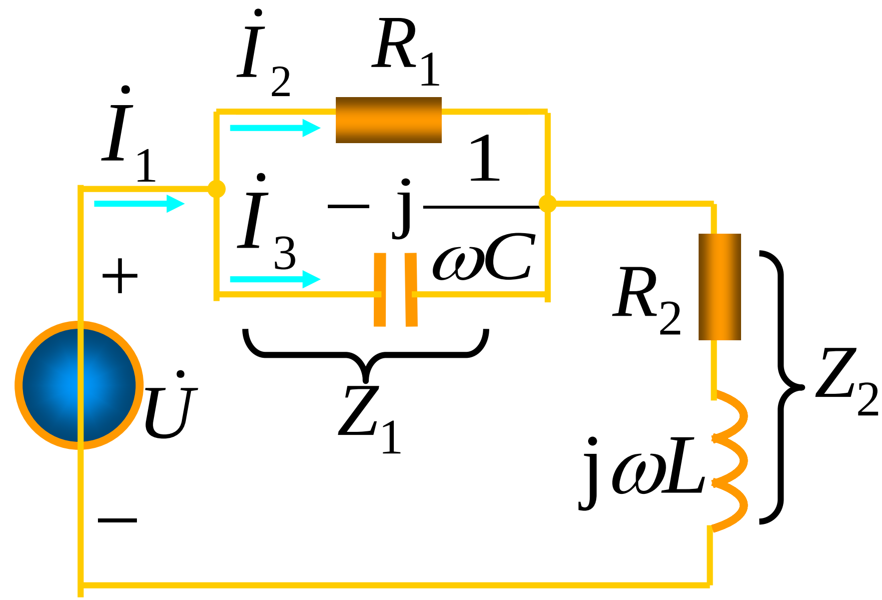
$$= 102.11 - \mathrm{j}132.13 = 166.99\angle -52.3^\circ\ \Omega$$



$$\dot{I}_1 = \frac{\dot{U}}{Z} = \frac{100\angle 0^\circ}{166.99\angle -52.3^\circ} = 0.6\angle 52.3^\circ \text{ A}$$

$$\dot{I}_2 = \frac{-\mathrm{j}\, 1/\omega C}{R_1 - \mathrm{j}\dfrac{1}{\omega C}}\dot{I}_1 = \frac{-\mathrm{j}318.47}{1049.5\angle -17.7^\circ} \times 0.6\angle 52.3^\circ$$

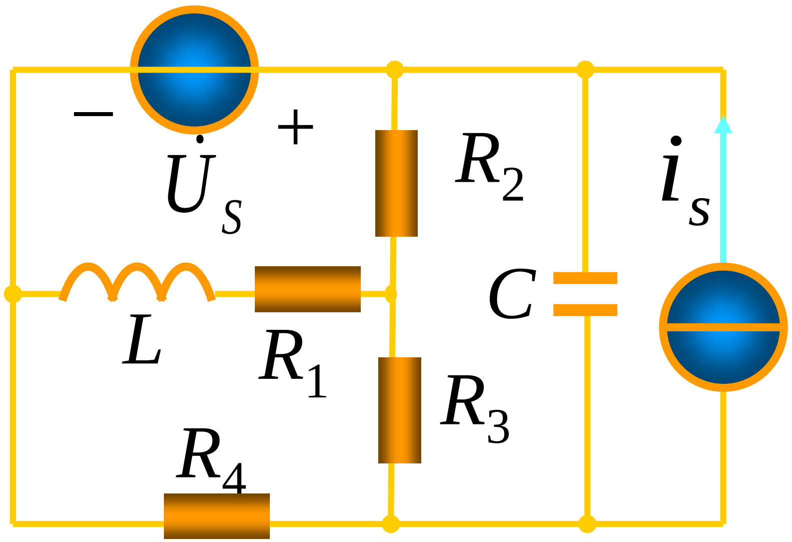
$$= 0.181\angle -20^\circ \text{ A}$$

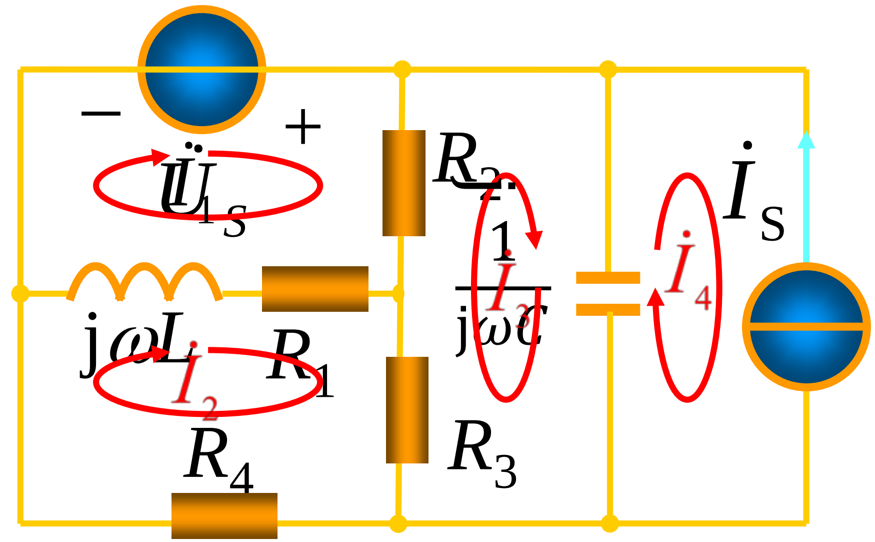


$$\dot{I}_3 = \frac{R_1}{R_1 - \mathrm{j}\dfrac{1}{\omega C}}\dot{I}_1$$

$$= \frac{1000}{1049.5\angle -17.7^\circ} \times 0.6\angle 52.3^\circ = 0.57\angle 70^\circ \text{ A}$$

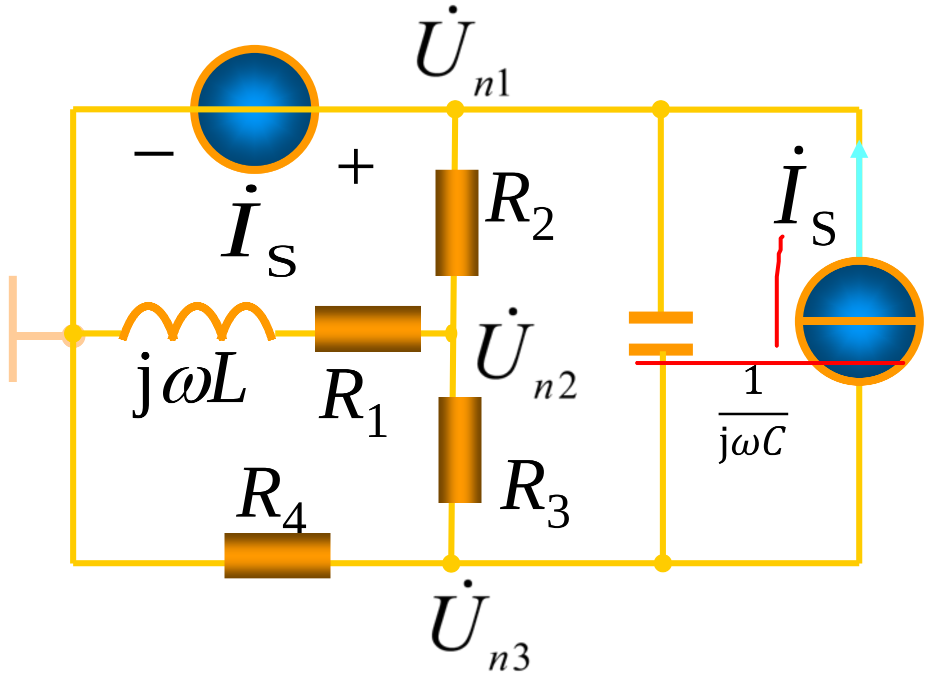
例2　列写电路的回路电流方程和结点电压方程





解　**回路方程**

$$
\begin{cases}
(R_1+R_2+\mathrm{j}\omega L)\dot{I}_1-(R_1+\mathrm{j}\omega L)\dot{I}_2-R_2\dot{I}_3=\dot{U}_S \\
(R_1+R_3+R_4+\mathrm{j}\omega L)\dot{I}_2-(R_1+\mathrm{j}\omega L)\dot{I}_1-R_3\dot{I}_3=0 \\
(R_2+R_3+\dfrac{1}{\mathrm{j}\omega C})\dot{I}_3-R_2\dot{I}_1-R_3\dot{I}_2+\mathrm{j}\dfrac{1}{\omega C}\dot{I}_4=0 \\
\dot{I}_4=-\dot{I}_S
\end{cases}
$$

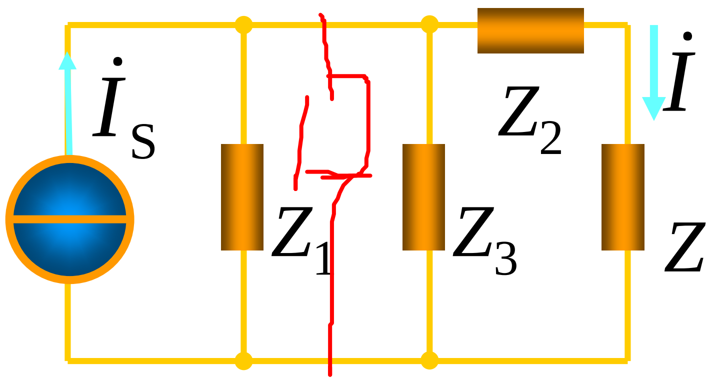


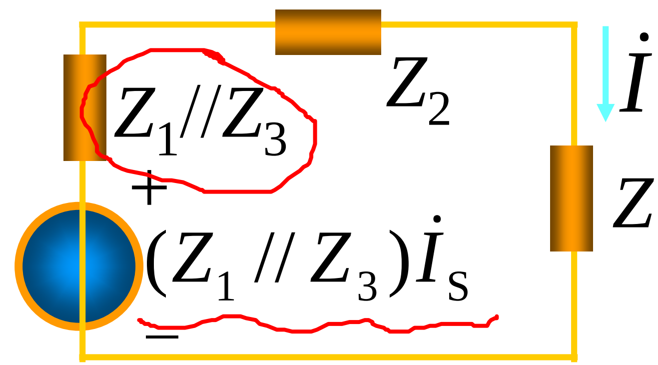
结点方程

$$\begin{cases} \dot{U}_{n1} = \dot{U}_S \\ \left(\frac{1}{R_1 + \mathrm{j}\omega L} + \frac{1}{R_2} + \frac{1}{R_3}\right)\dot{U}_{n2} - \frac{1}{R_2}\dot{U}_{n1} - \frac{1}{R_3}\dot{U}_{n3} = 0 \\ \left(\frac{1}{R_3} + \frac{1}{R_4} + \mathrm{j}\omega C\right)\dot{U}_{n3} - \frac{1}{R_3}\dot{U}_{n2} - \mathrm{j}\omega C\dot{U}_{n1} = -\dot{I}_S \end{cases}$$

4j

例3　已知：$\dot{I}_S = 4\angle 90^\circ \text{ A}$，$Z_1 = Z_2 = -\text{j}30\ \Omega$，$Z_3 = 30\ \Omega$，$Z = 45\ \Omega$，求电流 $\dot{I}$.





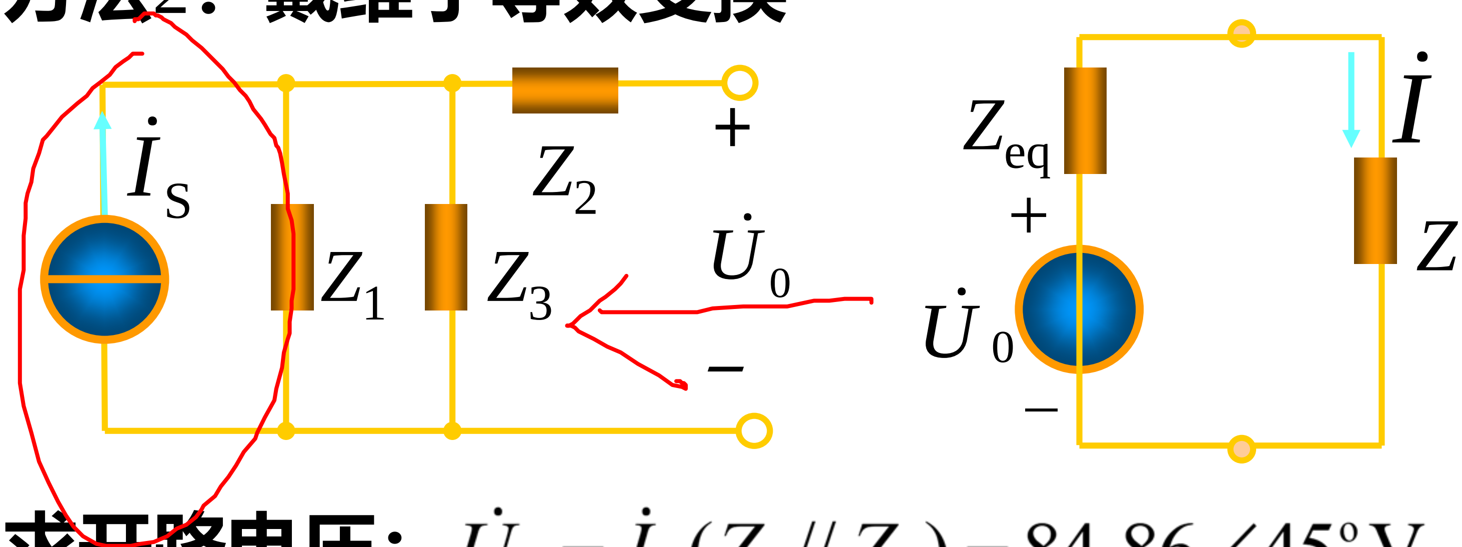
解　**方法1：电源变换**

$$Z_1 // Z_3 = \frac{30(-\text{j}30)}{30-\text{j}30} = 15-\text{j}15\Omega$$

$$\dot{I} = \frac{\dot{I}_S(Z_1//Z_3)}{Z_1//Z_3+Z_2+Z} = \frac{\text{j}4(15-\text{j}15)}{15-\text{j}15-\text{j}30+45}$$

$$= \frac{5.657\angle 45^\circ}{5\angle -36.9^\circ} = 1.13\angle 81.9^\circ \text{ A}$$

方法2：戴维宁等效变换

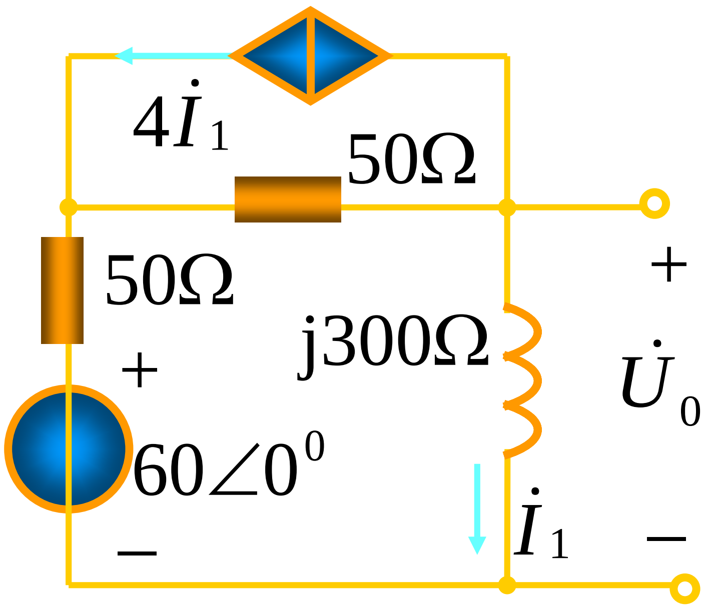


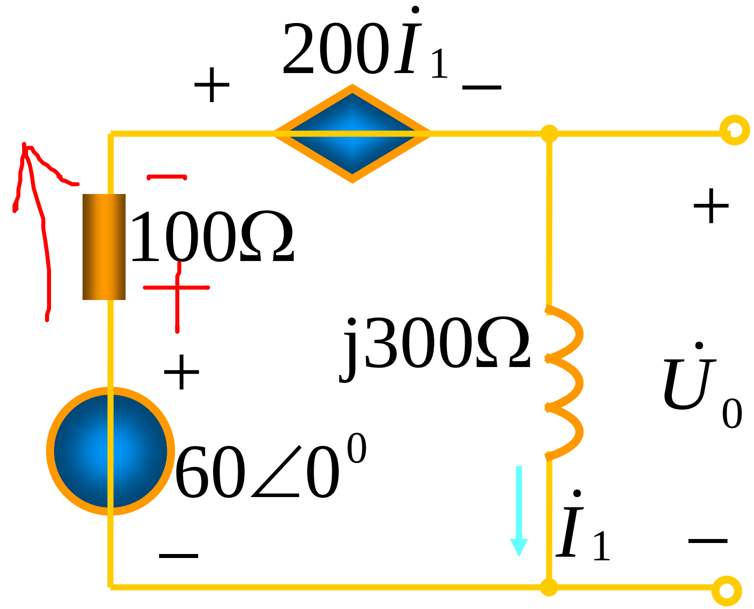
求开路电压： $\dot{U}_0 = \dot{I}_S(Z_1 // Z_3) = 84.86\angle 45^\circ \text{V}$

求等效电阻： $Z_{eq} = Z_1 // Z_3 + Z_2 = 15 - \text{j}45\Omega$

$$\dot{I} = \frac{\dot{U}_0}{Z_{eq} + Z} = \frac{84.86\angle 45^\circ}{15 - \text{j}45 + 45} = 1.13\angle 81.9^\circ \text{A}$$

例4 求图示电路的戴维宁等效电路。

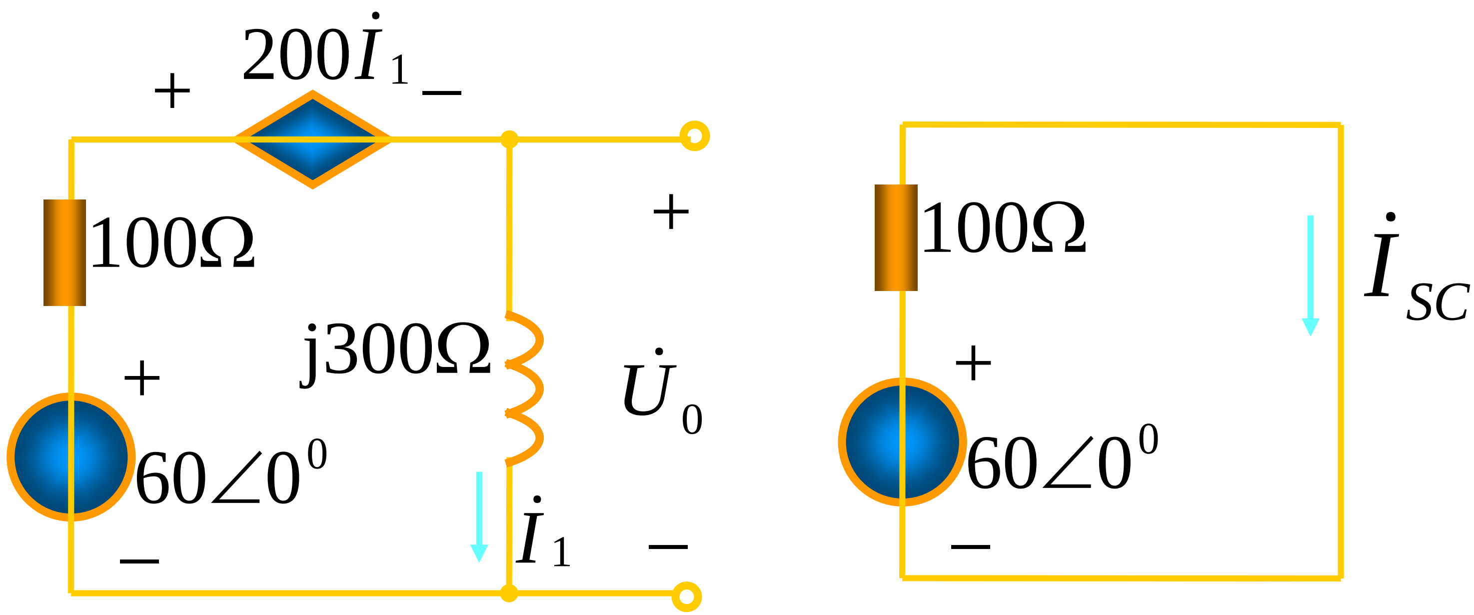




解 **求开路电压：**

$$\dot{U}_o = -200\dot{I}_1 - 100\dot{I}_1 + 60 = -300\dot{I}_1 + 60 = -300\frac{\dot{U}_0}{j300} + 60$$

$$\rightarrow \quad \dot{U}_o = \frac{60}{1-j} = 30\sqrt{2}\angle 45^0\,\text{V}$$



求短路电流：

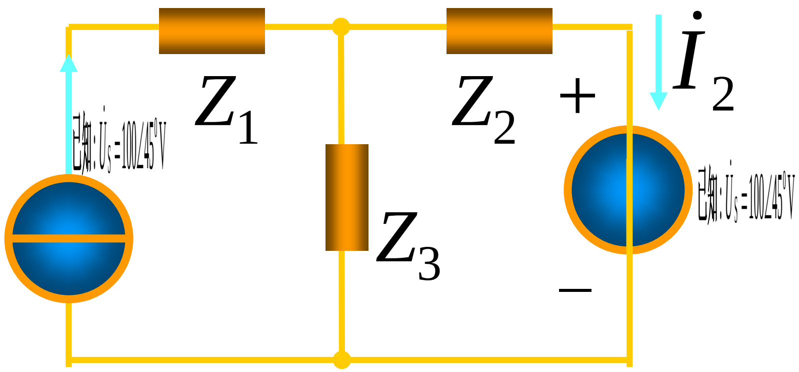
$$\dot{I}_{SC} = 60/100 = 0.6\angle 0^0 \text{ A}$$

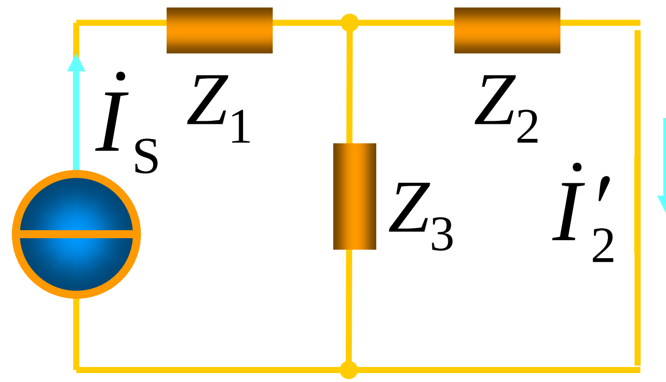
$$Z_{eq} = \frac{\dot{U}_0}{\dot{I}_{SC}} = \frac{30\sqrt{2}\angle 45^0}{0.6} = 50\sqrt{2}\angle 45^0 \Omega$$

例5　用叠加定理计算电流 $\dot{I}_2$

已知：$\dot{U}_S = 100\angle 45^\circ \mathrm{V}$

$\dot{I}_S = 4\angle 0^\circ \mathbf{A}, Z_1 = Z_3 = 50\angle 30^\circ \Omega, Z_2 = 50\angle -30^\circ \Omega.$

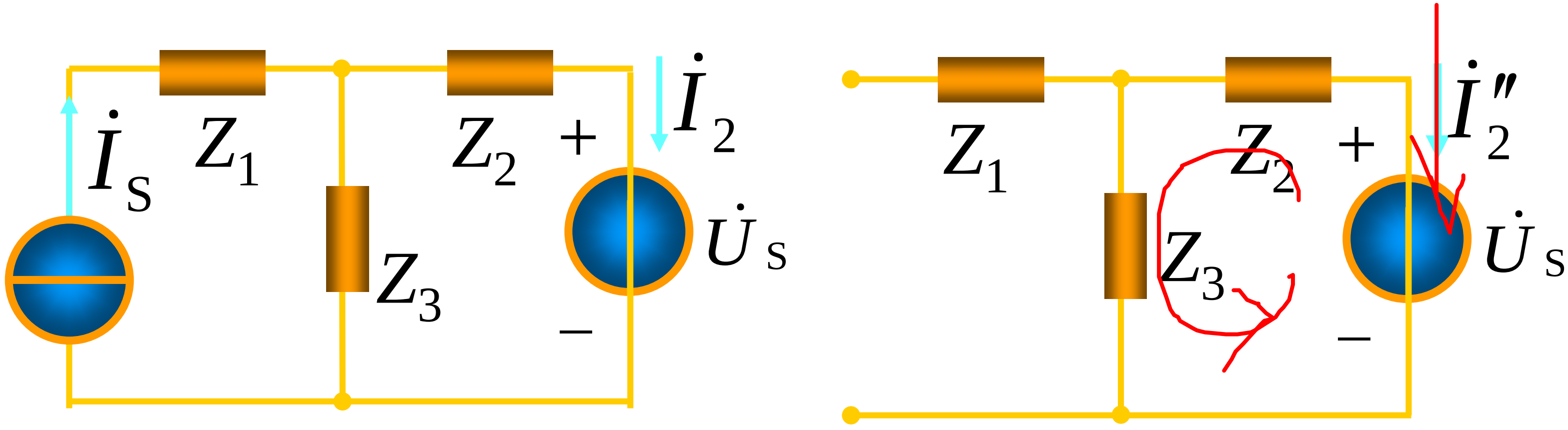




解　(1) $\dot{I}_S$单独作用($\dot{U}_S$短路)：

$$\dot{I}'_2 = \dot{I}_S \frac{Z_3}{Z_2 + Z_3} = 4\angle 0^\circ \times \frac{50\angle 30^\circ}{50\angle -30^\circ + 50\angle 30^\circ}$$

$$= \frac{200\angle 30^\circ}{50\sqrt{3}} = 2.31\angle 30^\circ \mathrm{A}$$



(2) $\dot{U}_S$单独作用($\dot{I}_S$开路):

$$\dot{I}_2'' = -\frac{\dot{U}_S}{Z_2 + Z_3} = \frac{-100\angle 45^\circ}{50\sqrt{3}} = 1.155\angle -135^\circ \text{ A}$$

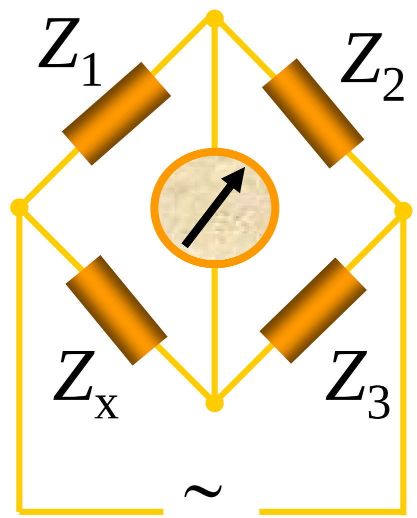
$$\dot{I}_2 = \dot{I}_2' + \dot{I}_2'' = 2.31\angle 30^\circ + 1.155\angle -135^\circ \text{ A}$$

例6 **已知平衡电桥** $Z_1=R_1$, $Z_2=R_2$, $Z_3=R_3+\mathrm{j}\omega L_3$。
求： $Z_x=R_x+\mathrm{j}\omega L_x$。

解 **平衡条件：** $Z_1 Z_3= Z_2 Z_x$ **得：**

$$|Z_1|\angle\varphi_1 \bullet |Z_3|\angle\varphi_3 = |Z_2|\angle\varphi_2 \bullet |Z_x|\angle\varphi_x$$

$$\Rightarrow \begin{cases} |Z_1|\,|Z_3| = |Z_2|\,|Z_x| \\ \varphi_1+\varphi_3 = \varphi_2+\varphi_x \end{cases}$$



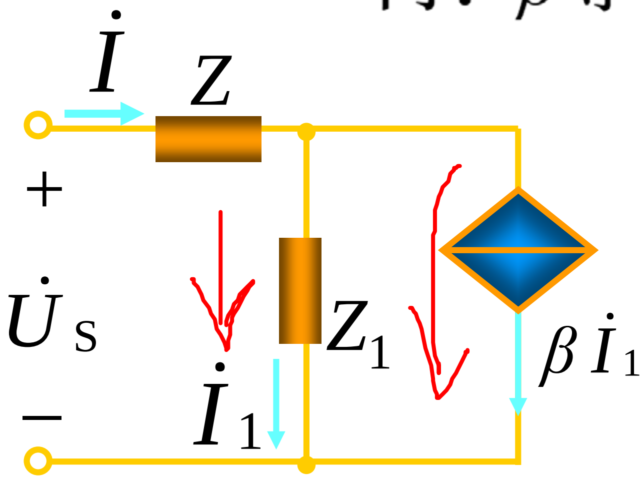
$$R_1(R_3+\mathrm{j}\omega L_3)=R_2(R_x+\mathrm{j}\omega L_x)$$

$$\therefore \quad R_x=R_1R_3/R_2\ , \quad L_x=L_3R_1/R_2$$

例7 **已知：** $Z=10+j50\Omega$, $Z_1=400+j1000\Omega$。

问：β等于多少时，$\dot{I}_1$和$\dot{U}_S$相位差90°？

±A



解 分析：

找出$\dot{I}_1$和$\dot{U}_S$关系：$\dot{U}_S=Z_{转}\dot{I}_1$，

$Z_{转}$实部为零，相位差为90°.

$$\dot{U}_S=Z\dot{I}+Z_1\dot{I}_1=Z(1+\beta)\dot{I}_1+Z_1\dot{I}_1$$

$$\frac{\dot{U}_S}{\dot{I}_1}=(1+\beta)Z+Z_1=410+10\beta+j(50+50\beta+1000)$$

令 $410+10\beta=0$ ，$\beta=-41$

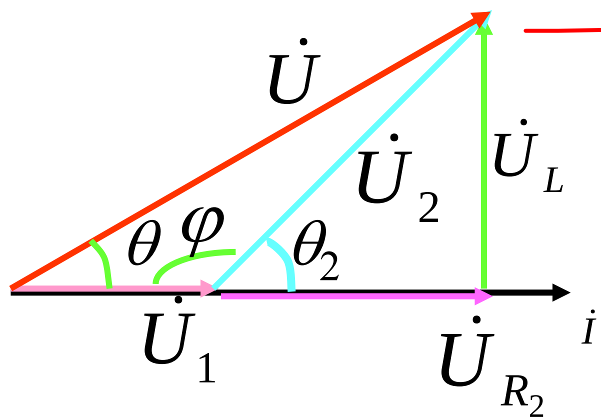
a+bj

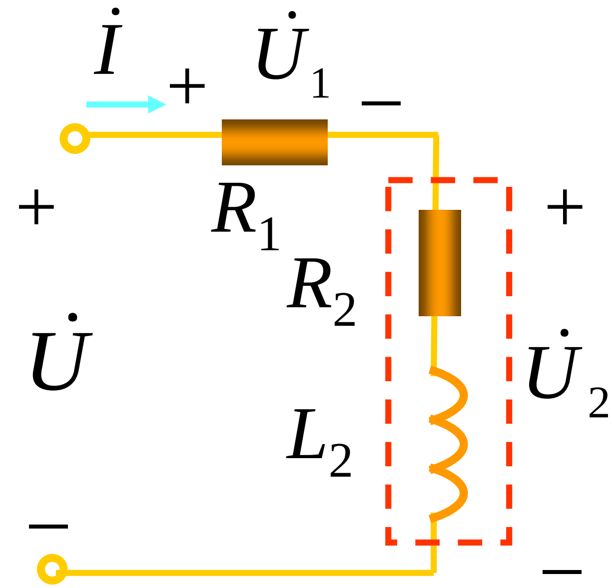
$$\frac{\dot{U}_S}{\dot{I}_1}=-j1000$$ 故电流领先电压90°.

例8 **已知：** U=115V, U_1=55.4V , U_2=80V, R_1=32Ω , f=50Hz。 **求：线圈的电阻R_2和电感L_2。**

解 **方法一、 画相量图分析。**

$$\dot{U}=\dot{U}_1+\dot{U}_2=\dot{U}_1+\dot{U}_{R_2}+\dot{U}_L$$





$$U^2=U_1^2+U_2^2+2U_1U_2\cos\varphi$$

$$\cos\varphi=-0.4237 \qquad \therefore \varphi=115.1^\circ$$

$$Z_2 = R_2 + j\omega L_2 = |Z_2|\angle\theta_2 = \frac{\dot{U}_2}{\dot{I}}$$

$$\theta_2 = 180^\circ - \varphi = 64.9^\circ$$

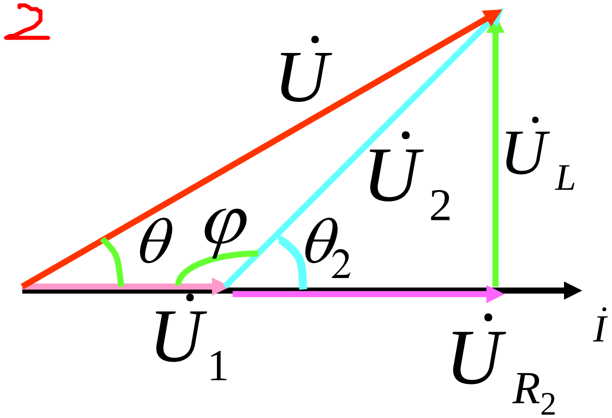
$$I = U_1 / R_1 = 55.4/32 = 1.73\text{A}$$

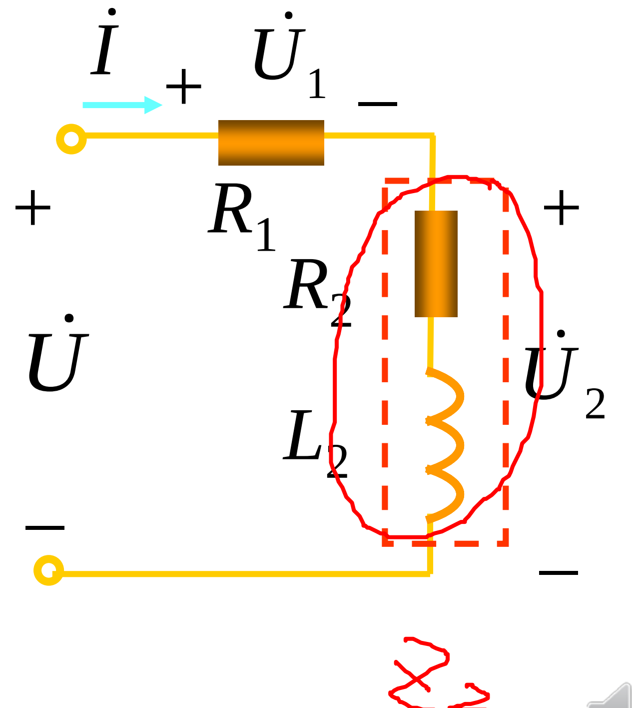
$$|Z_2| = U_2 / I = 80/1.73 = 46.2\Omega$$

$$R_2 = |Z_2|\cos\theta_2 = 19.6\Omega$$

$$X_2 = |Z_2|\sin\theta_2 = 41.8\Omega$$

$$L = X_2/(2\pi f) = 0.133\text{H}$$





方法二、

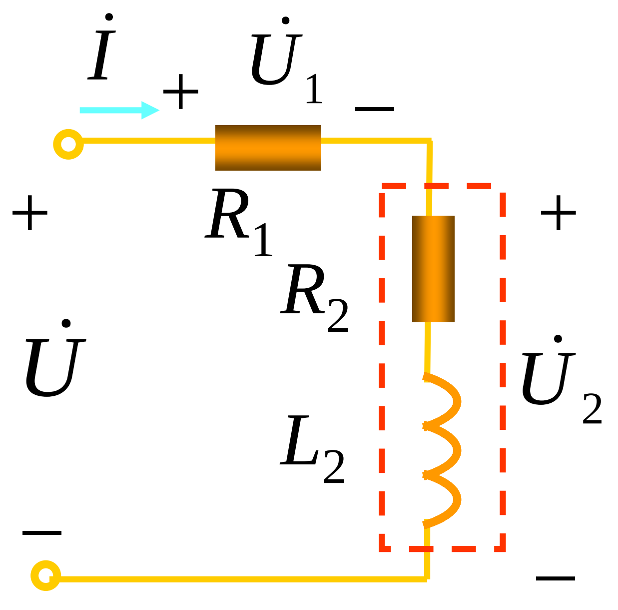
$$\dot{U}=\dot{U}_1+\dot{U}_2=55.4\angle 0^0+80\angle\varphi=115\angle\theta$$

$$\begin{cases} 55.4+80\cos\varphi=115\cos\theta \\ 80\sin\varphi=115\sin\theta \end{cases}$$

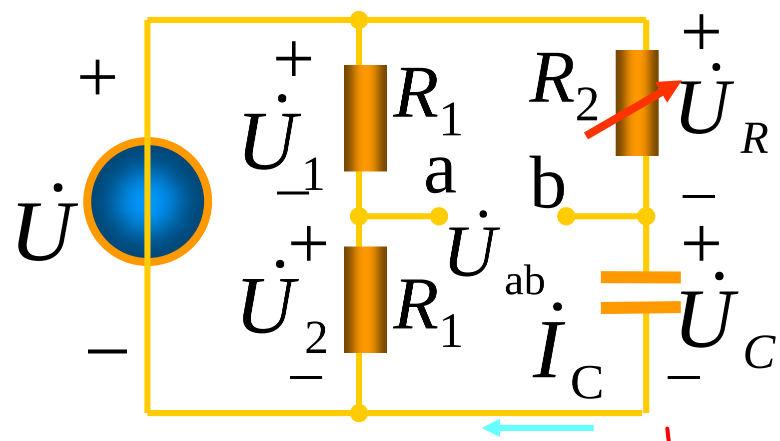
$$\Rightarrow \quad \cos\varphi=0.424$$

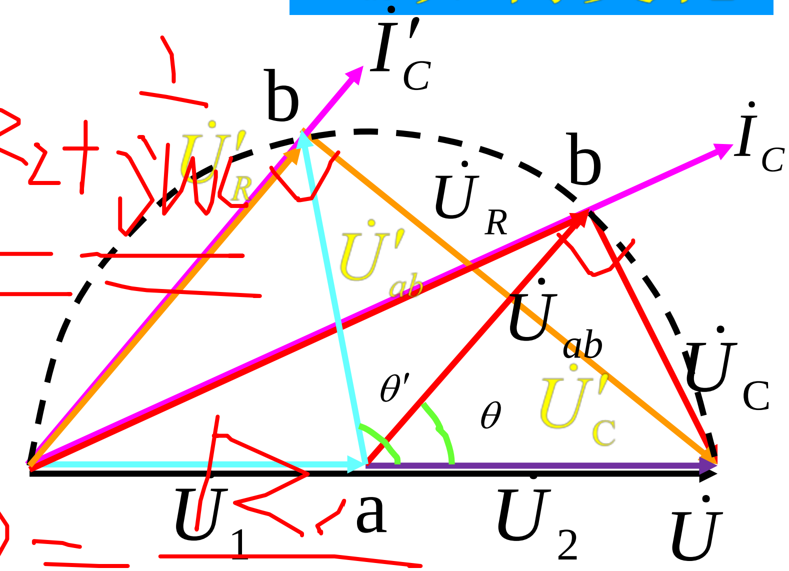
$$\varphi=64.93^0$$

其余步骤同解法一。



例9 移相桥电路。当R_2由$0\rightarrow\infty$时，$\dot{U}_{ab}$如何变化？





解 **用相量图分析**

$$\dot{U}=\dot{U}_1+\dot{U}_2,\quad \dot{U}_1=\dot{U}_2=\frac{\dot{U}}{2}$$

$$\dot{U}=\dot{U}_R+\dot{U}_C\qquad \dot{U}_{ab}=\dot{U}_R-\dot{U}_1$$

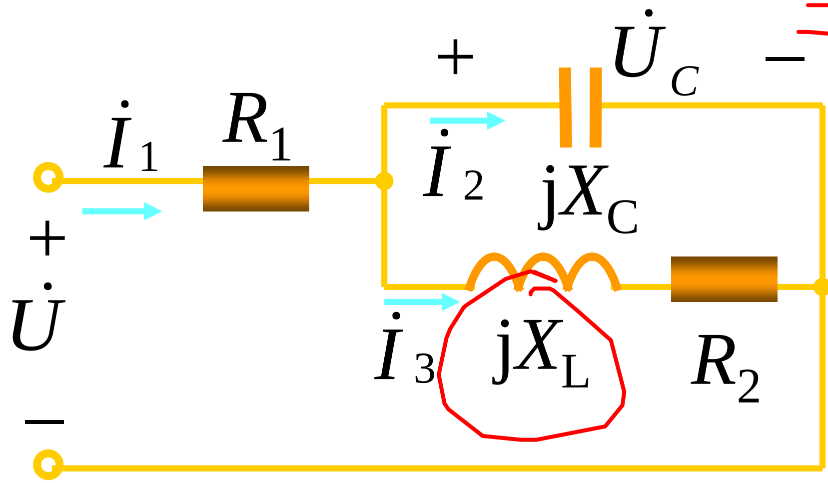
当$R_2=0$，$\theta=180°$；

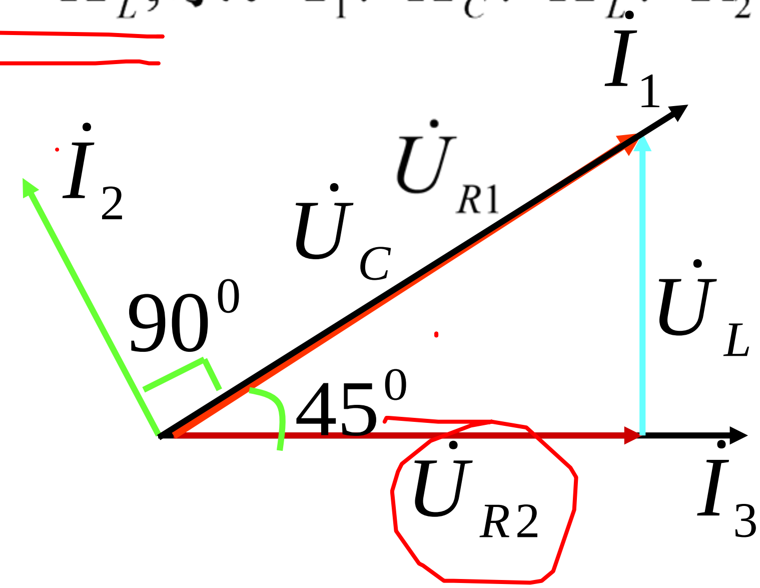
当$R_2\rightarrow\infty$，$\theta=0°$。

由相量图可知当R_2改变，$U_{ab}=\dfrac{1}{2}U$不变，相位改变；

θ为移相角，移相范围 $180^\circ\sim0^\circ$

例10 **图示电路，** $I_2 = 10\text{A}$、$I_3 = 10\sqrt{2}\text{A}$、$U = 200\text{V}$、$R_1 = 5\Omega$、$R_2 = X_L$，求：I_1、X_C、X_L、R_2。





解

$$\dot{I}_1 = \dot{I}_2 + \dot{I}_3 = 10\angle 135^0 + 10\sqrt{2}\angle 0^0 = 10\angle 45^0 \Rightarrow I_1 = 10\text{A}$$

$$\dot{U} = \dot{U}_{R1} + \dot{U}_C \Rightarrow 200 = 5\times 10 + U_C \Rightarrow U_C = 150\text{V}$$

$$\dot{U}_C = \dot{U}_{R2} + \dot{U}_L \Rightarrow U_C = \sqrt{2U_{R2}^2} \Rightarrow U_{R2} = U_L = 75\sqrt{2}$$

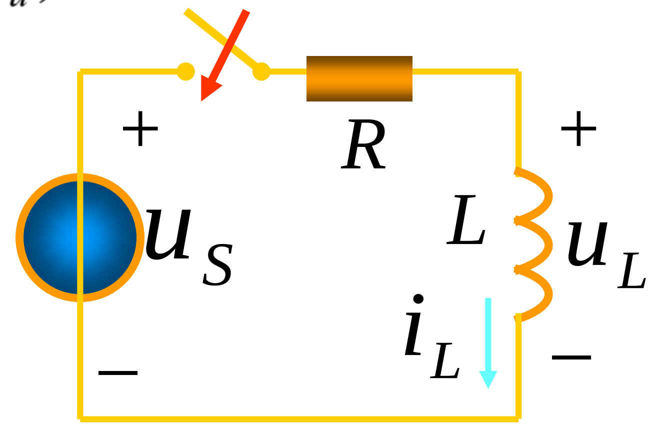
$$X_C = -\frac{150}{10} = -15\Omega \qquad R_2 = X_L = \frac{75\sqrt{2}}{10\sqrt{2}} = 7.5\Omega$$

$$\frac{U_{R2}}{I_3}$$

例11 **求RL串联电路在正弦输入下的零状态响应。**

已知：$u_S = \sqrt{2}U\cos(\omega t + \psi_u)$

$$f(t) = f'(t) + [f(0_+) - f'(0_+)]e^{-\frac{t}{\tau}}$$



解 **应用三要素法：**

$$i_L(0_+) = i_L(0_-) = 0 \qquad \tau = L/R$$

用相量法求正弦稳态解

$$\dot{I} = \frac{\dot{U}}{R + j\omega L} = \frac{U}{\sqrt{R^2 + (\omega L)^2}}\angle\psi_u - \psi_Z = I\angle\psi_i$$

$$i_L(t) = i_L(\infty) + \left[i_L(0_+) - i_L(\infty)\Big|_{0_+} \right] e^{-\frac{t}{\tau}}$$

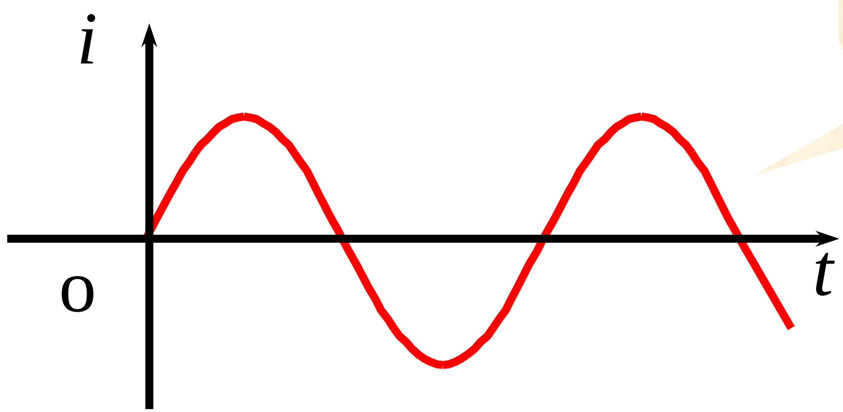
$$i_L(t) = I_m\cos(\omega t + \psi_i) - I_m\cos\psi_i e^{-\frac{t}{\tau}}$$

$$i_L(t) = I_m \cos(\omega t + \psi_i) - I_m \cos\psi_i e^{-\frac{t}{\tau}}$$

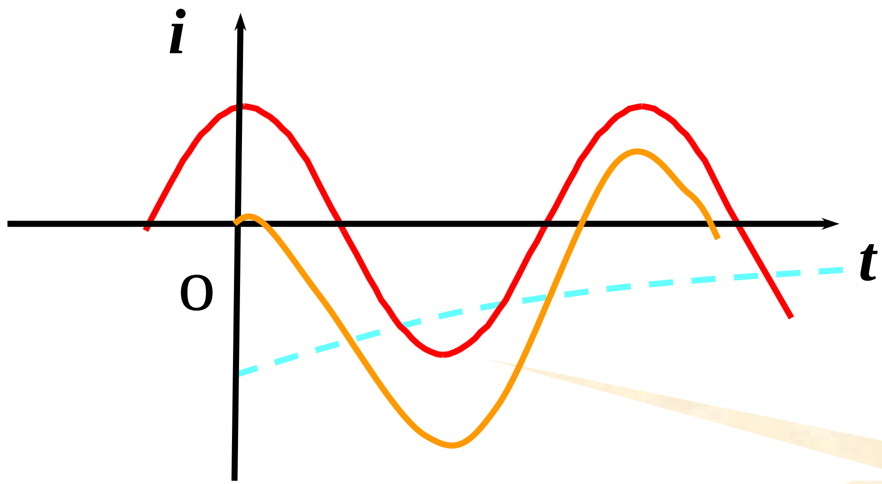
注意　过渡过程与接入时刻有关

当 $\psi_i = -\dfrac{\pi}{2}$，$i_L(t) = I_m \cos(\omega t + \psi_i)$

直接进入稳定状态



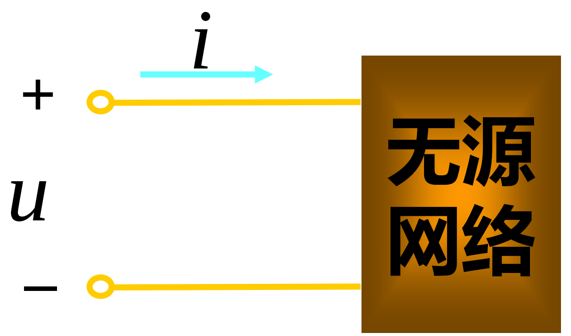
当 $\psi_i=0$，$i_L(t)=I_m\cos(\omega t)-I_m e^{-\frac{t}{\tau}}$



出现瞬时电流大于稳态电流现象

9.4 正弦稳态电路的功率

1. 瞬时功率 (instantaneous power)



$$u(t)=\sqrt{2}U\cos\omega t$$

$$i(t)=\sqrt{2}I\cos(\omega t-\varphi)$$

φ为u和i的相位差$\varphi=\Psi_u-\Psi_i$

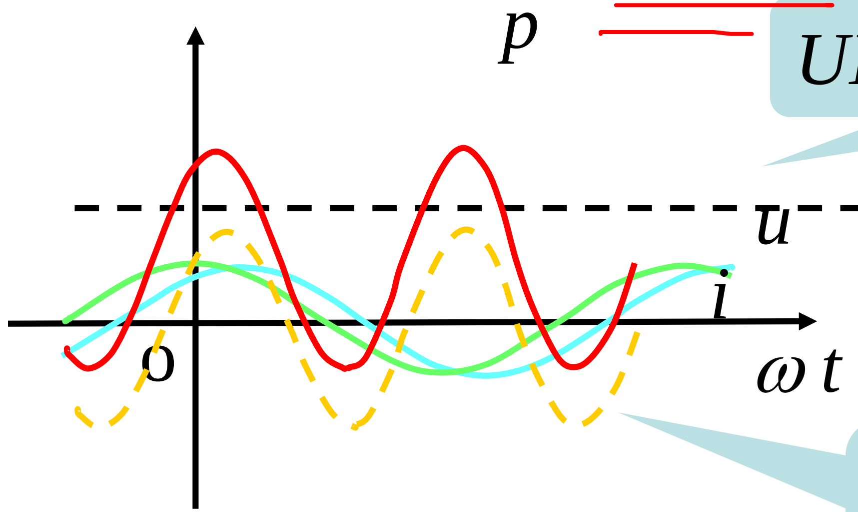
$$p(t)=ui=\sqrt{2}U\cos\omega t\cdot\sqrt{2}I\cos(\omega t-\varphi)$$

$$=UI[\cos\varphi+\cos(2\omega t-\varphi)]$$ **第一种分解方法；**

$$=UI\cos\varphi(1+\cos 2\omega t)+UI\sin\varphi\sin 2\omega t$$

第二种分解方法。

第一种分解方法： $p(t) = UI[\cos\varphi + \cos(2\omega t - \varphi)]$



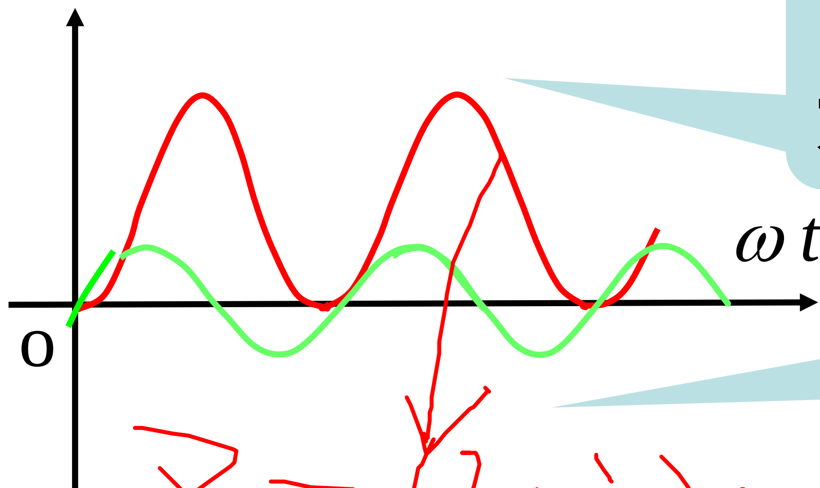
$UI\cos\varphi$ **恒定分量。**

$UI\cos(2\omega t - \varphi)$ **为正弦分量。**

- p **有时为正，有时为负；**
- $p>0$，**电路吸收功率；**
- $p<0$，**电路发出功率；**

第二种分解方法：

$$p(t) = UI\cos\varphi\,(1+\cos 2\omega t) + UI\sin\phi\,\sin 2\omega t$$



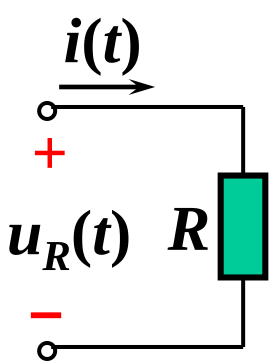
$UI\cos\varphi\,(1+\cos 2\omega t)$ **为不可逆分量。**

$UI\sin\varphi\,\sin 2\omega t$ **为可逆分量。**

$Z = R + jX$

- **部分能量在电源和一端口之间来回交换。**

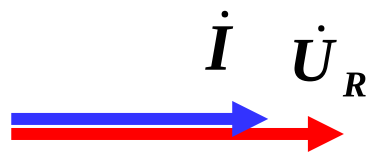
(1) 电阻元件的瞬时功率



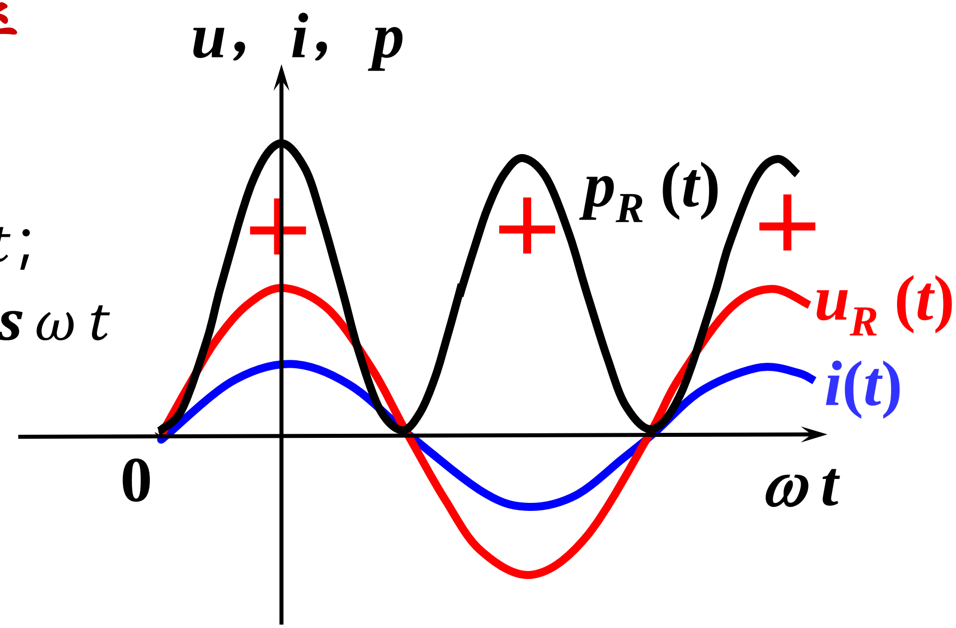
设

$$i(t) = \sqrt{2}I\cos\omega t;$$

$$u_R(t) = \sqrt{2}U_R\cos\omega t$$



相量图



$$2\cos^2\alpha = 1 + \cos 2\alpha$$

吸收的瞬时功率

$$p_R(t) = u_R(t)i(t) = \sqrt{2}U_R\cos\omega t\sqrt{2}I\cos\omega t$$

$$= U_R I(1 + \cos 2\omega t)$$

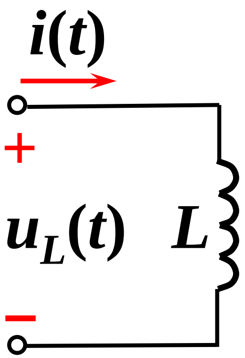
$$p(t) = UI[\cos\varphi + \cos(2\omega t - \varphi)]$$

- 瞬时功率的角频率为2ω
- $p_R \geq 0$

电阻总吸收功率

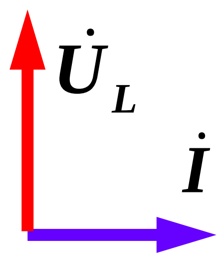
（2） 电感元件的瞬时功率

实际交替吸收和发出等量功率

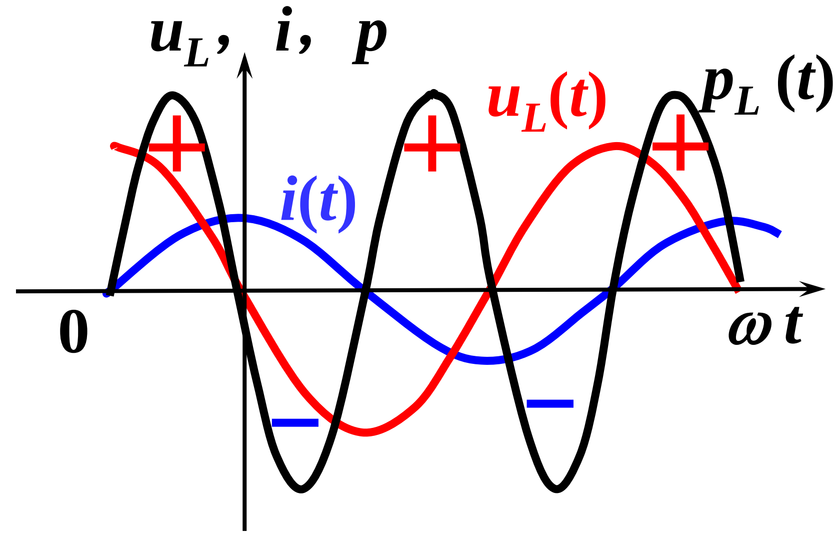


设

$$i(t)=\sqrt{2}I\cos(\omega t);$$
$$u_L(t)=\sqrt{2}U_L\cos(\omega t+90°)$$



相量图



按吸收功率列写表达式:

$$p_L(t)=u_L(t)i(t)$$
$$=\sqrt{2}U_L\cos(\omega t+90°)\sqrt{2}I\cos(\omega t)$$
$$=2U_L I\cos(\omega t+90°)\cos(\omega t)=-U_L I\sin(2\omega t)$$

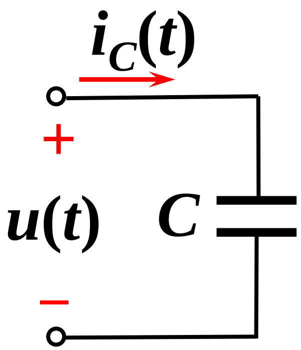
$$\cos(a+b)\times\cos(a-b)=2\cos a\cos b$$

$$p(t)=UI[\cos\varphi+\cos(2\omega t-\varphi)]$$

◆ 瞬时功率的角频率为2ω。

(3) 电容元件的瞬时功率

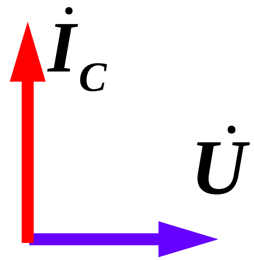
实际交替吸收和发出等量功率

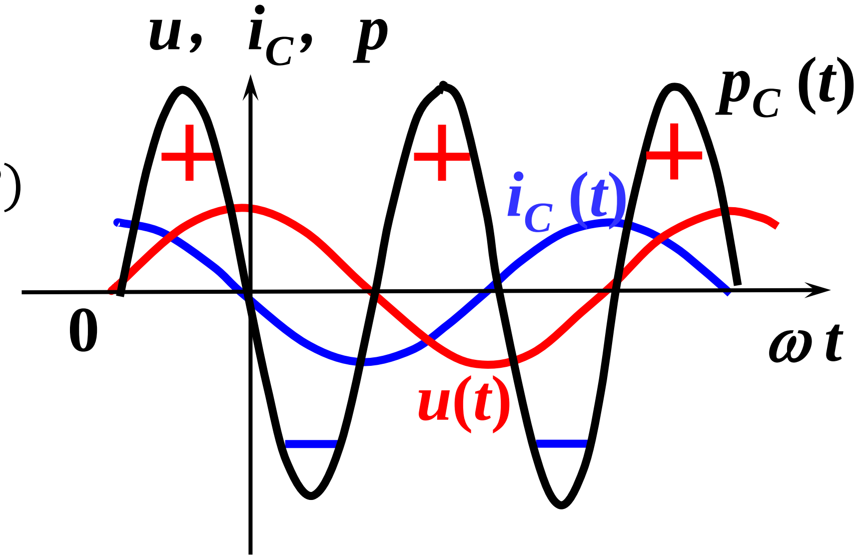


设：$u(t)=\sqrt{2}U\cos(\omega t)$；

$i_C(t)=\sqrt{2}I_C\cos(\omega t+90°)$

相量图





按吸收功率列写表达式：

$$p(t)=UI[\cos\varphi+\cos(2\omega t-\varphi)]$$

$$p_C(t)=u(t)i_C(t)$$

$$=\sqrt{2}U\cos(\omega t)\sqrt{2}I_C\cos(\omega t+90°)$$

$$=2UI_C\cos(\omega t+90°)\cos(\omega t)=-UI_C\sin(2\omega t)$$

◆ 瞬时功率的角频率为2ω。

2.平均功率 P (average power)

$$P = \frac{1}{T}\int_0^T p\,\mathrm{d}t = \frac{1}{T}\int_0^T [UI\cos\varphi + UI\cos(2\omega t - \varphi)]\mathrm{d}t$$

$$= UI\cos\varphi$$

$$\boxed{P = UI\cos\varphi}$$

P **的单位：** W **（瓦）**

$\varphi = \psi_u - \psi_i$： **功率因数角。对无源网络，为其等效阻抗的阻抗角。**

$\cos\varphi$： **功率因数。**

$$\cos\varphi \begin{cases} 1, & \text{纯电阻} \\ 0, & \text{纯电抗} \end{cases}$$

有功功率守恒：电路中所有元件吸收的有功功率的代数和为零。

一般地，有： $0 \le |\cos\varphi| \le 1$

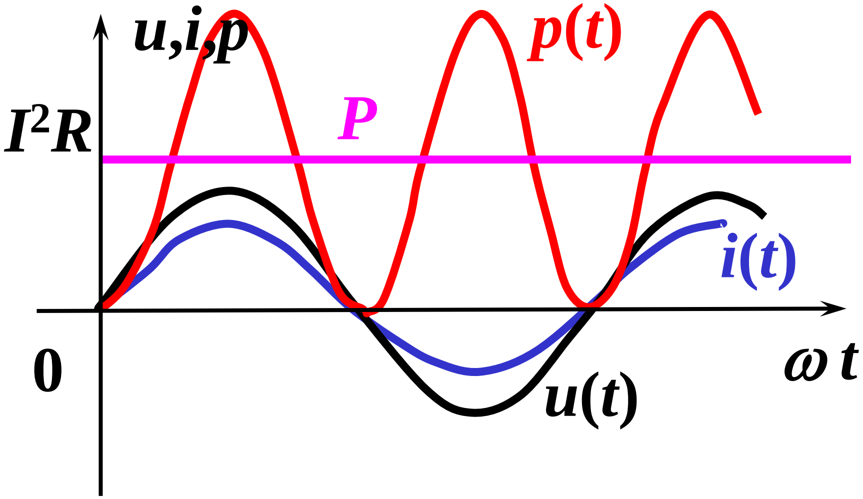
$X>0, \varphi>0$，感性， $X<0, \varphi<0$，容性，

$$P = UI\cos\varphi$$

结论

平均功率实际上是电阻消耗的功率，亦称为有功功率。表示电路实际消耗的功率，它不仅与电压电流有效值有关，而且与 $\cos\varphi$ 有关，这是交流和直流的很大区别，主要由于电压、电流存在相位差。

纯电阻(电阻元件或等效纯阻性网络) 条件下， $\varphi = 0^\circ$



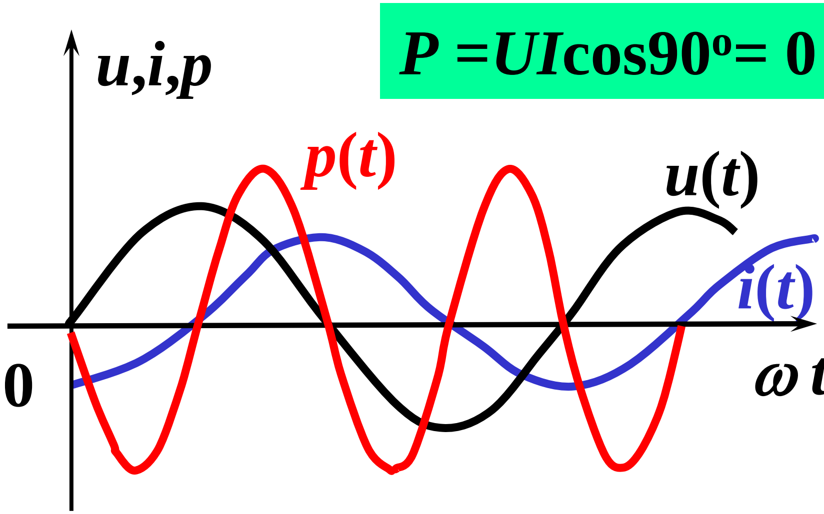
$$P = UI\cos\varphi = UI = I^2R = U^2/R$$

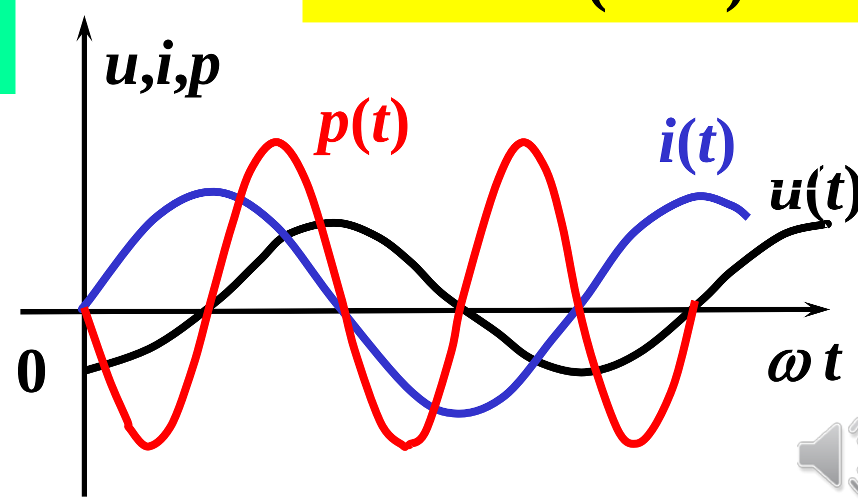
纯电感 (电感元件或等效纯感性网络) 条件下， $\varphi = 90^\circ$

纯电容(电容元件或等效纯容性网络) 条件下， $\varphi = -90^\circ$

$$P = UI\cos(-90^o) = 0$$

$$P = UI\cos 90^o = 0$$





3. 无功功率 Q (reactive power)

$$p(t) = UI\cos\varphi(1-\cos 2\omega t) + UI\sin\varphi\sin 2\omega t$$

$$\boxed{Q \overset{\text{def}}{=} UI\sin\varphi}$$

单位： var **(乏)。**

- Q **的大小反映网络与外电路交换功率的速率。是由储能元件**L、C**的性质决定的**
- $Q>0$，**表示网络吸收无功功率；**
- $Q<0$，**表示网络发出无功功率。**

无功功率守恒：电路中所有元件吸收无功功率的代数和为零。

4. 视在功率S (*apparent power*)

$$S \overset{\text{def}}{=} UI \qquad \text{单位: VA (伏安)}$$

表征电气设备的容量

（例如发电机的发电容量）

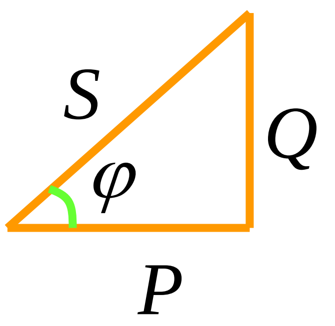
有功，无功，视在功率的关系：

有功功率：$P=UI\cos\varphi$　　单位：W

无功功率：$Q=UI\sin\varphi$　　单位：var

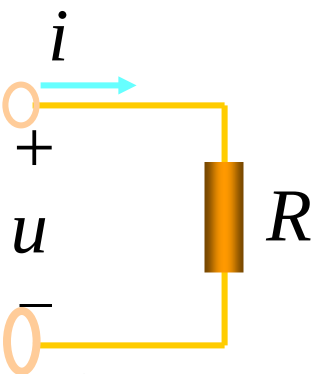
视在功率：$S=UI$　　单位：VA

$$S=\sqrt{P^2+Q^2}$$



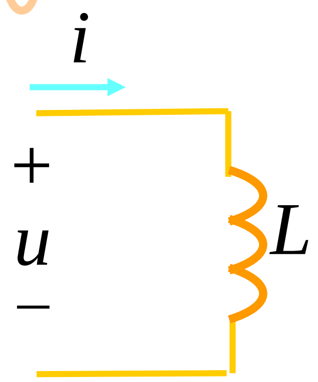
功率三角形

5. R、L、C元件的有功功率和无功功率



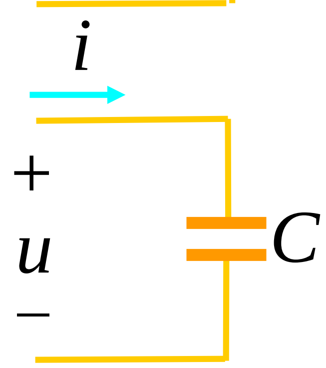
$$P_R = UI\cos\varphi = UI\cos 0° = UI = I^2R = U^2/R$$

$$Q_R = UI\sin\varphi = UI\sin 0° = 0$$



$$P_L = UI\cos\varphi = UI\cos 90° = 0$$

$$Q_L = UI\sin\varphi = UI\sin 90° = UI = I^2X_L$$

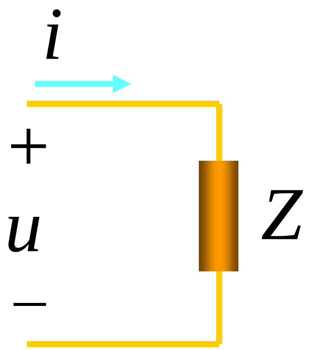


$$P_C = UI\cos\varphi = UI\cos(-90°) = 0$$

$-\frac{1}{\omega C}$

$$Q_C = UI\sin\varphi = UI\sin(-90°) = -UI = I^2X_C$$

6. 任意阻抗的功率计算



$$P_Z = UI\cos\varphi = I^2|Z|\cos\varphi = I^2R$$

$$Q_Z = UI\sin\varphi = I^2|Z|\sin\varphi = I^2X$$

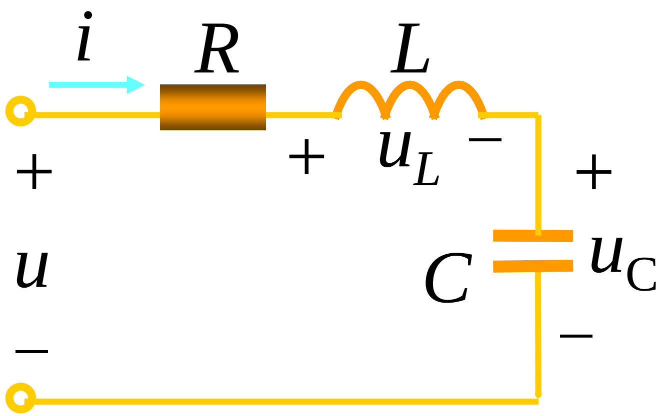
$$= I^2(X_L + X_C) = Q_L + Q_C$$

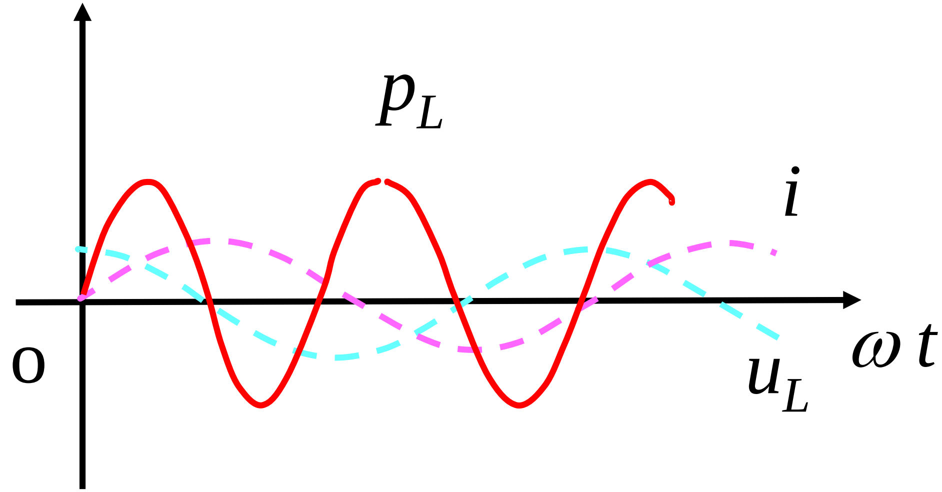
$Q_L = I^2 X_L > 0$　　吸收无功为正

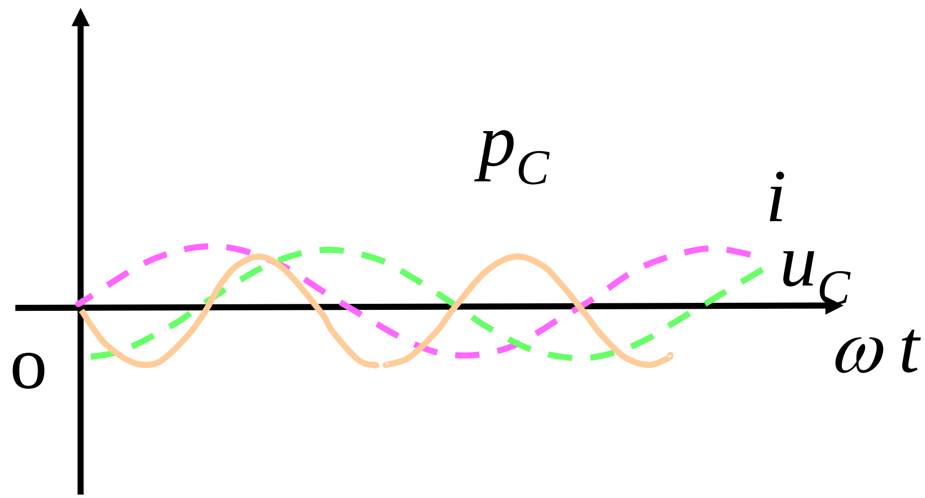
$Q_C = I^2 X_C < 0$　　吸收无功为负　**(发出无功)**

$$S = \sqrt{P^2 + Q^2} = I^2\sqrt{R^2 + X^2} = I^2|Z|$$

电感、电容的无功补偿作用





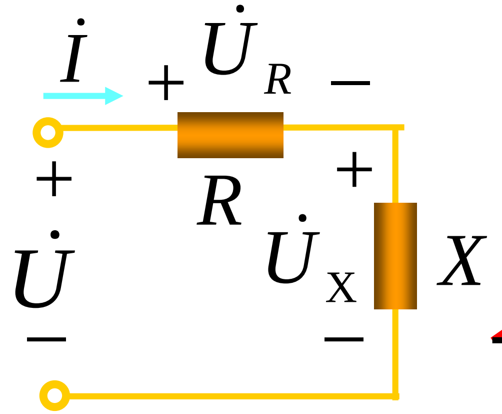


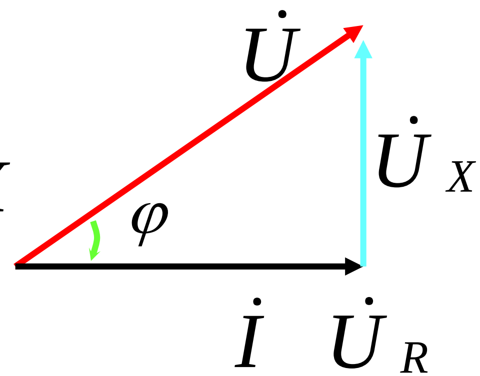
L发出功率时，C刚好吸收功率，与外电路交换功率为 p_L+p_C。

L、C的无功具有互相补偿的作用。

电压、电流的有功分量和无功分量：

以感性负载为例



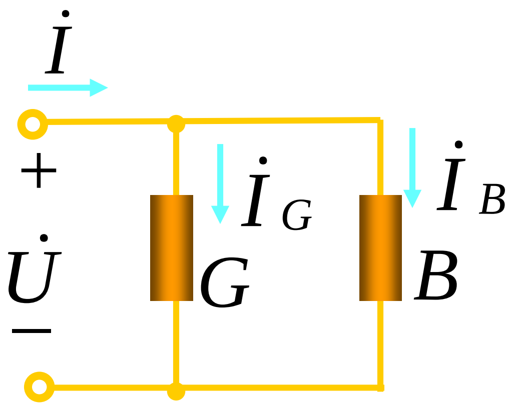


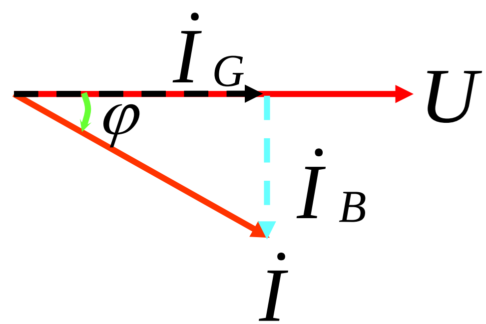
$$P = UI\cos\varphi = U_R I$$

$$Q = UI\sin\varphi = U_X I$$

称$\dot{U}_R$为$\dot{U}$的有功分量

称$\dot{U}_X$为$\dot{U}$的无功分量





$$P = UI\cos\varphi = UI_G$$

$$Q = UI\sin\varphi = UI_B$$

称$\dot{I}_G$为$\dot{I}$的有功分量

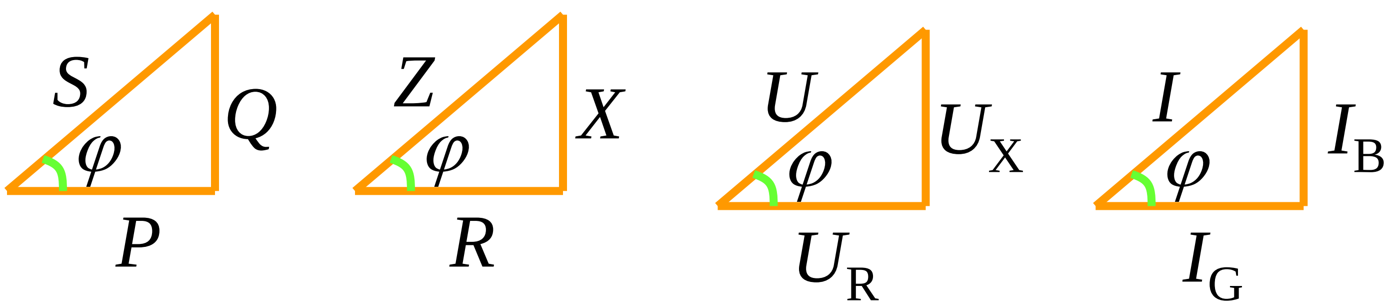
称$\dot{I}_B$为$\dot{I}$的无功分量

$$P = UI\cos\varphi = U_R I \qquad Q = UI\sin\varphi = U_X I$$

$$S = \sqrt{P^2 + Q^2} = I\sqrt{U_R^2 + U_X^2} = IU$$

$$P = UI\cos\varphi = UI_G \qquad Q = UI\sin\varphi = UI_B$$

$$S = \sqrt{P^2 + Q^2} = U\sqrt{I_G^2 + I_B^2} = IU$$



相似三角形

无功的物理意义:

反映电源和负载之间交换能量的速率。

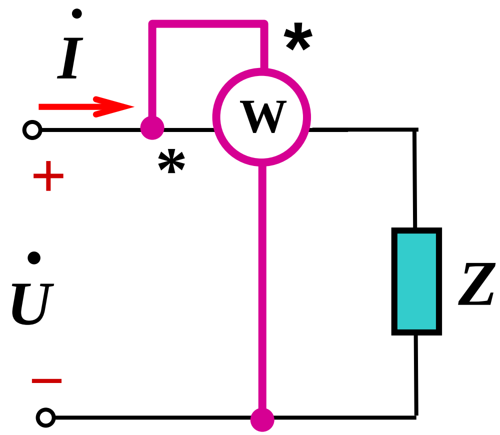
$$Q_L = I^2 X_L = I^2 \omega L = \omega \cdot \frac{1}{2} L(\sqrt{2} I)^2$$

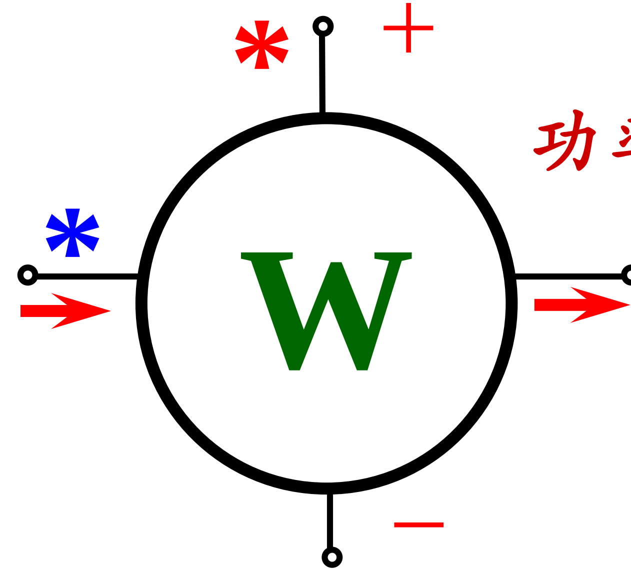
$$= \omega \cdot \frac{1}{2} L I_m^{\ 2} = 2\pi f W_{max} = \frac{2\pi}{T} \cdot W_{max}$$

许多用电设备均是根据电磁感应原理工作的，它们都是依靠建立交变磁场才能进行能量的转换和传递。

为建立交变磁场和感应磁通而需要的电功率称为无功功率，因此，所谓的"无功"并不是"无用"的电功率，只不过它的功率并不转化为机械能、热能而已；因此在供用电系统中除了需要有功电源外，还需要无功电源，两者缺一不可。

(2) 有功功率的测量

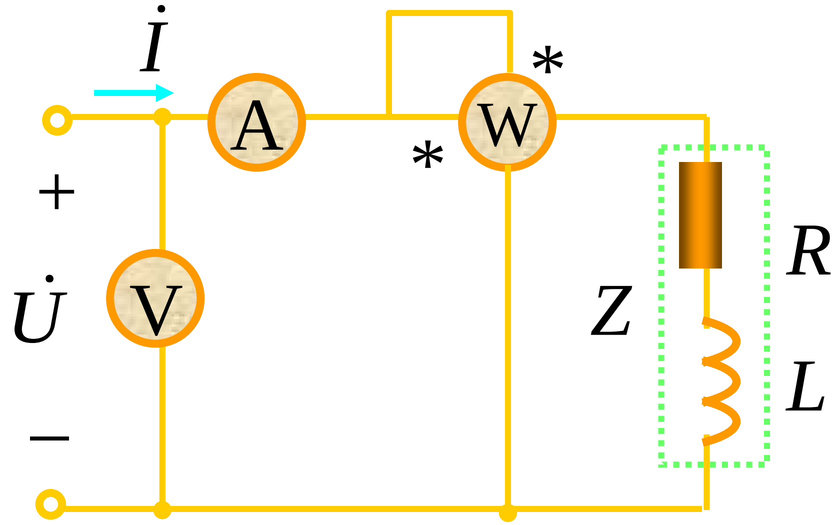




(1) 功率表接线：在负载电压和电流取关联参考方向下，让负载电流从功率表电流线圈标“*”端流入；功率表电压线圈的“*”端接负载电压的正端。如此接线下，功率表的示值反映的即为负载吸收的有功功率。 功率表有有功与无功之分。

(2) 功率表量程：测量有功功率时，*P*、*U*、*I* 均不能超量程，即除接功率表外，还应使用有效值电压表和有效值电流表。

例1 **三表法测线圈参数。** **已知：** f=50Hz， **且测得** U=50V，I=1A，P=30W。



解法 1

$$S = UI = 50 \times 1 = 50\,\mathrm{VA}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{50^2 - 30^2} = 40\,\mathrm{var}$$

$$R = \frac{P}{I^2} = \frac{30}{1} = 30\Omega \qquad X_L = \frac{Q}{I^2} = \frac{40}{1} = 40\Omega$$

$$L = \frac{X_L}{\omega} = \frac{40}{100\pi} = 0.127\,\mathrm{H}$$

解法 2 $P = I^2 R$ $\therefore R = \dfrac{P}{I^2} = \dfrac{30}{1^2} = 30\Omega$

$$|Z| = \frac{U}{I} = \frac{50}{1} = 50\Omega$$

又 $|Z| = \sqrt{R^2 + (\omega L)^2}$

$$\rightarrow L = \frac{1}{\omega}\sqrt{|Z|^2 - R^2} = \frac{1}{314}\sqrt{50^2 - 30^2} = \frac{40}{314} = 0.127\,\mathrm{H}$$

解法 3 $P = UI\cos\varphi$ $\rightarrow \cos\varphi = \dfrac{P}{UI} = \dfrac{30}{50 \times 1} = 0.6$

$$|Z| = \frac{U}{I} = \frac{50}{1} = 50\Omega$$

$$R = |Z|\cos\varphi = 50 \times 0.6 = 30\Omega$$

$$X_\mathrm{L} = |Z|\sin\varphi = 50 \times 0.8 = 40\Omega$$

7. 功率因数的提高

功率因数低带来的问题：

①设备不能充分利用，电流到了额定值，但功率容量还有；

$$P=UI\cos\varphi=S\cos\varphi$$

负载

$\cos\varphi=1$，　$P=S=75\text{kW}$

$\cos\varphi=0.7$，$P=0.7S=52.5\text{kW}$

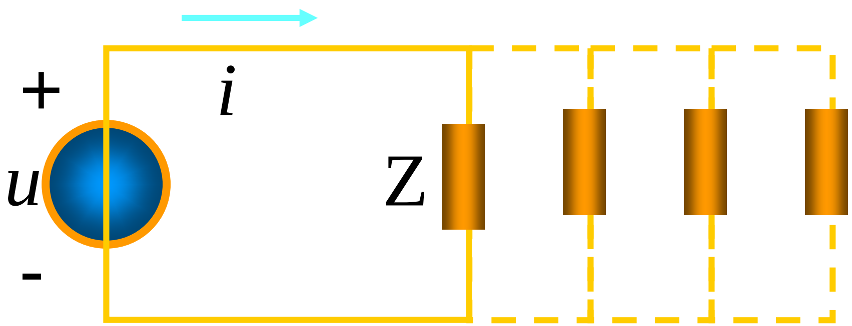
设备容量 S (额定)向负载送多少有功要由负载的阻抗角决定。

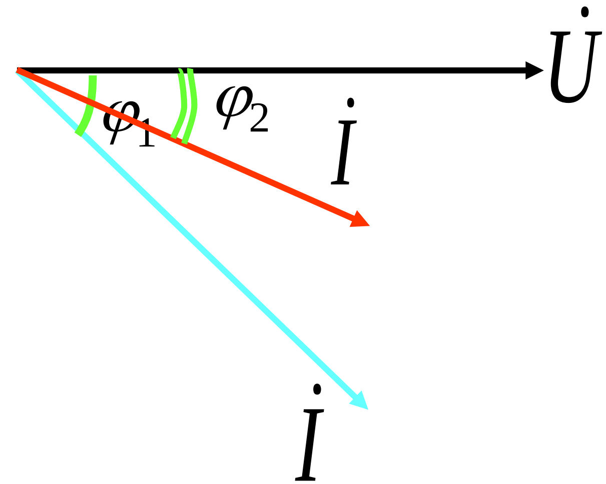
一般用户：　异步电机　空载　$\cos\varphi=0.2\sim0.3$

满载　$\cos\varphi=0.7\sim0.85$

日光灯　$\cos\varphi=0.45\sim0.6$

② 当输出相同的有功功率时，线路上电流大，$I=P/(U\cos\varphi)$，线路压降损耗大。





设： 电源电压有效值 $U_S=10V$，

负荷吸收的有功功率$P=10W$（恒定）。

- $\cos\varphi=1$　　$I=1A$
- $\cos\varphi=0.5$　　$I=2A$
- $\cos\varphi=0.1$　　$I=10A$

$$\boxed{P=UI\cos\varphi}$$

$$P = UI\cos\varphi$$

$$I\downarrow$$

$$I = P/(U\cos\varphi)$$

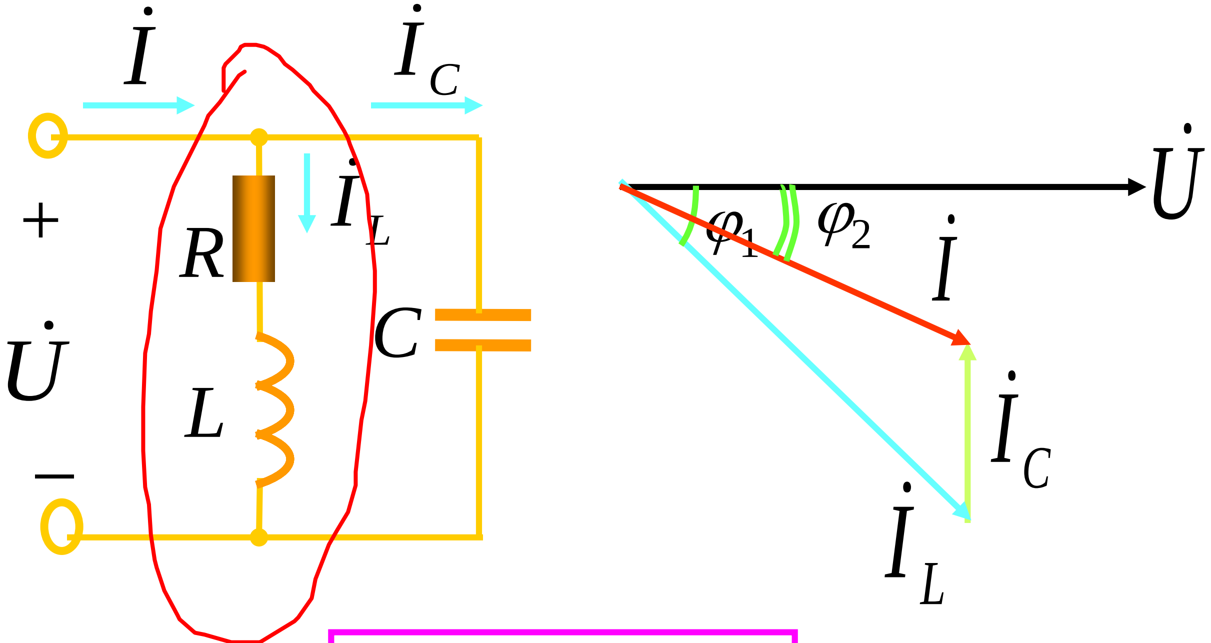
$$U\uparrow \qquad \cos\varphi\uparrow$$

解决办法：（1）高压传输

（2）改进自身设备

（3）并联电容，提高功率因数 。

分析



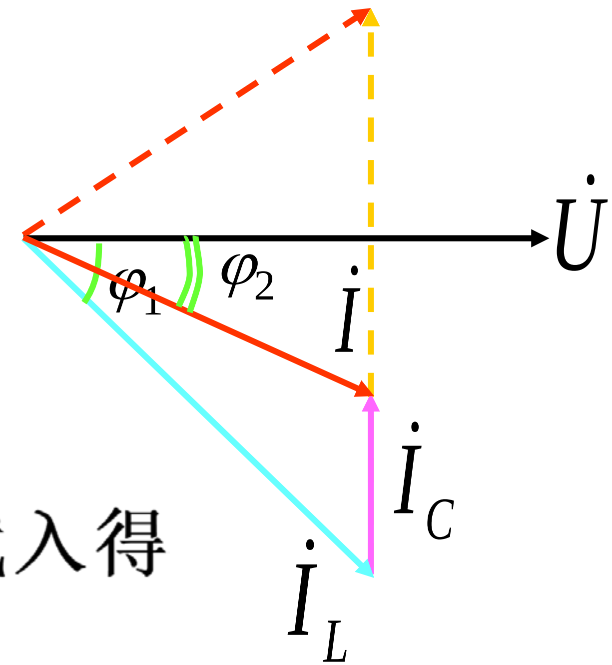
$$\boxed{P = UI\cos\varphi}$$

特点：

并联电容后，原负载的电压和电流不变，吸收的有功功率和无功功率不变。

即：负载的工作状态不变。但电路的功率因数提高了。

并联电容的确定：

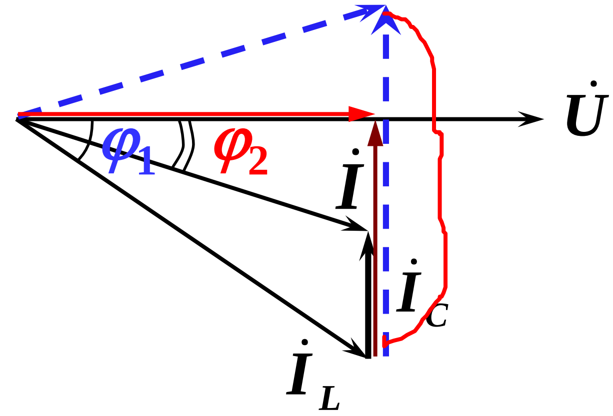


$$I_C = I_L \sin\varphi_1 - I\sin\varphi_2$$

将 $I = \dfrac{P}{U\cos\varphi_2}$ ， $I_L = \dfrac{P}{U\cos\varphi_1}$ 代入得

$$I_C = \omega CU = \frac{P}{U}(\mathrm{tg}\varphi_1 - \mathrm{tg}\varphi_2)$$

$$\rightarrow \quad C = \frac{P}{\omega U^2}(\mathrm{tg}\varphi_1 - \mathrm{tg}\varphi_2)$$



补偿容量不同

- 欠补偿
- 全补偿　　不要求(电容设备投资增加,经济效果不明显)
- 过补偿　　功率因数又由高变低(性质不同)

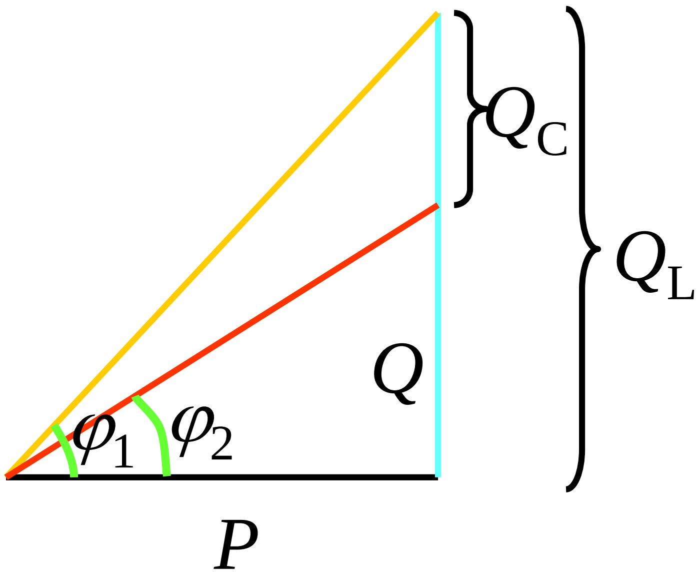
实际实施

- 性能
- 成本

一般补偿到$\cos\varphi$=0.9-0.95

(滞后，即仍为感性)

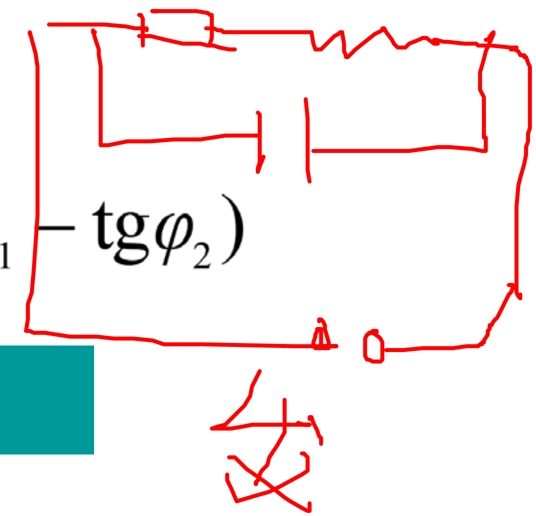
并联电容也可以用功率三角形确定：



$$|Q_C| = |Q_L - Q| = P(\mathrm{tg}\varphi_1 - \mathrm{tg}\varphi_2)$$

$$|Q_C| = \omega C U^2$$

$$\therefore \quad C = \frac{P}{\omega U^2}(\mathrm{tg}\varphi_1 - \mathrm{tg}\varphi_2)$$



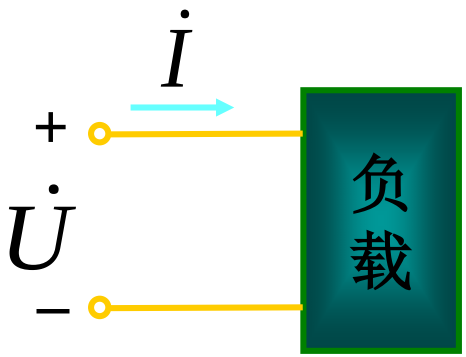
从功率角度看：

并联电容后，电源向负载输送的有功 $UI_L\cos\varphi_1 = UI\cos\varphi_2$ 不变，但是电源向负载输送的无功 $UI\sin\varphi_2 < UI_L\sin\varphi_1$ 减少了，减少的这部分无功由电容“产生”来补偿，使感性负载吸收的无功不变，而功率因数得到改善。

9.5 复功率

1. 复功率(complex power)

为了用相量$\dot{U}$和$\dot{I}$来计算功率，引入“复功率”



定义： $\overline{S} = \dot{U}\dot{I}^*$ 单位 VA

$$\overline{S} = UI\angle(\Psi_u - \Psi_i) = UI\angle\varphi = S\angle\varphi$$

$$= UI\cos\varphi + \mathrm{j}UI\sin\varphi = P + \mathrm{j}Q$$

也可表示为：

$$\overline{S} = \dot{U}\dot{I}^* = Z\dot{I}\cdot\dot{I}^* = ZI^2 = (R+\mathrm{j}X)I^2 = RI^2 + \mathrm{j}XI^2$$

$$\text{or} \quad \overline{S} = \dot{U}\dot{I}^* = \dot{U}(\dot{U}Y)^* = \dot{U}\cdot\dot{U}^*Y^* = U^2Y^*$$

结论

①$\overline{S}$是复数，而不是相量，它不对应任意正弦量；

② $\overline{S}$ 把 P、Q、S 联系在一起，它的实部是平均功率，虚部是无功功率，模是视在功率；

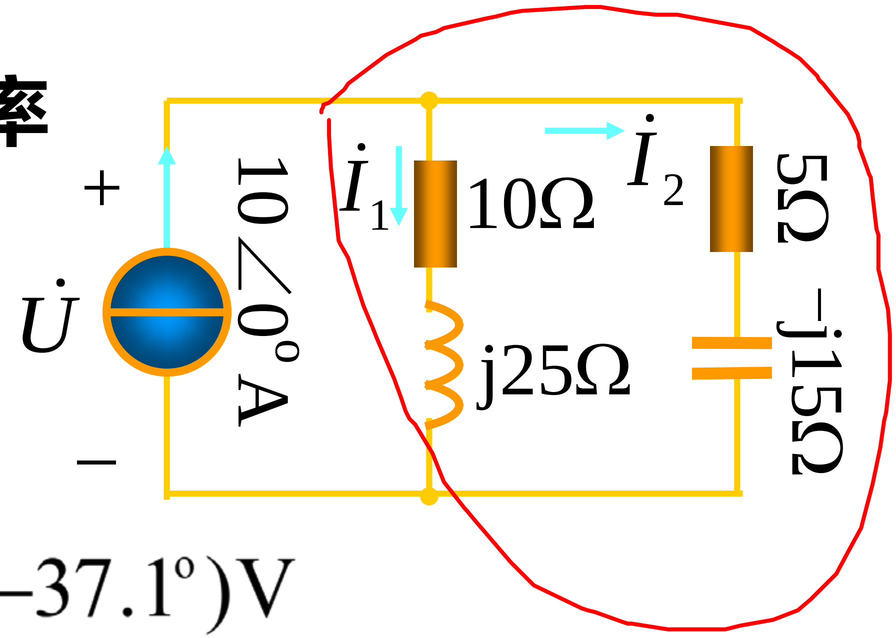
③ 复功率满足守恒定理：在正弦稳态下，任一电路的所有支路吸收的复功率之和为零。即

$$\begin{cases} \sum_{k=1}^{b} P_k = 0 \\ \sum_{k=1}^{b} Q_k = 0 \end{cases} \longrightarrow \sum_{k=1}^{b}(P_k + \mathrm{j}Q_k) = \sum_{k=1}^{b} \overline{S}_k = 0$$

$$\because U \neq U_1 + U_2 \quad \therefore S \neq S_1 + S_2$$

注意 复功率守恒，视在功率不守恒

例　**求电路各支路的复功率**



解1

$$Z = (10 + \mathrm{j}25)//(5 - \mathrm{j}15)$$

$$\dot{U} = 10\angle 0^{\circ} \times Z = 236\angle(-37.1^{\circ})\mathrm{V}$$

$$\overline{S}_{发} = 236\angle(-37.1^{\circ}) \times 10\angle 0^{\circ} = 1882 - \mathrm{j}1424\ \mathrm{VA}$$

$$\overline{S}_{1吸} = U^2 Y_1^* = 236^2 \left(\frac{1}{10 + \mathrm{j}25}\right)^* = 768 + \mathrm{j}1920\quad \mathrm{VA}$$

$$\overline{S}_{2吸} = U^2 Y_2^* = 1113 - \mathrm{j}3345\,\mathrm{VA}$$

$$\overline{S}_{1吸} + \overline{S}_{2吸} = \overline{S}_{发}$$

解2

$$\dot{I}_1 = 10\angle 0^\circ \times \frac{5 - \mathrm{j}15}{10 + \mathrm{j}25 + 5 - \mathrm{j}15} = 8.77\angle(-105.3^\circ)\quad \mathrm{A}$$

$$\dot{I}_2 = \dot{I}_S - \dot{I}_1 = 14.94\angle 34.5^\circ \quad \mathrm{A}$$

$$\overline{S}_{1吸} = I_1^{\ 2} Z_1 = 8.77^2 \times (10 + \mathrm{j}25) = 769 + \mathrm{j}1923\,\mathrm{VA}$$

$$\overline{S}_{2吸} = I_2^{\ 2} Z_2 = 14.94^2 \times (5 - \mathrm{j}15) = 1116 - \mathrm{j}3348\,\mathrm{VA}$$

$$\overline{S}_{发} = \dot{I}_1 Z_1 \cdot \dot{I}_S^{\ *} = 10 \times 8.77\angle(-105.3^\circ)(10 + \mathrm{j}25)$$

$$= 1885 - \mathrm{j}1423\ \mathrm{VA}$$

9.6 最大功率传输

正弦稳态电路中负载获得最大功率P_{max}的条件

$$Z_i = R_i + jX_i, \quad Z_L = R_L + jX_L$$

$$\dot{I} = \frac{\dot{U}_S}{Z_i + Z_L}, \quad I = \frac{U_S}{\sqrt{(R_i + R_L)^2 + (X_i + X_L)^2}}$$

有功功率 $P = R_L I^2 = \dfrac{R_L U_S^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$

讨论 正弦电路中负载获得最大功率$P_{\max}$的条件

$$P=\frac{R_L U_S^2}{(R_i+R_L)^2+\cancel{(X_i+X_L)^2}}$$

$$P_{\max}=\frac{U_S^2}{4R_i}$$

①若$Z_L=R_L+jX_L$可任意改变

a)先设 R_L 不变，X_L 改变

显然，当$X_i+X_L=0$，即$X_L=-X_i$时，P 获得最大值。

b)再讨论 R_L 改变时，P 的最大值

当$R_L=R_i$ 时，P 获得最大值

$$R_L=R_i \quad X_L=-X_i \quad \longrightarrow \quad Z_L=Z_i^*$$

最佳匹配条件

②若$Z_L = R_L + jX_L$只允许X_L改变

获得最大功率的条件是：$X_i + X_L = 0$，**即** $X_L = -X_i$

最大功率为 $P_{max} = \dfrac{R_L U_S^2}{(R_i + R_L)^2}$

③若$Z_L = R_L$为纯电阻

电路中的电流为：$\dot{I} = \dfrac{\dot{U}_S}{Z_i + R_L}$，$I = \dfrac{U_S}{\sqrt{(R_i + R_L)^2 + X_i^2}}$

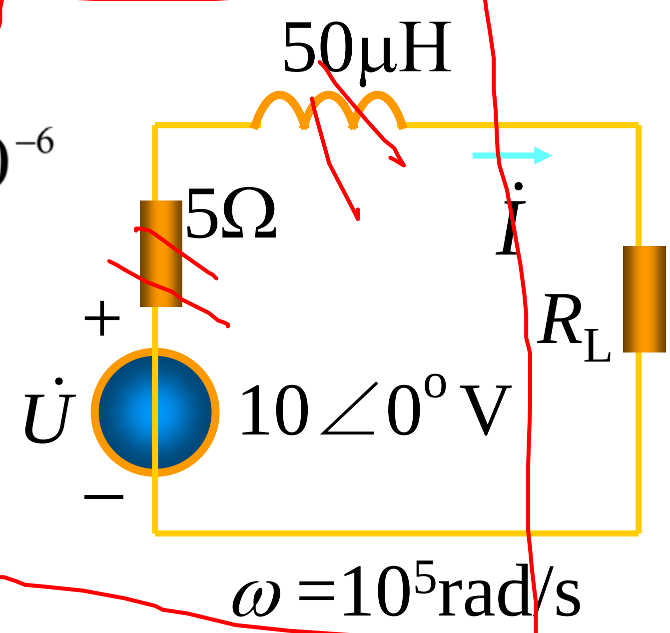
负载获得的功率为：$P = \dfrac{R_L U_S^2}{(R_i + R_L)^2 + X_i^2}$

模匹配

令 $\dfrac{dP}{dR_L} = 0 \Rightarrow$ 获得最大功率条件 $R_L = \sqrt{R_i^2 + X_i^2} = |Z_i|$

例

电路如图，求：1.R_L=5Ω时其消耗的功率；
2. R_L=?能获得最大功率，并求最大功率；
3.在R_L两端并联一电容，问R_L和C为多大时能与内阻抗最佳匹配，并求最大功率。



解

$$Z_i = R + \mathrm{j}X_L = 5 + \mathrm{j}10^5 \times 50 \times 10^{-6}$$
$$= 5 + \mathrm{j}5\ \Omega$$

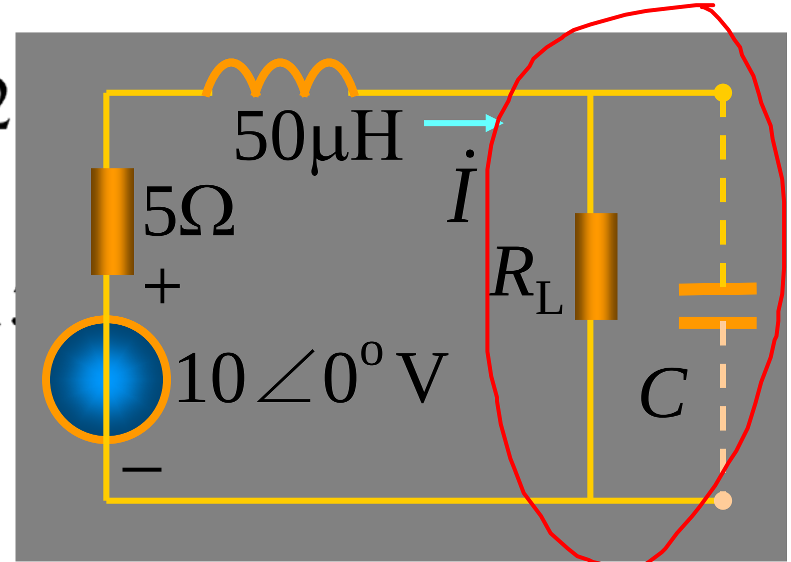
1. $$\dot{I} = \frac{10\angle 0^{\circ}}{5+\mathrm{j}5+5} = 0.89\angle(-26.6^{\circ})\mathrm{A}$$

$$P_L = I^2 R_L = 0.89^2 \times 5 = 4\mathrm{W}$$

2. 当 $R_L = \sqrt{R_i^2 + X_i^2} = \sqrt{5^2+5^2} = 7.07\Omega$ 获最大功率

$$\dot{I} = \frac{10\angle 0^\circ}{5 + \mathrm{j}5 + 7.07} = 0.766\angle(-22$$

$$P_L = I^2 R_L = 0.766^2 \times 7.07 = 4.1$$



$$3.\quad Y = \frac{1}{R_L} + \mathrm{j}\omega C$$

$$Z_{\mathrm{L}} = \frac{1}{Y} = \frac{R_L}{1 + \mathrm{j}\omega C R_L} = \frac{R_L}{1 + (\omega C R_L)^2} - \mathrm{j}\frac{\omega C R_L^2}{1 + (\omega C R_L)^2}$$

当 $\begin{cases} \dfrac{R_L}{1 + (\omega C R_L)^2} = 5 \\ \dfrac{\omega C R_L^2}{1 + (\omega C R_L)^2} = 5 \end{cases} \Rightarrow \begin{cases} R_L = 10\Omega \\ C = 1\mu F \end{cases}$ 获最大功率

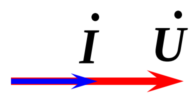
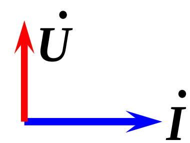
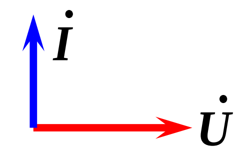
$$\dot{I} = \frac{10\angle 0^\circ}{10} = 1\mathrm{A}$$

$$P_{\max} = I^2 R_i = 1 \times 5 = 5\mathrm{W}$$

求解正弦稳态电路的相量法小结

1. 正弦量的三要素：I_m（U_m），ψ，ω

2.

	电阻	电感	电容
时域	$u=Ri$	$u=L\frac{\mathrm{d}i}{\mathrm{d}t}$	$i=C\frac{\mathrm{d}u}{\mathrm{d}t}$
频域(相量)	$\dot{U}=R\dot{I}$	$\dot{U}=\mathrm{j}\omega L\dot{I}$	$\dot{I}=\mathrm{j}\omega C\dot{U}$
相位			
有效值	$U=RI$	$U=X_LI$ $X_L=\omega L$	$U=-X_CI$ $X_C=-1/(\omega C)$
有功	$P=I^2R=U^2/R$	0	0
无功	0	$Q=I_LU_L$	$Q=-I_CU_C$
能量		$W=Li^2/2$	$W=Cu^2/2$

注：这里，电阻、电感、电容不限于单个理想元件，也可是等效的结果。

3. 以相量法分析求解线性时不变正弦稳态电路的基本步骤

①先确定电路的相量模型 $\begin{cases}\text{电压、电流} \rightarrow \text{相量} \\ \text{电阻、电感、电容} \rightarrow \text{复阻抗}\end{cases}$

②列写相量形式的KCL、KVL、元件约束方程

③之前学习的电路定理和相关计算方法等均适用

④画相量图：对单电源激励的电路，从距电源最远端画起相对容易；并联选电压、串联选电流作参考相量。

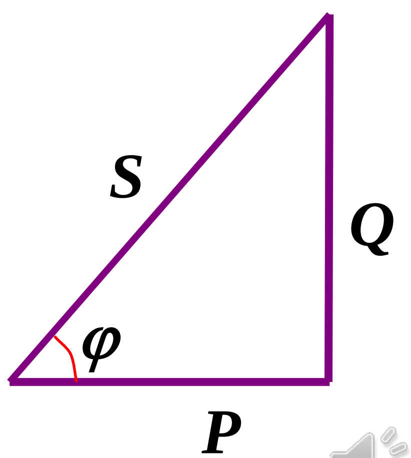
4. 功率关系

复功率 $\bar{S}$ $\bar{S} = P + \mathrm{j}Q = \sqrt{P^2 + Q^2}\angle\varphi$

视在功率 S $S = UI = |\bar{S}|$

有功功率 P $P = UI\cos\varphi = \mathrm{Re}[\bar{S}]$

无功功率 Q $Q = UI\sin\varphi = \mathrm{Im}[\bar{S}]$



Homework

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9-8
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