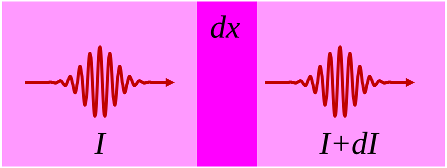
The laser idea



$$W = \sigma F = \sigma \frac{I}{h\nu}$$

$$dI = -W(N_1 - N_2)h\nu dx$$

$$dI = -\alpha I dx$$

Lambert-Beer's law

$N_1 > N_2$ → $dI < 0$ **ABSORPTION**

$$\alpha = \frac{W(N_1 - N_2)h\nu}{I} = \sigma(N_1 - N_2)$$

$N_2 > N_1$ → $dI > 0$ **GAIN**

$$g = \frac{W(N_2 - N_1)h\nu}{I} = \sigma(N_2 - N_1)$$

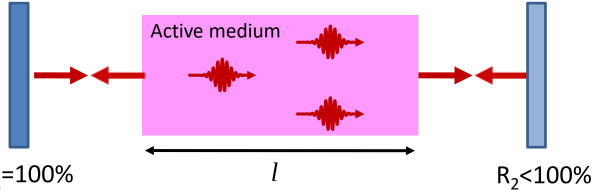
Population inversion

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$$

Boltzmann's constant

$$k = 1.38 \cdot 10^{-23} J/K$$

Threshold and critical inversion



$$\frac{F_{out}}{F_{in}} = e^{\sigma(N_2-N_1)l}$$

$$F' = Fe^{\sigma(N_2-N_1)l}(1-L_i)R_2 e^{\sigma(N_2-N_1)l}(1-L_i)R_1$$

Threshold $F' = F$

Critical inversion

$$\boxed{(N_2 - N_1)_c = N_c = -\frac{\ln R_1 R_2 + 2\ln(1-L_i)}{2\sigma l}}$$

$$N_c = \frac{\gamma}{\sigma l}$$

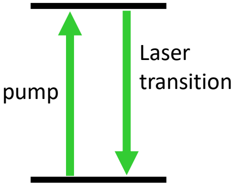
$$\boxed{\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2}}$$

Logarithmic cavity losses (for single pass)

How to get population inversion

$$dI = -W(N_1 - N_2)h\nu dx$$

Two-level system

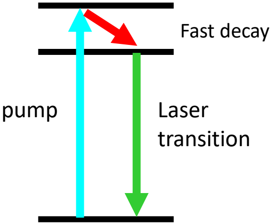


At most: N_2=N_1

No population inversion

No laser action!

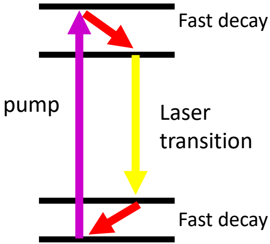
Three-level system



N_2>N_1 for N_2>N_t/2

Difficult laser action!

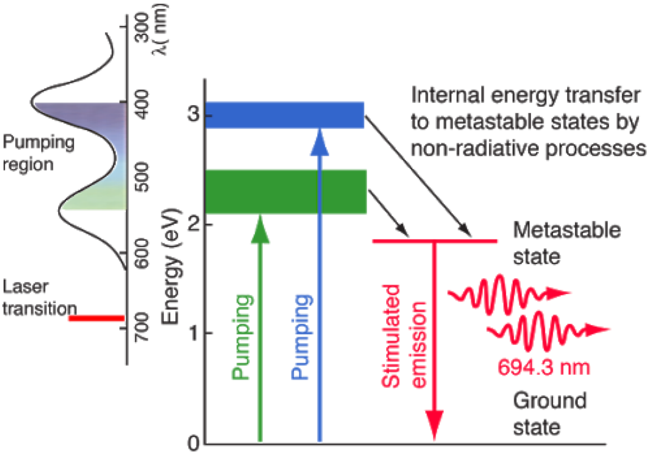
Four-level system

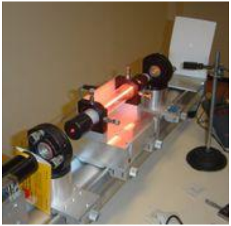


Always N_2>N_1

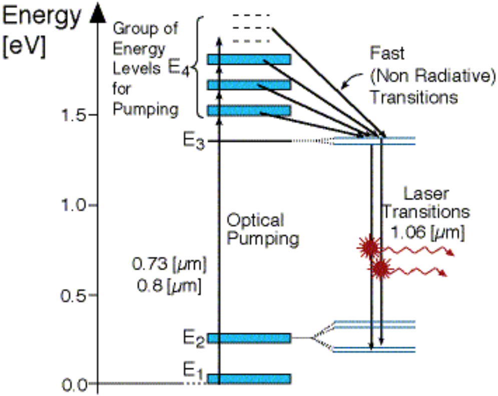
Efficient laser action!

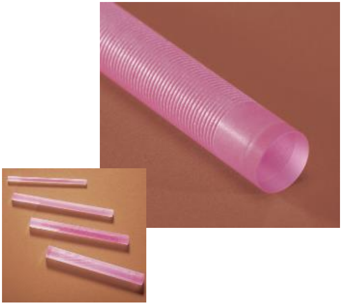
Three-level system: **Ruby laser**





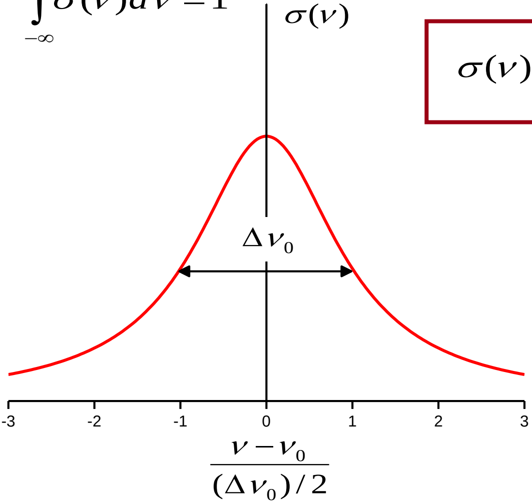
Four-level system: **Nd:YAG**





Homogeneous

$$\int_{-\infty}^{+\infty} \sigma(\nu)d\nu = 1$$



Lorentzian broadening (collisional)

$$\boxed{\sigma(\nu) = 2\tau_c \frac{1}{1+4\pi^2\tau_c^2(\nu-\nu_0)^2}}$$

$$\Delta\nu_0 = \frac{1}{\pi\tau_c}$$

He-Ne laser $\Delta\nu_0 \propto P \quad \tau_c \propto \frac{1}{P}$

$P = 1.0\ Torr \quad T = 300\ K$

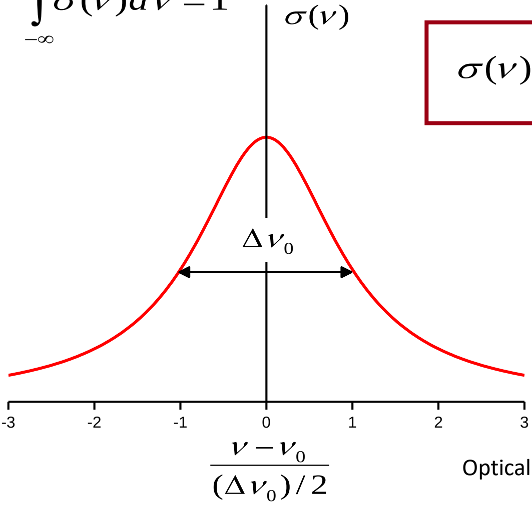
$\tau_c \sim 0.2\ \mu s \quad \Delta\nu_0 = 1.5\ MHz$

Nd:YAG laser $\Delta\nu_0 = 120\ GHz$

Ruby laser $\Delta\nu_0 = 330\ GHz$

Homogeneous

$$\int_{-\infty}^{+\infty} \sigma(\nu) d\nu = 1$$



Lorentzian broadening (collisional)

$$\boxed{\sigma(\nu) = 2\tau_c \frac{1}{1+4\pi^2\tau_c^2(\nu-\nu_0)^2}}$$

$$\Delta\nu_0 = \frac{1}{\pi\tau_c}$$

Natural broadening

$$\boxed{\Delta\nu_{nat} = \frac{1}{2\pi\tau_{sp}}}$$

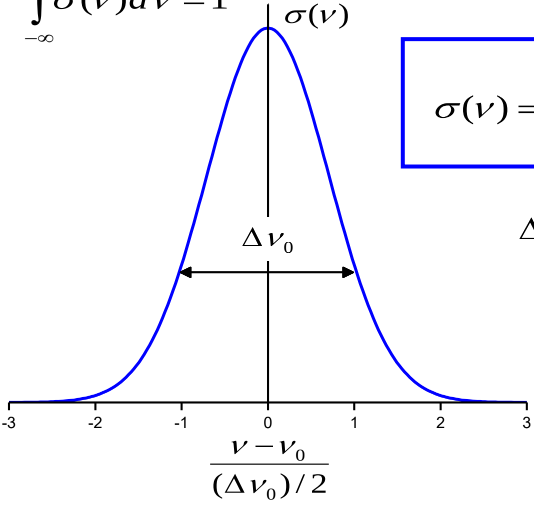
Optical transitions (λ in the visible, e.g. green)

$\tau_{sp} \sim 10\ ns$ $\quad\quad$ $\Delta\nu_0 = 16\ MHz$

Line broadening

Inhomogeneous

$$\int_{-\infty}^{+\infty} \sigma(\nu)\, d\nu = 1$$



Gaussian broadening (Doppler)

$$\boxed{\sigma(\nu) = \frac{1}{\nu_0}\left(\frac{Mc^2}{2\pi kT}\right)^{1/2} e^{-\frac{Mc^2}{2kT}\left(\frac{\nu-\nu_0}{\nu_0}\right)^2}}$$

$$\Delta\nu_0 = 2\nu_0\sqrt{\frac{2kT}{Mc^2}\ln 2}$$

He-Ne laser (@ 300 K)

$\Delta\nu_0 = 1.5\ GHz$ Doppler

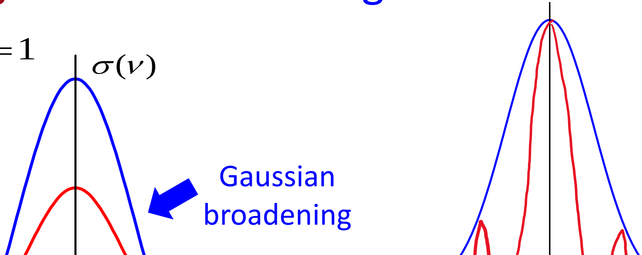
$\Delta\nu_0 = 1.5\ MHz$ Collisional

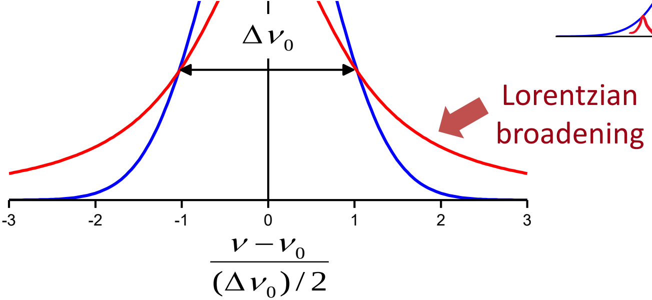
Stark effect (solid-state lasers)

$\Delta\nu_0 \sim 1\ GHz$

Homogeneous vs Inhomogeneous

$$\int_{-\infty}^{+\infty} \sigma(\nu) d\nu = 1$$




$\Delta\nu_0$
Lorentzian
broadening
-3
-2
-1
0
1
2
3
$\frac{\nu - \nu_0}{(\Delta\nu_0)/2}$