

## Small-step evaluation rules for the $L23_\tau$ internal language

**Corrections:** The boxed rules for conditionals have been corrected; Z and SZ were exchanged in the  $t_1$  position.

$$\frac{t_1 \rightarrow t_1'}{\mathbf{S}t_1 \rightarrow \mathbf{S}t_1'} \quad \overline{[\mathbf{Z} + t_2] \rightarrow t_2} \quad \overline{[\mathbf{S}v_1 + t_2] \rightarrow [v_1 + \mathbf{S}t_2]} \quad \frac{t_1 \rightarrow t_1'}{[t_1 + t_2] \rightarrow [t_1' + t_2]}$$

$$\overline{[\mathbf{Z} - v_2] \rightarrow \mathbf{Z}} \quad \overline{[v_1 - \mathbf{Z}] \rightarrow v_1} \quad \overline{[\mathbf{S}v_1 - \mathbf{S}v_2] \rightarrow [v_1 - v_2]}$$

$$\frac{t_1 \rightarrow t_1'}{[t_1 - t_2] \rightarrow [t_1' - t_2]} \quad \frac{t_2 \rightarrow t_2'}{[v_1 - t_2] \rightarrow [v_1 - t_2']}$$

$$\overline{[\mathbf{Z} < \mathbf{Z}] \rightarrow \mathbf{Z}} \quad \overline{[\mathbf{Z} < \mathbf{S}v_2] \rightarrow \mathbf{SZ}} \quad \overline{[v_1 < \mathbf{Z}] \rightarrow \mathbf{Z}} \quad \overline{[\mathbf{S}v_1 < \mathbf{S}v_2] \rightarrow [v_1 < v_2]}$$

$$\frac{t_1 \rightarrow t_1'}{[t_1 < t_2] \rightarrow [t_1' < t_2]} \quad \frac{t_2 \rightarrow t_2'}{[v_1 < t_2] \rightarrow [v_1 < t_2']}$$

$$\overline{[\mathbf{Z} == \mathbf{Z}] \rightarrow \mathbf{SZ}} \quad \overline{[\mathbf{Z} == \mathbf{S}v_2] \rightarrow \mathbf{Z}} \quad \overline{[\mathbf{S}v_1 == \mathbf{Z}] \rightarrow \mathbf{Z}} \quad \overline{[\mathbf{S}v_1 == \mathbf{S}v_2] \rightarrow [v_1 == v_2]}$$

$$\overline{[(v_1, v_2) == (v_3, v_4)] \rightarrow [[v_1 == v_3]?[v_2 == v_4] : \mathbf{Z}]}$$

$$\frac{t_1 \rightarrow t_1'}{[t_1 == t_2] \rightarrow [t_1' == t_2]} \quad \frac{t_2 \rightarrow t_2'}{[v_1 == t_2] \rightarrow [v_1 == t_2']}$$

$$\boxed{\overline{[\mathbf{SZ} ? t_2 : t_3] \rightarrow t_2}} \quad \boxed{\overline{[\mathbf{Z} ? t_2 : t_3] \rightarrow t_3}} \quad \frac{t_1 \rightarrow t_1'}{[t_1 ? t_2 : t_3] \rightarrow [t_1' ? t_2 : t_3]}$$

$$\frac{t_1 \rightarrow t_1'}{(t_1, t_2) \rightarrow (t_1', t_2)} \quad \frac{t_2 \rightarrow t_2'}{(v_1, t_2) \rightarrow (v_1, t_2')}$$

$$\frac{t_1 \rightarrow t_1'}{1\#t_1 \rightarrow 1\#t_1'} \quad \frac{t_1 \rightarrow t_1'}{2\#t_1 \rightarrow 2\#t_1'} \quad \overline{1\#(v_1, v_2) \rightarrow v_1} \quad \overline{2\#(v_1, v_2) \rightarrow v_2}$$