Estimating and Evaluating the Stochastic Volatility Model for Bitcoin

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Abstract. *Academic literature on Bitcoin although scant, suggests that the GARCH(1,1) model does a fair job of estimating Bitcoin volatility. However, there is little or no literature on the performance of stochastic volatility models on the same, even though this approach is often used to model currency volatility (Bates, 1996; Kastner et al, 2014). Stochastic volatility is therefore modeled for Bitcoin within the Bayesian framework, only to discover the superiority of the GARCH(1,1) model. Rolling-window GARCH(1,1) forecasts are also used to model Bitcoin volatility and this model too is found to outperform the stochastic volatility model.*

1. Introduction

Bitcoin is a peer-to-peer electronic cash transactions system that was coded up by the programmer(s) who went by the pseudonym Satoshi Nakamoto. Transactions are verified by network nodes and recorded in a public distributed ledger called the blockchain.

Bitcoin production is based on a fixed rate, controlled by an algorithm that expires in 2040. The algorithm is programmed to create a complex problem every ten minutes, approximately. Individuals, called Miners attempt to solve these problems.

Solving the problem by the miner also involves sorting all the transactions done during a specific time frame in a meaningful manner. Miners are also rewarded with Bitcoins. Financial markets trade the crypto currency alongside other reputable national currencies. Some nations, like Germany and India, have even declared Bitcoins to be a legal currency.

The price of Bitcoin has been fluctuating heavily from the time of its inception till date. In 2012 it was trading at \$5. By 2013 end it was at $\sim$ \$1200, and today it trades at \$1750. Bitcoin is quite similar to Gold, in that it has little intrinsic value and instead derives its value from the fact that it is scarce and difficult to mine. Understanding the price volatility therefore, is worth investigating.

2. Literature Review

Bouoiyour and Selmi (2014) use cointergration to test the relationship between the number of transactions and the price of Bitcoins using the ARDL bounds testing approach. They perform several estimations, across multiple samples, and find that the relationship between the number of transactions and the price of Bitcoins is significant and positive across all periods.

Kristoufek (2013) suggests that Bitcoins can be explained solely by speculative bubbles.

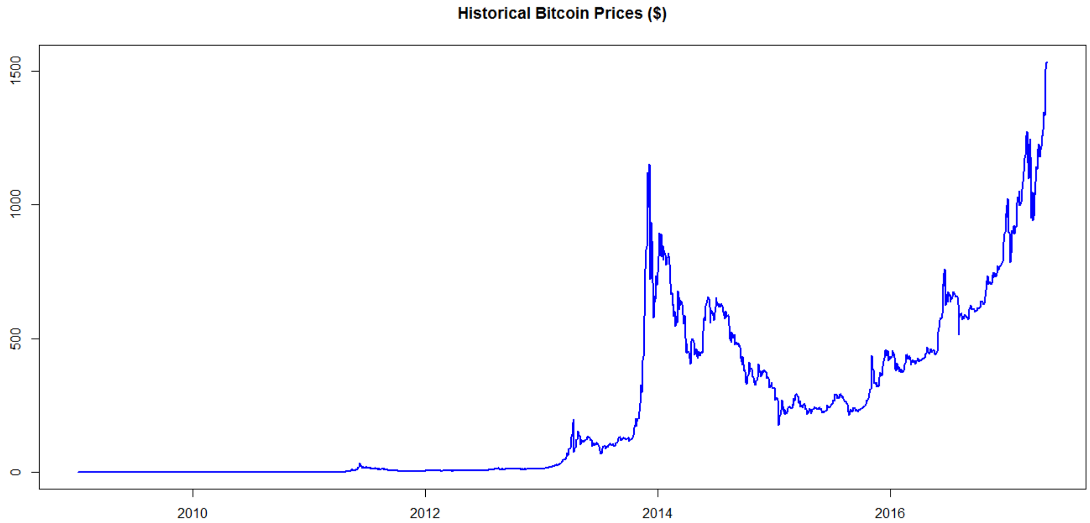
Tuck and Fry (2015) also claim that the fluctuations in Bitcoin prices can be explained entirely by speculative bubbles.

Anne Haubo Dyhrberg (2015) does a GARCH volatility analysis.

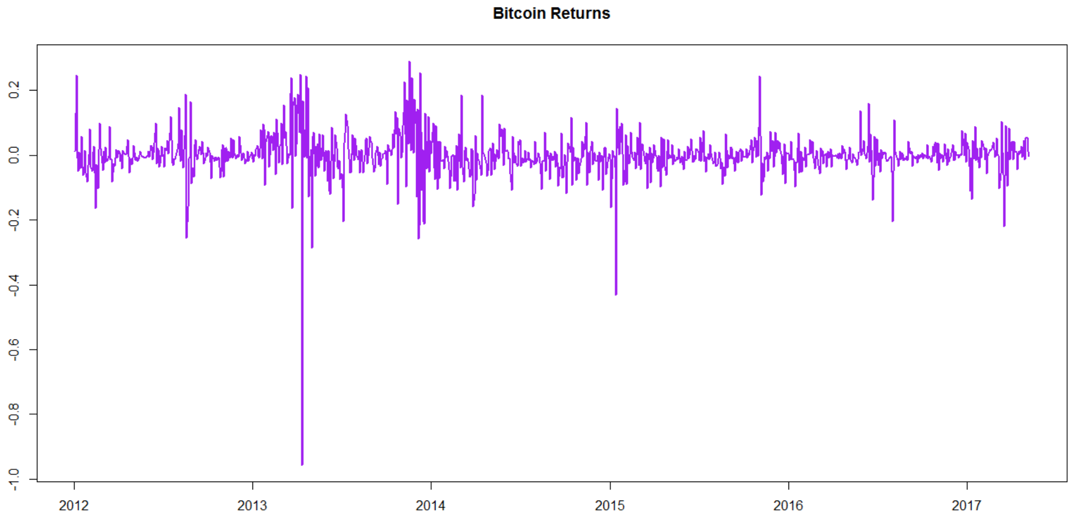
Interestingly enough, as pointed out in **Brian Vockathaler (2015)**, most studies on Bitcoin were written during or right after a 53-day period in which the value of Bitcoin ballooned to $1200 from $100.

3. Data and Empirical Analysis

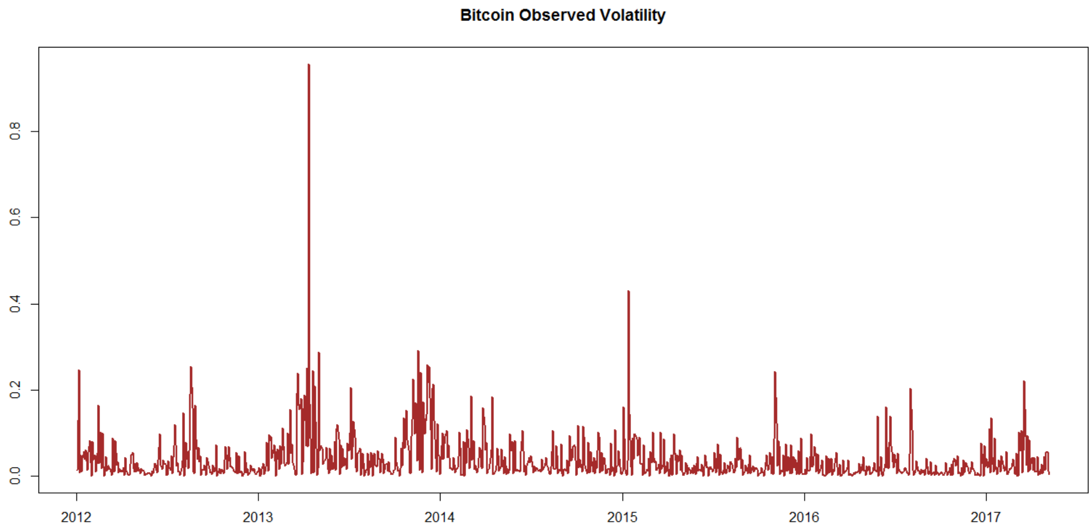
Bitcoin data on prices, daily volume traded on the network, transactions per day, etc. are available freely on https://blochchain.info



For the purpose of this analysis, the data from 2012 till date has been used. The returns for this period are stationary at the 1% level of significance according to the ADF test and the Phillips-Perron Unit Root Test.



The deterministic GARCH(1,1) model has been shown to explain Bitcoin volatility pretty well (Vockathaler, 2015). However there is no literature on modeling stochastic volatility of Bitcoin returns. The observed volatility in Bitcoin returns is shown below.



This paper draws on Kastner and Fruhwirth-Schnatter (2014) to perform a stochastic volatility analysis, which entails specifying prior distributions and configuration parameters (as per choices in literature), running a Markov Chain Monte Carlo (MCMC) sampler, and obtaining draws from the posterior distribution of parameters and latent variables to be then be used for predicting future volatility.

SV equations

$$r_t - \mu_t = e_t$$

$$e_t | h_t \sim N(0, \exp(h_t))$$

$$h_t | h_{t-1}, \mu, \phi, \sigma_\eta \sim N(\mu + \phi(h_{t-1} - \mu), \sigma_\eta^2)$$

$$h_0|\mu, \phi, \sigma_\eta \sim N(\mu, \frac{\sigma_\eta^2}{1-\phi^2})$$

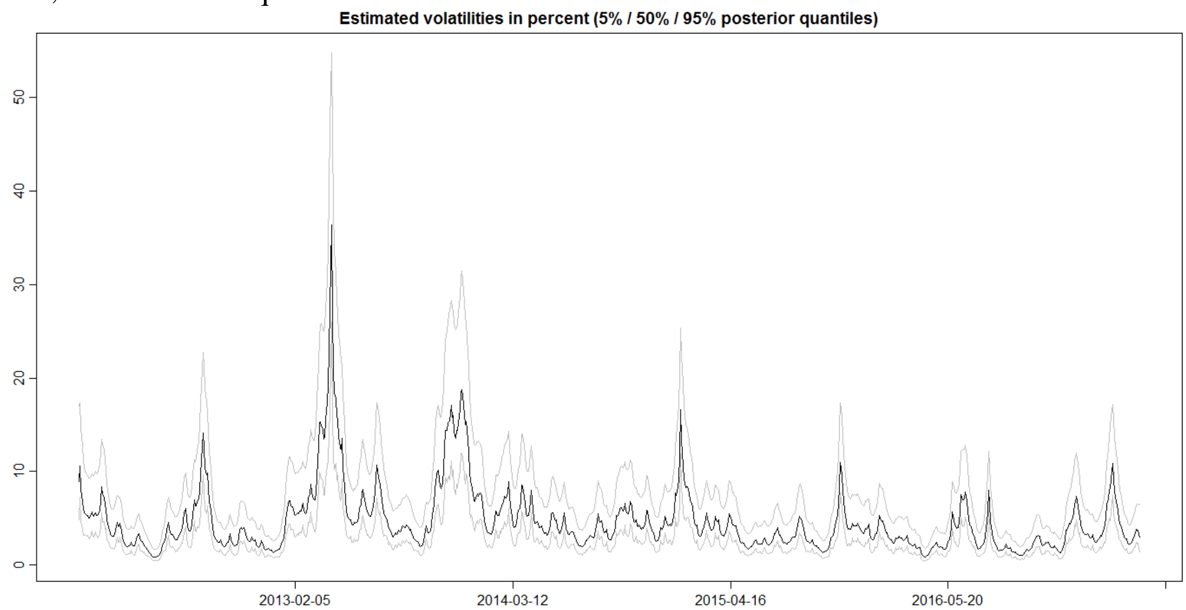
A prior distribution for the parameter vector θ has to be specified. As per Kim et al (1998) parameters are taken to be independent such that $p(\theta) = p(\mu)p(\phi)p(\sigma_\eta)$.
The persistance parameter $\phi \in (-1, 1)$ is chosen such that $(\phi + 1)/2 \sim B(a_0, b_0)$ implying

$$p(\phi) = \frac{1}{2B(a_0, b_0)}(\frac{1+\phi}{2})^{a_0-1}(\frac{1-\phi}{2})^{b_0-1}$$

where a_0 and b_0 are positive hyperparameters and $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ denotes the Beta function. The hyperparameters are set to $a_0 = 20, b_0 = 1.5$ in the context of this analysis, and are tuned to minimize RMSE of estimated volatility.

SV Results

This being a Bayesian approach, probablistic volatility estimates are shown for 5%, 50% and 95% quantiles.



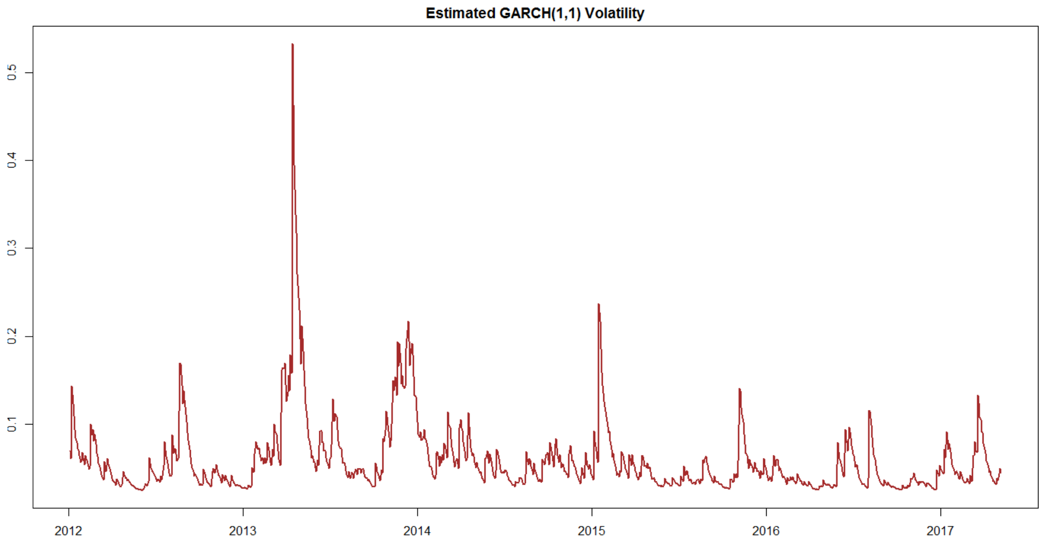
Stochastic Volatility Summary Statistics

Posterior draws of parameters (thinning = 1):

	mean	sd	5%	50%	95%	ESS
mu	-6.665	0.2354	-7.05	-6.657	-6.307	1144
phi	0.909	0.0228	0.87	0.910	0.944	384
sigma	0.592	0.0648	0.49	0.590	0.703	241
exp(mu/2)	0.036	0.0041	0.03	0.036	0.043	1144
sigma^2	0.355	0.0775	0.24	0.348	0.494	241

GARCH(1,1) Results

The GARCH model transforms the standard OLS residuals into an endogenous process that allows its variance to vary across periods. The GARCH(1,1) model fitted on the log returns have been documented in literature to provide a good fit of Bitcoin's volatility. The GARCH volatility estimates and summary statistics are shown below.



GARCH(1,1) Summary Statistics

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Coefficient(s):
         mu        omega       alpha1        beta1
-5.8516e-18   1.5427e-04   2.8975e-01   7.2723e-01

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu     -5.852e-18   1.328e-03    0.000        1
omega   1.543e-04   3.417e-05    4.514 6.35e-06 ***
alpha1  2.897e-01   3.977e-02    7.285 3.22e-13 ***
beta1   7.272e-01   2.765e-02   26.303  < 2e-16 ***
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 1511.723    normalized:  1.548897
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Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  2358.182  0
 Shapiro-Wilk Test  R    W      0.9105892 0
 Ljung-Box Test     R    Q(10)  32.64468  0.00031247
 Ljung-Box Test     R    Q(15)  40.5947   0.0003685824
 Ljung-Box Test     R    Q(20)  47.24959  0.000541565
 Ljung-Box Test     R^2  Q(10)  15.66809  0.1095361
 Ljung-Box Test     R^2  Q(15)  18.08838  0.2580477
 Ljung-Box Test     R^2  Q(20)  19.73359  0.474702
 LM Arch Test       R    TR^2   17.93885  0.1175599

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-3.089597 -3.069582  -3.08963  -3.081981
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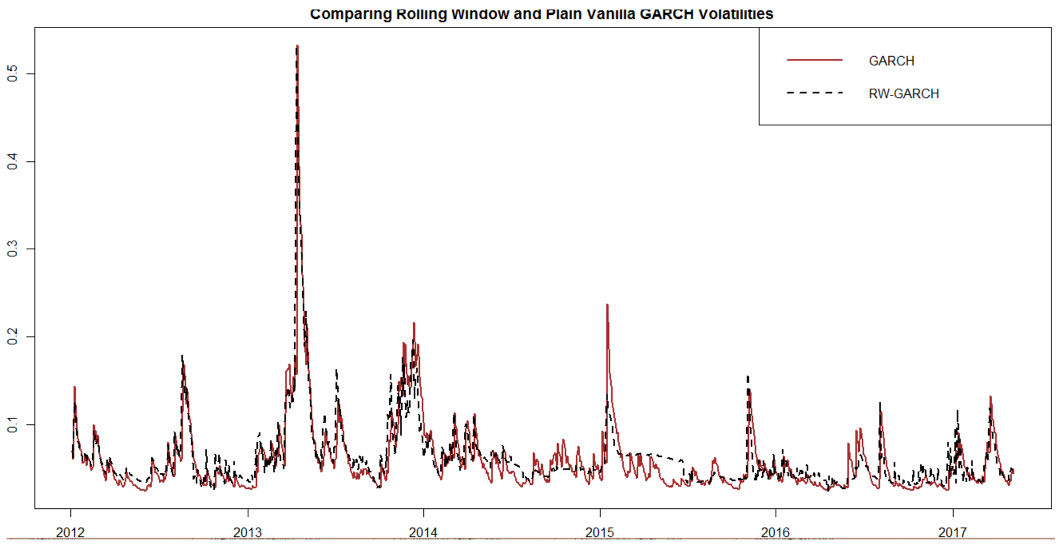
Rolling Window GARCH(1,1) Results

In addition to the above analysis, rolling window estimates of volatility have been calculated with an appropriately chosen window size (200) to account for seasonality in the data. This is a procedure wherein one-step ahead volatility is predicted using a GARCH(1,1) model fitted on the most recent 200 lags and iteratively appended to a vector of estimated volatility till the penultimate data point in the series.

Later in this analysis we calculate RMSE estimates for each method employed and window size is tuned to minimize RMSE in order to get optimal window size. The estimates from the plain-vanilla GARCH and the rolling-window GARCH are almost indistinguishable from each other upon visual inspection.



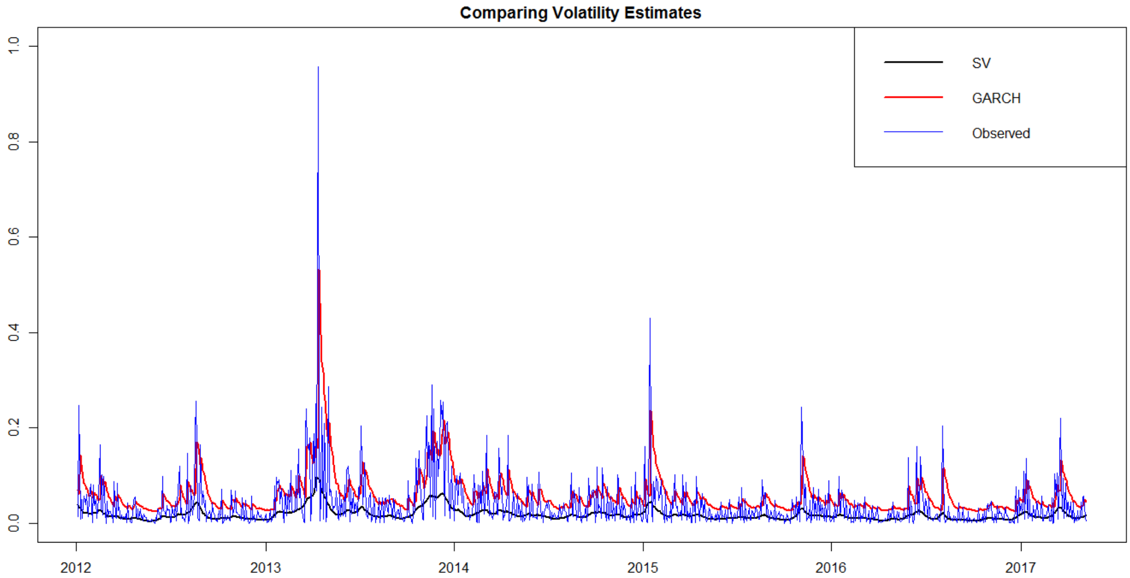
A Comparison of Methods

Finally, the GARCH(1,1) (and its rolling-window variant) are compared with the SV model by measuring the root mean squared errors for the respective models.

$$RMSE = \sqrt{\frac{1}{N}\sum(\hat{\sigma_i} - \sigma_i)^2}$$

where σ_i is the squared return for day i.

Model	RMSE
Stochastic Volatility Model	0.129
GARCH(1,1)	0.027
RW GARCH(1,1)	0.053



For the purpose of calculating RMSE of the SV model, the 50% quantile of the Bayesian volatility estimates are used. A visual inspection of the above volatility estimates clearly show the GARCH fit to be closer to observed volatility than the SV fit.The RMSE estimates for each model confirm the same.

4. Conclusions

Stochastic volatility models, unfortunately do not explain Bitcoin's volatility as well as the time-tested GARCH(1,1). In fact for the given data, the GARCH model does better than SV by a factor of ~ 4.8. However, it must be noted that since these results are based on 50% quantile estimates of stochastic volatility. The superiority of the GARCH(1,1) model nevertheless remains uncontested.

Between the plain-vanilla GARCH and a rolling window GARCH, the difference in the RMSE is a factor of 2. Given that the rolling-window estimates are a vector of predicted values, to expect its RMSE to outperform GARCH(1,1) fit on the entire data is meaningless. Nevertheless, the narrowness of this difference points to the robustness of GARCH(1,1) as a model for forecasting Bitcoin volatility.

5. References

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