$$O(x_0) \text{ - функция диф-мая в окрестности } x_0$$

$$v: \ O(x_0) \to \mathbb{R} \text{ - линейное}$$

$$v(fg) = v(f)g(x_0) + f(x_0)v(g)$$

T - множество всех дифференцируемых $O(x_0)$ в окр-ти x_0

$$\gamma(t): \ (-\epsilon, \epsilon) \to M \text{ гладкое отображение}$$

$$f \in 0(x_0)$$

$$\frac{f(\gamma(t))}{dt}|_{t=0} = v_\gamma(f)$$

$$\gamma(t) = (\gamma_1(t), \dots, \gamma_n(t)) \quad \gamma_i(t) = x_i(\gamma(t))$$

$$\frac{df(\gamma_1, \dots, \gamma_n)}{dt}|_{t=0} = \frac{\partial f}{\partial x_1}|_{x_0}\gamma_1'(0) + \dots + \frac{\partial f}{\partial x_n}|_{x_0}\gamma_n'(0) = v_\gamma(f)$$

$$v_1, \dots, v_n \quad v_i = \frac{\partial}{\partial x_i}|_{x_0} \in T_{x_0}$$

$$v \in T_{x_0}$$

$$v(c) = 0, \ c = const$$

$$v(1) = 0$$

$$v(1) = v(1^2) = ...$$

$$f(x_1, \dots, x_n) = f(0) + \sum_{i=1}^n \frac{\partial f}{\partial x_i}|_0 \cdot x_i + g$$

$$x_1^{\alpha_1} \dots x_n^{\alpha_n} = x^\alpha \quad \alpha_1 + \dots + \alpha_n = k+1$$

$$m = \{f \in O(x_0) \mid f(x_0) = 0\} \text{ - max идеал}$$

$$f \in m \implies \frac{1}{f} \in O_{x_0} \iff m \text{ - max идеал}$$

$$m^2 = \langle g \cdot h, \ g, h \in m \rangle$$

$$f = f(0) + \sum_{i=}^n \frac{\partial f}{\partial x_i}|_{x_0} x_i + g, \quad g \in m^2$$

$$v \in T_{x_0} \quad v(m^2) = 0$$

$$f, g \in m$$

$$v(fg) = v(f)\underbrace{g(x_0)}_{=0} + \underbrace{f(x_0)}_{=0}v(g) = 0$$

$$v(f) = v(f(0) + \sum_{i=1}^n \frac{\partial f}{\partial x_i}\Big|_0 x_i + g) = \sum_{i=1}^n \frac{\partial f}{\partial x_i}\Big|_0 \cdot v(x_i) = \sum_{i=1}^n a_i \frac{\partial f}{\partial x_i}\Big|_0 = \left(\sum_{i=1}^n a_i \frac{\partial}{\partial x_i}\Big|_0\right)(f)$$

$$v = \sum_{i=1}^n a_i \left(\frac{\partial}{\partial x_i}\right)_0$$

$$T_{x_0} = \langle \frac{\partial}{\partial x_i}\Big|_0, \ i = 1, \dots, n \rangle$$

$$\Phi: \ M \to N$$

$$\Phi(x_0) = y_0$$

$$d\Phi_{x_0}: \ \Gamma_{x_0} \to \Gamma_{y_0}$$

$$g \in O_{y_0} \quad v \in T_{x_0}$$

$$g \circ \Phi \in 0_{x_0}$$

g определена на N $g \circ \Phi$ определена

$$v(g \circ \Phi) = d\Phi_{x_0}(v)(y)$$

$$\{y_1, \dots, y_n\} \text{ - координаты в окр-ти т } y_0$$

$$\{x_1, \dots, x_m\} \text{ - координаты в окр-ти w } x_0$$

$$y_1 = \phi_1(x_1, \dots, x_m)$$

$$\dots$$

$$y_n = \phi_n(x_1, \dots, x_m)$$

$$J\left(\frac{y_1, \dots, y_n}{x_1, \dots, x_m}\right) = \begin{pmatrix} \frac{\partial \phi_1}{\partial x_1} & \dots & \frac{\partial \phi_1}{\partial x_m} \\ \dots & \dots & \dots \\ \frac{\partial \phi_n}{\partial x_1} & \dots & \frac{\partial \phi_b}{\partial x_m} \end{pmatrix}$$

$$T_{y_0} = \langle \frac{\partial}{\partial y_i}\big|_{y_0} \rangle$$

$$T_{x_0} = \langle \frac{\partial}{\partial x_j}\big|_{x_0} \rangle$$

$$(d\Phi_{x_0}(v)) = J\left(\frac{y_1, \dots, y_n}{x_1, \dots, x_m}\right)\big|_0(v)$$

$$\left(d\Phi_{x_0}\left(\frac{\partial}{\partial x_i}\right)_{x_0}\right)(y_i) = \sum_{i=1}^{n} a_i \frac{\partial}{\partial y_i}\big|_{y_0} = \left(\frac{\partial}{\partial x_1}(y \cdot \Phi)\right)_{x_0} = \frac{\partial \phi_i}{\partial x_1}\big|_{x_0}$$

Векторное поле

X на многообразии M

$$X(x_i) \in T_{x_0}$$

$$X(x_0) = \sum_{i=1}^{n} a_i \frac{\partial}{\partial x_i}\big|_{x_0}$$

В коорд.

$$X(x) = \sum_{i=1}^{n} a_i(x) \frac{\partial}{\partial x_i}\big|_{x}$$

$$X = \sum_{i=1}^{n} a_i(x) \frac{\partial}{\partial x_i}$$