MCMC I
11th Summer Institute in Statistics and Modeling in Infectious Diseases
Course Time Plan
July 15-17, 2019

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Course Description: This module is an introduction to Markov chain Monte Carlo methods with some simple applications in infectious disease studies. The course includes an introduction to Bayesian inference, Monte Carlo, MCMC, some background theory, and convergence diagnostics. Algorithms include Gibbs sampling and Metropolis-Hastings and combinations. Programming is in R. Familiarity with the R statistical package is assumed.

Course schedule: The course is composed of 10 90-minute sessions, for a total of 15 hours of instruction.

1 Introduction to Bayesian Inference

- Overview of the course.
- Bayesian inference: Likelihood, prior, posterior, normalizing constant
- Conjugate priors; Beta-binomial; Poisson-gamma; normal-normal
- Posterior summaries, mean, mode, posterior intervals
- Motivating examples: Chain binomial model (Reed-Frost), General Epidemic Model, SIS model.
- Lab:
  - Goals: Warm-up with R for simple Bayesian computation
  - Example: Posterior distribution of transmission probability with a binomial sampling distribution using a conjugate beta prior distribution
  - Summarizing posterior inference (mean, median, posterior quantiles and intervals)
  - Varying the amount of prior information

2 Introduction to Gibbs Sampling

- Chain binomial model and data augmentation
- Brief introduction to Gibbs sampling
- Lab
  - Goals: Simple data augmentation using MCMC
  - Example: Gibbs sampler for the chain binomial model.

3 Introduction to computation

- Random number generators
- Non-iterative Monte Carlo methods
  - Direct sampling, Monte Carlo integration (classical Monte Carlo)
  - Indirect methods: importance sampling, rejection sampling,
- Basic Markov Chain Theory
  - Definitions
  - Stationarity
  - The ergodic theorem
- Lab:
  - Goals: Importance sampling, Markov chain

4 Markov chain Monte Carlo methods

- Gibbs sampling
  - Background
  - Revisit simple Gibbs sampler for chain-binomial model
- Metropolis-Hasting algorithm
- Lab:
  - Goals: M-H: elementary missing data imputation on S-I model

5 Metropolis-Hasting and Gibbs combined

- Example: Hierarchical model
- Lab:
  - Goals: Combining Metropolis and Gibbs in one algorithm
  - Example: Beta-binomial hierarchical model with rat data

6 Chain binomial model revisited

- Hierarchical chain binomial model with hyperparameters
  - Model checking
  - Allowing for heterogeneity

- Lab:
  - Goals: Combined M-H and Gibbs and learning model checking
  - Example: Hierarchical beta-binomial chain binomial model

7 General Epidemic Model

- The general epidemic model and incompletely observed data
- Algorithm
- Lab: General epidemic model
  - Goals: parameter estimation with data augmentation
  - Example: smallpox transmission

8 Diagnostics, etc

- Assessing convergence (more or less), Coda
- Variance reduction, Monte Carlo error
- Poisson process
- Lab: Diagnostics
  - Goals: learn how to do basic diagnostics on chain and output
  - Coda
  - Diagnostics on previous examples

9 SIS model

- Binary Markov process model for a recurrent infection
- Likelihood
- Algorithm
- Lab: Estimating rates in simple SIS model
  - Goals: Data simulation and parameter estimation from complete data in a simple SIS model.
  - Example: simulate one long chain in one person
  - Estimating rates from complete data
  - Diagnostics