NYU Computer Science Bridge to Tandon Course	Winter 2021

Homework 6

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1 Question 5

a. Answer:

Proof.
Since n ≥ 0,
$5\text{n}^3 + 2n^2 + 3n \leq 5n^3 + 2n^2 + 3n^2$ (if n ≥ 1) $= 5\text{n}^3 + 5n^2 \leq 5n^3 + 5n^3 = 10n^3$
$5n^3 + 2n^2 + 3n \geq 5n^3$
Thus, $5n^3 \leq 5n^3 + 2n^2 + 3n \leq 10n^3$ when $n \geq 1$
Let f(n) $= 5n^3 + 2n^2 + 3n$, g(n) $= n^3$
Thus, $5 * g(n) \leq f(n) \leq 10 * g(n)$ when $n \geq 1$
Since there exists $c_1 = 10$, $c_2 = 5$, $n_0 = 1$, and for all n $\geq \text{n}_0$, we have $5 * g(n) \leq f(n) \leq 10 * g(n)$, so $5n^3 + 2n^2 + 3n = \Theta(n^3)$ ■

b. Answer:

Proof.
Since ≥ 0,
$\sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n} \leq \sqrt{7n^2 + 2n^2} = 3n$ (when n ≥ 1)
$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7n^2} = \sqrt{7}n$ (when $2n - 8 \geq 0$, namely $n \geq 4$)
Let f(n) $= \sqrt{7n^2 + 2n - 8}$, g(n)= n
Thus, $\sqrt{7} * g(n) \leq f(n) \leq 3 * g(n)$ when $n \geq 4$
Since there exists $c_1 = 3$, $c_2 = \sqrt{7}$, $n_0 = 4$, and for all n $\geq \text{n}_0$, we have $\sqrt{7} * g(n) \leq f(n) \leq 3 * g(n)$, so $\sqrt{7n^2 + 2n - 8} = \Theta(n)$ ■