Respuestas ejercicios propuestos para LaTex

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4 de agosto de 2023

1. Primer punto

Utilice los entornos `\array` o `\pmatrix` para recrear la siguiente matrix:

$$\begin{pmatrix} F[1,1] & \cdots & F[1,m] \\ \vdots & \ddots & \vdots \\ F[n,1] & \cdots & F[n,m] \end{pmatrix}$$

Tenga en cuenta que para los puntos puesto en anterior matrix los código dados fueron los siguientes:

```
\cdots   Horizontales (centados)
\ldots  Horizontales (al piso)
\vdots  Verticales
\ddots  Diagonales
```

2. Segundo Punto

Recrear exactamente el extracto dado.

Ejercicios sobre coordenadas cilíndricas

La ecuación del cardiole está dada por $r = k(1 + cos\theta)$ por lo tanto:

$$\dot{r} = -k\ddot{\theta}sen(\theta)$$
$$\ddot{r} = -k(\dot{\theta}^2 cos\theta + \ddot{\theta}sen\theta)$$

En coordenadas polares la velocidad de la partícula está dada por la expresión:

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

Como la rapidez es constante entonces

$$V_0^2 = \dot{r}^2 + r^2\dot{\theta}^2 = 2k^2\dot{\theta}^2(1 + cos\theta)$$
$$\Longrightarrow \boxed{\dot{\theta} = \tfrac{V_0}{\sqrt{2k^2(1+cos\theta)}} = \tfrac{V_0}{\sqrt{2kr}}}$$

Para calcular la aceleración radial $a_r = \ddot{r} - r\ddot{\theta}^2$ necesitamos calcular $\ddot{\theta}$

$$\dot{\theta} = \frac{V_0}{\sqrt{2kr}} \Longrightarrow \ddot{\theta} = -\frac{V_0\dot{r}}{2r\sqrt{2kr}} = \frac{ksen\theta}{2r}\dot{\theta}^2 = \frac{V_0^2 sen\theta}{4r^2}$$

Luego

$$a_r = \ddot{r} - r\dot{\theta}^2 = -kcos\theta\dot{\theta}^2 - ksen\theta\ddot{\theta} - r\dot{\theta}^2$$
$$\Longrightarrow a_r = -k\dot{\theta}^2[cos\theta + \frac{sin^2\theta}{2(1 + cos\theta)} + (1 + cos\theta)]$$
$$a_r = -k\dot{\theta}^2[\frac{2cos\theta + 2cos^2\theta + sen^2\theta + 2 + 4cos\theta + 2cos^2\theta}{2(1 + cos\theta)}]$$
$$a_r = \frac{-k\dot{\theta}^2}{2(1 + cos\theta)}(4cos^2\theta + sen^2\theta + 6cos\theta + 2) = \frac{-3k\dot{\theta}^2}{2(1 + cos\theta)}(1 + cos\theta)^2$$

$$\boxed{a_r = -\tfrac{3}{4}\tfrac{V_0^2}{k}}$$

3. Tercer Punto

Reproduzca el siguiente extracto de documento usando LaTex.

Let θ, β be $3x3$ skew-symmetric matrices and σ be a $3x3$ matrix. Find symmetric S, T such that:

$$(S-\theta)(T-\beta)=\sigma$$

After all previous consideratios, to find one solution we assume that T is diagonal.

Denote R= $\begin{pmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{pmatrix}$, A= $\begin{pmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \\ m_7 & m_8 & m_) \end{pmatrix}$, Q= $\begin{pmatrix} 0 & q_1 & q_2 \\ -q_1 & 0 & q_3 \\ -q_2 & -q_3 & 0 \end{pmatrix}$, T= $\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$.

Then we have TRT= $\begin{pmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{pmatrix}$, $A^\dagger T = \begin{pmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{pmatrix}$

Then we have the explicit form of the equation:

$$cxy + m_4 y - m_2 x = q_1 \tag{1}$$

$$bxz + m_7 z - m_3 x = q_2 \tag{2}$$

$$ayz + m_8 z - m_6 y = q_3 \tag{3}$$

This system of equations is solved by eliminate z (by (2) and (3)) then calculate y from x (by the identity of xy). Then we are left with a quadratic equation of x.
Have x we can solve y, z.
The explicit solution is obtainable but not worth calculated by hand.

4. Punto 4

Here are the formulas for counting in various ways:

	No Repetiton	Repetition Allowed
Not sensitive to order	$\binom{n}{r}$	$\left(\binom{n}{r}\right) = \binom{n+r-1}{r}$
Sensitive to order	$P(n, r)$	n^r

Cuadro 1: Primera tabla de la tarea :D

Here are examples to demonstrate:

	No Repetiton	Repetition Allowed
Not sensitive to order	5 distinct books choose 3 books to take home $\binom{5}{3}$	unlimited copies of 5 books choose 3 copies to take home $\left(\binom{5}{3}\right) = \binom{5+3-1}{3}$
Sensitive to order	5 distinct books give one to person A, B, and C $P(5, 3)$	unlimited copies of 5 books give one to person A, B, and C 5^3

Cuadro 2: Segunda tabla de la tarea :D

	No Repetiton	Repetition Allowed
Not sensitive to order	5 distinct books choose 3 books to take home $\binom{5}{3}$	unlimited copies of 5 books choose 3 copies to take home $\left(\binom{5}{3}\right) = \binom{5+3-1}{3}$
Sensitive to order	5 distinct books give one to person A, B, and C $P(5, 3)$	unlimited copies of 5 books give one to person A, B, and C 5^3

Cuadro 3: Segunda tabla de la tarea :D

And here are some more formulas:

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$$\binom{n}{r} = \binom{n}{n-r}$$

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

Fin

5. Punto Tarea Revelo

Reproduzca el documento a continuación.

Exercise 1: Sum rule for the Energy

- Show that the expectation value of the total kinetic energy in the ground state of a many-body fermion system is given by

$$\langle T\rangle = -i\int d\vec{x}\lim_{\vec{x'}\to\ \vec{x}}(-\frac{\hbar^2\nabla^2}{2m})\ \ Tr[G(\vec{x},t;\vec{x'},t^+)] \tag{4}$$

  where we consider the trapping potential $U(\vec{x})=0$, and $\hat{T}=-\frac{\hbar^2\nabla^2}{2m}$.

- Show that the total ground state energy is given by

$$\begin{aligned} E &= \langle\hat{H}\rangle \\ &= \langle\hat{T}+\hat{V}\rangle \\ &= \langle\hat{T}\rangle+\langle\hat{V} \\ &= -\tfrac{i}{2}\int d\vec{x}\lim_{\vec{x'}\to\ \vec{x}}(i\hbar\tfrac{\partial}{\partial t}-\tfrac{\hbar^2\nabla^2}{2m})\ \ Tr[G(\vec{x},t;\vec{x'},t^+)] \end{aligned} \tag{5}$$

Hint: To evaluate the mean value of potential energy

$$\langle\hat{V}\rangle = \frac{\langle\psi_0|\frac{1}{2}\int d\vec{x}\int d\vec{x'}\hat{\psi}^\dagger_\beta(\vec{x})\hat{\psi}^\dagger_\alpha(\vec{x'})V(|\vec{x}-\vec{x'}|)\hat{\psi_\alpha}(\vec{x'})\psi_\beta(\vec{x})|\psi_0\rangle}{\langle\psi_0|\psi_0\rangle} \tag{6}$$

use the result obtained in exercise 2, set 9:

$$\left[i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2\nabla^2}{2m}\right]\hat{\psi}_{H\alpha}(\vec{x},t)=\int d\vec{x'}\hat{\psi}^\dagger_{H\beta}(\vec{x}',t)V(|\vec{x}-\vec{x}'|\hat{\psi}_{H\beta}(\vec{x}',t)\hat{\psi}_{H\alpha}(\vec{x},t) \tag{7}$$

6. Punto 5

DRIVLE'S THEOREM AND THE R-O LEMMA

In this Section we will state and prove our main result. The fundamental equation of wet fish-pricing is that of Whackabath [1]:

$$f_{xxx}+3f_{xx}-2\cdot Ker(f)=0, \tag{8}$$

where Whackabath's equation (8) is hardly ever used.
It is an interesting question whether the Whackabath's equation (8) in standard topology can be applied without change in Gackworth's Ω-topologies. A very full discussion of Gackworth's work was given in [1, 2].
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[1] T. I. Strainer & B. J. M. Wilkins 1993, A new result on Drivle's Theorem, Proc. Iceland Cod Fish Soc. Lond. Ser. D, 134 (8678–8679).

[2] B. J. M. Wilkins, "Topological Dynamics and the Haddock Fishery", Unpublished, 1987.