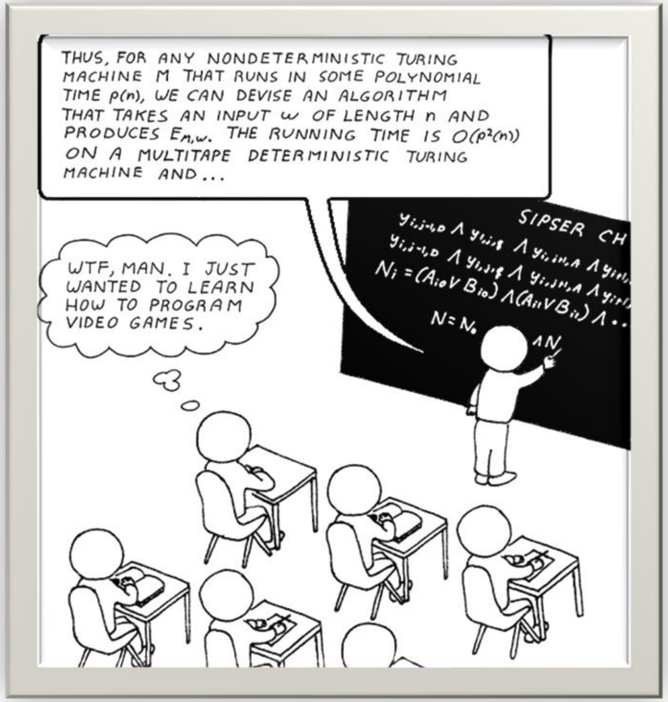
Game Theory
Meets
Computer Science

A Basic Introduction



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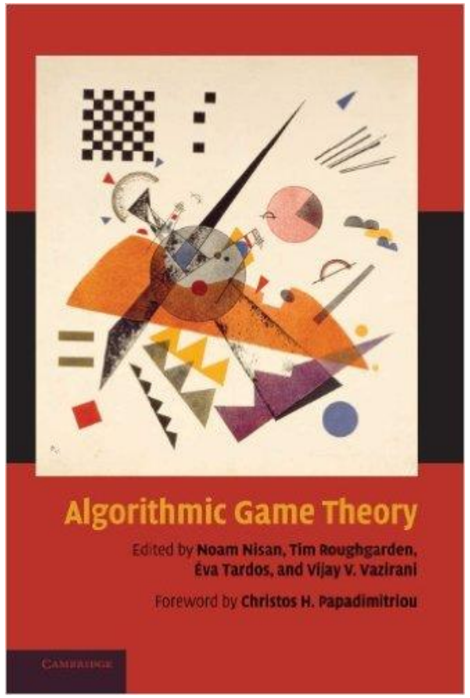
Algorithmic Game Theory

- **Algorithmic Game Theory** is an area in the intersection of game theory and algorithm design, whose objective is to design algorithms in strategic environments [Nisan et al. 2007].
- *Computing in Games*
  - *Algorithms for computing equilibria*
- *Algorithmic Mechanism Design*
  - *Design games that have both good game-theoretical and algorithmic properties*



Prisoner’s Dilemma

Column / Row	Deny	Confess
Deny	(-1, -1)	(-10, 0)
Confess	(0, -10)	(-8, -8)

Prisoner's Dilemma

Row \ Column	Deny	Confess
Deny	(-1, -1)	(-10, 0)
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Prisoner's Dilemma

Row \ Column	Deny	Confess
Deny	(-1, -1)	(-10, 0)
Confess	(0, -10)	(-8, -8)

- The equilibrium payoffs are (-8,-8) Pareto inefficient!!
- worse for both players than (–1, –1)

Generalized Second Price (GSP) Auction

Algorithmic Game Theory in Artificial Intelligence

- Game Playing: computation challenge, AlphaGo, Libratus, General Game Playing
- Mechanism Design: the allocation of scarce resources (security games), Ad/online auctions, Computational Social Choice
- **IJCAI Computers and Thought Award**: 7 out of the 16 winners (1999-2023) had worked on AGT
  - Nick Jennings (1999), Tuomas Sandholm (2003), Peter Stone (2007), Vice Conitzer (2011), Ariel Procaccia (2015), Piotr Skowron (2020), Fei Fang (2021).

Outline

- Game Theory Basics
- Game Playing Algorithms
  - Adversarial Search
  - Monte-Carlo Tree Search
  - Generalized Reinforcement Learning

Game Theory Meets Computer Science

PART ONE GAME THEORY BASICS

Let us play a game: The Grade Game

Without showing others what you are doing, write down on a form either the letter α or the letter β. Think of this as a ‘grade bid’. We will randomly pair your form with one other form. Neither you nor your pair will ever know with whom you were paired. Here is how grades may be assigned.

- If you put α and your pair puts β, then you will get grade A, and your pair grade C.
- If both you and your pair put α, then you both will get grade B-.
- If you put β and your pair puts α, then you will get grade C, and your pair grade A.
- If both you and your pair put β, then you both will get grade B+.

Outcome Matrix

you \ pair	α	β
α	(B-, B-)	(A, C)
β	(C, A)	(B+, B+)

Your outcome by (α, β)

Your pair's outcome by (α, β)

What a rational player choose?

you \ pair	α	β
α	(B-, B-)	(A, C)
β	(C, A)	(B+, B+)

- Payoff—each outcome yield for each player
- Preferences of players: not just you but also your opponents

Basic Concepts

- What is a game?

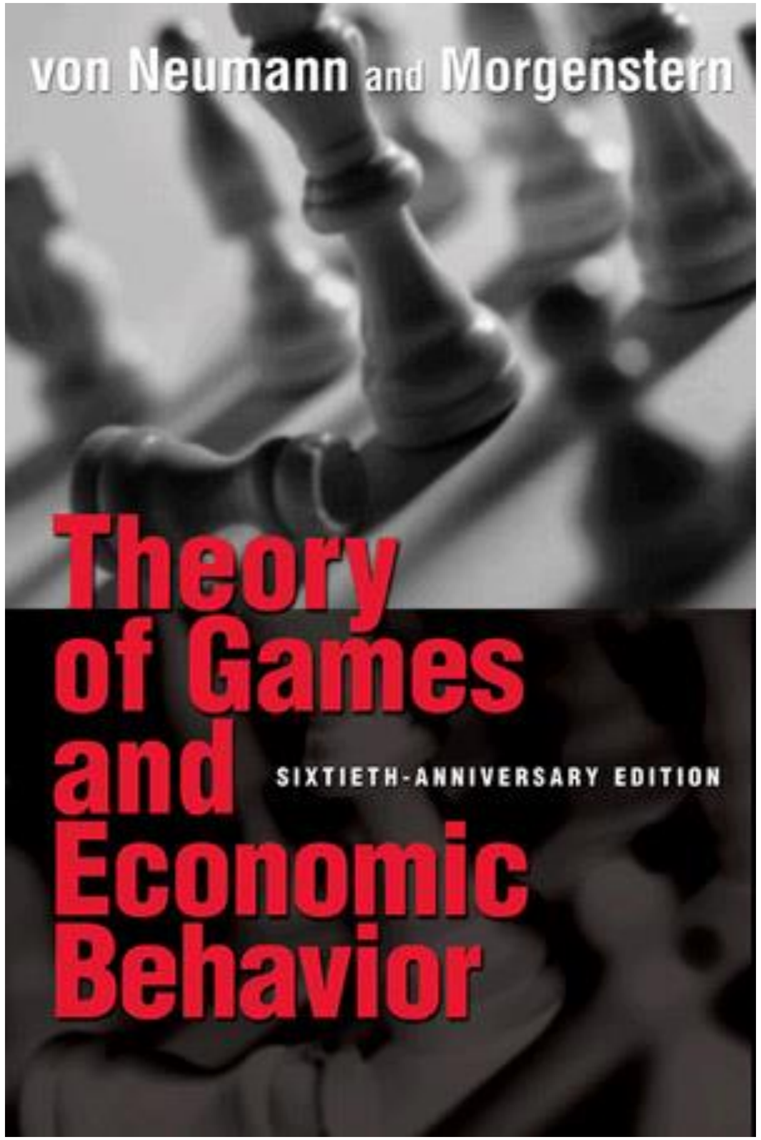
Basic Concepts

- A game: strategic decision-making situation
    - rational players
    - available actions
    - possible utilities
    - Information
- Equilibrium: a strategy profile consisting of a best strategy for each player
- We focus on noncooperative games.
    - No external force or agencies enforcing coalitions.

Player Strategy

- Pure strategy
  - Choose an action to play
  - E.g., "α", "β"
  - Simply an action.
    - In repeated or multi-move games (like Chess), need to choose an action to play at every step of the game based on history.
- Mixed strategy
  - Choose a probability distribution over actions
  - Randomize over pure strategies
  - E.g., "α with probability 0.7, and β with probability 0.3"

Game Theory



1944 by Princeton University Press

- The study of mathematical models of strategic interaction among rational decision-makers.
- Applications in all fields of social science, as well as in logic, systems science, and computer science.

----Wikipedia

"the classic work upon which modern-day game theory is based."

Possible payoff: Type 1

If every player only cares about her own grade then (assuming A > B+ > B- > C for each player)

you \ pair	α	β
α	(0, 0)	(3, -1)
β	(-1, 3)	(1, 1)

What should you choose in this case?

Possible payoffs: Type 1

If every player only cares about her own grade then (assuming A > B+ > B- > C for each player)

you \ pair	α	β
α	(0, 0)	(3, -1)
β	(-1, 3)	(1, 1)

- Your payoff from α is *strictly* higher than that from β regardless of others' choices.
- Your strategy α *strictly dominates* your strategy β.

Dominant Strategy

- For player *i*, strategy x dominates strategy y if playing x is better than playing y no matter what the other players do

Lesson 1： *You should never play a strictly dominated strategy*

What about the opponent?

you \ pair	α	β
α	($\underline{0}$, $\underline{0}$)	($\underline{3}$, -1)
β	(-1, $\underline{3}$)	(1, 1)

- Given the payoff, she will also choose strategy “α”.

What about the opponent?

you \ pair	α	β
α	(0, 0)	(3, -1)
β	(-1, 3)	(1, 1)

- Given the payoff, she will also choose strategy "α".
- Dominant-strategy equilibrium: (α,α).

Something goes wrong?

you \ pair	α	β
α	(0, 0)	(3, -1)
β	(-1, 3)	(1, 1)

- Dominant-strategy equilibrium: (α,α).

Something goes wrong?

you \ pair	α	β
α	($\underline{0}$, $\underline{0}$)	($\underline{3}$, -1)
β	(-1, $\underline{3}$)	(1, 1)

- Dominant-strategy equilibrium: (α,α) → (B-,B-) Pareto inefficient!!
- Games like this called Prisoners' Dilemmas.

Something goes wrong?

you \ pair	α	β
α	$(\underline{0}, \underline{0})$	$(\underline{3}, -1)$
β	$(-1, \underline{3})$	$(1, 1)$

- Dominant-strategy equilibrium: (α,α) → (B-,B-) Pareto inefficient!!
- Games like this called Prisoners' Dilemmas.

Recap: Prisoner's Dilemma

Column / Row	Deny	Confess
Deny	(-1, -1)	(-10, 0)
Confess	(0, -10)	(-8, -8)

- A dominant strategy equilibrium is (Confess, Confess)
- The equilibrium payoffs are (-8,-8)
- worse for both players than (–1, –1)

Lesson 2： *Rational play by rational players can lead to bad outcomes.*

Other possible payoff: Type 2

What if you are type 1 but you know your opponent is type 2 who cares not only about her own grade but also about yours?

you \ pair	α	β
α	(0, 0)	(3, -3)
β	(-1, -1)	(1, 1)

What should you choose in this case?

Type 1 versus Type 2

What if you are Type 1 *but you know your opponent is* Type 2?

you \ pair	α	β
α	(0, 0)	(3, -3)
β	(-1, -1)	(1, 1)

- What should you choose in this case?
  - Choose α

The Converse

What if you are Type 2 *but you know your opponent is* Type 1?

you \ pair	α	β
α	(0, 0)	(-1, -3)
β	(-3, 3)	(1, 1)

- Neither of your strategies dominates the other.

What should you choose in this case?

Put yourself in your opponents' shoes

pair / you	α	β
α	(0, 0)	(-1, -3)
β	(-3, 3)	(1, 1)

- Neither of your strategies dominates the other.

Put yourself in your opponents' shoes

you \ pair	α	β
α	(0, 0)	(-1, -3)
β	(-3, 3)	(1, 1)

- Neither of your strategies dominates the other.
- Your pair's strategy α strictly dominates her strategy β.

Put yourself in your opponents' shoes

you \ pair	α	β
α	(0, 0)	(-1, -3)
~~β~~	~~(-3, 3)~~	~~(1, 1)~~

- Neither of your strategies dominates the other.
- Your pair's strategy α strictly dominates her strategy β.
- Then you should play α.

Iterated Elimination

- What if there are no dominant strategies?
  - No single strategy dominates every other strategy
  - But some strategies might still be dominated

- Assume every knows everyone is rational.
  - Can remove their dominated strategies
  - Might reveal a newly dominant strategy

Iterated Elimination

you \ pair	α	β
α	(0, 0)	(-1, -3)
β	(-3, 3)	(1, 1)

Lesson 3: *If you do not have a dominated strategy, put yourself in your opponents' shoes to try to predict what they will do. For example, in their shoes, they would not choose a dominated strategy.*

Possible payoffs

Suppose that each person cares not only about her own grade but also about the grade of the person with whom she is paired.

you \ pair	α	β
α	(0, 0)	(-1, -3)
β	(-3, -1)	(1, 1)

What should you choose in this case?

Coordination Game

you \ pair	α	β
α	(0, 0)	(-1, -3)
β	(-3, -1)	(1, 1)

- No strategy is dominated.

Lesson 4: *To figure out what actions you should choose in a game, a good first step is to figure out what are your payoffs (what do you care about) and what are other players' payoffs.*

Nash Equilibrium

John Forbes Nash (June 13, 1928 -May 23, 2015)

- Game theory, differential geometry, and partial differential equations
- Laid the foundations for modern noncooperative game theory
- Awarded the 1994 Nobel Prize for Economics

Nash Equilibrium

- A set of strategies, one for each player such that no player has incentive to deviate from her strategy given that the other players do not deviate

you \ pair	α	β
α	(0, 0)	(-1, -3)
β	(-3, -1)	(1, 1)

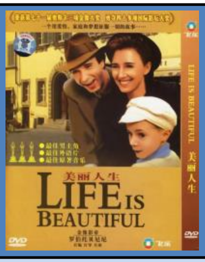
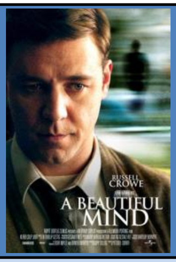
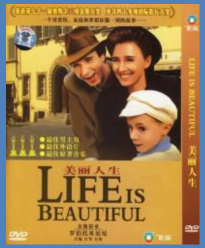
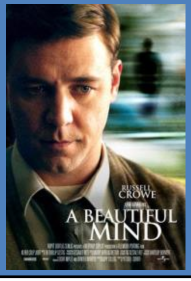
➢ Try to figure out the NEs

Nash Equilibrium

you \ pair	α	β
α	(0, 0)	(-1, -3)
β	(-3, -1)	(1, 1)

- (α,α) and (β,β) are Nash Equilibria.
- A Dominant-strategy equilibrium is a Nash Equilibrium.

The Battle of the Sexes

Boy / Girl		
	(2, 1)	(0, 0)
	(0, 0)	(1, 2)

- Two NEs (美丽人生,美丽人生), (美丽心灵,美丽心灵)
- Which one to choose?

Experimental Results

What do people do in real world?

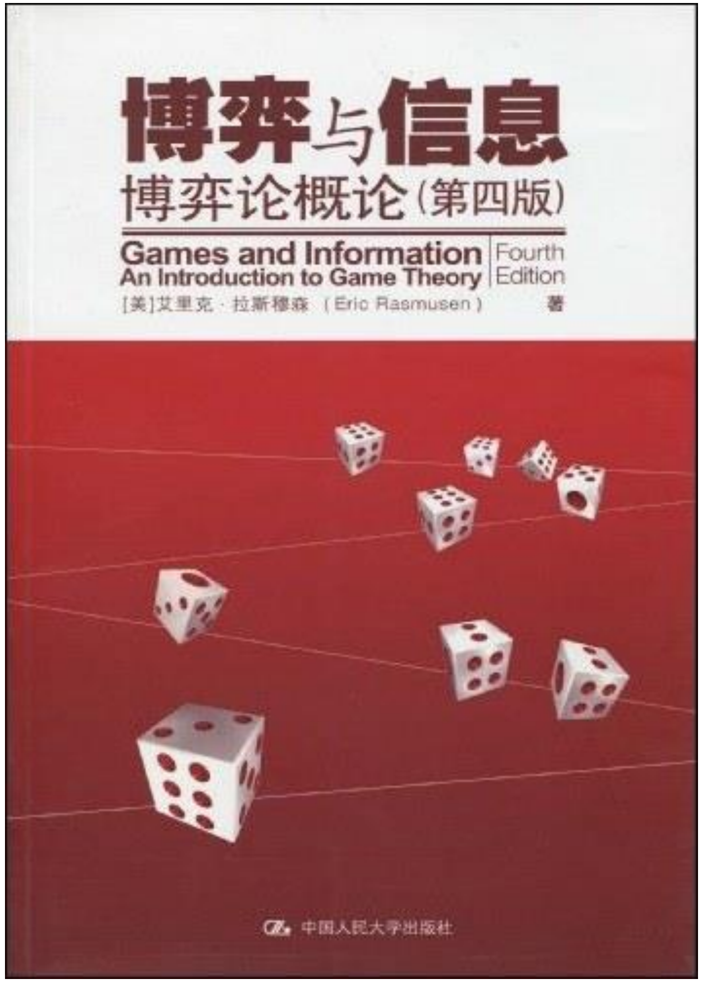
- *In larger experiments with ‘normal people’, roughly 70% of people choose Alpha and roughly 30% choose Beta.*

Mixed Strategies: Rock-Paper-Scissor

P1 \ P2			
	0,0	-1,1	1,-1
	1,-1	0,0	-1 , 1
	-1,1	1 , -1	0,0

- No dominant strategy; No NE in pure strategies
- Have a mixed-strategy NE

How to calculate the mixed-strategy NE?



Existence of Nash Equilibria

- Theorem (Nash,1951)

  Every finite game has a mixed-strategy Nash equilibrium.

Nash, J. (1951). Non-cooperative games. Annals of Mathematics, 54, 286–295

Knowledge Game: The Muddy Children Puzzle

1. At least one of you has mud on your forehead.
2. Can you tell for sure whether or not you have mud on your forehead.

Exercise: Pick up a number

- Without showing your neighbor what you‘re doing, put in the box below a whole number between 1 and a 100.
- We will calculate the average number chosen in the class.
- The winner in this game is the person whose number is closest to one-fifth times the average in the class.
- The winner will win the prize and all other players win nothing.

Conclusion

- Three Concepts
  - Dominant-strategy equilibrium
  - Nash equilibrium
  - Mixed-strategy Nash equilibrium
- Four Lessons
  - *Put yourself in your opponents' shoes*

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