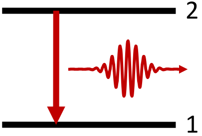
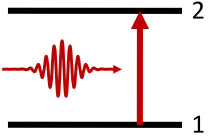
Thermodynamic equilibrium

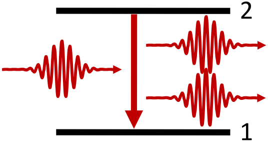
Spontaneous emission



Absorption



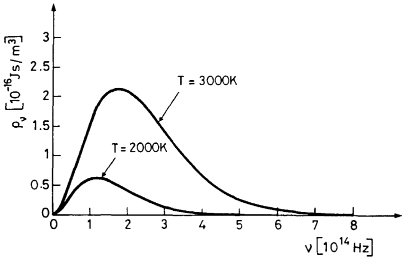
Stimulated emission



$$A_{21}N_2^e + B_{21}\rho(\nu)N_2^e = B_{12}\rho(\nu)N_1^e$$

$$\frac{N_2^e}{N_1^e} = e^{-(E_2-E_1)/kT} = e^{-h\nu/kT}$$

$$\rho(\nu) = \frac{8\pi\nu^2 n^3}{c^3}\frac{h\nu}{e^{h\nu/kT}-1} = \frac{A_{21}}{B_{12}e^{h\nu/kT}-B_{21}}$$



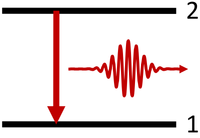
$$B_{21} = B_{12} = B$$

$$\sigma_{21} = \sigma_{12} \qquad W_{21} = W_{12} = W$$

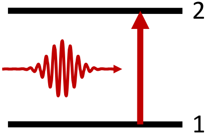
$$A_{21} = A = \left(\frac{8\pi h\nu^3 n^3}{c^3}\right)B$$

Thermodynamic equilibrium

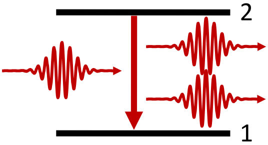
Spontaneous emission



Absorption



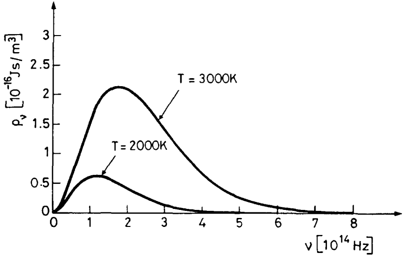
Stimulated emission



$$A_{21}N_2^e + B_{21}\rho(\nu)N_2^e = B_{12}\rho(\nu)N_1^e$$

$$\frac{N_2^e}{N_1^e} = \frac{g_2}{g_1}e^{-(E_2-E_1)/kT} = \frac{g_2}{g_1}e^{-h\nu/kT}$$

$$\rho(\nu) = \frac{8\pi\nu^2 n^3}{c^3}\frac{h\nu}{e^{h\nu/kT}-1} = \frac{A_{21}}{B_{12}e^{h\nu/kT}-B_{21}}$$



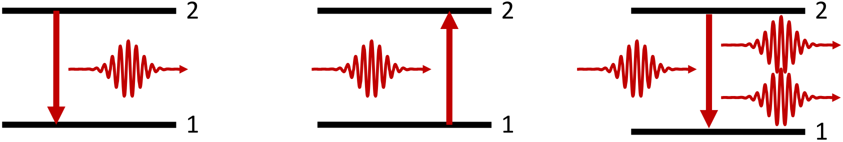
$$g_2B_{21} = g_1B_{12}$$

$$g_2\sigma_{21} = g_1\sigma_{12} \qquad g_2W_{21} = g_1W_{12}$$

Degenerate levels

Fermi's golden rule

Spontaneous emission | Absorption | Stimulated emission



Fermi's golden rule

$$\boxed{W_{21} = \frac{2\pi}{\hbar}|M_{12}|^2\delta(h\nu)}$$

Transition decay rate

Transition matrix element

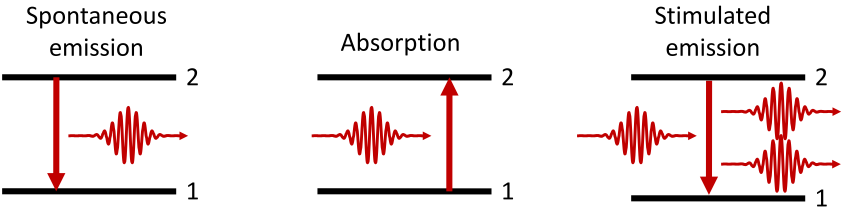
Local Density of final Optical States (LDOS)

$$M_{12} = \langle 2|H'|1\rangle = \int \psi_2^*(\vec{r})H'(\vec{r})\psi_1(\vec{r})d^3\vec{r}$$

Perturbation caused by the interaction with the light beam

O. Svelto, *Principles of Lasers*
B.H. Bransden and C.J. Joachain, *Physics of Atoms and Molecules*

Fermi's golden rule



Fermi's golden rule

$$\boxed{W_{21} = \frac{2\pi}{\hbar}|M_{12}|^2\delta(h\nu)}$$

Transition decay rate

Transition matrix element

Local Density of final States (LDOS)

$$M_{12} = \langle 2|H'|1\rangle = \int \psi_2^*(\vec{r})H'(\vec{r})\psi_1(\vec{r})d^3\vec{r}$$

$$M_{12} = -\vec{\mu}_{12}\cdot\vec{E}$$

electric dipole moment

$$\vec{\mu}_{12} = -e\big(\langle 2|x|1\rangle\hat{\imath} + \langle 2|y|1\rangle\hat{\jmath} + \langle 2|z|1\rangle\hat{k}\big)$$

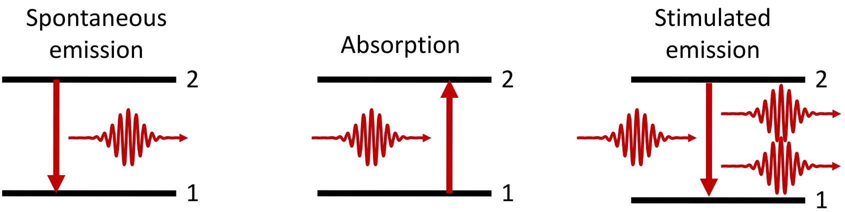
Electric-dipole transitions (E1)

$$H' = -\vec{p}\cdot\vec{E}$$

$$\vec{p} = -e\vec{r}$$

O. Svelto, *Principles of Lasers*
B.H. Bransden and C.J. Joachain, *Physics of Atoms and Molecules*

Fermi's golden rule

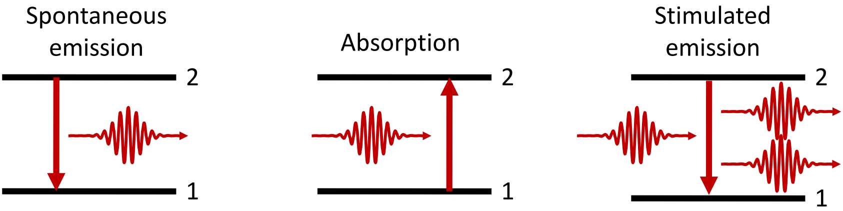


Fermi's golden rule

$$W_{21} = \frac{2\pi}{\hbar} |M_{12}|^2 \delta(h\nu)$$

Transition	Notation	A_{21} (s^{-1})	Radiative lifetime	Parity change
Electric dipole	E1	10^7 - 10^9	1 - 100 ns	yes
Magnetic dipole	M1	10^3 - 10^5	0.01 - 1 ms	no
Electric quadrupole	E2	10^3 - 10^5	0.01 - 1 ms	no
Magnetic quadrupole	M2	0.1 - 10	0.1 – 10 s	yes
Electric octupole	E3	0.1 - 10	0.1 – 10 s	yes
…				

Fermi's golden rule



Fermi's golden rule

$$\boxed{W_{21} = \frac{2\pi}{\hbar}|M_{12}|^2\delta(h\nu)}$$

Transition decay rate

Transition matrix element

Local Density of final States (LDOS)

$$A_{21} = \frac{16\pi^3\nu^3 n}{3h\epsilon_0 c^3}|\vec{\mu}_{12}|^2 = A$$

$$B_{21} = \frac{2\pi^2}{3h^2\epsilon_0 n^2}|\vec{\mu}_{12}|^2 = B$$

$$\frac{A}{B} = \frac{8\pi h\nu^3 n^3}{c^3}$$

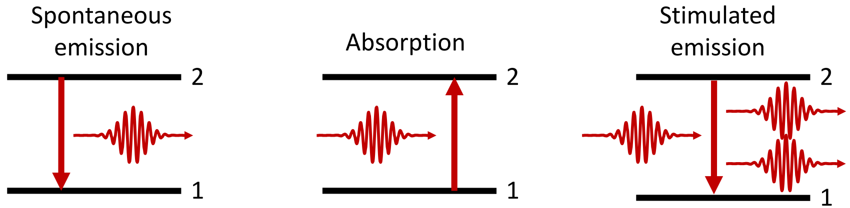
Electric-dipole transitions (E1)

$$H' = -\vec{p}\cdot\vec{E}$$

$$\vec{p} = -e\vec{r}$$

O. Svelto, *Principles of Lasers*
B.H. Bransden and C.J. Joachain, *Physics of Atoms and Molecules*

Fermi's golden rule



Fermi's golden rule

$$\boxed{W_{21} = \frac{2\pi}{\hbar}|M_{12}|^2\delta(h\nu)}$$

Transition decay rate

Transition matrix element

Local Density of final States (LDOS)

$$A_{ji} = \frac{16\pi^3 {\nu_{ji}}^3 n}{3h\epsilon_0 c^3}\frac{e^2}{g_j}\sum_{m_j,m_i}\left|\langle j,m_j|\vec{r}|i,m_i\rangle\right|^2$$

degeneracy of the upper state

$$f_{ji} = \frac{2m(2\pi)^2\nu_{ji}}{3he^2}\left|\vec{\mu}_{ij}\right|^2$$

Oscillator's strength

Electric-dipole transitions (E1)

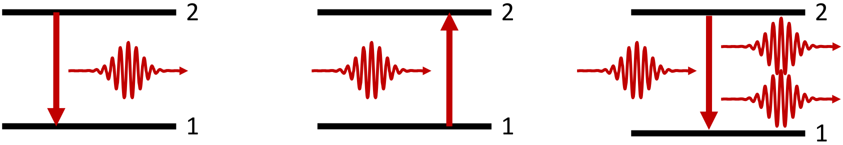
$$H' = -\vec{p}\cdot\vec{E}$$

$$\vec{p} = -e\vec{r}$$

O. Svelto, *Principles of Lasers*
B.H. Bransden and C.J. Joachain, *Physics of Atoms and Molecules*

Fermi's golden rule

Spontaneous emission — Absorption — Stimulated emission



Fermi's golden rule

$$W_{21} = \frac{2\pi}{\hbar} |M_{12}|^2 \delta(h\nu)$$

Transition decay rate

Transition matrix element

Local Density of final States (LDOS)

Selection rules

(1) The parity of the wave function must change

(2) $\Delta l = \pm 1$ for the changing electron

(3) $\Delta L = 0, \pm 1$ but $L = 0 \rightarrow 0$ is forbidden

(4) $\Delta J = 0, \pm 1$ but $J = 0 \rightarrow 0$ is forbidden

(5) $\Delta S = 0$

Electric-dipole transitions (E1)

$$H' = -\vec{p} \cdot \vec{E}$$

$$\vec{p} = -e\vec{r}$$

O. Svelto, *Principles of Lasers*
B.H. Bransden and C.J. Joachain, *Physics of Atoms and Molecules*

Radiative relaxation control:

emitter near a planar interface

K.H. Drexhage, Influence of a dielectric interface on fluorescence decay time *J. Lumin.* **1** 693-701 (1970)

Fermi's golden rule

$$W_{fi} = \frac{2\pi}{\hbar} \left| M_{if} \right|^2 \delta_f$$

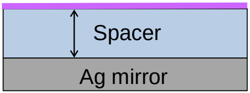
Transition decay rate

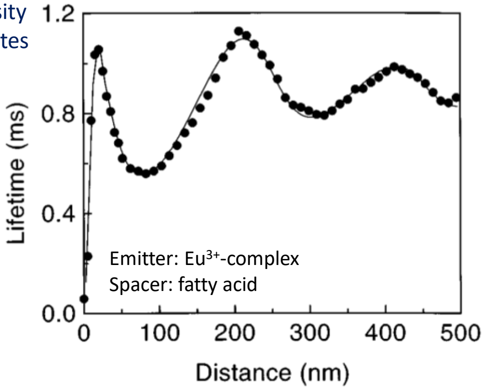
Interaction matrix element

Local density of final states (LDOS)

An interface in close proximity ($d < \lambda_{em}$) of an emitter changes the photonic local density of states.

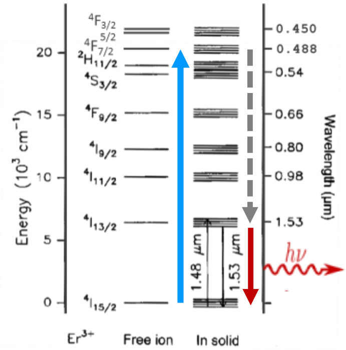
Emitters-containing monolayer



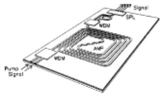


$Er:SiO_2$ for optical telecommunication

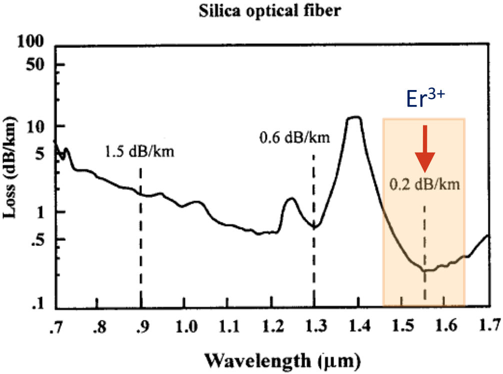
Er^{3+} energy levels



A. Polman, J. Appl. Phys. 82, 1 (1997).



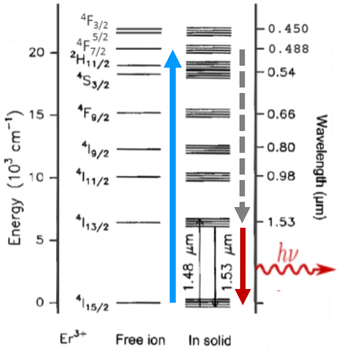
SiO_2 optical amplifier

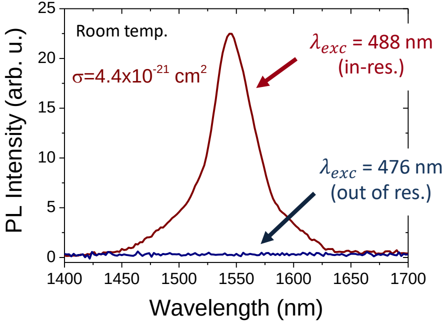


$Er:SiO_2$

for optical telecommunication

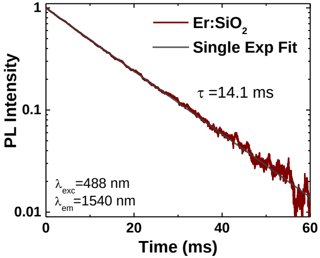
Er^{3+} energy levels





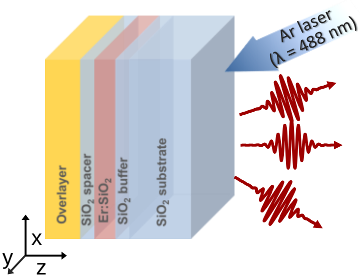
Limits:

- resonant excitation
- low excitation cross-section ($\sigma_{exc} \sim 10^{-21} \div 10^{-20}$ cm^2)
- long PL lifetime: $\tau_{em} \sim 14$ ms

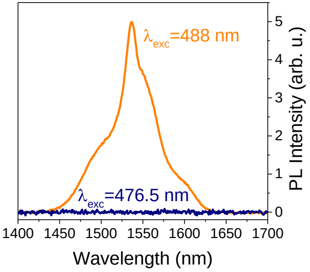


Radiative relaxation control:

emitter near a planar interface

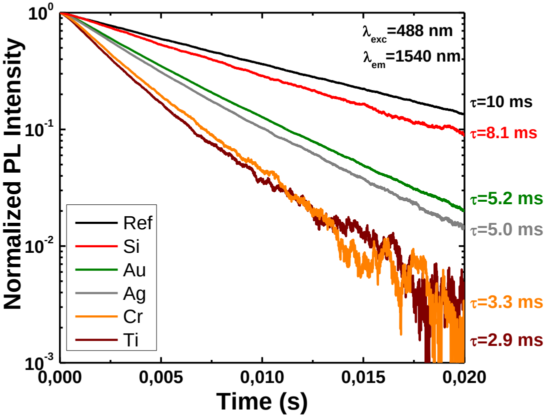


[Er^{3+}] = 0.1 at%



Dielectric function effect

Er:SiO_2 t = 75 nm; Spacer t = 0 nm



B. Kalinic, T. Cesca et al., J. Phys. Chem. C 119, 6728 (2015)

Radiative relaxation control:

emitter near an ordered array

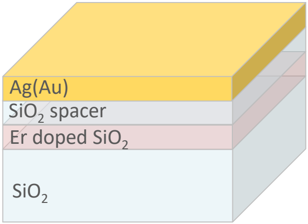
Optics and Laser Physics

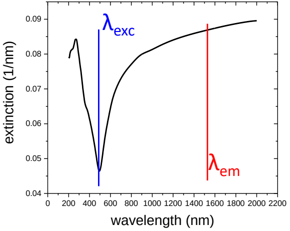
T. Cesca

Università degli Studi di Padova

1222·2022 800 ANNI

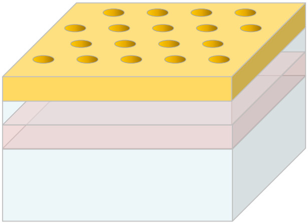
Thin Film

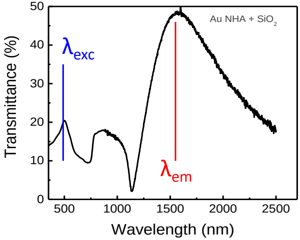




generic

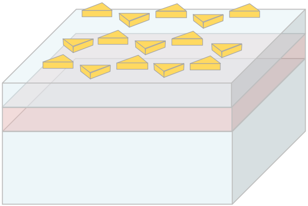
NanoHole Array

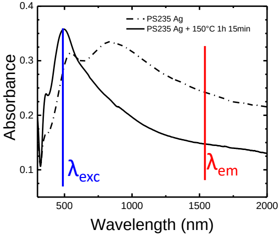




EOT resonant with Er^{3+} emission

NanoPrism Array





LSPR resonant with Er^{3+} excitation

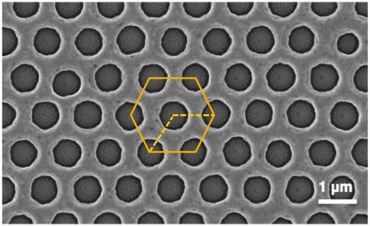
NanoHole Array (NHA)

Extraordinary optical transmission

EXTRAORDINARY OPTICAL TRANSMISSION (EOT)

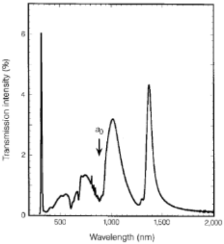
NHA transmittance higher than classical aperture theory for isolated holes

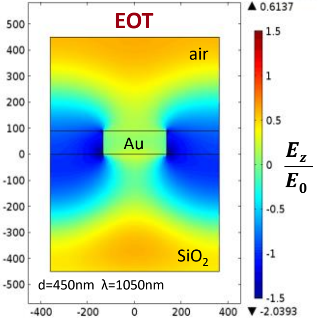
I_{INC}



$$I_{TR} > I_{INC} * f_{HOLE}$$

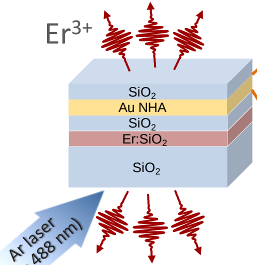
T. W. Ebbesen *et al.*, *Nature* **391**, 667 (1998)

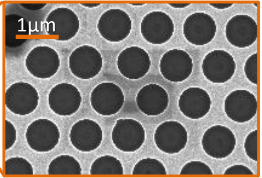


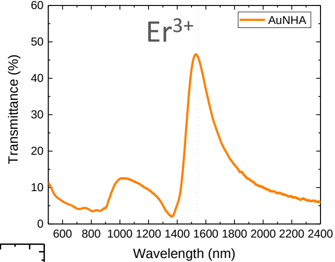


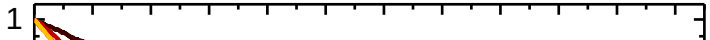
Er^{3+} - Au NHA

Emission lifetime control



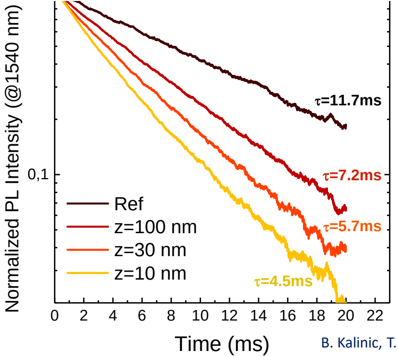






Er:SiO_2 layer:

t = 20 nm



Spacer thickness:

- t = 10 nm
- t = 30 nm
- t = 100 nm

B. Kalinic, T. Cesca et al., J. Phys. Chem. C 119, 6728 (2015)