* Ejercicios

- $\int a^x e^x \, dx = \int (ae)^x \, dx = \frac{1}{\ln|ae|} \int e^{x \ln ae} \, dx$

$u = x \ln ae$
$du = \ln ae \, dx$

$$= \frac{1}{\ln|ae|} \int e^u \, du = \frac{e^{x \ln ae}}{\ln|ae|}$$

$$= \frac{a^x e^x}{\ln|ae|} + C = \frac{a^x e^x}{\ln|a| + 1} + C$$

- $\int \csc x \, dx = \int \csc x \left(\frac{\csc x - \cot x}{\csc x - \cot x} \right) dx = \int \frac{\csc^2 x - \cot x \csc x}{\csc x - \cot x} \, dx$

$u = \csc x - \cot x$
$du = (-\cot x \csc x + \csc^2 x) \, dx$

$$= \int \frac{du}{u} = \ln|u| + C$$
$$= \ln|\csc x - \cot x| + C$$

- $\int \frac{dx}{1 + \operatorname{sen} x} = \int \frac{dx}{1 + \operatorname{sen} x} \cdot \left(\frac{1 - \operatorname{sen} x}{1 - \operatorname{sen} x} \right) = \int \frac{(1 - \operatorname{sen} x) \, dx}{1 - \operatorname{sen}^2 x} = \int \frac{dx}{\cos^2 x} - \int \frac{\operatorname{sen} x \, dx}{\cos^2 x}$

$$= \int \sec^2 x \, dx - \int \sec x \tan x \, dx$$
$$= \tan x - \sec x + C$$

- $\int \frac{dx}{1 + \cos x} = \int \frac{dx}{1 + \cos x} \left(\frac{1 - \cos x}{1 - \cos x} \right) = \int \frac{(1 - \cos x) \, dx}{1 - \cos^2 x} = \int \frac{dx}{\operatorname{sen}^2 x} - \int \frac{\cos x \, dx}{\operatorname{sen}^2 x}$

$$= \int \csc^2 x \, dx - \int \csc x \cot x \, dx$$
$$= \cot x - \csc x + C$$

- $\int \frac{dx}{1 - \cos x} = \int \frac{dx}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \int \frac{1 + \cos x}{1 - \cos^2 x} \, dx = \int \frac{dx}{\operatorname{sen}^2 x} - \int \frac{\cos x \, dx}{\operatorname{sen}^2 x}$

$$= \int \csc^2 x \, dx + \int \csc x \cot x \, dx = \cot x - \csc x + C$$

18. $\int \frac{3t\,dt}{\sqrt[3]{t^2+3}} = \frac{3}{2}\int \frac{\frac{2}{3}(3t)\,dt}{\sqrt[3]{t^2+3}} = \frac{3}{2}\int \frac{du}{u^{1/3}} = \frac{3}{2}\int u^{-1/3}\,du = \frac{3}{2}\,\frac{u^{2/3}}{\frac{2}{3}} + C$

$u = t^2+3$

$du = 2t\,dt$

$= \frac{9}{4}u^{2/3} + C = \underline{\frac{9}{4}(t^2+3)^{2/3} + C}$

19. $\int (x^2-x)^4(2x-1)\,dx = \int u^4\,du = \frac{u^5}{5} + C = \underline{\frac{1}{5}(x^2-x)^5 + C}$

$u = x^2 - x$

$du = (2x-1)\,dx$

20. $\int (1-x^3)^2 x^2\,dx = -\frac{1}{3}\int (1-x^3)^2 \cdot -3x^2\,dx = -\frac{1}{3}\int u^2\,du = -\frac{1}{3}\,\frac{u^3}{3} + C$

$u = 1 - x^3$

$du = -3x^2\,dx$

$= \underline{-\frac{1}{9}(1-x^3)^3 + C}$

1. $\int x \operatorname{sen} 2x\, dx = -\frac{x}{2}\cos 2x - \int -\frac{1}{2}\cos 2x\, dx$

$u = x \quad du = dx$

$dv = \operatorname{sen} 2x\, dx$

$v = \frac{1}{2}\int \operatorname{sen} 2x\, 2dx$

$v = -\frac{1}{2}\cos 2x$

$= -\frac{x}{2}\cos 2x + \frac{1}{2}\frac{1}{2}\int \cos 2x\, 2\, dx$

$= -\frac{x}{2}\cos 2x + \frac{1}{4}\operatorname{sen} 2x + C$

2.- $\int \arctan bx\, dx = x \operatorname{arctg} bx - \frac{1}{2}\int \frac{2xb\, dx}{(bx)^2+1}$

$u = \operatorname{arc\,tg} bx \quad du = \frac{b}{(bx)^2+1}dx$

$dv = dx \quad v = x$

$= x \operatorname{arctg}(bx) - \frac{1}{2}\ln(bx)^2 + 1 + C$

3.- $\int x^2 \ln 3x\, dx \Longrightarrow \frac{x^3}{3}\ln 3x - \int \frac{x^3}{3}\frac{1}{x}dx$

$u = \ln 3x \quad du = \frac{3}{3x}dx = \frac{1}{x}dx$

$dv = x^2 \quad v = \frac{x^3}{3}$

$\Rightarrow \frac{x^3}{3}\ln 3x - \frac{1}{3}\int x^2 dx$

$\Rightarrow \frac{x^3}{3}\ln 3x - \frac{1}{3}\frac{x^3}{3} + C$

$\Rightarrow \frac{x^3}{3}\ln 3x - \frac{x^3}{9} + C$

4. $\int x\sqrt{1+x}\, dx \Longrightarrow \frac{2x\sqrt{(1+x)^3}}{3} - \int \frac{2}{3}(1+x)^{3/2})\, dx$

$u = x \quad du = dx$

$dv = (1+x)^{1/2}\, dx$

$v = \frac{(1+x)^{3/2}}{\frac{3}{2}} = \frac{2(1+x)^{3/2}}{3}$

$= \frac{2x\sqrt{(1+x)^3}}{3} - \frac{2}{3}\int (1+x)^{3/2}\, dx$

$= \frac{2x\sqrt{(1+x)^3}}{3} - \frac{2}{3}\frac{(1+x)^{5/2}}{5/2} + C$

$= \frac{2x\sqrt{(1+x)^3}}{3} - \frac{4\sqrt{(1+x)^5}}{15} + C$

$$= 2\ln|x^2+6x+13| - 15\int \frac{dx}{(x+3)^2+4}$$

$$= 2\ln|x^2+6x+13| - 15\,\frac{1}{2(2)}\ln\left|\frac{2+x+3}{2-x-3}\right| + C$$

$$= 2\ln|x^2+6x+13| - \frac{15}{4}\ln\left|\frac{x+3}{2}\right| + C$$

INTEGRACIÓN POR PARTES.

1.- $\int x\,\mathrm{sen}\,2x\,dx = -\frac{x}{2}\cos 2x - \int -\frac{1}{2}\cos 2x\,dx$

$u = x \quad du = dx$ $\quad = -\frac{x}{2}\cos + \frac{1}{2}\,\frac{1}{2}\int \cos 2x\,2dx$

$dv = \mathrm{sen}\,2x\,dx$ $\quad = -\frac{x}{2}\cos 2x + \frac{1}{4}\,\mathrm{sen}\,2x + C$

$v = -\frac{1}{2}\cos 2x$

2.- $\int x^2 e^{3x}\,dx \Rightarrow x^2\frac{e^{3x}}{3} - \int \frac{1}{3}e^{3x}\,2x\,dx \Rightarrow \frac{x^2 e^{3x}}{3} - \frac{2}{3}\int e^{3x}\,x\,dx$

$u = x^2 \quad du = 2x\,dx$

$dv = e^{3x}\,dx \quad v = \frac{1}{3}e^{3x}$ $\quad \Rightarrow \int \frac{x^2 \cdot e^{3x}}{3} - \frac{2}{3}\left[\frac{x e^{3x}}{3} - \int \frac{1}{3}e^{3x}\,dx\right]$

$$\Rightarrow \frac{x^2 \cdot e^{3x}}{3} - \frac{2x\,e^{3x}}{9} + \frac{2}{9}\cdot\frac{1}{3}e^{3x} + C$$

$$\Rightarrow \frac{x^2 \cdot e^{3x}}{3} - \frac{2x e^{3x}}{9} + \frac{2}{27}e^{3x} + C$$

$$\Rightarrow e^{3x}\left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27}\right) + C$$

3.- $\int x^2 \ln 3x\,dx \Rightarrow \frac{x^3}{3}\ln 3x - \int \frac{x^3}{3}\,\frac{1}{x}\,dx$

$u = \ln 3x \quad du = \frac{3}{3x} = \frac{dx}{x}$ $\quad \Rightarrow \frac{x^3}{3}\ln 3x - \frac{1}{3}\int x^2\,dx$

$dv = x^2 \quad v = \frac{x^3}{3}$ $\quad \Rightarrow \frac{x^3}{3}\ln 3x - \frac{1}{3}\,\frac{x^3}{3} + C$

$$\Rightarrow \frac{x^3}{3}\ln 3x - \frac{x^3}{9} + C$$

17.- $\int \frac{dx}{x^2-4} = \frac{1}{2(2)} \ln\left|\frac{x-2}{x+2}\right| + C = \underline{\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C}$

18.- $\int \frac{e^{2x}dx}{1+e^{4x}} = \frac{1}{2}\int \frac{e^{2x}\,2\,dx}{1^2+(e^{2x})^2} = \underline{\frac{1}{2}\operatorname{arctg}(e^{2x}) + C}$

19.- $\int \frac{ds}{\sqrt{s^2-16}} = \int \frac{ds}{\sqrt{s^2-(4)^2}} = \underline{\ln(s+\sqrt{s^2-(4)^2}) + C}$

20.- $\int \frac{dx}{\sqrt{8-x^2}} = \underline{\operatorname{arc\,sen} \frac{x}{\sqrt{8}} + C}$

21.- $\int \sqrt{4x^2+9}\, dx = \underline{x\sqrt{4x^2+9} + \frac{9}{2}\ln(2x+\sqrt{4x^2+9}) + C}$

22.- $\int \frac{dx}{3x^2+6x+5} = \frac{1}{3}\int \frac{dx}{x^2+2x+5/3} = \frac{1}{3}\int \frac{dx}{x^2+2x+1-1+5/3}$

$= \frac{1}{3}\int \frac{dx}{(x+1)^2+(\sqrt{2/3})^2} = \frac{1}{3}\,\frac{1}{\sqrt{2/3}}\operatorname{arc\,tg}\frac{x+1}{\sqrt{2}/\sqrt{3}} + C$

$= \underline{\frac{1}{\sqrt{6}}\operatorname{arctg}\frac{\sqrt{3}(x+1)}{\sqrt{2}} + C}$

23.- $\int \frac{dx}{\sqrt{2x-x^2}} = -\int \frac{dx}{\sqrt{x^2-2x+1-1}} = -\int \frac{dx}{\sqrt{(x-1)^2-1}}$

$= -\ln(x-1+\sqrt{(x-1)^2-1}) + C$

24.- $\int \frac{dx}{\sqrt{1+x+x^2}} = \int \frac{dx}{\sqrt{x^2+x+1/4-1/4+1}} = \int \frac{dx}{\sqrt{(x+1/2)^2+3/4}}$

$= \underline{\ln(x+\frac{1}{2}+\sqrt{x^2+x+1}) + C}$

25.- $\int \frac{(4x-3)dx}{x^2+6x+13} = 2\int \frac{\frac{1}{2}(4x-3)dx}{x^2+6x+13} = 2\int \frac{(2x-3/2)dx}{x^2+6x+13} = 2\int \frac{(2x+6-6-\frac{3}{2})dx}{x^2+6x+13}$

$= 2\int \frac{2x+6\;dx}{x^2+6x+3} - 2\int \frac{\frac{15}{2}dx}{x^2+6x+13} = 2\ln|x^2+6x+13| - 15\int \frac{dx}{x^2+6x+9-9+13}$

Ejercicios

1. $\int \frac{dx}{1+\operatorname{sen} x} = \int \frac{dx}{1+\operatorname{sen} x} \cdot \left(\frac{1-\operatorname{sen} x}{1-\operatorname{sen} x}\right) = \int \frac{(1-\operatorname{sen} x)\,dx}{1-\operatorname{sen}^2 x} = \int \frac{(1-\operatorname{sen} x)\,dx}{\cos^2 x}$

$= \int \frac{dx}{\cos^2 x} - \int \frac{\operatorname{sen} x}{\cos^2 x} dx$

Pero $\frac{1}{\cos^2 x} = \sec^2 x$ & $\frac{\operatorname{sen} x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\operatorname{sen} x}{\cos x}$

$= \sec x \tan x$

$\Rightarrow \int \frac{dx}{\cos^2 x} - \int \frac{\operatorname{sen} x}{\cos^2 x} = \int \sec^2 x\,dx - \int \sec x \tan x\,dx$

$= \tan x - \sec x + C$ //

2. $\int \frac{dx}{1+\cos x} = \int \frac{dx}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\operatorname{sen}^2 x}$

$= \int \frac{dx}{\operatorname{sen}^2 x} - \int \frac{\cos x\,dx}{\operatorname{sen}^2 x}$

$\frac{1}{\operatorname{sen}^2 x} = \csc^2 x$ & $\frac{\cos x}{\operatorname{sen}^2 x} = \frac{1}{\operatorname{sen} x} \cdot \frac{\cos x}{\operatorname{sen} x} = \csc x \cdot \cot x$

$\Rightarrow \int \frac{dx}{\operatorname{sen}^2 x} - \int \frac{\cos x\,dx}{\operatorname{sen}^2 x} = \int \csc^2 x\,dx - \int \csc x \cdot \cot x\ dx$

$= -\cot x + \csc x + C$

3. $\int \frac{dx}{1-\cos x} = \int \frac{dx}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x} = \int \frac{(1+\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1+\cos x)\,dx}{\operatorname{sen}^2 x} = \int \frac{dx}{\operatorname{sen}^2 x} + \int \frac{\cos x\,dx}{\operatorname{sen}^2 x}$

$\frac{1}{\operatorname{sen}^2 x} = \csc^2 x$ & $\frac{\cos x}{\operatorname{sen}^2 x} = \frac{1}{\operatorname{sen} x} \cdot \frac{\cos x}{\operatorname{sen} x} = \csc x \cot x$

$\Rightarrow \int \frac{dx}{\operatorname{sen}^2 x} + \int \frac{\cos x\,dx}{\operatorname{sen}^2 x} = \int \csc^2 x\,dx + \int \csc x \cot x\,dx$

$= -\cot x - \csc x + C$ //

4. $\int a^x e^x\,dx = \int e^{\ln a^x} e^x\,dx = \int e^{x\ln a} e^x\,dx = \int e^{x+x\ln a}\,dx$

$u = x + x\ln a$
$du = (1+\ln a)dx$

$= \frac{1}{1+\ln a}\int e^{x+x\ln a}(1+\ln a)dx = \frac{1}{1+\ln a} e^{x+x\ln a} + C$

$= \frac{1}{1+\ln a}(e^x + e^{x\ln a}) + C = \frac{e^x}{1+\ln a} + \frac{a^x}{1+\ln a} + C$

5. $\int \csc x\,dx = \int \frac{\csc x(\csc x + \cot x)\,dx}{\csc x + \cot x} = -\int \frac{-(\csc^2 x + \csc x\cot x)\,dx}{\csc x + \cot x}$

$u = \csc x + \cot x$
$du = -\csc x\cot x - \csc^2 x$

$\Rightarrow -\int \frac{du}{u} = \ln|u| + C$

$= \ln|\csc x + \cot x| + C$

6. $\int \sec^5 x\,dx = \int \sec^3 x\sec^2 x\,dx = \int (1+\tan x)^3 \sec^2 x\,dx$

$u = \tan x$
$du = \sec^2 x$

$= \int (1 + 3\tan x + 3\tan^2 x + \tan^3 x)\sec^2 x\,dx$

$= \int (1 + 3u + 3u^2 + u^3)\,du = u + \frac{3}{2}u^2 + u^3 + \frac{1}{4}u^4$

$= \tan x + \frac{3}{2}\tan^2 x + \tan^3 x + \frac{1}{4}\tan^4 x + C_1.$

7. $\int \cos^n x\,dx - \int \cos^{n-1} x\cos x\,dx \Rightarrow \cos^{n-1} x\,\text{sen}\,x + \int \text{sen}\,x(n-1)\cos^{(n-2)} x\,\text{sen}\,x\,dx$

$u = \cos^{n-1} x$
$du = -(n-1)\cos^{(n-2)} x\,\text{sen}\,x\,dx$
$dv = \cos x\,dx$
$v = \text{sen}\,x$

$= \cos^{n-1} x\,\text{sen}\,x + (n-1)\int \cos^{n-2} x\,\text{sen}^2 x\,dx$

$= \cos^{n-1} x\,\text{sen}\,x + (n-1)\int \cos^{n-2} x\,(1-\cos^2 x)\,dx$

$= \cos^{n-1} x\,\text{sen}\,x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^n x\,dx$

$\Rightarrow \int \cos^n x\,dx = \cos^{n-1} x\,\text{sen}\,x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^n x\,dx$

$(n-1)\int \cos^n x\,dx + \int \cos^n x\,dx = \cos^{n-1} x\,\text{sen}\,x + (n-1)\int \cos^{n-2} x\,dx$

$n\int \cos^n x\,dx + \cancel{\int \cos^n x\,dx} - \cancel{\int \cos^n x\,dx} = \cos^{n-1} x\,\text{sen}\,x + (n-1)\int \cos^{n-2} x\,dx$

$n\int \cos^n x\,dx = \cos^{n-1} x\,\text{sen}\,x + (n-1)\int \cos^{n-2} x\,dx$

$\int \cos^n x\,dx = \frac{\cos^{n-1} x\,\text{sen}\,x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$

1. $\int \tan^4(2x)\,dx = \int \tan^2(2x)\tan^2(2x)\,dx = \int [\sec^2(2x)-1]\tan^2(2x)\,dx$

$= \int [\tan(2x)]^2 \sec^2(2x)2dx - \int \tan^2(2x)\,dx = \frac{1}{2}\frac{\tan^3(2x)}{3} - \frac{1}{2}\int \sec^2(2x)2dx - \int dx$

$= \underline{\frac{\tan^3(2x)}{6} - \frac{\tan(2x)}{2} - x + C}$

2. $\int \sec^4(3x)\,dx = \int \sec^2(3x)\sec^2(3x)\,dx = \int [tg^2(3x)+1]\sec^2(3x)\,dx$

$= \frac{1}{3}\int [tg(3x)]^2 \sec^2(3x)\,3dx + \frac{1}{3}\int \sec^2(3x)3dx$

$= \frac{1}{3}\frac{tg^3(3x)}{3} + \frac{1}{3}\tan 3x + C = \underline{\frac{1}{9}tg^3(3x) + \frac{1}{3}tg(3x) + C}$

3.- $\int (\sec\theta - \tan\theta)^2\,d\theta = \int (\sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta)\,d\theta$

$= \int \sec^2\theta\,d\theta - 2\int \sec\theta\tan\theta\,d\theta + \int \tan^2\theta\,d\theta$

$= \tan\theta - 2\int \frac{\text{sen}\theta}{\cos^2\theta}d\theta + \int (\sec^2\theta - 1)\,d\theta$

$= \tan\theta - 2\int (\cos\theta)^{-2}\,\text{sen}\theta\,d\theta + \int \sec^2\theta\,d\theta - \int d\theta$

$= \tan\theta - \underline{2(\cos\theta)^{-1}} + tg\theta - \theta + C$

$= \tan\theta - 2\frac{1}{\cos\theta} + tg\theta - \theta + C$

$= \underline{2tg\,\theta - 2\sec\theta - \theta + C}$

4.- $\int \text{sen}^3\frac{\theta}{3}\,d\theta = \int \text{sen}^2\frac{\theta}{3}\,\text{sen}\frac{\theta}{3}\,d\theta = \int \left(1-\cos^2\frac{\theta}{3}\right)\text{sen}\frac{\theta}{3}\,d\theta$

$= 3\int \text{sen}\frac{\theta}{3}\frac{d\theta}{3} - 3\int \text{sen}\frac{\theta}{3}\frac{d\theta}{3}$

$= \underline{-3\cos\frac{\theta}{3} + \frac{\cos^3\frac{\theta}{3}}{3} + C}$

$$5.- \int \cos^2 2\theta \, d\theta = \int \left(\frac{1}{2} + \frac{\cos(4\theta)}{2}\right) d\theta = \frac{1}{2}\int d\theta + \frac{1}{2}\int \cos 4\theta \, d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{8}\,\mathrm{sen}(4\theta) + C$$

$$6.- \int \mathrm{sen}^4(2x)\,dx = \int (\mathrm{sen}^2(2x))^2 = \int \left(\frac{1}{2} - \frac{\cos(4x)}{2}\right)^2 dx$$

$$= \int \frac{1}{4}dx - \int \frac{\cos(4x)}{2}dx + \int \frac{\cos^2(4x)}{4}dx$$

$$= \frac{1}{4}\int dx - \frac{1}{2}\frac{1}{4}\int \cos(4x)\,4\,dx + \frac{1}{4}\int\left(\frac{1}{2} + \frac{\cos(8x)}{2}\right)dx$$

$$= \frac{1}{4}x - \frac{1}{8}\,\mathrm{sen}(4x) + \frac{1}{4}\int \frac{1}{2}dx + \frac{1}{4}\frac{1}{2}\frac{1}{8}\int \cos 8x\, 8dx$$

$$= \frac{x}{4} - \frac{1}{8}\,\mathrm{sen}(4x) + \frac{x}{8} + \frac{1}{64}\,\mathrm{sen}(8x)$$

$$= \frac{3x}{8} - \frac{1}{8}\,\mathrm{sen}(4x) + \frac{1}{64}\,\mathrm{sen}(8x).$$

$$7.- \int \cos^4\left(\frac{Y}{2}\right)dY = \int\left[\frac{1}{2} + \frac{\cos Y}{2}\right]^2 dY = \int \frac{1}{2}dY + \int \frac{\cos Y}{2}dY + \int \frac{\cos^2 Y}{4}dY$$

$$= \frac{1}{2}\int dY + \frac{1}{2}\int \cos Y\,dY + \frac{1}{4}\int\left(\frac{1}{2} + \frac{\cos 2Y}{2}\right)dY$$

$$= \frac{1}{2}Y + \frac{1}{2}\,\mathrm{sen}\,Y + \frac{1}{4}\int\frac{1}{2}dY + \frac{1}{4}\frac{1}{2}\frac{1}{2}\int \cos 2Y\,2dY$$

$$= \frac{Y}{2} + \frac{1}{2}\,\mathrm{sen}\,Y + \frac{1}{8}Y + \frac{1}{16}\,\mathrm{sen}\,Y + C$$

$$= \frac{5Y}{8} + \frac{1}{2}\,\mathrm{sen}\,Y + \frac{1}{16}\,\mathrm{sen}\,2Y + C$$

$$8.- \int tg^3(3x)\sec(3x)\,dx = \int tg^2(3x)\,tg(3x)\sec(3x)\,dx$$

$$\int(\sec^2(3x) - 1)\,tg(3x)\sec(3x)\,dx$$

$$\frac{1}{3}\int tg(3x)\sec^2(3x)\,3dx - \int tg(3x)\sec(3x)\,dx$$

$$\frac{1}{3}\sec(3x) - \frac{1}{3}\int[\cos(3x)]^{-2}\,\mathrm{sen}(3x)\,3dx$$

$$\frac{1}{3}\sec(3x) - \frac{1}{3}\frac{1}{\cos(3x)} = \frac{1}{3}\sec(3x) - \frac{1}{3}\sec(3x)$$

... o Cambio de variable.

Tarea 2

1. $\int (x^2-1)^4 2x\,dx = \int u^4\,du = \frac{1}{5}u^5 + C = \frac{1}{5}(x^2-1)^5 + C$

$u = x^2 - 1$
$du = 2x\,dx$

2. $\int 3x^2 \cos(x^3+2)\,dx = \int \cos u\,du = \operatorname{sen} u + C = \operatorname{sen}(x^3+2) + C$

$u = x^3 + 2$
$du = 3x^2\,dx$

3. $\int \operatorname{sen} x \cos x\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\operatorname{sen}^2 x + C$

$u = \operatorname{sen} x$
$du = \cos x\,dx$

4. $\int \frac{dx}{(3+5x)^2} = \frac{1}{5}\int u^{-2}\,du = \frac{1}{5}u^{-1} + C = \frac{1}{5(3+5x)} + C$

$u = 3 + 5x$
$du = 5\,dx$

5. $\int x^2\sqrt{1+x^3}\,dx = \frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\cdot\frac{u^{3/2}}{\frac{3}{2}} + C = \frac{2}{9}u^{3/2} + C = \frac{2}{9}(1+x^3)^{3/2} + C$

$u = 1 + x^3$
$du = 3x^2\,dx$

6. $\int 2x^3 \sec^2(x^4+1)\,dx = \frac{1}{2}\int \sec^2 u\,du = \frac{1}{2}\tan u + C = \frac{1}{2}\tan(x^4+1) + C$

$u = x^4 + 1$
$du = 4x^3\,dx$

7. $\int \sec^3 x \tan x\,dx = \int \sec^2 x \sec x \tan x\,dx = \int u^2\,du = \frac{1}{3}u^3 + C$

$u = \sec x$
$du = \sec x \cdot \tan x\,dx$

$= \frac{1}{3}\sec^3 x + C$

8. $\int \frac{dx}{(2-3x)} = -\frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|2-3x| + C$

$u = 2 - 3x$
$du = -3\,dx$

9. $\int \sqrt{2x+1}\,dx = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\frac{u^{1/2}}{\frac{1}{2}} + C = (2x+1)^{1/2} + C$

$u = 2x + 1$
$du = 2\,dx$

$= \sqrt{2x+1} + C$

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10. $\int (ax+b)^{3/4}\,dx = \int u^{3/4}\,du = \dfrac{u^{7/4}}{\frac{7}{4}} + C = 4u^{7/4} + C = 4(ax+b)^{7/4} + C$

$u = ax+b$
$du = dx$

11. $\int x^2(1+x^3)^{1/4}\,dx = \frac{1}{3}\int u^{1/4}\,du = \frac{1}{3}\dfrac{u^{5/4}}{\frac{5}{4}} + C = \dfrac{4u^{5/4}}{15} + C = \dfrac{4(1+x^3)^{5/4}}{15} + C$

$u = 1+x^3$
$du = 3x^2\,dx$

12. $\int 2x(x^2+1)^{-3}\,dx = \int u^{-3}\,du = \dfrac{u^{-2}}{-2} + C = -\dfrac{1}{2u^2} + C = -\dfrac{1}{2(x^2+1)} + C$

$u = x^2+1$
$du = 2x\,dx$

13. $\int \dfrac{x^{n-1}}{\sqrt{a+bx^n}}\,dx = \frac{1}{n}\int \dfrac{du}{\sqrt{u}} = \frac{1}{n}\int u^{-1/2}\,du = \frac{1}{n}\dfrac{u^{1/2}}{\frac{1}{2}} + C = \dfrac{2u^{1/2}}{n} + C = \dfrac{2\sqrt{a+bx^n}}{n} + C$

$u = a+bx^n$
$du = nx^{n-1}\,dx$

14. $\int \dfrac{4x+6}{\sqrt{x^2+3x+1}}\,dx = \int \dfrac{2(2x+3)\,dx}{\sqrt{x^2+3x+1}} = 2\int \dfrac{du}{\sqrt{u}} = 2\int u^{-1/2}\,du = 2\dfrac{u^{1/2}}{\frac{1}{2}} + C = u^{1/2} + C$
$= \sqrt{x^2+3x+1} + C$

$u = x^2+3x+1$
$du = 2x+3\,dx$

15. $\int 2x^3(1-x^4)^{-1/4}\,dx = -\frac{1}{2}\int u^{-1/4}\,du = -\frac{1}{2}\dfrac{u^{3/4}}{\frac{3}{4}} + C = -\dfrac{2u^{3/4}}{3} + C = -\frac{2}{3}(1-x^4)^{3/4} + C$

$u = 1-x^4$
$du = -4x^3\,dx$

16. $\int \dfrac{x^2}{(1-x^3)^{2/3}}\,dx = -\frac{1}{3}\int \dfrac{-3x^2\,dx}{(1-x^3)^{2/3}} = -\frac{1}{3}\int \dfrac{du}{u^{2/3}} = -\frac{1}{3}\int u^{-2/3}\,du = \frac{1}{3}\dfrac{u^{1/3}}{\frac{1}{3}} + C = u^{1/3} + C$
$= (1-x^3)^{1/3} + C$

$u = 1-x^3$
$du = -3x^2\,dx$

17. $\int \dfrac{2s}{\sqrt[3]{6-5s^2}}\,ds = -\frac{1}{5}\int \dfrac{-10s\,ds}{(6-5s^2)^{1/3}} = -\frac{1}{5}\int \dfrac{du}{u^{1/3}} = -\frac{1}{5}\int u^{-1/3}\,du = -\frac{1}{5}\dfrac{u^{2/3}}{\frac{2}{3}} + C$
$= -\frac{3}{10}u^{2/3} + C = -\frac{3}{10}(6-5s^2)^{2/3} + C$

$u = 6-5s^2$
$du = -10s\,ds$

18. $\int \dfrac{b^3x^3}{\sqrt{1-a^4x^4}}\,dx = -b^3\dfrac{1}{4a^4}\int \dfrac{-a^4 4x^3\,dx}{\sqrt{1-a^4x^4}} = -b^3\dfrac{1}{4a^4}\int u^{-1/2}\,du = -b^3\dfrac{1}{4a^4}u^{1/2} + C$
$= -\dfrac{b^3}{4a^4}(1-a^4x^4)^{1/2} + C$

$u = 1-a^4x^4$
$du = -a^4 4x^3\,dx$

19. $\int \dfrac{s}{(1+s^2)^3}\,ds = \frac{1}{2}\int \dfrac{2s\,ds}{(1+s^2)^3} = \frac{1}{2}\int \dfrac{du}{u^3} = \frac{1}{2}\int u^{-3}\,du = \frac{1}{2}\dfrac{u^{-2}}{-2} + C = \dfrac{1}{4u^2} + C$
$= \dfrac{1}{4(1+s^2)^2} + C$

$u = 1+s^2$
$du = 2s\,ds$

20. $\int \dfrac{3t}{(t^2+1)^2}\,dt = 3\frac{1}{2}\int \dfrac{2t\,dt}{(t^2+1)^2} = \frac{3}{2}\int \dfrac{du}{u} = \frac{3}{2}\ln|u| + C = \frac{3}{2}\ln|t^2+1| + C$

$u = t^2+1$
$du = 2t\,dt$

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Integración por cambio de variable.

1. $\int \frac{b\,dx}{(a+bx)^{1/3}} = \frac{1}{b}\int \frac{du}{u^{1/3}} = \frac{1}{b}\int u^{-1/3}du = \frac{1}{b}\frac{u^{2/3}}{2/3} + C = \frac{1}{2b}(a+bx)^{2/3} + C$

$u = a+bx$
$du = b\,dx$

2. $\int \frac{3t}{(t^2+1)}dt = 3\,\frac{1}{2}\int \frac{2t\,dt}{(t^2+1)^2} = \frac{3}{2}\int \frac{du}{u} = \frac{3}{2}\ln|u| + C = \frac{3}{2}\ln|t^2+1| + C$

$u = t^2+1$
$du = 2t\,dt$

3. $\int \frac{s}{(1+s^2)^3}ds = \frac{1}{2}\int \frac{2s\,ds}{(1+s^2)^3} = \frac{1}{2}\int \frac{du}{u^3} = \frac{1}{2}\int u^{-3}du = \frac{1}{2}\frac{u^{-2}}{-2} + C = \frac{1}{4u^2} + C$

$u = 1+s^2$
$du = 2s\,ds$

$= \frac{1}{4(1+s^2)^2} + C$

4. $\int \frac{b^3x^3}{\sqrt{1-a^4x^4}}dx = -b^3\frac{1}{4a^4}\int \frac{-a4x^3dx}{\sqrt{1-a^4x^4}} = \frac{-b^3}{4a^4}\int u^{-1/2}du = \frac{-b^3}{4a^4}u^{1/2} + C$

$u = 1-a^4x^4$
$dv = -a^4\,4x^3dx$

$= -\frac{b^3}{4a^4}(1-a^4x^4)^{1/2} + C$

5. $\int \frac{(5)2s}{\sqrt[3]{6-5s^2}}ds = -\frac{1}{5}\int \frac{-10s\,ds}{(6-5s^2)^{1/3}} = -\frac{1}{5}\int \frac{du}{u^{1/3}} = -\frac{1}{5}\frac{u^{2/3}}{\frac{2}{3}} + C$

$u = 6-5s^2$
$du = -10s\,ds$

$= -\frac{3}{10}u^{2/3} + C = -\frac{3}{10}(6-5s^2)^{2/3} + C$

6. $\int \frac{x^2}{(1-x^3)^{2/3}}dx = -\frac{1}{3}\int \frac{-3x^2\,dx}{(1-x^3)^{2/3}} = -\frac{1}{3}\int \frac{du}{u^{2/3}} = -\frac{1}{3}\int u^{-2/3}du = \frac{1}{3}\frac{u^{1/3}}{1/3} + C$

$u = 1-x^3$
$du = -3x^2dx$

$= u^{1/3} + C = (1-x^3)^{1/3} + C$

7. $\int 2x^3(1-x^4)^{-1/4}dx = -\frac{1}{2}\int u^{-1/4}du = -\frac{1}{2}\frac{u^{3/4}}{\frac{3}{4}} + C = -\frac{2u^{3/4}}{3} + C = -\frac{2}{3}(1-x^4)^{3/4} + C$

$u = 1-x^4$
$du = -4x^3dx$

8. $\int \frac{4x+6}{\sqrt{x^2+3x+1}}\,dx = \int \frac{2(2x+3)\,dx}{\sqrt{x^2+3x+1}} = 2\int \frac{du}{\sqrt{u}} = 2\int u^{-1/2}du = 2\,\frac{u^{1/2}}{\frac{1}{2}} + c = 4u^{1/2} + c$

$u = x^2+3x+1$

$du = (2x+3)dx$

$= 4(x^2+3x+1)^{1/2} + C$

9. $\int \frac{x^{(n-1)}}{\sqrt{a+bx^n}}\,dx = \frac{1}{n}\int \frac{du}{\sqrt{u}} = \frac{1}{n}\int u^{-1/2}du = \frac{1}{n}\,\frac{u^{1/2}}{\frac{1}{2}} + C = \frac{2u^{1/2}}{n} + C$

$u = a+bx^n$

$du = nx^{n-1}dx$

$= \frac{2}{n}\sqrt{(a+bx^n)} + C$

10. $\int 2x(x^2+1)^{-3}\,dx = \int u^{-3}du = \frac{u^{-2}}{-2} + C = -\frac{1}{2u^2} + C = -\frac{1}{2(x^2+1)} + C$

$u = x^2+1$

$du = 2x\,dx$

11. $\int x^2(1+x^3)^{1/4}\,dx = \frac{1}{3}\int u^{1/4}du = \frac{1}{3}\,\frac{u^{5/4}}{\frac{5}{4}} + C = \frac{4}{15}u^{5/4} + C = \frac{4}{15}(1+x^3)^{5/4} + C$

$u = 1+x^3$

$du = 3x^2dx$

12. $\int (ax+b)^{3/4}\,dx = \int u^{3/4}du = \frac{u^{3/4}}{\frac{3}{4}} + C = 4u^{7/4} + C = 4(ax+b)^{7/4} + C$

$u = ax+b$

$du = dx$

13. $\int 3x^2\cos(x^3+2)\,dx = \int \cos u\,du = \operatorname{sen} u + C = \operatorname{sen}(x^3+2) + C$

$u = x^3+2$

$du = 3x^2\,dx$

14. $\int \operatorname{sen}x\,\cos x\,dx = \int u\,du = \frac{1}{2}u^2 + c = \frac{1}{2}\operatorname{sen}^2 x + C$

$u = \operatorname{sen}x$

$du = \cos x\,dx$

15. $\int \sec^3 x\tan x\,dx = \int \sec^2 x\,\sec x\tan x\,dx = \int u^2du = \frac{1}{3}u^3 + C$

$u = \sec x$

$du = \sec x\tan x\,dx$

$= \frac{1}{3}\sec^3 x + C$

16. $\int 2x^3\sec^2(x^4+1)\,dx = \frac{1}{2}\int \sec^2 u\,du = \frac{1}{2}\tan u + C = \frac{1}{2}\tan(x^4+1) + C$

$u = x^4+1$

$du = 4x^3\,dx$

17. $\int \frac{(x+1)\,dx}{\sqrt{x^2+2x-4}} = \frac{1}{2}\int \frac{(2x+2)\,dx}{\sqrt{x^2+2x-4}} = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\int u^{-1/2}du = \frac{1}{2}\,\frac{u^{1/2}}{\frac{1}{2}} + c$

$u = x^2+2x-4$

$du = (2x+2)\,dx$

$= u^{1/2} + C = (x^2+2x-4)^{1/2} + C$

INTEGRACIÓN POR PARTES

Tarea ②

1. $\int x e^x\,dx = xe^x - \int e^x\,dx = \underline{xe^x - e^x + C}$

$u = x \quad du = dx$
$dv = e^x dx \quad v = e^x + C$

2. $\int x \operatorname{sen} 2x\,dx = -x\frac{1}{2}\cos 2x + \int \frac{1}{2}\cos 2x\,dx = -\frac{x}{2}\cos 2x + \frac{1}{4}\int \cos 2x\, 2dx$
$= \underline{-\frac{x}{2}\cos 2x + \frac{1}{4}\operatorname{sen} 2x + C}$

$u = x \quad du = dx$
$dv = \operatorname{sen} 2x\,dx$
$v = \int \operatorname{sen} 2x\,dx$
$= \frac{1}{2}\int \operatorname{sen} 2x\, 2dx$
$= -\frac{1}{2}\cos 2x + C$

3. $\int x \ln x\,dx = \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2}\frac{dx}{x} = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx = \underline{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$

$u = \ln x \quad du = \frac{dx}{x}$

$dv = x\,dx \quad v = \frac{x^2}{2}$

4. $\int x^2 e^x\,dx = x^2 e^x - \int e^x 2x\,dx = x^2e^x - 2\int e^x dx = x^2 e^x - 2(xe^x - e^x + C)$

$u = x^2 \quad du = 2x\,dx$
$dv = e^x dx \quad v = e^x$

$= \underline{x^2 e^x - 2xe^x + 2e^x + C}$

5. $\int e^x \cos x\,dx = \cos x\, e^x + \int e^x \operatorname{sen} x\,dx = \cos x\, e^x + \operatorname{sen} e^x - \int e^x \cos dx$

$u = \cos x \quad du = -\operatorname{sen} x\,dx$
$dv = e^x dx \quad v = e^x$

$u = \operatorname{sen} x \quad du = \cos x\,dx$
$dv = e^x dx \quad v = e^x$

$\therefore \int e^x \cos x\,dx = e^x \cos x + \operatorname{sen} x\, e^x - \int e^x \cos dx + C$

$2\int e^x \cos x\,dx = e^x \cos x + \operatorname{sen} x\, e^x + C$

$\int e^x \cos x\,dx = \underline{\frac{1}{2}e^x(\cos x + \operatorname{sen} x) + C}$

6. $\int x^5 \cos(x^3)\,dx = \int x^3 \cos(x^3)\, x^2\,dx = x^3 \cdot \frac{1}{3}\operatorname{sen}(x^3) - \int \frac{1}{3}\operatorname{sen}(x^3)\, 3x^2 dx$
$= \underline{\frac{1}{3}x^3 \operatorname{sen}(x^3) + \frac{1}{3}\cos(x^3) + C}$

$u = x^3 \quad du = 3x^2\,dx$
$dv = \cos(x^3) x^2\,dx$
$v = \frac{1}{3}\operatorname{sen}(x^3) + C$

7. $\int x \operatorname{sen}(x)\,dx = -x\cos(x) - \int -\cos(x)\,dx = \underline{-\cos(x) + \operatorname{sen}(x) + C}$

$u = x \quad du = dx$
$dv = \operatorname{sen} x\,dx \quad v = -\cos(x)$

8. $\int x e^{2x}\,dx = xe^{2x} - \frac{1}{2}\int e^{2x}\, 2dx = xe^{2x} + C = \underline{e^{2x}(x - \frac{1}{2}) + C}$

$u = x \quad du = dx$
$dv = e^{2x} dx \quad v = \frac{1}{2}e^{2x}$

9. $\int x^2 \cos(3x)\,dx = \frac{1}{3}x^2 \operatorname{sen}(3x) - \int \frac{1}{3}\operatorname{sen}(3x)\cos(3x)\,dx$
$= \underline{\frac{1}{3}x^2 \operatorname{sen} 3x - \frac{1}{18}\operatorname{sen}^2(3x) + C}$

$u = x^2 \quad dv = 2x\,dx$
$dv = \cos(3x)\,dx$
$v = \frac{1}{3}\operatorname{sen}(3x)$

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10. $\int x^2 e^{bx}\,dx = x^2 \frac{1}{b} e^{bx} - \int \frac{1}{b} e^{bx} 2x\,dx = x^2 \frac{1}{b} e^{bx} - \frac{2}{b}\int x e^{bx}\,dx$

$u = x^2 \quad du = 2x$

$dv = e^{bx}\,dx \quad v = \int e^{bx}\,dx$

$v = \frac{1}{b} e^{bx}$

$u = x \quad du = dx$

$dv = e^{bx}\,dx \quad v = \frac{1}{b} e^{bx}$

$= x^2 \frac{1}{b} e^{bx} - \frac{2}{b}\left(x \frac{1}{b} e^{bx} - \int \frac{1}{b} e^{bx}\,dx\right)$

$= x^2 \frac{1}{b} e^{bx} - \frac{2}{b^2} x e^{bx} - \frac{2}{b^2}\frac{1}{b}\int e^{bx}\, b\,dx$

$= \frac{x^2}{b} e^{bx} - \frac{2x}{b^2} e^{bx} - \frac{2}{b^3} e^{bx} + C$

11. $\int x^2 \ln(5x)\,dx = \ln(5x)\left(\frac{x^3}{3}\right) - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx = \ln(5x)\left(\frac{x^3}{3}\right) - \frac{1}{3}\int x^2\,dx$

$u = \ln(5x) \quad du = \frac{5}{5x} = \frac{1}{x}\,dx$

$dv = x^2\,dx$

$v = \frac{x^3}{3}$

$= \ln(5x)\left(\frac{x^3}{3}\right) - \frac{1}{3}\frac{x^3}{3} = \frac{x^3}{3}\ln(5x) - \frac{x^3}{9} + C$

12. $\int x e^{-x}\,dx = -xe^{-x} - \int -e^{-x}\,dx = -xe^{-x} - e^{-x} + C$

$u = x \quad du = dx$

$dv = e^{-x}\,dx \quad v = -e^{-x}$

$= -e^{-x}(x+1) + C$

13. $\int x \ln x^2\,dx = \ln x^2 \frac{x^2}{2} - \int \frac{x^2}{2}\frac{2}{x}\,dx = \frac{1}{2}x^2 \ln x^2 - \int x\,dx$

$u = \ln x^2 \quad du = \frac{2x}{x^2} = \frac{2}{x}\,dx$

$dv = x\,dx \quad v = \frac{x^2}{2}$

$= \frac{1}{2}x^2 \ln x^2 - \frac{x^2}{2} + C$

$= \frac{1}{2}x^2\left[\ln x^2 - 1\right] + C$

14. $\int \ln x\,dx = x\ln x - \int x \frac{1}{x}\,dx$

$u = \ln x \quad du = \frac{1}{x}\,dx$

$dv = dx \quad v = x$

$= x\ln x - x + C = x[\ln x - 1] + C$

15. $\int \arctan(x)\,dx = \arctan(x)(x) - \int x \frac{1}{x^2+1}\,dx$

$u = \arctan(x) \quad du = \frac{1}{x^2+1}\,dx$

$dv = dx \quad v = x$

$= x\arctan(x) - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$

$= x\arctan(x) - \frac{1}{2}\ln|x^2+1| + C$

16. $\int \operatorname{arcsen}(x)\,dx = x\operatorname{arcsen}(x) - \int \frac{x\,dx}{\sqrt{1-x^2}} = x\operatorname{arcsen}(x) - \int (1-x^2)^{-1/2} x\,dx$

$u = \operatorname{arcsen}(x) \quad du = \frac{dx}{\sqrt{1-x^2}}$

$dv = dx \quad v = x$

$= x\operatorname{arcsen}(x) + \frac{1}{2}\int (1-x^2)^{-1/2} - 2x\,dx$

$= x\operatorname{arcsen}(x) + \frac{1}{2}\frac{(1-x^2)^{1/2}}{\frac{1}{2}} + C$

$= x\operatorname{arcsen}(x) + \sqrt{1-x^2} + C$

17. $\int \operatorname{arcsec}(x)\,dx = x\operatorname{arcsec}(x) - \int \frac{x\,dx}{x\sqrt{x^2-1}} = x\operatorname{arcsec}(x) - \int \frac{dx}{\sqrt{x^2-1}}$

$u = \operatorname{arcsec}(x) \quad du = \frac{dx}{x\sqrt{x^2-1}}$

$dv = dx \quad v = x$

$= x\operatorname{arcsec}(x) - \ln\left|x + \sqrt{x^2-1}\right| + C$

18. $\int \arccos(2x)\,dx$

$u = \arccos 2x \quad du = \frac{-2dx}{\sqrt{1-(2x)^2}}$

$dv = dx \quad v = x$

$= x \arccos 2x - \int \frac{-2x\,dx}{\sqrt{1-(2x)^2}}$

$= x \arccos 2x - {}_4\int (1-4x^2)^{-1/2}\, 4\, 2x\,dx$

$= x \arccos 2x - \frac{1}{4}\, \frac{\sqrt{1-4x^2}}{\frac{1}{2}} + C$

$= x \arccos 2x - \frac{1}{2}\sqrt{1-4x^2} + C$

19. $\int \text{arc ctg}(3x)\,dx = x\,\text{arcctg}(3x) - \int \frac{6 \cdot 3x\,dx}{9x^2+1}$

$u = \text{arcctg}(3x)$

$du = \frac{3dx}{(3x)^2+1}$

$= x\,\text{arcctg}(3x) - \frac{1}{6}\ln|9x^2+1| + C$

$dv = dx \quad v = x$

20. $\int \text{arc csc}(4x)\,dx = x\,\text{arccsc}(4x) - \int \frac{-\cancel{x}\,dx}{\cancel{x}\sqrt{(4x)^2-1}}$

$u = \text{arccsc}(4x)$

$du = \frac{-4\,dx}{4x\sqrt{(4x)^2-1}} = -\frac{dx}{x\sqrt{(4x)^2-1}}$

$= x\,\text{arccsc}(4x) + \frac{1}{4}\int \frac{4\,dx}{\sqrt{(4x)^2-1}}$

$dv = dx \quad v = x$

$= x\,\text{arccsc}\,4x + \frac{1}{4}\ln\left|4x + \sqrt{(4x)^2-1}\right| + C$

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INTEGRALES CON FUNCIONES TRIGONOMETRICAS.

Tipo $\int sen^n x\,dx$; $\int cos^n x\,dx$ CASO I (Con n entero positivo impar).

1. $\int sen^3 x\,dx = \int sen^2 x\ sen\,x\,dx = \int (1-cos^2 x) sen\,x\,dx = -\int (1-u^2)\,du$

$u = cos\,x$
$du = -sen\,x$

$= -u + \frac{u^3}{3} = -cos\,x + \frac{cos^3}{3}$

2. $\int cos^5 x\,dx = \int cos^4 x\ cos\,x\,dx = \int (1-sen^2 x)^2 cos\,x\,dx = \int (1-2sen^2 x + sen^4 x) cos\,x\,dx$

$u = sen\,x$
$du = cos\,x\,dx$

$= \int (1-2u^2+u^4)\,du = u - \frac{2u^3}{3} + \frac{u^5}{5} = sen\,x - \frac{2}{3}sen^3 x + \frac{sen^5 x}{5} + C$

3. $\int sen^5 x\,dx = \int sen\,x^4\,sen\,x\,dx = \int (1-cos^2 x)^2 sen\,x\,dx = \int (1-2cos^2 x + cos^4 x) sen\,x\,dx$

$u = cos\,x$
$du = -sen\,x\,dx$

$= -\int (1-2u^2+u^4)\,du = -u + \frac{2}{3}u^3 + \frac{u^5}{5} = -cos\,x + \frac{2}{3}cos^3 x + \frac{1}{5}cos^5 x$

4. $\int tan^4 (2x)\,dx = \int tan^2(2x)\,tan^2(2x)\,dx = \int [sec^2(2x)-1]\,tan^2(2x)\,dx$

$= \int (tan(2x))^2 sec^2(2x)\,2dx - \int tan^2(2x)\,dx$

$= \frac{1}{2}\frac{tan^3(2x)}{3} - \frac{1}{2}\int sec^2(2x)\,2\,dx - \int dx$

$= \frac{1}{6}tan^3(2x) - \frac{1}{2}tan(2x) - x + C$

5. $\int sec^4(3x)\,dx = \int sec^2(3x)\,sec^2(3x)\,dx = \int [tg^2(3x)+1]\,sec^2(3x)\,dx$

$= \frac{1}{3}\int [tg(3x)]^2\,sec^2(3x)\,3dx + \frac{1}{3}\int sec^2(3x)\,3\,dx$

$= \frac{1}{3}\,\frac{tg^3(3x)}{3} + \frac{1}{3}tan(3x) + C = \frac{1}{9}tg^3(3x) + \frac{1}{3}tg(3x) + C$

6. $\int (sec\,\theta - tan\,\theta)^2\,d\theta = \int (sec^2\theta - 2sec\,\theta\,tan\,\theta + tan^2\theta)\,d\theta$

$= \int sec^2\theta\,d\theta - 2\int sec\,\theta\,tan\,\theta\,d\theta + \int tan^2\theta\,d\theta$

$= tan\,\theta - 2\,sec\,\theta + \int (sec^2\theta - 1)\,d\theta$

$= tan\theta - 2sec\theta + \int sec^2\theta - \int d\theta$

$= tan\theta - 2sec\theta + tan\theta - \theta$

$= 2tan\,\theta - 2sec\,\theta - \theta$

7. $\int \operatorname{sen}^3 \frac{\theta}{3}\, d\theta = \int \operatorname{sen}^2 \frac{\theta}{3} \operatorname{sen}\frac{\theta}{3}\, d\theta = \int \left(1-\cos^2\frac{\theta}{3}\right) \operatorname{sen}\frac{\theta}{3}\, d\theta$

$= 3\int \operatorname{sen}\frac{\theta}{3} \frac{d\theta}{3} - 3\int \operatorname{sen}\frac{\theta}{3} \frac{d\theta}{3}$

$= -3\cos\frac{\theta}{3} + \frac{1}{3}\cos^3\frac{\theta}{3} + C$

8. $\int \cos^2 2\theta\, d\theta = \int \left(\frac{1}{2} + \frac{\cos(4\theta)}{2}\right) d\theta = \frac{1}{2}\int d\theta + \frac{1}{4}\cdot\frac{1}{2}\int \cos 4\theta\, 4\, d\theta$

$= \frac{1}{2}\theta + \frac{1}{8}\operatorname{sen}(4\theta) + C$

9. $\int \operatorname{sen}^4(2x)\,dx = \int (\operatorname{sen}^2(2x))^2 = \int\left(\frac{1}{2} - \frac{\cos(4x)}{2}\right)^2 dx$

$= \int \frac{1}{4}dx - \int \frac{\cos(4x)dx}{2} + \int \frac{\cos^2(4x)\,dx}{4}$

$= \frac{1}{4}\int dx - \frac{1}{2}\cdot\frac{1}{4}\int \cos(4x)\,4\,dx + \frac{1}{4}\int\left(\frac{1}{2}+\frac{\cos(8x)}{2}\right)dx$

$= \frac{1}{4}x - \frac{1}{8}\operatorname{sen}(4x) + \frac{1}{4}\int\frac{1}{2}dx + \frac{1}{4}\cdot\frac{1}{2}\cdot\frac{1}{8}\int \cos 8x\, 8dx$

$= \frac{x}{4} - \frac{1}{8}\operatorname{sen}(4x) + \frac{1}{8}x + \frac{1}{64}\operatorname{sen}(8x)$

$= \frac{3}{8}x - \frac{1}{8}\operatorname{sen}(4x) + \frac{1}{64}\operatorname{sen}(8x)$

10. $\int \cos^4\left(\frac{y}{2}\right)dy = \int\left[\frac{1}{2} + \frac{\cos y}{2}\right]^2 dy = \int\frac{1}{2}dy + \int\frac{\cos y}{2}dy + \int\frac{\cos^2 y}{4}dy$

$= \frac{1}{2}\int dy + \frac{1}{2}\int\cos y\, dy + \frac{1}{4}\int\left(\frac{1}{2}+\frac{\cos 2y}{2}\right)dy$

$= \frac{1}{2}y + \frac{1}{2}\operatorname{sen} y + \frac{1}{4}\int\frac{1}{2}dy + \frac{1}{4}\cdot\frac{1}{2}\cdot\frac{1}{2}\int\cos 2y\, 2dy$

$= \frac{1}{2}y + \frac{1}{2}\operatorname{sen} y + \frac{1}{8}y + \frac{1}{16}\operatorname{sen}2y + C$

$= \frac{5}{8}y + \frac{1}{2}\operatorname{sen} y + \frac{1}{16}\operatorname{sen}2y + C$

11. $\int \tan^3(3x)\sec(3x)dx = \int tg^2(3x)\,tg(3x)\sec(3x)dx$

$= \int(\sec^2(3x)-1)\,tg(3x)\sec(3x)dx$

$= \frac{1}{3}\int tg(3x)\sec^2(3x)3dx - \int tg(3x)\sec(3x)dx$

$= \frac{1}{3}\sec(3x) - \frac{1}{3}\int[\cos(3x)]^{-2}\operatorname{sen}(3x)\,3dx$

$= \frac{1}{3}\sec(3x) - \frac{1}{3}\frac{1}{\cos(3x)} = \frac{1}{3}\sec(3x) - \frac{1}{3}\sec(3x)$

12. $\int sen^2 x \cos^5 x \, dx = \int sen^2 x \cos^4 x \cos x \, dx$

$= \int sen^2 x (1 - sen^2 x)^2 \cos x \, dx$

$= \int (sen^2 x - 2 sen^4 x + sen^6 x) \cos x \, dx$

$= \frac{1}{3} sen^3 x - \frac{2}{5} sen^5 x + \frac{1}{7} sen^7 x + C$

13. $\int \cos^2 x \, dx = \int (\frac{1}{2} + \frac{1}{2} \cos 2x) \, dx$

$= \frac{1}{2} \int dx + \frac{1}{2} \int \cos 2x \, dx$

$= \frac{1}{2} x + \frac{1}{4} sen \, 2x + C$

14. $\int sen^2 x \cos^2 x \, dx = \int (sen\, x \cos x)^2 \, dx$

$= \frac{1}{4} \int sen^2 2x \, dx$

$= \frac{1}{4} \int (\frac{1}{2} - \frac{1}{2} \cos 4x) \, dx$

$= \frac{1}{8} \int dx - \frac{1}{8} \int \cos 4x \, dx$

$= \frac{1}{8} x - \frac{1}{32} sen \, 4x + C$

15. $\int sen \, 5x \, sen \, 3x \, dx = \int \frac{1}{2} [\cos(5x - 3x) - \cos(5x + 3)] \, dx$

$= \int \frac{1}{2} (\cos 2x - \cos 8x) \, dx$

$= \frac{1}{4} sen \, 2x - \frac{1}{16} sen \, 8x + C$

16) $\int \tan^6 x \, dx = \int (\tan^4 x \sec^2 x - \tan^4 x) \, dx$

$= \int (\tan^4 x \sec^2 x - \tan^2 x \sec^2 x + \tan^2 x) \, dx$

$= \int (\tan^4 x \sec^2 x - \tan^2 x \sec^2 x + \sec^2 x - 1) \, dx$

$= \int \tan^4 x \sec^2 x \, dx - \int \tan^2 x \sec^2 x \, dx + \int \sec^2 x \, dx - \int dx$

$= \frac{1}{5} \tan^5 x - \frac{1}{3} \tan^3 x + \tan x - x + C$

17. $\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \int \sec x \, dx$

$= \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C$

18. $\int \sec^6 x\,dx = \int \sec^4 x \sec^2 x\,dx = \int (\tan^2 x + 1)^2 \sec^2 x\,dx$

$= \int (u^2+1)^2\,du \qquad u = \tan x \qquad du = \sec^2 x\,dx$

$= \int (u^4 + 2u^2 + 1)\,du$

$= \frac{1}{5}u^5 + \frac{2}{3}u^3 + u + C$

$= \frac{1}{5}\tan^5 x + \frac{2}{3}\tan^3 x + \tan x + C$

19. $\int \tan^5 x \sec^4 x\,dx = \int \tan^5 x \sec^2 x \sec^2 x\,dx$

$= \int \tan^5 x (\tan^2 x + 1) \sec^2 x\,dx$

$= \int \tan^7 x \sec^2 x\,dx + \int \tan^5 x \sec^2 x\,dx$

$= \frac{1}{8}\tan^8 x + \frac{1}{6}\tan^6 x + C$

20. $\int \tan^5 x \sec^3 x\,dx = \int \tan^4 x \sec^2 x \sec x \tan x\,dx$

$= \int (\sec^2 x - 1)^2 \sec^2 x \sec x \tan x\,dx$

$= \int (\sec^6 x - 2\sec^4 x + \sec^2 x) \sec x \tan x\,dx$

$= \frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C$

21. $\int \tan^2 x \sec x\,dx = \int (\sec^2 x - 1)\sec x\,dx$

$= \int (\sec^3 x - \sec x)\,dx$

$= \int \sec^3 x\,dx - \int \sec x\,dx$

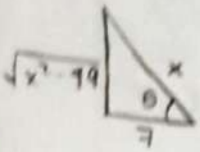
$= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + \ln|\sec x + \tan x| + C$

$= \frac{1}{2}\sec x \tan x - \frac{1}{2}\ln|\sec x + \tan x| + C$

SUSTITUCIÓN TRIGONOMETRICA

Tarea 5

1) $\int \frac{\sqrt{x^2-49}}{x}\,dx = \int \frac{7\tan\theta}{7\sec\theta}\, 7\sec\theta\tan\theta = \int 7\tan^2\theta$



$x = 7\sec\theta$

$\sqrt{x^2-49} = 7\tan\theta$

$dx = 7\sec\theta\tan\theta$

$\tan^2\theta = \sec^2\theta - 1$

$= \int 7(\sec^2\theta - 1)\,d\theta$

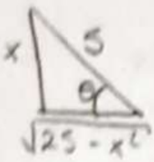
$= 7\int \sec^2\theta - 7\int d\theta$

$= 7\tan\theta - 7\theta$

$= 7\sqrt{x^2-49} - 7\,\text{arc sen}\left(\frac{7}{x}\right) + C$

$= 7\left(\sqrt{x^2-49} - \text{arc sen}\left(\frac{7}{x}\right) + C\right)$

2) $\int \frac{dx}{x^2\sqrt{25-x^2}} = \int \frac{du}{25\,\text{sen}^2 u} = \int \frac{\csc^2 u\,du}{5}$



$x = 5\,\text{sen}\,\theta$

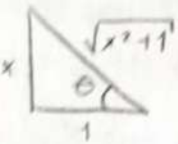
$dx = 5\cos\theta\,d\theta$

$\sqrt{25-x^2} = 5\cos\theta$

$= \frac{1}{5}\int \csc^2 u\,du = \frac{1}{5}(-\cot\theta) + C$

$\cot\theta = \frac{\sqrt{25-x^2}}{x} \Rightarrow \frac{-\sqrt{25-x^2}}{5x} + C$

3) $\int \frac{\sqrt{x^2+1}}{x}\,dx = \int \frac{\sec\theta}{\tan\theta}\sec^2\theta\,d\theta = \int \frac{\sec\theta}{\tan\theta}(1+\tan^2\theta)\,d\theta = \int \frac{\sec\theta + \sec\theta\cdot\tan^2\theta\,d\theta}{\tan\theta}$



$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\sec\theta = \sqrt{x^2+1}$

$= \int \frac{\sec\theta\,d\theta}{\tan\theta} + \int \sec\theta\tan\theta\,d\theta$

$= \ln(\csc\theta - \text{ctg}\,\theta) + \sec\theta$

$= \ln\left(\frac{\sqrt{x^2+1}}{x} - \frac{1}{x}\right) + \sqrt{x+1} + C$

4) $\int \frac{\sqrt{9-x^2}}{x^2}\frac{1}{(x)}dx = \int \frac{\cos z}{\text{sen}\,z}\cdot\frac{\cos z}{\text{sen}\,z}\,dz = \int \cot^2(z)\,dz = \int(\csc^2 \cancel{z} - 1)\,dz$

$x = 3\,\text{sen}\,z$

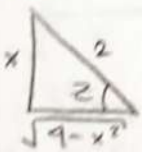
$dx = 3\cos z\,dz$

$= \int \csc^2 z\,dz - \int dz$

$= -\cot z - z + C$

$= \frac{-\sqrt{9-x^2}}{x} - \text{arcsen}\left(\frac{x}{3}\right) + C$

5) $\int \frac{dx}{\sqrt{4-x^2}} = \int \frac{2\cos z\,dz}{2\cos z} = \int dz$



$\frac{x}{2} = \text{sen}\,z$

$x = 2\,\text{sen}\,z$

$\frac{\sqrt{4-x^2}}{2} = \cos z$

$\sqrt{1-x^2} = 2\cos z$

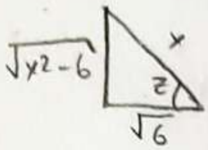
$dx = 2\cos z\,dz$

$= z + C$

$= \text{arc sen}\left(\frac{x}{2}\right) + C$

R.A.K.

6) $\int \frac{x^2\,dx}{\sqrt{x^2-6}} = \int \frac{6\sec^2 z\,(\sqrt{6}\sec z\tan z\,dz)}{\sqrt{6}\tan z} = 6\int \sec^3 z\,dz$



$= 6\left[\sec z\tan z + \frac{1}{2}\int \sec z\,dz\right]$

$= 6\left[\sec z\tan z + \frac{1}{2}\ln(\sec z + \tan z)\right] + C$

$x = \sqrt{6}\sec z$
$dx = \sqrt{6}\sec z\tan z\,dz$

$\sqrt{x^2-6} = \sqrt{6}\,dz$

$= 3\sec z\tan z + 3\ln\left(\frac{x}{\sqrt{6}} + \frac{\sqrt{x^2-6}}{6}\right) + C$

$= x\left(\frac{\sqrt{x^2-6}}{2}\right) + 3\ln\left(x + \frac{\sqrt{x^2-6}}{6}\right) + C$

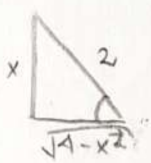
7) $\int \frac{t^2\,dt}{\sqrt{4-t^2}} = \int \frac{(4\,\mathrm{sen}^2 z)(2\cos z\,dz)}{2\cos z} = 4\int \mathrm{sen}^2 z\,dz$

$= 4\left[\frac{-\mathrm{sen}\,z\cos z}{2} + \frac{1}{2}\int \mathrm{sen}\,z\,dz\right.$

$t = 2\,\mathrm{sen}\,z$
$dt = z - \cos z$
$\sqrt{4-t^2} = \cos z$

$= 2\,\mathrm{sen}\,z\cos z + 2\int dz$
$= -2\,\mathrm{sen}\,z\cos z + 2z + C$
$= -t\left(\frac{\sqrt{4-t^2}}{2}\right) + 2\,\mathrm{arc\,sen}\left(\frac{t}{2}\right) + C$

8) $\int \frac{dx}{x^2\sqrt{4-x^2}} = \int \frac{2\cos\theta\,d\theta}{(2\,\mathrm{sen}\,\theta)^2\sqrt{4(1-\mathrm{sen}^2\theta)}} = \int \frac{2\cos\theta\,d\theta}{4\,\mathrm{sen}^2\theta\sqrt{4-4\,\mathrm{sen}^2\theta}}$



$= \int \frac{\cos\theta\,d\theta}{2\,\mathrm{sen}^2\theta\sqrt{4(1-\mathrm{sen}^2\theta)}} = \int \frac{\cos\theta\,d\theta}{2\,\mathrm{sen}^2\theta + 2\sqrt{\cos^2\theta}}$

$= \int \frac{\cos\theta\,d\theta}{4\,\mathrm{sen}^2\theta\cos\theta} = \frac{1}{4}\int \csc^2\theta\,d\theta = -\frac{1}{4}\cot\theta + C$

$x = 2\,\mathrm{sen}\,\theta$
$dx = 2\cos\theta\,d\theta$

$= \frac{\sqrt{4-x^2}}{4} + C$

9) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \int \frac{\cos\theta\,d\theta}{\sqrt{1-\mathrm{sen}^2\theta}} = \int \frac{\cos\theta\,d\theta}{\sqrt{\cos^2\theta}} = \int \frac{\cos\theta\,d\theta}{\cos\theta} = \int d\theta = \theta + C$

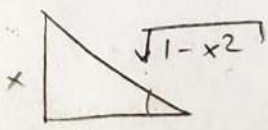
$x = \mathrm{sen}\,\theta$
$dx = \cos\theta\,d\theta$

$= \mathrm{sen}^{-1}x + C$

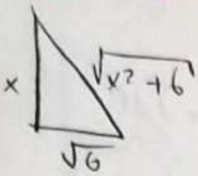
10) $\int \frac{dx}{\sqrt{4+x^2}} = \int \frac{2\sec^2\theta\,d\theta}{2\sec\theta} = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$

$x = 2\tan\theta$
$dx = 2\sec^2\theta\,d\theta$
$\sqrt{4+x^2} = 2\sec\theta$

$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$



11. $\int \frac{x^2\,dx}{\sqrt{x^2+6}} = \int \frac{6\tan^2\theta\sqrt{6}\sec^2\theta\,d\theta}{\sqrt{6}\sec\theta} = 6\int \tan^2\theta\sec\theta\,d\theta$



$x=\sqrt{6}\tan\theta$
$dx=\sqrt{6}\sec^2\theta\,d\theta$
$\sqrt{x^2+6}=\sqrt{6}\sec\theta$

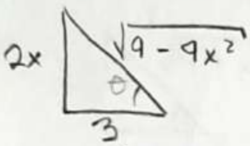
$= 6\int(\sec^2\theta-1)\sec\theta\,d\theta = 6\int(\sec^3\theta-\sec\theta)\,d\theta$

$= 6\left[\frac{1}{2}(\sec\theta\tan\theta)+\ln|\sec\theta+\tan\theta|\right]-6\ln|\sec\theta+\tan\theta|$

$= 3\cdot\frac{\sqrt{x^2+6}}{\sqrt{6}}\frac{x}{\sqrt{6}} - 3\ln\left|\frac{\sqrt{x^2+6}}{\sqrt{6}}+\frac{x}{\sqrt{6}}\right| + C$

$= \frac{x\sqrt{x^2+6}}{2} - 3\ln\left|\frac{\sqrt{x^2+6}+x}{\sqrt{6}}\right| + C$

12. $\int \frac{x\,dx}{(9+4x^2)^{3/2}} = \int \frac{\frac{3}{2}\tan\theta\,\frac{3}{2}\sec^2\theta\,d\theta}{27\sec^3\theta} = \frac{1}{12}\int\frac{\tan\theta}{\sec\theta}\,d\theta$



$= \frac{1}{12}\int\frac{\frac{\operatorname{sen}\theta}{\cos\theta}}{\frac{1}{\cos\theta}}\,d\theta = \frac{1}{12}\int \operatorname{sen}\theta\,d\theta = \frac{1}{12}(-\cos\theta)+C$

$x=\frac{3}{2}\tan\theta$
$dx=\frac{3}{2}\sec^2\theta\,d\theta$
$9+4x^2=9(1+\tan^2\theta)$

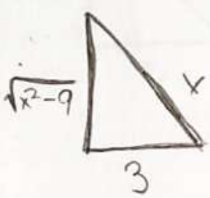
$\therefore = \frac{-1}{12}\cdot\frac{3}{\sqrt{9+x^2}}+C = \frac{-1}{4\sqrt{9+4x^2}}+C$

$x=\frac{3}{2}\tan\theta$
$dx=\frac{3}{2}\sec^2\theta\,d\theta$
$9+4x^2=9(1+\tan^2\theta)$

13. $\int\frac{x\,dx}{\sqrt{x^2-9}} = \int\frac{3\sec\theta\cdot3\sec\theta\tan\theta\,d\theta}{3\tan\theta}$

$= 9\int\sec^2\theta\,d\theta$

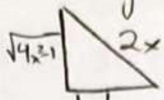
$= 9\tan\theta + C$



$x=3\sec\theta$
$dx=3\sec^2\theta\tan\theta\,d\theta$
$\sqrt{x^2-9}=3\tan\theta$

R. A. K.

14. $\int \frac{\sqrt{4x^2-1}}{x} dx = \int \frac{\tan\theta \cdot \frac{1}{2}\sec\theta \tan\theta d\theta}{1/2 \sec\theta} = \int \tan^2\theta d\theta = \int (\sec^2\theta - 1) d\theta$



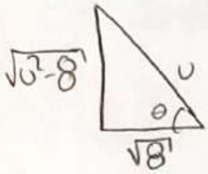
$x = \frac{1}{2}\sec\theta$

$dx = \frac{1}{2}\sec\theta \tan\theta d\theta$

$\sqrt{4x^2-1} = \tan\theta$

$= \int = \tan\theta - \theta + C = \sqrt{4x^2-1} - \operatorname{arcsec}(2x) + C$

15. $\int \frac{du}{u^2\sqrt{u^2-8}} = \int \frac{\sqrt{8}\sec\theta \tan\theta d\theta}{8\sec^2\theta \sqrt{8}\tan\theta} = \frac{1}{8}\int \frac{d\theta}{\sec\theta}$



$= \frac{1}{8}\int \cos\theta d\theta = \frac{1}{8}\operatorname{sen}\theta + C$

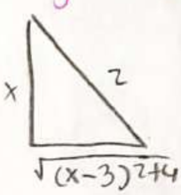
$u = \sqrt{8}\sec\theta$

$du = \sqrt{8}\sec\theta\tan\theta d\theta$

$\sqrt{u^2-8} =$

$\int \frac{du}{u^2\sqrt{u^2-8}} = \frac{1}{8}\frac{\sqrt{u^2-8}}{u} + C$

16. $\int \frac{dx}{\sqrt{x^2-6x+13}} = \int \frac{dx}{\sqrt{(x-3)^2+4}} = \int \frac{2\sec^2\theta d\theta}{2\sec\theta} = \int \sec\theta d\theta$



$x-3 = 2\tan\theta$

$dx = 2\sec^2\theta d\theta$

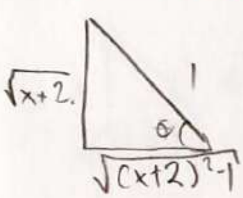
$\sqrt{(x-3)^2+4} = 2\sec\theta$

$= \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

$= \frac{1}{2}\ln\left|\frac{\sqrt{(x-3)^2+4}}{2} + \frac{x-3}{2}\right| + C$

$= \frac{1}{2}\ln\left|\frac{\sqrt{(x-3)^2+4} + x - 3}{2}\right| + C$

17. $\int \frac{2x\,dx}{\sqrt{x^2+4x+3}} = \int \frac{2x\,dx}{\sqrt{(x+2)^2-1}} = \int \frac{2(\sec\theta - 2)\sec\theta\tan\theta d\theta}{\tan\theta}$



$= 2\int (\sec^2\theta\ 2\sec\theta) d\theta$

$= 2\tan\theta - 4\ln|\sec\theta + \tan\theta| + C$

$= e\sqrt{(x+2)^2-1} - 4\ln|x+2+\sqrt{x^2+4x+3}| + C$

$x =$

$dx =$

$\sqrt{(x+2)^2-1}$

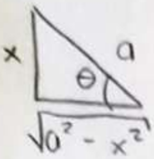
18. $\int \sqrt{a^2 - x^2}\, dx = a^2 \int \cos^2\theta \, d\theta = \frac{a^2}{2} \int (1 + \cos 2\theta)\, d\theta$

$= \frac{a^2}{2}\theta + \frac{a^2}{4} \sin 2\theta + C$

$= \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + \frac{a^2}{2} \sin\theta \cos\theta + C$

$= \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) \frac{a^2}{2}\left(\frac{x}{a}\right)\left(\frac{\sqrt{a^2 - x^2}}{a}\right) + C$

$= \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + C$



$\sin\theta = \frac{x}{a}$

$x = a \sin\theta$

$dx = a \cos\theta \, d\theta$

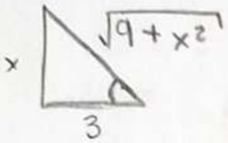
$\Rightarrow \sqrt{a^2 - x^2} = \sqrt{a^2 - a^2 \sin^2\theta}$

$\Rightarrow a\sqrt{1 - \sin^2\theta} = a \cos\theta$

19. $\int \frac{dx}{\sqrt{9 + x^2}} = \int \frac{3 \sec 2\theta \, d\theta}{3 \sec\theta} = \int \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{\sqrt{9 + x^2}}{3} + \frac{x}{3}\right| + C$

$= \ln\left|\frac{x + \sqrt{9 + x^2}}{3}\right| + C$



$x = 3\tan\theta$

$dx = 3\sec^2\theta \, d\theta$

$\sqrt{9 + x^2} = 3\sec\theta$

20. $\int \frac{dx}{\sqrt{x^2 + 2x + 26}} = \int \frac{5 \sec^2\theta \, d\theta}{5 \sec\theta} = \int \sec\theta \, d\theta$

$= \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{\sqrt{(x+1)^2 + 25} + (x+1)}{5}\right| + C$

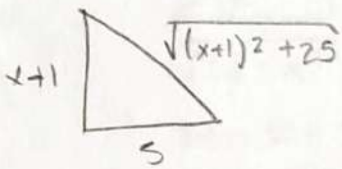
$x^2 + 2x + 1 + 25$

$= (x+1)^2 + 5^2$

$x + 1 = 5\tan\theta$

$d(x+1) = 5\sec^2\theta \, d\theta$

$\sqrt{(x+1)^2 + 25} = 5\sec\theta$



R. A. K.

FORMULARIO BASE :)

Transformada de Laplace de algunas funciones elementales.

$f(t)$	$F(s)$
$\mathcal{L}\{1\}$	$\frac{1}{s}$
$\mathcal{L}\{t\}$	$\frac{1}{s^2}$
$\mathcal{L}\{t^n\}$	$\frac{n!}{s^{n+1}}$; $n=0,1,2,3,\dots$
$\mathcal{L}\{e^{at}\}$	$\frac{1}{s \cdot a}$
$\mathcal{L}\{\text{sen } at\}$	$\frac{a}{s^2+a^2}$
$\mathcal{L}\{\cos at\}$	$\frac{s}{s^2+a^2}$
$\mathcal{L}\{\text{senh } at\}$	$\frac{a}{s^2-a^2}$
$\mathcal{L}\{\cosh at\}$	$\frac{s}{s^2-a^2}$
$\mathcal{L}\{u(t-a)\}$	$\frac{e^{-as}}{s}$
$\mathcal{L}\{e^{bt}\text{ senh } at\}$	$\frac{a}{(s-b)^2-a^2}$
$\mathcal{L}\{t^{\alpha}\}$	$\frac{\Gamma(\alpha+1)}{s^{(\alpha+1)}}$
$\mathcal{L}\{e^{at}f(t)\}$	$\mathcal{L}\{f(t)\}=F(s)\Big\vert_{s\to s-a}=F(s-a)$
$f(t)=\begin{cases} g(t); & 0\le t<a \\ h(t); & t\ge a\end{cases}$	$g(t)-g(t)u(t-a)+h(t)u(t-a)$
$\mathcal{L}\{f(t-a)u(t-a)\}$	$e^{-as}F(s)$ $F(s)=\mathcal{L}\{f(t)\}$ Sin desplazamiento
$\mathcal{L}\{g(t)u(t-a)\}$	$e^{-as}\mathcal{L}\{g(t+a)\}$
$\mathcal{L}\{t^n f(t)\}$	$(-1)^n \frac{d^n}{ds^n}F(s)$

- PRIMITIVAS O ANTIDERIVADAS.

La derivada es la operación inversa de la integral indefinida y viceversa. Es decir, la derivada y la integral indefinida son operaciones mutuamente inversas.

$$\int \frac{d}{dx}(f(x))\,dx = f(x) \qquad \frac{d}{dx}\int f(x)\,dx = f(x)$$

Reglas basicas de las derivadas.

$$\frac{d}{du}(C) = 0 \qquad \int 0\,dx = C$$

$$\frac{d}{du}(\text{sen}\,u) = \cos u \qquad \int \cos u\,du = \text{sen}\,u + C$$

$$\frac{d}{du}\cos u = -\text{sen}\,u \qquad \int \text{sen}\,u\,du = -\cos u + C$$

$$\frac{d}{du}\tan u = \sec^2 u \qquad \int \sec^2 u\,du = \tan u + C$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \qquad \int \sec u \tan u\,du = \sec u + C$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \qquad \int \csc u \cot u = -\csc u + C$$

$$\frac{d}{du}e^u = e^u \qquad \int e^u\,du = e^u + C$$

$$\frac{d}{du}\ln u = \frac{1}{u} \qquad \int \frac{1}{u}\,du = \ln|u| + C$$

- Tecnicas de integración

El objetivo de las tecnicas de integración o artificios de integración es el de llevar a la integral a su forma primitiva.

① INTEGRACIÓN POR CAMBIO DE VARIABLE. (20 Ejercicios)

- $\int \cos(2x)\,dx = \frac{1}{2}\int \cos 2x\; 2x\,dx = \frac{1}{2}\int \cos u\,du = \frac{1}{2}\text{sen}\,u + C = \underline{\frac{1}{2}\text{sen}(2x) + C}$

$u = 2x$
$du = 2dx$

- $\int x^2 e^{x^3}\,dx = \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \underline{\frac{1}{3}e^{x^3} + C}$

$u = x^3$
$du = 3x^2$

$\bullet \int \frac{3x^2+1}{\sqrt{x^3+x+1}}\,dx = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C = 2\sqrt{x^3+x+1} + C$

$u = x^3+x+1$
$du = (3x^2+1)\,dx$

$\bullet \int \frac{dx}{x\ln x} = \int \frac{du}{u} = \ln|u| + C$

$u = \ln x$
$du = \frac{dx}{x}$

$\bullet \int \tan x\,dx = -\int \frac{-\operatorname{sen} x}{\cos x}\,dx = -\int \frac{du}{u} = -\ln|u| + C$

$u = \cos x$
$du = -\operatorname{sen} x\,dx$

$= -\ln|\cos x| + C$
$= \ln|\cos^{-1}| + C$
$= \ln|\sec x| + C$

$\bullet \int a^x\,dx = \int e^{x\ln a}\,dx = \frac{1}{\ln a}\int e^{x\ln a}(\ln a)\,dx = \frac{1}{\ln a}\int e^u\,du = \frac{1}{\ln a}e^u + C$

$a^x = e^{\ln a^x} = e^{x\ln a}$

$u = x\ln a$
$du = \ln a\,dx$

$= \frac{1}{\ln a}e^{x\ln a} + C$
$= \frac{a^x}{\ln a} + C \quad a > 0,\ a \neq 1$

$\bullet \int \frac{dx}{1-e^{-x}} = \int \frac{dx}{1-e^{-x}}\left(\frac{e^x}{e^x}\right) = \int \frac{e^x\,dx}{e^x-1} = \int \frac{du}{u} = \ln|u| + C = \ln|e^x-1| + C$

$u = e^x - 1$
$du = e^x\,dx$

$\int a^x e^x\,dx$, $\int \csc x\,dx = \ln|\csc x + \cot x| + C$ ✓

$\bullet \int \frac{ae^\theta + b}{ae^\theta - b}\,d\theta = \int \frac{ae^\theta\,d\theta}{ae^\theta - b} + \int \frac{b\,d\theta}{ae^\theta - b}\left(\frac{e^{-\theta}}{e^{-\theta}}\right) = \int \frac{du}{u} + b\int \frac{e^{-\theta}\,d\theta}{a - e^{-\theta}b}$

$= \int \frac{du}{u} - \frac{\cancel{b}}{\cancel{b}}\int \frac{dv}{v} = \ln|u| - \ln|v| + C$

$u = ae^\theta - b$
$du = ae^\theta$
$v = a - e^{-\theta}b$
$dv = -e^{-\theta}b$

$= \ln\left|\frac{u}{v}\right| + C = \ln\left|\frac{ae^\theta - b}{a - e^{-\theta}b}\right| + C$

$\bullet \int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\int \sec x\,dx = \int \sec x\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)dx = \int \frac{\sec^2 x + \tan x\sec x}{\sec x + \tan x}\,dx$

$= \int \frac{du}{u} = \ln|u| + C$

$u = \sec x + \tan x$
$du = (\sec x\tan x + \sec^2 x)$

$= \ln|\sec x + \tan x| + C$

$\int \frac{dx}{1+\operatorname{sen} x}$; $\int \frac{dx}{1+\cos x}$; $\int \frac{dx}{1-\cos x}$

$$\int \frac{dx}{1-\operatorname{sen}x} = \int \frac{dx}{1-\operatorname{sen}x}\left(\frac{1+\operatorname{sen}x}{1+\operatorname{sen}x}\right) = \int \frac{(1+\operatorname{sen}x)\,dx}{1-\operatorname{sen}^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{\operatorname{sen}x\,dx}{\cos^2 x}$$

$$= \int \sec^2 x\,dx + \int \frac{1}{\cos x}\,\frac{\operatorname{sen}x}{\cos x}\,dx = \tan x + \int (\sec x \tan x) + C$$

$$= \underline{\tan x + \sec x + C}$$

② INTEGRACIÓN POR PARTES. (20 Ejercicios).

Pensamos en la aplicación de esta técnica cuando se observa en el Integrando un producto de una función por el diferencial de otra.

Teorema: Sean $u = f(x)$, $v = g(x)$; y $f'(x)$, $g'(x)$ funciones continuas, entonces.

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Teorema: De la regla de producto:

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)g'(x) + g(x)f'(x) \ldots ①$$

De ① $\longrightarrow$ $f(x)\cdot g'(x) = \frac{d}{dx}\left(f(x)\cdot g(x)\right) - g(x)f'(x) \ldots ②$

Integrando ambs miembros de ② con respecto de x, resulta.

$$\int f(x)g'(x)\,dx = \int \frac{d}{dx} f(x)g'(x)\,dx - \int g(x)f'(x)\,dx$$

Simplificando $\rightarrow$ $\int f(x)g'(x)\,dx = f(x)g'(x) - \int g(x)\cdot f'(x)\,dx$

$u = f(x)$ $\quad v = g(x)$

$du = f'(x)\,dx$ $\quad dv = g'(x)\,dx$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$\int x^n f(x)\,dx \Rightarrow u = x^n \quad dv = f(x)\,dx$$

- $\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx = x\tan x + \int \frac{-\operatorname{sen}x\,dx}{\cos x}$

  $u = x$ $\quad dv = \sec^2 x\,dx$

  $du = dx$ $\quad v = \tan x$

  $= \underline{x\tan x + \ln|\cos x| + C}$

- $\int x^2 e^{2x}\,dx = x^2 \frac{e^{2x}}{2} - \int \frac{e^{2x}}{\cancel{2}} \cancel{2}x\,dx = x^2\frac{e^{2x}}{2} - \left(x\frac{e^{2x}}{2} - \frac{1}{2}\int e^{2x}\,dx\right)$

  $u = x^2$ $\quad du = 2x\,dx$

  $dv = e^{2x}\,dx$ $\quad v = \frac{1}{2}e^{2x}$

  $w = x$ $\quad dw = dx$

  $dz = e^{2x}\,dx$

  $z = \frac{1}{2}e^{2x}$

  $= \underline{\frac{x^2 e^{2x}}{2} - \frac{xe^{2x}}{2} + \frac{1}{4}e^{2x} + C}$

$$\bullet \int \frac{x^3\,dx}{\sqrt{x^2+4}} = \int x^2 \frac{x\,dx}{\sqrt{x^2+4}} = 2x^2\sqrt{x^2+4} - \int 2\sqrt{x^2+4}\;2x\,dx$$

$v = x^2+4$
$dv = 2x\,dx$

$u = x^2 \quad du = 2x\,dx$

$dv = \frac{x\,dx}{\sqrt{x^2+4}} \quad v = 2\sqrt{x^2+4}$

$w = x^2+4$
$dw = 2x$

$$= 2x^2\sqrt{x^2+4} - 2\int v^{1/2}\,dv$$

$$= 2x^2\sqrt{x^2+4} - 2\frac{v^{3/2}}{\frac{3}{2}} + C$$

$$= 2x^2\sqrt{x^2+4} - \frac{4}{3}v^{3/2} + C$$

$$= 2x^2\sqrt{x^2+4} - \frac{4}{3}\sqrt{(x^2+4)^3} + C$$

$$\bullet \int x^n \ln|x|\,dx = \ln|x|\frac{x^{n+1}}{n+1} - \int \frac{x^{\cancel{n+1}}}{n+1}\frac{dx}{\cancel{x}} = \ln|x|\frac{x^{n+1}}{n+1} - \int \frac{x^n\,dx}{n+1}$$

$u = \ln|x| \quad du = \frac{dx}{x}$

$dv = x^n\,dx \quad v = \frac{x^{n+1}}{n+1}$

$$= \ln|x|\frac{x^{n+1}}{n+1} - \frac{1}{n+1}\int x^n\,dx = \ln|x|\frac{x^{n+1}}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C$$

¡$uv| - vu = vbu|$!

$$= \frac{x^{n+1}}{n+1}\left(\ln|x| - \frac{1}{n+1}\right) + C$$

$$\bullet \int \ln|x|\,dx = x\ln|x| - \int \cancel{x}\frac{dx}{\cancel{x}} = x\ln|x| - \int dx = x\ln|x| - x + C$$

$$= x(\ln|x| - 1) + C$$

$u = \ln|x| \quad du = \frac{dx}{x}$

$dv = dx \quad v = x$

$$\int e^{ax} f(x)\,dx$$

$u = f(x) \quad dv = e^{ax}\,dx$

¡$uv| - vu = vbu|$!

$$\bullet \int e^x \cos x\,dx = e^x\cos x + \int e^x \operatorname{sen} x\,dx = e^x\cos x + \left(\operatorname{sen} x\, e^x - \int e^x \cos x\,dx\right)$$

$u = \cos x \quad du = -\operatorname{sen} x\,dx$

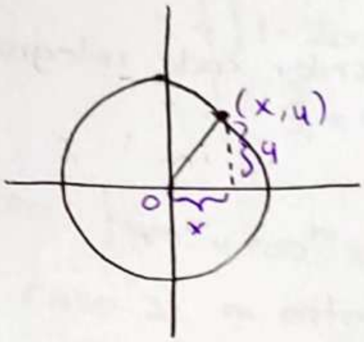
$dv = e^x\,dx \quad v = e^x$

$$\int e^x \cos x\,dx = \frac{e^x\cos x + \operatorname{sen} e^x}{2}$$

$u = \operatorname{sen} x \quad du = \cos x\,dx$

$dv = e^x dx \quad v = e^x$

FUNCIONES TRIGONOMETRICAS. (* Portafolio)



$$\operatorname{sen}\theta = \frac{co}{h} = \frac{y}{r} \Rightarrow y = r\operatorname{sen}\theta$$

$$\cos\theta = \frac{ca}{h} = \frac{x}{r} \Rightarrow x = r\cos\theta$$

$$\tan\theta = \frac{co}{ca} = \frac{y}{x} \Rightarrow \frac{r\operatorname{sen}\theta}{r\cos\theta} = \frac{\operatorname{sen}\theta}{\cos\theta}$$

$$\sec\theta = \frac{h}{ca} = \frac{r}{x} = \frac{r}{r\cos\theta} = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{h}{co} = \frac{r}{y} = \frac{r}{r\operatorname{sen}\theta} = \frac{1}{\operatorname{sen}\theta}$$

$$\cot\theta = \frac{ca}{co} = \frac{x}{y} = \frac{r\cos\theta}{r\operatorname{sen}\theta} = \frac{\cos\theta}{\operatorname{sen}\theta}$$

Por el teorema de Pitágoras.

$$x^2 + y^2 = r^2$$

Sustituyendo

$$\cancel{r^2}\cos^2\theta + \cancel{r^2}\operatorname{sen}^2\theta = \cancel{r^2}$$

$$\cos^2\theta + \operatorname{sen}^2\theta = 1 \ldots (1)$$

Multi. (1) por $\frac{1}{\cos^2\theta}$

$$1 + \tan^2\theta = \sec^2\theta$$

Multi. (1) por $\frac{1}{\operatorname{sen}^2\theta}$

$$\cot^2\theta + 1 = \csc^2\theta$$

$\int \sec^5 x\,dx$; $\int \cos^n x\,dx$.

- $\int \sec^3 x\,dx = \int \sec x \sec^2 x\,dx = \sec x\tan x - \int \tan^2 x\sec x\,dx$

$u = \sec x \quad du = \sec x\tan x\,dx$

$dv = \sec^2 x\,dx \quad v = \tan x$

$$= \sec x\tan x - \int \sec^3 x\,dx - \int \sec x\,dx$$

$$\int \sec^3 x\,dx = \frac{\sec x\operatorname{tg} x + \ln|\sec x + \operatorname{tg} x| + C}{2}$$

- Encuentra una fórmula para:

$$\int \operatorname{sen}^n x\,dx = \int \operatorname{sen}^{n-1}x\operatorname{sen}x\,dx = -\operatorname{sen}^{n-1}x\cos x + \int (n-1)\operatorname{sen}^{n-2}x\cos^2 x\,dx$$

$u = \operatorname{sen}^{n-1} x \quad du = (n-1)\operatorname{sen}^{n-2}x\cos x$

$dv = \operatorname{sen} x\,dx \quad v = -\cos x$

$$= -\operatorname{sen}^{n-1}x\cos x + (n-1)\int \operatorname{sen}^{n-2}x(1 - \operatorname{sen}^2 x)\,dx$$

$$= -\cos x\operatorname{sen}^{n-1}x + \int(n-1)\int \operatorname{sen}^{n-2}x\,dx - (n-1)\int \operatorname{sen}^n x\,dx$$

$$\int \operatorname{sen}^n x\,dx = \frac{1}{2}\left[-\cos x\operatorname{sen}^{n-1}x + (n-1)\int \operatorname{sen}^{n-2}x\,dx\right]$$

$$\int \operatorname{sen}^n x\,dx = -\frac{\cos x\operatorname{sen}^{n-1}}{n} + \frac{n-1}{n}\int \operatorname{sen}^{n-2}x\,dx.$$

③ INTEGRALES CON FUNCIONES TRIGONOMETRICAS.

Integrales tipo $\int sen^n x\, dx$; $\int cos^n x\, dx$

CASO I. Si n es entero positivo impar reescribir cada integral.

$$\int \underbrace{sen^{n-1}x}_{(1-cos^2x)}\, sen x\, dx \; ; \quad \int \underbrace{cos^{n-1}x}_{(1-sen^2x)}\, cos x\, dx$$

- $\int cos^5 x\, dx = \int cos^4 x\, cos x\, dx = \int (cos^2 x)^2 cos x\, dx$

$$= \int (1-sen^2x)^2 cos x\, dx = \int (1-2sen^2x + sen^4x)$$

$u = sen x \quad du = cos x\, dx$

$$= \int (1-2u^2+u^4)\, du = u - \frac{2}{3}u^3 + \frac{u^5}{5} + C$$

$$= sen x - \frac{2}{3} sen^3 x + \frac{sen^5 x}{5} + C$$

CASO II Si n es positivo y es par usar de manera directa la identidad para el coseno del doble ángulo.

$cos^2\theta + sen^2\theta = 1$

$cos(A \pm B) = cos A cos B \mp sen A sen B$

$sen(A \pm B) = sen A cos B \pm cos A sen B$

$$Cos\, 2\theta = Cos(\theta+\theta) = cos\theta\, cos\theta - sen\theta\, sen\theta$$
$$= cos^2\theta - sen^2\theta$$
$$= (1-sen^2\theta) - sen^2\theta$$
$$= 1 - 2sen^2\theta$$

$$\boxed{sen^2\theta = \frac{1-cos 2\theta}{2}}$$

$$cos^2\theta - (1-cos^2\theta)$$

$$cos 2\theta = 2cos^2\theta - 1$$

$$\boxed{cos^2\theta = \frac{1+cos 2\theta}{2}}$$

$\bullet \int sen^4 x\, dx = \int (sen^2 x)^2 dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int (1-2\cos 2x + \cos^2 2x) dx$

$= \frac{1}{4}\int 1 - 2\cos 2x + \left(\frac{1+\cos 4x}{2}\right) dx = \frac{1}{4}\int \left(\frac{3}{2} - 2\cos 2x + \frac{1}{4}\cos 4x\right) dx$

$= \frac{1}{4}\left(\frac{3}{2}x - sen 2x + \frac{1}{16}\cos 4x\right) + C$

TIPO $\int sen^n x \cos^m x\, dx$

CASO I m entero positivo impar, entonces

$$\int sen^n x \underbrace{\cos^{m-1} x}_{1-sen^2 x} \cos x\, dx$$

$\int sen^4 x \cos^3 x\, dx = \int sen^4 x \cos^2 x \cos x\, dx = \int sen^4 x (1 - sen^2 x) \cos x\, dx$

$= \int (sen^4 x - sen^6 x) \cos x\, dx = \int (u^4 - u^6) du$

$u = sen x \quad du = \cos x\, dx \qquad = \frac{u^5}{5} - \frac{u^7}{7} + C = \frac{sen^5 x}{5} + \frac{sen^7 x}{7} + C$

CASO II n entero positivo impar entonces

$$\int \underbrace{sen^{n-1}}_{1-\cos^2 x} x \cos x\, sen x\, dx$$

$\int sen^3 x \cos^4 x\, dx = \int sen^2 x \cos^4 x\, sen x\, dx = \int (1-\cos^2 x) \cos^4 x\, sen x\, dx$

$= \int (\cos^4 x - \cos^6 x)\, sen x\, dx = -\int (u^4 - u^6) du$

$u = \cos x$

$du = -sen x\, dx \qquad = -\frac{u^5}{5} + \frac{u^7}{7} + C = -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{5} + C$

CASO III m y n enteros positivos pares entonces utilizar las identidades.

$$sen^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$\cdot \int sen^2 x \cos^2 x\, dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx = \frac{1}{4}\int (1 - \cos^2 2x) dx$

$= \frac{1}{4}\int \left[1 - \left(\frac{1+\cos^2 4x}{2}\right)\right] dx = \frac{1}{4}\int \left[\frac{3}{4} + \frac{\cos 4x}{8}\right] dx$

$= \frac{1}{4}\left(\frac{3}{4}x - \frac{sen 4x}{8}\right) + C$

Tipo $\int tg^n x \sec^m x\, dx$

CASO I m entero positivo par, entonces:

$$\int \tan^n x \sec^{m-2} x \sec^2 x\, dx$$
$$\quad\quad\quad\quad 1+\tan^2 x$$

$\cdot \int tg^5 x \sec^4 x\, dx = \int tg^5 x \sec^2 x \sec^2 x\, dx = \int tg^5 x(1+tg^2 x)\sec^2 x\, dx$

$= \int (tg^5 x + tg^7 x)\sec^2 x\, dx = \int (u^5+u^7)\, du = \frac{u^6}{6} + \frac{u^8}{8} + C$

$u = tg x \quad du = \sec^2 x$

$$= \frac{tg^6 x}{6} + \frac{tg^8 x}{8} + C$$

CASO II n entero positivo impar entonces...

$$\int \tan^{n-1} x \sec^{m-1} \sec x \tan x\, dx$$
$$\sec^2 x - 1$$

$\int tg^3 x \sec^3 x\, dx = \int tg^2 x \sec^2 x\, tg x \sec x\, dx$

$= \int (\sec^2 x - 1)\sec^2 x\, tg x \sec x\, dx = \int (\sec^4 x - \sec^4 x)\sec x\, tg x\, dx$

$u = \sec x \quad = \int (u^4 - u^2)\, du = \frac{u^5}{5} - \frac{u^3}{3} + C$

$du = \sec x\, tg x$

$$= \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

CASO III: n potencia par y m potencia impar usar integración por partes.

$\int tg^2 x \sec^3 x\, dx$ (Tarea)

- Deducir 3 casos par integrales tipo $\int ctg^n x \csc^m x\, dx$ y un ejemplo por caso

* $\int \text{sen}\, nx \cos mx\, dx$
* $\int \cos nx\, \text{sen}\, mx\, dx$
* $\int \cos nx \cos mx\, dx$
* $\int \text{sen}\, nx\, \text{sen}\, mx\, dx$

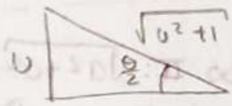
$$\cos(A+B) = \cos A \cos B - \operatorname{sen} A \operatorname{sen} B$$
$$\cos(A-B) = \cos A \cos B + \operatorname{sen} A \operatorname{sen} B$$

$$\cos(A+B) + \cos(A-B) = 2\cos A \cos B$$
$$\frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right] = \cos A \cos B$$
$$\frac{1}{2}\left[\int \cos(n+m)x\,dx + \int \cos(n-m)x\,dx\right]$$

- $\displaystyle\int \frac{\sec x}{tg^2 x}\,dx = \int \frac{\frac{1}{\cos x}}{\frac{\operatorname{sen}^2 x}{\cos^2 x}}\,dx = \int \frac{\cos x}{\operatorname{sen}^2 x}\,dx = \int \csc x \cot x = \underline{-\csc x + C}$
- $\displaystyle\int \frac{dx}{\operatorname{sen} ax + b} = \frac{1}{a}\int \frac{a\,dx}{\operatorname{sen} ax} + b = \frac{1}{a}\int \frac{d\theta}{\operatorname{sen}\theta + b}$

$\theta = ax \Rightarrow d\theta = a\,dx$

Sea $v = tg\left(\frac{\theta}{2}\right)$



$$\operatorname{sen}\left(\frac{\theta}{2}\right) = \frac{co}{h} = \frac{u}{\sqrt{1+u^2}}$$
$$\cos\left(\frac{\theta}{2}\right) = \frac{ca}{h} = \frac{1}{\sqrt{1+v^2}}$$

$$\cos(\theta) = \cos\left(\frac{\theta}{2} + \frac{\theta}{2}\right) = \cos\frac{\theta}{2}\cos\frac{\theta}{2} - \operatorname{sen}\frac{\theta}{2}\operatorname{sen}\frac{\theta}{2}$$
$$= \cos^2\frac{\theta}{2} - \operatorname{sen}^2\frac{\theta}{2} = \frac{1}{1+u^2} - \frac{u^2}{1+u^2} = \frac{1-u^2}{1+u^2}$$
$$\operatorname{sen}(\theta) = \operatorname{sen}\left(\frac{\theta}{2} + \frac{\theta}{2}\right) = \operatorname{sen}\frac{\theta}{2}\cos\frac{\theta}{2} + \cos\frac{\theta}{2} + \cos\frac{\theta}{2}\operatorname{sen}\frac{\theta}{2}$$
$$= 2\left[\frac{v}{\sqrt{1+u^2}}\right]\left[\frac{1}{\sqrt{1+v^2}}\right] = \underline{\frac{2v}{1+u^2}}$$

si $u = tg\frac{\theta}{2}$ $\quad du = \frac{1}{2}\sec^2\theta\,d\theta$ $\quad dv = \frac{1+v^2}{2}\,d\theta$
$$= \frac{1}{2}\frac{1}{\cos^2\frac{\theta}{2}}\,d\theta = \frac{\frac{1}{2}}{\frac{1}{1+u^2}}\,d\theta$$
$$\frac{1}{a}\int \frac{a\,dx}{\operatorname{sen} ax + b} = \frac{1}{a}\int \frac{\frac{2\,du}{1+u^2}}{\frac{2v}{1+u^2} + b} = \frac{1}{a}\int \frac{dv}{b(1+u^2)} = \frac{1}{a}\int \frac{dv}{b + \quad + bv^2}$$

(4) INTEGRACIÓN POR CAMBIO DE VARIABLE TRIGONOMETRICO.

Pensamos en la aplicación de la forma

$$(a^2-u^2)^n \; ; \; (a^2+u^2) \; ; \; (u^2-a^2)^n \quad n, a \in \mathbb{R}$$

Simplificar estos terminos por medio de un cambio de variable trigonometrico, el cual surge en forma natural a partir de las sumas o diferencias de cuadrados.

$\cos^2\theta + \operatorname{sen}^2\theta = 1$
$1 - \operatorname{sen}^2\theta = \cos^2\theta$
$a^2 - a^2\operatorname{sen}^2\theta = a^2\cos^2\theta$

$a^2 - u^2$
$u = a\operatorname{sen}\theta$

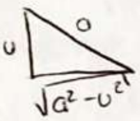
$1 + tg^2\theta = \sec^2\theta$
$a^2 + a^2 tg^2\theta = a^2\sec^2\theta$
$a^2 + u^2$
$u = a\,tg\,\theta$

$\sec^2\theta - 1 = \tan^2\theta$
$a^2\sec^2\theta - a^2 = a^2\tan^2\theta$
$u^2 - a^2$
$u = a\sec\theta$

CASO ESPECIAL: $n = \frac{1}{2}$

CASO I : $\sqrt{a^2-u^2}$

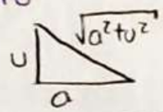
$u = a\operatorname{sen}\theta$
$du = a\cos\theta\, d\theta$



$\sqrt{a^2-a^2\operatorname{sen}^2\theta} =$
$\sqrt{a^2(1-\operatorname{sen}^2\theta)} = \sqrt{a^2\cos^2\theta} = a\cos\theta$

CASO II: $\sqrt{a^2+u^2}$

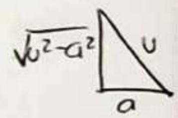
$u = a\,tg\,\theta$
$du = a\sec^2\theta\, d\theta$



$\sqrt{a^2+a^2tg^2\theta} = \sqrt{a^2(1+tg^2\theta)}$
$= \sqrt{a^2\sec^2\theta} = a\sec\theta$

CASO III : $\sqrt{u^2-a^2}$

$u = a\sec\theta$
$du = a\,tg\theta\sec\theta\, d\theta$



$\sqrt{a^2\sec^2\theta - a^2}$
$= \sqrt{a^2(\sec^2\theta-1)} = \sqrt{a^2tg^2\theta}$
$= a\,tg\,\theta$

- $\displaystyle\int \frac{dx}{x^2\sqrt{16-x^2}} = \int \frac{4\cos\theta\, d\theta}{16\operatorname{sen}^2\theta\, 4\cos\theta} = \frac{1}{16}\int \csc^2\theta = -\frac{1}{16}\,ctg\,\theta + C$
$\displaystyle = -\frac{1}{16}\left(\frac{\sqrt{16-x^2}}{x}\right) + C$

$x = 4\operatorname{sen}\theta$
$dx = 4\cos\theta\, d\theta$

- $\displaystyle\int \frac{dx}{\sqrt{4+x^2}} = \int \frac{2\sec^2\theta\, d\theta}{2\sec\theta} = \int \sec\theta\, d\theta = \ln|\sec\theta + tg\theta| + C$
$\displaystyle = \ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| + C$

$u = 2\,tg\,\theta$
$du = 2\sec^2\theta\, d\theta$

Luego $\sqrt{4+4tg^2\theta} = \sqrt{4(1+tg^2\theta)} = \sqrt{4\sec^2\theta} = 2\sec\theta$

$\displaystyle\int \frac{\sqrt{x^2-9}}{x}\,dx = \int \frac{3tg\theta\; 3tg\theta\sec\theta\, d\theta}{3\sec\theta} = \int 3tg^2\theta\, d\theta = 3\int(\sec^2\theta - 1)\, d\theta$
$\displaystyle = 3(tg\theta - \theta) + C = 3\left(\frac{\sqrt{x^2-9}}{x} - \sec^{-1}\left(\frac{x}{3}\right)\right) + C$

$x = 3\sec\theta$
$dx = 3\,tg\theta\sec\theta\, d\theta$

$\sqrt{x^2-9} = \sqrt{9\sec^2\theta - 9} = \sqrt{9(\sec^2\theta - 1)} = \sqrt{9tg^2\theta} = 3tg\theta$

$$\bullet \int \frac{dx}{\operatorname{sen}(ax)+b} = \frac{1}{a}\int \frac{d\theta}{\operatorname{sen}\theta + b}$$

$\theta = ax \Rightarrow d\theta = a\,dx$

$u = tg\left(\frac{\theta}{2}\right)$

$$= \frac{2}{ab}\int \frac{du}{u^2 + \frac{2}{b}u + 1} = \frac{2}{ab}\int \frac{du}{\left(u^2+\frac{2}{b}u+\frac{1}{b^2}\right) + \left(1-\frac{1}{b}\right)}$$

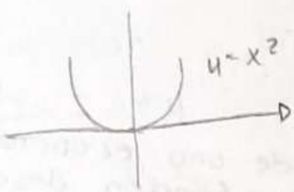
$$= \frac{2}{ab}\int \frac{du}{\left(u+\frac{1}{b}\right)^2 + \left(1-\frac{1}{b^2}\right)} \qquad \begin{matrix} w = u + \frac{1}{b} \\ dw = du \end{matrix} \qquad \alpha^2 \left(1 - \frac{1}{b^2}\right)$$

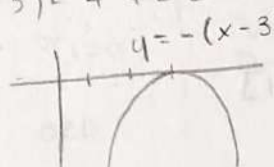
$$\frac{2}{ab}\int \frac{dw}{w^2+\alpha^2} = \frac{2}{ab\alpha} tg^{-1}\left(\frac{w}{\alpha}\right) + C$$

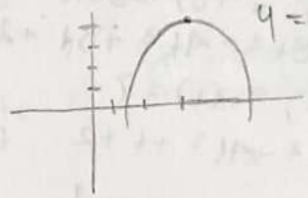
Investigar metodo de fracciones parciales.

Objetivo. Graficar función de

$$y(x) = -x^2 + 6x - 5$$
$$= (-(x+3)^2 - (3)^2 - 5 = 0$$
$$-(x-3)^2 + 4 = 0$$

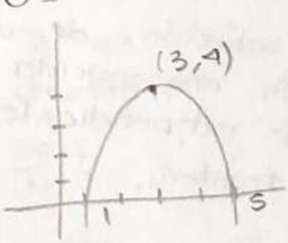






Los puntos de intersección de la grafica en el eje x ocurren cuando $y(x) = 0$, esto es:

$$0 = -x^2 + 6x - 5$$



Puntos de intersección de $q(x)$ son

$x_1 = 1 \qquad x_2 = 5$

Esto representa la solución de la ecuación.

Tarea.

Sea $ax^2 + bx + C = 0 \ldots (1)$

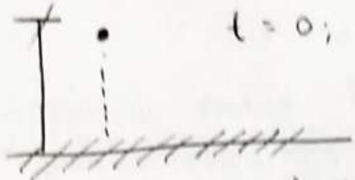
Demuestra que:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ es solución de (1)

CASO I: si $b^2 - 4ac > 0$; CASO II si $b^2 - 4ac < 0$; CASO III si $b^2 - 4ac = 0$

Ejemplo.

Caída libre de un objeto de masa m.



$t=0;\ y=h;\ v=0$

$F=ma$

$F=-mg=ma$

$-g=a$

$\frac{d^2y}{dt^2}=-g \qquad \int \frac{d}{dt}\left(\frac{dy}{dt}\right)dt=-g\int dt$

$v(t)=\frac{dy}{dt}=-gt+C_1$

$v(0)=0 \qquad 0=-g(0)+C_1 \qquad a=0$

$\frac{dy}{dt}=-gt \qquad y=\frac{1}{2}gt^2+C_2$

$y(0)=h$

$h=\frac{g(0)^2}{2}+C_2$

$y(t)=h-\frac{gt^2}{2}$

$t\in[0,t_1]$

$a(t)=16-24t$

a) $x(0)=2;\ x'(0)=3$

$x(t)=8t^2-4t^3+3t+2 \qquad t\in[0,\infty) \to$ inicial

b) $x(0)=1;\ x(1)=7$

$x(t)=8t^2-4t^3+t+2 \qquad t\in[0,1]$

Problema de valor inicial.

Es un problema que busca determinar la solución de una ecuación diferencial que está sujeta a condiciones sobre la función desconocida y sus derivadas están sujetas a un único valor en la variable independiente tales condiciones se llaman soluciones iniciales.

Problema de valor final

Es un problema que busca determinar la solución de una ecuación dif. sujeta a condiciones sobre la función desconocida y están sujetas a más de un valor en la variable independiente, tales como las soluciones, se llaman soluciones de la frontera.

UNIDAD I

Ecuación Diferencial.

Una ecuación diferencial es aquella que su estructura las derivadas de una función desconocida o variable dependiente con respecto de una o mas variables independientes.

Dada la definición anterior, podemos clasificar a las ec. dif. de acuerdo con lo siguiente.

Si la función desconocida o variable dependiente, depende de una sola variable, entonces la ecuación diferencial se llama "ecuación diferencial de tipo ordenada"

Sin embargo

- Si la función desconocida o variable dependiente depende de mas de una variable independiente se llama "ecuación dif. parcial"

Ecuación	Tipo	Variable Dependiente	Variable Independiente.
$\frac{dy}{dx} = 2x + y$	ORD.	y	x
$\frac{\partial^2 y}{\partial x^2} + 2\frac{\partial^2 v}{\partial y^2} = v$	PARCIAL	v	x, y
$y''' + 2(y'')^2 + y' = \cos x$	ORD	y	x.

Notación: Si $y = f(x)$ es una función doble de x, entonces se denotan por:

$$\frac{df(x)}{dx}\ ;\ \frac{dy}{dx}\ ;\ y'\ ;\ D \equiv \frac{d}{dx}$$

$$\frac{d^2f(x)}{dx^2}\ ;\ \frac{d^2y}{dx^2}\ ;\ y''\ ;\ D^2y$$

$$\vdots$$

$$\frac{d^nf(x)}{dx^n}\ ;\ \frac{d^ny}{dx^n}\ ;\ y^{(n)}\ ;\ D^ny$$

• Orden de una ecuación diferencial.

El orden que se asigna a una ecuación diferencial es el índice que aparece indicando en la derivada más alta.

$$\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 - 4y = e^x$$

• Grado de una ecuación diferencial.

El grado que se asigna a una ecuación diferencial es la potencia a la que aparece elevada la derivada mayor

	Tipo	VP	VI	Ord.	Grado
$(y'')^2 + (y')^3 + 3y = x^2$	Ord.	y	x	2	2

Ecuación dif. lineal de orden n

Una ecuación dif. lineal ordinaria de orden n es aquella que se expresa de la forma general siguiente.

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_0(x)y = f(x) \quad \dots (1)$$

$a_n(x), a_{n-1}(x), \dots, a_1(x), a_0(x)$ $\qquad a_n(x) \neq 0$

Son coeficientes de (1)

En (1) se pueden observar las siguientes características.

1. Coeficiente $a_n(x), \dots a_0(x)$ y $F(x)$ son funciones dadas de la variable independiente x.
2. En (1) la función desconocida y & sus derivadas están elevadas a la potencia uno.
3. En (1) no aparecen funciones trascendentes en cuyo argumento esté involucrada la función desconocida y y/o sus derivadas.

Ej.

	T.	Or.	Gr.	VD.	V.I.	L
$x^3\frac{d^3y}{dx^3} - \frac{dy}{dx} + 6y = e^x$	ORD	3	1	y	x	Si
$y''' + x(y'')^2 + 2y(y') + xy = 0$	ORD	3	1	y	x	No
$\frac{d^2\theta}{dt} + \frac{g}{l}\operatorname{sen}\theta = 0$	ORD	2	1	θ	t	No

Zill Pag 9 6ta Ed Sección 1.1 1-10

SOLUCIÓN DE UNA ECUACIÓN DIFERENCIAL.

La solución de una ecuación diferencial es una función o relación que la satisface, es decir, es una función o relación que la reduce a una identidad.

$y = y(t)$ Sol explícita

$\phi(x, y) = 0$ Sol implícita.

Solución explícita.

Es una función o relación que satisface a una ecuación diferencial y en la cual la variable dependiente, queda expresada explícitamente como una función de la variable independiente

$$y(x) = C_1 \cos x + C_2 \operatorname{sen} x + \ln|x| \; ; \quad y = y(x)$$

$$y(t) = h - \frac{gt^2}{2} \; ; \quad y = y(t)$$

Compruebe la q función dada es solución de la correspondiente ecuación diferencial.

$$y' + y = \operatorname{sen}(x) \ldots (1) \Rightarrow y = \frac{1}{2}\operatorname{sen} x - \frac{1}{2}\cos x + 10e^{-x}$$

$$y' = \frac{1}{2}\cos x + \frac{1}{2}\operatorname{sen} x + 10e^{-x}$$

sust. (1) con y y y'

$$\frac{1}{2}\operatorname{sen} x - \cancel{\frac{1}{2}\cos x} + \cancel{10e^{-x}} + \cancel{\frac{1}{2}\cos x} + \frac{1}{2}\operatorname{sen} x - \cancel{10e^{-x}} = \operatorname{sen} x$$

$\operatorname{sen} x = \operatorname{sen} x$ Identidad

$\therefore y(x) = \frac{1}{2}\operatorname{sen} x - \frac{1}{2}\cos x + 10e^{-x}$ Es solución explícita de (1)

$$y'' - 6y' + 13y = 0 \; ; \qquad y = e^{3x}\cos 2x$$

$$y' = 3e^{3x}\cos 2x - 2e^{3x}\operatorname{sen} 2x$$

$$y'' = 9e^{3x}\cos 2x - 12e^{3x}\operatorname{sen} 2x - 4e^{3x} x \operatorname{sen} 2x$$

Sustituyendo

$$9e^{3x}\cos 2x - 12e^{3x}\operatorname{sen} 2x - 4e^{3x} x \operatorname{sen} 2x - 6(3e^{3x}\cos 2x - 2e^{3x}\operatorname{sen} 2x)$$
$$+ 13(e^{3x}\cos 2x) = 0$$

$$9e^{3x}\cos 2x - 16e^{3x}\operatorname{sen} 2x - 18e^{3x}\cos 2x + 12e^{3x}\operatorname{sen} 2x$$
$$+ 13e^{3x}\cos 2x = 0$$

$y'' - 6y' + 13y = 0$ Identidad

$y(x) = e^{3x}\cos 2x$ es solución implícita.

$$\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial y^2} = u \quad ; \qquad u(x,y) = e^{3x}\,\text{sen}\,2y$$

$$\frac{\partial^2 u}{\partial x^2} = 9e^{3x}\text{sen}2y \qquad \frac{\partial^2 u}{\partial y} = -4e^{3x}\,\text{sen}\,2y$$

$$9e^{3x}\,\text{sen}2y + 2(4\,e^{3x}\,\text{sen}2y) = v$$

$$e^{3x}\text{sen}2y = e^{3x}\,\text{sen}2y \quad \underline{\text{identidad}}$$

$$e^{3x}\,\text{sen}2y = v\,(x,y) \text{ solución explícita de } (1)$$

Solución implícita.

Es una función o relación que satisface a una ec. dif. y en cuya estructura están involucrados tanto la variable dependiente como la independiente. En dado caso, decimos que la variable ind. está implícitamente como una función de la variable indepen diente.

$$x^2 + y^2 - 4 = 0 \ldots (1)$$

$$\phi(x,y) = 0$$

$$y'' + 2y(y')^3 = 0 \ ; \qquad y^3 - 3x + 3y = 5$$

$$3y^2y' - 3 + 3y' = 0$$

$$-y'(y^2+1) + 1$$

$$y' = \frac{1}{y^2+1}$$

$$y'' = \frac{-2y\,y'}{(y^2+1)^2} = \frac{-2y\,\frac{1}{y^2+1}}{(y^2+1)} = -\frac{2y}{(y^2+1)^3}$$

$$-\frac{2y}{(y^2+1)^3} + 2y\left(\frac{1}{(y^2+1)^3}\right) = 0$$

$$0 = 0$$

$$y^3 - 3x + 3y = 5 \longrightarrow \text{Sol. implícita de } (1)$$

Tarea 11-40 ⟶ impares. ✓

Problema de valor inicial y de frontera.

Ilustremos.

Una partícula se mueve sobre el eje x, de tal forma que su aceleración a cualquier tiempo de ≥ 0 está dada por $a(t) = 16 - 24t$

a) Encuentre la posición x de la partícula del origen o a $\forall\, t > 0$ asumiendo que al tiempo $t = 0$ está en $x = 2$ y viaja a una velocidad $v = 3$

b) trabaje con a) si solamente sabe que en $t = 0$ $x = 2$ y en $t = 1$ $x = 7$.

a) $a(t) = 16 - 24t$

$$\frac{d^2 t}{dt^2} = 16 - 24t$$

$$\frac{dx}{dt} = 16t - 12t^2 + C_1$$

$v(0) = 3 \quad C_1 = 0$

$$\frac{dx}{dt} = 16t - 12t^2 + 3$$

$$x = 8t^2 - 4t^3 + 3t + C_2$$

$C_2 = 2$

$$x = 8t^2 - 4t^3 + 3t + 2$$

b) $v(t) = \frac{dx}{dt} = 16t - 12t^2 + C_1$

$$x(t) = 8t^2 - 4t^3 + C_1 t + C_2$$

$x(0) = 2$

$$2 = 8(0)^2 - 4(0)^3 + C_1(0) + C_2$$

$C_2 = 2$

$x(1) = 7 \quad C_1 = 1$

$$7 = 8 - 4 + C_1 + 2$$

$$x(t) = 8t^2 - 4t^3 + t + 2$$

Solución general, Solución particular.

Ilustremos

Resolveremos

$$\frac{dy}{dx} = 2x; \; y(0) = 1$$

$$\int \frac{d}{dx}\, y \, dx = \int 2x \, dx$$

$y = x^2 + C$ → Solución General. (1)

en $y(0) = 1$ ó $(0,1)$ → Solución particular.

$$1 = (0)^2 + C$$

$$C = 1$$

$y(x) = x^2 + 1$ → (2)

Geometricamente la solución (1) representa una familia de parabolas y el vertice está localizado sobre el eje y.

Por otro lado la solución (2) representa la familia de parabolas dadas en t y que pasa por el punto (0, 1)

Solución General.

Es una función o relación que satisface una ec. dif. y que contiene en su estructura las constantes arbitrarias generadas de los procesos de integración así la solución general de una ecuación diferencial de orden n deberá contener en su estructura n constantes arbitrarias.

Solucion Particular.

Es una función o relación que satisface a una ecuación particular que contiene a partir de la solución general asignar valores a las constantes arbitrarias a partir de condiciones iniciales o de frontera apropiadas.

Nota: La solución de un problema de valor inicial es única.

UNIDAD II Ecuaciones diferenciales de primer orden

MÉTODOS DE SOLUCIÓN

- Método de separación de variable.

Considere $\frac{dy}{dx} = F(x,y) \dots (1)$

Nota: Adoptaremos como la forma general de primer orden.

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$N(x,y)\,dy - M(x,y)\,dx$$

$$\frac{dy}{dx} = -\frac{M(x,y)}{N(x,y)} = F(x,y)$$

Si en (1) es factorizable en sus variables $x = y$ esto es, si:

$$F(x,y) = f(x) - g(x)$$

Entonces decimos que la ec. dif. (1) es de variables separables.

Se puede llevar a $f(x)\,dx + g(y)\,dy = 0 \dots (2)$

Luego por integración inmediata se tiene que:

$$\int f(x)\,dx + \int g(y)\,dy = C \quad \text{Es la Sol. gral de (1)}$$

Resolver.

$$\frac{dy}{dx} = f(x,y) = \frac{x^2+1}{2-y} = (x^2+1)\left(\frac{1}{2-y}\right) \qquad y(-3)=4$$

$$\frac{dy}{dx} = (x^2+1)\left(\frac{1}{2-y}\right)$$

$$\int (2-y)\,dy = \int (x^2+1)\,dx$$

$$\int (x^2+1)\,dx - \int (2-y)\,dy = 0$$

$$\frac{x^3}{3} + x - 2y + \frac{y^2}{2} = C_1 \longrightarrow \text{implícita}$$

$$y(-3)=4 \quad \text{ó} \quad (-3,4)$$

$$\frac{(-3)^3}{3} - 3 - 2(4) + \frac{4^2}{2} = C_1$$

$$-9 - 3 - 8 + 8 = C_1$$

$$-12 = C_1$$

$$\Rightarrow \boxed{\frac{x^3}{3} + x - 2y + \frac{y^2}{2}} = -12 \longrightarrow \text{Sol. Particular Implícita}$$

$$\underset{a}{y^2} - \underset{b}{4y} + \underset{c}{\left(\tfrac{2}{3}x^3 + 2x + 24\right)} = 0$$

$$y(x) = \frac{4 \pm \sqrt{16 - 4\left(\frac{2}{3}x^3 + 2x + 24\right)}}{2}$$

$$y(x) = 2 \pm \sqrt{4 - \left(\tfrac{2}{3}x^3 + 2x + 24\right)}$$

$$y(x) = 2 + \sqrt{4 - \tfrac{2}{3}x^3 + 2x + 24}$$

$$\therefore\ y(x) = 2 \pm \sqrt{4 - \left(\tfrac{2}{3}x^3 + 2x + 24\right)} \quad \text{Sol. explícita.}$$

$$\frac{dy}{dx} = -\frac{x}{y} \qquad y\,dy = -x\,dy \qquad y(0)=?$$

$$x\,dy + y\,dy = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} = C \ldots (1)$$

$$2(1)$$

$$x^2 + y^2 = C_1$$

$$\text{en } y(0)=2 \quad \text{ó} \quad (0,2)$$

$$(0)^2 + (2)^2 = 4$$

$$C_1 = 4 \qquad x^2 + y^2 = 4$$

$$y = \sqrt{4 - x^2} \longrightarrow \text{part explícita}$$

12

$$x\frac{dy}{dx} - y = 2x^2y$$

$$\frac{2x^2y + y}{x} = y\frac{(2x^2+1)}{x}$$

$$-\int \frac{dy}{y} + \int \frac{2x^2+1}{x}\,dx = 0$$

$$-\ln|y| + \ln|x| + x^2 = C$$

$$\ln\left|\frac{x}{y}\right| + x^2 = C \rightarrow \text{Implícita}$$

$$\ln\left|\frac{x}{y}\right| = C - x^2$$

$$\frac{x}{y} = e^{C-x^2}$$

$$\frac{x}{y} = e^{C-x^2}$$

$$y(x) = \frac{x}{e^{C-x^2}}$$

$$= xe^{x^2} - C$$

$$xe^{x^2} - e^{-C}$$

$$y(x) = Axe^{x^2}$$ Sol. gral explícita.

$$y = Axe^{x^2}$$

$$y' = Ae^{x^3} + Ax(2xe^{x^2})$$

$$x(Ae^{x^2} + 2Ax^2e^{x^2}) - Axe^{x^2} = 2x^2(2Ax^2e^{x^2})$$

$$2Ax^2e^{x^2} = 2Ax^2e^{x^2}$$

Resolver:

$$(1+y^2)\,dx + (1+x^2)\,dy = 0$$

$$x + y = C(1-xy)$$

$$(1+x^2)\,dy = -(1+y^2)\,dx$$

$$\frac{dy}{dx} = -\frac{(1+y^2)}{1+x^2}$$

$$\frac{dy}{dx} = -\frac{(1+y^2)}{(1+x^2)}$$

$$\frac{dy}{1+y^2} = -\frac{dx}{1+x^2}$$

$$\frac{dy}{1+y^2} + \frac{dx}{1+x^2} = 0$$

$$\int \frac{dx}{1+y^2} + \int \frac{dx}{1+x^2}$$

$$\arctan(y) + \arctan(x) + C$$

$= C$ (Sol. gral.)

$$tg(tg^{-1}x + tg^{-1}y) = tg\,C$$

$$tg(A+B) = C$$

$$\frac{Tg(tg^{-1}x) + tg(tg^{-1}x)}{1 - tg(tg^{-1}x)\,tg(tg^{-1}y)}$$

$$\frac{x+y}{1-xy} = C$$

$$x + y = C(1-xy)$$

$$\tan(A+B) = \frac{\operatorname{sen}(A+B)}{\cos(A+B)} = \frac{\operatorname{sen} A \cos B + \cos A \operatorname{sen} B}{\cos A \cos B + \operatorname{sen} A \operatorname{sen} B}$$

$$= \frac{\cos A \cos B \left(\frac{\operatorname{sen} A}{\cos A} + \frac{\operatorname{sen} B}{\cos B} \right)}{\cos A \cos B \left(1 - \frac{\operatorname{sen} A \cos B}{\cos A \cos B} \right)} = \frac{\tan A + \operatorname{tg} B}{1 - \operatorname{tg} A \operatorname{tg} B}$$

$$\frac{dr}{d\phi} = \frac{\operatorname{sen}\phi + e^{2r}\operatorname{sen}\phi}{3e^r + e^r\cos\phi} \quad ; \quad r\left(\frac{\pi}{2}\right) = 0$$

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Solución:

$$\frac{dr}{d\phi} = \frac{\operatorname{sen}\phi(1+e^{2r})}{(3+\cos 2\phi)\, e^r}$$

$$\frac{e^r\, dr}{1+e^{2r}} = \frac{\operatorname{sen}\phi\, d\phi}{3+\cos 2\phi}$$

$$\underbrace{\int \frac{\operatorname{sen}\phi\, d\phi}{3+\cos 2\phi}}_{\mathrm{I}} - \underbrace{\int \frac{e^r dr}{1+e^{2r}}}_{\mathrm{II}} = 0$$

I) $$\int \frac{\operatorname{sen}\phi\, d\phi}{3+\cos 2\phi} = \int \frac{\operatorname{sen}\phi\, d\phi}{3+\cos^2\phi - \operatorname{sen}^2\phi}$$

$$= \int \frac{\operatorname{sen}\phi\, d\phi}{2-2\cos^2\phi} = \frac{1}{2}\int \frac{\operatorname{sen}\phi\, d\phi}{1-\cos^2\phi}$$

$$= -\frac{1}{2}\operatorname{arctg}(\cos\phi)$$

II) $$\int \frac{e^r dr}{1+e^{2r}} = \int \frac{e^r dr}{1+(e^r)^2} = \int \frac{dw}{1+w^2} = \operatorname{arctg}(e^r) + C$$

$$-\frac{1}{2}\operatorname{arctg}(\cos\phi) - \operatorname{arctg}(e^r) = C$$

Por -2

$$\operatorname{arctg}(\cos\phi) + 2\operatorname{arctg}(e^r) = C$$

$$r\left(\frac{\pi}{2}\right) = 0 \qquad \left(\frac{\pi}{2}, 0\right)$$

$$\operatorname{tg}^{-1}\left(\cos\frac{\pi}{2}\right)^{\to 0} + 2\operatorname{tg}^{-1}(e^r) = C_1$$

$$2\operatorname{tg}^{-1}(1) = C_1 \qquad \operatorname{tg}\phi = 1 = \frac{\operatorname{sen}\phi}{\cos\phi}$$

$$2\left(\frac{\pi}{4}\right) = C$$

$$C = \frac{\pi}{2}$$

$$\operatorname{tg}^{-1}(\cos\phi) + 2\operatorname{tg}^{-1}(e^r) = \frac{\pi}{2}$$

$$\frac{dy}{dx} = \operatorname{sen} 5x$$

$$dy = \operatorname{sen} 5x\, dx$$

$$\int \operatorname{sen} 5x\, dx - \int dy = 0$$

$$-\frac{1}{5}\cos 5x - y = C$$

$\cos 5x + 5y = C_1 \longrightarrow$ Sol gral Implícita

$$\frac{dx}{dy} = \frac{x^2y^2}{1+x}$$

$$\frac{(1+x)}{x^2}dx - y^2\, dy = 0$$

$$\int \frac{1}{x^2}dx + \int \frac{1}{x}dx - \int y^2 dy = 0$$

$$-\frac{1}{x} + \ln|x| - \frac{y^3}{3} = C$$

$\bullet\ (x+1)\dfrac{dy}{dx} = x+6$

$\dfrac{x+6}{x+1}dx - dy = 0$

$\dfrac{(x+1)+5}{x+1}dx - dy = 0$

$\int dx + \int \dfrac{5}{x+1}dx - \int dy = 0$

$x + 5\ln|x+1| - y = C$

$\underline{x + \ln|(x+1)^5| - y = C}$

$(1+x^2+y^2+x^2y^2)\,dy = y^2\,dx$

$dy = \dfrac{y^2}{1+x^2+y^2+x^2y^2}dx$

$dy = \dfrac{y^2}{(1+x^2)+y^2(1+x^2)}dx$

$dy = \dfrac{y^2}{(1+x^2)(1+y^2)}dx$

$\dfrac{1+y^2}{y^2}dy = \dfrac{dx}{1+x^2}$

$\int\dfrac{dy}{y^2} + \int dy - \int\dfrac{dx}{1+x^2} = 0$

$-\dfrac{1}{y} + y - tg^{-1}(x) = C$???

$tg^{-1}(x) + \dfrac{1}{y} - \dfrac{y^2}{2} = C$

$\bullet\ (4y+yx^2)dy - (2x+xy^2)dx = 0$

$(4y+yx^2)dy = (2x+xy^2)dx$

$dy = \dfrac{x(2+y^2)}{y(4+x^2)}dx$

$\dfrac{y}{2+y^2}dy = \dfrac{x}{4+x^2}dx$

$\dfrac{1}{2}\int\dfrac{x\,dx}{4+x^2} - \int\dfrac{y\,dy}{2+y^2} = 0$

$\dfrac{1}{2}\ln|4+x^2| - \dfrac{1}{2}\ln|2+y^2| + C$ (Por 2)

$\ln\left|\dfrac{4+x^2}{2+y^2}\right| = C_1$

$\underline{\dfrac{4+x^2}{2+y^2} = C_1}$

$\bullet\ y\ln|x|\dfrac{dy}{dx} = \dfrac{(y+1)^2}{x}$

$\dfrac{y\,dy}{(y+1)^2} = \dfrac{dx}{x\ln|x|}$

$\dfrac{y\,dy}{y^2+2y+1}$

$e^y(\operatorname{sen}2x)dx + \cos x(e^{2y}-y)dy = 0$

$e^y(\operatorname{sen}2x)dx = -\cos x(e^{2y}-y)dy$

$\dfrac{\operatorname{sen}2x}{\cos x}dx = -\dfrac{(e^{2y}-y)}{e^y}dy$

$\underbrace{\int\dfrac{\operatorname{sen}2x}{\cos x}dx}_{I} + \underbrace{\int\dfrac{e^{2y}-y}{e^y}dy}_{II} = 0$

$I\ \int\dfrac{\operatorname{sen}x\cancel{\cos x} + \cos x\operatorname{sen}x}{\cancel{\cos x}}dx$

II) $u = y \quad dv = \dfrac{dy}{e^y}$

$du = dy \quad v = -\dfrac{1}{e^y}$

$\int\left(e^y - \dfrac{y}{e^y}\right)dy$

$\Rightarrow -2\cos x + e^y - \dfrac{y}{e^y} + \int\dfrac{dy}{e^y} = 0$

$= -2\cos x + e^y - \dfrac{y}{e^y} - \dfrac{1}{e^y}$

$\underline{= -2\cos x + e^y - \dfrac{1}{e^y}(y+1) = C}$

$$\frac{dy}{dx} = \frac{xy + 2y - x - 2}{xy - 3y + x - 3}$$

$$\frac{dy}{dx} = \frac{y(x+2) - (x+2)}{y(x-3) + (x-3)}$$

$$\frac{dy}{dx} = \frac{(x+2)(y-1)}{(x-3)(y+1)}$$

$$\frac{(y+1)\,dy}{(y-1)} = \frac{(x+2)\,dx}{(x-3)}$$

$$\int \frac{x+2}{x-3}\,dx - \int \frac{(y+1)\,dy}{(y-1)} = 0$$

$$\sec y \frac{dy}{dx} + \operatorname{sen}(x-y) = \operatorname{sen}(x+y)$$

$$\sec y \frac{dy}{dx} + \operatorname{sen} x \cos y - \cos x \operatorname{sen} y = \operatorname{sen} x \cos y + \cos x \operatorname{sen} y$$

$$\sec y \, dy = 2 \cos x \operatorname{sen} y \, dx$$

$$\int 2\cos x \, dx - \int \frac{\sec y}{\operatorname{sen} y}\, dy$$

Ecuaciones de primer orden separables por cambio de variable algebraico

$$\frac{dy}{dx} = \underbrace{\operatorname{sen}(x-y)}_{f(x,y) \neq f(x)\cdot g(y)} \dots (1)$$

$$\frac{dy}{dx} = \operatorname{sen}(x+y) = \cos(x-y) = \cos(x+y)$$

Sea $t = x-y \quad t = t(x)$

$$\frac{dt}{dx} = 1 - \frac{dy}{dx} \qquad \boxed{\frac{dy}{dx} = 1 - \frac{dt}{dx}}$$

$$\Rightarrow 1 - \frac{dt}{dx} = \operatorname{sen} t$$

$$= (1-\operatorname{sen} t) = \frac{dt}{dx} \Rightarrow -\frac{dt}{(1-\operatorname{sen} t)} + dx = 0$$

$$= \int dx - \int \frac{dt}{(1-\operatorname{sen} t)} = 0$$

$$= x - \int \frac{1+\operatorname{sen} t}{(1-\operatorname{sen}^2 t)} dt = 0$$

$$= x - \int \frac{1+\operatorname{sen} t}{\cos^2 t} dt = 0$$

$$x - tg\, t + \sec t = C$$

$$x - tg(x-y) - \sec(x-y) = C.$$

$$\frac{dy}{dx} = \alpha x + \beta y + \gamma \;;\; \alpha, \beta, \gamma \text{ ctes.}$$

Sean $\alpha x + \beta y + \gamma = t$

$$\frac{dt}{dx} = \alpha + \beta \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\beta}\left(\frac{dt}{dx} - \alpha\right)$$

$$\frac{1}{\beta}\left(\frac{dt}{dx} - \alpha\right) = t \Rightarrow -\left(-\frac{dt}{\beta t + \alpha}\right) + dx = 0$$

$$-\frac{1}{\beta} \ln|\beta t + \alpha| + x = C$$

$$\ln|(\beta t + \alpha)^{-1/\beta}| + x = C$$

$$\ln|\beta(\alpha x + \beta y + \gamma) + \alpha)^{-1/\beta}| + x = C$$

Sol. Gral. implícita.

RESOLVER

$$\frac{dy}{dx} = a^{x+y} \qquad t = x+y$$

$$\frac{dt}{dx} = 1 + \frac{dy}{dx} \qquad \frac{dy}{dx} = -1 + \frac{dt}{dx}$$

$$\frac{dt}{dx} - 1 = a^t \qquad \int dx - \int \frac{dt}{a^t+1} = 0$$

$$\rightarrow \int \frac{dt}{a^t+1} = \int \frac{dt}{e^{t\ln a}+1} \qquad \frac{1}{\ln a}\int \frac{du}{e^u+1}$$

$$\frac{1}{\ln a}\int \frac{e^{-u}\,du}{1+e^{-u}} = -\frac{1}{\ln a}$$

$$\frac{x + \ln|1+a^{-t}|}{\ln|a|} = C$$

$$\frac{dy}{dx} = (x+y+1)^2$$

$$t = x+y+1$$

$$\frac{dt}{dx} = 1 + \frac{dy}{dx} \qquad \frac{dy}{dx} = \frac{dt}{dx} - 1$$

$$\frac{dt}{dx} - 1 = t^2$$

$$-\int \frac{dt}{t^2+1} + \int dx = 0$$

$$x - \operatorname{arctg}(t) = C$$

$$x - \operatorname{arctg}(x+y+1) = C$$

$$\frac{dy}{dx} = 2 + \sqrt{y-2x+3}$$

$$t = y - 2x + 3$$

$$\frac{dt}{dx} = \frac{dy}{dx} - 2$$

$$\frac{dy}{dx} = \frac{dt}{dx} + 2$$

$$\frac{dy}{dx} + \cancel{2} = \cancel{2} + \sqrt{t}$$

$$-\int \frac{dt}{\sqrt{t}} + \int dx = 0$$

$$-2(t)^{1/2} + x = C$$

$$-2\sqrt{y-2x+3} + x = C$$

$$\frac{dy}{dx} = 1 + e^{y-x+5}$$

sea $y - x + 5 = t$

$$\frac{dt}{dx} = \frac{dy}{dx} - 1$$

$$\frac{dt}{dx} + \cancel{1} = \cancel{1} + e^t$$

$$\int \frac{dt}{e^t} = \int dx$$

$$\int dx - \int \frac{dt}{e^t} = 0$$

$$\frac{1}{e^t} + x = C$$

$$\frac{1}{e^{y-x+5}} + x = C$$

$$\frac{dy}{dx} = \tan^2(x+y) \qquad t = x+y$$

$$\frac{dt}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dt}{dx} - 1 = \tan^2 t$$

$$\int \frac{dt}{\tan^2 t + 1} = \int dx$$

$$\int dx - \int \frac{dt}{\tan^2 t + 1} = 0$$

$$\int dx - \int \cos^2 t \, dt = 0$$

$$x - \tfrac{1}{2}\int [1 + \cos 2t] \, dt$$

$$x - \tfrac{1}{2}\int dt - \tfrac{1}{2}\tfrac{1}{2}\int \cos 2t \, 2dt$$

$$x - \tfrac{1}{2}t - \tfrac{1}{4}\operatorname{sen} 2t + C$$

$$x - \tfrac{1}{2}(x+y) - \tfrac{1}{4}\operatorname{sen}[2(x+y)] = C$$

$$\frac{dy}{dx} = \frac{2y}{x} + \frac{x^3}{y} + x\tan\left(\frac{y}{x^2}\right) \rightarrow \int \frac{dx}{x} - \int \frac{dt}{\frac{1}{t} + \tan(t)} = \ln x - \ln|\cos t + t\operatorname{sen} t| = \ln c$$

$$t = \frac{y}{x^2} \qquad y = x^2 t$$

$$= \ln\left|\frac{x}{\cos\left(\frac{y}{x^2}\right) + \frac{y}{x^2}\operatorname{sen}\frac{y}{x^2}}\right| = \ln c$$

$$\frac{x}{\cos\left(\frac{y}{x^2}\right) + \frac{y}{x^2}\operatorname{sen}\frac{y}{x^2}} = C$$

Implícita

$$\frac{dy}{dx} = x^2\frac{dt}{dx} + t\,2x$$

$$x^2\frac{dt}{dx} = 2xt = \frac{2x^2 t}{x} + \frac{x^3}{x^2} + x\operatorname{tg}(t)$$

$$\frac{dt}{dx} = 2xt + \frac{x}{t} + x\operatorname{tg}(t) = x\left(2t + \frac{1}{t} + \operatorname{tg}(t)\right)$$

Ecuación homogenea de primer orden

$$\underline{M(x,y)\,dx + N(x,y)\,dy = 0} \rightarrow \text{Ecuación no separable.}$$

Para establecer la homogeneidad de una ecuación diferencial de 1er orden es necesario establecer conocimientos de como asignar el grado a una función que depende de más de una variable.

$f(x) = ax^3 + bx^2 + cx + d$ 3er grado.

$f(x,y) = x^2y + 1$

$f(x,y) = x^4 + x^3y + x^2y^2 + 1$ 4° grado

$f(x,y) = \sqrt{x^2 - y^2} + x\,\text{sen}\left(\frac{x}{y}\right)$

$F(x,y) = x^4 + x^3y + x^2y^2 + y^4$ función homogenea de 4° grado.

Criterio de funciones homogenea.

DEF. Se dice que la funcion $f(x,y)$ es una función homogenea de grado en sus variables x & y si se verifica la igualdad.

$$\boxed{F(tx, ty) = t^n(x,y) \quad \forall\ x\ y\ y\ t}$$

$f(x,y) = \sqrt{x^2 - y^2} + x \text{ sen } \frac{x}{y}$

$f(tx, ty) = \sqrt{(tx)^2 - (ty)^2} + tx \text{ sen } \frac{tx}{ty}$

$= \sqrt{t^2x^2 - t^2y^2} + tx \text{ sen } \frac{x}{y}$

$= \sqrt{t^2(x^2 - y^2)} + tx \text{ sen } \frac{x}{y}$

$= t\sqrt{x^2 - y^2} + tx \text{ sen } \frac{x}{y}$

$= \underbrace{t}_{t^n} \underbrace{\left(\sqrt{x^2 - y^2} + x \text{ sen } \frac{x}{y}\right)}_{(x,y)}$

Ecuación homogenea.

$F(x,y) = \frac{2xy}{x^2 + y^2}$

$f(tx, ty) = \frac{2tx\,ty}{(tx)^2 + (ty)^2}$

$= \frac{t^2\,2xy}{t^2x^2 + t^2y^2}$

$= \frac{\cancel{t^2}\,2xy}{\cancel{t^2}(x^2 + y^2)}$

$= \underbrace{\frac{2xy}{x^2 + y^2}}$

Función homogenea de grado cero.

Def. Sea $f(x,y) = \dfrac{M(x,y)}{N(x,y)} \ldots (1)$

Si en (1) las funciones $M(x,y)$ y $N(x,y)$ son funciones homogeneas y del mismo grado entonces $f(x,y)$ es función de grado 0.

Ecuación diferencial de primer orden

Def. Se dice que la ecuación dif. si $M(x,y)dx + N(x,y) = 0$ es una ecuación dif. homogenea o si las funciones $M(x,y)$ y $N(x,y)$ son funciones homogeneas del mismo grado en sus variables x y Y

Se enuncia en la forma análoga:
Se dice que la ec. dif. $\dfrac{dy}{dx} = \dfrac{M(x,y)}{N(x,y)} = f(x,y)$ es homogenea.

Teorema Si la ec. dif. $M(x,y)dx + N(x,y)dy = 0$ es homogenea, entonces se puede escribir de la forma $\boxed{\dfrac{dy}{dx} = f\left(\dfrac{y}{x}\right)}$

Demostración

Suponga en la ecuación dif. $M(x,y)dx + N(x,y)dy = 0 \ldots (1)$
$M(x,y)$ y $N(x,y)$ son funciones homogeneas
De acuerdo con el criterio de funciones homogeneas

$$M(tx, ty) = t^n M(x,y) \ldots (2) \quad \forall x, y \in t$$
$$N(tx, ty) = t^n N(x,y)$$

$$\frac{dy}{dx} = -\frac{M(x,y)}{N(x,y)} = F(x,y) \ldots (3)$$

de (2) tenemos

$$M(x,y) = \frac{M(tx, ty)}{t^n} \Rightarrow N(x,y) = \frac{N(tx+ty)}{t^2}$$

Sustituyendo

$$\frac{dy}{dx} = -\frac{\frac{M(tx,ty)}{t^n}}{\frac{N(tx,ty)}{t^2}} = -\frac{M(tx,ty)}{N(tx,ty)}$$

Sea $t = \frac{1}{x}$

$$\frac{dy}{dx} = -\frac{M\left(\frac{x}{x}, \frac{y}{x}\right)}{N\left(\frac{x}{x}, \frac{y}{x}\right)} = -\frac{M\left(1 - \frac{y}{x}\right)}{N\left(1, \frac{y}{x}\right)} = f\left(\frac{y}{x}\right) \text{ q.e.d.}$$

∴ toda la ec. homogenea de primer orden se puede escribir

$$\boxed{\frac{dy}{dx} = f\left(\frac{y}{x}\right)} \ldots (5)$$

haciendo $u = \frac{y}{x}$ $\quad y = xu \Rightarrow \frac{dy}{dx} = u + x\frac{du}{dx}$

$$u + x\frac{du}{dx} = f(u) \qquad x\frac{du}{dx} = f(u) - u$$

$$\int \frac{du}{f(u)-u} = \int \frac{dx}{x} + C$$

$$\frac{dy}{dx} = \frac{x-y}{x+y} = \frac{(x-y)\frac{1}{x}}{(x+y)\frac{1}{x}} = \frac{1-\frac{y}{x}}{1-\frac{y}{x}} = f\left(\frac{y}{x}\right) \qquad u = \frac{y}{x}$$

$$\frac{dy}{dx} = x\frac{du}{dx} + u \rightarrow u + \frac{du}{dx} = \frac{x - xu}{x + xu} = \frac{x(1-u)}{x(1+u)}$$

$$x\frac{du}{dx} = \frac{1-u-u(1+u)}{1+2}$$

$$\int \frac{dx}{x} - \int \frac{(u+1)\,dx}{1-2u-u^2} = 0 \qquad \ln x + \frac{1}{2}\ln|u^2+2u-1| = \ln C$$

$$\ln|(u^2+2u-1)^{1/2}(x)| = C_1$$

$$(u^2+2u-1)^{1/2}\,x = C$$

$$(u^2+2u-1)x^2 = C_1$$

$$\left[\left(\frac{y}{x}\right)^2 + 2\left(\frac{y}{x}\right) - 1\right]x^2 = C_1$$

$$y^2 + 2yx - x^2 = C_1.$$

$$x\frac{dy}{dx} = y - \sqrt{x^2+y^2}$$

$\frac{dy}{dx} = \frac{y - \sqrt{x^2+y^2}}{x} \Rightarrow$ homogenea p/M

$f(x,y)$ = homogenea grado cero

$$u = \frac{y}{x} \Rightarrow \frac{dy}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = ux - \sqrt{x^2 + u x^2}$$

$$u + x\frac{du}{dx} = ux - x\sqrt{1+u^2}$$

$$\cancel{u}\ \cancel{x}\frac{du}{dx} = \cancel{x}(\cancel{u} - \sqrt{1+u^2})$$

$$\int \frac{dx}{x} - \int \frac{du}{\sqrt{1+u^2}} = 0$$

$$\ln|x| - \ln\left|\frac{\sqrt{1+u^2}}{1} + u\right| = c$$

$$\frac{x}{\sqrt{1+u^2}+u} = C_1$$

$$\frac{x}{\left(1+\frac{y^2}{x^2}\right)^{1/2} + \frac{y}{x}} = C_1$$

Resolver

$(3xy^2 + 2x^3)dx + y^3 dy = 0 \quad \dots (1)$

$u = \frac{x}{y} \qquad \frac{dy}{dx} = u + \frac{x\,du}{dx}$

$u + x\frac{du}{dx} = -\left[\frac{3xy^2 + 2x^3}{y^3}\right] = -\left[\frac{3x^3u^2 + 2x^3}{x^3u^3}\right]$

$u + x\frac{du}{dx} = -\left[\frac{3u^2+2}{u^3}\right] - u \qquad u^4 + 3u^2 + 2$

$(u^2+2)(u^2+1)$

$= -\left(\frac{3u^2+2+u^4}{u^3}\right)$

$\frac{-u^3\,du}{(u^2+2)(u^2+1)} = \frac{dx}{x}$

$\int \frac{dx}{x} + \int \frac{u^3\,du}{(u^2+2)(u^2+1)} = 0 \qquad \frac{u^3}{(u^2+2)(u^2+1)} = \frac{Ax+B}{u^2+2} + \frac{Cx+D}{(u^2+1)}$

$u^3 = (Au+B)(u^2+1) + (Cu+D)(u^2+2)$

$u^3 = Au^3 + Au + Bu^2 + B + Cu^3 + 2Cu + Du^2 + 2D$

$u^3 = u^3(A+C) + u^2(D+B) + u(2C+A) + 2D+B$

$A+C=1$

$B+D=0 \qquad A=-2C \qquad A=-2C \qquad 0=-D+2D$

$A+2C=0 \qquad 1=-2C+C \qquad A=2$

$B+2D=0 \qquad 1=-C \qquad D=0$

$C=-1 \qquad B=0$

$\therefore \frac{(2x^2+y^2)^2}{y^2+y^2} = C_1$

$\frac{dy}{dx} = \frac{6x^2 - 5xy - 2y^2}{6x^2 - 8xy + y^2}$

$\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2} \qquad y(1) = 1 \qquad \ln|x| + \frac{x}{y} = C \qquad (1,1)$

$u = \frac{y}{x} \qquad \frac{dy}{dx} = u + \frac{x\,du}{dx} \qquad \ln|1| + 1 = C \Rightarrow C = 1$

$u + x\frac{du}{dx} = u + u^2 \qquad \ln|x| + \frac{x}{y} = 1$

$\int\frac{dx}{x} - \int\frac{du}{u^2} = 0$

$\ln|x| + \frac{1}{u} = C$

Zill 2.4

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Ecuación diferencial exacta.

Def. Se dice que la ecuación diferencial $M(x,y)dx + N(x,y)dy = 0$ es exacta si existe una función $u(x,y)$ tal que

$$du(x,y) = M(x,y) + N(x,y)$$

Idea intuitiva de exactitud.

Para establecer la exactitud de una ecuación dif. es necesario tener pleno conocimiento de como dterminar el diferencial de una función que depende de más de una variable

$$y = f(x) \qquad dy = f'(x)\,dx$$

Sea u una función de $x \in y$ si $u(x,y)$ posee al menos primeras derivadas parciales continuas en alguna región del plano xy entonces el diferencial de $u(x,y)$ se denota $du(x,y)$ y se define como

$$du(x,y) = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$

Sea $u(x,y) = \frac{x^2y^2}{2} - x - y \qquad du(x,y)$

$$\frac{\partial u}{\partial x} = xy^2 - 1 \qquad \frac{\partial u}{\partial y} = x^2y - 1$$

$$\frac{x^2y^2}{2} - x - y$$

$$du(x,y) = (xy^2 - 1)dx + (x^2y - 1)dy$$

$$\int d\left(\frac{x^2y^2}{2} - x - y\right) = 0$$

$$\frac{x^2y^2}{2} - x - y = C$$

En donde $M(x,y)dx + N(x,y)dy$ representa al diferencial de la función $u(x,y) \ldots (1)$

$$du(x,y) = M(x,y)dx + N(x,y)dy$$

En tal caso la forma abreviada de (1)

$du(x,y) = 0$ forma integrable directa.

Esto es $\int du(x,y) = C$ Solución general de (1).

Teorema de exactitud.

Teorema:

> Parte I: Si $du(x,y) = M(x,y)\,dx + N(x,y)\,dy = 0$

$\therefore \dfrac{\partial M}{\partial x} = \dfrac{\partial N}{\partial x}$ • Condición necesaria para la exactitud.

> Parte II: Si $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial y}$

$\therefore$ Existe $u(x,y) \,/\, du(x,y) = N(x,y)\,dx + N(x,y)\,dy$

equivalente $u(x,y)$ existe $/ \dfrac{\partial u}{\partial x} = M(x,y)$

Condición Suficiencia $\longrightarrow \dfrac{\partial u}{\partial y} = N(x,y)$

• Demostración.

> Parte I: Si $M(x,y)\,dx + N(x,y)\,dy = 0$ ...(1) es exacta

$\rightarrow$ por definición existe una función $u(x,y)$ /

$$\partial u(x,y) = M(x,y)\,dx + N(x,y)\,dy \quad \ldots (2)$$

$$\partial u(x,y) = \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) \ldots (3)$$

Comparando (2) y (3)

$$\frac{\partial u}{\partial x} = M(x,y) = \frac{\partial u}{\partial y} = N(x,y) \ldots (4)$$

Derivando en forma crozada las ec. en 4

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial M}{\partial y} \Rightarrow \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial N}{\partial x} \ldots (5)$$

Si la solución $u(x,y)$ posee al menos segundas derivadas parciales continuas en algun punto del plano $xy \longrightarrow$ el orden de derivación carece de importancia.

Supongamos que $u(x,y)$ posee segunda derivada continua al menos hasta el segundo orden, entonces...

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x} \;; \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

la cual representa la condición para la exactitud.

> Parte II Si se verifica $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x} \ldots (1)$

entonces la ec. dif. (1) es exacta tal que

$du(x,y) = M(x,y)dx + N(x,y)dy$ ó $\dfrac{\partial u}{\partial x} = M(x,y) \,;\, \dfrac{\partial u}{\partial y} = N(x,y) \ldots (4)$

entonces de (4)

$$\partial u(x,y) = M(x,y)\partial x$$

Integrando ambos miembros

$$\int du(x,y) = \int M(x,y)\partial x$$

$$u(x,y) = \int M(x,y)\partial x + C \ldots (6)$$

La constante de integración no puede ser función de las variables de integración x pero si puede ser función de y a continuación supondremos y demostraremos que (6) es

$$C = f(y) \Rightarrow u(x,y) = \int M(x,y)\partial x + f(y) \ldots (7)$$

Sustituyendo (4) en (7)

$$\frac{\partial}{\partial y}\left[\int M(x,y)\partial x + f(y)\right] = N(x,y)$$

$$\frac{\partial}{\partial y}\int M(x,y)\partial x + f'(y) = N(x,y)$$

$$f'(y) = N(x,y) - \frac{\partial}{\partial y}\int M(x,y)\partial x$$

$$\frac{df(y)}{dy} = N(x,y) - \frac{\partial}{\partial y}\int M(x,y)dx \ldots (8)$$

Para integrar (8) es necesario que

$$N(x,y) - \frac{\partial}{\partial y}\int M(x,y)dx = g(y)$$

Se prueba del siguiente modo

$$\frac{\partial}{\partial x}\left[N(x,y) - \frac{\partial}{\partial y}\int M(x,y)\partial x\right] = \frac{\partial N}{\partial x} - \frac{\partial}{\partial x}\frac{\partial}{\partial x}\int M(x,y)\partial x$$

$$= \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0$$

$$\therefore \text{ en } (7) \quad c = f(y)$$

Resolver:

* $2xy\,dx + (x^2 + \cos y)\,dy = 0$

$\frac{\partial}{\partial y} M(x,y) = 2x$ $\qquad \therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial}{\partial x} N(x,y) = 2x$

$\frac{\partial U}{\partial x} = M(x,y);\ \frac{du}{dx} = N(x,y) \ldots (2)$

de $\partial U(x,y) = M(x,y)\,\partial x$

$u(x,y) = x^2y + f(y) \ldots (3)$

Sust. (3) en (2)

$\frac{\partial}{\partial y}[x^2y + f(y)] = x^2 + \cos y$

$x^2 + f'(y) = x^2 + \cos y$

$F(y) = \int \cos y\, dy$

$f(y) = \operatorname{sen} y \ldots (4)$

Sustituimos (4) en (3)

$u(x,y) = x^2y + \operatorname{sen} y$

$\partial(x^2y + \operatorname{sen} y) = 0$

$\underline{yx^2 + \operatorname{sen} y = C}$ Sol. gral de ec.

Resolver

$\underbrace{(\operatorname{sen} y - y\operatorname{sen} x)}_{M(x,y)}dx + \underbrace{(\cos x + x\cos y - y)}_{N(x,y)}dy = 0 \ldots (1)$

$\frac{\partial M}{\partial y} = \cos y - \operatorname{sen} x$ $\qquad \therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\qquad \frac{\partial U}{\partial x} = M(x,y),\ \frac{\partial U}{\partial y} = N(x,y) \ldots (2)$

$\frac{\partial N}{\partial x} = -\operatorname{sen} x + \cos y$

De la ec. (2)

$\partial U(x,y) = M(x,y)\,\partial x$

$\int \partial U(x,y) = \int (\operatorname{sen} y - y\operatorname{sen} x)\,\partial x$

$u(x,y) = \operatorname{sen} y \int dx - y\int \operatorname{sen} x\, dx$

$u(x,y) = x\operatorname{sen} y + y\cos x + f(y) \ldots (3)$

Sust. en (2)

$\frac{\partial}{\partial y}(x\operatorname{sen} y + y\cos x + f(y)) = \cos x + x\cos y - y$

$x\cos y + \cos x + f'(y) = \cos x + x\cos y - y$

$f'(y) = -y \Rightarrow -\frac{y^2}{2} \Rightarrow$

Sustituir

$u(x,y) = x\operatorname{sen} y + y\cos x - \frac{y^2}{2}$

$d(x\operatorname{sen} y + y\cos x - \frac{y^2}{2}) = 0$

$\int d(x\operatorname{sen} y + y\cos x - \frac{y^2}{2}) = 0$

$\underline{x\operatorname{sen} y + y\cos x - \frac{y^2}{2} = C}$

$$\underbrace{\left(1+\ln|x|+\frac{y}{x}\right)}_{M(x,y)}dx = \underbrace{(1-\ln|x|)}_{N(x,y)}dy$$

$$\frac{\partial M}{\partial y} = \frac{1}{x}$$

$$\frac{\partial N}{\partial x} = \frac{1}{x}$$

$$\int\left(1+\ln|x|+\frac{y}{x}\right)\partial x = x + x\ln x - x + y\ln|x| + f(y) \ldots (3)$$

$$\frac{\partial}{\partial y}\left[x + x\ln|x| - x + y\ln x + f(y)\right] = \ln x - 1$$

$$\ln x + f'(y) = \ln x - 1$$

$$f(y) = -y$$

$$\int \partial(x\ln x - x\ln x - y) = 0$$

$$x\ln x - y\ln x - y = 0$$

$$(2x-1)dx + (3y+7)dy = 0$$

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$(5x+4y)dx+(4x-8y^2)dy=0$

$\frac{\partial M}{\partial y}=4 \qquad \frac{\partial N}{\partial x}=4$

$\int(5x+4y)\partial x=\frac{5}{2}x^2+4xy+f(y)$

$\frac{\partial}{\partial y}\left(\frac{5}{2}x^2+4xy+f(y)\right)=4x-8y^2$

$4x+f'(y)=4x-8y^2$

$f'(y)=-8y^2 \Rightarrow f(y)=-\frac{8}{3}y^3$

$\int d\left(\frac{5}{2}x^2+4xy-\frac{8}{3}y^3\right)=0$

$\frac{5}{2}x^2+4xy-\frac{8}{3}y^3=C$

$(y^3-y^2\operatorname{sen}x-x)dx+(3xy^2+2y\cos x)dy=0$

$\frac{\partial M}{\partial y}=3y^2-2y\operatorname{sen}x$

$\frac{\partial N}{\partial x}=3y^2-2y\operatorname{sen}x$

$\int(y^3-y^2\operatorname{sen}x-x)\partial x=xy^3+y^2\cos x-\frac{x^2}{2}+f(y)$

$\frac{\partial}{\partial y}\left(xy^3+y^2\cos x-\frac{x^2}{2}+f(y)\right)=3xy^2+2y\cos x$

$3xy^2+2y\cos x+f'(y)=3xy^2+2y\cos x$

$f'(y)=0 \Rightarrow f(y)=C$

$\int d\left(xy^3+y^2\cos x-\frac{x^2}{2}\right)=0$

$\underline{xy^3+y^2\cos x-\frac{x^2}{2}=C}$

$(x^3+y^3)dx+(3xy^2)dy=0$

• Valor de m para la cual la ec. dif. 36/Apendice

$(12x - y\,sen\,xy + my^4)dx - (20xy^3 + x\,sen\,xy)dy = 0$

$\frac{\partial M}{\partial y} = -sen\,xy - xy\cos y + 4my^3 \quad \frac{\partial N}{\partial x} = -20y^3 - sen\,x - y - xy\cos xy$

$\underline{m = -5}$

$M(x,y)dx + (xe^{xy} + 2x + \frac{1}{x})dx$

$u = xy$
$du = xdy$

$\frac{\partial N}{\partial x} = e^{xy} + xye^{xy} + 2y - \frac{1}{x^2}$

$u = y \quad dv = e^{xy}$
$du = dy \quad v = \frac{1}{x}e^{xy}$

$\frac{\partial M}{\partial x} = e^{xy} + xye^{xy} + 2y - \frac{1}{x^2}$

$\frac{y}{x} - e^{xy} - \frac{1}{x}\int e^{xy}dy$

$\frac{y}{x}e^{xy} - \frac{1}{x^2}e^{xy}$

$M = \frac{1}{x}e^{xy} + y^2 - \frac{y}{x^2} + \frac{xy}{x}e^{xy} - \frac{x}{x^2}e^{xy} + f(x)$

$M(x,y) = y^2 + ye^{xy} - \frac{y}{x^2} + f(x)$

* Factor integrante...

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Ilustremos este concepto a partir del siguiente ejemplo...

La ecuación dif:

$6xy\,dx + (4y + 9x^2)dy = 0 \ldots (1)$ Apendice.

Multiplicar $M(x,y) = y^2$

$6xy^3dx + (4y^3 + 9x^2y^2)dy$

Suponga que la ec. dif.

$M(x,y)dx + N(x,y)dy = 0 \ \ldots (1)$

No es separable y tampoco exacta, si multiplicamos por la función $(x,y) \neq 0$ tal que:

$$\mu M(x,y)dx + \mu N(x,y)dy = 0$$

Ya es exacta entonces se dice que la función multiplicadora $\mu(x,y)$ es el factor integrante de la ec. dif. (1)

En otras palabras

El factor integrante es una función multiplicadora que reduce a una ec. dif. exacta, en una ec. dif. exacta.

Obtener de un Factor integrante apropiado.

Suponga que $M(x,y)\,dx + N(x,y) = 0 \ldots (1) \quad M(x,y) \neq 0$

Entonces por definición. $\mu M(x,y) + \mu N(x,y) = 0 \ldots (2)$

$$\frac{\partial(\mu M)}{\partial y} = \frac{\partial(\mu N)}{\partial x} \ldots (3)$$

• Dos casos:

CASO I: El factor integrante es solo función de x, esto es

$\mu = \mu(x)$ en tal caso de (3)

$$\mu \frac{\partial M}{\partial y} = \mu \frac{\partial N}{\partial x} + N \frac{d}{x}$$

$$N \frac{\partial \mu}{x} = \mu \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$\frac{\partial \mu}{\mu} = \underbrace{\frac{1}{N} \left(\frac{\partial M}{\partial N} - \frac{\partial N}{\partial x} \right)}_{f(x)} dx \ldots (4)$$

$$\int \frac{\partial \mu}{\mu} = \int f(x)\,dx$$

$$\ln|u| = f(x)\,dx \longrightarrow \mu(x) = e^{\int f(x)\,dx}$$

TI) Entonces $\quad S_1 = \frac{1}{N}\left(\frac{\partial N}{\partial y} - \frac{\partial u}{\partial x} \right)$

$$\mu(x) = e^{\int f(x)\,dx} = \mu(y) = e^{\int g(y)\,dy}$$

Resuelva

$y\,dx + (3 + 3x - y)\,dy = 0 \ldots (1)$

Solución.

TI) $\quad S_1 \ \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = f(x) \Rightarrow \cancel{\mu(x) = e^{\int f(x)\,dx}}$

$$\frac{1}{3-3x-y}[1-3] = \frac{-2}{3-3x-y} \neq f(x)$$

TII) $\quad S_1 \ \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) = g(y) \Rightarrow \mu(y)\,e^{\int g(y)\,dy}$

$$\frac{1}{y}[3-1] = \frac{2}{y} = g(y)$$

$$\mu(y) = e^{\int \frac{2}{y}\,dy} = e^{2\frac{dy}{y}} = e^{2\ln y} = e^{\ln y^2} = y^2$$

$$\text{en } (y) = y^2 \ldots (2)$$

$y^2(y\,dx + (3+3x-y))\,dy = 0$

$y^3\,dx + (3y^2 + 3xy^2 - y^3)\,dy = 0 \qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3y^2$

Una curva tiene una pendiente dada por $\frac{dy}{dx} = \frac{2xy}{x^2 y^2}$. Pasa por el punto (2,1) encontrar su ecuación.

$$(2xy)dx + (x^2 y^2)dy = 0$$ Apendice.

$$\frac{\partial M}{\partial y} = 2x \quad \frac{\partial N}{\partial x} = -2x$$

$$T1: \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{y^2 - x^2}(2x+2x) = \frac{4x}{y^2-x^2}$$

$$T2: \frac{1}{2xy}(-2x-2x) = \frac{-4\cancel{x}}{2\cancel{x}y} = -\frac{2}{y} = g(y)$$

$$M(y) = e^{\int -\frac{2}{y}dy} = y^{-2} = \frac{1}{y^2}$$

$$\frac{2x}{y}dx + \left(-\frac{x^2}{y^2} + 1\right)dy = 0$$

$$\frac{-2x}{y^2} = \frac{2x}{y^2}$$

$$\int \partial u(x,y) = \int \frac{2x}{y}\partial x = \frac{x^2}{y} + F(y)$$

$$\partial\left(\frac{x^2}{y} + F(y)\right) = -\frac{x^2}{y^2} + 1$$

$$-\frac{x^2}{y^2} + f'(y) = -\frac{x^2}{y^2} + 1 \Rightarrow y = f(y)$$

$$\left(\frac{x^2}{y} + y\right) = C \quad \text{Sol. gral. de (1)}$$

$$\left(\frac{x^2}{y} + y\right) = 5 \quad \text{Sol. particular (1)}$$

Murray p. 52 Seccion A A-J

Seccion B 1-4

$$(3x + 2y^2)dx + 2xyd = 0$$

$$T_1: \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{2xy}(4y)$$

$(y^2\cos x - y)\,dx + (x+y^2)\,dy = 0$

$f_1 = \frac{1}{x+y^2}\,(2y\cos x - 2)$

$f_2 = \frac{1}{y^2\cos x - y}\,(2y + y^2 \operatorname{sen} x) = \frac{2y + y^2\operatorname{sen} x}{y^2\cos y - y} = \frac{2\cancel{y}(1+y\operatorname{sen} x)}{-\cancel{y}(y\cos y - 1)}$

$= \operatorname{sen} x - \frac{x}{y} + y = C$

Zill Pag 9 6^{ta} Ed. Sección 1.1 1-10 / 11-40 (impares)

1) Lineal, orden n
2) lineal, orden 3
3) lineal, orden 1
4) lineal, orden 1
5) lineal, orden n
6) No-lineal orden 2
7) lineal, orden 2
8) lineal, orden 2
9) lineal, orden 1
10) lineal, orden 1

11-40

11. $2y' + y = 0$; $y = e^{-x/2}$ $= -\frac{1}{2} e^{-x/2}$

$$2\left(-e^{-x/2}\frac{1}{2}\right) + e^{-x/2} = 0$$
$$-e^{-x/2} + e^{x/2} = 0$$
$$0 = 0 \quad \therefore \text{Si es Solución}$$

13. $\frac{dy}{dx} - 2y = e^{3x}$; $y = e^{3x} + 10e^{2x}$ $\Rightarrow y' = 3e^{3x} + 20e^{2x}$

$$3e^{3x} + 20e^{2x} - 2(e^{3x} + 10e^{2x}) = e^{3x}$$
$$3e^{3x} + \cancel{20e^{2x}} - 2e^{3x} - \cancel{20e^{2x}} = e^{3x}$$
$$e^{3x} = e^{3x} \quad \therefore \text{Si es Solución}$$

15. $y' = 25 + y^2$; $y = 5\tan 5x \Rightarrow y' = 25\sec^2 5x$

$$25\sec^2 5x = 25 + (5\tan 5x)^2$$
$$25\sec^2 5x = 25 + 25\tan^2 5x$$
$$25\sec^2 5x = 25(1 + \tan^2 5x)$$
$$25\sec^2 5x = 25\sec^2 5x \quad \therefore \text{Si es solución}$$

17. $y' + y = \operatorname{sen} x$; $y = \frac{1}{2}\operatorname{sen} x - \frac{1}{2}\cos x + 10e^{-x}$

$$\Rightarrow y' = \frac{1}{2}\cos x + \frac{1}{2}\operatorname{sen} x - 10e^{-x}$$
$$\cancel{\frac{1}{2}\cos x} + \frac{1}{2}\operatorname{sen} x - \cancel{10e^{-x}} + \frac{1}{2}\operatorname{sen} x - \cancel{\frac{1}{2}\cos x} + \cancel{10e^{-x}} = \operatorname{sen} y$$
$$\frac{1}{2}\operatorname{sen} x + \frac{1}{2}\operatorname{sen} x = \operatorname{sen} x$$
$$\operatorname{sen} x = \operatorname{sen} x \quad \therefore \text{Si es Sol.}$$

19. $x^2 dy + 2xy\, dx = 0$; $y = \frac{1}{-x^2}$ $\qquad y' = 2x^{-3} = \frac{2}{x^3}$

$$x^2 dy = -2xy\, dx$$
$$\frac{dy}{dx} = -\frac{2xy}{x^2}$$
$$\frac{dy}{dx} + \frac{2y}{x} = 0$$
$$\Rightarrow \frac{2}{x^3} + \frac{2\left(\frac{1}{-x^2}\right)}{x}$$

$$\Rightarrow \frac{2}{x^3} - \frac{\frac{2}{x^2}}{x} = 0$$
$$\frac{\cancel{2}}{x^3} - \frac{\cancel{2}}{x^3} = 0$$
$$0 = 0 \quad \therefore \text{Si es Sol.}$$

21. $2xy' + y(y')^2 = y$; $y^2 = C_1(x + \frac{1}{4}C_1)$??

Sea $C_1(x + \frac{1}{4}C_1) = w$

$y^2 = w \quad y = \sqrt{w} \quad y' = \frac{1}{2\sqrt{w}}$

$2x\left(\frac{1}{2\sqrt{w}}\right) + \sqrt{w}\left(\frac{1}{2\sqrt{w}}\right)^2 = \sqrt{w}$

$\frac{x}{\sqrt{w}} + \sqrt{w}\frac{1}{4(\sqrt{w})^2} = \sqrt{w}$

$\frac{x}{\sqrt{w}} + \frac{1}{4\sqrt{w}} = \sqrt{w}$

$\frac{x\,4\sqrt{w} + \sqrt{w}}{4w} = \sqrt{w}$

$\frac{\sqrt{w}(4x+1)}{4w} = \sqrt{w}$

$\frac{4x+1}{4\sqrt{w}} \neq \sqrt{w}$

$\frac{4x+1}{4\sqrt{C_1(x+\frac{1}{4}C_1)}} \neq \sqrt{C_1(x+\frac{1}{4}C_1)}$

$\therefore$ No es Sol.

23. $y' - \frac{1}{x}y = 1$; $y = x\ln x$, $x > 0$

$y' = x\frac{1}{x} + \ln x(1)$

$y' = 1 + \ln x$

$1 + \ln x - \frac{1}{\cancel{x}}(\cancel{x}\ln x) = 1$

$1 + \cancel{\ln x} - \cancel{\ln x} = 1$

$1 = 1 \quad \therefore$ Si es Sol.

25. $\frac{dx}{dt} = (2-x)(1-x)$; $\ln\frac{2-x}{1-x} = t$

$\frac{dt}{dx} = \frac{1}{(2-x)(1-x)}$

$t = \ln|2-x| - \ln|1-x|$

$t' = -\frac{1}{(2-x)} - \frac{(-1)}{(1-x)}$

$\frac{dt}{dx} - \frac{1}{(2-x)(1-x)} = 0$

$t' = \frac{1}{(1-x)} - \frac{1}{2-x}$

$\Rightarrow \cancel{\frac{1}{(1-x)(2-x)}} - \cancel{\frac{1}{(2-x)(1-x)}} = 0$

$t' = \frac{2 - \cancel{x} - 1 + \cancel{x}}{(1-x)(2-x)} = \frac{1}{(1-x)(2-x)}$

$0 = 0 \quad \therefore$ Si es solución

27. $(x^2+y^2)dx + (x^2 - xy)dy = 0$; $c(x+y)^2 = xe^{y/5}$

29. $y'' - 6y' + 13y = 0$; $y = e^{3x}\cos 2x$

$y' = e^{3x}[-\text{sen}(2x)(2)] + \cos 2x(3e^{3x}$

$= -2e^{3x}\text{sen}(2x) + 3e^{3x}\cos 2x$

$y'' = -2e^{3x}\cos(2x)(2) + \text{sen}(2x)[-6e^{3x}] + 3e^{3x}[-\text{sen}(2x)(2)] + \cos(2x)9e^{3x}$

$= -4e^{3x}\cos(2x) - 6e^{3x}\text{sen}(2x) - 6e^{3x}\text{sen}(2x) + 9e^{3x}\cos(2x)$

$= 5e^{3x}\cos(2x) - 12e^{3x}\text{sen}(2x)$

$5e^{3x}\cos(2x) - 12e^{3x}\text{sen}(2x) - 6[-2e^{3x}\text{sen}(2x) + 3e^{3x}\cos(2x)] + 13e^{3x}\cos(2x) = 0$

$\cancel{5e^{3x}\cos(2x)} \cancel{-12e^{3x}\text{sen}(2x)} \cancel{+12e^{3x}\text{sen}(2x)} \cancel{-18e^{3x}\cos(2x)} + \cancel{13e^{3x}\cos(2x)} = 0$

$0 = 0$ ∴ Es Solución.

31. $y'' = y$; $y = \cosh x + \text{senh}\, x$

$y' = \sinh x + \cosh x$

$y'' = \cosh x + \text{senh}\, x$

$\cosh x + \text{senh}\, x = \cosh x + \text{senh}\, x$

∴ Si es solución

33. $y'' + (y')^2 = 0 \qquad y = \ln|x| + C_1 + C_2$

$$y' = \frac{1}{x} \qquad y'' = -\frac{1}{x^2}$$

$$-\frac{1}{x^2} + \left(\frac{1}{x}\right)^2 = 0$$

$$-\frac{1}{x^2} + \frac{1}{x^2} = 0$$

$$0 = 0 \quad \therefore \text{ Si es Sol.}$$

35. $x\dfrac{d^2y}{dx^2} + \dfrac{2dy}{dx} = 0 \; ; \qquad y = C_1 + C_2x^{-1}, \quad x > 0$

$$y' = -C_2x^{-2} = -\frac{C_2}{x^2}$$

$$y'' = 2C_2x^{-3} = \frac{2C_2}{x^3}$$

$$x\left(\frac{2C_2}{x^3}\right) + 2\left(\frac{-C_2}{x^2}\right) = 0$$

$$\cancel{\frac{2C_2}{x^2}} - \cancel{\frac{2C_2}{x^2}} = 0$$

$$0 = 0 \quad \therefore \text{ Si es Solu.}$$

37. $x^2y'' - 3xy' + 4y = 0 \; ; \qquad y = x^2 + x^2\ln x, \quad x > 0$

$$y' = 2x + x^2\frac{1}{x} + \ln x \, 2x$$

$$= 3x + 2x\ln x$$

$$y'' = 3 + 2x\frac{1}{x} + \ln x \, 2$$

$$= 5 + 2\ln x$$

$$\Rightarrow x^2(5 + 2\ln x) - 3x(3x + 2x\ln x) + 4(x^2 + x^2\ln x) = 0$$

$$\cancel{5x^2} + \cancel{2x^2\ln x} - \cancel{9x^2} - \cancel{6x^2\ln x} + \cancel{4x^2} + \cancel{4x^2\ln x} = 0$$

$$0 = 0 \quad \therefore \text{ Si es Sol.}$$

39. $y''' - 3y'' + 3y' - y = 0 \; ; \qquad y = x^2e^x$

$$y' = 2xe^x + x^2e^x$$

$$y'' = 2xe^x + 2e^x + 2xe^x + x^2e^x$$

$$= 4xe^x + 2e^x + x^2e^x$$

$$y''' = 4xe^x + 4e^x + 2e^x + 2xe^x + x^2e^x$$

$$4xe^x + 4e^x + 2e^x + 2xe^x + x^2e^x - 3(4xe^x + 2e^x + x^2e^x) + 3(2xe^x + x^2e^x) - x^2e^x = 0$$

$$\cancel{6xe^x} + \cancel{6e^x} + \cancel{x^2e^x} - \cancel{12xe^x} - \cancel{6e^x} - \cancel{3x^2e^x} + \cancel{6xe^x} + \cancel{3x^2e^x} - \cancel{x^2e^x} = 0$$

$$0 = 0 \quad \therefore \text{ Si es Sol.}$$

CAP 2   1-47 (impares)

1) $\frac{dy}{dx} = \operatorname{sen} 5x$ $dy = \operatorname{sen}(5x)\,dx$ $\operatorname{sen}(5x)\,dx - dy = 0$

$\int \operatorname{sen}(5x)\,dx - \int dy = 0$

$-\frac{1}{5}\cos(5x) - y = C$

$\frac{1}{5}\cos(5x) + y = C_1$

3) $dx + e^{3x}\,dy = 0$ $dx = -e^{3x}\,dy$ $\frac{dx}{e^{3x}} = -dy \longrightarrow \frac{dx}{e^{3x}} + dy = 0$

$\int e^{-3x}\,dx + \int dy = 0$ $-\frac{1}{3}e^{-3x} + y = C.$

5) $(x+1)\frac{dy}{dx} = x + 6$ $\frac{dy}{dx} = \frac{x+6}{x+1}$ $dy = \frac{x+6}{x+1}\,dx$ $\frac{x+6}{x+1}\,dx - dy = 0$

$\int \frac{(x+1)+5}{(x+1)}\,dx - \int dy = 0$ $\int\left(1 + \frac{5}{x+1}\right)dx - \int dy = 0$

$x + 5\ln|x+1| - y = C$ $x\,\ln|(x+1)^5| - y = C$

7) $x\frac{dy}{dx} = 4y$ $\frac{dy}{dx} = \frac{4y}{x}$ $\frac{dy}{4y} = x\,dx$ $x\,dx - \frac{dy}{4} = 0$

$\int x\,dx - \int\frac{dy}{4} = 0$ $\ln|x| - \frac{1}{4}\ln|y| = C$ $\ln|x| - \ln|y^{1/4}| = C$

$\ln\left|\frac{x}{y^{1/4}}\right| = C$

9) $\frac{dy}{dx} = \frac{y^3}{x^2}$ $\frac{dy}{y^3} = \frac{dx}{x^2}$ $\frac{dx}{x^2} - \frac{dy}{y^3} = 0$ $\int\frac{dx}{x^2} - \int\frac{dy}{y^3} = 0$

$\int x^{-2}\,dx - \int y^{-3}\,dy = 0$ $-\frac{1}{x} + \frac{1}{2}y^{-2} = C$

11) $\frac{dx}{dy} = \frac{x^2y^2}{1+x}$ $\frac{dx}{dy} = y^2 \cdot \frac{x^2}{1+x}$ $\frac{1+x}{x^2}\,dx = y^2\,dy$ $\int\frac{(1+x)}{x^2}\,dx - \int y^2\,dy = 0$

$\int\frac{dx}{x^2} + \int\frac{dx}{x} - \int y^2\,dy = 0$ $-\frac{1}{x} + \ln|x| - \frac{1}{3}y^3 = C$

13) $\frac{dy}{dx} = e^{3x+2y}$ $\frac{dy}{dx} = e^{3x}\cdot e^{2y}$ $\frac{dy}{e^{2y}} = e^{3x}\,dx$ $e^{3x}\,dx - \frac{dy}{e^{2y}} = 0$

$\int e^{3x}\,dx - \int e^{-2y}\,dy = 0$ $\frac{1}{3}e^{3x} + \frac{1}{2}e^{-2y} = C$

15) $(4y + yx^2)\,dy - (2x + xy^2)\,dx = 0$ $(4y+yx^2)\,dy = (2x+xy^2)\,dx$

$\frac{dy}{dx} = \frac{(2x+xy^2)}{(4y+yx^2)}$ $\frac{dy}{dx} = \frac{x(2+y^2)}{y(4+x^2)}$ $\frac{y\,dy}{(2+y^2)} = \frac{x\,dx}{4+x^2}$

$$\frac{1}{2}\int\frac{x\,dx}{4+x^2}-\frac{1}{2}\int\frac{y\,dy}{2+y^2}=0 \qquad \frac{1}{2}\ln|4+x^2|-\frac{1}{2}\ln|2+y^2|=C$$

$$\ln|4+x^2|-\frac{1}{2}\ln|2+y^2|=C_1 \qquad \ln\left|\frac{4+x^2}{2+y^2}\right|=C_1$$

17) $2y(x+1)\,dy = x\,dx \qquad 2y\,dy=\frac{x}{x+1}dx \qquad \frac{x}{x+1}dx-2y\,dy=0$

$$\int\frac{x}{x+1}dx-\int 2y\,dy=0 \qquad \begin{aligned}u&=x+1\\x&=u-1\\du&=dx\end{aligned} \Rightarrow \int\frac{u-1}{u}du-\int 2y\,dy=0$$

$$\int du-\int\frac{du}{u}-\int 2y\,dy=0 \qquad u-\ln|u|-y^2=C$$

$$x+1-\ln|x+1|-y^2=C \qquad \underline{x-\ln|x+1|-y^2=C_1}$$

19) $y\ln x\frac{dx}{dy}=\left(\frac{y+1}{x}\right)^2 \qquad \frac{dx}{dy}=\frac{\frac{(y+1)^2}{x^2}}{y\ln x} \qquad \frac{dx}{dy}=\frac{(y+1)^2}{y\,x\ln x}$

$$x\ln x\,dx=\frac{(y+1)^2}{y}dy \qquad x\ln x-\frac{(y^2+2y+1)}{y}dy$$

$$\int x\ln x\,dx-\int\left(y+2+\frac{1}{y}\right)dy$$

$$\begin{aligned}u&=\ln x & du&=\frac{dx}{x}\\dv&=x\,dx & v&=\frac{1}{2}x^2\end{aligned} \qquad \begin{aligned}\int x\ln x\,dx&=\ln x\cdot\frac{1}{2}x^2-\int\frac{x^2}{2}\frac{dx}{x}\\&=\frac{1}{2}x^2\ln x-\frac{1}{2}\int x\,dx\\&=\frac{1}{2}x^2\ln x-\frac{1}{4}x^2+C\end{aligned}$$

$$\underline{\frac{1}{2}x^2\ln x-\frac{1}{4}x^2-\frac{1}{2}y^2-2y-\ln|y|=C}$$

21) $\frac{ds}{dr}=ks \qquad \frac{ds}{ks}=dr \qquad \frac{ds}{ks}-dr=0 \qquad k\int\frac{ds}{s}-\int dr=0$

$$\underline{k\ln|s|-r=C}$$

23) $\frac{dP}{dt}=P-P^2 \qquad \frac{dP}{P-P^2}=dt \qquad \frac{dP}{P-P^2}-dt=0$

$$\int\frac{dP}{P(1-P)}=\int\frac{A}{P}+\frac{B}{1-P}dP=\int\frac{A(1-P)+BP}{P(1-P)}dP=\int\frac{A-AP+BP}{P(1-P)}dP$$

$$\int\frac{A+P(B-A)}{P(1-P)} \qquad \int\frac{dP}{P-P^2}-\int dt=\int\left(\frac{1}{P}+\frac{1}{1-P}\right)dP\int dt$$

$B-A=0$

$A=1$

$\Rightarrow B=1$

$$\underline{\ln|P|-\ln|1-P|-t=C}$$

25. $\sec^2 x\, dy + \csc y\, dx = 0 \qquad \sec^2 x\, dy = -\csc y\, dx \qquad \frac{dy}{-\csc y} = \frac{dx}{\sec^2 x}$

$$\frac{dx}{\sec^2 x} + \frac{dy}{\csc y} = 0 \qquad \int \frac{1}{\sec^2 x} dx + \int \frac{1}{\csc y}\, dy = 0$$

$$\int \cos^2 x\, dx + \int \operatorname{sen} y\, dy = 0 \qquad \int \left(\frac{1 + \cos 2x}{2}\right) dx + \int \operatorname{sen} y\, dy$$

$$\frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x\, dx + \int \operatorname{sen} y\, dy = 0$$

$$\underline{\frac{1}{2}x + \frac{1}{4}\cos 2x - \cos y = C}$$

27. $e^y \operatorname{sen} 2x\, dx + \cos x\,(e^{2y} - y)\, dy = 0$

$$e^y \operatorname{sen} 2x\, dx = -\cos x\,(e^{2y} - y)\, dy$$

$$\frac{dx}{dy} = \frac{-\cos x\,(e^{2y} - y)}{e^y \operatorname{sen} 2x}$$

$$-\frac{\operatorname{sen} 2x}{\cos x} dx = \frac{(e^{2y} - y)}{e^y} dy$$

$$\int \frac{\operatorname{sen}(2x)}{\cos x} dx + \int \frac{(e^{2y} - y)}{e^y} dy = 0$$

$\operatorname{sen} 2x = \operatorname{Sen}(x + x)$

$= \operatorname{sen} x \cos x + \cos x \operatorname{sen} x$

$= 2 \operatorname{sen} x \cos x$

$$\int 2 \operatorname{sen} x\, dx + \int (e^{2y} - y) e^{-y}\, dy = 0$$

$$\int 2 \operatorname{sen} x\, dx + \int (e^y - ye^{-y})\, dy = 0$$

$$\underline{e^y - ye^{-y} + e^{-y} - 2\cos x = C}$$

$u = y \quad v = e^{-y}$

$du = dy \quad dv = -e^y \qquad y = e^{-y} - e^y$

29. $(e^y + 1)^2 e^{-y}\, dx + (e^x + 1)^3 e^{-x}\, dy = 0$

$$(e^y + 1)^2 e^{-y}\, dx = -(e^x + 1)^3 e^{-x}\, dy$$

$$-\frac{dx}{(e^x + 1)^3 e^{-x}} = \frac{dy}{(e^y + 1)^2 e^{-y}} \qquad \frac{dy}{(e^y + 1)^2 e^{-y}} + \frac{dx}{(e^x + 1)^3 e^{-x}} = 0$$

$$\frac{dy}{(e^{2y} + 2e^y + 1)e^{-y}} + \frac{dx}{(e^{3x} + 3e^{2x} + 3e^x + 1)e^{-x}} = 0$$

$$\frac{dy}{e^y + 2 + e^{-y}} + \frac{dx}{e^{2x} + 3e^x + 3 + e^{-x}} = 0$$

31) $(y - yx^2)\frac{dy}{dx} = (y+1)^2$ $\quad y(1-x^2)\frac{dy}{dx} = (y+1)^2$

$\frac{dy}{dx} = \frac{(y+1)^2}{y(1-x^2)}$ $\quad (1-x^2)\frac{dy}{dx} = \frac{y^2+2y+1}{y}$

$(1-x^2)\frac{dy}{dx} = y+2+\frac{1}{y}$ $\quad \frac{dy}{y+2+\frac{1}{y}} = (1-x^2)\,dx$

33). $\frac{dy}{dx} = \frac{xy+3x-y-3}{xy-2x+4y-8}$ $\quad \frac{dy}{dx} = \frac{x(y+3)-(y+3)}{x(y-2)+4(y-2)}$

$\frac{dy}{dx} = \frac{(y+3)}{(y-2)}\frac{(x-1)}{(x+4)}$ $\quad \frac{y-2}{y+3}\,dy = \frac{(x-1)}{(x+4)}\,dx$

$\int\left(\frac{x-1}{x+4}\right)dx - \int\left(\frac{y-2}{y+3}\right)dy = 0$ $\quad \int\frac{x}{x+4}dx - \int\frac{dx}{x+4} - \int\frac{y\,dy}{y+3} - \int\frac{2\,dy}{y+3}$

$u = x+4 \quad v = y+3$
$du = dx \quad dv = dy$
$x = u-4 \quad y = v-3$

$\Rightarrow \int\frac{u-4}{u}\,du - \int\frac{du}{u} - \int\frac{v-3}{v}dv = \int\frac{2dv}{v} = 0$

$\int du - \int\frac{4du}{u} - \int\frac{du}{u} - \int dv + \int\frac{3dv}{v} - \int\frac{2dv}{v} = 0$

$u - 4\ln|u| - \ln|u| - v + 3\ln|v| - 2\ln|v| = c$

$u - 5\ln|u| - v + \ln|v| = c$

$\underline{x+4 - \ln|(x+4)^5| - (y+3) + \ln|y+3| = c}$

35. $\frac{dy}{dx} = \operatorname{sen}x(\cos 2y - \cos^2 y)$ $\quad \frac{dy}{\cos 2y - \cos^2 y} = \operatorname{sen}x\, dx$

$$\operatorname{sen}x\, dx - \frac{dy}{\cos 2y - \cos^2 y} = 0$$

$$\operatorname{sen}x\, dx - \frac{dy}{\cancel{\cos^2 y} - \operatorname{Sen}^2 y - \cancel{\cos^2 y}} = 0$$

$$\int \operatorname{sen}x\, dx + \int \csc^2 y\, dy = 0 \qquad \underline{-\cos x - \cot y = C}$$

37) $x\sqrt{1-y^2}\, dx = dy$ $\quad x\,dx = \frac{dy}{\sqrt{1-y^2}}$ $\quad \int x\,dx - \int \frac{dy}{\sqrt{1-y^2}} = 0$

$$\underline{\tfrac{1}{2}x^2 - \arcsin y = C}$$

39) $(e^x + e^{-x})\frac{dy}{dx} = y^2$ $\quad \frac{dy}{y^2} = \frac{dx}{e^x + e^{-x}}$

41) $(e^{-y}+1)\operatorname{sen}x\, dx = (1+\cos x)\, dy$, $\quad y(0) = 0$

$$\frac{\operatorname{sen}x}{1+\cos x}dx = (e^{-y}+1)\,dy \qquad \int \frac{\operatorname{sen}x}{1+\cos x}dx - \int (e^{-y}+1)\,dy = 0$$

$$\int \left(\frac{\operatorname{sen}x}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x}\right)dx - \int (e^{-y}+1)\,dy = 0$$

$$\int \left(\frac{\operatorname{sen}x - \cos x \operatorname{sen}x}{1-\cos^2 x}\right)dx - \int (e^{-y}+1)\,dy = 0$$

$$\int \left(\frac{\operatorname{sen}x - \cos x \operatorname{sen}x}{\operatorname{sen}^2 x}\right)dx - \int (e^{-y}+1)\,dy = 0$$

$$\int \frac{1}{\operatorname{sen}x}dx - \int \frac{\cos x}{\operatorname{sen}x}dx - \int (e^{-y}+1)\,dy = 0$$

$$\int \csc\, dx - \int \cot x\, dx - \int (e^{-y}+1)\,dy = 0$$

$\ln|\csc x - \cot x| - \ln|\operatorname{sen} x| + e^{-y} = C$

$y(0) = 0 \quad ó \quad (0,0)$

$\ln|\csc 0 - \cot 0| - \ln|\operatorname{sen} 0| + e^{0} = C$

$1 = C$

$\ln|\csc x - \cot x| - \ln|\operatorname{sen} x| + e^{-y} - 1 = 0$ Sol. Part.

43) $y\,dy = 4x(y^2+1)^{1/2}\,dx = 0 \quad y(1) = 0$

$\frac{y\,dy}{\sqrt{y^2+1}} = 4x\,dx \qquad \int 4x\,dx - \frac{1}{2}\int \frac{y\,dy}{\sqrt{y^2+1}} = 0$

$\frac{4}{2}x^2 - \frac{1}{2}\int u^{-1/2}\,du \qquad u = y^2+1, \; du = 2y$

$2x^2 - u^{1/2} = C \Rightarrow \underline{2x^2 - \sqrt{y^2+1} = C}$ Sol. Gral

$y(1) = 0 \quad ó \quad (1,0)$

$2(1)^2 - \sqrt{(0)^2+1} = C$

$2 - 1 = C$

$1 = C$

$\underline{2x^2 - \sqrt{y^2+1} - 1 = 0}$ Sol Part.

45) $\frac{dx}{dy} = 4(x^2+1), \quad x\left(\frac{\pi}{4}\right) = 1 \qquad \int \frac{dx}{x^2+1} = \int 4\,dy \qquad \int \frac{dx}{x^2+1} - 4\int dy = 0$

$\underline{\operatorname{arctg} x - 4y = C}$ Sol. Gral. $\quad x\left(\frac{\pi}{4}\right) = 1 \quad ó \quad \left(1, \frac{\pi}{4}\right)$

$\operatorname{arctg}(1) - 4\left(\frac{\pi}{4}\right) = C \qquad 45 - \pi = C \qquad 41.85 \approx C$

$\underline{\operatorname{arctg} x - 4y - 41.85 = 0}$ Sol. Part.

17 $x^2 \frac{dy}{dx} = y - xy$ $\qquad y(-1) = -1$

$x^2 \frac{dy}{dx} = y(1-x)$ $\qquad \frac{dy}{y} = \frac{(1-x)dx}{x^2}$ $\qquad \frac{(1-x)dx}{x^2} - \frac{dy}{y}$

$\int \frac{dx}{x^2} - \int \frac{dx}{x} - \int \frac{dy}{y} = 0$ $\qquad -\frac{1}{x} - \ln|x| - \ln|y| = C$

$-\frac{1}{x} - \ln\left|\frac{x}{y}\right| = C$ $\qquad y(-1) = -1$ ó $(-1, -1)$

$\frac{1}{-(-1)} - \ln\left|\frac{-1}{-1}\right| = C$ $\qquad 1 - 0 = C$

$1 = C$

$-\frac{1}{x} - \ln\left|\frac{x}{y}\right| - 1 = 0$ Sol: Parti.

Gill 2.4 Pag 67 1-14

1) $(x-y)dx + x\,dy = 0$

$x\,dy = -(x-y)dx$

$x\,dy = (y-x)dx$

$\frac{dy}{dx} = \frac{y-x}{x}$

$\frac{dy}{dx} = \frac{y}{x} - 1$

$\frac{dy}{dx} = v + \frac{dv}{dx}$ $\qquad$ Sea $\frac{y}{x} = v$

$v + \frac{x\,dv}{dx} = v - 1$

$dv = -\frac{dx}{x}$

$\int \frac{dx}{x} + \int dv = 0$

$\ln|x| + v = C$

$\ln|x| + \frac{y}{x} = C$

2. $(x+y)dx + x\,dy = 0$

Sea $v = \frac{y}{x}$

$x\,dy = -(x+y)dx$

$dy = -\frac{(x+y)}{x}dx$

$dy = -(1+v)dx$

$\frac{dy}{dx} = -(1+v)$

$v + x\frac{dv}{dx} = -1 - v$

$x\frac{dv}{dx} = -1 - 2v$

$x\,dv = -(1+2v)dx$

$\frac{dv}{1+2v} = -\frac{dx}{x}$

$\int \frac{dx}{x} + \int \frac{dv}{1+2v} = 0$

$\ln|x| + \frac{1}{2}\ln|1+2v| = C$

$\ln|x(1+2v)^{1/2}| = \ln C$

$x\left(1 + 2\frac{y}{x}\right)^{1/2} = C_1$

3. $x\,dx + (y-2x)\,dy = 0$

Sea $u = \frac{y}{x}$

$(y-2x)\,dy = -x\,dx$

$$\frac{dy}{dx} = -\frac{x}{y-2x}$$

$$\frac{du}{dx} = -\left(\frac{1}{u-2}\right)$$

$$u + x\frac{du}{dx} = -\left(\frac{1}{u-2}\right)$$

$$x\frac{du}{dx} = -\left(\frac{1}{u-2} - \frac{u}{1}\right)$$

$$x\frac{du}{dx} = -\left(\frac{1-u^2+2u}{u-2}\right)$$

$$\frac{u-2\,du}{1-u^2+2u} = -\frac{dx}{x}$$

$$\int\frac{(u-2)\,du}{1-u^2+2u} + \int\frac{dx}{x} = 0$$

4. $y\,dx = 2(x+y)\,dy$

$$\frac{dy}{dx} = \frac{y}{2x+2y} \qquad \text{Sea } u = \frac{y}{x}$$

$$\frac{du}{dx} = \frac{u}{2+2u}$$

$$u + x\frac{du}{dx} = \frac{u}{2(1+u)}$$

$$x\frac{du}{dx} = \frac{u}{2(1+u)} - u$$

$$x\frac{du}{dx} = \frac{u - 2u(1+u)}{2(1+u)}$$

$$\frac{(2+2u)\,du}{u-2u-2u^2} = \frac{dx}{x}$$

$$\int\frac{dx}{x} - \int\frac{2+2u\,du}{-u-2u^2}$$

$$\ln|x| + 2\int\frac{(1+u)\,du}{2u^2+u}$$

5. $(y^2+yx)dx - x^2dy = 0$

$(y^2+yx)dx = x^2 dy$

$$\frac{dy}{dx} = \frac{y^2+yx}{x^2}$$

$$\frac{du}{dx} = u^2 + \frac{u}{x} \qquad u^2 + \frac{u}{x}$$

$$u + x\frac{du}{dx} = u^2 + \frac{u}{x}$$

$$x\frac{du}{dx} = u^2 + \frac{u}{x} - u$$

$$0 = x\frac{du}{dx} + u^2 + \frac{u}{x} - u$$

6. $(y^2+yx)dx + x^2dy = 0$

$(y^2+yx)dx = -x^2dy$

$$\frac{dy}{dx} = -\frac{x^2}{(y^2+yx)}$$

$$\frac{dy}{dx} = \frac{-x^2}{y(y+x)}$$

$$\frac{du}{dx} = -\frac{x}{u(u+1)}$$

$$u + x\frac{du}{dx} = -\frac{x}{u(u+1)} - u$$

7.- $\frac{dy}{dx} = \frac{y-x}{y+x}$

$v = \frac{y}{x} \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx - x}{vx + x}$

$= \frac{(v-1)(v+1)v}{v+1} = \frac{v-1-v^2-v}{v+1}$

$\int \frac{(v+1)dv}{1+v^2} + \int \frac{dx}{x} = 0$

$\frac{1}{2}\ln|1+v^2| + \operatorname{arctg}(1+v^2) + \ln x = 0$

$\ln|(1+\frac{y^2}{x^2}| + \operatorname{arctg}(1+\frac{y^2}{x^2}) = C$

$\ln|x+\frac{y^2}{x}| + \operatorname{arctg}(1+\frac{y^2}{x^2}) = C$

8. $\frac{dy}{dx} = \frac{x+3y}{3x+y}$

$v + x\frac{dv}{dx} = \frac{1+3v}{3+v}$

$x\frac{dv}{dx} = \frac{1+3v}{3+v} - \frac{v}{1} = \frac{1+3v-3v+v^2}{3v} = \frac{1-v^2}{3v}$

$x\,dv = \frac{1-v^2}{3v}dx$

$\frac{3v\,dv}{1-v^2} = \frac{dx}{x}$

$\int\frac{dx}{x} - 3\int\frac{v\,dv}{(1-v^2)} = 0$

$\ln|x| + \frac{3}{2}\ln|1-v^2| = C$

$w = 1-v^2$
$dw = -2v$

$x(1-v^2)^{3/2} = C_1$

$x\left(1-\frac{y}{x}\right)^{3/2} = C_1$

~~INTEGRALES CON FUNCIONES TRIGONOMETRICAS~~

Pag 50. 2.2. 1 - 42 Impares

1. $\underbrace{(2x-1)}_{M}dx + \underbrace{(3y+7)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = 0 \qquad \frac{\partial N}{\partial x} = 0$

$\frac{\partial U}{\partial x} = 2x-1 \qquad \frac{\partial U}{\partial y} = 3y+7$

$U = \int (2x-1)\partial x + F(y) = x - x + F'(y)$

$\frac{\partial}{\partial y}(f'(y)) = 3y+7$

$f'(y) = 3y+7$

$f(y) = \frac{3}{2}y^2 + 7$

$f(y) = 3y^2 + 14y$

$3y^2 + 14y = C$

3. $\underbrace{(5x+4y)}_{M}dx + \underbrace{(4x-8y^3)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = 4 \qquad \frac{\partial N}{\partial x} = 4$

$U = \int (5x+4y)\partial x + F(y)$

$= \frac{5x^2}{2} + f'(y)$

$\frac{d}{\partial y}\left[\frac{5x^2}{2} + 4yx + f'(y)\right] = 4x - 8y^3$

$4x + f'(y) = 4x - 8y^3$

$f'(y) = -8y^3$

$f(y) = \frac{-8y^4}{4}$

$f(y) = -2y^4$

$\frac{5}{2}x^2 + 4yx - 2y^4 = C$

5) $\underbrace{(2y^2x-3)}_{M}dx + \underbrace{(2yx^2+4)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = 2xy \qquad \frac{\partial N}{\partial x} = 2yx$

$\int (2y^2x-3)\partial x + f(y) = \frac{\cancel{2}y^2x^2}{\cancel{2}} - 3x + f'(y)$

$\frac{\partial}{\partial y}\left[y^2x^2 - 3x + f'(y)\right] = 2yx^2 + 4$

$2x^2y + f'(y) = 2yx^2 + 4$

$f'(y) = 4$

$f(y) = 4y$

$y^2x^2 - 3x + 4y = C$

*7. $(x+y)(x-y)dx + x(x-2y)dy = 0$

$\underbrace{(x^2-y^2)}_{M}dx + \underbrace{(x^2-2xy)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = -2y \qquad \frac{\partial M}{\partial x} = 2x - 2y$

$\therefore$ No es exacta

9) $\underbrace{(y^3 - y^2 \operatorname{sen}x - x)}_{M}dx + \underbrace{(3xy^2 + 2y\cos x)}_{N}dy$

$\frac{\partial M}{\partial y} = 3y^2 - 2y \operatorname{sen}x \quad \frac{\partial N}{\partial x} = 3y^2 - 2y \operatorname{sen}x$

$U = \int (y^3 - y^2 \operatorname{sen}x - x)\partial x + F(y)$

$= y^3x + y^2\cos x - \frac{x^2}{2} + f'(y) = 3xy^2 + 2y\cos x$

$\frac{\partial}{\partial y}\left[xy^3 + y^2\cos x - \frac{x^2}{2} + f(y)\right] = 3xy^2 + 2y\cos x$

$3xy^2 + 2y\cos x + f'(y) = 3xy^2 + 2y\cos x$

$F'(y) = 0$

$f(y) = C$

$xy^3 + y^2\cos x - \frac{x^2}{2} + C$

$xy^3 + y^2\cos x - \frac{x^2}{2} = C$ //

11) $\underbrace{(y\ln y - e^{-xy})}_{M}dx + \underbrace{(\frac{1}{y} + x\ln y)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = y\frac{1}{y} + \ln - e^{-xy} \quad \frac{\partial M}{\partial x} = x\frac{1}{y}$

No es exacta.

13) $x\frac{dy}{dx} = 2xe^x - y + 6x^2$

$\underbrace{(2xe^x - y + 6x^2)}_{M}dx - \underbrace{x}_{N}\,dy = 0$

$\frac{\partial M}{\partial y} = -1 \qquad \frac{\partial N}{\partial x} = -1$

$\int -x\,\partial y + f(x)$

$-xy + f'(x) = 2xe^x - y + 6x^2$

$\frac{\partial}{\partial x}\left[-xy + f'(x)\right] = 2xe^x - y + 6x^2$

$\cancel{-y} + f'(x) = 2xe^x \cancel{-y} + 6x^2$

$f'(x) = 2xe^x + 6x^2$

$-xy + 2xe^x + 6x^2 = C$ //

15) $\underbrace{(1 - \frac{3}{x} + y)}_{M}dx + \underbrace{(1 - \frac{3}{y} + x)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = 1 \qquad \frac{\partial N}{\partial x} = 1$

$\int (1 - \frac{3}{x} + y)\partial x + F(y)$

$= x - 3\ln|x| + yx + f(y)$

$\frac{\partial}{\partial y}\left[x - 3\ln|x| + yx + f(y)\right] = 1 - \frac{3}{y} + x$

$\cancel{x} + f'(y) = 1 - \frac{3}{y} + \cancel{x}$

$f'(y) = 1 - \frac{3}{y}$

$f(y) = y - 3\ln|y|$

$x - 3\ln|x| + yx + y - 3\ln|y| = C$

$x - \ln|x^3| + xy + y - \ln|y^3| = C$

$x + xy + y - \ln\left|\frac{x^3}{y^3}\right| = C$ //

17) $\left(x^2y^3 - \frac{1}{1+9x^2}\right)\frac{dx}{dy} + x^3y^2 = 0$

$$\underbrace{\left(x^2y^3 - \frac{1}{1+9x^2}\right)}_{M}dx + \underbrace{(x^3y^2)}_{N}dy = 0$$

$$\frac{\partial M}{\partial y} = 3x^2y^2 \qquad \frac{\partial N}{\partial x} = 3y^2x^2$$

$$\int (x^3y^3)\,\partial y + f(x)$$
$$= \frac{x^3y^4}{4} + f(x)$$

$$\frac{\partial}{\partial x}\left[\frac{x^3y^4}{4} + f(x)\right] = x^2y^3 - \frac{1}{1+9x^2}$$

$$\frac{3y^4}{4}x^2 + f(x) = x^2y^3 - \frac{1}{1+9x^2}$$

$f(x) =$

19. $(\underbrace{\tan x - \operatorname{sen} x \operatorname{sen} y}_{M})dx + \underbrace{\cos x \cos y}_{N}\,dy = 0$

$$\frac{\partial M}{\partial y} = -\operatorname{sen} x \cos y \qquad \frac{\partial N}{\partial x} = -\operatorname{sen} x \cos y$$

$$\int (\cos x \cos y)\,\partial y + f(x)$$
$$= \cos x \operatorname{sen} y + f(x)$$

$$\frac{\partial}{\partial x}[\cos x \operatorname{sen} y + f(x)] = \tan x - \operatorname{sen} x \operatorname{sen} y$$

$$-\operatorname{sen} y \operatorname{sen} x + f'(x) = \tan x \cancel{- \operatorname{sen} x \operatorname{sen} y}$$
$$f'(x) = \tan x$$
$$f(x) = \ln|\sec x|$$

$$\cos x \operatorname{sen} y + \ln|\sec x| = C$$

21) $(1 - 2x^2 - 2y)\frac{dy}{dx} = 4x^3 + 4xy$

$$\underbrace{(4x^3 + 4xy)}_{M}dx + \underbrace{(-1 + 2x^2 + 2y)}_{N}dy = 0$$

$$\frac{\partial M}{\partial y} = 4x \qquad \frac{\partial N}{\partial x} = 4x$$

$$\int (4x^3 + 4xy)\,\partial x + f(y)$$
$$= \frac{4x^4}{4} + \frac{4yx^2}{2} + f(y)$$

$$\frac{d}{\partial y}[x^4 + 2yx^2 + f(y)] = 2x^2 + 2y - 1$$

$$\cancel{2x^2} + f'(y) = \cancel{2x^2} + 2y - 1$$
$$f'(y) = 2y - 1$$
$$f(y) = y^2 - y$$

$$x^4 + 2yx^2 + y^2 - y = C$$

23) $\underbrace{(4x^3y - 15x^2 - y)}_{M}dx + \underbrace{(x^4 + 3y - x)}_{N}dy = 0$

$$\frac{\partial M}{\partial y} = 4x^3 - 1 \qquad \frac{\partial N}{\partial x} = 4x^3 - 1$$

$$\int (4x^3y - 15x^2 - y)\,dx + f(y)$$
$$= \frac{\cancel{4}yx^4}{\cancel{4}} - \frac{15x^3}{3} - yx + f(y)$$

$$\frac{d}{\partial y}[yx^4 - 5x^3 - yx + f(y)] = x^4 + 3y - x$$

$$\cancel{x^4} \cancel{- x} + f'(y) = \cancel{x^4} + 3y \cancel{- x}$$
$$f'(y) = 3y$$
$$f(y) = \frac{3y^2}{2}$$

$$yx^4 - 5x^3 - yx + \frac{3}{2}y^2$$

25) $(x+y)^2dx+(2xy+x^2-1)dy=0$

$y(1)=1$

$\underbrace{(x^2+2xy+y^2)}_{M}dx+\underbrace{(2xy+x^2-1)}_{N}dy=0$

$\frac{\partial M}{\partial y}=2x+2y \quad \frac{\partial N}{\partial x}=2y+2x$

$\int(x^2+2xy+y^2)\partial x+f(y)$

$=\frac{x^3}{3}+\frac{2yx^2}{2}+y^2x+f(y)$

$\frac{\partial}{\partial y}\left[\frac{x^3}{3}+yx^2+y^2x+f(y)\right]=2xy+x^2-1$

$\cancel{x^2}+\cancel{2xy}+f'(y)=\cancel{2xy}+\cancel{x^2}-1$

$f'(y)=-1$

$f(y)=-y$

$\frac{x^3}{3}+yx^2+y^2x-y=C$

$y(1)=1 \quad (1,1)$

$\frac{1^3}{3}+(1)(1)^2+(1)^2(1)-1=C$

$\frac{1}{3}+1+\cancel{1}-\cancel{1}=C$

$\frac{4}{3}=C$

$\therefore \frac{x^3}{3}+yx^2+y^2x-y=\frac{4}{3}$

27) $(4y+2x-5)dx+(6y+4x-1)dy=0$

$y(-1)=2$

$\frac{\partial M}{\partial y}=4 \quad \frac{\partial N}{\partial x}=4$

$\int(4y+2x-5)\partial x+f(y)$

$=4yx+x^2-5x+f(y)$

$\frac{\partial}{\partial y}[4yx+x^2-5x+f(y)]=6y+4x-1$

$\cancel{4x}+f'(y)=6y+\cancel{4x}-1$

$f'(y)=6y-1$

$f(y)=\frac{6y^2}{2}-y=3y^2-y$

$(-1,2)$

$4yx+x^2-5x+3y^2-y=C$

$4(2)(-1)+(-1)^2-5(-1)+3(2)^2-(2)=C$

$-8+1+5+12-2=C$

$8=C$

$\rightarrow 4yx+x^2-5x+3y^2-y=8$

29) $\underbrace{(y^2\cos x-3x^2y-2x)}_{M}dx+\underbrace{(2y\operatorname{sen}x-x^3+\ln y)}_{N}dy=0$

$y(0)=e$

$\frac{\partial M}{\partial y}=2y\cos x-3x^2$

$\frac{\partial N}{\partial x}=2y\cos x-3x^2$

$\int(y^2\cos x-3x^2y-2x)\partial x+f(y)$

$y^2\operatorname{sen}x-\frac{3yx^3}{3}-\frac{2x^2}{2}+f(y)$

$\frac{\partial}{\partial y}[y^2\operatorname{sen}x-yx^3-x^2+f(y)]$

$\cancel{2y\operatorname{sen}x}-\cancel{x^3}+f'(y)=\cancel{2y\operatorname{sen}x}-\cancel{x^3}+\ln y$

$f'(y)=\ln y$

$\int\ln y\,dy \quad u=\ln y \quad du=\frac{1}{y}dy$

$dv=dy \quad v=y$

$\hookrightarrow y\ln y-\int dy=y\ln y-y$

$y^2\operatorname{sen}x-yx^3-x^2+y\ln y-y=C$

$y(0)=e \quad (0,e)$

$(e)^2\operatorname{sen}(0)-(e)(0)^3+e\ln e-e=C$

$e-e=C$

$0=C$

$\therefore y^2\operatorname{sen}x-yx^3-x^2+y\ln-y=0$

31) $\underbrace{(y^3 + Kxy^4 - 2x)}_{M}dx + \underbrace{(3xy^2 + 20x^2y^3)}_{N}dy = 0$

$$\frac{\partial N}{\partial x} = 3y^2 + 40y^3x$$

$$\frac{\partial M}{\partial y} = 3y^2 + 4Kxy^3$$

$$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y} \Rightarrow \cancel{3y^2} + 40y^3x = \cancel{3y^2} + 4Kxy^3$$

$$40\cancel{y^3}x = 4K\cancel{xy^3}$$

$$40 = 4K$$

$$K = 10$$

33. $(2xy^2 + ye^x)dx + (2x^2y + Ke^x - 1)dy = 0$

35. Deduzca una función tal que la ec. dif sea exacta.

$$M(x,y)\,dx + (xe^{xy} + 2xy + \frac{1}{x})dy = 0$$

$$\frac{\partial N}{\partial x} = xe^{xy} + e^{xy} + 2y - \frac{1}{x^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \frac{\partial M}{\partial y} = xye^{xy} + e^{xy} + 2y - \frac{1}{x^2}$$

$$\int \partial M(x,y) = \int (xye^{xy} + e^{xy} + 2y - \frac{1}{x^2})\partial y$$

$u = y \quad du = dy$

$dv = e^{xy} \quad v = \frac{1}{x}e^{xy}$

$$M(x,y) = x\left(\frac{y}{x}e^{xy} - \frac{1}{x^2}e^{xy}\right) + \frac{1}{x}e^{xy} + y^2 - \frac{y}{x^2} + f(x)$$

$$= ye^{xy} - \cancel{\frac{1}{x}e^{xy}} + \cancel{\frac{1}{x}e^{xy}} + y^2 - \frac{y}{x^2} + f(x)$$

$ye^{xy} - \frac{1}{x}\int e^{xy}dy$

$ye^{xy} - \frac{1}{x^2}e^{xy}$

$$M(x,y) = ye^{xy} + y^2 - \frac{y}{x^2} + f(x)$$

37) Compruebe que $\mu(x,y)$ (factor integrante) haga exacta la ecuación.

$$6xy\,dx + (4y + 9x^2)dy = 0 \qquad \mu(x,y) = y^2$$

Multiplicando por y^2

$$\underbrace{6xy^3}_{M}\,dx + (\underbrace{4y^3 + 9y^2x^2}_{N})$$

$$\frac{\partial M}{\partial y} = 18xy^2$$

$$\frac{\partial N}{\partial x} = 18y^2x$$

39. $(-xy\,\mathrm{sen}x + 2y\cos x)dx + 2x\cos x\,dy = 0 \qquad \mu(x,y) = xy$

Multiplicando por xy

$$(-x^2y^2\,\mathrm{sen}x + 2xy^2\cos x)dx + (2x^2y\cos x)dy = 0$$

$$\frac{\partial M}{\partial y} = -x^2\,\mathrm{sen}x\,2y + 2x\cos x\,2y$$
$$= -2x^2y\,\mathrm{sen}x + 4xy\cos x$$

$$\frac{\partial N}{\partial x} = -2x^2y\,\mathrm{sen}x + 4xy\cos x$$

41. $(2y^2 + 3x)dx + 2xy\,dy = 0 \qquad \mu(x,y) = x$

Multiplicamos por x

$$\underbrace{(2xy^2 + 3x^2)}_{M}dx + \underbrace{2x^2y}_{N}\,dy$$

$$\frac{\partial M}{\partial y} = 4xy \qquad \frac{\partial N}{\partial x} = 4xy$$

Murray p. 52

Sección A.

a) $\underbrace{(3x+2y^2)}_{M}dx + \underbrace{2xy}_{N}\,dy = 0$

$\frac{\partial M}{\partial y} = 4y \qquad \frac{\partial N}{\partial x} = 2y$

$T_I: \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{4y-2y}{2xy} = \frac{2\cancel{y}}{2x\cancel{y}}$

$= \frac{1}{x} \Rightarrow e^{\int \frac{1}{x}dx} = e^{\ln|x|}$

$= x$

Multiplicamos $M(x,y) + N(x,y)$ por x

$\rightarrow (3x^2+2xy^2)dx + 2x^2y\,dy = 0$

$\frac{\partial M}{\partial y} = 4xy \qquad \frac{\partial N}{\partial x} = 4xy$

$U = \int(3x^2+2xy^2)\partial x + f(x)$

$= \frac{3x^3}{3} + \frac{2y^2x^2}{2} + f(x)$

$\frac{\partial}{\partial y}[x^3+y^2x^2+f(x)]$

$\cancel{2x^2y} + F'(x) = \cancel{2x^2y}$

$F'(x) = 0$

$F(x) = C$

$\therefore x^3+y^2x^2+C = C_1$

$x^3+y^2x^2 = C_2$ //

b) $(2x^3-y)dx + x\,dy = 0 \quad y(1)=1$

$\frac{\partial M}{\partial y} = -1 \qquad \frac{\partial N}{\partial x} = 1$

$T_I: \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-1-1}{x} = -\frac{2}{x}$

$\Rightarrow e^{\int -\frac{2}{x}dx} = e^{-2\ln|x|} = \cancel{e}^{\cancel{\ln}|x^{-2}|}$

$= x^{-2} = \frac{1}{x^2}$

$\Rightarrow (2x - \frac{y}{x^2})dx + \frac{1}{x}dy$

$\frac{\partial M}{\partial y} = -\frac{1}{x^2} \qquad \frac{\partial N}{\partial x} = -\frac{1}{x^2}$

$\int(2x - \frac{y}{x^2})\partial x + f(y)$

$= x - \frac{y}{x} + f(y)$

$\frac{\partial}{\partial y}[x - \frac{y}{x} + f(y)] = \frac{1}{x}$

$\cancel{\frac{1}{x}} + f'(y) = \cancel{\frac{1}{x}}$

$f'(y) = 0$

$f(y) = C$

$\Rightarrow x - \frac{y}{x} = C_1$ //

$y(1)=1$

$1 - \frac{1}{1} = C_1$

$0 = C_1$

$x - \frac{y}{x} = 0$ //

c) $(y^2\cos x - y)dx + (x+y^2)dy$

$\frac{\partial M}{\partial y} = \cos x\, 2y - 1 \qquad \frac{\partial N}{\partial x} = 1$

$T_I: \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{\cos x\, 2y - 1 - 1}{x+y^2}$

$\frac{2y\cos x - 2}{x+y^2}$

$T_2: \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{1-\cos x\, 2y + 1}{y^2\cos x - y}$

$= \frac{2-2y\cos x}{y(y\cos x - 1)} = \frac{2(1-y\cos x)}{-y(1-y\cos x)}$

$= -\frac{2}{y} \Rightarrow e^{\int -\frac{2}{y}dy} = e^{-2\ln|y|}$

$= \cancel{e}^{\cancel{\ln}|y^{-2}|} = y^{-2} = \frac{1}{y^2}$

$\Rightarrow (\cos x - \frac{1}{y})dx + (\frac{x}{y^2}+1)dy$

$\frac{\partial M}{\partial y} = \frac{1}{y^2} \quad \frac{\partial N}{\partial x} = \frac{1}{y^2}$

$$\int \left(\cos x - \frac{1}{y}\right) \partial x + f(y)$$

c)

$$-\operatorname{sen} x - \frac{x}{y} + f(y)$$

$$\frac{d}{\partial y}\left[-\operatorname{sen} x - \frac{x}{y} + f(y)\right] = \frac{x}{y^2} + 1$$

$$\cancel{\frac{x}{y^2}} + f'(y) = \cancel{\frac{x}{y^2}} + 1$$

$$f'(y) = 1$$

$$f(y) = y$$

$$\underline{-\operatorname{sen} x - \frac{x}{y} + y = C}$$

d) $(x + x^3 \operatorname{sen}(2y))dy - 2y\,dx = 0$

$$\frac{\partial M}{\partial y} = x^3 2\cos(2y) \qquad \frac{\partial N}{\partial x} = 0$$

$$T_I: \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2x^3 \cos(2y)}{\cancel{2y}}$$

$$T_{II}: \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{-2x^3 \cos(2y)}{x + x^3 \operatorname{sen}(2y)}$$

e) $\frac{dy}{dx} = \frac{\operatorname{sen} y}{x\cos y - \operatorname{sen}^2 y}$; $y(0) = \frac{\pi}{2}$

$$(\operatorname{sen} y)dx - (x\cos y - \operatorname{sen}^2 y)dy = 0.$$

$$\frac{\partial M}{\partial y} = \cos y \qquad \frac{\partial N}{\partial x} = -\cos y$$

$$T_I: \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{\cos y + \cos y}{\operatorname{sen} y}$$

f) $(2y\,sen\,x - \cos^3 x)dx + \cos x\,dy = 0$

$\frac{\partial M}{\partial y} = 2sen\,x \qquad \frac{\partial N}{\partial x} = -sen\,x$

$T_I = \dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \dfrac{2sen\,x + sen\,x}{\cos x}$

$= \dfrac{3sen\,x}{\cos x} = 3\tan x$

$e^{\int 3\tan x\,dx} = e^{3\ln|\sec x|}$

$\sec^3 x$ //

$(2y\tan x\sec^2 x - 1)dx + \sec^2 x\,dy = 0$

$\frac{\partial M}{\partial y} = 2\tan x\sec^2 x \quad \frac{\partial N}{\partial x} = 2\tan x\sec^2 x$

$\int (2y\tan x\sec^2 x - 1)\partial x + f(y) =$

$2y$

26/Sep/13

LA ECUACIÓN DIF LINEAL DE PRIMER ORDEN.

Recordemos que una ecuación diferencial de orden n tiene la forma general...

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_1(x)\frac{dy}{dx} + a_0(x)y = F(x) \quad \dots (1)$$

En dónde $a_n(x)$, $a_{n-1}(x)$, ..., $a_0(x)$ con $a_n(x) \neq 0$ son coefic. de (1)

En la ec. dif. (1) se observan las sig. características...

- Los coeficientes $a_n(x)$, ... $a_0(x)$ y $F(x)$ son funciones dadas de la variable independiente x.
- La función desconocida y sus derivadas están elevadas a la potencia uno.
- En (1) no deben aparecer funciones trascendentes en cuyo argumento esté involucrada la función desconocida Y y/o sus derivadas.

Si en (1) $n = 1$, entonces la ecuación se reduce a:

$$a_1(x)\frac{dy}{dx} + a_0(x)y = F(x) \quad \dots (2)$$

La cual representa una ecuación dif. lineal de primer orden.

Multiplicando (2) por $\frac{1}{a_1(x)}$

$$\frac{1}{a_1(x)}\left[a_1(x)\frac{dy}{dx} + a_0(x)y = F(x)\right]$$

$$\frac{dy}{dx} + \underbrace{\frac{a_0(x)}{a_1(x)}}_{P(x)} y = \underbrace{\frac{F(x)}{a_1(x)}}_{Q(x)}$$

Por lo que (2) se reduce a: $\frac{dy}{dx} + P(x)y = Q(x) \quad \dots (3)$

La cual es la forma general que adoptaremos para representar a una ecuación diferencial lineal de primer orden.

- Si en (3) $P(x)$ y $Q(x)$ son constantes, entonces la ecuación diferencial es de variables separables.
- Si en (3) $P(x)$ es un múltiplo escalar de $Q(x)$, esto es; si $P(x) = K\,Q(x)$, entonces la ecuación (3) es de variables separables.

Supongamos pues que la ec. dif. (3) es no separable.
Supongamos pues que la ec. dif. (3) es no separable.

Entonces usaremos el metodo de exactitud para resolverla.

$$\Longrightarrow \frac{dy}{dx} = Q(x) - P(x)y$$

$$dy = (Q(x) - P(x)y)\,dx$$

$$(Q(x) - P(x)y) - dy = 0 \quad \ldots (4)$$

En donde

$$M(x,y) = Q(x) - P(x)y \qquad \frac{\partial M}{\partial y} = -P(x) \qquad \frac{\partial N}{\partial x} = 0$$

$$N(x,y) = -1$$

Esto es $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \rightarrow$ Con lo que se prueba que toda ec. dif. de primer orden es inexacta.

TI: Si $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y}\right) = f(x) = \mu(x) = e^{\int f(x)dx}$

$$-\frac{1}{1}(-P(x) - 0) = \frac{-P(x)}{(-1)} = P(x) = f(x)$$

$$\therefore \boxed{\mu(x) = e^{\int P(x)dx}} \quad \ldots (5)$$

es el factor integrante.

NOTA: El factor integrante de la ec. dif. lineal de primer orden es función de la variable independiente x.

Multiplicando (5) por (3)

$$e^{\int P(x)dx}\left[\frac{dy}{dx} + P(x)y = Q(x)\right]$$

$$e^{\int P(x)dx}\frac{dy}{dx} + e^{\int P(x)dx}P(x)y = e^{\int P(x)dx}Q(x)$$

$$\boxed{\frac{d}{dx}\left[e^{\int P(x)dx}\cdot y\right] = e^{\int P(x)dx}Q(x)}$$

Integrando ambos miembros con respecto de x resulta

$$\int \frac{d}{\cancel{dx}}\left[e^{\int P(x)dx}\cdot y\right]\cancel{dx} = \int e^{\int P(x)dx}Q(x)\,dx$$

$$\boxed{e^{\int P(x)dx}\cdot y = \int e^{\int P(x)dx}Q(x) + C}$$

es una solución general de (3)

$$\frac{d}{dx}\left[e^{\int P(x)dx}\cdot y\right] = \left(e^{\int P(x)dx}\frac{dy}{dx} + y\right)\cdot\frac{d}{dx}\left[e^{\int P(x)dx}\right]$$

$$= e^{\int P(x)dx}\frac{dy}{dx} + y\cdot e^{\int P(x)dx}\cdot\cancel{\frac{d}{dx}\left[\int P(x)dx\right]} \rightarrow P(x)$$

$$= e^{\int P(x)dx}\frac{dy}{dx} + e^{\int P(x)dx}\cdot P(x)y$$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$\frac{dy}{dx} = F(x,y) \rightarrow \frac{dy}{dx} + P(x)y = Q(x)$$

- $(1+x^2)dy + (xy + x^3 + x)dx = 0$

$$(1+x^2)dy = -(xy + x^3 + x)dx$$

$$\frac{dy}{dx} = \frac{-(xy + x^3 + x)}{1+x^2}$$

$$\frac{dy}{dx} = \frac{-xy}{1+x^2} - \frac{x^3+x}{1+x^2}$$

$$\frac{dy}{dx} = \frac{-xy}{1+x^2} - \frac{x\cancel{(x^2+1)}}{\cancel{(x^2+1)}}$$

$$\frac{dy}{dx} = \frac{-xy}{1+x^2} - x$$

$$\frac{dy}{dx} + \frac{xy}{x^2+1} = -x \ldots \textbf{(1)}$$

① $P(x) = \frac{x}{x^2+1}$

② Calcular factor integrante.

$$\mu(x) = e^{\int P(x)dx} = e^{\int \frac{x}{x^2+1}dx}$$

$$\mu(x) = e^{\frac{1}{2}\ln|x^2+1|}$$

$$\mu(x) = (x^2+1)^{1/2} \ldots \textbf{(2)}$$

②.① Multiplicar (2) por (1)

$$(x^2+1)^{1/2}\left[\frac{dy}{dx} + \frac{xy}{x^2+1} = -x\right]$$

$$(x^2+1)^{1/2}\frac{dy}{dx} + \underbrace{(x^2+1)^{1/2}}\underbrace{\frac{x}{x^2+1}y} = -x(x^2+1)^{1/2}$$

③ Factorizar

$$\frac{d}{dx}\left[(x^2+1)^{1/2} \cdot y\right] = -x(x^2+1)^{1/2}$$

④ Integrando

$$\int \frac{d}{dx}\left[(x^2+1)^{1/2} \cdot y\right]dx = -\int x(x^2+1)^{1/2}$$

$$(x^2+1)^{1/2} \cdot y = -\int (x^2+1)^{1/2}\, x\, dx$$

$u = x^2+1$

$du = 2x\,dx$

$$= \frac{1}{2}\int (x^2+1)^{1/2} 2x\,dx$$

$$= \frac{1}{2}\int u^{1/2}du = \frac{1}{2}\,\frac{u^{3/2}}{\frac{3}{2}} + C$$

$$\Rightarrow (x^2+1)^{1/2}y = -\frac{1}{3}(x^2+1)^{3/2} + C$$

Solucion general

$$y(x) = \frac{-\frac{1}{3}(x^2+1)^{3/2} + C}{(x+1)^{1/2}}$$

$$\frac{dx}{dy} + P(y)x = Q(y) \qquad \mu(y)$$

$$\frac{dy}{dx} = \frac{1}{x\,\text{sen}\,y + 2\,\text{sen}\,2y}$$

$$\frac{dx}{dy} = x\,\text{sen}\,y + 2\,\text{sen}\,2y$$

$$\frac{dx}{dy} - x\,\text{sen}\,y = 2\,\text{sen}\,2y$$

$$\therefore \frac{dx}{dy} - (\text{sen}\,y)x = 2\,\text{sen}\,2y \ldots (1)$$

① $P(y) = -\text{sen}\,y$

② Fact. Inte.

$$\mu(y) = e^{\int P(y)dy} = e^{-\int \text{sen}\,y\,dy}$$

$$= e^{\cos y}$$

$$\therefore \mu(y) = e^{\cos y} \ldots (2)$$

Multi. (2) por (1)

$$e^{\cos y}\left[\frac{dx}{dy} - (\text{sen}\,y)\cdot x = 2\,\text{sen}\,2y\right]$$

$$e^{\cos y}\frac{dx}{dy} - e^{\cos y}(\text{sen}\,y)x = 2\,\text{sen}\,2y\,e^{\cos y}$$

③ Factorizar

$$\frac{d}{dy}\left[e^{\cos y} \cdot x\right] = 2e^{\cos y}\,\text{sen}\,2y$$

④ Integrar

$$\int \frac{d}{dy} \cdot x = 2\int e^{\cos y}\,\text{sen}\,2y\,dy$$

$^*\text{sen}\,2y = \text{sen}(y+y)$

$= \text{sen}\,y\cos y + \cos y\,\text{sen}\,y$

$= 2\,\text{sen}\,y\cos y$

$e^{\cos y} \cdot x = 4\int e^{\cos y} \operatorname{sen} y \cos y \, dy$

$w = \cos y; \quad dw = -\operatorname{sen} y \, dx$

$= -4\int e^{\cos y} \cos y(-\operatorname{sen} y) dy$

$= -4\int e^{w} w \, dw \quad u = w \quad dv = e^{w} dw$
$\quad du = dw; \; v = e^{w}$

$= -4[we^{w} - \int e^{w} dw]$

$= -4[we^{w} - e^{w}] + C$

$\therefore e^{\cos y} \cdot x = -4[\cos y \, e^{\cos y} - e^{\cos y}]$

$\cos x \dfrac{dy}{dx} + y \operatorname{sen} x = 1$

$\dfrac{dy}{dx} = \dfrac{1 - y \operatorname{sen} x}{\cos x}$

$\dfrac{dy}{dx} = \dfrac{1}{\cos x} - \dfrac{y \operatorname{sen} x}{\cos x}$

$\dfrac{dy}{dx} + \dfrac{\operatorname{sen} x}{\cos x} y = \sec x$

1) $P(x) = \dfrac{\operatorname{sen} x}{\cos x}$

$e^{\int \frac{1}{\cos x} \operatorname{sen} x dx} = e^{\ln|\sec x|} = \sec x$

$\sec x [\dfrac{dy}{dx} + \dfrac{\operatorname{sen} x}{\cos x} y = \sec x]$

$\sec x \dfrac{dy}{dx} + \sec x \tan x \, y = \sec^2 x$

$\int \dfrac{d}{dx} [\sec x \cdot y) dx = \int \sec^2 x \, dx$

$x \dfrac{dy}{dx} + 4y = x^3 - x$

$\dfrac{dy}{dx} + \dfrac{4}{x} y = \dfrac{\cancel{x}(x^2-1)}{\cancel{x}}$

$\dfrac{dy}{dx} + \left(\dfrac{4}{x}\right) y = x^2 - 1 \quad \text{---} (1)$

$e^{\int \frac{4}{x} dx} = e^{4\ln|x|} = x^4 \quad (2)$

(2) por (1)

$x^4 \dfrac{dy}{dx} + (4x^3) y = x^6 - x^4$

$\int \dfrac{d}{dx} [x^4 \cdot y] = x^6 - x^4$

$x^4 \cdot y = \dfrac{x^7}{7} - \dfrac{x^5}{5} + C$

$$\frac{dI}{dt} + P(t)I = Q(t)$$

$$\frac{dI}{dt} + 2I = 10e^{-2t} ; \quad I(0) = 0$$

① $P(t) = 2$

② FACT. INTEGRANTE

$$M(t) = e^{\int P(t)dt}$$
$$= e^{2t}$$
$$\therefore M(t) = e^{2t} \quad \ldots(2)$$

$$e^{2t}\left[\frac{dI}{dt} + 2I = 10e^{-2t}\right]$$

$$e^{2t}\frac{dI}{dt} + 2e^{2t}I = 10$$

③ FACTORIZAR.

$$\frac{d}{dt}\left[e^{2t} \cdot I\right] = 10$$

INTEGRANDO

$$\int \frac{d}{dt}\left[e^{2t} \cdot I\right] = 10\int dt$$
$$e^{2t} \cdot I = 10t + C$$

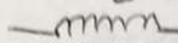
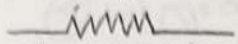
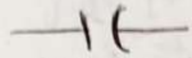
En $I(0) = 0$

$$e^{2(0)} \cdot (0) = 10(0) + C$$
$$\therefore C = 0$$
$$e^{2t} \cdot I = 10t$$
$$\therefore I(t) = 10te^{-2t} ; \forall t \geq 0$$

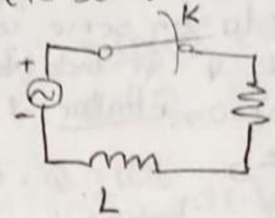
APLICACIÓN DE LA EC. DIF. LINEAL DE PRIMER ORDEN A CIRCUITOS ELECTRICOS EN SERIE RC Y RL

Este tipo de problemas se puede modelar a partir de la ley de Kirchhoff de los voltajes, que establece lo siguiente:

- La soma algebraica de todas las cargas de voltaje alrededor de un circuito electrico es cero, o equivalente.
- El voltaje suministrado por la fuente es igual a todas las caidas de voltaje

ELEMENTO	VOLTAJE
L  (INDUCTOR)	$V_L = L\frac{di}{dt}$
R  (RESISTOR)	$V_R = iR$
 (CAPACITOR)	$V_C = \frac{q}{C}$

- CTO. EN SERIE RL.



DE ACUERDO CON LA LEY DE KIRCHOFF

$$V_R + V_L = E(t)$$

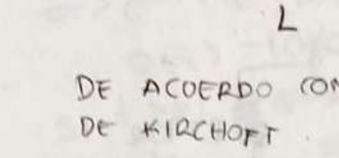
Sust. Voltajes

$$L\frac{di}{dt} + iR = E(t)$$

$$\therefore \boxed{\frac{di}{dt} + \left(\frac{R}{L}\right)i = \frac{E(t)}{L}}$$

MODELO PARA UN CTO EN SERI RL $\quad i = i(t)$

- CTO EN SERIE RC



Nuevamente por ley de Kirchhoff

$$V_R + V_C = E(t)$$

Sust. Voltajes

$$iR + \frac{Q}{C} = E(t)$$

Pero $i = \frac{dQ}{dt}$

$$\Rightarrow R\frac{dQ}{dt} + \frac{Q}{C} = E(t)$$

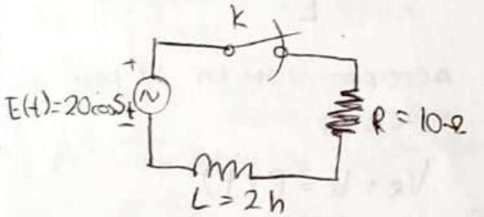
$$\therefore \boxed{\frac{dQ}{dt} + \left(\frac{1}{RC}\right)Q = \frac{E(t)}{R}}$$

MODELO PARA UN CTO. ELEC. EN SERIE RC

$$Q = Q(t)$$
$$i = \frac{dQ}{dt}$$

Ejemplos...

• Una fuente de voltaje dada por $E(t) = 20\cos 5t$ V. se conecta en serie con una resistencia $R = 10\ \Omega$ y un inductor $L = 2h$. Si $i(0) = 0$, Hallar $i = i(t)$ $\forall t \geq 0$



La ec. dif. que modela a un cto elec. en serie RL esta dada por:

$$\frac{di}{dt} + \left(\frac{R}{L}\right)i = \frac{E(t)}{L} \quad \ldots (1)$$

Sust. Datos:

$$\frac{di}{dt} + \left(\frac{10}{2}\right)i = \frac{20\cos 5t}{2}$$

Simplificando

$$\frac{di}{dt} + 5i = 10\cos 5t \quad \ldots (2)$$

(1) $P(t) = 5$

(2) Factor integrante.

$$\mu(t) = e^{\int P(t)dt} = e^{5\int dt}$$
$$= e^{5t}$$
$$\therefore \mu(t) = e^{5t} \quad \ldots (3)$$

MULT. (3) por (2)

$$e^{5t}\left[\frac{di}{dt} + 5i = 10\cos 5t\right]$$
$$e^{5t}\frac{di}{dt} + e^{5t}5i = 10\cos 5t\, e^{5t}$$
$$e^{5t}\frac{di}{dt} + 5e^{5t}i = 10e^{5t}\cos 5t$$

(3) FACTORIZAR

$$\frac{d}{dt}\left[e^{5t}\cdot i\right] = 10e^{5t}\cos 5t$$

(4) INTEGRAR.

$$\int \frac{d}{dt}\left[e^{5t}\cdot i\right]dt = 10\int e^{5t}\cos 5t\,dt$$

DONDE:

$$\int e^{5t}\cos 5t\,dt = \cancel{\tfrac{1}{5}e^{5t}}$$

$u = \cos t$; $du = -5\operatorname{sen} t\,dt$

$dv = e^{5t}dt$ $\quad v = \frac{1}{5}e^{5t}$

$$= \frac{1}{5}e^{5t}\cos t + \int e^{5t}\operatorname{sen} 5t\,dt$$

$u = \operatorname{sen} 5t$; $dv = e^{5t}\,dt$

$du = 5\cos t\,dt$; $v = \frac{1}{5}e^{5t}$

$$= \frac{1}{5}e^{5t}\cos t + \frac{1}{5}\operatorname{sen} 5t - \int e^{5t}\cos 5t\,dt$$

$$2\int e^{5t}\cos 5t\,dt = \frac{1}{5}e^{5t}\cos t + \frac{1}{5}\operatorname{sen} 5t$$

$$\therefore \int e^{5t}\cos 5t\,dt = \frac{1}{10}e^{5t}\left[\cos 5t + \operatorname{sen} 5t\right] + c \quad \ldots (5)$$

SUST. DE (5) EN (4)

$$e^{5t}\cdot i = \cancel{10}\left(\frac{1}{\cancel{10}}\right)e^{5t}(\cos 5t + \operatorname{sen} 5t) + c$$

En $e^{5t}\cdot i = e^{5t}(\cos 5t + \operatorname{sen} 5t) + c$

en $i(0) = 0$

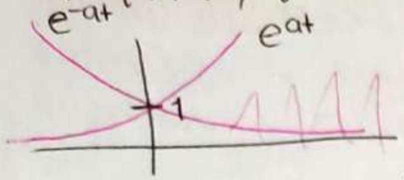
$$\overset{1}{e^{5(0)}}\,\overset{0}{(0)} = e^{5(0)}\left[\overset{1}{\cos(0)} + \overset{0}{\operatorname{sen}(0)}\right] + c$$

$$0 = 1 + c$$
$$\therefore c = -1$$

$$e^{5t}\cdot i = e^{5t}(\cos 5t + \operatorname{sen} 5t) - 1$$

$$\therefore i(t) = \cos 5t + \operatorname{sen} 5t - e^{-5t} \ ; \forall t \geq 0$$

Si $t >> 1$; $t \to +\infty$



$i(t) = \cos 5(t) + \operatorname{sen} 5t$; $\forall t >> 1$

Portafolio

Pag. 88 Murray 82

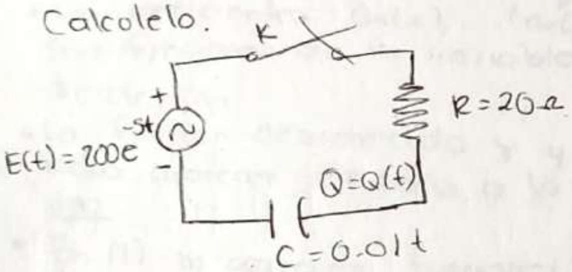
1 - 8 Sec. A.

Prog. ESP.

Zill 72

1 - 6 Ejemplos

- Una fuente decayente dada por $E(t) = 200e^{-5t}$ V, se conecta en serie con una resistencia $R = 20\Omega$ y un condensador $C = 0.01f$. Asumiendo que $Q(0) = 0$, encuentre $Q = Q(t)$ y $i = i(t)$ $\forall t > 0$. Muestre que la carga avanza un maximo. Calculelo.



$$\frac{dQ}{dt} + \left(\frac{1}{RC}\right)Q = \frac{E(t)}{R} \ldots (1)$$

SUST. DATOS:

$$\frac{dQ}{dt} + \left(\frac{1}{(20)(0.01)}\right)Q = \frac{200e^{-5t}}{20}$$

$$= \boxed{\frac{dQ}{dt} + 5Q = 10e^{-5t}} \ldots (2)$$

① $P(t) = 5$

② $M(t) = e^{5t} \ldots (3)$

MULT. (3) por (2)

$$e^{5t}\left[\frac{dQ}{dt} + 5Q = 10e^{-5t}\right]$$

$$e^{5t}\frac{dQ}{dt} + 5e^{5t}Q = 10$$

Factorizar

$$\frac{d}{dt}\left[e^{5t} \cdot Q\right] = 10$$

④ Integrando

$$\int \cancel{\frac{d}{dt}}\left[e^{5t} \cdot Q\right]\cancel{dt} = 10\int dt$$

$$\boxed{e^{5t} \cdot Q = 10t + C}$$ Sol Grl.

En $Q(0) = 0$

$$\cancel{e^{5(0)} \cdot (0)}^{0} = \cancel{10(0)}^{0} + C$$

$\therefore C = 0$

$$\Rightarrow e^{5t} \cdot Q = 10t$$

$$\therefore Q(t) = 10te^{-5t}; \forall t \geq 0$$

Si $t >> 1 \Rightarrow Q \to 0$.

$$i(t) = \frac{dQ}{dt} = 10\left[e^{-5t} - 5te^{-5t}\right]$$

$$\therefore i(t) = 10e^{-5t}[1 - 5t]; \forall t \geq 0$$

Si $t >> 1 \Rightarrow i \to 0$

$$Q' = \underbrace{10e^{-5t}}_{a}\underbrace{[1-5t]}_{b} = 0$$

Como $e^{-5t} \neq 0$ $\forall t \in \mathbb{R}$

$$\Rightarrow 1 - 5t = 0 \quad t = \frac{1}{5}$$

Pto critico

Criterio 2a derivada

Si $x = x_0$ es punto critico de $f(x)$, $\longrightarrow$

- Si $f''(x_0) > 0 \Rightarrow f(x_0)$ MIN
- Si $f''(x_0) < 0 \Rightarrow f(x_0)$ MAX.

$$Q'' = 10e^{-5t}\left[-5+(1-5t)(-5)\right]$$

$$Q''(t) = 10e^{-5t}\left[-10+25t\right] \Rightarrow Q''\left(\tfrac{1}{5}\right) < 0$$

$$\therefore\ Q_{max}\left(\tfrac{1}{5}\right) = \frac{2}{e} \approx \underline{0.74\,C}$$

UNIDAD III: Ecs. DIFERENCIALES LINEALES DE SEGUNDO ORDEN Y DE ORDEN SUPERIOR $(n \geq 2)$

• Restricción 1: Para la obtención de la solución general (por método analítico), la ec. dif. de orden superior debe ser lineal.

La ec. dif. de orden n.

Def. Una ec. dif. lineal de orden n, es aquella que se expresa en la forma general.

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \dots + a_1 x\frac{dy}{dx} + a_0(x)y = F(x)$$

$\dots (1)$

En donde $a_n(x), a_{n-1}(x), \dots a_1(x), a_0(x)$ con $a_n(x) \neq 0$ Son los coeficientes de (1)

En (1) podemos observar las sig.

- Los coeficientes $a_n(x), \dots (a_0(x)$ y $F(x)$ son funciones de la variable independiente x.
- La función desconocida Y y sus derivados aparecen elevadas a la potencia <u>uno</u>.
- En (1) no aparecen funciones trascendentes en cuyo argumento esté involucrada la función desconocida y y/o sus derivadas.

Si (1) $F(x) \neq 0$, entonces la ec. dif. se llama "Ecuación Diferencial lineal no homogenea de orden n" o ec. dif. con segundo miembro.

Por otro lado, si en (1), $F(x) = 0$, esto es;

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \dots + a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

$\dots (2)$

Entonces la ecuación dif. se llama "Ec. dif. Lineal homogenea de orden n" O ec. dif. sin segundo miembro o bien la ecuación complementaria de (1)

$$\frac{du}{dx} = f(x) \rightarrow \text{Metodo de integración} \rightarrow \text{Solución general}$$

$$\frac{d^n y}{dx^n} = f(x) \rightarrow y = y(x)$$

$$y = \boxed{y_c} + \boxed{y_p}$$

y_p → Sol part. de la ec. dif. No homogenea
→ Met. de coef. indet.
→ Met. vara. para.

y_c → Sol gral. de la ec. dif. homogenea.

Para la simplificación de análisis posteriores es necesario introducir el siguiente símbolo para indicar operaciones que involucren derivadas.

$$D \equiv \frac{d}{dx}$$

$$D^2 \equiv \frac{d^2}{dx^2}; \dots; D^n \equiv \frac{d^n}{dx^n}$$

El símbolo D recibe el nombre de operador diferencial <u>Lineal.</u>

Sean $f(x)$ y $g(x)$ funciones derivables de x y $K \in \mathbb{R}$

i) $D[f(x) + g(x)] = D[f(x)] + D[g(x)]$

ii) $D[K f(x)] = K D[f(x)]$

TRABAJO DERECHO A
SEGUNDO EXAMEN PARCIAL.

ECUACIÓN DE BERNOULLI

Un tipo común de ecuación dif. de primer orden no lineal es:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n \quad \ldots (1)$$

Donde $n \in \mathbb{R}$. Se llama ec. Dif. de Bernoulli.

Si en (1) $n=0$ o $n=1$ la ec. dif. es lineal.
En cualquier otro caso, la ec. dif. de Bernoulli es no lineal.

1: Demostrar que a partir del cambio de variable

$$u = y^{1-n} \;;\; u = u(x)$$

La ec. dif. de Bernoulli

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

Puede reducirse a la ec. dif. lineal de primer orden:

$$\frac{du}{dx} + \underbrace{(1-n)P(x)}_{P_1(x)}u = \underbrace{(1-n)Q(x)}_{Q_1(x)}$$

Sea $u = y^{1-n}$; $u = u(x)$

$$\frac{1}{1-n} - 1 = \frac{1-(1-n)}{1-n} = \frac{n}{1-n}$$

$$\Rightarrow y = u^{\frac{1}{1-n}} \Rightarrow \frac{dy}{dx} = \frac{1}{1-n} u^{\frac{1}{1-n}} \cdot \frac{du}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{1}{1-n} u^{\frac{n}{1-n}} \frac{du}{dx}$$

(Derecho a examen)

Resolver los problemas de Bernoulli del Zill, pag. 68
Del 15–20

INTRODUCIENDO ESTA NOTACIÓN EN (1)

$$a_n(x)D^n y + a_{n-1}(x)D^{n-1}y + \dots + a_1(x)Dy + a_0(x)y = F(x)$$

Como D es operador lineal.

$$\left[a_n(x)D^n + a_{n-1}(x)D^{n-1} + \dots + a_1(x)D + a_0(x)\right]y = F(x)$$

Sea:

$$a_n(x)D^n + a_{n-1}(x)D^{n-1} + \dots + a_1(x)D + a_0(x) \equiv \phi(D)$$

Con lo que la forma abreviada de (1) es:

$$\phi(D)y = F(x) \dots (1)$$

y la ec dif (2)

$$\phi(D)y = 0 \dots (2)$$

Donde $\phi(D)$ Representa al operador dif. lineal de orden n.

Esto es...

$$\phi(D)[\alpha f(x) + \beta y(x)]$$
$$= \phi(D)[\alpha f(x)] + \phi(D)[\beta y(x)]$$
$$= \alpha\phi(D)[f(x)] + \beta\phi(D)[y(x)]$$

Sol. gral. de la ec. dif. lineal no homogenea de orden n.

Def: La solución gral. de la ec. dif. no homogenea

$$\phi(D)y = F(x) \dots (1)$$

Se puede obtener a partir de la suma $y = y_c + y_p$

En donde y_c Representa la solución general de la ec. complementaria o homogenea de (1)

$$\phi(D)y = 0 \dots (2)$$

Mientras y_p es una solución particular de (1).

La definición anterior establece lo siguiente:

Suponga que la función y_p representa la solución particular de la ec. dif. no homogenea

$$\phi(D)y = F(x) \dots (1)$$

En tal caso

$$\phi(D)y_p = F(x) \quad \text{IDENTIDAD}$$

Por otro lado, ahora suponga que y_c es solución general de la ec. homogenea o complementaria de (1)

$$\phi(D)y = 0 \dots (2)$$

Esto es;

$$\phi(D)y_c = 0 \quad \text{IDENTIDAD}$$

Sumando miembro a miembro estas identidades.

$$\phi(D)y_c + \phi(D)y_p = 0 + F(x) \quad \text{IDENT.}$$

Pero $\phi(D)$ es lineal

$$\phi(D)[y_c + y_p] = F(x) \quad \text{IDENTIDAD}$$

Lo que significa que la suma

$$y = y_c + y_p$$

Es solución de (1) y por def. es una solución gral.

Ejemplo. Encontrar la solución gral de

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3x \dots (1)$$

Sabiendo que

$y_c = C_1e^{3x} + C_2e^{2x}$ es la solución general de la ec. dif. homogenea de (1)

$$\frac{d^2y}{dx^2} - \frac{5dy}{dx} + 6y = 0 \dots (2)$$

y $y_p = \frac{1}{2}x + \frac{5}{12}$, es sol. part. de (1)

Probemos primero que

$$y_c = C_1e^{3x} + C_2e^{2x} \text{ es sol gral de (2)}$$

$$y_c' = 3C_1e^{3x} + 2C_2e^{2x}$$

$$y''_c = 9C_1e^{3x} + 4C_2e^{2x}$$

Sust. y_c y sus derivadas

$$9C_1e^{3x} + 4C_2e^{2x} - 5(3C_1e^{3x} + 2C_2e^{2x}) + 6(C_1e^{3x} + C_2e^{2x}) = 0$$

$$9C_1e^{3x} + 4C_2e^{2x} - 15C_1e^{3x} - 10C_2e^{2x} + 6C_1e^{3x} + 6C_2e^{2x} = 0$$

$0 = 0$ IDENT.

$\therefore\ y_c = C_1e^{3x} + C_2e^{2x}$ es sol. gral de (2)

Por otro lado $y_p = \frac{1}{2}x + \frac{5}{12}$ es sol particular de (1)

$y' = \frac{1}{2}$; $y'' = 0$

$$0 - 5\left(\tfrac{1}{2}\right) + 6\left(\tfrac{1}{2}x + \tfrac{5}{12}\right) = 3x$$

$$-\frac{5}{2} + \frac{6}{2}x + \frac{30}{12}$$

$$-\frac{5}{2} + 3x + \frac{5}{2} = 3x$$

$3x = 3x$ ident.

$\therefore\ y_p = \frac{1}{2}x + \frac{5}{12}$ es sol part de (1)

Entonces por def. se tiene que la sol. general de (1) es

$$y = y_c + y_p = C_1e^{3x} + C_2e^{2x} + \frac{1}{2}x + \frac{5}{12}$$

$\phi(D)y_p = F(x)$

$y = \boxed{y_c} + y_p$ — sol part.

Sol. gral

$\phi(D)y = 0$

En efecto

$$y' = 3C_1e^{3x} + 2C_2e^{2x} + \frac{1}{2}$$

$$y'' = 9C_1e^{3x} + 4C_2e^{2x}$$

Sust y y sus derivadas en (1)

$$9C_1e^{3x} + 4C_2e^{2x} - 5\left(3C_1e^{3x} + 2C_2e^{2x} + \tfrac{1}{2}\right) + 6\left(C_1e^{3x} + C_2e^{2x} + \tfrac{1}{2}x + \tfrac{5}{12}\right) = 3x$$

$$9C_1e^{3x} + 4C_2e^{2x} - 15C_1e^{3x} - 10C_2e^{2x} - \frac{5}{2} + 6C_1e^{3x} + 6C_2e^{2x} + 3x + \frac{5}{2} = 3x$$

$3x = 3x$ Identidad.