Assignment 1

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Dobrow chapter 1 problems 1.6, 1.8, 1.10, 1.16, 1.22, 1.37, 1.44, 1.45

1.6.a: {(1,3), (2,2), (3,1)}

1.6.b.: {(3,6), (4,5), (5,4), (6,3)}

1.6.c.: {(1,3), (2,3), (3,3), (4,3), (5,3), (6,3)}

1.6.d.: {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

1.6.e.: {(3,1), (4,1), (5,1), (6,1), (5,2), (6,2)}

1.8: In terms of Boys (B) the space is B = {0,1,2,3,4,5,6} or in terms of Children (C) with Boys (B) and Girls (G) represented: C = {(G), (B,G), (B,B,G), (B,B,B,G), (B,B,B,B,G), (B,B,B,B,B,G), (B,B,B,B,B,B)}

1.10: Because $p + p^2 + p \leq 1$ using the Quadratic equation $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ we get that $p \leq \sqrt{2} - 1$ so one possible value of p would be approximately 0.414213

1.16.a:

$$\frac{1}{26^2 + 26^3 + 26^4 + 26^5} = \frac{1}{12356604} \approx 0.000000081$$

1.16.b:

$$\frac{26^4}{26^2 + 26^3 + 26^4 + 26^5} = \frac{456976}{12356604} \approx 0.036982329$$

1.16.c:

$$\frac{26 + 26^2 + 26^2 + 26^3}{26^2 + 26^3 + 26^4 + 26^5} = \frac{18954}{12356604} \approx 0.001533917$$

1.16.d:

$$\frac{25^2 + 25^3 + 25^4 + 25^5}{26^2 + 26^3 + 26^4 + 26^5} = \frac{10172500}{12356604} \approx 0.176756$$

1.22:

$$Pr(A \cup B) = 0.6$$
$$Pr(A \cup B^C) = 0.8$$
$$Pr(A) + Pr(B) - Pr(AB) = 0.6$$
$$Pr(A) + Pr(B^C) - Pr(AB^C) = 0.8$$
$$Pr(A) + 1 - Pr(B) - Pr(A(1 - Pr(B))) = 0.8$$

Thus

$$Pr(A) = 0.4$$

1.37:

$$\frac{5000}{4} = 1250$$

$$\frac{5000}{7} = 714$$
$$\frac{5000}{10} = 500$$
$$\frac{5000}{28} = 178$$
$$\frac{5000}{70} = 71$$
$$\frac{5000}{40} = 125$$
$$\frac{5000}{280} = 17$$

Using $Pr(A) + Pr(B) + Pr(C) - Pr(AB) - Pr(BC) - Pr(AC) + Pr(ABC)$

$$1250 + 714 + 500 - 178 - 71 - 125 + 17 = 2107$$

Thus the probability is

$$\frac{2107}{5000} = 0.4214$$

1.44:

```
trials = 1e+06
sum(replicate(trials, 2 %in% sample(c(1:4), size = 5, replace = TRUE)))/trials
```

```
## [1] 0.763067
```

1.45:

```
X <- sample(c(1,4,8,16), size = 1, replace = TRUE, prob = c(0.1, 0.2, 0.3, 0.4))
X
```

```
## [1] 8
```

Dobrow chapter 2 problems 2.10, 2.12, 2.14, 2.24, 2.26, 2.30

2.10: As there is a $\frac{1}{3}$ chance of the ball from A being White, and a $\frac{2}{3}$ chance of it being red, the probabilities are $\frac{1}{3}$ of it being $\frac{2}{5}$ white and $\frac{2}{3}$ of it being $\frac{1}{5}$ thus the total probability is $\frac{4}{15} = 0.2\bar{6}$

2.12: $Pr(A) = \frac{1}{2}$, $Pr(A^C) = \frac{1}{2}$, $Pr(B^C|AC) = \frac{1}{3}$, $Pr(B|AC) = \frac{2}{3}$, $Pr(C|A) = \frac{1}{4}$, $Pr(C^C|A) = \frac{3}{4}$
Given those:

$$Pr(B|AC) = \frac{2}{3} = \frac{Pr(ABC)}{Pr(AC)}$$

$$Pr(C|A) = \frac{Pr(AC)}{Pr(A)}$$

$$\frac{1}{4} = \frac{Pr(AC)}{\frac{1}{2}}$$

$$Pr(AC) = \frac{1}{8}$$

$$\frac{2}{3} \times \frac{1}{8} = Pr(ABC) = \frac{1}{12} = 0.8\bar{3}$$

2.14:

$$Pr(B) = 1 - Pr(B^C) = 1 - \prod_{i=1}^{n-1}(1 - \frac{i}{687}) > 0.5$$

$$1 - \frac{687!}{((687-n)! \times 687^n)} = 0.5$$

$$2(687!) = ((687-n)! \times 687^n)$$

$$2n! = 687^n$$

$$log(2n!) = n \times log(687)$$

Thus n is 32 if we wish for a greater than half chance that two Martians were born on the same day.

2.24: (Given the fix in the errata): $Pr(Cancer) = 0.0238$, $Pr(TestNegative|Cancer) = 0.15$, $Pr(TestPositive|NoCancer) = 0.05$, $Pr(TestPositive|Cancer) = 0.85$, $Pr(TestNegative|NoCancer) = 0.95$ Using Bayes we thus have

$$\frac{0.85 \times 0.0238}{0.85 \times 0.238 + 0.05 \times (1 - 0.0238)} \approx 0.293$$

Thus a woman who tested positive would have approximately a 29.3% chance of having cancer if they tested positive.

2.26: Using Bayes, we have

$$\frac{0.8 \times 0.05}{0.8 \times 0.05 + 0.2 \times 0.95} \approx 0.1739$$

There is approximately a 17.39% chance that the cab is actually blue.

2.30: The initial probability of selecting the correct envelope is $\frac{1}{4}$. Assuming that someone always picks envelope one, the chance of winning is roughly $\frac{1}{4}$ if they keep the envelope they stayed with. If they switch, the probability is $\frac{3}{4}$ thus you should switch it.

```
shoulda.stayed <- 0
shoulda.switched <- 0
trials <- 10000
for(n in 1:trials) {
  envelope.with.money <- sample(1:4,1)
  first.guess <- sample(1:4,1)
  # Increases the count depending on boolean test
  shoulda.stayed <- shoulda.stayed + (envelope.with.money == first.guess)
  shoulda.switched <- shoulda.switched + (envelope.with.money != first.guess)
}
shoulda.stayed/trials
```

```
## [1] 0.241
```

```
shoulda.switched/trials
```

```
## [1] 0.759
```

We can thus clearly see that switching is the better option.

Challenge Problems (Optional for up to one point each) Dobrow chapter 1 problem 1.32 Dobrow chapter 2 problems 2.20, 2.33 :

1.32: Using taylor expansons we know that

$$\sum_{k\geq 0} \frac{3^k}{k!} = e^3$$

thus c must equal $\frac{1}{e^3}$ or e^{-3}

2.20: Using Bayes we have

$$\frac{1 \times 0.5}{1 \times 0.5 + 0.5 \times 0.5} = \frac{2}{3}$$

Then we have

$$\frac{2}{3} \times 1 + \frac{1}{3} \times \frac{1}{2} = \frac{5}{6} = 0.8\bar{3}$$

2.33:

```r
box1 <- c('g', 'g')
box2 <- c('g', 's')
# Simulating the removal of one gold coin from the pool
boxes <- c(box1, box2)[-1]
trials <- 100000
sum(replicate(trials, 'g' == sample(boxes, 1)))/trials
```

```
## [1] 0.66425
```

We can thus see that there is a $\frac{2}{3}$ rds probability of the second coin being gold.