

# 1 Lecture 5

## Slide 1

**Fresnel's formulas**

Boundary conditions: continuity of tangential components at the interface

<b>TE (s) polarization</b> $\begin{cases} E_i + E_r = E_t \\ -B_i \cos\theta_i + B_r \cos\theta_r = -B_t \cos\theta_t \end{cases}$	<b>Fresnel's reflection and transmission coefficients</b> $r_s = \left( \frac{E_r}{E_i} \right)_{TE} \quad t_s = \left( \frac{E_t}{E_i} \right)_{TE}$
<b>TM (p) polarization</b> $\begin{cases} B_i - B_r = B_t \\ E_i \cos\theta_i + E_r \cos\theta_r = E_t \cos\theta_t \end{cases}$	
$r_p = \left( \frac{E_r}{E_i} \right)_{TM} \quad t_p = \left( \frac{E_t}{E_i} \right)_{TM}$	
$\theta_i = \theta_r \quad B = \frac{E}{v} = \frac{E}{c} n$	

We want to see what happens when TE and TM are impinging in the interface between two media for the amplitude.

Firstly, we have to consider the boundary condition at the interface assuming continuity for the tangential component.

We have also the reflection law and the relation between electric and magnetic field amplitudes.

If we solve this system we can determine the **Fresnel's reflection and transmission coefficients**.

## Slide 2

**Fresnel's formulas**

**Reflection coefficients**

$$r_s = \left( \frac{E_r}{E_i} \right)_{TE} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{\cos\theta_i - n \cos\theta_t}{\cos\theta_i + n \cos\theta_t}$$

$$r_p = \left( \frac{E_r}{E_i} \right)_{TM} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{-n \cos\theta_i + \cos\theta_t}{n \cos\theta_i + \cos\theta_t}$$

**Transmission coefficients**

$$t_s = \left( \frac{E_t}{E_i} \right)_{TE} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{2 \cos\theta_i}{\cos\theta_i + n \cos\theta_t}$$

$$t_p = \left( \frac{E_t}{E_i} \right)_{TM} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{2 \cos\theta_i}{n \cos\theta_i + \cos\theta_t}$$

You end up with different equations for **reflection coefficients** and **transmission coefficients**. All the different ways in which you can write these coefficients is related to the way in which you can write the trigonometric formulas.

## Slide 3

**Fresnel's formulas**

**Snell's law**

$$n = \frac{n_2}{n_1} = \frac{\sin\theta_i}{\sin\theta_t}$$

$$r_s = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \quad t_s = \frac{2 \cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t)}$$

$$r_p = -\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \quad t_p = \frac{2 \cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)}$$

At normal incidence ( $\theta_i = \theta_r = \theta_t = 0$ ):

$$r_s = r_p = \frac{E_r}{E_i} = \frac{1-n}{1+n} \quad t_s = t_p = \frac{E_t}{E_i} = \frac{2}{1+n}$$

$r < 0$  ( $n > 1$ , es. from air to glass) indicates that the phase of the reflected wave is varied of  $180^\circ$  with respect to the incident wave.

The reflections and transmission coefficients can be rewritten by applying also the Snell's law.

Let us see what are the expressions of the coefficients at normal incidence. The two polarization states are degenerate so we should expect the same coefficients for TE and TM.

The reflection coefficients could be positive or negative according to the value of  $n$ . A negative reflection coefficients means that the phase of your reflected wave is  $180^\circ$  wrt the incident beam.

## Slide 4

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Slightly increasing the intensity of a laser beam incident on a glass window, where will the window get damaged earlier (on the front or on the back)?

$$r = \frac{1-n}{1+n} = \frac{1-(1/1.5)}{1+(1/1.5)} = 0.2 \quad E_r = r E_i$$

$$I = |E_i + E_r|^2 = |(1+r)E_i|^2 = (1.2)^2 I_i = 1.44 I_i \quad 44\% \text{ higher!}$$

The last thing we note ( $r < 1$ ) has interesting consequences. Let us imagine to be able to increase the intensity of a laser beam incident on a glass window. Which side of the window will be damaged earlier?

The answer is the **backside**, this is because of the phase shift. The phase shift in the front surface is  $180^\circ$  (incidence wave and reflected wave are out of phase). In the back surface they are in phase. If you calculate the intensity of the beam on the back surface, you obtain that it is 44% higher than for the front side!

## Slide 5

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### Reflectance (R)

$$I = \frac{1}{2} n \epsilon_0 c |E|^2$$

$$R = \frac{I_r A_r}{I_i A_i} = \left| \frac{E_r}{E_i} \right|^2 = |r|^2$$

$$R_s = \left| \frac{E_r}{E_i}_{TE} \right|^2 = |r_s|^2 \quad R_p = \left| \frac{E_r}{E_i}_{TM} \right|^2 = |r_p|^2$$

At normal incidence ( $\theta_i = \theta_r = \theta_t = 0^\circ$ )

$$R_s = R_p = R = \left( \frac{1-n}{1+n} \right)^2$$

From air ( $n_1 = 1$ ) to glass ( $n_2 = 1.5$ ):

$$R = 0.04 \text{ (4%)}$$

At grazing incidence ( $\theta_i \sim 90^\circ$ )

$$R_s = R_p = R = 1$$

R is independent of n.

**Reflectance** is the ratio between the power of the reflected beam wrt the power of the incidence beam. Since we are in the same medium the angle of incidence and refraction are the same, the **footprint** of the incidence beam will be the same of the reflected beam, so you can cancel the area term in the reflectance. So that you obtain  $R = |r|^2$ .

We have to distinguish the reflectance for TE and TM. At normal incidence, for  $n_1 = 1$  and  $n_2 = 1.5$ , we obtain  $R = 0.04$ . This means that at the interface between air and common glass the percentage of the incidence power that is reflected is about 4 %. So you will lose 4 % of the incidence beam.

At grazing incidence, it is easy to calculate that the reflectance is independent on  $n$ . So, for any medium, if you look close to  $90^\circ$ , will have  $R$  of the order of 1, so it would be highly reflecting.

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### Transmittance (T)

$$I = \frac{1}{2} n \epsilon_0 c |E|^2$$

$$T = \frac{I_t A_t}{I_i A_i} = \frac{n_2}{n_1} \left| \frac{E_t}{E_i} \right|^2 = \frac{w_t}{w_i} = n |t|^2 \cos \theta_t / \cos \theta_i$$

At normal incidence ( $\theta_i = \theta_r = \theta_t = 0^\circ$ )

$$T_s = T_p = T = n \left( \frac{2}{1+n} \right)^2$$

From air ( $n_1 = 1$ ) to glass ( $n_2 = 1.5$ ):  $T = 0.96 \text{ (96%)}$

For any angle of incidence:

$$R + T = 1$$

The **Transmittance** coefficient is the power of the transmitted beam divided by the power of the incidence beam. Since we are passing from media with different refractive index, the footprint (beam diameter) of the beam will be different (they follow the Snell's law).

At normal incidence, for  $n_1 = 1$  and  $n_2 = 1.5$  you will get  $T = 0.96$ .

At any angle of incidence we have to satisfy the **conservation of energy**:

$$R + T = 1$$

because we are considering media which are not absorbing.

If we have a stack of optical elements, we have to be aware that we have a portion of the energy that can be reflected that it is not negligible!

## Slide 7

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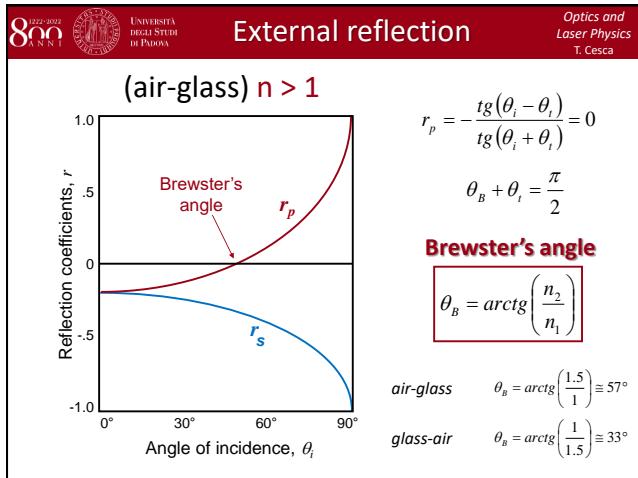
At night the windows seem mirrors when you are in an illuminated room!

$I_{in} >> I_{out}$

$R = 8\% \quad T = 92\%$

This happens because at the interface of the window you will get that a portion of the light inside is reflected and transmitted outside and the same for the light outside. But, in an illuminated room, the internal light reflected by the windows will be much larger than the light transmitted from outside. This is why the window look like a mirror at the night.

## Slide 8

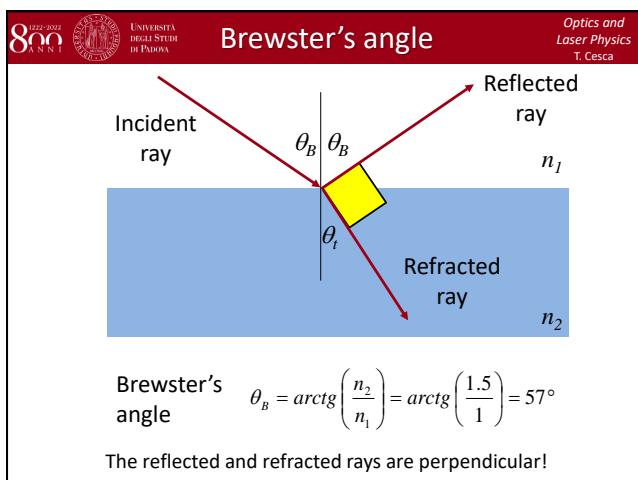


Let us look at the reflection coefficient  $r$  for the two polarization state as a function of the angle of incidence. The reflection coefficient for p polarization has a specific angle for which it goes to zero. This is called **Brewster's angle**: coefficient for which the  $r_p$  goes to zero.

We talk about **external reflection** when we are passing from a medium with a smaller refractive index toward a medium with a larger one.

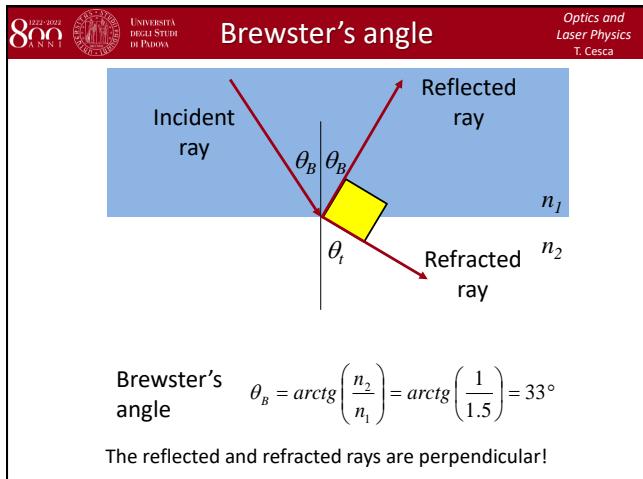
The Brewster angle can be observed also for an **internal reflecting** (from a larger to a smaller refractive index). The Brewster's angle is the angle for which the sum with the transmitted angle is equal to  $\pi/2$ .

## Slide 9

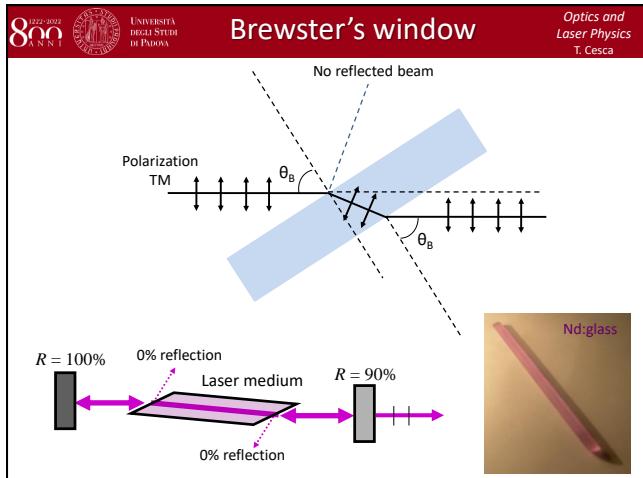


At Brewster, the refracted ray and the reflected are perpendicular! This happens both for external and internal reflection.

## Slide 10



## Slide 11

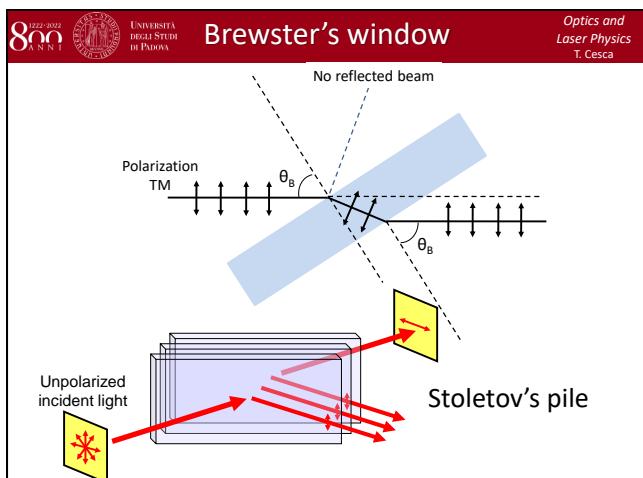


The concept of Brewster's angle is important because if you are impinging with TM polarization at  $\theta_B$ , the reflection coefficient is 0 (you have no reflection of the beam). This is a way to get negligible reflection.

It is easy to demonstrate that you will get no reflection also at the second interface!

This is widely used in laser media. This is a simple sketch of a laser cavity (solid state laser). The surfaces are cut at Brewster angle. In a laser road the beam will pass thousand of times in order to have amplification. If the reflectance is not null you will use a great amount of energy, so you have to minimize the losses due to reflection. This could be done in two ways: by using low reflectance coatings (but the problem is that they can be damaged easily) or place all the optical elements at the Brewster's angle.

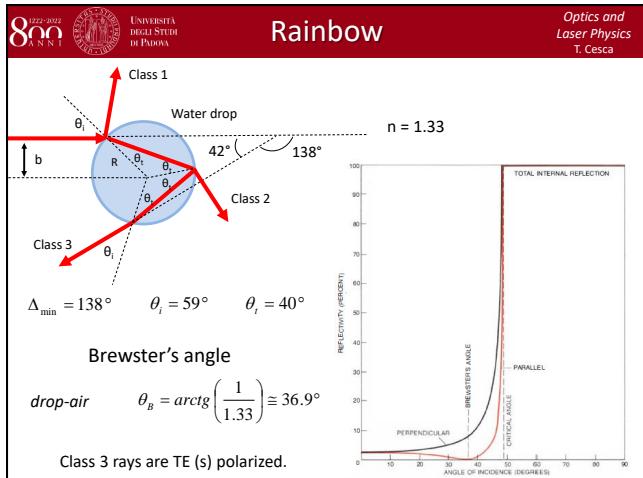
## Slide 12



This idea can be used also for a polarizer. Let us consider a stack of parallel plates. Let us consider an unpolarized incident beam (superposition of both waves polarized p and s) at Brewster's angle, you have no reflection for p, so the reflected beam will be now polarized s. So, we will have an electric field which is parallel to the interface (or perpendicular to the plane of incidence).

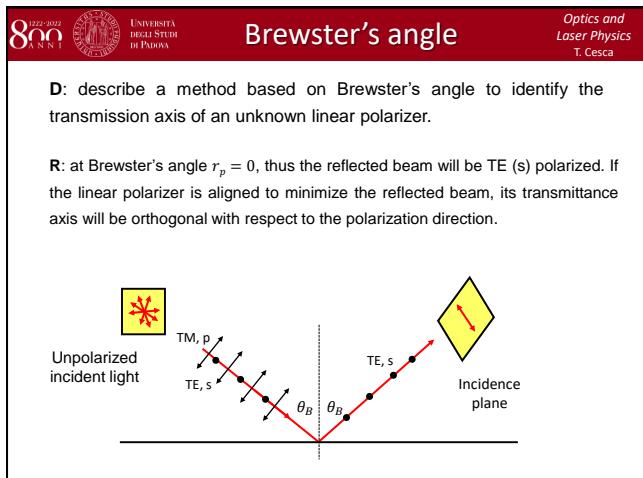
If you use a stack, every time of the reflection, the transmitted beam will be progressively more and more polarized p (**Stoletov's pile**).

## Slide 13



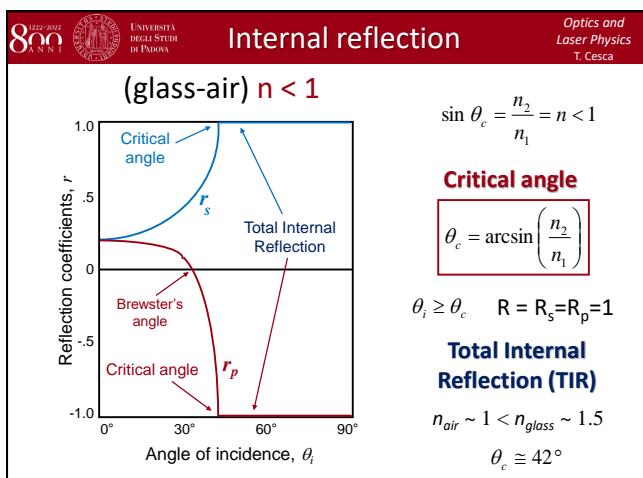
The light coming from the rainbow is highly polarized. For the primary arch, the angle of minimum deviation is  $138^\circ$  which corresponds to an angle of incidence of  $59^\circ$  and transmitted of  $40^\circ$ . You will see that the Brewster's angle is around  $36.9^\circ$ . This means that the beam is ray is coming in a condition of transmitted angle very close to the Brewster's angle. This means that the p component will not be reflected and the ray that is coming out will be s polarized.

## Slide 14



Let us make an exercise. When you are impinging with unpolarized light at a Brewster's angle, the reflected ray is s polarized. So, if you align your linear polarizer in order to minimize the reflected beam, this means that its transmission axis will be orthogonal to the polarization direction (parallel to the plane of incidence).

## Slide 15



We have that

- external reflection:  $n > 1$
- internal reflection:  $n < 1$

In the case of internal reflection, we have again the Brewster's angle but also it is interesting to note that we have a **critical angle**. The larger is the incidence angle, the more the refracted beam is going far away from the normal at the interface. You will end up in a condition in which the transmitted beam will be parallel to the interface. This is possible only when the relative refractive index is smaller than one.

At angle larger than  $\theta_c$ , we are in the **total internal reflection (TIR) condition**.

## Slide 16

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The diagram illustrates the critical angle at an interface between two media with refractive indices  $n_1$  and  $n_2$ . An incident ray enters from the left at an angle  $\theta_i$  relative to the normal. At the critical angle  $\theta_c$ , the refracted ray is parallel to the interface. For  $\theta_i > \theta_c$ , the refracted ray is reflected back into the first medium as a total internal reflection.

$$\text{Critical angle } \theta_c = \arcsin\left(\frac{n_2}{n_1}\right) = \arcsin\left(\frac{1}{1.5}\right) = 42^\circ$$

The refracted ray will be parallel to the interface.

## Slide 17

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$$n = \frac{n_2}{n_1} < 1 \quad \vartheta_i > \vartheta_c \quad \text{Critical angle}$$

$$\sin^2 \vartheta_t + \cos^2 \vartheta_t = 1 \quad \sin \vartheta_i = n \sin \vartheta_t$$

$$r_s = \frac{\cos \vartheta_i - n \cos \vartheta_t}{\cos \vartheta_i + n \cos \vartheta_t} = \frac{\cos \vartheta_i - i\sqrt{\sin^2 \vartheta_i - n^2}}{\cos \vartheta_i + i\sqrt{\sin^2 \vartheta_i - n^2}} = e^{i\phi_s}$$

$$r_p = \frac{-n \cos \vartheta_i + \cos \vartheta_t}{n \cos \vartheta_i + \cos \vartheta_t} = \frac{-n^2 \cos \vartheta_i + i\sqrt{\sin^2 \vartheta_i - n^2}}{n^2 \cos \vartheta_i + i\sqrt{\sin^2 \vartheta_i - n^2}} = -e^{i\phi_p}$$

$$R = R_s = R_p = |r_s|^2 = |r_p|^2 = 1 \quad \text{Total Internal Reflection}$$

For angles equal than the critical angle, the modulus of the reflection coefficients is 1 for both  $r_s$  and  $r_p$ . But for  $\vartheta_i > \vartheta_c$ , the coefficients become complex numbers.

## Slide 18

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A photograph of a sea turtle swimming in clear blue water above a coral reef. The turtle's body is partially submerged, and its head is above the surface, demonstrating how light rays travel from the sky through the water to form an image.

This is a nice example of total internal reflection, when you look at the sky from water.

## Slide 19

**800 ANNI** UNIVERSITÀ DEGLI STUDI DI PADOVA **Applications of TIR** Optics and Laser Physics T. Cesca

The slide shows four applications of TIR:

- Beam steers**: Two diagrams showing light being reflected off a prism at 45° angles.
- Porro's prisms**: A diagram of a binocular lens system using two prisms to invert the image.
- Optical fibers**: A photograph of a bundle of optical fibers with light points at the ends, and a schematic showing light traveling through a core with cladding, reflecting off the boundary at an angle greater than the critical angle  $\theta_c$ .
- Light path through a prism**: A diagram showing light entering a prism at an angle  $\theta_0$ , reflecting off the lateral face at an angle  $\theta_1$  (labeled "no TIR"), and reflecting off the bottom face at an angle  $\theta_2$  (labeled "TIR"). The angle  $\theta_2 = 90^\circ$  is labeled as "TIR".

There are several applications of this phenomenon. If you have a glass prism and you cut it at 45° (in the middle) and if you impinge perpendicular to the face, when you reach the lateral face you are impinging at an angle larger than the critical one, so the beam will be totally reflected.

This idea is used also for **Porro's prism** for binoculars, for seeing the image not reversed (in the right orientation). Another situation is in **optical fibers**.

## Slide 20

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**Snell's law**

$$n_0 \sin \theta_0 = n_1 \sin \theta_1 = n_1 \cos \theta_1^{\text{int}} = n_1 \sqrt{1 - (\sin \theta_1^{\text{int}})^2} = n_1 \sqrt{1 - \left( \frac{n_2 \sin \theta_2}{n_1} \right)^2} = n_1 \sqrt{1 - \left( \frac{n_2}{n_1} \right)^2} = \sqrt{n_1^2 - n_2^2}$$

$\theta_2 = 90^\circ$  (TIR)

**Optical fibers**

$n_1 = n_{\text{core}} > n_{\text{cladding}} = n_2$

$\theta_0$  = acceptance angle (Numerical Aperture, NA)

$$\sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

High index contrast ( $n_1/n_2$ ): high NA

The slide shows a diagram of an optical fiber with labels for Cladding, Core, and Buffer Coating. It includes a schematic of light entering from the left at an angle  $\theta_0$  and reflecting off the boundary between the core and cladding at an angle  $\theta_1$ . The angle  $\theta_2 = 90^\circ$  is labeled as "TIR". Below this, another schematic shows light entering at an angle  $\theta_0$  and reflecting off the boundary at an angle  $\theta_1$  (labeled "no TIR") and  $\theta_2$  (labeled "TIR").

You can calculate the **acceptance angle** (maximum angle in which you can enter for obtaining totally internal reflection).

## Slide 21

**800 ANNI** UNIVERSITÀ DEGLI STUDI DI PADOVA **Frustrated Total Internal Reflection (FTIR)** Optics and Laser Physics T. Cesca

The diagram illustrates FTIR with three parts:

- A prism in air (labeled air) with a gap between two faces. Light enters at angle  $\theta_i$  and reflects off the bottom face at angle  $\theta_r$ . Detectors D1 and D2 are shown. An arrow points to "to DMM".
- A prism with a gap, similar to (a), but with a red arrow pointing to the gap, indicating no reflection.
- A prism with a gap filled with index matching fluid, allowing light to pass through the gap.

Below the diagrams are three fluorescence images labeled (d), (e), and (f), showing light paths and reflections within the prisms.

Let us consider a glass prism and another prism of the same material with a gap. If you close completely the gap (you consider a unique block of material), you will not get reflection anymore. You use **index matching fluid** to frustrate total internal reflection.

## Slide 22

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Angle of incidence > critical angle for TIR

Where the two surfaces are in contact the total internal reflection is frustrated.

This is what is used to realize readers of fingerprints. Let us consider a glass and a beam coming from inside the glass. If you have a roughness in the material (air inside), when you are in contact you are frustrating total internal reflection, while when you have air you get total internal reflection.

## Slide 23

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### Sensors of fingerprints

Right angle prism

laser

See TIR from a fingerprint valley and FTIR from a ridge.  
This works because the ridges are higher than the evanescent wave penetration.

With this idea we can make sensors of fingerprints. Our skin is a medium with a refractive index close to glass, and is a rough surface. When we put our finger in contact with the sensors, there is a laser light and we can see that there will be positions in which we will have total internal reflection (white lines) and frustration (black lines).

## Slide 24

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TM (p): horizontal  
TE (s): vertical

Incidence plane: horizontal plane  $\begin{pmatrix} A \\ B \end{pmatrix}$  Jones vector

**Reflection matrix**  $M_R = \begin{pmatrix} -r_p & 0 \\ 0 & r_s \end{pmatrix}$       **Transmission matrix**  $M_T = \begin{pmatrix} t_p & 0 \\ 0 & t_s \end{pmatrix}$

$$\begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} -r_p & 0 \\ 0 & r_s \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} \quad \begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} t_p & 0 \\ 0 & t_s \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

At grazing incidence:  $M_R = \pm \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  (+): internal reflection,  $n < 1$   
(-): external reflection,  $n > 1$

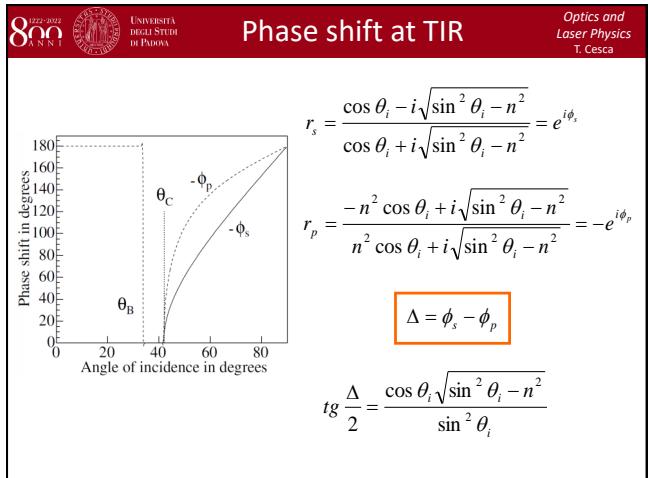
At TIR:  $\begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} e^{i\phi_p} & 0 \\ 0 & e^{i\phi_s} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = e^{i\phi_p} \begin{pmatrix} A \\ Be^{i\Delta} \end{pmatrix}$   $\Delta = \phi_s - \phi_p$   
**dephasing**

At TIR the reflected wave is typically elliptically polarized!

We can also apply Jones's notation for TE and TM and create what are called **reflection and transmission matrices**.

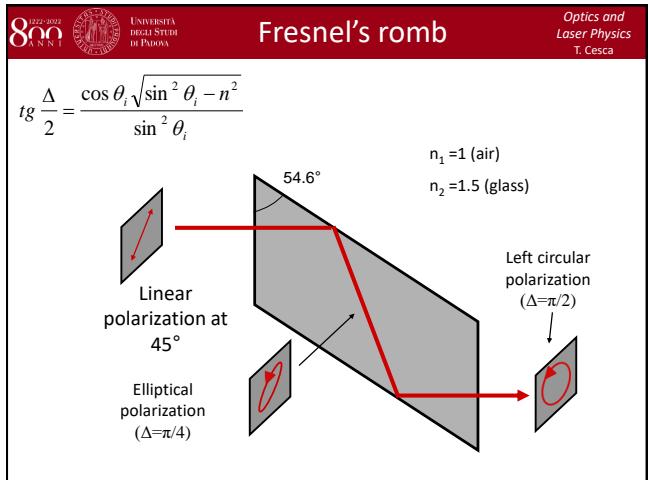
At grazing incidence the reflection matrix becomes simpler.

## Slide 25



The phase shift is related to the angle of incidence and to the relative refractive index.

## Slide 26



We can construct a polarizer. We have a romb of glass with refractive index 1.5 (the environment is air). We impinge linearly with a linear polarized beam (same amplitude) at 45°. You will get a phase shift of  $\pi/4$ . If the romb is long enough in order to get another phase shift, the outgoing beam will be dephased wrt the incidence polarization. You get a circularly polarized beam (same amplitude but with a phase shift).