Determine si la ecuación dada es lineal en las variables x y Y.

1. $2x - 3y = 4$ Si
2. $3x - 4xy = 0$ No
5. $2\operatorname{sen} x - y = 14$ No
3. $\frac{3}{y} + \frac{2}{x} - 1 = 0$ No
4. $x^2 + y^2 = 4$ No
6. $(\operatorname{sen} 2)x - y = 14$ Si

Encuentre una representación paramétrica de la ecuación lineal dada.

7. $2x - 4y = 0$

$\frac{1}{2}(2x - 4y) = 0$

$x - 2y = 0$

$x = 2y$

Sea $y = t$

$\therefore x = 2t$

$\forall t \in \mathbb{R}$

8. $3x + \frac{1}{2}y = 9$

$6x + y = 18$

$y = 18 - 6x$

Sea $x = t$

$\therefore y = 18 - 6t$

$\forall t \in \mathbb{R}$

9. $x + y + z = 1$

$x = 1 - y - z$

Sea $y = s$, $z = t$

$\begin{cases} y = s \\ z = t \\ x = 1 - s - z \end{cases}$

$\forall s \,\&\, z \in \mathbb{R}$

10. $\frac{1}{2}x - \frac{1}{3}y = \frac{1}{4}$

$2\left(\frac{1}{2}x - \frac{1}{3}y = \frac{1}{4}\right)$

$x - \frac{2}{3}y = \frac{1}{2}$

$x = \frac{1}{2} + \frac{2}{3}t$

Sea $y = t$

$\forall t \in \mathbb{R}$

11. $13x_1 - 26x_2 + 39x_3 = 13$

$\frac{1}{13}(13x_1 - 26x_2 + 39x_3 = 13)$

$x_1 - 2x + 3x_3 = 1$

$x_1 = 1 + 2x_2 - 3x_3$

Sea $x_2 = s$ $x_3 = t$

$\therefore x_1 = 1 + 2s - 3t$

$\forall s \,\&\, t \in \mathbb{R}$

12. $x_1 - x_2 + 2x_3 - x_4 = 14$

$x_1 = x_2 - 2x_3 + x_4 + 14$

sea $x_2 = r$ $x_3 = s$ $x_4 = t$

$\therefore x_1 = r - 2s + t + 14$

$\forall r, s \,\&\, t \in \mathbb{R}$

Use la sustitución hacia atrás para resolver el sistema de ecuaciones lineales dado.

13. $x_1 - x_2 = 2$

$x_2 = 3$

$x_1 - 3 = 2$

$x_1 = 5$

14. $2x_1 - 4x_2 = 6$

$3x_2 = 9$

$x_2 = 9/3$

$x_2 = 3$

$2x_1 - 4(3) = 6$

$2x_1 = 6 + 12$

$x_1 = 9$

15. $-x + y - z = 0$

$2y + z = 3$

$\frac{1}{2}z = 0$

$z = 0$

$2y + 0 = 3$

$y = 3/2$

$\therefore -x = y - z$

$x = \frac{3}{2} - 0$

$x = \frac{3}{2}$

16. $x - y = 4$

$2y + z = 6$

$3z = 6$

$z = 2$

$2y + 2 = 6$

$y = \frac{6-2}{2} =$

$x = 4 + 4$

$x = 6$

17. $5x + 2y = 0$

$-2y = 0$

$y = 0$

$5x = 0$

$x = 0$

18. $x_1 + x_2 + x_3 = 0$

$x_2 = 0$

$x_1 + x_3 = 0$

$x_1 = -x_3$

Resuelva el sistema de ecuaciones lineales que se da.

19. $$\begin{aligned} 2x_1 + x_2 &= 4 \\ x_1 - x_2 &= 2 \end{aligned} \sim \left(\begin{array}{cc|c} 2 & 1 & 4 \\ 1 & -1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & 2 \\ 2 & 1 & 4 \end{array}\right) \overset{R_2: -2R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 3 & 0 \end{array}\right)$$ Sin Sol.

20. $$\begin{aligned} x + 3y &= 2 \\ -x + 2y &= 3 \end{aligned} \sim \left(\begin{array}{cc|c} 1 & 3 & 2 \\ -1 & 2 & 3 \end{array}\right) \overset{R_2: R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 5 & 5 \end{array}\right) \overset{R_2: \frac{1}{5}R_2}{\sim} \left(\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 1 & 1 \end{array}\right)$$

$$\overset{R_1 = -3R_2 + R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 1 \end{array}\right)$$

21. $$\begin{aligned} x - y &= 1 \\ -2x + 2y &= 5 \end{aligned} \sim \left(\begin{array}{cc|c} 1 & -1 & 1 \\ -2 & 2 & 5 \end{array}\right) \overset{R_2: 2R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 0 & 7 \end{array}\right)$$ Sin Sol.

22. $\frac{1}{2}x - \frac{1}{3}y = 1 \qquad x = 2 + \frac{2}{3}y \qquad$ sea $y = t \quad \therefore x = 2 + \frac{2}{3}t \quad \forall t \in \mathbb{R}$.

23. $$\begin{aligned} x_1 - x_2 &= 0 \\ 3x_1 - 2x_2 &= -1 \end{aligned} \sim \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 3 & -2 & -1 \end{array}\right) \overset{R_2 = -3R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 1 & -1 \end{array}\right) \overset{R_1 = R_2 + R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & -1 \end{array}\right)$$

24. $$\begin{aligned} 3x + 2y &= 2 \\ 6x + 4y &= 14 \end{aligned} \sim \left(\begin{array}{cc|c} 3 & 2 & 2 \\ 6 & 4 & 14 \end{array}\right) \overset{R_2 = \frac{1}{2}R_2}{\sim} \left(\begin{array}{cc|c} 3 & 2 & 2 \\ 3 & 2 & 7 \end{array}\right) \overset{R_2 = -R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 3 & 2 & 2 \\ 0 & 0 & 5 \end{array}\right)$$ Sin Sol.

25. $$\begin{aligned} 9x - 3y &= -1 \\ \frac{1}{5}x + \frac{2}{5}y &= -\frac{1}{3} \end{aligned} \sim \left(\begin{array}{cc|c} 9 & -3 & -1 \\ \frac{1}{5} & \frac{2}{5} & -\frac{1}{3} \end{array}\right) \underset{R_2 = 5R_2}{\overset{R_1 = \frac{1}{9}R_1}{\sim}} \left(\begin{array}{cc|c} 1 & -\frac{1}{3} & -8/9 \\ 1 & 2 & -5/3 \end{array}\right) \overset{R_2 = -R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 1 & -\frac{1}{3} & -8/9 \\ 0 & 7/3 & -7/9 \end{array}\right)$$

$$\overset{R_2 = \frac{3}{7}R_2}{\sim} \left(\begin{array}{cc|c} 1 & -\frac{1}{3} & -8/9 \\ 0 & 1 & -3/9 \end{array}\right) \overset{R_1: \frac{1}{3}R_2 + R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & -8/9 \\ 0 & 1 & -1/9 \end{array}\right)$$ Sol. Única.

26. $$\begin{aligned} x - 2x_2 &= 0 \\ 6x_1 + 2x_2 &= 0 \end{aligned} \sim \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 6 & 2 & 0 \end{array}\right) \overset{R_2: -6R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 14 & 0 \end{array}\right) \overset{R_2: \frac{1}{14}R_2}{\sim} \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 1 & 0 \end{array}\right) \overset{R_1: 2R_2 + R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)$$ Sin S.

27. $$\begin{aligned} 2u + v &= 120 \\ u + 2v &= 120 \end{aligned} \sim \left(\begin{array}{cc|c} 2 & 1 & 120 \\ 1 & 2 & 120 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & 120 \\ 2 & 1 & 120 \end{array}\right) \overset{R_2: -2R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 1 & 2 & 120 \\ 0 & -3 & -120 \end{array}\right) \overset{R_2: -\frac{1}{3}R_2}{\sim} \left(\begin{array}{cc|c} 1 & 2 & 120 \\ 0 & 1 & 40 \end{array}\right)$$

$$\overset{R_1: -2R_2 + R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & 40 \\ 0 & 1 & 40 \end{array}\right)$$ Sol. Única

R.A.K

28. $\begin{aligned} 1.8x_1 + 1.2x_2 &= 4 \\ 9x_1 + 6x_2 &= 3 \end{aligned} \sim \left(\begin{array}{cc|c} 1.8 & 1.2 & 4 \\ 9 & 6 & 3 \end{array}\right) \underset{R_2:\ \frac{1}{3}R_2}{\overset{R_1:\ \frac{1}{1.8}R_1}{\sim}} \left(\begin{array}{cc|c} 1 & 0.6\bar{6} & 2.2\bar{2} \\ 3 & 2 & 1 \end{array}\right) \overset{R_2:\ -3R_1+R_2}{\sim} \left(\begin{array}{cc|c} 1 & 0.6\bar{6} & 2.2\bar{2} \\ 0 & 0 & -5.\bar{6} \end{array}\right)$

29. $\begin{aligned} 2.5x_1 - 3x_2 &= 1.5 \\ 10x_1 - 12x_2 &= 6 \end{aligned} \sim \left(\begin{array}{cc|c} 2.5 & -3 & 1.5 \\ 10 & -12 & 6 \end{array}\right) \underset{R_2:\ \frac{1}{2}R_2}{\overset{R_1:\ \frac{1}{2.5}R_1}{\sim}} \left(\begin{array}{cc|c} 1 & -1.2 & 0.6 \\ 5 & -6 & 3 \end{array}\right) \overset{R_2:\ -5R_1+R_2}{\sim} \left(\begin{array}{cc|c} 1 & -1.2 & 0.6 \\ 0 & 0 & 0 \end{array}\right)$

$\rightarrow x_1 = \frac{6}{10} + \frac{12}{10}x_2$ Sea $x_2 = t$ $\therefore$ $x = \frac{6}{10} + \frac{12}{10}t \ \ \forall t \in \mathbb{R}$

30. $\begin{aligned} \tfrac{2}{3}x_1 + \tfrac{1}{6}x_2 &= 0 \\ 4x_1 + x_2 &= 0 \end{aligned} \sim \left(\begin{array}{cc|c} \frac{2}{3} & \frac{1}{6} & 0 \\ 4 & 1 & 0 \end{array}\right) \overset{R_1:\ \frac{3}{2}R_1}{\sim} \left(\begin{array}{cc|c} 1 & \frac{1}{4} & 0 \\ 4 & 1 & 0 \end{array}\right) \overset{R_2:\ -4R_1+R_2}{\sim} \left(\begin{array}{cc|c} 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 \end{array}\right)$

$\rightarrow x_2 = -4x_1$ sea $x_1 = t$ $\therefore$ $x_2 = -4t \ \ \forall t \in \mathbb{R}$

31. $\begin{aligned} \tfrac{x-1}{2} + \tfrac{y+2}{3} &= 4 \\ x - 2y &= 5 \end{aligned} \sim \begin{aligned} \tfrac{x}{2} - \tfrac{1}{2} + \tfrac{y}{3} + \tfrac{2}{3} &= 4 \\ x - 2y &= 5 \end{aligned} \sim \begin{aligned} \tfrac{1}{2}x + \tfrac{1}{3}y + \tfrac{1}{6} &= 4 \\ x - 2y &= 5 \end{aligned} \sim \begin{aligned} \tfrac{1}{2}x + \tfrac{1}{3}y &= \tfrac{25}{6} \\ x - 2y &= 5 \end{aligned}$

$\sim \left(\begin{array}{cc|c} \frac{1}{2} & \frac{1}{3} & 25/6 \\ 1 & -2 & 5 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & 5 \\ 1/2 & 1/3 & 25/6 \end{array}\right) \overset{R_2:\ -R_1+2R_2}{\sim} \left(\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 8/3 & 10/3 \end{array}\right) \overset{R_2:\ \frac{3}{8}R_2}{\sim} \left(\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & 10/8 \end{array}\right)$

$\overset{R_1:\ 2R_2+R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & 15/2 \\ 0 & 1 & 10/8 \end{array}\right)$ Sol. única.

32. $\begin{aligned} \tfrac{x_1+3}{4} + \tfrac{x_2-1}{3} &= 1 \\ 2x_1 - x_2 &= 12 \end{aligned} \sim \begin{aligned} \tfrac{x_1}{4} + \tfrac{3}{4} + \tfrac{x_2}{3} - \tfrac{1}{3} &= 1 \\ 2x_1 - x_2 &= 12 \end{aligned} \sim \begin{aligned} \tfrac{1}{4}x_1 + \tfrac{1}{3}x_2 + \tfrac{5}{12} &= 1 \\ 2x_1 - x_2 &= 12 \end{aligned} \sim \begin{aligned} \tfrac{1}{4}x_1 + \tfrac{1}{3}x_2 &= \tfrac{7}{12} \\ 2x_1 - x_2 &= 12 \end{aligned}$

$\sim \left(\begin{array}{cc|c} \frac{1}{4} & \frac{1}{3} & \frac{7}{12} \\ 2 & -1 & 12 \end{array}\right) \overset{R_1:\ 4R_1}{\sim} \left(\begin{array}{cc|c} 1 & 4/3 & 7/3 \\ 2 & -1 & 12 \end{array}\right) \overset{R_2:\ -2R_1+R_2}{\sim} \left(\begin{array}{cc|c} 1 & 4/3 & 7/3 \\ 0 & -11/3 & 22/3 \end{array}\right) \overset{R_2:\ -\frac{3}{11}R_2}{\sim} \left(\begin{array}{cc|c} 1 & 4/3 & 7/3 \\ 0 & 1 & -2 \end{array}\right)$

$\overset{R_1:\ -\frac{4}{3}R_2+R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -2 \end{array}\right)$ Sol. única.

33. $\begin{aligned} 0.02x_1 - 0.05x_2 &= -0.19 \\ 0.03x_1 + 0.04x_2 &= 0.52 \end{aligned} \sim \left(\begin{array}{cc|c} 0.02 & -0.05 & -0.19 \\ 0.03 & 0.04 & 0.52 \end{array}\right) \overset{R_1:\ \frac{1}{0.02}R_1}{\sim} \left(\begin{array}{cc|c} 1 & -2.5 & -9.5 \\ 0.03 & 0.04 & 0.52 \end{array}\right)$

$\overset{R_2:\ -0.03R_1+R_2}{\sim} \left(\begin{array}{cc|c} 1 & -2.5 & -9.5 \\ 0 & 0.115 & 0.805 \end{array}\right) \overset{R_2:\ \frac{1}{0.115}R_2}{\sim} \left(\begin{array}{cc|c} 1 & -2.5 & -9.5 \\ 0 & 1 & 7 \end{array}\right) \overset{R_1:\ 2.5R_2+R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & 8 \\ 0 & 1 & 7 \end{array}\right)$

34. $\begin{aligned} 0.05x_1 - 0.03x_2 &= 0.21 \\ 0.07x_1 + 0.02x_2 &= 0.17 \end{aligned} \sim \left(\begin{array}{cc|c} 0.05 & -0.03 & 0.21 \\ 0.07 & 0.02 & 0.17 \end{array}\right) \overset{R_1:\ \frac{1}{0.05}R_1}{\sim} \left(\begin{array}{cc|c} 1 & -0.6 & 4.2 \\ 0.07 & 0.02 & 0.17 \end{array}\right)$

$\overset{R_2:\ -0.07R_1+R_2}{\sim} \left(\begin{array}{cc|c} 1 & -0.6 & 4.2 \\ 0 & 0.062 & -0.124 \end{array}\right) \overset{R_2:\ \frac{1}{0.062}R_2}{\sim} \left(\begin{array}{cc|c} 1 & -0.6 & 4.2 \\ 0 & 1 & -2 \end{array}\right) \overset{R_1:\ 0.6R_2+R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right)$ Sol. U.

35. $x + y + z = 6$
$2x - y + z = 3$
$3x - z = 0$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 1 & 3 \\ 3 & 0 & -1 & 0 \end{array}\right) \xrightarrow[R_3: -3R_1+R_3]{R_2: -2R_1+R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & -1 & -9 \\ 0 & -3 & -4 & -18 \end{array}\right) \xrightarrow{R_3 - R_2 + R_3} \left(\begin{array}{cc} 1 & \\ 0 & -3 \\ 0 & 0 \end{array}\right.$$

$$\xrightarrow[R_2: -\frac{1}{3}R_2]{R_3: -\frac{1}{3}R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1/3 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{R_2: -\frac{1}{3}R_3+R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{R_1: -R_3+R_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{R_1: -R_2+R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

36. $x + y + z = 2$
$-x + 3y + 2z = 8$
$4x + y = 4$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ -1 & 3 & 2 & 8 \\ 4 & 1 & 0 & 4 \end{array}\right) \xrightarrow[R_3: -4R_1+R_3]{R_2: R_1+R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 4 & 3 & 10 \\ 0 & -3 & -4 & -4 \end{array}\right) \xrightarrow{R_2: \frac{1}{4}R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 3/4 & 5/2 \\ 0 & -3 & -4 & -4 \end{array}\right)$$

$$\xrightarrow{R_3: -3R_2+R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 3/4 & 5/2 \\ 0 & 0 & -7/4 & 7/2 \end{array}\right) \xrightarrow{R_3: -\frac{4}{7}R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 3/4 & 5/2 \\ 0 & 0 & 1 & -2 \end{array}\right) \xrightarrow{R_2: -\frac{3}{4}R_3+R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right) \xrightarrow{R_1: -R_3+R_1}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right) \xrightarrow{R_1: -R_2+R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right) \text{ Sol. única}$$

37. $4x + y - 3z = 11$
$2x - 3y + 2z = 9$
$x + y + z = -3$

$$\sim \left(\begin{array}{ccc|c} 4 & 1 & -3 & 11 \\ 2 & -3 & 2 & 9 \\ 1 & 1 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 4 & 1 & -3 & 11 \\ 2 & -3 & 2 & 9 \end{array}\right) \xrightarrow[R_3: \frac{1}{2}R_3]{R_2: \frac{1}{4}R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 1 & 1/4 & -3/4 & 11/4 \\ 1 & -3/2 & 1 & 9/2 \end{array}\right)$$

$$\xrightarrow[R_3: -R_1+R_3]{R_2: -R_1+R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -3/4 & -7/4 & 23/4 \\ 0 & -5/2 & 0 & 15/2 \end{array}\right) \xrightarrow[-\frac{2}{5}R_3]{R_2: -\frac{4}{3}R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 1 & 7/3 & -23/3 \\ 0 & 1 & 0 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 1 & 0 & -3 \\ 1 & 1 & 7/3 & -23/3 \end{array}\right) \xrightarrow{R_3: -R_2+R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 7/3 & -14/3 \end{array}\right) \xrightarrow{R_3: \frac{3}{7}R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{array}\right) \xrightarrow{R_1: -R_3+R_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{array}\right) \xrightarrow{R_1: -R_2+R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

38. $2x + 2z = 2$
$5x + 3y = 4$
$3y - 4z = 4$

$$\sim \left(\begin{array}{ccc|c} 2 & 0 & 2 & 2 \\ 5 & 3 & 0 & 4 \\ 0 & 3 & -4 & 4 \end{array}\right) \xrightarrow{R_1: \frac{1}{2}R_1} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 5 & 3 & 0 & 4 \\ 0 & 3 & -4 & 4 \end{array}\right) \xrightarrow{R_2: -5R_1+R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 3 & -5 & -1 \\ 0 & 3 & -4 & 4 \end{array}\right)$$

$$\xrightarrow{R_2: \frac{1}{3}R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -5/3 & -1/3 \\ 0 & 3 & -4 & 4 \end{array}\right) \xrightarrow{R_3: -3R_2+R_3} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -5/3 & -1/3 \\ 0 & 0 & 1 & 5 \end{array}\right) \xrightarrow{R_2: \frac{5}{3}R_3+R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 24/3 \\ 0 & 0 & 1 & 5 \end{array}\right) \xrightarrow{R_1: -R_3+R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 24/3 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

39. $3x_1 - 2x_2 + 4x_3 = 1$
$x_1 + x_2 - 2x_3 = 3$
$2x_1 - 3x_2 + 6x_3 = 8$

$$\sim \left(\begin{array}{ccc|c} 3 & -2 & 4 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & -3 & 6 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 2 & -3 & 6 & 8 \\ 3 & -2 & 4 & 1 \end{array}\right) \xrightarrow[R_3: -3R_1+R_2]{R_2: -2R_1+R_2} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 0 & -5 & 10 & 2 \\ 0 & -5 & 10 & -8 \end{array}\right)$$

$$\xrightarrow{R_3: -R_2+R_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 0 & -5 & 10 & 2 \\ 0 & 0 & 0 & -10 \end{array}\right) \text{ Sin Sol}$$

40. $5x_1 - 3x_2 + 2x_3 = 3$
$2x_1 + 4x_2 - x_3 = 7$
$x_1 - 11x_2 + 4x_3 = 3$

$$\sim \left(\begin{array}{ccc|c} 5 & -3 & 2 & 3 \\ 2 & 4 & -1 & 7 \\ 1 & -11 & 4 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -11 & 4 & 3 \\ 2 & 4 & -1 & 7 \\ 5 & -3 & 2 & 3 \end{array}\right) \underset{\substack{R_2: -2R_1 + R_2 \\ R_3: -5R_1 + R_3}}{\sim} \left(\begin{array}{ccc|c} 1 & -11 & 4 & 3 \\ 0 & 26 & -9 & 1 \\ 0 & 52 & -18 & -12 \end{array}\right)$$

$$\underset{R_3: -2R_2 + R_3}{\sim} \left(\begin{array}{ccc|c} 1 & -11 & 4 & 3 \\ 0 & 26 & -9 & 1 \\ 0 & 0 & 0 & -14 \end{array}\right)$$ Sin. Sol.

41. $x_1 + 2x_2 - 7x_3 = -4$
$2x_1 + x_2 + x_3 = 13$
$3x_1 + 9x_2 - 36x_3 = -33$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -7 & -4 \\ 2 & 1 & 1 & 13 \\ 3 & 9 & -36 & -33 \end{array}\right) \underset{\substack{R_2: -2R_1 + R_2 \\ R_3: -3R_1 + R_3}}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & -7 & -4 \\ 0 & -3 & 15 & 21 \\ 0 & 3 & 15 & -21 \end{array}\right) \underset{R_3: R_2 + R_3}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & -7 & -4 \\ 0 & -3 & 15 & 21 \\ 0 & 0 & 30 & 0 \end{array}\right)$$ Sin Sol.

42. $2x_1 + x_2 - 3x_3 = 4$
$4x_1 + 2x_3 = 10$
$-2x_1 + 3x_2 - 13x_3 = -8$

$$\sim \left(\begin{array}{ccc|c} 2 & 1 & -3 & 4 \\ 4 & 0 & 2 & 10 \\ -2 & 3 & -13 & -8 \end{array}\right) \underset{R_1: \frac{1}{2}R_1}{\sim} \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & -\frac{3}{2} & 2 \\ 4 & 0 & 2 & 10 \\ -2 & 3 & -13 & -8 \end{array}\right) \underset{\substack{R_2: -4R_1 + R_2 \\ R_3: 2R_1 + R_3}}{\sim} \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & -\frac{3}{2} & 2 \\ 0 & -2 & 8 & 2 \\ 0 & 4 & -16 & -4 \end{array}\right)$$

$$\underset{\substack{R_3: 2R_2 + R_3 \\ R_2: \frac{1}{2}R_2}}{\sim} \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & -\frac{3}{2} & 2 \\ 0 & -1 & 4 & 1 \\ 0 & 0 & 0 & -2 \end{array}\right)$$ Sin Sol.

43. $x_1 + 4x_3 = 13$
$4x_1 - 2x_2 + x_3 = 7$
$2x_1 - 2x_2 - 7x_3 = -19$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 4 & 13 \\ 4 & -2 & 1 & 7 \\ 2 & -2 & -7 & 19 \end{array}\right) \underset{\substack{R_2: -4R_1 + R_2 \\ R_3: -2R_1 + R_3}}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 4 & 13 \\ 0 & -2 & -15 & -45 \\ 0 & -2 & -15 & 5 \end{array}\right) \underset{R_3: -R_2 + R_3}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 4 & 13 \\ 0 & -2 & -15 & 4 \\ 0 & 0 & 0 & 50 \end{array}\right)$$ Sin Sol

44. $x_1 - 2x_2 + 5x_3 = 2$
$3x_1 + 2x_2 - x_3 = -2$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 5 & 2 \\ 3 & 2 & -1 & -2 \end{array}\right) \underset{R_2: -3R_1 + R_2}{\sim} \left(\begin{array}{ccc|c} 1 & -2 & 5 & 2 \\ 0 & 8 & 16 & -8 \end{array}\right) \underset{R_2: \frac{1}{8}R_2}{\sim} \left(\begin{array}{ccc|c} 1 & -2 & 5 & 2 \\ 0 & 1 & 2 & -1 \end{array}\right)$$

$$\underset{R_1: 2R_2 + R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 9 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right)$$ Sin Sol.

45. $x - 3y + 2z = 18$
$5x - 15y + 10z = 18$

$$\sim \left(\begin{array}{ccc|c} 1 & -3 & 2 & 18 \\ 5 & -15 & 10 & 18 \end{array}\right) \underset{R_2: -5R_1 + R_2}{} \left(\begin{array}{ccc|c} 1 & -3 & 2 & 18 \\ 0 & 0 & 0 & -72 \end{array}\right)$$ Sin. Sol.

46. $x = 1$
$x + y = 10$
$2x - y = -5$

$$\sim \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 1 & 1 & 10 \\ 2 & -1 & -5 \end{array}\right) \underset{\substack{R_2: -R_1 + R_2 \\ R_3: -2R_1 + R_3}}{\sim} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 9 \\ 0 & -1 & -7 \end{array}\right) \underset{R_3: R_2 + R_3}{\sim} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 9 \\ 0 & 0 & 2 \end{array}\right)$$ Sin Sol.

47. $x + y + z + w = 6$
$2x + 3y - w = 0$
$-3x + 4y + z + 2w = 4$
$x + 2y - z + w = 0$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 2 & 3 & 0 & -1 & 0 \\ -3 & 4 & 1 & 2 & 4 \\ 1 & 2 & -1 & 1 & 0 \end{array}\right) \underset{\substack{R_2: -2R_1 + R_2 \\ R_3: 3R_1 + R_3 \\ R_4: -R_1 + R_4}}{\sim} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ 0 & 7 & 4 & 5 & 22 \\ 0 & 1 & -2 & 0 & -6 \end{array}\right) \underset{\substack{R_3: -7R_2 + R_3 \\ R_4: -R_2 + R_4}}{\sim}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ 0 & 0 & 18 & 26 & 106 \\ 0 & 0 & 0 & 3 & 6 \end{array}\right) \underset{\substack{R_3: \frac{1}{18}R_3 \\ R_4: \frac{1}{3}R_4}}{\sim} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ 0 & 0 & 1 & \frac{13}{9} & \frac{53}{9} \\ 0 & 0 & 0 & 1 & 2 \end{array}\right) \underset{R_3: -\frac{13}{9}R_4 + R_3}{\sim} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right)$$

$R_1: -R_4+R_1$
$R_2: 3R_4+R_2$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 4 \\ 0 & 1 & -2 & 0 & -6 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right) \underset{R_2: 2R_3+R_2}{\overset{R_1: -R_3+R_1}{\sim}} \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right) \overset{R_1: -R_2+R_1}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right)$$

48. $\begin{aligned} x_1 \qquad\qquad + 3x_4 &= 4 \\ 2x_2 - x_3 - x_4 &= 0 \\ \cancel{2x}\ 3x_2 \qquad - 2x_4 &= 1 \\ 2x_1 - x_2 + 4x_3 \qquad &= 5 \end{aligned}$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 4 \\ 0 & 2 & -1 & -1 & 0 \\ 0 & 3 & 0 & -2 & 1 \\ 2 & -1 & 4 & 0 & 5 \end{array}\right) \overset{R_4: -2R_1+R_4}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 4 \\ 0 & 2 & -1 & -1 & 0 \\ 0 & 3 & 0 & -2 & 1 \\ 0 & -1 & 4 & -6 & -3 \end{array}\right)$$

$$\overset{R_2: -R_2}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 4 \\ 0 & 1 & -4 & 6 & 3 \\ 0 & 2 & -1 & -1 & 0 \\ 0 & 3 & 0 & -2 & 1 \end{array}\right) \underset{R_4: -3R_2+R_4}{\overset{R_3: -2R_2+R_3}{\sim}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 4 \\ 0 & 1 & -4 & 6 & 3 \\ 0 & 0 & 7 & -13 & -6 \\ 0 & 0 & 12 & -20 & -8 \end{array}\right) \overset{R_3: \frac{1}{7}R_3}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 4 \\ 0 & 1 & -4 & 6 & 3 \\ 0 & 0 & 1 & -13/7 & -6/7 \\ 0 & 0 & 12 & -20 & -8 \end{array}\right)$$

$$\overset{R_4: -12R_3+R_4}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 4 \\ 0 & 1 & -4 & 6 & 3 \\ 0 & 0 & 1 & -13/7 & -6/7 \\ 0 & 0 & 0 & 16/7 & 16/7 \end{array}\right) \overset{R_4: \frac{7}{16}R_4}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 4 \\ 0 & 1 & -4 & 6 & 3 \\ 0 & 0 & 1 & -13/7 & -6/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \overset{R_3: \frac{13}{7}R_4+R_3}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 4 \\ 0 & 1 & -4 & 6 & 3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$\underset{R_2: -6R_4+R_2}{\overset{R_1: -3R_4+R_1}{\sim}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -4 & 0 & -3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \overset{R_2: 4R_3+R_2}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

49. $\begin{aligned} 4x + 3y + 17z &= 0 \\ 5x + 4y + 22z &= 0 \\ 4x + 2y + 19z &= 0 \end{aligned}$

$$\sim \left(\begin{array}{ccc|c} 4 & 3 & 17 & 0 \\ 5 & 4 & 22 & 0 \\ 4 & 2 & 19 & 0 \end{array}\right) \overset{R_1: \frac{1}{4}R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 3/4 & 17/4 & 0 \\ 5 & 4 & 22 & 0 \\ 4 & 2 & 19 & 0 \end{array}\right) \underset{R_3: -4R_1+R_3}{\overset{R_2: -5R_1+R_2}{\sim}}$$

$$\left(\begin{array}{ccc|c} 1 & 3/4 & 17/4 & 0 \\ 0 & 1/4 & 3/4 & 0 \\ 0 & -5 & 2 & 0 \end{array}\right) \overset{R_2: 4R_2}{\sim} \left(\begin{array}{ccc|c} 1 & 3/4 & 17/4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -5 & 2 & 0 \end{array}\right) \overset{R_3: 5R_2+R_3}{\sim} \left(\begin{array}{ccc|c} 1 & 3/4 & 17/4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 15/2 & 0 \end{array}\right) \text{Sin Sol.}$$

50. $\begin{aligned} 2x + 3y \qquad &= 0 \\ 4x + 3y - z &= 0 \\ 8x + 3y + 3z &= 0 \end{aligned}$

$$\sim \left(\begin{array}{ccc|c} 2 & 3 & 0 & 0 \\ 4 & 3 & -1 & 0 \\ 8 & 3 & 3 & 0 \end{array}\right) \overset{R_1: \frac{1}{2}R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 3/2 & 0 & 0 \\ 4 & 3 & -1 & 0 \\ 8 & 3 & 3 & 0 \end{array}\right) \underset{R_3: -8R_1+R_3}{\overset{R_2: -4R_1+R_2}{\sim}}$$

$$\left(\begin{array}{ccc|c} 1 & 3/2 & 0 & 0 \\ 0 & -3 & -1 & 0 \\ 0 & -9 & 3 & 0 \end{array}\right) \overset{R_3: -3R_2+R_3}{\sim} \left(\begin{array}{ccc|c} 1 & 3/2 & 0 & 0 \\ 0 & -3 & -1 & 0 \\ 0 & 0 & 6 & 0 \end{array}\right)$$

51. $\begin{aligned} 5x + 5y - z &= 0 \\ 10x + 5y + 2z &= 0 \\ 5x + 15y - 9z &= 0 \end{aligned}$

$$\sim \left(\begin{array}{ccc|c} 5 & 5 & -1 & 0 \\ 10 & 5 & 2 & 0 \\ 5 & 15 & -9 & 0 \end{array}\right) \begin{array}{l} R_1: \frac{1}{5}R_1 \\ R_2: \frac{1}{10}R_2 \\ R_3: \frac{1}{5}R_3 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & -1/5 & 0 \\ 1 & 1/2 & 1/5 & 0 \\ 1 & 3 & -9/5 & 0 \end{array}\right)$$

R.A.K.

Resuelva para x y Y es sistema de ecuaciones lineales dado.

55. $(\cos\theta)x + (\operatorname{sen}\theta)y = 1$
$(-\operatorname{sen}\theta)x + (\cos\theta)y = 0$

$$\sim \left(\begin{array}{cc|c} \cos\theta & \operatorname{sen}\theta & 1 \\ -\operatorname{sen}\theta & \cos\theta & 0 \end{array}\right) \underset{R_1: \frac{\cos\theta}{\cos\theta}}{\sim} \left(\begin{array}{cc|c} 1 & \tan\theta & \frac{1}{\cos\theta} \\ -\operatorname{sen}\theta & \cos\theta & 0 \end{array}\right)$$

$$\underset{R_2: \operatorname{sen}\theta R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 1 & \tan\theta & \sec\theta \\ 0 & \sec\theta & \tan\theta \end{array}\right) \underset{R_2: \frac{1}{\sec\theta}R_2}{\sim} \left(\begin{array}{cc|c} 1 & \tan\theta & \sec\theta \\ 0 & 1 & \operatorname{sen}\theta \end{array}\right) \underset{R_1: -\tan\theta R_2 + R_1}{\sim}$$

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \cos\theta \end{array}\right)$$

$\operatorname{sen}\theta\left(\frac{\operatorname{sen}\theta}{\cos\theta}\right) + \cos\theta$

$\frac{\operatorname{sen}^2\theta}{\cos\theta} + \frac{\cos\theta}{1} = \frac{\operatorname{sen}^2\theta + \cos^2\theta}{\cos\theta}$

$\frac{1}{\cos\theta} = \sec\theta$

$\frac{1}{\cos\theta}\operatorname{sen}\theta + 0$

$\frac{\operatorname{sen}\theta}{\cos\theta}$

$-\tan\theta(\operatorname{sen}\theta) + \sec\theta$

$\frac{-\operatorname{sen}^2\theta}{\cos\theta} + \frac{1}{\cos\theta}$

$\frac{-\operatorname{sen}^2\theta\cos\theta + \cos\theta}{\cos^2\theta}$

$\frac{\cos\theta(1 - \operatorname{sen}^2\theta)}{\cos^2\theta}$

$\cos\theta$

56. $(\cos\theta)x + (\operatorname{sen}\theta)y = 1$
$(-\operatorname{sen}\theta)x + (\cos\theta)y = 1$

$$\sim \left(\begin{array}{cc|c} \cos\theta & \operatorname{sen}\theta & 1 \\ -\operatorname{sen}\theta & \cos\theta & 1 \end{array}\right) \underset{R_2: \frac{1}{\operatorname{sen}\theta}R_2}{\overset{R_1: \frac{1}{\cos\theta}R_1}{\sim}} \left(\begin{array}{cc|c} 1 & \frac{\operatorname{sen}\theta}{\cos\theta} & \frac{1}{\cos\theta} \\ -1 & \frac{\cos\theta}{\operatorname{sen}\theta} & \frac{1}{\operatorname{sen}\theta} \end{array}\right)$$

$$\underset{R_2: R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 1 & \frac{\operatorname{sen}\theta}{\cos\theta} & \frac{1}{\cos\theta} \\ 0 & \frac{1}{\cos\theta\operatorname{sen}\theta} & \frac{\operatorname{sen}\theta + \cos\theta}{\cos\theta\operatorname{sen}\theta} \end{array}\right) \underset{R_2: \cos\theta\operatorname{sen}\theta R_2}{\sim} \left(\begin{array}{cc|c} 1 & \frac{\operatorname{sen}\theta}{\cos\theta} & \frac{1}{\cos\theta} \\ 0 & 1 & \operatorname{sen}\theta + \cos\theta \end{array}\right)$$

$\frac{\operatorname{sen}\theta}{\cos\theta} + \frac{\cos\theta}{\operatorname{sen}\theta}$

$\frac{\operatorname{sen}^2\theta + \cos^2\theta}{\cos\theta\operatorname{sen}\theta}$

$\frac{1}{\cos\theta\operatorname{sen}\theta}$

$$\underset{R_1: -\frac{\operatorname{sen}\theta}{\cos\theta}R_2 + R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & \sec\theta \\ 0 & 1 & \tan\theta(\operatorname{sen}\theta - \cos\theta \end{array}\right)$$

$$\begin{array}{l}R_2: -R_1+R_2\\ R_3: -R_1+R_3\end{array} \sim \left(\begin{array}{ccc|c}1 & 1 & -1/5 & 0\\ 0 & -1/2 & 0 & 0\\ 0 & 2 & -8/5 & 0\end{array}\right) \xrightarrow{R_2: -2R_2} \left(\begin{array}{ccc|c}1 & 1 & -1/5 & 0\\ 0 & 1 & 0 & 0\\ 0 & 2 & -8/5 & 0\end{array}\right) \xrightarrow{R_3: -2R_2+R_3} \left(\begin{array}{ccc|c}1 & 1 & -1/5 & \\ 0 & 1 & 0 & 0\\ 0 & 0 & -8/5 & 0\end{array}\right)$$

$$R_3: -\frac{8}{5}R_3 \left(\begin{array}{ccc|c}1 & 1 & -1/5 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{array}\right) \begin{array}{l}R_1: \frac{1}{5}R_3+R_1\\ R_1: -R_2+R_1\end{array} \sim \left(\begin{array}{ccc|c}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{array}\right)$$

52. $\begin{array}{l}12x + 5y + z = 0\\ 12x + 4y - z = 0\end{array} \sim \left(\begin{array}{ccc|c}12 & 5 & 1 & 0\\ 12 & 4 & -1 & 0\end{array}\right) \begin{array}{l}R_1: \frac{1}{12}R_1\\ R_2: \frac{1}{12}R_2\end{array} \sim \left(\begin{array}{ccc|c}1 & 5/12 & 1/12 & 0\\ 1 & 1/3 & -1/12 & 0\end{array}\right) \xrightarrow{R_2: -R_1+R_2}$

$$\left(\begin{array}{ccc|c}1 & 5/12 & 1/12 & 0\\ 0 & -1/12 & -2/12 & 0\end{array}\right) \xrightarrow{R_2: -12R_2} \left(\begin{array}{ccc|c}1 & 5/12 & 1/12 & 0\\ 0 & 1 & -2 & 0\end{array}\right) \sim \begin{array}{l}x + 5/12y + \frac{1}{12}z = 0\\ y - 2z = 0\end{array}$$

$y = 2z$ Sea $\boxed{z = t}$ $\boxed{y = 2t}$

Despejando $x_1 = -\frac{5}{12}y - \frac{1}{12}z$ $\quad x_1 = -\frac{5}{6}t - \frac{1}{12}t$

$x_1 = -\frac{5}{12}(2t) - \frac{1}{12}t$ $\quad \boxed{x_1 = -\frac{11}{12}t}$ $\quad \forall t \in \mathbb{R}$

En los sig. ejercicios, resuelva el sistema de ecuaciones dado con $X = 1/x$, $Y = 1/y$ y $Z = 1/z$.

53. $\frac{12}{\frac{1}{x}} - \frac{12}{\frac{1}{y}} = 7 \sim \begin{array}{l}12x - 12y = 7\\ 3x + 4y = 0\end{array} \sim \left(\begin{array}{cc|c}12 & -12 & 7\\ 3 & 4 & 0\end{array}\right) \begin{array}{l}R_1: \frac{1}{12}R_1\\ R_2: \frac{1}{3}R_2\end{array} \sim \left(\begin{array}{cc|c}1 & -1 & 7/12\\ 1 & 4/3 & 0\end{array}\right)$

$\frac{3}{\frac{1}{x}} + \frac{4}{\frac{1}{y}} = 0$

$$\xrightarrow{R_2: -R_1+R_2} \left(\begin{array}{cc|c}1 & -1 & 7/12\\ 0 & 7/3 & -7/12\end{array}\right) \xrightarrow{R_2: \frac{3}{7}R_2} \left(\begin{array}{cc|c}1 & -1 & 7/12\\ 0 & 1 & -1/4\end{array}\right) \xrightarrow{R_1: R_2+R_1} \left(\begin{array}{cc|c}1 & 0 & 1/3\\ 0 & 1 & -1/4\end{array}\right)$$

54. $\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 4 \qquad \begin{array}{l}2x + y - 3z = 4\\ 4x + 2z = 10\\ -2x + 3y - 13z = -8\end{array} \sim \left(\begin{array}{ccc|c}2 & 1 & -3 & 4\\ 4 & 0 & 2 & 10\\ -2 & 3 & -13 & -8\end{array}\right) \begin{array}{l}R_1: \frac{1}{2}R_1\\ R_2: \frac{1}{4}R_2\\ R_3: -\frac{1}{2}R_3\end{array}$

$\frac{4}{x} + \frac{2}{z} = 10$

$-\frac{2}{x} + \frac{3}{y} - \frac{13}{z} = -8$

$$\left(\begin{array}{ccc|c}1 & 1/2 & -3/2 & 2\\ 1 & 0 & 1/2 & 5/2\\ -1 & -3/2 & -13/2 & -4\end{array}\right) \begin{array}{l}R_2: -R_1+R_2\\ R_3: R_1+R_3\end{array} \sim \left(\begin{array}{ccc|c}1 & 1/2 & -3/2 & 2\\ 0 & -1/2 & 2 & 1/2\\ 0 & 2 & -8 & -2\end{array}\right) \begin{array}{l}R_2: -2R_2\\ R_3: \frac{1}{2}R_3\end{array} \sim \left(\begin{array}{ccc|c}1 & 1/2 & -3/2 & 2\\ 0 & 1 & -4 & -1\\ 0 & 1 & -4 & -1\end{array}\right) \xrightarrow{R_3: -R_2+R_3}$$

$$\left(\begin{array}{ccc|c}1 & 1/2 & -3/2 & 2\\ 0 & 1 & -4 & -1\\ 0 & 0 & 0 & 0\end{array}\right)$$

K.A.K.

De valores a k para que...

57. La ecuación tenga exactamente 1 solución.

$$\begin{matrix} 4x + ky = 7 \\ kx + y = 0 \end{matrix} \sim \left(\begin{array}{cc|c} 4 & k & 7 \\ k & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} k & 1 & 0 \\ 4 & k & 7 \end{array}\right) \xrightarrow{R_1: \frac{1}{k}R_1} \left(\begin{array}{cc|c} 1 & \frac{1}{k} & 0 \\ 4 & k & 7 \end{array}\right) \xrightarrow{R_2: -4R_1+R_2}$$

$$\left(\begin{array}{cc|c} 1 & \frac{1}{k} & 0 \\ 0 & \frac{k^2-4}{k} & 7 \end{array}\right) \xrightarrow{R_2: \frac{k}{k^2-4}R_2} \left(\begin{array}{cc|c} 1 & \frac{1}{k} & 0 \\ 0 & 1 & \frac{7k}{k^2-4} \end{array}\right) \xrightarrow{R_1: -\frac{1}{k}R_2+R_1} \left(\begin{array}{cc|c} 1 & 0 & \frac{-7}{k^2+4} \\ 0 & 1 & \frac{7k}{k^2-4} \end{array}\right)$$

Sea $k \in \mathbb{R} \mid k \neq 2$ & -2

58. Infinidad de soluciones.

$$\begin{matrix} kx + y = 4 \\ 2x - 3y = -12 \end{matrix} \sim \left(\begin{array}{cc|c} k & 1 & 4 \\ 2 & -3 & 12 \end{array}\right) \xrightarrow{R_1: \frac{1}{k}R_1} \left(\begin{array}{cc|c} 1 & \frac{1}{k} & 4/k \\ 2 & -3 & 12 \end{array}\right) \xrightarrow{R_2: -2R_1+R_2} \left(\begin{array}{cc|c} 1 & \frac{1}{k} & 4/k \\ 0 & \frac{2-3k}{k} & \frac{12k-8}{k} \end{array}\right)$$

$\frac{2-3k}{k} = 0$

$2-3k=0$

$3k=2$

$\underline{k=2/3}$

$\frac{12k-8}{k} = 0$

$12k-8=0$

$12k=8$

$k=\frac{8}{12}=\frac{4}{6}=\underline{\frac{2}{3}}$

59. Exacta una solución

$$\begin{matrix} x + ky = 0 \\ kx + y = 0 \end{matrix} \quad \left(\begin{array}{cc|c} 1 & k & 0 \\ k & 1 & 0 \end{array}\right) \xrightarrow{R_2: -kR_1+R_2} \left(\begin{array}{cc|c} 1 & k & 0 \\ 0 & 1-k^2 & 0 \end{array}\right)$$

$$R_2: \frac{1}{1-k^2}R_2 \left(\begin{array}{cc|c} 1 & k & 0 \\ 0 & 1 & 0 \end{array}\right) \xrightarrow{R_1: -kR_2+R_1} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)$$

x y $y = 0 \quad \forall k \in \mathbb{R}$

60. Ninguna solución

$$\begin{matrix} x + ky = 2 \\ kx + y = 4 \end{matrix} \sim \left(\begin{array}{cc|c} 1 & k & 2 \\ k & 1 & 4 \end{array}\right) \xrightarrow{R_2: -kR_1+R_2} \left(\begin{array}{cc|c} 1 & k & 2 \\ 0 & 1-k^2 & -2k+4 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & 1 & 4 \end{array}\right) \xrightarrow{-R_1+R_2} \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 2 \end{array}\right)$$

$1-k^2=0$

$-1 = k^2$

$\underline{k=1}$

61. Ninguna solución.

$$\begin{matrix} x+2y+kz=6 \\ 3x+6y+8z=4 \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 2 & k & 6 \\ 3 & 6 & 8 & 4 \end{array}\right) \xrightarrow{R_2:-3R_1+R_2} \left(\begin{array}{ccc|c} 1 & 2 & k & 6 \\ 0 & 0 & 8-3k & -14 \end{array}\right)$$

$$8-3K=0$$
$$3K=8$$
$$k=8/3$$

62. Exacta una solución

$$\begin{matrix} Kx+2Ky+3Kz=4K \\ x+y+z=0 \\ 2x+y+z=1 \end{matrix} \sim \left(\begin{array}{ccc|c} k & 2K & 3K & 4K \\ 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & 1 \end{array}\right) \xrightarrow{R_1:\frac{1}{k}R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & 1 \end{array}\right)$$

$$\xrightarrow[R_3:-2R_1+R_3]{R_2:-R_1+R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -4 \\ 0 & -5 & -5 & -7 \end{array}\right) \xrightarrow{R_3:-5R_2+R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -4 \\ 0 & 0 & 5 & 13 \end{array}\right) \xrightarrow[R_2:-R_2]{R_3:\frac{1}{5}R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 13/5 \end{array}\right)$$

$$\xrightarrow{R_2:-2R_3+R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & -6/5 \\ 0 & 0 & 1 & 13/5 \end{array}\right) \xrightarrow[R_1:-3R_3+R_1]{R_1:-2R_2+R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -7/5 \\ 0 & 1 & 0 & -6/5 \\ 0 & 0 & 1 & 13/5 \end{array}\right)$$

$$\forall K \in \mathbb{R}$$

63. Determine los valores de K tal que las ec. lineales no tenga sol.

$$\begin{matrix} x+y+kz=3 \\ x+Ky+z=2 \\ Kx+y+z=1 \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 1 & K & 3 \\ 1 & K & 1 & 2 \\ K & 1 & 1 & 1 \end{array}\right) \xrightarrow[R_3:-KR_1+R_3]{R_2:-R_1+R_2} \left(\begin{array}{ccc|c} 1 & 1 & K & 3 \\ 0 & K-1 & 1-K & -1 \\ 0 & 1-K & 1-K^2 & 1-3K \end{array}\right)$$

$$\xrightarrow{R_2:\frac{1}{K-1}R_2} \left(\begin{array}{ccc|c} 1 & 1 & k & 3 \\ 0 & 1 & -1 & -\frac{1}{K-1} \\ 0 & 1-K & 1-K^2 & 1-3K \end{array}\right) \xrightarrow{R_3:-(1-K)R_2+R_3} \left(\begin{array}{ccc|c} 1 & 1 & K & 3 \\ 0 & 1 & -1 & -\frac{1}{K-1} \\ 0 & 0 & K^2-K+2 & -3K \end{array}\right)$$

$$K^2-K+2=0$$
$$(K+2)(K-1)=0$$

Efectuar las operaciones indicadas, dado que $a = 3$, $b = -4$ y

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

1. $aA + bB = 3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + (-4)\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} + \begin{bmatrix} 0 & -4 \\ 4 & -8 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 13 & 4 \end{bmatrix}$

2. $(A+B) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

3. $ab(B) = (3)(-4)\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = -12\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -12 \\ 12 & -24 \end{bmatrix}$

4. $(a+b)B = [3+(-4)]\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = -1\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$

5. $(a-b)(A-B) = [3-(-4)]\left\{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}\right\} = (-1)\begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -4 & -2 \end{bmatrix}$

6. $(ab)O = 3(-4)\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = -12\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}$$

7. $3X + 2A = B$

$$3\begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \\ z_1 & z_2 \end{bmatrix} + 2\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}$$

$3y_1 + 2 = -2$
$3y_1 = -4$
$y_1 = -\frac{4}{3}$

$3z_1 - 6 = 4$
$3z_1 = 10$
$z_1 = \frac{10}{3}$

$$\begin{bmatrix} 3x_1 & 3x_2 \\ 3y_1 & 3y_2 \\ 3z_1 & 3z_2 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ 2 & -10 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}$$

$3x_2 = 2$
$x_2 = \frac{2}{3}$

$3y_2 - 10 = 1$
$3y_2 = 11$
$y_2 = \frac{11}{3}$

$3x_1 - 8 = 1$
$3y_1 + 2 = -2$
$3z_1 - 6 = 4$

$3x_2 = 2$
$3y_2 - 10 = 1$
$3z_2 + 4 = 4$

$3x_1 = 1 + 8$
$3x_1 = 9$
$x_1 = 9/3$
$x_1 = 3$

$3z_2 + 4 = 4$
$3z_2 = 0$
$z_2 = 0$

$$X = \begin{bmatrix} 3 & \frac{2}{3} \\ -\frac{4}{3} & \frac{11}{3} \\ \frac{10}{3} & 0 \end{bmatrix}$$

8. $2A - 5B = 3X$

$$2\begin{bmatrix}-4 & 0\\ 1 & -5\\ -3 & 2\end{bmatrix} - 5\begin{bmatrix}1 & 2\\ -2 & 1\\ 4 & 4\end{bmatrix} = 3\begin{bmatrix}x_1 & x_2\\ y_1 & y_2\\ z_1 & z_2\end{bmatrix}$$

$$\begin{bmatrix}-8 & 0\\ 2 & -10\\ -6 & 4\end{bmatrix} + \begin{bmatrix}-5 & -10\\ 10 & -5\\ -20 & -20\end{bmatrix} = \begin{bmatrix}3x_1 & 3x_2\\ 3y_1 & 3y_2\\ 3z_1 & 3z_2\end{bmatrix}$$

$$\begin{bmatrix}-13 & -10\\ 12 & -15\\ -26 & -16\end{bmatrix} = \begin{bmatrix}3x_1 & 3x_2\\ 3y_1 & 3y_2\\ 3z_1 & 3z_2\end{bmatrix}$$

$3x_1 = -13$ $\quad x_1 = -\frac{13}{3}$

$3x_2 = -10$ $\quad x_2 = -10/3$

$3y_1 = 12$ $\quad y_1 = 12/3 = 4$

$3y_2 = -15$ $\quad y_2 = -15/3 = -5$

$3z_1 = -26$ $\quad z_1 = -26/3$

$3z_2 = -16$ $\quad z_2 = -16/3$

$$X = \begin{bmatrix}-\frac{13}{3} & -\frac{10}{3}\\ 4 & -5\\ -\frac{26}{3} & -\frac{16}{3}\end{bmatrix}$$

9. $X - 3A + 2B = 0$

$$\begin{bmatrix}x_1 & x_2\\ y_1 & y_2\\ w_1 & w_2\end{bmatrix} - 3\begin{bmatrix}-4 & 0\\ 1 & -5\\ -3 & 2\end{bmatrix} + 2\begin{bmatrix}1 & 2\\ -2 & 1\\ 4 & 4\end{bmatrix}$$

$$\begin{bmatrix}x_1 & x_2\\ y_1 & y_2\\ w_1 & w_2\end{bmatrix} - \begin{bmatrix}-12 & 0\\ 3 & -15\\ -9 & 6\end{bmatrix} + \begin{bmatrix}2 & 4\\ -4 & 2\\ 8 & 8\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

$x_1 + 12 + 2 = 0$ $\quad x_1 = -14$

$x_2 + 4 = 0$ $\quad x_2 = -4$

$y_1 - 3 - 4 = 0$ $\quad y_1 = 7$

$y_2 + 15 + 2 = 0$ $\quad y_2 = -17$

$w_1 + 9 + 8 = 0$ $\quad w_1 = -17$

$w_2 - 6 + 8 = 0$ $\quad w_2 = -2$

$$X = \begin{bmatrix}-14 & -4\\ 7 & -17\\ -17 & -2\end{bmatrix}$$

10. $6X - 4A - 3B = 0$

$$6\begin{bmatrix}x_1 & x_2\\ y_1 & y_2\\ w_1 & w_2\end{bmatrix} - 4\begin{bmatrix}-4 & 0\\ 1 & -15\\ -3 & 6\end{bmatrix} - 3\begin{bmatrix}1 & 2\\ -2 & 1\\ 4 & 4\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}6x_1 & 6x_2\\ 6y_1 & 6y_2\\ 6w_1 & 6w_2\end{bmatrix} + \begin{bmatrix}16 & 0\\ -4 & 60\\ 12 & -24\end{bmatrix} + \begin{bmatrix}-3 & -6\\ 6 & -3\\ -12 & -12\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

$6x_1 + 16 - 3 = 0$ $\quad x_1 = \frac{13}{6}$

$6x_2 - 6 = 0$ $\quad x_2 = 1$

$6y_1 - 4 + 6 = 0$ $\quad y_1 = 2/6 = \frac{1}{3}$

$6y_2 + 60 - 3 = 0$ $\quad y_2 = 57/6$

$6w_1 + 12 - 12 = 0$ $\quad w_1 = 0$

$6w_2 - 24 - 12 = 0$ $\quad w_2 = \frac{36}{6} = 6$

$$X = \begin{bmatrix}\frac{13}{6} & 1\\ \frac{1}{3} & 57/6\\ 0 & 6\end{bmatrix}$$

Realice las operaciones indicadas dado que $c = -2$ 4

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

11. $B(CA) = \underset{2\times 2}{\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 0 & 1 & -1 \\ -1 & -2 & -3 \end{bmatrix}} = \begin{bmatrix} -3 & -5 & -10 \\ -2 & -5 & -5 \end{bmatrix}$

$CA = \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & -2 & -3 \end{bmatrix}$

12. $C(BC) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -3 & -1 \end{bmatrix}$

$BC = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$

13. $(B+C)A = \begin{bmatrix} 1 & 4 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 6 & -1 \\ -2 & -2 & -8 \end{bmatrix}$

$B+C = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -2 & 2 \end{bmatrix}$

14. $B(C+O) = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$

$C+O = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

15. $(cB)(C+C) = \begin{bmatrix} -2 & -6 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 12 & -4 \\ 8 & 4 \end{bmatrix}$

$cB = -2 \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -6 \\ 2 & -4 \end{bmatrix}$

$C+C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

16. $B(cA) = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -2 & -4 & -6 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -10 & -12 \\ 2 & 6 & 2 \end{bmatrix}$

$cA = -2 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -4 & -6 \\ 0 & -2 & 2 \end{bmatrix}$

Si $AC = BC$, entonces A no es necesariamente igual a B. Demu[illegible]
lo anterior con las matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 4 \\ 3 & -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -6 & 3 \\ 5 & 4 & 4 \\ -1 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 4 & -2 & 3 \end{bmatrix}$$

$$A \neq B$$

$$AC = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 4 & -2 & 3 \end{bmatrix} \qquad BC = \begin{bmatrix} 4 & -6 & 3 \\ 5 & 4 & 4 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 4 & -2 & 3 \end{bmatrix}$$

$$AC = \begin{bmatrix} 12 & -6 & 9 \\ 16 & -8 & 12 \\ 4 & -2 & 3 \end{bmatrix} \Longleftrightarrow BC = \begin{bmatrix} 12 & -6 & 9 \\ 16 & -8 & 12 \\ 4 & -2 & 3 \end{bmatrix}$$

$$\rightarrow AC = BC$$

Si $AB = 0$ entonces no necesariamente es cierto que $A = 0$ o $B = 0$
Demuestre con las matrices:

$$A = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} \quad y \quad B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Efectue las operaciones indicadas dado que: $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ y $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

19. $A^2 = AA = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

20. $A^4 = AAAA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

21. $A[I + A] = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$

$$I + A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix}$$

2. $A + IA = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0 & -2 \end{bmatrix}$

$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

Encuentre a) A^T b) A^TA c) AA^T para:

23. $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 3 & -5 \end{bmatrix}$ $A^T = \begin{bmatrix} 2 & 4 \\ -1 & 3 \\ 3 & -5 \end{bmatrix}$ $A^TA = \underbrace{\begin{bmatrix} 2 & 4 \\ -1 & 3 \\ 3 & -5 \end{bmatrix}}_{3\times 2}\underbrace{\begin{bmatrix} 2 & -1 & 3 \\ 4 & 3 & -5 \end{bmatrix}}_{2\times 3} = \underbrace{\begin{bmatrix} 20 & 10 & -14 \\ 10 & 10 & -18 \\ -14 & -18 & 34 \end{bmatrix}}_{3\times 3}$

$AA^T = \underbrace{\begin{bmatrix} 2 & -1 & 3 \\ 4 & 3 & -5 \end{bmatrix}}_{2\times 3}\underbrace{\begin{bmatrix} 2 & 4 \\ -1 & 3 \\ 3 & -5 \end{bmatrix}}_{3\times 2} = \underbrace{\begin{bmatrix} 14 & -10 \\ -10 & 50 \end{bmatrix}}_{2\times 2}$

24. $A = \begin{bmatrix} -7 & 11 & 12 \\ 4 & -3 & 1 \\ 6 & -1 & 3 \end{bmatrix}$ $A^T = \begin{bmatrix} -7 & 4 & 6 \\ 11 & -3 & -1 \\ 12 & 1 & 3 \end{bmatrix}$ $A^TA = \begin{bmatrix} -7 & 4 & 6 \\ 11 & -3 & -1 \\ 12 & 1 & 3 \end{bmatrix}\begin{bmatrix} -7 & 11 & 12 \\ 4 & -3 & 1 \\ 6 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 101 & -95 & -62 \\ -95 & 131 & 126 \\ -62 & 126 & 154 \end{bmatrix}$

$AA^T = \begin{bmatrix} -7 & 11 & 12 \\ 4 & -3 & 1 \\ 6 & -1 & 3 \end{bmatrix}\begin{bmatrix} -7 & 4 & 6 \\ 11 & -3 & -1 \\ 12 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 314 & -49 & -17 \\ -49 & 26 & 30 \\ -17 & 30 & 46 \end{bmatrix}$

25. $A = \begin{bmatrix} 6 & 0 \\ 0 & -4 \\ 7 & 5 \end{bmatrix}$ $A^T = \begin{bmatrix} 6 & 0 & 7 \\ 0 & -4 & 5 \end{bmatrix}$ $A^TA = \underbrace{\begin{bmatrix} 6 & 0 & 7 \\ 0 & -4 & 5 \end{bmatrix}}_{2\times 3}\underbrace{\begin{bmatrix} 6 & 0 \\ 0 & -4 \\ 7 & 5 \end{bmatrix}}_{3\times 2} = \begin{bmatrix} 85 & 35 \\ 35 & 41 \end{bmatrix}$

$AA^T = \underbrace{\begin{bmatrix} 6 & 0 \\ 0 & -4 \\ 7 & 5 \end{bmatrix}}_{3\times 2}\underbrace{\begin{bmatrix} 6 & 0 & 7 \\ 0 & -4 & 5 \end{bmatrix}}_{2\times 3} = \begin{bmatrix} 36 & 0 & 42 \\ 0 & 16 & -20 \\ 42 & -20 & 74 \end{bmatrix}$

26. $A = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$ $A^T = [2 \ -1 \ -3]$ $A^TA = \underbrace{[2 \ -1 \ -3]}_{1\times 3}\underbrace{\begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}}_{3\times 1} = [14]$

Explique por qué la fórmula no es valida.

27. $(A+B)(A-B) = A^2 - B^2$

$$(A+B)(A-B) = (A+B)A - (A+B)B$$
$$= A^2 + BA - AB - B^2$$

Como $AB \neq BA$ (ya que las matrices no tienen prop. conmutativa en la multiplica.) en general $\rightarrow$ $BA - AB$ no se puede eliminar, y por lo tanto la fórmula no es valida.

28. $(A+B)(A+B) = A^2 + 2AB + B^2$

$$(A+B)(A+B) = (A+B)A + (A+B)B$$
$$= A^2 + BA + AB + B^2$$

Como $AB \neq AB$ en general $\rightarrow$ $BA + AB$ no se pueden simplificar como $2AB$ y por lo tanto la fórmula no es valida.

Compruebe $(AB)^T = B^T A^T$ para las matrices: $A = \begin{bmatrix} -1 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 0 \\ 1 & 2 \\ 1 & -1 \end{bmatrix}$

$$AB = \begin{bmatrix} -1 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 1 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix} \rightarrow (AB)^T = \begin{bmatrix} 2 & -5 \\ 4 & -1 \end{bmatrix}$$

$$B^T = \begin{bmatrix} -3 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} -1 & 2 \\ 1 & 0 \\ -2 & 1 \end{bmatrix} \quad B^T A^T = \underbrace{\begin{bmatrix} -3 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}}_{2\times 3} \underbrace{\begin{bmatrix} -1 & 2 \\ 1 & 0 \\ -2 & 1 \end{bmatrix}}_{3\times 2} = \begin{bmatrix} 2 & -5 \\ 4 & -1 \end{bmatrix}$$

$$\therefore (AB)^T = B^T A^T$$

Compruebe $(AB)^T = B^T A^T$ para $A = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$ y $B = \begin{bmatrix} -3 & -1 \\ 2 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -3 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -4 & -2 \end{bmatrix} \quad (AB)^T = \begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 \\ 2 & -2 \end{bmatrix} \quad B^T = \begin{bmatrix} -3 & 2 \\ -1 & 1 \end{bmatrix} \quad B^T A^T = \begin{bmatrix} -3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix}$$

$$\therefore (AB)^T = B^T A^T$$

Dados $X=\begin{bmatrix}1\\2\\3\end{bmatrix}$ $Y=\begin{bmatrix}1\\0\\2\end{bmatrix}$ $Z=\begin{bmatrix}1\\4\\4\end{bmatrix}$ $W=\begin{bmatrix}0\\0\\1\end{bmatrix}$ $0=\begin{bmatrix}0\\0\\0\end{bmatrix}$

31. Encuentre escalares a y b tales que $Z = aX$ y bY

$Z = aX + bY$ $\begin{bmatrix}1\\4\\4\end{bmatrix} = a\begin{bmatrix}1\\2\\3\end{bmatrix} + b\begin{bmatrix}1\\0\\2\end{bmatrix}$

$$\begin{pmatrix}a\\2a\\3a\end{pmatrix} + \begin{pmatrix}b\\0\\2b\end{pmatrix} = \begin{pmatrix}1\\4\\4\end{pmatrix}$$

$$\left(\begin{array}{cc|c}1&1&1\\2&0&4\\3&2&4\end{array}\right) \underset{R_3:-3R_1+R_3}{\overset{R_2:-2R_1+R_2}{\sim}} \left(\begin{array}{cc|c}1&1&1\\0&-2&2\\0&-1&1\end{array}\right) \overset{R_2:-\frac{1}{2}R_2}{\sim} \left(\begin{array}{cc|c}1&1&1\\0&1&-1\\0&-1&1\end{array}\right)$$

$$\overset{R_3:R_2+R_3}{\sim} \left(\begin{array}{cc|c}1&1&1\\0&1&-1\\0&0&0\end{array}\right)$$ ∞ Sol o única sol.

$a+b=1 \Rightarrow a-1=1$

$\underline{b=-1}$ $\underline{a=2}$

Comprobamos

$$2\begin{bmatrix}1\\2\\3\end{bmatrix} + (-1)\begin{bmatrix}1\\0\\2\end{bmatrix} = \begin{bmatrix}1\\4\\4\end{bmatrix}$$

$$\begin{bmatrix}2\\4\\6\end{bmatrix} + \begin{bmatrix}-1\\0\\-2\end{bmatrix} = \begin{bmatrix}1\\4\\4\end{bmatrix}$$

$$\begin{pmatrix}1\\4\\4\end{pmatrix} = \begin{pmatrix}1\\4\\4\end{pmatrix}$$

33. Demuestre que si $aX + bY + cW = 0 \rightarrow a=b=c=0.$

$a\begin{pmatrix}1\\2\\3\end{pmatrix} + b\begin{pmatrix}1\\0\\2\end{pmatrix} + c\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

$a+b=0$
$2a=0$
$3a+2b+c=0$

$$\sim \left(\begin{array}{ccc|c}1&1&0&0\\2&0&0&0\\3&2&1&0\end{array}\right) \underset{R_3:-3R_1+R_3}{\overset{R_2:-2R_1+R_2}{\sim}} \left(\begin{array}{ccc|c}1&1&0&0\\0&-2&0&0\\0&-1&1&0\end{array}\right)$$

$$\overset{R_2:-\frac{1}{2}R_2}{\sim} \left(\begin{array}{ccc|c}1&1&0&0\\0&1&0&0\\0&-1&1&0\end{array}\right) \underset{R_1:-R_2+R_1}{\overset{R_3:R_2+R_3}{\sim}} \left(\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\end{array}\right)$$

$\therefore$ $\underline{a, b \;\&\; c = 0}$

32. Demuestre que no existen escalares a y b tales que

$W = aX + bY$

$a\begin{bmatrix}1\\2\\3\end{bmatrix} + b\begin{bmatrix}1\\0\\2\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$a+b=0$
$2a+0=0$
$3a+2b=1$

$$\sim \left(\begin{array}{cc|c}1&1&0\\2&0&0\\3&2&1\end{array}\right) \underset{R_3:-3R_1+R_3}{\overset{R_2:-2R_1+R_2}{\sim}} \left(\begin{array}{cc|c}1&1&0\\0&-2&0\\0&-1&1\end{array}\right) \overset{R_2:-\frac{1}{2}R_2}{\sim}$$

$$\begin{bmatrix}a\\2a\\3a\end{bmatrix} + \begin{bmatrix}b\\0\\2b\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\left(\begin{array}{cc|c}1&1&0\\0&1&0\\0&-1&1\end{array}\right) \underset{R_1:-R_2+R_1}{\overset{R_3:R_2+R_3}{\sim}} \left(\begin{array}{cc|c}1&0&0\\0&1&0\\0&0&1\end{array}\right)$$

Sin Solución.

34. Encuentr escalares no nulos a, b y c tales que $aX + bY + cZ = 0$

$a\begin{pmatrix}1\\2\\3\end{pmatrix} + b\begin{pmatrix}1\\0\\2\end{pmatrix} + c\begin{pmatrix}1\\4\\4\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

$a+b+c=0$
$2a+4c=0$
$3a+2b+4c=0$

$$\sim \left(\begin{array}{ccc|c}1&1&1&0\\2&0&4&0\\3&2&4&0\end{array}\right) \underset{R_3:-3R_1+R_3}{\overset{R_2:-2R_1+R_2}{\sim}} \left(\begin{array}{ccc|c}1&1&1&0\\0&-2&2&0\\0&-1&1&0\end{array}\right)$$

$$\overset{R_2:-\frac{1}{2}R_2}{\sim} \left(\begin{array}{ccc|c}1&1&1&0\\0&1&-1&0\\0&-1&1&0\end{array}\right) \overset{R_3:R_2+R_1}{\sim} \left(\begin{array}{ccc|c}1&1&1&0\\0&1&-1&0\\0&0&0&0\end{array}\right) \Rightarrow \begin{array}{l}a+b+c=0\\b-c=0\end{array}$$

Sea $t=c$ $\forall t \in \mathbb{R}$

$c = \underline{t}$
$b = \underline{t}$
$a = -b+c = \underline{-2t}$

Calcule la potencia indicada para $A = [\,0 \quad 0 \quad 1\,]$

35. $A^{19} = AA^{18} = AA^9A^9 = AA^9(A^3)^3 = AA^9(A\cdot A\cdot A)^3$

$$A^2 = AA = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$A^3 = A^2A = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}$$

$$A^3 = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix} \qquad A = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix} \longrightarrow (A^3)^3 = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix} = A^9$$

$$A^9A^9 = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = A^{18}$$

$$A^{19} = AA^{18} = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}$$

36. $A^{20} = A^{19}A \longrightarrow A^{20} = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

Encuentre una matriz A que satisfaga

37. $A^2 = \begin{bmatrix}9&0\\0&4\end{bmatrix} \qquad A^2 = AA = \begin{pmatrix}x&y\\z&w\end{pmatrix}\begin{pmatrix}x&y\\z&w\end{pmatrix} = \begin{pmatrix}9&0\\0&4\end{pmatrix}$

$\rightarrow$ ① $x^2 + yz = 9$

② $xy + wy = 0 \rightarrow y(x+w) = 0 \rightarrow y = 0$ ó $x + w = 0$

③ $zx + zw = 0$ $\rightarrow z(x+w) = 0 \quad z = 0$ ó $(x+w) = 0$

④ $zy + w^2 = 4$

si $y = 0 \rightarrow$ en ① $x^2 = 9$, $x = 3$

si $z = 0 \rightarrow$ en 4 $w^2 = 4$, $w = 2$

$\rightarrow A = \begin{pmatrix}3&0\\0&2\end{pmatrix}$

$A^2 = AA = \begin{pmatrix}3&0\\0&2\end{pmatrix}\begin{pmatrix}3&0\\0&2\end{pmatrix} = \begin{pmatrix}9&0\\0&4\end{pmatrix}$

Aplique la sig. def. para determinar $f(A)$: Si f es la función polinomial definida por $f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$ entonces para una matriz $A_{n\times n}$, $f(A)$ se define como

$$f(A) = a_0 I_n + a_1 A + a_2 A^2 + \dots + a_n A^n.$$

39. $f(x) = x^2 - 5x + 2 \qquad A = \begin{bmatrix} 2 & 0 \\ 4 & 5 \end{bmatrix}$

$\rightarrow f(A) = A^2 - 5A + 2$

$$A^2 = AA = \begin{bmatrix} 2 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 28 & 25 \end{bmatrix}$$

$$f(A = \begin{bmatrix} 4 & 0 \\ 28 & 25 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 \\ 4 & 5 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 28 & 25 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 20 & 25 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 8 & 2 \end{bmatrix} \#$$

40. $f(x) = x^2 - 7x + 6 \qquad A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \qquad f(A) = A^2 - 7A + 6$

$$A^2 AA = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 29 & 28 \\ 7 & 8 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 29 & 28 \\ 7 & 8 \end{bmatrix} - 7\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 28 \\ 7 & 8 \end{bmatrix} - \begin{bmatrix} 35 & 28 \\ 7 & 14 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

41. $f(x) = x^3 - 10x^2 + 31x - 30 \qquad A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

$$A^2 = AA = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 5 & 38 \\ 0 & 4 & 42 \\ 0 & 0 & 25 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 9 & 5 & 38 \\ 0 & 4 & 42 \\ 0 & 0 & 25 \end{bmatrix}\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 27 & 19 & 256 \\ 0 & 8 & 234 \\ 0 & 0 & 125 \end{bmatrix}$$

$$f(A) = A^3 - 10A^2 + 31x - 30$$

$$= \begin{bmatrix} 27 & 19 & 256 \\ 0 & 8 & 239 \\ 0 & 0 & 125 \end{bmatrix} - 10\begin{bmatrix} 9 & 5 & 38 \\ 0 & 4 & 42 \\ 0 & 0 & 25 \end{bmatrix} + 31\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix} - 30\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 19 & 256 \\ 0 & 8 & 239 \\ 0 & 0 & 125 \end{bmatrix} - \begin{bmatrix} 90 & 50 & 380 \\ 0 & 40 & 420 \\ 0 & 0 & 250 \end{bmatrix} + \begin{bmatrix} 93 & 31 & 124 \\ 0 & 62 & 186 \\ 0 & 0 & 155 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

42. $f(x) = x^2 - 10x + 24 \qquad A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

$$A^2 = AA = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 56 & -40 \\ 20 & -4 \end{bmatrix}$$

$$f(A) = A^2 - 10A + 24$$

$$= \begin{bmatrix} 56 & -40 \\ 20 & -4 \end{bmatrix} - 10\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix} + 24\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 56 & -40 \\ 20 & -4 \end{bmatrix} - \begin{bmatrix} 80 & -40 \\ 20 & 20 \end{bmatrix} + \begin{bmatrix} 24 & 0 \\ 0 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

43. Demuestre la propiedad asociativa de matrices $A + (B + C) = (A + B) + C$

$$\forall ij, [(a+b)+c]_{ij} = (a+b)_{ij} + c_{ij} = a_{ij} + b_{ij} + c_{ij}$$

$$= a_{ij} + (b_{ij} + c_{ij}) = a_{ij} + (b+c)_{ij} = [a + (b+c)]_{ij}$$

$$\therefore (A+B) + C \Longleftrightarrow A + (B+C)$$

44. Demuestre la propieda asociativa de la multiplicación por un escalar $(cd)A = c(dA)$

$$\forall_{ij} (ca_{ij}) = ca_{ij} \rightarrow \forall_{ij} [(cd)A]_{ij} = (cd)A_{ij} = c(dA_{ij}) = [c(dA)]_{ij}$$

Determine si la matriz dada es simétrica, simétrica sesgada o ninguna de las dos. Una matriz cuadrada se denomina simétrica sesgada si $A^t = -A$.

53. $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \boxed{-\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}}$ $\therefore$ es simétrica sesgada o antisimétrica.

54. $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ $A^T = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ Simétrica.

55. $A = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 0 \end{bmatrix}$ Simétrica.

56. $A = \begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & -3 \\ 1 & 3 & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 3 \\ -1 & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & -3 \\ 1 & 3 & 0 \end{bmatrix}$ simétrica sesgada

Demuestre que B es la inversa de A.

1. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

$$A^{-1} = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \overset{R_2: -3R_1 + R_2}{\sim} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right] \overset{R_2: -\frac{1}{2}R_2}{\sim} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right]$$

$$\overset{R_1: -2R_2 + R_1}{\sim} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} \end{array}\right] \;\&\; \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = B$$

2. $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

$$A^{-1} = \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right] \overset{R_2: -2R_1 + R_2}{\sim} \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 5 & -2 & 1 \end{array}\right] \overset{R_2: \frac{1}{5}R_2}{\sim} \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & -\frac{2}{5} & \frac{1}{5} \end{array}\right]$$

$$\overset{R_1 = R_2 + R_1}{\sim} \left[\begin{array}{cc|cc} 1 & 0 & \frac{3}{5} & \frac{1}{5} \\ 0 & 1 & -\frac{2}{5} & \frac{1}{5} \end{array}\right] \;\&\; \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} = B$$

3. $A = \begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$ $B = \frac{1}{3}\begin{bmatrix} -4 & -5 & 3 \\ -4 & -8 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

$$A^{-1} = \left[\begin{array}{ccc|ccc} -2 & 2 & 3 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \overset{R_2 \leftrightarrow R_1}{\sim} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ -2 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \overset{R_2: 2R_1 + R_2}{\sim}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \overset{R_2 \leftrightarrow R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \\ 0 & 0 & 3 & 1 & 2 & 0 \end{array}\right] \overset{R_3: \frac{1}{3}R_3}{\sim}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1/3 & 2/3 & 0 \end{array}\right] \overset{R_2: -4R_3 + R_2}{\sim} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -4/3 & -8/3 & 1 \\ 0 & 0 & 1 & 1/3 & 2/3 & 0 \end{array}\right] \overset{R_1: R_2 + R_1}{\sim}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4/3 & 5/3 & 0 \\ 0 & 1 & 0 & -4/3 & -8/3 & 1 \\ 0 & 0 & 1 & 1/3 & 2/3 & 0 \end{array}\right] \rightarrow \begin{bmatrix} -\frac{4}{3} & \frac{5}{3} & 0 \\ -\frac{4}{3} & -\frac{8}{3} & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -4 & -5 & 3 \\ -4 & -8 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \underline{B}$$

4. $A = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 2 & -17 & 11 & 1 & 0 & 0 \\ -1 & 11 & -7 & 0 & 1 & 0 \\ 0 & 3 & -2 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & -11 & 7 & 0 & -1 & 0 \\ 2 & -17 & 11 & 1 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{R_2 : -2R_1 + R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & -11 & 7 & 0 & -1 & 0 \\ 0 & 5 & -3 & 1 & 2 & 0 \\ 0 & 3 & -2 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{R_2 : \frac{1}{5} R_2} \left[\begin{array}{ccc|ccc} 1 & -11 & 7 & 0 & -1 & 0 \\ 0 & 1 & -3/5 & 1/5 & 2/5 & 0 \\ 0 & 3 & -2 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{R_3 : -3R_2 + R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & -11 & 7 & 0 & -1 & 0 \\ 0 & 1 & -3/5 & 1/5 & 2/5 & 0 \\ 0 & 0 & -1/5 & -3/5 & -6/5 & 1 \end{array}\right] \underset{\sim}{R_3 : -5R_3} \left[\begin{array}{ccc|ccc} 1 & -11 & 7 & 0 & -1 & 0 \\ 0 & 1 & -3/5 & 1/5 & 2/5 & 0 \\ 0 & 0 & 1 & 3 & 6 & -5 \end{array}\right] \underset{\sim}{\begin{matrix} R_2 : \frac{3}{5} R_3 + R_2 \\ R_1 : -7R_3 + R_1 \end{matrix}}$$

$$\left[\begin{array}{ccc|ccc} 1 & -11 & 0 & -21 & -43 & 35 \\ 0 & 1 & 0 & 2 & 4 & -3 \\ 0 & 0 & 1 & 3 & 6 & -5 \end{array}\right] \underset{\sim}{R_1 : 11R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 2 & 4 & -3 \\ 0 & 0 & 1 & 3 & 6 & -5 \end{array}\right] = B$$

5. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & -3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{\begin{matrix} R_1 : \frac{1}{2} R_1 \\ R_2 : -\frac{1}{3} R_2 \\ R_3 : \frac{1}{4} R_3 \end{matrix}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{4} \end{array}\right] = B$$

6. $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix} \quad B = \frac{1}{5} \begin{bmatrix} -7 & 6 & 4 & -8 \\ 6 & -3 & -2 & 4 \\ 4 & -2 & -3 & 6 \\ -8 & 4 & 6 & -7 \end{bmatrix}$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \underset{\sim}{R_2 : -2R_1 + R_2} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & -2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \underset{\sim}{R_2 : -\frac{1}{3} R_2} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2/3 & 0 & 2/3 & -1/3 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \underset{\sim}{R_3 : -2R_2 + R_3}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2/3 & 0 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 7/3 & 2 & -4/3 & 2/3 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \underset{\sim}{R_3 : \frac{3}{7} R_3} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2/3 & 0 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & 6/7 & -4/7 & 2/7 & 3/7 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \underset{\sim}{R_4 : -2R_3 + R_4} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2/3 & 0 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & 6/7 & -4/7 & 2/7 & \frac{3}{7} & 0 \\ 0 & 0 & 0 & -5/7 & 8/7 & -4/7 & -\frac{6}{7} & 1 \end{array}\right)$$

$$\underset{\sim}{R_3 : -\frac{7}{5} R_4} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2/3 & 0 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & 6/7 & -4/7 & 2/7 & 3/7 & 0 \\ 0 & 0 & 0 & 1 & -8/5 & 4/5 & 6/5 & -7/5 \end{array}\right) \underset{\sim}{R_3 : -\frac{6}{7} R_4 + R_3} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2/3 & 0 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 4/5 & -2/5 & -3/5 & 6/5 \\ 0 & 0 & 0 & 1 & -8/5 & 4/5 & 6/5 & -7/5 \end{array}\right)$$

$R_1: 3R_3+R_1$ $\left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 6/5 & -3/5 & -2/5 & 4/5 \\ 0 & 0 & 1 & 0 & 4/5 & -2/5 & -3/5 & 6/5 \\ 0 & 0 & 0 & 1 & -8/5 & 4/5 & 6/5 & -7/5 \end{array}\right)$ $R_1: -2R_2+R_1$ $\underbrace{\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -7/5 & 6/5 & 4/5 & -8/5 \\ 0 & 1 & 0 & 0 & 6/5 & -3/5 & -2/5 & 4/5 \\ 0 & 0 & 1 & 0 & 4/5 & -2/5 & -3/5 & 6/5 \\ 0 & 0 & 0 & 1 & -8/5 & 4/5 & 6/5 & -7/5 \end{array}\right)}$

$$B = \frac{1}{5}\begin{pmatrix} -7 & 6 & 4 & -8 \\ 6 & -3 & -2 & 4 \\ 4 & -2 & -3 & 6 \\ -8 & 4 & 6 & -7 \end{pmatrix}$$

Determine la inversa de la matriz dada (en caso de existir)

7. $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ $|A| = 7 - 6 = 1 \neq 0$ $A^{-1} = \frac{1}{|A|}\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$

8. $\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$ $A^{-1} = \frac{1}{|A|}\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$ $|A| = -3 + 4 = 1 \neq 0$

9. $\begin{bmatrix} -7 & 33 \\ 4 & -19 \end{bmatrix}$ $A^{-1} = \frac{1}{|A|}\begin{bmatrix} -19 & -33 \\ -4 & -7 \end{bmatrix}$ $|A| = 133 - 132 = 1 \neq 0$

10. $\begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix}$ $|A| = 3 - 3 = 0 \therefore$ No existe A^{-1}

11. $\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$ $|A| = 16 - 16 = 0$ $\therefore$ No existe A^{-1}

12. $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$ $|A| = 0 + 1 = 1 \neq 0 \rightarrow A^{-1} = \frac{1}{|A|}\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

13. $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ $A^{-1} = \frac{1}{|A|}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$ $|A| = 8 - 3 = 5 \neq 0$

14. $\begin{bmatrix} 2 & 7 \\ -3 & -9 \end{bmatrix}$ $A^{-1} = \frac{1}{|A|}\begin{bmatrix} -9 & -7 \\ 3 & 2 \end{bmatrix}$ $|A| = -18 + 21 = 3 \neq 0$

15. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$ $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 4 & 0 & 1 & 0 \\ 3 & 6 & 5 & 0 & 0 & 1 \end{array}\right]$ $\begin{array}{l} R_2: -3R_1 + R_2 \\ R_3: -3R_1 + R_2 \end{array}$ $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & -3 & 1 & 0 \\ 0 & 3 & 2 & -3 & 0 & 1 \end{array}\right]$ $R_2: \frac{1}{2}R_2$

$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & -3/2 & 1/2 & 0 \\ 0 & 3 & 2 & -3 & 0 & 1 \end{array}\right)$ $R_3: -3R_2 + R_3$ $\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & -3/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 3/2 & -3/2 & 1 \end{array}\right)$ $R_3: 2R_3$ $\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & -3/2 & 1/2 & 0 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right)$

$\begin{array}{l} R_2: -\frac{1}{2}R_3 + R_2 \\ R_1: -R_3 + R_1 \end{array}$ $\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -2 & 3 & -2 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right)$ $R_1: -R_2 + R_1$ $\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right)$

$AA^{-1} = I = A^{-1}A \Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

16. $A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{pmatrix}$

$$A^{-1}: \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{array}\right] \overset{\substack{R_2: -3R_1+R_2 \\ R_3: R_1+R_3}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & -2 & -5 & 1 & 0 & 1 \end{array}\right] \overset{R_3: 2R}{\sim}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right] \overset{\substack{R_2: -3R_3+R_2 \\ R_1: -2R_3+R_1}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 11 & -4 & -2 \\ 0 & 1 & 0 & 12 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right] \overset{R_1: -2R_2+R_1}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -13 & 6 & 4 \\ 0 & 1 & 0 & 12 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right]$$

$$\begin{pmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{pmatrix}\begin{pmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

17. $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ 7 & 16 & -21 \end{bmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 7 & -10 & 0 & 1 & 0 \\ 7 & 16 & -21 & 0 & 0 & 1 \end{array}\right) \overset{\substack{R_2: -3R_1+R_2 \\ R_3: -7R_1+R_3}}{\sim} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & -3 & 1 & 0 \\ 0 & 2 & -14 & -7 & 0 & 1 \end{array}\right) \overset{R_3: -2R_2+R_3}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & -3 & 1 & 0 \\ 0 & 0 & 0 & -1 & -2 & 1 \end{array}\right)$$

No tiene inversa.

18. $$\begin{bmatrix} 10 & 5 & -7 \\ -5 & 1 & 4 \\ 3 & 2 & -2 \end{bmatrix} \overset{R_1: \frac{1}{10}R_1}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1/2 & -7/10 & \frac{1}{10} & 0 & 0 \\ -5 & 1 & 4 & 0 & 1 & 0 \\ 3 & 2 & -2 & 0 & 0 & 1 \end{array}\right] \overset{\substack{R_2: 5R_1+R_2 \\ R_3: -3R_1+R_3}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1/2 & -7/10 & \frac{1}{10} & 0 & 0 \\ 0 & 7/2 & 1/2 & 1/2 & 1 & 0 \\ 0 & 1/2 & 1/10 & -3/10 & 0 & 1 \end{array}\right] \begin{array}{l} R_2: \frac{2}{7}R_2 \\ R_3: 2R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & -7/10 & \frac{1}{10} & 0 & 0 \\ 0 & 1 & 1/7 & 1/7 & \frac{2}{7} & 0 \\ 0 & 1 & 1/5 & -\frac{3}{5} & 0 & 2 \end{array}\right] \overset{R_3: -R_2+R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1/2 & -7/10 & 1/10 & 0 & 0 \\ 0 & 1 & 1/7 & 1/7 & 2/7 & 0 \\ 0 & 0 & 2/35 & -\frac{26}{35} & -2/7 & 2 \end{array}\right] \overset{R_3: \frac{35}{2}R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1/2 & -7/10 & \frac{1}{10} & 0 & 0 \\ 0 & 1 & 1/7 & 1/7 & 2/7 & 0 \\ 0 & 0 & 1 & -13 & -5 & 35 \end{array}\right]$$

$$\overset{\substack{R_2: -\frac{1}{7}R_3+R_2 \\ R_1: \frac{7}{10}R_3+R_1}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1/2 & 0 & -9 & -7/2 & 49/2 \\ 0 & 1 & 0 & 2 & 1 & -5 \\ 0 & 0 & 1 & -13 & -5 & 35 \end{array}\right] \overset{R_1: -\frac{1}{2}R_2+R_1}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -10 & -4 & 27 \\ 0 & 1 & 0 & 2 & 1 & -5 \\ 0 & 0 & 1 & -13 & -5 & 35 \end{array}\right]$$

$$\begin{pmatrix} 10 & 5 & -7 \\ -5 & 1 & 4 \\ 3 & 2 & -2 \end{pmatrix}\begin{pmatrix} -10 & -4 & 27 \\ 2 & 1 & -5 \\ -13 & -5 & 35 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

19. $$\begin{bmatrix} 1 & -2 & -1 & -2 \\ 3 & -5 & -2 & -3 \\ 2 & -5 & -2 & -5 \\ -1 & 4 & 4 & 11 \end{bmatrix} \quad \left(\begin{array}{cccc|cccc} 1 & -2 & -1 & -2 & 1 & 0 & 0 & 0 \\ 3 & -5 & -2 & -3 & 0 & 1 & 0 & 0 \\ 2 & -5 & -2 & -5 & 0 & 0 & 1 & 0 \\ -1 & 4 & 4 & 11 & 0 & 0 & 0 & 1 \end{array}\right) \overset{\substack{R_2: -3R_1+R_2 \\ R_3: -2R_1+R_3 \\ R_4: R_1+R_4}}{\sim} \left(\begin{array}{cccc|cccc} 1 & -2 & -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -2 & 0 & 1 & 0 \\ 0 & 2 & 3 & 9 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\overset{\substack{R_3: R_2+R_3 \\ R_4: -2R_2+R_4}}{\sim} \left(\begin{array}{cccc|cccc} 1 & -2 & -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -5 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 7 & -2 & 0 & 1 \end{array}\right) \overset{R_4: -R_3+R_4}{\sim} \left(\begin{array}{cccc|cccc} 1 & -2 & -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -5 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 12 & -3 & -1 & 1 \end{array}\right) \begin{array}{l} R_3: -2R_4+R_3 \\ R_2: -3R_4+R_2 \\ R_1: -2R_4+R_1 \end{array}$$

$$\left(\begin{array}{cccc|cccc} & -2 & -1 & 0 & -23 & 6 & 2 & -2 \\ & 1 & 1 & 0 & -39 & 10 & 3 & -3 \\ 0 & 0 & 1 & 0 & -29 & 7 & 3 & -2 \\ 1 & 0 & 0 & 1 & 12 & -3 & -1 & 1 \end{array}\right) \underset{\sim}{\substack{R_2: -R_3+R_2 \\ R_1: R_3+R_4}} \left(\begin{array}{cccc|cccc} 1 & -2 & 0 & 0 & -52 & 13 & 5 & -4 \\ 0 & 1 & 0 & 0 & -10 & 3 & 0 & -1 \\ 0 & 0 & 1 & 0 & -29 & 7 & 3 & -2 \\ 0 & 0 & 0 & 1 & 12 & -3 & -1 & 1 \end{array}\right) \underset{\sim}{R_1 - 2R_2 + R_1}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -72 & 19 & 5 & -6 \\ 0 & 1 & 0 & 0 & -10 & 3 & 0 & -1 \\ 0 & 0 & 1 & 0 & -29 & 7 & 3 & -2 \\ 0 & 0 & 0 & 1 & 12 & -3 & -1 & 1 \end{array}\right)$$

20. $\begin{bmatrix} 4 & 8 & -7 & 14 \\ 2 & 5 & -4 & 6 \\ 0 & 2 & 1 & -7 \\ 3 & 6 & -5 & 10 \end{bmatrix}$

$$\left(\begin{array}{cccc|cccc} 4 & 8 & -7 & 14 & 1 & 0 & 0 & 0 \\ 2 & 5 & -4 & 6 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & -7 & 0 & 0 & 1 & 0 \\ 3 & 6 & 5 & 10 & 0 & 0 & 0 & 1 \end{array}\right) \underset{\sim}{R_1: \tfrac{1}{4}R_1} \left(\begin{array}{cccc|cccc} 1 & 2 & -7/4 & 7/2 & \tfrac{1}{4} & 0 & 0 & 0 \\ 2 & 5 & -4 & 6 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & -7 & 0 & 0 & 1 & 0 \\ 3 & 6 & 5 & 10 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\underset{\sim}{\substack{R_2: -2R_1+R_2 \\ R_4: -3R_1+R_4}} \left(\begin{array}{cccc|cccc} 1 & 2 & -7/4 & 7/2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & -1 & -1/2 & 1 & 0 & 0 \\ 0 & 2 & 1 & -7 & 0 & 0 & 1 & 0 \\ 0 & 0 & 41/4 & -1/2 & -3/4 & 0 & 0 & 1 \end{array}\right) \underset{\sim}{R_3: -2R_2+R_3} \left(\begin{array}{cccc|cccc} 1 & 2 & -7/4 & 7/2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & -1 & -1/2 & 1 & 0 & 0 \\ 0 & 0 & 2 & -5 & 1 & -2 & 1 & 0 \\ 0 & 0 & 41/4 & -1/2 & -3/4 & 0 & 0 & 1 \end{array}\right) \underset{\sim}{R_3: \tfrac{1}{2}R_3}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & -7/4 & 7/2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & -1 & -1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -5/2 & 1/2 & -1 & 1/2 & 0 \\ 0 & 0 & 41/4 & -1/2 & -3/4 & 0 & 0 & 1 \end{array}\right) \underset{\sim}{R_4: -\tfrac{41}{4}R_3+R_4} \left(\begin{array}{cccc|cccc} 1 & 2 & -7/4 & 7/2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & -1 & -1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -5/2 & 1/2 & -1 & 1/2 & 0 \\ 0 & 0 & 0 & \tfrac{201}{8} & -47/8 & \tfrac{41}{4} & -\tfrac{41}{8} & \tfrac{41}{4} \end{array}\right) \underset{\sim}{R_4: \tfrac{8}{201}R_4}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & -7/4 & 7/2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & -1 & -1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -5/2 & 1/2 & -1 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & \tfrac{-47}{201} & \tfrac{82}{201} & -\tfrac{41}{201} & \tfrac{-82}{201} \end{array}\right) \underset{\sim}{\substack{R_1: -\tfrac{7}{2}R_4+R_1 \\ R_2: R_4+R_2 \\ R_3: \tfrac{5}{2}R_4+R_3}} \left(\begin{array}{cccc|cccc} 1 & 2 & -7/4 & 0 & \tfrac{731}{402} & -\tfrac{287}{201} & \tfrac{487}{402} & \tfrac{287}{201} \\ 0 & 1 & -1/2 & 0 & -\tfrac{283}{402} & \tfrac{283}{201} & -\tfrac{41}{201} & -\tfrac{82}{201} \\ 0 & 0 & 1 & 0 & -17/201 & 4/201 & -2/201 & -205/201 \\ 0 & 0 & 0 & 1 & -\tfrac{47}{201} & \tfrac{82}{201} & -\tfrac{41}{201} & -\tfrac{82}{201} \end{array}\right)$$

$$\underset{\sim}{\substack{R_2: \tfrac{1}{2}R_3+R_2 \\ R_1: \tfrac{7}{4}R_3+R_1}} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & \tfrac{1343}{804} & \tfrac{282}{201} & \tfrac{320}{268} & \tfrac{-287}{804} \\ 0 & 1 & 0 & 0 & \tfrac{-52}{67} & \tfrac{95}{67} & -\tfrac{14}{67} & \tfrac{-123}{134} \\ 0 & 0 & 1 & 0 & \tfrac{-17}{201} & \tfrac{4}{201} & \tfrac{-2}{201} & -\tfrac{205}{201} \\ 0 & 0 & 0 & 1 & \tfrac{-47}{201} & \tfrac{82}{201} & -\tfrac{41}{201} & \tfrac{-82}{201} \end{array}\right) \underset{\sim}{R_1: -2R_2+R_1} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \tfrac{2591}{804} & \tfrac{290}{201} & \tfrac{341}{268} & \tfrac{1189}{804} \\ 0 & 1 & 0 & 0 & \tfrac{-52}{67} & \tfrac{95}{67} & -\tfrac{14}{66} & \tfrac{-123}{134} \\ 0 & 0 & 1 & 0 & \tfrac{-17}{201} & \tfrac{4}{201} & \tfrac{-2}{201} & \tfrac{-205}{201} \\ 0 & 0 & 0 & 1 & \tfrac{-47}{201} & \tfrac{82}{201} & -\tfrac{41}{201} & \tfrac{-82}{201} \end{array}\right)$$

21 $\begin{bmatrix}1&1&2\\3&1&0\\-2&0&3\end{bmatrix}$ $\left(\begin{array}{ccc|ccc}1&1&2&1&0&0\\3&1&0&0&1&0\\-2&0&3&0&0&1\end{array}\right)\underset{\sim}{\begin{matrix}R_2:-3R_1+R_2\\R_3:2R_1+R_3\end{matrix}}\left(\begin{array}{ccc|ccc}0&-2&-6&-3&1&\\0&2&7&2&0&1\end{array}\right)$

$\left(\begin{array}{ccc|ccc}1&1&2&1&0&0\\0&1&3&3/2&-1/2&0\\0&2&7&2&0&1\end{array}\right)\underset{\sim}{R_3:-2R_2+R_3}\left(\begin{array}{ccc|ccc}1&1&2&1&0&0\\0&1&3&3/2&-1/2&0\\0&0&1&-1&1&1\end{array}\right)\underset{\sim}{\begin{matrix}R_2:-3R_3+R_2\\R_1:-2R_3+R_1\end{matrix}}$

$\left(\begin{array}{ccc|ccc}1&1&0&3&-2&-2\\0&1&0&9/2&-7/2&-3\\0&0&1&-1&1&1\end{array}\right)\underset{\sim}{R_1:-R_2+R_1}\left(\begin{array}{ccc|ccc}1&0&0&-3/2&3/2&1\\0&1&0&9/2&-7/2&-3\\0&0&1&-1&1&1\end{array}\right)$

$$AA^{-1}=\begin{pmatrix}1&1&2\\3&1&0\\-2&0&3\end{pmatrix}\begin{pmatrix}-3/2&3/2&1\\9/2&-7/2&-3\\-1&1&1\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$$A^{-1}A=\begin{pmatrix}-3/2&3/2&1\\9/2&-7/2&-3\\-1&1&1\end{pmatrix}\begin{pmatrix}1&1&2\\3&1&0\\-2&0&3\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$= I$

22. $\begin{bmatrix}3&2&5\\2&2&4\\-4&4&0\end{bmatrix}$ $\left[\begin{array}{ccc|ccc}3&2&5&1&0&0\\2&2&4&0&1&0\\-4&4&0&0&0&1\end{array}\right]\underset{\sim}{R_2\leftrightarrow R_1}\left[\begin{array}{ccc|ccc}2&2&4&0&1&0\\3&2&5&1&0&0\\-4&4&0&0&0&1\end{array}\right]\underset{\sim}{R_1:\tfrac{1}{2}R_1}$

$\left[\begin{array}{ccc|ccc}1&1&2&0&1/2&0\\3&2&5&1&0&0\\-4&4&0&0&0&1\end{array}\right]\underset{\sim}{\begin{matrix}R_2:-3R_1+R_2\\R_3:4R_1+R_3\end{matrix}}\left[\begin{array}{ccc|ccc}1&1&2&0&1/2&0\\0&-1&-1&1&-3/2&0\\0&8&8&0&2&1\end{array}\right]\underset{\sim}{R_2:-R_2}$

$\left[\begin{array}{ccc|ccc}1&1&2&0&1/2&0\\0&1&1&-1&3/2&0\\0&8&8&0&2&1\end{array}\right]\underset{\sim}{R_3:-8R_2+R_3}\left[\begin{array}{ccc|ccc}1&1&2&0&1/2&0\\0&1&1&-1&3/2&0\\0&0&0&8&10&1\end{array}\right]$ No tiene inversa

23. $\begin{bmatrix}0.1&0.2&0.3\\-0.3&0.2&0.2\\0.5&0.5&0.5\end{bmatrix}$ $\left[\begin{array}{ccc|ccc}0.1&0.2&0.3&1&0&0\\-0.3&0.2&0.2&0&1&0\\0.5&0.5&0.5&0&0&1\end{array}\right]\underset{\sim}{R_1:\tfrac{1}{0.1}R_1}\left[\begin{array}{ccc|ccc}1&2&3&10&0&0\\-0.3&0.2&0.2&0&1&0\\0.5&0.5&0.5&0&0&1\end{array}\right]$

$\underset{\sim}{\begin{matrix}R_2:0.3R_1+R_2\\R_3:-0.5R_1+R_3\end{matrix}}\left(\begin{array}{ccc|ccc}1&2&3&10&0&0\\0&0.8&1.1&3&1&0\\0&-0.5&-1&-5&0&1\end{array}\right)\underset{\sim}{\begin{matrix}R_2:\tfrac{1}{0.8}R_2\\R_3:\tfrac{1}{0.5}\end{matrix}}\left(\begin{array}{ccc|ccc}1&2&3&10&0&0\\0&1&1.375&3.75&1.25&0\\0&-1&-2&10&0&2\end{array}\right)\underset{\sim}{R_3:R_2+R_3}$

$\left(\begin{array}{ccc|ccc}1&2&3&10&0&0\\0&1&1.375&3.75&1.25&0\\0&0&-0.625&13.75&1.25&2\end{array}\right)\underset{\sim}{R_3:\tfrac{1}{-0.625}}\left(\begin{array}{ccc|ccc}1&2&3&10&0&0\\0&1&1.375&3.75&1.25&0\\0&0&1&-22&-200&-3.2\end{array}\right)\underset{\sim}{\begin{matrix}R_1:-3R_3+R_1\\R_2:-1.375R_3+R_2\end{matrix}}$

$\left(\begin{array}{ccc|ccc}1&2&0&76&600&9.6\\0&1&0&34&276.25&4.4\\0&0&1&-22&-200&-3.2\end{array}\right)\underset{\sim}{R_1:-2R_2+R_1}\left(\begin{array}{ccc|ccc}1&0&0&8&47.5&0.8\\0&1&0&34&276.25&4.4\\0&0&1&-22&-200&-3.2\end{array}\right)$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1: \frac{1}{2}R_1 \\ R_2: \frac{1}{3}R_2 \\ R_3: \frac{1}{5}R_3 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/5 \end{array}\right]$$

$$AA^{-1} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{-1}A = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

25. $\begin{bmatrix} -8 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix}$ No tiene inversa

26. $\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

$$\left[\begin{array}{cccc|cccc} 1 & 3 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 6 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_2: \frac{1}{2}R_2 \\ R_3: -\frac{1}{2}R_3 \\ R_4: \frac{1}{5}R_4 \end{array} \sim \left[\begin{array}{cccc|cccc} 1 & 3 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & -1/2 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1/5 \end{array}\right]$$

$$\begin{array}{l} R_2: -3R_4+R_2 \\ R_3: \frac{1}{2}R_4+R_3 \end{array} \sim \left(\begin{array}{cccc|cccc} 1 & 3 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1/2 & 0 & -3/5 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1/2 & 1/10 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1/5 \end{array}\right) \begin{array}{l} R_1: 2R_3+R_1 \\ R_2: -2R_3+R_2 \end{array} \sim \left(\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & 1 & 0 & -1 & 2/10 \\ 0 & 1 & 0 & 0 & 0 & 1/2 & 1 & -4/5 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 1/10 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1/5 \end{array}\right) R_1: -3R_2+R_1$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -3/2 & -4 & 13/5 \\ 0 & 1 & 0 & 0 & 0 & 1/2 & -1 & -4/5 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1/2 & 1/10 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1/5 \end{array}\right)$$

$$AA^{-1} = \begin{pmatrix} 1 & 3 & -2 & 0 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 & -3/2 & -4 & 13/5 \\ 0 & 1/2 & -1 & -4/5 \\ 0 & 0 & -1/2 & 1/10 \\ 0 & 0 & 0 & 1/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A^{-1}A = \begin{pmatrix} 1 & -3/2 & -4 & 13/5 \\ 0 & 1/2 & -1 & -4/5 \\ 0 & 0 & -1/2 & 1/10 \\ 0 & 0 & 0 & 1/5 \end{pmatrix}\begin{pmatrix} 1 & 3 & -2 & 0 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

27. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 5 & 5 \end{bmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 4 & 0 & 0 & 1 & 0 \\ 2 & 5 & 5 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} R_2: -3R_1+R_2 \\ R_3: -2R_1+R_3 \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & -3 & 1 & 0 \\ 0 & 5 & 5 & -2 & 0 & 1 \end{array}\right) R_2: \frac{1}{4}R_2 \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3/4 & \frac{1}{4} & 0 \\ 0 & 5 & 5 & -2 & 0 & 1 \end{array}\right)$$

$$R_3: -5R_2+R_3 \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3/4 & 1/4 & 0 \\ 0 & 0 & 5 & 7/4 & -5/4 & 1 \end{array}\right) R_3: \frac{1}{5}R_3 \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3/4 & 1/4 & 0 \\ 0 & 0 & 1 & 7/20 & -1/4 & 1/5 \end{array}\right)$$

$$AA^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 5 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -3/4 & 1/4 & 0 \\ 7/20 & -1/4 & 1/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{-1}A = \begin{pmatrix} 1 & 0 & 0 \\ -3/4 & 1/4 & 0 \\ 7/20 & -1/4 & 1/5 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 5 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

28. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 0 \\ 2 & 5 & 5 \end{bmatrix}$ $\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 & 0 \\ 2 & 5 & 5 & 0 & 0 & 1 \end{array}\right) \underset{\sim}{R_2: -3R_1+R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 2 & 5 & 5 & 0 & 0 & 1 \end{array}\right)$ No tiene inversa

29. $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 4 \\ 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 4 \end{bmatrix}$ $\left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 4 & 0 & 1 & 0 & 0 \\ 1 & 0 & 3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 4 & 0 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{\substack{R_3: -R_1+R_3 \\ R_2: \frac{1}{2}R_2}} \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 4 & 0 & 0 & 0 & 1 \end{array}\right]$ No tiene inverso

30. $\begin{bmatrix} \frac{1}{a} & 0 \\ a & a \end{bmatrix}$ $a \neq 0$ $\left[\begin{array}{cc|cc} \frac{1}{a} & 0 & 1 & 0 \\ a & a & 0 & 1 \end{array}\right] \underset{\sim}{\substack{R_1: aR_1 \\ R_2: \frac{1}{a}R_2}} \left[\begin{array}{cc|cc} 1 & 0 & a & 0 \\ 1 & 1 & 0 & \frac{1}{a} \end{array}\right] \underset{\sim}{R_2: -R_1+R_2} \left[\begin{array}{cc|cc} 1 & 0 & a & 0 \\ 0 & 1 & -a & \frac{1}{a} \end{array}\right]$

$a \neq 0$

Use una matriz inversa para resolver los sistemas de ecs. lineales dados.

31. a) $-x + y = 4$
$-2x + y = 0$ $\sim A = \begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix}$ $|A| = -1 - 2 = -3 \neq 0 \Rightarrow A^{-1} = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$

$AX = B$ $\begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}}_{2\times 2} \underbrace{\begin{pmatrix} 4 \\ 0 \end{pmatrix}}_{2\times 1}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

$A^{-1}AX = A^{-1}B$ $\therefore$ $x = 4$ $y = 8$

$X = A^{-1}B$

b) $-x + y = -3$
$-2x + y = +5$ $\begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$ $A = \begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix}$ $|A| = -1 - 2 = -3 \neq 0$

$A^{-1} = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ -11 \end{pmatrix}$ $x = -8$
$y = -11$

c) $-x + y = 0$
$-2x + y = 0$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $x = 0$ $y = 0$

32. a) $-2x + 3y = 5$
$x + 4y = 10$ $A = \begin{pmatrix} -2 & 3 \\ 1 & 4 \end{pmatrix}$ $|A| = -8 - 3 = -11 \neq 0 \Rightarrow A^{-1} = \frac{1}{|A|}\begin{pmatrix} 4 & -3 \\ -1 & -2 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{4}{11} & 3/11 \\ 1/11 & 2/11 \end{pmatrix}\begin{pmatrix} 5 \\ 10 \end{pmatrix} = \begin{pmatrix} 10/11 \\ 25/11 \end{pmatrix}$ $A^{-1} = \frac{-11}{-11}\begin{pmatrix} -4/11 & 3/11 \\ 1/11 & 2/11 \end{pmatrix}$

$x = \frac{10}{11}$ $y = \frac{25}{11}$

b) $2x + 3y = 0$
$x + 4y = 0$ $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ $|A| = 8 - 3 = 5$ $A^{-1} = \frac{1}{|A|}\begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix} = \frac{5}{5}\begin{pmatrix} 4/5 & -3 \\ -1 & 2/5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $x = 0$ $y = 0$

c) $2x + 3y = 1$
$x + 4y = -2$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad x=2 \quad y=-1$$

33. a) $3x + 2y + 2z = 0$
$2x + 2y + 2z = 5$
$-4x + 4y + 3z = 2$

$Ax = B$ $\begin{pmatrix} 3 & 2 & 2 \\ 2 & 2 & 2 \\ -4 & 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 3 & 2 & 2 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 1 & 0 \\ -4 & 4 & 3 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 2 & 2 & 2 & 0 & 1 & 0 \\ 3 & 2 & 2 & 1 & 0 & 0 \\ -4 & 4 & 3 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1: \frac{1}{2}R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1/2 & 0 \\ 3 & 2 & 2 & 1 & 0 & 0 \\ -4 & 4 & 3 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_3: 4R_1 + R_3]{R_2: -3R_1 + R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1/2 & 0 \\ 0 & -1 & -1 & 1 & -3/2 & 0 \\ 0 & 8 & 7 & 0 & 2 & 1 \end{array}\right) \xrightarrow{R_2: -R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1/2 & 0 \\ 0 & 1 & 1 & -1 & 3/2 & 0 \\ 0 & 8 & 7 & 0 & 2 & 1 \end{array}\right) \xrightarrow{R_3: -8R_2 + R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1/2 & 0 \\ 0 & 1 & 1 & -1 & 3/2 & 0 \\ 0 & 0 & -1 & 8 & -10 & 1 \end{array}\right)$$

$$\xrightarrow{R_3: -R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1/2 & 0 \\ 0 & 1 & 1 & -1 & 3/2 & 0 \\ 0 & 0 & 1 & -8 & 10 & -1 \end{array}\right) \xrightarrow[R_1: -R_3 + R_1]{R_2: -R_3 + R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 8 & -19/2 & 1 \\ 0 & 1 & 0 & 7 & -17/2 & 1 \\ 0 & 0 & 1 & -8 & 10 & -1 \end{array}\right) \xrightarrow{R_1: -R_2 + R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 7 & -17/2 & 1 \\ 0 & 1 & 1 & -8 & 10 & -1 \end{array}\right)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 7 & -17/2 & 1 \\ -8 & 10 & -1 \end{pmatrix}}_{3\times 3}\underbrace{\begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}}_{3\times 1} = \begin{pmatrix} -5 \\ -81/2 \\ 48 \end{pmatrix} \quad x=-5 \quad y=-\frac{81}{2} \quad z=48$$

b) $3x + 2y + 2z = -1$
$2x + 2y + 2z = 2$
$-4x + 4y + 3z = 0$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 7 & -17/2 & 1 \\ -8 & 10 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -24 \\ 28 \end{pmatrix} \quad x=-3 \quad y=-24 \quad z=28$$

c) $3x + 2y + 2z = 0$
$2x + 2y + 2z = 0$
$-4x + 4y + 3z = 0$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 7 & -17/2 & 1 \\ -8 & 10 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad x=0 \quad y=0 \quad z=0$$

34. $x_1 - 2x_2 - x_3 - 2x_4 = 0$
$3x_1 - 5x_2 - 2x_3 - 3x_4 = 1$
$2x_1 - 5x_2 - 2x_3 - 5x_4 = -1$
$-x_1 + 4x_2 + 4x_3 + 11x_4 = 2$

$$\begin{pmatrix} 1 & -2 & -1 & -2 \\ 3 & -5 & -2 & -3 \\ 2 & -5 & -2 & -5 \\ -1 & 4 & 4 & 11 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 2 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 1 & -2 & -1 & -2 & 1 & 0 & 0 & 0 \\ 3 & -5 & -2 & -3 & 0 & 1 & 0 & 0 \\ 2 & -5 & -2 & -5 & 0 & 0 & 1 & 0 \\ -1 & 4 & 4 & 11 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_4: R_1 + R_1]{R_2: -3R_1 + R_2,\ R_3: -2R_1 + R_3} \left(\begin{array}{cccc|cccc} 1 & -2 & -1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -2 & 0 & 1 & 0 \\ 0 & 2 & 3 & 9 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_4: -2R_2 + R_4]{R_3: R_2 + R_3}$$

$$\left(\begin{array}{cccc|cccc} 1 & -2 & -1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -5 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 7 & -2 & 0 & 1 \end{array}\right) \xrightarrow{R_4: -R_3 + R_4} \left(\begin{array}{cccc|cccc} 1 & -2 & -1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -5 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 12 & -3 & -1 & 1 \end{array}\right) \xrightarrow[R_2: -3R_4 + R_2,\ R_1: 2R_4 + R_1]{R_3: -2R_4 + R_3}$$

$$\left(\begin{array}{cccc|cccc} 1 & -2 & -1 & 0 & 25 & -6 & -2 & 2 \\ 0 & 1 & 1 & 0 & -39 & 10 & 3 & -3 \\ 0 & 0 & 1 & 0 & -29 & 7 & 3 & -2 \\ 0 & 0 & 0 & 1 & 12 & -3 & -1 & 1 \end{array}\right) \xrightarrow[R_1: R_3 + R_1]{R_2: -R_3 + R_2} \left(\begin{array}{cccc|cccc} 1 & -2 & 0 & 0 & -4 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -10 & 3 & 0 & -1 \\ 0 & 0 & 1 & 0 & -29 & 7 & 3 & -2 \\ 0 & 0 & 0 & 1 & 12 & -3 & -1 & 1 \end{array}\right) \xrightarrow{R_1: 2R_2 + R_1}$$

$$\left(\begin{array}{cccc|cccc} 1&0&0&0&-24&7&1&-2\\ 0&1&0&0&-10&3&0&-1\\ 0&0&1&0&-29&7&3&-2\\ 0&0&0&1&12&-3&-1&1\end{array}\right) \qquad \begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} = \begin{pmatrix}-24&7&1&-2\\-10&3&0&-1\\-29&7&3&-2\\12&-3&-1&1\end{pmatrix}\begin{pmatrix}0\\1\\-1\\2\end{pmatrix} = \begin{pmatrix}2\\1\\0\\0\end{pmatrix}$$

$x_1 = 2 \quad x_2 = 1 \quad x_3 = 0 \quad x_4 = 0$

b)
$$\begin{aligned} x_1 - 2x_2 - x_3 - 2x_4 &= 1\\ 3x_1 - 5x_2 - 2x_3 - 3x_4 &= -2\\ 2x_1 - 5x_2 - 2x_3 - 5x_4 &= 0\\ -x_1 + 4x_2 + 4x_3 + 11x_4 &= -3\end{aligned} \qquad \begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} = \begin{pmatrix}-24&7&1&-2\\-10&3&0&-1\\-29&7&3&-2\\12&-3&-1&1\end{pmatrix}\begin{pmatrix}1\\-2\\0\\-3\end{pmatrix} = \begin{pmatrix}-32\\-13\\-37\\15\end{pmatrix}$$

$x_1 = -32 \quad x_2 = -13 \quad x_3 = -37 \quad x_4 = 15$

Use las matrices para encontrar a) $(AB)^{-1}$ b) $(A^T)^{-1}$ c) A^{-2} d) $(2A)^{-1}$

35. $A^{-1} = \begin{pmatrix}2&5\\-7&6\end{pmatrix}, \quad B^{-1} = \begin{pmatrix}7&-3\\2&0\end{pmatrix}$

$A = \begin{pmatrix}6&-5\\7&2\end{pmatrix} \qquad B = \begin{pmatrix}0&3\\-2&7\end{pmatrix} \qquad AB = \begin{pmatrix}10&-17\\-4&35\end{pmatrix} \Rightarrow |AB| = 350 - 68 = 282$

$|AB| \neq 0$

$A^T = \begin{pmatrix}6&7\\-5&2\end{pmatrix} \quad |A^T| = 12 + 35 = 47 \qquad \Rightarrow (AB)^{-1} = \dfrac{1}{|AB|}\begin{pmatrix}35&17\\4&10\end{pmatrix}$

$|A^T| \neq 0$

$(A^T)^{-1} = \dfrac{1}{|A^T|}\begin{pmatrix}2&-7\\5&6\end{pmatrix} \qquad A^{-2} = A^{-1}A^{-1} = \begin{pmatrix}2&5\\-7&6\end{pmatrix}\begin{pmatrix}2&5\\-7&6\end{pmatrix} = \begin{pmatrix}-31&40\\-56&1\end{pmatrix}$

$2A = 2\begin{pmatrix}6&-5\\7&2\end{pmatrix} = \begin{pmatrix}12&-10\\14&4\end{pmatrix} \quad |2A| = 48 + 140 = 188$

$|2A| \neq 0$

$(2A)^{-1} = \begin{pmatrix}4&10\\-14&12\end{pmatrix}$

36. $A^{-1} = \begin{bmatrix}-\frac{2}{7}&\frac{1}{7}\\ \frac{3}{7}&\frac{2}{7}\end{bmatrix}, \quad B^{-1} = \begin{bmatrix}\frac{5}{11}&\frac{2}{11}\\ \frac{3}{11}&-\frac{1}{11}\end{bmatrix}$

$A = \begin{bmatrix}\frac{2}{7}&-\frac{1}{7}\\ -\frac{3}{7}&-\frac{2}{7}\end{bmatrix} \qquad B = \begin{bmatrix}-\frac{1}{11}&-\frac{2}{11}\\ -\frac{3}{11}&\frac{5}{11}\end{bmatrix}$

a) $(AB)^{-1}$

$$AB = \begin{pmatrix} \frac{2}{7} & -\frac{1}{7} \\ -\frac{3}{7} & -\frac{2}{7} \end{pmatrix}\begin{pmatrix} -\frac{1}{11} & -\frac{2}{11} \\ -\frac{3}{11} & \frac{5}{11} \end{pmatrix} = \begin{pmatrix} \frac{5}{77} & -\frac{9}{77} \\ \frac{9}{77} & -\frac{4}{77} \end{pmatrix} \Rightarrow |AB| = \left(\frac{5}{77}\right)\left(-\frac{4}{77}\right) - \left(-\frac{9}{77}\right)\left(\frac{9}{77}\right) = \frac{61}{5929} \neq 0$$

$$(AB)^{-1} = \begin{pmatrix} -\frac{4}{77} & \frac{9}{77} \\ -\frac{9}{77} & \frac{5}{77} \end{pmatrix}$$

b) $(A^T)^{-1}$ $\quad A^T = \begin{pmatrix} \frac{2}{7} & -\frac{3}{7} \\ -\frac{1}{7} & -\frac{2}{7} \end{pmatrix} \quad |A^T| = \left(\frac{2}{7}\right)\left(-\frac{2}{7}\right) - \left(-\frac{3}{7}\right)\left(-\frac{1}{7}\right) = -\frac{1}{7} \neq 0$

$$(A^T)^{-1} = \begin{pmatrix} -\frac{2}{7} & \frac{3}{7} \\ \frac{1}{7} & \frac{2}{7} \end{pmatrix}$$

c) $A^{-2} = A^{-1}A^{-1} = \begin{pmatrix} -\frac{2}{7} & \frac{1}{7} \\ \frac{3}{7} & \frac{2}{7} \end{pmatrix}\begin{pmatrix} -\frac{2}{7} & \frac{1}{7} \\ \frac{3}{7} & \frac{2}{7} \end{pmatrix} = \begin{pmatrix} \frac{1}{7} & 0 \\ 0 & \frac{1}{7} \end{pmatrix}$

d) $(2A)^{-1}$ $\quad 2A = 2\begin{pmatrix} \frac{2}{7} & -\frac{1}{7} \\ -\frac{3}{7} & -\frac{2}{7} \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & -\frac{2}{7} \\ -\frac{6}{7} & -\frac{4}{7} \end{pmatrix} \quad |2A| = -\frac{16}{49} - \frac{12}{49} = -\frac{4}{7} \neq 0$

$$(2A)^{-1} = \begin{pmatrix} -\frac{4}{7} & \frac{2}{7} \\ \frac{6}{7} & \frac{4}{7} \end{pmatrix}$$

37. $A^{-1} = \begin{pmatrix} 1 & -\frac{1}{2} & \frac{3}{4} \\ \frac{3}{2} & \frac{1}{2} & -2 \\ \frac{1}{4} & 1 & \frac{3}{2} \end{pmatrix} \quad B^{-1} = \begin{pmatrix} 2 & 4 & \frac{5}{2} \\ -\frac{3}{4} & 2 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 2 \end{pmatrix}$

$$A = \left(\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & \frac{3}{4} & 1 & 0 & 0 \\ \frac{3}{2} & \frac{1}{2} & -2 & 0 & 1 & 0 \\ \frac{1}{4} & 1 & \frac{3}{2} & 0 & 0 & 1 \end{array}\right) \begin{array}{l} R_2: -\frac{3}{2}R_1 + R_2 \\ R_3: -\frac{1}{4}R_1 + R_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & -1/2 & 3/4 & 1 & 0 & 0 \\ 0 & \frac{5}{4} & -\frac{25}{8} & -\frac{3}{2} & 1 & 0 \\ 0 & \frac{9}{8} & \frac{21}{16} & -\frac{1}{4} & 0 & 1 \end{array}\right) \begin{array}{l} R_2: \frac{4}{5}R_2 \\ R_3: \frac{8}{9}R_3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1/2 & 3/4 & 1 & 0 & 0 \\ 0 & 1 & -5/2 & -6/5 & 4/5 & 0 \\ 0 & 1 & 7/6 & -2/9 & 0 & 8/9 \end{array}\right) R_3: -R_2 + R_3 \left(\begin{array}{ccc|ccc} 1 & -1/2 & 3/4 & 1 & 0 & 0 \\ 0 & 1 & -5/2 & -6/5 & 4/5 & 0 \\ 0 & 0 & 11/3 & 44/45 & -4/5 & 8/9 \end{array}\right) R_3: \frac{3}{11}R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & -1/2 & 3/4 & 1 & 0 & 0 \\ 0 & 1 & -5/2 & -6/5 & 4/5 & 0 \\ 0 & 0 & 1 & 4/15 & -12/55 & \frac{8}{33} \end{array}\right) \begin{array}{l} R_1: -\frac{3}{4}R_3 + R_1 \\ R_2: \frac{5}{2}R_3 + R_2 \end{array} \left(\begin{array}{ccc|ccc} 1 & -1/2 & 0 & 4/5 & 9/55 & -2/11 \\ 0 & 1 & 0 & -8/15 & 14/55 & 20/33 \\ 0 & 0 & 1 & 4/15 & -12/55 & 8/33 \end{array}\right) R_1 = \frac{1}{2}R_2 + R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8/15 & 16/55 & 4/33 \\ 0 & 1 & 0 & -8/15 & 14/55 & 20/33 \\ 0 & 0 & 1 & 4/15 & -12/55 & 8/33 \end{array}\right)$$

$$AA^{-1} = \begin{pmatrix} 1 & -1/2 & 3/4 \\ 3/2 & 1/2 & -2 \\ 1/4 & 1 & 3/2 \end{pmatrix} \begin{pmatrix} 8/15 & 16/55 & 4/33 \\ -8/15 & 14/55 & 20/33 \\ 4/15 & -12/55 & 8/33 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B = \left(\begin{array}{ccc|ccc} 2 & 4 & \frac{5}{2} & 1 & 0 & 0 \\ -\frac{3}{4} & 2 & \frac{1}{4} & 0 & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & 2 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} R_1: \frac{1}{2}R_1 \\ R_2: 4R_2 \\ R_3: 4R_3 \\ \sim \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & \frac{5}{4} & \frac{1}{2} & 0 & 0 \\ -3 & 8 & 1 & 0 & 4 & 0 \\ 1 & 2 & 8 & 0 & 0 & 4 \end{array}\right) \begin{array}{l} R_2: 3R_1 + R_2 \\ R_3: -R_1 + R_3 \\ \sim \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & \frac{5}{4} & \frac{1}{2} & 0 & 0 \\ 0 & 14 & 19/4 & 3/2 & 4 & 0 \\ 0 & 0 & 27/4 & -\frac{1}{2} & 0 & 4 \end{array}\right)$$

$$\begin{array}{l} R_2: \frac{1}{14}R_2 \\ R_3: \frac{4}{27}R_3 \\ \sim \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 5/4 & 1/2 & 0 & 0 \\ 0 & 1 & 19/56 & 3/28 & 2/7 & 0 \\ 0 & 0 & 1 & -2/27 & 0 & 16/27 \end{array}\right) \begin{array}{l} R_2: -\frac{19}{56}R_3 + R_2 \\ R_1: -\frac{5}{4}R_3 + R_1 \\ \sim \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 16/27 & 0 & -20/27 \\ 0 & 1 & 0 & 25/189 & 2/7 & -38/189 \\ 0 & 0 & 1 & -2/27 & 0 & 16/27 \end{array}\right) \begin{array}{l} R_1: -2R_2 + R_1 \\ \sim \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 62/189 & -4/7 & -64/189 \\ 0 & 1 & 0 & 25/189 & 2/7 & -38/189 \\ 0 & 0 & 1 & -2/27 & 0 & 16/27 \end{array}\right)$$

R: $A^{-1} = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 6 & 5 & -3 \\ -2 & 4 & -1 \\ 1 & 3 & 4 \end{bmatrix}$

$$A = \left(\begin{array}{ccc|ccc} 1 & -4 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 4 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \overset{R_3: -4R_1 + R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & -4 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 18 & -7 & -4 & 0 & 1 \end{array}\right) \overset{R_3: -18R_2 + R_3}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & -4 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & -61 & -4 & -18 & 1 \end{array}\right) \overset{R_3: -\frac{1}{61}R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & -4 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 4/61 & 18/61 & -1/61 \end{array}\right) \overset{\substack{R_2: -3R_3 + R_2 \\ R_1: -2R_3 + R_1}}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & -4 & 0 & 53/61 & -36/61 & 2/61 \\ 0 & 1 & 0 & -12/61 & 7/61 & 3/61 \\ 0 & 0 & 1 & 4/61 & 18/61 & -1/61 \end{array}\right) \overset{R_1: 4R_2 + R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/61 & -8/61 & 14/61 \\ 0 & 1 & 0 & -12/61 & 7/61 & 3/61 \\ 0 & 0 & 1 & 4/61 & 18/61 & -1/61 \end{array}\right)$$

$$\underbrace{\begin{matrix} 5/61 & -8/61 & 14/61 \\ -12/61 & 7/61 & 3/61 \\ 4/61 & 18/61 & -1/61 \end{matrix}}_{A}$$

$$B = \left[\begin{array}{ccc|ccc} 6 & 5 & -3 & 1 & 0 & 0 \\ -2 & 4 & -1 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right] \overset{R_1 \leftrightarrow R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 0 & 0 & 1 \\ -2 & 4 & -1 & 0 & 1 & 0 \\ 6 & 5 & -3 & 1 & 0 & 0 \end{array}\right] \overset{\substack{R_2: 2R_1 + R_2 \\ R_3: -6R_1 + R_3}}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 0 & 0 & 1 \\ 0 & 10 & 7 & 0 & 1 & 2 \\ 0 & -13 & -27 & 1 & 0 & -6 \end{array}\right) \overset{R_2: \frac{1}{10}R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 0 & 0 & 1 \\ 0 & 1 & 7/10 & 0 & 1/10 & 1/5 \\ 0 & -13 & -27 & 1 & 0 & -6 \end{array}\right) \overset{R_3: 13R_2 + R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 0 & 0 & 1 \\ 0 & 1 & 7/10 & 0 & 1/10 & 1/5 \\ 0 & 0 & -\frac{179}{10} & 1 & 13/10 & -17/5 \end{array}\right)$$

$$\overset{R_3: -\frac{10}{179}R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 0 & 0 & 1 \\ 0 & 1 & 7/10 & 0 & 1/10 & 1/5 \\ 0 & 0 & 1 & -\frac{10}{179} & -\frac{13}{179} & \frac{34}{179} \end{array}\right) \overset{\substack{R_2: -\frac{7}{10}R_3 + R_2 \\ R_1: -4R_3 + R_1}}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & 40/179 & 52/179 & 43/179 \\ 0 & 1 & 0 & 7/179 & 27/179 & 12/179 \\ 0 & 0 & 1 & -10/179 & -13/179 & 34/179 \end{array}\right)$$

39. Encuentre x tal que la matriz A sea igual a su propia inversa.

$$A=\begin{bmatrix}3 & x\\ -2 & -3\end{bmatrix} \quad A^{-1}=\left(\begin{array}{cc|cc}3 & x & 1 & 0\\ -2 & -3 & 0 & 1\end{array}\right) \underset{R_2:\frac{1}{2}R_2}{\overset{R_1:\frac{1}{3}R_1}{}} \left(\begin{array}{cc|cc}1 & \frac{x}{3} & \frac{1}{3} & 0\\ -1 & -\frac{3}{2} & 0 & \frac{1}{2}\end{array}\right) \underset{\sim}{R_2: R_1+R_2}$$

$$\left(\begin{array}{cc|cc}1 & \frac{x}{3} & \frac{1}{3} & 0\\ 0 & \frac{2x-9}{6} & \frac{1}{3} & \frac{1}{2}\end{array}\right) \underset{\sim}{R_2: \frac{6}{2x-9}R_2} \left(\begin{array}{cc|cc}1 & \frac{x}{3} & \frac{1}{3} & 0\\ 0 & 1 & \frac{6}{6x-27} & \frac{6}{4x-18}\end{array}\right) \underset{\sim}{R_1: -\frac{x}{3}R_2+R_1} \left(\begin{array}{cc|cc}1 & 0 & \frac{-81}{54x-243} & -\frac{6x}{12x-54}\\ 0 & 1 & \frac{6}{6x-27} & \frac{6}{4x-18}\end{array}\right)$$

$$-\frac{6x}{12x-54}=x$$

$$12x^2-54x+6x=0$$
$$12x^2-48x=0$$
$$12x(x-4)=0$$
$$12x=0 \qquad x=0 \quad ó \quad x-4=0$$
$$\boxed{x=4}$$

$$\frac{-81}{54(4)-243}=3 \qquad \frac{6}{6(4)-27}=-2 \qquad \frac{6}{4(4)-18}=-3$$

$$A^{-1}=\begin{bmatrix}3 & 4\\ -2 & -3\end{bmatrix}$$

40. Encuentre x tal que la matriz A sea singular.

$$A=\begin{bmatrix}4 & x\\ -2 & -3\end{bmatrix} \quad |A| = 4(-3)-(-2x)$$

Para que sea SINGULAR $|A|=0 \rightarrow -12+2x=0$

$$2x=12$$
$$x=12/2 \Longrightarrow \quad 4(-3)-(6(-2))=0$$
$$x=6 \qquad -12+12=0$$
$$0=0$$

41. Encuentre A dado que: $(2A)^{-1}=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$

$$2A=\left(\begin{array}{cc|cc}1 & 2 & 1 & 0\\ 3 & 4 & 0 & 1\end{array}\right) \underset{\sim}{R_2: -3R_1+R_2} \left(\begin{array}{cc|cc}1 & 2 & 1 & 0\\ 0 & -2 & -3 & 1\end{array}\right) \underset{\sim}{R_2: -\frac{1}{2}R_2} \left(\begin{array}{cc|cc}1 & 2 & 1 & 0\\ 0 & 1 & \frac{3}{2} & -1/2\end{array}\right)$$

$$\underset{\sim}{R_1: -2R_2+R_1} \left(\begin{array}{cc|cc}1 & 0 & -2 & 1\\ 0 & 1 & \frac{3}{2} & -1/2\end{array}\right) \quad A=\begin{pmatrix}-1 & \frac{1}{2}\\ 3/4 & -\frac{1}{4}\end{pmatrix}$$

42. Demuestre que A es invertible y encuentre A^{-1}.

$$A=\begin{bmatrix} \operatorname{sen}\theta & \cos\theta \\ -\cos\theta & \operatorname{sen}\theta \end{bmatrix} \quad |A| = \operatorname{sen}^2\theta + \cos^2\theta = 1 \neq 0 \therefore \text{Existe } A^{-1}$$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} \operatorname{sen}\theta & -\cos\theta \\ \cos\theta & \operatorname{sen}\theta \end{bmatrix} = \frac{1}{1}\begin{bmatrix} \operatorname{sen}\theta & -\cos\theta \\ \cos\theta & \operatorname{sen}\theta \end{bmatrix} = \begin{bmatrix} \operatorname{sen}\theta & -\cos\theta \\ \cos\theta & \operatorname{sen}\theta \end{bmatrix}$$

¿En qué condiciones la matriz diagonal A es invertible? Encuentre su inversa.

$a_{ij} \neq 0$ Donde $i=j$

$$A=\begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix} \quad A^{-1} = \left[\begin{array}{ccccc|cccc} a_{11} & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn} & 0 & 0 & & 1 \end{array}\right]$$

$$\underset{\sim}{\frac{R_{ij}}{a_{ij}}} \left[\begin{array}{ccccc|cccc} 1 & 0 & 0 & \cdots & 0 & \frac{1}{a_{11}} & 0 & 0 \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & \frac{1}{a_{22}} & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & & & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & \frac{1}{a_{nn}} \end{array}\right]$$

Encuentre A^{-1} de...

a) $A=\begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $A^{-1}=\left[\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$ $\begin{matrix} R_1: \frac{1}{-1}R_1 \\ R_2: \frac{1}{3}R_2 \\ R_3: \frac{1}{2}R_3 \end{matrix}$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2} & 0 \end{array}\right]$ $\underbrace{}_{A^{-1}}$

b) $A=\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ $A^{-1}=\left[\begin{array}{ccc|ccc} \frac{1}{2} & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 1 \end{array}\right]$ $\begin{matrix} R_1: \frac{\frac{1}{2}}{\frac{1}{2}}R_1 \\ R_2: \frac{1/3}{1/3}R_2 \\ R_3: \frac{1/4}{1/4}R_3 \end{matrix}$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 4 \end{array}\right]$ $\underbrace{}_{A^{-1}}$

Demuestre que si C es una matriz invertible tal que $CA = CB \rightarrow A = B$

$$CA = CB$$
$$C^{-1}CA = C^{-1}CB$$
$$IA = IB$$
$$A = B$$

Demuestre que si A, B y C son matrices cuadradas y $ABC = I \rightarrow B$ es invertible y $B^{-1} = CA$

$$ABC = I \qquad B = B^{-1} I$$
$$A\, B^{-1} I\, C = I$$
$$A\, CA\, C\, I = I$$

Encuentre una representación paramétrica

1. $-4x + 2y - 6z = 1$ Sea $z = t$ y $x = s$ $\forall\, t \,\&\, s \in \mathbb{R}$.

$2y = 1 + 6z + 4x$

$y = \frac{1}{2} + 3z + 2x$

$\therefore$ $x = s$

$y = \frac{1}{2} + 3t + 2s$

$z = t$

Determine si la matriz dada está en forma escalonada y si es así determine si también está en la forma escalonada reducida.

3. $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Es escalonada pero no reducida

5. $\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Escalonada no reducida.

Encuentre el conjunto solución del sist. de ecs. lineales representado por la matriz dada.

7. $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $x + 2y = 0$, $z = 0$ $x = -2y$ Sea $y = t$ $\forall\, t \in \mathbb{R}$

$\therefore x = -2t$

Resuelva el sistema de ecs. dado.

9. $x + y = 2$, $x - y = 0$ $x = y$ $y + y = 2$

$2y = 2$ $\therefore \underline{x = 1}$

$\underline{y = 1}$

10. $y = 2x$, $y = x + 4$ $2x = x + 4$ $\underline{x = 4}$ $y = 2(4)$ $\underline{y = 8}$

13. $y + x = 0$, $2x + y = 0$ $\sim \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 1 & 0 \end{array}\right) \underset{\sim}{R_2: -2R_1 + R_2} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -1 & 0 \end{array}\right) \underset{\sim}{R_2: -R_2} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right) \underset{\sim}{R_1: -R_2 + R_1} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)$

$\therefore x = 0$

$y = 0$

15. $x - y = 9$, $-x + y = 1$ $\sim \left(\begin{array}{cc|c} 1 & -1 & 9 \\ -1 & 1 & 1 \end{array}\right) \underset{\sim}{R_2: R_1 + R_2} \left(\begin{array}{cc|c} 1 & -1 & 9 \\ 0 & 0 & 10 \end{array}\right)$ Sin Solución

17. $0.2x_1 + 0.3x_2 = 0.14$, $0.4x_1 + 0.5x_2 = 0.20$ $\sim \left(\begin{array}{cc|c} 0.2 & 0.3 & 0.14 \\ 0.4 & 0.5 & 0.2 \end{array}\right) \underset{R_2: \frac{1}{0.4}R_2}{\overset{R_1: \frac{1}{0.2}R_1}{\sim}} \left(\begin{array}{cc|c} 1 & 1.5 & 0.7 \\ 1 & 1.25 & 0.5 \end{array}\right)$

$\underset{\sim}{R_2: -R_1 + R_2} \left(\begin{array}{cc|c} 1 & 1.5 & 0.7 \\ 0 & -0.25 & -0.2 \end{array}\right) \underset{\sim}{R_2: \frac{1}{-0.25}} \left(\begin{array}{cc|c} 1 & 1.5 & 0.7 \\ 0 & 1 & 0.8 \end{array}\right) \underset{\sim}{R_1: -1.5R_2 + R_1} \left(\begin{array}{cc|c} 1 & 0 & -0.5 \\ 0 & 1 & 0.8 \end{array}\right)$

$x_1 = -0.5$ $x_2 = 0.8$

19. $\frac{1}{2}x - \frac{1}{3}y = 0$
$3x + 2(y+5) = 10$
$\sim$ $\frac{1}{2}x - \frac{1}{3}y = 0$
$3x + 2y + 10 = 10$
$\sim$ $\frac{1}{2}x - \frac{1}{3}y = 0$
$3x + 2y = 0$

$$\sim \left(\begin{array}{cc|c} \frac{1}{2} & -\frac{1}{3} & 0 \\ 3 & 2 & 0 \end{array}\right) \overset{R_1: 2R_1}{\sim} \left(\begin{array}{cc|c} 1 & -\frac{2}{3} & 0 \\ 3 & 2 & 0 \end{array}\right) \overset{R_2: -3R_1 + R_2}{\sim} \left(\begin{array}{cc|c} 1 & -\frac{2}{3} & 0 \\ 0 & 4 & 0 \end{array}\right) \overset{R_2: \frac{1}{4}R_2}{\sim} \left(\begin{array}{cc|c} 1 & -\frac{2}{3} & 0 \\ 0 & 1 & 0 \end{array}\right)$$

$$\overset{R_1 = \frac{2}{3}R_2 + R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right) \quad \begin{array}{l} x = 0 \\ y = 0 \end{array}$$

21. $-x + y + 2z = 1$
$2x + 3y + z = -2$
$5x + 4y + 2z = 4$

$$\sim \left(\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{array}\right) \overset{R_1: -R_1}{\sim} \left(\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{array}\right) \overset{\substack{R_2: -2R_1 + R_2 \\ R_3: -5R_1 + R_3}}{\sim}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 5 & 5 & 0 \\ 0 & 9 & 12 & 9 \end{array}\right) \overset{R_2: \frac{1}{5}R_2}{\sim} \left(\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 9 & 12 & 9 \end{array}\right) \overset{R_3: -9R_2 + R_3}{\sim} \left(\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 9 \end{array}\right)$$

$$\overset{R_3: \frac{1}{3}R_3}{\sim} \left(\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right) \overset{\substack{R_2: -R_3 + R_2 \\ R_1: 2R_3 + R_1}}{\sim} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right) \overset{R_1: R_2 + R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$x = 2 \quad y = -3 \quad z = 3$

23. $2x + 3y + 3z = 3$
$6x + 6y + 12z = 13$
$12x + 9y - z = 2$

$$\sim \left(\begin{array}{ccc|c} 2 & 3 & 3 & 3 \\ 6 & 6 & 12 & 13 \\ 12 & 9 & -1 & 2 \end{array}\right) \overset{R_1: \frac{1}{2}R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 3/2 & 3/2 & 3/2 \\ 6 & 6 & 12 & 13 \\ 12 & 9 & -1 & 2 \end{array}\right)$$

$$\overset{\substack{R_2: -6R_1 + R_2 \\ R_3: -12R_1 + R_3}}{\sim} \left(\begin{array}{ccc|c} 1 & 3/2 & 3/2 & 3/2 \\ 0 & -3 & 3 & 4 \\ 0 & -9 & -19 & -16 \end{array}\right) \overset{R_2: -\frac{1}{3}R_2}{\sim} \left(\begin{array}{ccc|c} 1 & 3/2 & 3/2 & 3/2 \\ 0 & 1 & -1 & -4/3 \\ 0 & -9 & -19 & -16 \end{array}\right) \overset{R_3: 9R_2 + R_3}{\sim}$$

$$\left(\begin{array}{ccc|c} 1 & 3/2 & 3/2 & 3/2 \\ 0 & 1 & -1 & -4/3 \\ 0 & 0 & -28 & -28 \end{array}\right) \overset{R_3: -\frac{1}{28}}{\sim} \left(\begin{array}{ccc|c} 1 & 3/2 & 3/2 & 3/2 \\ 0 & 1 & -1 & -4/3 \\ 0 & 0 & 1 & 1 \end{array}\right) \overset{\substack{R_2: R_3 + R_2 \\ R_1: -\frac{3}{2}R_3 + R_1}}{\sim} \left(\begin{array}{ccc|c} 1 & 3/2 & 0 & 0 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\overset{R_1: -\frac{3}{2}R_2 + R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad \begin{array}{l} x = \frac{1}{2} \\ y = -1/3 \\ z = 1 \end{array}$$

25.

25. $\cancel{2x + 3y + 3z = 3}$

$$\begin{aligned} 2x - 3y \quad &= -7 \\ -x + 3y - 3z &= 11 \end{aligned} \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & -6 \\ 2 & -3 & 0 & -7 \\ -1 & 3 & -3 & 11 \end{array}\right) \underset{\sim}{\substack{R_2: -2R_1+R_2 \\ R_3: R_1+R_3}} \left(\begin{array}{ccc|c} 1 & -2 & 1 & -6 \\ 0 & 1 & -2 & 5 \\ 0 & 1 & -2 & 5 \end{array}\right)$$

$$\underset{\sim}{R_3: -R_2+R_3} \left(\begin{array}{ccc|c} 1 & -2 & 1 & -6 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right) \infty \text{ de Sol.} \rightarrow \begin{aligned} x - 2y + z &= -6 \\ y - 2z &= 5 \end{aligned}$$

$y = 2z + 5$ Sea $t = z \quad \forall t \in \mathbb{R}$

$x = 2y - z - 6$

$x = 2(2z + 5) - 6$

$x = 4z + 10 - 6 - t$

$x = 3z + 4$

$\hookrightarrow \therefore \quad x = 3t + 4$

$y = 2t + 5$

$z = t$

27. $$\begin{aligned} 2x + y + 2z &= 4 \\ 2x + 2y \quad &= 5 \\ 2x - y + 6z &= 2 \end{aligned} \sim \left(\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{array}\right) \underset{\sim}{R_1: \tfrac{1}{2}R_1} \left(\begin{array}{ccc|c} 1 & 1/2 & 1 & 2 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{array}\right) \underset{\sim}{\substack{R_2: -2R_1+R_2 \\ R_3: -2R_1+R_3}} \left(\begin{array}{ccc|c} 1 & 1/2 & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & -2 & 4 & -2 \end{array}\right)$$

$$\underset{\sim}{R_3: 2R_2+R_3} \left(\begin{array}{ccc|c} 1 & 1/2 & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 2 & -1 \end{array}\right) \underset{\sim}{R_3: \tfrac{1}{2}R_3} \left(\begin{array}{ccc|c} 1 & 1/2 & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1/2 \end{array}\right) \underset{R_1: -R_3+R_1}{\overset{R_2: 2R_3+R_2}{\sim}} \left(\begin{array}{ccc|c} 1 & 1/2 & 0 & 5/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/2 \end{array}\right) \underset{\sim}{R_1: -\tfrac{1}{2}R_2+R_1}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/2 \end{array}\right) \text{ Sol única } \therefore \quad \begin{aligned} x &= 5/2 \\ y &= 0 \\ z &= -1/2 \end{aligned}$$

29. $$\begin{aligned} 2x_1 + x_2 + x_3 + 2x_4 &= -1 \\ 5x_1 - 2x_2 + x_3 - 3x_4 &= 0 \\ -x_1 + 3x_2 + 2x_3 + 2x_4 &= 1 \\ 3x_1 + 2x_2 + 3x_3 - 5x_4 &= 12 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 1 & 2 & -1 \\ 5 & -2 & 1 & -3 & 0 \\ -1 & 3 & 2 & 2 & 1 \\ 3 & 2 & 3 & -5 & 12 \end{array}\right) \underset{\sim}{R_1: \tfrac{1}{2}R_1} \left(\begin{array}{cccc|c} 1 & 1/2 & 1/2 & 1 & -1/2 \\ 5 & -2 & 1 & -3 & 0 \\ -1 & 3 & 2 & 2 & 1 \\ 3 & 2 & 3 & -5 & 12 \end{array}\right) \underset{\sim}{\substack{R_2: -5R_1+R_2 \\ R_3: R_1+R_2 \\ R_4: -3R_1+R_4}}$$

$$\left(\begin{array}{cccc|c} 1 & 1/2 & 1/2 & 1 & -1/2 \\ 0 & -9/2 & -3/2 & -8 & 5/2 \\ 0 & 7/2 & 5/2 & 3 & 1/2 \\ 0 & 1/2 & 3/2 & -8 & 27/2 \end{array}\right) \underset{\sim}{\substack{R_2: -\frac{2}{9}R_2 \\ R_3: 2R_3 \\ R_4: 2R_4}} \left(\begin{array}{cccc|c} 1 & 1/2 & 1/2 & 1 & -1/2 \\ 0 & 1 & 1/3 & 16/9 & -5/9 \\ 0 & 7 & 5 & 6 & 1 \\ 0 & 1 & 3 & -16 & 27 \end{array}\right) \underset{\sim}{\substack{R_3: -7R_2+R_3 \\ R_4: -R_2+R_4}}$$

$$\left(\begin{array}{cccc|c} 1 & 1/2 & 1/2 & 1 & -1/2 \\ 0 & 1 & 1/3 & 16/9 & -5/9 \\ 0 & 0 & 8/3 & -58/9 & 44/9 \\ 0 & 0 & 8/3 & -160/9 & 248/9 \end{array}\right) \underset{R_3: \frac{3}{8}R_3}{\overset{R_4: -R_3+R_4}{\sim}} \left(\begin{array}{cccc|c} 1 & 1/2 & 1/2 & 1 & -1/2 \\ 0 & 1 & 1/3 & 16/9 & -5/9 \\ 0 & 0 & 1 & -29/12 & 11/6 \\ 0 & 0 & 0 & -34/3 & 68/3 \end{array}\right) R_4: -\frac{3}{34}$$

$$\left(\begin{array}{cccc|c} 1 & 1/2 & 1/2 & 1 & -1/2 \\ 0 & 1 & 1/3 & 16/9 & -5/9 \\ 0 & 0 & 1 & -29/12 & 11/6 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right) \begin{array}{l} R_1: -R_4+R_1 \\ R_2: -\frac{16}{9}R_4+R_2 \\ R_3: \frac{29}{12}R_4+R_3 \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1/2 & 1/2 & 0 & 3/2 \\ 0 & 1 & 1/3 & 0 & 3 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right) \begin{array}{l} R_1: -\frac{1}{2}R_3+R_1 \\ R_2: -\frac{1}{3}R_3+R_2 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right) \overset{R_1: -\frac{1}{2}R_2+R_1}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right) \therefore \begin{array}{l} x = 1 \\ x_1 = 4 \\ x_2 = -3 \\ x_3 = -2 \end{array}$$

Resuelva el sistema homogeneo de ecuaciones lineales que se da.

31. $x_1 - 2x_2 - 8x_2 = 0$
$x_1 + 2x_2 + 12x_2 = 0$
$2x + x_2 + 4x_2 = 0$

$$\to \left(\begin{array}{ccc|c} 1 & -2 & -8 & 0 \\ 1 & 2 & 12 & 0 \\ 2 & 1 & 4 & 0 \end{array}\right) \overset{\begin{array}{l} R_2: -R_1+R_2 \\ R_3: -2R_1+R_2 \end{array}}{\sim} \left(\begin{array}{ccc|c} 1 & -2 & -8 & 0 \\ 0 & 4 & 20 & 0 \\ 0 & 5 & 20 & 0 \end{array}\right) \underset{R_3: -5R_2+R_3}{\overset{R_2: \frac{1}{4}R_2}{\sim}}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & -8 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & -5 & 0 \end{array}\right) \overset{R_3: -\frac{1}{5}R_3}{\sim} \left(\begin{array}{ccc|c} 1 & -2 & -8 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \overset{\begin{array}{l} R_1: 8R_3+R_1 \\ R_2: -5R_3+R_1 \end{array}}{\sim} \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \overset{R_1: 2R_2+R_1}{\sim}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

5. Determine el valor de K tal que el sistema sea inconsistente.

$$\begin{aligned} Kx + y &= 0 \\ x + Ky &= 1 \end{aligned} \sim \left(\begin{array}{cc|c} K & 1 & 0 \\ 1 & K & 1 \end{array}\right) \overset{R_1: \frac{1}{K}}{\sim} \left(\begin{array}{cc|c} 1 & \frac{1}{K} & 0 \\ 1 & K & 1 \end{array}\right) \overset{R_2: -R_1+R_2}{\sim}$$

$$\left(\begin{array}{cc|c} 1 & \frac{1}{K} & 0 \\ 0 & K-\frac{1}{K} & 1 \end{array}\right)$$

Para que el sist. sea inconsistente $K - \frac{1}{K}$ tiene que ser igual a 0

$\rightarrow$ $K - \frac{1}{K} = 0$ $\quad K = \frac{1}{K}$ $\quad K^2 = 1$

$\therefore K = 1$ //

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 1 \end{array}\right) \overset{R_2: -R_1+R_2}{\sim} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ \boxed{0 & 0 & 1} \end{array}\right)$$

* 7) Determine que las dos matrices sig. son equivalentes por renglones.

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & 2 \\ 3 & 1 & 2 \end{bmatrix} \text{ y } \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 6 \\ 5 & 5 & 10 \end{bmatrix}$$

~~Se dice que $A_{m \times n}$ es equivalente por filas (renglones) a $B_{m \times n}$ si partiendo de A podemos obtener B efectuando un número finito de operaciones elementales por columnas~~

$$\left(\quad \right)$$

③ Encuentre (de ser posible) condiciones sobre a, b y c de modo que el sig. sistema de ecs. lineales
a) no tenga solucion b) tenga exactamente una solución y
c) que tenga infinidad de soluciones.

$$\begin{aligned} 2x - y + z &= a \\ x + y + 2z &= b \\ 3y + 3z &= c \end{aligned} \sim \left(\begin{array}{ccc|c} 2 & -1 & 1 & a \\ 1 & 4 & 2 & b \\ 0 & 3 & 3 & c \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 4 & 2 & b \\ 2 & -1 & 1 & a \\ 0 & 3 & 3 & c \end{array}\right)$$

$$\xrightarrow{R_2 := -2R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 4 & 2 & b \\ 0 & -9 & -3 & -2b+a \\ 0 & 3 & 3 & c \end{array}\right) \xrightarrow[R_3 := \frac{1}{3}R_3]{R_2 := -\frac{1}{9}R_2} \left(\begin{array}{ccc|c} 1 & 4 & 2 & b \\ 0 & 1 & 1/3 & \frac{2b-a}{9} \\ 0 & 1 & 1 & c/3 \end{array}\right) \xrightarrow{R_3 := -R_2 + R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 2 & b \\ 0 & 1 & 1/3 & \frac{2b-a}{9} \\ 0 & 0 & 2/3 & \frac{3c-2b-a}{9} \end{array}\right) \xrightarrow{R_3 : \frac{3}{2}R_3} \left(\begin{array}{ccc|c} 1 & 4 & 2 & b \\ 0 & 1 & 1/3 & \frac{2b-a}{9} \\ 0 & 0 & 1 & \frac{3c-2b-a}{6} \end{array}\right) \xrightarrow[R_1 : -2R_3 + R_1]{R_2 : -\frac{1}{3}R_3 + R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 0 & \frac{a+5b-3c}{3} \\ 0 & 1 & 0 & \frac{6b-a-3c}{18} \\ 0 & 0 & 1 & \frac{3c-2b-a}{6} \end{array}\right) \xrightarrow{R_1 : -4R_2 + R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2a+6c-12b}{9} \\ 0 & 1 & 0 & \frac{6b-a-3c}{18} \\ 0 & 0 & 1 & \frac{3c-2b-a}{6} \end{array}\right)$$

a) No tenga Solución ✗
La matriz se pudo escalonar ∴ Debe de tener solución única o ∞ de soluciones.

b) Exactamente una solución

$$\begin{aligned} 2a + 6c - 12b &\neq 0 \\ 6b - a - 3c &\neq 0 \\ 3c - 2b - a &\neq 0 \end{aligned}$$

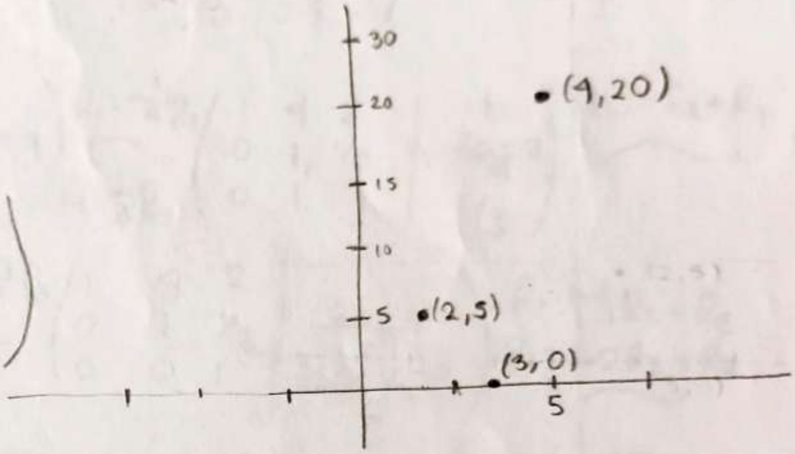
c) Infinidad de Soluciones

7. Encuentre un sist. inconsistente de ecs. lineales que tenga más variables que ecuaciones.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 2 \\ 2x_1 + 4x_2 + 6x_3 &= 12 \end{aligned} \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 12 \end{array}\right) \overset{R_2:-2R_1+R_2}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & 8 \end{array}\right)$$

9. Determine el polinomio cuya gráfica pasa por los puntos dados y bosqueje la gráfica del polinomio, mostrando los puntos dados.

$(2,5)(3,0)(4,20)$



$$\begin{aligned} 4x + 20y &= 0 \\ 2x + 5y &= 0 \\ 3x + 0y &= 0 \end{aligned}$$

$$\left(\begin{array}{cc|c} 4 & 20 & 0 \\ 2 & 5 & 0 \\ 3 & 0 & 0 \end{array}\right) \overset{R_1:\frac{1}{4}R_1}{\sim} \left(\begin{array}{cc|c} 1 & 5 & 0 \\ 2 & 5 & 0 \\ 3 & 0 & 0 \end{array}\right)$$

$$\overset{\substack{R_2:-2R_1+R_2 \\ R_3:-3R_1+R_3}}{\sim} \left(\begin{array}{cc|c} 1 & 5 & 0 \\ 0 & -5 & 0 \\ 0 & -15 & 0 \end{array}\right) \overset{R_2:-\frac{1}{5}R_2}{\sim} \left(\begin{array}{cc|c} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & -15 & 0 \end{array}\right)$$

$$\overset{R_3:15R_2+R_3}{\sim} \left(\begin{array}{cc|c} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \overset{R_1:-5R_2+R_1}{\sim} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

∞ de sol.

$$\begin{aligned} 4x_1 + 20y &= 0 \\ 2x_2 + 5y &= 0 \\ 3x + 0y &= 0 \end{aligned}$$

$XY = B$

$Ax = B$

Una empresa tiene ventas (medidas en millones) de 50, 60 y 75 dólares durante 3 años consecutivos. Encuentre una función cuadrática que se ajuste a estos datos y use el resultado para predecir las ventas durante el cuarto año.

$$f(1) = a(1)^2 + b(1) + c = 50$$
$$f(2) = a(2)^2 + b(2) + c = 60$$
$$f(3) = a(3)^2 + b(3) + c = 75$$

$$\Rightarrow \begin{aligned} a + b + c &= 50 \\ 4a + 2b + c &= 60 \\ 9a + 3b + c &= 75 \end{aligned} \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 4 & 2 & 1 & 60 \\ 9 & 3 & 1 & 75 \end{array}\right) \begin{array}{l} R_2: -4R_1 + R_2 \\ R_3: -9R_1 + R_3 \end{array} \sim$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & -2 & -3 & -140 \\ 0 & -6 & -8 & -375 \end{array}\right) \underset{\sim}{R_2: -\tfrac{1}{2}R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 1 & 3/2 & 70 \\ 0 & -6 & -8 & -375 \end{array}\right) \underset{\sim}{R_3: 6R_2 + R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 1 & 3/2 & 70 \\ 0 & 0 & 1 & 45 \end{array}\right)$$

$$\begin{array}{l} R_2: -\tfrac{3}{2}R_3 + R_2 \\ R_1: -R_3 + R_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 1 & 0 & 5/2 \\ 0 & 0 & 1 & 45 \end{array}\right) \underset{\sim}{R_1: -R_2 + R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5/2 \\ 0 & 1 & 0 & 5/2 \\ 0 & 0 & 1 & 45 \end{array}\right)$$

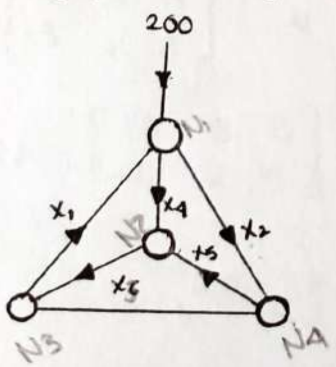
$$\Rightarrow f(x) = \frac{5}{2}x^2 + \frac{5}{2}x + 45$$

$$\therefore f(4) = \frac{5}{2}(4)^2 + \frac{5}{2}(4) + 45$$
$$= 40 + 10 + 45$$
$$= \underline{95 \text{ millones de dólares}}$$

13. En la Figura se muestra el flujo a traves de una red.

a) Resuelva el sist. para x_i, $i = 1, 2, \ldots 6$.

b) Encuentre el flujo cuando $x_3 = 100$, $x_5 = 50$ y $x_6 = 50$



$$\Sigma_{in} = \Sigma_{out}$$

$$200 - x_4 - x_2 + x_1 = 0$$
$$x_5 + x_4 - x_3 = 0$$
$$x_3 - x_1 = 0$$
$$x_2 - x_5 = 0$$

De 1 a 6 encuentre a) $A+B$ b) $A-B$ c) $2A$ d) $2A-B$

(1) $A=\begin{pmatrix}1 & -1\\ 2 & -1\end{pmatrix}$, $B=\begin{pmatrix}2 & -1\\ -1 & 8\end{pmatrix}$

$A+B=\begin{pmatrix}3 & -2\\ 1 & 7\end{pmatrix}$ $A-B=\begin{pmatrix}-1 & 0\\ 3 & -9\end{pmatrix}$ $2A=\begin{pmatrix}2 & -2\\ 4 & -2\end{pmatrix}$ $2A-B=\begin{pmatrix}0 & -1\\ 5 & -10\end{pmatrix}$

(3) $A=\begin{pmatrix}6 & -1\\ 2 & 4\\ -3 & 5\end{pmatrix}$ $B=\begin{pmatrix}1 & 4\\ -1 & 5\\ 1 & 10\end{pmatrix}$

$A+B=\begin{pmatrix}7 & 3\\ 1 & 9\\ -2 & 15\end{pmatrix}$ $A-B=\begin{pmatrix}5 & -5\\ 3 & -1\\ -4 & -5\end{pmatrix}$ $2A=\begin{pmatrix}12 & -2\\ 4 & 8\\ -6 & 10\end{pmatrix}$ $2A-B=\begin{pmatrix}11 & -6\\ 5 & 3\\ -7 & 0\end{pmatrix}$

(5) $A=\begin{pmatrix}2 & 2 & -1 & 0 & 1\\ 1 & 1 & -2 & 0 & -1\end{pmatrix}$ $B=\begin{pmatrix}1 & 1 & -1 & 1 & 0\\ -3 & 4 & 9 & -6 & -7\end{pmatrix}$

$A+B=\begin{pmatrix}3 & 3 & -2 & 1 & 1\\ -2 & 5 & 7 & -6 & -8\end{pmatrix}$ $A-B=\begin{pmatrix}1 & 1 & 0 & -1 & 1\\ 4 & -3 & -11 & 6 & 6\end{pmatrix}$

$2A=\begin{pmatrix}4 & 4 & -2 & 0 & 2\\ 2 & 2 & -4 & 0 & -2\end{pmatrix}$ $2A-B=\begin{pmatrix}3 & 3 & -1 & -1 & 2\\ 5 & -2 & -13 & 6 & 5\end{pmatrix}$

(7) Encuentre a) C_{21} y b) C_{13} donde $C=2A-3B$ y...

$A=\begin{bmatrix}5 & 4 & 4\\ -3 & 1 & 2\end{bmatrix}$ y $B=\begin{bmatrix}1 & 2 & -7\\ 0 & -5 & 1\end{bmatrix}$

$2A=\begin{bmatrix}10 & 8 & 8\\ -6 & 2 & 4\end{bmatrix}$ $3B=\begin{bmatrix}3 & 6 & -21\\ 0 & -15 & 3\end{bmatrix}$

$2A-3B=\begin{bmatrix}7 & 2 & 29\\ -6 & 17 & 1\end{bmatrix}=C$

$C_{21}=-6$

$C_{13}=29$

(9) Despeje x, y y z en la ecuación matricial.

$$4\begin{bmatrix} x & y \\ z & -1 \end{bmatrix} = 2\begin{bmatrix} y & z \\ -x & 1 \end{bmatrix} + 2\begin{bmatrix} 4 & x \\ 5 & -x \end{bmatrix}$$

$$2\begin{bmatrix} x & y \\ z & -1 \end{bmatrix} = \begin{bmatrix} y & z \\ -x & 1 \end{bmatrix} + \begin{bmatrix} 4 & x \\ 5 & -x \end{bmatrix}$$

$$2\begin{bmatrix} x & y \\ z & -1 \end{bmatrix} = \begin{bmatrix} 4+y & x+z \\ 5-x & 1-x \end{bmatrix}$$

(11) Encuentr a) AB y b) BA

11) $A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ -1 & 8 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 8 \end{pmatrix} = \begin{pmatrix} 0 & 15 \\ 6 & 12 \end{pmatrix} \qquad BA = \begin{pmatrix} 2 & -1 \\ -1 & 8 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 31 & 14 \end{pmatrix}$$

13) $A = \begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}$

$$AB = \begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -10 \\ 10 & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 0 & -10 \\ 10 & 0 \end{pmatrix}$$

15. $A = \begin{pmatrix} 1 & -1 & 7 \\ 2 & -1 & 8 \\ 3 & 1 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 2 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & -1 & 7 \\ 2 & -1 & 8 \\ 3 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 6 & -21 & 15 \\ 8 & -23 & 19 \\ 4 & -7 & 5 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 1 & -1 & 7 \\ 2 & -1 & 8 \\ 3 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 9 & 0 & 13 \\ 9 & -2 & 21 \\ 1 & 4 & -19 \end{pmatrix}$$

Encuentre a) AB y b) BA (si es que están definidos)

(17.) $A=\begin{pmatrix}2&1\\-3&4\\1&6\end{pmatrix}$ $B=\begin{pmatrix}0&-1&0\\4&0&2\\8&-1&7\end{pmatrix}$ × No está definida en AB

$\underbrace{3\times 2 \quad 3\times 3}$

$BA=\begin{pmatrix}0&-1&0\\4&0&2\\8&-1&7\end{pmatrix}\begin{pmatrix}2&1\\-3&4\\1&6\end{pmatrix}=\begin{pmatrix}3&-4\\6&16\\26&46\end{pmatrix}$

$\underbrace{3\times 3 \quad 3\times 2}$

(19) $A=\begin{pmatrix}-1&3\\4&-5\\0&2\end{pmatrix}$ $B=\begin{pmatrix}1&2\\0&7\end{pmatrix}$

$\underbrace{3\times 2 \quad 2\times 2}$

$AB=\begin{pmatrix}-1&3\\4&-5\\0&2\end{pmatrix}\begin{pmatrix}1&2\\0&7\end{pmatrix}=\begin{pmatrix}-1&19\\4&-27\\0&14\end{pmatrix}$

$BA=\begin{pmatrix}1&2\\0&7\end{pmatrix}\begin{pmatrix}-1&3\\4&-5\\0&2\end{pmatrix}$ → No está definido en BA

$\underbrace{2\times 2 \quad 3\times 2}$

(21) $A=\begin{pmatrix}5&0&0\\0&-8&0\\0&0&7\end{pmatrix}$ $B=\begin{pmatrix}\frac{1}{5}&0&0\\0&-\frac{1}{8}&0\\0&0&\frac{1}{7}\end{pmatrix}$

$AB=\begin{pmatrix}5&0&0\\0&-8&0\\0&0&7\end{pmatrix}\begin{pmatrix}\frac{1}{5}&0&0\\0&-\frac{1}{8}&0\\0&0&\frac{1}{7}\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

$BA=\begin{pmatrix}\frac{1}{5}&0&0\\0&-\frac{1}{8}&0\\0&0&\frac{1}{7}\end{pmatrix}\begin{pmatrix}5&0&0\\0&-8&0\\0&0&7\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

(23) $A=\begin{pmatrix}6\\-2\\1\\6\end{pmatrix}$, $B=(10 \quad 12)$

$\underbrace{4\times 1 \quad 1\times 2}$

$AB=\begin{pmatrix}6\\-2\\1\\6\end{pmatrix}(10 \quad 12)=\begin{pmatrix}60&72\\-20&-24\\10&12\\60&72\end{pmatrix}_{4\times 2}$

$BA=(10 \quad 12)\begin{pmatrix}6\\-2\\1\\6\end{pmatrix}$ No está definida en BA

$\underbrace{1\times 2 \quad 4\times 1}$

Escriba el sistema de ecs. lineales dado en la forma $AX = B^{\alpha}$, y despeje x de esa ecuación matricial

(25) $\begin{aligned} x_1 + 3x_2 &= -1 \\ 2x_1 - x_2 &= 3 \end{aligned}$ $\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$|A| = (-1-6) = -7 \neq 0 \therefore A$ es invertible

$$A^{-1}AX = A^{-1}B$$
$$IX = A^{-1}B \Rightarrow \begin{pmatrix} -1 & -3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 3 \end{pmatrix}$$
$$X = A^{-1}B$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underbrace{\begin{pmatrix} -1 & -3 \\ -2 & 1 \end{pmatrix}}_{2\times 2}\underbrace{\begin{pmatrix} -1 \\ 3 \end{pmatrix}}_{2\times 1}$$

$$\begin{pmatrix} -1 & -3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} = \frac{1}{|A|}\begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -8 \\ 5 \end{pmatrix}_{2\times 1}$$

$$= \frac{-7}{-7}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(27) $\begin{aligned} 8x_1 - 8x_2 &= 0 \\ -3x_1 + 2x_2 &= 0 \end{aligned}$ $\begin{pmatrix} 8 & -8 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$|A| = 16 - (24) = -8 \neq 0 \therefore A$ es inv.

$$A^{-1}AX = A^{-1}B \Rightarrow \begin{pmatrix} 2 & 8 \\ 3 & 8 \end{pmatrix}\begin{pmatrix} 8 & -8 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & 8 \\ 3 & 8 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$IX = A^{-1}B$$
$$X = A^{-1}B$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & 8 \\ 3 & 8 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 8 \\ 3 & 8 \end{pmatrix}\begin{pmatrix} 8 & -8 \\ -3 & 2 \end{pmatrix} = \frac{1}{|A|}\begin{pmatrix} -8 & 0 \\ 0 & -8 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$= \frac{-8}{-8}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

81)

$$\begin{aligned} x_1 \quad\quad + 2x_3 &= 5 \\ 3x_1 - 2x_2 + x_3 &= 8 \\ -2x_1 + 2x_2 - x_3 &= -3 \end{aligned} \qquad \begin{pmatrix} 1 & 0 & 2 \\ 3 & -2 & 1 \\ -2 & 2 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ -3 \end{pmatrix}$$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

$$[A \mid I] \sim [I \mid A^{-1}]$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 & 1 & 0 \\ -2 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \underset{R_3: 2R_1 + R_3}{\overset{R_2: -3R_1 + R_2}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -2 & -5 & -3 & 1 & 0 \\ 0 & 2 & 5 & 0 & 0 & 1 \end{array}\right) \overset{R_3: R_2 + R_3}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -2 & -5 & -3 & 1 & 0 \\ \boxed{0 \quad 0 \quad 0} & & & -3 & 1 & 1 \end{array}\right)$$ $\therefore$ A no tiene inversa y $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ No se puede factorizar.

31. Despeje a A de la ecuación matricial

$$\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$BA = I$$

$$B^{-1}BA = B^{-1}I$$

$$IA = B^{-1}I$$

$$A = B^{-1}I$$

$|B| = 5 - 6 = -1$ & $-1 \neq 0$

$\therefore \Rightarrow$ invertible

$$\begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} A = \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A = \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix}$$

7

33. Despeje a, b, c y d de la ecuación matricial.

$$\begin{bmatrix} a-b & 2b+c \\ c-2d & a+d \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 5 & -2 \end{bmatrix}$$

$$\begin{bmatrix} a & 2b \\ c & a \end{bmatrix} + \begin{bmatrix} -b & c \\ -2d & d \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 5 & -2 \end{bmatrix}$$

$A+B$

Donde $A = \begin{pmatrix} a & 2b \\ c & a \end{pmatrix}$

& $B = \begin{pmatrix} -b & c \\ -2d & d \end{pmatrix}$

35. Encuentre condiciones sobre w, x y y z de modo que $AB = BA$ para:

$$A = \begin{bmatrix} w & x \\ y & z \end{bmatrix} \quad y \quad B = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} w & x \\ y & z \end{pmatrix}$$

$AB = BA = I \qquad A^{-1}A = AA^{-1} = I \Rightarrow B^{-1} = A$

$B^{-1} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$AB = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$BA = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Encuentre el producto AA para la matriz diagonal dada. Una matriz cuadrada se denomina matriz diagonal si todos los elementos fuera de la diagonal principal son nulos.

$$A = \begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & \cdots & 0 \\ 0 & 0 & a_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix}$$

37. $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$AA = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

39. Demuestre que si A y B son diagonales del mismo orden entonces $AB = BA$

$$\begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}\begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

41. Encuentre la traza de la matriz proporcionada. La traza de una matriz $A_{n\times n}$ es la suma de los elementos de la diagonal principal. Es decir $T_r(A) = a_{11} + a_{22} + \dots + a_{nn}$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & 4 \\ 3 & 1 & 3 \end{pmatrix} \quad T_r(A) = 1 - 2 + 3 = 2$$

43. Demuestre que las expresiones siguientes son verdaderas si A y B son matrices cuadradas de orden n y c es un escalar.

a) $T_r(A+B) = T_r(A) + T_r(B)$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$T_r(A) = a_{11} + a_{22}$

$T_r(B) = b_{11} + b_{22}$

$$A+B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$$

$T_r(A) + T_r(B) = \underline{a_{11} + a_{22} + b_{11} + b_{22}}$

Tenemos que $a_{11} + b_{11} + a_{22} + b_{22} = a_{11} + a_{22} + b_{11} + b_{22}$

$\therefore T_r(A+B) = T_r(A) + T_r(B)$

$T_r(A+B) = \underline{a_{11} + b_{11} + a_{22} + b_{22}}$

45. Demuestre que la sig. ecuación matricial no tiene sol.

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}_{B} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|B| = 1 - 1 = 0$$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \overset{R_2 := -R_1 + R_2}{\sim} \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ \boxed{0 \quad 0} & & -1 & 1 \end{array}\right]$$

Sin Solución.

47. Demuestre que si el producto AB es una matriz cuadrada, entonces el producto BA está definido

$$AB = \begin{pmatrix} a_{11} & a_{21} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} a_{11} & a_{21} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

matriz $n \times n$ matriz $m \times n$

∴ Está definido.

49. Sean A y B dos matrices tales que el producto AB está definido Demuestre que si A tiene dos renglones identicos, entonces los dos renglones correspondientes de AB también son identicos.

$$A = \begin{pmatrix} \boxed{\begin{matrix} a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \end{matrix}} \\ \begin{matrix} c_1 & c_2 & c_3 \end{matrix} \end{pmatrix} \quad B = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$$AB = \begin{pmatrix} a_1a_1 + a_2b_1 + a_3c_1 & a_1a_2 + a_2b_2 + a_3c_2 & a_1a_3 + a_2b_3 + a_3c_3 \\ a_1a_1 + a_2b_1 + a_3c_1 & a_1a_2 + a_2b_2 + a_3c_3 & a_1a_3 + a_2b_3 + a_3c_3 \\ a_1c_1 + c_2b_1 + c_3c_1 & c_1a_2 + c_2b_2 + c_3c_2 & c_1a_3 + c_2b_3 + c_3c_3 \end{pmatrix}$$

*51. Cierta empresa cuenta con cuatro fábricas. Cada una produce dos productos. El número de unidades de producto i producido en la fabrica i en un dia se representa por a_n en la matriz.

Describa el inverso del vector en el espacio dado.

7. $\mathbb{R}^4$ $\quad \exists\ \alpha \in \mathbb{R}^4 \ni \alpha + a = 0?$

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) + (a_1, a_2, a_3, a_4) = (0, 0, 0, 0)$$
$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (0, 0, 0, 0) - (a_1, a_2, a_3, a_4)$$
$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (-a_1, -a_2, -a_3, -a_4)$$

8. $C(-\infty, \infty)$ $\quad \exists\ \alpha \in C \ni \alpha + a = 0?$

$$\alpha + a = 0$$
$$\alpha = 0 - a$$
$$\alpha = -a$$

9. $M_{2\times 3}$ $\quad \exists\ N_{2\times 3} \ni N_{2\times 3} + M_{2\times 3} = 0_{2\times 3}?$

$$\begin{pmatrix} n_1 & n_2 & n_3 \\ n_4 & n_5 & n_6 \end{pmatrix} + \begin{pmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\begin{pmatrix} n_1 & n_2 & n_3 \\ n_4 & n_5 & n_6 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \end{pmatrix}$$
$$\begin{pmatrix} n_1 & n_2 & n_3 \\ n_4 & n_5 & n_6 \end{pmatrix} = -\begin{pmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \end{pmatrix}$$

10. $M_{1\times 4}$ $\quad \exists\ N_{1\times 4} \ni N_{1\times 4} + M_{1\times 4} = 0_{1\times 4}?$

$$(n_1\ n_2\ n_3\ n_4) + (m_1\ m_2\ m_3\ m_4) = (0\ 0\ 0\ 0)$$
$$(n_1\ n_2\ n_3\ n_4) = (0\ 0\ 0\ 0) - (m_1\ m_2\ m_3\ m_4)$$
$$(n_1\ n_2\ n_3\ n_4) = -(m_1\ m_2\ m_3\ m_4)$$

11. P_3 $\quad \exists\ \alpha(x) \ni p(x) + \alpha(x) = 0(x)?$

$$p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \qquad \alpha(x) = b_0 + b_1x + b_2x^2 + b_3x^3$$
$$(a_0 + a_1x + a_2x^2 + a_3x^3) + (b_0 + b_1x + b_2x^2 + b_3x^3) = (0 + 0x + 0x^2 + 0x^3)$$
$$b_0 + b_1x + b_2x^2 + b_3x^3 = (0 + 0x + 0x^2 + 0x^3) - (a_0 + a_1x + a_2x^2 + a_3x^3)$$
$$b_0 + b_1x + b_2x^2 + b_3x^3 = -(a_0 + a_1x + a_2x^2 + a_3x^3)$$

12. $M_{2\times 2}$ $\quad \exists\ N_{2\times 3} \ni N_{2\times 2} + M_{2\times 3} = 0_{2\times 2}?$

$$\begin{pmatrix} n_1 & n_2 \\ n_3 & n_4 \end{pmatrix} + \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$
$$\begin{pmatrix} n_1 & n_2 \\ n_3 & n_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}$$
$$\begin{pmatrix} n_1 & n_2 \\ n_3 & n_4 \end{pmatrix} = -\begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}$$

Determine si el conjunto dado, junto con las operaciones indicadas, es un espacio vectorial. En caso negativo identifique uno de los 10 axiomas de espacios vectoriales que no se cumpla.

13. $M_{4,6}$ Operaciones normales.

S1) Cerradura ✓
S2) Conmutativa ✓
S3) Asociativa ✓
S4) Neutro Aditivo $\exists N_{4,6} \ \sqcap \ N_{4\times6} + N_{4\times6} = M_{46}$? ✓ $O_{4\times6}$
S5) Inverso Aditivo $\exists N_{46} \ \sqcap \ M_{46} + N_{46} = O_{4\times6}$? ✓ $-M_{4\times6}$
M1) Cerradura ✓ $\alpha M_{4\times6}$
M2) Distributiva ✓
M3) Distributiva (Escalares) ✓
M4) Asociativa (Multi de 2 esca) ✓
M5) Identidad Multiplicativa Matriz $I_{6\times6}$

Espacio Vectorial

14. $M_{1,1}$ con las operaciones normales.

S1) Cerradura ✓
S2) Conmutativa ✓
S3) Asociativa ✓
S4) Neutro aditivo
$\exists N_{1\times1} \ \sqcap \ N_{1\times1} + N_{1\times1} = M_{1\times1}$? ✓ $O_{1\times1}$
S5) Inverso Aditivo
$\exists N_{1\times1} \ \sqcap \ M_{1\times1} + N_{1\times1} = O_{1\times1}$? ✓ $-M_{1\times1}$

M1) Cerradura ✓
M2) Distributiva ✓
M3) Distri (Escalares) ✓
M4) Asociativa ✓
M5) Ident ✓

Espacio Vectorial.

15. El conjunto de todos los polinomios de quinto grado con las operaciones normales.

S1) Cerradura $p(x) + q(x)$ ✓
S2) Conmutativa $p(x) + q(x) = q(x) + p(x)$ ✓
S3) Asociativa $p(x) + [q(x) + r(x)] = [p(x) + q(x)] + r(x)$ ✓
S4) Neutro aditivo

Veamos $\exists q(x) \ \sqcap \ p(x) + q(x) = p(x)$?

$$p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$$
$$q(x) = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 + b_5x^5$$
$$p(x) + q(x) = p(x)$$
$$q(x) = p(x) - p(x)$$
$$q(x) = O(x)$$
$$q(x) = 0 + 0x + 0x^2 + 0x^3 + 0x^4 + 0x^5$$

S5) Inverso aditivo

Veamos $\exists p(x) \ \sqcap \ p(x) + q(x) = O(x)$?

$$p(x) + q(x) = O(x)$$
$$p(x) = O(x) - q(x)$$
$$p(x) = -q(x)$$

M1) Cerradura $\alpha p(x)$ ✓

M2) Distributiva $\alpha(p(x)+q(x)) = \alpha p(x) + \alpha q(x)$

Ucamos $\alpha(p(x)+q(x)) = \alpha(a_0+a_1x+a_2x^2+a_3x^3+a_4x^4-$

$\alpha(p(x)+q(x)) = \alpha[(a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5)+(b_0+b_1x+b_2x^2+b_3x^3+b_4x^4+b_5x^5)]$

$= \alpha[(a_0+b_0)+(a_1+b_1)x+(a_2+b_2)x^2+(a_3+b_3)x^3+(a_4+b_4)x^4+(a_5+b_5)x^5]$

$= \alpha(a_0+b_0)+\alpha(a_1+b_1)x+\alpha(a_2+b_2)x^2+\alpha(a_3+b_3)x^3+\alpha(a_4+b_4)x^4+\alpha(a_5+b_5)x^5$

$= \alpha a_0+\alpha a_1x+\alpha a_2x^2+\alpha a_3x^3+\alpha a_4x^4+\alpha a_5x^5+\alpha b_0+\alpha b_1x+\alpha b_2x^2+\alpha b_3x^3+ab_4x^4+\alpha b_5x^5$

$= \alpha(a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5)+\alpha(b_0+b_1x+b_2x^2+b_3x^3+b_4x^4+b_5x^5)$

$= \alpha p(x) + \alpha q(x)$ q.e.d.

M3) Distributiva de la suma de 2 escalares.

$(\alpha+\beta)p(x) = \alpha p(x) + \beta p(x)$

$(\alpha+\beta)p(x) = (\alpha+\beta)(a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5)$

$= (\alpha+\beta)a_0+(\alpha+\beta)a_1x+(\alpha+\beta)a_2x^2+(\alpha+\beta)a_3x^3+(\alpha+\beta)a_4x^4+(\alpha+\beta)a_5x^5$

$= \alpha a_0+\alpha a_1x+\alpha a_2x^2+\alpha a_3x^3+\alpha a_4x^4+\alpha a_5x^5+\beta a_0+\beta a_1x+\beta a_2x^2+\beta a_3x^3+\beta a_4x^4+\beta a_5x^5$

$= \alpha(a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5)+\beta(a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5)$

$= \alpha p(x) + \beta p(x)$ ✓ $\therefore$ q.e.d.

M4) Asociativa $\alpha(\beta p(x)) = \beta(\alpha p(x))$

Ucamos $\alpha(\beta p(x)) = \alpha(\beta(a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5)$

$= \alpha(\beta a_0+\beta a_1x+\beta a_2x^2+\beta a_3x^3+\beta a_4x^4+\beta a_5x^5)$

$= \alpha\beta a_0+\alpha\beta a_1x+\alpha\beta a_2x^2+\alpha\beta a_3x^3+\alpha\beta a_4x^4+\alpha\beta a_5x^5$

$= \beta(\alpha a_0+\alpha a_1x+\alpha a_2x^2+\alpha a_3x^3+\alpha a_4x^4+\alpha a_5x^5$

$= \beta(\alpha(a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5)$

$= \beta(\alpha p(x))$ ✓ q.e.d.

M5) Identidad multiplicativa

$I\,p(x) = p(x)$

$I = \frac{p(x)}{p(x)} = 1$ ✓

Espacio Vectoria

) Cerradu

) Conm

3)

, el conjunto de todos los polinomios, con las operaciones normales.

1) Cerradura ✓
2) Conmutativa ✓
S3) Asociativa ✓
S4) Neutro Aditivo

Veamos $\exists\, p(x) \ni p(x) + q(x) = q(x)$? ✓

$$p(x) + q(x) = q(x)$$
$$p(x) = q(x) - q(x)$$
$$p(x) = 0(x) = 0 + 0x + 0x^2 + \dots + 0x^{n-1} + 0x^n$$

S5) Inverso Aditivo ✓

Veamos $\exists\, p(x) \ni p(x) + q(x) = 0(x)$?

$$p(x) + q(x) = 0(x)$$
$$p(x) = 0(x) - q(x)$$
$$p(x) = -q(x)$$

M1) Cerradura $\alpha p(x)$ ✓

M2) Distributiva $\alpha[p(x) + q(x)] = \alpha p(x) + \alpha q(x)$

$$\alpha[p(x)+q(x)] = \alpha[(a_0 + a_1x + a_1x^2 + \dots + a_{n-1}x^{n-1} + a_nx^n) + (b_0 + b_1x + b_2x^2 + \dots + b_{n-1}x^{n-1} + b_nx^n)]$$
$$= \alpha[(a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2 + \dots + (a_{n-1}+b_{n-1})x^{n-1} + (a_n+b_n)x^n]$$
$$= \alpha(a_0+b_0) + \alpha(a_1+b_1)x + \alpha(a_2+b_2)x^2 + \dots + \alpha(a_{n-1}+b_{n-1})x^{n-1} + \alpha(a_n+b_n)x^n$$
$$= \alpha a_0 + \alpha a_1x + \alpha a_2x^2 + \dots + \alpha a_{n-1}x^{n-1} + \alpha a_nx^n + \alpha b_0 + \alpha b_1x + \alpha b_2x^2 + \dots + \alpha b_{n-1}x^{n-1} + \alpha b_nx^n$$
$$= \alpha(a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} + a_nx^n) + \alpha(b_0 + b_1x + b_2x^2 + \dots + b_{n-1}x^{n-1} + b_nx^n)$$
$$= \alpha p(x) + \alpha q(x) \quad \therefore \quad q.e.d.$$

M3) Distributiva (Suma de 2 escalares)

Veamos $(\alpha + \beta)p(x) = \alpha p(x) + \beta p(x)$ Sea $p(x) = (a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} + a_nx^n)$

$$(\alpha+\beta)p(x) = (\alpha+\beta)(a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} + a_nx^n)$$
$$= [(\alpha+\beta)a_0 + (\alpha+\beta)a_1x + (\alpha+\beta)a_2x^2 + \dots + (\alpha+\beta)a_{n-1}x^{n-1} + (\alpha+\beta)a_nx^n]$$
$$= \alpha a_0 + \alpha a_1x + \alpha a_2x^2 + \dots + \alpha a_{n-1}x^{n-1} + \alpha a_nx^n + \beta a_0 + \beta a_1x + \beta a_2x^2 + \dots + \beta a_{n-1}x^{n-1} + \beta a_nx^n$$
$$= \alpha(a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} + a_nx^n) + \beta(a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} + a_nx^n)$$
$$= \alpha p(x) + \beta p(x) \quad \therefore \quad q.e.d.$$

M4) Asociativa $\alpha(\beta p(x)) = \beta(\alpha p(x))$

$$\alpha(\beta p(x)) = \alpha[\beta(a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} + a_nx^n)]$$
$$= \alpha[\beta a_0 + \beta a_1x + \beta a_2x^2 + \dots + \beta a_{n-1}x^{n-1} + \beta a_nx^n]$$
$$= [\alpha\beta a_0 + \alpha\beta a_1x + \alpha\beta a_2x^2 + \dots + \alpha\beta a_{n-1}x^{n-1} + \alpha\beta a_nx^n]$$
$$= \beta(\alpha a_0 + \alpha a_1x + \alpha a_2x^2 + \dots + \alpha a_{n-1}x^{n-1} + \alpha a_nx^n]$$
$$= \beta[\alpha(a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} + a_nx^n)]$$
$$= \beta(\alpha(p(x))) \quad \therefore \text{ q.e.d.}$$

M5. Identidad multiplicativa

$$I\ p(x) = p(x)$$
$$I = \frac{p(x)}{p(x)} = 1 \text{ Escalar} \qquad \therefore \text{ Es un Espacio Vectorial.}$$

17. El Conjunto $\{(x,x) : x$ es un número real$\}$ con las operaciones normales.

S1) Cerradora ✓

S2) Conmutatividad $(x_1, x_1) + (x_2, x_2) = (x_2, x_2) + (x_1, x_1)$ ✓

S3) Asociativa $(x_1, x_1) + [(x_2, x_2) + (x_3, x_3)] = [(x_1, x_1) + (x_2, x_2)] + (x_3, x_3)$

S4) Neutro Aditivo

Veamos $\exists\ (y,y) \ni (y,y) + (x,x) = (x,x)$?

$$(y,y) = (x,x) - (x,x)$$
$$(y,y) = (0,0)$$

S5) Inverso Aditivo

Veamos $\exists\ (y,y) \ni (y,y) + (x,x) = (0,0)$?

$$(y,y) + (x,x) = (0,0)$$
$$(y,y) = (0,0) - (x,x)$$
$$(y,y) = (-x, -x)$$

M1) Cerradora $\alpha(x,x) = (\alpha x, \alpha x)$ ✓

M2) Distributiva $\alpha[(x,x) + (x_1, x_1)] = \alpha(x,x) + \alpha(x_1, x_1)$

$$\alpha[(x,x) + (x_1, x_1)] = \alpha[(x+x_1, x+x_1)]$$
$$= \alpha(x+x_1), \alpha(x+x_1) \quad \therefore \text{ q.e.d.}$$

M3) Distributiva 2 esca

Veamos $(\alpha+\beta)(x,x) = \alpha(x,x) + \beta(x,x)$

$$((\alpha+\beta)x, (\alpha+\beta)x)$$
$$(\alpha x + \beta x, \alpha x + \beta x)$$
$$(\alpha x, \alpha x) + (\beta x, \beta x)$$
$$\alpha(x,x) + \beta(x,x) \quad \therefore \text{ q.e.d.}$$

Asociativa $\alpha[\beta(x,x)] = \beta[\alpha(x,x)]$

Veamos $\alpha[\beta(x,x)] = \alpha[\beta x, \beta x] = (\alpha\beta x, \alpha\beta x) = (\beta\alpha x, \beta\alpha x)$

$= \beta[\alpha x, \alpha x]$

$= \beta[\alpha(x,x)]$

M5) $I(x,x) = (x,x)$ ∴ Espacio Vectorial.

$I = (1,1)$ ✓

18. El conjunto $\{(x,y): x \geq 0, y$ es un número real$\}$ Con las operaciones nor. de $\mathbb{R}^2$

S1) Cerradura ✓

S2) Conmutativa $(x,y)+(x_1,y_1) = (x_1,y_1)+(x,y)$ ✓

S3) Asociatividad $(x_1,y_1)+[(x_2,y_2)+(x_3,y_3)] = [(x_1,y_1)+(x_2,y_2)]+(x_3,y_3)$

S4) Neutro aditivo

Veamos $\exists (w,z) \sqcap (w,z)+(x,y) = (x,y)$?

$(w,z) = (x,y) - (x,y)$

$(w,z) = (x-x, y-y)$

$(w,z) = (0,0)$

S5) Inverso Aditivo

Veamos $\exists (w,z) \sqcap (w,z)+(x,y) = (0,0)$?

$(w,z) = (0,0) - (x,y)$

$(w,z) = (-x,-y)$

Pero $-x < 0$ ∴ No es un Esp. Vect.

19. El conjunto de todas las matrices 2×2 de la forma $N = \begin{bmatrix} a & b \\ c & 0 \end{bmatrix}$ con las operaciones normales.

S1) Cerradura ✓

S2) Conmutativa ✓

S3) Asociativo ✓

S4) Neutro aditivo

Veamos $\exists M_{2\times2} \sqcap M_{2\times2} + N_{2\times2} = N_{2\times2}$

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$$

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} - \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$$

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

S5) Inverso Aditivo

Veamos $\exists N_{2\times2} \sqcap M_{2\times2} + N_{2\times2} = O_{2\times2}$?

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} = \begin{pmatrix} -a & -b \\ -c & 0 \end{pmatrix}$$

M1) Cerradura $\alpha\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} = \begin{pmatrix} \alpha a & \alpha b \\ \alpha c & 0 \end{pmatrix}$ ✓

M2) Distributiva $\alpha\left[\begin{pmatrix} a_1 & b_1 \\ c_1 & 0 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & 0 \end{pmatrix}\right] = \alpha\begin{pmatrix} a_1 & b_1 \\ c_1 & 0 \end{pmatrix} + \alpha\begin{pmatrix} a_2 & b_2 \\ c_2 & 0 \end{pmatrix}$

$$\alpha\left[\begin{pmatrix} a_1 & b_1 \\ c_1 & 0 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & 0 \end{pmatrix}\right] = \alpha\begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & 0+0 \end{pmatrix} = \begin{pmatrix} \alpha(a_1+a_2) & \alpha(b_1+b_2) \\ \alpha(c_1+c_2) & \alpha(0+0) \end{pmatrix}$$

$$= \begin{pmatrix} \alpha a_1 + \alpha a_2 & \alpha b_1 + \alpha b_2 \\ \alpha a_1 + \alpha c_2 & \alpha 0 + \alpha 0 \end{pmatrix} = \begin{pmatrix} \alpha a_1 & \alpha b_1 \\ \alpha c_1 & \alpha 0 \end{pmatrix} + \begin{pmatrix} \alpha a_2 & \alpha b_2 \\ \alpha c_2 & \alpha 0 \end{pmatrix}$$

$$= \alpha\begin{pmatrix} a_1 & b_1 \\ c_1 & 0 \end{pmatrix} + \alpha\begin{pmatrix} a_2 & b_2 \\ c_2 & 0 \end{pmatrix} \quad \therefore \text{q.e.d.}$$

M3. Distributiva (2 esca.) Veamos...

$$(\alpha+\beta)\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} = \alpha\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} + \beta\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$$

$$(\alpha+\beta)\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} = \begin{pmatrix} (\alpha+\beta)a & (\alpha+\beta)b \\ (\alpha+\beta)c & (\alpha+\beta)0 \end{pmatrix} = \begin{pmatrix} \alpha a + \beta a & \alpha b + \beta b \\ \alpha c + \beta c & \alpha 0 + \beta 0 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha a & \alpha b \\ \alpha c & \alpha 0 \end{pmatrix} + \begin{pmatrix} \beta a & \beta b \\ \beta c & \beta(0) \end{pmatrix} = \alpha\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} + \beta\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} \text{ q.e.d.}$$

M4) Asociativa Veamos que $\alpha\left[\beta\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}\right] = \beta\left[\alpha\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}\right]$

$$\alpha\left[\beta\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}\right] = \alpha\begin{pmatrix} \beta a & \beta b \\ \beta c & \beta(0) \end{pmatrix} = \begin{pmatrix} \alpha\beta a & \alpha\beta b \\ \alpha\beta c & \alpha\beta(0) \end{pmatrix} = \begin{pmatrix} \beta\alpha a & \beta\alpha b \\ \beta\alpha c & \beta\alpha(0) \end{pmatrix} = \beta\begin{pmatrix} \alpha a & \alpha b \\ \alpha c & \alpha(0) \end{pmatrix}$$

$$= \beta\left[\alpha\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}\right] \quad \therefore \text{q.e.d}$$

M5) $I\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ Por definición

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\therefore$ Espacio Vectorial

El conjunto de todas las matrices de la forma $\begin{bmatrix} a & b \\ c & 1 \end{bmatrix}$ con las op. normales.

Cerradura $\begin{pmatrix} a & b \\ c & 1 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & 1 \end{pmatrix} = \begin{pmatrix} a+a_1 & b_1+b \\ c+c_1 & 1+1 \end{pmatrix}$ $1+1 \neq 2$ $\therefore$ No es espacio Vectorial

21. El conjunto de todas las matrices singulares 2×2 con las op. normales.

Singular: que no tiene inversa. Sea $M = \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix}$ Donde $a_1 a_4 = a_2 a_3$ para ser singular.

α multiplo de a_1 y β de a_2

S1) Cerradura $\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ \alpha b_1 & \beta b_2 \end{pmatrix}$

$$\begin{pmatrix} a_1+b_1 & a_2+b_2 \\ \alpha a_1+\alpha b_1 & \beta a_2+\beta b_2 \end{pmatrix} = \begin{pmatrix} a_1+b_1 & a_2+b_2 \\ \alpha(a_1+b_1) & \beta(a_2+b_2) \end{pmatrix} \checkmark$$

S2) Conmutativa $\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ \alpha b_1 & \beta b_2 \end{pmatrix} = \begin{pmatrix} b_1 & b_2 \\ \alpha b_1 & \beta b_2 \end{pmatrix} + \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} \checkmark$

S3) Asociativa $\checkmark$

S4) Neutro Aditivo Veamos $\exists \begin{pmatrix} c_1 & c_2 \\ \alpha c_1 & \beta c_2 \end{pmatrix} \sqcap \begin{pmatrix} c_1 & c_2 \\ \alpha c_1 & \beta c_2 \end{pmatrix} + \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix}$

$$\begin{pmatrix} c_1 & c_2 \\ \alpha c_1 & \beta c_2 \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} - \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix}$$

$$\begin{pmatrix} c_1 & c_2 \\ \alpha c_1 & \beta c_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

S5) Inverso aditivo $\exists \begin{pmatrix} d_1 & d_2 \\ \alpha d_1 & \beta d_2 \end{pmatrix} \sqcap \begin{pmatrix} d_1 & d_2 \\ \alpha d_1 & \beta d_2 \end{pmatrix} + \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}?$

$$\begin{pmatrix} d_1 & d_2 \\ \alpha d_1 & \beta d_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} = \begin{pmatrix} -a_1 & -a_2 \\ -\alpha a_1 & -\beta a_2 \end{pmatrix}$$

M1) $\gamma \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix}$ Cerradura

$$= \begin{pmatrix} \gamma a_1 & \gamma a_2 \\ \alpha \gamma a_1 & \beta \gamma a_2 \end{pmatrix} \checkmark$$

M2) Distributiva $\gamma \left[\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ \alpha b_1 & \beta b_2 \end{pmatrix} \right] = \gamma \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} + \gamma \begin{pmatrix} b_1 & b_2 \\ \alpha b_1 & \beta b_2 \end{pmatrix}$

$$\gamma \left[\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ \alpha b_1 & \beta b_2 \end{pmatrix} \right] = \gamma \left[\begin{pmatrix} a_1+b_1 & a_2+b_2 \\ \alpha a_1+\alpha b_1 & \beta a_2+\beta b_2 \end{pmatrix} \right] = \begin{pmatrix} \gamma(a_1+b_1) & \gamma a_2+\gamma b_2 \\ \gamma\alpha a_1+\gamma\alpha b_1 & \gamma\beta a_2+\gamma\beta b_2 \end{pmatrix}$$

$$= \begin{pmatrix} \gamma a_1 & \gamma a_2 \\ \gamma \alpha a_1 & \gamma\beta a_2 \end{pmatrix} + \begin{pmatrix} \gamma b_1 & \gamma b_2 \\ \gamma \alpha b_1 & \gamma\beta b_2 \end{pmatrix} = \gamma \begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} + \gamma \begin{pmatrix} b_1 & b_2 \\ \alpha b_1 & \beta b_2 \end{pmatrix}$$

$\therefore$ q.e.d

M3) Distri. (2 esc)

$$(\phi+\gamma)\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} = \phi\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} + \gamma\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix}$$

$$(\phi+\gamma)\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} = \begin{pmatrix} (\phi+\gamma)a_1 & (\phi+\gamma)a_2 \\ (\phi+\gamma)\alpha a_1 & (\phi+\gamma)\beta a_2 \end{pmatrix} = \begin{pmatrix} \phi a_1 + \gamma a_1 & \phi a_2 + \gamma a_2 \\ \phi\alpha a_1 + \gamma\alpha a_1 & \gamma\beta a_2 + \gamma\beta a_2 \end{pmatrix}$$

$$= \begin{pmatrix} \phi a_1 & \phi a_2 \\ \phi\alpha a_1 & \phi\beta a_2 \end{pmatrix} + \begin{pmatrix} \gamma a_1 & \gamma a_2 \\ \gamma\alpha a_1 & \gamma\beta a_2 \end{pmatrix} = \phi\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix} + \gamma\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix}$$

M4) Asociativa $\phi\left[\gamma\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix}\right] = \gamma\left[\phi\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix}\right]$

$$\phi\left[\gamma\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix}\right] = \phi\begin{pmatrix} \gamma a_1 & \gamma a_2 \\ \gamma\alpha a_1 & \gamma\beta a_2 \end{pmatrix} = \begin{pmatrix} \phi\gamma a_1 & \phi\gamma a_2 \\ \phi\gamma\alpha a_1 & \phi\gamma\beta a_2 \end{pmatrix} = \begin{pmatrix} \gamma\phi a_1 & \gamma\phi a_2 \\ \gamma\phi\alpha a_1 & \gamma\phi\beta a_2 \end{pmatrix}$$

$$= \gamma\begin{pmatrix} \phi a_1 & \phi a_2 \\ \phi\alpha a_1 & \phi\beta a_2 \end{pmatrix} = \gamma\left[\phi\begin{pmatrix} a_1 & a_2 \\ \alpha a_1 & \beta a_2 \end{pmatrix}\right] \therefore$$ q.e.d. Esp. Vecto.

22. El conjunto de todas las matrices no singulares 2×2 Op. Norm.

Sea $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Considerando que a y c & b y d no sean multiplos entre si.

S1) Cerradura $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ ✓

S2) Conmutativa $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ ✓

S3) Asociativa ✓

S4) Neutro aditivo $\exists \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \sqcap \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ ✓

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{pmatrix} a_1 & b_1 \\ a & d_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{pmatrix} a_1 & b_1 \\ a & d_1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

S5) Inverso aditivo $\exists \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \sqcap \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$?

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = \begin{pmatrix} -a & -b \\ -c & -d \end{pmatrix}$$ ✓

$$\alpha\begin{pmatrix} a_1 & b \\ c & d \end{pmatrix} = \begin{pmatrix} \alpha a_1 & \alpha b \\ \alpha c & \alpha d \end{pmatrix} ✓$$

) Distributiva

$$\alpha\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\right] = \alpha\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \alpha\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$\alpha\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\right] = \alpha\begin{pmatrix} a+a_1 & b+b_1 \\ c+c_1 & d+d_1 \end{pmatrix} = \begin{pmatrix} \alpha(a+a_1) & \alpha(b+b_1) \\ \alpha(c+c_1) & \alpha(d+d_1) \end{pmatrix}$$

$$= \begin{pmatrix} \alpha a_1 + \alpha a & \alpha b_1 + \alpha b \\ \alpha c + \alpha c_1 & \alpha d + \alpha d_1 \end{pmatrix} = \begin{pmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{pmatrix} + \begin{pmatrix} \alpha a_1 & \alpha b_1 \\ \alpha c_1 & \alpha d_1 \end{pmatrix}$$

$$= \alpha\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \alpha\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \quad \therefore \quad \text{q.e.d.}$$

M3) Distributiva $(\alpha+\beta)\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \alpha\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \beta\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$(\alpha+\beta)\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} (\alpha+\beta)a & (\alpha+\beta)b \\ (\alpha+\beta)c & (\alpha+\beta)d \end{pmatrix} = \begin{pmatrix} \alpha a + \beta a & \alpha b + \beta b \\ \alpha c + \beta c & \alpha d + \beta d \end{pmatrix}$$

$$= \begin{pmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{pmatrix} + \begin{pmatrix} \beta a & \beta b \\ \beta c & \beta d \end{pmatrix} = \alpha\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \beta\begin{pmatrix} a & b \\ c & d \end{pmatrix} \therefore \text{ q.e.d.}$$

M4) $\alpha\left[\beta\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = \beta\left[\alpha\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]$

$$\alpha\left[\beta\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = \alpha\begin{bmatrix} \beta a & \beta b \\ \beta c & \beta d \end{bmatrix} = \begin{bmatrix} \alpha\beta a & \alpha\beta b \\ \alpha\beta c & \alpha\beta d \end{bmatrix} = \begin{bmatrix} \beta\alpha a & \beta\alpha b \\ \beta\alpha c & \beta\alpha d \end{bmatrix}$$

$$= \beta\left(\begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix}\right) = \beta\left[\alpha\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] \therefore \text{ q.e.d.}$$

M5) Ident. $I\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Por def. $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\therefore$ Un espacio Vectorial

23. El conjunto de todas las matrices diagonales 2x2 (Op. normales).

S1) Cerradura ✓

S2) Conmutatividad ✓

S3) Asociativa ✓

S4) Neutro Aditivo $\exists \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \ni \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} + \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$?

$$\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} + \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$

$$\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} - \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

S5) Inverso Aditivo $\exists \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \ni \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} + \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} + \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$

$$\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} -a_1 & 0 \\ 0 & -a_2 \end{pmatrix}$$

M1) Cerradora $\alpha \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$ ✓

M2) Distributiva

$$\alpha\left[\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} + \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}\right] = \alpha\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} + \alpha\begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}$$

$$\alpha\left[\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} + \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}\right] = \alpha\begin{pmatrix} a_1+b_1 & 0+0 \\ 0+0 & a_2+b_2 \end{pmatrix} = \begin{pmatrix} \alpha(a_1+b_1) & \alpha(0+0) \\ \alpha(0+0) & \alpha(a_2+b_2) \end{pmatrix}$$

$$= \begin{pmatrix} \alpha a_1 & \alpha 0 \\ \alpha 0 & \alpha a_2 \end{pmatrix} + \begin{pmatrix} \alpha b_1 & \alpha 0 \\ \alpha 0 & \alpha b_2 \end{pmatrix} = \alpha\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} + \alpha\begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \therefore q.e.d.$$

M3) Distributiva 2 esc.

$$(\alpha+\beta)\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \alpha\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} + \beta\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$

$$(\alpha+\beta)\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \begin{pmatrix} (\alpha+\beta)a_1 & (\alpha+\beta)0 \\ (\alpha+\beta)0 & (\alpha+\beta)a_2 \end{pmatrix} = \begin{pmatrix} \alpha a_1+\beta a_1 & \alpha 0+\beta 0 \\ \alpha 0+\beta 0 & \alpha a_2+\beta a_2 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha a_1 & \alpha 0 \\ \alpha 0 & \alpha a_2 \end{pmatrix} + \begin{pmatrix} \beta a_1 & \beta 0 \\ \beta 0 & \beta a_2 \end{pmatrix} = \alpha\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} + \beta\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$

$\therefore$ q.e.d

M4) Asociativa $\alpha\left[\beta\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}\right] = \beta\left[\alpha\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}\right]$

$$\alpha\left[\beta\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}\right] = \alpha\begin{pmatrix} \beta a_1 & \beta(0) \\ \beta(0) & \beta a_2 \end{pmatrix} = \begin{pmatrix} \alpha\beta a_1 & \alpha\beta(0) \\ \alpha\beta(0) & \alpha\beta a_2 \end{pmatrix} = \begin{pmatrix} \beta\alpha a_1 & \beta\alpha(0) \\ \beta\alpha(0) & \beta\alpha(a_2) \end{pmatrix}$$

$$= \beta\begin{pmatrix} \alpha a_1 & \alpha 0 \\ \alpha 0 & \alpha a_2 \end{pmatrix} = \beta\left[\alpha\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}\right] \therefore q.e.d.$$

M5) Identidad mult.

$$I\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

9) En vez de aplicar las def. normales en $\mathbb{R}^2$ Suponga que se define como sigue y responda ¿$\mathbb{R}^2$ es un espacio vectorial? Justifique.

a) $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$
$c(x, y) = (cx, y)$

S1) Cerradura ✓
S2) Conmutatividad ✓
S3) Asociativa ✓
S4) Neutro Ad. $\exists (x,y) \ni (x,y) + (x_1, y_1) = (x_1, y_1)$?
$(x,y) = (x_1, y_1) - (x_1, y_1)$
$(x,y) = (0,0)$
S5) Inv. Ad. $\exists (x,y) \ni (x,y) + (x_1, y_1) = (0,0)$?
$(x,y) = (0,0) - (x_1, y_1)$
$(x,y) = (-x_1, -y_1)$
M1) Cerradura $c(x,y) = (cx, y)$ ✓
M2) Distributiva $c[(x,y) + (x_1, y_1)] = c(x,y) + c(x_1, y_1)$

$c[(x,y) + (x_1, y_1)] = c(x + x_1, y + y_1) = (c(x + x_1), y + y_1)$
$= cx + cx_1, y + y_1 = (cx, y) + (cx_1 + y_1)$
$= \underline{c(x,y) + c(x_1 + y_1)}$ ✓ ∴ q.e.d.

M3) Distributiva (2 escalares)
$(\alpha + \beta)(x,y) = \alpha(x,y) + \beta(x,y)$
$((\alpha + \beta)x, y)$ $\quad$ $(\alpha x, y) + (\beta x, y)$
$= (\alpha x + \beta x, \underline{y})$ $\quad$ $= (\alpha x + \beta x, \underline{y + y})$ ∴ No es un esp. vec. al no cumplir $\underline{M3}$.

b) $(x_1, y_1) + (x_2, y_2) = (x_1, 0)$
$c(x,y) = (cx, cy)$

S1) Cerradura ✓
S2) Conmutativa ✓ $(x_1, y_1) + (x_2 y_2) = (x_2, y_2) + (x_1, y_1)$
De la primera parte: $(x_1, y_1) + (x_2, y_2) = (x_1, 0)$
De la segunda parte: $(x_2, y_2) + (x_1, y_1) = (x_2, 0)$
$(x_1, 0) \neq (x_2, 0)$ ∴ No es un esp. vect. al no cumplir S2.

c) $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$
$c(x, y) = (\sqrt{c}\,x, \sqrt{c}\,y)$

S1) Cerradura ✓
S2) Conmutatividad $(x_1, y_1) + (x_2, y_2) = (x_2, y_2) + (x_1, y_1)$ ✓
S3) Asociatividad ✓
S4) Neutro aditivo $\exists\, (x, y) \ni (x, y) + (x_1, y_1) = (x_1, y_1)$?
Veamos
$$(x, y) + (x_1, y_1) = (x_1, y_1)$$
$$(x, y) = (x_1, y_1) - (x_1, y_1)$$
$$(x, y) = \underline{(0, 0)}$$

S5) Inverso aditivo $\exists\, (x, y) \in \mathbb{R}^2 \ni (x, y) + (x_1, y_1) = (0, 0)$?
Veamos
$$(x, y) + (x_1, y_1) = (0, 0)$$
$$(x, y) = (0, 0) - (x_1, y_1)$$
$$(x, y) = \underline{(-x_1, -y_1)} /\!/$$

M1) Cerradura ✓
M2) Distributiva $\alpha[(x, y) + (x_1, y_1)] = \alpha(x, y) + \alpha(x_1, y_1)$

De la primera parte

$= \alpha[(x + x_1, y + y_1)]$
$= (\sqrt{\alpha}(x + x_1), \sqrt{\alpha}(y + y_1))$

De la segunda parte

$= (\sqrt{\alpha}\,x, \sqrt{\alpha}\,y) + (\sqrt{\alpha}\,x_1, \sqrt{\alpha}\,y_1)$
$= (\sqrt{\alpha}\,x + \sqrt{\alpha}\,x_1, \sqrt{\alpha}\,y + \sqrt{\alpha}\,y_1)$
$= (\sqrt{\alpha}(x + x_1), \sqrt{\alpha}(y + y_1))$ $\therefore$ q.e.d.

M3) Distributiva $(\alpha + \beta)(x, y) = \alpha(x, y) + \beta(x, y)$

De la primera parte:

$(\alpha + \beta)(x, y) = (\sqrt{\alpha + \beta}\,x + \sqrt{\alpha + \beta}\,y)$

De la segunda parte

$\alpha(x, y) + \beta(x, y)$
$= (\sqrt{\alpha}\,x + \sqrt{\alpha}, y) + (\sqrt{\beta}\,x + \sqrt{\beta}\,y)$
$= (\sqrt{\alpha}\,x + \sqrt{\beta}\,x, \sqrt{\alpha}\,y + \sqrt{\beta}\,y)$
$= ((\sqrt{\alpha} + \sqrt{\beta})x, (\sqrt{\alpha} + \sqrt{\beta})y)$

Pero $\sqrt{\alpha + \beta} \neq \sqrt{\alpha} + \sqrt{\beta}$ $\therefore$ No es Esp. Vect. al no cumplir <u>M3</u>

16. Con las definiciones sig. de la suma y multiplicacion escalar en $\mathbb{R}^3$ justifique si $\mathbb{R}^3$ es o no un espacio vectorial.

a) $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$

$c(x, y, z) = (cx, cy, 0)$

S1) Cerradura ✓

S2) Conmutatividad ✓

S3) Asociatividad ✓

S4) Neutro Aditivo $\exists (x,y,z) \in \mathbb{R}^3 \ni (x,y,z) + (x_1, y_1, z_1) = (x_1, y_1, z_1)$?

Veamos $(x, y, z) + (x_1, y_1, z_1) = (x_1, y_1, z_1)$

$(x, y, z) = (x_1, y_1, z_1) - (x_1, y_1, z_1) = (0, 0, 0)$

S5) Inverso Aditivo $\exists (x,y,z) \in \mathbb{R}^3 \ni (x,y,z) + (x_1, y_1, z_1) = (0, 0, 0)$?

Veamos $(x, y, z) + (x_1, y_1, z_1) = (0, 0, 0)$

$(x, y, z) = (0, 0, 0) - (x_1, y_1, z_1)$

$(x, y, z) = (-x_1, -y_1, -z_1)$

M1) Cerradura ✓

M2) Distributiva $c[(x, y, z) + (x_1, y_1, z_1)] = c(x, y, z) + c(x_1, y_1, z_1)$

$c(x + x_1, y + y_1, z + z_1)$

$(c(x+x_1), c(y+y_1), 0)$

De la segunda parte

$= (cx, cy, 0) + (cx_1, cy_1, 0)$

$= (cx + cx_1, cy + cy_1, 0 + 0)$

$= (c(x + x_1), c(y+y_1), 0)$

∴ q.e.d.

M3) Distributiva (2 escalares)

$(\alpha + \beta)(x, y, z) = \alpha(x, y, z) + \beta(x, y, z)$

De la Primera parte

$((\alpha+\beta)x, (\alpha+\beta)y, 0)$

De la segunda parte.

$(\alpha x, \alpha y, 0) + (\beta x, \beta y, 0)$

$(\alpha x + \beta x, \alpha y + \beta y, 0 + 0)$

$((\alpha + \beta)x, (\alpha+\beta)y, 0)$ ∴ q.e.d.

M4) Asociativa $\alpha[\beta(x,y,z)] = \beta[\alpha(x,y,z)]$

$\alpha[\beta(x,y,z)] = \alpha(\beta x, \beta y, 0) = (\alpha\beta x, \alpha\beta y, 0) = (\beta\alpha x, \beta\alpha y, 0)$
$= \beta(\alpha x, \alpha y, 0) = \beta[\alpha(x,y,z)]$ $\therefore$ q.e.d.

M5) Identidad

b) $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (0,0,0)$
$c(x, y, z) = (cx, cy, cz)$

S1) Cerradura ✓
S2) Conmutatividad ✓
S3) Asociatividad ✓
S4) Neutro Aditivo $\exists (x,y,z) \in \mathbb{R}^3 \sqcap (x,y,z) + (x_1,y_1,z_1) = (x_1,y_1,z_1)$?

No, ya que la suma está definida para resultar $(0,0,0)$

$\therefore$ No es Esp. Vect. al no cumplir S4

c) $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1+x_2+1, y_1+y_2+1, z_1+z_2+1)$
$c(x, y, z) = (cx, cy, cz)$.

S1) Cerradura ✓
S2) Conmutatividad ✓ $(x_1, y_1, z_1) + (x, y, z) = (x, y, z) + (x_1, y_1, z_1)$
S3) Asociatividad ✓

$(x,y,z) + [(x_1,y_1,z_1) + (x_2,y_2,z_2)] \overset{?}{=} [(x,y,z) + (x_1,y_1,z_1)] + (x_2,y_2,z_2)$
$(x,y,z) + [x_1+x_2+1, y_1+y_2+1, z_1+z_2+1] = [x+x_1+1, y+y_1+1, z+z_1+1] + (x_2,y_2,z_2)$
$(x+x_1+x_2+1+1, y+y_1+y_2+1+1, z+z_1+z_2+1+1) \overset{?}{=} (x+x_1+1+x_2+1, y+y_1+1+y_2+1, z+z_1+1+z_2+1)$
$(x+x_1+x_2+2, y+y_1+y_2+2, z+z_1+z_2+2) = (x+x_1+x_2+2, y+y_1+y_2+2, z+z_1+z_2+2)$
$\therefore$ q.e.d.

S4) Neutro Aditivo $\exists (x,y,z) \in \mathbb{R}^3 \sqcap (x,y,z) + (x_1,y_1,z_1) = (x_1,y_1,z_1)$?

Veamos $(x,y,z) + (x_1,y_1,z_1) = (x_1,y_1,z_1)$
$(x+x_1+1, y+y_1+1, z+z_1+1) = (x_1,y_1,z_1)$
$(x,y,z) + (x_1+1, y_1+1, z_1+1) = (x_1,y_1,z_1)$
$(x,y,z) = (x_1,y_1,z_1) - (x_1+1, y_1+1, z_1+1)$
$(x,y,z) = (\cancel{x_1}-\cancel{x_1}-1, \cancel{y_1}-\cancel{y_1}-1, \cancel{z_1}-\cancel{z_1}-1)$
$(x,y,z) = (-1, -1, -1)$
$(-1,-1,-1) + (\alpha, \beta, \gamma) = (\cancel{-1}+\alpha+\cancel{1}, \beta\cancel{-1}+\cancel{1}, \gamma\cancel{-1}+\cancel{1})$
$= (\alpha, \beta, \gamma)$ $\therefore$ q.e.d.

Inverso Aditivo $\exists (x,y,z) \in \mathbb{R}^3 \ni (x,y,z)+(x_1,y_1,z_1) = (-1,-1,-1)$?

Veamos
$$(x,y,z)+(x_1,y_1,z_1) = (-1,-1,-1)$$
$$(x,y,z)+(x_1,y_1,z_1) = -(1,1,1)$$
$$(x,y,z) = -(1,1,1)+(-(x_1,y_1,z_1))$$
$$(x,y,z) = -(x_1+1+1,\ y_1+1+1,\ z_1+1+1)$$
$$(x,y,z) = -(x_1+2,\ y_1+2,\ z_1+2)$$
$$(x,y,z) = (-x_1-2,\ -y_1-2,\ -z_1-2)$$

$$(-x_1-2,-y_1-2,-z_1-2)+(x_1,y_1,z_1)$$
$$=(\cancel{-x_1}-2\cancel{+x_1}+1,\ -\cancel{y_1}+\cancel{y_1}-2+1,\ \cancel{z_1}-\cancel{z_1}-2+1)$$
$$=(-1,-1,-1) \quad \therefore \text{ q.c.d.}$$

M1) Cerradura ✓

M2) Distributiva $\alpha[(x,y,z)+(x_1,y_1,z_1)] = \alpha(x,y,z)+\alpha(x_1,y_1,z_1)$

De la primera parte

$$\alpha[x+x_1+1,\ y+y_1+1,\ z+z_1+1]$$
$$=(\alpha(x+x_1+1);\ \alpha(y+y_1+1)+\alpha(z+z_1+1))$$
$$=\alpha x+\alpha x_1+\alpha,\ \alpha y+\alpha y_1+\alpha,\ \alpha z+\alpha z_1+\alpha \neq$$

De la segunda parte

$$(\alpha x,\alpha y,\alpha z)+(\alpha x_1,\alpha y_1,\alpha z_1)$$
$$=(\alpha x+\alpha x_1+1,\ \alpha y+\alpha y_1+1,\ \alpha z+\alpha z_1+1)$$

$\therefore$ No es un Esp. Vect. por no cumplir M2

27. Demuestre que $M_{2\times2}$ con las Op. Norm. es un espacio Vectorial.

1) Cerradura $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}+\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}=\begin{pmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{pmatrix} M_{2\times2}$

2) Conmutativa $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}+\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}=\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}+\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$

$$\Downarrow$$

$$\begin{pmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{pmatrix}=\begin{pmatrix} b_1+a_1 & b_2+a_2 \\ b_3+a_3 & b_4+a_4 \end{pmatrix}=\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}+\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \therefore \text{qcd}$$

3) Asociativa $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}+\left[\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}+\begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}\right]=\left[\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}+\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}\right]+\begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}$

Veamos

$$\Rightarrow \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}+\begin{bmatrix} b_1+c_1 & b_2+c_2 \\ b_3+c_3 & b_4+c_4 \end{bmatrix}=\begin{bmatrix} a_1+(b_1+c_1) & a_2+(b_2+c_2) \\ a_3+(b_3+c_3) & a_4+(b_4+c_4) \end{bmatrix}=\begin{vmatrix} (a_1+b_1)+c_1 & (a_2+b_2)+c_2 \\ (a_3+b_3)+c_3 & (a_4+b_4)+c_4 \end{vmatrix}$$

$$=\begin{pmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{pmatrix}+\begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}=\left[\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}+\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}\right]+\begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}$$

S4) Neutro Aditivo $\exists \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \ni \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$?

$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$ $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} - \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$

$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

S5) Inverso Aditivo $\exists \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \ni \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$?

$\Rightarrow \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$ $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} -b_1 & -b_2 \\ -b_3 & -b_4 \end{pmatrix}$

M1) Cerradura $\alpha \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} \alpha a_1 & \alpha a_2 \\ \alpha a_3 & \alpha a_4 \end{pmatrix}$ ✓

M2) Distributiva $\alpha \left[\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \right] = \alpha \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \alpha \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$

$\alpha \begin{pmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{pmatrix} = \begin{pmatrix} \alpha(a_1+b_1) & \alpha(a_2+b_2) \\ \alpha(a_3+b_3) & \alpha(a_4+b_4) \end{pmatrix} = \begin{pmatrix} \alpha a_1+\alpha b_1 & \alpha a_2+\alpha b_2 \\ \alpha a_3+\alpha b_3 & \alpha a_4+\alpha b_4 \end{pmatrix}$

$= \begin{pmatrix} \alpha a_1 & \alpha a_2 \\ \alpha a_3 & \alpha a_3 \end{pmatrix} + \begin{pmatrix} \alpha b_1 & \alpha b_2 \\ \alpha b_3 & \alpha b_4 \end{pmatrix} = \alpha \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \alpha \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$

$\therefore$ q.e.d.

M3) Distributiva (2 escal.) $(\alpha+\beta)\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \alpha \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \beta \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$

$(\alpha+\beta)\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} (\alpha+\beta)a_1 & (\alpha+\beta)a_2 \\ (\alpha+\beta)a_3 & (\alpha+\beta)a_4 \end{pmatrix} = \begin{pmatrix} \alpha a_1+\beta a_1 & \alpha a_2+\beta a_2 \\ \alpha a_3+\beta a_3 & \alpha a_4+\beta a_4 \end{pmatrix} \Rightarrow$

$$\Rightarrow \begin{pmatrix} \alpha a_1 & \alpha a_2 \\ \alpha a_3 & \alpha a_4 \end{pmatrix} + \begin{pmatrix} \beta a_1 & \beta a_2 \\ \beta a_3 & \beta a_4 \end{pmatrix} = \alpha \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \beta \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \therefore \text{q.e.d.}$$

M4) Asociativa $\alpha\left[\beta\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}\right] = \beta\left[\alpha\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}\right]$

$$\alpha\left[\beta\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}\right] = \alpha\begin{pmatrix} \beta a_1 & \beta a_2 \\ \beta a_3 & \beta a_4 \end{pmatrix} = \begin{pmatrix} \alpha\beta a_1 & \alpha\beta a_2 \\ \alpha\beta a_3 & \alpha\beta a_4 \end{pmatrix} = \begin{pmatrix} \beta\alpha a_1 & \beta\alpha a_2 \\ \beta\alpha a_3 & \beta\alpha a_4 \end{pmatrix} = \beta\begin{pmatrix} \alpha a_1 & \alpha a_2 \\ \alpha a_3 & \alpha a_4 \end{pmatrix}$$

$$= \beta\left[\alpha\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}\right] \therefore \text{q.e.d.}$$

M5) Identidad $I\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$ Por definición

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\therefore$ M_{2x2} Es Un Espacio Vectorial

28. Demuestre que $\{(x, 2x) : x$ es un número real$\}$ con las operaciones normales de $\mathbb{R}^2$ es un Espacio Vectorial.

S1) Cerradura $(x, 2x) + (x_1, 2x_1) = (x + x_1, 2x + 2x_1)$
$= (x + x_1, 2(x + x_1))$ ✓

S2) Conmutatividad $(x, 2x) + (x_1, 2x_1) = (x_1, 2x_1) + (x, 2x)$

$(x + x_1, 2x + 2x_1) \overset{?}{=} (x_1 + x, 2x_1 + 2x)$

$(x + x_1, 2(x + x_1)) = (x_1 + x, 2(x_1 + x))$

S3) Asociativa $(x, 2x) + [(x_1, 2x_1) + (x_2, 2x_2)] = [(x, 2x) + (x_1, 2x_1)] + (x_2, 2x_2)$

Partiendo de la 1ª parte

$(x, 2x) + (x_1 + x_2, 2x_1 + 2x_2) = (x + (x_1 + x_2), 2x + (2x_1 + 2x_2))$
$= ((x + x_1) + x_2, (2x + 2x_1) + 2x_2) = (x + x_1, 2x + 2x_1)(x_2, 2x_2)$
$= [(x + 2x) + (x_1 + 2x_1)] + (x_2, 2x_2)$ $\therefore$ q.e.d

S4) Neutro Aditivo $\exists (x, y) \in \mathbb{R}^2$ ☐ $(x, y) + (x_1, y_1) = (x_1, y_1)$?

Veamos $(x, y) = (x_1, y_1) - (x_1, y_1)$
$(x, y) = (x_1 - x_1, y_1 - y_1)$
$(x, y) = (0, 0)$

S5) Inverso Aditivo $\exists (x, y) \in \mathbb{R}^2$ ☐ $(x, y) + (x_1, y_1) = (0, 0)$?

Veamos $(x, y) + (x_1 + y_1) = (0, 0)$ $(x, y) = (-x_1, -y_1)$
$(x, y) = (0, 0) - (x_1 + y_1)$
$(x, y) = (0 - x_1, 0 - y_1)$

M1) Cerradura $\alpha(x, 2x) = (\alpha x, \alpha 2x)$ ✓
$= (\alpha x, 2\alpha x$

M2) Distributiva $\alpha[(x, 2x) + (x_1, 2x_1)] = \alpha(x, 2x) + \alpha(x_1, 2x_1)$

Partiendo de la 1ª parte

$\alpha[(x+x_1, 2x+2x_1)] = (\alpha(x+x_1), \alpha(2x+2x_1)) = (\alpha x + \alpha x_1, \alpha 2x + \alpha 2x_1)$

$= (\alpha x, \alpha 2x) + (\alpha x_1, \alpha 2x_1) = \underline{\alpha(x, 2x) + \alpha(x_1, 2x_1)}$ ∴ q.e.d.

M3) Distributiva (2 escalares)

$\underline{(\alpha+\beta)(x, 2x)} = \alpha(x, 2x) + \beta(x, 2x)$

Partiendo de

$((\alpha+\beta)x, (\alpha+\beta)2x) = (\alpha x + \beta x, \alpha 2x + \beta 2x) = (\alpha x, \alpha 2x) + (\beta x, \beta 2x)$

$= \underline{\alpha(x, 2x) + \beta(x, 2x)}$ ∴ q.e.d.

M4) Asociativa $\alpha(\beta(x, 2x)) = \beta(\alpha(x, 2x))$

Partiendo de la 1ª parte

$\alpha(\beta(x, 2x)) = \alpha(\beta x, \beta 2x) = (\alpha\beta x, \alpha\beta 2x) = (\beta\alpha x, \beta\alpha 2x)$

$= \beta(\alpha x, \alpha 2x) = \underline{\beta(\alpha(x, 2x))}$ ∴ q.e.d.

M5) Identidad $I(x, 2x) = x, 2x$ $I = 1$ escalar

Como se cumplen los 10 axiomas q.e.d que $(x, 2x)$ es un E.V.

29. Determine si el conjunto $\mathbb{R}^2$ con las operaciones definidas a continuación es un Espacio Vectorial. Si sí compruebe los axiomas, si no, escriba los que no se cumplen.

$(x_1, y_1) + (x_2, y_2) = (x_1 x_2, y_1 y_2)$ $c(x_1, y_1) = (cx_1, cx_2)$

No se cumplen los axiomas:

M2) Distributiva $\alpha[(x, y) + (x_1, y_1)] = \alpha(x, y) + \alpha(x_1, y_1)$

De la primera parte

$\alpha(xx_1, yy_1)$
$= (\alpha x x_1, \alpha y y_1)$

De la Segunda Parte

$\alpha(x, y) + \alpha(x_1, y_1)$
$= (\alpha x, \alpha y) + (\alpha x_1, \alpha y_1)$
$= (\alpha x \alpha x_1, \alpha y \alpha y_1)$
$= (\alpha^2 x x_1, \alpha^2 y y_1)$

M3) Distributiva $(\alpha+\beta)(x, y) = \alpha(x, y) + \beta(x, y)$

1ª Parte

$(\alpha\beta)(x, y)$
$(\alpha\beta x, \alpha\beta y)$ $\neq$

2ª parte.

$(\alpha x, \alpha y) + (\beta x, \beta y)$
$(\alpha x \beta x, \alpha y \beta y)$

Compruebe que W es un subconjunto de V. Suponga a V con las ope. norm.

1. $W = \{(x_1, x_2, x_3, 0) : x_1, x_2, x_3$ numeros reales cualesquiera$\}$
$V = \mathbb{R}^4$

S1) Cerradura $(x_1, x_2, x_3, 0) + (y_1, y_2, y_3, 0) = (x_1+y_1, x_2+y_2, x_3+y_3, 0+0)$ ✓
S2) Conmutativa ✓
S3) Asociativa ✓
S4) Neutro Aditivo $\exists (x, y, z, w) \in \mathbb{R}^4 \ni (x, y, z, w) + (x_1, x_2, x_3, 0) = (x_1, x_2, x_3, 0)$

Veamos $(x, y, z, w) + (x_1, x_2, x_3, 0) = (x_1, x_2, x_3, 0)$
$(x, y, z, w) = (x_1, x_2, x_3, 0) - (x_1, x_2, x_3, 0)$

$(x, y, z, w) = (0, 0, 0, 0)$

S5) Inverso Aditivo $\exists (x, y, z, w) \ni (x, y, z, w) + (x_1, x_2, x_3, 0) = (0, 0, 0, 0)$?

Veamos $(x, y, z, w) = (0, 0, 0, 0) - (x_1, x_2, x_3, 0)$
$(x, y, z, w) = (-x_1, -x_2, -x_3, 0)$

M1) Cerradura $\alpha(x_1, x_2, x_3, 0) = (\alpha x_1, \alpha x_2, \alpha x_3, 0)$ ✓
M2) Distributiva: $\alpha[(x_1, x_2, x_3, 0) + (x, y, z, 0)] = \alpha(x_1, x_2, x_3, 0) + \alpha(x, y, z, 0)$
P. Primera Parte
$\alpha[(x_1+x), (x_2+y), (x_3+z), (0+0)]$
$= \alpha(x_1+x), \alpha(x_2+y), \alpha(x_3+z), \alpha(0+0)$
$= \alpha x_1 + \alpha x, \alpha x_2 + \alpha y, \alpha x_3 + \alpha z, \alpha 0 + \alpha 0$
$= (\alpha x_1, \alpha x_2, \alpha x_3, \alpha 0) + (\alpha x, \alpha y, \alpha z, \alpha 0)$
$= \alpha(x_1, x_2, x_3, 0) + \alpha(x, y, z, 0)$ $\therefore$ q.e.d.

M3) Distributiva (2 escalares) $(\alpha+\beta)(x_1, x_2, x_3, 0) = \alpha(x_1, x_2, x_3, 0) + \beta(x_1, x_2, x_3, 0)$
Partien. de la P.P.
$(\alpha+\beta)(x_1, x_2, x_3, 0) = ((\alpha+\beta)x_1, (\alpha+\beta)x_2, (\alpha+\beta)x_3, (\alpha+\beta)0)$
$= (\alpha x_1 + \beta x_1, \alpha x_2 + \beta x_2, \alpha x_3 + \beta x_3, \alpha(0) + \beta(0))$
$= (\alpha x_1, \alpha x_2, \alpha x_3, \alpha 0) + (\beta x_1, \beta x_2, \beta x_3, \beta(0))$
$= \alpha(x_1, x_2, x_3, 0) + \beta(x_1, x_2, x_3, 0)$

M4) Asociativa $\alpha[\beta(x_1, x_2, x_3, 0)] = \beta[\alpha(x_1, x_2, x_3, 0)]$
P. P. P.
$= \alpha(\beta x_1, \beta x_2, \beta x_3, \beta(0)) = (\alpha\beta x_1, \alpha\beta x_2, \alpha\beta x_3, \alpha\beta(0))$
$= (\beta\alpha x_1, \beta\alpha x_2, \beta\alpha x_3, \beta\alpha(0)) = \beta(\alpha x_1, \alpha x_2, \alpha x_3, \alpha(0))$
$= \beta[\alpha(x_1, x_2, x_3, 0)]$ $\therefore$ q.e.d.

M5) Identidad $I(x_1, x_2, x_3, 0) = (x_1, x_2, x_3, 0)$ Escalar 1

$\therefore$ Si es un Esp. Vect.

2. $W = \{(x, y, 2x-3y) : x \text{ y } Y$ números reales cualesquiera $\}$.
$V = \mathbb{R}^3$

S1) Cerradura ✓
S2) Conmutativa ✓
S3) Asociativa ✓ $(x, y, 2x-3y) + [(x_1, y_1, 2x_1-3y_1) + (x_2, y_2, 2x_2-3y_2)]$
$= [(x, y, 2x-3y) + (x_1, y_1, 2x_1-3y_1)] + (x_2, y_2, 2x_2-3y_2)$

Primera Parte | Segunda Parte

$(x, y, 2x-3y) + (x_1+x_2, y_1+y_2, 2x_1-3y_1+2x_2-3y_2)$ | $(x+x_1, y+y_1, 2x-3y+2x_1-3y_1) + (x_2, y_2, 2x_2-3y_2)$
$(x+x_1+x_2, y+y_1+y_2, 2x-3y+2x_1-3y_1+2x_2-3y_2)$ | $(x+x_1+x_2, y+y_1+y_2, 2x-3y+2x_1-3y+2x_2-3y_2)$

S4) Neutro Aditivo $\exists (x_1, y_1, z_1) \in \mathbb{R}^3 \sqcap (x_1, y_1, z_1) + (x, y, 2x-3y) = (x, y, 2x-3y)$?

Veamos $(x_1, y_1, z_1) = (x, y, 2x-3y) - (x, y, 2x-3y)$
$= (x-x, y-y, 2x-3y-2x+3y)$
$= (0, 0, 0)$

S5) Inverso Aditivo $\exists (x_1, y_1, z_1) \in \mathbb{R}^3 \sqcap (x_1, y_1, z_1) + (x, y, 2x-3y) = (0, 0, 0)$?

Veamos $(x_1, y_1, z_1) = (0, 0, 0) - (x, y, 2x-3y)$
$= (0-x, 0-y, 0-2x+3y)$
$= (-x, -y, 3y-2x)$

M1) Cerradura ✓ $\alpha(x, y, 2x-3y) = (\alpha x, \alpha y, \alpha 2x - \alpha 3y)$

M2) Distributiva $\alpha[(x, y, 2x-3y) + (x_1, y_1, 2x_1-3y_1)] = \alpha(x, y, 2x-3y) + \alpha(x_1, y_1, 2x_1-3y_1)$

Partiendo de la Prim. Part.

$\alpha(x+x_1, y+y_1, (2x-3y)+(2x_1-3y_1)) = \alpha(x+x_1), \alpha(y+y_1), \alpha[(2x-3y)+(2x_1-3y_1)]$
$= (\alpha x + \alpha x_1, \alpha y + \alpha y_1, \alpha(2x-3y) + \alpha(2x_1-3y_1))$
$= (\alpha x, \alpha y, \alpha(2x-3y)) + (\alpha x_1, \alpha y_1, \alpha(2x_1-3y_1))$
$= \alpha(x, y, 2x-3y) + \alpha(x_1, y_1, 2x_1-3y_1)$ $\therefore$ q.e.d.

M3) Distributiva $(\alpha+\beta)(x, y, 2x-3y) = \alpha(x, y, 2x-3y) + \beta(x, y, 2x-3y)$

P. P. P $(\alpha+\beta)(x, y, 2x-3y) = ((\alpha+\beta)x, (\alpha+\beta)y, (\alpha+\beta)(2x-3y))$
$= \alpha x + \beta x, \alpha y + \beta y, \alpha(2x-3y) + \beta(2x-3y)$
$= (\alpha x, \alpha y, \alpha(2x-3y)) + (\beta x, \beta y, \beta(2x-3y))$
$= \alpha(x, y, 2x-3y) + \beta(x, y, 2x-3y)$ $\therefore$ q.e.d.

M4) Asociativa $\alpha[\beta(x, y, 2x-3y)] = \beta[\alpha(x, y, 2x-3y)]$

P. P. P. $\alpha(\beta x, \beta y, \beta(2x-3y)) = (\alpha\beta x, \alpha\beta y, \alpha\beta(2x-3y))$

$= (\beta\alpha x, \beta\alpha y, \beta\alpha(2x-3y))$

$= (\beta(\alpha x, \alpha y, \alpha(2x-3y))$

$= \beta[\alpha(x, y, 2x-3y)]$ ∴ q.e.d.

M5) Identidad $I(x, y, 2x-3y) = (x, y, 2x-3y)$

Escalar $\underline{1 \checkmark}$

∴ Es Esp. Vectorial.

3. W es el conjunto de todas las matrices 2x2 de la forma $\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$ $V = M_{2x2}$

S1) Cerradura ✓

S2) Conmutativa ✓

S3) Asociativa ✓

S4) Neutro Aditivo $\exists\, N_{22} \ni M_{22} + N_{22} = M_{22}$?

$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$ $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} - \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$ $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

S5) Inverso Aditivo $\exists\, N_{2x2} \ni M_{2x2} + N_{2x2} = O_{2x2}$?

$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$ $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} 0 & -a \\ -b & 0 \end{pmatrix}$

M1) Cerradura ✓

M2) Distributiva $\alpha\left[\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} + \begin{pmatrix} 0 & a_1 \\ b_1 & 0 \end{pmatrix}\right] = \alpha\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} + \alpha\begin{pmatrix} 0 & a_1 \\ b_1 & 0 \end{pmatrix}$

P. P. P.

$\alpha\begin{bmatrix} 0+0 & a+a_1 \\ b+b_1 & 0+0 \end{bmatrix} = \begin{pmatrix} \alpha(0+0) & \alpha(a+a_1) \\ \alpha(b+b_1) & \alpha(0+0) \end{pmatrix} = \begin{pmatrix} \alpha 0+\alpha 0 & \alpha a+\alpha a_1 \\ \alpha b+\alpha b_1 & \alpha 0+\alpha 0 \end{pmatrix} = \begin{pmatrix} \alpha 0 & \alpha a \\ \alpha b & \alpha 0 \end{pmatrix} + \begin{pmatrix} \alpha 0 & \alpha a_1 \\ \alpha b_1 & \alpha 0 \end{pmatrix}$

$= \alpha\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} + \alpha\begin{pmatrix} 0 & a_1 \\ b_1 & 0 \end{pmatrix}$ ∴ q.e.d.

M3) Distributiva $(\alpha+\beta)\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} = \alpha\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} + \beta\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$

P.P.P $\begin{pmatrix} (\alpha+\beta)0 & (\alpha+\beta)a \\ (\alpha+\beta)b & (\alpha+\beta)0 \end{pmatrix} = \begin{pmatrix} \alpha 0+\beta 0 & \alpha a+\beta a \\ \alpha b+\beta b & \alpha 0+\beta 0 \end{pmatrix} = \begin{pmatrix} \alpha 0 & \alpha a \\ \alpha b & \alpha 0 \end{pmatrix} + \begin{pmatrix} \beta 0 & \beta a \\ \beta b & \beta 0 \end{pmatrix}$

$= \alpha\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} + \beta\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$ ∴ q.e.d.

M4) Asociativa $\alpha\left[\beta\begin{pmatrix}0 & a\\ b & 0\end{pmatrix}\right] = \beta\left[\alpha\begin{pmatrix}0 & a\\ b & 0\end{pmatrix}\right]$

P.P.P. $\alpha\left[\beta\begin{pmatrix}0 & a\\ b & 0\end{pmatrix}\right] = \alpha\begin{pmatrix}\beta 0 & \beta a\\ \beta b & \beta 0\end{pmatrix} = \begin{pmatrix}\alpha\beta 0 & \alpha\beta a\\ \alpha\beta b & \alpha\beta 0\end{pmatrix} = \begin{pmatrix}\beta\alpha 0 & \beta\alpha a\\ \beta\alpha b & \beta\alpha 0\end{pmatrix}$

$= \beta\begin{pmatrix}\alpha 0 & \alpha a\\ \alpha b & \alpha 0\end{pmatrix} = \beta\left[\alpha\begin{pmatrix}0 & a\\ b & 0\end{pmatrix}\right]$

M5) Identidad (Por definición) $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

4. W es el conjunto de todas las matrices 3×2 de la forma $\begin{bmatrix}a & b\\ a+b & 0\\ 0 & c\end{bmatrix}$

S1) Cerradura ✓

S2) Conmutativa ✓

S3) Asociativa ✓

S4) Neutro Aditivo $\exists\ M_{3x2}$ PT $M_{3x2} + N_{3x2} = N_{3x2}$?

Veamos $\begin{pmatrix}a & b\\ a+b & 0\\ 0 & c\end{pmatrix} + \begin{pmatrix}a_1 & b_1\\ a_1+b_1 & 0\\ 0 & c_1\end{pmatrix} = \begin{pmatrix}a_1 & b_1\\ a_1+b_1 & 0\\ 0 & c_1\end{pmatrix}$

$\begin{pmatrix}a & b\\ a+b & 0\\ 0 & c\end{pmatrix} = \begin{pmatrix}a_1 & b_1\\ a_1+b_1 & 0\\ 0 & c_1\end{pmatrix} - \begin{pmatrix}a_1 & b_1\\ a_1+b_1 & 0\\ 0 & c_1\end{pmatrix}$

$\begin{pmatrix}a & b\\ a+b & 0\\ 0 & c\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}$

S5) Inverso Aditivo $\exists\ M_{3x2}$ PT $M_{3x2} + N_{3x2} = 0_{3x2}$?

Veamos $\begin{pmatrix}a_1 & b_1\\ a_1+b_1 & 0\\ 0 & c_1\end{pmatrix} + \begin{pmatrix}a & b\\ a+b & 0\\ 0 & c\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}$

$\begin{pmatrix}a_1 & b_1\\ a_1+b_1 & 0\\ 0 & c_1\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix} - \begin{pmatrix}a & b\\ a+b & 0\\ 0 & c\end{pmatrix}$

$\begin{pmatrix}a_1 & b_1\\ a_1+b_1 & 0\\ 0 & c_1\end{pmatrix} = \begin{pmatrix}-a & -b\\ -a-b & 0\\ 0 & -c\end{pmatrix}$

M1) Cerradura ✓

M2) Distributiva $\alpha\left[\begin{pmatrix}a & b\\ a+b & 0\\ 0 & c\end{pmatrix} + \begin{pmatrix}a_1 & b_1\\ a_1+b_1 & 0\\ 0 & c_1\end{pmatrix}\right] = \alpha\begin{pmatrix}a & b\\ a+b & 0\\ 0 & c\end{pmatrix} + \alpha\begin{pmatrix}a_1 & b_1\\ a_1+b_1 & 0\\ 0 & c_1\end{pmatrix}$

P.P.P.

$\alpha\begin{pmatrix}a+a_1 & b+b_1\\ (a+b)+(a_1+b_1) & 0+0\\ 0+0 & c+c_1\end{pmatrix} = \begin{pmatrix}\alpha a+\alpha a_1 & \alpha b+\alpha b_1\\ \alpha(a+b)+\alpha(a_1+b_1) & \alpha 0+\alpha 0\\ \alpha 0+\alpha 0 & \alpha c+\alpha c_1\end{pmatrix} = \begin{pmatrix}\alpha a & \alpha b\\ \alpha(a+b) & \alpha 0\\ \alpha 0 & \alpha c\end{pmatrix} + \begin{pmatrix}\alpha a_1 & \alpha b_1\\ \alpha(a_1+b_1) & \alpha 0\\ \alpha 0 & \alpha c_1\end{pmatrix}$

$$= \alpha \begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix} + \alpha \begin{pmatrix} a_1 & b_1 \\ a_1+b_1 & 0 \\ 0 & c_1 \end{pmatrix} \therefore \text{q.e.d.}$$

M3) Distributiva $(\alpha+\beta)\begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix} = \alpha \begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix} + \beta \begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix}$

P.P.P.

$$= \begin{pmatrix} (\alpha+\beta)a & (\alpha+\beta)b \\ (\alpha+\beta)(a+b) & (\alpha+\beta)(0) \\ (\alpha+\beta)(0) & (\alpha+\beta)c \end{pmatrix} = \begin{pmatrix} \alpha a+\beta a & \alpha b+\beta b \\ \alpha(a+b)+\beta(a+b) & \alpha(0)+\beta(0) \\ \alpha(0)+\beta(0) & \alpha c+\beta c \end{pmatrix}$$

$$= \begin{pmatrix} \alpha a & \alpha b \\ \alpha(a+b) & \alpha(0) \\ \alpha(0) & \alpha c \end{pmatrix} + \begin{pmatrix} \beta a & \beta b \\ \beta(a+b) & \beta(0) \\ \beta(0) & \beta c \end{pmatrix} = \alpha \begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix} + \beta \begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix} \therefore \text{q.e.d.}$$

M4) Asociación $\alpha \left[\beta \begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix}\right] = \beta \left[\alpha \begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix}\right]$

$$\alpha \begin{bmatrix} \beta a & \beta b \\ \beta(a+b) & \beta(0) \\ \beta(0) & \beta c \end{bmatrix} = \begin{bmatrix} \alpha\beta a & \alpha\beta b \\ \alpha\beta(a+b) & \alpha\beta(0) \\ \alpha\beta(0) & \alpha\beta c \end{bmatrix} = \begin{pmatrix} \beta\alpha a & \beta\alpha b \\ \beta\alpha(a+b) & \beta\alpha(0) \\ \beta\alpha(0) & \beta\alpha c \end{pmatrix} = \beta \begin{pmatrix} \alpha a & \alpha b \\ \alpha(a+b) & \alpha(0) \\ \alpha(0) & \alpha c \end{pmatrix}$$

$$= \beta \left[\alpha \begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix}\right]$$

M5) Identidad $\underbrace{\begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix}}_{3\times2} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{2\times2} = \begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix} \therefore$ es un Sub. Vect.

3×2

- En los sig. ejercicios demuestre que no es subespacio por medio de un ejemplo específico.

1. W es el conjunto de todos los vectores $\mathbb{R}^3$ cuya 3ª compon. es -1

$(x, y, -1) \rightarrow$ Forma

S1) Cerradura $(x, y, -1) + (x_1, y_1, -1) = (x+x_1, y+y_1, \underline{-2}) \therefore$ No es Sub.

2. W es el conjunto de todos los vectores en $\mathbb{R}^2$ cuyas comp. son numeros racionales.

Sección 4.5 BASES.

• Escriba la base normal para el espacio vectorial dado.

1. $\mathbb{R}^6$

$B = \{(1,0,0,0,0,0), (0,1,0,0,0,0,0), (0,0,1,0,0,0), (0,0,0,1,0,0), (0,0,0,0,1,0)$
$(0,0,0,0,0,1)\}$

2: $M_{4,1}$

$$B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

3. M_{2x4}

$$B = \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\}$$

4. P_4 $B = \{1, x, x^2, x^3\}$.

• Explique por qué S no es una base de $\mathbb{R}^2$

5. $S = \{(1,2), (1,0), (0,1)\}$ Porque tiene más vectores que la dimensión
6. $S = \{(-4,5), (0,0)\}$ Porque los vectores son ld
7. $S = \{(6,-5), (12,-10)\}$ Porque los vectores so ld
8. $S = \{(-3,2)\}$ Porque el vector es ld.

• Explique por qué S no es una base de $\mathbb{R}^3$

9. $S = \{(1,3,0), (4,1,2), (-2,5,-2)\}$ Porque son ld (Determinante
10. $S = \{(7,0,3), (8,-4,1)\}$ Porque son ld. (Determinante)
11. $S = \{(0,0,0), (1,0,0), (0,1,0)\}$ Porque son ld.
12. $S = \{(6,4,1), (3,-5,1), (8,13,6), (0,6,9)\}$ Porque hay más vectores que la dim.

• Explique por qué S no es una base de P_2.

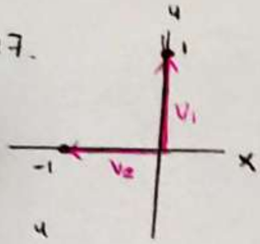
13. $S = \{1, 2x, x^2-4, 5x\}$ Porque es linealmente dependiente
14. $S = \{1-x, 1-x^2, 3x^2-2x-1\}$ Porque tiene infinidad de sols. el sist.

• Explique por que S no es una base de M_{2x2}

15. $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ Son li pero no generar a todo M_{2x2}

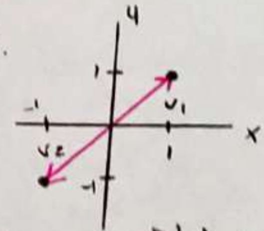
• Determine si el conjunto (v_1, v_2) es una base de $\mathbb{R}^2$

17.



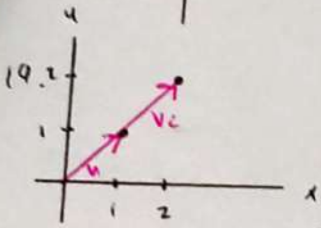
$v_1 = (0,1)$
$v_2 = (-1, 0)$
$\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = 1 \neq 0 \therefore$ li
Si es base de $\mathbb{R}^2$

18.

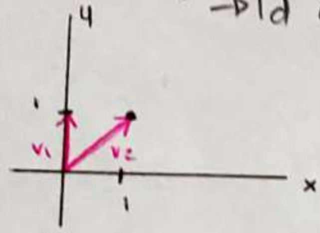


$v_1 = (1,1)$
$v_2 = (-1,-1)$
$\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix} = -1+1 = 0$
$\to$ ld $y \therefore$ No es base

19.



$v_1 = (1,1)$; $v_2 = (2,2)$
$\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2-2 = 0$ ld.
$\therefore$ No es base



$v_1 = (0,1)$; $v_2 = (1,1)$
$\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \neq 0$
Es base.

• Determine si S es una base del espacio vectorial dado.

21. $S = \{(3,-2), (4,5)\}$ para $\mathbb{R}^2$

$$\begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix} = 15 + 8 = 23 \neq 0$$
$\therefore$ li

Entonces si es una base.

22. $S = \{(1,5,3), (0,1,2), (0,0,6)\}$ para $\mathbb{R}^3$

$$\begin{vmatrix} 1 & 5 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 6 \end{vmatrix} = 6 \neq 0 \text{ li}$$
Si es base

23. $S = \{(0,3,-2), (4,0,3), (-8,15,-16)\}$ para $\mathbb{R}^3$

$$\begin{vmatrix} 0 & 3 & -2 \\ 4 & 0 & 3 \\ -8 & 15 & -16 \end{vmatrix} = -3(-64+24) - 2(60)$$
$$= -3(-40) - 120$$
$$= 120 - 120 = 0 \text{ ld.}$$
No es una bas

24. $S = \{(0,0,0), (1,5,6), (6,2,1)\}$ para $\mathbb{R}^3$

$$\begin{vmatrix} 0 & 0 & 0 \\ 1 & 5 & 6 \\ 6 & 2 & 1 \end{vmatrix} = 0$$ ld. No es base.

25. $S = \{(-1,2,0,0), (2,0,-1,0), (3,0,0,4), (0,0,5,0)\}$ para $\mathbb{R}^4$

$$\begin{vmatrix} -1 & 2 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & 4 \\ 0 & 0 & 5 & 0 \end{vmatrix} \sim \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 4 & -1 & 0 \\ 0 & 6 & 0 & 4 \\ 0 & 0 & 5 & 0 \end{vmatrix} = -1 \begin{vmatrix} 4 & -1 & 0 \\ 6 & 0 & 4 \\ 0 & 5 & 0 \end{vmatrix} \sim -1[4(-20)] = 80$$

$80 \neq 0$ li $\therefore$ Si es base

7. Determine si $S=\left\{\begin{pmatrix}2&0\\0&3\end{pmatrix},\begin{pmatrix}1&4\\0&1\end{pmatrix},\begin{pmatrix}0&1\\3&2\end{pmatrix},\begin{pmatrix}0&1\\2&0\end{pmatrix}\right\}$ es base de $M_{2\times 2}$.

$$c_1\begin{pmatrix}2&0\\0&3\end{pmatrix}+c_2\begin{pmatrix}1&4\\0&1\end{pmatrix}+c_3\begin{pmatrix}0&1\\3&2\end{pmatrix}+c_4\begin{pmatrix}0&1\\2&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

$$\begin{pmatrix}2c_1&0\\0&3c_1\end{pmatrix}+\begin{pmatrix}c_2&4c_2\\0&c_2\end{pmatrix}+\begin{pmatrix}0&c_3\\3c_3&2c_3\end{pmatrix}+\begin{pmatrix}0&c_4\\2c_4&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

$$\begin{pmatrix}2c_1+2c_2&4c_2+c_3+c_4\\3c_3+2c_4&3c_1+c_2+2c_3\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}\Rightarrow \begin{aligned}2c_1+2c_2&=0\\4c_2+c_3+c_4&=0\\3c_3+2c_4&=0\\3c_1+c_2+2c_3&=0\end{aligned}$$

$$\begin{vmatrix}2&2&0&0\\0&4&1&1\\0&0&3&2\\3&1&2&0\end{vmatrix}\sim\begin{vmatrix}1&1&0&0\\0&1&1/4&1/4\\0&0&1&2/3\\0&-2&2&0\end{vmatrix}\sim\begin{vmatrix}1&1&0&0\\0&1&1/4&1/4\\0&0&1&2/3\\0&0&5/2&2/4\end{vmatrix}$$

$$\sim\begin{vmatrix}1&1&0&0\\0&1&1/4&1/4\\0&0&1&2/3\\0&0&0&-7/6\end{vmatrix}=-\frac{7}{6}\neq 0 \therefore \text{li} \quad \text{Si es base}$$

- Determine si S es una base de P_3.

$S=\{t^3-2t^2+1,\ t^2-4,\ t^3+2t,\ 5t\}$

$$c_1(t^3-2t^2+1)+c_2(t^2-4)+c_3(t^3+2t)+c_4(5t)=0+0t+0t^2+0t^3$$

$$c_1t^3-2c_1t^2+c_1+c_2t^2-4c_2+c_3t^3+2c_3t+5c_4t)=0$$

$$\begin{aligned}c_1-4c_2&=0\\2c_3+5c_4&=0\\-2c_1+c_2&=0\\c_1+c_3&=0\end{aligned}\sim\left(\begin{array}{cccc|c}1&-4&0&0&0\\0&0&2&5&0\\-2&1&0&0&0\\1&0&1&0&0\end{array}\right)$$

$$\left(\begin{array}{cccc|c}1&-4&0&0&0\\0&0&2&5&0\\0&-7&0&0&0\\0&4&1&0&0\end{array}\right)\sim\left(\begin{array}{cccc|c}1&-4&0&0&0\\0&1&1/4&0&0\\0&0&7/4&0&0\\0&0&2&5&0\end{array}\right)\sim\left(\begin{array}{cccc|c}1&-4&0&0&0\\0&1&1/4&0&0\\0&0&1&0&0\\0&0&0&1&0\end{array}\right)$$

$$\left(\begin{array}{cccc|c}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\end{array}\right)\therefore \text{Si es base.}$$

• Determine si S es una base para $\mathbb{R}^3$, si es así escriba (8, 3, 8)
como una combinación lineal de los vectores en S.

31. $S = \{(4,3,2), (0,3,2), (0,0,2)\}$

$$\begin{vmatrix} 4 & 3 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{vmatrix} = (4)(3)(2) = 24 \neq 0 \quad \therefore \text{ li } \text{ Si es base}$$

$$(8,3,8) = \alpha(4,3,2) + \beta(0,3,2) + \gamma(0,0,2)$$
$$\Rightarrow = (4\alpha, 3\alpha, 2\alpha) + (0, 3\beta, 2\beta) + (0,0,2\gamma)$$

$$\begin{aligned} 4\alpha &= 8 \\ 3\alpha + 3\beta &= 3 \\ 2\alpha + 2\beta + 2\gamma &= 8 \end{aligned} \longrightarrow \boxed{\begin{aligned} \alpha &= 2 \\ \beta &= -1 \\ \gamma &= 3 \end{aligned}} \Rightarrow$$

$$\begin{aligned} (8,3,8) &= 2(4,3,2) - (0,3,2) + 3(0,0,2) \\ &= (8,6,4) - (0,3,2) + (0,0,6) \\ &= (8,3,8) \end{aligned}$$

32. $S = \{(0,0,0), (1,3,4), (6,1,-2)\}$

$$\begin{vmatrix} 0 & 0 & 0 \\ 1 & 3 & 4 \\ 6 & 1 & -2 \end{vmatrix} = 0 \quad \text{ld. No es base}$$

33 $S = \{(\frac{2}{3}, \frac{5}{2}, 1), (1, \frac{3}{2}, 0), (2, 12, 6)\}$

$$\begin{vmatrix} \frac{2}{3} & \frac{5}{2} & 1 \\ 1 & \frac{3}{2} & 0 \\ 2 & 12 & 6 \end{vmatrix} \sim \begin{vmatrix} 1 & 6 & 3 \\ 0 & -9/2 & -3 \\ 0 & -3/2 & -1 \end{vmatrix} \sim \begin{vmatrix} 1 & 6 & 3 \\ 0 & -9 & -6 \\ 0 & -3 & -2 \end{vmatrix} \sim \begin{vmatrix} 1 & 6 & 3 \\ 0 & -9 & -6 \\ 0 & 0 & 0 \end{vmatrix}$$

el det = 0 $\therefore$ son ld y no es base.

34. $S = \{(1,4,7), (3,0,1), (2,1,2)\}$

$$\begin{vmatrix} 1 & 4 & 7 \\ 3 & 0 & 1 \\ 2 & 1 & 2 \end{vmatrix} \sim \begin{vmatrix} 1 & 4 & 7 \\ 0 & -12 & -20 \\ 0 & -7 & -12 \end{vmatrix} = 1[144 - 140] = 4 \neq 0 \text{ li es base}$$

$$\begin{aligned} (8,3,8) &= \alpha(1,4,7) + \beta(3,0,1) + \gamma(2,1,2) \\ &= (\alpha, 4\alpha, 7\alpha) + (3\beta, 0, \beta) + (2\gamma, \gamma, 2\gamma) \end{aligned}$$

$$\begin{aligned} \alpha + 3\beta + 2\gamma &= 8 \\ 4\alpha + \quad + \gamma &= 3 \\ 7\alpha + \beta + 2\gamma &= 8 \end{aligned} \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 4 & 0 & 1 & 3 \\ 7 & 1 & 2 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 0 & -12 & -7 & -29 \\ 0 & -20 & -12 & -48 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 0 & 1 & 7/12 & 29/12 \\ 0 & 0 & -1/3 & 1/3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 0 & 1 & 7/12 & 29/12 \\ 0 & 0 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 0 & 10 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right) \Rightarrow (8,3,8) = 1(1,4,7)+3(3,0,1)-1(2,1,2)$$
$$= (1,4,7)+(9,0,1)+(-2,-1,-2)$$
$$= (8,3,8)$$

- Determine la dimensión del espacio vectorial dado.

35. $\mathbb{R}^6$ dim(6) 36. $\mathbb{R}$ dim(1) 37. P_7 dim(8) 38. M_{23} dim(6)

- Determine una base D_{3x3} (Espacio vectorial de todas las matrices diagonales 3x3) ¿Cuál es la dimensión de este espacio vectorial?

$$B = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right\} \quad \underline{Dim(3)}$$

- Determine todos los subconjunto de S que forman una base de $\mathbb{R}^2$

$S = \{(1,0),(0,1),(1,1)\}$

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \checkmark \quad \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 \checkmark \quad \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \checkmark$$

$B_1 = \{(1,0),(0,1)\}$ $B_2 = \{(1,0)(1,1)\}$ $B_3 = \{(0,1)(1,1)\}$

- Determine todos los subconjuntos de S que forman una base de $\mathbb{R}^3$

$S = \{(1,3,-2),(-4,1,1),(-2,7,-3),(2,1,1)\}$

$$\begin{vmatrix} 1 & 3 & -2 \\ -4 & 1 & 1 \\ -2 & 7 & -3 \end{vmatrix} = (-3-7)+4(-9+14)-2(3+2)$$
$$= -10+20-10 = 0 \quad \text{ld}$$

$$\begin{vmatrix} 1 & 3 & -2 \\ -4 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 4(3+2)+2(3+2)$$
$$= 20+12 = 32 \neq 0 \quad \text{li}$$

$B_1 = \{(1,3,-2),(-4,1,1),(2,1,1)$

$$\begin{vmatrix} 1 & 3 & -2 \\ -2 & 7 & -3 \\ 2 & 1 & 1 \end{vmatrix} = (7+3)+2(3+2)+2(-9+14)$$
$$= 10+10+10 = 30 \neq 0 \quad \text{li}$$

$B_2 = \{(1,3,-2),(-2,7,-3),(2,1,1)\}$

$$\begin{vmatrix} -4 & 1 & 1 \\ -2 & 7 & -3 \\ 2 & 1 & 1 \end{vmatrix} = -4(7+3)+2(-3-7)$$
$$= -40-20 = -60 \neq 0 \quad \text{li}$$

$B_3 = \{(-4,1,1),(-2,7,-3),(2,1,1)\}$

• Encuentre una base de $\mathbb{R}^2$ que contenga a $(1,1)$

$\begin{vmatrix} x & y \\ 1 & 1 \end{vmatrix} = x - y$ ∴ para ser li $x \neq y$ $\quad B = \{(1,2),(1,1)\}$

$x = y$

• Encuentre una base de $\mathbb{R}^3$ que contenga al conjunto $S = \{(1,0,2),(0,1,1)\}$

$\begin{vmatrix} a & b & c \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -2a - (b-c)$

$-2a \neq 0$ y $b \neq c$

$-2a \neq (b-c)$

$2a \neq c - b$

Con 2, 3, 4

$\begin{vmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 2[-2] - 1[3-4]$

$= -4 + 1 = -3$

li ∴ base.

• De una descripción geométrica, encuentre una base y determine la dimensión del subespacio W de $\mathbb{R}^2$

• $W = \{(2t, t) : t$ es un número real$\}$

• Es una recta que pasa por el origen

• Sea $t = 1$ $\quad W = \{(2,1)\}$

• $\text{Dim}(W) = 1$

• Encuentre una base y la dimensión de W en $\mathbb{R}^4$

49. $W = \{(2s - t, s, t, s\}$

$$\begin{pmatrix} 2s-t \\ s \\ t \\ s \end{pmatrix} = \begin{pmatrix} 2s \\ s \\ 0 \\ s \end{pmatrix} + \begin{pmatrix} -t \\ 0 \\ t \\ 0 \end{pmatrix} = s\begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$B_w = \{(2,1,0,1), (-1,0,1,0)\}$ $\quad$ Dim = 2

50. $W = \{(5t - 3t, t, t)\}$

$$\begin{pmatrix} 5t-3t \\ t \\ t \end{pmatrix} = t\begin{pmatrix} 5-3 \\ 1 \\ 1 \end{pmatrix} = t\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$B_w = \{(2,1,1)\}$

51. $W = \{(0, 6t, t, -t)\}$

$$\begin{pmatrix} 0 \\ 6t \\ t \\ -t \end{pmatrix} = t\begin{pmatrix} 0 \\ 6 \\ 1 \\ -1 \end{pmatrix}$$

$B_w = \{(0,6,1,-1)\}$

dim = 1

52. $W = \{(s + 4t, t, s, 2s - t)\}$

$$\begin{pmatrix} s+4t \\ t \\ s \\ 2s-t \end{pmatrix} = \begin{pmatrix} s \\ 0 \\ s \\ 2s \end{pmatrix} + \begin{pmatrix} 4t \\ t \\ 0 \\ -t \end{pmatrix} = s\begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

$B_w = \{(1,0,1,2), (4,1,0,-1)\}$

Dim = 2

Determine si la proposición es falsa o verdadera. Justifique su respuesta.

- Si $\dim(V) = n$ entonces hay un conjunto de $n-1$ vectores en V que generan a V. Falsa, porque V debe tener por lo menos n vectores
- Si $\dim(V) = n$, entonces hay un conjunto de $n+1$ vectores en V que genere a V. Verdadero
- Si $\dim(V) = n$, entonces cualquier conjunto de $n+1$ vectores en V debe ser dependiente. Verdadera
- Si $\dim(V) = n$, entonces cualquier conjunto de $n-1$ vectores en V debe de ser independiente. Falso porque si son dependientes todos son $0 \therefore$ ld.

~~Sección~~ 4.6 RANGO.

- Determine el rango de la matriz, una base del espacio renglón y una base del espacio columna.

1. $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \to B_R = \{(1,0),(0,1)\}$
$B_C = \{(1,0),(0,1)\} \leftarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

rag = (2)

2. $[1, 2\ 3]$ $B_R = \{(1,2,3)\}$

rag = (1) $B_C = \{(1,0,0),(2,0,0),(3,0,0)\}$

3. $\begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 \\ 4 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1/2 \\ 0 & 4 \end{bmatrix}$

Rang = 2 $B_R = \{(1,2),(0,1)\}$ $\sim \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}$

$B_C = \{(1, 1/2),(0,1)\}$

4. $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2 \\ 0 & 14 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -1/2 \end{bmatrix}$

Rango = 2 $B_R = \{(1,-3,2),(0,1,-1/2)\}$

$\begin{bmatrix} 1 & 4 \\ -3 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 14 \\ 0 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B_C = \{(1,4),(0,1)\}$

5. $\begin{bmatrix} 4 & 20 & 31 \\ 6 & -5 & -6 \\ 2 & -11 & -16 \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & 31/4 \\ 0 & -35 & -105/2 \\ 0 & -21 & -63/2 \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & 31/4 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{bmatrix}$

$B_R = \{(1, 5, \frac{31}{4}), (0, 1, \frac{3}{2})\}$

$\begin{bmatrix} 4 & 6 & 2 \\ 20 & -5 & -11 \\ 31 & -6 & -16 \end{bmatrix} \sim \begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & -35 & -21 \\ 0 & -105/2 & -63/2 \end{bmatrix} \sim \begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 3/5 \\ 0 & 0 & 0 \end{bmatrix}$

$B_C = \{(1, \frac{3}{2}, \frac{1}{2}), (0, 1, \frac{3}{5})\}$

7. $\begin{bmatrix} -2 & -4 & 4 & 5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -2 & -5/2 \\ 0 & 0 & 0 & 7/2 \\ 0 & 0 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -2 & -5/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$B_R = \{(1, 2, -2, -\frac{5}{2}), (0, 0, 0, 1)\}$

$\begin{bmatrix} -2 & 3 & -2 \\ -4 & 6 & -4 \\ 4 & -6 & 4 \\ 5 & -4 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & -3/2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 7/2 & 4 \end{bmatrix}$ $B_C = \{(1, -\frac{3}{2}, 1), (0, \frac{7}{2}, 4)\}$

Rango = 2

• Determine una base del subespacio de $\mathbb{R}^4$ generado por S.

9. $S = \{(2, 9, -2, 53), (-3, 2, 3, -2), (8, -3, -8, 17), (0, -3, 0, 15)\}$

$$\begin{pmatrix} 2 & 9 & -2 & 53 \\ -3 & 2 & 3 & -2 \\ 8 & -3 & -8 & 17 \\ 0 & -3 & 0 & 15 \end{pmatrix} \sim \begin{pmatrix} 1 & 9/2 & -1 & 53/2 \\ 0 & 31/2 & 0 & 155/2 \\ 0 & -39 & 0 & -195 \\ 0 & -3 & 0 & 15 \end{pmatrix} \sim \begin{pmatrix} 1 & 9/2 & -1 & 53/2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -234 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad B = \{(1,0,-1,0), (0,1,0,0), (0,0,0,1)\}$$

11. $S = \{(-3, 2, 5, 28), (-6, 1, -8, -1), (14, -10, 12, -10), (0, 5, 12, 50)\}$

$$\begin{pmatrix} -3 & 2 & 5 & 28 \\ -6 & 1 & -8 & -1 \\ 14 & -10 & 12 & 10 \\ 0 & 5 & 12 & 50 \end{pmatrix} \sim \begin{pmatrix} 1 & -2/3 & -5/3 & -28/3 \\ 0 & -3 & -18 & -57 \\ 0 & -2 & 106 & 422 \\ 0 & 5 & 12 & 50 \end{pmatrix} \sim \begin{pmatrix} 1 & -2/3 & -5/3 & -28/3 \\ 0 & 1 & 6 & 19 \\ 0 & 0 & -59 & -230 \\ 0 & 0 & -18 & -45 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2/3 & -5/3 & -28/3 \\ 0 & 1 & 6 & 19 \\ 0 & 0 & 1 & \frac{230}{59} \\ 0 & 0 & 0 & \frac{1485}{59} \end{pmatrix} \sim \begin{pmatrix} 1 & -2/3 & -5/3 & 0 \\ 0 & 1 & 6 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2/3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad B = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$$

• Determine una base y la dimension del espacio solución de $Ax = 0$

13. $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \quad \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\left(\begin{array}{cc|c} 1 & 3 & 0 \\ 2 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -7 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 1 & 0 \end{array}\right) \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad B = (0,0) \quad \text{Dim}(0)$$

15. $A = [1 \quad 2 \quad 3] \rightarrow a + 2b + 3c$

Despejamos a $\quad a = -2b - 3c$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -2b - 3c \\ b \\ c \end{pmatrix} = \begin{pmatrix} -2b \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} -3c \\ 0 \\ c \end{pmatrix} = b \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + c \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$$

$$B = \{(-2, 1, 0), (-3, 0, 1)\}$$

$$A=\begin{bmatrix}1&2&-3\\2&-1&4\\4&3&-2\end{bmatrix}\sim\begin{bmatrix}1&2&-3\\0&-5&10\\0&-5&10\end{bmatrix}\sim\begin{bmatrix}1&2&-3\\0&1&-2\\0&0&0\end{bmatrix}$$

$$a+2b-3c=0 \qquad a=3c-2b$$
$$b-2c=0 \qquad b=2c$$

$$\Rightarrow a=3c-4c=-c \qquad \begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}-c\\2c\\c\end{pmatrix}=c\begin{pmatrix}-1\\2\\1\end{pmatrix}$$

$$B=\{(-1,2,1)\} \quad Dim=1$$

19. Encuentre una base y la dimension de espacio sol. del sistema homogeneo de ecs lineales dado.

$$\begin{aligned}-x+y+z&=0\\3x-y&=0\\2x-4y-5z&=0\end{aligned} \qquad \begin{pmatrix}-1&1&1\\3&-1&0\\2&-4&-5\end{pmatrix}\sim\begin{pmatrix}1&-1&-1\\0&2&3\\0&-2&-3\end{pmatrix}\sim\begin{pmatrix}1&-1&-1\\0&2&3\\0&0&0\end{pmatrix}$$

$$x-y-z=0 \qquad x=z+y \rightarrow x=z-\frac{3}{2}z=-\frac{1}{2}z$$
$$2y+3z=0 \qquad y=-\frac{3}{2}z$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-\frac{1}{2}z\\-\frac{3}{2}z\\z\end{pmatrix}=z\begin{pmatrix}-\frac{1}{2}\\-\frac{3}{2}\\1\end{pmatrix} \quad B=\{(-1,-3,2)\} \quad \underline{Dim(1)}$$

21. $$\begin{aligned}x-2y+3z&=0\\-3x-6y-9z&=0\end{aligned} \qquad \begin{pmatrix}1&-2&3\\-3&-6&-9\end{pmatrix}\sim\begin{pmatrix}1&-2&3\\0&-12&0\end{pmatrix}$$

$$x-2y+3z=0 \qquad x=2y-3z \qquad x=-3z$$
$$y=0 \qquad y=0$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-3z\\0\\z\end{pmatrix}=z\begin{pmatrix}-3\\0\\1\end{pmatrix} \qquad B=\{(-3,0,1)\}$$

23. $$\begin{aligned}9x_1-4x_2-2x_3-20x_4&=0\\12x_1-6x_2-4x_3-29x_4&=0\\3x_1-2x_2-7x_4&=0\\3x_1-2x_2-x_3-8x_4&=0\end{aligned} \qquad \begin{pmatrix}9&-4&-2&-20\\12&-6&-4&-29\\3&-2&0&-7\\3&-2&-1&-8\end{pmatrix}\sim\begin{pmatrix}1&-4/9&-2/9&-\frac{20}{9}\\0&-2/3&-4/3&-7/3\\0&-2/3&2/3&-1/3\\0&-2/3&-1/3&-4/3\end{pmatrix}$$

$$\begin{pmatrix} 1 & -4/9 & -1/9 & -20/9 \\ 0 & 1 & -2 & 7/2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -4/9 & -1/9 & -20/9 \\ 0 & 1 & -2 & 7/2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$9x_1 - 4x_2 - x_3 - 20x_4 = 0$

$2x_2 - 2x_3 + 7x_4 = 0 \longrightarrow x_2 = -\frac{11}{2}x_4$

$x_3 + x_4 = 0 \longrightarrow x_3 = -x_4$

$2x_2 + 4x_4 + 7x_4 = 0$

$2x_2 = -11x_4$

$x_2 = -\frac{11}{2}x_4$

$9x_1 + 22x_4 + x_4 - 20x_4 = 0$

$x_1 = \frac{3}{9}x_4 = \frac{1}{3}x_4$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{3}x_4 \\ -\frac{11}{2}x_4 \\ -x_4 \\ x_4 \end{pmatrix} = x_4 \begin{pmatrix} 1/3 \\ -\frac{11}{2} \\ -1 \\ 1 \end{pmatrix}$$

$(1, -\frac{33}{2}, -3, 3)$

$(2, -33, -6, 6)$ Dim (1)

• Determine si el sistema no homogeneo $Ax = b$ es consistente, y si el sistema es consistente, escriba la solución en la forma $x = x_h + x_p$, donde x_h es una solución de $Ax = 0$ y x_p es una sol. particular de $Ax = b$.

25. $x + 3y + 10z = 18$

$-2x + 7y + 32z = 29$

$-x + 3y + 14z = 12$

$x + y + 2z = 8$

$$\left(\begin{array}{ccc|c} 1 & 3 & 10 & 18 \\ -2 & 7 & 32 & 29 \\ -1 & 3 & 14 & 12 \\ 1 & 1 & 2 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 10 & 18 \\ 0 & 13 & 52 & 65 \\ 0 & 6 & 24 & 30 \\ 0 & -2 & -8 & -10 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 10 & 18 \\ 0 & 1 & 4 & 5 \\ 0 & 1 & 4 & 5 \\ 0 & 13 & 52 & 65 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 10 & 18 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Sea $t = x_3$

$x_3 = 1$

$x_1 = 2$

$x_2 = -4$

$x_1 + 3x_2 + 10x_3 = 0$

$x_2 + 4x_3 = 0$

$x_2 = -4x_3$

$x_1 = -3x_2 - 10x_3$

$= 12x_3 - 10x_3$

$x_1 = 2x_3$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2x_3 \\ -4x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$$

Consistente. $x = t(2, 4, 1) + (1, 2, -4)$

$$\begin{aligned} 3w - 2x + 16y - 2z &= -7 \\ -w + 5x - 14y + 18z &= 29 \\ 3w - x + 14y + 2z &= 1 \end{aligned} \quad \left(\begin{array}{cccc|c} 3 & -2 & 16 & -2 & -7 \\ -1 & 5 & -14 & 18 & 29 \\ 3 & -1 & 14 & 2 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & -5 & 14 & -18 & -29 \\ 0 & 13 & -26 & 52 & 80 \\ 0 & 14 & -28 & 56 & 88 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -5 & 14 & -18 & -29 \\ 0 & 1 & -2 & 4 & 44/7 \\ 0 & 0 & 0 & 0 & -12/7 \end{array}\right)$$

Sistema inconsistente.

- Determine si $b \in$ al espacio columna de A. Si es así, escriba b como combinación lineal de los vectores columna de A.

$$A = \begin{bmatrix} -1 & 0 \\ 4 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

~~Pág~~ 229 - 230.

CONJUNTOS GENERADORES

1. Determine cuáles vectores u, v, w pueden expresarse como combinaciones lineales de los vectores en S.

$S = \{(2,-1,3), (5,0,4)\}$

a) $u = (0,-5,7)$ ✓

b) $v = (16, -\frac{1}{2}, \frac{27}{2})$ ✓

c) $w = (3,6,-2)$

a) $(0,-5,7) = \alpha(2,-1,3) + \beta(5,0,4)$

$= (2\alpha - \alpha, 3\alpha) + (5\beta, 0, 4\beta)$

$= (2\alpha + 5\beta, -\alpha, 3\alpha + 4\beta)$

$2\alpha + 5\beta = 0$

$-\alpha = -5$

$3\alpha + 4\beta = 7$

$$\left(\begin{array}{cc|c} 2 & 5 & 0 \\ -1 & 0 & -5 \\ 3 & 4 & 7 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 5 & -10 \\ 0 & 4 & -8 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow (0,-5,7) = 5(2,-1,3) - 2(5,0,4)$$

b) $(16, -\frac{1}{2}, \frac{27}{2}) = \alpha(2,-1,3) + \beta(5,0,4)$

$$\left(\begin{array}{cc|c} 2 & 5 & 16 \\ -1 & 0 & -1/2 \\ 3 & 4 & 27/2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 1/2 \\ 0 & 5 & 15 \\ 0 & 4 & 12 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 1/2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow (16, -\tfrac{1}{2}, \tfrac{27}{2}) = \tfrac{1}{2}(2,-1,3) + 3(5,0,4)$$

c) $w = (3,6,-2) = \alpha(2,-1,3) + \beta(5,0,4)$

$$\left(\begin{array}{cc|c} 2 & 5 & 3 \\ -1 & 0 & 6 \\ 3 & 4 & -2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & -6 \\ 0 & 5 & 15 \\ 0 & 4 & 16 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & -6 \\ 0 & 1 & -3 \\ 0 & 0 & 4 \end{array}\right)$$ No se puede escribir com comb lin.

3. $S = \{(2,0,7), (2,4,5), (2,-12,13)\}$

a) $u = (4,-20,24)$ ✓

b) $v = (-1,0,0)$ ✗ ✓

c) $w = (6,24,9)$ ✓

$\alpha(2,0,7) + \beta(2,4,5) + \gamma(2,-12,13)$

$= (2\alpha, 0, 7\alpha) + (2\beta, 4\beta, 5\beta) + (2\gamma, -12\gamma, 13\gamma)$

$\Rightarrow 2\alpha + 2\beta + 2\gamma$

$4\beta - 12\gamma$

$7\alpha + 5\beta + 13\gamma$

$$\left(\begin{array}{ccc|ccc} 2 & 2 & 2 & 4 & -1 & 6 \\ 0 & 4 & -12 & -20 & 0 & 24 \\ 7 & 5 & 13 & 24 & 0 & 9 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 2 & -1/2 & 3 \\ 0 & 1 & -3 & -5 & 0 & 6 \\ 0 & -2 & 6 & 10 & 7/2 & -12 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 2 & -1/2 & 3 \\ 0 & 1 & -3 & -5 & 0 & 6 \\ 0 & 0 & 0 & 0 & 7/2 & 0 \end{array}\right)$$

El segundo vector no puede escribirse como una comb. lin

$\frac{7}{5}(2,1) - \frac{1}{5}(-1,2)$

3

Determine si el conjunto S dado genera a $\mathbb{R}^2$. En caso negativo proporcionar una descripción geométrica del subespacio generado por S.

5. $S = \{(2,1), (-1,2)\}$ $(a,b) = \alpha(2,1) + \beta(-1,2)$

$\Rightarrow (a,b) = (2\alpha, \alpha) + (-\beta, 2\beta)$

$\Rightarrow 2\alpha - \beta = a$

$\alpha + 2\beta = b$

$$\left(\begin{array}{cc|c} 2 & -1 & a \\ 1 & 2 & b \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & b \\ 2 & -1 & a \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & b \\ 0 & -5 & a-2b \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & b \\ 0 & 1 & \frac{a-2b}{-5} \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & \frac{-2a+4b}{-5} + b \\ 0 & 1 & \frac{a-2b}{-5} \end{array}\right) \begin{array}{l} \Rightarrow \alpha \\ \Rightarrow \beta \end{array}$$

$\Rightarrow$ Cualquier vector (a,b) en $\mathbb{R}^2$ puede escribirse como una comb. lineal de $(2,1)$ y $(-7,2)$. con $\alpha = \frac{9b-2a}{-5}+b$ y $\beta = \frac{a-2b}{-5}$

7. $S=\{(-3,5)\}$ $\quad (a,b)=\alpha(-3,5)$

$$\begin{matrix}-3=a\\5=b\end{matrix} \quad \left(\begin{matrix}-3\\5\end{matrix}\middle|\begin{matrix}a\\b\end{matrix}\right)\sim\left(\begin{matrix}1\\0\end{matrix}\middle|\begin{matrix}-\frac{a}{3}\\ \frac{5a}{3}+b\end{matrix}\right)$$

Si $\frac{5}{3}a+b\neq 0$ No genera a $\mathbb{R}^2$ (genera una recta)

9. $S=\{(-1,2),(2,-4)\}$ $\quad (a,b)=\alpha(-1,2)+\beta(2,-4)$

$(-\alpha,2\alpha)+(2\beta,-4\beta)=(a,b)$

$$\begin{matrix}-\alpha+2\beta=a\\2\alpha-4\beta=b\end{matrix}\sim\left(\begin{matrix}-1&2\\2&-4\end{matrix}\middle|\begin{matrix}a\\b\end{matrix}\right)\sim\left(\begin{matrix}1&-2\\0&0\end{matrix}\middle|\begin{matrix}-a\\b+2a\end{matrix}\right)$$

Si $b+2a\neq 0$ No generan a $\mathbb{R}^2$ (genera una recta).

- Determine si el conjunto S dado, genera a $\mathbb{R}^3$.

11. $S=\{(4,7,3),(-1,2,6),(2,-3,5)\}$ $\quad$ Sea $(a,b,c)\in\mathbb{R}^3$

$$\begin{aligned}(a,b,c)&=\alpha(4,7,3)+\beta(-1,2,6)+\gamma(2,-3,5)\\&=(4\alpha,7\alpha,3\alpha)+(-\beta,2\beta,6\beta)+(2\gamma,-3\gamma,5\gamma)\\&=(4\alpha-\beta+2\gamma,\ 7\alpha+2\beta-3\gamma,\ 3\alpha+6\beta+5\gamma)\end{aligned}$$

$$\begin{matrix}4\alpha-\beta+2\gamma=a\\7\alpha+2\beta-3\gamma=b\\3\alpha+6\beta+5\gamma=c\end{matrix}\sim\left(\begin{matrix}4&-1&2\\7&2&-3\\3&6&5\end{matrix}\middle|\begin{matrix}a\\b\\c\end{matrix}\right)\underset{\substack{R_2:-7R_1+R_2\\R_3:-3R_1+R_3}}{\overset{R_1:\frac{1}{4}R_1}{\sim}}\left(\begin{matrix}1&-1/4&\frac{1}{2}\\0&15/4&-13/2\\0&27/4&7/2\end{matrix}\middle|\begin{matrix}\frac{1}{4}a\\b-\frac{7}{4}a\\c-\frac{3}{2}a\end{matrix}\right)$$

$$\left(\begin{matrix}1&-1/4&\frac{1}{2}\\0&1&\frac{-26}{15}\\0&0&\frac{76}{5}\end{matrix}\middle|\begin{matrix}\frac{1}{4}a\\ \frac{4}{15}b-\frac{7}{15}a\\ -\frac{9}{5}b+\frac{33}{20}a+c\end{matrix}\right)\sim\left(\begin{matrix}1&-1/4&0\\0&1&0\\0&0&1\end{matrix}\middle|\begin{matrix}\frac{119}{608}a-\frac{9}{152}b-\frac{5}{152}c\\ \frac{127}{456}a+\frac{7}{114}b+\frac{13}{114}c\\ \frac{33}{304}a-\frac{9}{76}b+\frac{5}{76}c\end{matrix}\right)$$

$$\left(\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\middle|\begin{matrix}\frac{121}{456}a-\frac{5}{114}b-\frac{1}{228}c\\ \frac{127}{456}a+\frac{7}{114}b+\frac{13}{114}c\\ \frac{33}{304}a-\frac{9}{76}b+\frac{5}{76}c\end{matrix}\right)\therefore S \text{ genera a } \mathbb{R}^3$$

13. $S=\{(-2,5,0),(4,6,3)\}$ $\quad (a,b,c)=\alpha(-2,5,0)+\beta(4,6,3)$

$$\begin{aligned}(a,b,c)&=(-2\alpha,5\alpha,0)+(4\beta,6\beta,3\beta)\\&=(-2\alpha+4\beta,\ 5\alpha+6\beta,\ 3\beta)\end{aligned}$$

$$\begin{matrix}-2\alpha+4\beta=a\\5\alpha+6\beta=b\\3\beta=c\end{matrix}\sim\left(\begin{matrix}-2&4\\5&6\\0&3\end{matrix}\middle|\begin{matrix}a\\b\\c\end{matrix}\right)\sim\left(\begin{matrix}1&-2\\0&-14\\0&3\end{matrix}\middle|\begin{matrix}-\frac{1}{2}a\\ \frac{5}{2}a+b\\c\end{matrix}\right)\sim\left(\begin{matrix}1&-2\\0&1\\0&0\end{matrix}\middle|\begin{matrix}-\frac{1}{2}a\\ -\frac{5}{28}a-\frac{1}{14}b\\ \frac{15}{28}a+\frac{3}{14}b+c\end{matrix}\right)$$

Si $\frac{15}{28}a+\frac{3}{14}b+c\neq 0$ $\therefore$ No generaría a $\mathbb{R}^3$ $\therefore$ No genera

$S=\{(1,-2,0),(0,0,1),(-1,2,0)\}$ $\quad (a,b,c)=\alpha(1,-2,0)+\beta(0,0,1)+\gamma(-1,2,0)$

$(a,b,c)=(\alpha,-2\alpha,0)+(0,0,\beta)+(-\gamma,2\gamma,0)$

$$\begin{aligned}\alpha - \gamma &= a\\ -2\alpha + 2\gamma &= b\\ \beta &= c\end{aligned} \sim \left(\begin{array}{ccc|c}1&0&-1&a\\-2&0&2&b\\0&1&0&c\end{array}\right)\sim\left(\begin{array}{ccc|c}1&0&-1&a\\0&1&0&c\\0&0&0&c+2a\end{array}\right)$$

Si $c+2a\neq 0$ No genera

- En los sig. ejercicios, determine si el conjunto S es li o ld.

17. $S=\{(-2,2),(3,5)\}$

$$\begin{vmatrix}-2&2\\3&5\end{vmatrix}=-10-6=-16\neq 0$$
$\therefore$ li

19. $S=\{(0,0),(1,-1)\}$

$$\begin{vmatrix}0&0\\1&-1\end{vmatrix}=0 \quad \therefore \text{ ld}$$

21. $S=\{(1,-4,1),(6,3,2)\}$

$$\left(\begin{array}{cc|c}1&6&0\\-4&3&0\\1&2&0\end{array}\right)\sim\left(\begin{array}{cc|c}1&6&0\\0&-9&0\\0&27&0\end{array}\right)\sim\left(\begin{array}{cc|c}1&6&0\\0&1&0\\0&0&0\end{array}\right)$$

$$\begin{vmatrix}1&6\\0&1\end{vmatrix}=1\neq 0 \therefore \text{ li}$$

23. $S=\{(1,1,1),(2,2,2),(3,3,3)\}$

$$\begin{vmatrix}1&1&1\\2&2&2\\3&3&3\end{vmatrix}=1(6-6)-2(3-3)+3(2-2)$$
$$=0-0+0$$
$$=0 \therefore \text{ ld}$$

25. $S=\{(-4,-3,4),(1,-2,3),(6,0,0)\}$

$$\begin{vmatrix}-4&-3&4\\1&-2&3\\6&0&0\end{vmatrix}=6(-9+8)=6(-1)=-6$$
$\neq 0 \therefore$ li

27. $S=\{(4,-3,6,2),(1,8,3,1),(3,-2,-1,0)\}$

$$\left(\begin{array}{ccc|c}4&1&3&0\\-3&8&-2&0\\6&3&-1&0\\2&1&0&0\end{array}\right)\sim\left(\begin{array}{ccc|c}1&1/2&0&0\\0&19/2&-2&0\\0&0&-1&0\\0&-1&3&0\end{array}\right)$$

$$\left(\begin{array}{ccc|c}1&1/2&0&0\\0&1&-3&0\\0&0&1&0\\0&0&0&0\end{array}\right)\quad\begin{vmatrix}1&1/2&0\\0&1&-3\\0&0&1\end{vmatrix}=1\neq 0 \therefore \text{ l}$$

- Demuestre que el conjunto dado es ld. al hallar una combinación lineal no trivial (de vectores en el conjunto) cuya suma sea el vector cero. Luego, exprese uno de los vectores del conjunto como una combinación lineal de los demás.

29. $S=\{(3,4),(-1,1),(2,0)\}$ $\quad \alpha(3,4)+\beta(-1,1)+\gamma(2,0)=(0,0)$

$(3\alpha,4\alpha)+(-\beta,\beta)+(2\gamma,0)=(0,0)$

$3\alpha-\beta+2\gamma=0$ Mas incognitas

$4\alpha+\beta \quad =0$ que ecs. $\therefore$ Sol. $\infty \Rightarrow$ ld

$(3,4)=\alpha(-1,1)+\beta(2,0)$

$\quad=(-\alpha,\alpha)+(2\beta,0)$ $\quad \Rightarrow (3,4)=4(-1,1)+\frac{7}{2}(2,0)$

$-\alpha+2\beta=3 \longrightarrow \beta=\frac{7}{2}$ $\quad =(-4,4)+(7,0)$

$\alpha \quad =4$ $\quad =(3,4)$

31. $S=\{(1,1,1),(1,1,0),(0,1,1),(0,0,1)\}$

$$x(1,1,1)+y(1,1,0)+z(0,1,1)+w(0,0,1)=(0,0,0)$$
$$(x,x,x)+(y,y,0)+(0,z,z)+(0,0,w)=(0,0,0)$$

$$\begin{aligned} x+y \quad\quad &= 0 \\ x+y+z \quad &= 0 \\ x+\quad z+w &= 0 \end{aligned}$$

Menos ecs. que variables $\therefore$ ∞ de Sols. $\Rightarrow$ ld.

$$(1,1,1)=\alpha(1,1,0)+\beta(0,1,1)+\gamma(0,0,1)$$

$$\begin{aligned} \alpha &= 1 \Rightarrow \alpha=1 \\ \alpha+\beta &= 1 \Rightarrow \beta=0 \\ \beta+\gamma &= 1 \Rightarrow \gamma=1 \end{aligned}$$

$$\Rightarrow (1,1,1)=1(1,1,0)+0(0,1,1)+1(0,0,1)$$
$$=(1,1,0)+(0,0,1)$$
$$=(1,1,1)$$

33. ¿Para qué valores de t, los siguientes conjuntos son li?

a) $S=\{(t,1,1),(1,t,1),(1,1,t)\}$

$$\left(\begin{array}{ccc|c} t & 1 & 1 & 0 \\ 1 & t & 1 & 0 \\ 1 & 1 & t & 0 \end{array}\right) \sim \left|\begin{array}{ccc} 1 & 1 & t \\ 1 & t & 1 \\ t & 1 & 1 \end{array}\right| \sim \left|\begin{array}{ccc} 1 & 1 & t \\ 0 & t-1 & 1-t \\ 0 & 1-t & 1-t^2 \end{array}\right| \sim \left|\begin{array}{ccc} 1 & 1 & t \\ 0 & t-1 & 1-t \\ 0 & 0 & -t^2-t+2 \end{array}\right|$$

$$\left|\begin{array}{ccc} 1 & 1 & t \\ 0 & 1 & \frac{1-t}{t-1} \\ 0 & 0 & t^2+t-2 \end{array}\right|$$

$$t^2+t-2=(t+2)(t-1)$$

Son li si $\det \neq 0 \Rightarrow t \neq -2$ y 1

b) $S=\{(t,1,1),(1,0,1),(1,1,3t)\}$

$$\left|\begin{array}{ccc} t & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 3t \end{array}\right| = -1(3t-1)-(t-1)$$
$$=-3t+1-t+1$$
$$-4t+2=0$$
$$2=4t$$
$$t=\frac{2}{4}=\frac{1}{2}$$

$\Rightarrow t \neq \frac{1}{2}$ Para ser li

Dadas $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$ y $B \begin{bmatrix} 0 & 5 \\ 1 & -2 \end{bmatrix}$ en M_{22} determine cuales de las matrices son combinaciones lineales de A y B

a) $\begin{bmatrix} 6 & -19 \\ 10 & 7 \end{bmatrix} = \alpha \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} + \beta \begin{bmatrix} 0 & 5 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 2\alpha & -3\alpha \\ 4\alpha & \alpha \end{bmatrix} + \begin{bmatrix} 0 & 5\beta \\ \beta & -2\beta \end{bmatrix}$

$= \begin{bmatrix} 2\alpha & -3\alpha+5\beta \\ 4\alpha+\beta & \alpha-2\beta \end{bmatrix}$

$2\alpha = 6 \longrightarrow \alpha = \frac{6}{2} = 3$

$-3\alpha + 5\beta = -19$

$4\alpha + \beta = 10$

$\alpha - 2\beta = 7 \longrightarrow 3 - 2\beta = 7 \quad \beta = -2$

$\begin{bmatrix} 6 & -19 \\ 10 & 7 \end{bmatrix} = 3 \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} - 2 \begin{bmatrix} 0 & 5 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 6 & -9 \\ 12 & 3 \end{bmatrix} + \begin{bmatrix} 0 & -10 \\ -2 & -4 \end{bmatrix}$

$\begin{bmatrix} 6 & -19 \\ 10 & 7 \end{bmatrix} = \begin{bmatrix} 6 & -19 \\ 10 & 7 \end{bmatrix}$ ✓

b) $\begin{bmatrix} 6 & 2 \\ 4 & 11 \end{bmatrix}$ $\quad 2\alpha = 6 \quad \alpha = 3$

$\alpha - 2\beta = 11 \quad 3 - 2\beta = 11 \quad \beta = -4$

$\begin{bmatrix} 6 & 2 \\ 4 & 11 \end{bmatrix} = 3 \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} - 4 \begin{bmatrix} 0 & 5 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 6 & -9 \\ 12 & 3 \end{bmatrix} + \begin{bmatrix} 0 & -20 \\ -4 & -8 \end{bmatrix} = \begin{bmatrix} 6 & -29 \\ 8 & -5 \end{bmatrix}$

No es una combinación lineal ✗

c) $\begin{bmatrix} -2 & 28 \\ 1 & -11 \end{bmatrix}$ $\quad 2\alpha = -2 \quad \alpha = -1$

$\alpha - 2\beta = -11 \quad \beta = 5$

$\begin{bmatrix} -2 & 28 \\ 1 & -11 \end{bmatrix} = -1 \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} + 5 \begin{bmatrix} 0 & 5 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ -4 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 25 \\ 5 & -10 \end{bmatrix} = \begin{bmatrix} -2 & 28 \\ 1 & -11 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad 2\alpha = 0 \quad \alpha = 0$

$\alpha - 2\beta = 0 \quad \beta = 0$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} + 0 \begin{bmatrix} 0 & 5 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ✓

37. Determine si los sig. polinomios son l.i.

a) $S = \{2 - x,\ 2x - x^2,\ 6 - 5x + x^2\}$

$C_1(2-x) + C_2(2x - x^2) + C_3(6 - 5x + x^2) = 0 + 0x + 0x^2$

$(2C_1 - C_1x) + (2C_2x - C_2x^2) + (6C_3 - 5C_3x + C_3x^2) = 0 + 0x + 0x^2$

$(2C_1 + 6C_3) + (2C_2 - C_1 - 5C_3)x + (C_3 - C_2)x^2 = 0 + 0x + 0x^2$

$$\begin{aligned} 2C_1 + 6C_3 &= 0 \\ 2C_2 - C_1 - 5C_3 &= 0 \\ C_3 - C_2 &= 0 \end{aligned} \sim \left(\begin{array}{ccc|c} 2 & 0 & 6 & 0 \\ -1 & 2 & -5 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Hay infinidad de sols $\therefore$ Son ld.

b) $S = \{(x^2-1,\ 2x+5)\}$

$C_1(x^2-1) + C_2(2x+5) = \vec{0}$

$C_1x^2 - C_1 + 2C_2x + 5C_2 = 0 + 0x + 0x^2$

$(5C_2 - C_1) + (2C_2)x + (C_1)x^2 = 0 + 0x + 0x^2$

$$\begin{aligned} -C_1 + 5C_2 &= 0 \\ 2C_2 &= 0 \rightarrow C_2 = 0 \\ C_1 &= 0 \end{aligned}$$

$\therefore$ Son lI

c) $S = \{x^2+3x+1,\ 2x^2+x-1,\ 4x\}$

$C_1(x^2+3x+1) + C_2(2x^2+x-1) + C_3 4x = \vec{0}$

$C_1 + 3C_1x + C_1x^2 + 2C_2x^2 + C_2x - C_2 + C_3 4x = 0 + 0x + 0x^2$

$$\begin{aligned} C_1 - C_2 &= 0 \\ 3C_1 + C_2 + 4C_3 &= 0 \\ C_1 + 2C_2 &= 0 \end{aligned} \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 3 & 1 & 4 & 0 \\ 1 & 2 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 3 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

Tiene la sol. trivial $\therefore$ Son li

41.

1 Sea $T: P_2 \to P_2$ una transformación definida por

$$T(p(x)) = x^2 \frac{d^2}{dx^2} p(x) + 2x \frac{d}{dx} p(x)$$

a) Demuestre que la transformación es lineal.

- Primero debemos mostrar que $T(p(x) + q(x)) = T(p(x)) + T(q(x))$

$\Rightarrow$ $p(x)$ y $q(x) \subset P_2$

$p(x) = a_1x^2 + b_1x + c_1 \qquad q(x) = a_2x^2 + b_2x + c_2$

$> T(p(x) + q(x)) = T((a_1x^2 + b_1x + c_1) + (a_2x^2 + b_2x + c_2))$

$= T((a_1+a_2)x^2 + (b_1+b_2)x + (c_1+c_2))$

Para la transformación: $(r(x) = (a_1+a_2)x^2 + (b_1+b_2)x + (c_1+c_2))$

$\frac{d^2}{dx^2} r(x) = 2(a_1+a_2) \qquad \frac{d}{dx} r(x) = 2(a_1+a_2)x + (b_1+b_2)$

$\Rightarrow T(p(x)+q(x)) = x^2(2(a_1+a_2)) + 2x(2(a_1+a_2)x + (b_1+b_2))$

$= 2x^2(a_1+a_2) + 4x^2(a_1+a_2) + 2x(b_1+b_2)$

$= 6x^2(a_1+a_2) + 2x(b_1+b_2)$ //

$> T(p(x)) + T(q(x)) = T(a_1x^2 + b_1x + c_1) + T(a_2x^2 + b_2x + c_2)$

$= x^2(2a_1) + 2x(2a_1x + b_1) + (x^2 2a_2) + 2x(2a_2x + b_2)$

$= 2x^2a_1 + 4x^2a_1 + 2xb_1 + 2x^2a_2 + 4x^2a_2 + 2xb_2$

$= 6x^2a_1 + 6x^2a_2 + 2x(b_1+b_2)$

$= 6x^2(a_1+a_2) + 2x(b_1+b_2)$ //

$\therefore T(p(x)+q(x)) = T(p(x)+q(x)) \Rightarrow$ La 1ª prop. q.e.d.

- Ahora debemos mostrar que $T(\alpha p(x)) = \alpha T(p(x))$

$> T(\alpha p(x)) = T(\alpha(ax^2 + bx + c))$

$= T(\alpha ax^2 + \alpha bx + \alpha c)$

$= x^2(2\alpha a) + 2x(2\alpha ax + \alpha b)$

$= 2\alpha x^2 a + 4\alpha x^2 a + 2\alpha xb$

$= 6\alpha x^2 a + 2\alpha xb$

$> \alpha T(p(x)) = \alpha\, T(ax^2 + bx + c)$

$= \alpha\,(x^2(2a) + 2x(2ax + b))$

$= \alpha\,(x^2 2a + 4x^2 a + 2xb)$

$= 2\alpha x^2 a + 4\alpha x^2 a + 2\alpha x b$

$= 6\alpha x^2 a + 2\alpha x b$ //

$\Rightarrow T(\alpha p(x)) = \alpha T(p(x))$

$\boxed{\text{Al cumplirse las dos propiedades q.e.d. que la transformación es lineal.}}$

b) Si $B_1 = \{2,\ x+1,\ 2x^2-1\}$ y $B_2 = \{-1,\ x-1,\ x^2-x+1\}$ son bases de P_2, determine la matriz asociada a la transformación lineal respecto a estas bases.

$-1 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix},\quad x-1 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix},\quad x^2-x+1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$T(2) = 0 = \begin{pmatrix} 0 \\ 8 \end{pmatrix},\quad T(x+1) = 2x \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix},\quad T(2x^2-1) = 12x^2 = \begin{pmatrix} 12 \\ 0 \\ 0 \end{pmatrix}$

$\left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 0 & 0 & 12 \\ 0 & 1 & -1 & 0 & 2 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 12 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 12 \end{array}\right)$

$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 12 \\ 0 & 1 & 0 & 0 & 2 & 12 \\ 0 & 0 & 1 & 0 & 0 & 12 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & 2 & 12 \\ 0 & 0 & 1 & 0 & 0 & 12 \end{array}\right)$

$\therefore\quad M = \begin{pmatrix} 0 & -2 & 0 \\ 0 & 2 & 12 \\ 0 & 0 & 12 \end{pmatrix}$ //

0

Para la matriz $A = \begin{pmatrix} 8 & 2 & -1 \\ 2 & 5 & 2 \\ -1 & 2 & 8 \end{pmatrix}$

Determinar

a) los eigenvalores

b) una matriz ortogonal $Q \mid Q^t A Q = D$

c) Verifique que $Q^t A Q = D$

a) $|A - \lambda I|$

$$\begin{vmatrix} 8-\lambda & 2 & -1 \\ 2 & 5-\lambda & 2 \\ -1 & 2 & 8-\lambda \end{vmatrix}$$

$(8-\lambda)(5-\lambda)(8-\lambda) + 2(2)(-1) + (-1)(2)(2) - (-1)(5-\lambda)(-1)$
$- (2)(2)(8-\lambda) - (8-\lambda)(2)(2)$
$= (8-\lambda)(40 - 8\lambda - 5\lambda - \lambda^2) - 4 - 4 - 5 + \lambda - 32 + 4\lambda - 32 + 4\lambda$
$= 320 - 104\lambda + 8\lambda^2 - 40\lambda + 13\lambda^2 - \lambda^3 - 77 - 9\lambda$
$= -\lambda^3 + 21 - 135\lambda + 243$

$(\lambda - 3)(-\lambda^2 + 18\lambda - 81)$
$= (\lambda - 3)(\lambda - 9)(\lambda - 9)$

$\therefore$ Los valores propios son $\lambda_1 = 3$ $\lambda_2 = 9$ multiplicidad 2.

b) $\begin{pmatrix} 8-3 & 2 & -1 \\ 2 & 5-3 & 2 \\ -1 & 2 & 8-3 \end{pmatrix} \sim \begin{pmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 5 & 2 & -1 \\ -1 & 2 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & -6 \\ 0 & 3 & 6 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad x_1 - x_3 = 0$, $x_2 + 2x_3 = 0$

sea $x_3 = s$

$x_1 = x_3$, $x_2 = -2x_3$ $\Rightarrow$ $x_1 = s$, $x_2 = -2s$, $x_3 = s$

$\Rightarrow \begin{pmatrix} s \\ -2s \\ s \end{pmatrix} = s \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

Cuando $\lambda = 9$

$\begin{pmatrix} 8-9 & 2 & -1 \\ 2 & 5-9 & 2 \\ -1 & 2 & 8-9 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & -1 \\ 2 & -4 & 2 \\ -1 & 2 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 \\ 1 & -2 & 1 \\ -1 & 2 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$x_1 - 2x_2 + x_3 = 0$

Donde
$x_2 = s$
$x_3 = t$
$x_1 = 2s - t$

$\Rightarrow \begin{pmatrix} 2s - t \\ s \\ t \end{pmatrix} = s \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$\vec{v}_1 = (1,-2,1)$

$\vec{v}_2 = (2,1,0)$

$\vec{v}_3 = (-1,0,1)$

$$\vec{v}_2 \cdot \vec{v}_3 = (2)(-1)+(2)(0)+(0)(1) = -2$$

$\therefore$ No son ortogonales.

Aplicamos Gram-Schmidt

$$e_1 = \frac{(1,-2,1)}{\sqrt{1^2+2^2+1^2}} = \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right) = \left(\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}\right)$$

$$v_2 = (2,1,0) - \left[(2,1,0)\cdot\left(\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}\right)\right]\left(\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}\right)$$

$$= (2,1,0) - \left[\frac{2\sqrt{6}}{6} - \frac{\sqrt{6}}{3}\right]\left(\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}\right)$$

$$= (2,1,0)$$

$$e_2 = \frac{(2,1,0)}{\sqrt{2^2+1^2}} = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right) = \left(\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, 0\right)$$

$$v_3' = (-1,0,1) - \left[(-1,0,1)\cdot\left(\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}\right)\right]\left(\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}\right) - \left[(-1,0,1)\cdot\left(\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, 0\right)\right]$$

$$v_3' = (-1,0,1) - \left[-\frac{\sqrt{6}}{6} + \frac{\sqrt{6}}{6}\right]\left(\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}\right) - \left[\frac{-2\sqrt{5}}{5}\right]\left(\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, 0\right)$$

$$= (-1,0,1) - \left(-\frac{4}{5}, -\frac{2}{5}, 0\right) = \left(-\frac{1}{5}, \frac{2}{5}, 1\right)$$

$$e_3 = \frac{\left(-\frac{1}{5}, \frac{2}{5}, 2\right)}{\sqrt{\frac{1}{5}^2 + \frac{2}{5}^2 + 2^2}} = \frac{\left(-\frac{1}{5}, \frac{2}{5}, 1\right)}{\frac{\sqrt{6}}{\sqrt{5}}} = -\frac{\sqrt{5}}{5\sqrt{6}}, \frac{2\sqrt{5}}{5\sqrt{6}}, \frac{\sqrt{5}}{\sqrt{6}}$$

$$= \left(-\frac{\sqrt{6}\sqrt{5}}{5\cdot 6}, \frac{2\sqrt{5}\sqrt{6}}{5\cdot 6}, \frac{\sqrt{5}\cdot\sqrt{6}}{6}\right) = \left(-\frac{\sqrt{30}}{30}, \frac{2\sqrt{30}}{30}, \frac{\sqrt{30}}{6}\right)$$

$$\Rightarrow Q = \begin{pmatrix} \frac{\sqrt{6}}{6} & \frac{2\sqrt{5}}{5} & -\frac{\sqrt{30}}{30} \\ \frac{\sqrt{6}}{3} & \frac{\sqrt{5}}{5} & \frac{2\sqrt{30}}{30} \\ \frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{30}}{6} \end{pmatrix}$$

c) $Q^t A Q = D$

$$Q^t = \begin{pmatrix} \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} \\ \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{30}}{30} & \frac{2\sqrt{30}}{30} & \frac{\sqrt{30}}{6} \end{pmatrix}$$

$$AQ = \begin{pmatrix} 8 & 2 & 1 \\ 2 & 5 & 2 \\ -1 & 2 & 8 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{6}}{6} & \frac{2\sqrt{5}}{5} & -\frac{\sqrt{30}}{30} \\ -\frac{\sqrt{6}}{3} & \frac{\sqrt{5}}{5} & \frac{2\sqrt{30}}{30} \\ \frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{30}}{6} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} \\ \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & 0 \\ \frac{\sqrt{30}}{30} & \frac{2\sqrt{30}}{30} & \frac{\sqrt{30}}{6} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{6}}{2} & \frac{18\sqrt{5}}{5} & -\frac{3\sqrt{30}}{10} \\ \sqrt{6} & \frac{9\sqrt{5}}{5} & \frac{3\sqrt{30}}{5} \\ \frac{\sqrt{6}}{2} & 0 & \frac{3\sqrt{30}}{2} \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$Q^t A Q = D$ q.e.d

Dado la matriz $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$ Demostrar que

$$A^n = \begin{pmatrix} \frac{2}{3} + \frac{1}{3}4^n & -\frac{2}{3} + \frac{2}{3}4^n \\ -\frac{1}{3} + \frac{1}{3}4^n & \frac{1}{3} + \frac{2}{3}4^n \end{pmatrix}$$

$\Rightarrow$ Dado que sus eigenvectores son $v_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ y $v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$P = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \qquad P^{-1} = \frac{1}{|A|}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

$$P^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Ahora $\begin{pmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{pmatrix} = (2-\lambda)(3-\lambda) - 2$

$$= 6 - 2\lambda - 3\lambda + \lambda^2 - 2$$

$$= \lambda^2 - 5\lambda + 4 = (\lambda - 4)(\lambda - 1)$$

$$\therefore (1, 4)$$

$\Rightarrow$ $D = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$; Por def: $A^n = P D^n P^{-1}$

$$A^n = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1^n & 0 \\ 0 & 4^n \end{pmatrix}\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$A^n = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3}4^n & \frac{2}{3}4^n \end{pmatrix}$$

$$A^n = \begin{pmatrix} \frac{2}{3} + \frac{1}{3}4^n & -\frac{2}{3} + \frac{2}{3}4^n \\ -\frac{1}{3} - \frac{1}{3}4^n & \frac{1}{3} + \frac{2}{3}4^n \end{pmatrix}$$ //

$\therefore$ q.e.d.

1

5. Considera la transformación lineal $T: \mathbb{R}^3 \to \mathbb{R}^4$ definida por

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x-y \\ y+z \\ 2x-y-z \\ -x+y+2z \end{pmatrix}$$

Determina el Kernel de T, su nulidad, la imagen y el rango de T.

$$\Rightarrow T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ -1 \end{pmatrix}, \quad T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \quad T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 2 \end{pmatrix}$$

$$A_T = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & -1 & -1 \\ -1 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & -1 & -1 \\ -1 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \rho(A) + \nu(A) = 3$$

$$\rho(A) = 3 \qquad \nu(A) = 3 - 3 = 0$$

$$I_T = \text{gen}\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 2 \end{pmatrix} \right\}, \ \nu(T) = 0 \ y \ \rho(T) = 3$$

Dada la siguiente transformación lineal $T: M_{22} \to P_3$ definida como;

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a+b+c+(b+c+d)x+(a-d)x^2+(a+2b+2c+d)x^3$$

determine.

a) el Kernel de la transformación, diga si la trans. es uno auno.

$$\Rightarrow \quad T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a+b+c+(b+c+d)x+(a-d)x^2+(a+2b+2c+d)x^3$$

$$\begin{aligned} a+b+c \quad &= 0 \\ b+c+d &= 0 \\ a \quad -d &= 0 \\ a+2b+2c+d &= 0 \end{aligned} \sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & -1 \\ 1 & 2 & 2 & 1 \end{pmatrix} \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{aligned} a-d&=0 \\ b+c+d&=0 \end{aligned}$$

Despejamos "a"

$$\begin{aligned} a &= d \\ d &= -b-c \end{aligned} \Rightarrow \quad \begin{aligned} a &= -t-s \\ b &= t \\ c &= s \\ d &= -t-s \end{aligned}$$

Sea $b=t$ y $c=s$

> Entonces el Ker(T) está dado por todas las matrices de la forma:

$$Ker(T) = \left\{ \begin{pmatrix} -t-s & t \\ s & -t-s \end{pmatrix} \middle| \ t,s \in R \right\}$$

> Para una base el Kernel...

$$\begin{pmatrix} -t-s & t \\ s & -t-s \end{pmatrix} = t\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} + s\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$\Rightarrow$ una base del Kernel es $\left\{ \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \right\}$

luego $Dim(Ker(T)) = 2$ y como una transformación uno a uno tiene $Dim(Ker(T)) = 0$ está no lo es.

b) La imagen de la transformación, diga si la transformación es sobre.

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a+b+c + (b+c+d)x + (a-d)x^2 + (a+2b+2c+d)x^3 \in \mathrm{im}(T)$$

$$\Rightarrow \quad a+b+c + (b+c+d)x + (a-d)x^2 + (a+2b+2c+d)x^3$$

$$= a(1+x^2+x^3) + b(1+x+2x^3) + c(1+x+2x^3) + d(x-x^2+x^3)$$

$$\therefore \{1+x^2+x^3, 1+x+2x^3, x-x^2+x^3\}$$ es la img de T.

Una transformación es sobre sii las dimensiones de V y W ($T: V \to W$) son iguales.

En este caso $\mathrm{Dim}(\mathrm{im}(T)) = 2 \neq \mathrm{Dim}(P_3) = 3$ $\therefore$ No es sobre

?

1

Use la inversa de la matriz de coef. para resolver el sist... 2

$$\begin{aligned} x - 2y - 2z &= a \\ x - y - z &= b \\ 3x - 8y + 13z &= c \end{aligned} \sim \begin{pmatrix} 1 & -2 & -2 \\ 1 & -1 & -1 \\ 3 & -8 & 13 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$A^{-1}AX = A^{-1}B$$
$$IX = A^{-1}B$$
$$X = A^{-1}B$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & -2 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 1 & 0 \\ 3 & -8 & 13 & 0 & 0 & 1 \end{array}\right) \underset{R_3: -3R_1+R_3}{\overset{R_2: -R_1+R_2}{\sim}} \left(\begin{array}{ccc|ccc} 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 19 & -3 & 0 & 1 \end{array}\right) \overset{R_3: 2R_2+R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 21 & -5 & 2 & 1 \end{array}\right)$$

$$\overset{R_3: \frac{1}{21}R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{5}{21} & \frac{2}{21} & \frac{1}{21} \end{array}\right) \underset{R_1: 2R_3+R_1}{\overset{R_2: -R_3+R_2}{\sim}} \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & \frac{11}{21} & \frac{4}{21} & \frac{2}{21} \\ 0 & 1 & 0 & -\frac{16}{21} & \frac{19}{21} & -\frac{1}{21} \\ 0 & 0 & 1 & -\frac{5}{21} & \frac{2}{21} & \frac{1}{21} \end{array}\right) \overset{R_1: 2R_2+R_1}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -\frac{16}{21} & \frac{19}{21} & -\frac{1}{21} \\ 0 & 0 & 1 & -\frac{5}{21} & \frac{2}{21} & \frac{1}{21} \end{array}\right)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} -1 & 2 & 0 \\ -\frac{16}{21} & \frac{19}{21} & -\frac{1}{21} \\ -\frac{5}{21} & \frac{2}{21} & \frac{1}{21} \end{pmatrix}}_{3\times 3} \underbrace{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}_{3\times 1} = \begin{pmatrix} -a + 2b \\ -\frac{16}{21}a + \frac{19}{21}b - \frac{1}{21}c \\ -\frac{5}{21}a + \frac{2}{21}b + \frac{1}{21}c \end{pmatrix}$$

2

⑤ Hallar A^{-1} utilizando el metodo de la matriz adjunta comprobar su resultado 4

$$A=\begin{pmatrix}1 & 0 & -1\\ 0 & 2 & 2\\ 1 & 1 & -1\end{pmatrix} \qquad A^{-1}=\frac{1}{|A|}\,adj(A)$$

1

Para la adj A ⟹

Sea B la matriz de cof. de A ⟶

$$\text{cof. de } 1=\begin{vmatrix}2 & 2\\ 1 & -1\end{vmatrix}=-4$$

$$'' \quad 0=\begin{vmatrix}0 & 2\\ 1 & -1\end{vmatrix}=2$$

$$'' \quad -1=\begin{vmatrix}0 & 2\\ 1 & 1\end{vmatrix}=-2$$

$$'' \quad 0=\begin{vmatrix}0 & -1\\ 1 & -1\end{vmatrix}=-1$$

$$'' \quad 2=\begin{vmatrix}1 & -1\\ 1 & -1\end{vmatrix}=0$$

$$'' \quad 2=\begin{vmatrix}1 & 0\\ 1 & 1\end{vmatrix}=-1$$

$$'' \quad 1=\begin{vmatrix}0 & -1\\ 2 & 2\end{vmatrix}=2$$

$$'' \quad 1=\begin{vmatrix}1 & -1\\ 0 & 2\end{vmatrix}=-2$$

$$'' \quad -1=\begin{vmatrix}1 & 0\\ 0 & 2\end{vmatrix}=2$$

$$\Rightarrow B=\begin{pmatrix}-4 & 2 & -2\\ -1 & 0 & -1\\ 2 & -2 & 2\end{pmatrix}$$

$$B^T=\begin{pmatrix}-4 & -1 & 2\\ 2 & 0 & -2\\ -2 & -1 & 2\end{pmatrix}=adj(A)$$

$$|A|=1\begin{vmatrix}2 & 2\\ 1 & -1\end{vmatrix}+1\begin{vmatrix}0 & -1\\ 2 & 2\end{vmatrix}$$

$$|A|=-4+2=-2$$

$$A^{-1}=\frac{1}{-2}\begin{pmatrix}-4 & -1 & 2\\ 2 & 0 & -2\\ -2 & -1 & 2\end{pmatrix}$$

$$A^{-1}=\begin{pmatrix}2 & \frac{1}{2} & -1\\ -1 & 0 & 1\\ 1 & \frac{1}{2} & -1\end{pmatrix}$$

Comprobamos

$$A^{-1}A=AA^{-1}=I$$

$$A^{-1}A=\begin{pmatrix}2 & \frac{1}{2} & -1\\ -1 & 0 & 1\\ 1 & \frac{1}{2} & -1\end{pmatrix}\begin{pmatrix}1 & 0 & -1\\ 0 & 2 & 2\\ 1 & 1 & -1\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$AA^{-1}=\begin{pmatrix}1 & 0 & -1\\ 0 & 2 & 2\\ 1 & 1 & -1\end{pmatrix}\begin{pmatrix}2 & \frac{1}{2} & -1\\ -1 & 0 & 1\\ 1 & \frac{1}{2} & -1\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

2

Sea el sistema (3)

$$2x_1 - 6x_2 + 8x_3 = 12$$
$$x_1 + 6x_2 - 19x_3 = 0$$
$$2x_1 - 10x_2 + (31+a)x_3 = 2+b$$

Determine los valores de a y b para los cuales . . .

a) No tenga solución.

$$\left(\begin{array}{ccc|c} 2 & -6 & 8 & 12 \\ 1 & 6 & -19 & 0 \\ 2 & -10 & 31+a & 2+b \end{array}\right) \xrightarrow{R_2 \leftrightarrow R_1} \sim \left(\begin{array}{ccc|c} 1 & 6 & -19 & 0 \\ 2 & -6 & 8 & 12 \\ 2 & -10 & 31+a & 2+b \end{array}\right) \underset{R_3: -2R_1+R_3}{\overset{R_2: -2R_1+R_2}{\sim}} \left(\begin{array}{ccc|c} 1 & 6 & -19 & 0 \\ 0 & -18 & 46 & 12 \\ 0 & -22 & 69+a & 2+b \end{array}\right)$$

$$R_2: -\tfrac{1}{18}R_2 \left(\begin{array}{ccc|c} 1 & 6 & -19 & 0 \\ 0 & 1 & -23/9 & -2/3 \\ 0 & -22 & 69+a & 2+b \end{array}\right) \overset{R_3: 22R_2+R_3}{\sim} \left(\begin{array}{ccc|c} 1 & 6 & -19 & 0 \\ 0 & 1 & 23/9 & -2/3 \\ 0 & 0 & \frac{115}{9}+a & -\frac{38}{3}+b \end{array}\right)$$

$$\frac{115}{9} + a = \frac{115+9a}{9}$$

Para que No tengo solución $\frac{115}{9} + a = 0$ & $-\frac{38}{3} + b \neq 0$

$\Rightarrow a = -\frac{115}{9}$ & $-\frac{38}{3} + b = 0$ $b = \frac{38}{3}$ b Cualquier valor diferente de $\frac{38}{3}$

b) Solución única // Continuando Con Gauss

$$R_3: \frac{9}{115+9a}R_3 \left(\begin{array}{ccc|c} 1 & 6 & -19 & 0 \\ 0 & 1 & 23/9 & -2/3 \\ 0 & 0 & 1 & \frac{-114+9b}{115+9a} \end{array}\right)$$

Para que tenga sol. única $115 + 9a \neq 0$
$\therefore$ b Puede tomar cualquier valor &
$115 + 9a \neq 0$

$115 + 9a = 0$ $9a = 115$
$a = \frac{115}{9}$
$a \approx 12.77$

$\therefore a \neq 12.77$
Puede tomar cualquier valor menos 12.77.

c) Para que tenga ∞ de Sol.

$$\frac{115+9a}{9} = 0 \quad \& \quad \frac{3b-38}{3} = 0$$

$\Rightarrow$ $9a = -115$ $3b = 38$

$a = -\frac{115}{9}$ $b = \frac{38}{3}$

$a = -12.77$ $b = 12.66$

(2)

4.

$R_2: -R_1 + R_2 \quad R_3: -R_1 + R_3$

$$A = \begin{pmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & c+b \end{pmatrix} \qquad |A| = \begin{vmatrix} 1 & a & b+c \\ 0 & b-a & c+a \\ 0 & c-a & c+b \end{vmatrix}$$

$$-\cancel{b}-\cancel{c}+\cancel{c}+\cancel{b} = 0$$

$$-b-\cancel{c}+\cancel{c}+a = a-b$$

$$|A| = \left(\frac{1}{c-a}\right)\left(\frac{1}{b-a}\right)\begin{vmatrix} 1 & a & b+c \\ 0 & 1 & \frac{c+a}{b-a} \\ 0 & 1 & \frac{c+b}{c-a} \end{vmatrix}$$

$R_3: -R_2 + R_3$

$$|A| = \left(\frac{1}{c-a}\right)\left(\frac{1}{b-a}\right)\begin{pmatrix} 1 & a & b+c \\ 0 & 1 & \frac{c+a}{b-a} \\ 0 & 0 & \frac{c+b}{c-a} - \frac{c+a}{b-a} \end{pmatrix}$$

$$|A| = \left(\frac{1}{c-a}\right)\left(\frac{1}{b-a}\right)\left(\frac{c+b}{c-a} - \frac{c+a}{b-a}\right)$$

$$= \left(\frac{1}{(c-a)(b-a)}\right)\left(\frac{(c+b)(b-a) - (c+a)(c-a)}{(c-a)(b-a)}\right)$$

$$= \frac{(c+b)(b-a) - (c+a)(c-a)}{[(c-a)(b-a)]^2}$$

$R_2: -R_1 + R_2 \quad R_3: -R_1 + R_3$

$$B = \begin{pmatrix} a_1^2 & a_1 & 1 \\ a_2^2 & a_2 & 1 \\ a_3^2 & a_3 & 1 \end{pmatrix} \qquad |B| = \begin{vmatrix} a_1^2 & a_1 & 1 \\ a_2^2-a_1^2 & a_2-a_1 & 0 \\ a_3^2-a_1^2 & a_3-a_1 & 0 \end{vmatrix} \sim - \begin{vmatrix} 1 & a_1 & a_1^2 \\ 0 & a_2-a_1 & a_2^2-a_1^2 \\ 0 & a_3-a_1 & a_3^2-a_1^2 \end{vmatrix}$$

$$|B| = -\left(\frac{1}{a_2-a_1}\right)\left(\frac{1}{a_3-a_1}\right)\begin{vmatrix} 1 & a_1 & a_1^2 \\ 0 & 1 & \frac{a_2^2-a_1^2}{a_2-a_1} \\ 0 & 1 & \frac{a_3^2-a_1^2}{a_3-a_1} \end{vmatrix} \sim -\left(\frac{1}{a_2-a_1}\right)\left(\frac{1}{a_3-a_1}\right)\begin{vmatrix} 1 & a_1 & a_1^2 \\ 0 & 1 & a_2+a_1 \\ 0 & 1 & a_3+a_1 \end{vmatrix}$$

$R_3: -R_2 + R_1$

$$|B| = -\left(\frac{1}{a_2-a_1}\right)\left(\frac{1}{a_3-a_1}\right)\begin{vmatrix} 1 & a_1 & a_1^2 \\ 0 & 1 & a_2+a_1 \\ 0 & 0 & a_3+a_1-a_2-a_1 \end{vmatrix}$$

$$|B| = (1)(1)(a_3-a_2) = a_3 - a_2$$

Muy bien Katherin

① Determine si los polinomios de 2° grado ~~dados~~ forman un conjunto ortogonal y, en caso negativo, aplique el proceso de ortonormalización de Gram-Schmidt para formar un conjunto ortonormal.

$\{x^2-1,\ x-1\}$

Sol $\vec{v}_1 = x^2-1,\ \vec{v}_2 = x-1$

Veamos si son unitarios $x^2-1 = \begin{pmatrix}1\\0\\-1\end{pmatrix} \Rightarrow \|x^2-1\|^2 = \begin{pmatrix}1\\0\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-1\end{pmatrix} = 1+1=2$

$\therefore \|x^2-1\| = \sqrt{2} \neq 1$

ahora $x-1 = \begin{pmatrix}0\\1\\-1\end{pmatrix} \Rightarrow \begin{pmatrix}0\\1\\-1\end{pmatrix}\cdot\begin{pmatrix}0\\1\\-1\end{pmatrix} = 1+1=2$

$\therefore \|x-1\| \neq 1$

Apliquemos el proceso de ortonormalización de G-S

$\vec{w}_1 = x^2-1$

$\vec{w}_2 = \vec{v}_2 - \text{Proy}_{\vec{w}_1}\vec{v}_2 = v_2 - \dfrac{\langle\vec{v}_2,\vec{w}_1\rangle}{\langle\vec{w}_1,\vec{w}_1\rangle}\vec{w}_1$

Donde $\langle\vec{v}_2,\vec{w}_1\rangle = (1)(0)+(0)(1)+1 = 1$

$\langle\vec{w}_1,\vec{w}_1\rangle = (1)(1)+(0)(0)+(-1)(-1) = 2$

por lo tanto

do

$\vec{w}_2 = (x-1) - \frac{1}{2}(x^2-1) = -\frac{1}{2}x^2 + x - \frac{1}{2}$

$\Rightarrow S' = \{x^2-1,\ -\frac{1}{2}x^2+x-\frac{1}{2}\}$ es ortogonal

Para hacerlo ortonormal necesito que c/vector sea unitario

$\|x^2-1\| = \sqrt{2}$

$\|-\frac{1}{2}x^2+x-\frac{1}{2}\| = \sqrt{\frac{1}{4}+1+\frac{1}{4}} = \sqrt{\frac{6}{4}} = \frac{\sqrt{6}}{2}$

$S'' = \left\{\dfrac{x^2-1}{\sqrt{2}},\ \dfrac{-\frac{1}{2}(x^2-2x+1)}{6}\right\}$.

(2) Encontrar la matriz de cambio de base $P_{B\to C}$ & $P_{C\to B}$ para la base

$\beta = \{1+x^2,\ 1,\ 1-x+x^2\}$

$C = \{x-x^2,\ x,\ 1+x+x^2\}$

Posteriormente, encuentre el vector coordenado $P(x) = 1+2x-x^2$ con respecto a la base C.

$P_{B\to C}$: $1+x^2 = \alpha(x-x^2)+\beta(x)+\gamma(1+x+x^2)$

$1+x^2 = \alpha x - \alpha x^2 + \beta x + \gamma + \gamma x + \gamma x^2$

$1+x^2 = \gamma + (\alpha+\beta+\gamma)x + (\gamma-\alpha)x^2$

$\gamma = 1$

$\gamma - \alpha = 1 \rightarrow \alpha = 0$

$\alpha+\beta+\gamma = 0 \rightarrow \beta = -1$

$$P_{B\to C} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

$1 = \gamma + (\alpha+\beta+\gamma)x + (\gamma-\alpha)x^2$

$\gamma = 1$

$\alpha+\beta-\gamma = 0 \rightarrow \beta = 0$

$\gamma - \alpha = 0 \rightarrow \alpha = 1$

$$P_{C\to B} = (P_{B\to C})^{-1} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ -1 & -1 & -1 \end{pmatrix}$$

$1-x+x^2 = \gamma + (\alpha+\beta+\gamma)x + (\gamma-\alpha)x^2$

$\gamma = 1$

$\alpha+\beta+\gamma = -1 \rightarrow \beta = -2$

$\gamma - \alpha = 1 \rightarrow \alpha = 0$

$x - x^2 = \alpha(1+x^2) + \beta + \gamma(1-x+x^2)$

$= \alpha + \alpha x^2 + \beta + \gamma - \gamma x + \gamma x^2$

$= (\alpha+\beta+\gamma) + (-\gamma)x + (\gamma+\alpha)x^2$

$\alpha+\beta+\gamma = 0 \rightarrow \beta = 1$

$-\gamma = 1 \rightarrow \gamma = -1$

$\gamma + \alpha = -1 \rightarrow \alpha = 0$

$$P_{C\to B} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$$

$x = (\alpha+\beta+\gamma) + (-\gamma)x + (\gamma+\alpha)x^2$

$-\gamma = 1 \rightarrow \gamma = -1$

$\alpha+\beta+\gamma = 0 \rightarrow \beta = 0$

$\gamma + \alpha = 0 \rightarrow \alpha = 1$

$1+x+x^2 = (\alpha+\beta+\gamma) + (-\gamma)x + (\gamma+\alpha)x^2$

$\alpha+\beta+\gamma = 1 \quad \beta = 0$

$-\gamma = 1 \rightarrow \gamma = -1$

$\gamma + \alpha = 1 \quad \alpha = 2$

$p(x) = 1+2x-x^2$

$$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$P(x)_C = 2 + x + 2x^2$

$\Rightarrow$

a) Encuentre una base para el espacio vectorial H formada por

$H = \{(x,y,z) \in \mathbb{R}^3 / 2x - y - z = 0\}$

b) Encuentra una base para el espacio solución del sistema homogéneo dado por

$$\begin{aligned} x - 3y + z &= 0 \\ -2x + 2y - 3z &= 0 \\ 4x - 8y + 5z &= 0 \end{aligned}$$

a) $H = \{(x,y,z) \in \mathbb{R}^3 / 2x - y - z = 0\}$

$2x - y - z = 0$

$y = 2x - z$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ 2x - z \\ z \end{pmatrix} = \begin{pmatrix} x \\ 2x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -z \\ z \end{pmatrix} = x\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + z\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$B = \{(1,2,0), (0,-1,1)\}$

Ahora vemos si es li

$\alpha(1,2,0) + \beta(0,-1,1) = (0,0,0)$

$(\alpha, 2\alpha, 0) + (0, -\beta, \beta) = (0,0,0)$

$\alpha = 0$

$2\alpha - \beta = 0 \to \alpha = \beta = 0$

$\beta = 0$

$\therefore B = \{(1,2,0), (0,-1,1)\}$ es base

b)

$$\left(\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ -2 & 2 & -3 & 0 \\ 4 & -8 & 5 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & -4 & -1 & 0 \\ 0 & 4 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 1 & 1/4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x - 3y + z = 0$

$y + 1/4 z = 0$

$x = 3y - z$

$y = -1/4 z$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3y - z \\ -1/4 z \\ z \end{pmatrix} = \begin{pmatrix} 3y \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -z \\ -1/4 z \\ z \end{pmatrix}$$

$$= y\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + z\begin{pmatrix} -1 \\ -1/4 \\ 1 \end{pmatrix}$$

$B = \{(3,0,0) + (-1, -1/4, 1)\}$

4) Dada la matriz $A = \begin{pmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 2 & 2 & 4 & 3 \end{pmatrix}$; Encuentra

a) $Ke(A)$, b) $\nu(A)$ (nulidad) c) $\rho(A)$ (rango) d) Una base para el espacio renglón

Dim = n - ran(A)

$$A = \begin{pmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 2 & 2 & 4 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

a) $Ker(A) = \{(1,1,2,2), (0,1,1,1), (0,0,0,1)\}$

b) $\nu(A) = 3$

c) $Rang = n - Dim(A) = 4 - 3 = 1$

d) $B = \{(1,1,2,2), (0,1,1,1), (0,0,0,1)\}$

$$x + y + 2z + 2w = 0$$
$$y + z + w = 0$$
$$w = 0$$

$x + y + 2z = 0$

$y + z = 0$

$x = -t$

$y = -t$

$z = t$

$w = 0$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -t \\ -t \\ t \\ 0 \end{pmatrix} = t\begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} \quad \forall t \in \mathbb{R}$$

$Ker(A) = gen\{(-1,-1,1,0)\}$

Base del espacio renglón

Dados $(x_1, y_1), (x_2, y_2) \in \mathbb{R}^2$, definimos $(x_1, y_1) + (x_2, y_2) \doteq (2y_1 + 2y_2, -x_1 - x_2)$
& $r(x_1, y_1) \doteq (2r x_1, -r y_1)$

¿Es $\mathbb{R}^2$ un espacio vectorial con las operaciones de suma y multiplicación por escalar definidas así?

Probemos con las propiedades:

① Cerradura bajo la suma ✓

$(x_1, y_1) + (x_2, y_2) \doteq (2y_1 + 2y_2, -x_1 - x_2) \in \mathbb{R}^2$

② Distributiva (Dos escalares)

$$(r+\beta)(x_1, y_1) = r(x_1, y_1) + \beta(x_1, y_1)$$

$$(2(r+\beta)\underline{x_1}, -(r+\beta)y_1)$$

$$(2rx_1, -ry_1) + (2\beta x_1, -\beta y_1)$$

$$(2(-r\underline{y_1}) + 2(-\beta \underline{y_1}), -(2rx_1), -(2\beta x_1))$$

No es un espacio vectorial al no cumplir una de las operaciones de suma y multiplicación definidas (No cumple la distribución multiplicativa con dos escalares)

2.5