1. 解：$\because R=\{<\phi,\{\phi\}>,<\{\phi\},\{\phi,\{\phi\}\}>\}$

$\therefore$ (1) $R^{-1}=\{<\{\phi\},\phi>,<\{\phi,\{\phi\}\},\{\phi\}>\}$

(2) $R\circ R=\{<\phi,\{\phi,\{\phi\}\}>\}$.

2. 解：$\because A=\{2,3,4\}$, $B=\{4,6,7\}$. $C=\{8,9,12,14\}$.

$R_1=\{<a,b>\mid a$ 是素数且 a 整除 $b\}$

$R_2=\{<b,c>\mid b$ 整除 $c\}$

$\therefore R_1=\{<2,4>,<2,6>,<3,6>\}$.

$R_2=\{<4,8>,<4,12>,<6,12>,<7,14>\}$.

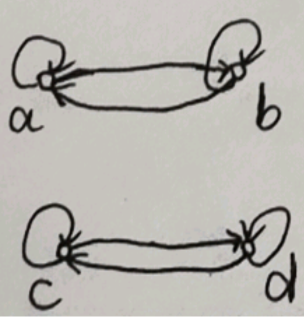
$\therefore R_1\circ R_2=\{<2,8>,<2,12>,<3,12>\}$

关系矩阵为：$\begin{array}{c c} & \begin{array}{cc} 8 & 12 \end{array} \\ \begin{array}{c} 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \end{array}$

8. 解：$\because R$ 是等价关系.

$\therefore R$ 自反.对称传递.

R 的关系图为：



等价类：$[a]=[b]=\{a,b\}$

$[c]=[d]=\{c,d\}$

9. $\because xRy \Leftrightarrow 2 \mid (x+y)$

x与y具有相同的奇偶性.轮换对称性.

$\therefore$ 令 $A = \{2x \mid x \in N\}$. 则划分为: $\{A, N-A\}$.

10. 证明: ① 自反性:

任取$\langle x,y\rangle$, 则 $\langle x,y\rangle \in Z^+ \times Z^+ \Rightarrow xy = yx$

$\Rightarrow \langle\langle x,y\rangle,\langle x,y\rangle\rangle \in R$

故: R是自反的.

② 对称性:

任取$\langle x,y\rangle, \langle u,v\rangle \in Z^+ \times Z^+$, 则:

$\langle\langle x,y\rangle,\langle u,v\rangle\rangle \in R \Rightarrow xv = yu \Rightarrow uy = vx$

$\Rightarrow \langle\langle u,v\rangle,\langle x,y\rangle\rangle \in R$

故: R是对称的.

③ 传递性:

任取$\langle x,y\rangle, \langle u,v\rangle, \langle w,t\rangle \in Z^+ \times Z^+$, 则:

$\langle\langle x,y\rangle,\langle u,v\rangle\rangle \in R \wedge \langle\langle u,v\rangle,\langle w,t\rangle\rangle \in R$

$\Rightarrow xv = yu \wedge ut = vw \Rightarrow \frac{x}{y} = \frac{u}{v} \wedge \frac{u}{v} = \frac{w}{t}$

$\Rightarrow \frac{x}{y} = \frac{w}{t} \Rightarrow xt = yw \Rightarrow \langle\langle x,y\rangle,\langle w,t\rangle\rangle \in R$

故: R是传递的.

综上所述, R是等价关系。

11. 模 6 同余关系：$R=\{\langle a,b\rangle \mid a,b\in Z,\ a\equiv b \pmod 6\}$

模 6 同余类：$[i]=[6z+i]$，$z\in Z$，$i=0,1,2,3,4,5$

即：$[0]=[\cdots -18,-12,-6,0,6,12,18\cdots]$

$[1]=[\cdots -17,-11,-5,1,7,13,19\cdots]$

$[2]=[\cdots -16,-10,-4,2,8,14,20\cdots]$

$[3]=[\cdots -15,-9,-3,3,9,15,21\cdots]$

$[4]=[\cdots -14,-8,-2,4,10,16,22\cdots]$

$[5]=[\cdots -13,-7,-1,5,11,17,23\cdots]$