# Atomic Spin Dynamics: Motion

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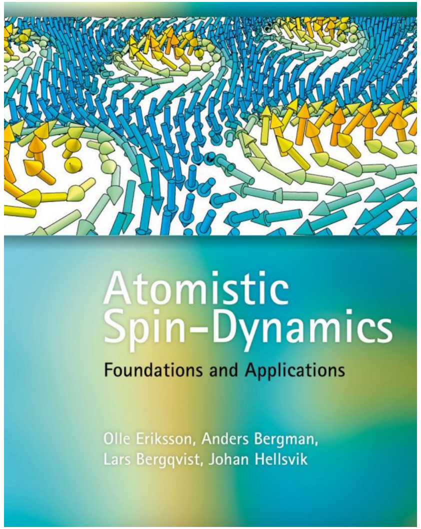


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**AIM:** To understand the contributions to the equations of motion for atomic magnetic moments and how the equations of motion describe magnetization dynamics in simple mono-layer systems.

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# Landau-Lifshitz-Gilbert Eq. (LLG)

AKA: Our EoM for atomic moments

$$\frac{d\mathbf{m}_i}{dt} = -\gamma_L \mathbf{m}_i \times (\mathbf{B}_i + \mathbf{B}_i^{\mathrm{fl}}) - \gamma_L \frac{\alpha}{m_i} \mathbf{m}_i \times [\mathbf{m}_i \times (\mathbf{B}_i + \mathbf{B}_i^{\mathrm{fl}})]$$

$m_i$ - magnetic moment of an atom at site (i) in the lattice

$\gamma_L$ - Renormalized gyromagnetic ratio derived as $= \dfrac{\gamma}{(1+\alpha^2)}$

$B_i$, $B_i^{fl}$ - Effective and stochastic (fl) magnetic field experience at site i by the atomic moment

α (scalar) - Gilbert damping constant

# Stochastic Magnetic Field ($B_i^{fl}$)

$B_i^{fl}$ describes temperature effects in the theory of atomistic spin dynamics in a Langevin dynamics approach, $B_i^{fl}$ can be referred to as a Langevin force.

Its inclusion in the LLG allows for description of “the dynamics of the natural building block of any material, atoms and their magnetic moments.” (p56)

Excluding the Langevin force can decompose LLG into the micromagnetic Landau-Lifshitz Eq. (MMLL), which describes magnetization density with a vector field.

# Damping

$$\frac{d\mathbf{m}_i}{dt} = -\gamma_L \mathbf{m}_i \times (\mathbf{B}_i + \mathbf{B}_i^{\mathrm{fl}}) - \gamma_L \frac{\alpha}{m_i} \mathbf{m}_i \times [\mathbf{m}_i \times (\mathbf{B}_i + \mathbf{B}_i^{\mathrm{fl}})]$$

Mechanisms describing dissipation of energy and angular momentum.

Understood to have emerged from a microscopic theory of quantum mechanical operators. For example: $\frac{d\langle\hat{\mathbf{s}}\rangle}{dt} = \langle\hat{\mathbf{s}}\rangle \times \mathbf{B}_{\mathrm{eff}} - \lambda\langle\hat{\mathbf{s}}\rangle \times (\langle\hat{\mathbf{s}}\rangle \times \mathbf{B}_{\mathrm{eff}})$ , the EoM for the expectation value of the spin operator. First term being the precessional and the second the damping term.

# Landau-Lifshitz Eq. (LL)

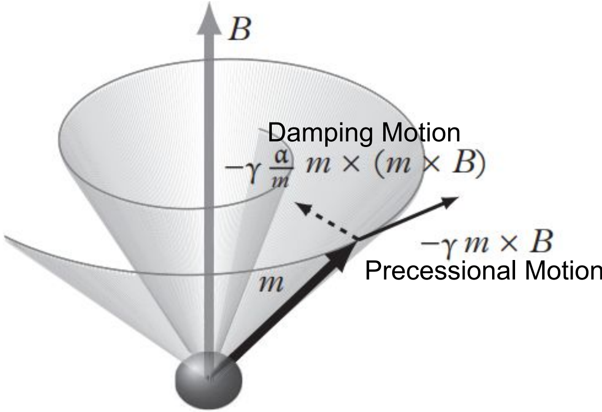
$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{B} - \frac{\lambda}{m} \mathbf{m} \times (\mathbf{m} \times \mathbf{B})$$

λ (parameter): determined by the damping torque being perpendicular to the precession torque

Note:

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{B} + \frac{\alpha}{m} \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}$$

As for the Landau–Lifshitz damping torque, The Gilbert damping torque is perpendicular to the precession torque, similar to the LL damping torque. At the small damping limit, solutions to the LL and the LLG equations are close. For large damping, the difference between LL and LLG solutions is greater.

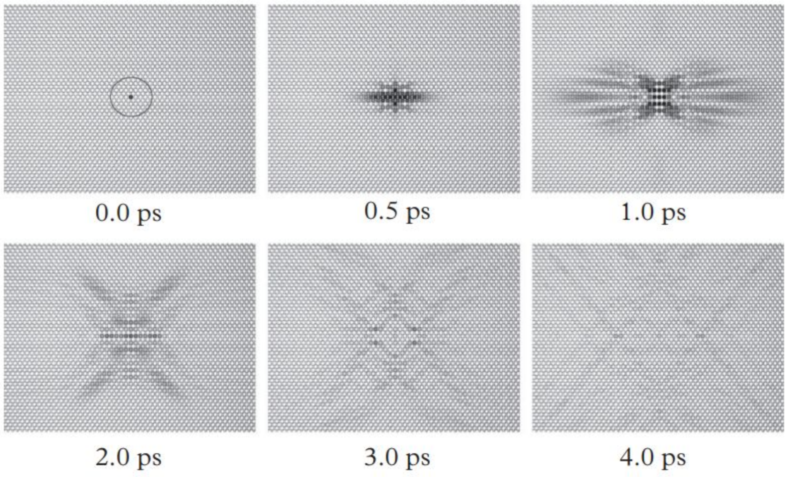


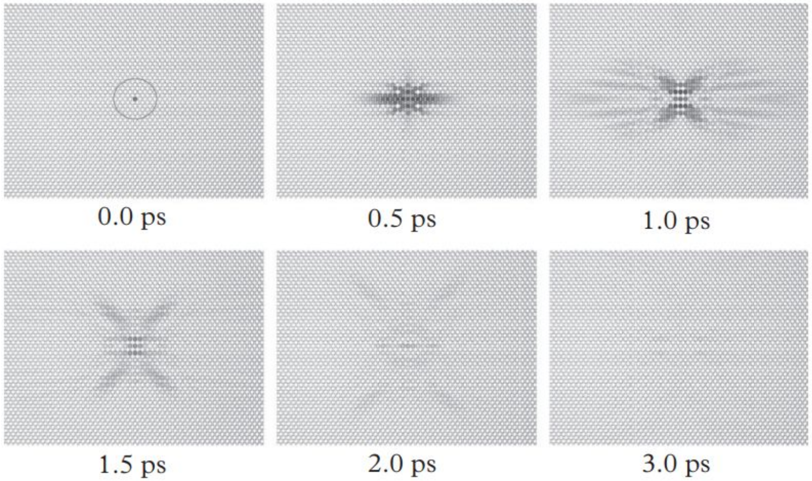

Motion described by LL

# Monolayer of Fe on a W(110) surface

A display of how precession torque and the damping term of the LL equation contribute to the time evolution of magnetism

Initially, this ferromagnetic system has one of its atomic spins reversed to align antiparallel to its neighbours




No damping included, the system does not relax to the equilibrium ferromagnetic state, but the excitation gets more and more spread out across the system

Finite damping included, the system relaxes to the equilibrium ferromagnetic state

# Connecting the LL and LLG Equations

- Both preserve the magnitude of magnetization.
- With isotropic damping, they are identical if a renormalized gyromagnetic ratio is introduced.
- For isotropic damping, the Landau–Lifshitz and the Landau–Lifshitz–Gilbert equation differ only in the regard that $\gamma_L = \gamma$
- In the limit of vanishing damping, $\alpha \to 0$, the gyromagnetic ratios become equal, $\gamma_L \to \gamma$

$$\lambda = \frac{\gamma\alpha}{(1+\alpha^2)} = \gamma_L \alpha;$$

Using the renormalized gyromagnetic ratio and expressing the LL relaxation parameter λ in units of the Gilbert damping parameter α, we can express the LL as so

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma_L \mathbf{m} \times \mathbf{B} - \gamma_L \frac{\alpha}{m} \mathbf{m} \times (\mathbf{m} \times \mathbf{B})$$

# Wrapping Up

- Damping effect on magnetic moments
- LLG Master Eq. for Spin Dynamics
- LL EoM for precessional motion and damping effects on magnetic moment