$$\underset{z=z_1}{\text{Res}}\, f(z) = \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}i\right)\left[\frac{\sqrt{2}}{4}(-1+i)\right]$$

$$= \frac{2\sqrt{2}}{16}(-1+i)i = \frac{\sqrt{2}}{8}(-i-1)$$

$$\int_{-\infty}^{\infty} \frac{dx}{x^4+1} = 2\pi i\left[-\frac{\sqrt{2}}{8}i - \frac{\sqrt{2}}{8} - \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4}i\right]$$

$$= 2\pi i\left[-\frac{3\sqrt{2}}{8} - \frac{3\sqrt{2}}{8}i\right] = \frac{\pi}{4}\left[3\sqrt{2} - 3\sqrt{2}i\right]$$

n) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{4+2x^4} = \oint_c \frac{dz}{4+2z^4} = \frac{1}{2}\oint_c \frac{dz}{z^4+2} = 2\pi i\left[\sum_{k=1}^{n} \underset{z=z_k\, sps}{\text{Res}}\, f(z)\right.$

$z^4 = -2 \quad z = (-2)^{1/4} \quad n = 4 \quad k = 0,1,2,3 \quad |z| = 2$

$z_0 = 2\left[\cos\frac{\pi}{4} + i\,\text{sen}\,\frac{\pi}{4}\right] = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}i = \sqrt{2} + \sqrt{2}i$

$z_1 = 2\left[\cos\frac{3}{4}\pi + i\,\text{sen}\,\frac{3}{4}\pi\right] = -\frac{2}{\sqrt{2}} + i\frac{2}{\sqrt{2}} = -\sqrt{2} + \sqrt{2}i$

$z_2 = 2\left[\cos\frac{5}{4}\pi + i\,\text{sen}\,\frac{5}{4}\pi\right] = -\frac{2}{\sqrt{2}} - i\frac{2}{\sqrt{2}} = -\sqrt{2} - \sqrt{2}i$

$z_3 = 2\left(\cos\frac{7}{4}\pi + i\,\text{sen}\,\frac{7}{4}\pi\right) = \frac{2}{\sqrt{2}} - i\frac{2}{\sqrt{2}} = \sqrt{2} - \sqrt{2}i$

$$\oint_c \frac{dz}{(z-z_0)(z-z_1)(z-z_2)(z-z_3)} =$$

$$\frac{1}{z-z_0}\left\{\left[a_0 + a_1(z-z_0) + \dots\right]\left[b_0 + b_1(z-z_0) + \dots\right]\left[c_0 + c_1(z-z_0) + \dots\right]\right\}$$

$\underset{z=z_0}{\text{Res}}\, f(z) = a_0 b_0 c_0$

$$a_0 = \frac{1}{z - z_1}\Big|_{z_0} = \frac{1}{\sqrt{2} + \cancel{\sqrt{2}i} + \sqrt{2} - \cancel{\sqrt{2}i}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$b_0 = \frac{1}{z - z_2}\Big|_{z_0} = \frac{1}{\sqrt{2} + \sqrt{2}i + \sqrt{2} + \sqrt{2}i} = \frac{1}{2\sqrt{2} + 2\sqrt{2}i} = \frac{1}{2\sqrt{2}(1+i)} = \frac{\sqrt{2}(1-i)}{8}$$

$$c_0 = \frac{1}{z - z_3}\Big|_{z_0} = \frac{1}{\cancel{\sqrt{2}} + \sqrt{2}i - \cancel{\sqrt{2}} + \sqrt{2}i} = \frac{1}{2\sqrt{2}i} = -\frac{\sqrt{2}}{4}i$$

$$\operatorname{Res}_{z=z_0} f(z) = \left(\frac{\sqrt{2}}{4}\right)\left[\frac{\sqrt{2}}{8}(1-i)\right]\left(-\frac{\sqrt{2}}{4}i\right)$$

$$= -\frac{\sqrt{2}}{64}(1+i)$$

$$\operatorname{Res}_{z=z_1} f(z) = a_0 b_0 c_0$$ Donde

$$a_0 = \frac{1}{z - \sqrt{2} - \sqrt{2}i}\Big|_{z_1} = \frac{1}{-\sqrt{2} + \cancel{\sqrt{2}i} - \sqrt{2} - \cancel{\sqrt{2}i}} = -\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

$$b_0 = \frac{1}{z + \sqrt{2} + \sqrt{2}i}\Big|_{z_1} = \frac{1}{-\cancel{\sqrt{2}} + \sqrt{2}i + \cancel{\sqrt{2}} + \sqrt{2}i} = \frac{1}{2\sqrt{2}i} = -\frac{\sqrt{2}}{4}i$$

$$c_0 = \frac{1}{z - \sqrt{2} + \sqrt{2}i}\Big|_{z_1} = \frac{1}{-\sqrt{2} + \sqrt{2}i - \sqrt{2} + \sqrt{2}i} = \frac{1}{-2\sqrt{2} + 2\sqrt{2}i} = \frac{1}{2\sqrt{2}(-1+i)} = \frac{\sqrt{2}}{8}(-1-i)$$

$$\int_{-\infty}^{\infty} \frac{dx}{4 + 2x^4} = \frac{1}{2}\oint_C \frac{dz}{z^4 + 2} = \left[\cancel{2}\pi i\left(-\frac{\sqrt{2}}{64} - \frac{\sqrt{2}}{64}i - \frac{\sqrt{2}}{8} - \frac{\sqrt{2}}{8}i\right)\right]\frac{1}{\cancel{2}}$$

$$= \pi i\left[-\frac{9\sqrt{2}}{64} - \frac{9\sqrt{2}}{64}i\right] = \frac{\pi}{64}\left(9\sqrt{2} - 9\sqrt{2}i\right)$$

i) Encuentre las series de potencias de $f(z) = \frac{5z-2}{z(z-1)}$ alrededor de $z_0 = 0$ y $z_0 = 1$ indicando el dominio.

$5z - 2 = A(z-1) + Bz$

$5z - 2 = z(A+B) - A$

$A = 2$

$A + B = 5 \Rightarrow B = 3$

$$\frac{1}{1-q} = \sum_{n=0}^{\infty} q^n \quad |q| < 1$$

$$\frac{5z-2}{z(z-1)} = \frac{2}{z} + \frac{3}{z-1} = 2\frac{1}{(z-1+1)} - 3\frac{1}{1-z}$$

$$= 2\frac{1}{1-[-(z-1)]} - 3\frac{1}{1-(z-0)}$$

$$= 2\sum_{n=0}^{\infty}(-1)^n(z-1)^n - 3\sum_{n=0}^{\infty}(z-0)^n$$

en $|z-1| < 1$ y $|z| < 1$

i.e $|z-1| < 1 \cap |z| < 1$

ii) Calcule $\int_C \frac{5z-2}{z(z-1)} dz = 2\pi i \left(\sum_{k=1}^{n} \operatorname{Res}_{z=z_k \in sps} f(z) \right)$

$$\frac{1}{z}\{\underbrace{[a_0 + a_1(z-0) + \ldots]}_{a_0}\underbrace{[b_0 + b_1(z-0)^2 \ldots]}_{b_0}\}$$

$$\operatorname{Res}_{z=0} f(z) = [5(0) - 2][0-1] = 2$$

$$\operatorname{Res}_{z=1} f(z) = [5(-1) - 2][-1] = 7$$

$$\Rightarrow \int_C \frac{5z-2}{z(z-1)} = 2\pi i(7+2) = \underline{18\pi i}$$

$e^{i\theta} = \cos\theta + i \operatorname{sen}\theta$

5. a) Calcule $\displaystyle\int_\gamma \frac{e^z}{(z^2+1)^2}\,dz$

$$\int_\gamma \frac{e^z}{(z+i)^2\underbrace{(z-i)^2}_{SPS}}\,dz = 2\pi i \sum_{k=1}^{n} \operatorname*{Res}_{z=z_k \in SPS} f(z)$$

$$\frac{1}{(z-i)^2}\left\{[a_0 + a_1(z-i) + \dots][b_0 + b_1(z-i) + \dots]\right\}$$

$$\frac{1}{(z-i)^2}\left\{\dots + a_0 b_1 (z-i) + a_1 b_0 (z-i) + \dots\right\}$$

$$\operatorname*{Res}_{z=z_k} f(z) = a_0 b_1 + a_1 b_0 \qquad \text{Siendo}$$

$$a_0 = \left.\frac{1}{(z+i)^2}\right|_i = \frac{1}{(2i)^2} = -\frac{1}{4}$$

$$a_1 = -\left.\frac{2}{(z+i)^3}\right|_i = -\frac{1}{(i)^3} = \frac{1}{i} = -i$$

$$b_0 = b_1 = e^z\big|_i = e^i$$

$$\Rightarrow \operatorname*{Res}_{z=z_k} f(z) = \left(-\frac{1}{4}\right)(e^i) + (-i)(e^i) = e^i\left[-\frac{1}{4} - i\right]$$

$$\int_\gamma \frac{e^z\,dz}{(z+i)^2(z-i)^2} = 2\pi i\left[e^i\left(-\frac{1}{4} - i\right)\right] = \underline{e^i\pi\left(2 - \frac{1}{2}i\right)}$$

b) Calcule $\displaystyle\int_\gamma \frac{\cos(z)}{(z^2+\sqrt{2})^3}\,dz$

$$\int_\gamma \frac{\cos(z)}{(z^2+\sqrt{2})^3}\,dz = \mathrm{Re}\left\{2\pi i \sum_{k=1}^{n} \underset{z=z_k \in SPS}{\mathrm{Res}}\, f(z)\right\}$$

$$\text{Sea } f(z) = \frac{1}{(z^2+\sqrt{2})^3}\, e^{iz} = \frac{e^{iz}}{(z-\underbrace{i\sqrt[4]{2}}_{z_1})^3(z+\underbrace{i\sqrt[4]{2}}_{z_2})^3}$$

$$\frac{1}{(z-z_1)^3}\left\{[a_0+a_1(z-z_1)+a_2(z-z_2)^2+\dots][b_0+b_1(z-z_1)+b_2(z-z_1)^2+\dots]\right\}$$

$$\underset{z=z_1}{\mathrm{Res}}\, f(z) = a_0b_2/2! + a_2b_0/2! + a_1b_1 \qquad \text{Donde.}$$

$$a_0 = e^{iz}\Big|_{z_1} = e^{i(i\sqrt[4]{2})} = \frac{1}{e^{\sqrt[4]{2}}}$$

$$a_1 = ie^{iz}\Big|_{z_1} = ie^{i(i\sqrt[4]{2})} = i\frac{1}{e^{\sqrt[4]{2}}}$$

$$a_2 = -e^{iz}\Big|_{z_1} = -e^{i(i\sqrt[4]{2})} = -\frac{1}{e^{\sqrt[4]{2}}}/2! = -\frac{1}{2e^{\sqrt[4]{2}}}$$

$$b_0 = \frac{1}{(z+i\sqrt[4]{2})^3}\Big|_{z_1} = \frac{1}{(i\sqrt[4]{2}+i\sqrt[4]{2})^3} = \frac{1}{(2i\sqrt[4]{2})^3} = -\frac{1}{8i\sqrt[4]{2^3}} = \frac{1}{8\sqrt[4]{2^3}}i$$

$$b_1 = -\frac{3}{(z+i\sqrt[4]{2})^4}\Big|_{z_1} = -\frac{3}{(i\sqrt[4]{2}+i\sqrt[4]{2})^4} = -\frac{3}{(2i\sqrt[4]{2})^4} = -\frac{3}{32}$$

$$b_2 = \frac{12}{(z+i\sqrt[4]{2})^5}\Big|_{z_1} = \frac{12}{(i\sqrt[4]{2}+i\sqrt[4]{2})^5} = \frac{12}{(2\sqrt[4]{2}\,i)^5} = \frac{12}{i\,32\sqrt[4]{2^5}} = -\frac{3i}{16\sqrt[4]{2^5}}$$

$$\underset{z=z_1}{\mathrm{Res}}\, f(z) = \left(\frac{1}{e^{\sqrt[4]{2}}}\right)\left(-\frac{3i}{16\sqrt[4]{2^5}}\right) + \left(-\frac{1}{2e^{\sqrt[4]{2}}}\right)\left(\frac{i}{8\sqrt[4]{2^3}}\right) + \left(\frac{i}{e^{\sqrt[4]{2}}}\right)\left(-\frac{3}{32}\right)$$

$$= -\frac{i}{16e^{\sqrt[4]{2}}}\left[\frac{3}{\sqrt[4]{2^5}} + \frac{1}{\sqrt[4]{2^3}} + \frac{3}{2}\right] = -\frac{i}{16e^{\sqrt[4]{2}}}\left[\frac{6\sqrt[4]{2^3}+2\sqrt[4]{2^5}+3\sqrt[4]{2^3}\sqrt[4]{2^5}}{2\sqrt[4]{2^3}\sqrt[4]{2^5}}\right]$$

$$\operatorname{Res}_{z=z_1} f(z) = -\frac{i}{16e^{\sqrt[4]{2}}}\left[\frac{3\sqrt[4]{2^3}+\sqrt[4]{2^5}+6}{4}\right] = -\frac{i}{64e^{\sqrt[4]{2}}}\left(3\sqrt[4]{2^3}+\sqrt[4]{2^5}+6\right)$$

$$\int_\gamma \frac{\cos z}{(z^2+\sqrt{2})^3}\,dz = \operatorname{Re}\left\{2\pi i\left[-\frac{i}{64e^{\sqrt[4]{2}}}\left(3\sqrt[4]{2^3}+\sqrt[4]{2^5}+6\right)\right]\right\}$$

$$= \operatorname{Re}\left\{\frac{\pi}{32e^{\sqrt[4]{2}}}\left(3\sqrt[4]{2^3}+\sqrt[4]{2^5}+6\right)\right\}$$

$$= \frac{\pi}{32e^{\sqrt[4]{2}}}\left(3\sqrt[4]{2^3}+\sqrt[4]{2^5}+6\right)$$

$$\int_\gamma \frac{\operatorname{sen}(\pi z)}{(z-i)^6}\,dz = \operatorname{Im}\left\{2\pi i \sum_{k=1}^{n} \operatorname{Res}_{z=z_k \in sps} f(z)\right\}$$

$$f(z) = \frac{1}{(z-i)^6}\, e^{i\pi z}$$

$$\frac{1}{(z-i)^6}\left[a_0+\dots+a_4(z-i)^4+a_5(z-i)^5+\dots\right]$$

$$\operatorname{Res}_{z=i} f(z) = f^{v}(z)/5!\Big|_i = \frac{i\pi^5 e^{i\pi z}}{5!}\Big|_i = \frac{i\pi^5 e^{i\pi(i)}}{120} = \frac{i\pi^5}{120e^{\pi}}$$

$a_0 = e^{i\pi z}$

$a_1 = i\pi e^{i\pi z}$

$a_2 = -\pi^2 e^{i\pi z}$

$= -i\pi^3 e^{i\pi z}$

$= \pi^4 e^{i\pi z}$

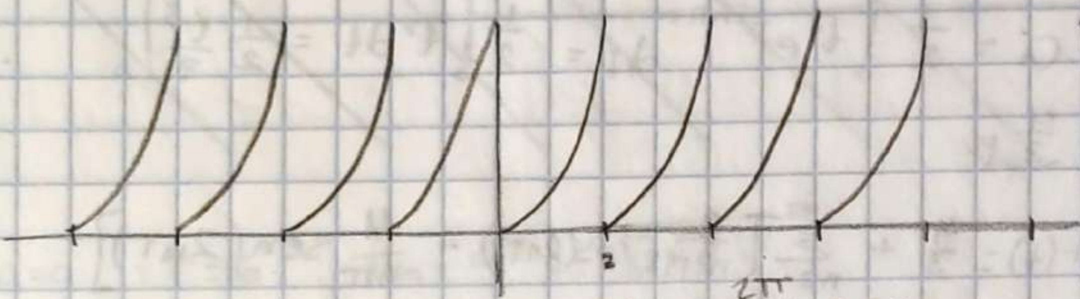
$= i\pi^5 e^{i\pi z}$

$$\int_\gamma \frac{\operatorname{sen}(\pi z)}{(z-i)^6}\,dz = \operatorname{Im}\left\{2\pi i\left(\frac{i\pi^5}{120e^{\pi}}\right)\right\}$$

$$= \operatorname{Im}\left\{-\frac{\pi^6}{60e^{\pi}}\right\} = 0$$

1. Encuentre la serie de Fourier compleja para las sig. funciones:

a) $f(t) = t^2$ para $0 \leq t < 2$ $\quad f(t+2) = f(t)$



$a = 0 \quad T = 2 \qquad\qquad w_0 = \frac{2\pi}{T} = \pi$

$$F(T) = \sum_{-\infty}^{\infty} c_n e^{inw_0T} \qquad c_n = \frac{1}{T}\int_a^{a+T} f(T)\, e^{-inw_0T}\, dT$$

$$\Rightarrow c_n = \frac{1}{T}\int_0^2 t^2 e^{-in\pi T}\, dT \qquad u = t^2 \quad du = 2t\,dt$$

$$dv = e^{-in\pi T}dt \quad v = -\frac{1}{in\pi} e^{-in\pi T}$$

$$c_n = \frac{1}{2}\left\{-t^2 \frac{1}{in\pi} e^{-in\pi T}\Big|_0^2 - \int_0^2 -\frac{1}{in\pi} 2t e^{-in\pi T}\, dT\right\}$$

$$= \frac{1}{2}\left\{\frac{-4}{in\pi} e^{-in\pi 2} + \frac{2}{in\pi}\int_0^2 t e^{-in\pi T}\, dT\right\} \qquad u = T \quad du = dT$$

$$dv = e^{-in\pi T} \quad v = -\frac{1}{in\pi} e^{-in\pi T}$$

$$= -\frac{2}{in\pi} e^{-2in\pi} + \frac{1}{in\pi}\left[-\frac{1}{in\pi} T e^{-in\pi T}\Big|_0^2 - \frac{1}{(in\pi)^2} e^{-in\pi T}\Big|_0^2\right]$$

$$= -\frac{2}{in\pi} e^{-2in\pi} + \frac{1}{in\pi}\left[-\frac{2}{in\pi} e^{-in2\pi} - \frac{1}{(in\pi)^2} e^{-in\pi 2} + \frac{1}{(in\pi)^2}\right]$$

$$\Rightarrow e^{-2in\pi} = \cos(2n\pi) - i\,\mathrm{sen}(2n\pi)^{\nearrow 0} = 1$$

$$\Rightarrow -\frac{2}{in\pi} + \frac{1}{in\pi}\left[-\frac{2}{in\pi} - \cancel{\frac{1}{(in\pi)^2}} + \cancel{\frac{1}{(in\pi)^2}}\right]$$

$$= -\frac{2}{in\pi} + \frac{2}{n^2\pi^2} = \frac{2}{n^2\pi^2} - \frac{2}{n\pi} i = \frac{1}{2}\left[\frac{4}{n^2\pi^2} - \frac{4}{n\pi} i\right]$$

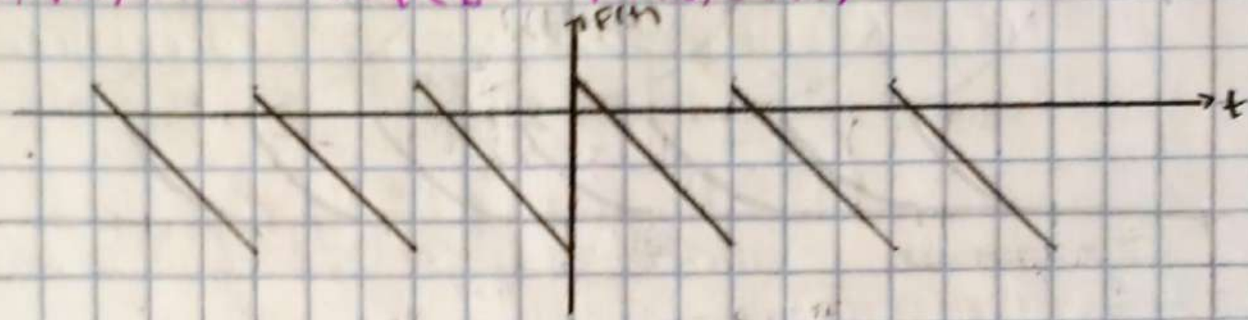
$$f(T) = \sum_{-\infty}^{\infty} C_n e^{in\omega_0 t}$$

$$a_0 = C_0 = \frac{1}{2}\int_0^2 t^2 e^{-in\pi T}\, dT = \frac{1}{2}\int_0^2 t^2\, dt = \frac{1}{2}\frac{t^3}{3}\Big|_0^2$$

$$= \frac{4}{3}$$

$$\Rightarrow f(t) = \frac{4}{3} + \sum_{n=1}^{\infty}\left[\frac{4}{n^2\pi^2}\cos(2nT) + \frac{4}{n\pi}\operatorname{sen}(2nT)\right]$$

b) $f(t) = 1 - t \quad 0 < t < 6 \quad f(t+6) = f(t)$



$a = 0 \quad T = 6 \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$

$f(t) = \sum_{-\infty}^{\infty} C_n e^{-in\omega_0 \tau} \qquad C_n = \frac{1}{T} \int_a^{a+T} f(\tau) e^{-in\omega_0 \tau} d\tau$

$C_n = \frac{1}{6} \int_0^6 (1-\tau) e^{-in\frac{\pi}{3}\tau} d\tau \qquad u = (1-\tau) \quad du = -d\tau$

$dv = e^{-in\frac{\pi}{3}\tau} d\tau \quad v = -\frac{3}{in\pi} e^{-in\frac{\pi}{3}\tau}$

$C_n = \frac{1}{6} \left\{ -(1-\tau) \frac{3}{in\pi} e^{-in\frac{\pi}{3}\tau} \Big|_0^6 - \frac{3}{in\pi} \int_0^6 e^{-in\frac{\pi}{3}\tau} d\tau \right\}$

$= \frac{1}{6} \left\{ \left[\frac{15}{in\pi} e^{-2in\pi} \right] + \left[\frac{3}{in\pi} \right] + \frac{3}{in\pi} \cdot \frac{3}{in\pi} e^{-in\frac{\pi}{3}\tau} \Big|_0^6 \right\}$

$= \frac{1}{6} \left\{ \frac{15}{in\pi} e^{-2in\pi} + \frac{3}{in\pi} + \frac{9}{(in\pi)^2} \left[e^{-2in\pi} - 1 \right]^{\to 0} \right\}$

$e^{-2in\pi} = \overset{\to 1}{\cos(2n\pi)} - i\overset{\to 0}{\operatorname{sen}(2n\pi)}$

$=$

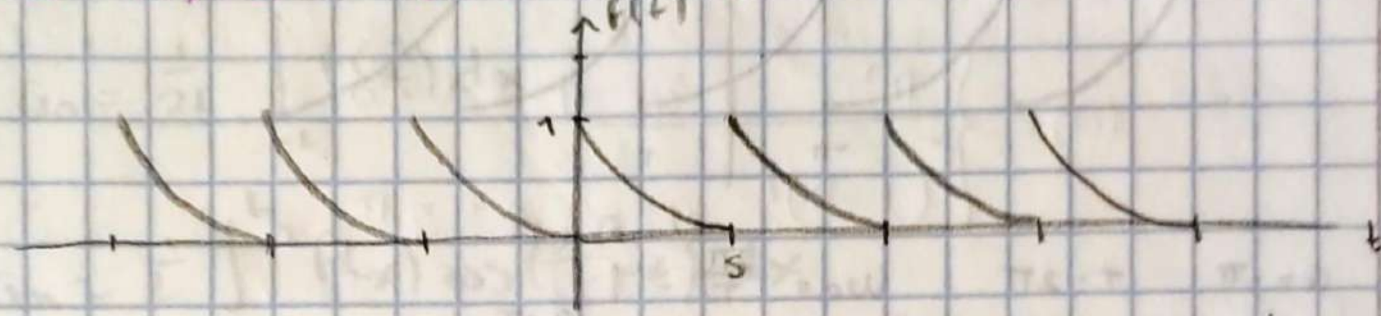
$\Rightarrow C_n = \frac{1}{6} \left\{ \frac{15}{in\pi} + \frac{3}{in\pi} \right\} = \frac{3}{in\pi} = -\frac{3}{n\pi} i$

$C_n = \frac{1}{2} \left\{ 0 - i \frac{6}{n\pi} \right\}$

$C_0 = a_0 \Rightarrow \frac{1}{6} \int_0^6 (1-\tau) d\tau = -\frac{1}{6} \frac{(1-\tau)^2}{2} \Big|_0^6 = -\frac{1}{6} \left[\frac{25}{2} - \frac{1}{2} \right] = -2$

$$\Rightarrow f(t) = -2 + \sum_{n=1}^{\infty} \left\{ \frac{6}{n\pi} \operatorname{sen}\left(n \frac{\pi}{3} T\right) \right\}$$

c) a $f(t)=e^{-t}$ $\quad 0<t<5 \quad$ $f(t+5)=f(t)$



$a=0 \quad T=5 \quad \omega_0=\frac{2\pi}{T}=\frac{2}{5}\pi$

$f(\tau)=\sum_{-\infty}^{\infty} C_n e^{in\omega_0\tau} \qquad C_n=\frac{1}{T}\int_a^{a+T} f(\tau)\,e^{-in\omega_0\tau}\,d\tau$

$$C_n=\frac{1}{5}\int_0^5 e^{-\tau}e^{-in\omega_0\tau}\,d\tau=\frac{1}{5}\int_0^5 e^{-(1+in\omega_0)\tau}\,d\tau$$

$$=-\frac{1}{5}\cdot\frac{1}{(1+in\omega_0)}\,e^{-(1+in\omega_0\tau)}\Big|_0^5=-\frac{1}{5+\cancel{5}in\frac{2}{\cancel{5}}\pi}\left[e^{-(1+in\frac{2}{\cancel{5}}\pi\cancel{5})}-1\right]$$

$$=-\frac{1}{5+2in\pi}\cdot\left[e^{-(1+2in\pi)}-1\right]$$

$$=-\frac{1}{5+2in\pi}\left[\frac{1}{e}\,e^{-2in\pi}-1\right] \qquad e^{-2in\pi}=\cos(2n\pi)-i\,sen(2n\pi)$$

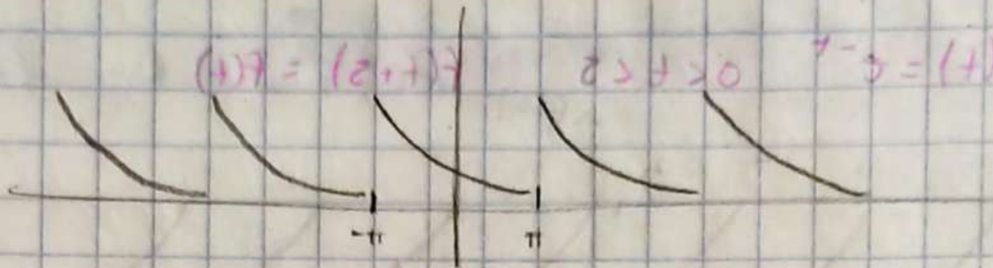
$$\Rightarrow C_n=-\frac{1}{5+2in\pi}\left[\frac{1}{e}-1\right] \qquad \frac{1}{-5+2in\pi}\cdot\frac{-5-2in\pi}{-5-2in\pi}=\frac{-5-2in\pi}{25+4n^2\pi^2}$$

$$\Rightarrow C_n=\frac{-5}{25+4n^2\pi^2}\left(\frac{1}{e}-1\right)-i\frac{2n\pi}{25+4n^2\pi^2}\left(\frac{1}{e}-1\right)$$

$$a_0=C_0=\frac{1}{5}\int_0^5 e^{-\tau}\,d\tau=-\frac{1}{5}e^{-\tau}\Big|_0^5=-\frac{1}{5}\left(e^{-5}-1\right)$$

$$=-\frac{1}{5e^5}+\frac{1}{5}$$

$$f(t)=\frac{1}{5}+\left[\sum_{n=1}^{\infty}\frac{-10}{25+4n^2\pi^2}\cos\left(\frac{2}{5}n\pi\tau\right)+\frac{4n\pi}{25+4n^2\pi}sen\left(\frac{2}{5}n\pi\tau\right)\right]\frac{1}{e}-1$$



$a = -\pi \qquad T = 2\pi \qquad \omega_0 = \frac{2\pi}{T} = 1$

$$f(t) = \sum_{-\infty}^{\infty} C_n e^{in\omega_0 t} \qquad C_n = \frac{1}{T}\int_a^{a+T} f(t)\, e^{-in\omega_0 t}\, dt$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-t}\, e^{-int}\, dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-(1+in)t}\, dt$$

$$C_n = -\frac{1}{2\pi}\,\frac{1}{1+in}\, e^{-(1+in)t}\Big|_{-\pi}^{\pi} = -\frac{1}{2\pi}\,\frac{1}{1+in}\left[e^{-\pi} e^{-in\pi} - e^{\pi} e^{in\pi}\right]$$

$$e^{\pm in\pi} = \cos(n\pi) \pm i\,\overset{0}{\mathrm{sen}(n\pi)} = (-1)^n$$

$$C_n = -\frac{1}{2\pi}\cdot\frac{1-in}{2}\left[e^{-\pi}(-1)^n - e^{\pi}(-1)^n\right]$$

$$C_n = \frac{1-in}{4\pi}(-1)^{n+1}\left(e^{-\pi} - e^{\pi}\right)$$

$$= \frac{1}{2}\left[\frac{1-in}{2\pi}(-1)^{n+1}\left(e^{-\pi} - e^{\pi}\right)\right] =$$

$$= \frac{1}{2}\left[\frac{(-1)^{n+1}\left(e^{-\pi} - e^{\pi}\right)}{2\pi} - i\,\frac{(-1)^{n+1}\left(e^{-\pi} - e^{\pi}\right)}{2\pi}\right]$$

$$f(t) = \frac{\left(e^{-\pi} - e^{\pi}\right)}{2\pi}\sum_{n=1}^{\infty} (-1)^{n+1}\cos(nt) + (-1)^{n+1}\,\mathrm{sen}(nt)$$

e^{-t}

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\, dx$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi}{L} x\right) dx$$

Pruebe que la serie de Fourier que representa la función periódica $f(t)$ definida por

$$f(t) = \begin{cases} \pi^2 & ; -\pi \leq t < 0 \\ (t-\pi)^2 & 0 < t < \pi \end{cases}$$

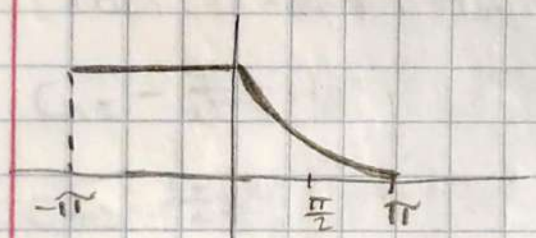
tal que $f(t) = f(t+2\pi)$ es:

$$f(t) = \frac{2}{3}\pi^2 + \sum_{n=1}^{\infty}\left[\frac{2}{n^2}\cos(nt) + \frac{(-1)^n}{n}\pi \operatorname{sen}(nt)\right] - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\operatorname{sen}(2n-1)t}{(2n-1)^3}$$

$$\Rightarrow f(T) = \sum_{-\infty}^{\infty} C_n e^{in\omega_0 T} \quad ; \quad C_n = \frac{1}{T}\int_a^{a+T} f(T)e^{-in\omega_0 T}\,dT$$

en $\underbrace{[-\pi, \pi]}_{2\pi}$ $\Rightarrow \omega_0 = \frac{2\pi}{T} = 1 \qquad a = -\pi \qquad T = 2\pi$

$$\Rightarrow C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(T)e^{-inT}$$



$$C_n = \frac{1}{2\pi}\left\{\int_{-\pi}^{0}\pi^2 e^{-in\omega_0 T}\,dT + \int_0^{\pi}(t-\pi)^2 e^{-in\omega_0 T}\,dT\right\}$$

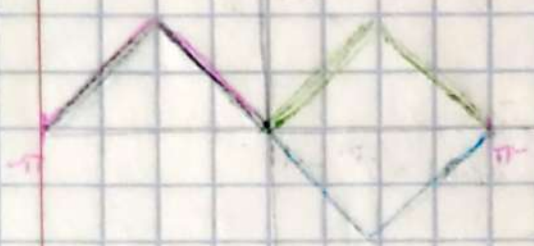
$$C_n = \frac{1}{2\pi}\left\{\pi^2\int_{-\pi}^{0} e^{-inT}\,dT + \int_0^{\pi}(T-\pi)^2 e^{-inT}\,dT\right\}$$

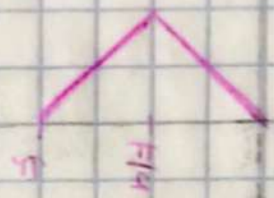
$$C_n = \frac{1}{2\pi}\left\{-\frac{\pi^2}{in}e^{-inT}\Big|_{-\pi}^{0} - \frac{1}{in}(T-\pi)^2 e^{-inT}\Big|_0^{\pi} + \frac{2}{in}\int_0^{\pi}(T-\pi)e^{-inT}\,d\right.$$

$$u = (T-\pi)^2 \qquad dv = e^{-inT}\,dT$$

$$du = 2(T-\pi)\,dT \qquad v = -\frac{1}{in}e^{-inT}$$

$$C_n = \frac{1}{2\pi}\left\{ -\frac{\pi^2}{in}\left[1 - e^{in\pi}\right]\right.$$



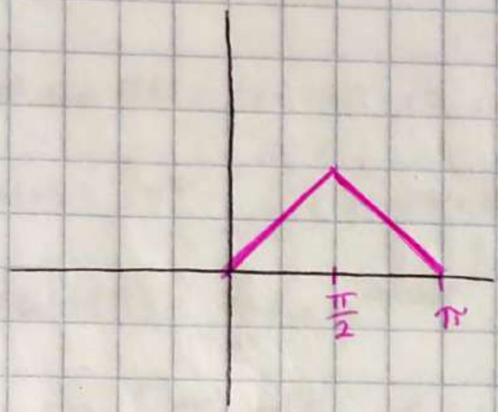


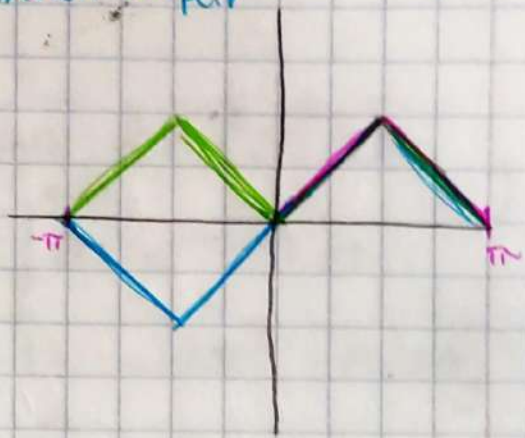
4) $f(t)$ con periodo 2π definida dentro del dom $0 \leq t < \pi$ por

$$f(t) = \begin{cases} t & \text{si } 0 \leq t < \frac{\pi}{2} \\ \pi - t & \text{si } \frac{\pi}{2} \leq t \leq \pi \end{cases}$$

$f(t)$ funcion par

$f(t)$ funcion impar





Para la función par...

$$a_0 = \frac{1}{L}\int_0^L f(T)\,dT = \frac{1}{\pi}\left\{\int_0^{\frac{\pi}{2}} T\,dT + \int_{\frac{\pi}{2}}^{\pi} (\pi - T)\,dT\right\}$$

$$= \frac{1}{\pi}\left\{ \frac{T^2}{2}\Big|_0^{\pi/2} - \frac{(\pi-T)^2}{2}\Big|_{\pi/2}^{\pi} \right\} = \frac{1}{\pi}\left\{\frac{\pi^2}{8} + \frac{\pi^2}{8}\right\} = \frac{2\pi^2}{8\pi} = \frac{\pi}{4}$$

$$a_n = \frac{2}{L}\int_0^L f(T)\cos\left(\frac{n\pi}{L}T\right)dT = \frac{2}{\pi}\left\{\int_0^{\pi/2} T\cos(nT)\,dT + \int_{\pi/2}^{\pi}(\pi - T)\cos(nT)\,dT\right\}$$

$$= \frac{2}{\pi}\left\{ \frac{T}{n}\,\mathrm{sen}(nT)\Big|_0^{\pi/2} - \frac{1}{n}\int_0^{\pi/2}\mathrm{sen}(nT)\,dT + \frac{(\pi - T)}{n}\,\mathrm{sen}(nT)\Big|_{\pi/2}^{\pi} + \frac{1}{n}\int_{\pi/2}^{\pi}\mathrm{sen}(nT)\,dT \right\}$$

$u = T;\ du = dT$

$dv = \cos(nT)\,dT$

$v = \frac{1}{n}\,\mathrm{sen}(nT)$

$u = (\pi - T)\ ;\ du = -dT$

$dv = \cos(nT)\,dT \quad v = \frac{1}{n}\,\mathrm{sen}(nT)$

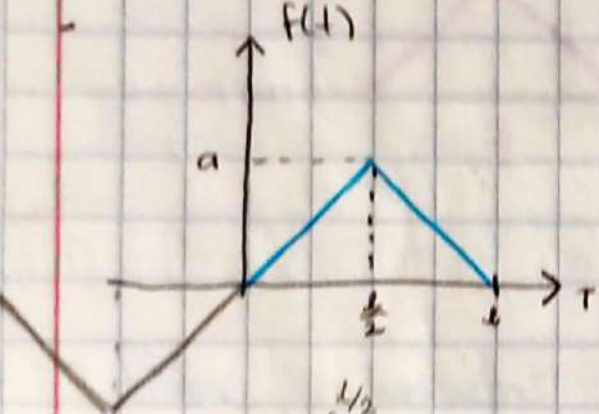
$$= \frac{2}{\pi}\left\{ \cancel{\frac{\pi}{2n}\,\mathrm{sen}\left(\frac{\pi}{2}n\right)} + \frac{1}{n^2}\cos(nT)\Big|_0^{\pi/2} - \cancel{\frac{\pi}{2n}\,\mathrm{sen}\left(\frac{\pi}{2}n\right)} - \frac{1}{n^2}\cos(nT)\Big|_{\pi/2}^{\pi} \right\}$$

$$= \frac{2}{\pi}\left\{ \frac{1}{n^2}\cos\left(\frac{\pi}{2}n\right) - \frac{1}{n^2} - \frac{1}{n^2}\cos(n\pi) + \frac{1}{n^2}\cos\left(\frac{\pi}{2}n\right) \right\}$$

$$= -\frac{2}{\pi n^2}\left\{ 1 - 2\cos\left(\frac{\pi n}{2}\right) + (-1)^n \right\}$$

5. Una cuerda uniforme está firmemente estirada y tiene sus extremos fijos en los puntos $x=0$ y $x=L$. El punto medio de la cuerda está desplazado a una distancia a. Si $f(t)$ denota el perfil de desplazamiento de la cuerda, exprese $f(t)$ como una expresión en la serie de Fourier que solamente está formada por términos senos.

$$f(t)=\begin{cases} f_1(T)=\frac{2a}{l}t & ; \; 0<t<\frac{l}{2} \\ f_2(T)=2a-\frac{2at}{l} & ; \; \frac{l}{2}<t<l \end{cases}$$



$$f(T)=\sum_{n=1}^{\infty} b_n \operatorname{sen}\left(\frac{n\pi}{l}T\right)$$

$$b_n=\frac{2}{l}\left\{\int_0^{l/2}\left(\frac{2a}{l}t\right)\operatorname{sen}\left(\frac{n\pi}{l}t\right)dt+\int_{l/2}^{l}\left(2a-\frac{2aT}{l}\right)\operatorname{sen}\left(\frac{n\pi}{l}T\right)dT\right\}$$

$u=\frac{2a}{l}t \quad du=\frac{2a}{l} \qquad u=2a-\frac{2a}{l}t \quad du=-\frac{2a}{l}$

$dv=\operatorname{sen}\left(\frac{n\pi}{l}T\right)dT \quad v=-\frac{l}{n\pi}\cos\left(\frac{n\pi}{l}T\right)$

$$b_n=\frac{2}{l}\left\{-\frac{2a}{n\pi}T\cos\left(\frac{n\pi}{l}T\right)\Big|_0^{l/2}+\frac{2a}{n\pi}\int_0^{l/2}\cos\left(\frac{n\pi}{l}T\right)dT\right.$$

$$\left.-\left(2a-\frac{2a}{l}T\right)\frac{l}{n\pi}\cos\left(\frac{n\pi}{l}T\right)\Big|_{l/2}^{l}-\frac{2a}{n\pi}\int_{l/2}^{l}\cos\left(\frac{n\pi}{l}T\right)dT\right\}$$

$$=\frac{2}{l}\left\{-\frac{2a}{n\pi}\frac{l}{2}\cos\left(\frac{n\pi}{2}\right)+\frac{2al}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{l}T\right)\Big|_0^{l/2}\right.$$

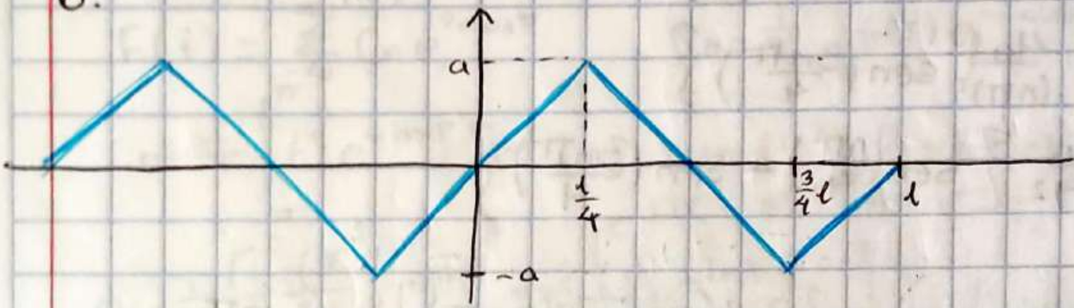
$$+\frac{al}{n\pi}\cos\left(\frac{n\pi}{2}\right)-\frac{2al}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{l}T\right)\Big|_{l/2}^{l}$$

$$b_n = \frac{2}{l}\left\{\frac{2al}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{2}\right) + \frac{2al}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{2}\right)\right\}$$

$$= \frac{8a}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{2}\right)$$

$$F(T) = \sum_{n=1}^{\infty} \frac{8a}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{2}\right)\operatorname{sen}\left(\frac{n\pi}{l}T\right)$$

6.-



$$F(t) = \begin{cases} \frac{4a}{l}t & ; \ 0 < t < \frac{l}{4} \\ 2a - \frac{4at}{l} & ; \ \frac{l}{4} < t < \frac{3}{4}l \\ \frac{4at}{l} - 4a & ; \ \frac{3}{4} < t < l \end{cases}$$

$$F(T) = \sum_{n=1}^{\infty} b_n \operatorname{sen}\left(\frac{n\pi}{l}T\right)$$

$$b_n = \frac{2}{l}\left\{\int_0^{l/4}\left(\frac{4a}{l}T\right)\operatorname{sen}\left(\frac{n\pi}{l}T\right) + \int_{l/4}^{3/4 l}\left(2a - \frac{4a}{l}T\right)\operatorname{sen}\left(\frac{n\pi}{l}T\right) + \int_{3/4 l}^{l}\left(\frac{4a}{l}T - 4a\right)\operatorname{sen}\left(\frac{n\pi}{l}T\right)\right\}$$

$u_1 = \frac{4a}{l}T$; $du_1 = \frac{4a}{l}dT$ $\quad u_2 = 2a - \frac{4a}{l}T$; $du_2 = -\frac{4a}{l}dT$ $\quad u_3 = \frac{4a}{l}T - 4a$ $\quad du_3 = \frac{4a}{l}dT$

$dv = \operatorname{sen}\left(\frac{n\pi}{l}T\right)dT$ $\quad v = -\frac{l}{n\pi}\cos\left(\frac{n\pi}{l}T\right)$

$$b_n = \frac{2}{l}\left\{-\frac{4aT}{n\pi}\cos\left(\frac{n\pi}{l}T\right)\Big|_0^{l/4} + \frac{4a}{n\pi}\int_0^{l/4}\cos\left(\frac{n\pi}{l}T\right)dT - \frac{2al - 4aT}{n\pi}\cos\left(\frac{n\pi}{l}T\right)\Big|_{l/4}^{3/4 l}\right.$$
$$\left. - \frac{4a}{n\pi}\int_{l/4}^{3/4 l}\cos\left(\frac{n\pi}{l}T\right)dT - \frac{4aT - 4al}{n\pi}\cos\left(\frac{n\pi}{l}T\right)\Big|_{3/4 l}^{l} + \frac{4a}{n\pi}\int_{3/4 l}^{l}\cos\left(\frac{n\pi}{l}T\right)dT\right\}$$

$$b_n = \frac{2}{l}\Big\{ -\cancel{\frac{al}{n\pi}\cos\left(\frac{n\pi}{4}\right)} + \frac{4al}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{l}T\right)\Big|_0^{l/4} + \cancel{\frac{al}{n\pi}\cos\left(\frac{3n\pi}{4}\right)} + \cancel{\frac{al}{n\pi}\cos\left(\frac{n\pi}{4}\right)}$$

$$-\frac{4al}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{l}T\right)\Big|_{l/4}^{3/4\,l} - \cancel{\frac{al}{n\pi}\cos\left(\frac{3n}{4}\pi\right)} + \frac{4al}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{l}T\right)\Big|_{3/4\,l}^{l}\Big\}$$

$$b_n = \frac{2}{l}\Big\{ \frac{4al}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{4}\right) - \frac{4al}{(n\pi)^2}\operatorname{sen}\left(\frac{3n\pi}{4}\right) + \frac{4al}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{4}\right)$$

$$+ - \frac{4al}{(n\pi)^2}\operatorname{sen}\left(\frac{3n\pi}{4}\right)\Big\}$$

$$b_n = \frac{16a}{(n\pi)^2}\Big\{\operatorname{sen}\left(\frac{n\pi}{4}\right) - \operatorname{sen}\left(\frac{3n\pi}{4}\right)\Big\}$$

$$f(T) = \sum_{n=1}^{\infty}\left[\frac{16a}{(n\pi)^2}\operatorname{sen}\left(\frac{n\pi}{4}\right) - \frac{16a}{(n\pi)^2}\operatorname{sen}\left(\frac{3n\pi}{4}\right)\right]\operatorname{sen}\left(\frac{n\pi}{l}T\right)$$

$$C_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dT$$

Pruebe que la forma compleja de la expansión de serie de Fourier de la función periodica $f(t) = t^2 \quad (-\pi \leq t < \pi)$ + $f(t) = f(t+2\pi)$ está dada por

$\longleftarrow \frac{2\pi}{T} \longrightarrow$

$$f(t) = \frac{\pi^2}{6} + \sum_{n=0}^{\infty} \frac{2}{n^2}(-1)^n e^{int} \qquad \omega_0 = \frac{2\pi}{T} = 1$$

$$F(t) = \sum_{-\infty}^{\infty} C_n e^{in\omega_0 T} \qquad C_n = \frac{1}{T}\int_a^{a+T} f(T) e^{-in\omega_0 T}\,dT$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2 e^{-inT}\,dT \qquad u = t^2 \quad du = 2t\,dt$$
$$dv = e^{-inT}\,dT \quad v = -\frac{1}{in} e^{-inT}$$

$$C_n = \frac{1}{2\pi}\left[-\frac{t^2}{in} e^{-inT}\Big|_{-\pi}^{\pi} + \frac{2}{in}\int_{-\pi}^{\pi} t e^{-inT}\,dT\right] \qquad u = t \quad du = dt$$

$$C_n = \frac{1}{2\pi}\left\{-\frac{\pi^2}{in} e^{-in\pi} + \frac{\pi^2}{in} e^{in\pi} + \frac{2}{in}\left[-\frac{t}{in} e^{-inT}\Big|_{-\pi}^{\pi} + \frac{1}{in}\int_{-\pi}^{\pi} e^{-inT}\,dT\right]\right\}$$

$$C_n = \frac{1}{2\pi}\left\{-\frac{\pi^2}{in} e^{-in\pi} + \frac{\pi^2}{in} e^{in\pi} + \frac{2}{in}\left[-\frac{\pi}{in} e^{-inT} - \frac{\pi}{in} e^{in\pi} - \frac{1}{(in)^2} e^{-inT}\Big|_{-\pi}^{\pi}\right]\right\}$$

$$C_n = \frac{1}{2\pi}\left\{-\frac{\pi^2}{in}\cancel{(e^{in\pi} - e^{in\pi})}^{0} + \frac{2}{in}\left[-\frac{\pi}{in}(e^{in\pi} + e^{-in\pi}) + \frac{1}{(in)^2}\cancel{(e^{in\pi} - e^{-in\pi})}^{0}\right]\right\}$$

$$C_n = \frac{1}{\cancel{2\pi}}\left\{\frac{\cancel{2}}{\cancel{i}n}\left[-\frac{\cancel{\pi}}{\cancel{i}n}\left((-1)^n + (-1)^n\right)\right]\right\}$$

$$= \frac{1}{n^2}\left[2(-1)^n\right] = \frac{2}{n^2}(-1)^n \qquad n \neq 0$$

$$\Rightarrow C_0 = a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2 e^{-i(0)T}\,dT = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2\,dt = \frac{1}{2\pi}\left[\frac{t^3}{3}\right]\Big|_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{\pi^3}{3} - \frac{(-\pi)^3}{3}\right] = \frac{1}{6\pi}(\pi^3 + \pi^3) = \frac{2\pi^2}{6} = \frac{\pi^2}{3}$$

$$F(t) = \frac{\pi^2}{3} + \sum_{n=0}^{\infty} \frac{2}{n^2}(-1)^n e^{int}$$

$$\frac{1}{\pi}\,\frac{t^3}{3}\Big|_0^{\pi} = \frac{1}{3\pi}(\pi^3) = \frac{\pi^2}{3}$$

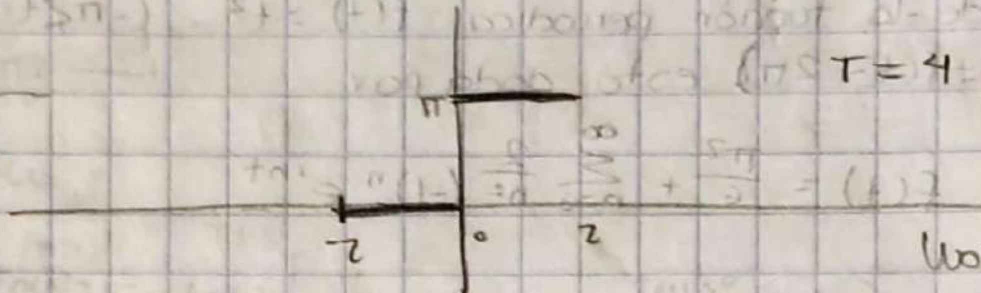
$C_n = \frac{1}{2}(a_n + ib_n)$

$C_n =$

$2C_n = a_n - ib_n$

$2C_0 = a_0 - ib_0$

8. Obtener s. de F. de $f(t)=\begin{cases} 0\ ;\ si & -2<t<0 \\ \pi\ ;\ si & 0<t<2 \end{cases}$



$T=4$

$w_0=\frac{2\pi}{T}=\frac{2\pi}{4}=\frac{\pi}{2}$

$$C_n=\sum_{-\infty}^{\infty} C_n e^{inw_0T} \qquad C_n=\frac{1}{T}\int_a^{a+T} f(T)e^{-inw_0T}\,dT$$

$$C_n=\frac{1}{4}\int_0^2 \pi e^{-in\frac{\pi}{2}T}\,dT=-\frac{2\pi}{4in\pi}e^{-in\frac{\pi}{2}T}\Big|_0^2$$

$$=-\frac{1}{2in}\left[e^{-in\pi}-1\right]=i\frac{1}{2n}\left[(-1)^n-1\right]$$

$$C_n=\frac{1}{2}(a_n-ib_n)$$

$$C_n=\frac{1}{2}\{0-[1-(-1)^n]n\}$$

$$\frac{1}{T}\int_a^{a+T}[f(t)]^2\,dt=\frac{1}{4}(a_0)^2+\frac{1}{2}\sum_{n=1}^{\infty}\left[(a_n)^2+(b_n)^2\right]$$

$$\frac{1}{4}\int_0^2 \pi^2\,dt=\frac{1}{2}\sum_{n=1}^{\infty}[1-(-1)^n]n$$

$$\frac{\pi^2}{4}t\Big|_0^2=\frac{1}{2} \qquad \frac{\pi^2}{2}$$

9) $f(t) = 500t \quad 0 < t < \frac{1}{15}$

$f(t) = f(t + \frac{1}{50})$

$\pi/16$

$$\int_0^{2\pi} \frac{d\theta}{(5+3\,sen\,\theta)^2} = \int_0^{2\pi} \frac{d\theta}{[(\frac{5}{3}+sen\theta)3]^2}$$

$$= \int_0^{2\pi} \frac{d\theta}{3^2[\frac{5}{3}+sen\theta]^2} = \frac{1}{9}\int_0^{2\pi} \frac{1}{[\frac{5}{3}+(z-\frac{1}{z})\frac{1}{2i}]^2}\frac{dz}{iz}$$

$$= \frac{1}{9}\int_{|z|=1} \frac{1}{\{[\frac{10iz}{3}+(z^2-1)]\frac{1}{2iz}\}^2}\frac{dz}{iz}$$

$$= \frac{1}{9}\int_{|z|=1} \frac{1}{(z^2+\frac{10iz}{3}-1)^2(\frac{1}{2iz})^2}\frac{dz}{iz}$$

$$= \frac{1}{9}\int_{|z|=1} \frac{dz}{(z^2+\frac{10iz}{3}-1)\left(-\frac{1}{4z^2}\right)}\frac{dz}{iz}$$

$$= -\frac{4}{9i}\int_{|z|=1} \frac{dz}{z(z^2+\frac{10iz}{3}-1)^2} = \frac{4}{9}i\int_\gamma \frac{dz}{z(z+z_1)^2(z+z_2)^2}$$

$$z_{1,2} = \frac{-\frac{10i}{3} \pm \sqrt{(\frac{10i}{3})^2+4}}{2} = \frac{-\frac{10i}{3} \pm \sqrt{-\frac{100}{9}+\frac{36}{9}}}{2}$$

$$= \frac{-\frac{10i}{3} \pm \sqrt{\frac{-64}{9}}}{3} = \frac{\frac{-10i}{3} \pm \frac{8i}{3}}{3} \quad \frac{-10i \pm 8i}{9}$$

$$z_1 = \frac{-10i+8i}{9} = \boxed{\frac{-2i}{9}} \quad z_2 = \frac{-10i-8i}{9} = -2i$$

$$\Rightarrow \frac{1}{(z-0)}\{[a_0+a_1(z-0)+\dots][b_0+b_1(z-0)+\dots]\}$$

Laurent $z=0$, $z=1$

⑥ $f(z) = \dfrac{1}{z^3(z-i)^2} = \dfrac{A}{z^3} + \dfrac{B}{(z-i)^2}$ $\dfrac{5}{32}\pi$

$A(z-i)^2 +$

2 9

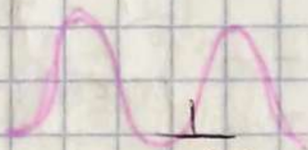
$\dfrac{1}{z^3}\left[\dfrac{1}{(z-i)^2}\right] = \dfrac{1}{z}\Big[$

$5k^2 - \dfrac{9-18k+9k^2+k^4}{1-2k+k^2} +$

$\dfrac{5k^2 - 10k^3 + 5k^4 \cancel{-9} + 18k - 9k^2 - k^4 + 4k^4 - 12k^2 \cancel{+9}}{1-2k+k^2}$

$\dfrac{8k^4 - 10k^3 - 16k^2 + 18k}{(1-k)^2}$

$\dfrac{k(8k^3 - 10k^2 - 16k + 18)}{(1-k)^4}$



$\dfrac{1}{z-4i}$

3/46 $\Big(\dfrac{1}{z-4i} = \dfrac{A}{4i}$

$\dfrac{1}{2i}\left(z - \dfrac{1}{z}\right)$ 1, 2, 7, 9, 12

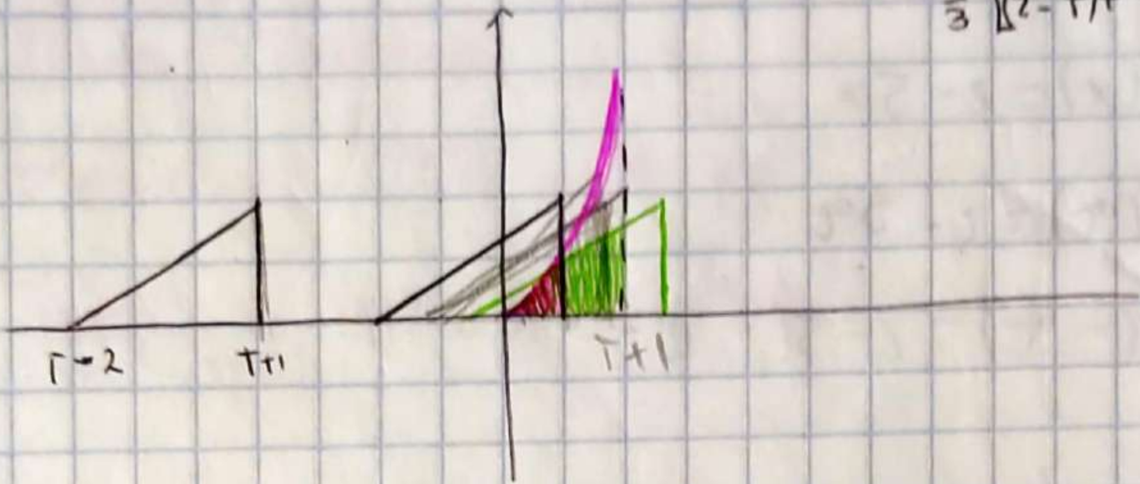
8/3 $\dfrac{1}{z-2i}$ $-2i$ $\dfrac{z^2-1}{z}$ 14

1 2 3 5

$\dfrac{1}{(z-i)-3i}$

$z - i(B-1)$

$\frac{2}{3}[(2-T)+\tau]$



$0 \quad ; \qquad T+1 < 0 \qquad T < -1$

$\int_0^{T+1} \tau^2 [(2-T)+\tau]\, d\tau = \frac{\tau^3}{3}\Big|_0^{T+1} = \frac{(T+1)^3}{3}$ $\quad -1 < T < 1$ en $f(1) = \frac{8}{3}$ e)

$\frac{2}{3}\int_0^2 \tau^2[(2-T)+\tau]\, d\tau = \frac{2}{3}\int_0^2 [\tau^2(2-T)+\tau^3]\, d\tau$

$\frac{2}{3}\left[(2-T)\frac{\tau^3}{3}\Big|_0^2 + \frac{\tau^4}{4}\Big|_0^2\right]$

$\frac{2}{3}\left[\frac{8}{3}(2-T) + \frac{12}{3}\right] = \frac{8}{9}[2(2-T)+3] = \frac{40}{9}$

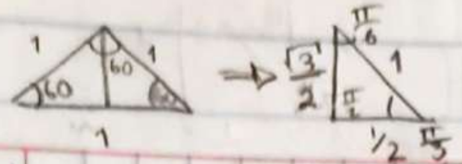
$\int_0^{T+1} \tau^2(2-T)+\tau^3$

$\frac{2}{3}\left[\frac{(2-T)(T+1)^3}{3} + \frac{(T+1)^4}{4}\right]$

$\frac{2}{3}\left[\frac{8}{3} + \frac{12}{3}\right] = \frac{40}{9}$

$\cos(x) = \frac{ca(x)}{hip(x)}$ $\cos(-x) = \cos(x)$

$\text{sen}(x) = \frac{co(x)}{hip(x)}$ $\text{sen}(-x) = -\text{sen}(x)$



$z^3 + 8i = 0$

$z = (-8i)^{1/3}$ $n = 3$ $\theta = -\frac{\pi}{2}$ $K = 0, 1, 2$ $r = |z| = \sqrt{(-8)^2} = 8$

$w_K = r^{1/n}\left[\cos\left(\frac{\theta + 2k\pi}{n}\right) + i\,\text{sen}\left(\frac{\theta + 2k\pi}{n}\right)\right]$

$w_0 = 8^{1/3}\left[\cos\left(\frac{-\frac{\pi}{2}}{3}\right) + i\,\text{sen}\left(\frac{-\frac{\pi}{2}}{3}\right)\right]$

$= 2\cos\left(-\frac{\pi}{6}\right) + i\,\text{sen}\left(-\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{6}\right) - i\,\text{sen}\left(\frac{\pi}{6}\right)$

$= 2\left(\frac{\sqrt{3}}{2}\right) - i\,\frac{1}{2}(2) = \sqrt{3} - i$

$w_1 = 2\left[\cos\left(\frac{-\frac{\pi}{2} + 2\pi}{3}\right) + i\,\text{sen}\left(\frac{-\frac{\pi}{2} + 2\pi}{3}\right)\right] = 2\left[\cos\left(\frac{\pi}{2}\right)^{\to 0} + i\,\text{sen}\left(\frac{\pi}{2}\right)\right]$

$= 2i$

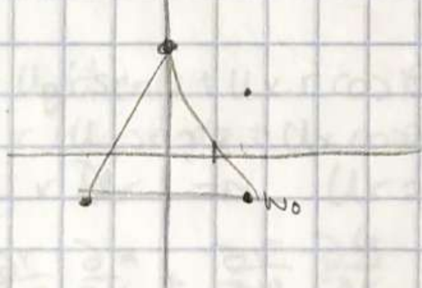
$w_2 = 2\left[\cos\left(\frac{-\frac{\pi}{2} + 4\pi}{3}\right) + i\,\text{sen}\left(\frac{-\frac{\pi}{2} + 4\pi}{3}\right)\right] = 2\left[\cos\left(\frac{7}{6}\pi\right) + i\,\text{sen}\left(\frac{7}{6}\pi\right)\right]$

$\cos\left(\pi + \frac{1}{6}\pi\right) = \cos\pi\cos\frac{\pi}{6} - \cancel{\text{sen}(\pi)}^{\to 0}\,\text{sen}\frac{\pi}{6} = (-1)\frac{\sqrt{3}}{2}$

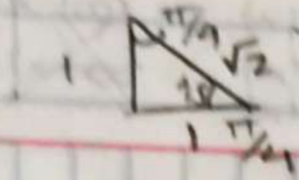
$= -\frac{\sqrt{3}}{2}$

$\text{sen}\left(\pi + \frac{\pi}{6}\right) = \cancel{\text{sen}\pi}^{\to 0}\cos\frac{\pi}{6} + \cos\pi\,\text{sen}\frac{\pi}{6} = (-1)\left(\frac{1}{2}\right) = -\frac{1}{2}$

$w_2 = \cancel{2}\left(-\frac{\sqrt{3}}{\cancel{2}}\right) + i\cancel{2}\left(-\frac{1}{\cancel{2}}\right) = -\sqrt{3} - i$



D:



$z^4+4=0$

$z=(-4)^{1/4}$ $\quad |z|=r=\sqrt{(-4)^2}=4 \quad n=4 \quad K=0,1,2,3 \quad \theta=\pi$

$w_k = r^{1/n}\left[\cos\left(\frac{\theta+2k\pi}{n}\right) + i\,sen\left(\frac{\theta+2k\pi}{n}\right)\right]$

$w_0 = 4^{1/4}\left[\cos\left(\frac{\pi}{4}\right) + i\,sen\left(\frac{\pi}{4}\right)\right] = 4^{1/4}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = 1+i$

$w_1 = 4^{1/4}\left[\cos\left(\frac{\pi+2\pi}{4}\right) + i\,sen\left(\frac{\pi+2\pi}{4}\right)\right] = 4^{1/4}\left[\cos\left(\frac{3}{4}\pi\right) + i\,sen\left(\frac{3}{4}\pi\right)\right]$

$\cos\left(\pi-\frac{\pi}{4}\right) = \cos\pi\cos\frac{\pi}{4} = (-1)\cdot\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$,

$sen\left(\pi-\frac{\pi}{4}\right) = sen\pi\cos\frac{\pi}{4} - \cos\pi\, sen\frac{\pi}{4} = -(-1)\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$w_1 = 4^{1/4}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = -1+i$

$w_2 = 4^{1/4}\left(\cos\left(\frac{5\pi}{4}\right) + i\,sen\left(\frac{5}{4}\pi\right)\right)$

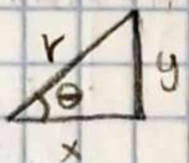
$z=\frac{1}{w}$ $\qquad x^2+y^2=r \quad \cdot \quad |z|=r$

$\left|\frac{1}{w}\right|=r$

$$u_x = v_y$$
$$v_x = -u_y$$
$$u_y = -v_x$$

Muestre q' las condiciones de Cauchy-Rieman en coordenadas polares están dados por

$$u_r = \frac{v_\theta}{r} \quad \& \quad v_r = -\frac{u_\theta}{r}$$



$$x = r\cos\theta$$
$$y = r\,\mathrm{sen}\,\theta$$

$$\frac{\partial x}{\partial r} = \cos\theta \ ; \ \frac{\partial x}{\partial \theta} = -r\,\mathrm{sen}\,\theta \ ; \ \frac{\partial y}{\partial r} = \mathrm{sen}\,\theta \ ; \ \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} = u_x\cos\theta + u_y\,\mathrm{sen}\,\theta \quad (*)$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta} = -u_x r\,\mathrm{sen}\,\theta + u_y r\cos\theta \quad (**)$$

$$\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta} = -v_x r\,\mathrm{sen}\,\theta + v_y r\cos\theta$$

$$v_\theta = u_y r\,\mathrm{sen}\,\theta + u_x r\cos\theta$$
$$= r\,(u_y\,\mathrm{sen}\,\theta + u_x\cos\theta)$$
$$v_\theta = r\,u_r \Rightarrow u_r = \frac{v_\theta}{r}$$

$$v_r = \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r} = v_x\cos\theta + v_y\,\mathrm{sen}\,\theta$$
$$= -u_y\cos\theta + u_x\,\mathrm{sen}\,\theta$$
$$-v_r = u_y\cos\theta - u_x\,\mathrm{sen}\,\theta$$
$$** = r\,(-u_x\,\mathrm{sen}\,\theta + u_y\cos\theta)$$
$$\Rightarrow \frac{u_\theta}{r} = -v_r \quad ó \quad u_r = -\frac{u_\theta}{r}$$

$$\left[\frac{A}{(z-2i)} + \frac{B}{(z-4)} = \frac{1}{(z-2i)(z-4)}\right](z-2i)(z-4)$$

$$A(z-4) + B(z-2i) = 1$$

$$f(z) = -\frac{1}{2i}\left[\frac{1}{z-2i}\right] + \frac{1}{4-2i}\left[\frac{1}{z-4}\right]$$

$$= -\frac{1}{2i}\frac{1}{z}\left[\frac{1}{1-\frac{2i}{z}}\right] + \frac{1}{4-2i}\cdot\frac{1}{z}\left[\frac{1}{1-\frac{4}{z}}\right]$$

$$= -\frac{1}{2iz}\sum\left(\frac{2i}{z}\right)^n + \frac{1}{4-2i}\frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{4}{z}\right)^n$$

$$= -\frac{1}{2i}\sum_{n=0}^{\infty}\frac{(2i)^n}{z^{n+1}}$$

$$\left|\frac{2i}{z}\right| < 1$$

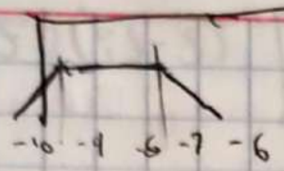
$$|2i| < |z|$$

$$2 < |z|$$

$4T^2+4T+1-[T^2-14T+49]+5$ $\quad$ $\begin{array}{r}28\\-9\\\hline 252\end{array}$

$3T^2+18T-43$ $\quad$ $\begin{array}{r}81\\3\\\hline 243\end{array}$

$\begin{array}{r}18\\9\\\hline 162\end{array}$



200

$-\begin{array}{r}81\\3\\\hline 243\end{array}-162$ $\quad$ $\begin{array}{r}243\\162\\\hline 081\end{array}$ $\quad$ $\begin{array}{r}81\\-43\\\hline 38\end{array}$

$$h_3=2\int_{-10}^{T-2}(3+T+\tau)\,d\tau+2\int_{T-2}^{T}d\tau+2\int_{T}^{T+1}(1+T-\tau)\,d\tau$$

$$=(3+T+\tau)^2\Big|_{-10}^{T-2}+2\tau\Big|_{T-2}^{T}-(1+T-\tau)^2\Big|_{T}^{T+1}$$

$$=(3+T+T-2)^2-(3+T-10)^2+2T-2(T-2)+1$$

$(1+2T)^2-(-7+T)^2+\cancel{2T}-\cancel{2T}+4+1$

$1+4T+4T^2-(49-14T+T^2)+5$

$3T^2+18T-43$ en -9 ?

$-2T+4$

$=(3+T+T-2)^2-(3+T-10)^2+2T-2(T-2)+1$

$=(2T+1)^2-(T-7)^2+5$

$\begin{array}{r}-17\\-17\\\hline 119\\17\\\hline 289\end{array}$ $\quad$ $\begin{array}{r}-16\\16\\\hline 96\\16\\\hline 256\end{array}$ $\quad$ $\begin{array}{r}289\\-256\\\hline 033\end{array}$:V 38

$(3-T+T-2)^2\Big|_{-10}^{T-2}+2\cancel{T}-2T+4+1$ $\quad$ $6-(7+T)^2$

$6-(3-T-10)^2+$ $\quad$ $6-(-7-T)^2$

$6-[(-1)^2(7+T)]^2$

0,1

$(2,0)+T[(3,0)-(2,1)]$

$(2,1)+(T,-T)$

$(-1,0)+T[(0,1)-(-1,0)]$

$(-1,0)+(T,T)$

$2+T,\ 1-T$

$f(t)$ $\boxed{\cancel{x+1}}$ $\boxed{3-x}$

$-1,\ T$

$x=\cancel{y}\ 1$

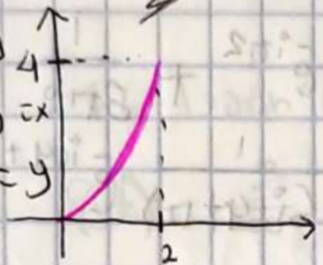
$y=x+1$

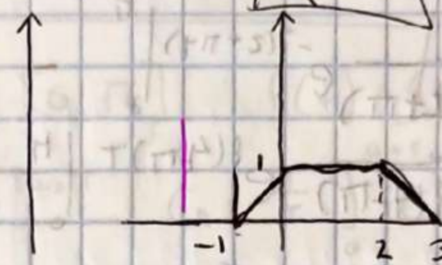
$y=1-T$

$T=1-y$

$2+1-y=x$

$3-x=y$





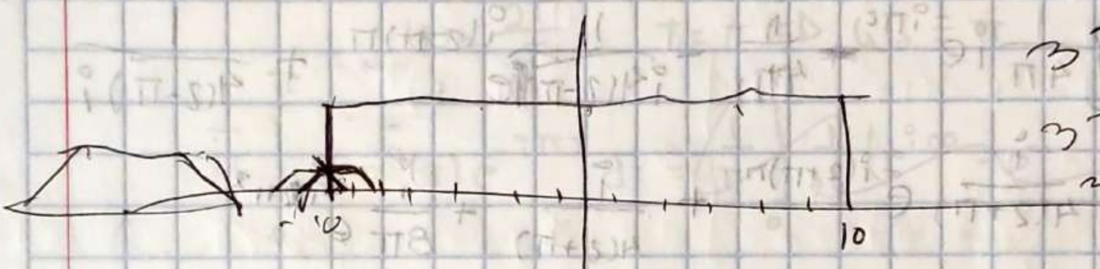
$3-(T+\alpha)$

$3-T-\alpha$

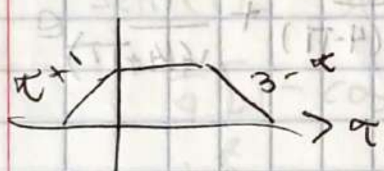
$3-T+(T-3)$

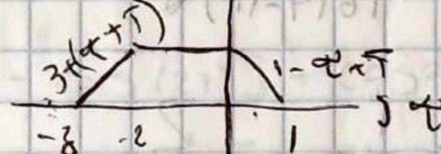
$3-T-T+3$

$3-T+\alpha$



$f(-\tau)$ $f(T-\alpha)$ $3-T+\alpha$





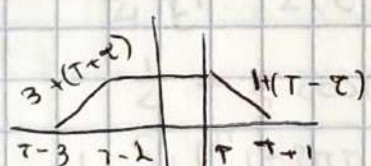
$3+T+T-3$

$3+T+(T+3)$

$(T-3,\ 0)$

$3-T$

$\cancel{3+T}+(T+3)$



$h_0(t)=\quad 0 \qquad t+1<-10 \quad \text{ie} \quad T<-11$

$h_1(t)=2\int_{-10}^{T+1}(1+T-\tau)\,d\tau=-\cancel{2}\dfrac{(1+T-\tau)^2}{\cancel{2}}\Bigg|_{-10}^{T+1}=+(11+T)^2$

$-10<t+1<-9$

en -10 vale $+1$

$-11<T<-10$

$h_2(t)=2\int_{-10}^{T}d\tau+2\int_{T}^{T+1}(1+T-\tau)\,d\tau=2\tau\Big|_{-10}^{T}-\cancel{2}\dfrac{(1+T-\tau)^2}{\cancel{2}}\Bigg|_{T}^{T+1}$

$=2T+20+1=2T+21 \qquad -9<T+1<-8$

en -9 $=3$ $\qquad -10<T<-9$

$$= \frac{1}{\pi}\Big\{ \frac{1}{4\pi} i e^{-i\pi^2} + \frac{1}{4\pi i} - \frac{1}{4(2-\pi)i} e^{i(2-\pi)\tau}\Big|_0^{\pi}$$

$$+ \frac{1}{i4(2+\pi)} e^{-i(2+\pi)\tau}\Big|_0^{\pi} - \frac{1}{8\pi i} e^{-i\pi^2} + \frac{1}{8\pi i}$$

$$+ \frac{1}{16i(4-\pi)} e^{i(4-\pi)\tau}\Big|_0^{\pi} - \frac{1}{16i(4+\pi)} e^{-i(4+\pi)\tau}\Big|_0^{\pi}\Big\}$$

$$= \frac{1}{\pi}\Big\{ \frac{1}{4\pi} i e^{-i\pi^2} - \frac{i}{4\pi} - \frac{1}{i4(2-\pi)} e^{i(2-\pi)\pi} + \frac{1}{4(2-\pi)} i$$

$$- \frac{i}{4(2+\pi)} e^{-i(2+\pi)\pi} + \frac{i}{4(2+\pi)} + \frac{i}{8\pi} e^{-i\pi^2}$$

$$- \frac{i}{16(4-\pi)} e^{i(4-\pi)\pi} + \frac{i}{16(4-\pi)} + \frac{i}{16(4+\pi)} e^{-i(4+\pi)\pi}$$

$$- \frac{i}{16(4+\pi)} \Big\}$$

$$\operatorname{sen} n\theta = \frac{1}{2i}\left(e^{in\theta} - e^{-in\theta}\right)$$

$$h_1(t) = 2\int_{-10}$$

$$f(t) = \operatorname{sen}^4 t \qquad [0, \pi] \qquad f(t) = f(T+\pi)$$

$$f(t) = \sum_{-\infty}^{\infty} C_n e^{i\omega_0 T} \qquad C_n = \frac{1}{T}\int_a^{a+T} f(T) e^{-i\omega_0 T}\, dT$$

$$\omega_0 = \frac{2\pi}{T} \qquad T = \pi \Rightarrow \omega_0 = \pi$$

$$\frac{1}{\pi}\int_0^{\pi} \operatorname{sen}^4(T) e^{-i\pi T}\, dT = \cancel{\frac{1}{\pi}\int_0^{\pi} \frac{1}{2i}\left(e^{in\theta} - e^{-i}\right.}$$

$$\operatorname{sen}^4(T) = \operatorname{sen}^2(T)\operatorname{sen}^2(T) = \frac{1}{2}[1 - \cos 2T]\,\frac{1}{2}[1 - \cos 2T]$$

$$\frac{1}{4}[1 - \cos(2T) - \cos(2T)] + \cos^2(2T)]$$

$$\frac{1}{4}\left\{1 - 2\cos(2T) + \frac{1}{2}\left[\cancel{\cos}\, 1 + \cos(4T)\right]\right\}$$

$$\frac{1}{4} - \frac{1}{2}\cos(2T) + \frac{1}{8} + \frac{1}{8}\cos(4T)$$

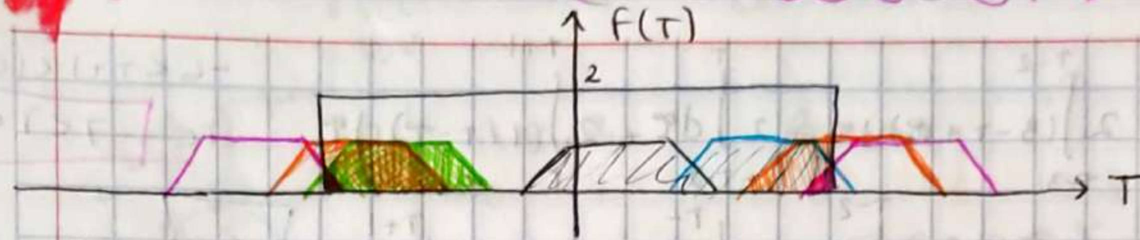
$$\Rightarrow \frac{1}{\pi}\left\{\frac{1}{4}\int_0^{\pi} e^{-i\pi T} dT - \frac{1}{4}\int_0^{\pi}\left(e^{i2T} + e^{-2iT}\right)e^{-i\pi T} + \frac{1}{8}\int e^{-i\pi T} dT\right.$$

$$\left. + \frac{1}{16}\int\left(e^{i4T} + e^{-4iT}\right)e^{-i\pi T} dT\right\}$$

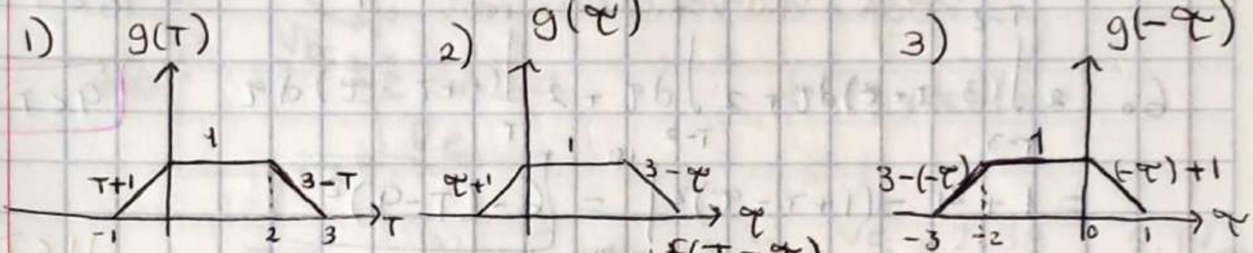
$$= \frac{1}{\pi}\left\{-\frac{1}{4i\pi} e^{-i\pi T}\Big|_0^{\pi} - \frac{1}{4}\left[\int_0^{\pi} e^{i(2-\pi)T} dT + \int_0^{\pi} e^{-i(2+\pi)T} dT\right]\right.$$

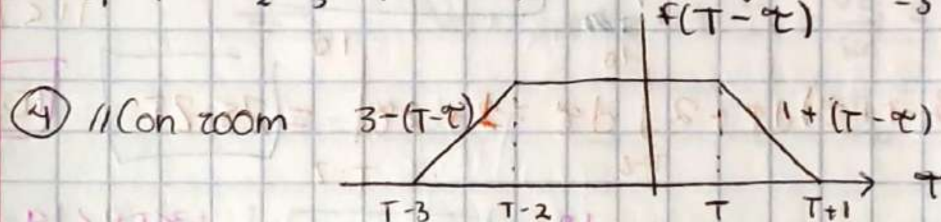
$$\left. - \frac{1}{8\pi i} e^{-i\pi T}\Big|_0^{\pi} + \frac{1}{16}\left[\int_0^{\pi} e^{i(4-\pi)T} dT + \int_0^{\pi} e^{-i(4+\pi)T} dT\right]\right\}$$

Ejercicio de Convolución.



Para construir el sistema móvil:





h(T):

1. $0; \quad -\infty < T+1 < -10$ i.e $\boxed{-\infty < T < -11}$

2. $2\int_{-10}^{T+1}(1+T-\tau)\,d\tau = -(1+T-\tau)^2\Big|_{-10}^{T+1} = \boxed{(11+T)^2}; \quad -10<T+1<-9 \quad \boxed{-11<T<-10}$

3. $2\int_{-10}^{T} dT + 2\int_{T}^{T+1}(1+T-\tau)\,d\tau = 2T\Big|_{-10} - (1+T-\tau)^2\Big|_{T} = \boxed{2T+21}$ $\quad -9<T+1<-7 \quad \boxed{-10<T<-8}$

4. $2\int_{-10}^{T-2} d\tau + 2\int_{T-2}^{T} d\tau + 2\int_{T}^{T+1}(1-T-\tau)\,d\tau$ $\quad -8<T+1<-6$

$= (3-T+\tau)^2\Big|_{-10}^{T-2} + 2\tau\Big|_{T-2}^{T} - (1+T-\tau)^2\Big|_{T}^{T+1}$ $\quad \boxed{-8<T<-7}$

$= 1-(7+T)^2 + 2T - 2(T-2) + 1 = \boxed{6-(7+T)^2}$

5. $2\int_{T-3}^{T-2}(3-T+\tau)\,d\tau + 2\int_{T-2}^{T} d\tau + 2\int_{T}^{T+1}(1+T-\tau)\,d\tau$

$-6 < T+1 < 10$ ie $\boxed{-7 < T < 9}$

$= (3-T+\tau)^2\Big|_{T-3}^{T-2} + 2\tau\Big|_{T-2}^{T} - (1+T-\tau)^2\Big|_{T}^{T+1} = \boxed{6}$

$10 < T+1 < 11$

6. $2\int_{T-3}^{T-2}(3-T+\tau)\,d\tau + 2\int_{T-2}^{T} d\tau + 2\int_{T}^{10}(1+T-\tau)\,d\tau$ $\boxed{9 < T < 10}$

$= 1 + 4 - (1+T-\tau)^2\Big|_{T}^{10} = \boxed{6-(T-9)^2}$

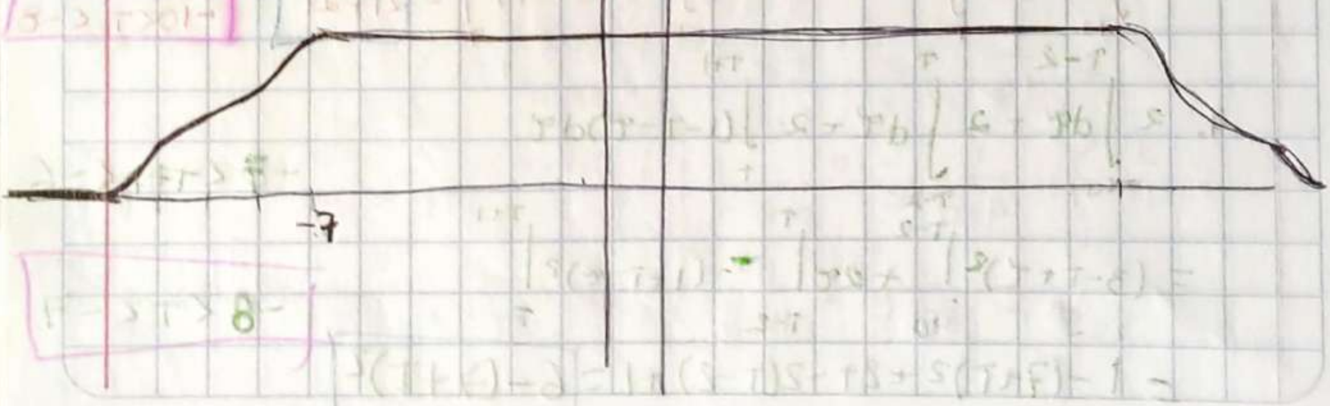
$11 < T+1 < 13$

7. $2\int_{T-3}^{T-2}(3-T+\tau)\,d\tau + 2\int_{T-2}^{10} d\tau = 1 + 2\tau\Big|_{T-2}^{10} = \boxed{25-2T}$ $\boxed{10 < T < 12}$

$13 < T+1 < 14$

8. $2\int_{T-3}^{10}(3-T+\tau)\,d\tau = (3-T+\tau)^2\Big|_{T-3}^{10} = \boxed{(13-T)^2}$ $\boxed{12 < T < 13}$

9. 0 ; $14 < T+1 < \infty$ $\boxed{13 < T < \infty}$



$$\mathcal{F}^{-1}\{W(T)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} W(T)\, e^{i\omega T}\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} f_1(\omega)\, e^{i\omega T} \ldots$$

$$\int_{-\infty}^{\infty} \delta'(T)\phi(T)\,dT = \phi(T)\delta(T)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \delta(T)\phi'(T)\,dT$$

$$\Rightarrow = -\phi'(0)$$

$u = \phi(T)$ $\quad du = \phi'(T)\,dT$

$dv = \delta'(T)$ $\quad v = \delta(T)$

$$\int_{-\infty}^{\infty} \delta^n(T)\phi(T)\,dT = (-1)^n \phi^n(0)$$

$$\int_{-\infty}^{\infty} \delta'(T)\phi(T)\,dT = -\int_{-\infty}^{\infty} \delta(T)\phi'(T)\,dT$$

$$\Rightarrow \text{Si} \quad \int_{-\infty}^{\infty} H'(T)\,\phi(T) = -\int_{-\infty}^{\infty} H(T)\,\phi'(T)\,dT$$

Usando $\int_{-\infty}^{\infty} H(T)\phi(T)\,dT = \int_{0}^{\infty} \phi(T)\,dT$

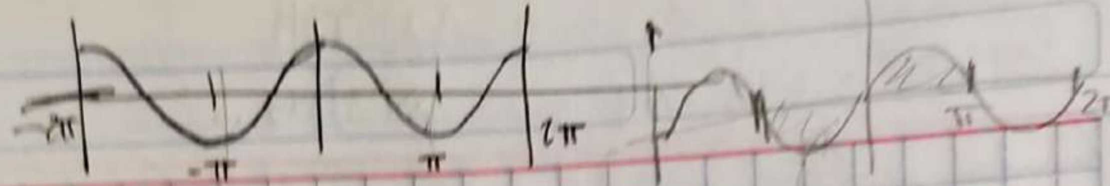
$$\Rightarrow -\int_{0}^{\infty} \phi'(T)\,dT = -\int_{0} \frac{\partial\phi}{\partial T}\,dT = -\int_{0}^{\infty} \partial\phi$$

$$-\phi(T)\Big|_{0}^{\infty}$$

$$= -[\phi(\infty) - \phi(0)]$$

$$= \phi(0) = \boxed{\int_{-\infty}^{\infty} \delta(T)\phi(T)\,dT = \delta(T)}$$

$$\Rightarrow H'(T) = \delta(T)$$



$sen(\tau) = 0$

$\tau = n\pi$

a) $\tau\, sen\, \tau \quad [-\pi, \pi]$

$$f(\tau) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi}{L}\tau\right) + b_n\, sen\left(\frac{n\pi}{L}\tau\right) \right]$$

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(\tau)\, d\tau$$

$u = \tau \quad du = d\tau$

$dv = sen\,\tau\, d\tau \quad v = -\cos\tau$

$$a_0 = \frac{1}{\pi} \int_0^{\pi} \tau\, sen\,\tau\, d\tau$$

$$a_0 = \frac{1}{\pi} \left\{ -\tau \cos\tau \Big|_0^{\pi} + \int_0^{\pi} \cos\tau\, d\tau \right\}$$

$$a_0 = \frac{1}{\pi} \left\{ -\pi(-1) + \cancel{sen} \Big|_0^{\pi} \overset{\nearrow 0}{} \right\} = 1$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(\tau) \cos\left(\frac{n\pi}{L}\tau\right) d\tau$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} \tau\, sen\,\tau\, \cos(n\tau)\, d\tau$$

$\cos A\, sen B$

$$sen(A+B) = sen A \cos B + \cos A\, sen B$$

$$sen(A-B) = sen A \cos B - \cos A\, sen B$$

$$sen A \cos B = \frac{1}{2}\left[sen(A+B) + sen(A-B) \right]$$

$$a_n = \frac{1}{\pi} \int_0^{\pi} \tau \left[sen(\tau + n\tau) + sen(\tau - n\tau) \right]$$

$$a_n = \frac{1}{\pi} \left\{ \int_0^{\pi} \tau\, sen[(n+1)\tau]\, d\tau + \int_0^{\pi} \tau\, sen[(1-n)\tau]\, d\tau \right\}$$

$u = \tau \quad du = d\tau$

$dv = sen[(n+1)\tau] \quad v = -\frac{1}{n\pm1} \cos[(n\pm1)\tau]$

$$a_n = \frac{1}{\pi} \left\{ -\frac{\tau}{n+1} \cos[(n+1)\tau] \Big|_0^{\pi} + \frac{1}{n+1} \int_0^{\pi} \cos[(n+1)\tau]\, d\tau - \frac{\tau}{n-1} \cos(\pi - n\pi) \Big|_0^{\pi} + \frac{1}{1-n} \int_0^{\pi} \cos[(1-n)\tau]\, d\tau \right\}$$

$$y(t) = f(t) * h(t)$$

$$\operatorname{sen}[\pi(\tau-4)]H(\tau-4) - \operatorname{sen}[\pi(\tau-8)]H(\tau-8)$$

$$\mathcal{F}\{\operatorname{sen}^2(t+2)\}$$

$$= \pi\delta(\omega) - \frac{1}{2}\{e^{4i}\pi\delta(\omega-2) + e^{-4i}\delta(\omega+2)\}$$

1(I).- Para la función $w = \frac{z}{(z-1)(z-2)}$ escriba dos desarrollos en series de potencias; uno de Taylor y otro de Laurent, indicando la región de validez en cada desarrollo:

1(D).- Dada la función $f(z) = \frac{1}{(z-2i)(z-4)}$

a) Escriba un desarrollo en serie de potencias en la región $4 < |z|$.

b) Escriba un desarrollo en serie de Taylor alrededor de un punto donde la función sea analítica, indicando el dominio de validez de dicho desarrollo.

2(I).- Resolver una, dije **una** de las siguientes integrales,

i) $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+\alpha^2)(x+\beta^2)}$; $\alpha > \beta > 0$ y ii) $\int_0^{2\pi} \frac{\cos(\theta)}{1+\frac{1}{4}\cos(\theta)} d\theta$,

2(D).- Resolver solo **una** de las siguientes integrales,

i) $\int_{-\infty}^{+\infty} \frac{x sen(ax)}{x^2+k^2} dx$; $a > 0$ & $k > 0$ y ii) $\int_0^{2\pi} \frac{sen(\theta)}{4+sen(\theta)} d\theta$,

3.- Calcule la siguiente integral,

I) $\int_\gamma \frac{sen(\pi z)}{(z-i)^6} dz$ D) $\int_\gamma \frac{\cos(\pi z)}{(z-i)^7} dz$

Siendo γ cualquier contorno cerrado que encierre a la singularidad.
NO USAR LA FÓRMULA INTEGRAL DE CAUCHY, APLIQUE EL TEOREMA DEL RESIDUO, CALCULANDO EL RESIDUO HACIENDO EL DESARROLLO EN SERIES LAURENT RESPECTIVO.

4(I&D).- i) Encuentre las series de potencia de $f(z) = \frac{5z-2}{z(z-1)}$ alrededor de $z_0 = 0$ y $z_0 = 1$ indicando los dominios en cada desarrollo.

ii) Calcule la siguiente integral $\int_C \frac{5z-2}{z(z-1)} dz$, donde el contorno C, es cualquier contorno que encierra los polos de la función. Obsérvese que el resultado es inmediato si identifica los residuos de los desarrollos en series de potencia en el inciso i).

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(1) $f(z) = \dfrac{1}{(z-2i)(z-4)} = \dfrac{A}{z-2i} + \dfrac{B}{(z-4)}$

Aplicando frac. parciales

a) $A = -\dfrac{1}{2i}$; $B = \dfrac{1}{4-2i}$ $\quad f(z) = \dfrac{-\frac{1}{2i}}{z-2i} + \dfrac{\frac{1}{4-2i}}{(z-4)}$

$$f(z) = -\frac{1}{2i(z-2i)} + \frac{1}{(z-4)(4-2i)}$$

$$= \frac{1}{2i} \cdot \frac{1}{2i}\left[\frac{1}{1-\frac{z}{2i}}\right] + \frac{1}{4-2i} \frac{1}{z}\left[\frac{1}{1-\frac{z}{4}}\right]$$

$$= \frac{1}{2i} \cdot \frac{1}{2i}\left[\sum_{n=0}^{\infty} \frac{z^n}{(2i)^n}\right] + \frac{1}{4-2i} \frac{1}{z} \sum_{n=0}^{\infty} \frac{z^n}{4^n}$$

en $\left|\frac{z}{2i}\right| < 1$ & $\left|\frac{z}{4}\right| < 1$

i.e $|z| < 2$ & $|z| < 4$

pero hiciste la de Taylor!.

---

b) $z_0 = 0$

$$f(z) = -\frac{1}{2i}\left[\frac{1}{z-2i}\right] + \frac{1}{4-2i}\left[\frac{1}{z-4}\right]$$

$$= \frac{1}{2i}\left(\frac{1}{1-\frac{z}{2i}}\right) - \frac{1}{4}\frac{1}{4-2i}\left(\frac{1}{1-\frac{z}{4}}\right)$$

$$= \frac{1}{2i}\left(\sum_{n=0}^{\infty} \frac{z^n}{(2i)^n}\right) - \frac{1}{4-2i}\frac{1}{4}\left(\sum_{n=0}^{\infty} \frac{z^n}{4^n}\right)$$

en $\left|\frac{z}{2i}\right| < 1$ & $\left|\frac{z}{4}\right| < 1$

$|z| < 1$ & $|z| < 4$ i.e $|z| < 1$

$$f(z) = \sum_{n=0}^{\infty} \frac{z^n}{(2i)^{n+1}} - \frac{1}{4-2i} \sum_{n=0}^{\infty} \frac{z^n}{4^{n+1}}$$

2. $f(z) = \int_0^{2\pi} \frac{\operatorname{sen}\theta}{4+\operatorname{sen}\theta}\, d\theta = \int_{\gamma: |z|=1} \frac{\frac{1}{2i}\left[\frac{z^2-1}{z}\right]}{4+\left[\frac{z^2-1}{z}\right]} \frac{dz}{iz}$

$\operatorname{sen}\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$

$\operatorname{sen}\theta = \frac{1}{2i}\left[\frac{z^2-1}{z}\right]$

$z = e^{i\theta}$

$dz = ie^{i\theta}\, d\theta$

$d\theta = \frac{dz}{ie^{i\theta}} = \frac{dz}{iz}$

$f(z) = \int_\gamma \frac{\frac{z^2-1}{2i\cancel{z}}}{\frac{4z+z^2-1}{\cancel{z}}} \frac{dz}{iz}$

$= \int_\gamma \frac{z^2-1}{(z^2+4z-1)2i} \frac{dz}{iz}$

$= -\frac{1}{2}\int_\gamma \frac{z^2-1}{(z^2+4z-1)(z)}\, dz$

$z_{1,2} = \frac{-4 \pm \sqrt{16+4}}{2} = -2 \pm \sqrt{5}$ $\quad z_1 = -2+\sqrt{5}$ $\quad z_2 = -2-\sqrt{5}$

$f(z) = -\frac{1}{2}\int_\gamma \frac{z^2-1}{(z-z_1)(z-z_2)z}\, dz = 2\pi i\left[\operatorname{Res}_{z=z_2} f(z) + \operatorname{Res}_{z=0} f(z)\right]$

$\frac{1}{(z-z_2)}\{a_0 + a_1(z-z_2) + \cdots$

$a_0 = \frac{z^2-1}{z-z_1}\cdot z\Big|_{z_2} \Big/ 0! = \frac{-2-\sqrt{5}-1}{-2\sqrt{5}} \Big/ 0! = \frac{-(\sqrt{5}+3)}{-2\sqrt{5}} = \frac{3+\sqrt{5}}{2\sqrt{5}}$

$\frac{1}{(z-0)}\{b_0 + b_0(z-0) + \cdots\}$

$b_0 = \frac{z^2-1}{(z-z_1)(z-z_2)}\Big|_0 \Big/ 0! = \frac{-1}{(2-\sqrt{5})(2+\sqrt{5})} = \frac{-1}{4+5} = -\frac{1}{9}$

$F(z) = \int_0^{2\pi} \frac{\operatorname{sen}\theta}{4+\operatorname{sen}\theta}\, d\theta = 2\pi i\left[\frac{3+\sqrt{5}}{2\sqrt{5}} - \frac{1}{9}\right]$

$= \cancel{2}\pi i\left[\frac{27+9\sqrt{5}-2\sqrt{5}}{9\cdot\cancel{2}\sqrt{5}}\right] = \pi i\left[\frac{27+7\sqrt{5}}{9\sqrt{5}}\right]$

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1.- a) Encontrar todos los valores de z, y localizarlos en el plano complejo si

i) **(Fila izquierda)** $z^3+8i=0$ ii) **(Fila derecha)** $z^4+4=0$.

b) i) **(Fila izquierda)** Dada $u(x,y)=e^{-y}(x\cdot\cos x - y\cdot sen y)$, encuentre una función $v(x,y)$ tal que la función $f(z)=u+iv$ sea analítica.

ii) **(Fila derecha)** Dada $v(x,y)=e^{-y}(y\cdot\cos x + x\cdot sen y)$, encuentre una función $u(x,y)$ tal que la función $f(z)=u+iv$ sea analítica.

c) Para la función encontrada en el inciso b), calcule la $f'(z)=\dfrac{df}{dz}$ y exprese la función $f(z)$ y la derivada en términos de la variable z (**ambas filas**).

2.-

i) **(Fila izquierda)** Muestre bajo qué condiciones, el mapeo de inversión $w=\dfrac{1}{z}$ mapea círculos del plano z en rectas o círculos en el plano w.

ii) **(Fila derecha)** Muestre bajo qué condiciones, el mapeo de inversión $w=\dfrac{1}{z}$ mapea rectas del plano z en rectas o círculos en el plano w.

3.- Para el mapeo $w=\dfrac{z+i}{z+1}$ encuentre las imágenes en el plano w de,

i) **(Fila izquierda)** $y=x+1$ &

ii) **(Fila derecha)** $(x-1)^2+y^2=1$.

4.- Muestre que las condiciones de Cauchy-Riemann en coordenadas polares están dadas por (**ambas filas**):

$$u_r=\frac{v_\theta}{r} \quad \& \quad v_r=-\frac{u_\theta}{r}$$

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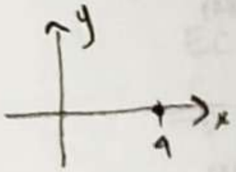
$z^4 + 4 = 0$

$z = (4)^{1/4} \qquad r = |z| = 4 \qquad \theta = 0 \qquad K = 0,1,2,3 \qquad n = 4$

$$w_K = r^{1/n}\left[\cos \frac{\theta + 2\pi K}{n} + i \operatorname{sen} \frac{\theta + 2\pi K}{n}\right]$$



$$w_0 = 4^{1/4}(\cos 0 + i \operatorname{sen} 0) = 4^{1/4}(1 + i0) = 4^{1/4}$$

$$w_1 = 4^{1/4}\left(\cos \frac{2\pi}{4} + i \operatorname{sen} \frac{2\pi}{4}\right)$$

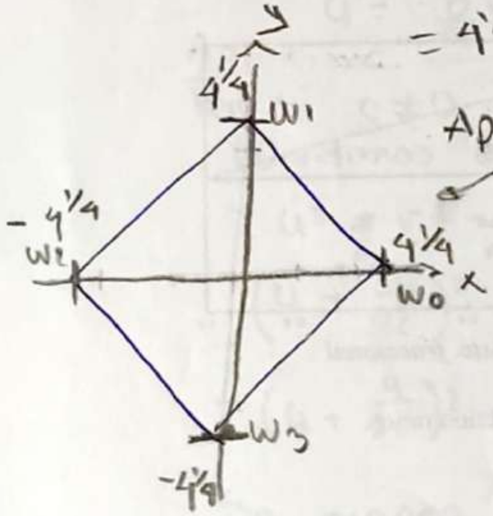
$$= 4^{1/4}\left(\cos \frac{\pi}{2} + i \operatorname{sen} \frac{\pi}{2}\right) = 4^{1/4}(0 + i)$$

$$w_2 = 4^{1/4}\left(\cos \frac{4\pi}{4} + i \operatorname{sen} \frac{4\pi}{4}\right)$$

$$= 4^{1/4}(\cos \pi + i \operatorname{sen} \pi) = 4^{1/4}(-1 + 0i)$$

$$w_3 = 4^{1/4}\left(\cos \frac{6\pi}{4} + i \operatorname{sen} \frac{6\pi}{4}\right)$$

$$= 4^{1/4}\left(\cos \frac{3}{2}\pi + i \operatorname{sen} \frac{3}{2}\pi\right) = 4^{1/4}(0 - i)$$



Aproximación

Polígono Regular de 4 lados

Rodriguez Aguilar Kathia

$w = \frac{1}{z} \Rightarrow z = \frac{1}{w}$

$z = \frac{1}{u+iv} \cdot \frac{u-iv}{u-iv} = \frac{u-iv}{u^2+v^2}$

$\frac{u}{u^2+v^2} - i\frac{v}{u^2+v^2} = x+iy \quad ; \quad x = \frac{u}{u^2+v^2} \qquad y = -\frac{v}{u^2+v^2}$

Ec. de la recta. $\boxed{ax+by+c=0}$

$\Rightarrow a\left(\frac{u}{u^2+v^2}\right) - b\left(\frac{v}{u^2+v^2}\right) + c = 0$

Mult por (u^2+v^2)

$\boxed{au - bv + c(u^2+v^2) = 0}$

$cu^2+cv^2 + au - bv = 0$

Si $c=0$ se mapea en una recta de la forma $au-bv=0$

$u = \frac{b}{a}v$ ó $v = \frac{a}{b}u$

Pero si $c \neq 0 \Rightarrow$
(Dividimos entre c)

$u^2 + v^2 + \frac{a}{c}u - \frac{b}{c}v = 0$

$\left(u + \frac{a}{2c}\right)^2 + \left(v - \frac{b}{2c}\right)^2 = \frac{a^2}{4c^2} + \frac{b^2}{4c^2}$

$\left(u + \frac{a}{2c}\right)^2 + \left(v - \frac{b}{2c}\right)^2 = \left(\frac{\sqrt{a^2+b^2}}{2c}\right)^2$

Se mapea en un circulo con centro en $\left(-\frac{a}{2c}, \frac{b}{2c}\right)$ y radio $\frac{\sqrt{a^2+b^2}}{2c}$

Para el mapeo $w = \frac{z+i}{z+1}$ img de $(x-1)^2 + y^2 = 1$

$$wz + w = z + i$$
$$wz - z = i - w$$
$$z(w-1) = i - w$$
$$z = \frac{i-w}{w-1}$$

$$z = \frac{-u + i(1-v)}{(u-1)+iv} \cdot \frac{(u-1)-iv}{(u-1)-iv}$$

$$z = \frac{-u(u-1) + iuv + i(1-v)(u-1) + v - v^2}{(u-1)^2 + v^2}$$

$$z = \frac{u - u^2 + i(u-1+v) + v - v^2}{(u-1)^2 + v^2}$$

$$z = \underbrace{\frac{u - u^2 + v - v^2}{(u-1)^2 + v^2}}_{x} + i \underbrace{\frac{u-1+v}{(u-1)^2+v^2}}_{y}$$

$$\left(\frac{u-u^2+v-v^2}{(u-1)^2+v^2} - 1\right)^2 + \left(\frac{u-1+v}{(u-1)^2+v^2}\right)^2 = 1$$

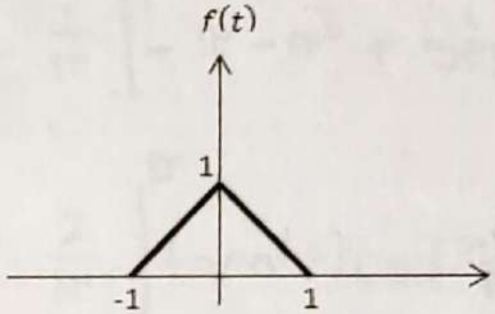
$$(3u - 2u - 2v^2 - v - 1)^2 + (u-1+v)^2 = [(u-1)^2 + v^2]^2$$

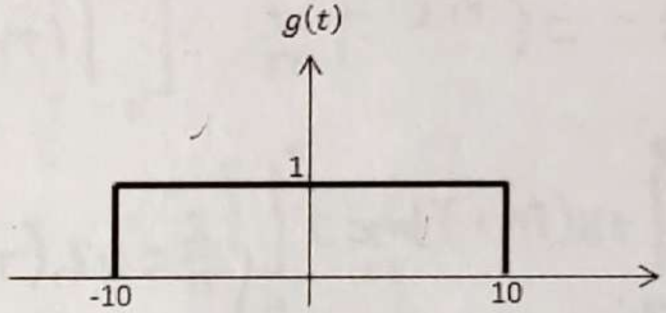
$$+6v^2 + 3u^4 - 8u^3 + 8u^2 - 4u + 1 + 4u^2v^2 - 8uv^2 - 2v^2 + 3v^4 - 4uv^3$$
$$+ 4vu^2 + 5v^3 + 6v^2 + 2$$

$3(u^2 -$

1.- Encuentre la serie de Fourier de la función $f(t) = t[sen(t)]$ definida en el intervalo $(-\pi, \pi)$

2.- a) Calcule $h(t) = g(t) * f(t)$, ilustrando geométricamente los pasos que se siguen en el procedimiento de convolución. Grafique $h(t)$.





b) Dadas las funciones,

$$f(t) = sen(\pi t)H(t)$$
$$h(t) = \delta(t-4) - \delta(t-8)$$

calcular $g(t) = f(t) * h(t)$. Grafique cada una de las funciones, así como el resultado final de la Convolución.

3.- a) Calcular la Transformada de Fourier de la función $f(t) = sen^2(t+2)$

b) Calcule la transformada inversa de Fourier de $F(w) = \dfrac{sen(w)}{w(2+jw)}$

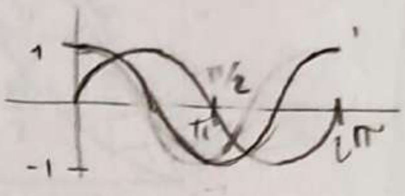
Rodriguez Aguilar Kathia

$$f(t) = \overset{I}{t}\,\overset{I}{sen}(t) \qquad (-\pi, \pi)$$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi}{L}x\right) + b_n\, sen\left(\frac{n\pi}{L}x\right)\right]$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(T)\,dT \qquad a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\left(\frac{n\pi}{L}T\right)dT$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\, sen\left(\frac{n\pi}{L}T\right)dT$$



$u = t \quad du = dt \quad dv = sen(t)\,dt \quad v = -\cos(t)$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} t\, sen(t)\,dT = \frac{1}{\pi}\left[-t\cos(t)dt\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi}\cos(t)\,dt\right]$$

$$a_0 = \frac{1}{\pi}\left[-\pi - \pi + \cancel{sen(T)}^{\,0}\Big|_{-\pi}^{\pi}\right] = \frac{1}{\pi}(-2\pi) = -2$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} t\, sen(t)\cos\left(\frac{n\pi}{\pi}T\right)dT = \frac{2}{\pi}\left\{\int_0^{\pi} t\, sen(T+nT)\,dt + \int_0^{\pi} t\, sen(T-nT)\,dT\right\}$$

$$sen(A+B) = senA\cos B + \cos A\, senB$$

$$-1 \quad -sen(A-B) = -senA\cos B + \cos A\, senB$$

$$\Rightarrow \quad senA\cos B = \left[sen(A+B) + sen(A-B)\right]\frac{1}{2}$$

$\cos A$

$u = t \quad du = dt$

$dv = sen[(1 \pm n)T]$

$v = -\frac{1}{1 \pm n}\cos[(1 \pm n)T]$

$$a_n = \frac{2}{\pi}\left\{-\frac{t}{1+n}\cos(T+nT)\Big|_0^{\pi} + \frac{1}{1+n}\int_0^{\pi}\cos(T+nT)\,dT - \frac{t}{1-n}\cos(T-nT)\Big|_0^{\pi}\right.$$

$$\left. + \frac{1}{1-n}\int_0^{\pi}\cos(T-nT)\,dT\right\}$$

$$a_n = \frac{2}{\pi}\left\{-\frac{\pi}{1+n}\cos(\pi+n\pi)^{-1} + \frac{1}{(1+n)^2}\, sen(T+nT)^{0}\Big|_0^{\pi} - \frac{\pi}{1-n}\cos(\pi - n\pi)^{-1}\right.$$

$$\left. + \frac{1}{(1-n)^2}\, sen(T-nT)^{0}\Big|_0^{\pi}\right\}$$

$$\frac{2}{\pi}\left\{-\frac{\pi}{1+n}-\frac{\pi}{1-n}\right\}=-\frac{2}{\pi}\left\{\frac{\pi-\cancel{n\pi}+\pi+\cancel{n\pi}}{1-n^2}\right\}=-\frac{2}{\cancel{\pi}}\left(\frac{2\cancel{\pi}}{1-n^2}\right)$$

$$=-\frac{4}{1-n^2}$$

$u=t \quad du=dt$

$dr=\operatorname{sen}(2T)dT \quad r=-\frac{1}{2}\cos(2T)$

para

$$a_1=\frac{2}{\pi}\left\{\int_0^{\pi} t\operatorname{sen}(2T)dT\right\}=\frac{2}{\pi}\left\{-\frac{t}{2}\cos(2T)\Big|_0^{\pi}+\frac{1}{2}\int_0^{\pi}\cos(2T)dT\right\}$$

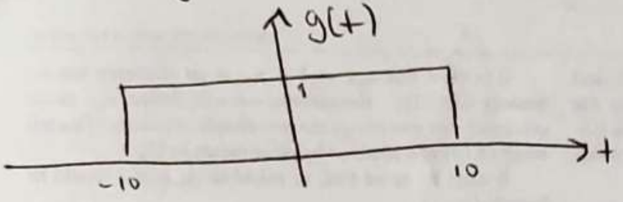
$$=\frac{2}{\pi}\left\{-\frac{\pi}{2}+\cancel{\frac{1}{4}\operatorname{sen}(2T)\Big|_0^{\pi}}\right\}=-\frac{2}{\pi}\left(\frac{\pi}{2}\right)=-1$$

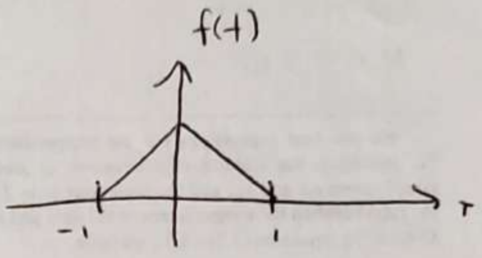
$b_n=0$

$$\Rightarrow f(x)=-2+\sum_{n=1}^{\infty}-\frac{4}{1-n^2}\cos\left(\frac{n\pi}{2}x\right)$$

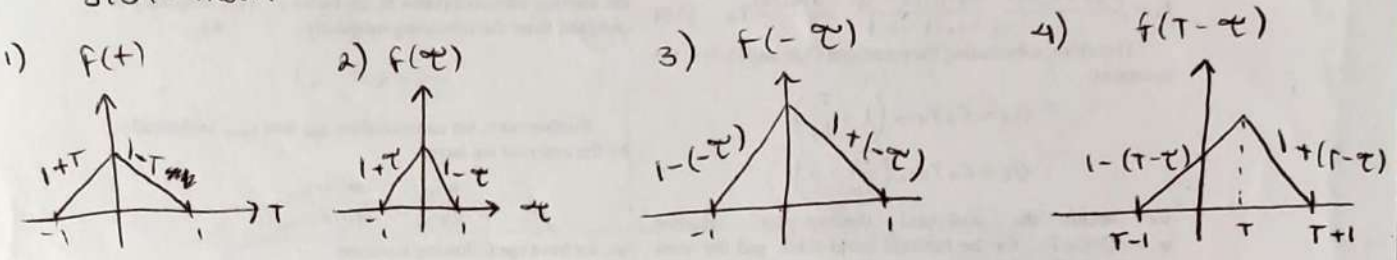
$n\neq 1$

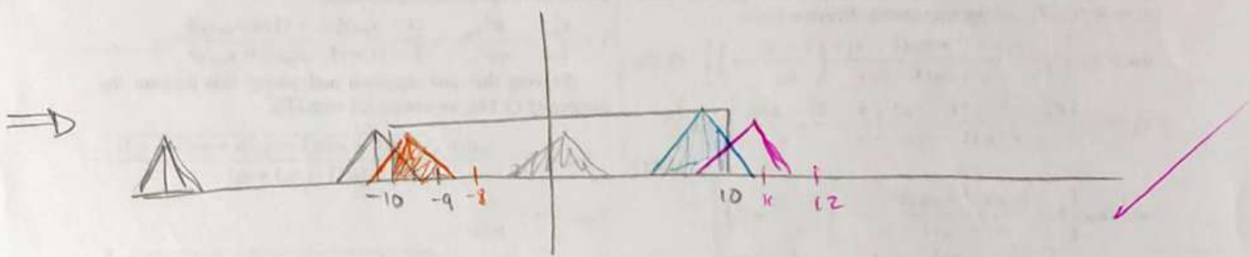
② $h(t) = g(t) * f(t)$





sist. movil





$h(T) =$

~~h1~~ $0; \quad -\infty < T+1 < -10 \quad$ i.e $\quad \boxed{-\infty < T < -11}$

$$h_1(T) = \int_{-10}^{T+1} (1+T-\tau)\,d\tau = -\frac{(1+T-\tau)^2}{2}\Bigg|_{-10}^{T+1} = \boxed{\frac{(11+T)^2}{2}}$$

$-10 < T+1 < -9$

$\boxed{-11 < T < -10}$

0 y $\frac{1}{2}$

$$h_2(T) = \int_{-10}^{T} (1-T+\tau)\,d\tau + \int_{T}^{T+1} (1+T-\tau)\,d\tau = \frac{(1-T+\tau)^2}{2}\Bigg|_{-10}^{T} - \frac{(1+T-\tau)^2}{2}\Bigg|_{T}^{T+1}$$

en $-9 < T+1 < -8$

$\boxed{-10 < T < -9}$

$$= \frac{1}{2} - \frac{(9+T)^2}{2} + \frac{1}{2} = \boxed{1 - \frac{(9+T)^2}{2}}$$

$\frac{1}{2}$ y 1

$$h_3(T) = \int_{T-1}^{T} (1-T+\tau)\,d\tau + \int_{T}^{T+1} (1+t-\tau)\,d\tau = \frac{(1-T+\tau)^2}{2}\Bigg|_{T-1}^{T} + \frac{1}{2}$$

en $-8 < T+1 < 10$

$\boxed{-9 < T < 9}$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$h_4(T) = \int_{T-1}^{T} (1-T+\tau)\,d\tau + \int_{T}^{10} (1+T-\tau)\,d\tau = \frac{1}{2} - \frac{(1+T-\tau)^2}{2}\Bigg|_{T}^{10}$$

$10 < T+1 < 11$

$\boxed{9 < T < 10}$

$$= \frac{1}{2} - \frac{(T-9)^2}{2} + \frac{1}{2} = 1 - \frac{(T-9)^2}{2}$$

1 y $\frac{1}{2}$

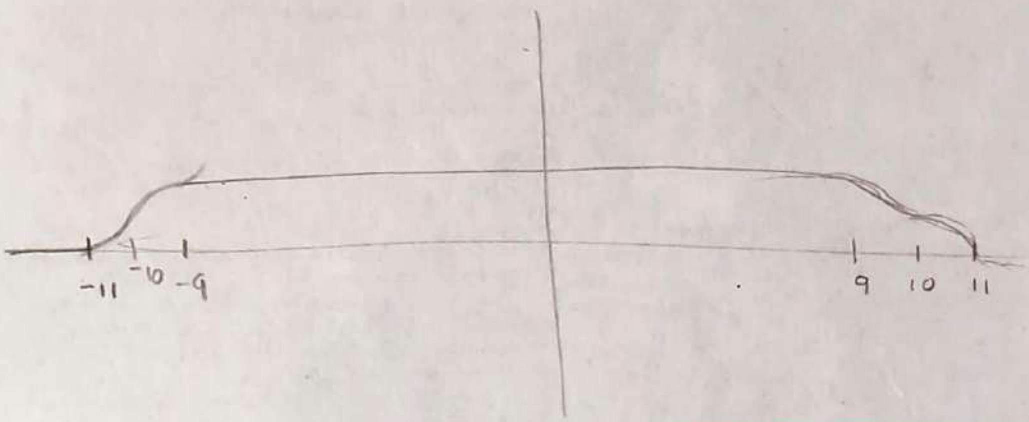
$$h_5(T) = \int_{T-1}^{10} (1-T+\tau)\,d\tau = \left.\frac{(1-T+\tau)^2}{2}\right|_{T-1}^{10}$$

en $11 < T+1 < 12$

$10 < T < 11$

$$= \boxed{\frac{(11-T)^2}{2}}$$

$h_6(T) = 0 \qquad 11 < T < \infty$



(3)

$f(t) = \text{sen}^2(T+2)$

$$\mathcal{F}\{\text{sen}^2(T+2)\} = \mathcal{F}\left\{\frac{1+\cos(2T+4)}{2}\right\} = \frac{1}{2}\mathcal{F}\{1+\cos(2T+4)\}$$

$$= \frac{1}{2}\mathcal{F}\{1\} + \frac{1}{2}\mathcal{F}\{\cos(2T+4)\} \qquad \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\,e^{-i\omega T}\,dT$$

$$\Rightarrow \mathcal{F}\{\text{sen}^2(T+2)\} = \frac{1}{2}\int_{-\infty}^{\infty} e^{-i\omega T}\,dT + \frac{1}{2}\int_{-\infty}^{\infty} \cos(2T+4)\,e^{-i\omega T}\,dT$$

Usando

$$\int_{-\infty}^{\infty} e^{ixy}\,dx = 2\pi\delta(y)$$

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$

$$\Rightarrow = \frac{1}{\cancel{2}}\cancel{2}\pi\,\delta(-\omega) + \frac{1}{2\cdot 2}\int_{-\infty}^{\infty}\left[e^{i(2T+4)} + e^{-i(2T+4)}\right]e^{-i\omega T}\,dT$$

$$= \pi\delta(\omega) + \frac{1}{4}\int_{-\infty}^{\infty} e^{i2T}e^{4i}e^{-i\omega T}\,dT + \frac{1}{4}\int_{-\infty}^{\infty} e^{-i2T}e^{-4i}e^{-i\omega T}\,dT$$

$$= \pi\delta(\omega) + \frac{e^{4i}}{4}\int_{-\infty}^{\infty} e^{i(2-\omega)T}\,dT + \frac{e^{-4i}}{4}\int_{-\infty}^{\infty} e^{-i(2+\omega)T}\,dT$$

$$= \pi\delta(\omega) + \frac{e^{4i}}{4}2\pi\,\delta(2-\omega) + \frac{e^{-4i}}{4}2\pi\,\delta[-(2+\omega)]$$

$$= \pi\delta(\omega) + \frac{\pi}{2}\left\{e^{4i}\,\delta(\omega-2) + e^{-4i}\,\delta(\omega+2)\right\}$$