

# HW1

Problem 3

int i=2 while loop  
 $\downarrow$   $\downarrow$

Part (a)  $T(n) = \theta(1) + \sum_{i=2}^{n-1} (\theta(1))$

first iteration  $i = 4 = 2^2$

second iteration  $i = 16 = 2^4$

$\vdots$

$k^{\text{th}}$  iteration  $i = 2^{2k}$

$$2^{2k} = n$$

$$k = \log_2 (2^{2k}) = 2 \log_2 (2^k) \approx \log(n)$$

$$\therefore T(n) = \theta(1) + \theta(\log(n)) \approx \theta(\log(n))$$

Part (b)  $T(n) = \sum_{i=1}^{n-1} (\theta(1) + \theta(\sum_{k=0}^{i-1} \theta(1)))$

$\uparrow$  Outer For loop  $\uparrow$  inner For loop Assuming it is always True

if statement:  $i \% \sqrt{n} == 0$

$n=1$   $i=1,$

$n=2$   $i=1,2$

$n=4$   $i=2,4,$

$\vdots$

$n$   $i = \sqrt{n}, 2\sqrt{n}, \dots, n$

# of iterations  $= \frac{n}{\sqrt{n}} = \sqrt{n}$

$$\therefore \theta\left(\sum_{k=0}^{i-1} \theta(1)\right) \Rightarrow \sum_{c=1}^{\sqrt{n}} \sum_{k=0}^{i-1} \theta(1) = \sum_{c=1}^{\sqrt{n}} \theta(\sqrt{n} \cdot i^2) = \theta(\sqrt{n} \cdot i^2)$$

$$\therefore i = c\sqrt{n}$$

$$\theta(\sqrt{n} \cdot i^2) = \theta(n^{\frac{1}{2}} \cdot n^{\frac{1}{2}}) = \theta(n)$$

$$\therefore T(n) = (n-1) \theta(n)$$

$$= \theta(n^2)$$

$$\begin{array}{ccc}
 \text{outer for loop} & & \text{for } i = 1; i \leq n; i++ \\
 \Downarrow & & \Downarrow \\
 & & \text{inner for loop} \\
 \Downarrow & & \\
 \text{Part (c)} \quad T(n) = \sum_{i=1}^n (\Theta(1)) + \sum_{k=1}^n (\Theta(\sum_{i=1}^k \Theta(1)) + \Theta(1))
 \end{array}$$

Assume if  $(A[k] == i)$  is always True

Since  $m$  doubles until  $m > n$  in the inner loop  $\therefore$  for last iteration  $k = n$   
 $k = \log_2(n)$   
 $O(\sum_{k=1}^n \Theta(1) + \Theta(1)) = O(\sum_{k=1}^{\log_2(n)} \Theta(1)) = O(\log(n))$

Since for any Array  $A[i]$ , every index  $k$  can only point to one value  $i$  and every value  $i$  can point to one or more than one indices, making the function described to be injective

$\therefore$  The if statement if  $(A[k] == i)$  is True  $n$  times for a set of  $(k, i)$  s.t.  $A[k] == i$  as there is only  $n$  set of  $(k, i)$  that satisfies  $A[k] == i$

$$\therefore T(n) = \sum_{i=1}^n (\sum_{k=1}^n O(\log n)) = \Theta(n \log n)$$

$$\begin{array}{ccc}
 & \text{inner for loop} \\
 & \Downarrow \\
 \text{Part (d)} \quad T(n) = \sum_{i=0}^{n-1} (\Theta(1) + O(\Theta(1) + \sum_{j=0}^{\text{size}} \Theta(1))) \\
 \uparrow \quad \quad \quad \uparrow \quad \quad \quad \uparrow \\
 \text{outer for loop} \quad \text{Assume if } (i == \text{size}) \text{ is always True}
 \end{array}$$

For 1st Resize	size = 10	newSize = 15
2nd	size = 15	newSize = 22
3rd	size = 22	newSize = 33
4th	size = 33	newSize = 49

$$\vdots \\
 k^{\text{th}} \quad \text{size} = 10 \times 1.5^{k-1} \quad \text{newSize} = 10 \times 1.5^k$$

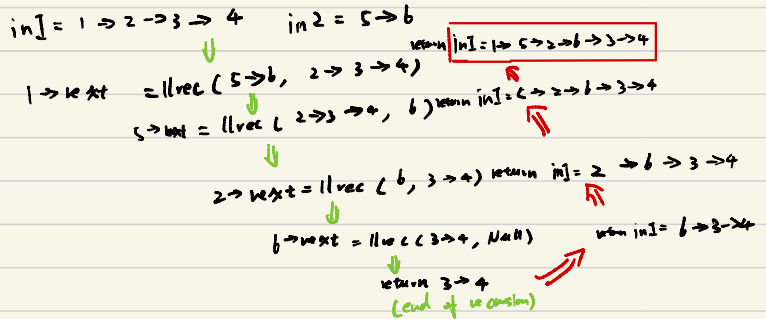
$$k-1 = \log_{1.5} \left( \frac{\text{size}}{10} \right) \approx \log \left( \frac{\text{size}}{10} \right) \Rightarrow \log(n)$$

$\therefore$  For an integer  $n \geq 10$ ,  $\log(n)$  times of resizes are needed

$$\begin{aligned}
 \therefore T(n) &= \sum_{i=0}^{n-1} (\Theta(1) + \log(n)) \\
 &= \Theta(n \log n)
 \end{aligned}$$

# Problem 4

Question A:



Question b:

$in1 = nullptr$      $in2 = 2$

$llrec(nullptr, [2])$

$if (in1 == nullptr) == True \Rightarrow return in2;$

$\therefore in2 = [2];$  is returned by  $llrec(in1, in2)$  when  $in1 = nullptr$   
 $in2 = [2]$