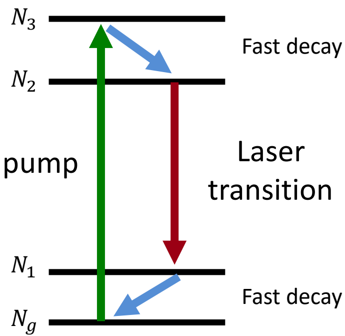
CW behavior



$$N_1 \cong N_3 \cong 0 \qquad \frac{dN_g}{dt} \cong 0$$

$$N \equiv N_2 - N_1 \cong N_2$$ **Population inversion**

$$\begin{cases} \dfrac{dN}{dt} = R_p - B\phi N - \dfrac{N}{\tau} & (*) \\ \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} & (**) \end{cases}$$

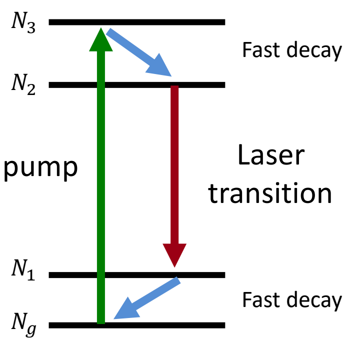
Let's assume that at $t = 0$ an arbitrarily small number of photons (e.g., $\phi_i = 1$) is present within the cavity due to spontaneous emission.

From equation (**) we get that to have **laser action** (i.e., the amplification of the number of photons in the cavity) it should be:

$$\frac{d\phi}{dt} \geq 0 \quad \Longrightarrow \quad V_a B N \geq \frac{1}{\tau_c} \quad \Longrightarrow$$

Laser action starts when it is reached a **critical population inversion**

CW behavior



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Population inversion

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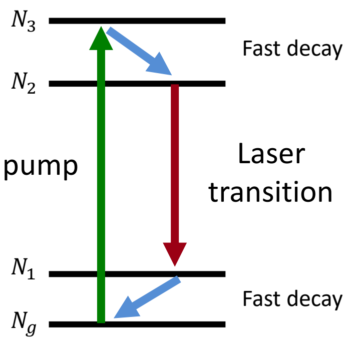
$$\frac{d\phi}{dt} = 0 \quad \Longrightarrow \quad \boxed{N_C = \frac{1}{V_a B \tau_c} = \frac{\gamma}{\sigma l}}$$

$$B = \frac{\sigma l c}{V_a L_e} \qquad \tau_c = \frac{L_e}{\gamma c}$$

$$\frac{d\phi}{dt} \geq 0 \quad \Longrightarrow \quad V_a B N \geq \frac{1}{\tau_c} \quad \Longrightarrow$$

Laser action starts when it is reached a **critical population inversion**

CW behavior



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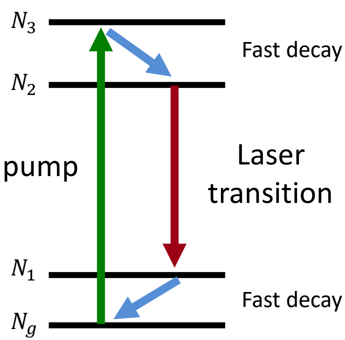
The corresponding **critical pumping rate** (R_{PC}) is obtained from (*) imposing:

$$\frac{dN}{dt} = 0 \quad \text{(steady-state)} \qquad N = N_C \qquad \phi = 0$$

CW behavior



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$$\Longrightarrow \quad \boxed{R_{PC} = \frac{N_C}{\tau}}$$

The **critical pumping rate** corresponds to the situation in which the rate of pump transitions is equal to the spontaneous transitions from level 2.

The corresponding **critical pumping rate** (R_{PC}) is obtained from (*) imposing:

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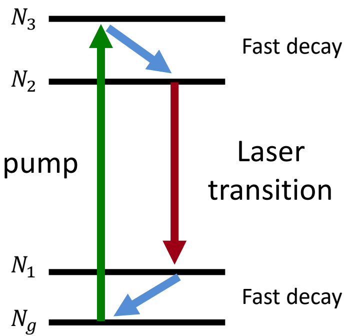
CW behavior



$N_1 \cong N_3 \cong 0$ $\qquad \dfrac{dN_g}{dt} \cong 0$

$N \equiv N_2 - N_1 \cong N_2$ **Population inversion**

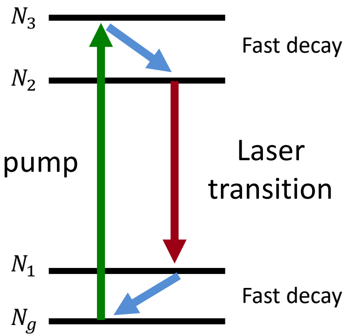
$$\begin{cases} \dfrac{dN}{dt} = R_p - B\phi N - \dfrac{N}{\tau} & (*) \\ \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} & (**) \end{cases}$$

$R_P > R_{PC}$ The number of photons ϕ increases from its initial value determined by spontaneous emission.

If R_P is independent of time, ϕ reaches a steady-state value ϕ_0 which corresponds to a steady-state value of population inversion N_0.

ϕ_0 and N_0 can be determined imposing: $\dfrac{d\phi}{dt} = 0 = \dfrac{dN}{dt}$

CW behavior



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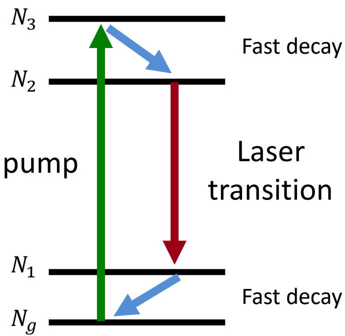
$R_P > R_{PC}$

$$\frac{d\phi}{dt} = 0 \quad \Longrightarrow \quad N_0 = \frac{1}{V_a B \tau_c} = \frac{\gamma}{\sigma l} = N_C \; !$$

$$\frac{dN}{dt} = 0 \quad \Longrightarrow \quad \phi_0 = V_a \tau_c \left[R_P - \frac{N_0}{\tau} \right]$$

CW behavior



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$R_P < R_{PC}$ Below threshold $\Longrightarrow$ $\phi = 0$ No laser action!

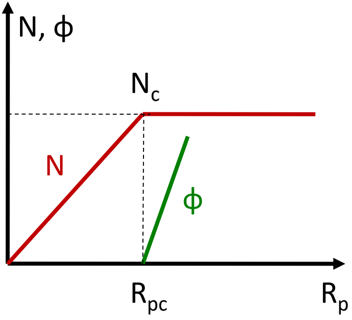
$$\Longrightarrow \frac{dN}{dt} = R_p - \frac{N}{\tau} \Longrightarrow \frac{dN}{N - R_p\tau} = -\frac{1}{\tau}dt \Longrightarrow \int \frac{dN}{N - R_p\tau} = \int_0^t -\frac{1}{\tau}dt'$$

$$\Longrightarrow N(t) = R_p\tau\left(1 - e^{-t/\tau}\right) \Longrightarrow t \longrightarrow \infty \quad N_\infty = R_p\tau$$

Linear function of R_P

CW behavior

Steady-state regime



$$R_P < R_{PC} \implies \begin{cases} N = R_p \tau \\ \phi = 0 \end{cases}$$

$$R_P = R_{PC} \implies \begin{cases} N = N_C = \dfrac{\gamma}{\sigma l} \\ \phi = 0 \end{cases}$$

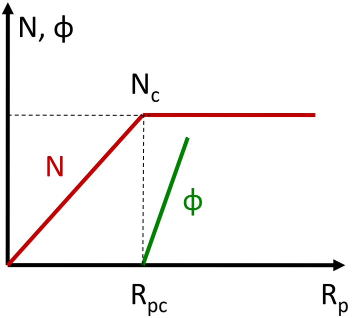
$$R_P > R_{PC} \implies \begin{cases} N = N_0 = N_C \\ \phi_0 = V_a \tau_c \left[R_P - \dfrac{N_0}{\tau} \right] \end{cases}$$

$$x = \frac{R_P}{R_{PC}}$$ **over-threshold factor**

The pumping rate increases population inversion **below threshold** (**energy stored in the active medium**), while it increases the number of photons in the cavity **above threshold** (**energy stored in the cavity**).

CW behavior

Steady-state regime



$$\phi_0 = V_a\tau_c\left[R_P - \frac{N_0}{\tau}\right] = V_a N_0 \frac{\tau_c}{\tau}\left[\frac{R_P\tau}{N_0} - 1\right]$$

$$R_{PC} = \frac{N_C}{\tau} = \frac{N_0}{\tau} \qquad N_C = \frac{\gamma}{\sigma l} = N_0$$

$$\phi_0 = V_a N_0 \frac{\tau_c}{\tau}(x-1) \qquad A_b = \frac{V_a}{l}$$

transverse section of the mode (beam area)

$$x = \frac{R_P}{R_{PC}}$$ **over-threshold factor** $$x = \frac{P_P}{P_{th}}$$

$$A_b < A = \frac{V}{l}$$

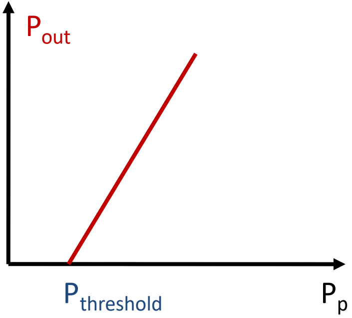
section of the active medium

$$\phi_0 = \frac{A_b\gamma}{\sigma}\frac{\tau_c}{\tau}\left(\frac{P_P}{P_{th}} - 1\right)$$

Remembering that $P_{out} = \left(\frac{\gamma_2 c}{2L_e}\right)h\nu\phi$

CW behavior

Steady-state regime



$$P_{out} = \frac{\gamma_2}{2}\frac{c}{L_e}\frac{A_b\gamma}{\sigma}\frac{\tau_c}{\tau}\left(\frac{P_p}{P_{th}} - 1\right)$$

$$I_s = \frac{h\nu}{\sigma\tau}$$ Saturation intensity for a 4-level system

$$\tau_c = \frac{L_e}{\gamma c}$$ Photon lifetime

$$\boxed{P_{out} = A_b I_s \frac{\gamma_2}{2}\left(\frac{P_p}{P_{th}} - 1\right)}$$

$$\phi_0 = \frac{A_b\gamma}{\sigma}\frac{\tau_c}{\tau}\left(\frac{P_P}{P_{th}} - 1\right)$$

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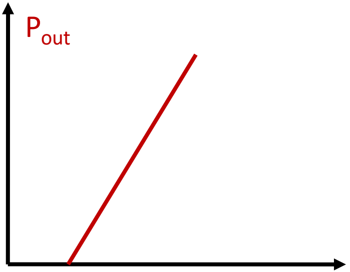
Steady-state regime



$$P_{out} = \frac{\gamma_2}{2}\frac{c}{L_e}\frac{A_b\gamma}{\sigma}\frac{\tau_c}{\tau}\left(\frac{P_p}{P_{th}} - 1\right)$$

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$$\boxed{P_{out} = A_b I_s \frac{\gamma_2}{2}\left(\frac{P_p}{P_{th}} - 1\right)}$$

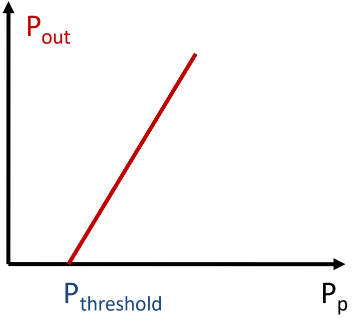
Laser slope efficiency

$$\eta_s = \frac{dP_{out}}{dP_p} \implies \eta_s = \frac{A_b h\nu}{\sigma\tau}\frac{\gamma_2}{2}\frac{1}{P_{th}}$$

Remembering that for optical and electrical pumping: $R_p = \eta_p\left(\frac{P_p}{Alh\nu_{mp}}\right)$

CW behavior

Steady-state regime



$$R_{pc} = \frac{N_C}{\tau} = \frac{\gamma}{\sigma l \tau} = \eta_p \left(\frac{P_{th}}{Alh\nu_{mp}}\right)$$

$$\Downarrow$$

$$P_{th} = \left(\frac{\gamma}{\eta_p}\right)\left(\frac{h\nu_{mp}}{\tau}\right)\left(\frac{A}{\sigma}\right)$$

$$\Downarrow$$

$$\eta_s = \eta_p \left(\frac{\gamma_2}{2\gamma}\right)\left(\frac{h\nu}{h\nu_{mp}}\right)\left(\frac{A_b}{A}\right)$$

$$\Downarrow$$

$$\eta_s = \eta_p \eta_c \eta_q \eta_T$$

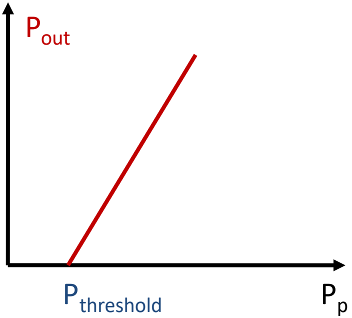
Laser slope efficiency

$$\eta_s = \frac{dP_{out}}{dP_p} \quad \Longrightarrow \quad \eta_s = \frac{A_b h\nu}{\sigma\tau}\frac{\gamma_2}{2}\frac{1}{P_{th}}$$

Remembering that for optical and electrical pumping: $R_p = \eta_p \left(\frac{P_p}{Alh\nu_{mp}}\right)$

CW behavior

Steady-state regime



$$\eta_p = \text{ pumping efficiency}$$

$$\eta_c = \frac{\gamma_2}{2\gamma} = \text{ outcoupling efficiency}$$

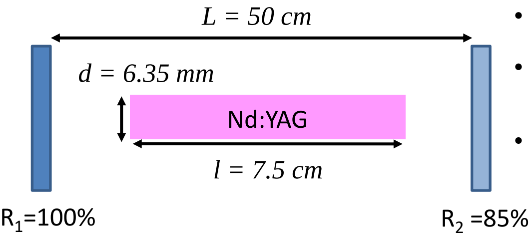
$$\eta_c = \frac{h\nu}{h\nu_{mp}} = \text{ laser quantum efficiency}$$

$$\eta_T = \frac{A_b}{A} = \text{ transverse efficiency}$$

Laser slope efficiency

$$\eta_s = \eta_p \left(\frac{\gamma_2}{2\gamma}\right)\left(\frac{h\nu}{h\nu_{mp}}\right)\left(\frac{A_b}{A}\right) = \eta_p \eta_c \eta_q \eta_T$$

CW laser: Nd:YAG



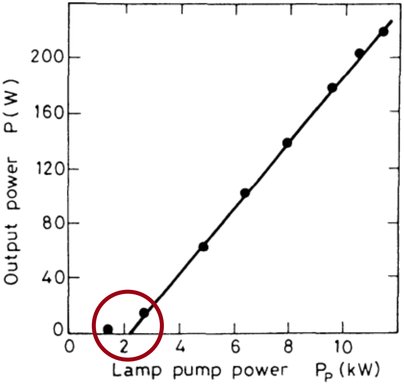
- Optical pumping with elliptical cavity
- High output power in cw: $\sim 100 - 200\ W$
- Multimodal oscillation: longitudinal and transverse

$$P_{out}(W) = 53\left(\frac{P_p}{P_{th}} - 1\right)$$ output power

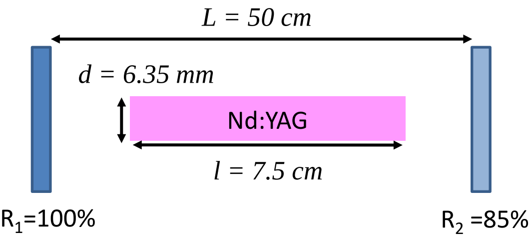
$P_{th} = 2.2\ kW$ threshold power

$$\eta_s = \frac{dP_{out}}{dP_p} = \frac{53\ W}{2.2\ kW} = 2.4\%$$

laser slope efficiency



CW laser: Nd:YAG



$$\lambda = 1.064\ \mu m$$

$$h\nu = 1.87\ 10^{-19}\ J$$

$$\sigma = 2.8\ 10^{-19}\ cm^2$$

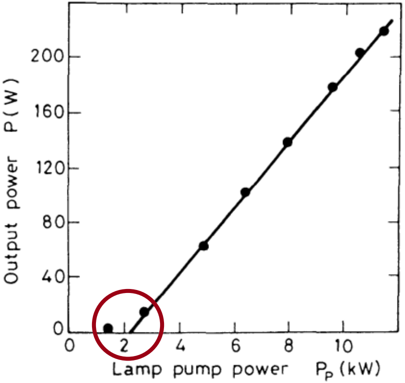
$$\tau = 230\ \mu s$$

$$P_{out}(W) = 53\left(\frac{P_p}{P_{th}} - 1\right)$$ output power

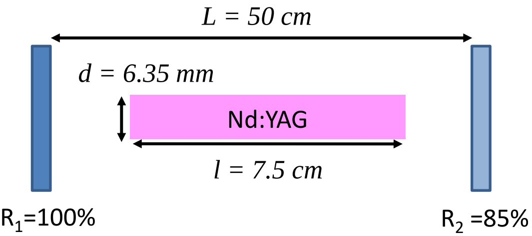
$$P_{out} = A_b I_s \frac{\gamma_2}{2}\left(\frac{P_p}{P_{th}} - 1\right)$$

$$I_s = \frac{h\nu}{\sigma\tau} = 2.9\ \frac{kW}{cm^2} \qquad \gamma_2 = -lnR_2 = 0.162$$

$$A_b \cong 0.23\ cm^2 < A = 0.317 cm^2$$



CW laser: Nd:YAG



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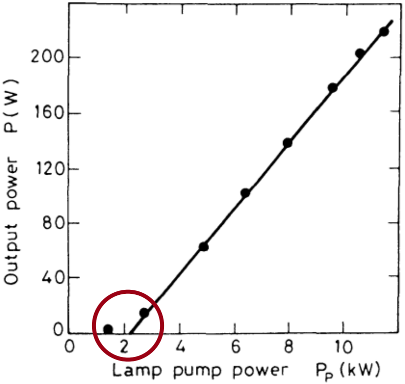
$$P_{th} = 2.2\ kW$$

$$\gamma_2 = -lnR_2 = 0.162$$

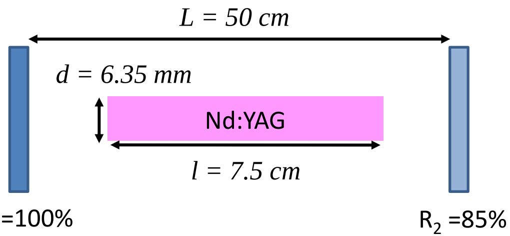
$$P_{th} = \left(\frac{\gamma}{\eta_p}\right)\left(\frac{h\nu_{mp}}{\tau}\right)\left(\frac{A}{\sigma}\right)$$

To compare the experimental values of P_{th} and η_s it is necessary to know γ.

$$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2} \qquad \gamma_1 = -lnR_1 = 0$$



CW laser: Nd:YAG



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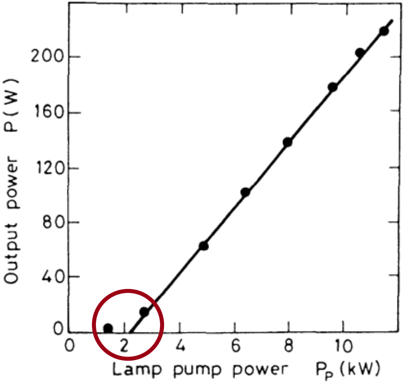
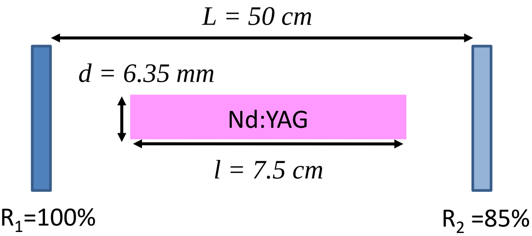
$$P_{th} = \left(\frac{\gamma}{\eta_p}\right)\left(\frac{h\nu_{mp}}{\tau}\right)\left(\frac{A}{\sigma}\right)$$

$$\gamma = \eta_p\left(\frac{\sigma}{A}\right)\left(\frac{\tau}{h\nu_{mp}}\right)P_{th} = \frac{-lnR_2}{2} + \gamma_i$$

$$-lnR_2 = -2\gamma_i + 2\eta_p\left(\frac{\sigma}{A}\right)\left(\frac{\tau}{h\nu_{mp}}\right)P_{th}$$



CW laser: Nd:YAG



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$$\tau = 230\ \mu s$$

$$P_{th} = 2.2\ kW \qquad \gamma_2 = -lnR_2 = 0.162$$

$$P_{th} = \left(\frac{\gamma}{\eta_p}\right)\left(\frac{h\nu_{mp}}{\tau}\right)\left(\frac{A}{\sigma}\right)$$

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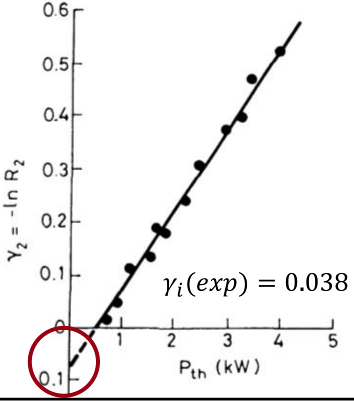
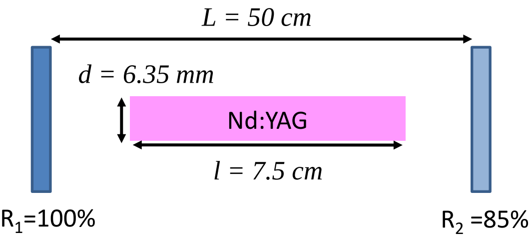


CW laser: Nd:YAG



$$\lambda = 1.064\ \mu m$$

$$h\nu = 1.87\ 10^{-19}\ J$$

$$\sigma = 2.8\ 10^{-19}\ cm^2$$

$$\tau = 230\ \mu s$$

$$\eta_s(exp) = 2.4\%$$

$$\eta_s = \eta_p \left(\frac{\gamma_2}{2\gamma}\right)\left(\frac{h\nu}{h\nu_{mp}}\right)\left(\frac{A_b}{A}\right) = \eta_p\eta_c\eta_q\eta_T$$

$$\eta_c = \frac{\gamma_2}{2\gamma} \cong 0.68$$

$$\gamma_i(exp) = 0.038$$

$$\gamma_2 = 0.162$$

$$\gamma = \gamma_i + \frac{\gamma_1+\gamma_2}{2} \cong 0.12$$

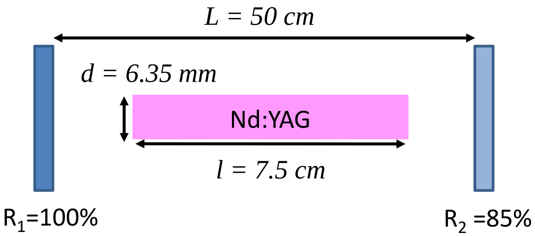
$$\eta_q = \frac{h\nu}{h\nu_{mp}} = \frac{\lambda_{mp}}{\lambda} \cong 0.89$$

$$\lambda_{mp} = 0.94\ \mu m$$

$$h\nu_{mp} = 2.11\ 10^{-19}\ J$$

$$\eta_T = \frac{A_b}{A} \cong \frac{0.23\ cm^2}{0.317\ cm^2} \cong 0.72 \quad \Rightarrow \quad \eta_p = \frac{\eta_s}{\eta_c\eta_q\eta_T} \cong 5.5\%$$

CW laser: Nd:YAG



$\lambda = 1.064\ \mu m$

$h\nu = 1.87\ 10^{-19}\ J$

$\sigma = 2.8\ 10^{-19}\ cm^2$

$\tau = 230\ \mu s$

$$N_C = \frac{\gamma}{\sigma l} = 5.7\ 10^{16}\ \frac{ions}{cm^3}$$

$\gamma \cong 0.12$

$$N_t = 1.38\ 10^{20}\ \frac{ions}{cm^3}$$

Total population (Nd doping: 1%)

$$\frac{N_C}{N_t} = 4.4\ 10^{-4}$$

The population inversion is a very small fraction of the total population!