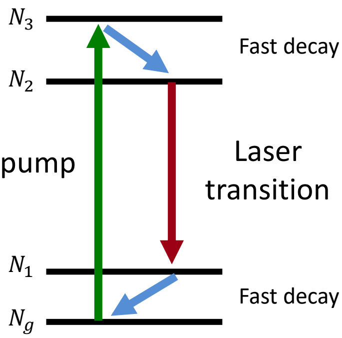
Self-terminating lasers



Working hypotheses:

- **Four-level laser:** $N_1 \cong 0$ always

What happens if the lower laser level has a finite lifetime ($\tau_1 \neq 0$)?

At equilibrium conditions (steady-state):

$$\frac{N_1}{\tau_1} = \frac{N_2}{\tau_{21}} \quad \Longrightarrow \quad \frac{N_2}{N_1} = \frac{\tau_{21}}{\tau_1}$$

To get laser action $\quad N_2 > N_1 \quad \Longrightarrow \quad \tau_{21} > \tau_1$

If $\quad \tau_{21} < \tau_1 \quad \Longrightarrow$ laser action is possible with pulsed pumping only

with $\quad \Delta t_p < \tau$

laser action ends when the accumulation of population in the lower laser level destroys population inversion

Δt_p: pulse duration

τ: total lifetime of the upper laser level

Self-terminating lasers

CW Nd:glass

A Nd:glass laser ($n = 1.54$) oscillating at the fundamental line ($\lambda = 1054\,nm$) emits in cw an output power $P_{out} = 320\,mW$. The length of the active medium is $l = 8\,cm$. The resonant cavity is a Fabry-Perot cavity of length $L = 50\,cm$, made of a first mirror with reflectivity $R_1 = 95\%$ and a second mirror (outcoupling mirror) with $R_2 = 75\%$.

Assuming that the internal losses for single pass are $L_i = 16\%$, determine:

1. the photon lifetime
2. the number of photons in the cavity
3. the critical population inversion
4. the saturation intensity

$h = 6.63\ 10^{-34}\,Js$

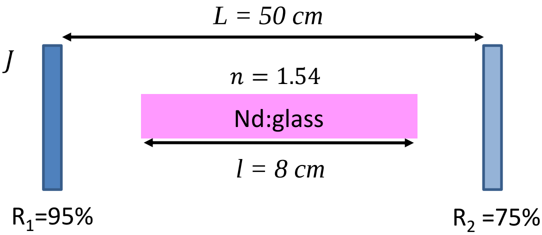
$c = 3\ 10^{8}\,m/s$

$\lambda = 1054\,nm$ $\quad h\nu = 1.89\ 10^{-19}\,J$

$\sigma = 4.0\ 10^{-20}\,cm^2$

$\tau = 300\,\mu s$

$P_{out} = 320\,mW$ $\quad L_i = 16\%$



CW Nd:glass

1. the photon lifetime

$$\tau_c = \frac{L_e}{\gamma c}$$

$$\gamma_1 = -lnR_1 \cong 0.05 \qquad \gamma_2 = -lnR_2 \cong 0.2877$$

$$\gamma_i = -ln(1-L_i) \cong 0.1744 \qquad \gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2} = 0.3433$$

$$L_e = L + (n-1)l = 54.32\ cm$$

$$h = 6.63\ 10^{-34}\ Js$$

$$c = 3\ 10^8\ m/s$$

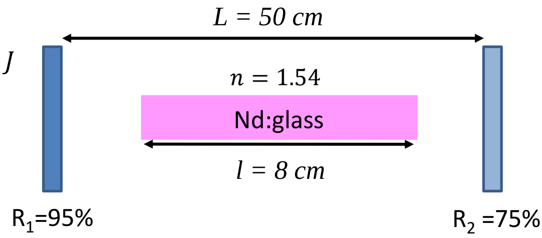
$$\tau_c = \frac{L_e}{\gamma c} = 7.86\ 10^{-9}\ s = 7.86\ ns$$

$\lambda = 1054\ nm \qquad h\nu = 1.89\ 10^{-19}\ J$

$\sigma = 4.0\ 10^{-20}\ cm^2$

$\tau = 300\ \mu s$

$P_{out} = 320\ mW \qquad L_i = 16\%$



CW Nd:glass

2. the number of photons in the cavity

$$P_{out} = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu\phi \qquad \phi = \left(\frac{2L_e}{\gamma_2 c}\right)\frac{P_{out}}{h\nu} \cong 2.13\ 10^{10}$$

$$L_e = L + (n-1)l = 54.32cm$$

$$\gamma_2 = -lnR_2 \cong 0.2877$$

$$h = 6.63\ 10^{-34}\ Js$$

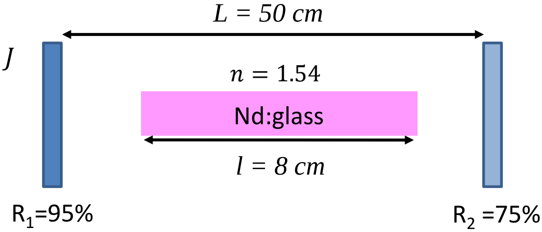
$$c = 3\ 10^8\ m/s$$

$\lambda = 1054\ nm$ $\qquad h\nu = 1.89\ 10^{-19}\ J$

$\sigma = 4.0\ 10^{-20}\ cm^2$

$\tau = 300\ \mu s$

$P_{out} = 320\ mW$ $\qquad L_i = 16\%$



CW Nd:glass

3. the critical population inversion

$$N_t = 3.2\ 10^{20}\ \frac{ions}{cm^3} \qquad \frac{N_C}{N_t} = 0.3\%$$

$$N_C = \frac{\gamma}{\sigma l} = 1.07\ 10^{18}\ \frac{ions}{cm^3}$$

$$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2} = 0.3433$$

4. the saturation intensity

$$I_s = \frac{h\nu}{\sigma\tau} = 15.75\ \frac{kW}{cm^2}$$

$$h = 6.63\ 10^{-34}\ Js$$

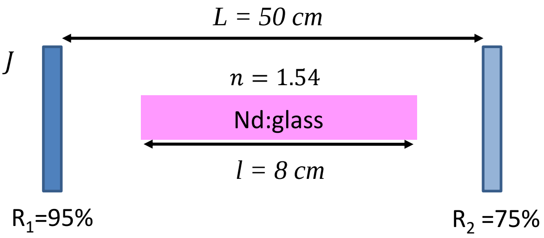
$$c = 3\ 10^{8}\ m/s$$

$$\lambda = 1054\ nm \qquad h\nu = 1.89\ 10^{-19}\ J$$

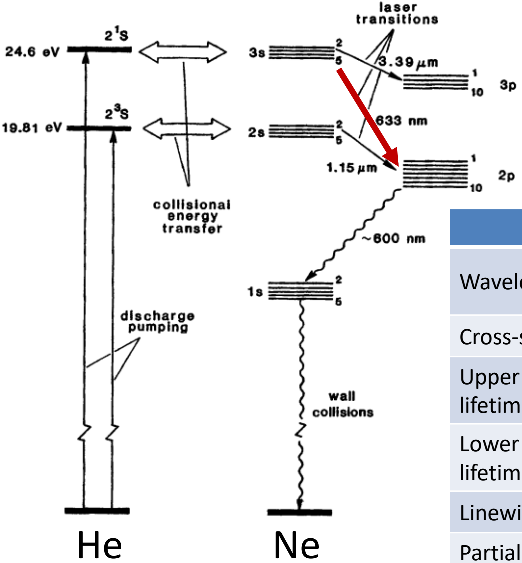
$$\sigma = 4.0\ 10^{-20}\ cm^2$$

$$\tau = 300\ \mu s$$

$$P_{out} = 320\ mW \qquad L_i = 16\%$$



He-Ne laser



Property		values
Wavelength	λ	633 nm
Cross-section	σ	30×10^{-14} cm^2
Upper laser level lifetime	τ	150 ns
Lower laser level lifetime	τ_1	10 ns
Linewidth	$\Delta\nu_0$	1.5 GHz
Partial pressure gas mixture		4 Torr (He) 0.8 Torr (Ne)

He-Ne laser

A He-Ne laser ($n \cong 1$) oscillating at the red line ($\lambda = 633\ nm$) emits in cw an output power $P_{out} = 30\ mW$. The resonant cavity is a Fabry-Perot cavity of length $L = 60\ cm$, totally filled with the gas and made of a first mirror with reflectivity $R_1 = 98\%$ and a second mirror (outcoupling mirror) with $R_2 = 80\%$.
Assuming that the internal losses for single pass are $L_i = 12\%$, determine:

1. the photon lifetime
2. the number of photons in the cavity
3. the critical population inversion
4. the saturation intensity

$h = 6.63\ 10^{-34}\ Js$

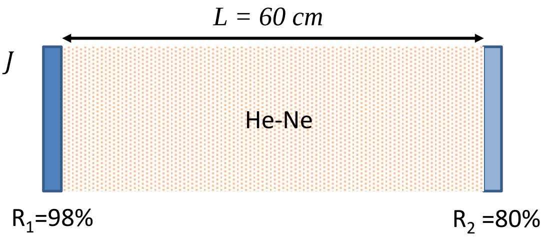
$c = 3\ 10^{8}\ m/s$

$\lambda = 633\ nm$ $\qquad h\nu = 3.14\ 10^{-19}\ J$

$\sigma = 3.0\ 10^{-13}\ cm^2$

$\tau = 150\ ns$

$P_{out} = 30\ mW$ $\qquad L_i = 12\%$



1. the photon lifetime

$$\tau_c = \frac{L_e}{\gamma c}$$

$$\gamma_1 = -lnR_1 \cong 0.02 \qquad \gamma_2 = -lnR_2 \cong 0.223$$

$$\gamma_i = -ln(1 - L_i) \cong 0.1278 \qquad \gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2} = 0.2493$$

$$L_e = nL = 60\ cm$$

$$h = 6.63\ 10^{-34}\ Js$$

$$c = 3\ 10^{8}\ m/s$$

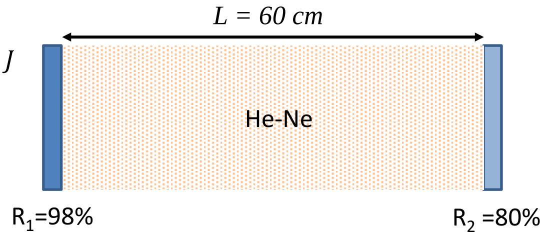
$$\tau_c = \frac{L_e}{\gamma c} = 7.86\ 10^{-9}\ s \cong 8\ ns$$

$\lambda = 633\ nm$ $\qquad hv = 3.14\ 10^{-19}\ J$

$\sigma = 3.0\ 10^{-13}\ cm^2$

$\tau = 150\ ns$

$P_{out} = 30\ mW$ $\qquad L_i = 12\%$



2. the number of photons in the cavity

$$P_{out} = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu\phi \qquad \phi = \left(\frac{2L_e}{\gamma_2 c}\right)\frac{P_{out}}{h\nu} \cong 1.71\ 10^9$$

$$L_e = nL = 60\ cm$$

$$h = 6.63\ 10^{-34}\ Js$$

$$c = 3\ 10^8\ m/s$$

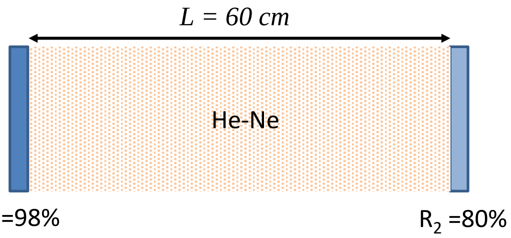
$$\gamma_2 = -lnR_2 \cong 0.223$$

$$\lambda = 633\ nm \qquad h\nu = 3.14\ 10^{-19}\ J$$

$$\sigma = 3.0\ 10^{-13}\ cm^2$$

$$\tau = 150\ ns$$

$$P_{out} = 30\ mW \qquad L_i = 12\%$$



He-Ne laser

3. the critical population inversion

$$N_C = \frac{\gamma}{\sigma l} = 1.385\ 10^{10}\ \frac{ions}{cm^3}$$

$$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2} = 0.2493$$

$$L = l = 60\ cm$$

4. the saturation intensity

$$I_s = \frac{h\nu}{\sigma\tau} = 6.98\ \frac{W}{cm^2}$$

$$h = 6.63\ 10^{-34}\ Js$$

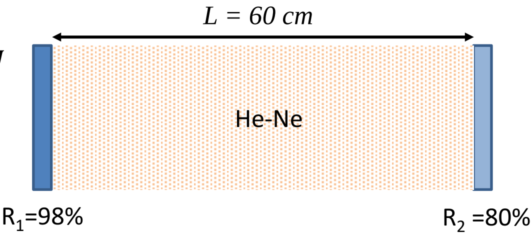
$$c = 3\ 10^{8}\ m/s$$

$\lambda = 633\ nm$ $\qquad h\nu = 3.14\ 10^{-19}\ J$

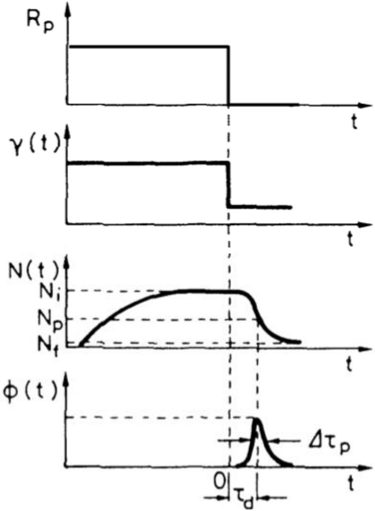
$\sigma = 3.0\ 10^{-13}\ cm^2$

$\tau = 150\ ns$

$P_{out} = 30\ mW$ $\qquad L_i = 12\%$

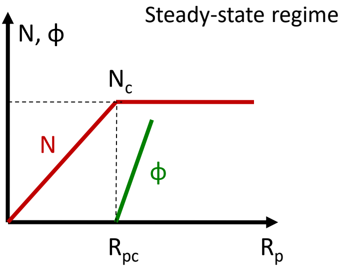


Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time



$$N_C = \frac{1}{V_a B \tau_c} = \frac{\gamma}{\sigma l}$$

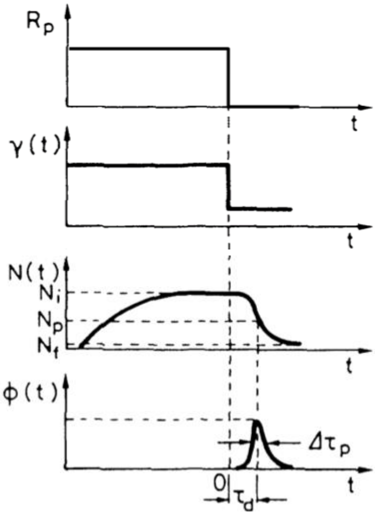
$R_P < R_{PC} \implies \phi = 0$ No laser action!

$$N(t) = R_p\tau\left(1 - e^{-t/\tau}\right) \qquad N_\infty = R_p\tau$$

$$Q = 2\pi\nu\,\tau_c \qquad \tau_c = \frac{L_e}{c\gamma}$$

Q-switch

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

- **Active Q-switch**
- **Fast Q-switch**
- **Pulsed pumping**

The duration of the **pump pulse** (t_p) should be $\boldsymbol{t_p \leq \tau}$:

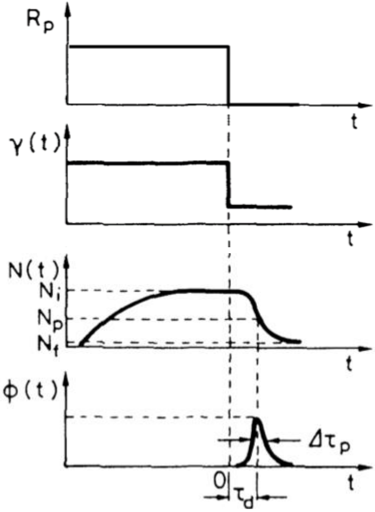
for $t_p \gg \tau$, $N(t)$ doesn't differ much from its asymptotic value and the pump power would be lost through spontaneous decays instead of being accumulated as population inversion.

$$N(t) = R_p\tau\left(1 - e^{-t/\tau}\right) \qquad N_\infty = R_p\tau$$

$$Q = 2\pi\nu\,\tau_c \qquad \tau_c = \frac{L_e}{c\gamma}$$

Q-switch

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

- **Active Q-switch**
- **Fast Q-switch**
- **Pulsed pumping**

To obtain a sufficiently **large population inversion**, it is necessary to have **long τ (ms).**

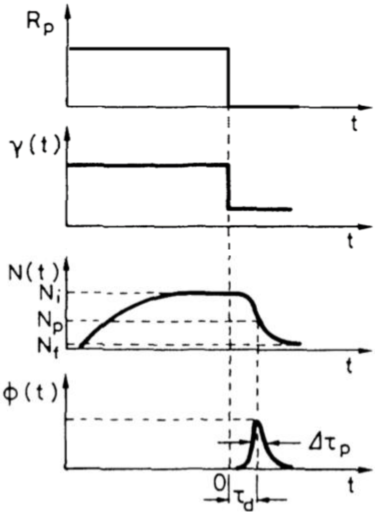
This condition is well satisfied by solid-state lasers (Nd, Er, Yb…) and some gas lasers (CO_2 or I_2).
For dye lasers or gas lasers as He-Ne or Ar τ is of the order of few ns and the population inversion that can be obtained is too low to make it worth operating in Q-switch mode.

$$N(t) = R_p\tau\left(1 - e^{-t/\tau}\right) \qquad N_\infty = R_p\tau$$

$$Q = 2\pi\nu\,\tau_c \qquad \tau_c = \frac{L_e}{c\gamma}$$

Q-switch

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

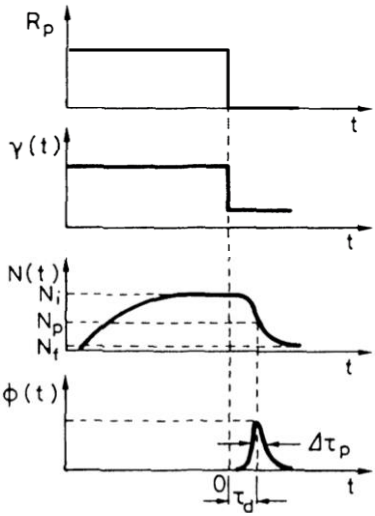
- **Active Q-switch**
- **Fast Q-switch**
- **Pulsed pumping**

- **Space-independent rate equations**

Working hypotheses:

- **Four-level laser:** $N_1 \cong N_3 \cong 0 \qquad \frac{dN_g}{dt} \cong 0$
- **Single mode** (longitudinal and transverse)
- **Homogeneous broadening**
- **Uniform energy density** of the mode
- **Uniform pumping**

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$N \equiv N_2 - N_1 \cong N_2$ Population inversion

$$\begin{cases} \dfrac{dN}{dt} = R_p - B\phi N - \dfrac{N}{\tau} \\ \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

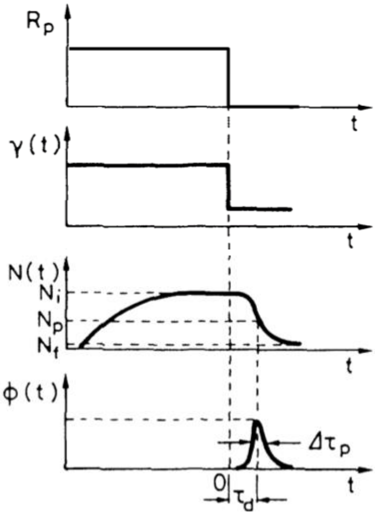
$N_i = R_p(0)\tau$

E_P = pump energy corresponding to a given pumping rate R_P

$$E_P \propto \int R_P\, dt \quad \Longrightarrow \quad E_P \propto R_p(0)$$

$$\Longrightarrow \quad E_P \propto N_i$$

Q-switch



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$N \equiv N_2 - N_1 \cong N_2$ Population inversion

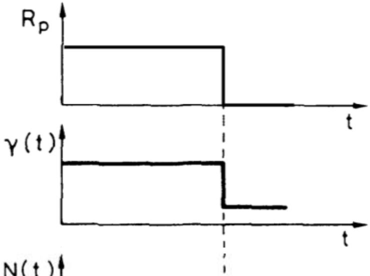
$$\begin{cases} \dfrac{dN}{dt} = R_p - B\phi N - \dfrac{N}{\tau} \\ \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$

$$x = \frac{E_P}{E_{PC}} = \frac{N_i}{N_C}$$ over-threshold factor (*)

$$N_C = \frac{\gamma}{\sigma l}$$ critical inversion for normal laser action (at Q-switch element open!)

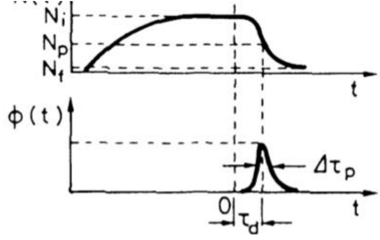
If N_C and x are known, equation (*) allows us to calculate the initial inversion N_i

Q-switch



$N \equiv N_2 - N_1 \cong N_2$ Population inversion

$$\begin{cases} \dfrac{dN}{dt} = \cancel{R_p} - B\phi N - \cancel{\dfrac{N}{\tau}} \\ \dfrac{d\phi}{dt} = V_a B\phi N - \dfrac{\phi}{\tau_c} \end{cases}$$



$\Delta\tau_p$ = pulse duration

τ_d = build-up time

$$N(0) = N_i$$

$$\phi(0) = \phi_i \approx 1$$ for laser action to start

For instantaneous Q-switch, the temporal evolution of $N(t)$ and $\phi(t)$ occurs on time scales so short that in the rate equations the terms related to the pumping rate R_p and the spontaneous emission $\frac{N}{\tau}$ can be neglected