NYU Computer Science Bridge to Tandon Course Winter 2021

Homework 7

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1 Question 3

8.2.2 b. Answer:

1. $f = \mathcal{O}(n^3)$
Proof.
Let c=8 and $n_0 = 1$. We will show that for any $n \geq 1$, $f(n) \leq 8n^3$
For $n \geq 1, 4 \leq 4n^2$
$\therefore$f(n)=n$^3 + 3n^2 + 4 \leq n^3 + 3n^2 + 4n^2 = n^3 + 7n^2$
For $n \geq 1, 7n^2 \leq 7n^3$
$\therefore$f(n)=n$^3 + 3n^2 + 4 \leq n^3 + 7n^2 \leq n^3 + 7n^3 = 8n^3$
Therefore, $f = \mathcal{O}(n^3)$.
2. $f = \Omega(n^3)$
Proof.
Let c=1 and $n_0 = 1$. We will show that for any $n \geq 1$, $f(n) \geq n^3$
For any $n \geq 1, 3n^2 + 4 > 0$
$\therefore$f(n)=n$^3 + 3n^2 + 4 \geq n^3$
Therefore, $f = \Omega(n^3)$
Combining 1 and 2, $f = \Theta(n^3)$

8.3.5 a. Answer:

The number in the sequence that is smaller than p will all be moved to the one side of the sequence, and the number that is larger than p will be all moved to the other side of the sequence.

8.3.5 b. Answer:

It depends on the length of the sequence.
Given any input, the total number of i:=i+1 or j:=j+1 will run n-1 times.
For example, (5,4,3,2,1,0) with p=0, j:=j+1 will run 5 times, with n=6.
Similarly,(7,6,1,3,4,-1,-2) with p=0, i:=i+1 will run 2 times and j:=j+1 will run 4 times, with n=7.

8.3.5 c. Answer:

It depends on the actual values of the numbers in the sequence, and the length of the sequence.

1.Minimum: 0 times
For example, (5,4,3,2,1,0) with p=0, there will be no swap.

2.Maximum: n/2 times when n is an even number and (n-1)/2 times when n is an odd number.
If n is an even number, the maximum swap is n/2. For example, (7,6,1,-4,-1,-2) with p=0, there will be 3 swaps.
If n is an odd number, the maximum swap is (n-1)/2. For example, (7,6,1,3,-4,-1,-2) with p=0, there will be 3 swaps.

8.3.5 d. Answer:

The worst case is the number of swap is at the highest level, which means there will be n/2 or (n-1)/2 swaps, with n being the length of the sequence.
Therefore, for the while loop and the if statement, it will run (n-1)+n/2 times or (n-1)+(n-1)/2 times. The value of the sum is 3/2*n-1 or 3/2*n-3/2, which is $\Omega(n)$

8.3.5 e. Answer:

For any input of size n, the number of the while loop will execute n-1 times, and swaps will execute n/2 or (n-1)/2 times in the worst case. The total number of the entire loop is at most c*n, for some constant c. There are at most d operations performed before and after the nested loop, so the total number of operations performed is at most cn+d, which is $\mathcal{O}(n)$.

2 Question 4

5.1.1 b. Answer: $40^7 + 40^8 + 40^9$

5.1.1 c. Answer: $14 \times (40^6 + 40^7 + 40^8)$

5.3.2 a. Answer: $3 \times 2^9 = 1536$

5.3.3 b. Answer: $10 \times 26^4 \times 9 \times 8 = 329022720$

5.3.3 c. Answer: $10 \times 9 \times 8 \times 26 \times 25 \times 24 \times 23 = 258336000$

5.2.3 a. Answer:

Define the function $f : B^9 \to E_{10}$ such that if x $\in$B^9, then f(x) is obtained by adding a parity bit to the end of B^9 so that we can have an even number of 1's. eg: f(101100000)=1011000001.

Reasons for being a bijection.
1. one-to-one:
For every element in B^9, we can only add either 1 or 0 to the end of B^9 to ensure that we have an even number of 1's, and this gives E_{10}. In other words, for every element in B^9, we only have one element in E_{10} that meets the requirements of having an even number of 1's. Therefore, the function is one-to-one.
2. onto:
Every element in E_{10} begins with a prefix of 9 numbers, which is the element in B^9. Therefore, for each of the element in E_{10}, there is one element in B^9 to map it. Therefore, the function is onto.
Combining 1 and 2, this function is a bijection.

5.2.3 b. Answer:

$|E_{10}| = |B^9| = 2^9$

3 Question 5

5.4.2 a. Answer: $10^4 \times 2 = 20000$

5.4.2 b. Answer: $2 \times P(10,4) = 10080$

5.5.3 a. Answer: $2^{10} = 1024$

5.5.3 b. Answer: $2^7 = 128$

5.5.3 c. Answer: $2^7 + 2^8 = 384$

5.5.3 d. Answer: $2^8 = 256$

5.5.3 e. Answer: $\binom{10}{6} = 210$

5.5.3 f. Answer: $\binom{9}{6} = 84$

5.5.3 g. Answer: $\binom{5}{1} \times \binom{5}{3} = 50$

5.5.5 a. Answer: $\binom{35}{10} \times \binom{30}{10}$

5.5.8 c. Answer: $\binom{26}{5} = 65780$

5.5.8 d. Answer: $\binom{13}{1} \times \binom{48}{1} = 624$

5.5.8 e. Answer: $\binom{13}{1} \times \binom{4}{2} \times \binom{12}{1} \times \binom{4}{3} = 3744$

5.5.8 f. Answer: $\binom{13}{5} \times 4^5 = 1317888$

5.6.6 a. Answer: $\binom{44}{5} \times \binom{56}{5}$

5.6.6 b. Answer: $P(44,2) \times P(56,2)$

4 Question 6

5.7.2 a. Answer: $\binom{52}{5} - \binom{39}{5} = 2023203$

5.7.2 b. Answer: $\binom{52}{5} - \binom{13}{5} \times 4^5 = 1281072$

5.8.4 a. Answer: 5^{20}

5.8.4 b. Answer: $\binom{20}{4} \times \binom{16}{4} \times \binom{12}{4} \times \binom{8}{4} \times \binom{4}{4}$

5 Question 7

a. Answer: 0

As there is 5 elements in the domain and only 4 elements in the range, there does not exist a function to be one-to-one, since two elements in the domain will have the same output.

b. Answer: 120

$5 \times 4 \times 3 \times 2 \times 1 = 120$

c. Answer: 720

$6 \times 5 \times 4 \times 3 \times 2 = 720$

d. Answer: 2520

$7 \times 6 \times 5 \times 4 \times 3 = 2520$