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## Hoja de Trabajo #6

25 de Marzo, 2018

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### Resolución Ejercicio #1

#### Definición Recursiva

- Dado un numero  $n$  que pertenece a los numero naturales unitarios y que este sea un  $n$  sucesor de cero ( $\sigma n(0) = n$ ).
- Y cero ( $0$ ) el primer numero de los numeros naturales unitarios.

#### Suma de dos números

Caso base:

$$\begin{aligned}n + 0 &= n \\ \sigma(n) + m &= \sigma(n + m)\end{aligned}$$

Caso inductivo:

$$\begin{aligned}\sigma(\sigma(\sigma(0))) + \sigma(\sigma(0)) \\ \sigma(\sigma(\sigma(0))) + \sigma(\sigma(0)) \\ \sigma(\sigma(\sigma(0) + \sigma(\sigma(0)))) \\ \sigma(\sigma(\sigma(0 + \sigma(\sigma(0)))) \\ \sigma(\sigma(\sigma(\sigma(0))))\end{aligned}$$

De manera que:

$$\sigma(n) = a, m = b, \sigma(n + m) = c$$

#### Multiplicación de dos números

Caso base:

$$\begin{aligned}n * 0 &= 0 \\ \sigma(n) * m &= \sigma((n) * m) + m \\ \sigma(0) * n &= \sigma(0 + n)\end{aligned}$$

Caso inductivo:

$$\begin{aligned}(\sigma(0) * \sigma(\sigma(0))) \\ \sigma(0) + \sigma(0) + \sigma(\sigma(0)) \text{ veces...} + \sigma(0) \\ \sigma(0) + [\sigma(0) + \sigma(\sigma(0)) \text{ veces...} + \sigma(0)] \\ \sigma(0) + [\sigma(0) * (\sigma(0))] \\ \sigma(\sigma(0))\end{aligned}$$

De manera que:

$$\sigma(n) = a, m = b, \sigma(n * m) = c$$

## Mayor que para números unitarios

Caso base:

- $\sigma(0) > 0$
- $\sigma(\sigma(n)) > n$

Caso inductivo:

- $\sigma(\sigma(0)) > \sigma(0)$   
 $\sigma(0) > 0$
- $\sigma(\sigma(n)) > n$   
 $\sigma(\sigma(\sigma(n))) > \sigma(n)$   
 $\sigma(\sigma(n)) > n$

## Resolución Ejercicio #2

### Propiedades con Inducción

#### 0.1. Demostracion 1

$$\begin{aligned}n + 0 &= n: \\ \sigma(n) + 0 &= \sigma(n) \\ \sigma(n + 0) &= \sigma(n) \\ \sigma(n) &= \sigma(n)\end{aligned}$$

#### 0.2. Demostracion 2

$$\begin{aligned}n + m &= m + n: \\ \sigma(\sigma(0)) + \sigma(\sigma(\sigma(0))) &= \sigma(\sigma(\sigma(0))) + \sigma(\sigma(0)) \\ \sigma(\sigma(0) + \sigma(\sigma(\sigma(0)))) &= \sigma(\sigma(\sigma(0)) + \sigma(\sigma(0))) \\ \sigma(\sigma(0 + \sigma(\sigma(\sigma(0)))))) &= \sigma(\sigma(\sigma(0) + \sigma(\sigma(0)))) \\ \sigma(\sigma(\sigma(\sigma(\sigma(0)))))) &= \sigma(\sigma(\sigma(0 + \sigma(\sigma(0)))))) \\ \sigma(\sigma(\sigma(\sigma(\sigma(0)))))) &= \sigma(\sigma(\sigma(\sigma(\sigma(0))))))\end{aligned}$$

#### 0.3. Demostracion 3

$$\begin{aligned}n * \sigma(\sigma(0)) &= n + n: \\ \sigma(n) * \sigma(\sigma(0)) &= \sigma(n) + \sigma(n) \\ \sigma(\sigma(0) + n) &= \sigma(n) + \sigma(n) \\ \sigma(\sigma(0 + n)) &= \sigma(n + \sigma(n)) \\ \sigma(\sigma(n)) &= \sigma(\sigma(n)) \\ \sigma(n) + \sigma(n) &= \sigma(n) + \sigma(n)\end{aligned}$$