PRINCIPIO DE SUPERPOSICIÓN

TEOREMA: Sean $y_1(x), y_2(x), \ldots y_k(x)$; K soluciones de la ec. dif. homogenea de orden n $[\phi(D)y=0]$ (Por simplicidad denotaremos estas k soluciones por $y_1, y_2, y_3 \ldots y_k$) en donde x está definida en un intervalo abierto I, entonces la combinación lineal

$$y = C_1y_1 + C_2y_2 + \ldots + C_ky_k$$

Es también solución si $x \in I$

Las constantes $C_1, C_2, \ldots C_k$; son constantes arbitrarias.

- Demostración

Suponga que la función D $y_1, y_2, \ldots y_k$; son soluciones de la ec. dif. homogenea de orden n

$$\phi(D)y = 0 \ldots (1)$$

Entonces

$$\phi(D)y_1 = 0;\ \phi(D)y_2 = 0; \ldots; \phi(D)y_k = 0$$

(K IDENTIDAD) ...(2)

Multiplicando en formas respectivas estas K identidades por las constantes arbitrarias $C_1, C_2, \ldots C_k$.

$$C_1\phi(D)y = 0; C_2\phi(D)y_2 = 0; \ldots C_k\phi(D)y_k = 0$$

(K IDENTIDAD) ...(3)

Por la linealidad del operador $\phi(D)$

$$\phi(D)[C_1y_1] = 0; \phi(D)[C_2y_2] = 0; \ldots \phi(D)[C_ky_k] = 0$$

K IDENTIDAD ...(4)

- Sumando miembro a miembro estas K identidades resulta:

$$\phi(D)[C_1y_1] + \phi(D)[C_2y_2] + \ldots + \phi(D)[C_ky_k] = 0 + 0 + \ldots + 0$$

Nuevamente la linealidad de $\phi(D)$

$$\phi(D)[C_1y_1 + C_2y_2 + \ldots + C_ky_k] = 0 \quad \underline{\text{IDENTIDAD}}$$

Por tanto concluimos que, la combinación li.

$$y = C_1y_1 + C_2y_2 + \ldots + C_ky_k$$

También es solución de (1)

NOTA: De (2) y (4) se puede destacar lo siguiente:

Si y_i es solución de la ec. dif. homogenea de orden n, entonces su multiplo escalar C_iy_i, tambien es una solución.

Ejemplo:

Probar que $y_1 = x^2$ & $y_2 = x^2 \ln|x|$ son soluciones de ...(1)

$$x^3y''' - 2xy' + 4y = 0 \ldots (1)$$

Entonces la combinación lineal $y = C_1x^2 + C_2x^2\ln|x|$ tambien es una solución.

- Probemos primero que $y_1 = x^2$ es sol. de (1)

$$y_1 = x^2 \quad y_1' = 2x \quad y_1'' = 2 \quad y_1''' = 0$$

$\Rightarrow$ Sustituyendo en (1)

$$x^3(0) - 2x(2x) + 4(x^2) = 0$$

$$0 - 4x^2 + 4x^2 = 0$$

$$0 = 0 \quad \text{IDENTIDAD}$$

$\therefore$ y_1 es Sol de (1)

Ahora probemos para $y_2 = x^2\ln|x|$

$$y_2' = x + 2x\ln|x|$$

$$y_2'' = 1 + 2 + 2\ln|x|$$

$$y_2''' = \frac{2}{x}$$

Sustituyendo en (1)

$$x^3\left(\frac{2}{x}\right) - 2x(x + 2x\ln|x|) + 4(x^2\ln|x|) = 0$$

$$2x^2 - 2x^2 - 4x^2\ln|x| + 4x^2\ln|x| = 0$$

$$0 = 0 \quad \text{IDENTIDAD}$$

$\therefore$ y_2 es Solución de (1)

Como y_1 & y_2 son Sol. de 1, entonces por el principio de superposición la combinación lineal $y = C_1x^2 + C_2x^2\ln|x|$, también es sol. de (1).

En efecto

$y = C_1x^2 + C_2x^2\ln|x|$

$y' = 2C_1x + 2C_2x\ln|x| + C_2x$

$y'' = 2C_1 + 2C_2\ln|x| + 2C_2 + C_2$

$y''' = \frac{2C_2}{x}$

• Sustituyendo en (1)

$x^3\left(\frac{2C_2}{x}\right) - 2x(2C_1x + 2C_2x\ln|x| + C_2x) + 4(C_1x^2 + C_2x^2\ln|x|)$

$= 2C_2x^2 - 4C_1x^2 - 4C_2x^2\ln|x| - 2C_2x^2 + 4C_1x^2 + 4C_2x^2\ln|x|$

$0 = 0$ IDENTIDAD

$\therefore$ la combinación lineal $y = C_1x^2 + C_2x^2\ln|x|$ es solución de (1)

DEPENDENCIA E INDEPENDENCIA LINEAL

Definición: Se dice que el conjunto de funciones $y_1(x), y_2(x), \ldots y_n(x)$, es linealmente dependiente [esto es l.d] $\forall x \in I$, si existen constantes arbitrarias $C_1, C_2, \ldots C_n$ NO TODAS CERO, tal que $C_1y_1 + C_2y_2 + \ldots + C_ny_n = 0$ ...(1) Se satisface.

En caso contrario se dice que el conjunto de funciones es linealmente independiente [esto es l.i]

En otras palabras se dice que el conjunto de funciones es l.i. si las únicas constantes que satisfacen (1) son $C_1 = C_2 = C_3 = \ldots = C_n = 0$

Ejemplo:

Suponga que las funcione $f_1(x)$ y $f_2(x)$ son l.d., entonces por definición existen constantes arbitrarias C_1 y C_2 (no ambas cero) tales que:

$C_1f_1(x) + C_2f_2(x) = 0$

• Supongamos que $C_1 \neq 0$

$$f_1(x) = -\frac{C_2}{C_1}f_2(x)$$

$$\therefore f_1(x) = K f_2(x)$$

• Si las funciones f_1 y f_2 son l.d. entonces se puede expresar a una de ellas como un multiplo escalar de

Probar que las funciones

$f_1(x) = \text{sen } 2x$; $f_2(x) = \text{sen } x \cos x$ son l.d.

$C_1 \text{sen } 2x + C_2 \text{sen } x \cos x = 0$

$2C_1\cos 2x + C_2(\cos^2x - \text{sen}^2x) = 0$

$f_1(x) = \text{sen } 2x = \text{sen}(x + x)$

$= \text{sen } x\cos x + \cos x \text{ sen } x$

$= 2\text{sen } x\cos x$

$= 2f_2(x)$

$\Rightarrow f_1(x) = 2f_2(x)$

$\therefore f_1$ y f_2 son l.d.

EL WRONSKIANO

Definición: Suponga que cada una de las funciones $y_1, y_2 \ldots y_n$ poseen $n-1$ derivadas al menos entonces el determinante

$$W = (y_1, y_2 \ldots y_n) = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \vdots & & & \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix}$$

Se llama el wronskiano de las fnes. ó slnes.

CONJUNTO FUNDAMENTAL DE SOLUCIONES.

Definición: Todo conjunto $y_1, y_2 \dots y_n$ de n soluciones linealmente independientes de la ec. dif. homogenea de orde n, se llama: "Conjunto Fundamental de Soluciones"

Solución general de la ec. dif. homogenea de orden n.

TEOREMA: Sean $y_1, y_2 \dots y_n$ un conjunto fundamental de soluciones de la ec. dif. homogene de orde n; Entonces la sol. gral. de la ec. dif. es:

$$y_c = C_1y_1 + C_2y_2 + \dots + C_ny_n$$

Donde $C_1, C_2 \dots C_n$ son ctes. arbitrarias

Demostración Suponga que

$$y_c = C_1y_1(x) + C_2y_2(x) + \dots + C_ny_n(x) \dots (1)$$

Es la solución general de la ec. dif. homogenea de orden n

$$a_n(x)y^n + a_{n-1}(x)y^{n-1} + \dots + a_1(x)y' + a_0(x)y = 0 \dots (2)$$

La cual a su vez está sujeta las condiciones iniciales siguientes:

$$y(x_0) = K;\ y'(x_0) = K_1;\ y''(x_0) = K_2; \dots; y^{n-1}(x_0) = K_{n-1}$$

Derivando n-1 veces (1) y aplicando las condiciones iniciales resulta:

$$\left.\begin{array}{l} C_1y_1(x_0) + C_2y_2(x_0) + \dots + C_ny_n(x_0) = K_0 \\ C_1y_1'(x_0) + C_2y_2'(x_0) + \dots + C_ny_n'(x_0) = K_1 \\ C_1y_1''(x_0) + C_2y_2''(x_0) + \dots + C_ny_n''(x_0) = K_2 \\ \quad \vdots \\ C_1y_1^{(n-1)}(x_0) + C_2y_2^{(n-1)}(x_0) + \dots + C_ny_n^{(n-1)}(x_0) = K_{n-1} \end{array}\right\} 3$$

Resolviendo el sistema para $C_1 C_2 \dots C_n$ por Kramen resulta que:

$$C_i = \frac{\Delta_i}{\Delta};\ i = 1, 2, \dots n$$

DONDE

$$\Delta_i = \begin{vmatrix} y_1 & y_2 & \dots & K_0 & \dots & y_n \\ y_1' & y_2' & \dots & K_1 & \dots & y_n' \\ \vdots & & & \vdots & & \vdots \\ y_1^{n-1} & y_2^{(n-1)} & \dots & K_{n-1} & \dots & y_n^{(n-1)} \end{vmatrix}$$

$$\Delta = \begin{vmatrix} y_1 & y_2 & \dots & y_n \\ y_1' & y_2' & \dots & y_n' \\ \vdots & & & \\ y_1^{n-1} & y_2^{n-1} & \dots & y_n^{n-1} \end{vmatrix} = W(y_1, y_2 \dots y_n)$$

$$\Rightarrow C_i = \frac{W_i}{W};\ i = 1, 2 \dots n$$

- Si $w \neq$ entonces $y_1; y_2; \dots; y_n$ son l.i
- Si $w \neq 0$ entonces la sol. del sist. (3) es única.
- Si $w \neq 0$ entonces la sol. del problema de valor inicial es única.

- Si $w = 0$ entonces $y_1, \dots y_n$ son l.d.
- Si $w = 0$ entonces la sol. del sist. (3) no es única.
- Si $w = 0$ entonces la sol. del problema de valor inicial no es única.

Ejemplo:

Considere la ec. dif.

$$y''' = 0;\ y(1) = 0;\ y'(1) = 1;\ y''(1) = 2 \dots (1)$$

Compruebe si con las funciones:

$$y_1 = x;\ y_2 = x^2;\ y_3 = 4x - 3x^2$$

Se puede construir la solución gral de la ec. dif. (1)

$y_1' = 1 \quad y'' = 0 \quad y''' = 0$

$0 = 0$ IDENTIDAD

$y_2' = 2x \quad y_2'' = 2 \quad y''' = 0$

$0 = 0$ IDENTIDAD

Criterio para soluciones l.i.

TEOREMA: Sean n soluciones $y_1, y_2 \dots y_n$ de la ec. dif. homogenea de orden n, en el intervalo I, entonces el conjunto de soluciones es l.i. en I, si y solo si $W(y_1, y_2, \dots y_n) \neq 0$ $\forall x \in I$

En caso contrario, esto es si;

Entonces se dice que el conjunto de soluciones es l.d. $y_1, y_2, \dots y_n$

$$C_1y_1 + C_2y_2 + \dots + C_ny_n = 0$$

$$C_1y_1' + C_2y_2' + \dots + C_ny_n' = 0$$

$$\vdots$$

$$C_1y_1^{(n-1)} + C_2y_2^{(n-1)} + \dots + C_ny_n^{(n-1)} = 0$$

Resolviendo el sistema para $C_1, C_2 \dots C_n$ por Kramer.

$$C_i = \frac{\Delta i}{\Delta} \; ; \; i = 1, 2, \dots n.$$

$$\Delta i = \begin{vmatrix} y_1 & y_2 & \dots & 0 & \dots & y_n \\ y_1' & y_2' & \dots & 0 & \dots & y_n' \\ \vdots & & & \vdots & & \\ y_n^{(n-1)} & y_2^{(n-1)} & \dots & 0 & \dots & y_n^{(n-1)} \end{vmatrix} = 0 \quad 0 \forall i$$

$$W(y_1, y_2 \dots y_n) = \begin{vmatrix} y_1 & y_2 & \dots & y_n \\ y_1' & y_2' & \dots & y_n' \\ \vdots & & & \\ y_1^{(n-1)} & y_2^{(n-1)} & \dots & y_n^{(n-1)} \end{vmatrix}$$

n (Raices de grado n)

$$C_1 = \frac{0}{W} \quad y_1, y_2 \dots y_n$$

Si $W \neq 0 \Rightarrow C_1 = C_2 = \dots = C_n = 0$

Sol. única

① $\phi(D)y = 0$

Sol. gral. de la ec. dif. homogenea de orden n

$$a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0 = 0$$

n raices, $x_1, x_2, \dots x_n \in \mathbb{R}$ y/o $\mathbb{C}$

Si $x \in \mathbb{Z}$ y/o Δ Division Sintética

$$(x - x_1)(x - x_2) \dots (x - x_n) = 0$$

$$y_c = C_1y_1 + \dots + C_ny_n$$

② $ay'' + by' + cy = 0$

$$y_c = C_1y_1 + C_2y_2 \qquad y_1, y_2$$

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

CI) $b^2 - 4ac = 0$
$\Rightarrow x_1 = x_2 \in \mathbb{R}$

CII) $b^2 - 4ac < 0$
$\Rightarrow x_1 \; \& \; x_2 \in \mathbb{C}$

CIII) $b^2 - 4ac > 0$
$\Rightarrow x_1 \neq x_2 \in \mathbb{R}$

$y_3' = 4-6x \quad y_3'' = -6 \quad y_3''' = 0$

$0 = 0$ IDENTIDAD.

$\therefore y_1 = x \,;\ y_2 = x^2 \ \& \ y_3 = 4x - 3x^2$

son soluciones de (1)

Como y_1, y_2 & y_3 son soluciones de (1), entonces por el principio de superposición la combinación lineal

$y = C_1 x + C_2 x^2 + C_3(4x - 3x^2)$

también es una solución.

$$W(y_1,y_2,y_3) = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = \begin{vmatrix} x & x^2 & 4x-3x \\ 1 & 2x & 4-6x \\ 0 & 2 & -6 \end{vmatrix}$$

$= x[-12x - 8 + 12x] - 1[-6x^2 - 8x + 6x^2]$

$= -8x + 6x^2 + 8x - 6x^2$

$= 0$

$\Rightarrow W = 0 \ \therefore y_1, y_2, y_3$ son l. d.

$y''' = 0 \,;\ y(1) = 0 \,;\ y'(1) = 1 \,;\ y''(1) = 2$

$y(x) = C_1 x + C_2 x^2 + C_3(4x - 3x^2)$ . . (a)

$y'(x) = C_1 + 2C_2 x + C_3(4 - 6x)$ . . . (b)

$y''(x) = 2C_2 - 6C_3$ . . . (c)

· Sust. en (ab & c las cond. iniciales

$0 = C_1(1) + C_2(1)^2 + C_3[4(1) - 3(1)^2]$

$1 = C_1 + 2C_2(1) + C_3[4 - 6(1)]$

$2 = 2C_2 - 6C_3$

· Simplificando el sistema:

$C_1 + C_2 + C_3 = 0$

$C_1 + 2C_2 - 2C_3 = 1$

$2C_2 - 6C_3 = 2$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \\ 0 & 2 & -6 \end{vmatrix} = 1[-12+4] - 1[-6-2]$$

$= -8 + 8 = 0$

$\therefore$ No es Sol. gral.

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*Zill. pag 128 Cap 4 4.1.2 15-30

Considere la ec. dif...

$y_1 = 5 \quad y_2 = \cos^2 x \quad y_3 = \operatorname{sen}^2 x$

$$W = \begin{vmatrix} 5 & \cos^2 x & \operatorname{sen}^2 x \\ 0 & -2\cos x \operatorname{sen} x & 2\operatorname{sen} x \cos x \\ 0 & -2(\cos^2 x - \operatorname{sen}^2 x) & 2(\cos^2 x - \operatorname{sen}^2 x) \end{vmatrix}$$

$= 5(-4\cos x \operatorname{sen} x(\cos^2 x - \operatorname{sen}^2 x) + 4\operatorname{sen} x \cos x(\cos^2 x - \operatorname{sen}^2 x))$

$= 5(-4\cos^3 x \operatorname{sen} x + 4\operatorname{sen}^3 x \cos x + 4\cos^3 x \operatorname{sen} x - 4\operatorname{sen}^3 x \cos x)$

$= 5(0) = 0$ l. d.

$y_1 = \cos 2x \quad y_2 = 1 \quad y_3 = \cos^2 x$

$$W = \begin{vmatrix} \cos 2x & 1 & \cos^2 x \\ -2\operatorname{sen} 2x & 0 & -2\cos x \operatorname{sen} x \\ -4\cos 2x & 0 & -2(\cos^2 x - \operatorname{sen}^2 x) \end{vmatrix}$$

$W = -1[4\operatorname{sen} 2x(\cos^2 x - \operatorname{sen}^2 x) - 8\cos 2x(\cos x \operatorname{sen} x)$

$= -1[4\operatorname{sen} 2x \cos 2x - 4\operatorname{sen} 2x \cos 2x$

$= -1(0) = 0 \ \therefore$ l.d.

$y'' - y' - 12y = 0 \,;\ y_1 = e^{-3x} \,;\ y_2 = e^{4x}$

$$W = (y_1, y_2) = \begin{vmatrix} e^{-3x} & e^{4x} \\ -3e^{-3x} & 4e^{4x} \end{vmatrix} = 4e^x + 3e^x = 7e^x \neq 0$$

$\therefore y_1, y_2$ & y_3 son l.i

$y_c = C_1 e^{-3x} + C_2 e^{4x}$ Es sol. gral de la ec. dif.

$\phi(D)y=0$

$\{y_1, y_2, \dots y_n\}$ l.i.

$y_c = C_1 y_1 + C_2 y_2 + \dots + C_n y_n$ Sol. gnrl.

UNICIDAD

$W \neq 0 \Rightarrow y_1, y_2 \dots y_n$ l.i

$W = 0 \Rightarrow y_1 ; \dots ; y_n$ l.d.

RESTRICCIÓN 2

Para la obtención de una solución gral. EXACTA, los coeficientes de la ec. dif. lineal de orden n, deben ser todos cte.

$$a_n(x)^n + \dots + a_1(x)y' + a_0(x)y = F(x) \dots (1)$$

Donde los coeficientes

$a_n(x), \dots, a_1(x), a_0(x)$ con $a_n(x) \neq 0$

son los coeficientes de (1)

Definición: Si todos los coeficientes de la ec. dif

$$a_n y^{n} + a_{n-1}y^{n-1} + \dots + a_1 y' + a_0 y = 0$$

son constantes, entonces la ec. dif se llama: "Ecuación diferencial homogenea de orden n con coeficientes constantes" *

$y_1, \dots y_n$ $\quad y = e^{mx}$ $\quad m \in \mathbb{R}$ y/o $\mathbb{C}$

La función generadora del conjunto $y_1, y_2 \dots y_n$ de n soluciones de la ec. dif. lineal homogenea con coeficientes constantes es $y = e^{mx}$ y se dice que esta función es solución de la ec. dif. para todos los valores de $m \in \mathbb{R}$ y/o $\mathbb{C}$ que la reduzca a una identidad.

Esto es:

Suponga que la función

$$y = e^{mx} \dots (1)$$

Es sol. de la ec. dif. lineal homogenea de orden n con coeficientes constantes.

$$a_n y^{(n)} + \dots + a_1 y' + a_0 y = 0 \dots (2)$$

Derivando n veces (1)

$y = e^{mx}$; $y' = me^{mx}$; $y'' = m^2 e^{mx} \dots y^n = m^n e^{mx}$

sustituyendo y y sus derivadas en (2)

$$a_n m^n e^{mx} + a_{n-1} m^{n-1} e^{mx} + \dots + a_1 m e^{mx} + a_0 e^{mx} = 0$$

$$\Rightarrow e^{mx}[a_n m^n + a_{n-1} m^{n-1} + \dots + a_1 m + a_0] = 0$$

Como $e^{mx} \neq 0 \quad \forall m, x$

Luego, para que la ec. dif. se reduzca a una identidad

$$a_n m^n + a_{n-1} m^{n-1} + \dots + a_1 m + a_0 = 0 \dots (3)$$

La cual representa una ec. algebraica de grado n, y que por el teorema fundamental del algebra, cuenta con las raices $m_1, m_2 \dots m_n \in \mathbb{R}$ y/o $\mathbb{C}$ y se dice entonces que la función $y = e^{mx}$. Es la sol. de la ec. dif. (2) para todo valor de m que satisface la ecuación algebraica (3).

La ecuación algebraica (3) se llama el "polinomio característico" o "ecuación característica" asociada a la ec. dif.

METODO PARA LA CONSTRUCCIÓN DE LA SOLUCIÓN GENERAL DE LA EC. DIF. HOMOGENEA CON COEF. CONSTANTES.

Dada la ec. dif:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 y' + a_0 y = 0$$

donde $a_n, a_{n-1}, \dots, a_1, a_0$ con $a_n \neq 0$ son cts.

① Escribir la ec. característica asociada con la ec. dif

$$a_n m^n + a_{n-1} m^{n-1} + \dots + a_1 m + a_0 = 0$$

② Calcular la n raices $m_1, m_2, \dots, m_n$ de la ec. caracteristica.

③ De acuerdo con el tipo de raices escribimos al conjunto $y_1, y_2, \dots y_n$ de n soluciones l.i de acuerdo con lo siguiente:

i) A toda raiz real simple m_i corresponde una sol. de la forma:

$$y_i = e^{m_i x}$$

ii) A toda raiz real de multiplicidad r, corresponden r soluciones de la forma:

$e^{mx}, xe^{mx}, x^2 e^{mx}, \dots, x^n e^{mx}$

iii) A todo par de raices complejos conjugadas de la forma $m = \alpha$ $n = \beta$

④ A todo par de raices complejas conjugadas de la forma $m = \alpha + i\beta$ y con multiplicidad M corresponde M soluciones de la forma

$e^{\alpha x}\operatorname{sen}\beta x;\ xe^{\alpha x}\operatorname{sen}\beta x;\ x^2 e^{\alpha x}\operatorname{sen}\beta x \dots$
$e^{\alpha x}\cos\beta x;\ xe^{\alpha x}\cos\beta x;\ x^2 e^{\alpha x}\cos\beta x \dots$

⑤ A genera el conjunto de n soluciones $y_1, y_2, \dots y_n$ l.i formamos la sol. gral.

$y_c = C_1 y_1 + C_2 y_2 + \dots + C_n y_n$

Donde $C_1, C_2, \dots C_n$ son cts. arbitrarias

$y'' - 3y' + 2y = 0$ --- (1)

① Ec. característica

$m^2 - 3m + 2 = 0$ --- (2)

② Raices

$(m-1)(m-2)$ $m_1 = 1$ $m_2 = 2$

③ Conjunto sol.

$y_1 = e^x$ $y_2 = e^{2x}$

④ Sol gral.

$y_c = C_1 e^{2x} + C_2 e^x$

METODO DE COEFICIENTES INDETERMINADOS PARA LA OBTENCIÓN DE LA SOLUCION PARTICULAR y_p DE LA ECUACIÓN DIF. LINEAL NO HOMOGENEA DE ORDEN N.

- Este metodo basicamente consiste en proponer una solución particular y_p de la ec. dif. lineal no homogenea de orden n, que sea semejante en forma a la función $f(x)$ que aparece en el segundo miembro de la ec. dif.

Cada termino de la sol. particular y_p propuesta deberá ir acompañado de una constante arbitraria (o coeficiente indeterminado) y que denotaremos con letras mayúsculas del alfabeto (A, B, C, D, ... ETC) las cuales se determinarán a partir de condiciones de identidad (criterios de comparación de terminos semejantes)

* Aunque el metodo es directo, está limitado a:

- Evaluaciones diferenciales lineales con coeficientes constantes
- Ecuaciones diferenciales cuya función $f(x)$ sea una función polinomial $P_n(x)$, o exponencial $e^{\alpha x}$, funciones trigonometricas de tipo $\cos\beta x$ y/o $\operatorname{sen}\beta x$ o bien sumas y productos finitos de las anteriores.

$F(x) = x^2 + 3x + 6$

$\rightarrow y_p = Ax^2 + Bx + C.$

Dada la funcion $F(x)$

¿Cómo proponer la solución particular y_p?

• Funciones Polinomiales

$F(x) = P_0(x)$

$$\left.\begin{array}{l} F(x)=5 \\ F(x)=\sqrt{\pi} \\ F(x)=-\frac{1}{2} \end{array}\right\} y_p = A$$

$F(x) = P_1(x)$

$$\left.\begin{array}{l} F(x)=x \\ F(x)=x-1 \\ F(x)=2x+3 \end{array}\right\} y_p = Ax+B$$

$F(x) = P_2(x)$

$$\left.\begin{array}{l} F(x)=x^2 \\ F(x)=x^2-4 \\ F(x)=4x^2+3x-1 \end{array}\right\} Ax^2+Bx+C$$

$$\boxed{F(x)=P_n(x) \rightarrow y_p = \tilde{P}_n(x)}$$

• Funcion Polinomial $P_n(x)$ por función exponencial $e^{\alpha x}$

$F(x) = P_0(x)e^{\alpha x}$

$$\left.\begin{array}{l} F(x)=(5)e^{\alpha x} \\ F(x)=(\sqrt{\pi})e^{\alpha x} \\ F(x)=(-\frac{1}{2})e^{\alpha x} \end{array}\right\} y_p = (A)e^{\alpha x}$$

$F(x) = P_1(x)$

$$\left.\begin{array}{l} F(x)=(x)e^{\alpha x} \\ F(x)=(x-1)e^{\alpha x} \\ F(x)=(2x+3)^{\alpha x} \end{array}\right\} y_p = (Ax+B)e^{\alpha x}$$

$F(x) = P_2(x)$

$$\left.\begin{array}{l} F(x)=(x^2)e^{\alpha x} \\ F(x)=(x^2-4)e^{\alpha x} \\ F(x)=(4x^2+3x-1)e^{\alpha x} \end{array}\right\} y_p = (Ax^2+Bx+C)e^{\alpha x}$$

$$\boxed{F(x)=P_n(x)e^{\alpha x} \rightarrow y_p = \tilde{P}_n(x)e^{\alpha x}}$$

• Funcion Polinomial $P_n(x)$ por función trigonométrica $\cos \beta x$ y/o $\operatorname{sen} \beta x$

$P_0(x)\operatorname{sen}\beta x$ ó $P_0(x)\cos\beta x$ ó

$P_0(x)\cos\beta x + Q_0(x)\operatorname{sen}\beta x$

$$\left.\begin{array}{l} F(x)=5\cos\beta x \text{ ó} \\ F(x)=\frac{1}{2}\operatorname{sen}\beta x \text{ ó} \\ F(x)=5\cos\beta x-\frac{1}{2}\operatorname{sen}\beta x \end{array}\right\} y_p = A\cos\beta x + B\operatorname{sen}\beta x$$

$P_1(x)\cos\beta x$ ó $Q_1(x)\operatorname{sen}\beta x$ ó

$P_1(x)\cos\beta x + Q_1(x)\operatorname{sen}\beta x$

$$\left.\begin{array}{l} F(x)=x\cos\beta x \\ F(x)=(x-1)\operatorname{sen}\beta x \\ F(x)=x\cos\beta x+(x-1)\operatorname{sen}\beta x \end{array}\right\} y_p = (Ax+B)\cos\beta x+(Cx+D)\operatorname{sen}\beta x$$

$P_2(x)\cos\beta x$ ó $Q_2(x)\operatorname{sen}\beta x$ ó

$P_2(x)\cos\beta x + Q_2(x)\operatorname{sen}\beta x$.

$$\underbrace{\begin{array}{l} F(x)=x^2\cos\beta x \text{ ó} \\ F(x)=(x^2-4)\operatorname{sen}\beta x \text{ ó} \\ F(x)=(3x^2+2x+1)\cos\beta x + x^2\operatorname{sen}\beta x \end{array}}$$

$y_p = (Ax^2+Bx+C)\cos\beta x + (Dx^2+Ex+F)\operatorname{sen}\beta x$

$$\boxed{\begin{array}{l} F(x)=P_n(x)\cos\beta x \text{ ó } Q_n(x)\operatorname{sen}\beta x \text{ ó} \\ P_n(x)\cos\beta x + Q_n(x)\operatorname{sen}\beta x \\ \rightarrow y_p = \tilde{P}_n(x)\cos\beta x + \tilde{Q}_n(x)\operatorname{sen}\beta x \end{array}}$$

* CASO ESPECIAL.

Si $F(x) = \underbrace{P_n(x)\cos\beta x + Q_m(x)}_{\max(m,n)}\operatorname{sen}\beta x$

$F(x) = x\cos\beta x + x^2\operatorname{sen}\beta x$

$y_p = (Ax^2+Bx+C)\cos\beta x + (Dx^2+Ex+F)\operatorname{sen}\beta x$

• CASO GENERAL.

$F(x) = P_n(x)e^{\alpha x}\cos\beta x$ ó

$F(x) = Q_n(x)e^{\alpha x}\operatorname{sen}\beta x$ ó

$F(x) = [P_n(x)\cos\beta x + Q_n\operatorname{sen}\beta x]e^{\alpha x}$

$y_p = [\tilde{P}_n\cos\beta x + \tilde{Q}_n\operatorname{sen}\beta x]e^{\alpha x}$

si $\phi(x) = f_1(x) + f_2(x) + \dots + f_n(x)$

$y_p = y_{p1} + y_{p2} + \dots + f_n(x)$

$y\ F(x) = \underbrace{x^2-3}_{f_1} + \underbrace{xe^{\alpha x}}_{f_2} + \underbrace{3\operatorname{sen}\beta x}_{f_3}$

$y_{p1} = Ax^2 + Bx + C$

$y_{p2} = (Dx+E)e^{\alpha x}$

$y_{p3} = F\cos\beta x + G\,\text{sen}\,\beta x$

$y_p = y_{p1} + y_{p2} + y_{p3}$

$F(x) = \underbrace{5x\cos x}_{f_1} + \underbrace{x\,\text{sen}\,2x}_{f_2}$

$y_{p1} = (Ax+B)\cos x + (Cx+D)\,\text{sen}\,x$

$y_{p2} = (Ex+F)\cos 2x + (Gx+H)\,\text{sen}\,2x$

$y_p = y_{p1} + y_{p2}$

Resolver:

• $y'' - 9y = 54 \ldots (1)$

- la ec dif. homogenea de (1)

$y'' - 9y = 0 \quad \ldots (2)$

- ec característica

- $m^2 - 9 = 0$

$m_{1,2} = \pm 3$

③ Sols. l.i.

$y_1 = e^{3x} \quad y_2 = e^{-3x}$

④ Sol gral. de (2)

$y_c = C_1e^{3x} + C_2e^{-3x}$

$F(x) = 54 \quad y_p = A$

$y'_p = y''_p = 0$

Sustituyendo y_p y sus derivadas en (1)

$0 - 9A = 54$

$A = -6$ es sol. part. de (1)

$\therefore y = y_c + y_p = C_1e^{3x} + C_2e^{-3x} - 6$

ES SOL. GRAL. DE (1)

• $y'' + 4y' - 2y = 2x^2 - 3x + 6 \ldots (1)$

• Ec. dif. homo de (1)

$y'' + 4y' - 2y = 0 \ldots (2)$

• Ec. Carac. de (2)

$m^2 + 4m - 2 = 0$

• Raíces

$m_{1,2} = \dfrac{-4 \pm \sqrt{16 - 4(1)(-2)}}{2(1)}$

$m_{1,2} = \dfrac{-4 \pm \sqrt{24}}{2} = -2 \pm \sqrt{6} \in \mathbb{R}$

• Soluciones l.i.

$y_1 = e^{(-2-\sqrt{6})x} \quad y_2 = e^{(-2+\sqrt{6})x}$

• Sol gral. de (2)

$y_c = C_1e^{(-2-\sqrt{6})x} + C_2e^{(-2+\sqrt{6})x}$

• $F(x) = 2x^2 - 3x + 6$

$y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

• Sustituyendo y_p y sus derivadas en (1)

$2A + 4(2Ax + B) - 2(Ax^2 + Bx + C) = 2x^2 - 3x + 6$

$2A + 8Ax + 4B - 2Ax^2 - 2Bx - 2C = 2x^2 - 3x + 6$

$(-2A)x^2 + (8A - 2B)x + (2A + 4B - 2C) = 2x^2 - 3x + 6$

COND. DE IDENTIDAD.

$-2A = 2 \longrightarrow A = -1$

$8A - 2B = -3 \longrightarrow B = -\frac{5}{2}$

$2A + 4B - 2C = 6 \rightarrow C = -9$

• $y_p = -x^2 - \frac{5}{2}x - 9$

• $y = y_c + y_p = C_1e^{(-2-\sqrt{6})x} + C_2e^{(-2+\sqrt{6})x} - x^2 - \frac{5}{2}x - 9$

sol. gral. de (1).

* $y'' + 3y' + 2y = 4x^2 \ldots (1)$

• Ec. homo. $y'' + 3y' + 2y = 0 \ldots (2)$

• Ec caracte. $m^2 + 3m + 2 = 0$

• Raíces $(m+2)(m+1) = 0 \quad m_1 = -2 \quad m_2 = -1$

• Sols. l.i. $y_1 = e^{-2x} \quad y_2 = e^{-x}$

• Sol gral de (2)

$y_c = C_1e^{-2x} + C_2e^{-x}$

• $F(x) = 4x^2$

• $y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

• Sustituyendo y_p y sus derivadas en (1)

$2A + 3(2Ax + B) + 2(Ax^2 + Bx + C) = 4x^2$

$2A + 6Ax + 3B + 2Ax^2 + 2Bx + 2C = 4x^2$

$(2A)x^2 \; (6A + 2B)x + (3B + 2C + 2A) = 4x^2$

$2A=4 \rightarrow A=2$

$6A+2B=0 \longrightarrow B=-6$

$3B+2C+2A=0 \longrightarrow C=7$

$\therefore y_p = 2x^2-6x+7$

$y = y_c + y_p = C_1e^{-2x} + C_2e^{-x} + 2x^2 - 6x + 7$ Sol gral. de (1)

$y'' + y' - 6y = -5e^{2x} \ldots 1$

Ec homo de (1)

$y'' + y' - 6y = 0$

Ec carac.

$m^2 + m - 6 = 0$

2 Raíces

$(m+3)(m-2)$

$m_1 = -3 \quad m_2 = 2$

Sols l.i

$y_1 = e^{-3x}; y_2 = e^{2x}$

Sol. gral de (2)

$y = C_1e^{-3x} + C_2e^{2x}$

$F(x) = -5e^{2x} \longrightarrow y_p = Ae^{2x}$

$y'_p = 2Ae^{2x}$

$y'' = 4Ae^{2x}$

Sust. y_p y sus der. en (1)

$4Ae^{2x} + 2Ae^{2x} - 6Ae^{2x} = -5e^{2x}$

$0 = -5e^{2x}$

$\rightarrow F(x) = -5e^{2x} \quad y_p = Axe^{2x}$

$y'_p = Ae^{2x} + 2Axe^{2x}$

$y''_p = 2Ae^{2x} + 4Axe^{2x}$

sustituyendo y_p y sus derivadas

$4Ae^{2x} + 4Axe^{2x} + Ae^{2x} + 2Axe^{2x} - 6Axe^{2x} = -5e^{2x}$

$5Ae^{2x} = -5e^{2x}$

$5A = -5 \rightarrow A = -1 \quad y_p = -xe^{2x}$

$\therefore y = y_c + y_p = C_1e^{-3x} + C_2e^{2x} - xe^{2x}$

- $y''' = x^2 \ldots (1)$
- Ec. homo $y''' = 0 \ldots (2)$
- Ec. carac. $m^3 = 0$
- Raíces $m_1 = m_2 = m_3 = 0$
- Sols li. $y_1 = e^{0x} \quad y_2 = xe^{0x} \quad y_3 = x^2e^{0x}$

$\rightarrow y_1 = 1 \quad y_2 = x \quad y_3 = x^2$

- Sol gral de (2)

$y_c = C_1 + C_2x + C_3x^2$

$\rightarrow F(x) = x^2 \rightarrow y_p = (Ax^2 + Bx + C)x^3$

$y_p = Ax^5 + Bx^4 + Cx^3$

$y'_p = 5Ax^4 + 4Bx^3 + 3Cx^2$

$y''_p = 20Ax^3 + 12Bx^2 + 6Cx$

$y'''_p = 60Ax^2 + 24Bx + 6C$

Sustituyendo y'''_p en (1)

$60Ax^2 + 24Bx + 6C = x^2$

$60A = 1 \rightarrow A = \frac{1}{60}$

$24B = 0 \rightarrow B = 0$

$6C = 0 \longrightarrow C = 0$

$\therefore y_p = \frac{1}{60}x^5$ Sol. part. de (1)

$\therefore y = y_c + y_p = C_1 + C_2x + C_3x^2 + \frac{1}{60}x^5$ //

Sol. gral. de (1)

- $y'' + 4y = 12\,\text{sen}\,2x \ldots (1)$
- Ec homo. $y'' + 4y = 0 \ldots (2)$
- Ec. carac. $m^2 + 4 = 0$
- Raíces $m_{12} = \pm\sqrt{-4} = \pm 2i$
- Sols li $y_1 = \cos 2x \quad y_2 = \text{sen}\,2x$
- Sol. gral. (2)

$y_c = C_1\cos 2x + C_2\,\text{sen}\,2x$

* $F(x) = 12\,\text{sen}\,2x$

$\rightarrow y_p = (A\cos 2x + B\,\text{sen}\,2x)x$

$y_p = Ax\cos 2x + Bx\,\text{sen}\,2x$

$y'_p = A\cos 2x - 2Ax\,\text{sen}\,2x + B\,\text{sen}\,2x + 2Bx\cos 2x$

$y''_p = -4A\,\text{sen}\,2x - 4Ax\cos 2x + 4B\cos 2x - 4Bx\,\text{sen}\,2x$

Sust. y_p y sus derivadas en (1)

$-4A\,\text{sen}\,2x - 4Ax\cos 2x + 4B\cos 2x - 4Bx\,\text{sen}\,2x$

$+4(Ax\cos 2x + Bx\,\text{sen}\,2x) = 12\,\text{sen}\,2x$

$-4A\,\text{sen}\,2x + 4B\cos 2x = 12\,\text{sen}\,2x$

$-4A = 12 \quad A = -3$

$4B = 0 \quad B = 0$

$\therefore y_p = -3x\cos 2x$ Sol part. de (1)

$\therefore y = y_c + y_p$
$= C_1\cos 2x + C_2 \operatorname{sen} 2x - 3x\cos 2x$ Sol. grl. de (1)

$y'' - 2y' - 3y = 4x - 5 + 6xe^{-x}$ ... (1)

Ec homogenea

$y'' - 2y' - 3y = 0$ ... (2)

• Ec. carac.

$m^2 - 2m - 3 = 0$

$(m-3)(m+1) = 0$

$m_1 = 3 \quad m_2 = -1$

• Sols. l.i.

$y_1 = e^{3x}$; $y_2 = e^{-x}$

• Sol. gral. de (2)

$y_c = C_1 e^{3x} + C_2 e^{-x}$

• $F(x) = \underbrace{4x - 5}_{F_1} + \underbrace{6xe^{-x}}_{F_2}$

$\rightarrow y_{p_1} = Ax + B$

$y_{p_2} = \underline{(Cx + D)e^{-x}}\, x$

$y_p = y_{p_1} + y_{p_2} = Ax + B + Cx^2e^{-x} + Dxe^{-x}$

$y'p = A + 2Cxe^{-x} - Cx^2e^{-x} + De^{-x} - Dxe^{-x}$

$y''p = 2Ce^{-x} - 4Cxe^{-x} + Cx^2e^{-x} - 2De^{-x} + Dxe^{-x}$

Sust. yp y sus derivadas en (1)

$2Ce^{-x} - \cancel{4Cxe^{-x}} + \cancel{Cx^2e^{-x}} - \cancel{2De^{-x}} + \cancel{Dxe^{-x}} - 2A - \cancel{4Cxe^{-x}}$
$+ \cancel{2Cx^2e^{-x}} - \cancel{2De^{-x}} + \cancel{2Dxe^{-x}} - \cancel{3Ax} - 3B - \cancel{3Cx^2e^{-x}}$
$- 3Dxe^{-x} = 4x - 5 + 6xe^{-x}$

$(2C - 4D)e^{-x} + (-8C)xe^{-x} + (-3A)x + (-2A - 3B) = 4x - 5 + 6xe^{-x}$

$2C - 4D = 0 \longrightarrow D = -\frac{3}{8}$

$-8C = 6 \longrightarrow C = -\frac{3}{4}$

$-3A = 4 \longrightarrow A = -\frac{4}{3}$

$-2A - 3B = -5 \longrightarrow B = \frac{23}{9}$

$\therefore y_p = -\frac{4}{3}x + \frac{23}{9} - \frac{3}{4}x^2e^{-x} - \frac{3}{8}xe^{-x}$

$y = y_c + y_p = C_1e^{3x} + C_2e^{-x} - \frac{4}{3}x + \frac{23}{9} - \frac{3}{4}x^2e^{-x} - \frac{3}{8}xe^{-x}$

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1 - 38 Impares.

$y'' + 3y' + 2y = 6$

• $y'' + 3y' + 2y = 0$

• $m^2 + 3m + 2 = 0$

• $(m+2)(m+1) = 0 \quad m_1 = -2 \quad m_2 = -1$

• $y_1 = e^{-2x} \quad y_2 = e^{-x}$

• $y_c = C_1e^{-2x} + C_2e^{-x}$

+ $F(x) = 6 \rightarrow y_p = A$

$y'_p = 0 = y''p$

$0 + 3(0) + 2A = 6$

$A = 3$

$y = y_c + y_p = C_1e^{-2x} + C_2e^{-x} + 3$

$y'' - 10y' + 25y = 30x + 3$

• $y'' - 10y' + 25y = 0$

• $m^2 - 10m + 25 = 0$

$(m-5)(m-5) = 0 \quad m_{1,2} = 5$

• $y_1 = e^{5x} \quad y_2 = xe^{5x}$

$y_c = C_1e^{5x} + C_2xe^{5x}$

$F(x) = 30x + 3 \rightarrow y_p = Ax + B$

$y'p = A$

$y''p = 0$

$0 - 10A + 25(Ax + B) = 30x + 3$

$-10A + 25Ax + 25B = 30x + 3$

$(25A) = 30 \longrightarrow \frac{6}{5}$

$25B - 10A = 3$

$25B - 10\left(\frac{6}{5}\right) = 3$

$25B = 15$

$B = \frac{3}{5} \qquad y_p = \frac{6}{5}x + \frac{3}{5}$

$y = C_1e^{5x} + C_2xe^{5x} + \frac{6}{5}x + \frac{3}{5}$

$\frac{1}{4}y'' + y' + y = x^2 - 2x$

- $\frac{1}{4}y'' + y' + y = 0$
- $\frac{1}{4}m^2 + m + 1 = 0$
- $m^2 + 4m + 4 = 0$

$(m+2)(m+2) = 0$

$m_1 = -2 \quad m_2 = -2$

- $y_1 = e^{-2x} \quad y_2 = xe^{-2x}$

$F(x) = x^2 - 2x \rightarrow y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A \rightarrow \frac{1}{2}A$

$\cancel{\frac{1}{4}(2A)} + 2Ax + B + Ax^2 + Bx + C = x^2 - 2x$

$(A)x^2 + (2A+B)x + (B + C + \frac{1}{2}A) = x^2 - 2x$

$A = 1$

$2A + B = -2 \quad B = -4$

$B + C + \frac{1}{2}A = 0$

$-4 + C + \frac{1}{2} = 0$

$C = \frac{7}{2}$

$y_p \; x^2 - 4x + \frac{7}{2}$

$y = x^2 - 4x + \frac{7}{2} + C_1e^{-2x} + C_2xe^{-2x}$

$y'' - y' = -3$

$y'' - y' = 0$

$m^2 - m = 0$

$m(m-1) = 0$

$m_1 = 0 \quad m_2 = 1$

$y_1 = 1 \quad y_2 = e^x$

$y_c = C_1 + C_2e^x$

$F(x) = -3 \quad y_p = Ax$

$y'_p = A \quad y'' = 0$

$0 - A = -3$

$A = 3 \quad y_p = 3x$

$y = C_1 + C_2e^x + 3x$

$y'' - 8y' + 20y = 100x^2 - 26xe^x$

- $y'' - 8y' + 20y = 0$
- $m^2 - 8m + 20 = 0$

$m_{1,2} = \dfrac{8 \pm \sqrt{64 - 4(1)(20)}}{2(1)}$

$m_{1,2} = 4 \pm 2i$

- $y_1 = e^{4x}\cos 2x \quad y_2 = e^{4x}\operatorname{sen} 2x$
- $y_c = C_1e^{4x}\cos 2x + C_2e^{4x}\operatorname{sen} 2x$

$F(x) = \underbrace{100x^2}_{f_1} - \underbrace{26xe^x}_{f_2}$

$y_{p1} = Ax^2 + Bx + C$

$y_{p2} = (Dx + E)e^x$

$y = y_{p1} + y_{p2} = Ax^2 + Bx + C + Dxe^x + Ee^x$

$y'_p = 2Ax + B + Dxe^x + De^x + Ee^x$

$y''_p = 2A + Dxe^x + De^x + De^x + Ee^x$

$2A + Dxe^x + 2e^x + Ee^x$

$\cancel{2A} + Dxe^x + 2e^x + Ee^x - \cancel{16Ax} - \cancel{16B} + 16xe^x - 16e^x - 16e^x$

$+ \cancel{20Ax^2} + \cancel{20B}x + 20C + 20Dxe^x + 20e^x$

$= 100x^2 - 26xe^x$

$(20A)x^2 + (20B - 16A)x + (2A - 16B$

$y'' + 3y = -48x^2 e^{3x}$

- $y'' + 3y = 0$
- $m^2 + 3 = 0$

$m_{12} = \pm\sqrt{-3} = \pm\sqrt{3}\, i$

$y_1 = \cos\sqrt{3}x \quad y_2 = \operatorname{sen}\sqrt{3}x$

- $y_c = C_1\cos\sqrt{3}x + C_2\operatorname{sen}\sqrt{3}x$

$F(x) = (Ax^2 + Bx + C)e^{3x}$

$y_p = Ax^2e^{3x} + Bxe^{3x} + Ce^{3x}$

$y'_p = 2Axe^{3x} + 3Ax^2e^{3x} + Be^{3x} + 3Be^{3x} + 3Ce^{3x}$

$y'' = 2Ae^{3x} + \underline{6Axe^{3x}} + \underline{6Axe^{3x}} + 9Ax^2e^{3x} + 3Be^{3x} + 9Be^{3x} + 9Ce^{3x}$

$= 2Ae^{3x} + 12Axe^{3x} + 9Ax^2e^{3x} + 6Be^{3x} + 9Bxe^{3x} + 9Ce^{3x}$

$\cancel{2Ae^{3x}} + \cancel{12Axe^{3x}} + \cancel{9Ax^2e^{3x}} + \cancel{6Be^{3x}} + \cancel{9Bxe^{3x}} + \cancel{9Ce^{3x}} + \cancel{3Ax^2e^{3x}} + \cancel{3Bxe^{3x}} + \cancel{3Ce^{3x}}$

$= -48x^2e^{3x}$

$(2A + 12C + 6B)e^{3x} + (12A + 9B + 3Bx)xe^{3x} + (9A + 3A)x^2e^{3x} = -48x^2e^{3x}$

$2A + 12C + 6B = 0$

$12A + 12B = 0 \qquad A + B = 0 \qquad A = -4$

$12A = -48 \qquad B = 4 \qquad A + 3B + 6C = 0$

$-4 + 12 + 6C = 0$

$C = -\frac{8}{6} = -\frac{4}{3}$

$y = C_1\cos\sqrt{3}x + C_2\operatorname{sen}\sqrt{3}x - 4x^2e^{3x} + 9xe^{3x} - \frac{4}{3}e^{3x}$

$y'' - 2y' + 5y = e^x\cos 2x$ Apendice

- ec h. $y'' - 2y' + 5y = 0$
- ec. c. $m^2 - 2m + 5 = 0$

$m_{12} = \dfrac{2 \pm\sqrt{4 - 4(1)(5)}}{2(1)} = \dfrac{2 \pm\sqrt{-16}}{2}$

$m_{12} = 1 \pm 2i \quad \alpha = 1 \quad \beta = 2$

- $y_1 = e^x\cos 2x \quad y_2 = e^x\operatorname{sen}2x$
- $y_c = C_1e^x\cos 2x + C_2e^x\operatorname{sen}2x$

$F(x) = (1)\cos 2x \cdot e^x$

$y_p = (A\cos 2x + B\operatorname{sen}2x)e^x(x)$

$Axe^x\cos 2x + Bxe^x\operatorname{sen}2x$

$y'_p = Ae^x\cos 2x + Axe^x\cos 2x - 2Axe^x\operatorname{sen}2x + Be^x\operatorname{sen}2x + Bxe^x\operatorname{sen}2x + 2Bxe^x\cos 2x$

$\ast$ $12y'' - 5y' - 2y = 0$

① $12m^2 - 5m - 2 = 0$

② $\dfrac{5 \pm \sqrt{25 - 4(12)(-2)}}{2(12)}$

$m_{1,2} = \dfrac{5 \pm \sqrt{121}}{24} = \dfrac{5 \pm 11}{24}$

$m_1 = \dfrac{16}{24} \quad m_2 = \dfrac{-6}{24}$

③ $y_1 = e^{(16/24x)} \quad y_2 = e^{-6/24x}$

④ $y_c = C_1 e^{\frac{16}{24}x} + C_2 e^{-\frac{6}{24}x}$

$\ast$ $12m^2 - 5m - 2 = 0$

$(-24) = (-8)(3)$

$12m^2 + 3m - 8m - 2 = 0$

$3m(4m+1) - 2(4m+1) = 0$

$(4m+1)(3m-2) = 0$

$m_1 = -\frac{1}{4} \quad m_2 = \frac{2}{3}$

$\ast$ $y'' - 2y' + 5y = 0$

① $m^2 - 2m + 5 = 0$

② $m_{1,2} = \dfrac{2 \pm \sqrt{4 - 4(1)(5)}}{2(1)} = \dfrac{2 \pm \sqrt{-16}}{2}$

$= 1 \pm 2i \quad \alpha = 1 \quad \beta = 2$

③ $y_1 = e^x \cos 2x \quad y_2 = e^x \operatorname{sen} 2x$

④ $y_c = C_1 e^x \cos 2x + C_2 e^x \operatorname{sen} 2x$

$\ast$ $y'' + y = 0$

- $m^2 + 1 = 0$
- $m_{1,2} = \pm\sqrt{-1} = \pm i \quad \alpha = 0 \quad \beta = 1$
- $y_1 = \cos x \quad y_2 = \operatorname{sen} x$
- $y_c = C_1 \cos x + C_2 \operatorname{sen} x$

$\ast$ $y''' - 3y'' + 9y' + 13y = 0$

- $m^3 - 3m^2 + 9m + 13 = 0 \qquad \{\pm 1; \pm 13\}$

$$\begin{array}{r|rrrr} & 1 & -3 & 9 & 13 \\ -1 & & -1 & 4 & -13 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$

$m^2 - 4m + 13 = 0$

$m_{2,3} = \dfrac{4 \pm \sqrt{16 - 4(1)(13)}}{2(1)} = 2 \pm 3i \quad \alpha = 2 \ \beta = 3$

- $y_1 = e^{2x} \cos 3x \quad y_2 = e^{2x} \operatorname{sen} 3x \quad y_3 = e^{-x}$
- $y_c = C_1 e^{2x} \cos 3x + C_2 e^{2x} \operatorname{sen} 3x + C_3 e^{-x}$

$\ast$ $y^{(iv)} + 8y'' + 16y = 0$

- $m^4 + 8m^2 + 16m = 0$
- $(m^2 + 4)(m^2 + 4) = 0$

$(m = \pm\sqrt{-4} = \pm 2i \quad \alpha = 0 \quad \beta = 2$

- $y_1 = \cos 2x \quad y_2 = \operatorname{sen} 2x \quad y_3 = x\cos 2x \quad y_4 = x \operatorname{sen} 2x$
- $y_c = C_1 \cos 2x + C_2 \operatorname{sen} 2x + C_3 x \cos 2x + C_4 x \operatorname{sen} 2x$

$\ast$Condiciones iniciales:

$y'' + 6y' + 5y = 0 \quad y(0) = 0; \quad y'(0) = 3$

- $m^2 + 6m + 5)$
- $(m+5)(m+1) \quad m_1 = -5 \quad m_2 = -1$
- $y_1 = e^{-5x} \quad y_2 = e^{-x}$
- $y_c = C_1 e^{-5x} + C_2 e^{-x}$

$y'_c = -5C_1 e^{-5x} - C_2 e^{-x}$

$0 = C_1 + C_2$

$3 = -5C_1 - C_2$

$C_2 = -5C_1 - 3$

$0 = C_1 - 5C_1 - 3 \qquad C_1 = -\frac{3}{4}$

$C_2 = -5(-\frac{3}{4}) - 3 = \frac{3}{4}$

$\therefore y_c = \frac{3}{4} e^{-x} - \frac{3}{4} e^{-5x}$

$y^{(vi)} - 6y^{(v)} + 12y^{(iv)} - 6y''' - 9y'' + 12y' - 4y = 0$

① Ecuacion Característica:

$m^6 - 6m^5 + 12m^4 - 6m^3 - 9m^2 + 12m - 4 = 0$

$a_0 = -4 \ ; \ \{\pm 1; \pm 2; \pm 4\}$

$a_n = 1 \ ; \ \{\pm 1\}$

$\frac{a_0}{a_n} = \{\pm 1; \pm 2 \pm; 4\}$

	1	-6	12	-6	-9	12	-4		
1		1	-5	7	1	-8	4	$m_1 = 1$	$(m-1)$
	1	-5	7	1	-8	4	0		
1		1	-4	3	4	-4		$m_2 = 1$	$(m-1)$
	1	-4	3	4	-4	0			
1		1	-3	0	4			$m_3 = 1$	$(m-1)$
	1	-3	0	4	0				
-1		-1	4	-4				$m_4 = -1$	$(m+1)$
	1	-4	4	0					
2		2	-4					$m_5 = 2$	$(m-2)$
	1	-2	0						
2		2						$m_6 = 2$	$(m-2)$
	1	0							

$\rightarrow (m-1)^3 (m+1)(m-2)^2$

③ Conjunto Sols.

$y_1 = e^x ; y_2 = xe^x ; y_3 = x^2 e^x ; y_4 = e^{-x} ; y_5 = e^{2x} ; y_6 = xe^{2x}$

④ Sol. Gral.

$y_c = C_1 e^x + C_2 x e^x + C_3 x^2 e^x + C_4 e^{-x} + C_5 e^{2x} + C_6 x e^{2x}$

$y''' = 0$

① Ec. carac. $\rightarrow m^3 = 0 \quad y''' = 0$

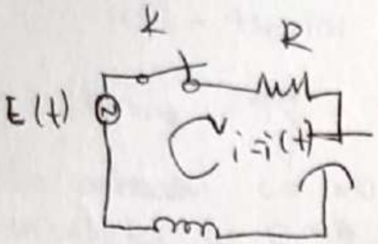
② Raices $\rightarrow m_1 = m_2 = m_3 = 0$

③ Sols. l.i. $\rightarrow y_1 = e^{0x} = 1 ; y_2 = xe^{0x} ; y_3 x^2 e^{0x}$

④ Solución gral $\rightarrow y_c = C_1 + C_2 x + C_3 x^2$

APLICACIONES CON EVALUACIONES DIF. LINEALES DE SEGUNDO ORDEN.

A CTOS ELECTRICOS EN SERIE RCL.



ELEMENTO	VOLTAJE
L INDUCTOR	$V_L = L\frac{di}{dt}$
R resistor	$V_R = iR$
c CAPACITOR	$V_C = \frac{Q}{C}$

Por la ley de Kirchhoff de los voltajes

$$V_L + V_R + V_C = E(t)$$

Sust.

$$L\frac{di}{dt} + iR + \frac{Q}{C} = E(t)$$

pero

$$i = \frac{dQ}{dt}$$

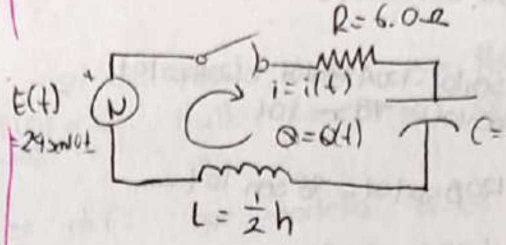
$$= L\frac{d}{dt}\left(\frac{dQ}{dt}\right) + R\left(\frac{dQ}{dt}\right) + \frac{Q}{C} = E(t)$$

$$\boxed{\therefore L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E(t)}$$

MODELO p/cto elec en serie RCL.

Un inductor $C = 0.5$ h es conectado en serie con una resistencia $R = 6.0\,\Omega$, un condensador $C = 0.02$ f y una fuente de voltaje ~~para~~ dada por $E(t) = 24\operatorname{sen}10t$

Si $Q(0) = 0$ y $i(0) = 0$, hallar $Q(t)$ e $i(t)$ $\forall\, t \geq 0$



La ec. dif que modela a un cto electrico en serie rcl está dado por:

$$LQ'' + RQ' + \frac{Q}{C} = E(t) \quad \ldots (1)$$

sust. datos en (1)

$$\frac{1}{2}Q'' + 6Q' + \frac{Q}{0.02} = 24\operatorname{sen}10t$$

simplificando

mult. por 2

$$Q'' + 12Q' + 100Q = 48\operatorname{sen}10t$$

- La ec homo. de (2)

$$Q'' + 2Q' + 100Q = 0 \quad \ldots (3)$$

(1) Ec. carac.

$$m^2 + 2m + 100 = 0$$

(2) Raíces

$$m_{1,2} = \frac{-12 \pm \sqrt{144 - 4(1)(100)}}{2(1)}$$

$$m_{1,2} = \frac{-12 \pm 16i}{2}$$

$$\therefore m_{1,2} = -6 \pm 8i; \quad \alpha = 6; \quad \beta = 8$$

(3) Sol. l.i:

$$Q_1 = e^{-6t}\cos 8t$$

$$Q_2 = e^{-6t}\operatorname{sen}8t$$

(4) Sol gral. de (3)

$$Q_h = C_1e^{-6t}\cos 8t + C_2e^{-6t}\operatorname{sen}8t$$

$F(t) = 48 \operatorname{sen} 10t$

$Q_p = A\cos 10t + B\operatorname{sen} 10t$

$Q'_p = -10A\operatorname{sen} 10t + 10B\cos 10t$

$Q''_p = -100A\cos 10t - 100B\operatorname{sen} 10t$

Sust. Q_p y sus derivadas en (2)

$-100A\cos 10t - 100B\operatorname{sen} 10t - 120A\operatorname{sen} 10t + 120B\cos 10t$
$+100A\cos 10t + 100B\operatorname{sen} 10t = 48\operatorname{sen} 10t$

$-120A\operatorname{sen} 10t + 120B\cos 10t = 48\operatorname{sen} 10t$

Cond. de ident.

$-120A = 48$

$120B = 0 \therefore B = 0$

$A = -\frac{48}{120} = -\frac{2}{5}$

$\therefore \boxed{Q_p = -\frac{2}{5}\cos 10t}$

sol. part. de (2)

$\therefore Q = Q_c + Q_p = e^{-6t}[C_1\cos 8t + C_2\operatorname{sen} 8t] - \frac{2}{5}\cos 10t$

Sol. gral de (2)

$\Rightarrow Q = e^{-6t}[C_1\cos 8t + C_2\operatorname{sen} 8t] - \frac{2}{5}\cos 10t \quad \triangleleft Q(0) = 0$

$i = Q' = e^{-6t}[-6C_1\cos 8t - 6C_2\operatorname{sen} 8t] - 8C_1\operatorname{sen} 8t + 8C_2\cos 8t] + 4\operatorname{sen} 10t$

- $i(0) = Q'(0) = 0$

- $e^{-6(0)}[C_1\cos 8(0) + C_2\operatorname{sen} 8(0)] - \frac{2}{5}\cos 10(0)$

$C_1 - \frac{2}{5} = 0 \qquad \boxed{C_1 = \frac{2}{5}}$

- $0 = e^{-6(0)}[-6C_1\cos 8(0) - 6C_2\operatorname{sen} 8(0) - 8C_1\operatorname{sen} 8(0)] + 8C_2\cos 8(0)] + 4\operatorname{sen} 10(0)$

$= -6C_1 + 8C_2 = 0$

$\boxed{C_2 = \frac{3}{10}}$

$\therefore Q(t) = e^{-6t}\left[\frac{2}{5}\cos 8t + \frac{3}{10}\operatorname{sen} 8t\right] - \frac{2}{5}\cos 10t, \; \forall t \geq 0$

Si $t >> 1$ ó $t \longrightarrow +\infty$

$Q(t) = -\frac{2}{5}\cos 10t \qquad \forall t \to +\infty$ Sol. de estado estacionario

$i(t) = e^{-6t}\left[-\frac{12}{5}\cos 8t - \frac{9}{5}\operatorname{sen} 8t - \frac{16}{5}\operatorname{sen} 8t + \frac{12}{5}\cos 8t\right] + 4\operatorname{sen} 10t$

$\therefore\ i(t) = -5e^{-6t}\operatorname{sen} 8t + 4\operatorname{sen} 10t \ ; \ \forall t \geq 0$

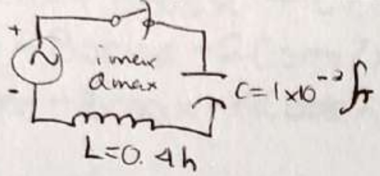
Si $t >> 1$ ó $t \longrightarrow +\infty$

$i(t) = 4\operatorname{sen} 10t \quad \forall t \longrightarrow \infty$

Murray 249 Sec. A $\longrightarrow$ 4 proble.

Un condensador $C = 1\times10^{-3} f$ se conecta en serie con una fuente dada por $E(t) = 20V$ y un inductor $L = 0.4h$. Si $Q(0) = 0$ e $i(0) = 0$. Hallar la carga Q y la corriente electrica i pero maximas.

$\hookrightarrow Q'$



La ec dif. que modela a un cto. electrico en serie LC esta dada por:

~~La ec~~

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = E(t) \ldots (1)$$

sust. datos en (1)

$$(0.4)\,Q'' + \frac{Q}{1\times10^{-3}} = 20$$

Mult. po $\frac{1}{0.4}$

$$Q'' + 2500\,Q = 50 \ldots (2)$$

Homo de (2)

$$Q'' + 2500\,Q = 0 \ldots (3)$$

1) Ec. carac

$$m^2 + 2500 = 0$$

Raices

$$m_{1,2} = \pm\sqrt{-2500} = \pm 50i$$

$\alpha = 0 \quad \beta = 50$

3) Sols. li

$Q_1 = \cos 50t \quad Q_2 = \operatorname{sen} 50t$

4) Sol gral de (3)

$$Q_c = C_1\cos 50t + C_2\operatorname{sen} 50t$$

$F(t) = 50 \rightarrow Q_p = A$

$Q_p' = Q_p'' = 0$

Sust. Q_p y sus deri. en (2) $\Rightarrow$

$$0 + 2500A = 50$$

$$\Rightarrow A = \frac{50}{2500} = \frac{1}{50}$$

$\therefore Q_p = \frac{1}{50}$ Sol. Part. de (2)

$\therefore Q = Q_c + Q_p\ C_1\cos 50t + C_2\operatorname{sen} 50t + \frac{1}{50} \rightarrow$ Sol gral de (2)

$$i = Q' = -50C_1\operatorname{sen} 50t + 50C_2\cos 50t$$

$Q(0) = 0 \qquad i(0) = Q'(0) = 0$

Aplicando condiciones

$$0 = C_1\cos 50(0) + C_2\operatorname{sen}(50(0)) + \frac{1}{50}$$

$$C_1 + \frac{1}{50} = 0 \qquad \therefore\ \boxed{C_1 = -\frac{1}{50}}$$

$$0 = -50C_1\operatorname{sen} 50(0) + 50C_2\cos 50(0) =$$

$$50C_2 = 0 \qquad \therefore\ C_2 = 0$$

$\therefore Q(t) = -\frac{1}{50}\cos 50t + \frac{1}{50} \ ; \ \forall t \geq 0$

$\therefore\ i(t) = \operatorname{sen} 50t \ ; \ \forall t \geq 0$

Q es maxima $\overset{\text{sii}}{\longleftrightarrow}$ $\cos 50t = -1$

$$\Rightarrow Q_{max} = \left(-\frac{1}{50}\right)(-1) + \frac{1}{50} = \frac{2}{50} = \frac{1}{25} = 0.04C$$

i es maxima sii el $\operatorname{sen} 50t = 1$

$\Rightarrow i_{max} = 1$ amp.

$y'' + 2y' + y = \operatorname{sen} x + 3\cos 2x$

$\cdot\ y'' + 2y' + y = 0$

$\cdot\ m^2 + 2m + 1 = 0$

$(m+1)(m+1) = 0$

$m_{1,2} = -1$

$y_1 = e^{-x} \qquad y_2 = xe^{-x}$

$y_c = C_1 e^{-x} + C_2 x e^{-x}$

$F(x) = \operatorname{sen} x + 3\cos 2x$

$y_p = A\operatorname{sen} x + B\cos x + C\cos 2x + D\operatorname{sen} 2x$

$y'_p = A\cos x - B\operatorname{sen} x - 2C\operatorname{sen} 2x + 2D\cos 2x$

$y''_p = -A\operatorname{sen} x - B\cos x - 4C\cos 2x - 4D\operatorname{sen} 2x$

$$\cancel{-A\operatorname{sen} x} - \cancel{B\cos x} - \underline{4C\cos 2x} - \underline{4D\operatorname{sen} 2x} + 2A\cos x - 2B\operatorname{sen} x - \underline{4C\operatorname{sen} 2x} + \underline{4D\cos 2x}$$
$$+ \cancel{A\operatorname{sen} x} + \cancel{B\cos x} + \underline{C\cos 2x} + \underline{D\operatorname{sen} 2x} = F(x)$$

$$(-4C + 4D + C)\cos 2x + (-4D - 4C + D)\operatorname{sen} 2x + 2A\cos x - 2B\operatorname{sen} x$$
$$= \operatorname{sen} x + 3\cos 2x$$

$-3C + 4D = 3$

$-3D + 4C = 0$

$2A = 0 \longrightarrow A = 0$

$-2B = 1 \longrightarrow B = -\frac{1}{2}$

$4D = \frac{3 + 3C}{4}$

$\frac{-9 - 9C}{4} + 4C = 0$

$-9 - 9C + 16C = 0$

$-9 + 7C = 1$

$C = \frac{9}{7}$

$y'' + 9y = 3\cos 3x$

$y'' + 9y = 0$

$m^2 + 9 = 0$

$m_{1,2} = \pm 3i$

$y_1 = \cos 3x \quad y_2 = \operatorname{sen} 3x$

$y_c = C_1 \cos 3x + C_2 \operatorname{sen} 3x$

$f(x) = (3\cos 3x)x$

$y_p = (A\cos 3x + B\operatorname{sen} 3x)\, x$

$= Ax\cos 3x + Bx\operatorname{sen} 3x$

$y'_p = A\cos 3x - 3Ax\operatorname{sen} 3x + B\operatorname{sen} 3x + 3Bx\operatorname{sen} 3x$

$y''_p = \underbrace{-3A\operatorname{sen} 3x - 3A\operatorname{sen} 3x}_{} - 9Ax\cos 3x + 3B\cos 3x + 3B\operatorname{sen} 3x + 9Bx\cos 3x$

$= -6A\operatorname{sen} 3x$

$-6A\operatorname{sen} 3x \cancel{-9Ax\cos 3x} + 3B\cos 3x + 3B\operatorname{sen} 3x \cancel{+9Bx\cos 3x} + \cancel{9Ax\cos 3x} + \cancel{9Bx\operatorname{sen} 3x} = f(x)$

$-6A\operatorname{sen} 3x + 3B\cos 3x + 3B\operatorname{sen} 3x$

MÉTODO DE VARIACIÓN DE PARÁMETROS PARA LA OBTENCIÓN DE LA SOLUCIÓN PARTICULAR y_p DE LA ECUACIÓN DIF. LINEAL NO HOMOGÉNEA DE ORDEN N.

Este método consiste en proponer como solución particular y_p de la ec. dif. no homogenea de orden n, a la combinación lineal:

$$y_p = U_1(x)y_1(x) + U_2(x)y_2(x) + \dots + U_n(x)y_n(x)$$

en donde $y_1, y_2, \dots, y_n$ son el conjunto de n soluciones li de la respectiva ec. dif. lineal homogenea de orden n y las funciones $U_1, U_2, \dots, U_n$ son parametros variables de identidad.

NOTA: A diferencia del método de coeficientes indeterminados, este método es mas general pues el requisito que se exige a la función f(x) en el segundo miembro de la ec. dif. es que esta sea una función integrante

Adaptación del método a la ec. dif. lineal de primer orden.

Considere la ec. dif. lineal de primer orden:

$$\frac{dy}{dx} + P(x) = Q(x) \quad \dots (1)$$

Suponga que:

$$y_p = U_1 y_1 \quad \dots (2)$$

Es sol. part. de (1), en donde $y_1 \neq 0$ es sol. de la ec. dif. homogenea de (1).

$$\frac{dy}{dx} + P(x)y = 0 \quad \dots (3)$$

y U_1 es un parametro desconocido por calcular. Derivando (2)

$$y_p' = U_1 y_1' + U_1' y_1$$

Sust. y_p y y'_p en (1)

Resulta

$$U_1 y_1' + U_1' y_1 + P(x) U_1 y_1 = Q(x)$$

$$U_1[y_1' + P(x)y_1] + U_1' y_1 = Q(x)$$

Pues $y_1 \neq 0$ es sol. de (3)

$$U_1' y_1 = Q(x)$$

$$\frac{dU_1}{dx} = U_1' = \frac{Q(x)}{y_1(x)}$$

Integrando ambos miembros de la ec. anterior, con respecto de x, resulta:

$$\int \frac{dU_1}{dx}\,dx = \int \frac{Q(x)}{y_1(x)}\,dx$$

$$\boxed{U_1(x) = \int \frac{Q(x)}{y_1(x)}\,dx}$$

Sust. (4) en (3) se tiene por tanto que

$$y_p = y_1(x) \int \frac{Q(x)}{y_1(x)}\,dx$$

es sol. part. de (1).

Resolver

$$\frac{dI}{dt} + 2I = 10e^{-2t} \; ; I(0) = 0 \quad \ldots (1)$$

La ec. homo de (1)

$$\frac{dI}{dt} + 2I = 0 \quad \ldots (2)$$

La ec. carac. de (2)

$m + 2 = 0$

② Raiz $m_1 = -2$

③ $I_1 = e^{-2t}$

④ $y_c = C_1 e^{-2t}$

Proponemos

$I_p = U_1 I_1$

Donde $U_1 \int \frac{Q(t)}{I_1(t)} dt$

$$I_p = I_1 \int \frac{Q(t)}{I_1(t)} dt$$

$$= e^{-2t} \int \frac{10\cancel{e^{-2t}}}{\cancel{e^{-2t}}} dt$$

$$= e^{-2t} \int 10 dt$$

$= e^{-2t}(10t)$ Sol. part. de (1)

$\therefore I = I_c + I_p = C_1 e^{-2t} + 10te^{-2t}$

Sol. gral de (1)

pero $I(0) = 0$

$0 = C_1 e^{-2(0)} + \cancel{10(0)e^{-2(0)}}$

$C_1 = 0$

$\therefore I(t) = 10te^{-2t} \; ; \forall t > 0$

Considere la ec. dif. lineal de orden dos

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = F(x) \ldots (1)$$

mult. por $\frac{1}{a_2(x)}$

$$\frac{1}{a_2(x)}\left[a_2(x)y'' + a_1(x)y' + a_0(x)y = F(x)\right]$$

$$y'' + \underbrace{\frac{a_1(x)}{a_2(x)}}_{P(x)}y' + \underbrace{\frac{a_0(x)}{a_2(x)}}_{Q(x)}y = \underbrace{\frac{F(x)}{a_2(x)}}_{G(x)}$$

Con lo que la ec. dif (1) se puede expresar en la forma general

$$y'' + P(x)y' + Q(x)y = G(x) \ldots (2)$$

Donde $\boxed{G(x) = \frac{F(x)}{a_2(x)}}$

Propongamos como solución part. y_p de (2) a

$$y_p = U_1 y_1 + U_2 y_2 \ldots (3)$$

en donde y_1, y_2 son soluciones li de ~~y2~~

$$y'' + P(x)y' + Q(x)y = 0 \ldots (3)$$

y U_1, U_2 son parametros variables desconocidos que obtendremos por cond de identidad.

Resolviendo (3)

$$y_p = U_1 y_1 + U_2 y_2$$

$$y'_p = U_1 y'_1 + U'_1 y_1 + U_2 y'_2 + U'_2 y_2$$

$$y''_p = U'_1 y'_1 + U_1 y''_1 + U''_1 y_1 + U'_1 y'_1 + U'_2 y'_2 + U_2 y''_2 + U''_2 y_2 + U'_2 y'_2$$

TRABAJO DERECHO A EXAMEN
(TERCER PARCIAL)

- Caso Especial de la ec. dif. lineal de orden n con coef. variables.

Ec. dif. de Euler-Cauchy

Una ec. dif. que se expresa en la forma general

$$a_n x^n \frac{d^n y}{dx^n} + a_{n-1}x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1 x\frac{dy}{dx} + a_0 y = F(x) \quad \ldots (1)$$

Donde $a_n, a_{n-1}, \ldots a_1, a_0$ con $a_n \neq 0$ son ctes. se llama "Ec. dif de Cauchy-Euler" ó "Ec. dif. de Euler-Cauchy" ó "Ec. dif. de Euler" ó "Ec dif de Cauchy" ó bien "Ec. equidimensional.

La ec. dif (1) se puede reducir a una ec. dif. con coef. constantes a partir de la sig. transformación

$$\boxed{x = e^z} \quad \ldots (2)$$

Ejemplo:

Resolver

$$x^2y'' + xy' + 4y = 1 \quad \ldots (1)$$

(z, y) $\quad \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} \to e^{-z}$

$$\boxed{z = \ln|x| \Rightarrow \frac{dz}{dx} = \frac{1}{x} = \frac{1}{e^z} = e^{-z} \quad \therefore \frac{dz}{dx} = e^{-z}}$$

$$\therefore \frac{dy}{dx} = e^{-z} \cdot \frac{dy}{dz}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left[e^{-z} \cdot \frac{dy}{dz}\right]$$

$$= \frac{d}{dx}\left[e^{-z}\frac{dy}{dz}\right] \cdot \frac{dz}{dx} \to e^{-z}$$

$$= e^{-z} \cdot \frac{d}{dz}\left[e^{-z} \cdot \frac{dy}{dz}\right]$$

Jueves 7 Nov.

$$= e^{-z} \cdot \left[e^{-z} \cdot \frac{d^2y}{dz} - e^{-z}\frac{dy}{dz}\right]$$

$$\boxed{\frac{d^2y}{dz^2} = e^{-2z}\left[\frac{d^2y}{dz} - \frac{dy}{dz}\right]}$$

Haciendo los cambios en (1)

$$\cancel{e^{2z}} \cdot \cancel{e^{-2z}}\left[\frac{d^2y}{dz^2} - \frac{dy}{dz}\right] + \cancel{e^{z}} \cdot e^{-z}\frac{dy}{dz} + 4y = 1$$

$$\frac{d^2y}{dz^2} - \cancel{\frac{dy}{dz}} + \cancel{\frac{dy}{dz}} + 4y = 1$$

$$\frac{d^2y}{dz} + 4y = 1 \quad \ldots (2)$$

- Ec. homo

$$y'' + 4y = 0 \quad \ldots (3)$$

- Ec. carac.

$$m^2 + 4 = 0$$

- Raices

$$m_{1,2} = \pm\sqrt{4} = \pm 2i$$

- Sols. li

$$y_1 = \cos 2z \qquad y_2 = \operatorname{sen} 2z$$

- Sol gral de 3

$$y_c = C_1\cos 2z + C_2 \operatorname{sen} 2z$$

$F(z) = A$

$y_p = 1 \qquad y_p' = y''_p = 0$

Sust. en (2)

$4A = 1 \qquad A = 1/4$

$y_p = 1/4 \qquad y = y_c + y_p = C_1\cos 2z + C_2\operatorname{sen} 2z + 1/4$

$$y = C_1\cos 2\ln|x| + C_2 \operatorname{sen} 2\ln|x| + 1/4$$

TRABAJO:

Desarrollar y''' y $y^{(iv)}$ en terminos de (x, y)

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$$

- Resolver

a) $x^3y''' + xy' - y = x\ln x$

b) $x^4y^{(iv)} + 6x^3y''' + 7x^2y'' + xy' - y = 1$

Jueves 7 Nov.

UNIDAD IV. LA TRANSFORMADA DE LAPLACE

La transformada de Laplace es un método matemático "Alternativo" a los metodos convencionales utilizados para resolver problemas de valor inicial (i.e. una ec. dif. de condiciones iniciales).

- Ventajas del método de Laplace VS metodos convencionales.

- Reduce la tarea de encontrar la sol. de un problema de valor inicial (un problema de calculo) a un problema 100% algebraico
- Proporciona la sol. de un problema de valor inicial en forma directa.

$\phi(D)y = F(x)$ P.V.I

$\phi(D)y = 0$

ec. carac., raices, soluciones

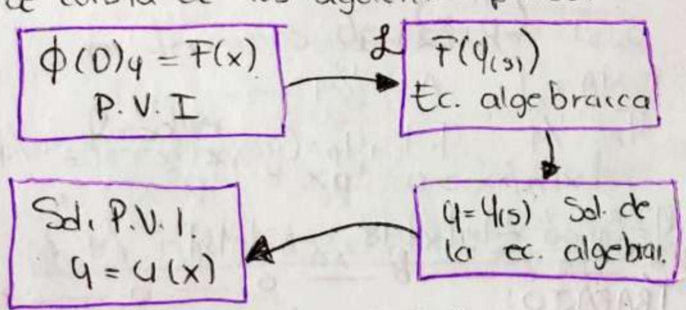
$y_c = C_1y_1 + C_2y_2 + \dots + C_ny_n$

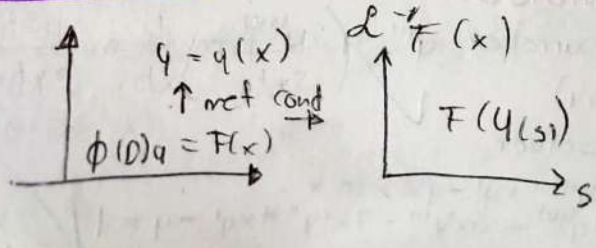
y_p — Coef. ind / Var. para.

$y = y_c + y_p = C_1y_1 + C_2y_2 + \dots + C_ny_n + y_p$

$y = y(t)$ - Sol. P.V.I.

Esquematicamente, el método de Laplace consta de los siguientes pasos:





La transformada de Laplace.

Definición: Sea f una función definida para $t \geq 0$. Entonces la transformada de laplace de $f(t)$ se define como:

$$\boxed{\mathcal{L}\{f(x)\} = \int_0^{+\infty} e^{-st} f(t) = dt = F(s)}$$

donde s es un parámetro real.

$$\boxed{\operatorname{sen}\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} ; \quad \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}}$$

El simbolo $\mathcal{L}$ se llama operador lineal de la transformada de laplace

NOTA: Para evaluar correctamente la integral en (1) es necesario introducir la sig. definición:

Integral impropio, definición:

Una integral con un limite infinito se llama integral impropia y se define de la sig. forma:

$$\int_a^{+\infty} f(x)\,dx = \lim_{K \to +\infty} \int_a^K f(x)\,dx$$

Dado lo anterior, la integral en (1), toma la forma:

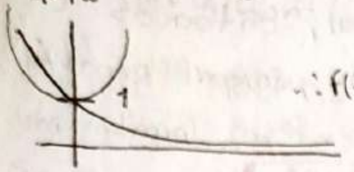
$$\mathcal{L}\{f(t)\} \int_0^{+\infty} e^{-st} f(t)dt = \lim_{K\to+\infty} \int_0^K e^{-st} f(t)dt = F(s)$$

y decimos que la integral converge sii el limite existe. En tal caso decimos que la transformada de Laplace de $f(x)$ es la Función $F(s)$.

En otros palabras, si la integral converge, entonces la transformada de Laplace de $f(t)$ existe y $f(s)$, en tal caso decimos que la función $f(t)$ es de orden exponencial.

Si $\lim_{t\to+\infty} e^{-ct} f(t) =$ existe, entonces $f(t)$ es de orden exponencial $f(t)=e^{t^2}$

$$\lim_{t\to+\infty} e^{-ct}\cdot e^{t^2}$$



$\therefore f(t)=e^{t^2}$ NO ES DE ORDEN EXPONENCIAL.

NOTACIÓN: En la descripción gral. emplearemos letras minúsculas para denotar la función que se desea transformar a la correspondiente mayúscula para denotar su transformada de Laplace, esto es:

$$\mathcal{L}\{f(t)\}=F(s);\ \mathcal{L}\{g(t)\}=G(s)$$
$$\mathcal{L}\{q(t)\}=Q(s);\ \mathcal{L}\{i(t)\}=I(s)$$

Determinar la transformada de Laplace de la sig. función

$f(t)=1$

$$\mathcal{L}\{f(t)\}=\int_0^{+\infty} e^{-st}(1)\,dt=$$
$$\mathcal{L}\{1\}=\int_0^{+\infty}e^{-st}(1)\,dt=\lim_{k\to+\infty}\int_0^k e^{-st}\,dt$$
$$=\lim_{k\to+\infty}\left[-\frac{e^{-st}}{s}\right]_0^k=\lim_{k\to+\infty}\left[\cancel{-\frac{e^{-sk}}{s}}^{\to 0}+\frac1s\right]=\frac1s$$

$$\boxed{\therefore \mathcal{L}\{1\}=\frac1s;\ s>0}$$

$$\mathcal{L}\{f(t)\}=\int_0^{+\infty}e^{-st}f(t)\,dt$$

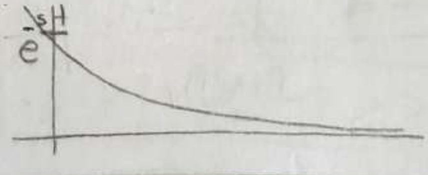
Determine la transformada de Laplace de las sig. funciones.

$f(t)=t$

$$\mathcal{L}\{t\}=\int_0^{+\infty}e^{-st}t\,dt=\lim_{k\to+\infty}\int_0^k e^{-st}t\,dt$$
$$=\lim_{k\to+\infty}\left[-\frac{te^{-st}}{s}\Big|_0^k+\frac1s\int_0^k e^{-st}\,dt\right]=\lim_{k\to+\infty}\left[\frac{-te^{-st}}{s}-\frac{e^{-st}}{s^2}\right]_0^k$$

$u=t;\ dv=-\frac{e^{-st}}{s}$ $\quad du=dt;\ v=\frac{-e^{-st}}{s}$

$$=\lim_{k\to+\infty}\left[-\frac{ke^{-sk}}{s}-\frac{e^{-sk}}{s^2}+\frac{1}{s^2}\right]=\frac1{s^2}$$



$$\boxed{\therefore \mathcal{L}\{t\}=\frac{1}{s^2};\ s>0}$$

$f(t)=e^{at}$

$$\mathcal{L}\{e^{at}\}=\int_0^{+\infty}e^{-st}\cdot e^{at}\,dt=\lim_{k\to+\infty}\int_0^k e^{-(s-a)t}\,dt$$
$$=\lim_{k\to+\infty}\left[\frac{e^{-(s-a)t}}{s-a}\right]_0^k$$
$$\mathcal{L}\{e^{at}\}=\lim_{k\to+\infty}\left[-\frac{e^{-(s-a)k}}{s-a}+\frac1{s-a}\right]=\frac1{s-a}$$

$$\boxed{\mathcal{L}\{e^{at}\}=\frac1{s-a};\ s-a>0 \text{ ó } s>a}$$

$f(t)=\text{sen } at$

$$\mathcal{L}\{\text{sen } at\}=\int_0^{+\infty}e^{-st}\,\text{sen }at\,dt=\lim_{k\to+\infty}\int_0^k e^{-st}\,\text{sen }at\,dt$$
$$=\lim_{k\to+\infty}\left[\frac{-e^{-st}\,\text{sen }at}{s}\Big|_0^k+\frac{a}{s}\int_0^k e^{-st}\cos at\,dt\right]$$

$u=\text{sen }at;\ dv=e^{-st}dt$

$du=a\cos at\,dt;\ v=\frac{-e^{-st}}{s}$

$w=\cos at;\ dz=e^{-st}dt$

$dw=-a\,\text{sen }at\,dt;\ v=\frac{-e^{-st}}{s}$

$$\mathcal{L}\{\text{sen }at\}=\lim_{k\to+\infty}\left[\frac{-e^{-st}\,\text{sen }at}{s}\Big|_0^k-\frac{ae^{-st}\cos at}{s^2}\right]_0^k$$
$$=\lim_{k\to+\infty}\left[-\frac{e^{-st}\,\text{sen }at}{s}-\frac{ae^{-st}\cos at}{s^2}\right]_0^k-\frac{a^2}{s^2}\int_0^{+\infty}e^{-st}\,\text{sen }at\,dt$$
$$\left[1+\frac{a^2}{s^2}\right]\mathcal{L}\{\text{sen }at\}=\lim_{k\to+\infty}\left[\cancel{-\frac{e^{-sk}\,\text{sen }ak}{s}}-\frac{ae^{-sk}\cos ak}{s^2}+\frac{a}{s^2}\right]$$
$$\left[1+\frac{a^2}{s^2}\right]\mathcal{L}\{\text{sen }at\}=\frac{a}{s^2}$$
$$\left[\frac{s^2+a^2}{s^2}\right]\mathcal{L}\{\text{sen }at\}=\frac{a}{s^2}$$

$$\boxed{\therefore\mathcal{L}\{\text{sen }at\}=\frac{a}{s^2+a^2}}$$

ó $\mathcal{L}\left\{\frac{\text{sen }at}{a}\right\}=\frac{1}{s^2+a^2}$

$f(t)=\cosh at$

$$\mathcal{L}\{\cosh at\}=\int_0^{+\infty}e^{-st}\cosh at\,dt$$
$$\cosh at=\frac{e^{at}+e^{-at}}{2}$$
$$\mathcal{L}\{\cosh at\}=\lim_{k\to+\infty}\int_0^k e^{-st}\cdot\left[\frac{e^{at}+e^{-at}}{2}\right]dt$$
$$=\frac12\lim_{k\to+\infty}\left[\int_0^k e^{-(s-a)t}\,dt+\int_0^k e^{-(s+a)t}\,dt\right]$$
$$=\frac12\lim_{k\to+\infty}\left[-\frac{e^{-(s-a)t}}{s-a}-\frac{e^{-(s+a)t}}{s+a}\right]_0^k$$
$$=\frac12\lim_{k\to+\infty}\left[-\cancel{\frac{e^{-(s-a)k}}{s-a}}^{\to0}-\cancel{\frac{e^{-(s+a)k}}{s+a}}^{\to0}+\frac1{s-a}+\frac1{s+a}\right]$$
$$=\frac12\left[\frac1{s-a}+\frac1{s+a}\right]=\frac12\left[\frac{2s}{s^2-a^2}\right]=\frac{s}{s^2-a^2}$$

$$\boxed{\therefore \mathcal{L}\{\cosh at\} = \frac{s}{s^2 - a^2}}$$

$f(t) = e^{bt} \operatorname{senh} at$

$$\mathcal{L}\{e^{bt} \operatorname{senh} at\} = \int_0^{+\infty} e^{-st}[e^{bt} \operatorname{senh} at]\, dt$$

$$\operatorname{senh} at = \frac{e^{at} - e^{-at}}{2}$$

$$= \lim_{k\to+\infty} \int_0^k e^{-st} \cdot e^{bt}\left[\frac{e^{at} - e^{-at}}{2}\right] dt$$

$$= \frac{1}{2}\lim_{k\to+\infty}\left[\int_0^k e^{-(s-b-a)t}\, dt - \int_0^k e^{-(s-b+a)t}\, dt\right]$$

$$= \frac{1}{2}\lim_{k\to+\infty}\left[-\frac{e^{-(s-b-a)t}}{s-b-a} + \frac{e^{-(s-b+a)t}}{s-b+a}\right]_0^k$$

$$= \frac{1}{2}\lim_{k\to+\infty}\left[-\frac{e^{-(s-b-a)k}}{s-b-a} + \frac{e^{-(s-b-a)k}}{s-b+a} + \frac{1}{s-b-a} - \frac{1}{s-b+a}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(s-b)-a} - \frac{1}{(s-b)+a}\right] = \frac{1}{2}\left[\frac{s-b+a-(s-b-a)}{(s-b)^2 - a^2}\right]$$

$$= \frac{1}{2}\left[\frac{2a}{(s-b)^2 - a^2}\right] = \frac{a}{(s-b)^2 - a^2}$$

$$\therefore \mathcal{L}\left\{\frac{e^{bt}\operatorname{senh} at}{a}\right\} = \frac{1}{(s-b)^2 - a^2} \quad \text{ó}$$

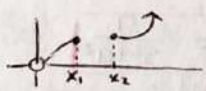
$$\boxed{\mathcal{L}\{e^{bt} \operatorname{senh} at\} = \frac{a}{(s-b)^2 - a^2}}$$

Sea...

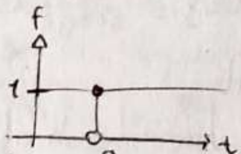
$$f(t) = \begin{cases} 0; & 0 \le t < a \quad \dots (1) \\ 1; & t \ge a \end{cases} \qquad D_f[0, +\infty)$$

Hallar

$\mathcal{L}\{f(x)\}$



$\mathcal{L}\{f(t)\}$



$$\mathcal{L}\{f(t)\} = \int_0^{+\infty} e^{-st} f(t)\, dt = \int_0^{+\infty} e^{-st}(0)\, dt + \int_0^{+\infty} e^{-st}(1)\, dt$$

$$\mathcal{L}\{f(t)\} = \lim_{a\to+\infty}\int_a^k e^{-st}\, dt = \lim_{k\to+\infty}\left[\frac{-e^{-st}}{s}\right]_a^k$$

$$= \lim_{k\to+\infty}\left[\frac{-e^{-sk}}{s} + \frac{e^{-as}}{s}\right] = \frac{e^{-as}}{s}$$

$$\boxed{\therefore \mathcal{L}\{f(t)\} = \frac{e^{-as}}{s}}$$

~~USO DE TABLAS.~~

La función dada en (1) se llama función de Heaviside o función escalón unitario y se denota por $u(t-a)$, entonces

$$u(t-a) = \begin{cases} 0; & 0 \le t < a \\ 1; & t > a \end{cases}$$

$$\boxed{\therefore \mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}}$$

USO DE TABLAS.

Para un manejo adecuado de tablas de transformada de Laplace, es importante notar que el operador $\mathcal{L}$ de Laplace es un operador lineal.

En efecto

Sean $f(t)$ y $g(t)$ funciones de orden exponencial (que la transformada de Laplace exista) y $\alpha, \beta \in \mathbb{R}$, entonces.

$$\mathcal{L}\{\alpha f(t) + \beta g(t)\} = \int_0^{+\infty} e^{-st}[\alpha f(t) + \beta g(t)]\, dt$$

$$= \int_0^{+\infty} e^{-st} \cdot \alpha f(t)\, dt + \int_0^{+\infty} e^{-st} \beta g(t)\, dt$$

$$= \alpha \int_0^{+\infty} e^{-st} f(t)\, dt + \beta \int_0^{+\infty} e^{-st} g(t)\, dt$$

$$= \alpha \mathcal{L}\{f(t)\} + \beta \mathcal{L}\{g(t)\}$$

EVALUAR:

• $\mathcal{L}\{\frac{1}{6}t^3 - 4t^2 - 3\}$

$$= \mathcal{L}\{\tfrac{1}{6}t^3\} - \mathcal{L}\{4t^2\} - \mathcal{L}\{3\}$$

$$= \frac{1}{6}\mathcal{L}\{t^3\} - 4\mathcal{L}\{t^2\} - 3\mathcal{L}\{1\}$$

$$= \frac{1}{6}\frac{3!}{s^{3+1}} - 4\frac{2!}{s^{2+1}} - 3\left(\frac{1}{s}\right)$$

$$= \frac{1}{6}\frac{6}{s^4} - 4\frac{2}{s^3} - \frac{3}{s}$$

$$= \frac{1}{s^4} - \frac{8}{s^3} - \frac{3}{s}$$

• $\mathcal{L}\{2e^{3t} - e^{-1/2t}\}$

$= 2\mathcal{L}\{e^{3t}\} - \mathcal{L}\{e^{-1/2t}\}$

$= 2\frac{1}{s-3} - \frac{1}{s+\frac{1}{2}}$ //

• $\mathcal{L}\{4e^{5t} + 6t^4 - 3\,\text{sen}\sqrt{2}t + 2\cosh\sqrt{3}t\}$

$= 4\mathcal{L}\{e^{5t}\} + 6\mathcal{L}\{t^4\} - 3\mathcal{L}\{\text{sen}\sqrt{2}t\} + 2\mathcal{L}\{\cosh\sqrt{3}t\}$

$= 4\frac{1}{s-5} + 6\frac{4!}{s^{4+1}} - 3\frac{\sqrt{2}}{s^2+2} + 2\frac{s}{s^2-3}$

$= \frac{4}{s-5} + \frac{144}{s^5} - \frac{3\sqrt{2}}{s^2+2} + \frac{2s}{s^2-3}$ //

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}};\quad n = 0, 1, 2, 3, \ldots$$

LA FUNCIÓN GAMMA

Def.

La definición de Euler de la función gamma es la siguiente:

$$\Gamma(n) = \int_0^{+\infty} t^{n-1} e^{-t}\, dt$$

NOTA: Para que la integral converja $n > 0$

Propiedades de la función gamma.

$\Gamma(n+1) = n\Gamma(n) \quad (n \in \mathbb{Q} > 0)$

si $n = \frac{1}{2} \Rightarrow \boxed{\Gamma(\tfrac{1}{2}) = \sqrt{\pi}}$ Probarlo (Apendice)

Demostrar que si $f(t) = t^\alpha$

$\Rightarrow \mathcal{L}\{t^\alpha\} = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}};\ \alpha > -1$

$\mathcal{L}\{t^\alpha\} = \int_0^{+\infty} e^{-st} t^\alpha dt = \lim_{k\to+\infty} \int_0^k e^{-st} t^\alpha dt$

$= \lim_{k\to+\infty}\left[-\frac{t^\alpha e^{-st}}{s}\Big|_0^k + \frac{\alpha}{s}\int_0 e^{-st} t^{\alpha-1} dt\right]$

$= \lim_{k\to+\infty}\left[\frac{-k^\alpha e^{-sk}}{s}\right] + \frac{\alpha}{s}\int_0^{+\infty} e^{-st} t^{\alpha-1} dt$

$u = t^\alpha \quad dv = e^{-st} dt$

$du = \alpha t^{\alpha-1} dt \quad v = -\frac{e^{-st}}{s}$

Sea $w = st$; $dw = s\,dt \quad t = \frac{w}{s} \quad dt = \frac{dw}{s}$

$\mathcal{L}\{t^\alpha\} = \frac{\alpha}{s}\int_0^{+\infty} e^{-w}\left(\frac{w}{s}\right)^{\alpha-1}\frac{dw}{s}$

$= \frac{\alpha}{s^{\alpha+1}}\int_0^{+\infty} e^{-w} w^{\alpha-1} dw = \frac{\alpha\Gamma(\alpha)}{s^{\alpha+1}} = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}};$

$\alpha > -1$

$$\boxed{\therefore\ \mathcal{L}\{t^\alpha\} = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}};\quad \alpha > -1}$$

EVALUE...

• $\mathcal{L}\{t^{-1/2}\} = \frac{\Gamma(-\frac{1}{2}+1)}{s^{-1/2+1}} = \frac{\Gamma(\frac{1}{2})}{s^{1/2}} = \frac{\sqrt{\pi}}{\sqrt{s}} = \sqrt{\frac{\pi}{s}}$

• $\mathcal{L}\{t^{7/2}\} = \frac{\Gamma(\frac{7}{2}+1)}{s^{7/2+1}} = \frac{\frac{7}{2}\Gamma(\frac{7}{2})}{s^{9/2}} = \frac{\frac{7}{2}\Gamma(\frac{5}{2}+1)}{s^{9/2}} = \frac{\frac{7}{2}\cdot\frac{5}{2}\Gamma(\frac{5}{2})}{s^{9/2}}$

$= \frac{\frac{7}{2}\cdot\frac{5}{2}\Gamma(\frac{3}{2}+1)}{s^{9/2}} = \frac{\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\Gamma(\frac{3}{2})}{s^{9/2}}$

$= \frac{\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\Gamma(\frac{1}{2}+1)}{s^{9/2}} = \frac{\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma(\frac{1}{2})}{s^{9/2}}$

$= \frac{105}{16}\sqrt{\frac{\pi}{s^9}}$

» PRIMER TEOREMA DE TRASLACIÓN

Teorema: Si $\mathcal{L}\{f(t)\} = F(s)$, entonces

$\mathcal{L}\{e^{at} f(t)\} = F(s-a);\quad s > a$

$$\boxed{\mathcal{L}\{e^{at}f(t)\} = \mathcal{L}\{f(t)\} = F(s)\Big|_{s\to s-a} = F(s-a)}$$

EVALUE...

• $\mathcal{L}\{e^{5t} t^3\} = \mathcal{L}\{t^3\} = \frac{3!}{s^{3+1}} = \frac{6}{s^4}\Big|_{s\to s-5} = \frac{6}{(s-5)^4}$ //

• $\mathcal{L}\{e^{-2t}\cos 4t\} = \mathcal{L}\{\cos 4t\} = \frac{s}{s^2+16}\Big|_{s\to s-(-2) \hookrightarrow s+2}$

$= \frac{s+2}{(s+2)^2+16}$

Cap. 7.1 Zill Pag. 304, 321 7.3

1-9 (Todos)

1-37 (Impares)

1-43 (Impares)

EVALUE...

$\mathcal{L}\{(e^t + e^{2t})^2 t\} = \mathcal{L}\{(e^{2t} + 2e^{3t} + e^{4t})t\}$

$= \mathcal{L}\{e^{2t}t\} + 2\mathcal{L}\{e^{3t}t\} + \mathcal{L}\{e^{4t}t\}$

$= \mathcal{L}\{t\}\Big|_{s\to s-2} + 2\mathcal{L}\{t\}\Big|_{s\to s-3} + \mathcal{L}\{t\}\Big|_{s\to s-4}$

$= \frac{1}{s^2}\Big|_{s\to s-2} + \frac{2}{s^2}\Big|_{s\to s-3} + \frac{1}{s^2}\Big|_{s\to s-4}$

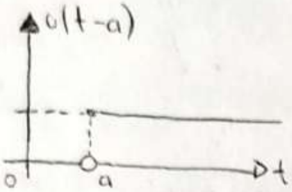
$= \frac{1}{(s-2)^2} + \frac{2}{(s-3)^2} + \frac{1}{(s-4)^4}$ //

Función Escalón Unitario

Definición.

La función escalón unitario o función de Heaviside se denota por $u(t-a)$ y se define como:

$$u(t-a) = \begin{cases} 0; & 0 \le t < a \\ 1; & t \ge a \end{cases} \quad D_f = [0, +\infty) \quad \ldots (1)$$



Grafica

Su transformada de Laplace...

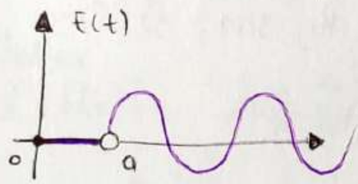
$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

Si mult. (1) por la función $g(t)$

$$g(t)u(t-a) = \begin{cases} 0; & 0 \le t < a \\ g(t); & t \ge a \end{cases}$$

Suponemos que $g(t) = \operatorname{sen} t$

$$\operatorname{sen} t \, u(t-a) = \begin{cases} 0; & 0 \le t < a \\ \operatorname{sen} t; & t \ge a \end{cases}$$



De lo anterior podemos concluir que toda función definida por tramos se puede expresar en terminos de la función escalón unitario.

En efecto; sea.

$$f(t) = \begin{cases} g(t); & 0 \le t < a \\ h(t); & t \ge a \end{cases} = g(t) + \begin{cases} 0; & 0 \le t < a \\ [-g(t) + h(t)]; & t \ge a \end{cases}$$

$$= g(t) + [-g(t) + h(t)] \begin{cases} 0; & 0 \le t < a \\ 1; & t \ge a \end{cases}$$

$$= g(t) + [-g(t) + h(t)]u(t-a)$$

$$\boxed{\therefore f(t) = \begin{cases} g(t); & 0 \le t < a \\ h(t); & t \ge a \end{cases} \\ = g(t) - g(t)u(t-a) + h(t)u(t-a)}$$

Analogamente

$$f(t) = \begin{cases} g(t); & 0 \le t < a \\ h(t); & a \le t < b \\ g(t); & t \ge b \end{cases} \qquad + g(t)u(t-b)$$

$$= g(t) - g(t)u(t-a) + h(t)u(t-a) - h(t)u(t-b) + g(t)u(t-b)$$

Ejemplo:

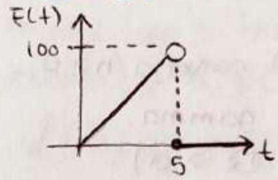
El voltaje de un circuito eléctrico está dado por:

$$E(t) = \begin{cases} 20t; & 0 \le t < 5 \\ 0; & t \ge 5 \end{cases}$$

Grafique y exprese $E(t)$ en terminos de funciones escalón unitario

Solución.

$y = mx + b \leftarrow$ ordenada al origen cuando $b=0$



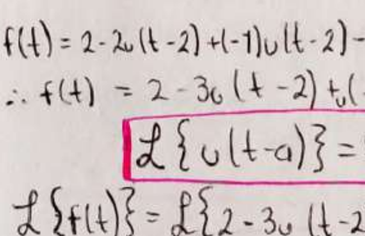
$$E(t) = \begin{cases} 20t; & 0 \le t < 5 \\ 0; & t \ge 5 \end{cases}$$

$g(t) = 20 \quad h(t) = 0 \quad a = 5$

$$E(t) = 20t - 20t\,u(t-5) + (0)\,u(t-5)$$

Ejemplo:

Determine la transformada de Laplace de la función $f(t)$.



$$f(t) = \begin{cases} 2; & 0 \le t < 2 \\ -1; & 2 \le t < 3 \\ 0; & t \ge 3 \end{cases}$$

$$f(t) = 2 - 2u(t-2) + (-1)u(t-2) - (-1)u(t-3) + \overset{0}{\overrightarrow{(0)u(t-3)}}$$

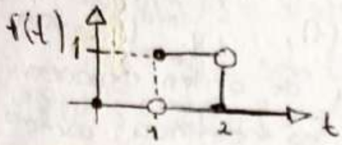
$$\therefore f(t) = 2 - 3u(t-2) + u(t-3)$$

$$\boxed{\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}}$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2 - 3u(t-2) + u(t-3)\}$$

$$= 2\mathcal{L}\{1\} - 3\mathcal{L}\{u(t-2)\} + \mathcal{L}\{u(t-3)\}$$

$$\mathcal{L}\{f(t)\} = \frac{2}{s} - \frac{3e^{-2s}}{s} + \frac{e^{-3s}}{s}$$



$$f(t) = \begin{cases} \cos t; & 0 \le t < \frac{\pi}{2} \\ \operatorname{sen}(t - \frac{\pi}{2}); & t \ge \frac{\pi}{2} \end{cases}$$

$$= \cos t - \cos u(t - \tfrac{\pi}{2}) + \operatorname{sen}(t - \tfrac{\pi}{2}) u(t - \tfrac{\pi}{2})$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\cos t - \cos u(t - \tfrac{\pi}{2}) + \operatorname{sen}(t - \tfrac{\pi}{2}) u(t - \tfrac{\pi}{2})\}$$

$$= \mathcal{L}\{\cos t\} - \mathcal{L}\{\cos u(t - \tfrac{\pi}{2})\} + \mathcal{L}\{\operatorname{sen}(t - \tfrac{\pi}{2}) u(t - \tfrac{\pi}{2})\}$$

» Segundo teorema de traslación

Teorema: Si $\mathcal{L}\{f(t)\} = F(s)$, entonces

$$\mathcal{L}\{f(t-a) u(t-a)\} = e^{-as} F(s); \quad a > 0$$

NOTA: $\mathcal{L}\{f(t-a) u(t-a)\} = e^{-as} \mathcal{L}\{f(t)\} = e^{-as} F(s)$

Para la aplicación de este teorema, la función moltiplicadora $f(t-a)$ debe tener el mismo desplazamiento que la función escalón unitario $u(t-a)$

Ejemplos

- $\mathcal{L}\{(t-2)^3 u(t-2)\} = e^{-2s} \mathcal{L}\{t^3\} = e^{-2s} \frac{3!}{s^{3+1}} = \frac{6e^{-2s}}{s^4}$

$$f(t) = \begin{cases} 0; & 0 \le t < 2 \\ (t-2)^3, & t \ge 2 \end{cases}$$

Procedimiento largo:

$$\mathcal{L}\{f(t)\} = \int_0^2 e^{-st}(0)\,dt + \int_2^\infty e^{-st}(t-2)^3\,dt$$

Ejemplo

- $\mathcal{L}\{\operatorname{sen}(t - \frac{\pi}{2}) u(t - \frac{\pi}{2})\} = e^{-\pi/2 s} \mathcal{L}\{\operatorname{sen} t\}$

$$= e^{-\pi/2 s} \frac{1}{s+1} = \frac{e^{-\pi/2 s}}{s^2+1}$$

$$\mathcal{L}\{e^{at} f(t)\} = \mathcal{L}\{f(t)\} = F(s)\Big|_{s \to s-a} = F(s-a)$$

$$\mathcal{L}\{(t-1)^3 e^{t-1} u(t-1)\} = \mathcal{L}\{e^{t-1}(t-1)^3 u(t-1)\}$$

$$= \mathcal{L}\{e^t \cdot e^{-1} (t-1)^3 u(t-1)\}$$

$$= e^{-1}\mathcal{L}\{e^t (t-1)^3 u(t-1)\} = \mathcal{L}\{(t-1)^3 u(t-1)\}\Big|_{s \to s-1}$$

$$= e^{-1}\left[\frac{6e^{-s}}{s^4}\right]\Big|_{s \to s-1}$$

Donde

$$\mathcal{L}\{(t-1)^3 u(t-1)\} = e^{-s}\mathcal{L}\{t^3\} = \frac{6e^{-s}}{s^4}$$

$$\Rightarrow e^{-1} \cdot \left[\frac{6e^{-(s-1)}}{(s-1)^4}\right] = \frac{6e^{-s}}{(s-1)^4}$$

$$f(t) = \begin{cases} 0; & 0 \le t < 1 \\ (t-1)^3 e^{t-1}; & t \ge 1 \end{cases}$$

» SEGUNDO TEOREMA DE TRASLACIÓN EN SU FORMA ALTERNATIVA

Si la función $g(t)$ carece de la forma $f(t-a)$ desplazada que se requiere en el teorema anterior, entonces.

$$\mathcal{L}\{g(t) u(t-a)\} = e^{-as} \mathcal{L}\{g(t-a)\}$$

$$\mathcal{L}\{g(t) u(t-a)\} = e^{-as} \mathcal{L}\{g(t)\big|_{t \to t+a}\} = e^{-as} \mathcal{L}\{g(t+a)\}$$

Ejemplos:

- $\mathcal{L}\{t\, u(t-2)\} = e^{-2s} \mathcal{L}\{t+2\} = e^{-2s}\left[\frac{1}{s^2} + \frac{2}{s}\right]$
- $\mathcal{L}\{\cos t\, u(t - \frac{\pi}{2})\} = e^{-\pi/2 s} \mathcal{L}\{\cos(t + \frac{\pi}{2})\}$

$f(t) = \cos t$ Pero $\cos(t + \frac{\pi}{2}) =$

$f(t + \frac{\pi}{2}) = \cos(t + \frac{\pi}{2})$ $\quad \cancel{\cos t \cos \frac{\pi}{2}} - \operatorname{sen} t \operatorname{sen} \frac{\pi}{2}$

$= -\operatorname{sen} t$

$$\Rightarrow e^{-\frac{\pi}{2}s} \mathcal{L}\{-\operatorname{sen} t\} = -e^{-\pi/2 s} \mathcal{L}\{\operatorname{sen} t\} = -e^{-\pi/2 s} \frac{1}{s^2+1}$$

$$= -\frac{e^{-\pi/2 s}}{s^2+1}$$

$$f(t) = \begin{cases} 0; & 0 \le t < \frac{\pi}{2} \\ \cos t; & t \ge \pi/2 \end{cases}$$

Con

$$\mathcal{L}\{f(t)\} = \int_0^{\pi/2} e^{-st}(0)\,dt + \int_{\frac{\pi}{2}}^{+\infty} e^{-st} \cos t\, dt$$

$$\boxed{\begin{aligned} &\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s} \\ &\mathcal{L}\{f(t-a) u(t-a)\} = e^{-as} F(s) \\ &\mathcal{L}\{g(t) u(t-a)\} = e^{-as} \mathcal{L}\{g(t+a)\} \end{aligned}}$$

*29 de Nov. Examen S: D:

Derivadas de transformadas.

Teorema: Si $\mathcal{L}\{f(t)\}=F(s)$ y $n=1,2,3,\dots$

$$\boxed{\mathcal{L}\{t^n f(t)\}=(-1)^n \frac{d^n}{ds^n}F(s)}$$

$$\mathcal{L}\{t^n f(t)\}=(-1)^n \frac{d^n}{ds^n}\mathcal{L}\{f(t)\}$$
$$=(-1)^n \frac{d^n}{ds^n}F(s)$$

$$\mathcal{L}\{te^{3t}\}=(-1)^1\frac{d}{ds}\mathcal{L}\{e^{3t}\}=-\frac{d}{ds}\left[\frac{1}{s-3}\right]$$

$f(t)=e^{3t}$ $$=-[-(s-3)^{-2}(1)]=\frac{1}{(s-3)^2}$$

$n=1$

$$*\mathcal{L}\{e^{3t}t\}=\mathcal{L}\{t\}=\frac{1}{s^2}\Big|_{s\to s-3}=\frac{1}{(s-3)^2}$$

CASO ESPECIAL.

$$\mathcal{L}\{e^{at}t^n\}=(-1)^n\frac{d^n}{ds^n}\mathcal{L}\{e^{at}\}$$
$$=(-1)^n\frac{d^n}{ds^n}\left[\frac{1}{s-a}\right]$$

Al revés.

$$\mathcal{L}\{t^n e^{at}\}=\mathcal{L}\{t^n\}=\frac{n!}{s^{n+1}}\Big|_{s\to s-a}=\frac{n!}{(s-a)^{n+1}}$$

$$\cdot\ \mathcal{L}\{t\underbrace{e^{-t}\cos t}_{f(t)}\}=(-1)^1\frac{d}{ds}\mathcal{L}\{e^{-t}\cos t\}$$

$n=1$

$$=-\frac{d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right]\Rightarrow$$

$$\mathcal{L}\{e^{-t}\cos t\}=\mathcal{L}\{\cos t\}=\frac{s}{s^2+1}\Big|_{s\to s+1}=\frac{s+1}{(s+1)^2+1}$$

$$\Rightarrow \mathcal{L}\{te^{-t}\cos t\}=\left[\frac{(s+1)^2+1-(s+1)\cdot 2(s+1)(1)}{[(s+1)^2+1]^2}\right]$$

$$=-\left[\frac{-(s+1)^2+1}{[(s+1)^2+1]^2}\right]=\frac{(s+1)^2+1}{[(s+1)^2+1]^2}$$

$\mathcal{L}\{e^{-t}t\cos t\}$ Ejercicio!

$$\Rightarrow\mathcal{L}\{(t-1)^3e^{t-1}u(t-1)\}$$ Con

$$\mathcal{L}\{t^n f(t)\}=(-1)^n\frac{d^n}{ds^n}\mathcal{L}\{e^{at}\}$$

$$\mathcal{L}\{t^3e^{t-1}u(t-1)\}=(-1)^3\frac{d^3}{ds^3}\mathcal{L}\{e^{t-1}u(t-1)\}$$

(Apendice.

TRANSFORMADA DE UNA DERIVADA.

Teorema: Si $y(t), y'(t),\dots,y^{(n-1)}(t)$ son continuas en $[0,\infty)$ y de orden exponencial y si la $y^{(n)}(t)$ es continua, parte por parte en $[0,\infty)$ entonces

$$\mathcal{L}\{y^{(n)}(t)\}=s^nY(s)-s^{n-1}y(0)-s^{n-2}y'(0)\dots$$
$$-y^{n-1}(0)$$

Donde

$$Y(s)=\mathcal{L}\{y(t)\}$$

Ejemplo

Suponga que $f(t)$ cuenta con las propiedades $y(0)=2 \quad y'(2)=3$ Despeje la transformada de Laplace $\mathcal{L}\{y(t)\}=Y(s)$ en la expresión $y''+y=1$

Solución

$$\mathcal{L}\{y''+y\}=\mathcal{L}\{1\}$$

$$\mathcal{L}\{y''\}+\mathcal{L}\{y\}=\mathcal{L}\{1\}$$

$$\mathcal{L}\{y''\}=s^2Y(s)-sy(0)-y'(0)$$
$$=s^2Y(s)-2s-3$$

$n=2$

$$\mathcal{L}\{y\}=Y(s)\qquad \mathcal{L}\{1\}=\frac{1}{s}$$

$$s^2Y(s)-2s-3+Y(s)=\frac{1}{s}$$

$$Y(s)=\frac{\frac{1}{s}+2s-3}{s^2+1}=\frac{1+2s^2-3s}{s(s^2+1)}$$

$$=\frac{2s^2-3s+1}{s(s^2+1)}$$

$$\frac{2s^2+3s+1}{s(s^2+1)}=\frac{A}{s}+\frac{Bs+C}{s^2+1}$$

$$=A\left(\frac{1}{s}\right)+B\left(\frac{s}{s^2+1}\right)+C\left(\frac{1}{s^2+1}\right)$$

$$Y(t)=A+B\cos t+C\operatorname{sen} t$$

Ejemplo: $y(0)=0 \quad y'(0)=0$

$$\mathcal{L}\{y(t)\} = Y(s)$$

$$y'' + y = \operatorname{sen} t$$

$$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \mathcal{L}\{\operatorname{sen} t\}$$

$$\mathcal{L}\{y''\} = s^2 Y(s) - s y(0) - y'(0) = s^2 Y(s)$$

$$\mathcal{L}\{y\} = Y(s) \quad \mathcal{L}\{\operatorname{sen} t\} = \frac{1}{s^2+1}$$

$$s^2 Y(s) + y(s) = \frac{1}{s^2+1}$$

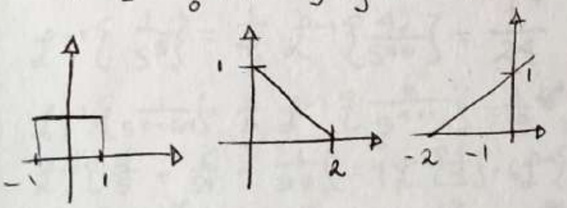
$$y(s) = \frac{1}{(s^2+1)^2}$$

» CONVOLUSIÓN.

Es una herramienta matemática utilizada en el análisis armónico.

Def. Si las soluciones $f(t)$ y $g(t)$ son continuas parte por parte en $[0,\infty)$, entonces la convolución de f y g se representa por $f*g$ y se define:

$$f*g = \int_0^t f(\tau) g\, g(t-\tau)\, d\tau$$



Sea $f(t) = e^{at} \quad g(t) = \operatorname{sen} t$

1) Determine la convolución de $e^t * \operatorname{sen} t$
2) Evalue $\mathcal{L}\{e^t * g(t)\}$

1) $f(t) = e^t \to f(\tau) = e^\tau$

$g(t) = \operatorname{sen} t \to g(t-\tau) = \operatorname{sen}(t-\tau)$

$$e^t * \operatorname{sen} t = \int_0^t e^\tau \operatorname{sen}(t-\tau)\, d\tau$$

$$= \left(e^\tau \operatorname{sen}(t-\tau)\Big|_0^t + e^\tau \cos(t-\tau)\Big|_0^t\right)\frac{1}{2}$$

$u = \operatorname{sen}(t-\tau) \quad dv = e^\tau dt$

$du = -\cos(t-\tau)d\tau \quad v = e^\tau$

$u = \cos(t-\tau) \quad dv = e^\tau dt$

$du = \operatorname{sen}(t-\tau)d\tau \quad v = e^\tau$

$$e^t \operatorname{sen} t = \frac{e^t \operatorname{sen}(t) + e^t - \cos t}{2}$$

$$= \frac{e^t - \operatorname{sen} t - \cos t}{2}$$

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$$\mathcal{L}\{e^t * \operatorname{sen} t\} = \mathcal{L}\{\tfrac{1}{2}(e^t - \operatorname{sen} t - \cos t)\}$$

$$= \frac{1}{2}\left(\frac{1}{s-1} - \frac{1}{s^2+1} - \frac{s}{s^2+1}\right)$$

$$= \frac{1}{2}\left(\frac{(s^2+1) + (s-1) - (1-s)}{(s-1)(s^2+1)}\right)$$

$$= \frac{2}{2(s-1)(s^2+1)} = \frac{1}{(s-1)(s^2+1)}$$

TEOREMA DE CONVOLUSIÓN.

Teorema Si $f(t)$ y $g(t)$ son continuas en tramos en $[0,\infty)$ y de orden exponencial, entonces la $\mathcal{L}$

$$\mathcal{L}\{f(t) * g(t)\} = F(s)\, g(s)$$

$$F(s) \cdot G(s) = \mathcal{L}\{F(s)\} \cdot \mathcal{L}\{g(t)\}$$

$$\mathcal{L}\{e^t * \operatorname{sen} t\} = \mathcal{L}\{e^t\}\mathcal{L}\{\operatorname{sen} t\}$$

$$= \frac{1}{s-1} \cdot \frac{1}{s^2+1}$$

$$\mathcal{L}\{e^{-t} * e^t \cos t\} = \mathcal{L}\{e^{-t}\}\mathcal{L}\{e^t \cos t\}$$

$$= \frac{1}{s+1} \cdot \frac{s-1}{(s-1)^2+1}$$

$$\mathcal{L}\left\{\int_0^t \operatorname{sen}\tau \cos(t-\tau)\, d\tau\right\}$$

$$= \mathcal{L}\{\operatorname{sen}\tau * \cos t\}$$

$$= \mathcal{L}\{\operatorname{sen} t\} \cdot \mathcal{L}\{\cos t\}$$

$$= \frac{1}{s^2+1} \cdot \frac{s}{s^2+1} = \frac{s}{(s^2+1)^2}$$

Si $g(t) = 1$ entonces

$$f * 1 = \int_0^t f(\tau)(1)d\tau = \int_0^t f(\tau)d\tau$$

$$\mathcal{L}\{f * 1\} = \mathcal{L}\{\int_0^t f(t)dt\} = \mathcal{L}\{f(t)\}$$

$$- \mathcal{L}\{1\} = \frac{F(s)}{s}$$

$$- \mathcal{L}\{\int_0^t f(\tau)d\tau\} = \frac{F(s)}{s}$$

>> Transformada de una integral

Si $f(t)$ es una función continua tramo a tramo de $[0, +\infty)$ y de orden exponencial, entonces,

$$\mathcal{L}\{\int_0^t f(t)d\tau\} = \frac{F(s)}{s}$$

$$\mathcal{L}\{\int_0^t f(\tau)d\tau\} = \frac{\mathcal{L}\{f(t)\}}{s} = \frac{F(s)}{s}$$

$$\mathcal{L}\{\int_0^t \cos\tau \, d\tau = \frac{\mathcal{L}\{\cos t\}}{s} = \frac{1}{s} \frac{s}{s^2+1}$$

$$= \frac{1}{s^2+1}$$

$$\mathcal{L}\{\int_0^t \tau \operatorname{sen}\tau d\tau\} = \frac{\mathcal{L}\{t \operatorname{sen} t\}}{s}$$

$$\mathcal{L}\{t \operatorname{sen} t\} = \frac{d}{ds}\mathcal{L}\{\operatorname{sen} t\}$$

$$= -(s^2+1)^{-1} = (-(s^2+1)^{-2} \cdot 2s$$

$$= \frac{2s}{(s^2+1)^2}$$

TRANSFORMADA INVERSA.

Ahora invertiremos nuestro problema, es decir, dada la función $F(s)$, nuestro objetivo será encontrar la función $f(t)$ que le corresponde.

Definición

Se dice que $f(t)$ es la transformada inversa de $F(s)$ y se expresa por

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

Donde $\mathcal{L}^{-1}$ es el operador inverso de Laplace y es un operador lineal. Esto es

$$\mathcal{L}^{-1}\{\alpha F(s) + \beta G(s)\} = \alpha \mathcal{L}^{-1}\{F(s)\} + \beta \mathcal{L}^{-1}\{G(s)\}$$

NOTA: Aunque la transformada inversa no define una operación como tal, por simplicidad es necesario introducir al operador lineal de $\mathcal{L}^{-1}$ que representa el operador de la transformada inversa de Laplace, esto es, si tenemos

$$\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} = \frac{1}{4!}\mathcal{L}^{-1}\left\{\frac{4!}{s^{4+1}}\right\} = \frac{t^4}{24}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+64}\right\} = \frac{1}{8}\mathcal{L}^{-1}\left\{\frac{8}{s^{4+1}}\right\} = \frac{1}{8}\operatorname{sen} 8t.$$

$$\mathcal{L}^{-1}\left\{\frac{4}{s} + \frac{6}{s^4} - \frac{1}{s+8}\right\} = 4\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + 6\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+8}\right\}$$

$$= 4t + \frac{\cancel{6}}{\cancel{6}}t^3 - e^{-8t} = \underline{4 + t^3 - e^{-8t}}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{4s^2+1}\right\} = \frac{1}{4}\left\{\frac{1}{s^2-\frac{1}{4}}\right\} = \frac{1}{4}2\mathcal{L}^{-1}\left\{\frac{\frac{1}{2}}{s^2-\frac{1}{4}}\right\}$$

$$= \frac{1}{2}\operatorname{senh}\frac{1}{2}t.$$

$$\mathcal{L}^{-1}\left\{\frac{4s}{4s^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+\frac{1}{4}}\right\} = \underline{\cos\frac{1}{2}t}$$

$$\mathcal{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\} = 2\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} - 6\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\}$$

$$= 2\cos 3t - 2\operatorname{sen} 3t.$$

>> Forma inversa del primer Teorema de traslación.

Si $f(t) = \mathcal{L}^{-1}\{F(s)\}$, entonces

$$\mathcal{L}\{e^{at}f(t)\} = \mathcal{L}\{f(t)\} = F(s)\Big|_{s\to s-a} = F(s-a)$$

$$\mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t)$$

$$\mathcal{L}^{-1}\{F(s-a)\} = e^{at}\mathcal{L}^{-1}\{F(s)\} = e^{at}f(s)$$

Evaluar $\mathcal{L}^{-1}\left\{\frac{s-9}{(s-9)^2+2}\right\}$

$$= e^{9t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+2}\right\} = e^{9t}\cos\sqrt{2}t$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+6s+11}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+6s+9)+2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{(s+3)^2+2}\right\} = \mathcal{L}^{-1}\left\{\frac{(s+3)-3}{(s+3)^2+2}\right\}$$

$$= e^{-3t}\mathcal{L}^{-1}\left\{\frac{s-3}{s^2+2}\right\} = e^{-3t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+2}\right\}$$

$$-\frac{3}{\sqrt{2}}\mathcal{L}^{-1}\left\{\frac{\sqrt{2}}{s^2+2}\right\}$$

$$= e^{-3t}\left(\cos\sqrt{2}t - \frac{3}{\sqrt{2}}\operatorname{sen}\sqrt{2}t\right)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-9)^3}\right\} \quad e^{9t}\mathcal{L}^{-1}\left\{\frac{1}{t^3}\right\}$$

$$\frac{e^{-9t}}{2}\{t^2\}$$

$$\mathcal{L}^{-1}\left\{\frac{2s+5}{s^2+6s+34}\right\} = 2\mathcal{L}^{-1}\left\{\frac{s+\frac{5}{2}}{s^2+6s+25}\right\}$$

$$= 2\mathcal{L}^{-1}\left\{\frac{s+\frac{5}{2}}{s^2+6s+9+25}\right\}$$

$$= 2\mathcal{L}^{-1}\left\{\frac{s+\frac{5}{2}}{(s+3)^2+25}\right\} = 2\mathcal{L}^{-}$$

$$= 2\mathcal{L}^{-1}\left\{\frac{(s+3)-3+\frac{5}{2}}{(s+3)^2+25}\right\}$$

$$= 2\mathcal{L}^{-1}\left\{\frac{(s+3)-\frac{1}{2}}{(s+3)^2+25}\right\}$$

$$= 2e^{-3t}\mathcal{L}^{-1}\{\ s-$$

$$\mathcal{L}^{-1}\left\{\frac{5s}{(s-2)^2}\right\} = 5\mathcal{L}^{-1}\left\{\frac{s}{(s-2)^2}\right\}$$

$$5\mathcal{L}^{-1}\left\{\frac{(s-2)+2}{(s-2)^2}\right\} = 5e^{2t}\mathcal{L}^{-1}\left\{\frac{s+2}{s^2}\right\}$$

$$= 5e^{2t}(1+2t)$$

Forma inversa del segundo Teorema de Traslación.

Si $\mathcal{L}^{-1}\{F(s)\} = f(t)$

$$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\,u(t-a)$$

$$\mathcal{L}^{-1}\{e^{-as}F(s)\} = \mathcal{L}^{-1}\{F(s)\}\Big|_{t\to t-a} u(t-a)$$

$$= f(t)\Big|_{t\to t-a} u(t-a) = f(t-a)u(t-a)$$

Evaluar

$$\mathcal{L}^{-1}\left\{\frac{e^{-\pi/2\, s}}{(s^2+9)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\}u\left(t-\frac{\pi}{2}\right)$$

$$= \frac{1}{3}\operatorname{sen} 3t\Big|_{t\to t-\frac{\pi}{2}} u\left(t-\frac{\pi}{2}\right)$$

$a = \frac{\pi}{2}$

$F(s) = \frac{1}{s^2+9}$

$$= \frac{1}{3}\operatorname{sen} 3\left(t-\frac{\pi}{2}\right)u\left(t-\frac{\pi}{2}\right) = A$$

$$\operatorname{sen}\left(3t-\frac{3\pi}{2}\right) = \operatorname{sen} 3t\cos\frac{3\pi}{2} - \cos 3t\operatorname{sen}\frac{3\pi}{2}$$

$$= \cos 3t$$

$$= \frac{1}{3}\cos 3t\, u\left(t-\frac{\pi}{2}\right)$$

$$= \begin{cases} 0; & 0 \le t < \frac{\pi}{2} \\ \frac{1}{3}\cos 3t; & t \ge \frac{\pi}{2} \end{cases}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\}\Big|_{t\to t-1} u(t-1)$$

$a = 1$

$F(s) = \frac{1}{s(s+1)}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+s-\frac{1}{4}} - \frac{1}{4}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+\frac{1}{2})^2-\frac{1}{4}}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{(s+\frac{1}{2})^2-\frac{1}{4}}\right\}$$

$$= e^{-1/2t}\mathcal{L}^{-1}\left\{\frac{1}{s^2-\frac{1}{4}}\right\}$$

$$= 2e^{-\frac{1}{2}t}\mathcal{L}^{-1}\left\{\frac{\frac{1}{2}}{s^2-\frac{1}{4}}\right\}$$

$$= 2e^{-1/2t}\operatorname{senh}\frac{1}{2}t$$

$$*\left(2e^{-\frac{1}{2}t}\operatorname{senh}\frac{1}{2}t\right)\Big|_{t\to t-1} u(t-1)$$

$$= 2e^{-\frac{1}{2}(t-1)}\operatorname{senh}\frac{1}{2}(t-1)u(t-1)$$

$$\begin{cases} 0; & 0 \le t < 1 \\ 2e^{-\frac{1}{2}(t-1)}\operatorname{senh}\frac{1}{2}(t-1) & t \ge 1 \end{cases}$$

Teorema de Convolusión

Si $f(t)$ y $g(t)$ son transformadas inversas de $F(s)$ y $G(s)$ respectivamente;

$$\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f*g = \int_0^t f(\tau)g(t-\tau)\,d\tau$$

$$\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = \mathcal{L}^{-1}\{F(s)\} * \mathcal{L}^{-1}\{G(s)\}$$

$$= f*g = \int_0^t f(\tau)\,g(t-\tau)\,d\tau$$

Ejemplo.

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s+4)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s-1}\cdot\frac{1}{s+4}\right\}$$

$$= e^{t} * e^{-4t} = 1$$

$$= \int_0^t e^{\tau}\cdot e^{-4(t-\tau)}\,d\tau = \int_0^t e^{\tau-4t+4\tau}\,d\tau$$

$$= \int_0^t e^{3\tau}\cdot e^{-4t}\,dt$$

$$= e^{-4t}\left(\frac{e^{3t}}{5}\right)\Big|_0^t = e^{-4t}\left(\frac{e^{5t}}{5}-\frac{1}{5}\right)$$

$$= \frac{1}{5}(e^{t}-e^{4t})$$

El metodo de Laplace aplicado a un problema de valor inicial, consta de los sig. pasos:

$$\boxed{\begin{array}{c}P.V.I\\ \phi(D)y=F(t)\end{array}} \xrightarrow{\mathcal{L}} \boxed{\begin{array}{c}\text{Ec algebraica}\\ F(Y(s))=0\end{array}} \xrightarrow[\text{rapido}]{\text{valido}} \boxed{\begin{array}{c}\text{Solución}\\ \text{ec alge.}\\ Y=Y(s)\end{array}} \xrightarrow{\mathcal{L}^{-1}} \boxed{\begin{array}{c}y=y(t)\\ \text{sol P.V.I}\end{array}}$$

$\frac{dI}{dt}+2I=10e^{-2t}$; $I(0)=0$

- Factor, exactitud
- por met ec. lineal.
- Var. de param.
- Laplace. ✓

1 Punto.

Resolver

$y'-3y=e^{2t}$; $y(0)=1$

① $\mathcal{L}\{y'\}-3\mathcal{L}\{y\}=\mathcal{L}\{e^{2t}\}$

Donde

$\mathcal{L}\{y'\}=sY(s)-y(0)=sY(s)-1$

$n=1$

$\mathcal{L}\{y\}=Y(s)$

$\mathcal{L}\{e^{2t}\}=\frac{1}{s-2}$

Sustituy.

$sY(s)-1-3Y(s)=\frac{1}{s-2}$ ec. algebraica

② Despeja $Y(s)$

$Y(s)(s-3)=\frac{1}{s-2}+1$

$Y(s)=\left(\frac{1+s-2}{s-2}\right)\left(\frac{1}{s-3}\right)$

$Y(s)=\frac{s-1}{(s-2)(s-3)}$ ⇒ Sol. ec. algebraica.

③ $\mathcal{L}^{-1}\{\quad\}=\mathcal{L}\{Y(s)\}=\mathcal{L}^{-1}\left\{\frac{s-1}{(s-2)(s-3)}\right\}$ ---(2)

Donde

$\mathcal{L}^{-1}\{Y(s)\}=y(t)$ $\quad\mathcal{L}^{-1}\left\{\frac{s-1}{(s-2)(s-3)}\right\}$ $\quad\frac{s-1}{(s-2)(s-3)}=\frac{A}{s-2}+\frac{B}{s-3}$

$=\frac{A(s-3)+B(s-2)}{(s-2)(s-3)}$

...CIONES PARCIALES.

EVALUE:

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s+1)(s+4)}\right\}$$

(1)

$$\frac{1}{(s-1)(s+1)(s+4)} = \frac{A}{s-1} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$= \frac{A(s+2)(s+4) + B(s-1)(s+4) + C(s-1)(s+2)}{(s-1)(s+2)(s+4)}$$

$$s^3(s-1)^{3}$$

$$1 = A(s+2)(s+4) + B(s-1)(s+4) + C(s-1)(s+2)$$

$$= A(s^2+6s+8) + B(s^2+3s-4) + C(s^2+s-2)$$

$$1 = (A+B+C)s^2 + (6A+3B+C)s + (8A-4B-2C)$$

COND. de ident.

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$$\begin{cases} A+B+C=0 \\ 6A+3B+C=0 \\ 8A-4B-2C=1 \end{cases} \quad -3B-5C=0 \quad 10C=1 \therefore C=\frac{1}{10}$$

$$\rightarrow -12B-10C=1 \qquad B=-\frac{1}{6}$$

$$\frac{1}{10}-\frac{1}{6}+A=0 \Rightarrow A=\frac{1}{6}-\frac{1}{10}=\frac{10-6}{60}=\frac{4}{60}=\frac{2}{30}=\frac{1}{15}$$

$$\mathcal{L}^{-1}\left\{\frac{A}{s-1}+\frac{B}{s+2}+\frac{C}{s+4}\right\} = \frac{1}{15}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \frac{1}{10}\mathcal{L}\left\{\frac{1}{s+4}\right\}$$

$$= \frac{1}{15}e^{t} - \frac{1}{6}e^{-2t} + \frac{1}{10}e^{-4t}$$

$s=1;\ s=-2;\ s=-4$

Si $s=1 \Rightarrow 1 = A(1+2)(1+4) \Rightarrow 1 = A(15) \therefore A = \frac{1}{15}$

Si $s=-2 \Rightarrow 1 = B(-2-1)(-2+4) \rightarrow 1 = B(-6) \therefore B = -\frac{1}{6}$

Si $s=-4 \Rightarrow 1 = C(-4-1)(-4+2) \Rightarrow 1 = C(10) \Rightarrow C = \frac{1}{10}$

$1\ s^2+s) = (s+1)(s-1)$

7.1-7.4

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\}\Bigg|_{t\to t-1} u(t-1) = (1-e^{-(t-1)})u(t-1) = \begin{cases} 0; & 0\le t<1 \\ 1-e^{-(t-1)}; & t\ge 1\end{cases}$$

$a=1$

$$F(s) = \frac{1}{s(s+1)}$$

Donde

$$\mathcal{L}\left\{\frac{1}{s(s+1)}\right\} \Rightarrow \mathcal{L}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = 1-e^{-t}$$

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \frac{A(s+1)+Bs}{s(s+1)}$$

$$1 = A(s+1)+Bs$$

Si $s=0 \Rightarrow 1 = A(0+1) \therefore A=1$

Si $s=-1 \Rightarrow 1 = B(-1) \therefore B=-1$

$$\mathcal{L}^{-1}\left\{\frac{s+1}{s^2(s+2)^3}\right\}$$

Proba

Economicos

Financieras.

Si $s=2 \Rightarrow 2-1 = A(2-3)$ $\quad A=-1$

Si $s=3 \Rightarrow 3-1 = B(3-2)$ $\quad B=2$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s-3)}\right\} = -\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + 2\,\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$\therefore$ Sust en (2)

$$= \cancel{-e^{2t}+2e^{3t}} \quad y(t) = -e^{2t}+2e^{3t}$$

$$Q(s)(s+5) = 2\,\frac{e^{-3s}}{s}$$

$$\boxed{Q(s) = \frac{2e^{-3s}}{s(s+5)}}$$ Sol. ec. alge.

③ $\mathcal{L}^{-1}$

$$\mathcal{L}^{-1}\{Q(s)\} = 2\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s(s+5)}\right\} \quad \ldots (3)$$

Donde

$$\mathcal{L}^{-1}\{Q(s)\} = q(t)$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s(s+5)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+5)}\right\}\Bigg|_{t\to t-3} u(t-3) = \frac{1}{5}(1-e^{-5t})\Bigg|_{t\to t-3} u(t-3)$$

$$= \frac{1}{5}(1-e^{-5(t-3)})u(t-3)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s+5)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s+5}\right\} \cdot \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = e^{-5t} * 1$$

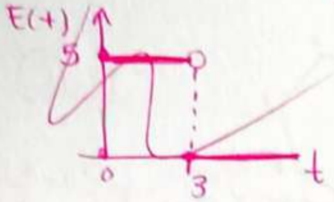
$$= \int_0^t e^{-5\tau}\,d\tau = \frac{e^{-5\tau}}{-5}\Bigg|_0^t = \frac{-e^{-5t}}{5} + \frac{1}{5}$$

$$= \frac{1}{5}(1-e^{-5t})$$

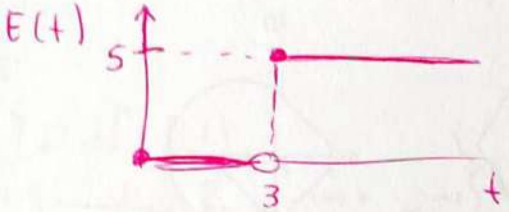
Sust. en 3

$$q(t) = \frac{2}{5}(1-e^{-5(t-3)})u(t-3) = \begin{cases} 0; & 0 \le t < 3 \\ \frac{2}{5}(1-e^{-5(t-3)}); & t \ge 3 \end{cases}$$

Use la transformada de Laplace para determinar la carga ~~de~~ $q(t)$ en el capacitor de un cto en serie RC, cuando $q(0)=0$; $R=2.5\,\Omega$, $C=0.08F$ y $E(t)$ el que se muestra en la figura



$$\cancel{\begin{cases} 5; & 0 \le t < 3 \\ 0; & t \ge 3 \end{cases}}$$



$$E(t) = \begin{cases} 0; & 0 \le t < 3 \\ 5; & t \ge 3 \end{cases} = \cancel{0 - (0)u(t-3)} + 5u(t-3)$$

$$\therefore E(t) = 5u(t-3)$$

Por otro lado, la ec. dif. que modela a un cto en serie RL

$$\frac{dq}{dt} + \frac{1}{RC}q = \frac{E(t)}{R} \quad \ldots (1)$$

Sust datos

$$q' + \frac{1}{(2.5)(0.08)}q = \frac{5}{2.5}u(t-3)$$

Simplificando

$$\boxed{q' + 5q = 2u(t-3)}$$

(1) $\mathcal{L}$

$$\mathcal{L}\{q'\} + 5\mathcal{L}\{q\} = 2\mathcal{L}\{u(t-3)\}$$

Donde

$$\mathcal{L}\{q'\} = sQ(s) - \cancel{q(0)}^{\,0} = sQ(s)$$

$$\mathcal{L}\{q\} = Q(s)$$

$$\mathcal{L}\{u(t-3)\} = \frac{e^{-3s}}{s}$$

Sust. en (2)

$$sQ(s) + 5Q(s) = 2\frac{e^{-3s}}{s} \quad \text{ec. algeb.}$$

$I(s)(s^2+200s+10000) = 20000[1-e^{-s}]$

$I(s)(s+100)^2 = 20000[1-e^{-s}]$

$\therefore I(s) = 20000\left[\frac{1}{(s+100)^2} - \frac{e^{-s}}{(s+100)^2}\right]$ sol. de ec algeb.

③ $\mathcal{L}^{-1}$

$$\mathcal{L}^{-1}\{I(s)\} = 20000\left[\mathcal{L}^{-1}\left\{\frac{1}{(s+100)^2}\right\} - \mathcal{L}^{-1}\left\{\frac{e^{-s}}{(s+100)^2}\right\}\right] \quad \cdots (3)$$

DONDE:

$\mathcal{L}^{-1}\{I(s)\} = i(t)$

$\mathcal{L}^{-1}\underbrace{\left\{\frac{1}{(s+100)^2}\right\}}_{} = e^{-100t}\ \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = \underline{te^{-100t}}$ ✓

$F(s-(-100))$

$a = -100$

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{(s+100)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+100)^2}\right\}\Bigg|_{t\to t-1} u(t-1)$$

$$= te^{-100t}\Big|_{t\to t-1} u(t-1)$$

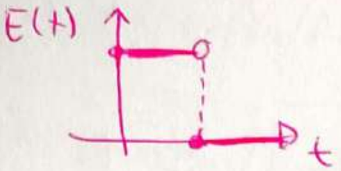
$$= (t-1)e^{-100(t-1)}\ \underline{u(t-1)}$$ ✓

Sust. en (3)

$$i(t) = 20000\left[te^{-100t} - (t-1)e^{-100(t-1)}u(t-1)\right]$$

$$= \begin{cases} 2\times10^4\, t e^{-100t}; & 0 \le t < 1 \\ 2\times10^4\left(te^{-100t} - (t-1)e^{-100(t-1)}\right); & t \ge 1 \end{cases}$$

Mediante la ec. integro-diferencial, determine la corriente $i(t)$ en un cto. eléctrico en serie RCL, cuando $L=0.005h$, $R=1.0\ \Omega$, $C=0.002\,F$; $i(0)=0$ y $E(t)$ el que se muestra en la fig.



$$E(t)\begin{cases} 100; & 0\le t<1 \\ 0; & t\ge 1 \end{cases} = 100 - 100u(t-1) + (0)u(t-1)$$

$$\therefore E(t) = 100[1-u(t-1)]$$

Sust. en (1)

$$(0.005)i' + (1.0)i + \frac{1}{(0.02)}\int_0^t i(\tau)\,d\tau = 100[1-u(t-1)]$$

mult. por $\frac{1}{0.005}$

$$i' + \frac{1}{0.005}i + \frac{1}{(0.02)(0.005)}\int_0^t i(\tau)\,d\tau = \frac{100}{0.005}[1-u(t-1)]$$

Simplificando

$$i' + 200i + 10000\int_0^t i(\tau)\,d\tau = 20000[1-u(t-1)]$$

① $\mathcal{L}$

$$\mathcal{L}\{i'\} + 200\mathcal{L}\{i\} + 1000\,\mathcal{L}\left\{\int_0^t i(\tau)\,d\tau\right\} = 20000\left[\mathcal{L}\{1\} - \mathcal{L}\{u(t-1)\}\right] \text{---}(2)$$

DONDE

$$\mathcal{L}\{i'\} = sI(s) - \cancel{i(0)}^{\,0} = sI(s)$$

$$\mathcal{L}\{i\} = I(s)$$

$$\mathcal{L}\left\{\int_0^t i(\tau)\,d\tau\right\} = \frac{\mathcal{L}\{i(t)\}}{s} = \frac{I(s)}{s}$$

$$\mathcal{L}\{1\} = \frac{1}{s};\quad \mathcal{L}\{u(t-1)\} = \frac{e^{-s}}{s}$$

sust. en (2)

$$sI(s) + 200I(s) + 10000\frac{I(s)}{s} = 20000\left[\frac{1}{s} - \frac{e^{-s}}{s}\right]$$ ec. algebraica.

② Despejar $I(s)$ Mult. por s.

$$s^2I(s) + 200sI(s) + 10000I(s) = 20000[1-e^{-s}]$$

Zill Prob. 39 al 43 sec. 7.5.

Segundo teorema de traslación

Teorema: Si la $\mathcal{L}\{f(t)\}=F(s)$,

$\Rightarrow$ $\mathcal{L}\{f(t-a)u(t-a)\}=e^{-as}F(s)$; $a>0$

$\mathcal{L}\{f(t-a)u(t-a)\}=e^{-as}\mathcal{L}\{f(t)\}=e^{-as}F(s)$

NOTA: Para la aplicación de este teorema, la función multiplicadora $f(t-a)$ debe tener el mismo desplazamiento que la función escalón unitario $u(t-a)$

Ejemplos:

$\mathcal{L}\{(t-2)^3u(t-2)\}=e^{-2s}\mathcal{L}\{t^3\}=\frac{3!}{s^{3+1}}=\frac{6e^{-2s}}{s^4}$

$f(t)=\begin{cases}0; & 0\le t<2\\(t-2)^3, & t\ge 2\end{cases}$

$\left\{\mathcal{L}\{f(t)\}=\int_0^2 e^{-st}(0)\,dt+\int_2^\infty e^{-st}(t-2)^3dt\right\}$ Proced. larg.

Ejemplos

$\mathcal{L}\{\operatorname{sen}(t-\frac{\pi}{2})u(t-\frac{\pi}{2})\}=e^{-\pi/2\, s}\mathcal{L}\{\operatorname{sen}t\}=e^{-\frac{\pi}{2}s}\frac{1}{s+1}=\frac{e^{-\pi/s}}{s^2+1}$

$\mathcal{L}\{e^{at}f(t)\}=\mathcal{L}\{f(t)\}=F(s)\Big|_{s\to s-a}=F(s-a)$

$\mathcal{L}\{(t-1)^3e^{t-1}u(t-1)\}=\mathcal{L}\{e^{t-1}\underbrace{(t-1)^3u(t-1)}_{f(t)}\}=\mathcal{L}\{e^t\cdot e^{-1}(t-1)^3u(t-1)\}$

$=e^{-1}\{e^t(t-1)^3u(t-1)\}=\mathcal{L}\{(t-1)^3u(t-1)\}\Big|_{s\to s-1}=e^{-1}\left[\frac{6e^{-s}}{s^4}\right]\Big|_{s\to s-1}\Rightarrow$

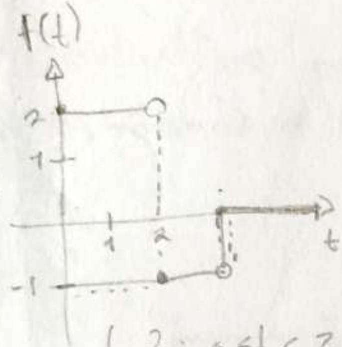
Donde

$\mathcal{L}\{(t-1)^3u(t-1)\}=e^{-s}\mathcal{L}\{t^3\}=\frac{6e^{-s}}{s^4}$

$\Rightarrow e^{-1}\cdot\left[\frac{6e^{-(s-1)}}{(s-1)^4}\right]=\frac{6e^{-s}}{(s-1)^4}$ //

Ejemplo

Determine la transformada de Laplace de la función $f(t)$



$$f(t)=\begin{cases} 2 ; & 0 \le t < 2 \\ -1 ; & 2 \le t < 3 \\ 0 ; & t \ge 3 \end{cases} = 2 - 2u(t-2) + (-1)u(t-2) - (-1)u(t-3) + \cancel{(0)u(t-3)}$$

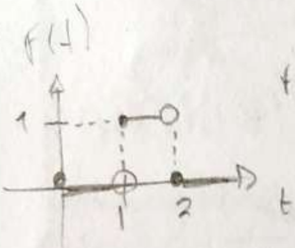
$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

$$\therefore f(t) = 2 - 3u(t-2) + u(t-3)$$

$a=2$ $\quad a=3$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2 - 3u(t-2) + u(t-3)\} = 2\mathcal{L}\{1\} - 3\mathcal{L}\{u(t-2)\} + \mathcal{L}\{u(t-3)\}$$

$$= \frac{2}{s} - \frac{3e^{-2s}}{s} + \frac{e^{-3s}}{s}$$



$$f(t)=\begin{cases} 0 ; & 0 \le t < 1 \\ 1 ; & 1 \le t < 2 \\ 0 ; & t \ge 2 \end{cases} = \cancel{0} - \cancel{(0)u(t-1)} + (1)u(t-1) - (1)u(t-2) + \cancel{(0)u(t-2)}$$

$$\boxed{f(t) = u(t-1) - u(t-2)}$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{u(t-1) - u(t-2)\} = \mathcal{L}\{u(t-1)\} - \mathcal{L}\{u(t-2)\} = \frac{e^{-s}}{s} - \frac{e^{-2s}}{s}$$

$$f(t)=\begin{cases} \cos t ; & 0 \le t < \frac{\pi}{2} \\ \operatorname{sen}(t-\frac{\pi}{2}) ; & t \ge \frac{\pi}{2} \end{cases} = \cos t - \cos u(t-\tfrac{\pi}{2}) + \operatorname{sen}(t-\tfrac{\pi}{2})u(t-\tfrac{\pi}{2})$$

$$f(t) = \cos t - \cos t\, u(t-\tfrac{\pi}{2}) + \operatorname{sen}(t-\tfrac{\pi}{2})u(t-\tfrac{\pi}{2})$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\cos t\} - \mathcal{L}\{\cos u(t-\tfrac{\pi}{2})\} + \mathcal{L}\{\operatorname{sen}(t-\tfrac{\pi}{2})u(t-\tfrac{\pi}{2})\} = \frac{s}{s^2+1} -$$

$$\mathcal{L}\{f(t)\} = \int_0^{+\infty} e^{-st} f(t)\,dt = \int_0^a e^{-st}(0)\,dt + \int_a^{+\infty} e^{-st}(1)\,dt$$

$$\mathcal{L}\{f(t)\} = \lim_{a \to \infty} \int_a^k e^{-st}\,dt = \lim_{k \to +\infty} \left[-\frac{e^{-st}}{s} \right|_a^k = \lim_{k \to \infty^+} \left[-\frac{e^{-k}}{s} + \frac{e^{-as}}{s} \right]$$

$$= \frac{e^{-as}}{s} \qquad \therefore \mathcal{L}\{f(t)\} = \frac{e^{-as}}{s}$$

La función dada en (1) se llama función de Heaviside o función escalón unitario y se denota por $u(t-a)$, entonces

$$u(t-a) = \begin{cases} 0 ; & 0 \leq t < a \\ 1 ; & t > a \end{cases} \qquad \therefore \mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

USO DE TABLAS

Para un manejo adecuado de tablas de tranformada de Laplace es importante notar que el operador $\mathcal{L}$ de laplace es un operador lineal.

En efecto

Sean $f(t)$ y $g(t)$ funciones de orden exponencial (q' la trans. de L. existe) y $\alpha, \beta \in \mathbb{R}$ entonces

$$\mathcal{L}\{\alpha f(t) + \beta g(t)\} = \int_0^{+\infty} e^{-s}[\alpha f(t) + \beta g(t)]\,dt$$

$$= \int_0^{+\infty} e^{-st} \cdot \alpha f(t)\,dt + \int_0^{+\infty} e^{-st} \beta\, g(t)\,dt$$

$$= \alpha \int_0^{+\infty} e^{-st} f(t)\,dt + \beta \int_0^{+\infty} e^{-st} \cdot g(t)\,dt$$

$$= \alpha \mathcal{L}\{f(t)\} + \beta \mathcal{L}\{g(t)\}$$

$$= \frac{1}{2}\lim_{k\to+\infty}\left[-\frac{e^{-(s-a)t}}{s-a} - \frac{e^{-(s+a)t}}{s+a}\right]_0^k = \frac{1}{2}\lim_{k\to+\infty}\left[-\cancel{\frac{e^{-(s-a)k}}{s-a}} - \cancel{\frac{e^{-(s+a)k}}{s+a}} + \frac{1}{s-a} + \frac{1}{s+a}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right] = \frac{1}{2}\left[\frac{2s}{s^2-a^2}\right] = \frac{s}{s^2-a^2}$$

$$\therefore \mathcal{L}\{\cosh at\} = \frac{s}{s^2-a^2} \qquad \boxed{\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}}$$

$f(t) = e^{bt}\,\text{senh}\ at$

$$\mathcal{L}\{e^{bt}\,\text{senh}\ at\} = \int_0^{+\infty} e^{-st}\cdot e^{bt}\,\text{senh}\ at\,dt$$

Ap. - $\mathcal{L}\{e^{bt}\cosh at\}$

$\text{senh}\ at = \frac{e^{at}-e^{-at}}{2}$

$$= \lim_{k\to\infty}\int_0^k e^{-st}\cdot e^{bt}\left[\frac{e^{at}-e^{-at}}{2}\right]dt = \frac{1}{2}\lim_{k\to+\infty}\left[\int_0^k e^{-(s-b-a)t}\,dt - \int_0^k e^{-(s-b+a)t}\,dt\right]$$

$$= \frac{1}{2}\lim_{k\to+\infty}\left[-\frac{e^{-(s-b-a)t}}{s-b-a} + \frac{e^{-(s-b+a)t}}{s-b+a}\right]_0^k$$

$$= \frac{1}{2}\lim_{k\to+\infty}\left[-\frac{e^{-(s-b-a)k}}{s-b-a} + \frac{e^{-(s-b-a)k}}{s-b+a} + \frac{1}{s-b-a} - \frac{1}{s-b+a}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(s-b)-a} - \frac{1}{(s-b)+a}\right] = \frac{1}{2}\left[\frac{s-b+a-(s-b-a)}{(s-b)^2-a^2}\right] = \frac{1}{2}\left[\frac{2a}{(s-b)^2-a^2}\right]$$

$$= \frac{a}{(s-b)^2-a^2}$$

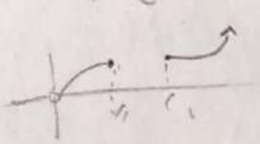
$$\mathcal{L}\left\{\frac{e^{bt}\,\text{senh}\ at}{a}\right\} = \frac{1}{(s-b)^2-a^2} \quad 7(x)$$

$$\mathcal{L}\{e^{bt}\,\text{senh}\ at\} = \frac{a}{(s-b)^2-a^2}$$

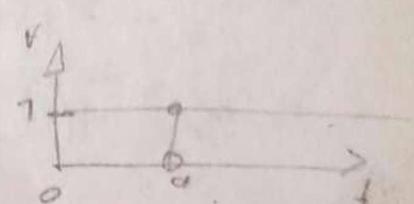
Sea $f(t) = \begin{cases} 0; & 0 \le t < a \\ 1; & t \ge a \end{cases}$ ... (i) $\quad D_f = [0, +\infty)$

HALLAR xD

$\mathcal{L}\{f(x)\}$



$\mathcal{L}\{f(t)\}$



$\cdot\ \mathcal{L}\left\{\int_0^t \tau \operatorname{sen}\tau \, d\tau\right\} = \dfrac{\mathcal{L}\{t \operatorname{sen} t\}}{s} = \dfrac{1}{s} \cdot \dfrac{1}{(s^2+1)^2} = \dfrac{1}{s(s^2+1)^2}$

Pero

$$\mathcal{L}\{t \operatorname{sen} t\} = (-1)^1 \frac{d}{ds} \mathcal{L}\{\operatorname{sen} t\} = -\frac{d}{ds}\left[\frac{1}{s^2+1}\right] = -\frac{d}{ds}(s^2+1)^{-1} = -$$

$F(t) = \operatorname{sen} t$

$n = 1$

$$= -\left(-(s^2+1)^{-2} \cdot (1)\right) = \frac{1}{(s^2+1)^2}$$

La transformada inversa

Ahora invertiremos nuestro problema, es decir, dada la función $F(s)$, nuestro objetivo será encontrar la función $f(t)$ que le corresponde, ~~def~~

- Def.

Se dice que $f(t)$ es la transformada inversa de $F(s)$ y se expresa por

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

En Donde: $\mathcal{L}^{-1}$ ~~se llama~~ es el operador inverso de Laplace el es un operador <u>lineal</u>.

Esto es

$$\mathcal{L}^{-1}\{\alpha F(s) + \beta G(s)\} = \alpha \mathcal{L}^{-1}\{F(s)\} + \beta \mathcal{L}^{-1}\{G(s)\}$$

TRANSFORMADA INVERSA DE ALGUNAS FUNCIONES ELEMENTALES

$F(s)$	$f(t)$	$F(s)$	$F(t)$
$\frac{1}{s}$	1	$\frac{s}{s^2+a^2}$	$\cos t$
$\frac{1}{s^2}$	t	$\frac{a}{s^2-a^2}$	$\operatorname{senh} at$
$\frac{n!}{s^{n+1}}$	t^n	$\frac{s}{s^2-a^2}$	$\cosh at$
$\frac{1}{s-a}$	e^{at}	$\frac{e^{-as}}{s}$	$u(t-a)$
$\frac{a}{s^2+a^2}$	$\operatorname{sen} t$		

d

Evalúe

Secc. 7.2 (impares) 1 - 33

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- $\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\} = \frac{1}{3!}\mathcal{L}^{-1}\left\{\frac{3!}{s^{3+1}}\right\} = \frac{t^3}{6}$

$n=3$

- $\mathcal{L}^{-1}\left\{\frac{1}{s^2+64}\right\} = \frac{1}{8}\mathcal{L}\left\{\frac{8}{s^2+64}\right\} = \frac{1}{8}\operatorname{sen} 8t$

$a=8$

- $\mathcal{L}^{-1}\left\{\frac{4}{s} + \frac{6}{s^5} - \frac{1}{s+8}\right\} = 4\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + 6\,\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} - \mathcal{L}^{-1}\left\{\underbrace{\frac{1}{s+8}}_{s-(-8)}\right\}$

$= 4(1) + 6\left(\frac{t^4}{24}\right) - e^{-8t} = 4 + \frac{t^4}{4} - e^{-8t}$

$\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} = \frac{1}{4!}\mathcal{L}^{-1}\left\{\frac{4!}{s^{4+1}}\right\} = \frac{t^4}{24}$

$n=4$

- $\mathcal{L}^{-1}\left\{\frac{10s}{s^2+16}\right\} = 10\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+16}\right\} = 10\cos 4t$

- $\mathcal{L}^{-1}\left\{\frac{1}{4s^2+1}\right\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s^2+\frac{1}{4}}\right\} = \frac{1}{4}\cdot\cancel{\frac{1}{2}}2\,\mathcal{L}^{-1}\left\{\frac{\frac{1}{2}}{s^2+\frac{1}{4}}\right\}$

$= \cancel{\frac{2}{8}}\ \frac{1}{2}\operatorname{sen}\frac{1}{2}t$

- $\mathcal{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\} = 2\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} - 6\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\}$

$-\frac{6}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$

$= 2\cos 3t - 2\operatorname{sen} 3t$

Teorema inversa del primer teorema de traslación

Si $\mathcal{L}^{-1}\{F(s)\} = f(t) \Rightarrow$

1TT $\left(\mathcal{L}\{e^{at}f(t)\} = \mathcal{L}\{f(t)\} = F(s)\Big|_{s\to s-a} = F(s-a)\right)$

$\Rightarrow \mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t)$

$\mathcal{L}^{-1}\{F(s-a)\} = e^{at}\mathcal{L}^{-1}\{F(s)\} = e^{at}f(t)$

$\mathcal{L}\{e^{at}f(t)\} = $

Evalue

- $\mathcal{L}^{-1}\left\{\frac{s-4}{(s-4)^2+2}\right\} = e^{4t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+2}\right\} = e^{4t}\cos\sqrt{2}\,t$

$a = 4$

- $\mathcal{L}^{-1}\left\{\frac{s}{s^2+6s+11}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+6s+9)+2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+3)^2+2}\right\}$

$= \mathcal{L}^{-1}\left\{\overbrace{\frac{(s+3)-3}{(s+3)^2+2}}^{F(s-(-3))}\right\}$

$= e^{-3t}\mathcal{L}^{-1}\left\{\frac{s-3}{s^2+2}\right\} = e^{-3t}\left[\mathcal{L}^{-1}\left\{\frac{s}{s^2+2}\right\} - \frac{3}{\sqrt{2}}\mathcal{L}^{-1}\left\{\frac{\sqrt{2}}{s^2+2}\right\}\right]$

$= e^{-3t}\left[\cos\sqrt{2}t - \frac{3}{\sqrt{2}}\operatorname{sen}\sqrt{2}\,t\right]$ 7.2 7.3 Tran inv.

- $\mathcal{L}^{-1}\left\{\frac{2s+5}{s^2+6s+34}\right\} = 2\mathcal{L}^{-1}\left\{\frac{s+\frac{5}{2}}{s^2+6s+34}\right\} = 2\mathcal{L}^{-1}\left\{\frac{s+\frac{5}{2}}{(s^2+6s+9)+25}\right\}$

$= 2\mathcal{L}^{-1}\left\{\frac{s+\frac{5}{2}}{(s+3)^2+25}\right\} = 2\mathcal{L}^{-1}\left\{\frac{(s+3)-3+\frac{5}{2}}{(s+3)^2+25}\right\}$

$s-(-3)$

$= 2\mathcal{L}^{-1}\left\{\underbrace{\frac{(s+3)-\frac{1}{2}}{(s+3)^2+25}}_{F(s-(-3))}\right\} = 2e^{-3t}\mathcal{L}^{-1}\left\{\frac{s-\frac{1}{2}}{s^2+25}\right\}$

$= 2e^{-3t}\left[\mathcal{L}^{-1}\left\{\frac{s}{s^2+25}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s^2+25}\right\}\right]$ $\frac{1}{10}$ 25

$= 2e^{-3t}\left[\cos 5t - \frac{1}{10}\operatorname{sen} 5t\right]$

e

$y' + y = f(t)$; $y(0) = 0$

Donde

$$f(t) = \begin{cases} 1; & 0 \le t < 1 \\ -1; & t \ge 1 \end{cases}$$

$= 1 - (1)u(t-1) + (-1)u(t-1)$

$f(t) = 1 - 2u(t-1)$

$y' + y = 1 - 2u(t-1)$; $y(0) = 0$

① $\mathcal{L}$

$\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{1\} - 2\mathcal{L}\{u(t-1)\}$ ...(1)

Donde

$\mathcal{L}\{y'\} = sY(s) - \cancel{y(0)}^{0} = sY(s)$

$\mathcal{L}\{y\} = Y(s)$; $\mathcal{L}\{1\} = \frac{1}{s}$

$\mathcal{L}\{u(t-1)\} = \frac{e^{-s}}{s}$

Sust. en (1)

$sY(s) + Y(s) = \frac{1}{s} - 2\frac{e^{-s}}{s}$ Ec Alg.

② Despejar $Y(s)$

$Y(s)(s+1) = \frac{1}{s} - \frac{2^{-s}}{s}$

$\therefore Y(s) = \frac{1}{s(s+1)} - 2\frac{e^{-s}}{s(s+1)}$ Sol. ec. alge.

③ $\mathcal{L}^{-1}$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\} - 2\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$ ...(2)

Donde

$\mathcal{L}^{-1}\{Y(s)\} = y(t)$

$\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} \cdot \mathcal{L}\left\{\frac{1}{s+1}\right\} = 1 * e^{-t}$

$= e^{-t} * 1 = \int_0^t e^{-\tau} d\tau = -e^{-\tau}\Big|_0^t = e^{-t} + 1$

$= 1 - e^{-t}$

$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\}\Big|_{t \to t-1} u(t-1)$

$F(s) = \frac{1}{s(s+1)}$ $= (1-e^{-t})\Big|_{t \to t-1} u(t-1)$

$a = 1$

$= (1 - e^{-(t-1)})u(t-1)$

$\therefore y(t) = 1 - e^{-t} - 2(1 - e^{-(t-1)})u(t-1)$

$$= \begin{cases} 1 - e^{-t}; & 0 \le t < 1 \\ 1 - e^{-t} - 2 + e^{-(t-1)}; & t \ge 1 \end{cases}$$

RESOLVER:

$y' + 2y = f(t)$; $y(0) = 0$

Donde:

$$f(t) = \begin{cases} t; & 0 \le t < 1 \\ 0; & t \ge 1 \end{cases}$$

$= t - tu(t-1) \cancel{t(0)u(t-1)}$

$f(t) = t - tu(t-1)$

① $\mathcal{L}$

$\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{t\} - \mathcal{L}\{tu(t-1)\}$ ---(1)

DONDE

$\mathcal{L}\{y'\} = sY(s) - \cancel{y(0)}^{0} = sY(s)$

$\mathcal{L}\{y\} = Y(s)$; $\mathcal{L}\{t\} = \frac{1}{s^2}$

$\mathcal{L}\{tu(t-1)\} = e^{-s}\mathcal{L}\{t+1\}$

$= e^{-s}\left[\frac{1}{s^2} + \frac{1}{s}\right]$

SUST. en (1)

$sY(s) + 2Y(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$

(Sol. ec. alge.)

② Despegar a $Y(s)$

$Y(s)(s+2) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$

$\therefore \boxed{Y(s) = \frac{1}{s^2(s+2)} - \frac{e^{-s}}{s^2(s+2)} - \frac{e^{-s}}{s(s+2)}}$

EVALUAR

$y'' - 6y' + 9y = t^2 e^{3t}; \quad y(0)=2; \quad y'(0)=6$

R: $y(t) = 2e^{3t}\left[\frac{t^4}{24} - 1\right]$ //

$$\mathcal{L}^{-1}\left\{\underbrace{\frac{1}{(s-3)^5}}_{F(s-3)}\right\} = e^{3t}\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} = \frac{e^{3t}}{4!}\mathcal{L}^{-1}\left\{\frac{4!}{s^{4+1}}\right\}$$

$a = 3$

$$= \frac{e^{3t}}{24}t^4$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} = e^{3t}$$

SUST. EN (2)

$$y(t) = 2\left[\frac{e^{3t}}{24}t^4 - e^{3t}\right]$$

$$\therefore y(t) = 2e^{3t}\left[\frac{t^4}{24} - 1\right]$$

SUST. EN (1)

$$s^2Y(s) - 2s - 6 - 6[sY(s) - 2] + 9Y(s) = \frac{2}{(s-3)^3} \quad \text{Ec. Alge.}$$

② Despejar $Y(s)$

$$Y(s)[s^2 - 6s + 9] - 2s + 6 = \frac{2}{(s-3)^3}$$

$$Y(s)[s^2 - 6s + 9] = \frac{2}{(s-3)^3} + 2s - 6$$

$Y(s)$

★ APENDICE:

$y'' + y = \operatorname{sen} t; \quad y(0)=0; \quad y'(0)=0$

EVALUAR

$y'' + y = \cos t; \quad y(0)=0; \quad y'(0)=0$

① $\mathcal{L}$

$$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \mathcal{L}\{\cos t\} \ldots (1)$$

Donde

$$\mathcal{L}\{y''\} = s^2Y(s) - \cancel{sy(0)}^{0} - \cancel{y'(0)}^{0}$$

$$= s^2Y(s)$$

$$\mathcal{L}\{y\} = Y(s)$$

$$\mathcal{L}\{\cos t\} = \frac{s}{s^2+1}$$

Sust. en (1)

$$s^2Y(s) + Y(s) = \frac{s}{s^2+1} \quad \text{Ec. Algc.}$$

② Despejar $Y(s)$

$$Y(s)(s^2+1) = \frac{s}{s^2+1}$$

$$\therefore \boxed{Y(s) = \frac{s}{(s^2+1)^2} \quad \text{Sol. ec. alg.}}$$

③ $\mathcal{L}^{-1}$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} \ldots (2)$$

Donde: $\mathcal{L}^{-1}\{Y(s)\} = Y(t)$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\cdot\frac{s}{s^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} \cdot \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}$$

$$= \operatorname{sen} t * \cos t = \int_0^t \underbrace{\operatorname{sen}\tau}_{A}\,\cos\underbrace{(t-\tau)}_{B}\,d\tau$$

$$\boxed{\begin{array}{l} \operatorname{sen}(A+B) = \operatorname{sen}A\cos B + \cos A \operatorname{sen} B \\ \operatorname{sen}(A-B) = \operatorname{sen}A\cos B - \cos A\operatorname{sen} B \\ \hline \operatorname{sen}(A+B) + \operatorname{sen}(A-B) = 2\operatorname{sen}A\cos B \\ \operatorname{sen}A\cos B = \frac{1}{2}[\operatorname{sen}(A+B) + \operatorname{sen}(A-B)] \end{array}}$$

$$= \frac{1}{2}\left[\operatorname{sen} t\int_0^t d\tau + \int_0^t \operatorname{sen}(2\tau - t)\,d\tau\right]$$

$$= \frac{1}{2}\left[\tau\Big|_0^t \operatorname{sen} t - \frac{1}{2}\cos(2\tau - t)\Big|_0^t\right]$$

$$= \frac{1}{2}\left[t\operatorname{sen} t - \frac{1}{2}(\cos t - \underbrace{\cos(-t)}_{\cos t})\right]$$

$$= \frac{1}{2}\left[t\operatorname{sen} t - \frac{1}{2}\cancel{(\cos t - \cos t)}^{0}\right]$$

$$= \frac{1}{2}t\operatorname{sen} t$$

$\therefore$ Sust. en (2)

$$Y(t) = \frac{1}{2}t\operatorname{sen} t$$ //

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23 $\mathcal{L}\{(t-1)u(t-1)\}$

$$e^{-s}\mathcal{L}\{t\} = e^{-s}\frac{1}{s^2}$$

25. $\mathcal{L}\{t\,u(t-2)\}$

$$e^{-2s}\mathcal{L}\{t+2\}$$

$$e^{-2s}\frac{1}{s^2} + \frac{2e^{-2s}}{s}$$

$$(t-1)^2 = (t^2-2t+1)(t-1)$$

$$\begin{array}{l} t^3 - t^2 \\ -2t^2 + 2t \\ \quad + t - 1 \\ \hline t^3 - 3t^2 + 3t - 1 \end{array}$$

27. $\mathcal{L}\{\cos 2t\,u(t-\pi)\}$

$$e^{-\pi s}\mathcal{L}\{\cos 2t\}$$

$$e^{-\pi s}\frac{s}{s^2+4}$$

*29. $\mathcal{L}\{(t-1)^3 e^{t-1} u(t-1)\} \Rightarrow e^{-s}\mathcal{L}\{e^{t-1}\}$

$$t^3 - t$$

$$\mathcal{L}\{t^3e^{t-1} - 3t^2e^{t-1} + 3te^{t-1}$$

31 $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\}$ $\frac{1}{2}(t-2)^2u(t-2)$

33 $\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\}$ $\cos(t-\pi)u(t-\pi)$

③ $\mathcal{L}^{-1}$

$$\mathcal{L}^{-1}\{Y(s)\}=\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+2)}\right\}-\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s^2(s+2)}\right\}-\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+2)}\right\}\cdots(2)$$

DONDE $\mathcal{L}^{-1}\{Y(s)\}=y(t)$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+2)}\right\}=\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}\cdot\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$$

$$=t*e^{-2t}=\int_0^t \tau e^{-2(t-\tau)}\,d\tau$$

$$\Rightarrow e^{-2t}\int_0^t \tau e^{2\tau}\,d\tau$$

$u=\tau;\ dv=e^{2\tau}d$

$du=d\tau;\ v=\frac{e^{2\tau}}{2}$

$$\Rightarrow e^{-2t}\left[\frac{\tau e^{2\tau}}{2}\Big|_0^t-\frac{1}{2}\int e^{2\tau}d\tau\right]$$

$$=e^{-2t}\left[\frac{\tau e^{2\tau}}{2}-\frac{e^{2\tau}}{4}\right]_0^t$$

$$=e^{-2t}\left[\frac{te^{2t}}{2}-\frac{e^{2t}}{4}+\frac{1}{4}\right]$$

$$=\frac{t}{2}-\frac{1}{4}+e^{-2t}\ //$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s^2(s+2)}\right\}=\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+2)}\right\}\Big|_{t\to t-1}u(t-1)$$

$$=\left(\frac{1}{2}t-\frac{1}{4}+e^{-2t}\right)\Big|_{t\to t-1}u(t-1)$$

$$=\left(\frac{1}{2}(t-1)-\frac{1}{4}+e^{-2(t-1)}\right)u(t-1)\ //$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+2)}\right\}=\mathcal{L}^{-1}\left\{\frac{1}{s(s+2)}\right\}\Big|_{s\to s-1}u(t-1)$$

DONDE

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s+2)}\right\}=\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$$

$$=1*e^{-2t}=e^{-2t}*1=\int_0^t e^{-2\tau}d\tau$$

$$=\frac{-e^{-2\tau}}{2}\Big|_0^t=\frac{-e^{-2t}}{2}+\frac{1}{2}=\frac{1}{2}\left[1-e^{-2t}\right]$$

$$=\frac{1}{2}(1-e^{-2t})\Big|_{t\to t-1}u(t-1)=\frac{1}{2}(1-e^{-2(t-1)})u(t-1)$$

Sost. en (2)

$$y(t)=\frac{1}{2}t-\frac{1}{4}+e^{-2t}-\left(\frac{1}{2}(t-1)-\frac{1}{4}+e^{-2(t-1)}\right)u(t-1)-\frac{1}{2}(1-e^{-2(t-1)})u(t-1)$$

$$=\begin{cases}\frac{1}{2}t-\frac{1}{4}+e^{-2t}; & 0\le t<1\\ \cancel{\frac{1}{2}t}-\cancel{\frac{1}{4}}+e^{-2t}-\cancel{\frac{1}{2}(t-1)}+\cancel{\frac{1}{4}}-\cancel{e^{-2(t-1)}}-\frac{1}{2}+\cancel{e^{-2(t-1)}}; & t\ge 1\end{cases}$$

$$\begin{cases}\frac{1}{2}t-\frac{1}{4}+e^{-2t}; & 0\le t<1\\ e^{-2t}-\frac{1}{4}e^{-2(t-1)}; & t\ge 1\end{cases}$$

$$f(t) = \begin{cases} 0; & 0 \leq t < 1 \\ (t-1)^3 e^{t-1}; & t \geq 1 \end{cases}$$

Segundo teorema de traslación en su forma alternativa

Si la función $g(t)$ carece de la forma $f(t-a)$ desplazada que se requiere en el teorema anterior, entonces

$$\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$$

$$\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t)\Big|_{t \to t+a}\} = e^{-as}\mathcal{L}\{g(t+a)\}$$

Ejemplos:

- $\mathcal{L}\{t\,u(t-2)\} = e^{-2s}\mathcal{L}\{t+2\} = e^{-2s}\left[\frac{1}{s^2} + \frac{2}{s}\right]$

- $\mathcal{L}\{\cos t\, u(t-\frac{\pi}{2})\} = e^{-\frac{\pi}{2}s}\mathcal{L}\{\cos(t+\frac{\pi}{2})\} \Rightarrow$

$f(t) = \cos t$

$f(t+\frac{\pi}{2}) = \cos(t+\frac{\pi}{2})$

Pero $\cos(t+\frac{\pi}{2})$

$= \cancel{\cos t \cos\frac{\pi}{2}} - \operatorname{sen} t\, \cancel{\operatorname{sen}\frac{\pi}{2}}$

$= -\operatorname{sen} t$

$\Rightarrow e^{-\frac{\pi}{2}s}\mathcal{L}\{-\operatorname{sen} t\} = -e^{-\pi/2}\mathcal{L}\{\operatorname{sen} t\} = -e^{-\pi/2\, s}\frac{1}{s^2+1}$

$= -\frac{e^{-\pi/2\, s}}{s^2+1}$

$$f(t) = \begin{cases} 0; & 0 \leq t < \frac{\pi}{2} \\ \cos t; & t \geq \frac{\pi}{2} \end{cases}$$

Con $\mathcal{L}$

$$\mathcal{L}\{f(t)\} = \int_0^{\pi/2} e^{-st}(0)\,dt + \int_{\pi/2}^{+\infty} e^{-st}\cos t\,dt$$

$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$

$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$

$\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}g(t+a)\}$

Mañana en 15

Examen :0

Derivadas de transformadas

Teorema:

Si $\mathcal{L}\{f(t)\} = F(s)$ y $n = 1, 2, 3, \dots$

$$\boxed{\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)}$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} \mathcal{L}\{f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$\mathcal{L}\{t e^{3t}\} = (-1)^1 \frac{d}{ds} \mathcal{L}\{e^{3t}\} = -\frac{d}{ds}\left[\frac{1}{s-3}\right]$$

$f(t) = e^{3t}$

$n = 1$

$$= -\left[-(s-3)^{-2} \cdot (1)\right] = \frac{1}{(s-3)^2}$$

$$\mathcal{L}\{e^{3t} t\} = \mathcal{L}\{t\} = \frac{1}{s^2}\Bigg|_{s \to s-3} = \frac{1}{(s-3)^2}$$

$f(t) = t$

— caso ESPECIAL

$$\mathcal{L}\{t^n e^{at}\} = (-1)^n \frac{d^n}{ds^n} \mathcal{L}\{e^{at}\} = (-1)^n \frac{d^n}{ds^n}\left[\frac{1}{s-a}\right]$$

Al reves

$$\mathcal{L}\{e^{at} t^n\} = \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}\Bigg|_{s \to s-a} = \frac{n!}{(s-a)^{n+1}}$$

$\mathcal{L}\{t$

$n = 1$

donde

$\mathcal{L}\{e^{-t}$

$$\frac{d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] =$$

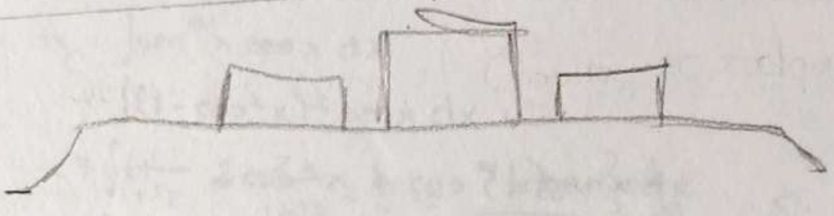
$$= -\left[\frac{(s+1)^2 + 1 - (s+1) \cdot 2(s+1)(1)}{[(s+1)^2+1]^2}\right]$$

$\mathcal{L}\{e^{-t} t \cos t\}$ Ejercicio

$\mathcal{L}\{(t-1)^3 e^{t-1} u(t-1)\}$ Con $\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} \ldots$

$\mathcal{L}\{t^3 e^{t-1} u(t-1)\} = (-1)^3 \frac{d^3}{ds^3} \mathcal{L}\{e^{t-1} u(t-1)\}$

Aprendice.



2 clases después a D

CASO ESPECIAL DE CONVOLUCIÓN

Si en (1), $g(t) = 1$

$$F*(1) = \int_0^t f(\tau)\cdot(1)\,d\tau = \int_0^t f(t)\,dt$$

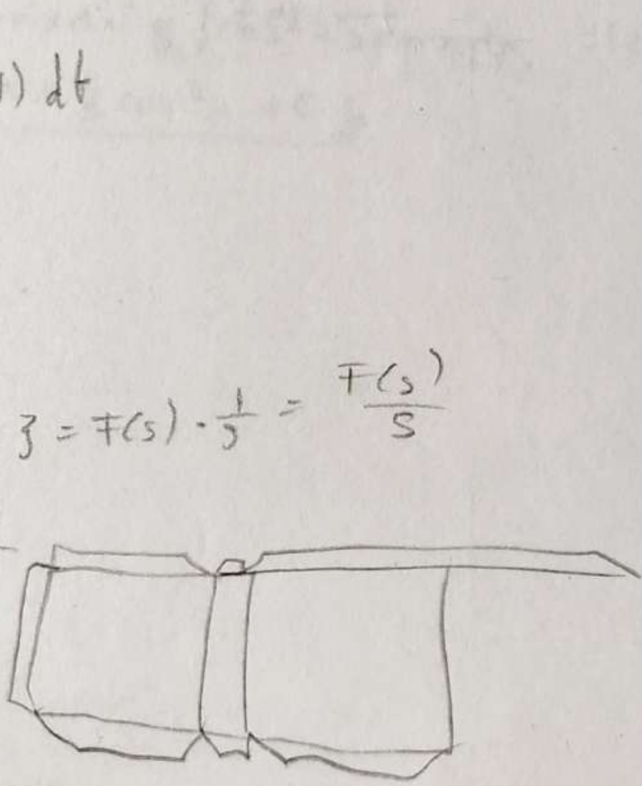
$g(t) = 1$

$g(t-\tau) = 1$

Por otro lado:

$$\mathcal{L}\{f*1\} = \mathcal{L}\{f(t)\}\cdot\mathcal{L}\{1\} = F(s)\cdot\frac{1}{s} = \frac{F(s)}{s}$$

$$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$



C

Transformada de una integral

Teorema Si $f(t)$ es una función continua tramo por tramo en $[0,+\infty)$ y de orden exponencial, entonces:

$$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}$$

$$\left\{\int_0^t f(\tau)d\tau\right\} = \frac{\mathcal{L}\{f(t)\}}{s} = \frac{F(s)}{s}$$

Ejemplos

Evaluar

$$\mathcal{L}\left\{\int_0^t \cos\tau\, d\tau\right\} = \frac{\mathcal{L}\{\cos t\}}{s} = \frac{1}{s}\cdot\frac{s}{s^2+1} = \frac{1}{s^2+1}$$

Donde

$$\mathcal{L}\{\cos t\} = \frac{s}{s^2+1}$$

$$F(s) = \frac{1}{s^2+1} \longrightarrow \operatorname{sen} t$$

Tarea 3 y 4

1. $\int sen^m x \cos^n x \, dx$ Con m ó n enteros positivos impares

Supongamos que n es impar Si n = 1, la integral es de la forma:

i) $\int sen^m x \cos x \, dx$

1 $\int sen^2 x \cos^5 x \, dx = \int sen^2 x \cos^4 x \cos x \, dx$

$= \int sen^2 x (1 - sen^2 x)^2 \cos x \, dx$

$= \int (sen^2 x - 2 sen^4 x + sen^6 x) \cos x \, dx$

$= \frac{1}{3} sen^3 x - \frac{2}{5} sen^5 x + \frac{1}{7} sen^7 x + C$

2. $\int sen^5 x \, dx = \int sen^4 x \, sen x \, dx$

$= \int (1 - \cos^2 x)^2 sen x \, dx$

$= \int (1 - 2\cos^2 x + \cos^4 x) sen x \, dx$

$= \int sen x \, dx - 2\int \cos^2 x \, sen x \, dx + \int \cos^4 x \, sen x \, dx$

$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

3. $\int sen^3 x \, dx = \int sen^2 x \, sen x \, dx = \int (1 - \cos^2 x) sen x \, dx$

$= \int sen x \, dx + \int \cos^2 x \, sen x \, dx$

$= -\cos x + \frac{1}{3}\cos^3 x + C$

R.A.K.

Respuestas Pag. 9 Zill, 6ta Ed. Sec. 1.1 1-10

1) lineal de orden n
2) lineal de orden 3
3) lineal, orden 1
4) lineal, orden 1
5) lineal orden n
6) No-lineal, orden 2
7) lineal, orden 2
8) lineal, orden 2
9) lineal, orden 1
10) lineal, orden 1.

Ej. 11 al 40 (impares)

11. $2y' + y = 0; \quad y = e^{-x/2} \Rightarrow y' = -e^{-x/2}\,\frac{1}{2}$

$$\cancel{2}\left(-\frac{1}{\cancel{2}}e^{-x/2}\right) + e^{-x/2} = 0$$

$$-\cancel{e^{-x/2}} + \cancel{e^{-x/2}} = 0$$

$$\underline{0 = 0}$$

$\therefore$ Si es solución.

13. $\frac{dy}{dx} - 2y = e^{3x}; \quad y = e^{3x} + 10e^{2x} \Rightarrow y' = 3e^{3x} + 20e^{2x}$

$$3e^{3x} + 20e^{2x} - 2(e^{3x} + 10e^{2x}) = e^{3x}$$

$$3e^{3x} + \cancel{20e^{2x}} - 2e^{3x} - \cancel{20e^{2x}} = e^{3x}$$

$$\underline{e^{3x} = e^{3x}}$$

$\therefore$ Si es Sol.

15. $y' = 25 + y^2; \quad y = 5\tan 5x \Rightarrow y' = 25\sec^2 5x$

$$25\sec^2 5x = 25 + (5\tan 5x)^2$$

$$25\sec^2 5x = 25 + 25\tan^2 5x$$

$$25\sec^2 5x = 25[1 + \tan^2 5x)$$

$$\underline{25\sec^2 5x = 25\sec^2 5x}$$

$\therefore$ Si es sol.

17. $y' + y = \operatorname{sen} x; \quad y = \frac{1}{2}\operatorname{sen} x - \frac{1}{2}\cos x + 10e^{-x}$

$$\Rightarrow y' = \frac{1}{2}\cos x + \frac{1}{2}\operatorname{sen} x - 10e^{-x}$$

$$\cancel{\tfrac{1}{2}\cos x} + \frac{1}{2}\operatorname{sen} x - \cancel{10e^{-x}} + \frac{1}{2}\operatorname{sen} x - \cancel{\tfrac{1}{2}\cos x} + \cancel{10e^{-x}} = \operatorname{sen} x$$

$$\frac{1}{2}\operatorname{sen} x + \frac{1}{2}\operatorname{sen} x = \operatorname{sen} x$$

$$\underline{\operatorname{sen} x = \operatorname{sen} x}$$

$\therefore$ Si es sol.

19) $x^2 dy + 2xy\,dx = 0$; $\quad y = -\frac{1}{x^2}$ $\quad y' = 2x^{-3} = \frac{2}{x^3}$

$\frac{1}{dx}\left(x^2 dy + 2xy\,dx = 0\right)$

$$x^2 \frac{dy}{dx} + 2xy = 0$$

$$x^2\left(\frac{2}{x^3}\right) + 2x\left(-\frac{1}{x^2}\right) = 0 \quad \therefore \text{ Si es sol.}$$

$$\frac{2}{x} - \frac{2}{x} = 0$$

2) $y = 2xy' + y(y')^2$; $\quad y^2 = C_1\left(x + \frac{1}{4}C_1\right)$

23. $y' - \frac{1}{x}y = 1$; $\quad y = x\ln x, \; x > 0$

$$y' = x\frac{1}{x} + \ln x = 1 + \ln x$$

$$1 + \ln x - \frac{1}{x}(x\ln x) \stackrel{?}{=} 1$$

$$1 + \ln x - \cancel{\ln x} = 1$$

$$1 = 1 \quad \text{Si es sol.}$$

25. $\frac{dx}{dt} = (2-x)(1-x)$; $\quad \ln\frac{2-x}{1-x} = t \Rightarrow \ln(2-x) - \ln(1-x) = t$

$$t' = \frac{-1}{2-x} - \left(\frac{-1}{1-x}\right)$$

$$t' = \frac{-1 + \cancel{x} - (-2 - \cancel{x})}{(2-x)(1-x)}$$

$$t' = \frac{1}{(2-x)(1-x)}$$

$$(2-x)(1-x)\frac{dt}{dx} = 1$$

$$\cancel{(2-x)(1-x)}\left(\frac{1}{\cancel{(2-x)(1-x)}}\right) = 1$$

$$1 = 1$$

27. $(x^2+y^2)\,dx + (x^2 - xy)\,dy = 0 \qquad c_1(x+y)^2 = x e^{y/x}$

29. $y'' - 6y' + 13y = 0; \qquad y = e^{3x}\cos(2x)$

$y' = e^{3x}[-\operatorname{sen}(2x)(2) + \cos 2x\,(3e^{3x})$

$y' = -2e^{3x}\operatorname{sen}(2x) + 3e^{3x}\cos 2x$

$y'' = -2e^{3x}\cos(2x)(2) + \operatorname{sen}(2x)(-6e^{3x}) + 3e^{3x}[-\operatorname{sen}(2x)(2)] + \cos(2x)\,9e^{3x}$

$= -4e^{3x}\cos(2x) - 6e^{3x}\operatorname{sen}(2x) - 6e^{3x}\operatorname{sen}(2x) + 9e^{3x}\cos(2x)$

$= 5e^{3x}\cos(2x) - 12e^{3x}\operatorname{sen}(2x)$

$5e^{3x}\cos(2x) - 12e^{3x}\operatorname{sen}(2x) - 6[-2e^{3x}\operatorname{sen}(2x) + 3e^{3x}\cos(2x)] + 13e^{3x}\cos(2x) = 0$

$5e^{3x}\cos(2x) - 12e^{3x}\operatorname{sen}(2x) + 12e^{3x}\operatorname{sen}(2x) - 18e^{3x}\cos(2x) + 13e^{3x}\cos(2x) = 0$

$\underline{0=0}$ $\therefore$ Si es sol.

31. $y'' = y; \qquad y = \cosh x + \operatorname{senh} x$

$y' = \sinh x + \cosh x$

$y'' = \cosh x + \operatorname{senh} x$

$\cosh x + \operatorname{senh} x = \cosh x + \operatorname{senh} x$

$\therefore$ si es sol.

33. $y'' + (y')^2 = 0 \quad y = \ln|x| + C_1 + C_2$

$y' = \frac{1}{x} \quad y'' = -\frac{1}{x^2}$

$-\frac{1}{x^2} + \left(\frac{1}{x}\right)^2 = 0$

$-\frac{1}{x^2} + \frac{1}{x^2} = 0 \qquad \therefore$ Si es Sol.

$\underline{0 = 0}$

35. $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0; \quad y = C_1 + C_2x^{-1}, \quad x > 0$

37. $x^2y'' - 3xy' + 4y = 0 \quad y = x^2 + x^2\ln x, \quad x > 0$

$y' = 2x + 2x\ln x + x^2\frac{1}{x}$

$= 2x + 2x\ln x + x$

$y'' = 2 + 2x\frac{1}{x} + 2\ln x + 1$

$= 3 + 2 + 2\ln x$

$5 + 2\ln x$

$x^2(5 + 2\ln x) - 3(2x + 2x\ln x + x) + 4(x^2 + x^2\ln x) = 0$

$5x^2 + \cancel{2x^2\ln x} - \cancel{6x^2} - 6x\ln x - \cancel{3x^2} + \cancel{4x^2} + 4x^2\ln x = 0$

$0 = 0 \quad \therefore$ Si es Sol.

39. $y''' - 3y'' + 3y' - y = 0; \quad y = x^2e^x$

$y' = x^2e^x + 2xe^x$

$y'' = 2xe^x + x^2e^x + 2e^x + 2xe^x$

$= 4xe^x + 2e^x + x^2e^x$

$y''' = 4xe^x + 4e^x + 2e^x + x^2e^x + 2xe^x$

$= 6xe^x + 6e^x + x^2e^x$

$\Rightarrow 6xe^x + 6e^x + x^2e^x - 3(4xe^x + 2e^x + x^2e^x) + 3(x^2e^x + 2xe^x) - x^2e^x = 0$

$\cancel{6xe^x} + \cancel{6e^x} - 12xe^x - \cancel{6e^x} - \cancel{3x^2e^x} + \cancel{3x^2e^x} + 6xe^x = 0$

$\underline{0 = 0} \quad \therefore$ Si es Sol.

CAP. 2 1-47 (impares)

1) $\frac{dy}{dx} = \text{sen } 5x$

$dy = \text{sen}(5x)\,dx$

$\text{sen}(5x)dx - dy = 0$

$\int \text{sen}(5x)dx - \int dy = 0$

$-\frac{1}{5}\cos(5x) - y = C$

$\frac{1}{5}\cos(5x) + y = C_1$

3) $dx + e^{3x}\,dy = 0$

$dx = -e^{3x}\,dy$

$\frac{dx}{e^{3x}} = -dy \rightarrow \frac{dx}{e^{3x}} + dy = 0$

$\int e^{-3x}\,dx + \int dy = 0$

$-\frac{1}{3}e^{-3x} + y = C$

5) $(x+1)\frac{dy}{dx} = x+6$

$\frac{dy}{dx} = \frac{x+6}{x+1}$ $\quad dy = \left(\frac{x+6}{x+1}\right)dx$

$\int\left(\frac{x+6}{x+1}\right)dx - \int dy = 0$

$\int \frac{(x+1)+5}{(x+1)}dx \int dy = 0$ $\quad \int\left(1 + \frac{5}{x+1}\right)dx - \int dy = 0$

$x + 5\ln|x+1| - y = C$

$x \ln|(x+1)^5| - y = C$

7. $x\frac{dy}{dx} = 4y$

$\frac{dy}{dx} = \frac{4y}{x}$ $\quad \frac{dy}{4y} = \frac{dx}{x}$ $\quad \frac{dx}{x} - \frac{dy}{4y} = 0$

$\int\frac{dx}{x} - \int\frac{dy}{4y} = 0$ $\quad \ln|x| - \frac{1}{4}\ln|y| = 0$ $\quad \ln|x| - \ln|y^{1/4}| = 0$

$\ln\left|\frac{x}{y^{1/4}}\right| = 0$

9. $\frac{dy}{dx} = \frac{y^3}{x^2}$

$\frac{dy}{y^3} = \frac{dx}{x^2}$ $\quad \int\frac{dx}{x^2} - \frac{dy}{y^3} = 0$ $\quad \int x^{-2}dx - \int y^{-3}dy = 0$

$-\frac{1}{x} + \frac{1}{2}y^{-2} + C$

11. $\frac{dx}{dy} = \frac{x^2y^2}{1+x}$

$\frac{dx}{dy} = \frac{y^2}{1}\cdot\frac{x^2}{(1+x)}$ $\quad \frac{(1+x)}{x^2}dx = y^2\,dy$

$\int\frac{(1+x)dx}{x^2} - y^2\,dy = 0$ $\quad \int\frac{dx}{x^2} + \int\frac{dx}{x} - \int y^2\,dy = 0$

$\int x^{-2}\,dx + \int \frac{dx}{x} - \int y^2\,dy = 0 \qquad -\frac{1}{x} + \ln|x| - \frac{1}{3}y^3 = C$

13. $\frac{dy}{dx} = e^{3x+2y} \qquad \frac{dy}{dx} = e^{3x}\cdot e^{2y} \qquad \frac{dy}{e^{2y}} = e^{3x}\,dx \qquad e^{3x}\,dx - \frac{dy}{e^{2y}} = 0$

$\int e^{3x}\,dx - \int e^{-2y}\,dy = 0 \qquad \frac{1}{3}e^{3x} + \frac{1}{2}e^{-2y} = C$

15.

19. $f(t) = 2t^4$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2t^4\}$$
$$= 2\,\mathcal{L}\{t^4\}$$
$$= \frac{2 \cdot 4!}{s^{4+1}} = \frac{48}{s^5}$$

20. $f(t) = t^5$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^5\}$$
$$= \frac{5!}{s^{5+1}} = \frac{120}{s^6}$$

21. $f(t) = 4t - 10$

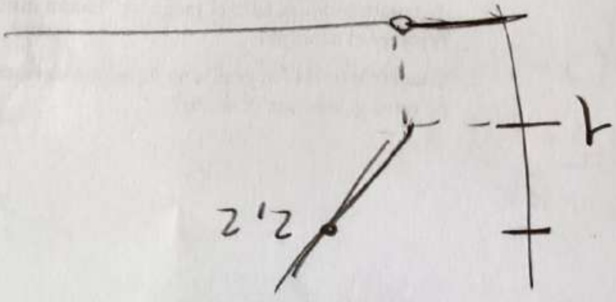
$$\mathcal{L}\{f(t)\} = \mathcal{L}\{4t - 10\}$$
$$= 4\,\mathcal{L}\{t\} - 10\,\mathcal{L}\{1\}$$
$$= \frac{4}{s^2} - \frac{10}{s}$$

22. $f(t) = 7t + 3$

$$\mathcal{L}\{f(t)\} = 7\,\mathcal{L}\{t\} + 3\,\mathcal{L}\{1\}$$
$$= \frac{7}{s^2} + \frac{3}{s}$$

23. $f(t) = t^2 + 6t - 3$

$$\mathcal{L}\{f(t)\} = \frac{2!}{s^{2+1}} + \frac{6}{s^2} - \frac{3}{s}$$



Respuestas del 1 - 10

Tarea 6

1) Lineal de orden n
2) lineal, orden 3
3) Lineal, orden 1
4) lineal, orden 1
5) lineal, orden n
6) No-lineal, orden 2
7) lineal orden 2
8) lineal, orden 2
9) lineal, orden 1
10) lineal, orden 1

11 - 33

11. $2y' + y = 0; \quad y = e^{-x/2} \Rightarrow y' = -e^{-x/2}\frac{1}{2}$

$$\cancel{2}(-e^{-x/2}\tfrac{1}{\cancel{2}}) + e^{-x/2} = 0$$
$$-e^{-x/2} + e^{x/2} = 0$$
$$0 = 0$$ Si es Solución

12. $y' + 4y = 32; \quad y = 8 \Rightarrow y' = 0$

$$0 + 4(8) = 32$$
$$32 = 32$$ Si es solución

13. $\frac{dy}{dx} - 2y = e^{3x}; \quad y = e^{3x} + 10e^{2x} \Rightarrow y' = 3e^{3x} + 20e^{2x}$

$$3e^{3x} + 20e^{2x} - 2(e^{3x} + 10e^{2x}) = e^{3x}$$
$$3e^{3x} + \cancel{20e^{2x}} - 2e^{3x} - \cancel{20e^{2x}} = e^{3x}$$
$$e^{3x} = e^{3x}$$ Si es Solución.

14. $\frac{dy}{dt} + 20y = 24; \quad y = \frac{6}{5} - \frac{6}{5}e^{-20t} \Rightarrow y' = 24e^{-20t}$

$$24e^{-20t} + 20(\tfrac{6}{5} - \tfrac{6}{5}e^{-20t}) = 24$$
$$24e^{-20t} + 20(\tfrac{6}{5})[1 - e^{-20t}) = 24$$
$$24[\cancel{e^{-20t}} + 1 - \cancel{e^{-20t}}) = 24$$
$$24(1) = 24$$
$$24 = 24$$ Sí es solución.

15. $y' = 25 + y^2; \quad y = 5\tan 5x \quad y' = 25\sec^2 5x$

$$25\sec^2 5x = 25 + (5\tan 5x)^2$$
$$25\sec^2 5x = 25 + 25\tan^2 5x$$
$$25\sec^2 5x = 25[1 + \tan^2 5x)$$
$$25\sec^2 5x = 25\sec^2 5x$$ Si es Sol.

16. $\frac{dy}{dx} = \sqrt{\frac{y}{x}}; \quad y = (\sqrt{x} + C)^2, \quad x > 0, C_1 > 0 \Rightarrow y' = \cancel{2}(\sqrt{x} + C)[\frac{1}{\cancel{2}\sqrt{x}}]$

$$y' = \frac{\sqrt{x} + C}{\sqrt{x}}$$

$$\frac{\sqrt{x} + C}{\sqrt{x}} = \sqrt{\frac{(\sqrt{x} + C)^{\cancel{2}}}{x}}$$
$$\frac{\sqrt{x} + C}{\sqrt{x}} = \frac{\sqrt{x} + C}{\sqrt{x}}$$ Es Sol.

R. A. K.

17. $y' + y = \operatorname{sen} x$; $y = \frac{1}{2}\operatorname{sen} x - \frac{1}{2}\cos x + 10e^{-x}$

$\Rightarrow y' = \frac{1}{2}\cos x + \frac{1}{2}\operatorname{sen} x - 10e^{-x}$

$\cancel{\frac{1}{2}\cos x} + \frac{1}{2}\operatorname{sen} x \cancel{- 10e^{-x}} + \frac{1}{2}\operatorname{sen} x \cancel{- \frac{1}{2}\cos x} \cancel{+ 10e^{-x}} = \operatorname{sen} x$

$\frac{1}{2}\operatorname{sen} x + \frac{1}{2}\operatorname{sen} x = \operatorname{sen} x$

$\operatorname{sen} x = \operatorname{sen} x$ Si es Sol.

18. $2xy\,dx + (x^2 + 2y)\,dy = 0$ $x^2y + y^2 = C_1$

$d(x^2y + y^2 = C_1)$

$= 2xy\,dx + x^2\,dy + 2y\,dy = 0$

Es Sol. $\Rightarrow = 2xy\,dx + (x^2 + 2y)\,dy = 0$

19. $x^2 dy + 2xy\,dx = 0$; $y = -\frac{1}{x^2} = -y^{-2} = y$

$x^2dy + 2xy\,dx = 0$ $y' = 2y$ $x^2y + 2x\left(-\frac{1}{x^2}\right) = 0$

$x^2 dy = -2xy\,dx$ $x^2y - \frac{2}{x}$

$x^2\frac{dy}{dx} = -2xy$ $x^2\frac{dy}{dx} + 2xy = 0$

20. $(y')^3 + xy' = y$; $y = x + 1$ $y' = 1$

$1^3 + x(1) = y$

$x + 1 = y$

$y = y$ Si es solución

$x^{1/2}$

21. $2xy' + y(y')^2 = y$; $y^2 = C_1(x + \frac{1}{4}C_1)$ $y = \sqrt{C_1(x + \frac{1}{4}C_1)}$

$2x\left(\frac{C_1}{2\sqrt{x}}\right.$ $\Rightarrow y = \sqrt{C_1(x + \frac{1}{4}C_1)}$ $y = C_1\sqrt{x} + C$

$\frac{C_1x}{\sqrt{x}} +$ $y' = \frac{C_1}{2\sqrt{C_1(x + \frac{1}{4}C_1)}}$ $y = \frac{C_1}{2\sqrt{x}} + 0$

$C_1\sqrt{x} +$

$2x\left(\frac{C_1}{2\sqrt{C_1(x + \frac{1}{4}C_1)}}\right) + \sqrt{C_1(x + \frac{1}{4}C_1)}\left[\frac{C_1}{2\sqrt{C_1(x + \frac{1}{4}C_1)}}\right]^2$

$\frac{xC_1}{\sqrt{C_1(x + \frac{1}{4}C_1)}} + \frac{C_1^2\sqrt{C_1(x + \frac{1}{4}C_1)}}{4[C_1(x + \frac{1}{4}C_1)]} = \frac{x4C_1[C_1(x + \frac{1}{4}C_1)] + C_1^2(C_1(x + \frac{1}{4}C_1)}{\sqrt{C_1(x + \frac{1}{4}C_1)}\;4[C_1(x + \frac{1}{4}C_1)]}$

$= \frac{C_1^3x}{\sqrt{C_1(x + \frac{1}{4}C_1)}}$ No es solución

28. $y'' + y' - 12y = 0 \; ; \quad y = C_1 e^{3x} + C_2 e^{-4x}$

$y' = 3C_1 e^{3x} - 4C_2 e^{-4x}$

$y'' = 9C_1 e^{3x} + 16 C_2 e^{-4x}$

$$9C_1 e^{3x} + 16C_2 e^{-4x} + 3C_1 e^{3x} - 4C_2 e^{-4} - 12C_1 e^{3x} - 12C_2 e^{-4x} \stackrel{?}{=} 0$$

$$12C_1 e^{3x} + 12C_2 e^{-4x} - 12C_1 e^{3x} - 12C_2 e^{-4x} = 0$$

$0 = 0 \quad \therefore$ Si es Sol.

29. $y'' - 6y' + 13y = 0 \; ; \quad y = e^{3x} \cos(2x)$

$y' = e^{3x}[-\operatorname{sen}(2x)(2)] + \cos 2x\,(3e^{3x})$

$y' = -2e^{3x} \operatorname{sen}(2x) + 3e^{3x} \cos 2x$

$$y'' = -2e^{3x}\cos(2x)(2) + \operatorname{sen}(2x)[-6e^{3x}] + 3e^{3x}[-\operatorname{sen}(2x)(2)] + \cos(2x)\,9e^{3x}$$

$$= -4e^{3x}\cos(2x) - 6e^{3x}\operatorname{sen}(2x) - 6e^{3x}\operatorname{sen}(2x) + 9e^{3x}\cos(2x)$$

$$= 5e^{3x}\cos(2x) - 12e^{3x}\operatorname{sen}(2x)$$

$$5e^{3x}\cos(2x) - 12e^{3x}\operatorname{sen}(2x) - 6[-2e^{3x}\operatorname{sen}(2x) + 3e^{3x}\cos(2x)] + 13e^{3x}\cos(2x) = 0$$

$$\cancel{5e^{3x}\cos(2x)} - \cancel{12e^{3x}\operatorname{sen}(2x)} + \cancel{12e^{3x}\operatorname{sen}(2x)} - \cancel{18e^{3x}\cos(2x)} + \cancel{13e^{3x}\cos(2x)} = 0$$

$0 = 0 \quad \therefore$ Si es Sol.

30. $\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 4y = 0 \qquad y = e^{2x} + xe^{2x}$

$y' = 3e^{2x} + 2xe^{2x} \qquad y'' = 4e^{2x} + 4xe^{2x} + 2e^{2x} + 2e^{2x}$

$\qquad = 8e^{2x} + 4xe^{2x}$

$$8e^{2x} + 4xe^{2x} - 4[3e^{2x} + 2xe^{2x}] + 4[e^{2x} + xe^{2x}] = 0$$

$$8e^{2x} + 4xe^{2x} - 12e^{2x} - 8xe^{2x} + 4e^{2x} + 4e^{2x} = 0$$

$$\cancel{12e^{2x}} + \cancel{8xe^{2x}} - \cancel{12e^{2x}} - \cancel{8xe^{2x}} = 0$$

$0 = 0 \quad \therefore$ Si es Sol.

31. $y'' = y \; ; \quad y = \cosh x + \operatorname{senh} x$

$y' = \sinh x + \cosh x$

$y'' = \cosh x + \operatorname{senh} x$

$$\cosh x + \operatorname{senh} x = \cosh x + \operatorname{senh} x$$

$\therefore$ Sí es Solución.

R. A. K.

32. $y'' + 25y = 0$ $\quad y = C_1 \cos 5x$

$y' = -5C_1 \operatorname{sen} 5x$

$y'' = -25C_1 \cos(5x)$

$-25 C_1 \cos 5x + 25 C_1 \cos 5x = 0$

$0 = 0 \quad \therefore$ Si es Solución

33. $y'' + (y')^2 = 0$ $\quad y = \ln|x| + C_1 + C_2$

$y' = \frac{1}{x} \quad y'' = -\frac{1}{x^2}$

$-\frac{1}{x^2} + \left(\frac{1}{x}\right)^2 = 0$

$-\frac{1}{x^2} + \frac{1}{x^2} = 0 \quad \therefore$ Si es Solución

$0 = 0$

$y' = 2x + x^2 \frac{1}{x} + \ln x \, 2x$

$y' = 2x + x + 2x \ln x$

$y'' = 2 + 1 + 2x \frac{1}{x} + 2\ln x$

$y'' = 5 + 2\ln x$

$x^2(5 + 2\ln x) - 3x(2x + x + 2x\ln x) + 4(x^2 + x^2 \ln x)$

$5x^2 + 2x^2 \ln x - 6x^2 - 3x^2 - 6x^2 \ln x + 4x^2 + 4x^2 \ln x$

$0 = 0$

$x y'' + 2y' = 0 \quad y = C_1 + C_2 x^{-1}$

$y' = -C_2 x^{-2} = -\frac{C_2}{x^2}$

$y'' = 2C_2 x^{-3} = \frac{2C_2}{x^3}$

$x\left(\frac{2C_2}{x^3}\right) + 2\left(-\frac{C_2}{x^2}\right) = 0$

$\frac{2C_2}{x^2} - \frac{2C_2}{x^2} = 0 \quad \therefore$ Si es Sol.

$y' = 2xe^x + x^2 e^x$

$y'' = 2xe^x + 2e^x + 2xe^x + x^2e^x = \underline{4xe^x + 2e^x + x^2e^x}$

$y''' = 4xe^x + 4e^x + 2e^x + 2xe^x + x^2e^x$

$4xe^x + 4e^x + 2e^x + 2xe^x + x^2e^x - 3(4xe^x + 2e^x + x^2e^x) + 3(2xe^x + x^2e^x) - x^2e^x = 0$

$6xe^x + 6e^x + x^2e^x - 12xe^x - 6e^x - 3x^2e^x + 6xe^x + 3x^2e^x - x^2 e^x = 0$

$0 = x^2e^x - x^2e^x = 0$

ECUACIÓN DE EULER-CAUCHY

- La ecuación de Cauchy-Euler de orden n, tiene la siguiente forma general...

$$a_n x^n \frac{d^n y}{dx^n} + a_{n-1} x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_1 x \frac{dy}{dx} + a_0 y = g(x)$$

Este tipo de Ecuacion Diferencial se puede a una ED. de coef. cts. si se realiza el sig. cambio de variable.

$$x = e^z$$

>> Derivamos usando la regla de la cadena

$$y' = \frac{dy}{dz} = \frac{\frac{dy}{dz}}{\frac{dx}{dz}} = \frac{\frac{dy}{dz}}{\frac{d}{dz}(e^z)} = \frac{\frac{dy}{dz}}{e^z} = e^{-z}\frac{dy}{dz}$$

>> Al calcular la segunda derivada.

$$y'' = \frac{d^2y}{dx^2} = \frac{\frac{dy'}{dz}}{\frac{dx}{dz}} = \frac{\frac{d}{dz}\left(e^{-z}\frac{dy}{dz}\right)}{\frac{d}{dz}(e^{z})} = \frac{e^{-z}\frac{d^2y}{dz^2} - e^{-z}\frac{dy}{dz}}{e^z} = e^{-2z}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)$$

>> Al calcular la tercer derivada.

$$y''' = \frac{dy''}{dx} = \frac{\frac{dy''}{dz}}{\frac{dx}{dz}} = \frac{\frac{d}{dz}\left[e^{-2z}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)\right]}{\frac{d}{dz}(e^z)}$$

$$= \frac{e^{-2z}\left(\frac{d^3y}{dz^3} - \frac{d^2y}{dz^2}\right) - 2e^{-2z}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)}{e^z} = e^{-3z}\left(\frac{d^3y}{dz^3} - 3\frac{d^2y}{dz^2} + 2\frac{dy}{dz}\right)$$

>> Al calcular la cuarta derivada.

$$y^{iv} = \frac{dy'''}{dx} = \frac{\frac{dy'''}{dz}}{\frac{dx}{dz}} = e^{-4z}\left(\frac{d^4y}{dz^4} - 6\frac{d^3y}{dz^3} + 11\frac{d^2y}{dz^2} - 6\frac{dy}{dz}\right)$$

• Resolver:

$x^3y''' + xy' - y = x \ln x$

Como

$y' = e^{-z}\frac{dy}{dz}$

$y''' = e^{-3z}\left(\frac{d^3y}{dz^3} - 3\frac{d^2y}{dz^2} + 2\frac{dy}{dz}\right)$

Sust. $e^{3t}\left[e^{-3z}\left(\frac{d^3y}{dz^3} - 3\frac{d^2y}{dz^2} + 2\frac{dy}{dz}\right) + \cancel{e^{z}}\left(e^{-z}\frac{dy}{dz}\right) - y = e^{-z}\ln|e^z|\right.$

$= \frac{d^3y}{dz^3} - 3\frac{d^2y}{dz^2} + 2\frac{dy}{dz} + \frac{dy}{dz} - y = ze^z$

$= \frac{d^3y}{dz^3} - 3\frac{d^2y}{dz^2} + 3\frac{dy}{dz} - y = ze^z$

• Ecuación homogénea

$y''' - 3y'' + 3y' - y = 0$

• Ecuación característica

$m^3 - 3m^2 + 3m - 1 = 0$

• Raices

$(m-1)^3 \quad m_{123} = 1$

• Sols. li

$y_1 = e^z;\ y_2 = ze^z;\ y_3 = z^2e^z$

• Sol. gral.

$y_c = C_1e^z + C_2ze^z + C_3z^2e^z$

$$W = \begin{vmatrix} e^z & ze^z & z^2e^z \\ e^z & ze^z + e^z & z^2e^z + 2ze^z \\ e^z & 2e^z + ze^z & z^2e^z + 4ze^z + 2e^z \end{vmatrix}$$

$$= e^z\begin{vmatrix} ze^z + e^z & z^2e^z + 2ze^z \\ ze^z + 2e^z & z^2e^z + 4ze^z + 2e^z \end{vmatrix} - ze^z\begin{vmatrix} e^z & z^2e^z + 2ze^z \\ e^z & z^2e^z + 4ze^z + 2e^z \end{vmatrix}$$

$$+ z^2e^z\begin{vmatrix} e^z & ze^z + e^z \\ e^z & 2e^z + ze^z \end{vmatrix}$$

$e^z[(ze^z+e^z)(z^2e^z+4ze^z+2e^z)-(z^2e^z+2ze^z)(ze^z+2e^z)]$

$=e^z\{[z^3e^{2z}+4z^2e^{2z}+2ze^{2z}+z^2e^{2z}+4ze^{2z}+2e^{2z}]-[z^3e^{2z}+2z^2e^{2z}+2z^2e^{2z}+4ze^{2z}]$

$=e^z[(z^3e^{2z}+5z^2e^{2z}+6ze^{2z}+2e^{2z})-(z^3e^{2z}+4z^2e^{2z}+4ze^{2z})]$

$=e^z[z^2e^{2z}+2ze^{2z}+2e^{2z}]=e^{3z}[z^2+2z+2]$ //

- $ze^z[(e^z)(z^2e^z+4ze^z+2e^z)-(e^z)(z^2e^z+2ze^z)]$

  $=ze^z[(z^2e^{2z}+4ze^{2z}+2e^{2z})-(z^2e^{2z}+2ze^{2z})]$

  $=ze^z[2ze^{2z}+2e^{2z}]=ze^{3z}(2z^2+2z)$ //

- $z^2e^z[(e^z)(2e^z+ze^z)-(e^z)(ze^z+e^z)]$

  $=z^2e^z[(2e^{2z}+ze^{2z})-(ze^{2z}+e^{2z})]=z^2e^z[2e^{2z}-e^{2z}]=e^{3z}[z^2]$

$\Rightarrow W=e^{3z}[z^2+2z+2-2z^2-2z+z^2]=\underline{\underline{2e^{3z}}}$ ///

$$W_1=\begin{vmatrix} 0 & ze^z & z^2e^z \\ 0 & ze^z+e^z & z^2e^z+2ze^z \\ ze^t & ze^z+2e^z & z^2e^z+4ze^z+2e^z \end{vmatrix}$$

$$-ze^z\begin{vmatrix} 0 & z^2e^z+2ze^z \\ ze^t & z^2e^z+4ze^z+2e^z \end{vmatrix}+z^2e^z\begin{vmatrix} 0 & ze^z+e^z \\ ze^t & ze^z+2e^z \end{vmatrix}$$

- $-ze^z[-(ze^t)(z^2e^z+2ze^z)]=ze^z[zz^2e^{z+t}+2tze^{z+t}]$

  $=e^{2z}[tz^3e^t+2z^2te^t]$

- $z^2e^z[-(te^t)(ze^z+e^z)]=-z^2e^z[zte^{z+t}+te^{z+t}]=-e^{2z}[z^3te^t+z^2te^t]$

  $e^{2z}[tz^3e^t+2z^2te^t-z^3te^t-z^2te^t]=e^{2z+t}\,z^2t=e^{3z}z^3$

$$W_2 = \begin{vmatrix} e^z & 0 & z^2e^z \\ e^z & 0 & z^2e^z+2ze^z \\ e^z & ze^z & z^2e^z+4ze^z+2e^z \end{vmatrix}$$

$$= e^z \begin{vmatrix} 0 & z^2e^z+2ze^z \\ ze^z & z^2e^z+4ze^z \end{vmatrix} + z^2e^z \begin{vmatrix} e^z & 0 \\ e^z & ze^z \end{vmatrix}$$

$$\cdot\ e^z\left[-(z^3e^{2z}+2z^2e^{2z})\right] = -e^{3z}\left[z^3+2z^2\right]$$

$$\cdot\ z^2e^z(e^z \cdot ze^z) = z^2e^z(ze^{2z}) = e^{3z}z^3 \quad = \underline{-2e^{3z}z^2}$$

$$W_3 = \begin{vmatrix} e^z & ze^z & 0 \\ e^z & ze^z+e^z & 0 \\ e^z & ze^z+2e^z & ze^z \end{vmatrix} = e^z \begin{vmatrix} ze^z+e^z & 0 \\ ze^z+2e^z & ze^z \end{vmatrix} + ze^z \begin{vmatrix} e^z & 0 \\ e^z & ze^z \end{vmatrix}$$

$$\cdot\ e^z\left[(ze^z+e^z)(ze^z)\right] = e^z\left[z^2e^{2z}+ze^{2z}\right] = \underline{e^{3z}\left[z^2+z\right]}$$

$$\cdot\ ze^z\left[e^z \cdot ze^z\right] = ze^z \cdot ze^{2z} = e^{3z}z^2$$

$$\Rightarrow e^{3z}\left[z^2+z-z^2\right] = \underline{ze^{3z}}$$

$$u_1 = \int \frac{W_1}{W}\,dz = \int \frac{e^{3z}z^3}{2e^{3z}} = \frac{1}{2}\int z^3\,dz = \frac{1}{2}\cdot\frac{z^4}{4} = \underline{\frac{1}{8}z^4}$$

$$u_2 = \int \frac{W_2}{W}\,dz = \int \frac{-\cancel{2}e^{3z}z^2}{\cancel{2}e^{3z}} = -\int z^2\,dz = \underline{-\frac{1}{3}z^3}$$

$$u_3 = \int \frac{W_3}{W}\,dz = \int \frac{e^{3z}z^2}{2e^{3z}} = \frac{1}{2}\int z^2\,dz = \underline{\frac{1}{6}z^3}$$

$$y_p = \frac{1}{8}z^4 - \frac{1}{3}z^3 + \frac{1}{6}z^3 = \frac{1}{8}z^4 - \frac{1}{6}z^3 = \frac{1}{8}\ln^4|x| - \frac{1}{6}\ln^3|x|$$

$$y_c = c_1e^z + c_2ze^z + c_3z^2e^z = c_1e^{\ln|x|} + c_2\ln|x|e^{\ln|x|} + c_3\ln^2|x|e^{\ln|x|}$$

$$= c_1x + c_2\ln|x|\,x + c_3x\ln^2|x|$$

$$\underline{y = c_1x + c_2x\ln|x| + c_3x\ln^2|x| + \frac{1}{8}\ln^4|x| - \frac{1}{6}\ln^3|x|}$$

$x^4 y^{(iv)} + 6x^3 y''' + 7x^2 y'' + xy' - y = 1$

Sust. $e^{4z}\left[e^{-4z}\left(\frac{d^4y}{dz^4} - 6\frac{d^3y}{dz^3} + 11\frac{d^2y}{dz^2} - 6\frac{dy}{dz}\right)\right] + 6e^{3z}\left[e^{-3z}\left(\frac{d^3y}{dz^3} - 3\frac{d^2y}{dz^2} + 2\frac{dy}{dz}\right)\right]$

$+ 7e^{2z}\left[e^{-2z}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)\right] + e^{z}\left[e^{-z}\frac{dy}{dz}\right] - y = 1$

$= \frac{d^4y}{dz^4} - 6\frac{d^3y}{dz^3} + 11\frac{d^2y}{dz^2} - 6\frac{dy}{dz} + \frac{6d^3y}{dz^3} - 18\frac{d^2y}{dz^2} + 12\frac{dy}{dz} + 7\frac{d^2y}{dz^2} - \frac{7dy}{dt} + \frac{dy}{dt} - y = 1$

$= \frac{d^4y}{dz^4} - y = 1$

- Ec. homogenea

$y^{iv} - y = 1$

- Ec. característica.

$m^4 - 1 = 0$

- Raices

$(m^2-1)(m^2+1) = 0$

$(m-1)(m+1)(m^2+1)$

$m_1 = 1 \quad m_2 = -1 \quad m_{23} = \pm i$

- Sols. li

$y_1 = e^z;\ y_2 = e^{-z};\ y_3 = \cos z;\ y_4 = \text{sen}(z)$

- Sol. gral.

$y_c = C_1e^z + C_2e^{-z} + C_3\cos z + C_4 \text{sen } z$

$F(x) = 1$

$y_p = A$

$y^{iv} = 0$

Sust.

$(0) + 6(0) + 7(0) + (0) - A = 1$

$-A = 1 \Rightarrow A = -1$

$\therefore y_p = -1$

$y_c = C_1e^{\ln|x|} + C_2e^{-\ln|x|} + C_3\cos(\ln|x|) + C_4 \text{ sen}(\ln|x|)$

$y = C_1x + C_2x + C_3\cos(\ln|x|) + C_4\text{sen}(\ln|x|) - 1$

Zill 2.3 Pag 60 1-50 impares.

(1) $\frac{dy}{dx} = 5y$

$\frac{dy}{dx} - 5y = 0$

$\mu = e^{\int -5\,dx} = e^{-5x}$

$e^{-5x}\left[\frac{dy}{dx} - 5y = 0\right]$

$\int \cancel{\frac{d}{dx}}\left[e^{-5x}y\right] = \int 0$

$e^{-5x}y = C$

$y = Ce^{5x}$

(3) $3\frac{dy}{dx} + 12y = 4 \quad \ldots (1)$

Mult (1) por $\frac{1}{3}$

$\frac{dy}{dx} + 4y = \frac{4}{3}$

$e^{\int 4\,dx} = e^{4x}$

$e^{4x}\left[\frac{dy}{dx} + 4y = \frac{4}{3}\right]$

$\int \cancel{\frac{d}{dx}}\left[e^{4x}\cdot y\right] = \int \frac{4}{3}e^{4x}\,dx$

$e^{4x}y = \frac{\cancel{4}}{3}\frac{1}{\cancel{4}}\int e^{4x}\,4\,dx$

$e^{4x}y = \frac{1}{3}e^{4x} + C$

$y = \frac{1}{3} + Ce^{-4x}$

(5) $\frac{dy}{dx} + y = e^{3x}$

$e^{x}\left[\frac{dy}{dx} + y = e^{3x}\right]$

$\int \cancel{\frac{d}{dx}}\left[e^{x}y\right] = \int e^{4x}$

$e^{x}y = \frac{1}{4}e^{4x} + C$

$y = \frac{1}{4}e^{3x} + Ce^{-x}$

(7) $y' + 3x^2y = x^2$

$\frac{dy}{dx} + 3x^2y = x^2$

$e^{\int 3x^2\,dx} = e^{x^3}$

$e^{x^3}\left[\frac{dy}{dx} + 3x^2y = x^2\right]$

$\int \cancel{\frac{d}{dx}}\left[e^{x^3}y\right] = \int e^{x^3}x^2$

$e^{x^3}y = \frac{1}{3}e^{x^3} + C$

$\therefore y = \frac{1}{3} + Ce^{-x^3}$

(9) $x^2y' + xy = 1$

$x^2\frac{dy}{dx} + xy = 1$ Mult por $\frac{1}{x^2}$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{1}{x^2}$

$e^{\int \frac{1}{x}dx} = e^{\ln|x|} = x$

$x\left[\frac{dy}{dx} + \frac{1}{x}y = \frac{1}{x^2}\right]$

$\int \cancel{\frac{d}{dx}}\left[xy\right] = \int\left[\frac{1}{x}\right]dx$

$xy = \ln|x| + C$

$y = x^{-1}\ln|x| + Cx^{-1}$

(11) $(x + 4y^2)\,dy + 2y\,dx = 0$

$\frac{dx}{dy} + \frac{x + 4y^2}{2y} = 0$

$\frac{dx}{dy} + \frac{1}{2y}x = -\frac{4y^2}{2y}$

$e^{\int \frac{1}{2}\frac{1}{y}dy} = e^{\frac{1}{2}\ln|y|} = e^{\ln|y^{1/2}|} = y^{1/2}$

$y^{1/2}\left[\frac{dx}{dy} + \frac{1}{2y}x = -2y\right]$

$\int \cancel{\frac{d}{dy}}\left[y^{1/2}x\right] = \int -2y^{3/2}$

$y^{1/2}x = -2\frac{y^{5/2}}{5/2}$

$y^{1/2}x = -\frac{1}{5}y^{5/2} + C$

$x = -\frac{1}{5}y^{4/2} + Cy^{-1/2}$

$x = -\frac{1}{5}y^{2} + Cy^{-1/2}$

13 $x\,dy = (x\,\text{sen}\,x - y)\,dx$

$\frac{dy}{dx} - \frac{(x\,\text{sen}\,x - y)}{x} = 0$

$\frac{dy}{dx} - \text{sen}\,x + \frac{1}{x}y = 0$

$\frac{dy}{dx} + \frac{1}{x}y = \text{sen}\,x$

$e^{\int \frac{1}{x}dx} = e^{\ln|x|} = x$

$x\left[\frac{dy}{dx} + \frac{1}{x}y = \text{sen}\,x\right]$

$\int \frac{d}{dx}[xy] = \int x\,\text{sen}\,x$

$xy =$

15 $(1+e^x)\frac{dy}{dx} + e^x y = 0$

$\frac{dy}{dx} + \frac{e^x}{1+e^x}y = 0$

$e^{\int \frac{e^x}{1+e^x}dx} = e^{\ln|1+e^x|} = (1+e^x)$

$1+e^x\left[\frac{dy}{dx} + \frac{e^x}{1+e^x}y = 0\right]$

$\int \frac{d}{dx}[(1+e^x)y] = \int 0$

$(1+e^x)y = C$

$y = \frac{C}{1+e^x}$

17. $\cos x\frac{dy}{dx} + y\,\text{sen}\,x = 1$

$\frac{dy}{dx} + y\frac{\text{sen}\,x}{\cos x} = \frac{1}{\cos x}$

$\frac{dy}{dx} + \tan x\,y = \sec x$

$e^{\int \tan x\,dx} = e^{\ln|\sec x|} = \sec x$

$\sec x\left[\frac{dy}{dx} + \tan x\,y = \sec x\right]$

$\int \frac{d}{dx}[\sec x\,y] = \int \sec^2 x$

$\sec x\,y = \tan x + C$

$y = \frac{\tan x}{\sec x} + \frac{C}{\sec x}$

$y = \text{sen}\,x + C\cos x$

19. $x\frac{dy}{dx} + 4y = x^3 - x$

$\frac{dy}{dx} + \frac{4}{x}y = x^2 - 1$

$e^{\int \frac{4}{x}dx} = e^{4\ln|x|} = e^{\ln|x^4|} = x^4$

$x^4\left[\frac{dy}{dx} + \frac{4}{x}y = x^2 - 1\right]$

$\int \frac{d}{dx}[x^4 y] = \int x^6 - x^4$

$x^4 y = \frac{x^7}{7} - \frac{x^5}{5} + C$

$y = \frac{1}{7}x^3 - \frac{1}{5}x + Cx^{-4}$

21. $x^2y' + x(x+2)y = e^x$

$\frac{dy}{dx} + \frac{(x+2)y}{x} = \frac{e^x}{x^2}$

$e^{\int (1 + \frac{2}{x})dx} = e^{x + \ln|x^2|} = x^2e^x$

$x^2e^x\left[\frac{dy}{dx} + \left(\frac{x+2}{x}\right)y = \frac{e^x}{x^2}\right]$

$\int \frac{d}{dx}[x^2e^x y] = \int e^{2x}$

$$e^x x^2 y = \frac{1}{2}e^{2x} + C$$

$$y = \frac{\frac{1}{2}e^{2x} + C}{e^x x^2}$$

$$y = \frac{1}{2}e^x x^{-2} + Ce^{-x}x^{-2}$$

(23) $\cos^2 x \,\text{sen}\, x \, dy + (y\cos^3 x - 1)dx = 0$

$$\frac{dy}{dx} + \frac{y\cos^3 x - 1}{\cos^2 x \,\text{sen}\, x} = 0$$

$$\frac{dy}{dx} + \frac{y\cos^3 x}{\cos^2 x \,\text{sen}\, x} = \frac{1}{\cos^2 x \,\text{sen}\, x}$$

$$\frac{dy}{dx} + \underbrace{\frac{\cos x}{\text{sen}\, x}}_{\cot x} y = \sec^2 x \frac{1}{\text{sen}\, x}$$

$$e^{\int \cot x \, dx} = e^{\ln|\text{sen}\, x|} = \text{sen}\, x$$

$$\text{sen}\, x\left[\frac{dy}{dx} + \cot x \cdot y = \sec^2 x \frac{1}{\text{sen}\, x}\right]$$

$$\int \frac{d}{dx}[\text{sen}\, x \, y] = \int \sec^2 x$$

$$\text{sen}\, x \, y = \tan x + C$$

$$y = \frac{\tan x}{\text{sen}\, x} + \frac{1}{\text{sen}\, x}C$$

$$y = \sec x + C\csc x$$

(25) $y\,dx + (xy + 2x - ye^y)dy = 0$

$$\frac{dx}{dy} + \frac{xy + 2x - ye^y}{y} = 0$$

$$\frac{dx}{dy} + x + \frac{2x}{y} = e^y$$

$$e^{\int(1 + \frac{2}{y})dy} = e^{y + \ln|y^2|} = e^y y^2$$

$$e^y y^2\left[\frac{dx}{dy} + \left(1 + \frac{2}{y}\right)x = e^y\right]$$

$$\int \frac{d}{dy}[e^y y^2 x] = \int e^{2y} y^2 \, dy$$

$u = y^2 \quad du = 2y$

$dv = e^{2y}dy \quad v = \frac{1}{2}e^{2y}$

$$e^y y^2 x = y^2 \frac{1}{2}e^{2y} - \int \frac{1}{2}e^{2y} 2y \, dy$$

$w = y \quad dw = 1$

$dz = e^{2y} \quad z = e^{2y}$

$$\to e^y y^2 x = \frac{1}{2}y^2 e^{2y} - \frac{1}{2}\left[ye^{2y} - \int e^{2y}dy\right]$$

$$e^y y^2 x = \frac{1}{2}y^2 e^{2y} - \frac{1}{2}ye^{2y} + \frac{1}{4}e^{2y} + C$$

$$\to x = \frac{1}{2}e^y - \frac{1}{2y}e^y + \frac{1}{4y^2}e^y + \frac{C}{y^2}e^{-y}$$

(27). $x\frac{dy}{dx} + (3x + 1)y = e^{-3x}$

$$\frac{dy}{dx} + \left(\frac{3x+1}{x}\right)y = e^{-3x}x^{-1}$$

$$e^{\int(3 + \frac{1}{x})dx} = e^{3x}x$$

$$\int \frac{d}{dx}[e^{3x}x\,y] = \int 1$$

$$e^{3x}x\,y = x + C$$

$$y = e^{-3x} + Ce^{-3x}x^{-1}$$

(29) $y\,dx - 4(x + y^6)dy = 0$

$$\frac{dx}{dy} - \frac{4x}{y} + \frac{4y^6}{y} = 0 \qquad -4y^5$$

$$e^{\int -\frac{4}{y}dy} = e^{\ln|y^{-4}|} = y^{-4}$$

$$\int \frac{d}{dy}[y^{-4}x] = \int -4y$$

$$y^{-4}x = -\frac{4y^2}{2} = -2y^2 + C$$

$$x = -2y^6 + Cy^4$$

(31) $\frac{dy}{dx} + y = \frac{1 - e^{-2x}}{e^x + e^{-x}}$

$$e^x\left[\frac{dy}{dx} + y = \frac{1 - e^{-2x}}{e^x + e^{-x}}\right]$$

$$\int \frac{d}{dx}[e^x y] = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$e^x y = \ln|-2(e^{2x} + 1)| - x + C$$

$y = e^{-x}\ln|-2(e^{2x}+1)| - xe^{-x} + Ce^{-x}$

33) $y\,dx + (x + 2xy^2 - 2y)\,dy = 0$

$$\frac{dx}{dy} + \frac{x + 2xy^2 - 2y}{y} = 0$$

$$\frac{dx}{dy} + \frac{x}{y} + 2xy = 2y$$

$$\frac{dx}{dy} + x\left(\frac{1}{y} + 2y\right) = 2y$$

$$e^{\int\left(\frac{1}{y} + 2y\right)dy} = e^{\ln|y| + y}$$

$$ye^{y}\left[\frac{dx}{dy} + \left(\frac{1}{y} + 2y\right)x = 2y\right]$$

$$\int \frac{d}{dy}\left[ye^{y}x\right] = 2y^2e^{y}$$

$u = y^2 \quad du = 2y\,dy$

$dv = e^{y}dy \quad v = e^{y}$

$$ye^{y}x = 2e^{y}y^2 - 2\int e^{y}\,2y\,dy$$

$u = y \quad du = dy$

$dv = e^{y}dy \quad v = e^{y}$

$$ye^{y}x = 2e^{y}y^2 - 4ye^{y} + 4\int e^{y}dy$$

$$ye^{y}x = 2e^{y}y^2 - 4ye^{y} + 4e^{y} + C$$

$$x = 2y - 4 + \frac{4}{y} + Cy^{-1}e^{-y}$$

35) $\frac{dr}{d\theta} + r\sec\theta = \cos\theta$

$$e^{\int\sec\theta\,d\theta} = e^{\ln|\sec\theta + \tan\theta|}$$

$$= \sec\theta + \tan\theta$$

$$\int \frac{d}{d\theta}\left[(\sec\theta + \tan\theta)r\right] = \int 1 + \operatorname{sen}\theta$$

$$r(\sec\theta + \tan\theta) = \theta - \cos\theta$$

37) $(x+2)^2\frac{dy}{dx} = 5 - 8y - 4xy$

$$\frac{dy}{dx} = \frac{5 - 8y - 4xy}{(x+2)^2}$$

$$\frac{dy}{dx} + \left(\frac{8 + 4x}{(x+2)^2}\right)y = \frac{5}{(x+2)^2}$$

$$\frac{dy}{dx} + \left(\frac{4}{x+2}\right)y = \frac{5}{(x+2)^2}$$

$$e^{\int\left(\frac{4}{x+2}\right)dx} = e^{4\int\frac{dx}{x+2}} = e^{4\ln|x+2|}$$

$$= e^{\ln|(x+2)^4|} = (x+2)^4$$

$$(x+2)^4\left[\frac{dy}{dx} + \left(\frac{4}{x+2}\right)y = \frac{5}{(x+2)^2}\right]$$

$$\int \frac{d}{dx}\left[(x+2)^4y\right] = \int 5(x+2)^2dx$$

$$(x+2)^4y = 5\int(x+2)^2dx$$

$$= \frac{5}{3}(x+2)^3 + C$$

$$y = \frac{5}{3}(x+2)^{-1} + C(x+2)^{-4}$$

39. $\frac{dy}{dx} = (10 - y)\cosh x$

$$\frac{dy}{dx} = 10\cosh x - y\cosh x$$

$$\frac{dy}{dx} + \cosh x \cdot y = 10\cosh x$$

$$e^{\int\cosh x} = e^{\operatorname{senh} x}$$

$$\int \frac{d}{dx}\left[e^{\operatorname{senh} x}y\right] = \int 10\cosh x \cdot e^{\operatorname{senh} x}$$

$$e^{\operatorname{senh} x}y = 10e^{\operatorname{senh} x} + C$$

$$y = 10 + Ce^{-\operatorname{senh} x}$$

1. $\frac{dy}{dx} + 5y = 20, \quad y(0) = 2$

$e^{\int 5\,dx} = e^{5x}$

$\int \frac{d}{dx}\left[e^{5x}y\right] = \int 20e^{5x}$

$e^{5x}y = 4e^{5x} + C$

$y = 4 + \frac{C}{e^{5x}}$

$y(0) = 2$

$2 = 4 + \frac{C}{e^{5(0)}}$

$2 = 4 + C$

$C = -2$

$\therefore y = 4 + 2e^{-5x}$ //

43. $L\frac{di}{dt} + Ri = E \quad L, R\ y\ E = \text{cts.}$

$i(0) = i_0$

$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$

$e^{\int \frac{R}{L}dt} = e^{\frac{R}{L}t}$

$\frac{d}{dx}\int e^{R/L\,t}\,i = \int \frac{E}{L}e^{R/L\,t}\,dt$

$e^{R/L\,t}i = \frac{E}{\cancel{L}}\frac{\cancel{L}}{R}e^{R/L\,t} + C$

$e^{R/L\,t}i = \frac{E}{R}e^{R/L\,t} + C$

$i = \frac{E}{R} + \frac{C}{e^{R/L\,t}}$

$i_0 = \frac{E}{R} + \frac{C}{e^{R/L(0)}} = \frac{E}{R} + C$

$C = i_0 - \frac{E}{R}$

$i = \frac{E}{R} + \frac{i_0 - \frac{E}{R}}{e^{R/L\,t}}$ //

45. $y' + (\tan x)y = \cos^2 x, \quad y(0) = -1$

$\frac{dy}{dx} + \tan x\ y = \cos^2 x$

$e^{\int \tan x\,dx} = e^{\ln|\sec x|} = \sec x$

$\int \frac{d}{dx}\left[\sec x \cdot y\right] = \int \cos x$

$\sec x \cdot y = \operatorname{sen} x + C$

$y = \operatorname{sen} x \cos x + C\cos x$

$-1 = \operatorname{sen}(0)\cos(0) + C\cos(0)$

$-1 = C$

$y = \operatorname{sen} x \cos x - \cos x$

47. $\frac{dT}{dt} = K(T-50); \quad K = \text{cte}, \quad T(0) = 200$

$\frac{dT}{dt} = KT - 50K$

$\frac{dT}{dt} - KT = -50K$

$e^{-\int K\,dt} = e^{-Kt}$

$\int \frac{d}{dt}\left[e^{-Kt}T\right] = \int -50Ke^{-Kt}\,dt$

$e^{-Kt}T = +50\cancel{K}e^{-Kt} + C$

$e^{-Kt}T = 50e^{-Kt} + C$

$T = 50 + Ce^{Kt}$

$200 = 50 + Ce^{K(0)}$

$200 - 50 = C$

$C = 150$

$\therefore T = 50 + 150e^{Kt}$ //

49. $(x+1)\frac{dy}{dx} + y = \ln x, \quad y(1) = 10$

$\frac{dy}{dx} + \frac{y}{(x+1)} = \frac{\ln x}{(x+1)}$

$e^{\int \frac{dx}{(x+1)}} = e^{\ln|x+1|} = (x+1)$

$\int \frac{d}{dx}\left[(x+1)y\right] = \int \ln x\,dx$

$(x+1)y =$

$u = \ln x \quad du = \frac{1}{x}$

$dv = dx \quad v = x$

$$(x+1)y = x\ln x - \int dx$$

$$(x+1)y = x\ln x - x + C$$

$$y = \frac{x\ln x}{x+1} - \frac{x}{x+1} + \frac{C}{x+1}$$

$$10 = \frac{\ln(1)}{2} - \frac{1}{2} + \frac{C}{2}$$

$$10 = -\frac{1}{2} + \frac{1}{2}C$$

$$\frac{21}{2} = \frac{1}{2}C$$

$$C = 21$$

$$\therefore \underline{y = (x\ln x - x + 21)(x+1)^{-1}}$$

~~Zill~~. Pag. 72   1 - 6 Ejemplos.

EJEMPLO 1. Un cultivo tiene una cantidad inicial N_0 de bacterias. Cuando $t = 1$ h, la cantidad medida de bacterias es $\frac{3}{2}N_0$. Si la razón de reproducción es proporcional a la cantidad de bacterias presentes, calcule el tiempo necesario para triplicar la cantidad inicial de los microorganismos.

$$\frac{dN}{dt} = KN \qquad \mu = e^{\int -k\,dt} = e^{-Kt}$$

$$\frac{dN}{dt} - KN = 0 \qquad \to \frac{d}{dt}\left[e^{-Kt}N\right] = 0$$

$$\to e^{-Kt}N = C \quad \Rightarrow N(t) = Ce^{-Kt}$$

Cuando $t=0$, $N_0 = Ce^0 = C \quad \to N(t) = N_0e^{Kt}$

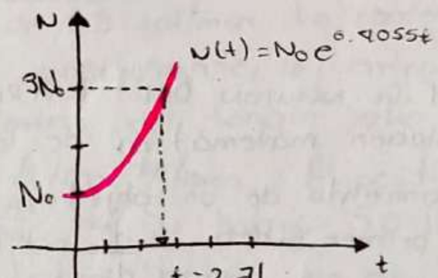
Cuando $t=1$, $\frac{3}{2}N_0 = N_0e^K \quad \to e^K = \frac{3}{2} \qquad K = \ln\frac{3}{2} = 0.4055$

$$N(t) = N_0e^{0.4055t}$$

Para triplicar

$$3N_0 = N_0e^{0.4055t} \qquad .4055t = \ln 3$$

$$t = \frac{\ln 3}{0.4055} \approx 2.71 \text{ h}$$



EJEMPLO 2. Un reactor de cría convierte al uranio 238, relativamente estable, a plutonio 239, un isótopo radiactivo. Al cabo de 15 años, se ha desintegrado el 0.043% de la cantidad inicial, A_0, de una muestra de plutonio. Calcule el periodo medio de ese isótopo, si la razón de desintegración es proporcional a la cantidad presente.

$A(t) = A_0e^{Kt}$ Si se desintegró 0.043%

queda 99.957%

$$\to 0.99957A_0 = A(15)$$

$$0.99957A_0 = A_0e^{15K}$$

$$K = \frac{1}{15}\ln 0.99957 = -0.0002867$$

$$\to A(t) = A_0e^{-0.0002867}$$

$$A(t) = A_0/2 \qquad t = \frac{\ln 2}{0.00002867} \approx 24180 \text{ años.}$$

$$A_0/2 = A_0e^{-0.00002867}$$

$$\frac{1}{2} = e^{-0.0002867t}$$

EJEMPLO 3. El analisis de un hueso fosilizado se encontró que contenía la centésima parte de la cantidad original de C-14. Determine la edad del fósil.

$$A(t) = A_0 e^{kt}$$

$$\frac{A_0}{2} = A(5600) \qquad \frac{A_0}{2} = A_0 e^{5600K} \rightarrow 5600K = \ln\frac{1}{2} = -\ln 2$$

$$\text{Donde } K = -(\ln 2)/5600 = -0.00012378$$

$$\rightarrow A(t) = A_0 e^{-0.00012378t}$$

$$A(t) = A_0/1000 \qquad A_0/1000 = A_0 e^{-0.00012378t}$$

$$-0.00012378t = \ln\frac{1}{100} = -\ln 1000$$

$$\rightarrow \frac{\ln 1000}{0.00012378} \approx \underline{55{,}800 \text{ años}}$$

LEY DE NEWTON DEL ENFRIAMIENTO. Anteriormente vimos que la formulación matemática de la ley empírica de Newton, relativa al enfriamiento de un objeto, se expresa con la ecuación diferencial lineal de primer orden $\frac{dT}{dt} = k(T - T_m)$ Donde k es una constante de proporcionalidad, T(t) es la temperatura del objeto cuando $t > 0$ y T_m es la temperatura ambiente; o sea la temperatura del medio que rodea el objeto. En el sig. ejemplo supondremos que T_m es constante.

EJEMPLO 4. Al sacar un pastel del horno su temperatura es 300°F. Después de 3 min, 200°F, ¿En cuánto tiempo? se enfriará hasta la temperatura ambiente de 70°F?

$$T_m = 70 \qquad \frac{dT}{dt} = K(T-70), \quad T(0) = 300 \qquad T(3) = 200$$

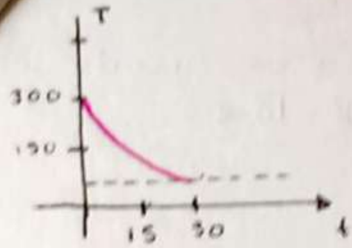
$$\int \frac{dT}{T-70} = \int K\,dt \longrightarrow \ln|T-70| = Kt + C_1$$

$$T - 70 = C_2 e^{Kt} \quad \therefore \quad T = C_2 e^{Kt} + 70$$

$$\text{Cuando } t=0,\ T=300 \rightarrow 300 = 70 + C_2 \qquad C_2 = 230$$

$$T = 70 + 230e^{Kt} \ ; \quad T(3) = 200 \quad e^{3K} = \frac{13}{23} \quad K = \frac{1}{3}\ln\frac{13}{23}$$

$$K = -0.19018 \quad \rightarrow T(t) = 70 + 230e^{-0.19018t}$$



$T(t)$	t (min)
75°	20.1
74°	21.3
73°	22.8
72°	24.9
71°	28.6
70.5°	32.3

MEZCLAS. Al mezclar dos fluidos, a veces se originan ecs. dif. lineales de primer orden. Cuando describimos la mezcla de dos salmueras, supusimos que la razón con que cambia la cantidad de sal $A'(t)$, en el tanque de mezcla es una razón neta: $\frac{dA}{dt} = \left(\text{razón con que entra la sustancia}\right) - \left(\text{razón con que sale la sustancia}\right) = R_1 - R_2$

EJEMPLO 5: En un tanque grande se contenía inicialmente 300 galones de una solución de salmuera. Al tanque entraba y salía sal porque se le bombeaba una solución a un flujo de 3 gal/min, se mezclaba con la solución original, y salía del tanque con un flujo de 3 gal/min. La concentración de la solución entrante era 2 lb/gal; por consiguiente, la entrada de sal era $R_1 = (2\ \text{lb/gal}) \cdot (3\ \text{gal/min}) = 6\ \text{lb/min}$; del tanque salía con una razón $R_2 = (3\ \text{gal/min}) \cdot (A/300\ \text{lb/gal}) = A/100\ \text{lb/min}$. A partir de esos datos y de la ec. obtuvimos $\frac{dA}{dt} = 6 - \frac{A}{100}$. → ¿Si había 50 lb de sal disueltas en los 300 galones iniciales, ¿Cuánta sal habrá en el tanque pasado mucho tiempo?

$$\frac{dA}{dt} = 6 - \frac{A}{100} \qquad A(0) = 50$$

$$\frac{dA}{dt} + \frac{1}{100}A = 6 \rightarrow M = e^{\int \frac{1}{100} dt} = e^{\frac{t}{100}}$$

$$\frac{dA}{dt} e^{t/100} + \frac{A}{100} e^{t/100} = 6e^{t/100}$$

$$\int \frac{d}{dt}\left[e^{t/100} A\right] = \int 6e^{t/100} dt$$

$$e^{t/100} \cdot A = 100 \cdot 6 \int e^{t/100} \frac{1}{100} dt$$

$$e^{t/100} \cdot A = 600 \left(e^{t/100}\right)$$

$$A = 600 + Ce^{-t/100}$$

Cuando $t = 0$, $A = 50$ $\qquad C = -550$

$$\rightarrow A(t) = 600 - 550e^{-t/100}$$

EJEMPLO 6. Un acumulador de 12 volts. se conecta a un circuito en serie LR, con una inductancia de $\frac{1}{2}$ henry y una $R = 10\Omega$. Determinar i, si la $i_0 = 0$

$$\frac{1}{2}\frac{di}{dt} + 10i = 12 \qquad i(0) = 0$$

$$\frac{di}{dt} + 20i = 24 \qquad \mu(x) = e^{\int 20\, dt} = e^{20t}$$

$$\int \frac{d}{dt}\left[e^{20t} i\right] = \int 24 e^{20t}\, dt$$

$$e^{20t} i = 24 \frac{1}{20} \int e^{20t}\, 20\, dt$$

$$e^{20t} i = \frac{6}{5} e^{20t} + C$$

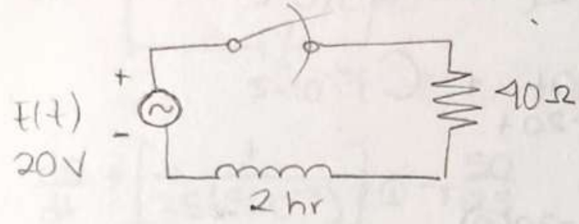
$$i = \frac{6}{5} + Ce^{-20t} \qquad \text{Si } i(0) = 0$$

$$\rightarrow 0 = \frac{6}{5} + C \longrightarrow C = -\frac{6}{5}$$

$$\therefore \quad i(t) = \frac{6}{5} - \frac{6}{5} e^{-20t}$$

Pag. 88 Murray 1-8 SECCIÓN A.

1. En $t=0$ una fem de 20 voltios se aplica a un circuito consistente de un inductor de 2 henrios en serie con una resistencia de 40 ohmios. Si la corriente es cero en $t=0$, ¿cuál es en cualquier tiempo $t \geqq 0$?



$$I(0)=0$$

$$\frac{di}{dt}+\left(\frac{40}{2}\right)i=\frac{20}{2}V$$

$$\frac{di}{dt}+20i=10$$

$$P(t)=20$$
$$e^{\int 20dt}=e^{20t} \implies e^{20t}\left[\frac{di}{dt}+20i=10\right]$$

$$\int\cancel{\frac{d}{dt}}\left[e^{20t}i\right]=\int 10e^{20t}$$

$$e^{20t}i=10\frac{1}{20}\int e^{20t}\,20dt$$

$$e^{20t}i=\frac{1}{2}e^{20t}+C$$

$$\implies i(t)=\frac{1}{2}+Ce^{-20t}$$

Pero $i(0)=\frac{1}{2}+Ce^{-20(0)}$

$$=\frac{1}{2}+C$$

$$\therefore C=-\frac{1}{2} \implies i(t)=\frac{1}{2}-\frac{1}{2}e^{-20t}$$

$$=\frac{1}{2}\left(1-e^{-20t}\right)$$

2. Trabaje con el ejercicio anterior si la fem es 100 sen. (10t)

$$\frac{di}{dt}+20i=100\text{ sen }10t \longrightarrow e^{20t}i=\int 100e^{20t}\text{ sen }10t$$

Para $\int 100e^{20t}\text{ sen }10t\,dt$ $\quad u=\text{sen }10t \quad du=10\cos 10t\,dt$

$$dv=\int 100e^{20t}dt \quad v=5e^{20t}$$

$$\int 100e^{20t}\cdot\text{sen }10t\,dt=5e^{20t}\text{sen}10t-\int 5e^{20t}\,10\cos 10t\,dt$$

$$u=10\cos 10t \quad du=-100\text{ sen }10t$$

$$dv=5e^{20t}dt \quad v=\frac{1}{4}e^{20t}$$

$$\Rightarrow \int 100 e^{20t} \operatorname{sen} 10t\,dt = 5e^{20t} \operatorname{sen} 10t - \left[10\cos 10t \cdot \tfrac{1}{4} e^{20t} - \int -100 \operatorname{sen}(10t) \tfrac{1}{4} e^{20t}\,dt\right.$$

$$= 5e^{20t} \operatorname{sen} 10t - \tfrac{5}{2} e^{20t} \cos 10t - \tfrac{1}{4}\int 100 e^{20t} \operatorname{sen} 10t\,dt$$

$$\int 100e^{20t} \operatorname{sen} 10t\,dt + \tfrac{1}{4}\int 100 e^{20t} \operatorname{sen} 10t\,dt = 5e^{20t} \operatorname{sen} 10t - \tfrac{5}{2} e^{20t} \cos 10t$$

$$\left(\tfrac{5}{4} \int 100e^{20t} \operatorname{sen} 10t\,dt = 5e^{20t} \operatorname{sen} 10t - \tfrac{5}{2} e^{20t} \cos 10t\right) \tfrac{4}{5}$$

$$\int 100e^{20t} \operatorname{sen} 10t\,dt = 4e^{20t} \operatorname{sen} 10t - 2e^{20t} \cos 10t$$

$$\Rightarrow e^{20t} i = 4e^{20t} \operatorname{sen} 10t - 2e^{20t} \cos 10t + C$$

$$i(t) = 4 \operatorname{sen} 10t - 2\cos 10t + Ce^{-20t}$$

$$i(0) = 0$$

$$0 = 4\operatorname{sen} 10(0)^{\to 0} - 2\cos 10(0)^{1} + Ce^{-20(0)^{1}}$$

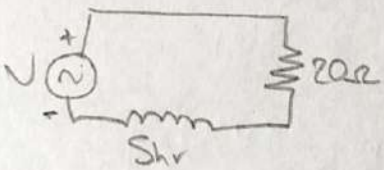
$$0 = -2 + C$$

$$\therefore C = 2$$

$$\Rightarrow \quad i(t) = [4 \operatorname{sen} 10t - 2\cos 10t + 2e^{-20t}]\ \tfrac{1}{2}$$

$$= 2\operatorname{sen} 10t - \cos 10t + e^{-20t}$$

(3) Una resistencia de 20 ohmios y un inductor de 5 henrios se conectan en serie en un circuito electrico en el cual hay un flujo de corriente de 20 amperios en tiempo $t=0$ Encuentre la corriente para $t>0$.



$$I(0) = 20$$

$$\frac{di}{dt} + \left(\frac{R}{L}\right) i = \frac{E(t)}{L}$$

$$\frac{di}{dt} + 4i = 0 \qquad e^{4t}\left[\frac{di}{dt} + 4i = 0\right]$$

$$e^{\int 4dt} = e^{4t} \longrightarrow \qquad \int \frac{d}{dt}\left[e^{4t} i\right] = \int 0$$

$$e^{4t} i = C$$

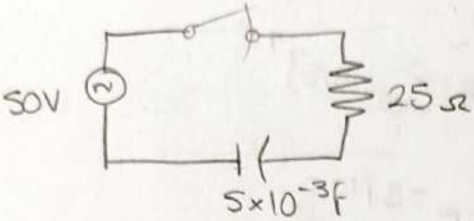
$$i = Ce^{-4t}$$

$$I(0) = 20 \longrightarrow \quad 20 = Ce^{-4(0)^{1}}$$

$$\therefore C = 20$$

$$y \longrightarrow \quad i = 20e^{-4t}$$

④ Un condensador de 5×10^{-3} faradios está en serie con una resistencia de 25 ohmios y una fem de 50 voltios, el interruptor se cierra en $t=0$. Asumiendo que la carga en el condensador es cero en $t=0$, determine la carga y la corriente en cualquier tiempo.



$$\frac{dQ}{dt} + \left(\frac{1}{RC}\right)Q = \frac{E(t)}{R}$$

$$\frac{dQ}{dt} + \left[\frac{1}{25(5\times10^{-3})}\right]Q = \frac{50}{25} \qquad i(t) = \frac{dQ}{dt}$$

$$\frac{dQ}{dt} + 8Q = 2 \xrightarrow{\text{Despejando}} \frac{dQ}{dt} = 2 - 8Q$$

$$e^{8t}Q = \int 2e^{8t}\,dt$$

$$e^{8t}Q = \frac{1}{4}e^{8t} + C$$

$$Q = \frac{1}{4} + Ce^{-8t}$$

$$Q(0) = 0$$

$$0 = \frac{1}{4} + Ce^{-8(0)}$$

$$\therefore C = -\frac{1}{4}$$

$$Q = \frac{1}{4} - \frac{1}{4}e^{-8t}$$

$$Q(t) = \frac{1}{4}\left(1 - e^{-8t}\right)$$

Sustituyendo

$$i(t) = 2 - 8\left(\frac{1}{4} - \frac{1}{4}e^{-8t}\right)$$

$$i(t) = \cancel{2 - 2} + 2e^{-8t}$$

$$i(t) = 2e^{-8t}$$

⑤ Trabaje el ejercicio anterior si la fem es $50\cos 6t$, $t \geq 0$

$$\frac{dQ}{dt} + 8Q = \frac{50\cos 6t}{25} \qquad \frac{dQ}{dt} + 8Q = 2\cos 6t$$

$$e^{8t}Q = \int 2e^{8t}\cos 6t\,dt \longrightarrow 2\int e^{8t}\cos 6t\,dt$$

$$\int e^{8t}\cos 6t\,dt = \frac{1}{8}e^{8t}\cos 6t + \int \frac{3}{4}e^{8t}\operatorname{sen} 6t\,dt$$

$u = \cos 6t \quad du = -6\operatorname{sen} 6t$

$dv = e^{8t}dt \quad v = \frac{1}{8}e^{8t}$

$w = \operatorname{sen} 6t \quad dw = 6\cos 6t$

$dz = e^{8t}dt \quad z = \frac{1}{8}e^{8t}$

$$= \frac{1}{8}e^{8t}\cos 6t + \frac{3}{4}\left[\frac{1}{8}e^{8t}\operatorname{sen}6t - \int \frac{3}{4}e^{8t}\cos 6t\right]$$

$$= \frac{1}{8}e^{8t}\cos 6t + \frac{3}{32}e^{8t}\operatorname{sen}6t - \frac{9}{16}\int e^{8t}\cos 6t\,dt$$

$$\int e^{8t}\cos 6t\,dt + \frac{9}{16}\int e^{8t}\cos 6t\,dt = \frac{1}{8}e^{8t}\cos 6t + \frac{3}{32}e^{8t}\operatorname{sen}6t$$

$$\left[\frac{25}{16}\int \cos 6t\, e^{8t}\, dt + \frac{1}{8}e^{8t}\cos 6t + \frac{3}{32}e^{8t}\operatorname{sen} 6t\right] 2$$

$$\Rightarrow e^{8t}Q = \frac{4}{25}e^{8t}\cos 6t + \frac{6}{50}e^{8t}\operatorname{sen} 6t + C$$

$$Q = \frac{4}{25}\cos 6t + \frac{6}{50}\operatorname{sen} 6t + Ce^{-8t}$$

$Q(0) = 0$

$$0 = \frac{4}{25}\cos 6(0)^{\nearrow 1} + \frac{6}{50}\operatorname{sen} 6(0)^{\nearrow 0} + Ce^{-8(0)^{\nearrow 1}}$$

$$0 = \frac{4}{25} + C \qquad \therefore \quad C = -\frac{4}{25}$$

$$\underline{Q = 0.16\cos 6t + 0.12\operatorname{sen} 6t - 0.16e^{-8t}} \;//$$

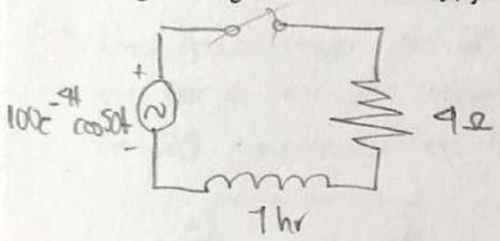
Luego $i(t) = \frac{dQ}{dt}$ $\qquad \frac{dQ}{dt} = 2\cos 6t - 8Q$

$$i(t) = 2\cos 6t - 8\left[0.16\cos 6t + 0.12\operatorname{sen} 6t - 0.16e^{-8t}\right]$$

$$i(t) = 2\cos 6t - 1.28\cos 6t - 0.96\operatorname{sen} 6t + 1.28e^{-8t}$$

$$i(t) = \underline{0.72\cos 6t - 0.96\operatorname{sen} 6t + 1.28e^{-8t}} \;//$$

⑥ Un circuito consiste en una resistencia de 4 ohmios y un inductor de 1 henrio, se conecta en serie con un voltaje dado por $100e^{-4t}\cos 50t$ $t \geq 0$. Encontrar $i(t)$ si $i = 0$ en $t = 0$



$$\frac{di}{dt} + \left(\frac{R}{L}\right)i = \frac{E(t)}{L}$$

$$\frac{di}{dt} + 4i = 100e^{-4t}\cos 50t$$

$$e^{\int 4\,dt} = e^{4t}$$

$$\int \frac{d}{dt}\left[e^{4t}i\right] = 100\cancel{e^{-4t}}\cos 50t\,\cancel{e^{4t}}$$

$$e^{4t}i = 100\int \cos 50t\, dt$$

$$e^{4t}i = 100\,\frac{1}{50}\operatorname{sen} 50t + C$$

$$e^{4t}i = 2\operatorname{sen} 50t + C$$

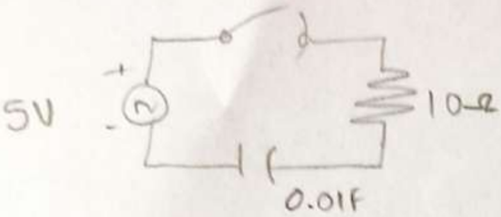
$$i(t) = 2e^{-4t}\operatorname{sen} 50t + Ce^{-4t}$$

$i(0) = 0$

$$0 = 2e^{-4(0)}\operatorname{sen} 50(0)^{\nearrow 0} + Ce^{-4(0)^{\nearrow 1}} \qquad \therefore\; \underline{i(t) = 2e^{-4t}\operatorname{sen} 50t} \;//$$

$C = 0$

Un circuito consiste de una resistencia de 10 ohmios y un condensador de 0.01 faradios en serie. La carga en el condensador es de 0.05 C. Encontrar la carga en el condensador y la corriente en tiempo t, después de cerrar el interruptor.



$$V = \frac{Q}{C} = \frac{0.05\,C}{0.01\,F} = 5V$$

$$\frac{dQ}{dt} + \left(\frac{1}{RC}\right)Q = \frac{E(t)}{R}$$

$$\frac{dQ}{dt} + \left[\frac{1}{10(0.01)}\right]Q = \frac{5}{10}$$

$$\frac{dQ}{dt} + 10Q = \frac{1}{2}$$

$$e^{10t}\left[\frac{dQ}{dt} + 10Q = \frac{1}{2}\right]$$

$$\int \frac{d}{dt}\left[e^{10t}Q\right] = \int \frac{1}{2}e^{10t}\,dt$$

$$e^{10t}Q = 0.05e^{10t} + C$$

8) Una resistencia de 20 ohmios se conecta en serie con un condensador de 0.01 faradios y una fem en voltios dada por $40e^{-3t} + 20e^{-6t}$. Si $Q = 0$ en $t = 0$, muestre que la caída máxima en el condensador es de 0.25 C.

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★ Compruebe si las funciones sig. son l.d. o l.i. en $(-\infty, \infty)$

✓ 15. $f_1(x) = x, \quad f_2(x) = x^2 \quad f_3(x) = 4x - 3x^2$

$$w = (f_1, f_2, f_3) = \begin{vmatrix} x & x^2 & 4x - 3x^2 \\ 1 & 2x & 4 - 6x \\ 0 & 2 & -6 \end{vmatrix}$$

$$w = x[2x(-6) - (8 - 12x)] - 1[\cancel{-6x^2} - 8x + \cancel{6x^2}]$$
$$= x[\cancel{-12x} - 8 + \cancel{12x}] - [-8x]$$
$$= -8x + 8x = 0 \therefore \text{l.d}$$

19. $f_1(x) = x, \; f_2(x) = x - 1, \; f_3(x) = x + 3$

$$w = \begin{vmatrix} x & x-1 & x+3 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0 \therefore \text{l.d.}$$

20. $f_1(x) = 2 + x, \; f_2(x) = 2 + |x|$

$$w = \begin{vmatrix} 2 + x & 2 + |x| \\ 1 & |1| \end{vmatrix} = 2 + |x| - (2 + x) = 0 \therefore \text{ld}$$

16. $f_1(x) = 0, \; f_2(x) = x, \; f_3(x) = e^x$

$$w = \begin{vmatrix} 0 & x & e^x \\ 0 & 1 & e^x \\ 0 & 0 & e^x \end{vmatrix} = 0 \therefore \text{l.d.}$$

21. $f_1(x) = 1 + x \quad f_2(x) = x \quad f_3(x) = x^2$

$$w = \begin{vmatrix} 1 + x & x & x^2 \\ 1 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix} = 2(1 + x - x) = 2 \neq 0 \therefore \text{l.i.}$$

17. $f_1(x) = 5, \; f_2(x) = \cos^2 x, \; f_3(x) = \operatorname{sen}^2 x$

$$w = \begin{vmatrix} 5 & \cos^2 x & \operatorname{sen}^2 x \\ 0 & -2\cos x \operatorname{sen} x & 2 \operatorname{sen} x \cos x \\ 0 & -2(\cos^2 x - \operatorname{sen}^2 x) & 2(\cos^2 x - \operatorname{sen}^2 x) \end{vmatrix}$$

$$w = 5[\cancel{-4\cos x \operatorname{sen} x(\cos^2 x - \operatorname{sen}^2 x)} + \cancel{4\cos x \operatorname{sen} x(\cos^2 x - \operatorname{sen}^2 x)}]$$
$$= 5(0) = 0 \therefore \text{l.d.}$$

22. $f_1(x) = e^x \quad f_2(x) = e^{-x} \quad f_3(x) = \operatorname{senh} x$

$$W = \begin{vmatrix} e^x & e^{-x} & \operatorname{senh} x \\ e^x & -e^{-x} & \cosh x \\ e^x & e^{-x} & \operatorname{senh} x \end{vmatrix}$$

$$w = e^x[-e^{-x} \operatorname{senh} x - e^{-x} \cosh x]$$
$$- e^x[e^{-x} \operatorname{senh} x - e^{-x} \operatorname{senh} x]$$
$$+ e^x[e^{-x} \cosh x + e^{-x} \operatorname{senh} x]$$
$$= e^0 \operatorname{senh} x - e^0 \cosh x - e^0 \operatorname{senh} x + e^0 \operatorname{senh} x$$
$$+ e^0 \cosh x + e^0 \operatorname{senh} x$$
$$= \operatorname{senh} x - \cancel{\cosh x} - \cancel{\operatorname{senh} x} + \operatorname{senh} x + \cancel{\cosh x}$$
$$+ \operatorname{senh} x \quad = 2 \operatorname{senh} x \neq 0 \therefore \text{l.i.}$$

18. $f_1(x) = \cos 2x, \; f_2(x) = 1, \; f_3(x) = \cos^2 x$

$$W = \begin{vmatrix} \cos 2x & 1 & \cos^2 x \\ -2\operatorname{sen}(2x) & 0 & -2\cos x \operatorname{sen} x \\ -4\cos(2x) & 0 & -2(\cos^2 x - \operatorname{sen}^2 x) \end{vmatrix}$$

$$= -1[4\operatorname{sen}(2x)(\cos^2 x - \operatorname{sen}^2 x) - 8\cos(2x)(\cos x \operatorname{sen} x)]$$
$$= -1[4\operatorname{sen}(2x)\cos(2x) - 4\cos(2x) 2\cos x \operatorname{sen} x]$$
$$= -1[\cancel{4\operatorname{sen}(2x)\cos(2x)} - \cancel{4\cos(2x)\operatorname{sen} 2x}]$$
$$= -1(0) = 0 \therefore \text{l.d}$$

23. $y'' - y' - 12y = 0 \quad (1) \qquad e^{-3x},\ e^{4x} \quad (-\infty,\infty)$

$y_1 = e^{-3x} \qquad y_2 = e^{4x}$

$y_1' = -3e^{-3x} \qquad y_2' = 4e^{4x}$

$y_1'' = 9e^{3x} \qquad y_2'' = 16e^{4x}$

Con y_1 y sus derivadas...

$9e^{-3x} + 3e^{-3x}) - 12(e^{-3x}) = 0$

$12e^{-3x} - 12e^{-3x} = 0$

$0 = 0$ IDENTIDAD.

Con y_2 y sus derivadas...

$16e^{4x} - 4e^{4x} - 12(e^{4x}) = 0$

$16e^{4x} - 16e^{4x} = 0$

$0 = 0$

Entonces la comb. lineal debe ser sol.

$y = C_1 e^{-3x} + C_2 e^{4x} = 0$

$y' = -3C_1 e^{-3x} + 4C_2 e^{4x} = 0$

$y'' = 9C_1 e^{3x} + 16C_2 e^{4x} = 0$

$\to 9C_1e^{3x} + 16C_2e^{4x} + 3C_1e^{-3x} - 4C_2e^{4x} - 12C_1e^{-3x} - 12C_2e^{4x} = 0 \implies$

$0 = 0$ IDENTIDAD

$\therefore$ la combinación lineal $y = C_1e^{-3x} + C_2e^{4x}$ es sol. de 1.

24. $y'' - 4y = 0;\ \cosh 2x,\ \operatorname{senh} 2x$

$y_1 = \cosh 2x \quad (1) \qquad y_2 = \operatorname{senh} 2x \quad (2)$

$y_1' = 2\operatorname{senh} 2x \qquad y_2' = 2\cosh 2x$

$y_1'' = 4\cosh 2x \qquad y_2'' = 4\operatorname{senh} 2x$

Con (1)

$4\cosh 2x - 4\cosh 2x = 0$

$0 = 0$ IDENT.

Con (2)

$4\operatorname{senh} 2x - 4\operatorname{senh} 2x = 0$

$0 = 0$ IDENT.

La Comb. lin. debe ser sol.

$y = C_1\cosh 2x + C_2\operatorname{senh} 2x$

$y' = 2C_1\operatorname{senh} 2x + 2C_2\cosh 2x$

$y'' = 4C_1\cosh 2x + 4C_2\operatorname{senh} 2x$

$\to 4C_1\cosh 2x + 4C_2\operatorname{senh} 2x - 4C_1\cosh 2x - 4C_2\operatorname{sen}$

$0 = 0$ IDENTIDAD.

$\therefore$ la comb. lineal $y = C_1\cos 2x + C_2\operatorname{senh} 2x$ es sol. de $y'' - 4y = 0$.

25. $y'' - 2y' + 5y = 0 \ldots (1)$

$e^x\cos 2x;\quad e^x\operatorname{sen} 2x$

$y = e^x\cos 2x$

$y' = e^x\cos 2x - 2e^x\operatorname{sen} 2x$

$y'' = -4e^x\operatorname{sen} 2x - 3e^x\cos 2x$

$\implies$

$-4e^x\operatorname{sen}2x - 3e^x\cos 2x - 2(e^x\cos 2x - 2e^x\operatorname{sen}2x) + 5e^x\cos 2x = 0$

$-4e^x\operatorname{sen}2x - 3e^x\cos 2x - 2e^x\cos 2x + 4e^x\operatorname{sen}2x + 5e^x\cos 2x = 0$

$0 = 0$ IDENTIDAD.

$y = e^x\operatorname{sen}2x$

$y' = 2e^x\cos 2x + e^x\operatorname{sen}2x$

$y'' = 2e^x\cos 2x - 3e^x\operatorname{sen}2x$

$\implies$

$2e^x\cos 2x - 3e^x\operatorname{sen}2x - 4e^x\cos 2x - 2e^x\operatorname{sen}2x + 5e^x\operatorname{sen}2x = 0$

$0 = 0$ IDENTIDAD.

$\therefore$ La combinación lineal $C_1e^x\cos 2x + C_2e^x\operatorname{sen}2x$ es solución de $y'' - 2y' + 5y = 0$.

$4y''-4y'+y=0$; $e^{x/2}$, $xe^{x/2}$

$y=e^{x/2}$

$y'=\frac{1}{2}e^{x/2}$

$y''=\frac{1}{4}e^{x/2}$

$\rightarrow 4\left(\frac{1}{4}e^{x/2}\right)-4\left(\frac{1}{2}e^{x/2}\right)+e^{x/2}=0$

$e^{x/2}-2e^{x/2}+e^{x/2}=0$

$0=0$ IDENT.

$y=xe^{x/2}$

$y'=\frac{1}{2}xe^{x/2}+e^{x/2}$

$y''=\frac{1}{4}xe^{x/2}+\frac{1}{2}e^{x/2}+\frac{1}{2}e^{x/2}$

$=\frac{1}{4}xe^{x/2}+e^{x/2}$

$\rightarrow 4\left(\frac{1}{4}xe^{x/2}+e^{x/2}\right)-4\left(\frac{1}{2}xe^{x/2}+e^{x/2}\right)+xe^{x/2}=0$

$xe^{x/2}+4e^{x/2}-2xe^{x/2}-4e^{x/2}+xe^{x/2}=0$

$0=0$ IDENT.

$\rightarrow$ La Com. lineal $C_1e^{x/2}+C_2xe^{x/2}$ es solución de $4y''-4y'+y=0$

27. $x^2y''-6xy'+12y=0$; x^3, x^4

$y=x^3$	$y=x^4$
$y'=3x^2$	$y'=4x^3$
$y''=6x$	$y''=12x^2$

$x^2\cdot 6x-6x\cdot 3x^2+12x^3=0$

$6x^3-18x^3+12x^3=0$

$0=0$ IDENT.

$x^2\,12x^2-6x\,4x^3+12x^4=0$

$12x^4-24x^4+12x^4=0$

$0=0$ IDENT.

$\therefore$ la Comb. lineal $C_1x^3+C_2x^4$ es sol. de $x^2y''-6xy'+12y=0$.

28. $x^2y''+xy'+y=0$; $\cos(\ln x)$, $\text{sen}(\ln x)$

$y=\cos(\ln x)$

$y'=-\frac{1}{x}\,\text{sen}(\ln x)$

$y''=\frac{1}{x^2}\left(\text{sen}(\ln x)-\cos(\ln(x))\right)$

$x^2\left[\frac{1}{x^2}(\text{sen}\ln x-\cos\ln x)\right]+x\left(-\frac{1}{x}\,\text{sen}\ln x\right)+\cos\ln x=0$

$\text{sen}\ln x-\cos\ln x-\text{sen}\ln x+\cos\ln x=0$

$0=0$ IDENT.

$y=\text{sen}(\ln x)$

$y'=\frac{1}{x}\cos(\ln x)$

$y''=-\frac{1}{x^2}\left(\text{sen}(\ln x)+\cos(\ln x)\right)$

$\rightarrow x^2\left(-\frac{1}{x^2}(\text{sen}\ln x+\cos\ln x)\right)+x\left(\frac{1}{x}\cos\ln x\right)+\text{sen}\ln x=0$

$-\text{sen}(\ln x)-\cos(\ln x)+\cos(\ln x)+\text{sen}(\ln x)=0$

$0=0$ IDENT

$\therefore$ la combinación lineal $C_1\cos(\ln x)+C_2\,\text{sen}(\ln x)$ es solución de $x^2y''+xy'+y=0$

29. $x^3y'''+6x^2y''+4xy'-4y=0$

$x,\ x^{-2},\ x^{-2}\ln x$

$y=x$	$y=x^{-2}$	$y=x^{-2}\ln x$
$y'=1$	$y'=-2x^{-3}$	$y'=x^{-3}+\ln x(-2x^{-3})$
$y''=0$	$y''=6x^{-4}$	$y''=-5x^{-4}+6\ln x\,x^{-4}$
$y'''=0$	$y'''=-24x^{-5}$	$y'''=26x^{-5}-24x^{-5}\ln x$

$* \ x^3(0)+6x^2(0)+4x(1)-4(x)=0$

$0+0+4x-4x=0$

$0=0$ IDENT.

$* \ x^3(-24x^{-5})+6x^2(6x^{-4})+4x(-2x^{-3})-4(x^{-2})=0$

$-24x^{-2}+36x^{-2}-8x^{-2}-4x^{-2}=0$

$0=0$ IDENT.

$* \ x^3(26x^{-5}-24x^{-5}\ln x)+6x^2(-5x^{-4}+6\ln x\,x^{-4})+4x(x^{-3}+\ln x(-2x^{-3}))$

$-4x^{-2}\ln x=0$

$\Rightarrow 26x^{-2}-24x^{-2}\ln x-30x^{-2}+36x^{-2}\ln x+4x^{-2}-8x^{-2}\ln x$

$-4x^{-2}\ln x=0$

$0=0$ IDENTIDAD.

30. $y^{4}+y''=0$; 1, x, $\cos x$, $\text{sen}\,x$

$y=1$	$y=x$	$y=\cos x$	$y=\text{sen}\,x$
$y''=0$	$y=1$	$y'=-\text{sen}\,x$	$y'=\cos x$

$\bullet\ y^{4}+$

$y'' - y' - 12y = 0 \quad e^{-3x}, e^{4x} \; (-\infty, \infty)$

$y_1 = e^{-3x} \qquad y_2 = e^{4x}$

$y_1' = -3e^{-3x} \qquad y_2' = 4e^{4x}$

$$W = \begin{vmatrix} e^{-3x} & e^{4x} \\ -3e^{-3x} & 4e^{4x} \end{vmatrix} = 4e^{x} + 3e^{x} = 7e^{x}$$

$7e^{x} \neq 0$

$\therefore y = C_1 e^{-3x} + C_2 e^{4x}$

24. $y'' - 4y = 0$; $\cosh 2x$; $\operatorname{senh} 2x$

$y_1 = \cosh 2x \qquad y_2 = \operatorname{senh} 2x$

$y_1' = 2\operatorname{senh} 2x \qquad y_2' = 2\cosh 2x$

$$W \begin{vmatrix} \cosh 2x & \operatorname{senh} 2x \\ 2\operatorname{senh} 2x & 2\cosh 2x \end{vmatrix}$$

$2\cosh^2 2x - 2\operatorname{senh} 2x \neq 0$

25. $y'' - 2y' + 5y = 0$; $e^{x}\cos 2x$; $e^{x}\operatorname{sen} 2x$

$y_1 = e^{x}\cos 2x \qquad y_2 = e^{x}\operatorname{sen} 2x$

$y_1' = e^{x}\cos 2x - 2e^{x}\operatorname{sen} 2x \qquad y_2' = e^{x}\operatorname{sen} 2x + 2e^{x}\cos 2x$

$$W = \begin{vmatrix} e^{x}\cos 2x & e^{x}\operatorname{sen} 2x \\ e^{x}\cos 2x - 2e^{x}\operatorname{sen} 2x & e^{x}\operatorname{sen} 2x + 2e^{x}\cos 2x \end{vmatrix}$$

$W = \cancel{e^{x}\cos 2x \operatorname{sen} 2x} + 2e^{2x}\cos^2 2x - \cancel{e^{2x}\operatorname{sen} 2x \cos 2x} + 2e^{2x}\operatorname{sen}^2 2x$

$2e^{2x}(\cos^2 2x + \operatorname{sen}^2 2x) = 2e^{2x}(1)$

$= 2e^{2x} \neq 0 \quad$ l.i

26. $4y'' - 4y' + y = 0$; $e^{x/2}$; $xe^{x/2}$

$$W = \begin{vmatrix} e^{x/2} & xe^{x/2} \\ \frac{1}{2}e^{x/2} & e^{x/2} + \frac{1}{2}xe^{x/2} \end{vmatrix}$$

$W = e^{x} + \cancel{\frac{1}{2}xe^{x}} - \cancel{\frac{1}{2}xe^{x}} = e^{x} \neq 0$

$\therefore y = C_1 e^{x/2} + C_2 xe^{x/2}$

27. $x^2y'' - 6xy' + 12y = 0$; x^3, x^4

$$W = \begin{vmatrix} x^3 & x^4 \\ 3x^2 & 4x^3 \end{vmatrix} = 4x^6 - 3x^6$$

$= x^6 \neq 0 \therefore y = C_1x^3 + C_2x^4$

28. $x^2y'' + xy' + y = 0$; $\cos(\ln x)$; $\operatorname{sen}(\ln x)$

$$W = \begin{vmatrix} \cos(\ln x) & \operatorname{sen}(\ln x) \\ -\frac{1}{x}\operatorname{sen}(\ln x) & \frac{1}{x}\cos(\ln x) \end{vmatrix}$$

$W = \frac{1}{x}\cos^2(\ln x) + \frac{1}{x}[\operatorname{sen}^2(\ln x)]$

$W = \frac{1}{x}[\cos^2(\ln x) + \operatorname{sen}^2(\ln x)]$

$= \frac{1}{x}(1) = \frac{1}{x} \neq 0 \quad$ l.i

$\therefore y = C_1\cos(\ln x) + C_2\operatorname{sen}(\ln x)$

29. $x^3y''' + 6x^2y'' + 4xy' - 4y = 0$

$x, \; x^{-2}, \; x^{-2}\ln x$

$$W = \begin{vmatrix} x & x^{-2} & x^{-2}\ln x \\ 1 & -2x^{-3} & -2x^{-3}\ln x + x^{-3} \\ 0 & 6x^{-4} & 6x^{-4}\ln x - 5x^{-4} \end{vmatrix}$$

$W = x[\cancel{-12x^{-7}\ln x} + 10x^{-7} + \cancel{12x^{-7}\ln x} - 6x^{-7}]$

$-1[\cancel{6x^{-6}\ln x} - 5x^{-6} - \cancel{6x^{-6}\ln x}]$

$= 10x^{-6} - 5x^{-6} = 5x^{-6} \neq 0 \quad$ li

$\therefore y = C_1x + C_2x^{-2} + C_3x^{-2}\ln x$

30 $y^{(4)} + y'' = 0$, 1, x, $\cos x$, $\operatorname{sen} x$

$$W = \begin{vmatrix} 1 & x & \cos x & \operatorname{sen} x \\ 0 & 1 & -\operatorname{sen} x & \cos x \\ 0 & 0 & -\cos x & -\operatorname{sen} x \\ 0 & 0 & \operatorname{sen} x & -\cos x \end{vmatrix}$$

$W = \cos^2 x + \operatorname{sen}^2 x = 1 \neq 0$

$\therefore y = C_1 + C_2x + C_3\cos x + C_4\operatorname{sen} x.$

$y = C_1e^{2(0)} + C_2e^{(0)}\cancel{\cos\sqrt{3}(0)}^{1} + C_3e^{-(0)}\cancel{\text{sen}\sqrt{3}(0)}^{0}$

$0 = C_1 + C_2$

y') $-1 = 2C_1e^{2(0)} - (C_2e^0\cancel{\cos\sqrt{3}(0)}^{1} - \sqrt{3}C_2e^0\cancel{\text{sen}\sqrt{3}(0)}^{0}$
$-C_3e^0\cancel{\text{sen}\sqrt{3}(0)}^{0} + \sqrt{3}C_3e^0\cancel{\cos\sqrt{3}(0)}^{1}$

$= 2C_1 - C_2 + \sqrt{3}C_3$

$y''c$) $4C_1e^{2(0)} + C_2e^0\cos\sqrt{3}(0) + \cancel{\sqrt{3}C_2e^0\text{sen}\sqrt{3}(0)}^{0}$
$-3C_2e^0\cos\sqrt{3}(0) - \sqrt{3}C_3e^0\cos\sqrt{3}(0) - \sqrt{3}C_3e^0\cos\sqrt{3}(0)$

$4C_1 + C_2 - 3C_2 = \sqrt{3}C_3 - \sqrt{3}C_3$

$4C_1 - 2C_2 - 2\sqrt{3}C_3 = 0$

$y = -\frac{1}{6}e^{2x} + \frac{1}{6}e^{-x}\cos\sqrt{3}x - \frac{\sqrt{3}}{6}e^{-x}\text{sen}\sqrt{3}x$ //

(57) $y^{(iv)} - 3y''' + 3y'' - y' = 0$

$y(0) = y'(0) = 0$
$y''(0) = y'''(0) = 1$

• $m^4 - 3m^3 + 3m^2 - m = 0$

$m(m^3 - 3m^2 + 3m - 1) = 0$

$m(m-1)^3 \quad m_1 = 0 \quad m_{234} = 1$

• $y_1 = e^{0x} \quad y_2 = e^x \quad y_3 = xe^x$
$y_4 = x^2e^x$

• $y_c = C_1 + C_2e^x + C_3xe^x + C_4x^2e^x$

$y'_c = C_2e^x + C_3e^x + C_3xe^x + 2C_4xe^x$
$+ C_4x^2e^x$

$y'' = C_2e^x + 2C_3e^x + C_3xe^x + 2C_4e^x + 4C_4xe^x$
$+ C_4x^2e^x$

$y''' = C_2e^x + 3C_3e^x + C_3xe^x + 6C_4e^x + 4C_4xe^x$
$+ 2C_4xe^x + C_4x^2e^x$

$0 = C_1 + C_2$
$0 = C_2 + C_3$
$1 = C_2 + 2C_3 + 2C_4$
$1 = C_2 + 3C_3 + 6C_4$

Resolviendo el sistema:
$C_1 = 2 \quad C_2 = -2 \quad C_3 = 2 \quad C_4 = -\frac{1}{2}$

$\therefore y = 2 - 2e^x + 2xe^x - \frac{1}{2}x^2e^x$ //

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(1) $4y'' + y' = 0$

- $4m^2 + m = 0$
- $m(4m+1) = 0$

$m_1 = 0 \quad m_2 = -\frac{1}{4}$

- $y_1 = e^{0x}$; $y_2 = e^{-1/4x}$
- $y_c = C_1 + C_2 e^{-\frac{1}{4}x}$

(3) $y'' - 36y = 0$

- $m^2 - 36m = 0$
- $(m-6)(m+6) = 0$

$m_1 = 6 \quad m_2 = -6$

- $y_1 = e^{6x} \quad y_2 = e^{-6x}$
- $y_c\, C_1 e^{6x} + C_2 e^{-6x}$

(5) $y'' + 9y = 0$

- $m^2 + 9 = 0$
- $m_2 = \pm\sqrt{-9} = 3i$
- $y_1 = e^{0x} \cos 3x$

$y_2 = e^{0x} \operatorname{sen} 3x$

- $y_c = C_1 \cos 3x + C_2 \operatorname{sen} 3x$

(7) $y'' - y' - 6y = 0$

- $m^2 - m - 6 = 0$
- $(m-3)(m+2) = 0$

$m_1 = 3 \quad m_2 = -2$

- $y_1 = e^{3x} \quad y_2 = e^{-2x}$
- $y_c = C_1 e^{3x} + C_2 e^{-2x}$

(9) $y'' + 8y' + 16y = 0$

- $m^2 + 8m + 16 = 0$
- $(m+4)^2 = 0$

$m_{1,2} = -4$

- $y_1 = e^{-4x} \quad y_2 = x e^{-4x}$
- $y_c = C_1 e^{-4x} + C_2 x e^{-4x}$

(11) $y'' + 3y' - 5y = 0$

- $m^2 + 3m - 5 = 0$
- $m_{1,2} = \frac{-3 \pm \sqrt{9 - 4(1)(-5)}}{2(1)} = \frac{-3 \pm \sqrt{29}}{2}$

$m_1 = \frac{-3+\sqrt{29}}{2} \quad m_2 = \frac{-3-\sqrt{29}}{2}$

- $y_1 = e^{\left(\frac{-3+\sqrt{29}}{2}\right)x} \quad y_2 = e^{\left(\frac{-3-\sqrt{29}}{2}\right)x}$
- $y_c = C_1 e^{\left(\frac{-3+\sqrt{29}}{2}\right)x} + C_2 e^{\left(\frac{-3+\sqrt{29}}{2}\right)x}$

(13) $12y'' - 5y' - 2y = 0$

- $12m^2 - 5m - 2 = 0$
- $m_{1,2} = \frac{5 \pm \sqrt{25 - 4(12)(-2)}}{2(12)} = \frac{5 \pm \sqrt{121}}{24} = \frac{5 \pm 11}{24}$

$m_1 = \frac{5+11}{24} = \frac{16}{24} = \frac{2}{3} \quad m_2 = \frac{5-11}{24} = -\frac{1}{4}$

- $y_1 = e^{2/3x} \quad y_2 = e^{-1/4x}$
- $y_c = C_1 e^{2/3x} + C_2 e^{-1/4x}$

(15) $y''' - 4y'' - 5y' = 0$

- $m^3 - 4m^2 - 5m = 0$
- $m(m^2 - 4m - 5) = 0$

$m_1 = 0 \quad (m-5)(m+1) = 0 \quad m_2 = 5 \quad m_3 = -1$

- $y_1 = e^{0x} \quad y_2 = e^{5x} \quad y_3 = e^{-x}$
- $y_c = C_1 + C_2 e^{5x} + C_3 e^{-x}$

(17) $3y'' + 2y' + y = 0$

- $3m^2 + 2m + 1 = 0$
- $m_{1,2} = \frac{-2 \pm \sqrt{4 - 4(3)(1)}}{2(3)} = \frac{-2 \pm 2\sqrt{2}\, i}{6}$

$m_{1,2} = -\frac{1}{3} \pm \frac{\sqrt{2}}{3} i \quad \alpha = -\frac{1}{3} \quad \beta = \frac{\sqrt{2}}{3}$

- $y_1 = e^{-1/3x} \cos \frac{\sqrt{2}}{3}x \quad y_2 = e^{-1/3x} \operatorname{sen} \frac{\sqrt{2}}{3}x$
- $y_c = C_1 e^{-x/3} \cos \frac{\sqrt{2}}{3}x + C_2 e^{-x/3} \operatorname{sen} \frac{\sqrt{2}}{3}x$

(19) $y'' - 4y' - 5y = 0$

- $m^2 - 4m - 5 = 0$
- $(m-5)(m+1) = 0 \quad m_1 = 5 \quad m_2 = -1$
- $y_1 = e^{5x} \quad y_2 = e^{-x}$
- $y_c = C_1 e^{5x} + C_2 e^{-x}$

21) $y''' - y = 0$

- $m^3 - 1 = 0$
-
$$\begin{array}{r|rrrr} & 1 & 0 & 0 & 1 \\ -1 & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & 0 \end{array} \quad m_1 = -1$$

$m^2 - m + 1 = 0$

$$m_{34} = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

- $y_1 = e^{x/2} \cos \frac{\sqrt{3}}{2} x$; $y_2 = e^{x/2} \operatorname{sen} \frac{\sqrt{3}}{2} x$
- $y_c = C_1 e^{x/2} \cos \frac{\sqrt{3}}{2} x + C_2 e^{x/2} \operatorname{sen} \frac{\sqrt{3}}{2} x$

23) $y''' - 5y'' + 3y' + 9y = 0$

- $m^3 - 5m^2 + 3m + 9 = 0$
- $a_0 = 9 \quad a_n = 1 \quad (\pm 1 \;\; \pm 3 \;\; \pm 9)$

$$\begin{array}{r|rrrr} & 1 & -5 & 3 & 9 \\ -1 & & -1 & 6 & -9 \\ \hline & 1 & -6 & 9 & 0 \end{array} \quad m_1 = -1$$

$m^2 - 6m + 9 = 0$

$(m-3)^2 \quad m_{2,3} = 3$

- $y_1 = e^{-x} \quad y_2 = e^{3x} \quad y_3 = xe^{3x}$
- $y_c = C_1 e^{-x} + C_2 e^{3x} + C_3 x e^{3x}$

25) $y''' + y'' - 2y = 0$

- $m^3 + m^2 - 2 = 0$
- $(\pm 1, \pm 2)$

$$\begin{array}{r|rrrr} & 1 & 1 & 0 & -2 \\ 1 & & 1 & 2 & 2 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

$m^2 + 2m + 2$

$$m_{1,2} = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = -1 \pm i$$

- $y_1 = e^{-x} \cos x \quad y_2 = e^{-x} \operatorname{sen} x$
- $y_c = C_1 e^{-x} \cos x + C_2 e^{-x} \operatorname{sen} x$

27) $y''' + 3y'' + 3y' + y = 0$

- $y^3 + 3m^2 + 3m + 1 = 0$
- $(y+1)^3 = 0 \quad y_{1,2,3} = -1$
- $y_1 = e^{-x} \quad y_2 = xe^{-x} \quad y_3 = x^2 e^{-x}$
- $y_c = C_1 e^{-x} + C_2 x e^{-x} + C_3 x^2 e^{-x}$

29) $y^{(iv)} + y''' + y'' = 0$

- $m^4 + m^3 + m^2 = 0$
- $m^2 (m^2 + m + 1) = 0$

$$m_1 = m_2 = 0 \quad m_{34} = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)}$$

$$m_{34} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

- $y_1 = e^{0x} \quad y_2 = xe^{0x}$

$y_3 = e^{-1/2 x} \cos \frac{\sqrt{3}}{2} x$

$y_4 = e^{-1/2 x} \operatorname{sen} \frac{\sqrt{3}}{2} x$

- $y_c = C_1 + C_2 x + C_3 e^{-x/2} \cos \frac{\sqrt{3}}{2} x + C_4 e^{-x/2} \operatorname{sen} \frac{\sqrt{3}}{2} x$

31) $16y^{(iv)} + 24y'' + 9y = 0$

- $16m^4 + 24m^2 + 9 = 0$
- $(4m^2 + 3)^2 = 0$

$m_{1234} = \sqrt{-\frac{3}{4}} = \sqrt{\frac{3}{4}} i$

- $y_1 = \cos \sqrt{\frac{3}{4}} x$

$y_2 = \operatorname{sen} \sqrt{\frac{3}{4}} x$

$y_3 = x \cos \sqrt{\frac{3}{4}} x$

$y_4 = x \operatorname{sen} \sqrt{\frac{3}{4}} x$

- $y_c = C_1 \cos \sqrt{\frac{3}{4}} x + C_2 \operatorname{sen} \sqrt{\frac{3}{4}} x + C_3 x \cos \sqrt{\frac{3}{4}} x + C_4 x \operatorname{sen} \sqrt{\frac{3}{4}} x$

33) $y^{(v)} - 16y' = 0$

- $m^5 - 16m = 0$
- $m(m^4 - 16) = 0 \quad m_1 = 0$

$m(m^2 - 4)(m^2 + 4)$

$m(m-2)(m+2)(m^2 + 4) = 0$

$m_1 = 0 \quad m_2 = 2 \quad m_3 = -2 \quad m_{45} = \pm 2i$

- $y_1 = e^{0x} \quad y_2 = e^{2x} \quad y_3 = e^{-2x}$

$y_4 = e^{0x} \cos 2x \quad y_5 = e^{0x} \operatorname{sen} 2x$

- $y_c = C_1 + C_2 e^{2x} + C_3 e^{-2x} + C_4 \cos 2x + C_5 \operatorname{sen} 2x$

$+5y^{(iv)} - 2y''' - 10y'' + y' + 5y = 0$

$+5m^4 - 2m^3 - 10m^2 + m + 5 = 0$

$\pm 1;\ \pm 5$

	1	5	-2	-10	1	5
1		1	6	4	-6	-5
	1	6	4	-6	-5	0
1		1	7	11	5	
	1	7	11	5	0	
-1		-1	-6	-5		
	1	6	5	0		
-1		-1	-5			
	1	5	0			
-5		-5				
	1	0				

$m_{12} = 1 \quad m_{34} = -1 \quad m_5 = -5$

- $y_1 = e^x \quad y_2 = xe^x \quad y_3 = e^{-x} \quad y_4 = xe^{-x}$
  $y_5 = e^{-5x}$
- $y_c = C_1e^x + C_2xe^x + C_3e^{-x} + C_4xe^{-x} + C_5e^{-5x}$ //

(37) $y'' + 16y = 0;\ y(0) = 2;\ y'(0) = -2$

- $m^2 + 16 = 0$
- $m_{12} = \pm\sqrt{-16} = \pm 4i$
- $y_1 = e^{0x}\cos 4x \quad y_2 = e^{0x}\operatorname{sen} 4x$
- $y_c = C_1\cos 4x + C_2\operatorname{sen} 4x$
- $y'_c = -4C_1\operatorname{sen} 4x + 4C_2\cos 4x$

$\to -2 = -4\operatorname{sen} 4(0) + 4C_2\cos 4(0)$

$\to 2 = C_1\cos 4(0) + C_2\operatorname{sen} 4(0)$

$-2 = -4C_2 \qquad C_2 = \frac{1}{2}$

$2 = C_1$

$\therefore y_c = 2\cos 4x - \frac{1}{2}\operatorname{sen} 4x$ //

(34) $y'' + 6y' + 5y = 0 \quad y(0) = 0 \quad y'(0) = 3$

- $m^2 + 6m + 5 = 0$
- $(m+5)(m+1) = 0$
  $m_1 = -5 \quad m_2 = -1$
- $y_1 = e^{-5x} \quad y_2 = e^{-x}$
- $y_c = C_1e^{-5x} + C_2e^{-x}$

$y'_c = -5C_1e^{-5x} - C_2e^{-x}$

$\to C_1e^{-5(0)} + C_2e^{-(0)} = 0$

$-5C_1e^{-5(0)} - C_2e^{-(0)} = 3$

$\to C_1 + C_2 = 0$

$-5C_1 - C_2 = 3$

$C_2 = -5C_1 - 3 \longrightarrow C_2 = -5\left(\frac{3}{4}\right) - 3$

$= \frac{3}{4}$

$C_1 - 5C_1 - 3 = 0$

$-4C_1 = 3$

$C_1 = -\frac{3}{4}$

$\therefore y_c = -\frac{3}{4}e^{-5x} + \frac{3}{4}e^{-x}$ //

(41) $2y'' - 2y' + y = 0;\ y(0) = -1;\ y'(0) = 0$

- $2m^2 - 2m + 1 = 0$
- $m_{12} = \dfrac{2 \pm \sqrt{4 - 4(2)(1)}}{2(2)} = \frac{1}{2} \pm \frac{1}{2}i$

- $y_1 = e^{1/2x}\cos\frac{1}{2}x \quad y_2 = e^{1/2x}\operatorname{sen}\frac{1}{2}x$
- $y_c = C_1e^{1/2x}\cos\frac{1}{2}x + C_2e^{1/2x}\operatorname{sen}\frac{1}{2}x$
- $y'_c = -\frac{1}{2}C_1e^{1/2x}\operatorname{sen}\frac{1}{2}x + \frac{1}{2}C_1e^{1/2x}\cos\frac{1}{2}x$
  $+ \frac{1}{2}C_2e^{1/2x}\cos\frac{1}{2}x + \frac{1}{2}C_2e^{1/2x}\operatorname{sen}\frac{1}{2}x$

Sustituyendo

- $C_1e^{1/2(0)}\cos\frac{1}{2}(0) + C_2e^{1/2(0)}\operatorname{sen}\frac{1}{2}(0) = -1$

$C_1 = -1$

- $-\frac{1}{2}C_1e^{1/2(0)}\operatorname{sen}\frac{1}{2}(0) + \frac{1}{2}C_1e^{1/2(0)}\cos\frac{1}{2}(0) + \frac{1}{2}C_2e^{1/2(0)}$
$\cos\frac{1}{2}(0) + \frac{1}{2}C_2e^{1/2(0)}\operatorname{sen}\frac{1}{2}(0) = 0$

$\frac{1}{2}C_1 + \frac{1}{2}C_2 = 0$

$C_2 = 2(-\frac{1}{2}C_1)$

$= -C_1$

$C_2 = -(-1) = 1$

$\therefore y_c = -e^{1/2x}\cos\frac{1}{2}x + e^{1/2x}\operatorname{sen}\frac{1}{2}x$ //

43) $y'' + y' + 2y = 0 \; ; \; y(0) = y'(0) = 0$

- $m^2 + m + 2 = 0$

$$m_{1,2} = \frac{-1 \pm \sqrt{1 - 4(1)(2)}}{2} = -\frac{1}{2} \pm \frac{\sqrt{7}}{2} i$$

- $y_1 = e^{-1/2x} \cos \frac{\sqrt{7}}{2} x \qquad y_2 = e^{-1/2x} \operatorname{sen} \frac{\sqrt{7}}{2} x$
- $y_c = C_1 e^{-1/2x} \cos \frac{\sqrt{7}}{2} x + C_2 e^{-1/2x} \operatorname{sen} \frac{\sqrt{7}}{2} x$

$y'_c = -\frac{1}{2} C_1 e^{-1/2x} \cos \frac{\sqrt{7}}{2} x - \frac{\sqrt{7}}{2} C_1 e^{-1/2x} \operatorname{sen} \frac{\sqrt{7}}{2} x$
$\quad - \frac{1}{2} C_2 e^{-1/2x} \operatorname{sen} \frac{\sqrt{7}}{2} x + \frac{\sqrt{7}}{2} C_2 e^{-1/2x} \cos \frac{\sqrt{7}}{2} x$

$\rightarrow C_1 e^{-1/2(0)} \cos \frac{\sqrt{7}}{2}(0) + C_2 e^{-1/2(0)} \operatorname{sen} \frac{\sqrt{7}}{2}(0) = 0$

$C_1 = 0$

$\rightarrow -\frac{1}{2} C_1 e^{-1/2(0)} \cos \frac{\sqrt{7}}{2}(0) - \frac{\sqrt{7}}{2} C_1 e^{-1/2(0)} \operatorname{sen} \frac{\sqrt{7}}{2}(0) - \frac{1}{2} C_2 e^{-1/2(0)} \operatorname{sen} \frac{\sqrt{7}}{2} x +$
$\frac{\sqrt{7}}{2} C_2 e^{-1/2(0)} \cos \frac{\sqrt{7}}{2}(0) = 0$

$-\frac{1}{2} C_1 + \frac{\sqrt{7}}{2} C_2 = 0$

$C_2 = 0$

$\therefore y_c = 0$

45) $y'' - 3y' + 2y = 0 \; ; \; y(1) = 0 ; \; y'(1) = 1$

- $m^2 - 3m + 2 = 0$
- $(m-2)(m-1) = 0$

$m_1 = 2 \quad m_2 = 1$

- $y_1 = e^{2x} \qquad y_2 = e^x$
- $y_c = C_1 e^{2x} + C_2 e^x$

$y'_c = 2C_1 e^{2x} + C_2 e^x$

$C_1 e^2 + C_2 e = 0 \qquad C_2 e = -C_1 e^2$
$2C_1 e^2 + C_2 e = 1$

$2C_1 e^2 + (-C_1 e^2) = 1$

$C_1 e^2 = 1$

$C_1 = \frac{1}{e^2}$

$C_2 = -\left(\frac{1}{e^2}\right)(e^2)$
$= -1$

$y_c = \frac{1}{e^2} e^{2x} - 1 e^x$
$= e^{2(x-1)} - e^x$

47) $y''' + 12y'' + 36y' = 0 \quad y(0) = 0 \quad y'(0) = 1 \quad y''(0) = -7$

- $m^3 + 12m^2 + 36m = 0$
- $m(m^2 + 12m + 36) = 0$

$m(m+6)(m+6) = 0$

$m_1 = 0 \quad m_2 = -6 \quad m_3 = -6$

- $y_1 = e^{0x} \quad y_2 = e^{-6x} \quad y_3 = xe^{-6x}$
- $y_c = C_1 + C_2 e^{-6x} + C_3 x e^{-6x}$

$y'_c = -6C_2 e^{-6x} - 6C_3 x e^{-6x} + C_3 e^{-6x}$

$y''_c = 36C_2 e^{-6x} + 36C_3 x e^{-6x} - 6C_3 e^{-6x} - 6C_3 e^{-6x}$

$C_1 + C_2 = 0$
$-6C_2 + C_3 = 1$
$36C_2 - 12C_3 = -7$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -6 & 1 & 1 \\ 0 & 36 & -12 & -7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{6} & -\frac{1}{6} \\ 0 & 0 & -6 & -7 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 1 & \frac{1}{6} \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -5/6 \\ 0 & 1 & 0 & 5/36 \\ 0 & 0 & 0 & 1/6 \end{array}\right)$$

$y_c = -\frac{5}{6} - \frac{5}{36} e^{-6x} + \frac{1}{6} x e^{-6x}$

49. $y''' - 8y = 0, \quad y(0) = 0 \quad y'(0) = -1 \quad y''(0) = 0$

- $m^3 - 8 = 0$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -8 \\ 2 & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array} \qquad m_1 = 2$$

$m^2 + 2m + 4$

$$m_{2,3} = \frac{-2 \pm \sqrt{4 - 4(1)(4)}}{2(1)} = \frac{-2 \pm \sqrt{-12}}{2} = -1 \pm \sqrt{3} i$$

- $y_1 = e^{2x} \quad y_2 = e^{-x} \cos \sqrt{3} x \quad y_3 = e^{-x} \operatorname{sen} \sqrt{3} x$
- $y_c = C_1 e^{2x} + C_2 e^{-x} \cos \sqrt{3} x + C_3 e^{-x} \operatorname{sen} \sqrt{3} x$
- $y'_c = 2C_1 e^{2x} - C_2 e^{-x} \cos \sqrt{3} x - \sqrt{3} C_2 e^{-x} \cos \sqrt{3} x$
$\quad - C_3 e^{-x} \operatorname{sen} \sqrt{3} x + \sqrt{3} C_3 e^{-x} \cos \sqrt{3} x$
- $y'' = 4C_1 e^{2x} + C_2 e^{-x} \cos \sqrt{3} x + \sqrt{3} C_2 e^{-x} \operatorname{sen} \sqrt{3} x + \sqrt{3} C_2 e^{-x} \cos \sqrt{3} x$
$\quad + 3C_2 e^{-x} \operatorname{sen} \sqrt{3} x + C_3 e^{-x} \operatorname{sen} \sqrt{3} x - \sqrt{3} C_3 e^{-x} \cos \sqrt{3} x$
$\quad - \sqrt{3} C_3 e^{-x} \cos \sqrt{3} x + 3\sqrt{3} C_3 e^{-x} \cos \sqrt{3} x$

2# Cap. 4 Pag. 152 Seccion 4.4 1-38 impares.

1. $y'' + 3y' + 2y = 6 \ldots (1)$

- Ec. homogenea

$y'' + 3y' + 2y = 0 \ldots (2)$

- Ec. caracte.

$m^2 + 3m + 2 = 0$

- Raices

$(m+1)(m+2)$

$m_1 = -1 \quad m_2 = -2$

- Sols. 1º

$y_1 = e^{-x} \quad y_2 = e^{-2x}$

- Sol. gral de (2)

$y_c = C_1 e^{-x} + C_2 e^{-2x}$

- $F(x) = 6$

Proponemos $y_p = A$

$y'_p = 0$

$y''_p = 0$

Sustituimos y, y' & y'' en (1)

$0 + 3(0) + 2A = 6$

$2A = 6 \rightarrow A = 3$

$y_p = 3$

$\therefore y = y_c + y_p = \underline{C_1 e^{-x} + C_2 e^{-2x} + 3}$ //

3. $y'' - 10y' + 25y = 30x + 3 \ldots (1)$

- $y'' - 10y' + 25y = 0 \ldots (2)$
- $m^2 - 10m + 25 = 0$
- $(m-5)(m-5) = 0$
- $y_1 = e^{-5x} \quad y_2 = xe^{-5x}$
- $y_c = C_1 e^{-5x} + C_2 x e^{-5x}$

- $F(x) = 30x + 3$

$y_p = Ax + B$

$y'_p = A$

$y''_p = 0$

Sustituimos en (1)

$-10A + 25(Ax + B) = 30x + 3$

$-10A + 25Ax + 25B = 30x + 3$

$(25A) = 30 \quad A = \frac{6}{5}$

$25B - 10A = 3 \qquad 25B = 15$

$25B - 10\left(\frac{6}{5}\right) = 3 \qquad B = \frac{3}{5}$

$\Rightarrow y_p = \frac{6}{5}x + \frac{3}{5}$

$\therefore y = y_c + y_p = \underline{C_1 e^{-5x} + C_2 x e^{-5x} + \frac{6}{5}x + \frac{3}{5}}$ //

(5) $\frac{1}{4}y'' + y' + y = x^2 - 2x \quad \cdots (1)$

- $\frac{1}{4}y'' + y' + y = 0 \quad \cdots (2)$
- $\frac{1}{4}m^2 + m + 1 = 0$
- $m^2 + 4m + 4 = 0$

$(m+2)(m+2) \qquad m_1 = -2 = m_2$

- $y_1 = e^{-2x} \qquad y_2 = xe^{-2x}$
- $y_c = C_1e^{-2x} + C_2xe^{-2x}$

- $F(x) = x^2 - 2x$

$y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

Sustituyendo y & sus der. en (1)

$\frac{1}{4}(2A) + 2Ax + B + Ax^2 + Bx + C = x^2 - 2x$

$\frac{1}{2}A + 2Ax + Bx + Ax^2 + B + C = x^2 - 2x$

$Ax^2 + (2A + B)x + (\frac{1}{2}A + B + C) = x^2 - 2x$

$A = 1$

$2A + B = -2 \longrightarrow B = -4$

$\frac{1}{2}A + B + C = 0 \longrightarrow C = \frac{7}{2}$

$\rightarrow y_p = x^2 - 4x + \frac{7}{2}$

$\therefore y = y_c + y_p = C_1e^{-2x} + C_2xe^{-2x} + x^2 - 4x + \frac{7}{2}$

(7) $y'' + 3y = -48x^2e^{3x}$

- $y'' + 3y = 0$
- $m^2 + 3 = 0$
- $m_{1,2} = \pm\sqrt{-3} = \pm\sqrt{3}\,i$
- $y_1 = \cos\sqrt{3}x \qquad y_2 = \operatorname{sen}\sqrt{3}x$
- $y_c = C_1\cos\sqrt{3}x + C_2\operatorname{sen}\sqrt{3}x$

- $F(x) = -48x^2e^{3x}$
- $y_p = (Ax^2 + Bx + C)e^{3x}$

$= Ax^2e^{3x} + Bxe^{3x} + Ce^{3x}$

$y'_p = 2Axe^{3x} + 3Ax^2e^{3x} + Be^{3x} + 3Bxe^{3x} + 3Ce^{3x}$

$y''_p = 2Ae^{3x} + 6Axe^{3x} + 6Axe^{3x} + 9Ax^2e^{3x} + 3Be^{3x} + 3Be^{3x} + 9Bxe^{3x} + 9Ce^{3x}$

Sustituimos

$2Ae^{3x} + 12Axe^{3x} + 9Ax^2e^{3x} + 6Be^{3x} + 9Bxe^{3x} + 9Ce^{3x} + 3Ax^2e^{3x} + 3Bxe^{3x} + 3Ce^{3x} = -48x^2e^{3x}$

$(12A)x^2e^{3x} + (12A + 9B + 3B)xe^{3x} + (2A + 6B + 12C)e^{3x} = -48x^2e^{3x}$

$12A = -48 \longrightarrow A = -4$

$12A + 12B = 0 \longrightarrow B = 4$

$2A + 6B + 12C = 0 \longrightarrow C = -\frac{4}{3}$

$\rightarrow y_p = (-4x^2 + 4x - \frac{4}{3})e^{3x}$

$\therefore y = y_c + y_p = C_1\cos\sqrt{3}x + C_2\operatorname{sen}\sqrt{3}x + (-4x^2 + 4x - \frac{4}{3})e^{3x}$

$y'' - y' = -3 \ldots (1)$

- Ec. homo

$y'' - y' = 0 \ldots (2)$

- Ec. carac.

$m^2 - m = 0$

- Raices

$m(m-1) = 0$

$m_1 = 0 \quad m_2 = 1$

- Sols. li

$y_1 = 1 \quad y_2 = e^x$

- Sol. gral. de (2)

$y_c = C_1 + C_2 e^x$

$F(x) = -3$

$y_p = Ax$

$y'p = A$

$y''p = 0$

- Sust. y, y', y'' en (1)

$-A = -3$

$A = 3$

$\therefore y_p = 3x$

$y = y_c + y_p = \underline{C_1 + C_2 e^x + 3x}$

ii. $y'' - y' + \frac{1}{4}y = 3 + e^{x/2} \ldots (1)$

- Ec homo

$y'' - y' + \frac{1}{4}y = 0 \ldots (2)$

- Ec carac.

$m^2 - m + \frac{1}{4} = 0$

- Raices

$(m - \frac{1}{2})^2 \quad m_1 = \frac{1}{2} = m_2$

- Sols. li

$y_1 = e^{x/2} \quad y_2 = xe^{x/2}$

- Sol. gral. de (2)

$y_c = C_1 e^{x/2} + C_2 x e^{x/2}$

- $F(x) = 3 + e^{x/2}$

$y_p = A + Bx^2 e^{x/2}$

$y'p = 2Bxe^{x/2} + \frac{1}{2}Bx^2 e^{x/2}$

$y''p = 2Be^{x/2} + 2Bxe^{x/2} + \frac{1}{4}Bx^2 e^{x/2}$

Sust. y Y sus derivadas en (1)

$2Be^{x/2} + \cancel{2Bxe^{x/2}} + \cancel{\frac{1}{4}Bx^2e^{x/2}} - \cancel{2Bxe^{x/2}} - \cancel{\frac{1}{2}Bx^2e^{x/2}} + \frac{1}{4}A + \cancel{\frac{1}{4}Bx^2e^{x/2}} = 3 + e^{x/2}$

$\frac{1}{4}A + 2Be^{x/2} = 3 + e^{x/2}$

$\frac{1}{4}A = 3 \longrightarrow A = 12$

$2B = 1 \longrightarrow B = \frac{1}{2}$

$\therefore y_p = 12 + \frac{1}{2}x^2 e^{x/2}$

$y = y_c + y_p = C_1 e^{x/2} + C_2 x e^{x/2} + 12 + \frac{1}{2}x^2 e^{x/2}$

13. $y'' + 4y = 3 \operatorname{sen} 2x \ldots (1)$

- Ec. homo $y'' + 4y = 0 \ldots (2)$
- Ec. carac. $m^2 + 4 = 0$
- Raices $m_{1,2} = \pm\sqrt{-4} = \pm 2i$
- Sols li $y_1 = \operatorname{sen} 2x \quad y_2 = \cos 2x$
- Sol. gral. de (2)

$y_c = C_1 \operatorname{sen} 2x + C_2 \cos 2x$

$F(x) = 3 \operatorname{sen} 2x$

$y_p = (A\cos 2x + B\operatorname{sen} 2x)x$

$y' = Ax\cos 2x + Bx\operatorname{sen} 2x$

$y'p = A\cos 2x - 2Ax\operatorname{sen} 2x + B\operatorname{sen} 2x + 2Bx\cos 2x$

$y''p = -4A\operatorname{sen} 2x - 4Ax\cos 2x + 4B\operatorname{sen} x - 4Bx\operatorname{sen} x$

Sustituyendo y Y sus deriv. en (1)

$-4A\operatorname{sen}2x - \cancel{4Ax\cos 2x} + 4B\cos 2x - \cancel{4Bx\operatorname{sen} x} + \cancel{4Ax\cos 2x} + \cancel{4Bx\operatorname{sen} x}$

$4B\cos 2x - 4A\operatorname{sen} 2x = 3\operatorname{sen} 2x \quad = F(x)$

$-4A = 3 \quad A = -\frac{3}{4}$

$4B = 0 \quad \rightarrow B = 0$

$\therefore y_p = -\frac{3}{4}x\cos 2x$

$\Rightarrow y = y_c + y_p = \underline{C_1 \operatorname{sen} 2x + C_2 \cos 2x - \frac{3}{4}x\cos 2x}$