

Страхование. Неправенство Крамера-Лундберта. Процессы восстановления.

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$$X_0 = u > 0$$

Поступают взносы $\Delta t \rightsquigarrow C \cdot \Delta t$

$$0 < T_1 < T_2 < \dots < T_n < \dots$$

страховые случаи

$$T_i \rightarrow \xi_i$$

$$X_t = u + ct - \sum_{i=1}^{N_t} \xi_i = u + ct - \sum_{i=1}^{\infty} \xi_i \cdot I(T_i < t)$$

$$N_t = \sum_{i=1}^{\infty} I(T_i < t)$$

$$X_t - X_0 = ct - \sum_{k=1}^N \xi_k$$

$T = \inf\{t > 0 : X_t \leq 0\}$ - момент разорения фирмы

$$P(T < t) = q(t, u) \text{ и } P(T < \infty) = q$$

Предположение

$$KL_1 : \tau = T_i - T_{i-1}, \quad \{\tau_i\} \text{ - н. о. р. случайные величины}$$

$$P(\tau_i > t) = e^{-\lambda t}, \quad t > 0$$

$$KL_2 : \{\xi_i\} \text{ - н. о. р. случайные величины}$$

$$P(\xi_i < x) = F(x)$$

$$\mu = \int_{-\infty}^{\infty} x dF(x) < \infty$$

$$KL_3 : \{\tau_i\}, \{\xi_i\} \text{ - независимые}$$

$$T_k = \tau_1 + \dots + \tau_k \text{ - имеет гамма распределение}$$

$$f_k(x) = \lambda^k \frac{x^{k-1}}{\Gamma(k)} e^{-\lambda x}$$

$$P(T_k \geq t) = P(N_t \leq k) = P(\tau_1 + \dots + \tau_k \geq t) = \int_t^{\infty} f_k(x) dx = \sum_{i=0}^k \frac{(\lambda t)^i}{i!} e^{-\lambda t}$$

$$P(N_t = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad \{N_t\}_{t \geq 0} \text{ - пуассоновские}$$

$$\text{Условие: существует } R > 0$$

$$\frac{\lambda}{c} \int_0^{\infty} e^{Rx} (1 - F(x)) dx = 1$$

$$E(X_t - X_0) = ct - E\left(\sum_{k=1}^{N_t} \xi_k\right) = ct - E\left(\sum_{k=1}^{\infty} \xi_k \cdot I(T_k < t)\right) = ct - \sum_{k=1}^{\infty} E(\xi_k \cdot I(T_k \leq t)) =$$

$$ct - \sum_{k=1}^{\infty} E(\xi_k) E(I(T_k \leq t)) = ct - \mu \sum_{k=1}^{\infty} P(T_k \leq t) = ct - \mu \sum_{k=1}^{\infty} P(N_t \geq k) = ct - \mu E(N_t) =$$

$$ct - \lambda \mu t = t(c - \lambda \mu) > 0$$

$$c > \lambda \mu$$

$$h(z) = \int_0^{\infty} (1 - e^{zx}) dF(x) = 1 - \int_0^{\infty} e^{zx} dF(x) = 1 - \varphi(z)$$

$$\varphi(z) = 1 - h(z), \quad h(0) = 0$$

$$g(z) = cz + \lambda h(z) = cz + \lambda - \lambda \varphi(z)$$

$$g(0) = 0$$

$$E(e^{-r(X_t - X_0)}) = E\left(e^{-r(ct - \sum_{k=0}^{N_t} \xi_k)}\right) = e^{-rct} E\left(e^{r \sum_{k=0}^{N_t} \xi_k}\right) = e^{-rct} \sum_{n=0}^{\infty} E\left(e^{r \sum_{k=1}^n \xi_k}\right) P(N_t = n)$$

$$= e^{-rct} \sum_{n=0}^{\infty} \frac{(\lambda t \varphi(r))^n}{n!} e^{-\lambda t} = e^{-rct - \lambda t + \lambda t \varphi(r)} = e^{-tg(r)}$$

$$\mathcal{F}_t = \sigma(X_u, u \leq t)$$

$$E(e^{-r(X_t - X_0)}) = e^{-tg(r)}$$

$$e^{-tg(r)} = E(e^{-r(X_t - u)}) = E(e^{-r(X_t - X_s) - r(X_s - u)}) = E(e^{-r(X_t - X_s)}) E(e^{-r(X_s - u)}) = E(e^{-r(X_t - X_s)} \cdot e^{-sg(r)})$$

$$E(e^{-r(X_t - X_s)}) = e^{-(t-s)g(r)}$$

$$E(e^{-r(X_t - X_s)} \mid \mathcal{F}_s) = E(e^{-r(X_t - X_s)}) = e^{-(t-s)g(r)}$$

$$z_t = e^{-X_t + tg(r)}$$

$$E(e^{-rX_t + tg(r)} \mid \mathcal{F}_s) = E(e^{-r(X_t - X_s)} \cdot e^{-rX_s + tg(r)} \mid \mathcal{F}_s) = e^{-rX_s + tg(r)}$$

$$E(e^{-r(X_t - X_s)} \mid \mathcal{F}_s) = e^{...}$$

$$E(z_t \mid \mathcal{F}_s) = z_s \Rightarrow \{z_t\} \text{ - мартингал}$$

$$E(z_t \mid \mathcal{F}_0) = E(z_t) = e^{-ru}$$

Теорема 1. $\{z_t, \mathcal{F}_t\}$ - мартингал

τ - момент остановки случайной величины, то $\{z_{\tau \wedge t}\}$

$$g(r) = cr + \lambda h(r)$$

$$g(0) = 0$$

$$g'(0) = cr - \lambda \varphi'(0) = c - \lambda \mu > 0$$

$$g''(r) = -\lambda \varphi''(r) = -\lambda \int_0^\infty x^2 e^{rx} dF(x)$$

Условия Крамера

$$g''(r) \leq 0$$

Теория восстановления

Прибор $[0, \tau_1]$, τ_1 - момент отказа

Второй $(\tau_1, T_2]$, $\tau_2 = T_2 - T_1$, $T_1 = \tau_1$

$$T_n = \tau_1 + \tau_2 + \dots + \tau_n \quad P(T_n = \tau_1 + \dots + \tau_n \geq t) = G_k(t)$$

$$P(T_n \geq t) = P(N_t \leq k) \quad P(N_t = k) = -(G_k(t) - G_{k-1}(t)) = p_k(t)$$

$$X_t = \max \left(k : \sum_{i=1}^k \tau_i < t \right)$$

$$E(X(t)) = \sum_{k=1}^{\infty} k p_k(t) = G_1(t) - G_2(t) + 2G_2(t) - 3G_3(t) + \dots = \sum_{k=1}^{\infty} G_k(t)$$

Теорема 2.

$$\frac{X(t)}{t} \xrightarrow[t \rightarrow 0]{} \frac{1}{a}$$