NYU Computer Science Bridge to Tandon Course Winter 2021

Homework 1

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1 Problem 1

A.1 Answer: 155

$10011011_2 = 1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 + 1 \cdot 2^7 = 1 + 2 + 8 + 16 + 128 = 155$

A.2 Answer: 237

$456_7 = 6 \cdot 7^0 + 5 \cdot 7^1 + 4 \cdot 7^2 = 6 + 35 + 196 = 237$

A.3 Answer: 906

$38A_16 = 1 \cdot 16^0 + 1 \cdot 16^1 + 0 \cdot 16^2 = 10 + 128 + 768 = 906$

A.4 Answer: 309

$2214_5 = 4 \cdot 5^0 + 1 \cdot 5^1 + 2 \cdot 5^2 + 2 \cdot 5^3 = 4 + 5 + 50 + 250 = 309$

B.1 Answer: $\mathbf{1000101_2}$

$69_{10} = 64 + 4 + 1 = 2^6 + 2^2 + 2^0 = 1000101_2$

B.2 Answer: $\mathbf{111100101_2}$

$485_{10} = 256 + 128 + 64 + 32 + 4 + 1 = 2^8 + 2^7 + 2^6 + 2^5 + 2^2 + 2^0 = 111100101_2$

B.3 Answer: $\mathbf{110110100011010_2}$

$6D1A_{16} = 110110100011010_2$, since $6_{16} = 0110_2, D_{16} = 1101_2, 1_{16} = 0001_2, A_{16} = 1010_2$

C.1 Answer: $\mathbf{6B_{16}}$

$1101011_2 = 6B$, since $0110_2 = 6_{16}, 1011_2 = B_{16}$

C.2 Answer: $\mathbf{37F_{16}}$

$895_{10} = 15 + 112 + 768 = 15 \cdot 16^0 + 7 \cdot 16^1 + 3 \cdot 16^2 = 37F$

2 Problem 2

1 Answer: $\mathbf{14303_8}$

$$\begin{array}{cccccc} & 7 & 5 & 6 & 6 \\ + & 4_1 & 5_1 & 1_1 & 5 \\ \hline 1 & 4 & 3 & 0 & 3 \end{array}$$

2 Answer: $\mathbf{11000000_2}$

$$\begin{array}{ccccccccc} & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ + & 0 & 0_1 & 0_1 & 0_1 & 1_1 & 1_1 & 0_1 & 1 \\ \hline & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

3 Answer: $\mathbf{C02B_{16}}$

$$\begin{array}{ccccc} & 7 & A & 6 & 6 \\ + & 4_1 & 5_1 & C & 5 \\ \hline & C & 0 & 2 & B \end{array}$$

4 Answer: $\mathbf{34_5}$

$$\begin{array}{ccccc} & 3_{-1} & 0_{-1} & 2_{-1} & 2 \\ - & 2 & 4 & 3 & 3 \\ \hline & & & 3 & 4 \end{array}$$

3 Problem 3

A.1 Answer: $\mathbf{01111100_{8\,bit\,2}}$

$124_{10} = 64 + 32 + 16 + 8 + 4 = 2^6 + 2^5 + 2^4 + 2^3 + 2^2 = 01111100_{8\,bit\,2}$

A.2 Answer: $\mathbf{10000100_{8\,bit\,2}}$

$$\begin{array}{ccccccccc}
 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
+ & 1_1 & 0_1 & 0_1 & 0_1 & 0_1 & 1 & 0 & 0 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

A.3 Answer: $\mathbf{01101101_{8\,bit\,2}}$

$109_{10} = 64 + 32 + 8 + 4 + 1 = 2^6 + 2^5 + 2^3 + 2^2 + 2^0 = 01101101_{8\,bit\,2}$

A.4 Answer: $\mathbf{10110001_{8\,bit\,2}}$

$79_{10} = 64 + 8 + 4 + 2 + 1 = 2^6 + 2^3 + 2^2 + 2^1 + 2^0 = 01001111_{8\,bit\,2}$

$$\begin{array}{ccccccccc}
 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\
+ & 1_1 & 0_1 & 1_1 & 1_1 & 0_1 & 0_1 & 0_1 & 1 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

B.1 Answer: $\mathbf{30}_{10}$

$00011110_2 = 1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 = 2 + 4 + 8 + 16 = 30$

B.2 Answer: $\mathbf{-26}_{10}$

$$\begin{array}{ccccccccc}
 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
+ & 0_1 & 0_1 & 0_1 & 1_1 & 1_1 & 0_1 & 1 & 0 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$00011010_2 = 1 \cdot 2^1 + 1 \cdot 2^3 + 1 \cdot 2^4 = 2 + 8 + 16 = 26$

B.3 Answer: $\mathbf{45}_{10}$

$00101101_2 = 1 \cdot 2^0 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^5 = 1 + 4 + 8 + 32 = 45$

B.4 Answer: -98$_{10}$

$$\begin{array}{ccccccccc} & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ + & 0_1 & 1_1 & 1_1 & 0_1 & 0_1 & 0_1 & 1_1 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$1100010_2 = 1 \cdot 2^1 + 1 \cdot 2^5 + 1 \cdot 2^6 = 2 + 32 + 64 = 98$

4 Problem 4

1.2.4.b Answer:

p	q	$\neg(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

1.2.4.c Answer:

p	q	r	$r \vee (p \wedge \neg q)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

1.3.4.b Answer:

p	q	$(p \to q) \to (q \to p)$
T	T	T
T	F	T
F	T	F
F	F	T

1.3.4.d Answer:

p	q	$(p \leftrightarrow q) \bigoplus (p \leftrightarrow \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T

5 Problem 5

1.2.7.b Answer: $(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$

1.2.7.c Answer: $B \vee (D \wedge M)$

1.3.7.b Answer: $(s \vee y) \rightarrow p$

1.3.7.c Answer: $p \rightarrow y$

1.3.7.d Answer: $p \leftrightarrow (s \wedge y)$

1.3.7.e Answer: $p \rightarrow (s \vee y)$

1.3.9.c Answer: $c \rightarrow p$

1.3.9.d Answer: $c \rightarrow p$

6 Problem 6

1.3.6.b Answer: If Joe is eligible for the honors program, then Joe maintains a B average.

1.3.6.c Answer: If Rajiv can go on the roller coaster, then he is at least four feet tall.

1.3.6.d Answer: If Rajiv is at least four feet tall, then he can go on the roller coaster.

1.3.10.c Answer: False.

The left hand side expression is true and the right hand side expression is false.

1.3.10.d Answer: Unknown.

If r is true, then the expression is false and if r is false, then the expression is true.

1.3.10.e Answer: Unknown.

If r is true, then the expression is true and if r is false, then the expression is false.

1.3.10.f Answer: True.

The hypothesis is false.

7 Problem 7

1.4.5.b Answer:

$\neg j \to (l \vee \neg r)$
$(r \wedge \neg l) \to j$
Logically equivalent

j	l	r	$\neg j \to (l \vee \neg r)$	(r $\wedge \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

1.4.5.c Answer:

j $\to \neg l$
$\neg j \to l$
Not logically equivalent

j	l	j $\to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

1.4.5.d Answer:

(r $\vee \neg l) \to j$
$j \to (r \wedge \neg l)$
Not logically equivalent

j	l	r	(r $\vee \neg l) \to j$	j $\to (r \wedge \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

8 Problem 8

1.5.2.c Answer:

(p $\rightarrow q) \wedge (p \rightarrow r)$	
$(\neg p \vee q) \wedge (p \rightarrow r)$	conditional identity
$(\neg p \vee q) \wedge (\neg p \vee r)$	conditional identity
$\neg p \vee (q \wedge r)$	distributive law
p $\rightarrow (q \wedge r)$	conditional identity

1.5.2.f Answer:

$\neg(p \vee (\neg p \wedge q))$	
$\neg p \wedge \neg(\neg p \wedge q)$	De Morgan's Law
$\neg p \wedge (\neg\neg p \vee \neg q)$	De Morgan's Law
$\neg p \wedge (p \vee \neg q)$	double negation law
$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$	distributive law
F $\vee(\neg p \wedge \neg q)$	complement law
$\neg p \wedge \neg q$	identity law

1.5.2.i Answer:

(p $\wedge q) \rightarrow r$	
$\neg(p \wedge q) \vee r$	conditional identity
$\neg p \vee \neg q \vee r$	De Morgan's Law
$\neg p \vee r \vee \neg q$	commutative law
$(\neg p \vee r) \vee \neg q$	associative law
$(\neg p \vee \neg\neg r) \vee \neg q$	double negation law
$\neg(p \wedge \neg r) \vee \neg q$	De Morgan's law
(p $\wedge\neg r) \rightarrow \neg q$	conditional identity

1.5.3.c Answer:

$\neg r \vee (\neg r \rightarrow p)$	
$\neg r \vee (\neg\neg r \vee p)$	conditional identity
$\neg r \vee (r \vee p)$	double negation law
$\neg r \vee r \vee p$	associative law
T $\vee p$	complement law
T	domination law

1.5.3.d Answer:

$\neg(p \to q) \to \neg q$	
$\neg(\neg p \vee q) \to \neg q$	conditional identity
$\neg\neg(\neg p \vee q) \vee \neg q$	conditional identity
$(\neg p \vee q) \vee \neg q$	double negation law
$\neg p \vee q \vee \neg q$	associative law
$\neg p \vee (q \vee \neg q)$	associative law
$\neg p \vee T$	complement law
T	domination law

9 Problem 9

1.6.3.c Answer:

$\exists x(x = x^2)$

1.6.3.d Answer:

$\forall x(x <= x^2)$

1.7.4.b Answer:

$\forall x\ (\neg S(x) \wedge W(x))$

1.7.4.c Answer:

$\forall x\ (S(x) \rightarrow \neg W(x))$

1.7.4.d Answer:

$\exists x\ (S(x) \wedge W(x))$

10 Problem 10

1.7.9.c Answer:True

1.7.9.d Answer:True

1.7.9.e Answer:True

1.7.9.f Answer:True

1.7.9.g Answer:False. Counter-example: c

1.7.9.h Answer:True

1.7.9.i Answer:True

1.9.2.b Answer:True, Q(2, 1), Q(2, 2) and Q(2, 3) are all true .

1.9.2.c Answer:True, P(1, 1), P(1, 2) and P(1, 3) are all true .

1.9.2.d Answer:False, all S(x,y) are false

1.9.2.e Answer:False, There is no y such that any of Q(1, y), Q(2, y) and Q(3, y) is true

1.9.2.f Answer:True, for any x, P(x,1) is true

1.9.2.g Answer:False, P(3,3) is false.

1.9.2.h Answer:True, Q(2,2) is true

1.9.2.b Answer:True, $\neg Q(x, y)$ is always true

11 Problem 11

1.10.4.c Answer:

$\exists x \exists y\ (x + y = xy)$

1.10.4.d Answer:

$\forall x \forall y\ ((x > 0) \wedge (y > 0) \rightarrow (x/y > 0))$

1.10.4.e Answer:

$\forall x\ ((x < 1) \wedge (x > 0) \rightarrow (1/x > 1))$

1.10.4.f Answer:

$\forall x \exists y\ (y < x)$

1.10.4.g Answer:

$\forall x \exists y\ (\neg(x = 0) \rightarrow xy = 1)$

1.10.7.c Answer:

$\exists x\ (D(x) \wedge N(x))$

1.10.7.d Answer:

$\forall x\ (D(x) \rightarrow P(Sam, x))$

1.10.7.e Answer:

$\exists x \forall y\ (N(x) \wedge P(x, y))$

1.10.7.f Answer:

$\exists x \forall y\ ((D(x) \wedge N(x)) \wedge ((D(y) \wedge N(y)) \rightarrow (y = x)))$

1.10.10.c Answer:

$\forall x \exists y\ (T(x, \mathrm{Math101}) \wedge T(x, y) \wedge \neg(y = \mathrm{Math101}))$

1.10.10.d Answer:

$\exists x \forall y\ (\neg(y = \mathrm{Math101}) \rightarrow T(x, y))$

1.10.10.e Answer:

$\forall x \exists y \exists z\ (\neg(x = \mathrm{Sam}) \rightarrow (T(x, y) \wedge T(x, z) \wedge \neg(y = z)))$

1.10.10.f Answer:

$\exists x \exists y \forall z\ (T(\mathrm{Sam}, x) \wedge T(\mathrm{Sam}, y) \wedge \neg(x = y) \wedge T(Sam, z) \rightarrow (z = x) \vee (z = y))$

12 Problem 12

1.8.2.b Answer:

$\forall x\ (P(x) \lor D(x))$
Negation: $\neg\forall x\ (P(x) \lor D(x))$
Applying De Morgan's Law: $\exists x\ (\neg P(x) \land \neg D(x))$
English: Some patient was given neither medication nor placebo.

1.8.2.c Answer:

$\exists x\ (D(x) \land M(x))$
Negation: $\neg\exists x\ (D(x) \land M(x))$
Applying De Morgan's Law: $\forall x\ (\neg D(x) \lor \neg M(x))$
English: For all patients, either they were not given medication or they did not have migraines.

1.8.2.d Answer:

$\forall x\ (\neg P(x) \lor M(x))$
Negation: $\neg\forall x\ (\neg P(x) \lor M(x))$
Applying De Morgan's Law: $\exists x\ (P(x) \land \neg M(x))$
English: Some patient had placebo but did not have migraines.

1.8.2.e Answer:

$\exists x\ (P(x) \land M(x))$
Negation: $\neg\exists x\ (P(x) \land M(x))$
Applying De Morgan's Law: $\forall x\ (\neg P(x) \lor \neg M(x))$
English: For all patients, either they were not given placebo or they did not have migraines.

1.9.4.c Answer:

$\forall x\exists y\ (P(x,y) \land \neg Q(x,y))$

1.9.4.d Answer:

$\forall x\exists y\ (P(x,y) \land \neg P(y,x)) \lor (P(y,x) \land \neg P(x,y))$

1.9.4.e Answer:

$\forall x\forall y\ (\neg P(x,y)) \lor \exists x\exists y\ (\neg Q(x,y))$