

1 Lecture 13

Slide 1

CW behavior

Optics and
Laser Physics
T. Cesca

The diagram shows a three-level system with levels N_3 , N_2 , N_1 , and N_g . A green arrow labeled "pump" points from N_g to N_2 . Blue arrows labeled "Fast decay" point from N_3 to N_2 and from N_1 to N_g . A red arrow labeled "Laser transition" points from N_2 to N_1 .

Population inversion

$$N \equiv N_2 - N_1 \cong N_2$$

$$\frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \quad (*)$$

$$\frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \quad (**)$$

Let's assume that at $t = 0$ an arbitrarily small number of photons (e.g., $\phi_i = 1$) is present within the cavity due to spontaneous emission.

From equation (**) we get that to have **laser action** (i.e., the amplification of the number of photons in the cavity) it should be:

$$\frac{d\phi}{dt} \geq 0 \Rightarrow V_a B N \geq \frac{1}{\tau_c} \Rightarrow \text{Laser action starts when it is reached a critical population inversion}$$

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Slide 2

CW behavior

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The diagram is identical to the one in Slide 1, showing a three-level system with levels N_3 , N_2 , N_1 , and N_g . A green arrow labeled "pump" points from N_g to N_2 . Blue arrows labeled "Fast decay" point from N_3 to N_2 and from N_1 to N_g . A red arrow labeled "Laser transition" points from N_2 to N_1 .

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$$\frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \quad (*)$$

$$\frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \quad (**)$$

$\frac{d\phi}{dt} = 0 \Rightarrow N_C = \frac{1}{V_a B \tau_c} = \frac{\gamma}{\sigma l}$

$$B = \frac{\sigma lc}{V_a L_e} \quad \tau_c = \frac{L_e}{\gamma c}$$

$\frac{d\phi}{dt} \geq 0 \Rightarrow V_a B N \geq \frac{1}{\tau_c} \Rightarrow \text{Laser action starts when it is reached a critical population inversion}$

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Slide 3

CW behavior

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T. Cesca

The diagram is identical to the one in Slide 1, showing a three-level system with levels N_3 , N_2 , N_1 , and N_g . A green arrow labeled "pump" points from N_g to N_2 . Blue arrows labeled "Fast decay" point from N_3 to N_2 and from N_1 to N_g . A red arrow labeled "Laser transition" points from N_2 to N_1 .

Population inversion

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$$B = \frac{\sigma lc}{V_a L_e} \quad \tau_c = \frac{L_e}{\gamma c}$$

The corresponding **critical pumping rate** (R_{PC}) is obtained from (*) imposing:

$$\frac{dN}{dt} = 0 \quad (\text{steady-state}) \quad N = N_C \quad \phi = 0$$

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The expression for the output power is the same for a three-level system.

Let us assume that at $t = 0$ there is a small number of photons within the cavity given by spontaneous emission. In this case, from the second rate equation, in order to get laser action we need an amplification of the number of photons. We obtain a condition on the population inversion: we have to overcome a **critical population inversion**.

This critical value can be computed. The result obtained is already being obtained calculating the threshold condition when we have the same number of photon at the beginning and between a passage back and forth (?).

We can also determine the **critical pumping rate** in order to obtain the *critical population inversion*. At the threshold there are no photon in the cavity (we neglect the single photon that we assume should be present by spontaneous emission).

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CW behavior

$N_3 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$

$N \equiv N_2 - N_1 \cong N_2 \quad \text{Population inversion}$

$\left\{ \begin{array}{l} \frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \\ \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \end{array} \right. \quad (*)$

$\Rightarrow R_{PC} = \frac{N_c}{\tau}$

The critical pumping rate (R_{PC}) corresponds to the situation in which the rate of pump transitions is equal to the spontaneous transitions from level 2.

The corresponding critical pumping rate (R_{PC}) is obtained from (*) imposing:

$$\frac{dN}{dt} = 0 \quad (\text{steady-state}) \quad N = N_c \quad \phi = 0$$

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CW behavior

$N_3 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$

$N \equiv N_2 - N_1 \cong N_2 \quad \text{Population inversion}$

$\left\{ \begin{array}{l} \frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \\ \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \end{array} \right. \quad (*)$

$R_p > R_{PC}$ The number of photons ϕ increases from its initial value determined by spontaneous emission.

If R_p is independent of time, ϕ reaches a steady-state value ϕ_0 which corresponds to a steady-state value of population inversion N_0 .

ϕ_0 and N_0 can be determined imposing: $\frac{d\phi}{dt} = 0 = \frac{dN}{dt}$

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CW behavior

$N_3 \cong N_3 \cong 0 \quad \frac{dN_g}{dt} \cong 0$

$N \equiv N_2 - N_1 \cong N_2 \quad \text{Population inversion}$

$\left\{ \begin{array}{l} \frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \\ \frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \end{array} \right. \quad (*)$

$R_p > R_{PC} \quad \frac{d\phi}{dt} = 0 \Rightarrow N_0 = \frac{1}{V_a B \tau_c} = \frac{\gamma}{\sigma l} = N_c !$

$\frac{dN}{dt} = 0 \Rightarrow \phi_0 = V_a \tau_c \left[R_p - \frac{N_0}{\tau} \right]$

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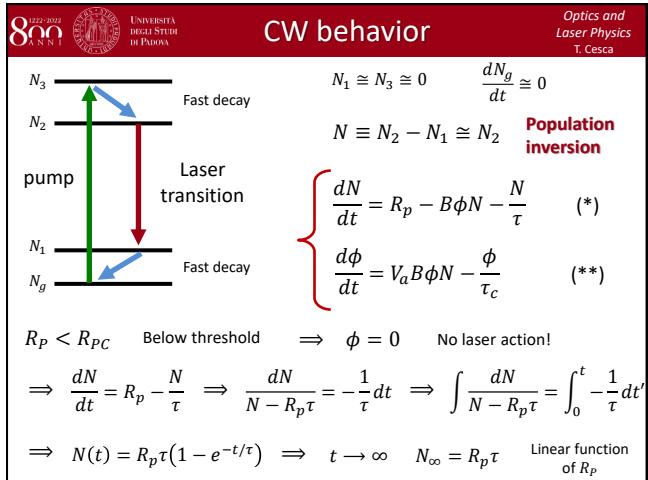
The critical pumping rate is the pumping rate that you need to apply to your system in order to balance spontaneous emission transition from the upper laser level.

If the pumping rate is larger than the critical pumping rate, the number of photons start to increase. As we suppose it is independent on time, the number of photon in the cavity reach a steady-state Φ_0 and also a steady-state of population inversion N_0 .

Let us compute the rate equations at the steady-state. We have $N_0 = N_c$, so the population inversion remains equal to the initial population inversion when the laser starts! So it is important to remind that the **population inversion does not change anymore!**

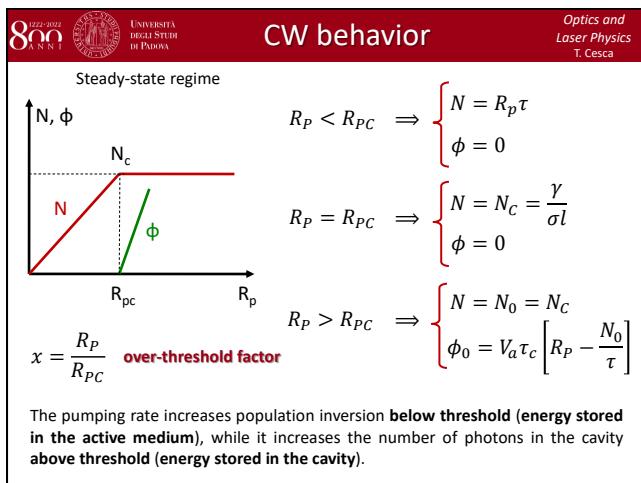
The number of photon above threshold at the steady-state increases linearly with the pumping rate.

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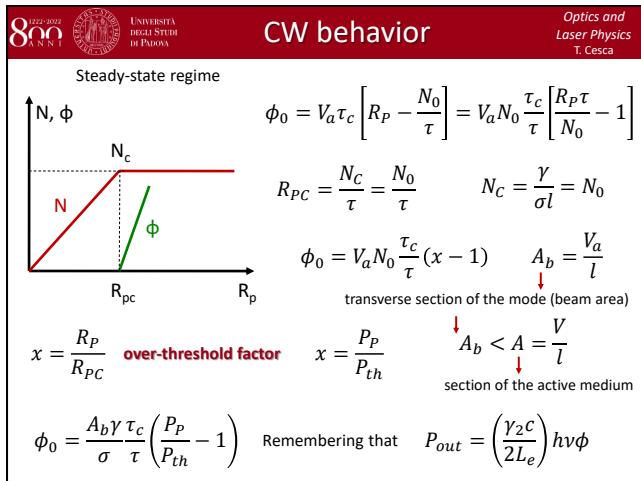
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Instead, below the threshold (there is no laser action), there are no photons by stimulated emission.

The evolution over time of the population inversion has an exponential trend with an asymptotic behavior. It is a linear function of the pumping rate.

Let us sum up the different regimes.

We can introduce the **over-threshold factor**: the ratio between the pumping rate and the critical pumping rate, so how much you are pumping above threshold.

So, when you pump the material **below threshold** the energy that you are giving to your system is stored in the active medium and is used to increase the population inversion.

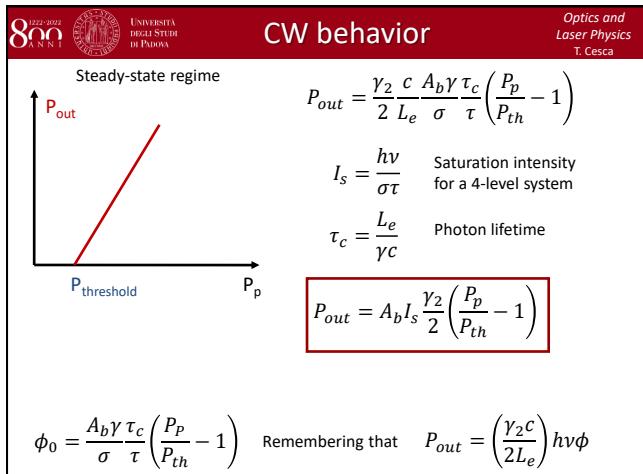
Instead, **above threshold**, the energy is stored in the cavity, because its the number of photon within the cavity which is increasing when laser action starts.

Let us suppose we are above threshold.

The **beam area** is in general smaller than the **section of the active medium**.

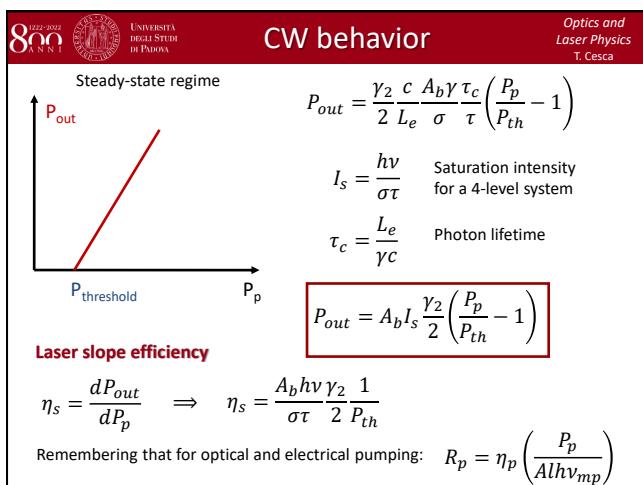
We can rewrite the number of photons as a function of the power.

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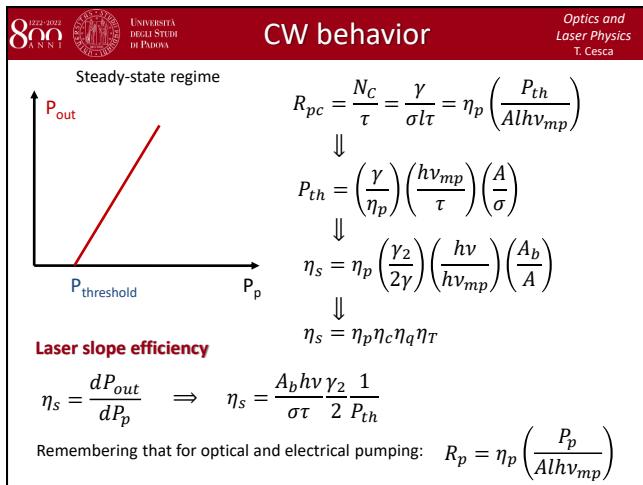
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If you are designing a laser, the behavior is a **threshold one**.

We introduced for a four-level system the **saturation intensity** and we can simplify the expression for the output power.

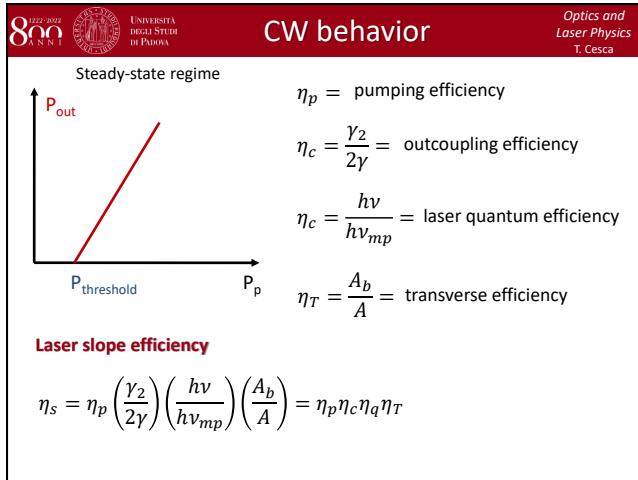
The **laser slope efficiency** is the slope of the line in the plot.

We remember also the expression for the **optical and electrical pumping**.

We can use these expression to rewrite the critical pumping rate.

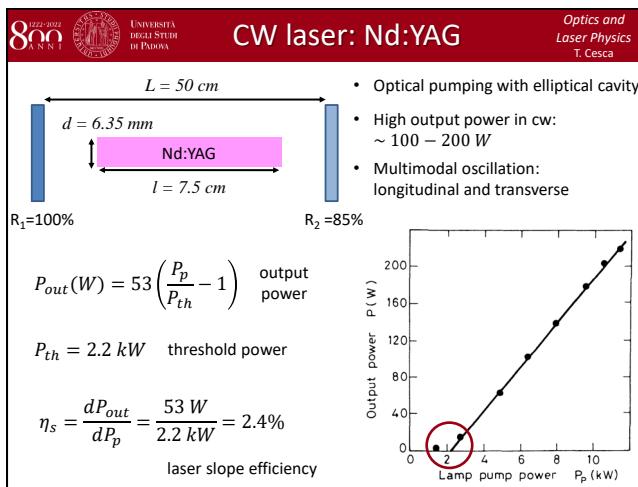
We can write an explicit expression for the **threshold power** P_{th} .

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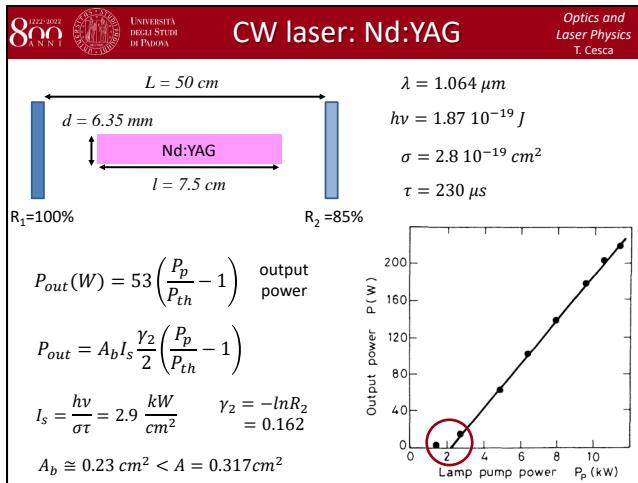
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These are the experimental parameters.

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The laser slope efficiency can be also rewritten in another form as the product of the **pumping efficiency**, **out-coupling efficiency** (fraction of photons that are extracted from the cavity, so is related to the losser of the outcoupling mirror wrt the total losses of the cavity), **laser quantum efficiency** (ratio between the energy of your photon and the minimum energy that you need to give to your system in order to pump to the upper laser level) and **transverse efficiency** (ratio between the beam size of the mode on the active medium divided by the section of the active medium, it say you what is the fraction of the active medium you are using with your mode).

The larger is the slope, the large is the power that you can obtain from a given pumping power.

Let us try to describe a real system and let us see how much we can describe it.

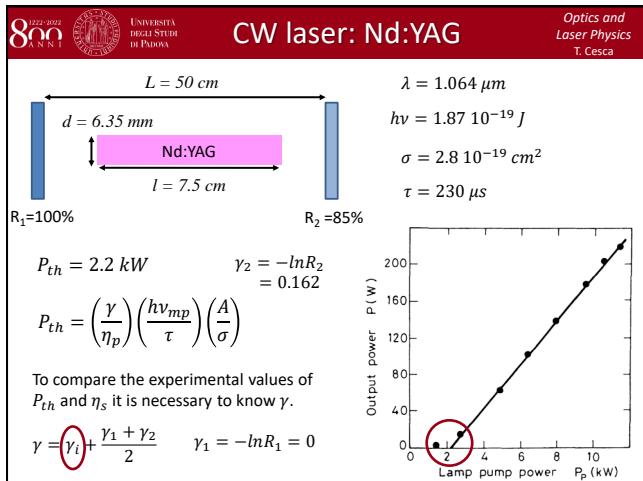
Let us consider a Fabry-Perot cavity. We have an **optical pumping with elliptical cavity**. There are no elements to select the mode of oscillations, so we have **multimodal oscillation**.

If you relax the first hypothesis of having a single mode, you end up with an intensity distribution which is homogeneous both in the transverse and longitudinal direction. So, it is possible in any case to get a good agreement between the experimental results and what we calculate phenomenologically.

Let us compare the experimental values with the analytical one.

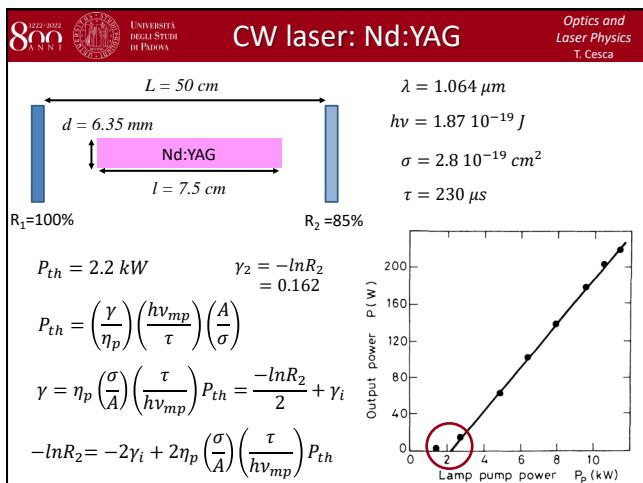
Here there are the physical parameters of the system. With the physical parameters of our system we can calculate the saturation intensity. We have to calculate also the logarithmic losses given by the outcoupling mirror. If we compare the experimental expression with what we can calculate theoretically, we can obtain the size of the medium. It is smaller wrt we can calculate by the transverse section of the rod and the diameter.

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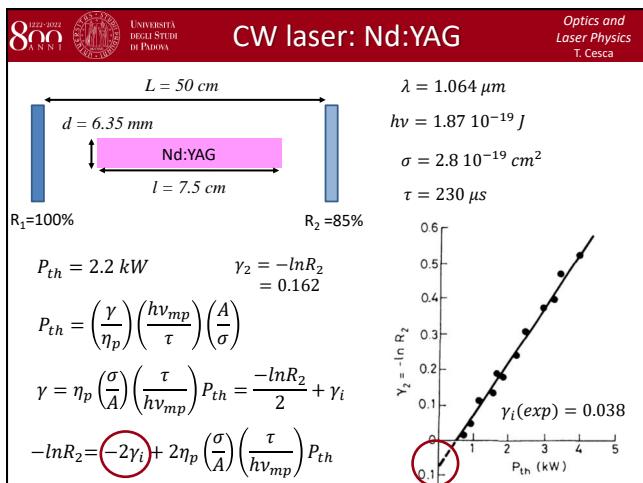
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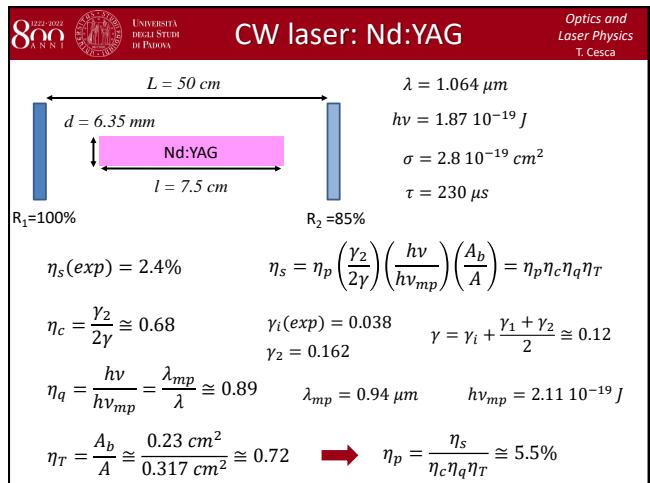
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We can also compare the power theoretical threshold with the experimental one. But, we need the logarithmic losses.

We need γ_i and we can compute it experimentally. So, you can do an experiment in which you vary the outcoupling mirror and you measure the threshold power of your system. In this way, you can obtain a linear relationship.

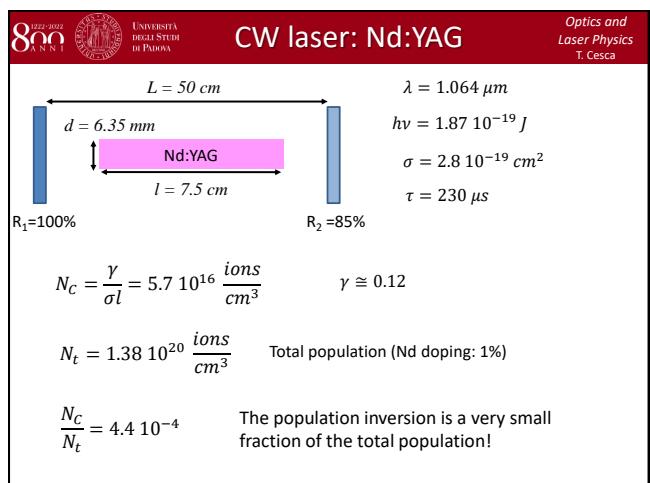
The intercept is $-2\gamma_i$. This is the most efficient way to determine experimentally the losses. Otherwise, you have to analyze your elements what are teh sizes and so on...

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We can compare the experimental value of the laser slope efficiency with the value that we have calculated.
We can compute the pumping efficiency.

Let us compute the ratio between the critical inversion and the total population in our material.