УМФ. Лекция

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$$u_{tt}(x,t) - a^2 u_{xx}(x,t) = 0 \quad x \geq 0,\ t \geq 0 \tag{1}$$

$$u_x(x,t)|_{x=0} = 0 \tag{2}$$

$$u(x,t)|_{t=0} = \phi(x) \tag{3}$$

$$u_t(x,t)|_{t=0} = \psi(x) \tag{4}$$

$$-\infty < x < \infty,\ t \geq 0$$

$$U_{tt}(x,t) - a^2 u_{xx}(x,t) = 0 \tag{5}$$

$$U(x,t)|_{t=0} = \Phi(x) \tag{6}$$

$$U_t(x,t)|_{t=0} = \Psi(x) \tag{7}$$

$$\Phi(x) = \begin{cases} \phi(x), & x \geq 0 \\ \phi(-x), & x < 0 \end{cases} \tag{8}$$

$$\Psi(x) = \begin{cases} \psi(x), & x \geq 0 \\ \psi(-x), & x < 0 \end{cases} \tag{9}$$

$$U(x,t) = \frac{\Phi(x-at) + \Phi(x+at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} \Psi(z)dz \tag{10}$$

$$F(x) = \int_{\lambda(x)}^{\beta(x)} f(x,\xi)d\xi$$

$$F'(x) = \int_{\lambda(x)}^{\beta(x)} + f(x,\beta(x))\beta'(x) - f(x,\alpha(x))\alpha'(x)$$

$$U_x(x,t) = \frac{\Phi'(x-at)\cdot 1 + \Phi'(x+at)\cdot 1}{2} + \frac{1}{2a}\cdot(\Psi(x+at)\cdot 1 - \Psi(x-at))\cdot 1$$

$$U_x|_{x=0} = \frac{\Phi'(-at) + \Phi'(at)}{2} + \frac{1}{2a}(\Psi(at) - \Psi(-at)) = 0$$

$$(2),\ (3) \implies \phi'(x)|_{x=0} = 0$$

$$u_{tt}(x,t) - a^2 u_{xx}(x,t) = 0 \quad x \geq 0,\ t \geq 0 \tag{11}$$

$$u|_{t=0} = 0,\ x \geq 0 \tag{12}$$

$$u_t|_{t=0} = 0,\ x \geq 0 \tag{13}$$

$$u(0,t) = \mu(t),\ t \geq 0 \tag{14}$$

$$u(x,t) = F(x-at) + G(x+at) \tag{15}$$

Подставим (15) в начальные условия

$$F(x) + G(x) = 0$$

$$u_t(x,t) = F'(x-at)\cdot(-a) + G'(x-at)\cdot a$$

$$u_t|_{t=0} = -aF'(x) + aG'(x) = 0$$

$$\begin{cases} F(x) + G(x) & = 0,\ x \geq 0 \\ -F'(x) + G'(x) & = 0,\ x \geq 0 \end{cases}$$

$$\begin{cases} G(x) = \frac{C}{2}, \\ F(x) = -\frac{C}{2}, \end{cases} \quad x \geq 0$$

$$u(x,t) = 0 \quad \begin{cases} x - at & \geq 0 \\ x + ar & \geq 0 \end{cases}$$

$x - at < 0$

$$u(x,t) = F(x-at) + \frac{c}{2}, \quad F(-at) + \frac{c}{2} = \mu(t)$$

Замена:

$$-at = z,\ z < 0,\ t = -\frac{z}{a}$$

$$F(z) = \mu(-\frac{z}{a}) - \frac{c}{2}$$

$$x - at < 0: \quad u(x,t) = \mu(t - \frac{x}{a}) - \frac{c}{2} + \frac{c}{2}$$

$$u(x,t) = \begin{cases} \mu(t - \frac{x}{a}), & t > \frac{x}{a} \\ 0, & t \leq \frac{x}{a} \end{cases}$$

Постановка основных задач для ур-й колебаний ограниченной струны

$$u_{tt}(x,t) + a^2 u_{xx}(x,t) = f(x,t) \quad x \in [0;l],\ t \geq 0 \tag{16}$$

$$u|_{t=0} = \phi(x),\ x \in [0;l] \tag{17}$$

$$u_t|_{t=0} = \psi(x),\ x \in [0;l] \tag{18}$$

$$x = 0 \qquad x = l \tag{19a}$$

$$u(0,t) = \mu_0(t),\ t \geq 0 \qquad u(l,t) = \mu_l(t),\ t \geq 0 \tag{19b}$$

$$u_x(O,t) = \nu_0(t),\ t \geq 0 \qquad u_x(l,t) = \nu_l(t),\ t \geq 0 \tag{19c}$$

$$-u_x(O,t) + h_0 u(0,t) = \varkappa_0(t),\ h_0 > 0 \qquad u_x(l,t) + h_l u(l,t) = \varkappa_l(t),\ h_l > 0 \tag{19d}$$

$$u_{tt}(x,t) + a^2 u_{xx}(x,t) = f(x,t)$$

$$u(x,t) = F(x-at) + G(x+at)$$

$$\begin{cases} F(x) - G(x) & = \phi(x) \\ -aF'(x) + aG'(x) & = \psi(x) \end{cases} \quad x \in [0,l]$$

$$\begin{cases} 0 \leq x - at \leq l \\ 0 \leq x + at \leq l \end{cases}$$

$$x \in \mathbb{R},\ t \geq 0$$

$$u_{tt}(x,t) + a^2 u_{xx}(x,t) = f(x,t)$$

$$u|_{t=0} = \phi(x),\ x \in \mathbb{R}$$

$$u_t|_{t=0} = \psi(x),\ x \in \mathbb{R}$$

$$u(x,t) = \frac{\phi(x-at) + \phi(x+at)}{2} + \frac{1}{2a} \int_0^\infty \int_{x-at}^{x+at} \psi(z) dz$$