Mode-locking

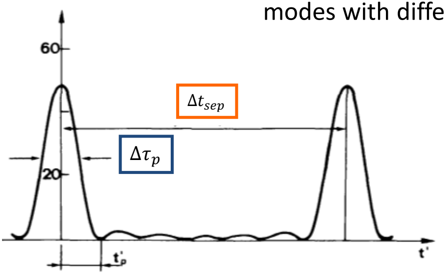
The **Q-switch** mode allows to obtain short pulses with very high intensity (peak photon number $\phi_P \approx 10^{17}$) but the **pulse duration** is of the order of **a few ns**:

$$\Delta\tau_p = \eta_E \tau_c \frac{x}{x - 1 - \ln(x)} \propto \tau_c$$

A technique that allows to obtain ultra-short pulses, down to 5 fs, is:

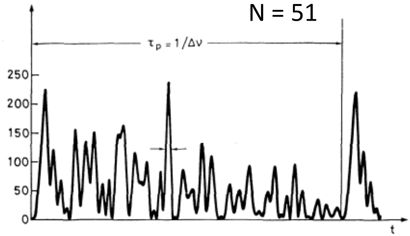
Mode-locking

This is obtained **combining in phase** (mode-locking) a given (large) number of different longitudinal modes with different frequencies.



when the modes of an em wave with different frequencies are combined **in phase**, they give rise to an electric field with amplitude and intensity with a characteristic **repetitive nature**

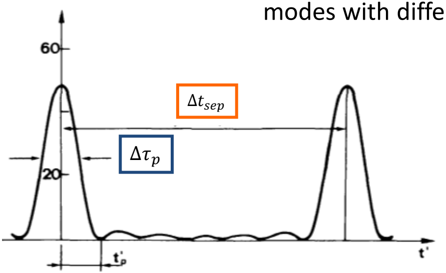
Mode-locking



when the modes of an em wave with different frequencies but **random phases** are added up they give rise to an electric field with amplitude and intensity **randomly distributed in the time domain**.

Mode-locking

This is obtained **combining in phase** (mode-locking) a given (large) number of different longitudinal modes with different frequencies.



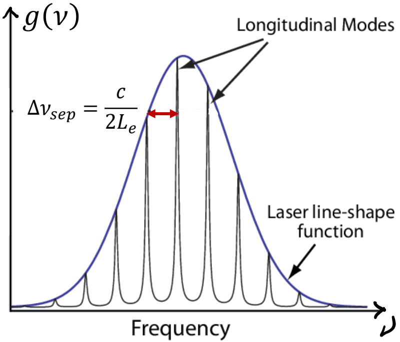
when the modes of an em wave with different frequencies are combined **in phase**, they give rise to an electric field with amplitude and intensity with a characteristic **repetitive nature**

Mode-locking

The **longitudinal modes** of a laser cavity are separated in frequency by:

$$\Delta\nu_{sep} = \frac{c}{2L_e}$$

If the gain bandwidth is larger than $\Delta\nu_{sep}$ it is possible to have more than one longitudinal mode oscillating in the cavity



$E(t) = E_m e^{i(\omega_m t + \phi_m)}$ Electric field amplitude of the m-th mode

Let's assume that N modes with the same amplitude E_0 are oscillating at the same time in the laser cavity. The total amplitude of the resulting field will be:

$$E(t) = E_0 \sum_{m=0}^{N-1} e^{i(\omega_m t + \phi_m)} \qquad \omega_{m+1} - \omega_m = \Delta\omega = 2\pi\Delta\nu_{sep} = \frac{\pi c}{L_e}$$

Mode-locking

If the modes oscillate with **random phases** (and the number is sufficiently large) the total intensity results (on average the mixed terms cancel out):

$$I(t) \propto |E(t)|^2 = E_0^2 \sum_{m=0}^{N-1} e^{i(\omega_m t+\phi_m)} e^{-i(\omega_m t+\phi_m)} = N E_0^2 \propto N I_0$$

Let's see what happens if we make the modes oscillate **in phase** (i.e., if they are **mode-locked**):

$$\phi_m = \phi_0 \,, \forall\, m \qquad E(t) = E_0 \sum_{m=0}^{N-1} e^{i(\omega_m t+\phi_0)} = E_0 e^{i\phi_0} \sum_{m=0}^{N-1} e^{i\omega_m t}$$

Defining $\quad \omega_m = \omega_0 + m\Delta\omega$

$$E(t) = E_0 e^{i\phi_0} \sum_{m=0}^{N-1} e^{i(\omega_0+m\Delta\omega)t} = E_0 e^{i(\omega_0 t+\phi_0)} \sum_{m=0}^{N-1} e^{i(m\Delta\omega)t}$$

Mode-locking

$$E(t) = E_0 e^{i\phi_0} \sum_{m=0}^{N-1} e^{i(\omega_0 + m\Delta\omega)t} = E_0 e^{i(\omega_0 t + \phi_0)} \sum_{m=0}^{N-1} e^{i(m\Delta\omega)t} =$$

$$= E_0 e^{i(\omega_0 t + \phi_0)} \left[1 + e^{i\Delta\omega t} + e^{2i\Delta\omega t} + \cdots\right]$$

$$= E_0 e^{i(\omega_0 t + \phi_0)} \left[\frac{1 - e^{iN\Delta\omega t}}{1 - e^{i\Delta\omega t}}\right]$$

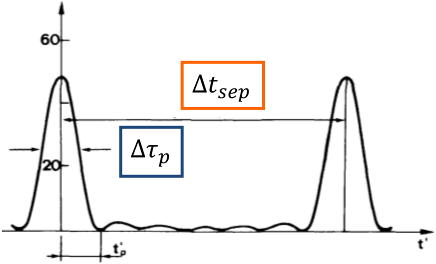
$$1 - cos(\vartheta) = 2 sin^2\left(\frac{\vartheta}{2}\right)$$

$$I(t) \propto |E(t)|^2 = E_0^2 \left|\frac{1 - e^{iN\Delta\omega t}}{1 - e^{i\Delta\omega t}}\right|^2 = E_0^2 \frac{sin^2\left(\frac{N\Delta\omega t}{2}\right)}{sin^2\left(\frac{\Delta\omega t}{2}\right)} \propto I_0 \frac{sin^2\left(\frac{N\Delta\omega t}{2}\right)}{sin^2\left(\frac{\Delta\omega t}{2}\right)}$$

N-slits interference!

Mode-locking

$$I(t) = I_0 \frac{\sin^2\left(\frac{N\Delta\omega t}{2}\right)}{\sin^2\left(\frac{\Delta\omega t}{2}\right)}$$



Maxima $(Den = 0)$ $\quad \frac{\Delta\omega t}{2} = m\pi \quad \Longrightarrow \quad t_{MAX} = \frac{m2\pi}{\Delta\omega}$

$$I(t_{MAX}) = N^2 I_0$$

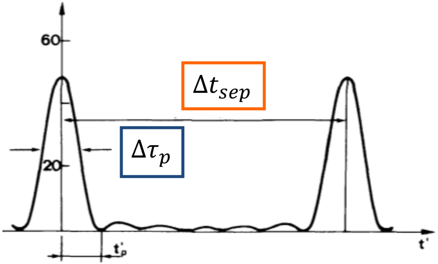
$$\Delta t_{sep} = \frac{2\pi}{\Delta\omega} = \frac{1}{\Delta\nu_{sep}} = \frac{2L_e}{c}$$

time needed to make a complete lap of the cavity (back and forth)

temporal difference between two consecutive maxima

Mode-locking

$$I(t) = I_0 \frac{\sin^2\left(\frac{N\Delta\omega t}{2}\right)}{\sin^2\left(\frac{\Delta\omega t}{2}\right)}$$



Minima $(Num = 0)$ $\quad \frac{N\Delta\omega t}{2} = m\pi \Longrightarrow t_{min} = \frac{m2\pi}{N\Delta\omega}$

The pulse width ($\Delta\tau_p$) is given by the temporal separation between two minima around a maximum

$$\Delta\tau_p = \frac{2\pi}{N\Delta\omega} = \frac{1}{N\Delta\nu_{sep}} = \frac{1}{\Delta\nu_0}$$

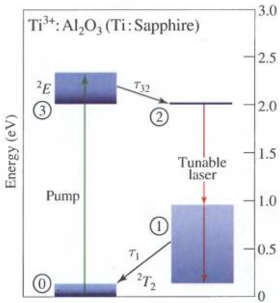
gain bandwidth

The width of the gain curve determines the pulse duration: the larger the gain bandwidth, the shorter the pulse duration.

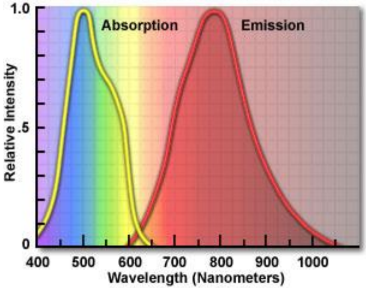
Nd lasers

Property		Nd:YAG	Nd:glass
Nd doping		1 % at.	3.8 wt. % (Nd_2O_3)
Nd^{3+} concentration	N_t	1.38 x 10^{20} ions/cm^3	3.2 x 10^{20} ions/cm^3
Wavelength	λ	1.064 μm	1.054 μm
Upper laser level lifetime	τ	230 μs	300 μs
Linewidth	$\Delta\nu_0$	4.0 cm^{-1} (~120 GHz)	180 cm^{-1} (~5.4THz)
Stimulated emission cross-section	σ_e	2.8 x 10^{-19} cm^2	0.4 x 10^{-19} cm^2
Refractive index	n	1.82	1.54

Ti:Al_2O_3 laser

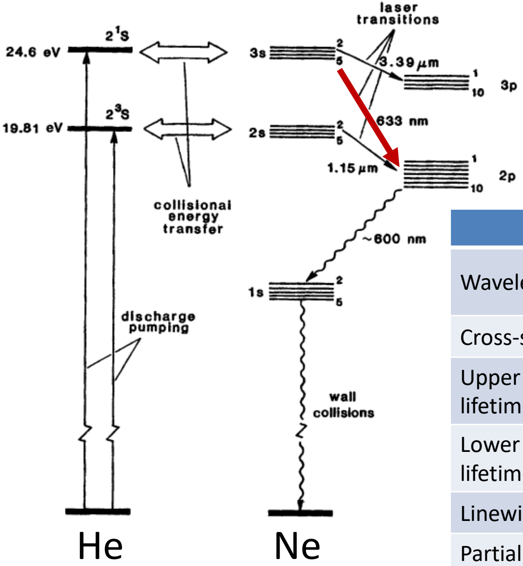


Ti:Sapphire Absorption/Emission Spectra



Proprietà		valore
Ti^{3+} concentration	N_t	3.3×10^{19} ions/cm^3
Wavelength	λ	660-1180 nm
Upper laser level lifetime	τ	3.8 μs
Linewidth	$\Delta\nu_0$	100 THz
Stimulated emission cross-section	σ_e	3.4×10^{-19} cm^2
Refractive index	n	1.76

He-Ne laser



Property		values
Wavelength	λ	633 nm
Cross-section	σ	30×10^{-14} cm^2
Upper laser level lifetime	τ	150 ns
Lower laser level lifetime	τ_1	10 ns
Linewidth	$\Delta\nu_0$	1.5 GHz
Partial pressure gas mixture		4 Torr (He) 0.8 Torr (Ne)

Mode-locking

How do we obtain the mode-locking of longitudinal modes?

with an **optical shutter** in the cavity

when the shutter is temporarily open, the electrical field of all the modes is maximized at the same time and thus they are in phase.

Active shutter

Electro-optic device: it has to open exactly when the pulse arrives, at temporal intervals of Δt_{sep}

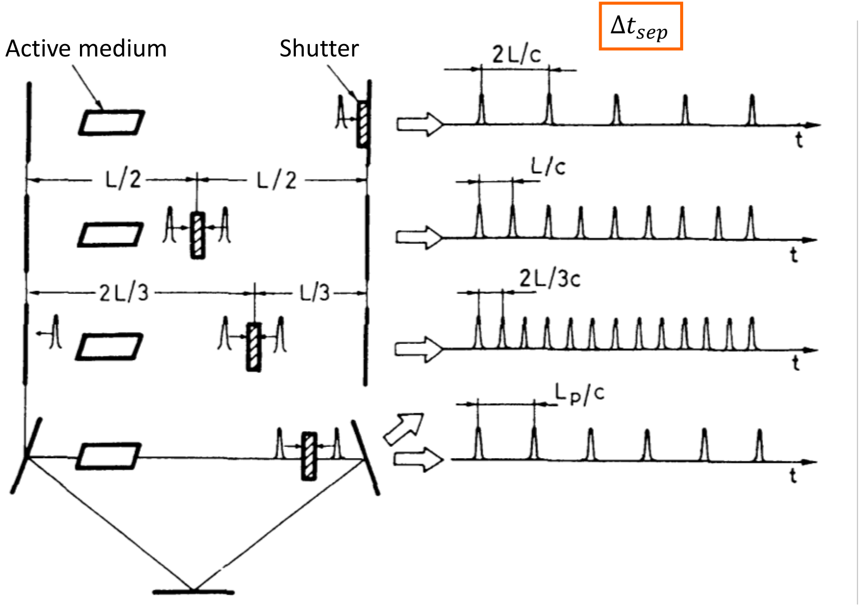
Passive shutter

Nonlinear optical element (**saturable absorber**): it opens automatically at the right moment by the arrival of the pulse itself.

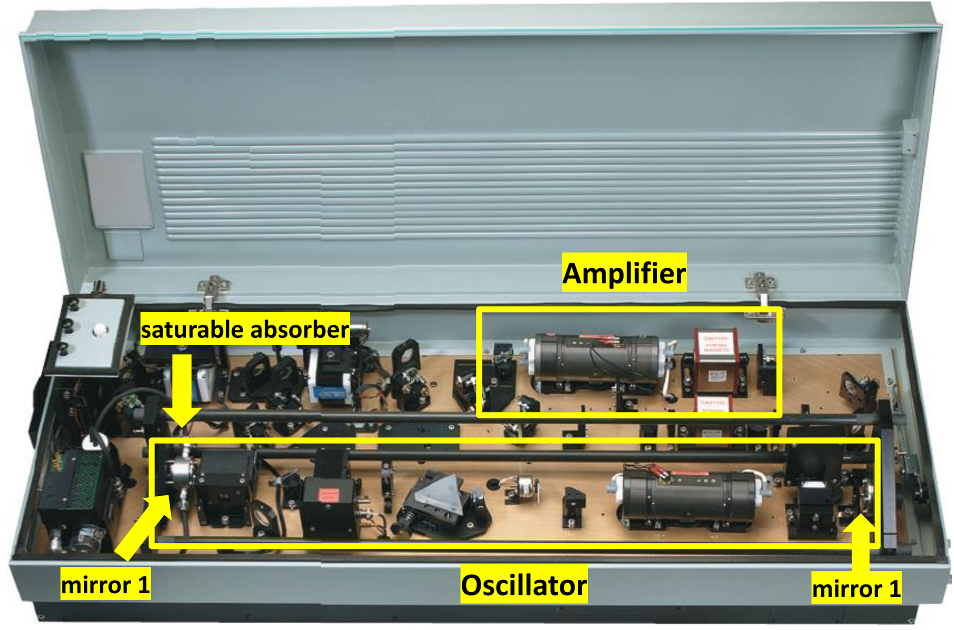
Passive shutters are typically faster than active ones.

Some noise, as an intensity spike (statistically always present), is needed to trigger the shutter opening when the beam has low intensity.

Harmonic mode-locking

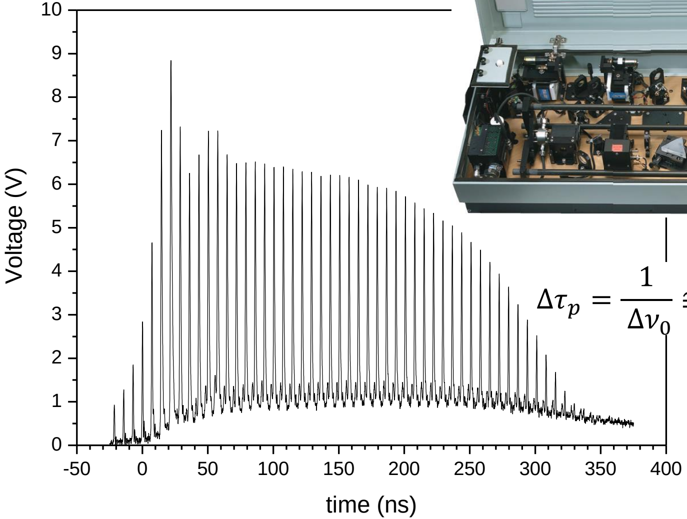


Mode-locked Nd:YAG (15 ps)



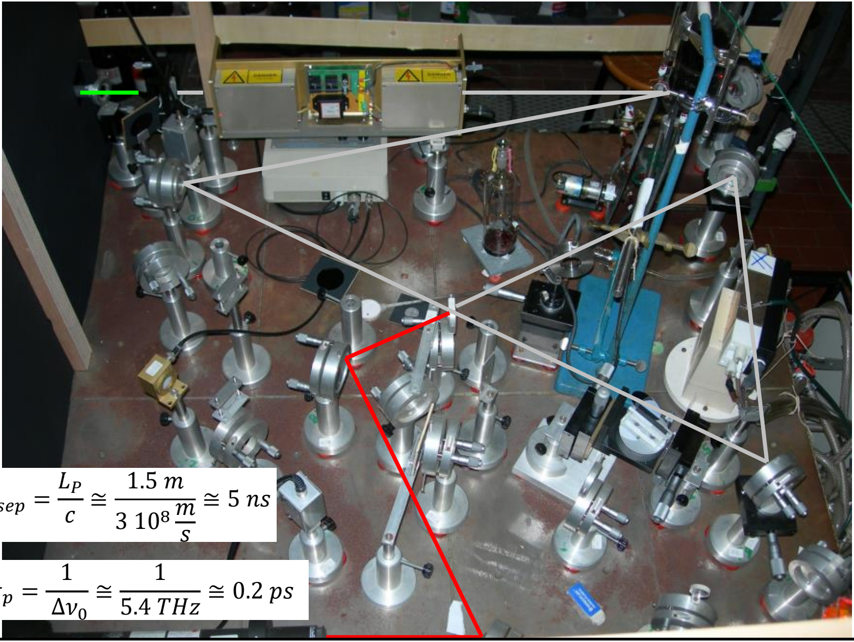
Mode-locked Nd:YAG (15 ps)

$$\Delta t_{sep} = \frac{2L_e}{c} \cong \frac{2\ 0.5\ m}{3\ 10^8\ \frac{m}{s}} \cong 3.3\ ns$$



$$\Delta\tau_p = \frac{1}{\Delta\nu_0} \cong \frac{1}{120\ GHz} \cong 8.3\ ps$$

Mode-locked Nd:glass (6 ps)
Ring cavity



$$\Delta t_{sep} = \frac{L_P}{c} \cong \frac{1.5\ m}{3\ 10^8\ \frac{m}{s}} \cong 5\ ns$$

$$\Delta \tau_p = \frac{1}{\Delta \nu_0} \cong \frac{1}{5.4\ THz} \cong 0.2\ ps$$

Mode-locking

Compute the mode-locked pulse width $\Delta\tau_p$ and the separation between pulses Δt_{sep} for the following mode-locked lasers:

a. A He-Ne laser operating at 632.8 nm, with a mirror cavity spacing of $L = 0.5\ m$

b. A Rh6G dye laser operating over its entire gain bandwidth (570-640 nm) with the cavity mirrors separated by 2 m. The index of refraction of a laser dye in a typical solvent is approximately 1.4

a.

For He-Ne we can assume a refractive index $n \cong 1 \quad \Longrightarrow \quad L_e = L = 0.5\ m$

$$\Delta t_{sep} = \frac{2L_e}{c} \cong \frac{2\ 0.5\ m}{3\ 10^8\ \frac{m}{s}} \cong 3.3\ ns$$

$$\Delta\nu_0 \cong 1.5\ GHz$$

$$\Delta\tau_p = \frac{1}{\Delta\nu_0} \cong \frac{1}{1.5\ GHz} \cong 6.67\ 10^{-10}\ s \cong 667\ ps$$

Doppler broadening (inhomogeneous)

Mode-locking

Compute the mode-locked pulse width $\Delta\tau_p$ and the separation between pulses Δt_{sep} for the following mode-locked lasers:

a. A He-Ne laser operating at 632.8 nm, with a mirror cavity spacing of $L = 0.5\ m$

b. A Rh6G dye laser operating over its entire gain bandwidth (570-640 nm) with the cavity mirrors separated by 2 m. The index of refraction of a laser dye in a typical solvent is approximately 1.4

b.

We can neglect the dye cell thickness with respect to the entire cavity length

$$\Longrightarrow \quad L_e = L = 2\,m \qquad \Delta t_{sep} = \frac{2L_e}{c} \cong \frac{2 \cdot 2\,m}{3\;10^8\,\frac{m}{s}} \cong 1.33\;10^{-8}\,s \cong 13.3\,ns$$

$$\Delta\nu_0 = \frac{c}{\lambda^2}\Delta\lambda \cong \frac{3\;10^8\,\frac{m}{s}}{(605\,nm)^2}(70\,nm) \cong 5.7\;10^{13} \cong 57\,THz$$

$$\Delta\tau_p = \frac{1}{\Delta\nu_0} \cong \frac{1}{57\,THz} \cong 1.75\;10^{-14}\,s \cong 17.5\,fs$$