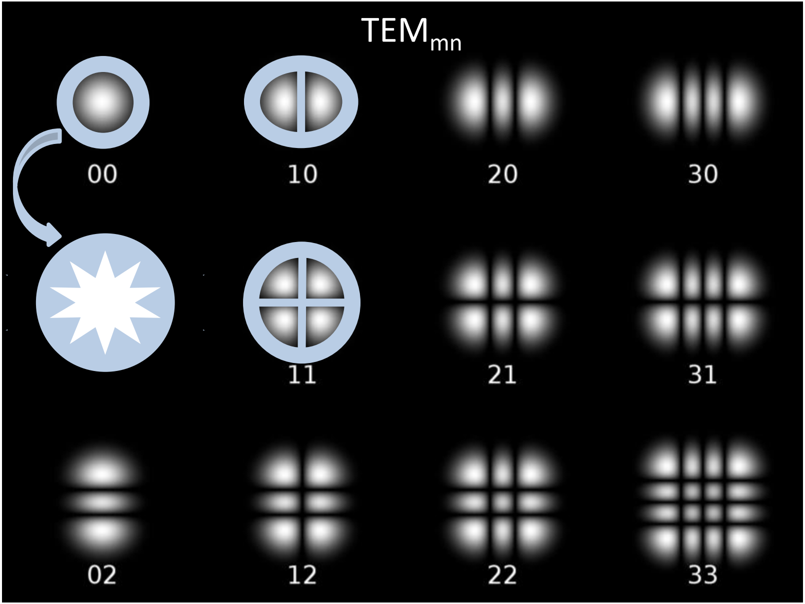
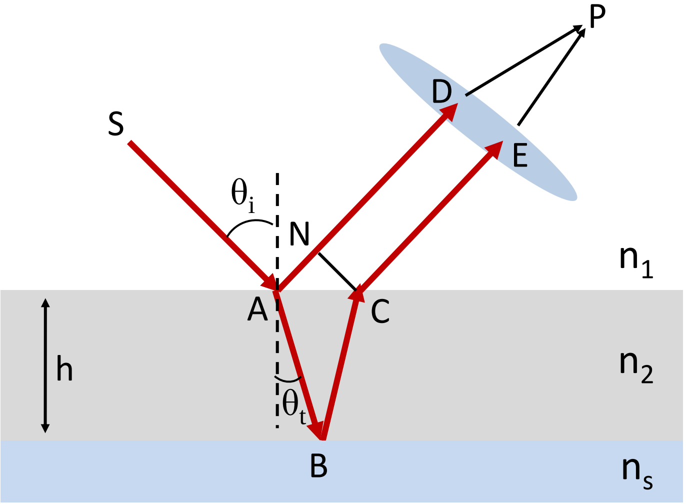
How to get single mode selection?



Thin-film interference



$$\delta = \frac{4\pi}{\lambda_0} n_2 h \cos\theta_t \; (+\pi)$$

Fringes with equal inclination
(Haidenger's fringes)

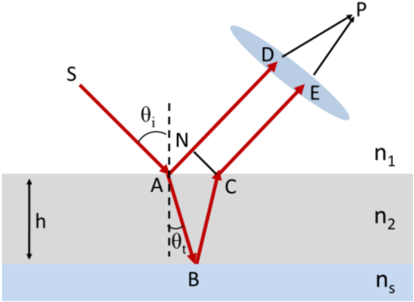
Thin-film interference

The difference of optical path is given by:

$$\Delta\mathcal{L} = n_2\left(\overline{AB} + \overline{BC}\right) - n_1\overline{AN}$$

$$\overline{AN} = \overline{AC}sin\theta_i = 2h\,\frac{sin\theta_t}{cos\theta_t}sin\theta_i$$

$$\overline{AB} = \overline{BC} = \frac{h}{cos\theta_t}$$



$$\Delta\mathcal{L} = n_2\frac{2h}{cos\theta_t} - n_1 2h\,\frac{sin\theta_t}{cos\theta_t}sin\theta_i \; = n_2\frac{2h}{cos\theta_t} - 2h\,\frac{sin\theta_t}{cos\theta_t}n_2 sin\theta_t =$$

$$= n_2\frac{2h}{cos\theta_t}\left(1 - sin^2\theta_t\right) = 2h\,n_2 cos\theta_t$$

$$\Delta\phi_{path} = k\Delta\mathcal{L} = \frac{2\pi}{\lambda_0}\Delta\mathcal{L} = \frac{4\pi}{\lambda_0}h\,n_2 cos\theta_t$$

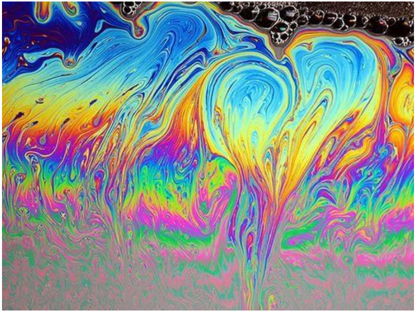
$$\Delta\phi_{refl} = \pi \qquad n_2 > n_1$$

$$\Delta\phi_{refl} = 0 \qquad n_2 < n_1$$

Exercise:

Some oil (with refractive index $n_2 = 1.25$) is on the surface of a water puddle with refractive index $n_s = 1.33$.

1. At normal incidence, what is the minimum oil film thickness reflecting most the red light at $\lambda = 650\ nm$?
2. At which angle of incidence the same film reflects most the blue light at $\lambda = 450\ nm$?



1. $n_1 < n_2 < n_s \quad \Longrightarrow$ each interface produce a π phase shift in reflection

$$\Longrightarrow \quad \Delta\phi_{refl} = 0 \qquad \delta = \frac{4\pi}{\lambda_0} n_2 h \cos\theta_t = 2\pi$$

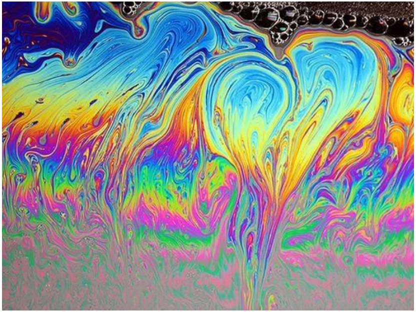
for constructive interference

1 at normal incidence

$$h = \frac{\lambda_0}{2n_2} = \frac{650\ nm}{2\ 1.25} = 260\ nm$$

Thin-film interference

Exercise:



Some oil (with refractive index $n_2 = 1.25$) is on the surface of a water puddle with refractive index $n_s = 1.33$.

1. At normal incidence, what is the minimum oil film thickness reflecting most the red light at $\lambda = 650\ nm$?
2. At which angle of incidence the same film reflects most the blue light at $\lambda = 450\ nm$?

2.

$$2\pi = \frac{4\pi}{\lambda_0} n_2 h \cos\theta_t \quad \Longrightarrow \quad \cos\theta_t = \frac{\lambda_0}{2n_2 h} = \frac{450\ nm}{2\ 1.25\ 260\ nm} = 0.692$$

$$\Longrightarrow \quad \theta_t = 42.2° \quad \Longrightarrow \quad \sin\theta_i = \frac{n_2}{n_1}\sin\theta_t = 0.902$$

$$\Longrightarrow \quad \theta_i = 64.4°$$

Thin-film interference

A film of soapy water suspended vertically on a metallic ring can get very thin before breaking

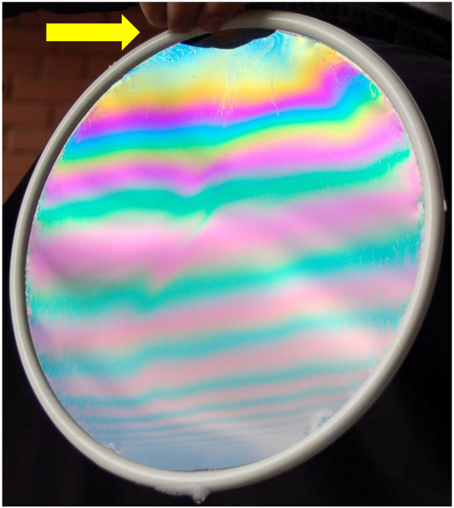
$$\Delta\phi_{path} = k\Delta\mathcal{L} = 0$$

$$n_1 = n_s = 1$$

$$\Delta\phi_{refl} = \pi$$

$$\delta = \Delta\phi_{path} + \Delta\phi_{refl} = \pi$$

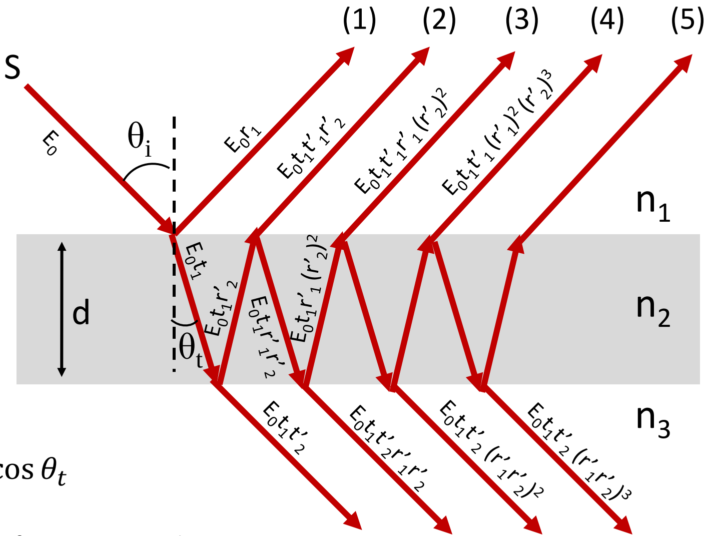
Destructive interference!



Multiple beams interference

$$r_1' = -r_1$$

$$r_2' = -r_2$$



$$\delta = \frac{4\pi}{\lambda_0} n_2 d \cos\theta_t$$

reflection phase shifts are accounted into the reflection coefficients

Multiple beams interference

Let’s consider the transmitted beam:

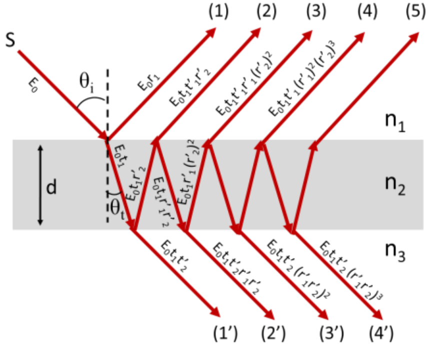
$$E_t = E_0 t_1 t_2' + E_0 t_1 t_2' r_2' r_1' e^{i\delta} + E_0 t_1 t_2' (r_2' r_1')^2 e^{i2\delta} + \cdots + E_0 t_1 t_2' (r_2' r_1')^N e^{iN\delta}$$

$$= E_0 t_1 t_2' [1 + x + x^2 + \cdots x^N] \qquad x = r_2' r_1' e^{i\delta}$$

$$\sum_{n=1}^{N} x^n \rightarrow \frac{1}{1-x}$$

$$|x| < 1, \quad N \text{ large}$$

$$E_t = E_0 t_1 t_2' \frac{1}{1 - r_2' r_1' e^{i\delta}}$$

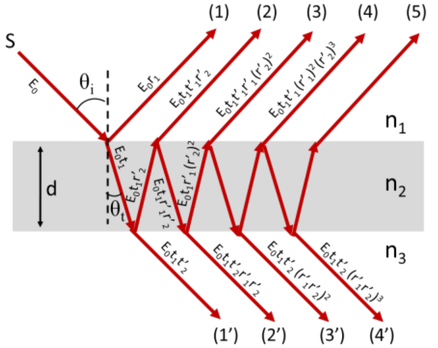


Analogously for the reflected beam:

$$E_r = E_0 r_1 + E_0 t_1 t_1' r_2' e^{i\delta} + E_0 t_1 t_1' r_1' (r_2')^2 e^{i2\delta} + \cdots + \\ + \cdots + E_0 t_1 t_1' (r_1')^{N-2} (r_2')^{N-1} e^{i(N-1)\delta}$$

$$= E_0 r_1 + E_0 t_1 t_1' r_2' e^{i\delta} [\underbrace{1 + x + x^2 + \cdots x^N}_{\frac{1}{1-x}}] \qquad x = r_2' r_1' e^{i\delta}$$

$$E_r = E_0 \left(r_1 + \frac{t_1 t_1' r_2' e^{i\delta}}{1 - r_2' r_1' e^{i\delta}} \right)$$



Multiple beams interference

Assuming $n_1 = n_3 \quad \Longrightarrow \quad r_1 = r_2 = r \qquad t_1 = t_2 = t$

$$E_t = E_0 t t' \frac{1}{1-(r')^2 e^{i\delta}} \qquad\qquad E_r = E_0 \left(r + \frac{t t' r' e^{i\delta}}{1-(r')^2 e^{i\delta}} \right)$$

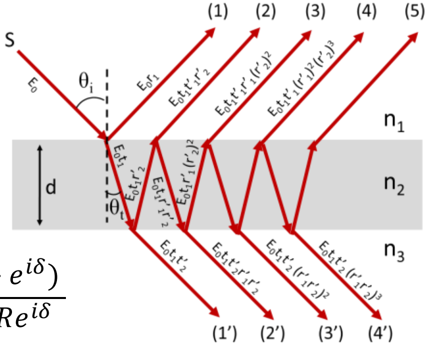
Defining $|r|^2 = |r'|^2 = R \qquad tt' = T$

$$\Longrightarrow \quad R + \frac{n_3}{n_1} T = 1 \quad \Longrightarrow \quad R + T = 1$$

$$E_t = E_0 (1-R) \frac{1}{1-Re^{i\delta}}$$

$r' = -r$

$$E_r = E_0 \left(r + \frac{(1-R) r' e^{i\delta}}{1-Re^{i\delta}} \right) = E_0 \frac{r(1-e^{i\delta})}{1-Re^{i\delta}}$$



Multiple beams interference

Given $\quad I_t \propto |E_t|^2 \qquad I_r \propto |E_r|^2$

$$I_t = I_0(1-R)^2 \frac{1}{1-Re^{i\delta}} \frac{1}{1-Re^{-i\delta}} = I_0(1-R)^2 \frac{1}{1+R^2-2R\cos\delta}$$

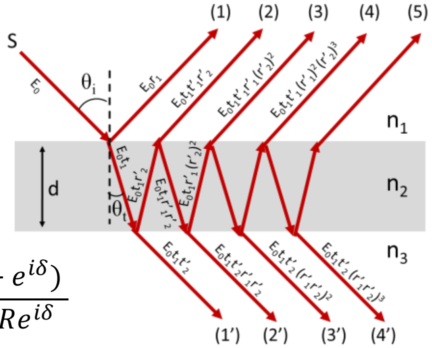
$$= I_0 \frac{(1-R)^2}{(1-R)^2 + 4R\left(\sin\frac{\delta}{2}\right)^2} \qquad \cos\delta = 1 - 2\left(\sin\frac{\delta}{2}\right)^2$$

$$= I_0 \frac{1}{1 + \frac{4R}{(1-R)^2}\left(\sin\frac{\delta}{2}\right)^2}$$

$$E_t = E_0(1-R)\frac{1}{1-Re^{i\delta}}$$

$$r' = -r$$

$$E_r = E_0\left(r + \frac{(1-R)r'e^{i\delta}}{1-Re^{i\delta}}\right) = E_0 \frac{r(1-e^{i\delta})}{1-Re^{i\delta}}$$



Multiple beams interference

Given $I_t \propto |E_t|^2$ $\quad I_r \propto |E_r|^2$

$$I_t = I_0 \frac{1}{1 + \frac{4R}{(1-R)^2}\left(sin\frac{\delta}{2}\right)^2}$$

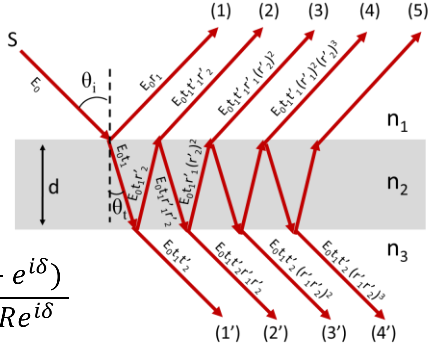
$$cos\delta = 1 - 2\left(sin\frac{\delta}{2}\right)^2$$

$$I_r = I_0 \frac{\frac{4R}{(1-R)^2}\left(sin\frac{\delta}{2}\right)^2}{1 + \frac{4R}{(1-R)^2}\left(sin\frac{\delta}{2}\right)^2}$$

$$E_t = E_0(1-R)\frac{1}{1-Re^{i\delta}}$$

$$r' = -r$$

$$E_r = E_0\left(r + \frac{(1-R)r'e^{i\delta}}{1-Re^{i\delta}}\right) = E_0\frac{r(1-e^{i\delta})}{1-Re^{i\delta}}$$



Multiple beams interference

Defining $F = \dfrac{4R}{(1-R)^2}$ **coefficient of finesse** (of the slab)

$$I_t = I_0 \frac{1}{1 + \frac{4R}{(1-R)^2}\left(sin\frac{\delta}{2}\right)^2} = I_0 \frac{1}{1 + F\left(sin\frac{\delta}{2}\right)^2}$$

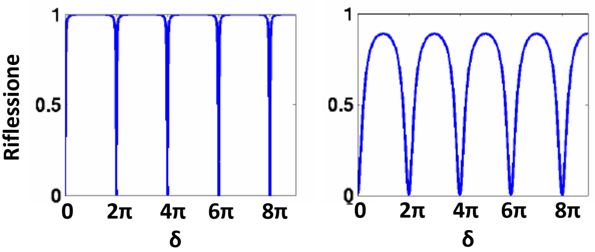
$$I_r = I_0 \frac{\frac{4R}{(1-R)^2}\left(sin\frac{\delta}{2}\right)^2}{1 + \frac{4R}{(1-R)^2}\left(sin\frac{\delta}{2}\right)^2} = I_0 \frac{F\left(sin\frac{\delta}{2}\right)^2}{1 + F\left(sin\frac{\delta}{2}\right)^2}$$

Airy's formulas

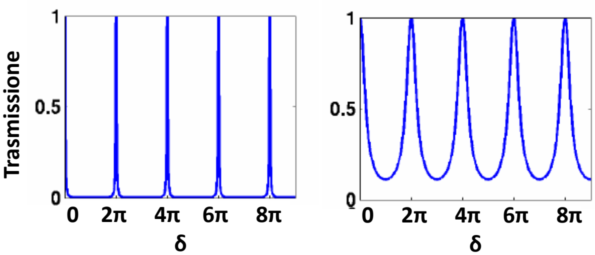
Airy's formulas

R = 0.95 | R = 0.05

$$\frac{I_r}{I_0} = \frac{F\sin^2(\delta/2)}{1+F\sin^2(\delta/2)}$$



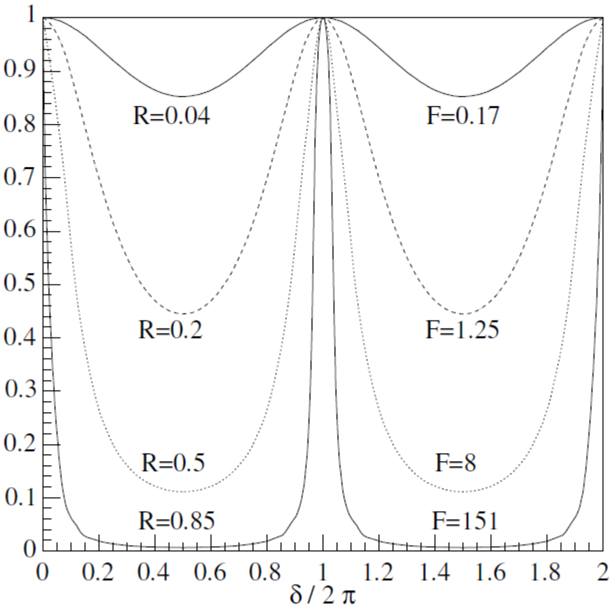
$$\frac{I_t}{I_0} = \frac{1}{1+F\sin^2(\delta/2)}$$



Airy's formulas

$$\frac{I_t}{I_0} = \frac{1}{1 + F\sin^2(\delta/2)}$$

$$F = \frac{4R}{(1-R)^2}$$



Airy's formulas

A useful parameter to evaluate the degree of definition of a fringe is its **full width at half maximum** (FWHM, γ)

$$\frac{I_t}{I_0} = \frac{1}{1 + F\left(\sin\frac{\delta_{1/2}}{2}\right)^2} = \frac{1}{2} \quad \Longrightarrow \quad \left(\sin\frac{\delta_{1/2}}{2}\right)^2 = \frac{1}{F}$$

$$\Longrightarrow \quad \sin\frac{\delta_{1/2}}{2} = \frac{1}{\sqrt{F}} \cong \frac{\delta_{1/2}}{2} \qquad F \gg 1$$

$$\boxed{\gamma = 2\delta_{1/2} = \frac{4}{\sqrt{F}}}$$

FWHM

$$\boxed{\mathcal{F} = \frac{2\pi}{\gamma} = \frac{\pi\sqrt{F}}{2} = \frac{\pi\sqrt{R}}{(1-R)}}$$

Finesse (of the fringes)

Airy's formulas

Reflection

$$\frac{I_r}{I_0} = \frac{F\sin^2(\delta/2)}{1+F\sin^2(\delta/2)}$$

Transmission

$$\frac{I_t}{I_0} = \frac{1}{1+F\sin^2(\delta/2)}$$

$$F = \frac{4R}{(1-R)^2}$$

Coefficient of finesse (of the slab)

$$\mathscr{F} = \frac{2\pi}{\gamma} = \frac{\pi\sqrt{F}}{2} = \frac{\pi\sqrt{R}}{(1-R)}$$

Finesse (of the fringes)

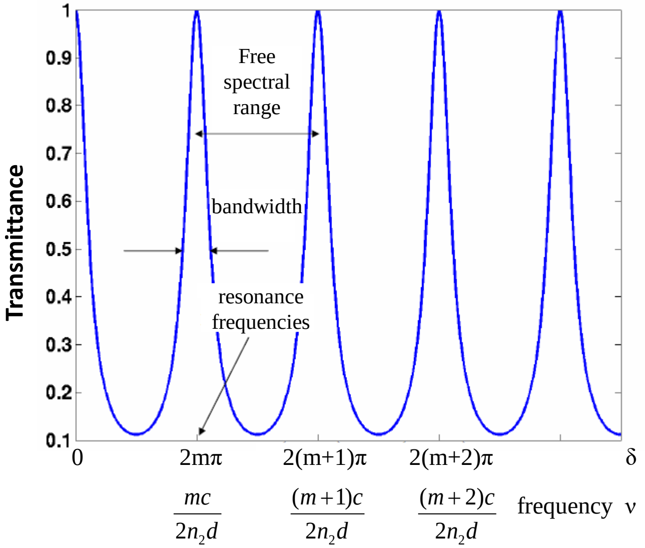
$$\gamma = \frac{4}{\sqrt{F}}$$

FWHM

For $R_1 \neq R_2$ replace $R \rightarrow \sqrt{R_1 R_2}$

Fabry-Perot etalon

$$\delta = \frac{4\pi}{\lambda_0} n_2 d \cos\theta_t \quad \Longrightarrow \quad \text{at normal incidence} \quad \delta = \frac{4\pi}{\lambda_0} n_2 d = 2m\pi \quad \text{maxima}$$



$$\nu = \frac{c}{\lambda_0}$$

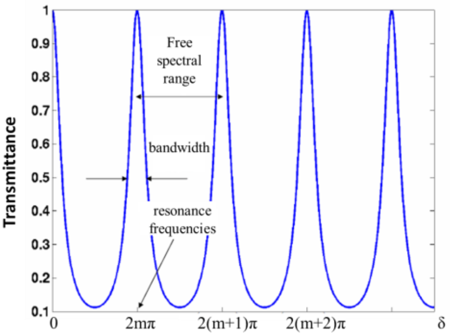
Fabry-Perot etalon

Fabry-Perot bandwidth, $\Delta\nu_{FP}$

$$\delta = \frac{4\pi}{\lambda_0} n_2 d = \frac{4\pi n_2 d}{c} \nu \qquad \nu = \frac{c}{\lambda_0}$$

$$\Longrightarrow \gamma = \Delta\delta = \frac{4\pi n_2 d}{c} \Delta\nu_{FP} = \frac{2\pi}{\mathscr{F}}$$

$$\boxed{\Delta\nu_{FP} = \frac{c}{2n_2 d} \frac{1}{\mathscr{F}}}$$

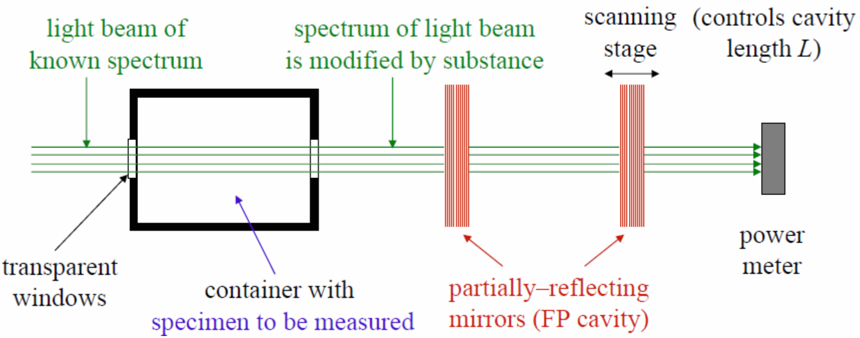


Free spectral range, $\Delta\nu_{FSR}$

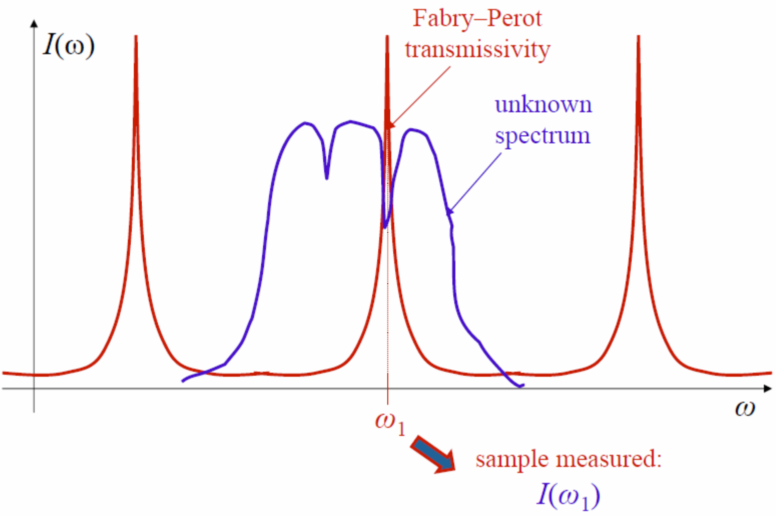
$$\Longrightarrow \quad \Delta\delta = 2\pi$$

$$\boxed{\Delta\nu_{FSR} = \frac{c}{2n_2 d}}$$

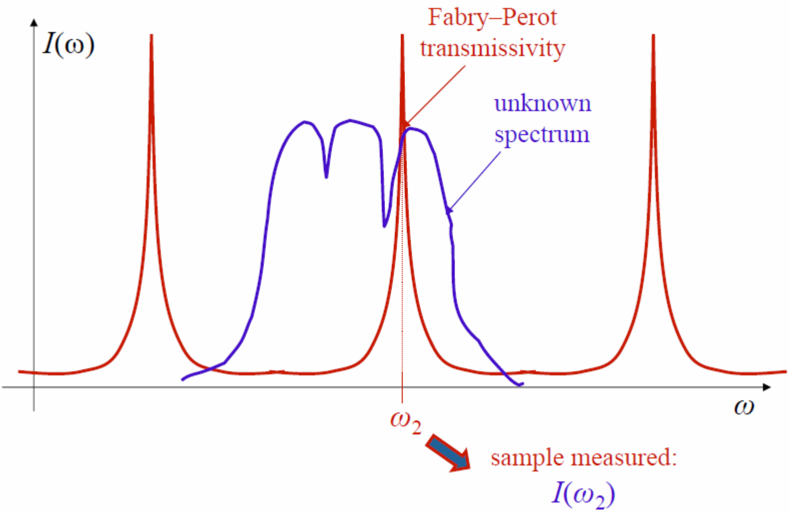
Spectroscopic applications



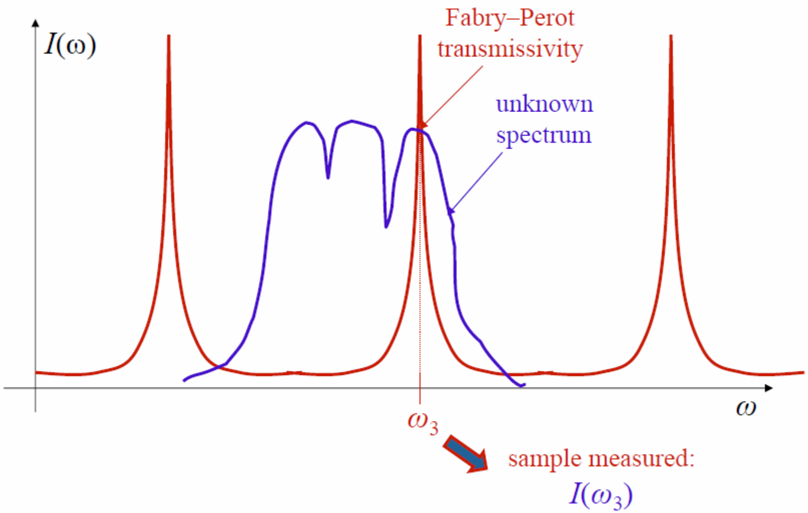
Spectroscopic applications



Spectroscopic applications



Spectroscopic applications

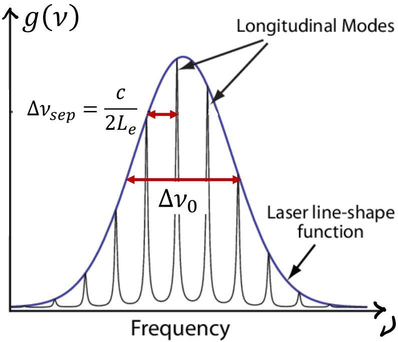


Longitudinal modes selection

To have only one longitudinal mode oscillating in the cavity it has to be:

$$\Delta\nu_{sep} = \frac{c}{2L_e} \geq \frac{\Delta\nu_0}{2}$$

$$\Longrightarrow \quad L_e \leq \frac{c}{\Delta\nu_0}$$



He-Ne laser @ 633 nm $\quad \Delta\nu_0 \cong 1.5\ GHz \quad \Longrightarrow \quad L_e \leq \frac{c}{\Delta\nu_0} = 20\ cm$

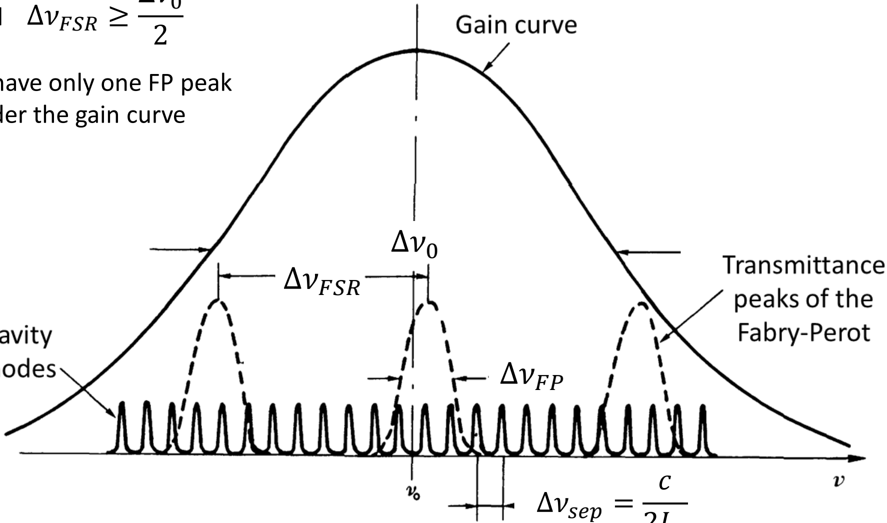
Nd:YAG laser @ 1064 nm $\quad \Delta\nu_0 \cong 120\ GHz \quad \Longrightarrow \quad L_e \leq \frac{c}{\Delta\nu_0} = 2.5\ mm$ **!**

Longitudinal modes selection

If a Fabry-Perot etalon is inserted in the cavity: $\Delta\nu_{sep} = \dfrac{c}{2L_e} \geq \dfrac{\Delta\nu_{FP}}{2} = \dfrac{\Delta\nu_{FSR}}{2\mathscr{F}}$

and $\Delta\nu_{FSR} \geq \dfrac{\Delta\nu_0}{2}$

to have only one FP peak under the gain curve



Longitudinal modes selection

If a Fabry-Perot etalon is inserted in the cavity: $\Delta\nu_{sep} = \dfrac{c}{2L_e} \geq \dfrac{\Delta\nu_{FP}}{2} = \dfrac{\Delta\nu_{FSR}}{2\mathscr{F}}$

and $\Delta\nu_{FSR} \geq \dfrac{\Delta\nu_0}{2}$

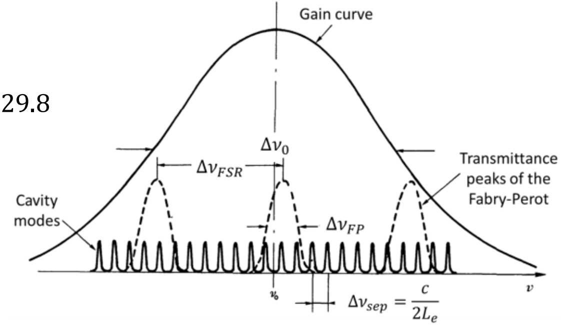
to have only one FP peak under the gain curve $\Longrightarrow$ $\dfrac{\Delta\nu_0}{2} \leq \Delta\nu_{FSR} \leq 2\mathscr{F}\Delta\nu_{sep} = 2\mathscr{F}\dfrac{c}{2L_e}$

$\Longrightarrow \quad L_e \leq 2\mathscr{F}\dfrac{c}{\Delta\nu_0}$

$R = 90\% \qquad \mathscr{F} = \dfrac{\pi\sqrt{R}}{(1-R)} \cong 29.8$

$R = 95\%$

$\mathscr{F} = \dfrac{\pi\sqrt{R}}{(1-R)} \cong 61.2$



Longitudinal modes selection

Ar laser @ 514.5 nm (green line) $\Delta\nu_0 \cong 3.5\ GHz$

without the FP etalon: $L_e \leq \dfrac{c}{\Delta\nu_0} \cong 8.6\ cm$

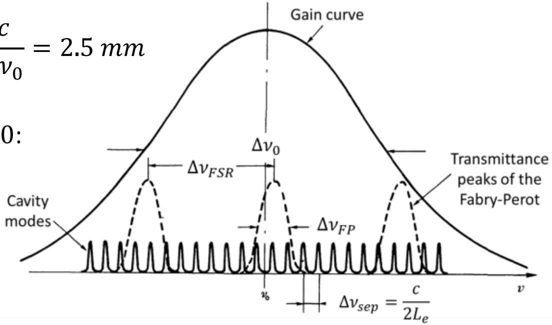
with one FP etalon with $\mathscr{F} = 30$: $L_e \leq 2\,\mathscr{F}\dfrac{c}{\Delta\nu_0} \cong 5.16\ m$ **!**

Nd:YAG laser @ 1064 nm $\Delta\nu_0 \cong 120\ GHz$

without the FP etalon: $L_e \leq \dfrac{c}{\Delta\nu_0} = 2.5\ mm$

with one FP etalon with $\mathscr{F} = 30$:

$$L_e \leq 2\,\mathscr{F}\frac{c}{\Delta\nu_0} \cong 15\ cm$$



If the condition $L_e \leq 2\,\mathscr{F}\dfrac{c}{\Delta\nu_0}$ is not satisfied

$\Longrightarrow$ it is not possible to select a single longitudinal mode with one FP only

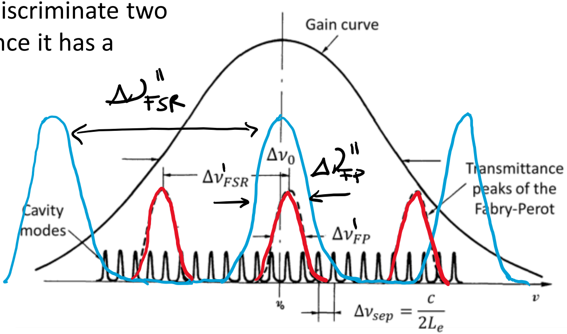
$\Longrightarrow$ two or more FPs are needed

Let's consider the case of 2 FP etalons with the same finesse $\mathscr{F}$ and different thickness.

The **thickest etalon** is used to discriminate two adjacent longitudinal modes since it has a smaller bandwidth:

$$\Delta\nu'_{FP} = \frac{c}{2n_2 d'}\frac{1}{\mathscr{F}}$$

The **thinnest etalon** is used to discriminate two transmission peaks of the first etalon



Longitudinal modes selection

$$\Longrightarrow \quad \Delta\nu_{sep} = \frac{c}{2L_e} \geq \frac{\Delta\nu'_{FP}}{2} = \frac{\Delta\nu'_{FSR}}{2\mathscr{F}}$$

to discriminate two adjacent longitudinal modes

$$\Delta\nu'_{FSR} \geq \frac{\Delta\nu''_{FP}}{2} = \frac{\Delta\nu''_{FSR}}{2\mathscr{F}}$$

to discriminate two adjacent transmission peaks of the first FP with the second FP

$$\Delta\nu''_{FSR} \geq \frac{\Delta\nu_0}{2}$$

to have only one transmission peak of the second FP under the gain curve

$$\Longrightarrow \quad 2\mathscr{F}\Delta\nu_{sep} \geq \Delta\nu'_{FSR} \geq \frac{\Delta\nu''_{FSR}}{2\mathscr{F}}$$

$$(2\mathscr{F})^2\Delta\nu_{sep} \geq \Delta\nu''_{FSR} \geq \frac{\Delta\nu_0}{2}$$

$$\Longrightarrow \quad L_e \leq (2\mathscr{F})^2 \frac{c}{\Delta\nu_0}$$