Trayectorias ortogonales monocromáticas ajenas

C. J. Rodrigo Guadalupe[1]

[1]Instituto de Matemáticas
Universidad Nacional Autónoma de México

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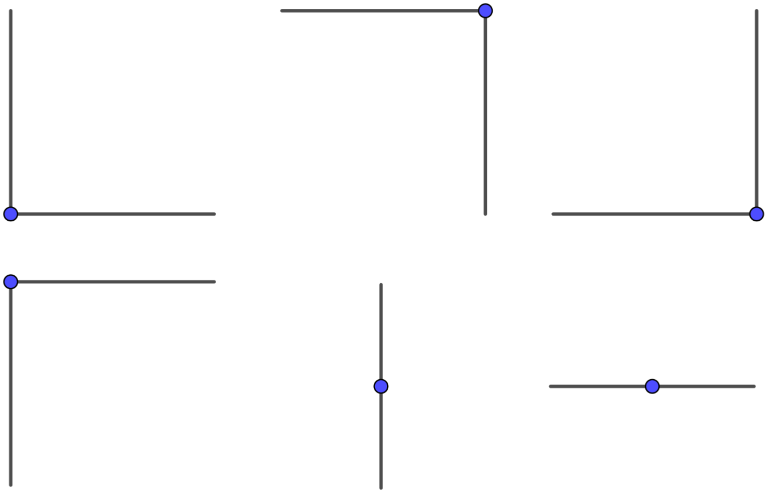
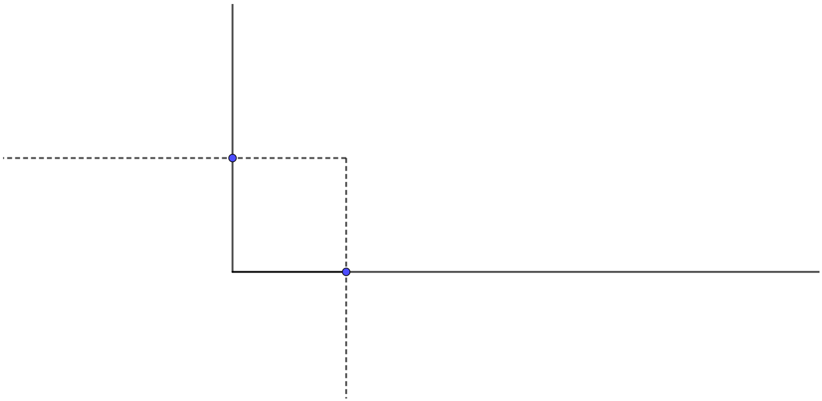
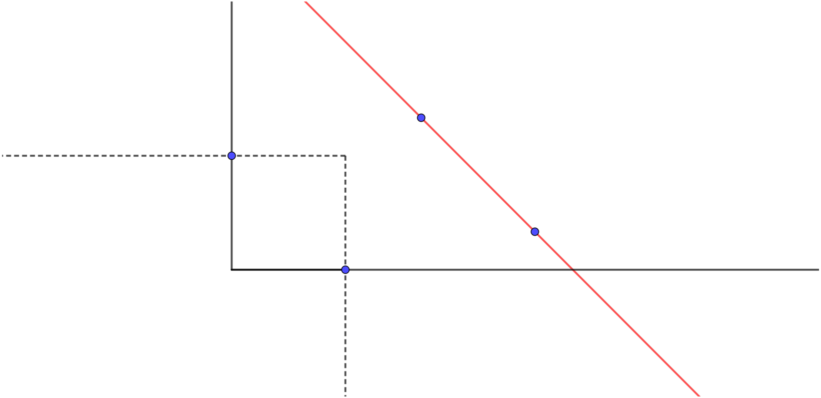
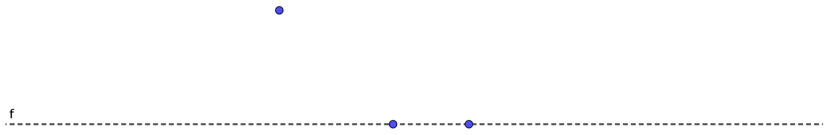
Para un punto x en el plano una línea en forma de L consistente de dos rayos, uno vertical y otro horizontal emanentes de x es llamado L-línea con *esquina en* x





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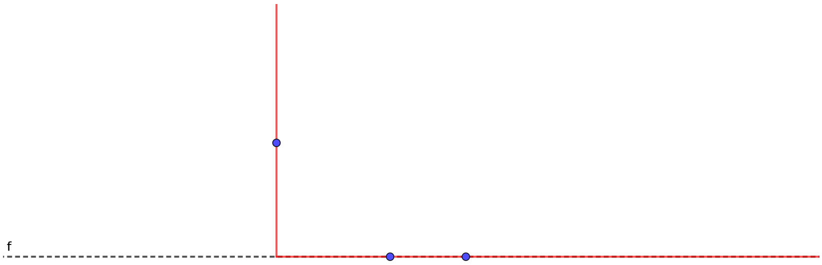
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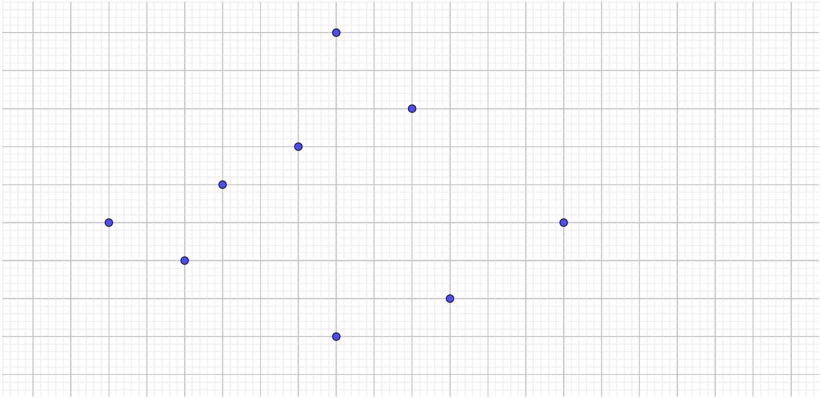
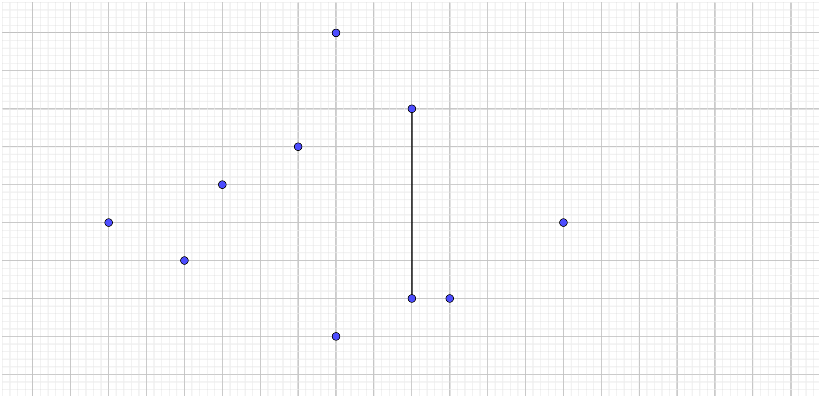
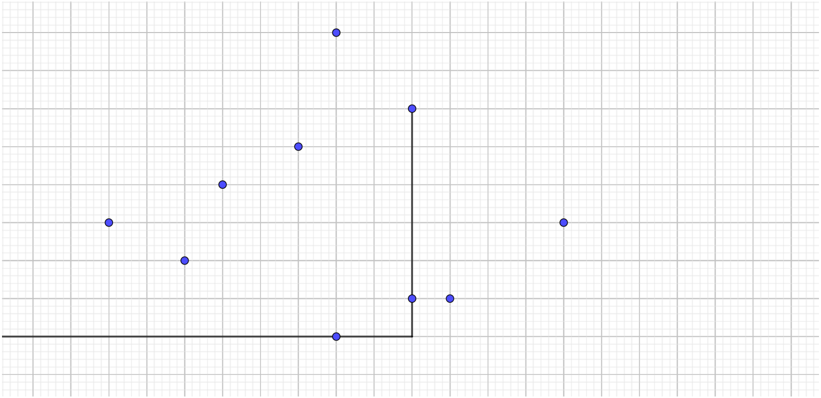






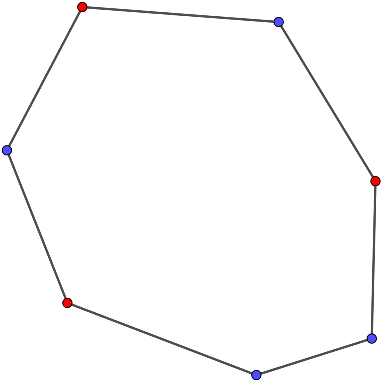
Teorema 1

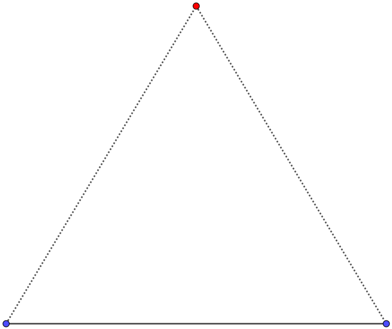
Sean R y B dos conjuntos ajenos de puntos rojos y azules tales que $R \cup B$ estan en posición general. Sea $\tau(R, B)$ el número de aristas xy en el cierre convexo de $(R \cup B)$ tal que uno de $\{x, y\}$ es rojo y el otro es azul. Entonces el número de cruces en $T_R \cup T_B$ está dado por

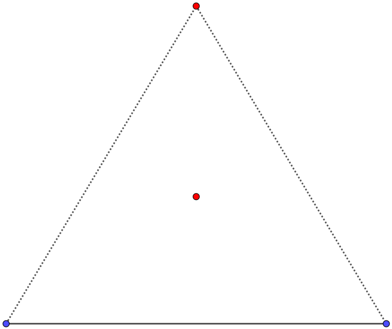
$$max\left\{\frac{\tau(R, B)}{2}, 0\right\}$$

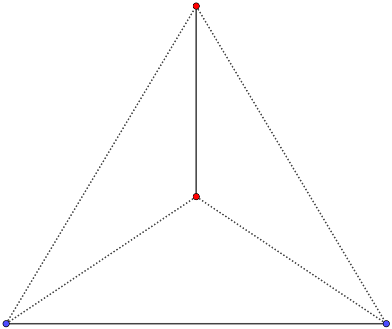
.

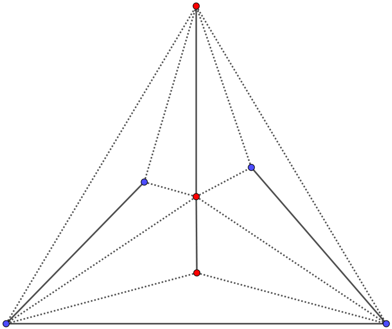
$\tau(R, B) = 6$

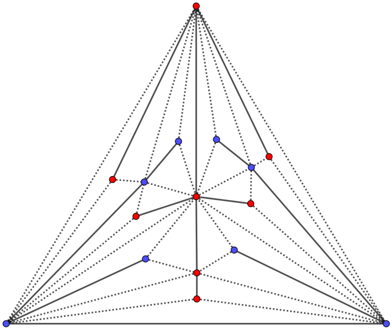


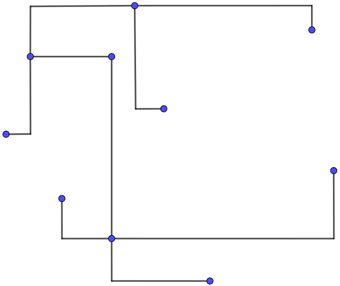


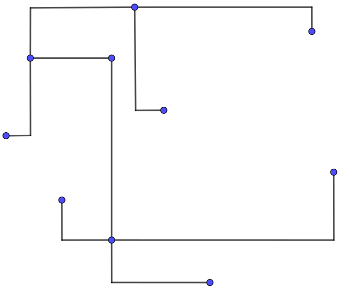


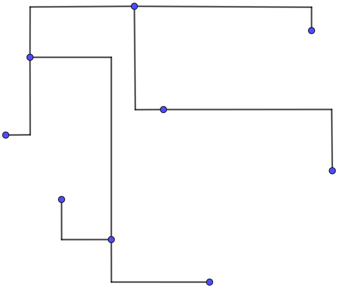




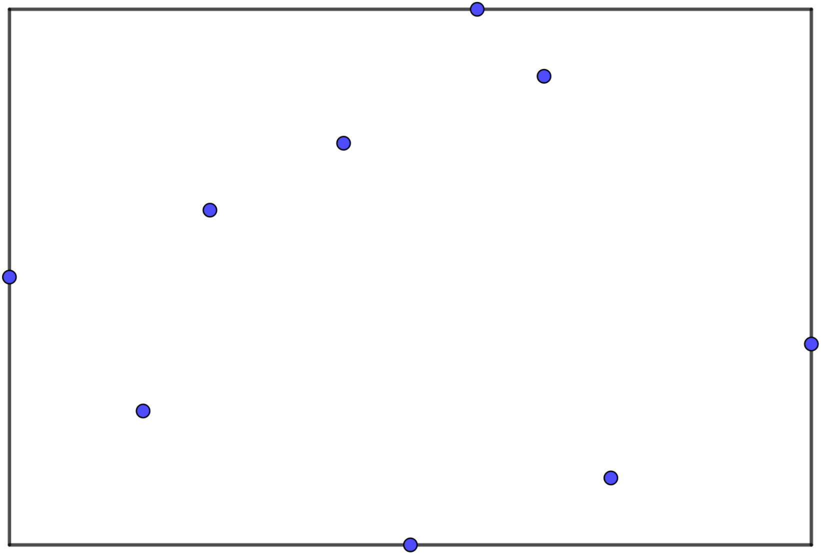


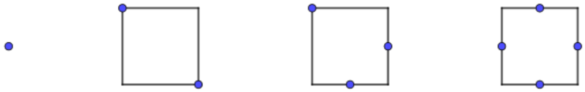






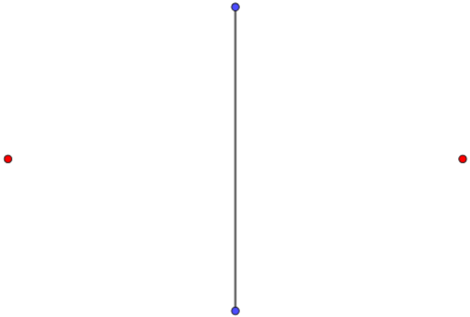
$rect(S)$





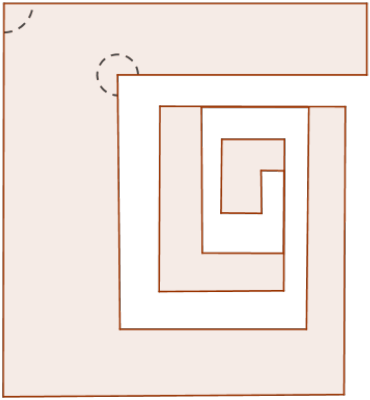
Teorema 2

Sean R y B conjuntos ajenos de puntos en el plano lattice, $R \cup B$ en posición general. Sea $\tau^*(R, B)$ el número de segmentos xy de L-líneas en la frontera de $rect(R \cup B)$ tal que uno de $\{x, y\}$ es rojo y el otro es azul. Entonces $\tau^*(R, B)$ es 0, 2 o 4 y el máximo número de cruces entre T_R y T_B es 1 cuando $\tau^*(R, B) = 4$. Además si $\tau^*(R, B) \leq 2$ podemos dibujar los árboles sin cruces con $\Delta(T_R) \leq 3$ y $\Delta(T_B) \leq 3$. [**Kano2013**]



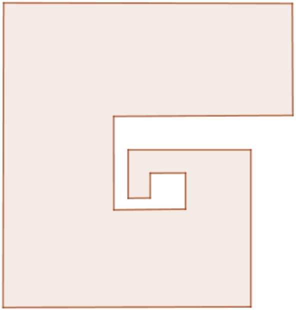
Polígono espiral ortogonal

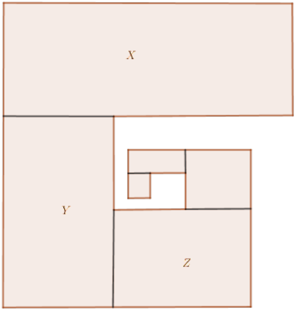
Un polígono espiral ortogonal es un polígono cuya frontera consiste de dos cadenas de aristas llamadas interna y externa. Cada ángulo interno de la cadena exterior es de $\frac{\pi}{2}$ y cada ángulo externo de la cadena interna es de $\frac{3\pi}{2}$.

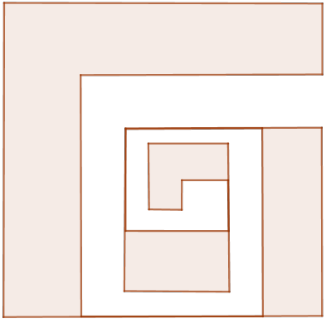


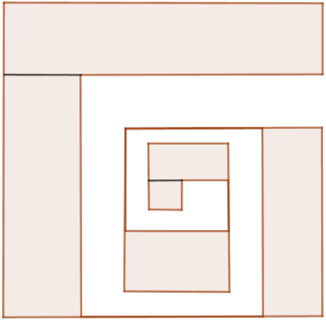
Lema 1

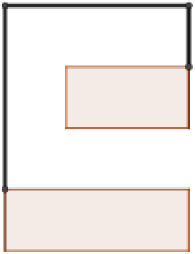
Sea P un polígono espiral ortogonal en el plano lattice y S un conjunto de puntos en posición general contenido en P y asumamos que cada arista de la cadena exterior tiene exactamente un punto y la cadena interior tambien tiene exactamente un punto o esta incluida en alguna cadena exterior. Entonces existe un arbol generador T tal que $\Delta(T) \leq 3$ y T está dentro de P.

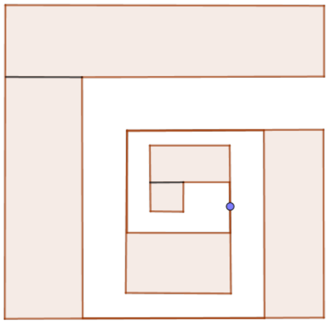


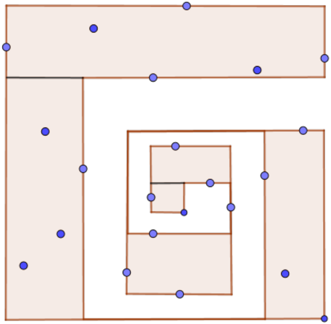

X
Y
Z











Overview de las trayectorias

Las trayectorias que construiremos en los rectángulos no planos:

- Comienzan en el punto superior y terminan en el inferior.

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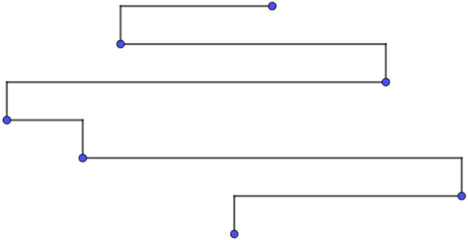
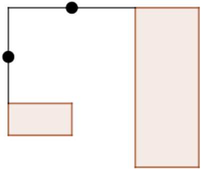
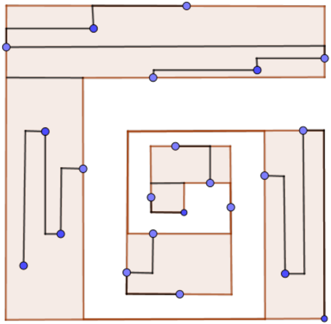
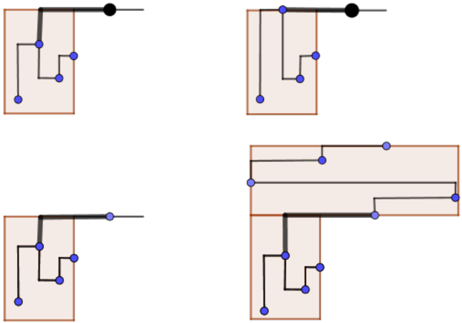
- Comienzan en el punto superior y terminan en el inferior.
- Pasan por todos los puntos del rectángulo.
- Cada segmento de L-línea xy tal que x está arriba de y empieza en x hacia un lado y termina en y por arriba.



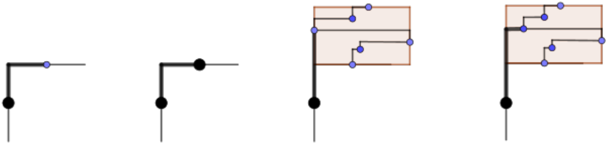


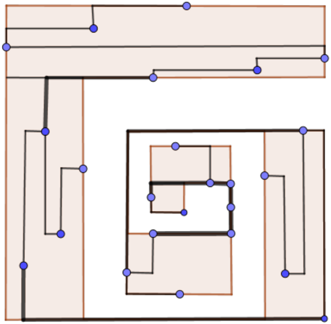


Caso 1. $Tr_{i+1} \neq \emptyset$



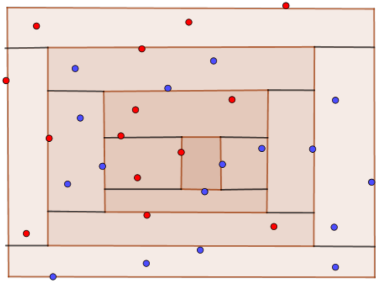
Caso 2. $Tr_{i+1} = \emptyset$

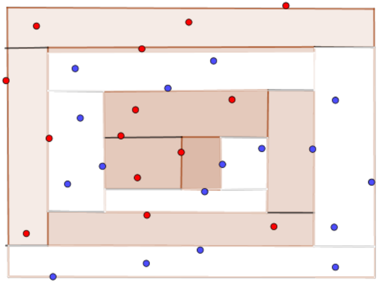


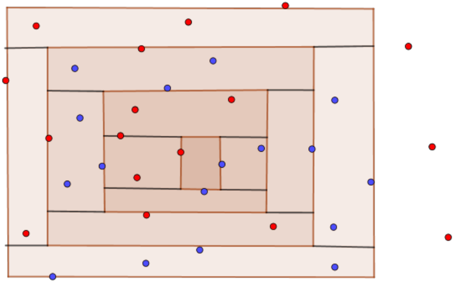


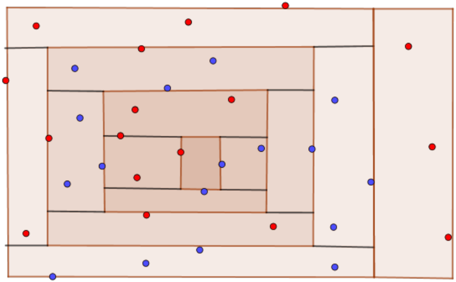
El arbol que construimos cumple que:

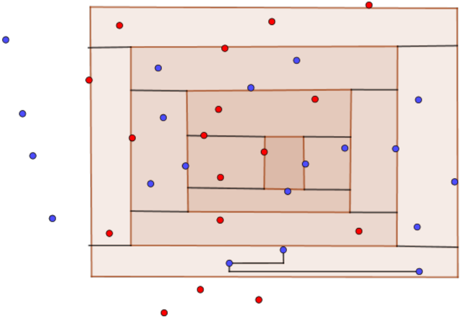
1. $\Delta(T) \leq 3$
2. está contenido en el polígono espiral

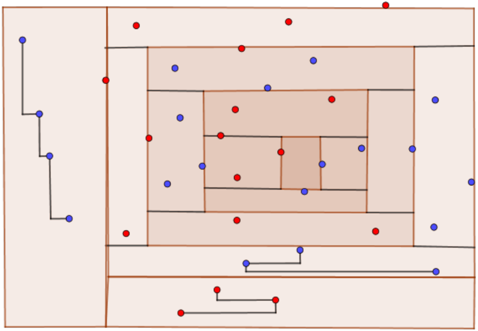


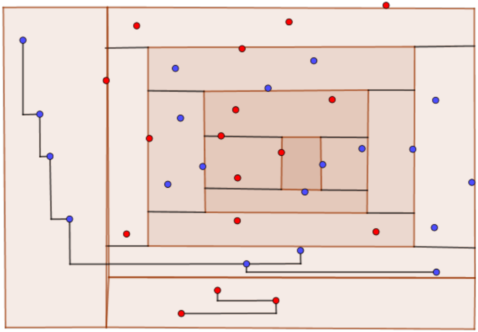


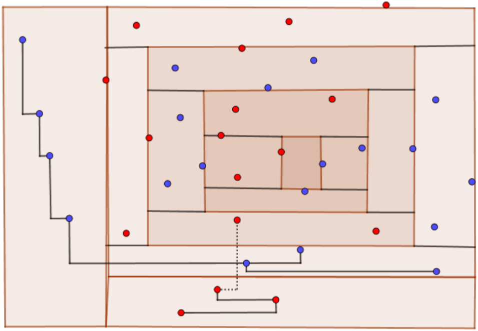


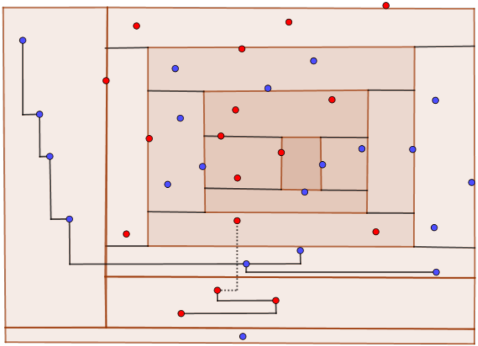


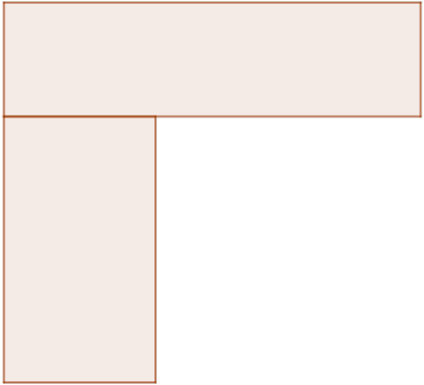


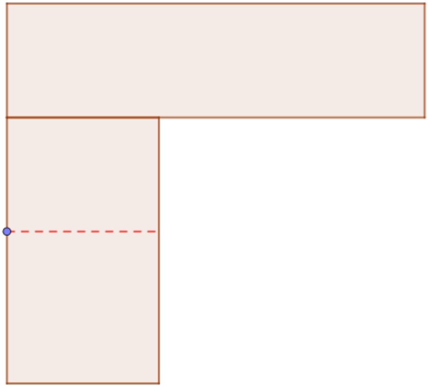


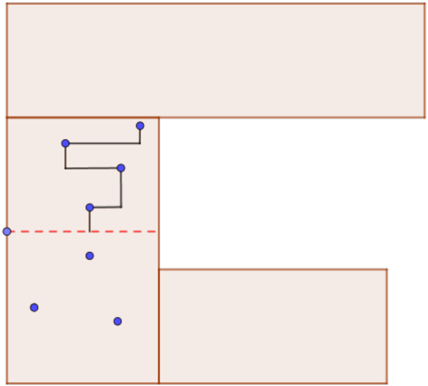


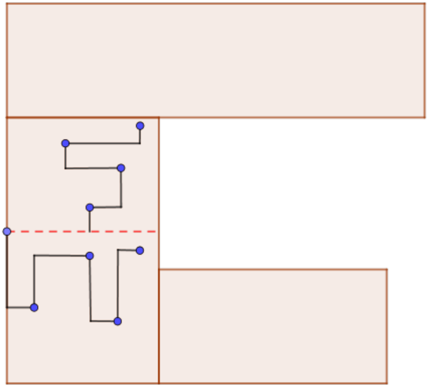


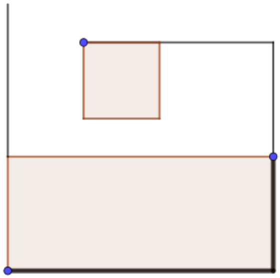


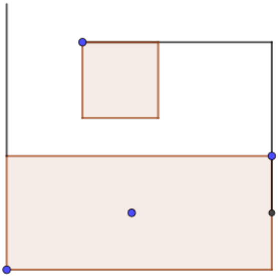


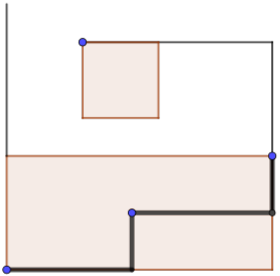


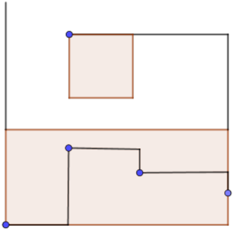


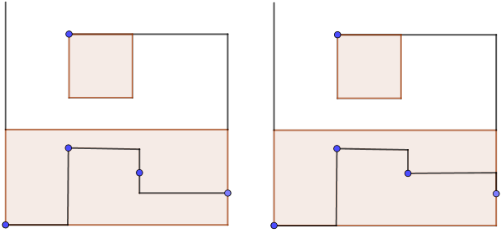


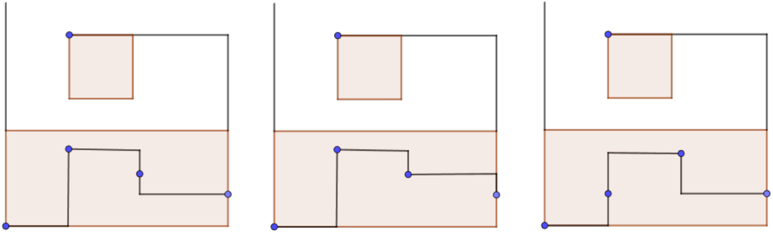


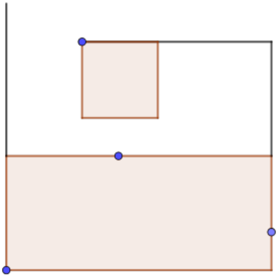


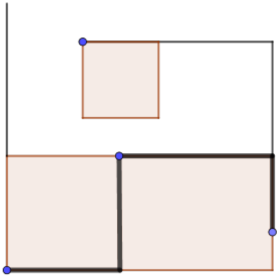


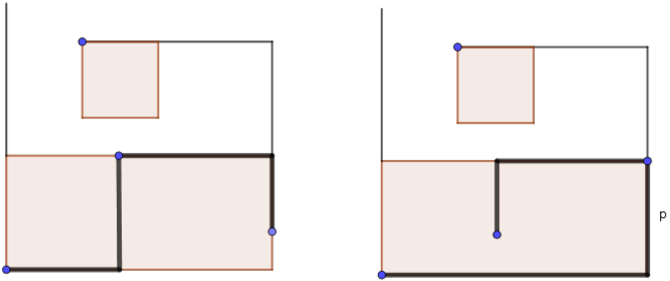








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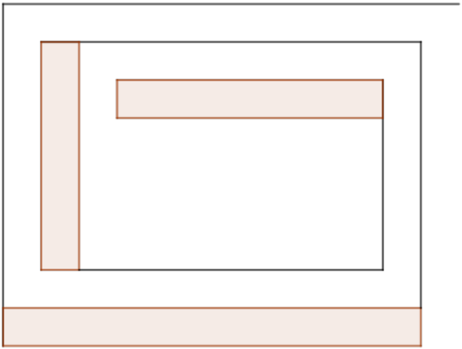

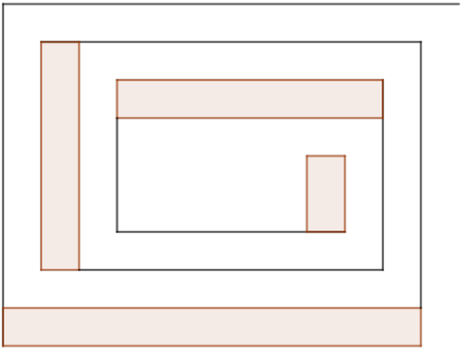

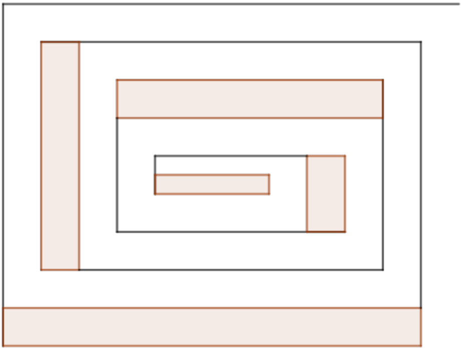

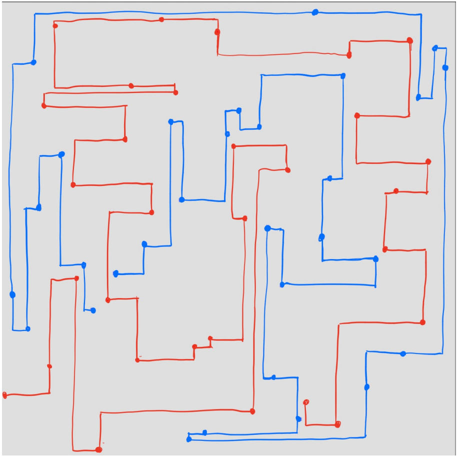

Gracias!