15. $y'' + y = 2x \operatorname{sen} x \ldots (1)$

• Ec. homo

$y'' + y = 0 \ldots (2)$

• Ec. carac.

$m^2 + 1 = 0$

• Raices

$m_{12} = \pm\sqrt{-1} = \pm i$

• Sols. li

$y_1 = \cos x \quad y_2 = \operatorname{sen} x$

• Sol. gral. de (2)

$y_c = C_1 \cos x + C_2 \operatorname{sen} x$

$F(x) = 2x \operatorname{sen} x$

$y_p = (Ax + B)\operatorname{sen} x + (Cx + D)\cos x$

$= Ax\operatorname{sen} x + B\operatorname{sen} x + Cx\cos x + D\cos x$

$y_p' = A\operatorname{sen} x + Ax\cos x + B\cos x + C\cos x - Cx\operatorname{sen} x - D\operatorname{sen} x$

$y_p'' = 2A\cos x - Ax\operatorname{sen} x - B\operatorname{sen} x - 2C\operatorname{sen} x - Cx\cos x$

Sust. y & sus deri en (1)

$2A\cos x - \cancel{Ax\operatorname{sen} x} - \cancel{B\operatorname{sen} x} - 2C\operatorname{sen} x - \cancel{Cx\cos x} + \cancel{Ax\operatorname{sen} x} + \cancel{B\operatorname{sen} x} + \cancel{Cx\cos x} + D\cos x = F(x)$

$(2A + D)\cos x - 2C\operatorname{sen} x = 2x\operatorname{sen} x$

17. $y'' - 2y' + 5y = e^x \cos 2x$

• Ec. homo

$y'' - 2y' + 5y = 0 \ldots (2)$

• Ec. caracter.

$m^2 - 2m + 5 = 0$

• R. $m_{12} = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2(1)}$

$m_{12} = 1 \pm 2i$

• Sls. li

$y_1 = e^x \cos 2x \quad y_2 = e^x \operatorname{sen} 2x$

• Sol. gral de (2)

$y_c = C_1 e^x \cos 2x + C_2 e^x \cos 2x$

$F(x) = e^x \cos 2x$

$y_p =$

$\overline{3)}\ y'' - y' = -3$

- $y'' - y' = 0$
- $m^2 - m = 0$
- $m(m-1) = 0$
- $m_1 = 0 \quad m_2 = 1$
- $y_1 = 1 \quad y_2 = e^x$
- $y_c = C_1 + C_2 e^x$

- $F(x) = -3$

$y_p = Ax$

$y'_p = A$

$y''_p = 0$

Sust.

$-A = -3$

$y_p = 3x$

$y = y_c + y_p = \underline{C_1 + C_2 e^x + 3x}$

(11) $y'' - y' + \frac{1}{4}y = 3 + e^{x/2}$

- $y'' - y + \frac{1}{4}y = 0$
- $m^2 - m + \frac{1}{4} = 0$

$(m - \frac{1}{2})^2$

$m_1 = m_2 = \frac{1}{2}$

- $y_1 = e^{1/2x} \quad y_2 = xe^{1/2x}$
- $y_c = C_1 e^{1/2x} + C_2 x e^{1/2x}$

- $F(x) = 3 + e^{x/2}$

$y_p = A + Bx^2 e^{x/2}$

$y'_p = 2xe^{x/2} + \frac{1}{2}Bx^2 e^{x/2}$

$y''_p = 2Be^{x/2} + 2Bxe^{x/2} + \frac{1}{4}Bx^2 e^{x/2}$

Sostituyendo

$2Be^{x/2} + \cancel{2xe^{x/2}} + \cancel{\frac{1}{4}x^2 e^{x/2}} - \cancel{2xe^{x/2}} - \cancel{\frac{1}{2}x^2 e^{x/2}} + \frac{1}{4}A + \cancel{\frac{1}{4}x^2 e^{x/2}} = 3 + e^{x/2}$

$\frac{1}{4}A + 2Be^{x/2} = 3 + e^{x/2}$

$\frac{1}{4}A = 3 \longrightarrow A = 12$

$2B = 1 \longrightarrow B = \frac{1}{2}$

$\Longrightarrow y_p = 12 + \frac{1}{2}x^2 e^{x/2}$

$y = y_c + y_p = \underline{C_1 e^{1/2x} + C_2 x e^{1/2x} + 12 + \frac{1}{2}x^2 e^{x/2}}$

(13) $y'' + 4y = 3 \text{ sen } 2x$

- $y'' + 4y = 0$
- $m^2 + 4 = 0$
- $m_{1,2} = \pm\sqrt{-4} = \pm 2i$
- $y_1 = \text{sen } 2x \quad y_2 = \cos 2x$
- $y_c = C_1 \text{ sen } 2x + C_2 \cos 2x$

- $F(x) = 3 \text{ sen } 2x$
- $y_p = (A \text{ sen } 2x + B\cos 2x)x$

$= Ax \text{ sen } 2x + Bx\cos 2x$

- $y'_p = A\text{sen}2x + 2Ax\cos 2x + B\cos 2x - 2Bx\,\text{sen}\,2x$
- $y''_p = 4A\cos 2x - 4Ax\,\text{sen}2x - 4B\,\text{sen}2x - 4Bx\cos 2x$

Sostituyendo

$4A\cos 2x - \cancel{4Ax\,\text{sen}2x} - 4B\text{sen}2x - \cancel{4Bx\cos 2x} + \cancel{4Ax\,\text{sen}2x} + \cancel{4Bx\cos 2x} = 3 \text{ sen } 2x$

$4A\cos 2x - 4B\text{sen}2x = 3 \text{ sen } 2x$

$-4B = 3 \rightarrow B = -\frac{3}{4}$

$4A = 0 \quad A = 0$

$\longrightarrow y_p = -\frac{3}{4}x\cos 2x$

$y = y_c + y_p = \underline{C_1 \text{sen}2x + C_2\cos 2x - \frac{3}{4}x\cos 2x}$

$y'' + y = 2x \text{ sen } x$

- $y'' + y = 0$
- $m^2 + 1 = 0$
- $m_{12} = \pm\sqrt{-1} = \pm i$
- $y_1 = \cos x \qquad y_2 = \text{sen} x$
- $y_c = C_1 \cos x + C_2 \text{sen} x$

- $F(x) = 2x \text{ sen} x$

$y_p = (Ax + B)x \text{sen} x + (Cx + D)x \cos x$

$y_p = Ax^2 \text{sen} x + Bx \text{sen} x + Cx^2 \cos x + Dx \cos x$

$y'_p = 2Ax \text{sen} x + Ax^2 \cos x + B \text{sen} x + Bx \cos x + 2Cx \cos x - Cx^2 \text{sen} x + D \cos x - Dx \text{sen} x$

$y''_p = 2A \text{sen} x + 2Ax \cos x + 2Ax \cos x - Ax^2 \text{sen} x + 2B \cos x - B \text{sen} x + 2C \cos x - 2Cx \cos x - 2D \text{sen} x - Dx \cos x$

$2A \text{sen} x + 2Ax \cos x + 2Ax \cos x \cancel{- Ax^2 \text{sen} x} + 2B \cos x \cancel{- Bx \text{sen} x} + 2C \cos x - 2Cx \cos x - 2D \text{sen} x \cancel{- Dx \cos x} \cancel{+ Ax^2 \text{sen} x} + \cancel{Bx \text{sen} x} + \cancel{Cx^2 \cos x} + \cancel{Dx \cos x} = 2x \text{ sen} x$

$2A \text{sen} x + 4Ax \cos x + 2B \cos x + 2C \cos x - 4Cx \text{sen} x - 2D \text{sen} x = F(x)$

$(2A - 2D) \text{sen} x + (2B + 2C) \cos x + (4A) x \cos x + (-4C) x \text{sen} x = 2x \text{sen} x$

$-4C = 2 \qquad C = -\frac{1}{2}$

$4A = 0 \qquad A = 0$

$2B + 2C = 0 \qquad B = \frac{1}{2}$

$2A - 2D = 0 \qquad D = 0$

$\therefore \; y_p = -\frac{1}{2} x^2 \cos x + \frac{1}{2} x \text{sen} x$

$y = y_c + y_p$

$\underline{y = C_1 \cos x + C_2 \text{sen} x - \frac{1}{2} x^2 \cos x + \frac{1}{2} x \text{sen} x}$

35 $y'' - 2y' + 5y = e^x \cos 2x$ (17)

- Ecuación homogenea

$$y'' - 2y' + 5y = 0 \quad \dots (2)$$

- Ecuación característica

$$m^2 - 2m + 5 = 0$$

- Raices

$$m_{1,2} = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2(1)} = \frac{2 \pm \sqrt{-16}}{2}$$

$$m_{1,2} = 1 \pm 2i$$

- Sols. l.i.

$y_1 = e^x \cos 2x \quad y_2 = e^x \operatorname{sen} 2x$

- Sol. gral. de (2)

$$y_c = C_1 e^x \cos 2x + C_2 e^x \operatorname{sen} 2x$$

$F(x) = e^x \cos 2x$

$\rightarrow y_p = (Ae^x \cos 2x + Be^x \operatorname{sen} 2x)x$

$\therefore y_p = Axe^x \cos 2x + Bxe^x \operatorname{sen} 2x$

$y_p = Axe^x \cos 2x + Bxe^x \operatorname{sen} 2x$

$y_p' = Ae^x \cos 2x + Axe^x \cos 2x - 2Axe^x \operatorname{sen} 2x + Be^x \operatorname{sen} 2x + Bxe^x \operatorname{sen} 2x + 2Bxe^x \cos 2x$

$y_p'' = Ae^x \cos 2x - 2Ae^x \operatorname{sen} 2x + Ae^x \cos 2x + Axe^x \cos 2x - 2Axe^x \operatorname{sen} 2x - 2Ae^x \operatorname{sen} 2x - 2Axe^x \operatorname{sen} 2x$
$- 4Axe^x \cos 2x + Be^x \operatorname{sen} 2x + 2Be^x \cos 2x + Be^x \operatorname{sen} 2x + Bxe^x \operatorname{sen} 2x + 2Bxe^x \cos 2x$
$+ 2Be^x \cos 2x + 2Bxe^x \cos 2x - 4Bxe^x \operatorname{sen} 2x$

$= \cancel{2Ae^x \cos 2x} - 4Ae^x \operatorname{sen} 2x \cancel{- 4Axe^x \operatorname{sen} 2x} \cancel{- 3Axe^x \cos 2x} + \cancel{2Be^x \operatorname{sen} 2x}$
$+ 4Be^x \cos 2x \cancel{- 3Bxe^x \operatorname{sen} 2x} + \cancel{4Bxe^x \cos 2x} \cancel{- 2Ae^x \cos 2x} \cancel{- 2Axe^x \cos 2x} + \cancel{4Axe^x \operatorname{sen} 2x}$
$\cancel{- 2Be^x \operatorname{sen} 2x} \cancel{- 2Bxe^x \operatorname{sen} 2x} \cancel{- 4Bxe^x \cos 2x} + \cancel{5Axe^x \cos 2x} + \cancel{5Bxe^x \operatorname{sen} 2x} = F(x)$

$\rightarrow y_p = 4Be^x \cos 2x - 4Ae^x \operatorname{sen} 2x = e^x \cos 2x$

$$4B = 1 \longrightarrow B = \frac{1}{4}$$

$$-4A = 0 \longrightarrow A = 0$$

$\therefore y_p = \frac{1}{4} x e^x \operatorname{sen} 2x$

$$y = y_c + y_p = C_1 e^x \cos 2x + C_2 e^x \operatorname{sen} 2x + \frac{1}{4} x e^x \operatorname{sen} 2x$$

(19) $y'' + 2y' + y = \text{sen}\,x + 3\cos 2x$

- $y'' + 2y + y = 0$
- $m^2 + 2m + 1 = 0$
- $(m+1)^2 = 0$

$m_{1,2} = -1$

- $y_1 = e^{-x} \quad y_2 = xe^{-x}$
- $y_c = C_1 e^{-x} + C_2 x e^{-x}$

- $F(x) = \text{sen}\,x + 3\cos 2x$
- $y_p = A\,\text{sen}\,x + B\cos x + C\,\text{sen}\,2x + D\cos 2x$
- $y'_p = A\cos x - B\,\text{sen}\,x + 2C\cos 2x - 2D\,\text{sen}\,2x$

$y''_p = -A\,\text{sen}\,x - B\cos x - 4C\,\text{sen}\,2x - 4D\cos 2x$

$\cancel{-A\,\text{sen}\,x} - \cancel{B\cos x} - \cancel{4C\,\text{sen}\,2x} - \cancel{4D\cos 2x} + 2A\cos x - 2B\,\text{sen}\,x + 4C\cos 2x - 4D\,\text{sen}\,2x + \cancel{A\,\text{sen}\,x} + \cancel{B\cos x} + \cancel{C\,\text{sen}\,2x} + \cancel{D\cos 2x} = F(x)$

$(2A)\cos x + (-2B)\,\text{sen}\,x + (4C - 3D)\cos 2x + (-4D - 3C)\,\text{sen}\,2x = \text{sen}\,x + 3\cos 2x$

$2A = 0 \rightarrow A = 0$

$-2B = 1 \rightarrow B = -\frac{1}{2}$

$4C - 3D = 3 \longrightarrow C = \frac{3+3D}{4} \longrightarrow C = \frac{3 + 3\left(-\frac{9}{25}\right)}{4}$

$-4D - 3C = 0 \longrightarrow -4D = 3\left[\frac{3+3D}{4}\right]$

$D = -\frac{9}{16} - \frac{9}{16}D$

$\frac{25}{16}D = -\frac{9}{16}$

$D = -\frac{9}{25}$

$C = \frac{3 + \left(-\frac{27}{25}\right)}{4}$

$C = \frac{12}{25}$

$\therefore\ y_p = -\frac{1}{2}\cos x + \frac{12}{25}\,\text{sen}\,2x - \frac{9}{25}\cos 2x$

$y = y_c + y_p = C_1 e^{-x} + C_2 x e^{-x} - \frac{1}{2}\cos x + \frac{12}{25}\,\text{sen}\,2x - \frac{9}{25}\cos 2x$

(21) $y''' - 6y'' = 3 - \cos x$

- $y''' - 6y'' = 0$
- $m^3 - 6m^2 = 0$
- $m^2(m-6) = 0$

$m_1 = m_2 = 0 \quad m_3 = 6$

- $y_1 = 1 \quad y_2 = x \quad y_3 = e^{6x}$
- $y_c = C_1 + C_2 x + C_3 e^{6x}$

$F(x) = 3 - \cos x$

$y_p = Ax^4 + Bx^3 + Cx^2 + D\,\text{sen}\,x + E\cos x$

$y'_p = 4Ax^3 + 3Bx^2 + 2Cx + D\cos x - E\,\text{sen}\,x$

$y''_p = 12Ax^2 + 6Bx + 2C - D\,\text{sen}\,x - E\cos x$

$y'''_p = 24Ax + 6B - D\cos x + E\,\text{sen}\,x$

$24Ax + 6B - D\cos x + E\,\text{sen}\,x - 72Ax^2 - 36Bx - 12C + 6D\,\text{sen}\,x + 6E\cos x = 3 - \cos x$

$(-72A)x^2 + (24A - 36B)x + (6B - 12C) + (6D + E)\,\text{sen}\,x + (6E - D)\cos x = 3 - \cos x$

$-72A = 0 \quad A = 0$

$24A - 3B = 0$

$6B - 12C = 3$

$C = -\frac{1}{4}$

$6D + E = 0$

$6E - D = 1$

$D = 6E - 1 \longrightarrow D = \frac{36}{37} - 1 = -\frac{1}{37}$

$36E - 6 + E = 0 \quad E = \frac{6}{37}$

$\overline{y_p} = -\frac{1}{4}x^2 - \frac{1}{37}\operatorname{sen}x + \frac{6}{37}\cos x$

$y = y_c + y_p = C_1 + C_2x + C_3e^{6x} - \frac{1}{4}x^2 - \frac{1}{37}\operatorname{sen}x + \frac{6}{37}\cos x$

(23) $y''' - 3y'' + 3y' - y = x - 4e^x$

- $y''' - 3y'' + 3y' - y = 0$
- $m^3 - 3m^2 + 3m - 1 = 0$
- $(m-1)^3 \quad m_1 = m_2 = m_3 = 1$
- $y_1 = e^x \quad y_2 = xe^x \quad y_3 = x^2e^x$
- $y_c = C_1e^x + C_2xe^x + C_3x^2e^x$

$F(x) = x - 4e^x$

$y_p = Ax + B + Cx^3e^x$

$y'_p = A + 3Cx^2e^x + Cx^3e^x$

$y''_p = 6Cxe^x + 6Cx^2e^x + Cx^3e^x$

$y'''_p = 6Ce^x + 12Cxe^x + 3Cx^2e^x + Cx^3e^x$

$6Ce^x + 12Cxe^x + 3Cx^2e^x + Cx^3e^x - 18Cxe^x - 18Cx^2e^x - 6Cx^3e^x + 3A + 9Cx^2e^x + 3x^3e^x - Ax - B - Cx^3e^x = F(x)$

$(6C)e^x + (-6C)xe^x + (-6C)x^2e^x + (-6C)x^3e^x + (-A)x + (3A - B) = x - 4e^x$

$6C = -4 \quad C = -\frac{2}{3}$

$-A = 1 \quad A = -1$

$3A - B = 0 \quad -3 = -B \quad B = 3$

$y_p = -x + 3 - \frac{2}{3}x^3e^x$

$y = y_c + y_p = C_1e^x + C_2xe^x + C_3x^2e^x - x + 3 - \frac{2}{3}x^3e^x$

(25) $y^{(4)} + 2y'' + y = (x-1)^2$

- $y^{(4)} + 2y'' + y = 0$
- $m^4 + 2m^2 + 1 = 0$
- $(m^2+1)^2$

$(m^2+1)(m^2+1)$

$y_1 = \operatorname{sen}x \quad y_2 = \cos x \quad y_3 = x\operatorname{sen}x$

$y_4 = x\cos x$

$y_c = C_1\operatorname{sen}x + C_2\cos x + C_3x\operatorname{sen}x + C_4x\cos x$

$F(x) = x^2 - 2x + 1$

$y_p = Ax^2 - Bx + C$

$y'_p = 2Ax - B$

$y'' = 2A$

$y''' = 0 \quad y^{(4)} = 0$

$4A + Ax^2 - Bx + C = x^2 - 2x + 1$

$(A)x^2 + (-B)x + (C + 4A) = x^2 - 2x + 1$

$A = 1$

$-B = -2 \quad B = 2$

$C + 4A = 1 \quad C = -3$

$y_p = x^2 - 2x - 3$

$y = y_c + y_p = C_1\operatorname{sen}x + C_2\cos x + C_3x\operatorname{sen}x + C_4x\cos x + x^2 - 2x - 3$

27 $y'' + 4y = -2, \quad y\left(\frac{\pi}{8}\right) = \frac{1}{2}, \quad y'\left(\frac{\pi}{8}\right) = 2$

- $y'' + 4y = 0$
- $m^2 + 4 = 0$
- $m_{1,2} = \pm\sqrt{-4}\,i = \pm 2i$
- $y_1 = \text{sen}\,2x \quad y_2 = \cos 2x$
- $y_c = C_1 \text{sen}\,2x + C_2 \cos 2x$

- $F(x) = -2$

$y_p = A$

$y'_p = 0$

$y''_p = 0$

$4A = -2 \quad A = -\frac{1}{2}$

$y = C_1 \text{sen}\,2x + C_2 \cos 2x - \frac{1}{2}$

$y\left(\frac{\pi}{8}\right) = \frac{1}{2}$

$\frac{1}{2} = C_1 \text{sen}\,2\left(\frac{\pi}{8}\right) + C_2 \cos 2\left(\frac{\pi}{8}\right)$

$\frac{1}{2} = 0.7C_1 + 0.7C_2$

$C_1 = \dfrac{\frac{1}{2} - 0.7C_2}{0.7} \longrightarrow$

$C_1 = \dfrac{\frac{1}{2} - 0.7(0.3)}{0.7}$

$C_1 = 0.41$

$y'\left(\frac{\pi}{8}\right) = 2$

$2 = 2C_1 \cos 2\left(\frac{\pi}{8}\right) - 2C_2\,\text{sen}\,2\left(\frac{\pi}{8}\right)$

$2 = 1.41C_1 - 1.41C_2$

$2 = 1.41\left(\dfrac{\frac{1}{2} - 0.7C_2}{0.7}\right) - 1.41C_2$

$2 = 1 - 1.41C_2 - 1.41C_2$

$1 = 2.82C_2$

$C_2 = 0.3$

$y = \underline{0.41\,\text{sen}\,2x + 0.3\cos 2x - \frac{1}{2}}$

28 $5y'' + y' = -6x \quad y(0) = 0 \quad y'(0) = -10$

- $5y'' + y' = 0$
- $5m^2 + m = 0$
- $m(5m + 1) = 0$

$m_1 = 0 \quad m_2 = -\frac{1}{5}$

- $y_1 = 1 \quad y_2 = e^{-1/5x}$
- $y_c = C_1 + C_2 e^{-1/5x}$

- $F(x) = -6x$

$y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

$10A + 2Ax + B = -6x$

$(2A)x + (10A + B) = -6x$

$2A = -6 \quad A = -3$

$10A + B = 0 \quad B = 30$

$y_p = -3x^2 + 30x$

$y = C_1 + C_2 e^{-x/5} - 3x^2 + 30x \longrightarrow 0 = C_1 + C_2 e^{(0)/5} - 3(0)^2 + 30(0)$

$y' = -\frac{1}{5}C_2 e^{-x/5} - 6x + 30 \longrightarrow -10 = -\frac{1}{5}C_2 e^{0/5} - 6(0) + 30$

$C_1 + C_2 = 0 \longrightarrow C_1 = -200$

$-\frac{1}{5}C_2 + 30 = -10$

$C_2 = 200$

$\therefore y = \underline{-200 + 200e^{-x/5} - 3x^2 + 30x}$

33 $y'' + 4y' + 5y = 35e^{-4x}, \quad y(0) = -3 \quad y'(0) = 1$

- $y'' + 4y' + 5y = 0$
- $m^2 + 4m + 5 = 0$
- $m_{12} = \dfrac{-4 \pm \sqrt{16 - 4(1)(5)}}{2(1)}$

$m_{12} = \dfrac{-4 \pm \sqrt{-4}}{2}$

$m_{12} = -2 \pm 2i$

- $y_1 = e^{-2x} \cos 2x$

$y_2 = e^{-2x} \operatorname{sen} 2x$

- $y_c = C_1 e^{-2x} \cos 2x + C_2 e^{-2x} \operatorname{sen} 2x$

$F(x) = 35e^{-4x}$

$y_p = Ae^{-4x}$

$y'_p = -4Ae^{-4x}$

$y''_p = 16Ae^{-4x}$

$16Ae^{-4x} - 16Ae^{-4x} + 5Ae^{-4x} = 35e^{-4x}$

$5A = 35 \quad A = 7$

$y_p = 7e^{-4x}$

$y = C_1 e^{-2x} \cos 2x + C_2 e^{-2x} \operatorname{sen} 2x + 7e^{-4x}$

$y' = -2C_1 e^{-2x} \cos 2x - 2C_1 e^{-2x} \operatorname{sen} 2x - 2C_2 e^{-2x} \operatorname{sen} 2x + 2C_2 e^{-2x} \cos 2x - 28e^{-4x}$

$-3 = C_1 e^{-2(0)} \cos 2(0) + \cancel{C_2 e^{-2(0)} \operatorname{sen} 2(0)}^{0} + 7e^{-4(0)}$

$1 = -2C_1 e^{-2(0)} \cos 2(0) - \cancel{2C_1 e^{-2(0)} \operatorname{sen} 2(0)}^{0} - \cancel{2C_2 e^{-2(0)} \operatorname{sen} 2(0)} + 2C_2 e^{-2(0)} \cos 2(0)^{1} - 28e^{-4(0)}$

$-3 = C_1 + 7 \rightarrow C_1 = -10$

$1 = -2C_1 + C_2 - 28$

$29 = -2(-10) + C_2 \qquad C_2 = 9$

$\therefore y = -10e^{-2x} \cos 2x + 9e^{-2x} \operatorname{sen} 2x + 7e^{-4x}$

33 $\dfrac{d^2x}{dt^2} + \omega^2 x = F_0 \operatorname{sen} \omega t \qquad x(0) = 0, \quad x'(0) = 0$

- $x'' + \omega^2 x = 0$
- $m^2 + \omega^2 = 0$
- $m_{12} = \pm\sqrt{-\omega^2} = \omega i$
- $y_1 = \operatorname{sen} \omega t \quad y_2 = \cos \omega t$
- $y_c = C_1 \operatorname{sen} \omega t + C_2 \cos \omega t$

$F(x) = F_0 \operatorname{sen} \omega t$

$y_p = At \operatorname{sen} \omega t + Bt \cos \omega t$

$y'_p = A \operatorname{sen} \omega t + At \cos \omega t + B \cos \omega t - Bt \operatorname{sen} \omega t$

$A \operatorname{sen} \omega t + At \cos \omega t + B \cos \omega t - Bt \operatorname{sen} \omega t + A\omega^2 t \operatorname{sen} \omega t + B\omega^2 \cos \omega t = F_0 \operatorname{sen} \omega t$

35) $y''' - 2y'' + y' = 2 - 24e^{x} + 40e^{5x} \quad y(0) = \frac{1}{2};\ y'(0) = \frac{5}{2} \quad y''(0) = -\frac{9}{2}$

- $y''' - 2y'' + y' = 0$
- $m^3 - 2m^2 + m = 0$
- $m(m^2 - 2m + 1)$

$m_1 = 0 \quad m_2 = 1 \quad m_3 = 1$

- $y_1 = 1 \quad y_2 = e^x \quad y_3 = xe^x$
- $y_c = C_1 + C_2e^x + C_3xe^x$

$F(x) = 2 - 24e^x + 40e^{5x}$

$y_p = Ax + B + Cx^2e^x + De^{5x}$

$y'_p = A + 2Cxe^x + Cx^2e^x + 5De^{5x}$

$y''_p = 2Ce^x + 4Cxe^x + Cx^2e^x + 25De^{5x}$

$y''' = 2Ce^x + 4Cxe^x + 4Cxe^x - 2Cxe^x + Cx^2e^x + 125De^{5x}$

⇒ Sustituimos

$$2Ce^x + \cancel{4Cxe^x} + \cancel{4Cxe^x} - \cancel{2Cxe^x} + \cancel{Cx^2e^x} + 125De^{5x} - \cancel{4Ce^x} - \cancel{8Cxe^x} - \cancel{2Cx^2e^x} - \cancel{50De^{5x}} + A$$
$$+ \cancel{2Cxe^x} + \cancel{Cx^2e^x} + 5De^{5x} = F(x)$$

$$A + 2Ce^x + 80De^{5x} = 2 - 24e^x + 40e^{5x}$$

$A = 2 \qquad 2C = -24 \qquad 80D = 40$

$\qquad\qquad C = 12 \qquad D = \frac{1}{2}$

$\therefore y_p = 2x - 12Cx^2e^x + \frac{1}{2}e^{5x}$

$y = C_1 + C_2e^x + C_3xe^x + 2x - 12x^2e^x + \frac{1}{2}e^{5x}$

$y' = C_2e^x + C_3e^x + C_3xe^x + 2 - 24xe^x - 12x^2e^x + \frac{5}{2}e^{5x}$

$y'' = C_2e^x + 2C_3e^x + C_3xe^x - 24e^x - 48xe^x - 12x^2e^x + \frac{25}{2}e^{5x}$

$y(0) = \frac{1}{2}\ ; \quad y(0) = \frac{5}{2} \quad y''(0) = -\frac{9}{2}$

$\Rightarrow$

$\frac{1}{2} = C_1 + C_2 + \frac{1}{2}$

$\frac{5}{2} = C_2 + C_3 + 2 + \frac{5}{2}$

$-\frac{9}{2} = C_2 + 2C_3 - 24 + \frac{25}{2}$

$C_1 + C_2 = 0$

$C_2 + C_3 = -2$

$C_2 + 2C_3 = 7$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & 1 & 2 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 9 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & 9 \end{array}\right) \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}$$

$C_1 = 11 \qquad C_2 = -11 \qquad C_3 = 9$

$\therefore y = 11 - 11e^x + 9xe^x + 2x - 12x^2e^x + \frac{1}{2}e^{5x}$

33. $f(t) = \text{senh } kt$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\text{senh } kt\}$$

$$= \frac{k}{s^2 - k^2}$$

35. $f(t) = e^t \text{ senh } t$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^t \text{ senh } t\}$$

$$= \mathcal{L}\{\text{senh } t\}\Big|_{s \to s-1}$$

$$= \frac{1}{s-1}\Big|_{s \to s-1} = \frac{1}{(s-1)^2 - 1}$$

37. $f(t) = \text{sen } 2t \cos 2t$

7.3 Zill Pag. 321.

1. $$\mathcal{L}\{t e^{10t}\} = \mathcal{L}\{t\}\Big|_{s \to s-10} = \frac{t}{s^2}\Big|_{s \to s-10} = \frac{t}{(s-10)^2}$$

3. $$\mathcal{L}\{t^3 e^{-2t}\} = \mathcal{L}\{t^3\}\Big|_{s \to s+2} = \frac{3!}{s^{3+1}}\Big|_{s \to s+2} = \frac{6}{(s+2)^4}$$

5. $$\mathcal{L}\{e^t \text{sen } 3t\} = \mathcal{L}\{\text{sen } 3t\}\Big|_{s \to s-1} = \frac{3}{s^2+9}\Big|_{s \to s-1} = \frac{3}{(s-1)^2+9}$$

7. $\mathcal{L}\{e^{5t}\operatorname{senh} 3t\} = \mathcal{L}\{\operatorname{senh} 3t\}\Big|_{s\to s-5} = \frac{3}{s^2-9}\Big|_{s-5} = \frac{3}{(s-5)^2-9}$ //

9. $\mathcal{L}\{t(e^t+e^{2t})^2\} = \mathcal{L}\{e^{2t}t + 2t\,e^{3t} + te^{4t}\}$

$= \mathcal{L}\{t\}\Big|_{s\to s-2} + 2\mathcal{L}\{t\}\Big|_{s\to s-3} + \mathcal{L}\{t\}\Big|_{s\to s-4}$

$= \frac{1}{(s-2)^2} + \frac{2}{(s-3)^2} + \frac{1}{(s-4)^2}$ //

11. $\mathcal{L}\{e^{-t}\operatorname{sen}^2 t\} = \mathcal{L}\{e^{-t}\frac{1}{2} - \frac{1}{2}e^{-t}\cos 2t\}$

$\frac{1}{2}\left[\frac{1}{s+1} - \mathcal{L}\{\cos 2t\}\Big|_{s\to s+1}\right] = \frac{1}{2}\left[\frac{1}{s+1} - \frac{s}{(s+1)^2+4}\right]$ //

13. $\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^3}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2!\cdot 1}{(s+2)^{2+1}}\right\} = \frac{1}{2}e^{-2t}t^2$ //

15. $\mathcal{L}^{-1}\left\{\frac{1}{s^2-6s+10}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s^2-6s+9)-9+10}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2+1}\right\}$

$= \operatorname{sen} t\, e^{3t}$ //

17. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+4-4+5}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+1}\right\}$

$= e^{-2t}\cos t$

19. $\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2}\right\}$

$s = A(s-1)+B$ $\qquad \mathcal{L}\{f(t)\} = \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\}$

$s = As - A + B$ $\qquad = e^t + te^t$ //

$B - A = 0$

$A = 1$

$B = 1$

$$\mathcal{L}^{-1}\left\{\frac{2s-1}{s^2(s+1)^3}\right\}$$

$$2s-1 = As(s+1)^3 + B(s+1)^3 + C(s^2)(s+1)^2 + D(s^2)(s+1) + E(s^2)$$

$$= -\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = 5\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - 4\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^3}\right\} + 5\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$$

$$= -t - 5e^{t} - 4te^{-t} + 5 - \frac{3}{2}t^2e^{-t}$$

23. $\mathcal{L}\{(t-1)\,\mathcal{U}(t-1)\} = e^{-s}\mathcal{L}\{t\} = e^{-s}\frac{1}{s^2}$

25. $\mathcal{L}\{t\,\mathcal{U}(t-2)\} = e^{-2s}\mathcal{L}\{t+2\} = e^{-2s}\left(\frac{1}{s^2} + \frac{2}{s}\right)$

27. $\mathcal{L}\{\cos 2t\,\mathcal{U}(t-\pi)\} = e^{-\pi s}\mathcal{L}\{\cos 2(t+\pi)\} = e^{-\pi s}\frac{s}{s^2+(t+\pi)^2}$

29. $\mathcal{L}\{(t-1)^3 e^{t-1}\,\mathcal{U}(t-1)\}$

31. $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\} = \frac{1}{2}(t-2)\,\mathcal{U}(t-2)$

$$\frac{e^{-2s}}{s^3}$$

7.2 Zill Pag. 311 1-33.

1. $\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^{2+1}}\right\}$

$\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^{2+1}}\right\} = \underline{\frac{1}{2}t^2}$ //

3. $\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{48}{s^5}\right\} = \underline{t - 2t^4}$ //

Donde $\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$

$\mathcal{L}^{-1}\left\{\frac{48}{s^5}\right\} = \frac{1}{4!}\mathcal{L}^{-1}\left\{\frac{4!}{s^{4+1}}\right\}$

$= \frac{48}{24}t^4$

5. $\mathcal{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{s^3+3s^2+3s+1}{s^4}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\}$

$= \underline{1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3}$ //

7. $\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s-2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$

$= \underline{t - 1 + e^{2t}}$ //

9. $\mathcal{L}^{-1}\left\{\frac{1}{4s+1}\right\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+\frac{1}{4}}\right\} = \underline{\frac{1}{4}e^{-1/4t}}$ //

11. $\mathcal{L}^{-1}\left\{\frac{5}{s^2+49}\right\} = \frac{5}{7}\mathcal{L}^{-1}\left\{\frac{7}{s^2+(7)^2}\right\} = \underline{\frac{5}{7}\operatorname{sen} 7t}$ //

13. $\mathcal{L}^{-1}\left\{\frac{4s}{4s^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+1/4}\right\} = \underline{\cos\frac{1}{4}t}$ //

15. $\mathcal{L}^{-1}\left\{\frac{1}{s^2-16}\right\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{4}{s^2-16}\right\} = \underline{\frac{1}{4}\operatorname{senh} 4t}$ //

17. $\mathcal{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\} = 2\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} - 6\left\{\frac{1}{s^2+9}\right\}$

$= \underline{2\cos 3t - \frac{6}{3}\operatorname{sen} 3t}$ //

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+3s}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{3s}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$\frac{1}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3} \qquad = \frac{1}{3} - \frac{1}{3}e^{-3t}$$

$$1 = A(s+3) + B(s)$$

$s=-3 \quad s=0$

$B = \frac{1}{3} \quad A = -\frac{1}{3}$

21. $$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s-3}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+3)(s-1)}\right\} = \frac{3}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$$

$$\frac{s}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1} = \frac{3}{4}(s+3) + \frac{1}{4}\left(\frac{1}{s-1}\right) \qquad = \frac{3}{4}e^{-3t} + \frac{1}{4}e^{t}$$

$$s = A(s-1) + B(s+3)$$

$s=-1 \quad B = -\frac{1}{2} \quad s=3 \quad A = \frac{3}{4}$

23. $$\mathcal{L}^{-1}\left\{\frac{0.9s}{(s-0.1)(s+0.2)}\right\} = 0.3e^{0.1t} + 0.6e^{-0.2t}$$

$$= \mathcal{L}^{-1}\left\{\frac{0.3}{s-0.1}\right\} + \mathcal{L}^{-1}\left\{\frac{0.6}{0.02}\right\}$$

$$(0.9s) = (s+0.2)A + B(s+0.1)$$

$s=-0.2 \qquad s=0.1$

$\frac{0.18}{0.3} = B = -0.6 \qquad 0.09 = (0.3)A$

$A = 0.3$

$$= 0.3e^{0.1t} + 0.06e^{-0.2t}$$

25. $$\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s-3)(s-6)}\right\}$$

$$s = A(s-3)(s-6) + B(s-2)(s-6) + C(s-2)(s-3)$$

$s=3 \quad B=-1 \qquad s=2 \quad A = \frac{1}{2}$

$s=6 \quad C = \frac{1}{2}$

$$\mathcal{L}\{f(t)\} = -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \mathcal{L}\left\{\frac{1}{s-3}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s-6}\right\}$$

$$= \frac{1}{2}e^{2t} - e^{3t} + \frac{1}{2}e^{6t}$$

27. $$\mathcal{L}^{-1}\left\{\frac{2s+4}{(s-2)(s^2+4s+3)}\right\}$$

$$\frac{2s+4}{(s-2)(s+1)(s+3)} = \frac{A}{(s-2)} + \frac{B}{(s+1)} + \frac{C}{(s+3)}$$

$$(2s+4) = A(s+1)(s+3) + B(s-2)(s+3) + C(s-2)(s+1)$$

$s=2 \quad \frac{8}{15} = A \qquad s=-1 \quad -\frac{2}{3} = B = -\frac{1}{3} \qquad s=-3 \quad -\frac{1}{5} = C$

$$\frac{8}{15}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} = \frac{8}{15}e^{2t} + \frac{1}{3}e^{t} - \frac{1}{5}e^{-3t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+4)}\right\}$$

$= A(s)(s^2+4) + B(s^2+4) + (Cs+D)(s^2)$

$1 = As^3 + 4As + Bs^2 + 4B + Cs^3 + Ds^2$

$1 = (A+D)s^3 + (B+D)s^2 + (4A)s + 4B$

$B = \frac{1}{4} \qquad B + D = 0$

$D = -\frac{1}{4}$

$\frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}$

$\mathcal{L}^{-1}\{f(t)\} = \frac{1}{4}t - \frac{1}{8}\operatorname{sen} 2t$

31. $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)(s+2)}\right\}$

$s = (As+B)(s+2) + C(s^2+4)$

$s = As^2 + 2As + Bs + 2B + Cs^2 + 4C$

$s = (A+C)s^2 + (2A+B)s + (2B+4C)$

$$\begin{aligned} 2A + B &= 1 \\ A + C &= 0 \\ 2B + 4C &= 0 \end{aligned} \quad \left(\begin{array}{ccc|c} 2 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & -2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & 0 & 1/2 \\ 1 & 0 & 0 & 1/4 \\ 0 & 0 & 1 & -1/4 \end{array}\right)$$

$\frac{1}{4}e^{-2t} + \frac{1}{4}\cos 2t + \frac{1}{4}\operatorname{sen}^2 t$

$\frac{1}{4}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\} - \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$

$= \frac{1}{4}\cos 2t + \frac{1}{4}\operatorname{sen} 2t - \frac{1}{4}e^{-2t}$

33. $\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)(s^2+4)}\right\}$

$1 = (As+B)(s^2+4) + (Cs+D)(s^2+1)$

$1 = As^3 + 4As + Bs^2 + 4B + Cs^3 + Cs + Ds^2 + D$

$1 = (A+C)s^3 + (B+D)s^2 + (4A+C) + (4B+D)$

$A + C = 0$

$B + D = 0$

$4A + C = 0$

$4B + D = 1$

$D = -\frac{1}{3} \quad B = \frac{1}{3} \quad C = 0 \quad A = 0$

$\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\} - \frac{1}{3}\left\{\frac{1}{s^2+1}\right\}$

$= \frac{1}{6}\operatorname{sen} 2t - \frac{1}{3}\operatorname{sen} t$

Cap. 7.1 Pag. 304 Zill.

1. $f(t)=\begin{cases}-1 \; ; \; 0 \le t < 1 \\ 1 \; ; \; t \ge 1\end{cases}$ $\quad g(t)=-1$, $h(t)=1$, $a=1$ $\Rightarrow$ $(-1)-(-1)u(t-1)+(1)u(t-1)$

$(-1)+u(t-1)+u(t-1)$

$(-1)+2u(t-1)$

$$\Rightarrow \mathcal{L}\{-1+2u(t-1)\} = -\mathcal{L}\{1\}+2\mathcal{L}\{u(t-1)\}$$

$$= -\frac{1}{s}+2\frac{e^{-s}}{s}$$

$$= \frac{2}{s}e^{-s}-\frac{1}{s}$$

2. $f(t)=\begin{cases}4 \; ; \; 0 \le t < 2 \\ 0 \; ; \; t \ge 2\end{cases}$ $\quad g(t)=4$, $h(t)=0$, $a=2$ $\Rightarrow$ $4-4u(t-2)+\underbrace{(0)u(t-2)}_{=0}$

$4-4u(t-2)$

$$= \mathcal{L}\{4-4u(t-2)\} = 4\mathcal{L}\{1\}-4\mathcal{L}\{u(t-2)\}$$

$$= \frac{4}{s}-\frac{4e^{-2s}}{s}$$

3. $f(t)=\begin{cases}t \; ; \; 0 \le t < 1 \\ 1 \; ; \; t \ge 1\end{cases}$ $\quad g(t)=t$, $h(t)=1$, $a=1$ $\Rightarrow$ $t-tu(t-1)+(1)u(t-1)$

$= t-tu(t-1)+u(t-1)$

$$\mathcal{L}\{t-tu(t-1)+u(t-1)\} = \mathcal{L}\{t\}-\mathcal{L}\{t\,u(t-1)\}+\mathcal{L}\{u(t-1)\}$$

$$= \frac{1}{s^2}-e^{-s}\mathcal{L}\{t+1\}+\frac{e^{-s}}{s}$$

$$= \frac{1}{s^2}-\frac{e^{-s}}{s^2}-\cancel{\frac{e^{-s}}{s}}+\cancel{\frac{e^{-s}}{s}}$$

$$= \frac{1}{s^2}-\frac{e^{-s}}{s^2}$$

$$f(t) = \begin{cases} 2t+1 & ; \quad 0 \le t < 1 \\ 0 & ; \quad t \ge 1 \end{cases} \qquad \begin{aligned} g(t) &= 2t+1 \\ h(t) &= 0 \\ a &= 1 \end{aligned} \qquad \begin{aligned} &(2t+1) - (2t+1)u(t-1) + 0u(t-1) \\ \Rightarrow\ &(2t+1) - (2t+1)u(t-1) \end{aligned}$$

$$\mathcal{L}\{(2t+1) - (2t+1)u(t-1)\} = \frac{2}{s^2} + \frac{1}{s} - e^{-s}\mathcal{L}\{2(t+1)+1\}$$

$$= \frac{2}{s^2} + \frac{1}{s} - \frac{2e^{-s}}{s^2} - \frac{3e^{-s}}{s}$$

5.- $f(t) = \begin{cases} \text{sen}\,t & ; \quad 0 \le t < \pi \\ 0 & ; \quad t \ge \pi \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^{\pi} \text{sen}\,t\, e^{-st}\,dt$$

$u = \text{sen}\,t \qquad dv = e^{-st}dt \qquad u = \cos t \qquad dv = e^{-st}\,dt$

$du = \cos t\,dt \qquad v = \frac{1}{s}e^{-st} \qquad du = -\text{sen}\,t\,dt \qquad v = -\frac{1}{s}e^{-st}$

$$= -\frac{1}{s}\,\text{sen}\,t\, e^{-st}\Big|_0^{\pi} + \frac{1}{s}\int_0^{\pi}\cos t\, e^{-st}dt = -\frac{\cos t\, e^{-st}}{s^2}\Big|_0^{\pi} - \int_0^{\pi}\text{sen}\,t\, e^{-st}\,dt\,\frac{1}{s^2}$$

$$\frac{1}{s^2} + 1 = \frac{1+s^2}{s^2} \Rightarrow -\frac{\cos t\, e^{-st}}{s}\Big|_0^{\pi} = \left(\frac{-e^{-s\pi} + e^{-s\pi}}{s^2}\right)\left(\frac{s^2}{1+s^2}\right)$$

$$= \frac{1+e^{-s\pi}}{1+s^2}$$

6. $f(t) = \begin{cases} 0 & ; \quad 0 \le t < \frac{\pi}{2} \\ \cos t & ; \quad t \ge \pi/2 \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_{\pi/2}^{\infty} e^{-st}\cos t\,dt = \lim_{k\to\infty}\int_{\pi/2}^{k} e^{-st}\cos t\,dt$$

$u = \cos t \qquad dv = e^{-st}dt$

$du = -\text{sen}\,t\,dt \qquad v = -\frac{1}{s}e^{-st}$

$u = \text{sen}\,t \qquad dv = e^{-st}dt$

$du = \cos t \qquad v = -\frac{1}{s}e^{-st}$

$$\int_{\pi/2}^{\infty} e^{-st}\cos t\,dt = -\frac{1}{s}\cos t\, e^{-st} - \frac{1}{s}\int_{\pi/2}^{\infty}\text{sen}\,t\, e^{-st}dt = -\frac{1}{s}\cos t\, e^{-st}\Big|_{\pi/2}^{\infty} + \frac{1}{s^2}\,\text{sen}\,t$$

$$e^{-st}\Big|_{\pi/2}^{\infty} - \frac{1}{s^2}\int_{\pi/2}^{\infty}\cos t\, e^{-st}dt$$

$$= -\frac{\cos t\, e^{-st} + \text{sen}\,t\, e^{-st}}{s^2}\left(\frac{s^2}{s^2+1}\right) = -\frac{s\cos t\, e^{-st} + \text{sen}\,t\, e^{-st}}{s^2+1}\Big|_{\pi/2}^{\infty} = \frac{e^{-\frac{s}{2}\pi}}{s^2+1}$$

$f(t) = e^{t+7}$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{t+7}\} = \mathcal{L}\{(e^t)(e^7)\}$

$= e^7 \mathcal{L}\{e^t\}$

$= \dfrac{e^7}{s-1}$

12. $f(t) = e^{-2t-5}$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-2t-5}\}$

$= \mathcal{L}\{(e^{-2t})(e^{-5})\}$

$= \dfrac{e^{-5}}{s-(-2)} = \dfrac{e^{-5}}{s+2}$

13. $f(t) = te^{4t}$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{te^{4t}\} = \mathcal{L}\{t\}\Big|_{s \to s-4}$

$a = 4$

$= \dfrac{1}{s^2}\Big|_{s \to s-4} = \dfrac{1}{(s-4)^2}$

14. $f(t) = t^2 e^{3t}$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2 e^{3t}\} = \mathcal{L}\{t^2\}\Big|_{s \to s-3}$

$a = 3$

$= \dfrac{2!}{s^{2+1}}\Big|_{s \to s-3} = \dfrac{2}{(s-3)^3}$

15. $f(t) = e^{-t} \operatorname{sen} t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-t} \operatorname{sen} t\}$

$= \mathcal{L}\{\operatorname{sen} t\}\Big|_{s \to s-(-1)} = \dfrac{1}{s^2+1}\Big|_{s \to s+1}$

$= \dfrac{1}{(s+1)^2+1}$

16. $f(t) = e^t \cos t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^t \cos t\}$

$= \mathcal{L}\{\cos t\}\Big|_{s \to s-1}$

$= \dfrac{s}{s^2+1}\Big|_{s \to s-1} = \dfrac{s}{(s-1)^2+1}$

17. $f(t) = t \cos t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{t \cos t\} = \dfrac{s^2-a}{(s^2+b)^2}$

$= \dfrac{s^2-1}{(s^2+1)^2}$

18. $f(t) = t \operatorname{sen} t$

$f(t) = 4t - 10$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{4t - 10\}$

$= 4\mathcal{L}\{t\} - 10\mathcal{L}\{1\}$

$= \frac{4}{s^2} - \frac{10}{s}$

23. $f(t) = t^2 + 6t - 3$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2 + 6t - 3\}$

$= \mathcal{L}\{t^2\} + 6\mathcal{L}\{t\} - 3\mathcal{L}\{1\}$

$= \frac{2!}{s^{2+1}} + \frac{6}{t^2} - \frac{3}{s}$

25. $f(t) = (t+1)^3$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{(t+1)^3\}$

$= \mathcal{L}\{t^3 + 3t^2 + 3t + 1\}$

$= \mathcal{L}\{t^3\} + 3\mathcal{L}\{t^2\} + 3\mathcal{L}\{t\} + \mathcal{L}\{1\}$

$= \frac{3!}{s^{3+1}} + \frac{3 \cdot 2!}{s^{2+1}} + \frac{3}{s^2} + \frac{1}{s}$

$= \frac{6}{s^4} + \frac{6}{s^3} + \frac{3}{s^2} + \frac{1}{s}$

27. $f(t) = 1 + e^{4t}$

$\mathcal{L}\{1 + e^{4t}\} = \mathcal{L}\{1\} + \mathcal{L}\{e^{4t}\}$

$= \frac{1}{s} + \frac{1}{s-4}$

29. $f(t) = (1 + e^{2t})^2$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{(1 + e^{2t})^2\}$

$= \mathcal{L}\{1 + 2e^{2t} + e^{4t}\}$

$= \mathcal{L}\{1\} + 2\mathcal{L}\{e^{2t}\} + \mathcal{L}\{e^{4t}\}$

$= \frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}$

31. $f(t) = 4t^2 - 5\,\text{sen}\,3t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{4t^2 - 5\,\text{sen}\,3t\}$

$= 4\mathcal{L}\{t^2\} - 5\mathcal{L}\{\text{sen}\,3t\}$

$= \frac{4(2!)}{s^{2+1}} - \frac{5(3)}{s^2+9}$

$= \frac{8}{s^3} - \frac{15}{s^2+9}$

35. $f(t) = e^t\,\text{senh}\,t$

$\mathcal{L}\{e^t\,\text{senh}\,t\} = \frac{s-1}{(s-1)^2 - 1}$

37. $f(t) = \text{sen}\,2t\,\cos 2t$

$\mathcal{L}\{\text{sen}\,2t\cos 2t\} = \int_0^\infty e^{-st}\,\text{sen}\,2t\cos 2t\,dt$

$= \frac{a^2 + b^2 + s^2}{a^4 + 2a^2(s^2 - b^2) + (b^2 + s^2)^2}$

$\frac{-a^2 - b^2 + s^2}{a^4 + 2a^2(s^2 - b^2) + (b^2 + s^2)^2}$

11. $f(t) = e^{t+7}$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{t+7}\}$$
$$= \mathcal{L}\{(e^{t})(e^{7})\}$$
$$= e^{7}\mathcal{L}\{e^{t}\}$$
$$= \frac{e^{7}}{s-1}$$

12. $f(t) = e^{-2t-5}$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-2t-5}\}$$
$$= \mathcal{L}\{(e^{-2t})(e^{-5})\}$$
$$= \frac{e^{-5}}{s+2}$$

13. $f(t) = te^{4t}$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{te^{4t}\}$$
$$= \mathcal{L}\{t\}\Big|_{s\to s-4}$$
$$= \frac{1}{s^2}\Big|_{s\to s-4} = \frac{1}{(s-4)^2}$$

15. $f(t) = e^{-t}\operatorname{sen} t$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-t}\operatorname{sen} t\}$$
$$= \mathcal{L}\{\operatorname{sen} t\}\Big|_{s\to s+1} = \frac{1}{s^2+1}\Big|_{s\to s+1}$$
$$= \frac{1}{(s+1)^2+1}$$

17. $f(t) = t \cos t$

$f(t) = t \rightarrow f(\tau) = \tau$

$g(t) = \operatorname{sen} t \rightarrow g(t-\tau) = \operatorname{sen}(t-\tau)$

$$t * \operatorname{sen} t = \int_0^t \tau \operatorname{sen}(t-\tau)\, d\tau$$

19. $f(t) = 2t^4$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2t^4\}$$
$$= 2\mathcal{L}\{t^4\}$$
$$= \frac{2\,4!}{s^{4+1}}$$
$$= \frac{48}{s^5}$$

Zill 7.4 3 - 27 Pag. 331

$$\mathcal{L}\{y'' + 3y'\} = s^2 Y(s) + 3sY(s)$$
$$= Y(s)\,(s^2 + 3s)$$

$$\mathcal{L}\{y'' - 4y' + 5y\} = s^2 Y(s) - 4sY(s) + 5Y(s)$$
$$= Y(s)\,(s^2 - 4 + 5)$$

$$\mathcal{L}\{y'' - 2y' + y = 0\}$$
$$s^2 Y(s) - 2sY(s) + Y(s) = 0$$
$$Y(s)\,(s^2 - 2s + 1) = 0$$

$$\mathcal{L}\{y'' + y = 1\}$$
$$s^2 Y(s) + Y(s) = \frac{1}{s}$$
$$Y(s)\,(s^2 + 1) = \frac{1}{s}$$
$$Y(s) = \frac{1}{(s^2+1)s}$$

$$\mathcal{L}\left\{\int_0^t e^{-\tau}\cos\tau\, d\tau\right\}$$
$$\mathcal{L}\{f(t)\} - \mathcal{L}\{e^{-t}\cos t\} \over s = \frac{s+1}{(s+1)^2+1}$$

$$\mathcal{L}\left\{\int_0^t \tau e^{(t-\tau)}\, d\tau\right\} = \mathcal{L}\{t * e^t\}$$
$$\mathcal{L}\{t\}\cdot\mathcal{L}\{e^t\} = \frac{1}{s}\cdot\frac{1}{s-1} = \frac{1}{s\cdot(s-1)}$$

$$\mathcal{L}\left\{t\int_0^t \operatorname{sen}\tau\, d\tau\right\} = \mathcal{L}\{-t\cos t - t\}$$
$$= \mathcal{L}\{t\cos t\} - \mathcal{L}\{t\} = (-1)\frac{d}{ds}\mathcal{L}\{\cos t\} = (-1)\frac{d}{ds}\left(\frac{s}{s^2+1}\right)$$
$$= \left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right) = \frac{s^2-1}{(s^2+1)^2} - \frac{1}{s^2} = \frac{3s+1}{s^2(s^2+1)^2}$$

$\mathcal{L}\{1 * t^3\} = \mathcal{L}\{1\} \cdot \mathcal{L}\{t^3\} = \frac{1}{s} \cdot \frac{16}{s^4} = \frac{6}{s^5}$

- $\mathcal{L}\{e^{-t} * e^t \cos t\} = \mathcal{L}\{e^{-t}\} \cdot \mathcal{L}\{e^t \cos t\} =$

$\frac{1}{s-1} \cdot \frac{(s-1}{(s-1)^2+1} = \frac{s-1}{(s-1)^3+(s-1)}$

- $\mathcal{L}\{t^2 * t^4\} = \mathcal{L}\{t^2\} \cdot \mathcal{L}\{t^4\}$

$= \frac{2}{s^3} \cdot \frac{24}{s^5} = \frac{48}{s^8}$

- $\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$

$1 = A(s+1) + Bs$

$s=-1 \quad s=0$

$B=-1 \quad A=1$

$= 1 - e^{-t}$

$\mathcal{L}\{\operatorname{senh} at\}$

$$\mathcal{L}\{\operatorname{senh} at\} = \int_0^\infty e^{-st}\operatorname{senh} at\, dt = \int_0^\infty e^{-st}\left(\frac{e^{at}-e^{-at}}{2}\right)dt$$

$$= \frac{1}{2}\int_0^\infty e^{-(s-a)t}\,dt - \frac{1}{2}\int_0^\infty e^{-(s+a)t}\,dt = \frac{1}{2}\lim_{b\to\infty}\left(\int_0^b e^{-(s-a)t}dt - \int_0^b e^{-(s+a)t}dt\right)$$

$$= \frac{1}{2}\lim_{b\to\infty}\left(-\frac{e^{-(s-a)t}}{s-a} + \frac{e^{-(s+a)t}}{s+a}\right)\Big|_0^b$$

$$= \frac{1}{2}\lim_{b\to\infty}\left(-\frac{e^{-(s-a)b}\to 0}{s-a} + \frac{e^{-(s+a)b}\to 0}{s+a} + \frac{e^{-(s-a)(0)}\to 1}{s-a} - \frac{e^{-(s+a)(0)}\to 1}{s+a}\right)$$

$$= \frac{1}{2}\left(\frac{s+a-(s-a)}{s^2-a^2}\right) = \frac{1}{2}\left(\frac{2a}{s^2-a^2}\right) = \frac{a}{s^2-a^2}$$

$\mathcal{L}\{e^{bt}\cosh at\}$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\left\{e^{bt}\left(\frac{e^{at}+e^{-at}}{2}\right)\right\} = \mathcal{L}\left\{\frac{e^{(b+a)t}+e^{-(a-b)t}}{2}\right\}$$

$$= \frac{1}{2}\int_0^\infty e^{-st}\left(e^{(b+a)t}+e^{-(a-b)t}\right)dt = \frac{1}{2}\left(\int_0^\infty e^{-(s-a-b)}dt + \int_0^\infty e^{-(s+a-b)t}dt\right)$$

$$= \frac{1}{2}\lim_{c\to\infty}\int_0^c e^{-(s-a-b)t}dt + \int_0^c e^{-(s+a-b)t}dt = \frac{1}{2}\lim_{c\to\infty}\left(\frac{-e^{-(s-a-b)t}}{s-a-b} - \frac{e^{-(s+a-b)t}}{s+a-b}\right)\Big|_0^c$$

$$= \frac{1}{2}\lim_{c\to\infty}\left(-\frac{e^{-(s-a-b)c}}{s-a-b} - \frac{e^{-(s+a-b)c}}{s+a-b} + \frac{e^{-(s-a-b)0}}{s-a-b} + \frac{e^{-(s-a-b)(0)}}{s+a-b}\right)$$

$$= \frac{1}{2}\left(\frac{1}{s-a-b} + \frac{1}{s+a-b}\right) = \frac{1}{2}\left(\frac{s+a-b-(s-a-b)}{(s-b)^2-a^2}\right) = \frac{\cancel{2}a}{\cancel{2}(s-b)^2-a^2}$$

$$\mathcal{L}\{f(t)\} = \frac{a}{(s-b)^2-a^2}$$

$$\mathcal{L}\{\cos at\} = \int_0^\infty e^{-st}\cos at\,dt = \lim_{k\to\infty}\int_0^k e^{-st}\,\text{sen}\,at\,dt$$

$u = \cos at \qquad dv = e^{-st}dt$

$du = -a\,\text{sen}\,at \qquad v = -\frac{e^{-st}}{s}$

$$-\frac{e^{-st}\cos at}{s}\Big|_0^k - \frac{a}{s}\int_0^k e^{-st}\,\text{sen}\,at\,dt \qquad \begin{matrix} u = \text{sen}\,at \\ du = a\cos at \end{matrix} \qquad \begin{matrix} dv = e^{-st}dt \\ v = \frac{-e^{-st}}{s} \end{matrix}$$

$$\frac{-e^{-st}\cos at}{s}\Big|_0^k - \frac{a}{s}\left[\frac{-e^{-st}\,\text{sen}\,at}{s}\Big|_0^k + \frac{a}{s}\int e^{-st}\cos at\right.$$

$$\frac{-e^{-st}\cos at}{s}\Big|_0^k + \frac{a e^{-st}\,\text{sen}\,at}{s^2}\Big|_0^k - \frac{a^2}{s^2}\int e^{-st}\cos at$$

$$\mathcal{L}\{\cos at\}\left(1 + \frac{a^2}{s^2}\right) = \lim_{k\to\infty}\left[-\frac{e^{-st}\cos at}{s} + \frac{a e^{-st}\,\text{sen}\,at}{s^2}\right]\Big|_0^k$$

$$\mathcal{L}\{\cos at\}\left(1 + \frac{a^2}{s^2}\right) = \lim_{k\to\infty}\left[\cancel{\frac{-e^{-sk}\cos at}{s}}^{\,0} + \cancel{\frac{a e^{-sk}\,\text{sen}\,at}{s^2}}^{\,0} + \frac{1}{s}\right]$$

$$\mathcal{L}\{\cos at\}\left[\frac{s^2+a^2}{s^2}\right] = \frac{a}{s} \Longrightarrow \mathcal{L}\{\cos at\} = \frac{\frac{1}{s}}{\frac{s^2+a^2}{s^2}}$$

$$\therefore\ \mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$$

Por Laplace.

$$\frac{di}{dt} + 2i = 10e^{-2t} \quad ; \quad i(0) = 0$$

$$\mathcal{L}\{i' + 2i = 10e^{-2t}\}$$

$$= sY(s) + 2Y(s) = \mathcal{L}\{10e^{-2t}\}$$

$$= Y(s)(s+2) = \frac{10}{s+2}$$

Despejando...

$$Y(s) = \frac{10}{(s+2)^2}$$

$$\mathcal{L}^{-1}\left\{Y(s) = \frac{10}{(s+2)^2}\right\}$$

$$= Y(t) = 10te^{-2t} \quad \forall t \geq 0$$

Rodriguez Aguilar Kathia

x

$$\frac{dI}{dt} + 2I = 10e^{-2t}; \quad I(0) = 0$$

① $P(t) = 2$

② FACTOR INTEGRANTE

$$\mu(t) = e^{\int P(t)\,dt}$$
$$= e^{2t}$$

$\therefore \mu(t) = e^{2t} \ldots (2)$

$$e^{2t}\left[\frac{dI}{dt} + 2I = 10e^{-2t}\right]$$

$$e^{2t}\frac{dI}{dt} + 2e^{2t}I = 10$$

③ FACTORIZAR.

$$\frac{d}{dt}\left[e^{2t}\cdot I\right] = 10$$

INTEGRANDO

$$\int \frac{d}{dt}\left[e^{2t}\cdot I\right] = 10\int dt$$

$$e^{2t}\cdot I = 10t + C$$

En $I(0) = 0$

$$e^{2(0)}\cdot(0) = 10(0) + C$$

$$\therefore C = 0$$

$$e^{2t}\cdot I = 10t$$

$$\therefore I(t) = 10te^{-2t}; \ \forall t \geq 0$$

$$\frac{dI}{dt} + 2I = 10e^{-2t}; \quad I(0) = 0$$

① Ec. HOMOGENEA

$$\frac{dI}{dt} + 2I = 0 \ldots (2)$$

② Ec. CARACTERÍSTICA DE (2)

$$m + 2 = 0$$

③ RAIZ $m_1 = -2$

④ $I_1 = e^{-2t}$

$$y_c = c_1e^{-2t}$$

Proponemos

$$I_p = U_1 I_1$$

Donde $U_1 \int \frac{Q(t)}{I_1(t)}\,dt$

$$I_p = I_1 \int \frac{Q(t)}{I_1(t)}\,dt$$

$$= e^{-2t}\int \frac{10\cancel{e^{-2t}}}{\cancel{e^{-2t}}}\,dt$$

$$= e^{-2t}\int 10\,dt$$

$= e^{-2t}(10t)$ Sol. part. (1)

$\therefore I = I_c + I_p = c_1e^{-2t} + 10te^{-2t}$

Sol. gral. (1)

Pero $I(0) = 0$

$$0 = c_1e^{-2(0)} + 10(0)e^{-2(0)}$$

$$0 = c_1$$

$$\therefore I(t) = 10te^{-2t}; \ \forall t \geq 0$$

$$\frac{d^2x}{dt^2} + w^2 x = \cancel{F_0 Sen\,wt}$$
$$F_0\, Sen(Wt) \quad \ldots (1)$$

$x'' + w^2 x = F_0\, sen(Wt)$

• Ec. homo

$x'' + w^2 x = 0 \quad \ldots (2)$

• Ec. carac

$m^2 + w^2 = 0$

• Raíces

$m_{1,2} = \pm\sqrt{-4w^2}$

$= \pm wi$

• Sols I_1

$y_1 = sen\,Wt \quad y_2 = \cos wt$

• Sol. gral. de (2)

$y_c = C_1 sen\,wt + C_2 \cos wt$

$F(x) = F_0\, sen\, w\, t$

Sol prop.

$y_p = (A\, sen\, Wt + B \cos Wt)t$

$\Rightarrow y_p = At\, sen\, Wt + Bt \cos Wt$

$y'_p = A\, sen Wt + WAt \cos Wt + B\cos Wt - WBt\, sen\, Wt$

$y''_p = \underline{WA\cos Wt} + \underline{WA\cos Wt} - W^2 At\, sen Wt - \underline{WB\, sen\, Wt} - WB\, sen Wt$

$- W^2 Bt \cos Wt$

$= 2WA\cos Wt - 2WB\, sen\, Wt - w^2 At\, sen Wt - w^2 Bt \cos Wt = F_0\, sen\, Wt$

Sust. en (1)

$2WA\cos Wt - 2WB\, sen\, Wt \cancel{- w^2 At\, sen\, Wt} \cancel{- w^2 Bt\cos Wt} \cancel{+ w^2 At\, sen Wt} \cancel{+ w^2 Bt \cos Wt} = F(t)$

$2WA\cos(Wt) - 2WB\, sen(Wt) = F_0\, sen(wt)$

$2WA = 0 \qquad -2WB = F_0$

$A = 0 \qquad B = -\frac{F_0}{2W}$

$y_p = -\frac{F_0}{2W} t\cos Wt$

$y = y_c + y_p = C_1 sen\, Wt + C_2 \cos Wt - \frac{F_0}{2W} t\cos Wt$

$x(0) = 0$

$x'(0) = 0$

$y = C_1 \operatorname{sen} \omega t + C_2 \cos \omega t - \frac{F_0}{2\omega} t \cos \omega t$

$y' = \omega C_1 \cos \omega t - \omega C_2 \operatorname{sen} \omega t - \frac{F_0}{2\omega} \cos \omega t + \frac{F_0}{2} x \operatorname{sen} \omega t$

$x(0) = 0 = x'(0)$

$0 = C_2$

$0 = \omega C_1 - \frac{F_0}{2\omega} \qquad C = \frac{F_0}{2\omega^2}$

$x = \frac{F_0}{2\omega^2} \operatorname{sen} \omega t - \frac{F_0}{2\omega} t \cos \omega t$

3.3

$L = 0.5h$

$R = 6.0\ \Omega$

$C = 0.02F$

$F(t) = 24 \operatorname{sen} 10t \quad \forall t \geq 0$

Si $Q(0) = 0$

$i(0) = 0$

$Q = Q(t)$

$i = i(t)$

$$L \frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E(t)$$

$$0.5Q'' + 6.0Q' + 50Q = 24 \operatorname{sen} 10t \quad \ldots (1)$$

- Ec homo.

$0.5Q'' + 6Q' + 50Q = 0 \quad \ldots (2)$

- Ec. caract

$0.5m^2 + 6m + 50 = 0$

- Raices

$$m_{1,2} = \frac{-6 \pm \sqrt{36 - 4(0.5)(50)}}{\cancel{2(0.5)} = 1}$$

$= -6 \pm \sqrt{-64}$

$= -6 \pm 8i$

- Sols. l.i

$y_1 = e^{-6x}\cos 8x$

$y_2 = e^{-6x}\operatorname{sen} 8x$

Sol. gral de (2)

$y_c = C_1 e^{-6x}\cos 8x + C_2 e^{-6x}\operatorname{sen} 8x$

$F(x) = 24 \operatorname{sen} 10t$

Sol. Prop.

$y_p = A\operatorname{sen} 10t + B\cos 10t$

$y'_p = 10A\cos 10t - 10B\operatorname{sen} 10t$

$y''_p = -100A\operatorname{sen} 10t - 100B\cos 10t$

Sust. en (1)

$0.5(-100A\operatorname{sen}10t - 100B\cos 10t) + 6(10A\cos 10t - 10B\operatorname{sen}10t) + 50A\operatorname{sen}10t + 50B\cos 10t = F(x)$

$\cancel{-50A\operatorname{sen}10t} \cancel{- 50B\cos 10t} + 60A\cos 10t - 60\operatorname{sen}10t \cancel{+ 50A\operatorname{sen}10t} \cancel{+ 50B\cos 10t} = 24\operatorname{sen}10t$

$60A\cos 10t - 60B\operatorname{sen}10t = 24\operatorname{sen}10t$

$60A = 0 \qquad A = 0$

$-60B = 24$

$B = -0.4 \qquad \therefore y_p = -0.4\cos 10t$

$y = y_c + y_p = C_1 e^{-6x}\cos 8x + C_2 e^{-6x}\operatorname{sen} 8x - 0.4\cos 10t$

2.5

¿ $Q(0) = 0$

$i(0) = 0$?

Usando la definición determine la Transformada de Laplace de la función $f(t) = \begin{cases} 0 ; & 0 \le t < \pi/2 \\ \cos t ; & t \ge \pi/2 \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^{\pi/2} \cos t \, e^{-st} \, dt \qquad \begin{array}{ll} u = \cos t & du = -\operatorname{sen} t \, dt \\ dv = e^{-st} dt & v = -\dfrac{e^{-st}}{s} \end{array}$$

$$\mathcal{L}\{f(t)\} = -\frac{e^{-st}\cos(t)}{s} - \frac{1}{s}\int \operatorname{sen} t \, e^{-st} \, dt \qquad \begin{array}{ll} u = \operatorname{sen}(t) & dv = e^{-st} dt \\ du = \cos(t)\, dt & v = -\frac{1}{s} e^{-st} \end{array}$$

$$\Rightarrow \mathcal{L}\{f(t)\} = -\frac{e^{-st}\cos t}{s} - \frac{1}{s}\left[-\frac{e^{-st}\operatorname{sen} t}{s} + \frac{1}{s}\int e^{-st}\cos(t)\, dt\right]$$

$$\int_0^{\pi/2} \cos t \, e^{-st} dt = -\frac{e^{-st}\cos t}{s} + \frac{e^{-st}\operatorname{sen} t}{s^2} - \frac{1}{s^2}\int_0^{\pi/2} e^{-st}\cos(t)\, dt$$

$$\int_0^{\pi/2} \cos t \, e^{-st} dt + \frac{1}{s^2}\int_0^{\pi/2} e^{-st}\cos(t) = -\frac{e^{-st}\cos t}{s} + \frac{e^{-st}\operatorname{sen} t}{s^2}$$

15

$$\int_0^{\pi/2} \cos t \, e^{-st} dt \left(\frac{s^2+1}{s^2}\right) = -\frac{e^{-st}\cos t}{s} + \frac{e^{-st}\operatorname{sen} t}{s^2}$$

$$\int_0^{\pi/2} \cos t \, e^{-st} dt = \left[-\frac{s e^{-st}\cos t}{s^2+1} + \frac{e^{-st}\operatorname{sen} t}{s^2+1}\right]_0^{\pi/2}$$

$$= -\frac{s}{s^2+1}\cancel{\cos\left(\tfrac{\pi}{2}\right)} e^{-\pi/2 s} + \frac{1}{s^2+1}e^{-s\pi/2}\operatorname{sen}\left(\tfrac{\pi}{2}\right) + \frac{s}{s^2+1}\cos(0) e^{-s(0)} - \frac{1}{s^2+1}e^{-s(0)}\operatorname{sen}(0)$$

$$= \frac{s}{s^2+1} - \frac{s}{s^2+1}e^{-\pi/2 s} = \frac{s}{s^2+1}\left(1 - e^{-\pi/2 s}\right)$$

$$\therefore \mathcal{L}\{F(t)\} = \cancel{\frac{s}{s^2+1}\left(1 - e^{-\pi/2 s}\right)}$$

$$= -\frac{e^{-\frac{\pi}{2}s}}{s^2+1}$$

2. Evalúe $\mathcal{L}^{-1}\left\{\dfrac{s+1}{(s^2+2s+2)^2}\right\}$

$$\mathcal{L}^{-1}\left\{\frac{s+1}{(s^2+2s+2)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s+1}{(s^2+2s+1+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s+1}{((s+1)^2+1)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\} \cdot \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\} = e^{-t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} \cdot \cancel{e}^{t}\left\{\frac{1}{s^2+1}\right\}$$

$$= e^{-2t}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} \cdot \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}$$

>> De donde: $\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} \cdot \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} = \int_0^t \cos t \ \operatorname{sen}(t-\tau)\, d\tau$

$$= \frac{1}{2}\int \operatorname{sen} t - \operatorname{sen}(2\tau - t)\, d\tau$$

$$u = 2\tau - t \qquad du = 2\,d\tau$$

$$= \frac{1}{2}\operatorname{sen} t\,(\tau)\Big|_0^t + \frac{1}{4}\cos(2\tau - t)\Big|_0^t$$

$$= \frac{1}{2}t\operatorname{sen} t + \frac{1}{4}\cos t - \frac{1}{4}\cos(-t)$$

$$= \frac{1}{2}t\operatorname{sen} t + \cancel{\frac{1}{4}\cos t} - \cancel{\frac{1}{4}\cos t}$$

$$= \frac{1}{2}t\operatorname{sen} t$$

$$\therefore \ \mathcal{L}^{-1}\left\{\frac{s+1}{(s^2+2s+2)^2}\right\} = \frac{1}{2}e^{-2t}\,t\operatorname{sen} t$$

2

(5) Use la transformada de Laplace para resolver

$$y'' + y = \operatorname{sen} t, \quad y(0) = 0, \quad y'(0) = 0$$

$$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \mathcal{L}\{\operatorname{sen} t\}$$

>> Donde:

$$\mathcal{L}\{y''\} = s^2 Y(s) - s\,Y(0) - y'(0) = s^2 Y(s)$$

$$\mathcal{L}\{y\} = Y(s)$$

$$\mathcal{L}\{\operatorname{sen} t\} = \frac{1}{s^2+1}$$

>> Sustituyendo

$$s^2 Y(s) + Y(s) = \frac{1}{s^2+1}$$

$$Y(s)(s^2+1) = \frac{1}{s^2+1}$$

$$Y(s) = \frac{1}{(s^2+1)^2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}}_{\operatorname{sen} t_\tau} \cdot \underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}}_{\operatorname{sen} t} \quad t = t - \tau$$

$$*$$

$$\Rightarrow \mathcal{L}^{-1}\{Y(s)\} = \int \operatorname{sen} \tau \cdot \operatorname{sen}(t-\tau)\, d\tau$$

$$\operatorname{sen}\tau \cdot \operatorname{sen}(t-\tau) = -\frac{1}{2}\int [\cos t - \cos(2\tau - t)]\, d\tau$$

$$\Rightarrow \mathcal{L}^{-1}\{Y(s)\} = -\frac{1}{2}\left[\int_0^t \cos t \, d\tau - \int_0^t \cos(2\tau - t)\, d\tau\right]$$

$$= -\frac{1}{2}\left[\cos(t)\tau\Big|_0^t - \frac{1}{2}\operatorname{sen}(2\tau - t)\right]_0^t$$

$$= -\frac{1}{2}\cos(t)\tau\Big|_0^t + \frac{1}{4}\operatorname{sen}(2\tau - t)\Big|_0^t$$

$$= -\frac{1}{2}\cos(t)t + \frac{1}{4}\operatorname{sen}(t) + \frac{1}{2}\cos(t)(0) - \frac{1}{4}\operatorname{sen}(-t)$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t)$$

$$\Rightarrow \quad y(t) = -\frac{1}{2}t\cos(t) + \frac{1}{2}\operatorname{sen} t$$

$$= \frac{1}{2}(\operatorname{sen} t - t\cos t)$$

1) Use la transformada de Laplace para resolver:

$$y' + y = f(t); \quad y(0) = 0 \qquad f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t, & t \ge 1 \end{cases}$$

$f(t) = (0) - (0)\,u(t-1) + t\,u(t-1)$

$f(t) = t\ u(t-1)$

$g(t) = 0$
$h(t) = t$
$a = 1$

>> $\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{t\ u(t-1)\}$

$sY(s) + Y(s) = e^{-s}\mathcal{L}\{t+1\}$

$Y(s)(s+1) = e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right)$

>> Despejamos a $Y(s)$

$Y(s)(s+1) = e^{-s}\left(\frac{s + s^2}{s^3}\right)$

$Y(s)(s+1) = e^{-s}\left(\frac{s(1+s)}{s^3}\right)$

$Y(s) = \frac{e^{-s}\,\cancel{s}\,\cancel{(s+1)}}{s^{\cancel{3}}\,\cancel{(s+1)}}$

$Y(s) = \frac{e^{-s}}{s^2}$

>> $\mathcal{L}^{-1}$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{e^{-s}}{s^2}\right\}$

$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}\Big|_{t \to t-1} u(t-1)$

$y(t) = t\Big|_{t \to t-1} u(t-1)$

$y(t) = (t-1)\,u(t-1)$ //

2