Частная и множественная корреляция

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{pmatrix}\right)$$

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \to \begin{pmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{pmatrix}$$

Корреляция X_1, X_2 при условии $X_3 = x_3$ - фикс

$$\rho_{12\cdot 3} = \frac{\rho_{12} - \rho_{13}\rho_{23}}{\sqrt{1-\rho_{13}^2}\sqrt{1-\rho_{23}^2}}$$

$$\mathbb{X} \in N(0, \Sigma)$$

$$\mathbb{X} = \begin{bmatrix} \mathbb{X}_1 \\ \mathbb{X}_2 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

$$\Sigma_{11} = E(\mathbb{X}_1 \mathbb{X}_1^T)$$

$$\Sigma_{12} = E(\mathbb{X}_1 \mathbb{X}_2^T)$$

$$\Sigma_{22} = E(\mathbb{X}_2 \mathbb{X}_2^T)$$

$$\mathbb{Y}_1 = \mathbb{X}_1 - A \cdot \mathbb{X}_2$$

$$\mathbb{Y}_2 = \mathbb{X}_2$$

$$\begin{bmatrix} \mathbb{Y}_1 \\ \mathbb{Y}_2 \end{bmatrix} \in N(0, \dots)$$

$$E(\mathbb{Y}_1 \cdot \mathbb{Y}_2^T) = 0$$

$$E(\mathbb{Y}_1) = 0 \quad E(\mathbb{Y}_2) = 0$$

$$E((\mathbb{X}_1 - A\mathbb{X}_2)\mathbb{X}_2^T) = 0 = \Sigma_{12} - A\Sigma_{22} = 0$$

$$\Sigma_{12} = A\Sigma_{22} \;\Big|\; \cdot \Sigma_{22}^{-1}$$

$$A = \Sigma_{12}\Sigma_{22}^{-1}$$

$$\mathbb{Y}_1 = \mathbb{X}_1 - \Sigma_{12}\Sigma_{22}^{-1}\mathbb{X}_2$$

$$E(\mathbb{Y}_1|\mathbb{X}_2) = E(\mathbb{X}_1|\mathbb{X}_2) - \Sigma_{12}\Sigma_{22}^{-1}\mathbb{X}_2 = 0$$

$$E(\mathbb{X}_1|\mathbb{X}_2) = \Sigma_{12}\Sigma_{22}^{-1}\mathbb{X}_2$$

$$E(\mathbb{X}_1) = \mu_1 \quad E(\mathbb{X}_1 - \mu_1) = 0$$

$$E(\mathbb{X}_2) = \mu_2 \quad E(\mathbb{X}_2 - \mu_2) = 0$$

$$E(\mathbb{X}_1|\mathbb{X}_2) = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(\mathbb{X}_2 - \mu_2)$$

$$\mathbb{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_{12} \\ \rho\sigma_{12} & \sigma_2^2 \end{pmatrix}\right)$$

$$E(X_1|X_2) = \mu_1 + \rho\sigma_1\sigma_2 \cdot \frac{1}{\sigma_2^2}(x_2 - \mu_2) = \mu_1 + \rho\frac{\sigma_1}{\sigma_2}(x_2 - \mu_2)$$

$$\mathbb{X} = \begin{pmatrix} \mathbb{X}_1 \\ \mathbb{X}_2 \end{pmatrix}, \quad \mathbb{X}_1 = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, \quad \mathbb{X}_2 = X_3$$

$$\Sigma = \begin{pmatrix} 1 & \rho_{12} & \vdots & \rho_{13} \\ \rho_{12} & 1 & \vdots & \rho_{23} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{13} & \rho_{23} & \vdots & 1 \end{pmatrix}$$

$$\mathbb{Y}_1 = \mathbb{X}_1 - \Sigma_{12}\Sigma_{22}^{-1}\mathbb{X}_2$$

$$E(\mathbb{Y}_1) = 0$$

$$E(\mathbb{Y}_1 \cdot \mathbb{Y}_1^T) = \Sigma_{12\cdot 3} = \dots = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$

Σ balls

$$(X_1, X_2, X_3, X_4)^T$$

$$\rho_{12\cdot 34} = \frac{\rho_{12\cdot 4} - \rho_{13\cdot 4}\rho_{23\cdot 4}}{\sqrt{1-\rho_{13\cdot 4}^2}\sqrt{1-\rho_{23\cdot 4}^2}}$$

$$\rho_{1j\cdot q_j} = \frac{-\mathbb{R}_{1j}}{\sqrt{\mathbb{R}_{11}}\sqrt{\mathbb{R}_{jj}}}$$

$$q_j = \{1, 2\dots, m\} \setminus \{1, j\}$$

$\mathbb{R}$ - матрица корреляции

$\mathbb{R}_{1j},\ \mathbb{R}_{11}, \mathbb{R}_{jj}$ - алгебраческие дополнения

$$E(X_1 | \mathbb{X}_2 = \mathbf{x}_2)$$

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Множественный коэффициент корреляции

$$1 - R_{1(2\dots n)}^2 = \frac{\sigma_{1\cdot 2\dots m}^2}{\sigma_1^2}$$

$R_{1(2\dots n)}^2$ - множественный коэффициент корреляции