1. Cross-sections ($R \ll \lambda,\ l = 1$)

$$\left\{\begin{aligned} \sigma_{sca} &= \frac{2\pi}{k^2}\sum_{l=1}^{\infty}(2l+1)\left(|a_l|^2+|b_l|^2\right) \\ \sigma_{ext} &= \frac{2\pi}{k^2}\sum_{l=1}^{\infty}(2l+1)\,\Re e(a_l+b_l) \\ \sigma_{abs} &= \sigma_{ext}-\sigma_{sca} \end{aligned}\right.$$

l = multipolarity
l = 1 dipole
l = 2 quadrupole
l = 3 octupole
…

$$k^2 \equiv \omega^2\varepsilon\mu = \left(\frac{\omega}{c}\right)^2 n^2 = \left(\frac{2\pi}{\lambda_0}\right)^2 n^2$$

$$\left\{\begin{aligned} a_1 &= -i\,\frac{2}{3}(kR)^3\,\frac{\varepsilon-\varepsilon_m}{\varepsilon+2\varepsilon_m} \\ b_1 &= -i\,\frac{1}{45}(kR)^5\,\frac{\varepsilon-\varepsilon_m}{\varepsilon_m} \end{aligned}\right.$$

$$\sigma_{ext} \cong \frac{2\pi}{k^2}\,3\,\Re e(a_1) = \frac{6\pi}{k^2}\,\Re e\left(-i\,\frac{2}{3}(kR)^3\,\frac{\varepsilon-\varepsilon_m}{\varepsilon+2\varepsilon_m}\right)$$

$$\sigma_{ext} = 9\,\frac{\omega}{c}\,\varepsilon_m^{\ 3/2}\,\mathrm{V}\,\frac{\varepsilon_2}{(\varepsilon_1+2\varepsilon_m)^2+(\varepsilon_2)^2}$$

2. Internal field ($R \ll \lambda,\ l = 1$)

$$\bar{E}_s = E_0 e^{-i\omega t} \sum_{l=1}^{\infty} i^l \frac{2l+1}{l(l+1)} (i a_l \bar{N}_{el1} - b_l \bar{M}_{ol1}) \cong E_0 e^{-i\omega t} \frac{3}{2} (-a_1 \bar{N}_{e11} - i b_1 \bar{M}_{o11})$$

$$\bar{E}_t = E_0 e^{-i\omega t} \sum_{l=1}^{\infty} i^l \frac{2l+1}{l(l+1)} (c_l \bar{M}_{ol1} - i d_l \bar{N}_{el1}) \cong E_0 e^{-i\omega t} \frac{3}{2} (d_1 \bar{N}_{e11} + c_1 \bar{M}_{o11})$$

$$\bar{E}_{in} = \bar{E}_t = \frac{3\varepsilon_m}{\varepsilon + 2\varepsilon_m} \bar{E}_0 = f_e \bar{E}_0$$

Local field enhancement

$$\varepsilon_1 + 2\varepsilon_m = 0$$

Resonance condition (Frölich)

Perfect agreement with the quasi-static case

$$\left\{ \begin{array}{l} \alpha = 4\pi R^3 \dfrac{\varepsilon - \varepsilon_m}{\varepsilon + 2\varepsilon_m} \xrightarrow{\omega \to 0} \alpha_{stat} = 4\pi R^3 \\ \varepsilon_1(\omega) \xrightarrow{\omega \to 0} -\infty \end{array} \right.$$

Perfect conductor

For metals

3. Scattering Rayleigh ($R \ll \lambda$)

$$\sigma_{sca} = \frac{2\pi}{k^2}\sum_{l=1}^{\infty}(2l+1)\left(|a_l|^2 + |b_l|^2\right)$$

If $R \ll \lambda$ then $kR \ll 1$

$$\begin{cases} a_1 = -i\,\dfrac{2}{3}(kR)^3\,\dfrac{\varepsilon - \varepsilon_m}{\varepsilon + 2\varepsilon_m} \\ b_1 = -i\,\dfrac{1}{45}(kR)^5\,\dfrac{\varepsilon - \varepsilon_m}{\varepsilon_m} \end{cases}$$

$$\sigma_{sca,1} = \frac{6\pi}{k^2}\left(|a_1|^2 + |b_1|^2\right) \approx \frac{6\pi}{k^2}|a_1|^2 =$$

$$= \frac{8\pi}{3}k^4R^6\left|\frac{\varepsilon - \varepsilon_m}{\varepsilon + 2\varepsilon_m}\right|^2 = \frac{8\pi}{3}\left(\frac{\omega}{c}\right)^4\varepsilon_m^2R^6\left|\frac{\varepsilon - \varepsilon_m}{\varepsilon + 2\varepsilon_m}\right|^2$$

if $\left|\dfrac{\varepsilon - \varepsilon_m}{\varepsilon + 2\varepsilon_m}\right|$ is weakly dependent on λ (not for metals…)

$$\sigma_{sca,1} \approx \omega^4$$

Why is the sky blu?

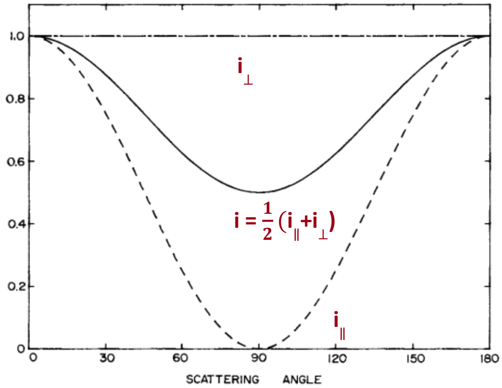
3. Scattering Rayleigh ($R \ll \lambda$)

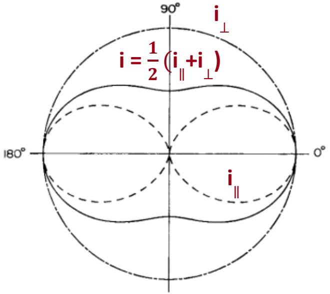
Normalized angular distribution vs. polarization

$$i_{\|} = \frac{9}{4}\frac{|a_1|^2}{k^2 r^2}\cos^2\theta$$

$$i_{\perp} = \frac{9}{4}\frac{|a_1|^2}{k^2 r^2}$$

$$i = \frac{1}{2}\left(i_{\|} + i_{\perp}\right)$$





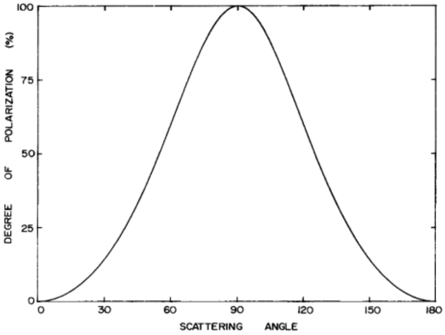
$i_{\|}$ Polar. ∥ to the scattering plane (p o TM)

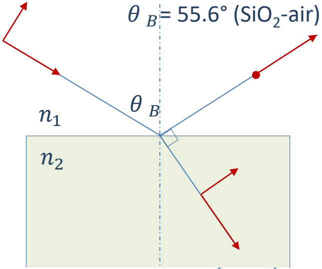
$i_{\perp}$ Polar. ⊥ to the scattering plane (s o TE)

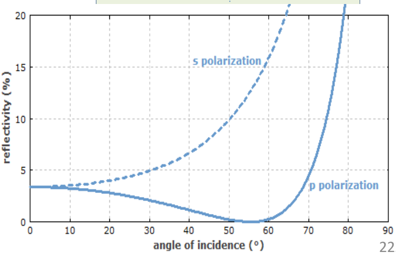
3. Scattering Rayleigh ($R \ll \lambda$)

Scattering-induced polarization of the light

$$\theta_B = arctan\left(\frac{n_2}{n_1}\right)$$

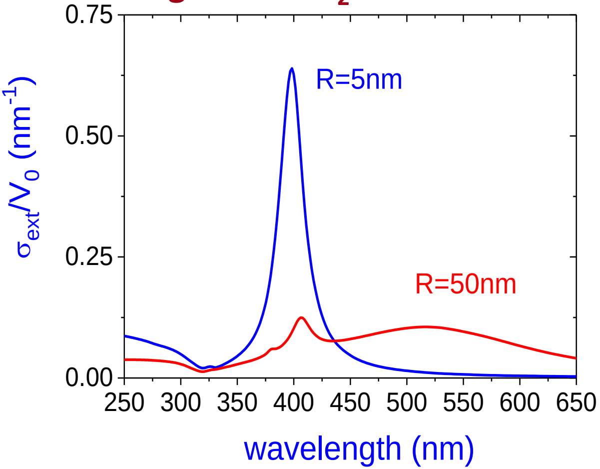






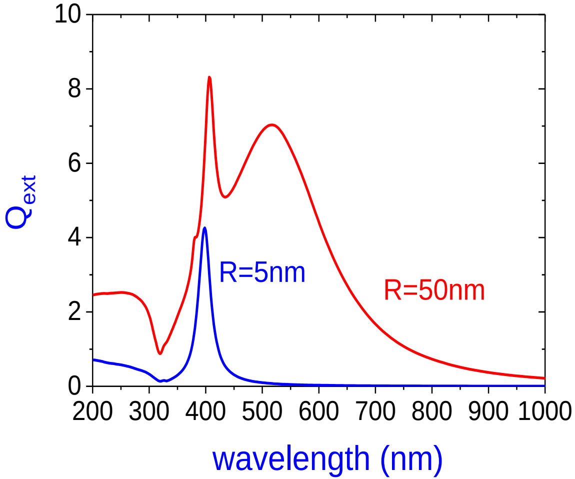
Similar to the Brewster angle for planar interfaces

Ag NP in SiO_2 – far-field



λ = 520 nm λ = 406 nm λ = 380 nm

Ag NP in SiO_2 – far-field (eff.)

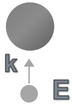


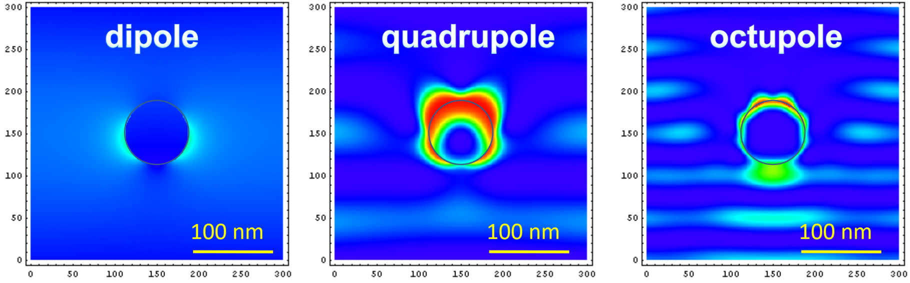
λ = 520 nm

λ = 406 nm

λ = 380 nm

Ag NP in SiO_2 – near-field





λ = 520 nm   λ = 406 nm   λ = 380 nm

1. L-SPR Physics

Size-dependent dielectric function

$$\varepsilon = \varepsilon(\omega, R)$$

What is the color of a Au NC ?