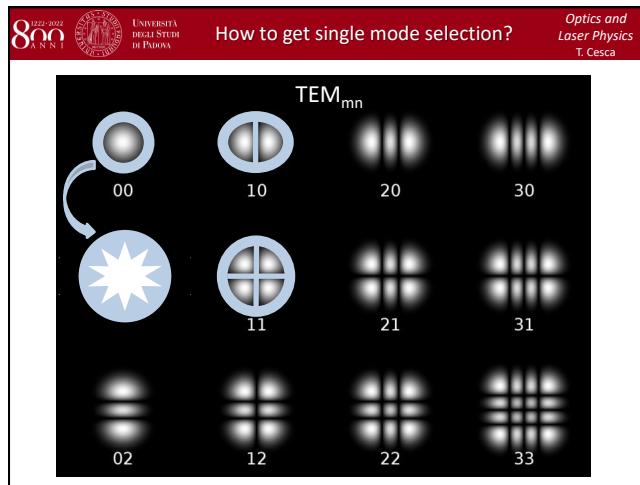


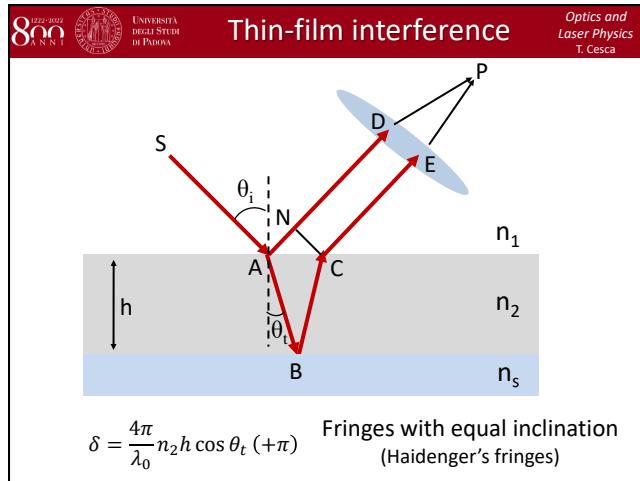
1 Lecture 18

Slide 1



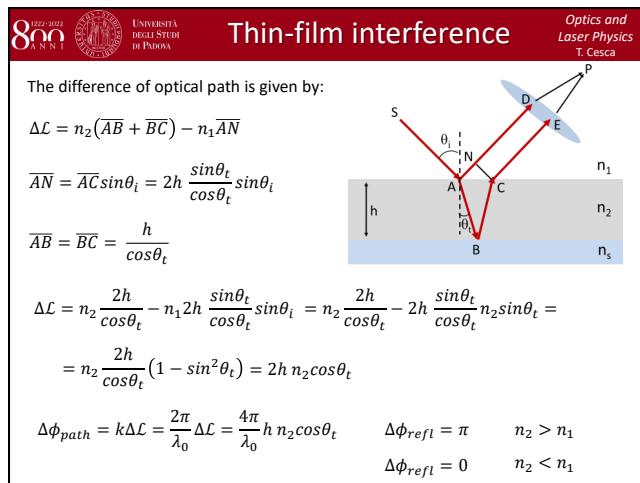
1

Slide 2



2

Slide 3



3

For mode locking we want multiple modes oscillating. Instead, one of the working hypothesis for Q-switch and CW mode is to have one single mode oscillating in the cavity. How can we select one single mode?

For the transverse mode we have seen that the method to isolate modes is by using **diaframs**. The higher is the order of the mode that you want to isolate, the difficult is to construct a diagram which match the shape of the mode.

For transverse mode you typically isolate the TEM₀₀ mode.

To conclude, a diafram induce high losses for all the mode which have a distribution in intensity different in space wrt to the mode you want to select.

For longitudinal mode selection the strategy is different. Firstly, we need to talk about **thin-film interference**. We have created a thin film of a material with n₂ on a substrate n_s.

The two beam of reflection can interfere and create fringes of equal inclination. The phase shift between these two beams is δ (it is given by the path difference between these two rays). We should consider the possibility to have further contribution for reflection which depends on the relative refractive index.

The different of phase is $k \Delta L$, where k is the wavevector in vacum.

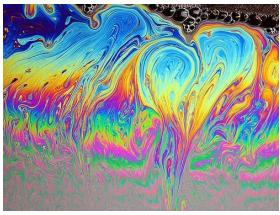
Slide 4

800 ANNI UNIVERSITÀ DEGLI STUDI DI PADOVA **Thin-film interference** Optics and Laser Physics T. Cesca

Exercise:
Some oil (with refractive index $n_2 = 1.25$) is on the surface of a water puddle with refractive index $n_s = 1.33$.

- At normal incidence, what is the minimum oil film thickness reflecting most the red light at $\lambda = 650 \text{ nm}$?
- At which angle of incidence the same film reflects most the blue light at $\lambda = 450 \text{ nm}$?

1. $n_1 < n_2 < n_s \implies$ each interface produce a π phase shift in reflection
 $\implies \Delta\phi_{refl} = 0 \quad \delta = \frac{4\pi}{\lambda_0} n_2 h \cos \theta_t = 2\pi \quad \text{for constructive interference}$
 $h = \frac{\lambda_0}{2n_2} = \frac{650 \text{ nm}}{2 \cdot 1.25} = 260 \text{ nm}$



4

Since each interface produce a π phase shift, we have no term of $\Delta\Phi_{refl}$. We want that δ is equal to 2π because we want to reflect most.

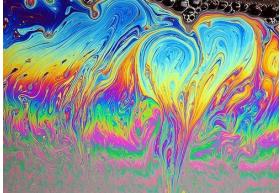
Slide 5

800 ANNI UNIVERSITÀ DEGLI STUDI DI PADOVA **Thin-film interference** Optics and Laser Physics T. Cesca

Exercise:
Some oil (with refractive index $n_2 = 1.25$) is on the surface of a water puddle with refractive index $n_s = 1.33$.

- At normal incidence, what is the minimum oil film thickness reflecting most the red light at $\lambda = 650 \text{ nm}$?
- At which angle of incidence the same film reflects most the blue light at $\lambda = 450 \text{ nm}$?

2. $2\pi = \frac{4\pi}{\lambda_0} n_2 h \cos \theta_t \implies \cos \theta_t = \frac{\lambda_0}{2n_2 h} = \frac{450 \text{ nm}}{2 \cdot 1.25 \cdot 260 \text{ nm}} = 0.692$
 $\implies \theta_t = 42.2^\circ \implies \sin \theta_t = \frac{n_2}{n_1} \sin \theta_t = 0.902$
 $\implies \theta_i = 64.4^\circ$



5

Slide 6

800 ANNI UNIVERSITÀ DEGLI STUDI DI PADOVA **Thin-film interference** Optics and Laser Physics T. Cesca

A film of soapy water suspended vertically on a metallic ring can get very thin before breaking

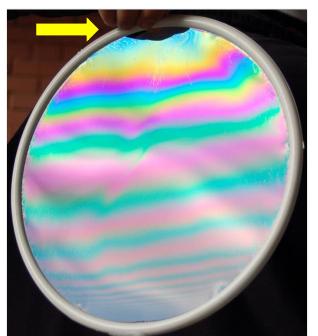
$$\Delta\phi_{path} = k\Delta\mathcal{L} = 0$$

$$n_1 = n_s = 1$$

$$\Delta\phi_{refl} = \pi$$

$$\delta = \Delta\phi_{path} + \Delta\phi_{refl} = \pi$$

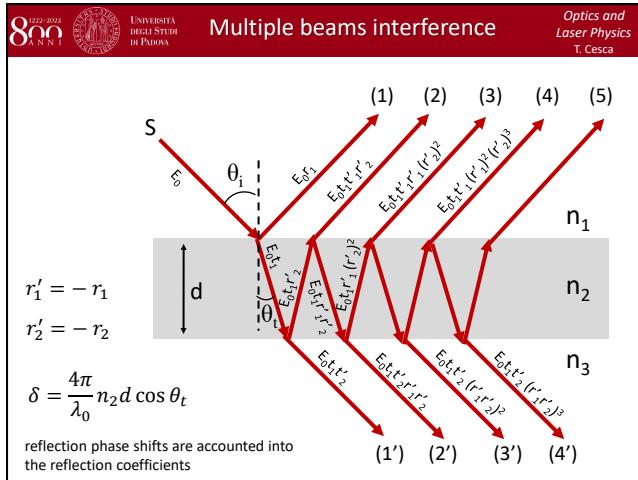
Destructive interference!



In the upper part of the film there is a black part. Before breaking the film can become very thin, the phase shift at this position has no contribution to the difference of path. The total phase shift is at this position equal to π : we have destructive interference, that is why we do not see any light in this point!

6

Slide 7



8

Slide 8

800 ANNI UNIVERSITÀ DEGLI STUDI DI PADOVA Multiple beams interference Optics and Laser Physics T. Cesca

Let's consider the transmitted beam:

$$E_t = E_0 t_1 t'_2 + E_0 t_1 t'_2 r'_1 r'_1 e^{i\delta} + E_0 t_1 t'_2 (r'_1)^2 e^{i2\delta} + \dots + E_0 t_1 t'_2 (r'_1)^N e^{iN\delta}$$

$$= E_0 t_1 t'_2 [1 + x + x^2 + \dots x^N] \quad x = r'_1 r'_1 e^{i\delta}$$

$$\sum_{n=1}^N x^n \rightarrow \frac{1}{1-x}$$

$$|x| < 1, \quad N \text{ large}$$

$$E_t = E_0 t_1 t'_2 \frac{1}{1 - r'_1 r'_1 e^{i\delta}}$$

9

Let us suppose that the slab is sufficiently large and we have to consider the interference not only between the first two beams but also the contribution of multiple beams. We want to see what happens.

Any phase shift due to reflection is already present inside the reflection coefficient!

Let us compute the field of the total transmitted beam.

800 ANNI UNIVERSITÀ DEGLI STUDI DI PADOVA Multiple beams interference Optics and Laser Physics T. Cesca

Analogously for the reflected beam:

$$E_r = E_0 r_1 + E_0 t_1 t'_1 r'_2 e^{i\delta} + E_0 t_1 t'_1 r'_1 (r'_2)^2 e^{i2\delta} + \dots +$$

$$+ \dots + E_0 t_1 t'_1 (r'_1)^{N-2} (r'_2)^{N-1} e^{i(N-1)\delta}$$

$$= E_0 r_1 + E_0 t_1 t'_1 r'_2 e^{i\delta} [1 + x + x^2 + \dots x^N] \quad x = r'_1 r'_1 e^{i\delta}$$

$$\frac{1}{1-x}$$

$$E_r = E_0 \left(r_1 + \frac{t_1 t'_1 r'_2 e^{i\delta}}{1 - r'_1 r'_1 e^{i\delta}} \right)$$

10

We can do the same for the reflected beam.

Slide 10

800 ANNI UNIVERSITÀ DEGLI STUDI DI PADOVA Multiple beams interference Optics and Laser Physics T. Cesca

Assuming $n_1 = n_3 \Rightarrow r_1 = r_2 = r \quad t_1 = t_2 = t$

$$E_t = E_0 tt' \frac{1}{1 - (r')^2 e^{i\delta}} \quad E_r = E_0 \left(r + \frac{tt' r' e^{i\delta}}{1 - (r')^2 e^{i\delta}} \right)$$

Defining $|r'|^2 = R \quad tt' = T$

$$\Rightarrow R + \frac{n_3}{n_1} T = 1 \Rightarrow R + T = 1$$

$$E_t = E_0 (1 - R) \frac{1}{1 - Re^{i\delta}} \quad r' = -r$$

$$E_r = E_0 \left(r + \frac{(1 - R)r' e^{i\delta}}{1 - Re^{i\delta}} \right) = E_0 \frac{r(1 - e^{i\delta})}{1 - Re^{i\delta}}$$

11

Slide 11

800 ANNI UNIVERSITÀ DEGLI STUDI DI PADOVA Multiple beams interference Optics and Laser Physics T. Cesca

Given $I_t \propto |E_t|^2 \quad I_r \propto |E_r|^2$

$$I_t = I_0 (1 - R)^2 \frac{1}{1 - Re^{i\delta}} \frac{1}{1 - Re^{-i\delta}} = I_0 (1 - R)^2 \frac{1}{1 + R^2 - 2R \cos \delta}$$

$$= I_0 \frac{(1 - R)^2}{(1 - R)^2 + 4R \left(\sin \frac{\delta}{2}\right)^2} \quad \cos \delta = 1 - 2 \left(\sin \frac{\delta}{2}\right)^2$$

$$= I_0 \frac{1}{1 + \frac{4R}{(1 - R)^2} \left(\sin \frac{\delta}{2}\right)^2}$$

$$E_t = E_0 (1 - R) \frac{1}{1 - Re^{i\delta}} \quad r' = -r$$

$$E_r = E_0 \left(r + \frac{(1 - R)r' e^{i\delta}}{1 - Re^{i\delta}} \right) = E_0 \frac{r(1 - e^{i\delta})}{1 - Re^{i\delta}}$$

12

Let us assume that $n_1 = n_3$ to simplify the problem.
Let us rewrite E_t and E_r as a function of the reflectance R .

Now, let us consider the intensity.
This is the expression for the transmitted intensity.

Slide 12

800 ANNI UNIVERSITÀ DEGLI STUDI DI PADOVA Multiple beams interference Optics and Laser Physics T. Cesca

Given $I_t \propto |E_t|^2 \quad I_r \propto |E_r|^2$

$$I_t = I_0 \frac{1}{1 + \frac{4R}{(1 - R)^2} \left(\sin \frac{\delta}{2}\right)^2} \quad \cos \delta = 1 - 2 \left(\sin \frac{\delta}{2}\right)^2$$

$$I_r = I_0 \frac{\frac{4R}{(1 - R)^2} \left(\sin \frac{\delta}{2}\right)^2}{1 + \frac{4R}{(1 - R)^2} \left(\sin \frac{\delta}{2}\right)^2}$$

$$E_t = E_0 (1 - R) \frac{1}{1 - Re^{i\delta}} \quad r' = -r$$

$$E_r = E_0 \left(r + \frac{(1 - R)r' e^{i\delta}}{1 - Re^{i\delta}} \right) = E_0 \frac{r(1 - e^{i\delta})}{1 - Re^{i\delta}}$$

We can do the same for the reflected intensity.

13

Slide 13

800 ANNI UNIVERSITÀ DEGLI STUDI DI PADOVA Multiple beams interference Optics and Laser Physics T. Cesca

Defining $F = \frac{4R}{(1-R)^2}$ coefficient of finesse (of the slab)

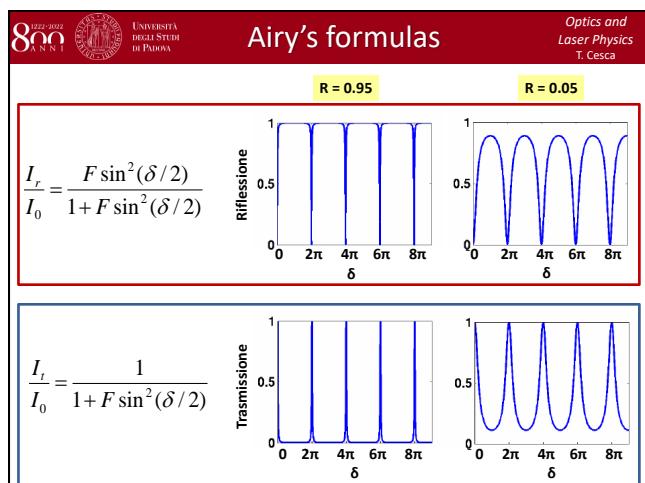
$$I_t = I_0 \frac{1}{1 + \frac{4R}{(1-R)^2} \left(\sin \frac{\delta}{2}\right)^2} = I_0 \frac{1}{1 + F \left(\sin \frac{\delta}{2}\right)^2}$$

$$I_r = I_0 \frac{\frac{4R}{(1-R)^2} \left(\sin \frac{\delta}{2}\right)^2}{1 + \frac{4R}{(1-R)^2} \left(\sin \frac{\delta}{2}\right)^2} = I_0 \frac{F \left(\sin \frac{\delta}{2}\right)^2}{1 + F \left(\sin \frac{\delta}{2}\right)^2}$$

Airy's formulas

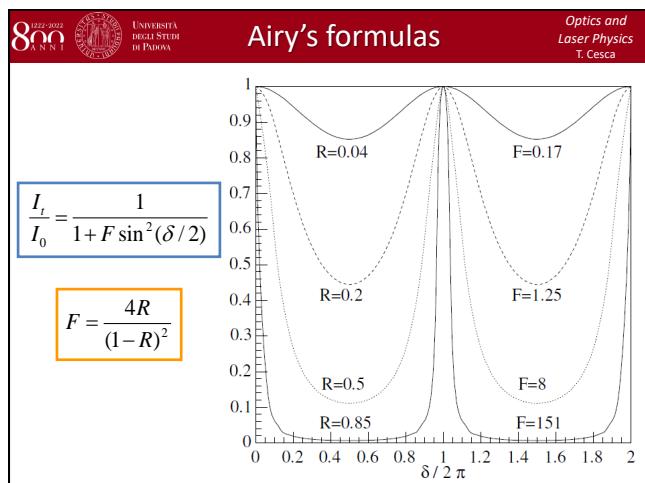
14

Slide 14



15

Slide 15



16

Let us simplify these expressions by introducing the coefficient of finesse (of the slab of the material with refractive index n_2).

These expressions are known as **Airy's formulas**.

If you shine a beam of light on a slab of material, in reflection you will get the minimum at certain position.. or if you want you have fringes in the intensity that is transmitted or you have minima in the reflected pattern. If R is very large you have very narrow fringes in transmission (or very narrow black lines in reflection), for small value of R they are definitely less defined and broadened.

This is an example for different value of R for the transmitted intensity.

Slide 16

Airy's formulas

A useful parameter to evaluate the degree of definition of a fringe is its **full width at half maximum (FWHM, γ)**

$$\frac{I_t}{I_0} = \frac{1}{1 + F \left(\sin \frac{\delta_{1/2}}{2} \right)^2} = \frac{1}{2} \Rightarrow \left(\sin \frac{\delta_{1/2}}{2} \right)^2 = \frac{1}{F}$$

$$\Rightarrow \sin \frac{\delta_{1/2}}{2} = \frac{1}{\sqrt{F}} \approx \frac{\delta_{1/2}}{2} \quad F \gg 1$$

$$\gamma = 2\delta_{1/2} = \frac{4}{\sqrt{F}} \quad \text{FWHM}$$

$$\mathcal{F} = \frac{2\pi}{\gamma} = \frac{\pi\sqrt{F}}{2} = \frac{\pi\sqrt{R}}{(1-R)} \quad \text{Finesse (of the fringes)}$$

17

Slide 17

Airy's formulas

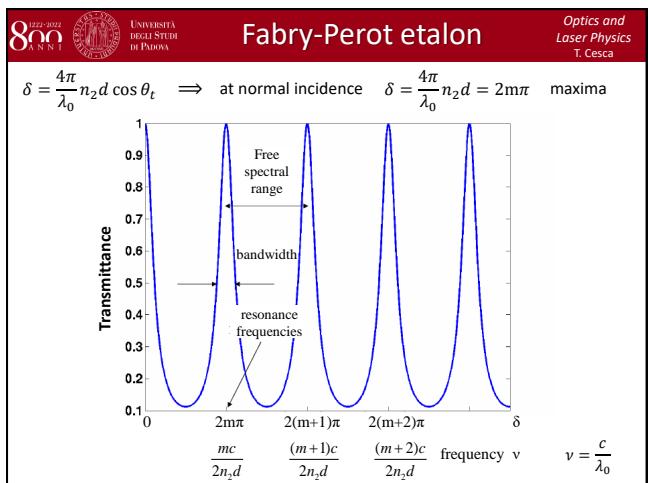
Reflection	Transmission
$\frac{I_r}{I_0} = \frac{F \sin^2(\delta/2)}{1 + F \sin^2(\delta/2)}$	$\frac{I_t}{I_0} = \frac{1}{1 + F \sin^2(\delta/2)}$
$F = \frac{4R}{(1-R)^2}$ Coefficient of finesse (of the slab)	
$\mathcal{F} = \frac{2\pi}{\gamma} = \frac{\pi\sqrt{F}}{2} = \frac{\pi\sqrt{R}}{(1-R)}$ Finesse (of the fringes) $\gamma = \frac{4}{\sqrt{F}}$ FWHM	
For $R_1 \neq R_2$ replace $R \rightarrow \sqrt{R_1 R_2}$	

18

A useful parameter to evaluate the degree of definition of a fringe is its **full width at half maximum (FWHM)**. If F is sufficiently large (so the reflectivity of the slab is large), we can approximate.. Another useful parameter is \mathcal{F} which is the **finesse (of the fringes)**. It is the difference between the difference in phase among two consecutive fringes divided by the full width at half maximum γ .

To sum up. We have assumed $n_1 = n_2$.

Slide 18



This is called **Fabry-Perot etalon**. In a cavity of a laser we have a slab of air between two mirrors. In two dimension is called a Fabry-Perot etalon.

19

Slide 19

Fabry-Perot etalon

Fabry-Perot bandwidth, $\Delta\nu_{FP}$

$$\delta = \frac{4\pi}{\lambda_0} n_2 d = \frac{4\pi n_2 d}{c} v \quad v = \frac{c}{\lambda_0}$$

$$\Rightarrow \gamma = \Delta\delta = \frac{4\pi n_2 d}{c} \Delta\nu_{FP} = \frac{2\pi}{\mathcal{F}}$$

$$\Delta\nu_{FP} = \frac{c}{2n_2 d} \frac{1}{\mathcal{F}}$$

Free spectral range, $\Delta\nu_{FSR}$ $\Rightarrow \Delta\delta = 2\pi$

$$\Delta\nu_{FSR} = \frac{c}{2n_2 d}$$

20

Slide 20

Fabry-Perot interferometer

Spectroscopic applications

23

Slide 21

Fabry-Perot interferometer

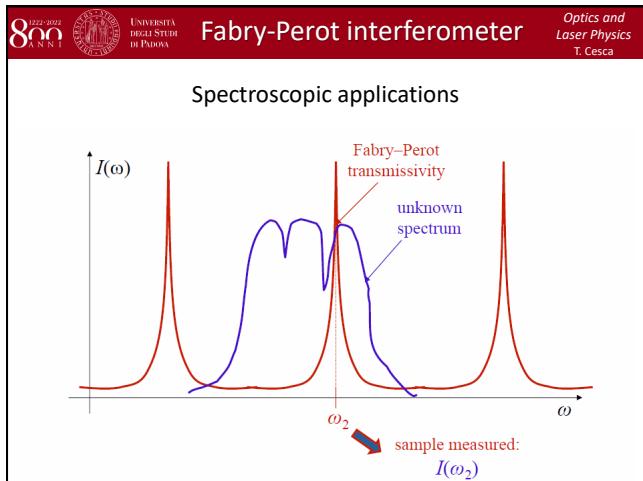
Spectroscopic applications

24

We can determine the **Fabry-Perot bandwidth**. Another parameter is the **free spectral range**. It is the difference in frequency that correspond to a difference in phase of 2π .

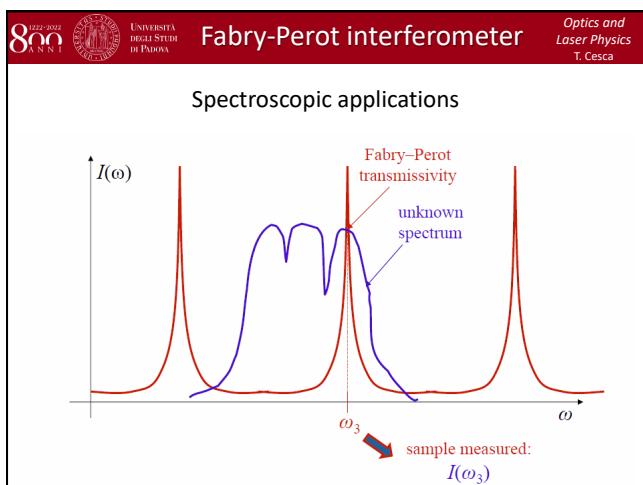
We call it etalon any time the slab as a given and fixed thickness. You can also realize in this way **Fabry-Perot interferometer** if instead of having fixed thickness for the slab you can vary it. Or for instance we are able to move one of the two mirrors (the thick of the slab hence is varying).

Slide 22



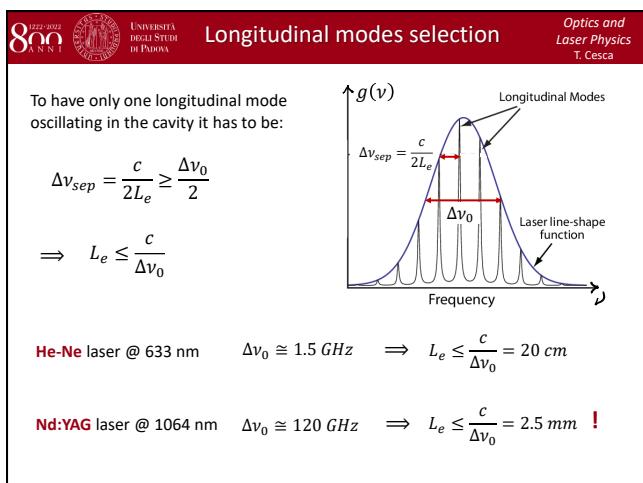
25

Slide 23



26

Slide 24



27

How to use all of these in order to get longitudinal modes selection?

It is possible to use a Fabry-Perot etalon to select. We have to satisfy this condition for the separation in frequency to have just one longitudinal mode that is amplified by the active medium. This condition become a condition in term of the effective length.

Let us try to make some calculation. We see that we should use too small effect length for the active medium. That is why we use Fabry-Perot etalon to solve the problem.

Slide 25

Longitudinal modes selection

If a Fabry-Perot etalon is inserted in the cavity: $\Delta\nu_{sep} = \frac{c}{2L_e} \geq \frac{\Delta\nu_{FP}}{2} = \frac{\Delta\nu_{FSR}}{2\mathcal{F}}$

and $\Delta\nu_{FSR} \geq \frac{\Delta\nu_0}{2}$

to have only one FP peak under the gain curve

Gain curve
Transmittance peaks of the Fabry-Perot
Cavity modes
 $\Delta\nu_0$
 $\Delta\nu_{FSR}$
 $\Delta\nu_{FP}$
 $\Delta\nu_{sep} = \frac{c}{2L_e}$

28

We have a material with a gain curve and we insert in the cavity a Fabry-Perot etalon. Now, the condition to satisfy to have one single mode is this one. The second condition is that we want just one peak.

Slide 26

Longitudinal modes selection

If a Fabry-Perot etalon is inserted in the cavity: $\Delta\nu_{sep} = \frac{c}{2L_e} \geq \frac{\Delta\nu_{FP}}{2} = \frac{\Delta\nu_{FSR}}{2\mathcal{F}}$

and $\Delta\nu_{FSR} \geq \frac{\Delta\nu_0}{2}$

to have only one FP peak $\Rightarrow \frac{\Delta\nu_0}{2} \leq \Delta\nu_{FSR} \leq 2\mathcal{F}\Delta\nu_{sep} = 2\mathcal{F}\frac{c}{2L_e}$

$\Rightarrow L_e \leq 2\mathcal{F}\frac{c}{\Delta\nu_0}$

$R = 90\% \quad \mathcal{F} = \frac{\pi\sqrt{R}}{(1-R)} \cong 29.8$

$R = 95\% \quad \mathcal{F} = \frac{\pi\sqrt{R}}{(1-R)} \cong 61.2$

Gain curve
Transmittance peaks of the Fabry-Perot
Cavity modes
 $\Delta\nu_0$
 $\Delta\nu_{FSR}$
 $\Delta\nu_{FP}$
 $\Delta\nu_{sep} = \frac{c}{2L_e}$

29

We obtain a condition for the effective length.

Slide 27

Longitudinal modes selection

Ar laser @ 514.5 nm (green line) $\Delta\nu_0 \cong 3.5 \text{ GHz}$

without the FP etalon: $L_e \leq \frac{c}{\Delta\nu_0} \cong 8.6 \text{ cm}$

with one FP etalon with $\mathcal{F} = 30$: $L_e \leq 2\mathcal{F}\frac{c}{\Delta\nu_0} \cong 5.16 \text{ m} !$

Nd:YAG laser @ 1064 nm $\Delta\nu_0 \cong 120 \text{ GHz}$

without the FP etalon: $L_e \leq \frac{c}{\Delta\nu_0} = 2.5 \text{ mm}$

with one FP etalon with $\mathcal{F} = 30$: $L_e \leq 2\mathcal{F}\frac{c}{\Delta\nu_0} \cong 15 \text{ cm}$

Gain curve
Transmittance peaks of the Fabry-Perot
Cavity modes
 $\Delta\nu_0$
 $\Delta\nu_{FSR}$
 $\Delta\nu_{FP}$
 $\Delta\nu_{sep} = \frac{c}{2L_e}$

30

Let us see some examples.

Slide 28

Longitudinal modes selection

If the condition $L_e \leq 2\mathcal{F} \frac{c}{\Delta\nu_0}$ is not satisfied

⇒ it is not possible to select a single longitudinal mode with one FP only
⇒ two or more FPs are needed

Let's consider the case of 2 FP etalons with the same finesse \mathcal{F} and different thickness.

The **thickest etalon** is used to discriminate two adjacent longitudinal modes since it has a smaller bandwidth:

$$\Delta\nu'_{FP} = \frac{c}{2n_2 d' \mathcal{F}}$$

The **thinnest etalon** is used to discriminate two transmission peaks of the first etalon

31

Slide 29

Longitudinal modes selection

$\Rightarrow \Delta\nu_{sep} = \frac{c}{2L_e} \geq \frac{\Delta\nu'_{FP}}{2} = \frac{\Delta\nu'_{FSR}}{2\mathcal{F}}$ to discriminate two adjacent longitudinal modes

$\Delta\nu'_{FSR} \geq \frac{\Delta\nu''_{FP}}{2} = \frac{\Delta\nu''_{FSR}}{2\mathcal{F}}$ to discriminate two adjacent transmission peaks of the first FP with the second FP

$\Delta\nu''_{FSR} \geq \frac{\Delta\nu_0}{2}$ to have only one transmission peak of the second FP under the gain curve

$\Rightarrow 2\mathcal{F}\Delta\nu_{sep} \geq \Delta\nu'_{FSR} \geq \frac{\Delta\nu''_{FSR}}{2\mathcal{F}}$

$(2\mathcal{F})^2\Delta\nu_{sep} \geq \Delta\nu''_{FSR} \geq \frac{\Delta\nu_0}{2}$

$\Rightarrow L_e \leq \frac{c}{(2\mathcal{F})^2 \Delta\nu_0}$

32

If just one Fabry-Perot etalon is not enough, we can use two or even more Fabry-Perot etalon.

Let us see what happens when we use 2 Fabry-Perot etalon with same finesse and different thickness.

The thickest etalon (the largest) is used to discriminate two adjacent longitudinal modes since it has a smaller bandwidth.

The thinnest etalon is used to have only one peak of the first etalon within the transmittance peak of the second etalon.

So we have three condition!

In this case we can increase a lot the effective length of the cavity.

The interference filter are the Fabry-Perot etalon.