Cavity quality factor

$$\phi_\nu(t) = \phi_\nu(0)e^{-(t/\tau_c)}$$

τ_c = lifetime of a photon in the cavity

$$\boxed{\tau_c = \frac{L_e}{c\gamma}}$$

$$\kappa = \frac{1}{\tau_c}$$ cavity photon decay rate

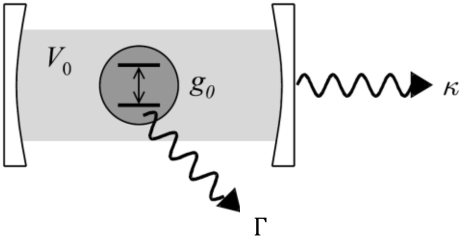
$$\boxed{Q = 2\pi\nu\tau_c = \frac{\nu}{\Delta\nu_c} = \frac{\omega}{\Delta\omega_c}}$$

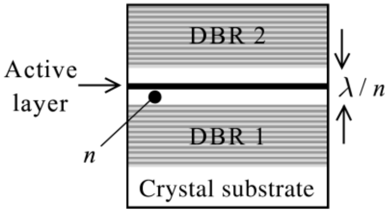
Cavity quality factor

$$\Delta\nu_c = \frac{1}{2\pi\tau_c} = \frac{\nu}{Q}$$

Bandwidth of the cavity resonance modes

Atoms in cavities





In resonance, the atom-cavity interaction is controlled by three parameters:

κ: photon decay rate of the cavity

Γ: non-resonant decay rate

g_0: atom-cavity coupling parameter

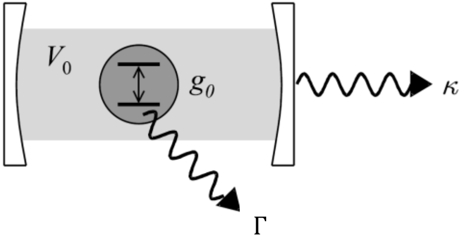
$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

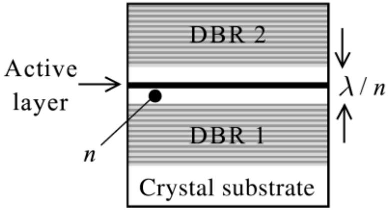
$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

In the limit of **strong coupling**, the atom-photon interaction is faster than the irreversible processes due to loss of photons out of the cavity mode

$\Longrightarrow$ the photon emission becomes a **reversible** process in which the photon is re-absorbed by the atom before being lost from cavity

Atoms in cavities





In resonance, the atom-cavity interaction is controlled by three parameters:

κ: photon decay rate of the cavity

Γ: non-resonant decay rate

g_0: atom-cavity coupling parameter

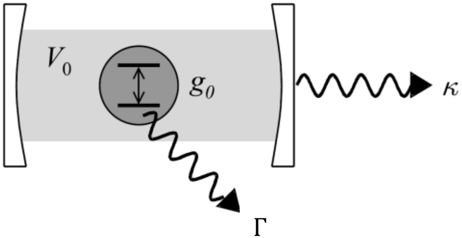
$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

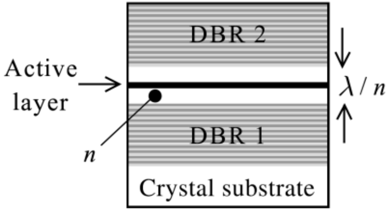
$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

In the limit of **weak coupling**, photons are lost from the atom-cavity system faster than the characteristic interaction time between the atom and the cavity

$\Longrightarrow$ the emission of light from the atom is an **irreversible** process, as for free-space spontaneous emission, but the emission rate is affected by the cavity

Atoms in cavities





In resonance, the atom-cavity interaction is controlled by three parameters:

κ: photon decay rate of the cavity

Γ: non-resonant decay rate

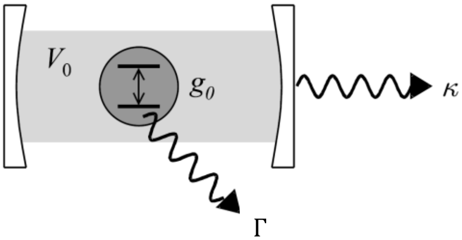
g_0: atom-cavity coupling parameter

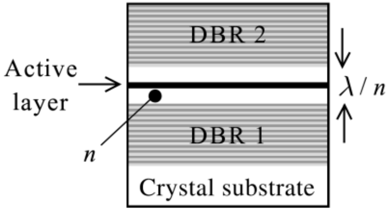
$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

$$\kappa = \frac{1}{\tau_c} = \frac{2\pi\nu}{Q} = \frac{\omega}{Q}$$

Atoms in cavities





In resonance, the atom-cavity interaction is controlled by three parameters:

κ: photon decay rate of the cavity

Γ: non-resonant decay rate

g_0: atom-cavity coupling parameter

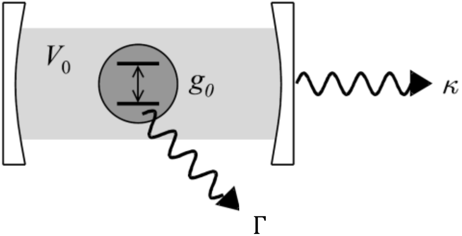
$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

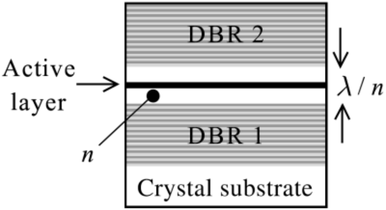
$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

$\Gamma = \tilde{\Gamma} + \Gamma'_{rad} + \Gamma_{nr}$:

- emission of a resonant photon in a direction that does not coincide with a cavity mode ($\tilde{\Gamma}$);
- radiative emission at different frequencies (not resonant) (Γ'_{rad});
- non-radiative processes (Γ_{nr})

Atoms in cavities





In resonance, the atom-cavity interaction is controlled by three parameters:

κ: photon decay rate of the cavity

Γ: non-resonant decay rate

***g_0*: atom-cavity coupling parameter**

$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

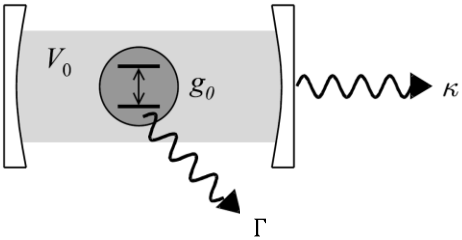
$$g_0 = \left(\frac{\mu_{12}^2 \omega}{2\epsilon_0 \hbar V_0} \right)^{1/2}$$

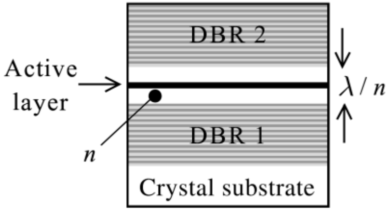
V_0 = modal volume

$$\vec{\mu}_{12} = -e\big(\langle 2|x|1\rangle \hat{\imath} + \langle 2|y|1\rangle \hat{\jmath} + \langle 2|z|1\rangle \hat{k}\big)$$

electric dipole moment of the transition

Atoms in cavities





In resonance, the atom-cavity interaction is controlled by three parameters:

κ: photon decay rate of the cavity

Γ: non-resonant decay rate

g_0: atom-cavity coupling parameter

$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

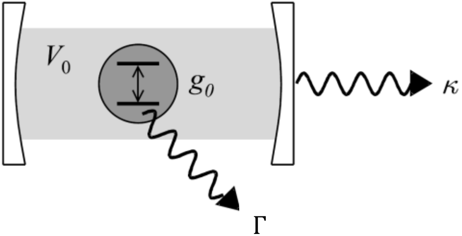
$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

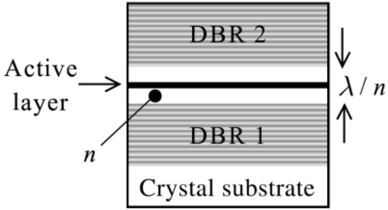
If we assume that the cavity decay rate κ is the dominant loss mechanism ($\kappa > \Gamma$)

$\Longrightarrow$ **strong coupling** occurs when: $g_0 \gg \kappa = \dfrac{\omega}{Q} \quad \Longrightarrow \quad Q \gg \left(\dfrac{2\epsilon_0 V_0 \hbar\omega}{\mu_{12}^2}\right)^{1/2}$

very strict!

Atoms in cavities





In resonance, the atom-cavity interaction is controlled by three parameters:

κ: photon decay rate of the cavity

Γ: non-resonant decay rate

g_0: atom-cavity coupling parameter

$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

If N atoms are present in the cavity $\Longrightarrow$ The **strong coupling** condition becomes:

$\sqrt{N} g_0 \gg (\kappa, \Gamma)$ $\Longrightarrow$ the factor $\sqrt{N}$ makes it easier to observe strong coupling

Atoms in cavities

Q: An air-spaced symmetric planar cavity of length $L = 60\ \mu m$ and modal volume $V_0 = 5 \times 10^{-14}\ m^3$ is locked in resonance with a cesium transition at $\lambda = 852\ nm$ which has an electric dipole moment $|\vec{\mu}_{12}| = 3 \times 10^{-29}\ C\ m$. The radiative lifetime of the transition is $\tau_R = 32\ ns$.

Estimate the smallest values of the cavity Q factor, the cavity finesse $\mathscr{F}$ and the mirror reflectivity R required for strong coupling for a single atom, assuming that the cavity loss rate is the dominant loss mechanism.

$h = 6.63\ 10^{-34}\ Js$ $\qquad c = 3\ 10^8\ m/s$ $\qquad \epsilon_0 = 8.854\ 10^{-12}\ F/m$

A:

$$\omega = 2\pi\nu = \frac{2\pi c}{\lambda} = 2.2 \times 10^{15}\ rad/s \qquad g_0 = \left(\frac{\mu_{12}^2\omega}{2\epsilon_0\hbar V_0}\right)^{1/2} = 1.46 \times 10^8\ rad/s$$

$$g_0 \gg \kappa = \frac{\omega}{Q} \quad \Longrightarrow \quad Q \gg \frac{\omega}{g_0} = \left(\frac{2\epsilon_0 V_0\hbar\omega}{\mu_{12}^2}\right)^{1/2} = 1.51 \times 10^7$$

$$Q = \frac{\omega}{\Delta\omega_c} \gg \frac{\omega}{g_0} \quad \Longrightarrow \quad \Delta\omega_c = 2\pi\Delta\nu_c \ll g_0 \quad \Longrightarrow \quad \Delta\nu_c \ll \frac{g_0}{2\pi}$$

Atoms in cavities

Q: An air-spaced symmetric planar cavity of length $L = 60\ \mu m$ and modal volume $V_0 = 5 \times 10^{-14}\ m^3$ is locked in resonance with a cesium transition at $\lambda = 852\ nm$ which has an electric dipole moment $|\vec{\mu}_{12}| = 3 \times 10^{-29}\ C\ m$. The radiative lifetime of the transition is $\tau_R = 32\ ns$.

Estimate the smallest values of the cavity Q factor, the cavity finesse $\mathscr{F}$ and the mirror reflectivity R required for strong coupling for a single atom, assuming that the cavity loss rate is the dominant loss mechanism.

$h = 6.63\ 10^{-34}\ Js$ $\qquad c = 3\ 10^{8}\ m/s$ $\qquad \epsilon_0 = 8.854\ 10^{-12}\ F/m$ $\qquad n_2 = 1$

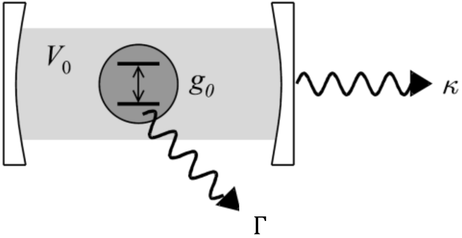
A:

$$\Delta\nu_c \ll \frac{g_0}{2\pi} \qquad \Delta\nu_c = \frac{c}{2n_2 L}\frac{1}{\mathscr{F}} \qquad \Longrightarrow \qquad \mathscr{F} \gg \frac{\pi c}{n_2 L g_0} = 5 \times 10^5$$

$$g_0 \gg \kappa = \frac{1}{\tau_c} \qquad \tau_c = \frac{L_e}{c\gamma} \qquad \gamma = \cancel{\gamma_i} + \frac{\gamma_1 + \gamma_2}{2} = \gamma_1 = \gamma_2 = -lnR$$

$$\Longrightarrow \quad \tau_c = \frac{L_e}{c\gamma} \gg \frac{1}{g_0} \qquad \Longrightarrow \quad lnR \gg -\frac{L_e g_0}{c} = -3 \times 10^{-5} \qquad \Longrightarrow \quad R \gg 99.997\%$$

Weak coupling

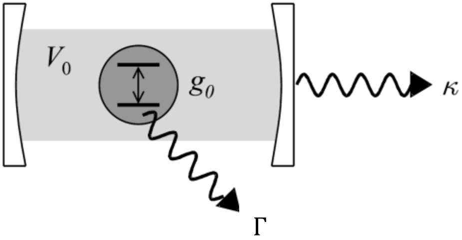


$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

In the limit of **weak coupling**, the effect of the cavity is small, so that the atom-cavity interaction can be treated by perturbation theory.

The main effect of the cavity is to enhance or suppress the photon density of states compared with the free-space value, depending on whether the cavity mode is resonant with the atomic transition or not.

Free-space spontaneous emission



$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

Fermi's golden rule

$$\boxed{W_{21} = \frac{2\pi}{\hbar}|M_{12}|^2\delta(h\nu)}$$

W_{21}: Transition decay rate

M_{12}: Transition matrix element

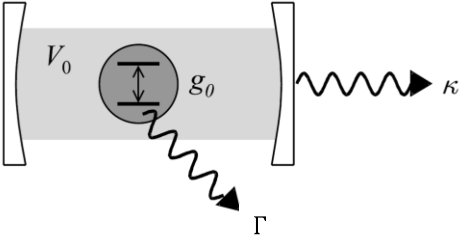
$\delta(h\nu)$: Local Density of final States (LDOS)

$$M_{12} = \langle 2|H'|1\rangle = \int \psi_2^*(\vec{r})H'(\vec{r})\psi_1(\vec{r})d^3\vec{r}$$

$$M_{12} = -\vec{\mu}_{12} \cdot \vec{E}$$

$\vec{\mu}_{12}$: electric dipole moment

Free-space spontaneous emission



$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

Fermi's golden rule

$$\boxed{W_{21} = \frac{2\pi}{\hbar} |M_{12}|^2 \delta(h\nu)}$$

Transition decay rate

Transition matrix element

Local Density of final States (LDOS)

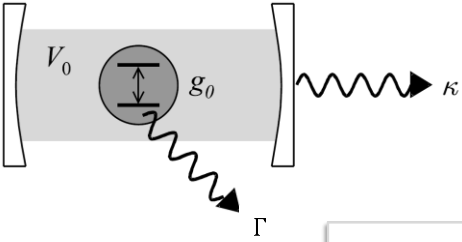
$$W_{21} = \frac{1}{\tau_R} = A_{21} = \frac{8\pi h\nu^3 n^3}{c^3} B_{21}$$

A_{21}= Eintstein's coefficient for spontaneous emission

B_{21}= Eintstein's coefficient for stimulated emission (absorption)

The Purcell effect

E. M. Purcell, 1946



$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

The Purcell factor:

$$\boxed{F_P = \frac{W_{21}^{cav}}{W_{21}^{free}} = \frac{\tau_R^{free}}{\tau_R^{cav}}}$$

For a single mode cavity (modal volume V_0) at exact resonance with the emission frequency of the atom (with the dipoles oriented along the field direction):

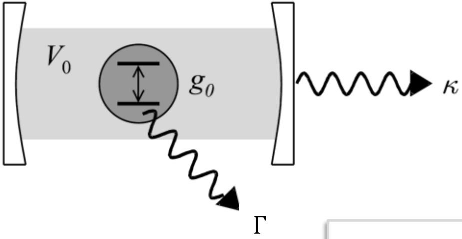
$$F_P = \frac{3Q(\lambda/n)^3}{4\pi^2 V_0}$$

λ is the free-space wavelength of the radiation

n is the refractive index of the medium inside the cavity

The Purcell effect

E. M. Purcell, 1946



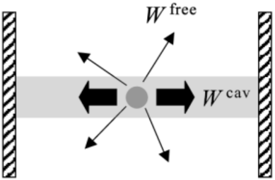
$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

The Purcell factor:

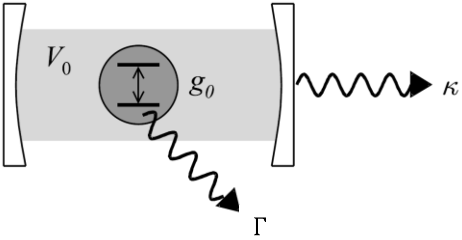
$$\boxed{F_P = \frac{W_{21}^{cav}}{W_{21}^{free}} = \frac{\tau_R^{free}}{\tau_R^{cav}}}$$

Spontaneous emission coupling factor

$$\beta = \frac{W_{21}^{cav}}{W_{21}^{free} + W_{21}^{cav}} = \frac{F_P}{1 + F_P}$$



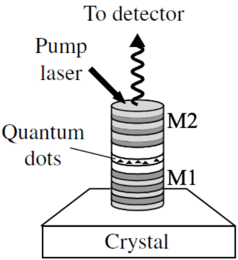
The Purcell effect

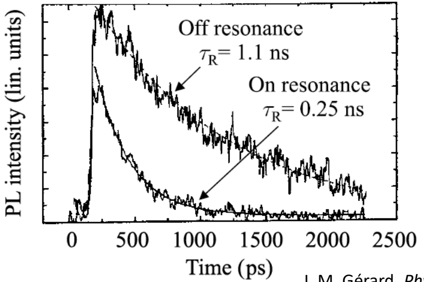


E. M. Purcell, 1946

$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

$$F_P = \frac{W_{21}^{cav}}{W_{21}^{free}} = \frac{\tau_R^{free}}{\tau_R^{cav}}$$

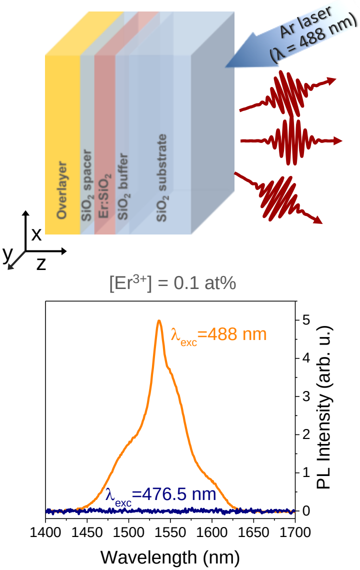




Micropillar cavities

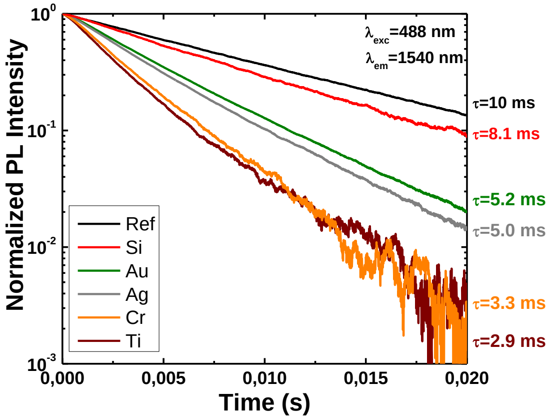
J. M. Gérard, *Phys. Rev. Lett.*, **81**, 1110 (1998)

emitter near a planar interface



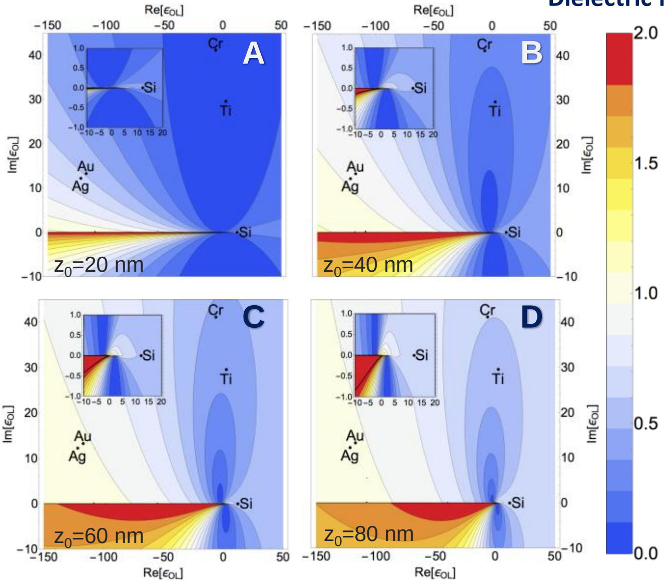
Dielectric function effect

Er:SiO$_2$ t = 75 nm; Spacer t = 0 nm



B. Kalinic, T. Cesca et al., J. Phys. Chem. C 119, 6728 (2015)

Dielectric function effect

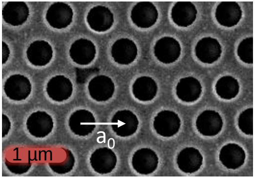


$$\frac{\tau}{\tau_0} = \frac{1}{F_P}$$

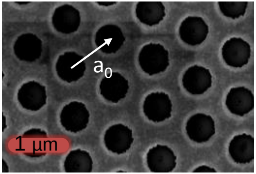
B. Kalinic, T. Cesca et al., J. Phys. Chem. C 119, 6728 (2015)

Er^{3+} - Au NHA

In-resonance

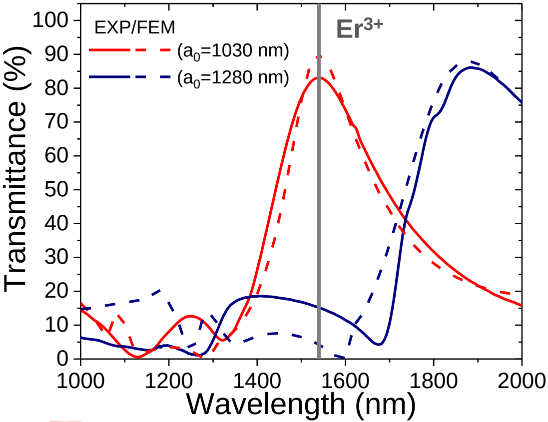


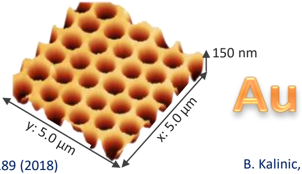
a_0=1030 nm



a_0=1280 nm

Out-of-resonance

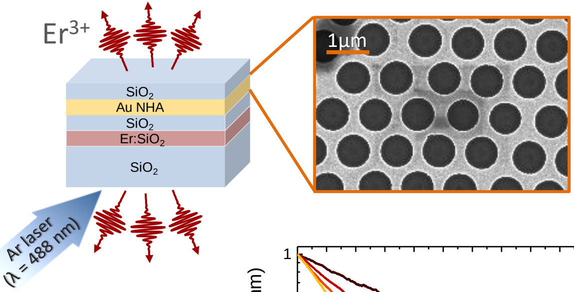


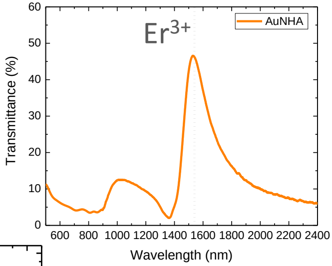


N. Michieli,... T. Cesca and G. Mattei, ACS Photonics, 5, 2189 (2018)

B. Kalinic, T. Cesca et al., J. Phys. Chem. C 119, 6728 (2015)

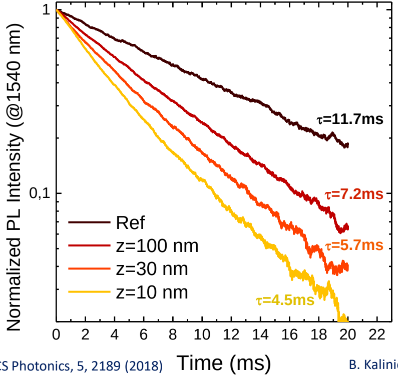
Er^{3+} - Au NHA





$Er:SiO_2$ layer:

t = 20 nm



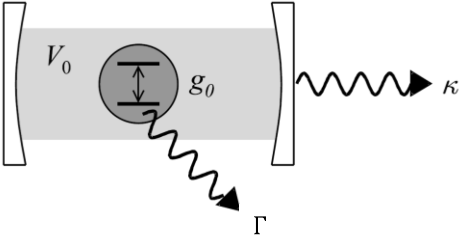
Spacer thickness:

- t = 10 nm
- t = 30 nm
- t = 100 nm

N. Michieli,… T. Cesca and G. Mattei, ACS Photonics, 5, 2189 (2018)

B. Kalinic, T. Cesca et al., J. Phys. Chem. C 119, 6728 (2015)

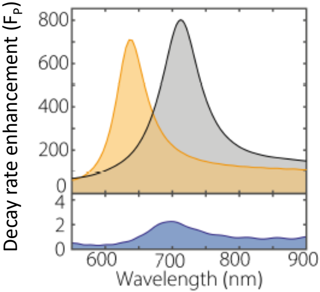
The Purcell effect



E. M. Purcell, 1946

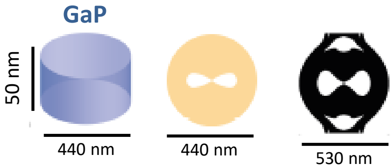
$g_0 \ll (\kappa, \Gamma)$: **weak coupling**

$$F_P = \frac{W_{21}^{cav}}{W_{21}^{free}} = \frac{\tau_R^{free}}{\tau_R^{cav}}$$

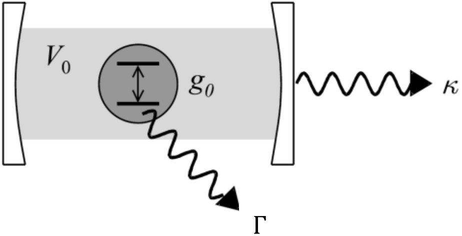


Nanoscale design of the local density of optical states

S. Mignuzzi et al., *NanoLett.*, 19, 1613 (2019)



Strong coupling



$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

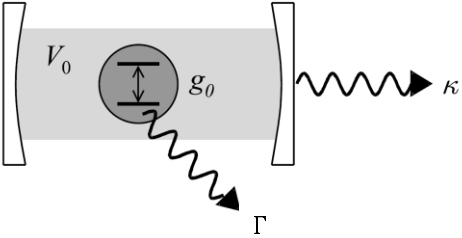
In the conditions of **strong coupling**, the interaction between the photons in the cavity mode and the atom is **reversible**.

The atom emits a photon into the resonant mode, which bounces between the mirrors and it is re-absorbed by the atom faster than it is lost from the mode.

The reversible interaction between the atom and the cavity field is thus faster than the irreversible processes due to loss of photons.

This regime of reversible light-atom interactions is called **cavity quantum electrodynamics (CQED)**

Strong coupling



$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

Jaynes-Cummings model
(1963)

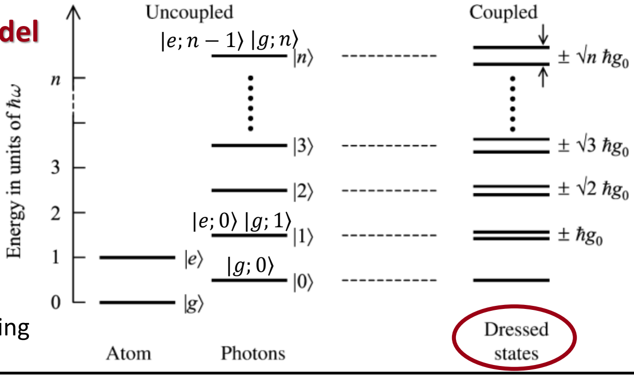
$$\Psi = |\psi; n\rangle$$

ψ: state of the atom

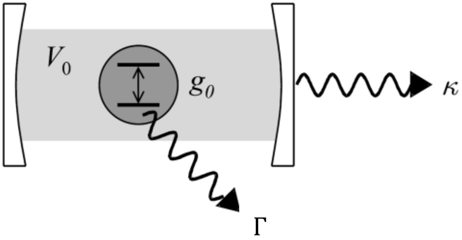
n: number of photons

$$\Delta E_n = 2\sqrt{n}\hbar g_0$$ splitting

Jaynes-Cummings ladder



Strong coupling



$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

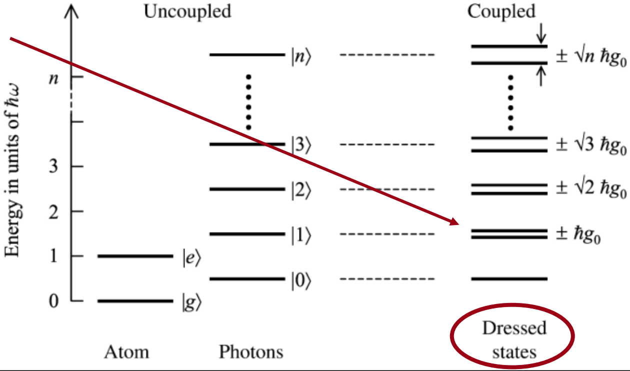
vacuum Rabi splitting

$$\Delta E^{vac} \equiv 2\hbar g_0$$

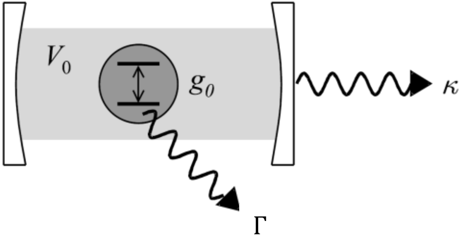
$$g_0 = \left(\frac{\mu_{12}^2 \omega}{2\epsilon_0 \hbar V_0}\right)^{1/2}$$

$$\Delta E^{vac} = \left(\frac{2\mu_{12}^2 \hbar\omega}{\epsilon_0 V_0}\right)^{1/2}$$

Jaynes-Cummings ladder



Strong coupling



$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

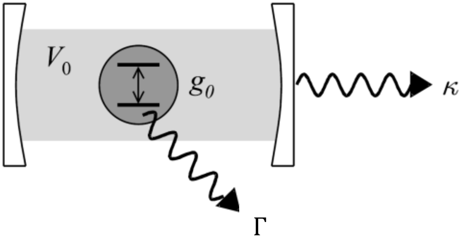
For N atoms in the cavity:

$$\Delta E^{vac}(N) = \sqrt{N}\left(\frac{2\mu_{12}^2 \hbar\omega}{\epsilon_0 V_0}\right)^{1/2}$$

If the medium within the cavity has a relative permittivity ϵ_r, we should replace ϵ_0 with $\epsilon_r \epsilon_0 \equiv n^2 \epsilon_0$, where n is the refractive index:

$$\Delta E^{vac}(N) = \sqrt{N}\left(\frac{2\mu_{12}^2 \hbar\omega}{n^2 \epsilon_0 V_0}\right)^{1/2}$$

Strong coupling



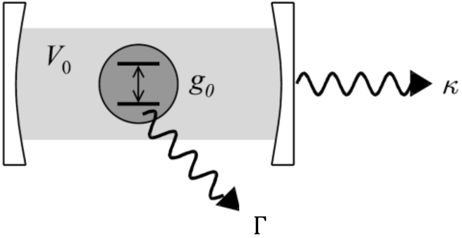
$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

In order to observe strong coupling it is necessary to have:

- cavities with small volumes to enhance the coupling constant g_0
- high Q-factors to reduce the photon loss rate
- other dissipative rates due to dephasing and non-resonant emission should be minimized
- the cavity should support only a single mode in resonance with the atom

challenging requirements!

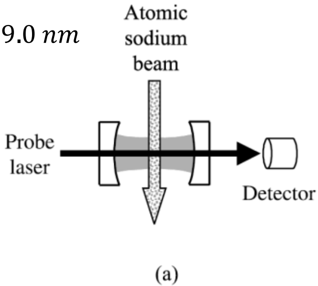
Strong coupling

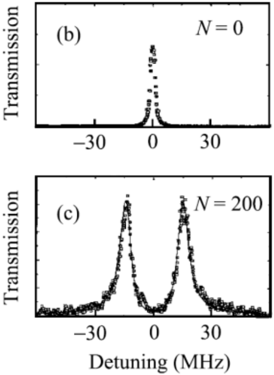


$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

$\lambda = 589.0\ nm$

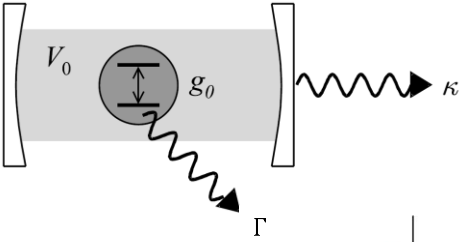
Tunable





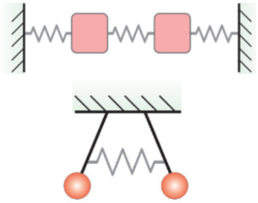
$\mathscr{F} = 26000 \qquad L = 3.2\ mm$

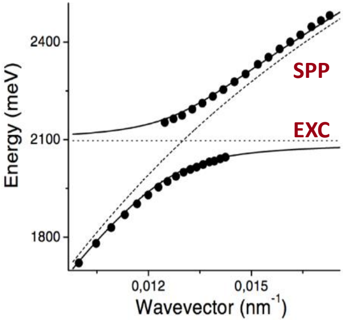
Strong coupling



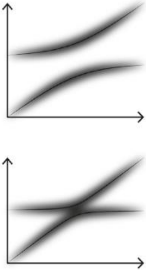
$g_0 \gg (\kappa, \Gamma)$: **strong coupling**

Classical model:
coupled oscillators





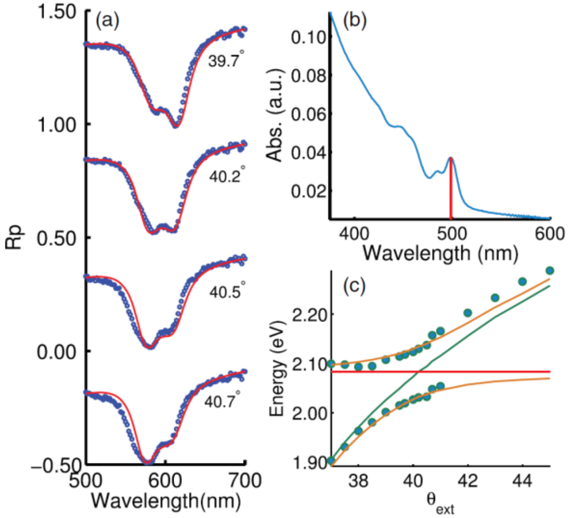
Rabi splitting

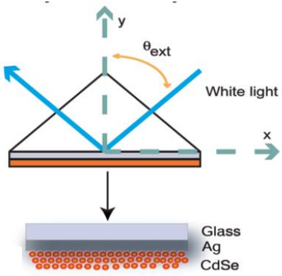


P. Törmä, W. L. Barnes, *Strong Coupling between Surface Plasmon Polaritons and Emitters: A Review*. *Rep. Prog. Phys.* **2015**, *78*, 013901.

Strong coupling

D. E. Gomez, K. C. Vernon, P. Mulvaney, and T. J. Davis, *Surface Plasmon Mediated Strong Exciton-Photon Coupling in Semiconductor Nanocrystals*. Nano Lett. **2010**, 10, 274.

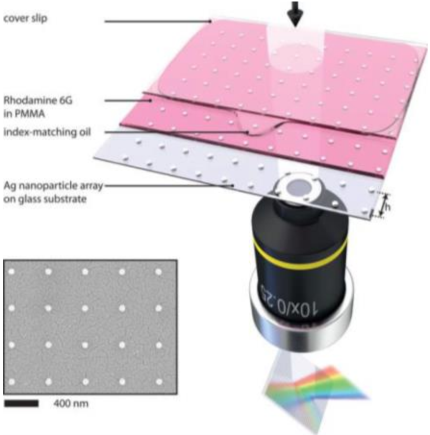




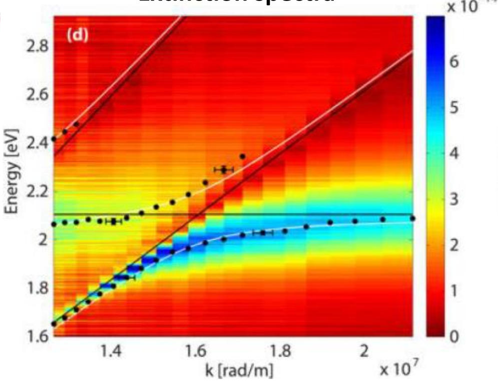
Ag/CdSe film

Strong coupling

A. I. Väkeväinen, R. J. Moerland, H. T. Rekola, A.-P. Eskelinen, J.-P. Martikainen, D.-H. Kim, and P. Törmä, *Plasmonic Surface Lattice Resonances at the Strong Coupling Regime*. Nano Lett. **2014**, 14, 1721.

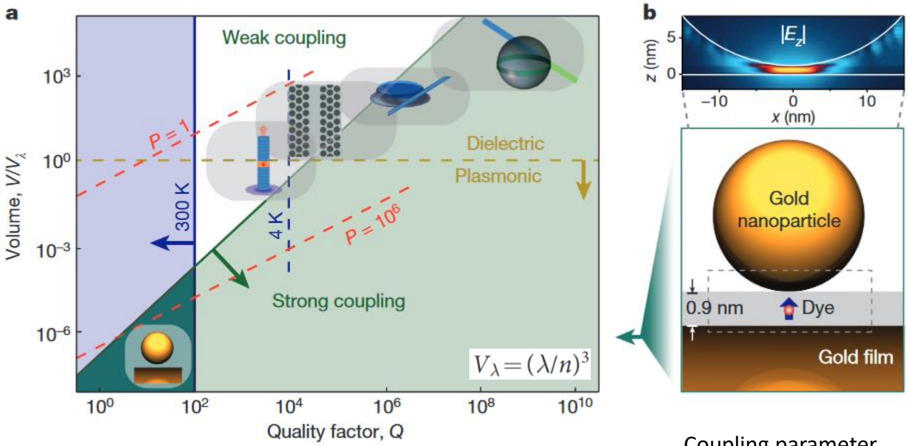


Extinction spectra



Strong coupling

R. Chikkaraddy, R et al, *Single-Molecule Strong Coupling at Room Temperature in Plasmonic Nanocavities*. Nature **2016**, 535, 127–130



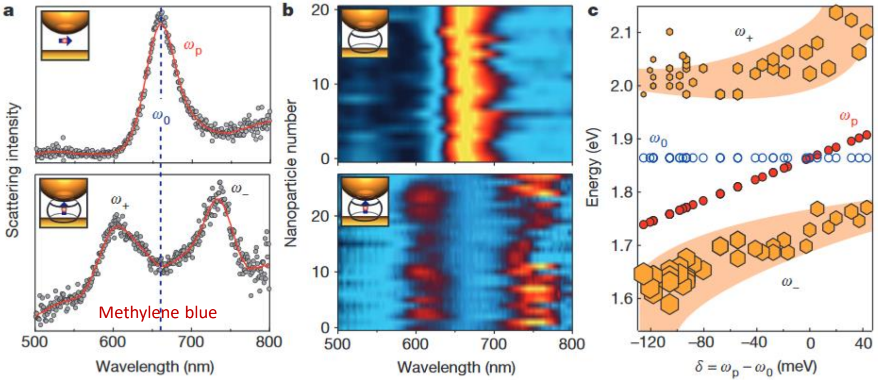
Coupling parameter

Quality factor $Q \propto \dfrac{1}{\kappa}$ Purcell factor $P = F_P \propto \dfrac{Q}{V}$ $g_0 \propto \dfrac{1}{\sqrt{V}}$

Strong coupling

R. Chikkaraddy, R et al, *Single-Molecule Strong Coupling at Room Temperature in Plasmonic Nanocavities*. Nature **2016**, 535, 127–130

Plexciton = hybrid plasmon-exciton



$g_0 = 380\ meV$

Quality factor $Q \propto \frac{1}{\kappa}$

Purcell factor $P = F_P \propto \frac{Q}{V}$

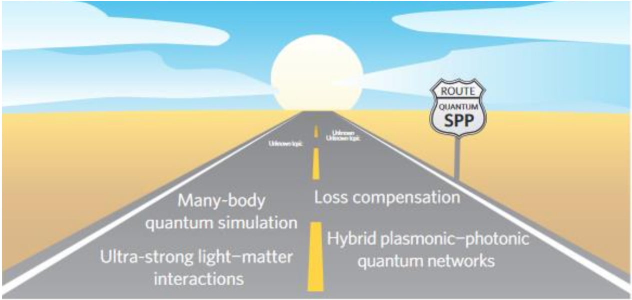
Coupling parameter $g_0 \propto \frac{1}{\sqrt{V}}$

Quantum Optics

Plasmonics

Quantum Plasmonics

Quantum Plasmonics Roadmap



Tame, M. S. et al., Quantum Plasmonics. *Nat Phys* **2013**, *9*, 329–340.