ТВиМС. Лекция

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Problem1 С целью исследования влияния погоды на урожайность сена

$$x_1 \text{ - урожайность}$$

$$x_2 \text{ - весеннее кол-во осадков}$$

$$x \text{ - сумма тем } > 5.5^\circ C$$

$$\overline{x}_1 = 35.146$$

$$\overline{x}_2 = 2.5\,cm$$

$$\overline{x}_3 = 312^\circ\,C$$

$$s_1^2 = 30.74 \qquad r_{12} = 0.8$$
$$s_2^2 = 7.8 \qquad r_{13} = -0.4$$
$$s_3^2 = 2230 \qquad r_{23} = -0.56$$

$$R = \begin{pmatrix} 1 & 0.8 & -0.4 \\ 0.8 & 1 & -0.56 \\ -0.4 & -0.56 & 1 \end{pmatrix}$$

$$\mathbb{X} = (X_1, X_2, \dots, X_p, X_{p+1}, \dots, X_{p+m})^T$$

$$X \in N(0, \Sigma)$$

$$\mathbb{X}_1 = (X_1, \dots, X_p)^T,\ \mathbb{X}_2 = (X_{p+1}, \dots, X_{p+m})^T$$

$$E(\mathbb{X}_1) = 0,\ E(\mathbb{X}_2) = 0$$

$$\Sigma_{11} = E(\mathbb{X}_1, \mathbb{X}_1^T)$$

$$\Sigma_{22} = E(\mathbb{X}_2, \mathbb{X}_2^T)$$

$$\Sigma_{12} = E(\mathbb{X}_1, \mathbb{X}_2^T) = \Sigma_{21}^T$$

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

$$\mathbb{Y}_2 = \mathbb{X}_2 \quad \mathbb{Y}_1 = \mathbb{X}_1 + A\mathbb{X}_2$$

$$\Sigma,\ \Sigma_{ij} \text{ - пол опред}$$

$$E((\mathbb{X}_1 + A\mathbb{X}_2)\mathbb{X}_2^T) = E(\mathbb{X}_1 \cdot \mathbb{X}_2^T) + A \cdot E(\mathbb{X}_2, \mathbb{X}_2^T) = \Sigma_{12} + A\Sigma_{22} = 0$$

$$\Sigma_{12}\Sigma_{22}^{-1} + A = 0$$

$$A = -\Sigma_{12}\Sigma_{22}^{-1}$$

$$\mathbb{Y}_1 \wedge \mathbb{Y}_2 \text{ - нез}$$

$$E(\mathbb{Y}_1 \mid \mathbb{Y}_2) = E(\mathbb{X}_1 - \Sigma_{12}\Sigma_{22}^{-1}E(\mathbb{X}_2|\mathbb{X}_2) = E(\mathbb{X}...$$

$$E(\mathbb{X}_1|\mathbb{X}_2 = x_2) = -\Sigma_{12}\Sigma_{22}^{-1}x_2$$

$$E(\mathbb{Y}_1\mathbb{Y}_2^T) = 0 \implies \mathbb{Y}_1,\ \mathbb{Y}_2 \text{ - независимы}$$

Example

$$\mathbb{X} = (X_1, X_2)^T$$

$$E(X_1)=0,\ E(X_2)=0$$
$$D(X_1)=0,\ D(X_2)=\sigma_2^2$$
$$\rho(X_1,X_2)=\rho$$
$$\Sigma=\begin{pmatrix}\sigma_1^2 & \rho\sigma_1\sigma_2\\ \rho\sigma_1\sigma_2 & \sigma_2^2\end{pmatrix}$$
$$\Sigma_{11}=\sigma_1^2\quad \Sigma_{22}=\sigma_2^2\quad \Sigma_{12}=\rho\sigma_1\sigma_2$$
$$E(X_1)=\mu_1\quad E(X_2)=\mu_2$$

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Example

$$\Sigma=\begin{pmatrix}1 & \rho_{12} & \rho_{13}\\ \rho_{12} & 1 & \rho_{23}\\ \rho_{13} & \rho_{23} & 1\end{pmatrix}$$
$$\Sigma_{11}=\begin{pmatrix}1 & \rho_{12}\\ \rho_{12} & 1\end{pmatrix},\quad \Sigma_{12}=\begin{pmatrix}\rho_{13}\\ \rho_{23}\end{pmatrix},\quad \Sigma_{22}=1$$
$$\mathbb{X}_1=\begin{pmatrix}X_1\\ X_2\end{pmatrix}\quad \mathbb{X}_2=X_2$$
$$\mathbb{Y}_1=\mathbb{X}_1-\Sigma_{12}\Sigma_{22}^{-1}\mathbb{X}_2$$
$$E(\mathbb{Y}_1\cdot\mathbb{Y}_1^T)=E((\mathbb{X}_1-\Sigma_{12}\Sigma_{22}^{-1}\mathbb{X}_2)(\mathbb{X}_1^T-(\Sigma_{12}\Sigma_{22}^{-1}\mathbb{X}_2)^T))=\dots=\Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}+\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{22}\Sigma_{22}^{-1}\Sigma_{21}=$$
$$=\Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{22}=\begin{pmatrix}1 & \rho_{12}\\ \rho_{12} & 1\end{pmatrix}-\begin{pmatrix}\rho_{13}^2 & \rho_{13}\rho_{23}\\ \rho_{13}\rho_{23} & \rho_{23}^2\end{pmatrix}=\begin{pmatrix}1-\rho_{13}^2 & \rho_{12}-\rho_{13}\rho_{23}\\ \rho_{12}-\rho_{13}\rho_{23} & 1-\rho_{23}^2\end{pmatrix}$$
$$\begin{pmatrix}1 & \frac{\rho_{12}-\rho_{13}\rho_{23}}{\sqrt{1-\rho_{13}^2}\sqrt{1-\rho_{23}^2}}\\ \dots & 1\end{pmatrix}$$
$$\rho_{12\cdot 3}=\frac{\rho_{12}-\rho_{13}\rho_{23}}{\sqrt{1-\rho_{13}^2}\sqrt{1-\rho_{23}^2}}\text{ - частный коэф. кор.}$$
$$\rho_{12\cdot 34}=\frac{\rho_{12\cdot 3}-\rho_{13\cdot 4}\cdot\rho_{23\cdot 4}}{\sqrt{1-\rho_{12\cdot 4}^2}\sqrt{1-\rho_{23\cdot 4}^2}}$$
$$\rho_{1j\cdot q_j}=-\frac{\mathbb{R}_{1j}}{\sqrt{\mathbb{R}_{11}\mathbb{R}_{jj}}}\quad \mathbb{R}_{ij}\text{ - алгебраическое дополнение до }\rho_{ij}$$
$$q_j=\{1,2\dots,m\}\setminus\{1,j\}$$
$$(X_1,X_2,\dots,X_m)^T\quad D(X_i)=\sigma_i^2$$
$$E(X_1|X_2,\dots,X_m)=-\sum_{j=2}^m\frac{\mathbb{R}_{1j}}{\mathbb{R}_{11}}\frac{\sigma_1}{\sigma_j}X_j$$
$$E(X_1-E(X_1|X_2\dots X_m)^2)=E\left((X_1+\sum_{j=2}^m\frac{\mathbb{R}_{1j}}{\mathbb{R}_{11}}\frac{\sigma_1}{\sigma_j}X_j)(X_1+\sum_{j=2}^m\frac{\mathbb{R}_{1j}}{\mathbb{R}_{11}}\frac{\sigma_1}{\sigma_j}X_j)\right)=\dots=\sigma_1^2+\sum_{j=2}^m\frac{\mathbb{R}_{1j}}{\mathbb{R}_{11}}\frac{\sigma_1}{\sigma_2}\rho_{1j}=\frac{|R|}{\mathbb{R}_{11}}$$
$$\frac{|\mathbb{R}|}{\mathbb{R}_{11}}=1-\mathbb{R}^2_{1(2\dots m)}$$
$$\mathbb{R}^2_{1(2\dots m)}\text{ - множественный коэф. корреляции}$$
$$\widehat{\mathbb{R}}^2_{1(2\dots m)}=r^2_{1(2\dots m)}$$
$$H_0:\ \mathbb{R}^2_{1(2\dots m)}=0\quad n\text{ - объём выборки}$$
$$F=\frac{n-m}{n-1}\frac{r^2_{1(2\dots m)}}{1-r^2_{1(2\dots m)}}\in F(m-1,n-m)\text{ - распределение Фишера}$$
$$F>F_{1-\alpha}(m-1,n-m)\text{ - отвергаем }H_0$$
$$m=2\quad r^2_{1(2\dots m)}=r^2_{12}$$
$$\frac{(n-2)r_{12}^2}{\sqrt{1-r_{12}^2}}>F(1,n-2)$$
$$\frac{\sqrt{n-2}|r_{12}|}{\sqrt{1-r_{12}^2}}>\sqrt{F(1,n-2}=t(n-2)$$