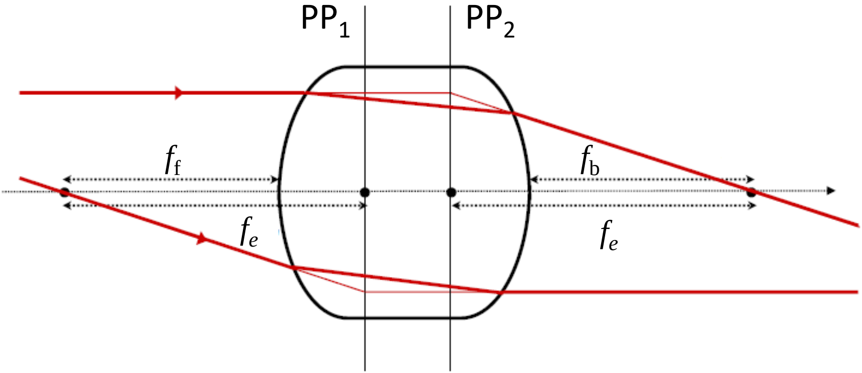
Thick lens



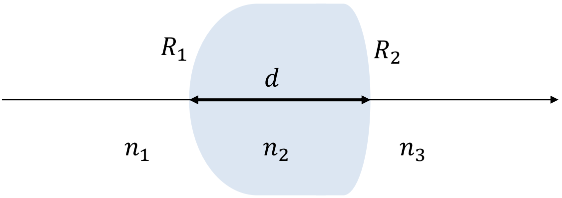
f_e = effective focal length (EFL)

f_f = forward focal length (FFL)

f_b = back focal length (BFL)

PP_1 e PP_2 = principal planes

Thick lens



Spherical diopter of radius R_2

Propagation in a medium with thickness d

Spherical diopter of radius R_1

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{n_2 - n_3}{n_3 R_2} & \dfrac{n_2}{n_3} \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{n_1 - n_2}{n_2 R_1} & \dfrac{n_1}{n_2} \end{bmatrix}$$

$n_1 = n_3 = 1$

$n_2 = n$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{n - 1}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1 - n}{n R_1} & \dfrac{1}{n} \end{bmatrix}$$

Thick lens

$$
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{n-1}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1-n}{nR_1} & \dfrac{1}{n} \end{bmatrix}
$$

$$
= \begin{bmatrix} 1 + \dfrac{d}{R_1}\dfrac{1-n}{n} & \dfrac{d}{n} \\ -\underbrace{\left((n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) + \dfrac{(n-1)^2}{n}\dfrac{d}{R_1 R_2} \right)}_{\dfrac{1}{f_e}} & 1 + \dfrac{d}{R_2}\dfrac{n-1}{n} \end{bmatrix}
$$

$$
\boxed{f_e = -\frac{1}{C}}
$$

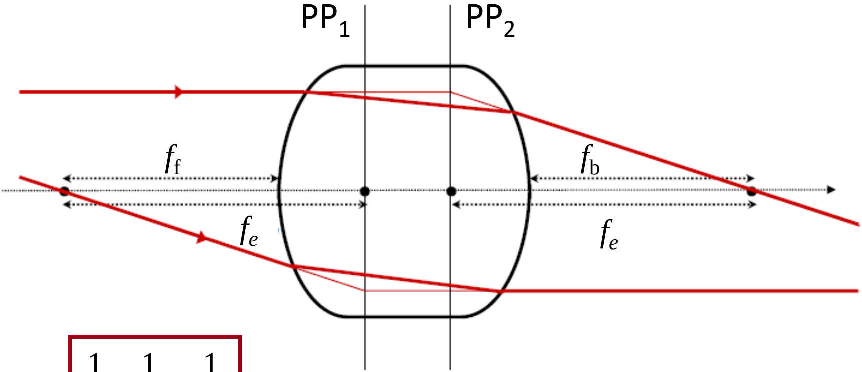
Effective focal length (EFL)

$$
\begin{cases} A f_e = -\dfrac{A}{C} = f_b \\[2ex] D f_e = -\dfrac{D}{C} = f_f \end{cases}
$$

Back focal length (BFL)

Forward focal length (FFL)

Thick lens



$$\boxed{\frac{1}{p}+\frac{1}{q}=\frac{1}{f_e}}$$

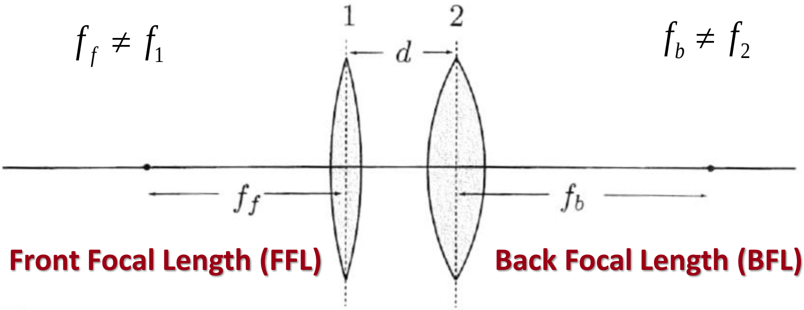
p = distance object-PP$_1$

q = distance image-PP$_2$

$$\boxed{\frac{1}{f_e}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{(n-1)^2}{n}\frac{d}{R_1R_2}}$$

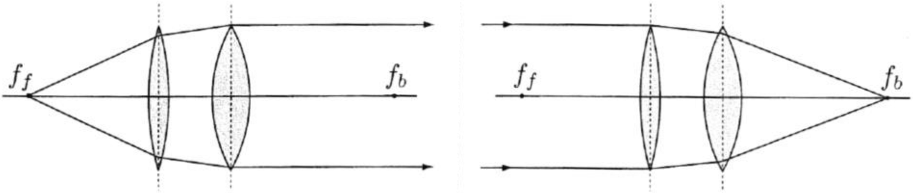
Optical systems

$$f_f \neq f_1 \qquad\qquad f_b \neq f_2$$



Front Focal Length (FFL)

Back Focal Length (BFL)



Effective focal length

$$\boxed{\frac{1}{p}+\frac{1}{q}=\frac{1}{f_e}}$$

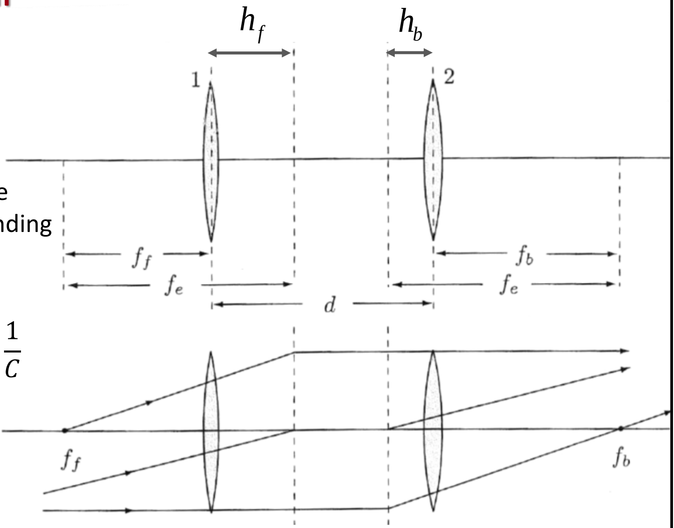
The distances p and q are referred to the corresponding **principal planes**.

$$f_e=\frac{f_1 f_2}{f_1+f_2-d}=-\frac{1}{C}$$

$$I=\frac{q}{p}$$

Transverse magnification

Principal planes



Effective focal length (EFL)

Effective focal length

$$\boxed{\frac{1}{p}+\frac{1}{q}=\frac{1}{f_e}}$$

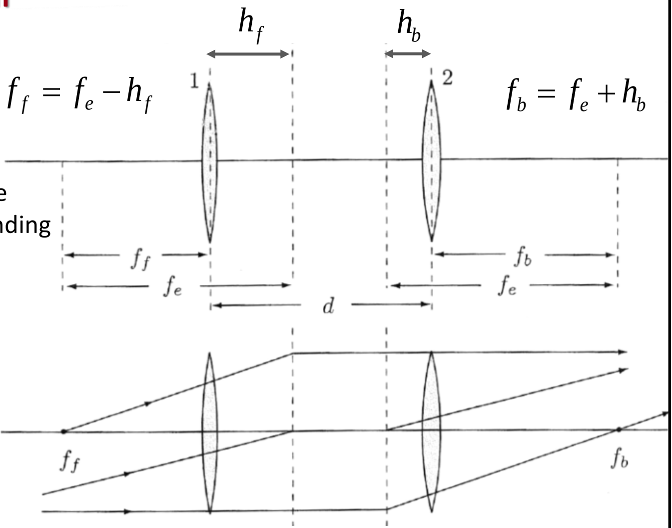
The distances p and q are referred to the corresponding **principal planes**.

$$h_f=\frac{f_e d}{f_2}=\frac{D-1}{C}$$

$$h_b=-\frac{f_e d}{f_1}=-\frac{A-1}{C}$$

Principal planes

$$f_f=f_e-h_f \qquad f_b=f_e+h_b$$

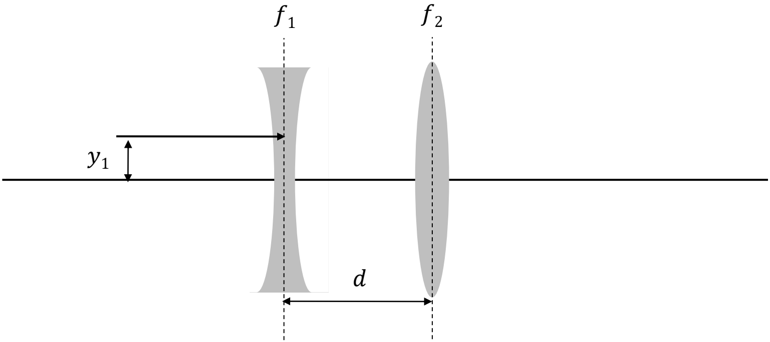


The principal planes on the **left** of the corresponding lens have **negative distance**.

Optical systems

Q: a ray parallel to the optical axis, at a height y_1 from it, propagates in air ($n = 1$) through an optical system made of a thin diverging lens with focal length $f_1 = -10\ cm$ and a thin converging lens with focal length $f_2 = 20\ cm$ at a distance d.

Determine: the distance d so that the ray will emerge from the converging lens still parallel to the optical axis, and the height y_2 of the emerging ray.



Q: a ray parallel to the optical axis, at a height y_1 from it, propagates in air $(n = 1)$ through an optical system made of a <u>thin diverging lens</u> with focal length $f_1 = -10\ cm$ and a <u>thin converging lens</u> with focal length $f_2 = 20\ cm$ at a distance d.

Determine: the distance d so that the ray will emerge from the converging lens still parallel to the optical axis, and the height y_2 of the emerging ray.

A:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f_1} & 1 \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{d}{f_1} & d \\ -\left(\dfrac{1}{f_2} + \dfrac{1}{f_1}\right) + \dfrac{d}{f_1 f_2} & 1 - \dfrac{d}{f_2} \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ 0 \end{bmatrix}$$

incident ray parallel to the optical axis

$$\begin{cases} y_2 = \left(1 - \dfrac{d}{f_1}\right) y_1 \\ \theta_2 = \left(-\left(\dfrac{1}{f_2} + \dfrac{1}{f_1}\right) + \dfrac{d}{f_1 f_2}\right) y_1 \end{cases}$$

Q: a ray parallel to the optical axis, at a height y_1 from it, propagates in air ($n = 1$) through an optical system made of a thin diverging lens with focal length $f_1 = -10\ cm$ and a thin converging lens with focal length $f_2 = 20\ cm$ at a distance d.

Determine: the distance d so that the ray will emerge from the converging lens still parallel to the optical axis, and the height y_2 of the emerging ray.

A:

In order to have an emerging beam parallel to the optical axis:

$$\theta_2 = \left(-\left(\frac{1}{f_2} + \frac{1}{f_1}\right) + \frac{d}{f_1 f_2}\right) y_1 = 0 \quad \Longrightarrow \quad \frac{d}{f_1 f_2} = \left(\frac{1}{f_2} + \frac{1}{f_1}\right) = \frac{f_1 + f_2}{f_1 f_2}$$

$$\Longrightarrow \quad d = f_1 + f_2 \quad \Longrightarrow \quad d = -10\ cm + 20\ cm = 10\ cm$$

$$y_2 = \left(1 - \frac{d}{f_1}\right) y_1 = \left(1 - \frac{10\ cm}{-10\ cm}\right) y_1 = 2 y_1$$

Beam expander

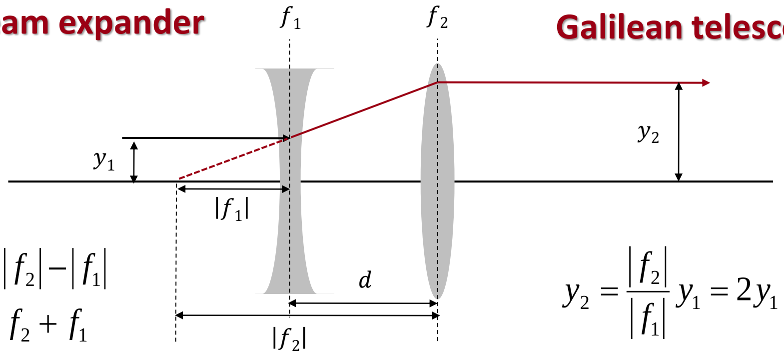
Optical systems

Q: a ray parallel to the optical axis, at a height y_1 from it, propagates in air ($n = 1$) through an optical system made of a <u>thin diverging lens</u> with focal length $f_1 = -10\ cm$ and a <u>thin converging lens</u> with focal length $f_2 = 20\ cm$ at a distance d.

Determine: the distance d so that the ray will emerge from the converging lens still parallel to the optical axis, and the height y_2 of the emerging ray.

Beam expander

Galilean telescope



$$d = |f_2| - |f_1|$$

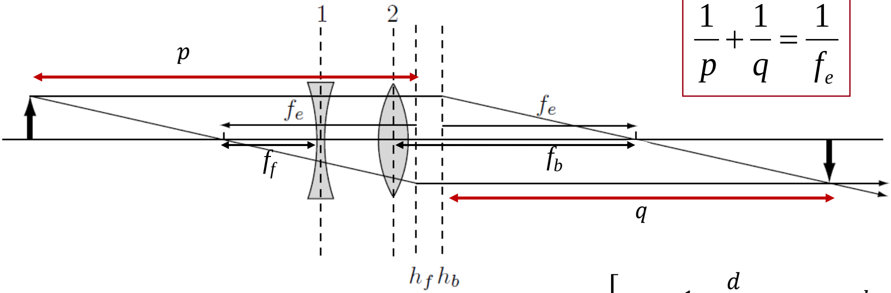
$$d = f_2 + f_1$$

$$y_2 = \frac{|f_2|}{|f_1|} y_1 = 2y_1$$

Optical systems

Q: an optical system consists of a diverging thin lens with focal length $f_1 = -20.0\ cm$ and a converging thin lens with focal length $f_2 = 10.0\ cm$ at a distance $d = 5.0\ cm$. Determine: effective focal length (f_e), forward focal length (f_f) and back focal length (f_b) of the optical system; position of the principal planes and transversal magnification of an object at a distance $\mathrm{z} = 20\ cm$ from lens 1.

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f_e}}$$



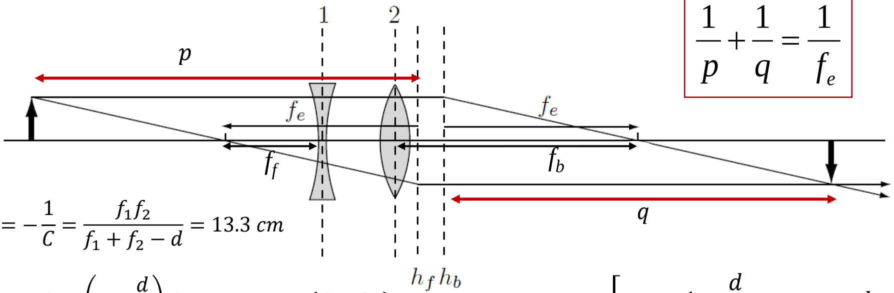
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f_1} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{d}{f_1} & d \\ -\left(\dfrac{1}{f_2} + \dfrac{1}{f_1}\right) + \dfrac{d}{f_1 f_2} & 1 - \dfrac{d}{f_2} \end{bmatrix}$$

Optical systems

Q: an optical system consists of a diverging thin lens with focal length $f_1 = -20.0\ cm$ and a converging thin lens with focal length $f_2 = 10.0\ cm$ at a distance $d = 5.0\ cm$. Determine: effective focal length (f_e), forward focal length (f_f) and back focal length (f_b) of the optical system; position of the principal planes and transversal magnification of an object at a distance $\mathrm{z} = 20\ cm$ from lens 1.

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f_e}}$$



$$f_e = -\frac{1}{C} = \frac{f_1 f_2}{f_1 + f_2 - d} = 13.3\ cm$$

$$f_b = A f_e = \left(1 - \frac{d}{f_1}\right) f_e = 16.7 cm = (f_e + h_b)$$

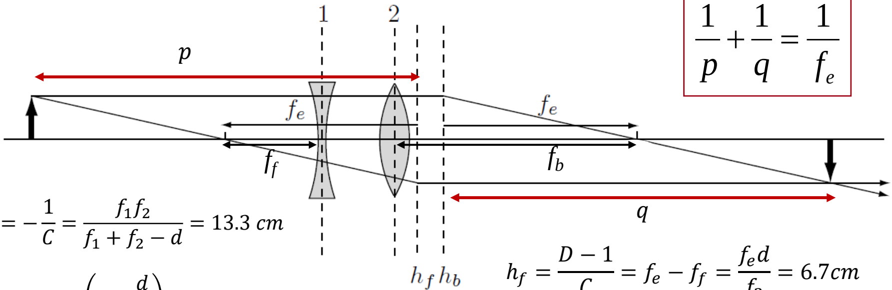
$$f_f = D f_e = \left(1 - \frac{d}{f_2}\right) f_e = 6.7 cm = (f_e - h_f)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{d}{f_1} & d \\ -\left(\dfrac{1}{f_2} + \dfrac{1}{f_1}\right) + \dfrac{d}{f_1 f_2} & 1 - \dfrac{d}{f_2} \end{bmatrix}$$

Optical systems

Q: an optical system consists of a diverging thin lens with focal length $f_1 = -20.0\ cm$ and a converging thin lens with focal length $f_2 = 10.0\ cm$ at a distance $d = 5.0\ cm$. Determine: effective focal length (f_e), forward focal length (f_f) and back focal length (f_b) of the optical system; position of the principal planes and transversal magnification of an object at a distance $\mathrm{z} = 20\ cm$ from lens 1.

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f_e}}$$



$$f_e = -\frac{1}{C} = \frac{f_1 f_2}{f_1 + f_2 - d} = 13.3\ cm$$

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$$f_f = D f_e = \left(1 - \frac{d}{f_2}\right) f_e = 6.7 cm = (f_e - h_f)$$

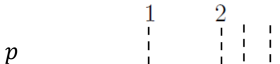
$$h_f = \frac{D-1}{C} = f_e - f_f = \frac{f_e d}{f_2} = 6.7 cm$$

$$h_b = -\frac{A-1}{C} = -(f_e - f_b) = -\frac{f_e d}{f_1} = 3.3\ cm$$

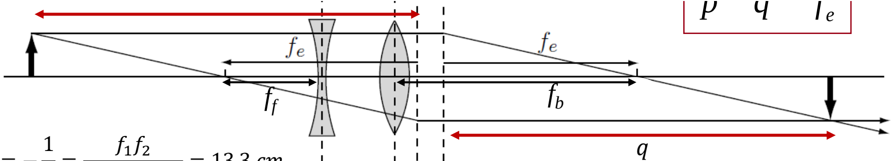
positive: PP on the right of the corresponding lens

Optical systems

Q: an optical system consists of a diverging thin lens with focal length $f_1 = -20.0\ cm$ and a converging thin lens with focal length $f_2 = 10.0\ cm$ at a distance $d = 5.0\ cm$. Determine: effective focal length (f_e), forward focal length (f_f) and back focal length (f_b) of the optical system; position of the principal planes and transversal magnification of an object at a distance $\mathrm{z} = 20\ cm$ from lens 1.



$$\boxed{\frac{1}{p}+\frac{1}{q}=\frac{1}{f}}$$



$$f_e = -\frac{1}{C} = \frac{f_1 f_2}{f_1 + f_2 - d} = 13.3\ cm$$

$$h_f = \frac{D-1}{C} = f_e - f_f = \frac{f_e d}{f_2} = 6.7cm$$

$$p = (z + h_f) = (20 + 6.7)\ cm = 26.7\ cm$$

$$h_b = -\frac{A-1}{C} = -(f_e - f_b) = -\frac{f_e d}{f_1} = 3.3\ cm$$

$p = q$ from the lens equation

$$I = \frac{q}{p} = 1$$

positive: PP on the right of the corresponding lens