STA304A1

Qizheng Wang

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Question 1

Question 1 (a)

$\mu = \frac{1}{N}\sum_{i=1}^{N} y(e_i) = \frac{1}{4} * (3+1+0+5) = 2.25$
$\sigma_y^2 = \frac{1}{4} * ((4-2.25)^2 + (3-2.25)^2 + (0-2.25^2) + (5-2.25)^2) = \frac{65}{16}$

Question 1 (b)

(i) The number of possible SRSs of size n=2 is $\binom{4}{2} = 6$
Possible sample is 12,13,14,23,24,34(by label).
For each sample,the probability that being selected is $\frac{1}{6}$.

(ii) For $y_1 = 3, y_2 = 1$:$\bar{y} = \frac{1}{2} * (3+1) = 2$,$s^2 = \frac{1}{2-1}((3-2)^2 + (1-2)^2) = 2$
For $y_1 = 3, y_3 = 0$:$\bar{y} = \frac{1}{2} * (3+0) = 1.5$,$s^2 = \frac{1}{2-1}((3-1.5)^2 + (0-1.5)^2) = 4.5$
For $y_1 = 3, y_4 = 5$:$\bar{y} = \frac{1}{2} * (3+5) = 4$,$s^2 = \frac{1}{2-1}((3-4)^2 + (5-4)^2) = 2$
For $y_2 = 1, y_3 = 0$:$\bar{y} = \frac{1}{2} * (1+0) = 0.5$,$s^2 = \frac{1}{2-1}((1-0.5)^2 + (0-0.5)^2) = 0.5$
For $y_2 = 1, y_4 = 5$:$\bar{y} = \frac{1}{2} * (1+5) = 3$,$s^2 = \frac{1}{2-1}((1-3)^2 + (5-3)^2) = 8$
For $y_3 = 0, y_4 = 5$:$\bar{y} = \frac{1}{2} * (0+5) = 2.5$,$s^2 = \frac{1}{2-1}((0-2.5)^2 + (5-2.5)^2) = 12.5$

(iii) $E(\bar{y}) = \frac{2+1.5+4+0.5+3+2.5}{6} = 2.25$
$Var(\bar{y}) = \frac{1}{6^2} * (2+4.5+2+0.5+8+12.5) = 0.82$
$\mu = \frac{3+1+0+5}{4} = \frac{9}{4} = 2.25$
$E(\bar{y}) = \mu$
Thus $\bar{y}$ is unbiased estimator of $\bar{Y}$,
$Bias(\bar{y}) = 0$
$MSE(\bar{y}) = V(\bar{y}) + [Bias(\bar{y})]^2 = V(\bar{y}) = 0.82$

(iv) $E(s^2) = \frac{2+4.5+2+0.5+8+12.5}{6} = \frac{29.5}{6} = 4.916667$
$V(s^2) = \frac{(2-4.92)^2+(4.5-4.92)^2+(2-4.92)^2+(0.5-4.92)^2+(8-4.92)^2+(12.5-4.92)^2}{6} = 17.23$

(v) $\sigma^2 = \frac{\sum_{i=1}^{N}(y_i-\mu)^2}{N} = \frac{(2-2.25)^2+(1.5-2.25)^2+(4-2.25)^2+(0.5-2.25)^2+(3-2.25)^2+(2.5-2.25)^2}{6} = 3.48$
Thus $\bar{y}$ is unbiased estimators for μ, but s^2 is not unbiased for σ^2

Question 1(c)

(i) The total number of possible SRSs of size n=2 is 16 The possible sample by label is 11,12,13,14,22,23,24,33,34,44. Probability for 11,22,33,44 is $\frac{1}{16}$,probability for 12,13,14,23,24,34 is $\frac{1}{8}$

(ii) For $y_1 = 3, y_1 = 3$:$\bar{y} = \frac{1}{2} * (3+3) = 3$,$s^2 = \frac{1}{3-3}((3-3)^2 + (3-3)^2) = 0$
For $y_1 = 3, y_2 = 1$:$\bar{y} = \frac{1}{2} * (3+1) = 2$,$s^2 = \frac{1}{2-1}((3-2)^2 + (1-2)^2) = 2$
For $y_1 = 3, y_3 = 0$:$\bar{y} = \frac{1}{2} * (3+0) = 1.5$,$s^2 = \frac{1}{2-1}((3-1.5)^2 + (0-1.5)^2) = 4.5$
For $y_1 = 3, y_4 = 5$:$\bar{y} = \frac{1}{2} * (3+5) = 4$,$s^2 = \frac{1}{2-1}((3-4)^2 + (5-4)^2) = 2$
Fpr $y_2 = 1, y_2 = 1$:$\bar{y} = \frac{1}{2} * (1+1) = 1$,$s^2 = \frac{1}{2-1}((1-1)^2 + (1-1)^2) = 0$
For $y_2 = 1, y_3 = 0$:$\bar{y} = \frac{1}{2} * (1+0) = 0.5$,$s^2 = \frac{1}{2-1}((1-0.5)^2 + (0-0.5)^2) = 0.5$
For $y_2 = 1, y_4 = 5$:$\bar{y} = \frac{1}{2} * (1+5) = 3$,$s^2 = \frac{1}{2-1}((1-3)^2 + (5-3)^2) = 8$
For $y_3 = 0, y_3 = 0$:$\bar{y} = \frac{1}{2} * (0+0) = 0$,$s^2 = \frac{1}{2-1}((0-0)^2 + (0-0)^2) = 0$
For $y_3 = 0, y_4 = 5$:$\bar{y} = \frac{1}{2} * (0+5) = 2.5$,$s^2 = \frac{1}{2-1}((0-2.5)^2 + (5-2.5)^2) = 12.5$
For $y_4 = 5, y_4 = 5$:$\bar{y} = \frac{1}{2} * (5+5) = 5$,$s^2 = \frac{1}{2-1}((5-5)^2 + (5-5)^2) = 0$

(iii)

$E(\bar{y}) = \frac{2*2+1.5*2+4*2+0.5*2+3*2+2.5*2+0*4}{16} = 1.6875$
$\mu = \frac{3+1+0+5}{4} = \frac{9}{4} = 2.25$
$E(\bar{y}) = \mu + Bias(\bar{y})$=2.25-0.5625
Thus $Bias(\bar{y}) = -0.5625$
$Var(\bar{y}) = \frac{1}{16^2} * (0*4 + 2 + 2 + 4.5 + 4.5 + 2 + 2 + 0.5 + 0.5 + 8 + 8 + 12.5 + 12.5) = 0.2305$
$MSE(\bar{y}) = V(\bar{y}) + [Bias(\bar{y})]^2 = 0.2305 - 0.5625 = -0.332$

(iv) $E(s^2) = \frac{0*4+2+2+4.5+4.5+2+2+0.5+0.5+8+8+12.5+12.5}{16} = \frac{59}{16} = 3.6875$
$V(s^2) = \frac{(0-3.7)^2*4+(2-3.7)^2*4+(4.5-3.7)^2*2+(0.5-3.7)^2*2+(8-3.7)^2*2+(12.5-3.7)^2*2}{16} = 14.93$

(v) $\sigma^2 = \frac{\sum_{i=1}^{N}(y_i-\mu)^2}{N} = \frac{(0-2.25)^2*3+(2-2.25)^2*2+(1.5-2.25)^2*2+(4-2.25)^2*2+(0.5-2.25)^2*2+(3-2.25)^2*2+(2.5-2.25)^2*2}{16} =$ 1.3554
Thus $Bias(\bar{y}) = -0.5625$
Thus $\bar{y}$ is biased estimator of μ
And s^2 is not unbiased for σ^2

(d)

Sampling plan in b is better, because it is the unbiased estimator.