Mount Kenya University

P.O. Box 342-01000 Thika

Email: Info@mku.ac.ke

Web: www.mku.ac.ke

# DEPARTMENT OF MANAGEMENT

# COURSE CODE: BMA3201

# COURSE TITLE: OPERATIONS RESEARCH

*Instructional Material for BBM - Distance Learning*

## COURSE OUTLINE

**Pre-requisites BMA3201**

Purpose: To introduce quantitative methods to solve problems encountered in management of business

**Course Objectives**: By the end of the course unit the student will be able to:

- Describe importance and application of operations research in management of business
- Use operations research methods to solve a number of classical business management problems
- Develop algorithms based on operations research problems solving methods
- Demonstrate competence in the applications research in business management

**Course Content:**

**1. Introduction - Week1&2**

- Factors facilitating the growth of operation research
- Definitions of operations research
- Characteristics of O.R
- Problems encountered when applying O.R techniques
- Modeling in Operational research

**2. Transportation - Week 3**

- Transportation technique
- Unequal availability and requirement quantities
- Maximization and the transportation technique
- Degeneracy and the transportation technique
- Dealing with degeneracy

**3. Games Theory - Week 4**

- The prisons dilemma
- Zero-sum games
- Mixed strategies

- The queuing game

**4. Network Analysis - Week 5 & 6**

- Introduction and terminology
- Time Analysis
- Cost Scheduling
- Resource Scheduling
- Activity on nodes

**5. Queuing System and Models - Week 7**

- Modes of arrivals
- Timing of arrivals
- General schematic representative of queuing process
- Behavior factors in queues
- Classification of queuing models

**6. Cat - Week 8**

**7. Inventory Control Models - Week 9 & 10**

- Terminology
- Types of control system
- Economic order Quantity
- Safety stock and Reorder levels

**8. Sequencing Models/Assignment - Week 11**

- Three conditions of assignment problem
- Assignment method
- Method for determining an optimum solution in an assignment problem
- Integer programming
- Dynamic programming

**9. Simulation - Week 12 & 13**

- Definition
- Why is simulation used
- Business models
- Assessing a models suitability
- Random selection
- Status variable

**10. Main Exam - Week 14**

**Recommended Text Books:**

- Thierauf and Klekamp (1990), *Decision Making through Operations Research*, McGraw Hill London.
- Taha H.A (1992), *Operations Research an Introduction*, Macmillan London.

*Module Author: Emma Anyika*

# TABLE OF CONTENT

## TOPIC ONE: INTRODUCTION TO OPERATION RESEARCH (O.R.)

### 1.0 Back-ground and Development

Discipline comes from a convergence of an interest in some class of problems and the development of scientific methods, techniques and tools which are adequate to solve these problems. O.R. is no exception. However, the name O.R dates back to 1937 during the World War II. During this period of war U.K. military executives were faced with waging a war of which they had very little experience, as the military experience is never continuous, many years pass before any war develops, consequently past experience is never adequate.

The military Executives then turned to the scientists who had been closely involved with technological development. They invited the scientists from different disciplines to assist in solving strategic and tactical problems associated with air and land defence of United Kingdom.

The scientists were asked to assist the military executives in:

a) Learning how to use newly developed radar in locating enemy aircraft, and

b) To do research on military operations.

Consequently, O.R. got its name because the first team formed was dealing with research on military operations. Other countries quickly picked up the newly developed discipline.

United States of America (U.S.A) successfully applied it on:

i) Complex logistical problems.

ii) Invention of new flight patterns.

iii) Planning sea mining.

iv) Effective utilization of electronic equipment.

After the war, some of those who served with O.R. teams moved into business, industry and civil government and began spreading word about the effectiveness of the new discipline in solving executive type of problems.

### 1.2 Factors which Facilitated the Growth of Operations Research

a) Differentiation, Specialization and segmentation of Management responsibilities in an organization.

b) Substantial progress in improving available Operations Research techniques.

c) Computer Revolution.

a) Differentiation, Specialization and Segmentation of management responsibilities

Before the industrial Revolution most businesses and industries consisted of small enterprises each directed by a single boss who did virtually everything:-

-Purchasing.

-Planning and supervision of production.

-Selling and marketing.

-Hiring and firing of personnel etc.

Since the advent of Industrial Revolution, the world experienced rapid growth of industrial enterprises both in size and complexity. Hence, it became impossible for one man to perform all the managerial functions.

The growth resulted in:

i) Division of management functions or functional specialization.
ii) Segmentation of management responsibilities.
iii) Decentralization of operations.
iv) Increased attention by scientists to the problems generated in the various functional divisions.

Growth and its attendant results created executives type problem which started to assert themselves as follows:-

Each functional unit in the organizations grew into autonomous empires with their own goals and value systems. Consequently each unit lost sight of the fact that each part is necessary for the accomplishment of the over-all objectives.

They did not see how their individual activities and objectives mesh with those of the over-all organization. From this industrial revolution, the objective of O.R. emerged. This is to provide managers of an organization with a scientific basis for solving problems involving the interaction of components of the organization in the best interest, as a whole.

Decisions best for the whole organization are called optimum decisions but those best relative to functions of one or more part of an organization are called sub-optimal decisions. O.R tries to find the best decisions relative to a large portion of total organization as is possible. O.R has a goal of an overall understanding of optimal solutions to executive – type problems in organizations.

It is the duty of O.R. person to use the systems approach by:

I) Integrating the policies and operations of all the departments in order to obtain an overall operation and policy that are best for the organization as a whole and the interest of any particular departments. He must, however consider the effects of the policy on each department.

II) Allocating the organizations available resources to their various activities in a way that is most effective for the whole organization. Operations Research is used when scarce resources are to be allocated effectively to the various activities.

Its use helps executives to understand and determine:

- The required resources.
- The way such resources interact.
- Consequences of their alternative uses.
- Consequences of attaining the optimal goals.

b). Improvement of O.R. techniques

There has been substantial progress in the development and improvements of Operations Research techniques. In 1947, the American mathematician George Dantzig developed simplex method for solving linear programming problems.

Before the end of 1950s many standard tools of O.R had been developed e.g. linear programming, Queuing theory, Inventory theory. These and many others that were developed helped further in spreading the usefulness and need for Operations Research.

c). Computer revolution

To deal with simple computations is not a problem as they can be done manually.

But – dealing with complex problems effectively may not be easily done manually, they do require some assistance and the development of electronic digital computers with ability to perform arithmetic calculations with speed and with tremendous capabilities of information storage and retrieval have been welcome news to operations researchers. Moreover, computers having grown through several generations have improved on speed, weight and value. The current generation computer is smaller, less expensive and more powerful. Hence, most institutions can afford them and start operations research work.

### 1.3 Definition of Operations Research

Several attempts have been made at defining Operation Research. Some of the definitions are either too general or just misleading.

O.R. may be defined as “an application of scientific methods, techniques and tools to problems involving the operations of systems so as to provide those in control of the operations with optimum solutions to the problems”.

The British Operational Research Society has adopted the following definitions:

“Operational Research is the Application of the methods of science to complex problems arising in the direction and management of large systems of men, machines, material and money in industry, business, government and defence”.

The distinctive approach is to develop a scientific model of the system incorporating measurements of factors such as chance and risk with which to predict and compare the outcomes of alternatives decisions, strategies and control.

The purpose is to help management determine its policy and actions scientifically.

It is an aid to a decision maker to better exploit the applied science technology combination of:

a) Quantitative methods and
b) High –speed digital computers

The discipline uses both experience and intuition while encouraging rational decision making based on the best available approaches and techniques.

### 1.4 Essential Characteristics of O.R.

- The essential characteristics of O.R are
- Systems Orientation
- Uses of inter-disciplinary team
- Adopt Scientific method
- Uncover new problems for study
- Need for complete conviction to the findings and recommendations

- Systems Orientation

The basic idea here is that “the activity of any part of an organization has some effect on the activity of every other part”.

The principle connects each and every part of the organization to every other part. Hence to evaluate any decision or action in an organization it is necessary to identify all significant interactions and evaluate their combined effect on the performance of the organization as a whole. A systems approach consists of covering the entire area under a manager’s control and not to concentrate on some specific and special region, section, or department of an organization. The objective of O.R is to provide managers of an organization with a scientific basis for solving problems involving the interaction of components of the organization in the best interest of the organization as a whole. They want an optimal decision which is best for the organization as a whole, not a sub-optimal decision which is best for part of an organization. O.R attempts to consider the interactions or chain of effects as far as these effects are significant. Hence, O.R approach must be comprehensive. O.R. is concerned with finding an optimum decision, policy or design. It does not merely seek to find a better solution to a problem than the one in use; it seeks the best solution possible. It may not always find it because of the limitations imposed by:

- The current state of science
- Lack of funds or
- Lack of opportunities.

But O.R’s efforts are continually directed to getting the optimum or as close to it as possible.

**1.5 Interdisciplinary Team**

In the years of O.R. formation, it emerged from other sciences from which it borrowed heavily. There is a great overlap in methods, techniques and tools with other sciences, but this is because of the way it was initially carried out.

It is a research performed by teams of scientists whose individual members have been drawn from different scientific and engineering disciplines. The team might be comprised of a mathematician, physicist, chemist, biologist, psychologist, sociologist and an economist working together on a problem. Each individual member of the group represents an alternative way of looking at the problem.

### 1.6 Adopt Scientific Method

O.R. uses updated and technological advanced tools e.g. computers. But O.R. problems cannot be analyzed under controlled conditions because the O.R person cannot manipulate the system he studies. Hence, he must build mathematical models to represent the actual problem. Such models can then be manipulated and analyzed. Certain variables can be changed while holding others constant in order to find out how the system can be affected. O.R. person are able to simulate the real world and also experiment with it in abstract terms. Solutions to such mathematical models represent the relationship that exists between controlled and uncontrolled variables. Objective functions are supplemented by a set of restive statements on the possible values of the controlled variables. Objectives of O.R models are basically of two types:

- Minimizing costs.
- Maximizing benefits.

The updated scientific method requires.

- Mathematical modeling
- Use of standard O.R techniques
- Establishing controls
- Utilization of computer capabilities.

Most of the solutions of an O.R problem contain uncovered new problems. All inter-related problems uncovered by the O.R approach do not have to be solved at the same time. However, each must be solved with consideration for other problems, if maximum benefits are to be obtained. Hence, O.R is not effectively used if it is restricted to one-shot projects. Greatest benefits can be obtained through continuity of interrelated problems and solutions.

### 1.7 Problems Encountered when Applying O.R. Techniques

a) Abstraction problem or use of models

Models require gross simplification of real world situations; hence they inaccurately reflect the real world. They fail to take into account all the aspects of the situations being modeled. Moreover, it is difficult to define all elements of the real world situation in a model, and then convert them in mathematical terms.

b) O.R. programs fail to gain acceptance and implementation due to lack of proper organization. This occurs when O.R. group members do not seek the participation of those affected. By not letting the affected to participate from the beginning, the O.R. team may not learn the technical aspects of the operations or define unrealistic objectives. Hence, they cast aside the important knowledge which would be incorporated into the constraints, assumptions, and the model itself. Not making use of the experience and judgment of line managers and their subordinates in an O.R. problem is a fraud. It is essential to gain the participation of, the managers who must act on recommendations, those who have final word concerning the functions under study, those who are affected by the final recommendations, operating management and their subordinates. Some other items that can cause resistance to the acceptance of an O.R. solution are managers who do not understand the model prejudice against quantitative techniques in general reluctance to change the status quo.

c) Communication problem.

Personnel who do not understand the models have `difficult accepting the models results.

Sometimes there is rigidity in interpretations of the results. O.R. team must see the models only as decision making tools, not as the know-all techniques .The team must effectively communicate the results of the study to all involved in the study. Let the presentation not be too highly technical as this would encounter communication problem.

d) Costs problem

Sometimes the models can turn out to be too expensive compared to the expected return from their use. Hence they get abandoned.

**1.8 The Phases of O.R.**

The major phases of an O.R project are:-

a) Formulating the problem.
b) Constructing a mathematical model to represent the systems under study.
c) Deriving solution from the model.
d) Testing the model and solution derived from it.
e) Establishing controls over the solution.
f) Implementing the solution.

Formulating the problem

To formulate the problem involves accepting that there is a problem then gives analysis to the system under study, objectives of the study, and alternative courses of action. Identify also those affected by the decisions under study and their pertinent objectives.

Also specify the measure and suitability of the effectiveness of the study results.

Constructing a mathematical model

The model should express the effectiveness of the system under study as a function of a set of variables, at least one of which is subject to control.

The general form of an O.R. model is:-

$E = f\left(x_i\, y_i\right)$

Where,

$E$ = Effectiveness of the system.

$x_i$ = Variables subject to control.

$y_i$ = Variables not subject to control.

Restrictions on values of the variables may be expressed as a set of equations and/or in equations.

Deriving model solution

Use either analytical or numerical procedure in deriving an optimal solution to the model.

Analytical procedures require the use of mathematical deductions by applying various branches of mathematics such as Calculus or Matrix Algebra.

Numerical procedures consist of trying various values of the control variables, comparing results obtained, and selecting that set of values of control variables yielding the best solution.

Such procedures may vary from simple trial and error to complex iteration. An iterative procedure is one whose successive trials tends to approach an optimum solution and is able to identify optimum solution which it is obtained.

Testing the model and solution

A model is a partial representation of reality. It is a good model if it can accurately predict the effect of changes in the system on the systems over-all effectiveness. The adequacy of the model can be tested by determining how well it does predict the effect of these changes. The solution can be evaluated by comparing results obtained without applying the solution with results obtained when the solution is used. Testing requires careful analysis as to what are and what are not valid data.

Establishing controls over the solution

A solution derived from a model remains a solution only as long as the uncontrolled variables retain their values and the relationship between variables in the model remains constant otherwise, the situation goes out of control. To establish controls over the solution develop tools for determining when significant changes occur and establish rules for modifying the solution to take account of changes.

Implementing the solution

Translate the solution into a set of operating procedures which are understood and applied by the personnel responsible for their use. Specify and carry out the required changes.

**1.9 Modeling in Operational Research**

It might be impossible or too expensive to carry out real experiments in order to find out which of a number of plans is optimum, such as implementing all the possible plans in succession and observing which provides the optimum value for the performance measure. All that is necessary in order to deal with the problem is a representation of the variables and the relationships between them, such a representation is called a "model" hence, and a model here is defined as an idealized representation of a real-life system. This system could be already in existence or it may be still a conceived idea awaiting execution.

Steps to follow:-

1$^{st}$ objective is to analyze the behavior of the system with a view to improving its performance.

2$^{nd}$ objective is to define the ideal structure of the future system which includes the functional relationships among its components.

The reliability of the solution thus obtained depends on the validity of the model in representing the real system.

System scientists have classified models into the following types:-

a) Iconic (physical) model – represent the system as it is, look like what they represent, though physical dimensions are usually scaled down or up i.e. toy airplane or car is an iconic model of a real one.

b) Analogue Models (Diagrammatic) – the relationship between the variables in the system under investigation are represented by similar relationships between variables in a different system. This basically requires the substitution of one property for another for the ultimate purpose of achieving convenience in manipulating the model to represent different states of the real system. Observed states of the analogue system can then be translated back to the real system e.g. graphs, charts etc.

c) Symbolic (Mathematical models) – employ a set of mathematical symbols to represent the decision variables of the system, and the relationship between them, i.e. Algebraic functions, linear programming, Inventory models, etc.

- Assumes solutions are defined for all real number – remedy thus use $x_i \geq 0$ the non – negativity constraint,
- Assumes the functions have a continuous domain that is used as an approximation.
- Methods have been developed which use only discreet domains i.e. poison probability distribution used in queuing theory.
- Verification problems when the system is new or even when it is old, historical data may not be the best. Fortunately we use the experience obtained from standard O.R. models to guide us to the application of such techniques to new situations.
- Implementation of solutions may not be perfect, so specialists and managers must work together from the inception stage to the end of implementation for its success. Advantages are that they are very adaptable, they are used very often and routing have been established for solving them called algorithm, many of which are iterative.
- The above order follows the order of importance in O.R applications-from the least abstract to the most abstract.

The introduction of digital computers has led to the introduction of two more types of models in O.R.

d) Simulation models – are digital representations which imitate the behavior of a system using the digital computer or analyze manually the statistic measuring the performance of the system are accumulated as the simulator advances, which are used to make decisions more flexible than symbolic models, hence can be applied in more complex situations, but has the disadvantages of not yielding general solutions like those of the symbolic models due to experimental error, and there is no reliable way for measuring such impression. Its application may also be uneconomical hence you apply this model when everything else has failed

e) Heuristic Models – are mainly used to explore alternative strategies which have been over looked previously, thus differing from mathematical and simulation models which are used to represent systems having well - defined strategies (courses of action). The heuristic models do not claim to find the best solution to the problem. Rather, by applying some intuitive rules or guidelines, new strategies can be generated which will yield improved solutions to the model.

## 1.10 The Structure of Mathematical Models

Unlike simulation and heuristic models for which no fixed structures can be suggesting, a mathematical model includes mainly three basic elements, namely:-

a) Decision variables and Parameters

Decision variables are the unknowns which lead to the solution of the problem –controllable. Parameters may be deterministic or probabilistic and are the prices, costs or unit resource usage, etc - which are (short run) uncontrollable variables.

b) Constraints or Restrictions

This allows for solving the problem, because they limit the decision variables to their feasible or permissible values. Non-negativity constraints allow for real life system which concerns the operation researcher.

c) Objective Function

This defines the measure of effectiveness of the system as a mathematical function of its decision variables. It lead to optimum results and a poor formulation of it can only lead to a poor solution of the problem, yielding sub - optimal results. Here you must use systems approach in formulating the objective functions, i.e. you must not reflect only on the sales, and finance departments and not the production department goals.

**Reference**:

Kumawar D.S and Kongere T.O (2003). *Fundamentals of Operations Research,* India Bishen Singh Mahendra Pal Singh (pg 1-21).

# TOPIC TWO: TRANSPORTATION

***2.0 Purpose***

***It mainly minimizes the transportation costs of supplying quantities of a commodity from the warehouses to the shops.***

***2.1 Objectives***

***After studying this topic the student will be able to:***

***a) recognize Transportation problems***
***b) Know how to set up the initial Transportation table***
***c) Be able to make the initial feasible allocations***
***d) Understand how to improve the initial allocation by calculating the shadow costs***
***e) Know when to include a dummy destination***
***f) Be able to use the Transportation technique for maximizing problems.***

## 2.2 Transportation Problems Defined

The typical transportation problem deals with a number of sources of supply (e.g. warehouses) and a number of destinations (e.g. retail shops). The usual objective is to minimize the transportation cost of supplying quantities of a commodity from the warehouses to the shops. The major requirement is that there must be a constant transportation cost per unit i.e. if one unit costs ksh 700 to transport from warehouse A to shop X, five units will cost ksh 3500. This will be recognized as the linearity requirement fundamental to all forms of Linear Programming (LP).

## 2.3 The Transportation Technique

Although the method of solving transportation problems described below differs in appearance from the Simplex method it has some basic similarities, as follows:

a) It is an iterative, step by step, process.
b) It starts with a feasible solution and each succeeding solution is also feasible.

c) At each stage a test is made to see whether transportation costs can be reduced

d) Optimum is reached when no further cost reductions are possible.

**2.4 Transportation Example**

The following simple example will be used as a basis for the step-by-step explanation of the transportation technique.

Example 1

A firm of office equipment suppliers has three depots located in various towns. It receives orders for a total of 15 special filing cabinets from four customers. In total in the three depots there are 15 of the correct filing cabinets available and the management wish to minimize delivery costs by dispatching the filing cabinets from the appropriate depot for each customer.

Details of the availabilities, requirements and transport costs per filing cabinet are given in the following table.

Note: The body of the table contains the transportation costs per cabinet from the depots to the customer. For example, it costs ksh14 to send 1 cabinet from Deport Y to customer B (and ksh 28 for 2, ksh 42 for 3 etc.)

Table 1

| | | | Customer A | Customer B | Customer C | Customer D | Total |
|---|---|---|---|---|---|---|---|
| Cabinets | | | 3 | 3 | 4 | 5 | 15 |
| | | | ksh | ksh | ksh | | |
| Available | Depot X | 2 | 13 | 11 | 15 | 20 | |
| | Depot Y | 6 | 17 | 14 | 12 | 13 | transportation |
| | Depot Z | 7 | 18 | 18 | 15 | 12 | cost per unit |
| | Total | 15 | | | | | |

Step1. Make an initial feasible allocation of deliveries. The method used for this initial solution does not affect the value of the optimum but a careful initial choice may reduce the number of iterations that have to be made. The method to be used is to select the cheapest route first, and allocate as many as possible then the next cheapest and so on. The result of such an allocation is as follows

Table 2

| | | Requirements | | | |
|---|---|---|---|---|---|
| | | A | B | C | D |
| | | 3 | 3 | 4 | 5 |
| Available | X 2 units | | 2(1) | | |
| | Y 6 units | 1(4) | 1(3) | 4(2) | |
| | Z 7 units | 2(5) | | | 5(2) |

Note: The numbers in the table represent deliveries of cabinets and the numbers in the brackets (1), (2), etc represent the sequence in which they are inserted, lowest cost first i.e.

| | | ksh |
|---|---|---|
| 1. 2 units X → B ksh11/ unit | Total cost | 22 |
| 2. 4 units Y → C ksh12/ unit | Total cost | 48 |
| 5 units Z → D ksh12/ unit | Total cost | 60 |
| 3. The next lowest cost move which is feasible i.e. does not exceed row or column totals is 1 unit Y → B ksh14/ unit | | 14 |
| 4. Similarly the next lowest feasible allocation 1 unit Y→ A ksh17/ unit | | 17 |
| 5. Finally to fulfill the row / column totals 2 units Z → A ksh18 /unit | | 36 |

Step2. Check solution obtained to see if it represents the minimum cost possible. This is done by calculating what are known as ‘shadow costs’ (i.e. imputed cost of not using a particular route) and comparing these with the real transport costs to see whether a change of allocation is desirable.

This is done as follows:

Calculate a nominal ‘dispatch’ and ‘reception’ cost for each occupied cell by making the assumption that the transport cost per unit is capable of being split between dispatch and reception costs thus:

| | | | | |
|---|---|---|---|---|
| D (X) | + | R (B) | = | 11 |
| D (Y) | + | R (A) | = | 17 |
| D (Y) | + | R (B) | = | 14 |
| D (Y) | + | R (C) | = | 12 |
| D (Z) | + | R (A) | = | 18 |
| D (Z) | + | R (D) | = | 12 |

Where D(X), D(Y) and D(Z) represent Dispatch cost from depots X,Y and Z, and R(A), R(B), R(C) and R(D) represent Reception costs at customers A,B,C,D.

By convention the first depot is assigned the value of zero i.e. D(X) = 0 and this value is substituted in the first equation and then all the other values can be obtained thus

| |
|---|
| R (A) = 14 |
| R (B) = 11 |
| R (C) = 9 |
| R (D) = 8 |
| D(X) = 0 |
| D(Y) = 3 |

| D(Z) = 4 |
|---|

Using these values the shadow costs of the unoccupied cells can be calculated. The unoccupied cells are X: A, X: C, X: D, Y: D, Z: B, Z: C.

| | | | | | | | Shadow cost |
|---|---|---|---|---|---|---|---|
| D(X) | + | R (A) | = | 0 | + | 14 | = I4 |
| D(X) | + | R (C) | = | 0 | + | 9 | = 9 |
| D(X) | + | R (D) | = | 0 | + | 8 | = 8 |
| D(Y) | + | R (D) | = | 3 | + | 8 | = I1 |
| D(Z) | + | R (B) | = | 4 | + | 11 | = I5 |
| D(Z) | + | R (C) | = | 4 | + | 9 | = I3 |

These computed 'shadow costs' are compared with the actual transport costs (from Table1), where the actual costs are less than shadow costs, overall costs can be reduced by allocating units into that cell.

| | | Actual Cost | - | Shadow cost | | +cost increase -cost reduction |
|---|---|---|---|---|---|---|
| Cell | X: A | 13 | - | 14 | = | -1 |
| | X: C | 15 | - | 9 | = | +6 |
| | X: D | 20 | - | 8 | = | +12 |
| | Y: D | 13 | - | 11 | = | +2 |
| | Z: B | 18 | - | 15 | = | +3 |
| | Z: C | 15 | - | 13 | = | +2 |

The meaning of this is that total costs could be reduced by ksh1 for every unit that can be transferred into cell X: A. As there is a cost reduction that can be made the solution in Table 2 is not optimum.

Step3. Make the maximum possible allocation of deliveries into the cell where actual costs are less than shadow costs using occupied cells i.e.

Cell X: A from Step 2, the number that can be allocated is governed by the need to keep within the column totals. This is done as follows:

Table 3

| | | Requirements | | | |
|---|---|---|---|---|---|
| | | A 3 | B 3 | C 4 | D 5 |
| Available | X 2 units | + | 2- | | |
| | Y 6 units | -1 | +1 | 4 | |
| | Z 7 units | 2 | | | 5 |

Table 3 is a reproduction of table 2 with a number of + and – inserted. These were inserted for the following reasons.

Cell X: A + indicates a transfer in as indicated in Step 2.

Cell X: B – indicates a transfer out to maintain Row X total.

Cell Y: B + indicates a transfer in to maintain Column B total.

Cell Y: A – indicates a transfer out to maintain Row Y and Column A totals

The maximum number than can be transferred into Cell X: A is the lowest number in the minus cells i.e. cells Y: A, and X: B which is 1 unit.

Therefore 1 unit is transferred in the + and – sequence described above resulting in the following table.

Table 4

Requirements

| | | A | B | C | D |
|---|---|---|---|---|---|
| | | 3 | 3 | 4 | 5 |
| | X 2 units | 1 | 1 | | |
| Available | Y 6 units | | 2 | 4 | |
| | Z 7 units | 2 | | | 5 |

The total cost of this solution is

| | | ksh |
|---|---|---|
| Cell X: A | 1unit @ ksh13 = | 13 |
| Cell X: B | 1unit @ ksh11 = | 11 |
| Cell X: B | 2unit @ ksh14 = | 28 |
| Cell X: C | 4unit @ ksh12 = | 48 |
| Cell X: A | 2unit @ ksh18 = | 36 |
| Cell X: D | 5unit @ ksh12 = | 60 |
| | | **ksh196** |

The new total cost is ksh1 less than the total cost established in step 1. This is the result expected because it was calculated in step 2 that ksh1 would be saved for every unit we were able to transfer to Cell X: A and we were able to transfer 1 unit only.

Notes: Always commence the + and – sequence with a + in the cell indicated by the (actual cost-shadow cost) calculation. Then put a – in the occupied cell in the same row which has an occupied cell in its column. Proceed until a – appears in the same column as the original +.

Step 4. Repeat step 2 i.e. checks that solution represents minimum cost. Each of the processes in step 2 is repeated using the latest solution (Table 4) as a basis, thus:

Nominal dispatch and reception costs for each occupied cell.

| |
|---|
| D (X) + R (A) = 13 |
| D (X) + R (B) = 11 |
| D (Y) + R (B) = 14 |
| D (Y) + R (C) = 12 |
| D (Z) + R (A) = 18 |
| D (Z) + R (D) = 12 |

Setting D (X) at zero the following values are obtained

| |
|---|
| R (A) = 13 |
| R (B) = 11 |
| R (C) = 9 |
| R (D) = 7 |
| D(X) = 0 |
| D(Y) = 3 |
| D (Z) = 5 |

Using these values the shadow costs of the unoccupied cells are calculated. The unoccupied cells are X: C, X: D, Y: A, Y: D, Z: B, And Z: C.

| | | | |
|---|---|---|---|
| D (X) + | R (C) | = | 9 |
| D (X) + | R (D) | = | 7 |
| D (Y) + | R (A) | = | 16 |
| D (Y) + | R (D) | = | 10 |
| D (Z) + | R (B) | = | 16 |
| D (Z) + | R (C) | = | 14 |

The computed shadow costs are compared with actual costs to see if any reduction in cost is possible.

| Actual cost | - | Shadow cost | = | +Cost increase<br>- Cost reduction |
|---|---|---|---|---|
| Cell X: C 15 | - | 9 | = | +6 |
| X: D 20 | - | 7 | = | +13 |
| X: A 17 | - | 16 | = | +1 |
| Y: D 13 | - | 10 | = | +3 |
| Z: B 18 | - | 16 | = | +2 |
| Z: C 15 | - | 14 | = | +1 |

It will be seen that all the answers are positive, therefore no further cost reduction is possible and optimum has been reached.

Optimum solution

1 unit X→ A

1 unit X→ B

2 units Y→ B

4 units X→ C

2 units X→ A

5 units Y→ D

With a total cost of ksh196

This solution is shown in the following tableau.

Table 5

| | A | B | C | D |
|---|---|---|---|---|
| X | 1 | 1 | | |
| Y | | 2 | 4 | |
| Z | 2 | | | 5 |

*Note*: In this example only one iteration was necessary to produce an optimum solution mainly because a good initial solution was chosen. The principles explained above would, of course, be equally suitable for much iteration.

**2.5 Unequal Availability and Requirement Quantities**

Example 1 above had equal quantities of units available and required. Obviously this is not always the case and the most common situation is that there are more units available to be dispatched than are required. The transportation technique can be used in such circumstances with only a slight adjustment to the initial table. A dummy destination, with zero transport costs, is inserted in the table to absorb the surplus available. Thereafter the transportation technique is followed. The following example explains the procedure.

**2.6 Transportation Example with a Dummy Destination**

Example 2

A firm of wholesale domestic equipment suppliers, with 3 warehouses, received orders for a total of 100 deep freezers from 4 retail shops. In total in the 3 warehouses there are 110 freezers available and the management wishes to minimize transport costs by dispatching the freezers required from the appropriate warehouses. Details of availabilities, requirements, and transport costs are given in the following table.

Table 6

| Freezers | Required | | | | |
|---|---|---|---|---|---|
| | Shop A 25 | shop B 25 | shop C 42 | shop D 8 | Total 100 |
| Available | ksh | ksh | ksh | ksh | |
| Warehouse I 40 | 3 | 16 | 9 | 2 | |
| Warehouse II 20 | 1 | 9 | 3 | 8 | transport |
| Warehouse III 50 | 4 | 5 | 2 | 5 | cost |
| Total 110 | | | | | per freezer |

Step1**.** Add a dummy destination to Table 6 with a zero transport costs and a requirement equal to the surplus available.

Dummy requirement = 110 – 100 =10 Freezers

Table 7

| Freezers | Required | | | | | |
|---|---|---|---|---|---|---|
| | Shop A 25 | shop B 25 | shop C 42 | shop D 8 | Dummy 10 | Total 110 |
| Warehouse I 40 | ksh3 | 16 | 9 | 2 | 0 | |
| Warehouse II 20 | ksh1 | 9 | 3 | 8 | 0 | transport cost |
| Warehouse III 50 | ksh4 | 5 | 2 | 5 | 0 | per freezer |
| Total 110 | | | | | | |

Step2. Now that the quantity available equals the quantity required (because of the insertion of the dummy) the solution can proceed in exactly the same manner described in Para.4 example 7. First set up an initial feasible solution.

Table 8

| | | Requirements | | | | |
|---|---|---|---|---|---|---|
| | | A | B | C | D | Dummy |
| | | 25 | 25 | 42 | 8 | 10 |
| | I 40 | 5(4) | 17(6) | | 8(3) | 10 (7) |
| Available | II 20 | 20 (1) | | | | |
| | III 50 | | 8(5) | 42(2) | | |

The numbers in the table represent the allocation made and the numbers in brackets represent the sequence they were inserted based on lowest cost and the necessity to maintain row/ column totals. The residue of 10 was allocated to the dummy.

The cost of this allocation is

| | | | | |
|---|---|---|---|---|
| | | Ksh | | Ksh |
| I - A | 5 units @ | 3 | = | 15 |
| I - B | 17 units @ | 16 | = | 272 |
| I - D | 8 units @ | 2 | = | 16 |
| I – Dummy | 10 units @ | zero cost | | |
| II - A | 20 units @ | 1 | = | 20 |
| III - B | 8 units @ | 5 | = | 40 |
| III - C | 42 units @ | 2 | = | 84 |
| | | | | **ksh 447** |

Step3. Check solution to see if it represents the minimum cost possible in the same manner as previously described i.e.

Dispatch & Reception Costs of used routes:

| | | | | |
|---|---|---|---|---|
| D (I) | + | R (A) | = | 3 |
| D (I) | + | R (B) | = | 16 |
| D (I) | + | R (D) | = | 2 |
| D (I) | + | R(Dummy) | = | 0 |
| D (II) | + | R (A) | = | 1 |
| D(III) | + | R (B) | = | 5 |
| D(III) | + | R (C) | = | 2 |

Setting D (I) at zero the following values can be obtained

| | | |
|---|---|---|
| R (A) | = | 3 |
| R (B) | = | 16 |
| R (C) | = | 13 |
| R (D) | = | 2 |
| R (Dummy) | = | 0 |
| D (I) | = | 0 |
| D (II) | = | -2 |
| D (III) | = | -11 |

Using these values the shadow costs of the unused routes can be calculated. The unused routes are I: C, II: B, II: C, II: D, II: Dummy, III: A, III: D and III: Dummy.

| | | | | | | | | Shadow costs Ksh |
|---|---|---|---|---|---|---|---|---|
| D (I) | + | R(C) | = | 0 | + | 13 | = | 13 |
| D (II) | + | R (B) | = | -2 | + | 16 | = | 14 |
| D (II) | + | R(C) | = | -2 | + | 13 | = | 11 |
| D (II) | + | R(D) | = | -2 | + | 2 | = | 0 |
| D (II) | + | R (Dummy) | = | -2 | + | 0 | = | -2 |
| D (III) | + | R (A) | = | -11 | + | 3 | = | -8 |
| D (III) | + | R (D) | = | -11 | + | 2 | = | -9 |
| D (III) | + | R (Dummy) | = | -11 | + | 0 | = | -11 |

The shadow costs are then deducted from actual costs.

| | Actual Cost | - | Shadow cost | = | + cost increase<br>- cost reduction |
|---|---|---|---|---|---|
| Cell I: C | 9 | - | 13 | = | -4 |
| II: B | 9 | - | 14 | = | -5 |
| II: C | 3 | - | 11 | = | -8 |
| II: D | 8 | - | 0 | = | +8 |
| II: Dummy | 0 | - | (-2) | = | +2 |
| III: A | 4 | - | (-8) | = | +12 |
| III: D | 5 | - | (-9) | = | +14 |
| II: Dummy | 0 | - | (-11) | = | +11 |

It will be seen that total costs can be reduced by ksh8 per unit for every unit that can be transferred into Cell II: C.

Step4. Make the maximum possible allocation of deliveries into Cell II: C. this is done by inserting a sequence of + and -, maintaining row and column totals.

Table 9

| | | Requirements | | | | |
|---|---|---|---|---|---|---|
| | | A | B | C | D | Dummy |
| | | 25 | 25 | 42 | 8 | 10 |
| Available | I 40 | 5+ | 17- | | 8 | 10 |
| | II 20 | 20 - | | + | | |
| | III 50 | | 8+ | 42- | | |

Note: This is a reproduction of table 7 with a sequence of + and – added starting with a + in Cell II : C and then – and + as necessary to maintain row and column balances. The maximum number that can be transferred is the lowest number in the minus cells i.e. 17 units in cell 1: B.

This transfer is made and the following table results.

Table 10

| | | | Requirements | | | | |
|---|---|---|---|---|---|---|---|
| | | | A | B | C | D | D |
| | | | 25 | 25 | 42 | 8 | 10 |
| Available | I | 40 | 22 | | | 8 | 10 |
| | II | 20 | 3 | | 17 | | |
| | III | 50 | | 25 | 25 | | |

The cost of this allocation is

(22x3) + (8x2) + (10x0) + (3x1) + (17x3) + (25x5) + (25x2) = ksh 311

This can be verified by deducting from the cost of the original allocation (Table 7) the savings of ksh8 per unit for the 17 units transferred i.e.

Ksh447- (17x8) = Ksh 311

Step 5.Repeat Step 3 to check if cost is at a minimum. After setting D (I) = 0, the following values can be obtained.

| | | |
|---|---|---|
| R (A) | = | 3 |
| R (B) | = | 8 |
| R (C) | = | 5 |
| R (D) | = | 2 |
| R (Dummy) | = | 0 |
| D (I) | = | 0 |
| D (II) | = | -2 |
| D (III) | = | -3 |

These values are used to calculate the shadow costs of the unused routes.

| | | Shadow costs Ksh |
|---|---|---|
| Cell I: B | = | 8 |
| I: C | = | 5 |
| II: B | = | 6 |
| II: D | = | 0 |
| II: Dummy | = | -2 |
| I: A | = | 0 |
| I: D | = | -1 |
| I: Dummy | = | -3 |

When these shadow costs are deducted from the actual costs no negative values result: the allocation shown in Table 10 is optimum with a minimum transportation cost of ksh 311.

### 2.7 Maximization and the Transportation Technique

Although transportation problems are usually minimizing problems, on occasion's problems are framed so that the objective is to make the allocations from sources to destinations in a manner which maximizes contribution or profit. These problems are dealt with relatively easy as follows.

### 2.8 Steps Involved in Maximization

Using Example 1, for comparison the following procedures should be followed.

| | Minimizing technique | |
|---|---|---|
| Step1 | Make initial feasible allocation Lowest cost first and so on | Shadow costs on unused routes for Optimality i.e. all positive; If not, Make allocation into cell with largest Negative difference |
| Step2 | Test differences between actual and | Make initial feasible allocation on basis of |

Maximizing technique
Maximum contribution first, then next highest and so on

For optimum, the differences between actual and shadow contributions for the unused routes should be all negative. If not, make allocation into cell with the largest positive difference. Apart from the differences above the transportation technique can be followed as usual.

**2.9 Maximizing and Dummies**

It will be recalled from example 2 that where availability and requirements are unequal a dummy destination with zero transport costs was introduced and thereafter the normal technique was followed. In a maximizing problem where there are more items available than are required, a dummy destination with zero contribution should also be introduced and the maximizing procedure in example 2 followed.

**2.10 Degeneracy and the Transportation Technique**

If Tables 3, 4, 5, 8, 9 and 10 are examined it will be seen in each case that the number of allocations made is less than the number of rows added to the number of columns. For example, Table 10 (3 rows and 5 columns) has 7 allocations. To be able to calculate the shadow dispatch and receiving costs this condition is essential but on occasions the number of allocations turns out to be less than rows + columns -1. This condition is known as degeneracy.

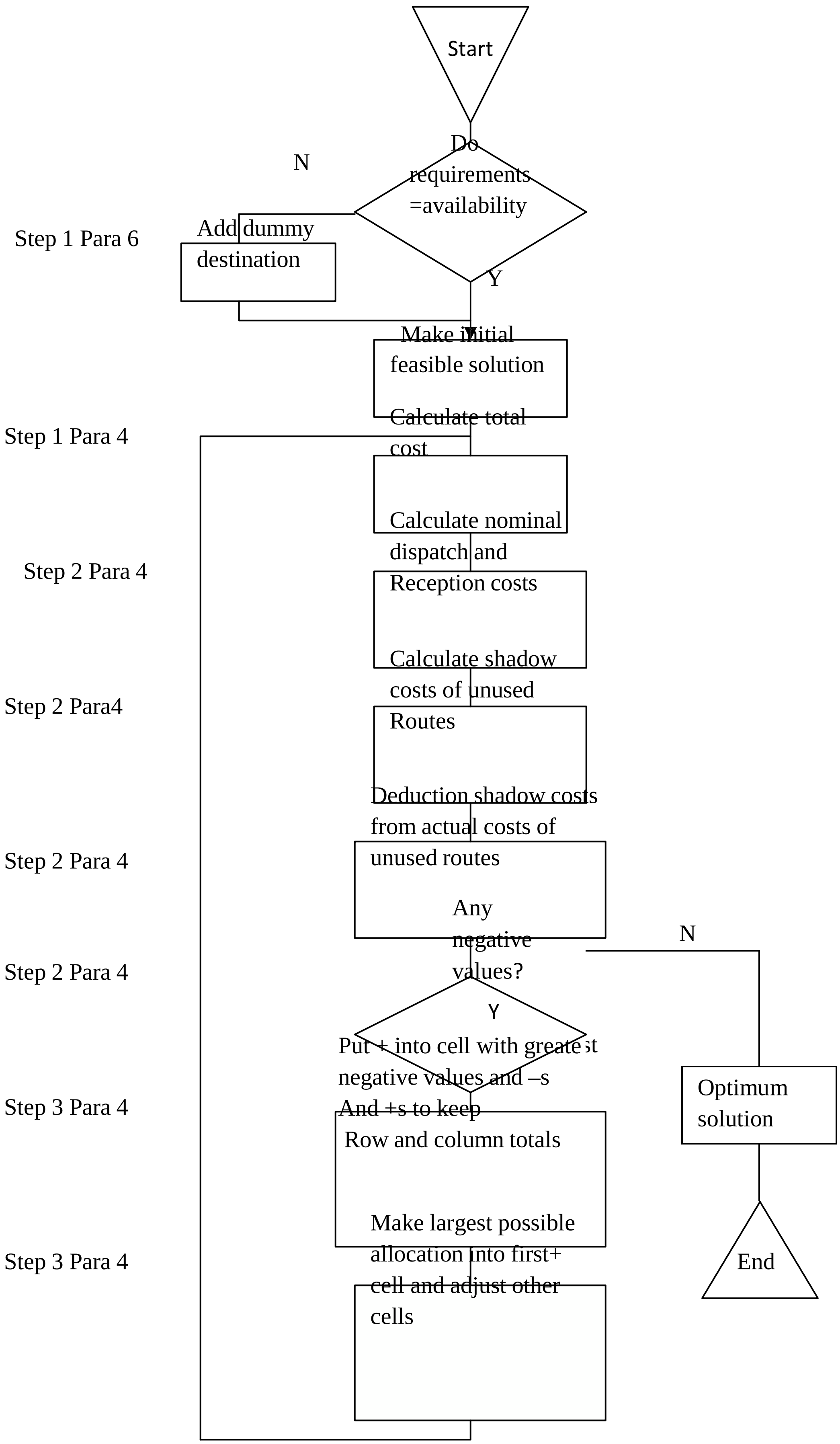
**2.11 Dealing with Degeneracy**

If the number of allocations is less than, No. of rows + columns - 1, then it is necessary to make one or more zero allocations to routes to bring up the number of allocations to rows+ columns-1. This means that, for shadow cost calculation purposes, one or more cells with zero allocation are treated as occupied. For example in a 3 row, 5 column tables if 6 allocations had been made which satisfied the requirements, any 1 cell would be given a zero allocations and treated as occupied. For moves following a zero allocation the cell with zero allocation is treated as a positive allocation and can therefore be moved to and from as with other allocations. The allocation must be into an independent cell.

## 2.12 Summary

- A Flowchart gives an outline of the transportation method for minimizing problems and is cross referenced to appropriate parts of the chapter.
- Maximizing problems follow the same general pattern except that the initial feasible solution is made on the basis of maximum contribution first and optimum is recognized when all the values are negative after deducting shadow from actual contribution.
- The number of allocations made must be one less than the number of rows plus the number of columns. If this does not happen after making the necessary actual allocations one or more cells, with zero allocation, must be treated as occupied

Outline of transportation method flowchart. Figure 1


Start
Do requirements =availability
N
Y
Step 1 Para 6
Add dummy destination
Make initial feasible solution
Step 1 Para 4
Calculate total cost
Step 2 Para 4
Calculate nominal dispatch and Reception costs
Step 2 Para4
Calculate shadow costs of unused Routes
Step 2 Para 4
Deduction shadow costs from actual costs of unused routes
Step 2 Para 4
Any negative values?
N
Y
Step 3 Para 4
Put + into cell with greatest negative values and –s And +s to keep Row and column totals
Step 3 Para 4
Make largest possible allocation into first+ cell and adjust other cells
Optimum solution
End

**Exercise**

***1. A firm has three shops with a total of 80 televisions. An order is received from the local Authority for 70 sets to be delivered to 4 schools, the transport costs from shop to schools are shown below together with the availabilities and requirements.*** ***Table 11***

| | | Schools | | | | |
|---|---|---|---|---|---|---|
| | | ***A*** | ***B*** | ***C*** | ***D*** | |
| | ***Sets*** | ***20*** | ***30*** | ***1 5*** | ***5*** | |
| ***Available*** | Shop I 40 | ***2*** | ***4*** | ***1*** | ***6*** | Costs |
| | Shop II 20 | ***4*** | ***3*** | ***3*** | ***3*** | |
| | Shop III 20 | ***1*** | ***2*** | ***5*** | ***2*** | |

***It is required to make the most economic deliveries.***

***Set up the initial table and make the initial feasible deliveries.***

***2. Work out the shadow prices for the tableau in question one***

***3. Solve the problem in question 1.***

**References:**

Lucey T. (2000). *Quantitative techniques 5th edition London*: Ashford colour press (page 286-305)

## TOPIC THREE: GAMES THEORY

***3.0 Purpose***

***In the language of Game Theory each player tries to maximize their payoff irrespective of what other players are doing.***

***3.1 Objectives***

***After studying this topic the student will be able to;***

- ***Choose strategies rationally when outcomes depend on the strategies chosen by others and when information is incomplete?***
- ***Rationalize whether to cooperate to realize the mutual gain (or avoid the mutual loss) or to act aggressively in seeking individual gain regardless of mutual gain or loss.***
- ***Know the circumstances under which aggression is rational and in what circumstances cooperation rational?***
- ***determine whether ongoing relationships differ from one-off encounters in this***

Game Theory can be regarded as a multi-agent decision problem. Which means there are many people contending for limited rewards/payoffs? They have to make certain moves on which their payoff depends. These people have to follow certain rules while making these moves. Each player is supposed to behave rationally.

**Rationality:**

In essence each player has to decide a set of moves which are in accordance with the rules of the game and which maximize his/her rewards.

Game Theory can be classified in two branches

- Non co-operative game theory: In this case the players work independently without assuming anything about what other players are doing.
- Co-operative game theory: Here players may co-operate with one another.

Game Theory has found applications in Economic, Evolutionary Biology, Sociology, Political

Science etc, now and its finding applications in Computer Science.

A game has the following:-

- Set of Players.

The two players who are choosing either Head or Tail in the Coin Matching Game form the set of players i.e. $P = \{P_1, P_2\}$

- Set of Rules.

There are certain rules which each player has to follow while playing the game. Each player can safely assume that others are following these rules. In coin matching game each player can choose either Head or Tail. The player has to act independently and made his selection only once. Player 1 wins if both selections are the same otherwise player 2 wins. These form the Rule set R for the coin matching game

- Set Strategies $S_i$ for each player $P_i$.

For example in Matching coins $S_1 = \{H, T\}$ and $S_2 = \{H, T\}$ are the strategies of the two Players. Which means each of them can choose either Head or Tail.

- Set of Outcomes.

In matching Coins its {Loss, Win} for both players. This is a function of the strategy profile selected. In our example $S_1 \times S_2 = \{(H, H), (H, T), (T, H), (T, T)\}$ is the strategy profile. clearly first and last are win situation for first player while the middle two are win cases for the second player.

-Pay off $u_i$ (o) for each player i and for each outcome.

This is the amount of benefit a player derives if a particular outcome happens. In general its different for different players.

Let the payoffs in coin matching game be,

$u_1$ (Win) = 100

$u_1$ (Loss) = 0

$u_2$ (Win) = 100

$u_2$ (Loss) = 0

Both the players would like to maximize their payoffs (rationality) so both will try to win. Now let's consider a slightly different case. We redefine the payoffs as,

Player 1 is competitor so

$u_1$ (Win) = 100

$u_1$ (Loss) = 0

While player 2 is a very concerned about seeing player 1 happy (player 1 is his little brother) so for him

$u_2$ (Win) = 10

$u_2$ (Loss) = 100

In this situation only player 1 would try hard to win while player 2 will try to lose. The point to note is that each player tries maximizing his payoff for which he/she would like to get the outcome which gives him maximum payoff.

Informally we can say the players sit across a table and play the game according to the set of rules. There is an outcome for each player when the game ends each player derives a pay off from this outcome. For example an outcome of victory brings payoff in terms of awards and fame to the cricket players, while loss means no payoff. Because all the players are rational beings they will try to maximize their payoffs. In non co-operative games players don't know what other players are doing. So they have to make the moves without looking at what others are doing. Each player chooses a strategy i.e. set of moves he would play.

Strategy

It is the set of moves that a player would play in a game. Being rational a player would chose the strategy in such a way as to maximize his/her payoff.

### 3.2 What is a Game?

A game has the following:

| | |
|---|---|
| Set of players | $D = \{P_i \mid 1 <= i <= n\}$ |
| Set of rules | R |
| Set of Strategies | $S_i$ for each player $P_i$ |
| Set of Outcomes | O |

Pay off $u_i$ (o) for each player i and for each outcome O.

Example; Coin Matching Game: Two players choose independently either Head or Tail and report it to a central authority. If both choose the same side of the coin, player 1 wins, otherwise 2 wins.

## 3.3 The Prisoners' Dilemma

### The Game

Tucker began with a little story, like this: two burglars, Bob and Al, are captured near the scene of a burglary and are given the "third degree" separately by the police. Each has to choose whether or not to confess and implicate the other. If neither man confesses, then both will serve one year on a charge of carrying a concealed weapon. If each confesses and implicates the other, both will go to prison for 10 years. However, if one burglar confesses and implicates the other, and the other burglar does not confess, the one who has collaborated with the police will go free, while the other burglar will go to prison for 20 years on the maximum charge. The strategies in this case are: confess or don't confess. The payoffs (penalties, actually) are the sentences served. We can express all this compactly in a "payoff table" of a kind that has become pretty standard in game theory. Here is the payoff table for the Prisoners' Dilemma game:

Table 12

| | | Al | |
|---|---|---|---|
| | | confess | don't |
| Bob | confess | 10,10 | 0,20 |
| | don't | 20,0 | 1,1 |

The table is read like this: Each prisoner chooses one of the two strategies. In effect, Al chooses a column and Bob chooses a row. The two numbers in each cell tell the outcomes for the two prisoners when the corresponding pair of strategies is chosen. The number to the left of the comma tells the payoff to the person who chooses the rows (Bob) while the number to the right of the column tells the payoff to the person who chooses the columns (Al). Thus (reading down the first column) if they both confess, each gets 10 years, but if Al confesses and Bob does not, Bob gets 20 and Al goes free. So: how to solve this game? What strategies are "rational" if both men want to minimize the time they spend in jail? Al might reason as follows: "Two things can happen: Bob can confess or Bob can keep quiet. Suppose Bob confesses. Then I get 20 years if I don't confess, 10 years if I do, so in that case it's best to confess. On the other hand, if Bob doesn't confess, and I don't either, I get a year; but in that case, if I confess I can go free. Either way, it's best if I confess. Therefore, I'll confess."But Bob can and presumably will reason in the same way so that they both confess and go to prison for 10 years each. Yet, if they had acted "irrationally," and kept quiet, they each could have gotten off with one year each.

Dominant Strategies

What has happened here is that the two prisoners have fallen into something called “dominant strategy equilibrium."

Dominant Strategy: Let an individual player in a game evaluate separately each of the strategy combinations he may face, and, for each combination, choose from his own strategies the one that gives the best payoff. If the same strategy is chosen for each of the different combinations of strategies the player might face, that strategy is called a "dominant strategy" for that player in that game.

Dominant Strategy Equilibrium: If, in a game, each player has a dominant strategy, and each player plays the dominant strategy, then that combination of (dominant) strategies and the corresponding payoffs are said to constitute the dominant strategy equilibrium for that game.

In the Prisoners' Dilemma game, to confess is a dominant strategy, and when both prisoners confess, that is dominant strategy equilibrium.

Issues With Respect to the Prisoners' Dilemma

This remarkable result that individually rational action results in both persons being made worse off in terms of their own self-interested purposes is what has made the wide impact in modern social science. For there are many interactions in the modern world that seem very much like that, from arms races through road congestion and pollution to the depletion of fisheries and the overexploitation of some subsurface water resources. These are all quite different interactions in detail, but are interactions in which (we suppose) individually rational action leads to inferior results for each person, and the Prisoners' Dilemma suggests something of what is going on in each of them. That is the source of its power. Having said that, we must also admit candidly that the Prisoners' Dilemma is very simplified and abstract if you will, an "unrealistic" conception of many of these interactions. A number of critical issues can be raised with the Prisoners' Dilemma, and each of these issues has been the basis of a large scholarly literature:

- The Prisoners' Dilemma is a two-person game, but many of the applications of the idea are really many-person interactions.
- We have assumed that there is no communication between the two prisoners. If they could communicate and commit themselves to coordinated strategies, we would expect a quite different outcome.
- In the Prisoners' Dilemma, the two prisoners interact only once. Repetition of the interactions might lead to quite different results.
- Compelling as the reasoning that leads to the dominant strategy equilibrium may be, it is not the only way this problem might be reasoned out. Perhaps it is not really the most rational answer after all.

**3.4 Zero-Sum Games**

By the time Tucker invented the Prisoners' Dilemma, Game Theory was already a going concern. But most of the earlier work had focused on a special class of games: zero-sum games.

For example, consider the children's game of "Matching coins." In this game, the two players agree that one will be "even" and the other will be "odd." Each one then shows a coin. The coin is shown simultaneously, and each player may show either a head or a tail. If both show the same side, then "even" wins the coin from "odd;" or if they show different sides, "odd" wins the coin from "even". Here is the payoff table for the game.

Table 13

| | | Odd | |
|---|---|---|---|
| | | Head | Tail |

| | | | |
|---|---|---|---|
| Even | Head | 1,-1 | -1,1 |
| | Tail | -1,1 | 1,-1 |

If we add up the payoffs in each cell, we find 1-1=0. This is a "zero-sum game."

Definition: If we add up the wins and losses in a game, treating losses as negatives, and wins as positives and find that the sum is zero for each set of strategies chosen, then the game is a "zero-sum game."

In less formal terms, a zero-sum game is a game in which one player's winnings equal the other player's losses. Do notice that the definition requires a zero sum for every set of strategies. If there is even one strategy set for which the sum differs from zero, then the game is not zero sum.

**3.5 Mixed Strategies**

Definition of mixed strategy: If a player in a game chooses among two or more strategies at random according to specific probabilities, this choice is called a "mixed strategy."

The game of matching coins has a solution in mixed strategies, and it is to offer heads or tails at random with probabilities 0.5 each way. Here is the reasoning: if odd offers heads with any probability greater than 0.5, then even can have better than even odds of winning by offering heads with probability 1. On the other hand, if odd offers heads with any probability less than 0.5, then even can have better than even odds of winning by offering tails with probability 1. The only way odd can get even odds of winning is to choose a randomized strategy with probability 0.5, and there is no way odd can get better than even odds. The 0.5 probability maximizes the minimum payoff over all pure or mixed strategies. And even can reason the same way (reversing heads and tails) and come to the same conclusion, so both players choose 0.5.

The maximum strategy is a "rational" solution to all two-person zero sum games. However, it is not a solution for any constant sum games. The difficulty is that there are a number of different solution concepts for no constant sum games, and no one is clearly the "right" answer in every case. The different solution concepts may overlap, though. We have already seen one possible solution concept for no constant sum games: the dominant strategy equilibrium. Let's take another look at the example of the two mineral water companies. Their payoff table was:

Table 14

| | | Perry | |
|---|---|---|---|
| | | Price=ksh10 | Price=ksh20 |
| Apollo | Price=ksh10 | 0,0 | 5000, -5000 |
| | Price=ksh20 | -5000, 5000 | 0,0 |

We saw that the maximum solution was for each company to cut price to ksh10. That is also dominant strategy equilibrium. It's easy to check that: Apollo can reason that either Perry cuts to ksh10 or not. If they do, Apollo is better off cutting to ksh10 to avoid a loss of ksh5000. But if Perrier doesn't cut, Apollo can earn a profit of ksh 5000 by cutting. And Perrier can reason in the same way, so cutting is a dominant strategy for each competitor. But this is, of course, a very simplified, even unrealistic, conception of price competition. Let's look at a more complicated, perhaps more realistic pricing example:

### 3.6 The Queuing Game

This section presents a "game" which extends the Prisoners' Dilemma in some interesting ways. The Prisoners' Dilemma is often offered as a paradigm for situations in which individual self-interested rationality leads to bad results, so that the participants may be made better off if an authority limits their freedom to choose their strategies independently. Powerful as the example is, there is much missing from it. Just to take one point: the Prisoners' Dilemma game is a two-person game, and many of the applications are many-person interactions. The game considered in this example extends the Prisoners' Dilemma sort of interaction to a group of more than two people. I believe it gives somewhat richer implications about the role of authority, and as we will see in a later section, its N-person structure links it in an important way to cooperative game theory.

As usual, let us begin with a story. Perhaps the story will call to mind some of the reader's experience. We suppose that six people are waiting at an airline boarding gate, but that the clerks have not yet arrived at the gate to check them in. Perhaps these six unfortunates have arrived on a connecting flight with a long layover. Anyway, they are sitting and awaiting their chance to check in, and one of them stands up and steps to the counter to be the first in the queue. As a result the others feel that they, too, must stand in the queue, and a number of people end up standing when they could have been sitting.

Here is a numerical example to illustrate a payoff structure that might lead to this result. Let us suppose that there are six people, and that the gross payoff to each passenger depends on when they are served, with payoffs as follows in the second column of Table 16. Order of service is listed in the first column.

Table 15

| Order served | Gross Payoff | Net Payoff |
|---|---|---|
| First | 20 | 18 |
| Second | 17 | 15 |
| Third | 14 | 12 |
| Fourth | 11 | 9 |
| Fifth | 8 | 6 |
| Sixth | 5 | 3 |

These payoffs assume, however, that one does not stand in line. There is a two-point effort penalty for standing in line, so that for those who stand in line, the net payoff to being served is two less than what is shown in the second column. These net payoffs are given in the third column of the table.

Those who do not stand in line are chosen for service at random, after those who stand in line have been served. (Assume that these six passengers are risk neutral.) If no-one stands in line, then each person has an equal chance of being served first, second, ..., sixth, and an expected payoff of 12.5. In such a case the aggregate payoff is 75.

But this will not be the case, since an individual can improve her payoff by standing in line, provided she is first in line. The net payoff to the person first in line is 18, so someone will get up and stand in line.

This leaves the average payoff at 11 for those who remain. Since the second person in line gets a net payoff of 15, someone will be better off to get up and stand in the second place in line.

This leaves the average payoff at 9.5 for those who remain. Since the third person in line gets a net payoff of 12, someone will be better off to get up and stand in the third place in line.

This leaves the average payoff at 8 for those who remain. Since the fourth person in line gets a net payoff of 9, someone will be better off to get up and stand in the fourth place in line.

This leaves the average payoff at 6.5 for those who remain. Since the fifth person in line gets a net payoff of 6, no-one else will join the queue. With 4 persons in the queue, we have arrived at Nash equilibrium of the game. The total payoff is 67, less than the 75 that would have been the total payoff if, somehow, the queue could have been prevented.

Two people are better off: the first two in line with the first gaining an assured payoff of 5.5 above the uncertain average payoff she would have had in the absence of queuing and the second gaining 2.5. But the rest are worse off. The third person in line gets 12, losing 0.5; the fourth 9, losing 3.5, and the rest get average payoffs of 6.5, losing 6 each. Since the total gains from queuing are 8 and the losses 16, we can say that, in one fairly clear sense, queuing is inefficient.

We should note that it is in the power of the authority (the airline, in this case) to prevent this inefficiency by the simple expedient of not respecting the queue. If the clerks were to ignore the queue and, let us say, pass out lots for order of service at the time of their arrival, there would be no point for anybody to stand in line, and there would be no effort wasted by queuing (in an equilibrium information state).

**3.7 Summary**

- Game theory. It's a multi-agent decision problem.
- Rationality. It implies that each player tries to maximize pay off irrespective of what other Players are doing
- Strategy. Set of moves that a player would play in a game.
- Prisoners' Dilemma. Each has to choose whether or not to confess and implicate the other.
- Zero sum games. If the wins and losses in a game are added treating losses as negative and gains as positive and the Sum is zero for each set of strategies**.**
- Mixed strategy. A game where a player chooses two or more strategies at random according to

specific probabilities

- Queuing game. It extends the prisoners' Dilemma in some interesting ways.

**Exercise**

1. ***We will think of two companies that sell mineral water. Each company has a fixed cost of Ksh 50000 per period, regardless whether they sell anything or not. We will call the companies Perry and Apollo, just to take two names at random.***

   ***The two companies are competing for the same market and each firm must choose a high price (Ksh 20 per bottle) or a low price (Ksh 10 per bottle). Here are the rules of the game:***

   ***a) At a price of ksh20, 5000 bottles can be sold for total revenue of ksh100000.***

   ***b) At a price of 10, 10000 bottles can be sold for total revenue of ksh100000.***

   ***c) If both companies charge the same price, they split the sales evenly between them.***

   ***d) If one company charges a higher price, the company with the lower price sells the whole***

   ***amount and the company with the higher price sells nothing.***

   ***e) Payoffs are profits revenue minus the Ksh 50000 fixed cost.***

   ***i) Draw the payoff table for these two companies***

   ***ii) What is the solution to the game?***

2. ***Following a long tradition in economics, we will think of two companies selling "widgets" at a price of ten, twenty, or thirty shillings per widget. The payoffs are profits after allowing for costs of all kinds and are shown in the table below. The general idea behind the example is that the company that charges a lower price will get more customers and thus, within limits, more profits than the high-price competitor.***

   ***Table 16***

| | | ***Acme Widgets*** | | |
|---|---|---|---|---|
| | | ***p=10*** | ***p=20*** | ***p=30*** |

| | | | | |
|---|---|---|---|---|
| ***Widgeon Widgets*** | ***p=10*** | ***0,0*** | ***50, -10*** | ***40,-20*** |
| | ***p=20*** | ***-10,50*** | ***20,20*** | ***90,10*** |
| | ***p=30*** | ***-20, 40*** | ***10,90*** | ***50,50*** |

***Find a solution for this game.***

**Reference:**

Roger McCain's Game Theory. *A None Technical Introduction to Analysis of strategy*. World Scientific Publishing Company (Pg 35-51).

# TOPIC FOUR: NETWORK ANALYSIS

***4.0 Purpose***

***Plan and control complex projects***

***4.1 Objectives***

***After studying this topic the student will be able to:***

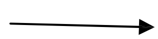
- ***Have been introduced to the control and planning technique called Network Analysis***
- ***Know what is meant by Activity, Event and Dummy Activity***
- ***Understand the key rules for drawing networks***
- ***Know the various ways activities are identified***
- ***Be able to use dummy activities correctly***

Network analysis is a generic term for a family of related techniques developed to aid management to plan and control projects. They can provide planning and control information on the time, cost and resource aspects of a project. Network analysis is likely to be of most value where projects are: complex, large, and restrictions exist.

## 4.2 Basic Network Terminology

Activity

This is a task or job of work which takes time and resources e.g. build a wall. It is represented in a network by an arrow as follows:



The head of the arrow indicates where the task ends and the tail where the tasks begin. It points from left to right and is not drawn to scale. An essential preliminary to the use of network analysis is establishing:

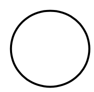
i) What activities are involved in the project?

ii) Their logical relationship.

iii) An estimate of the time the activity is expected to take.

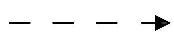
Event

This is a point in time and indicates the start or finish of an activity, or activities, e.g. wall built. It is represented in a network by a circle or node as follows:



Dummy activity

This is an activity which does not consume time or resources. It is used merely to show clear, logical dependencies between activities so as not to violate the rule for drawing networks. It is represented on a network by a dotted arrow as follows:

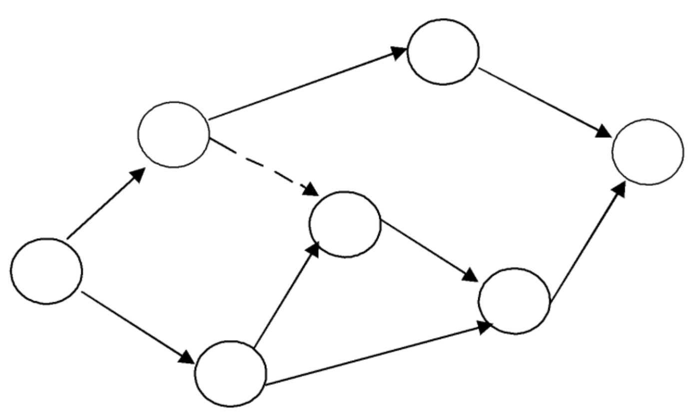


Dummy activities are not usually listed with the real activities but become necessary as the network is drawn.

Network

This is the combination of activities, dummy activities and events in logical sequence according to the rules for drawing networks.

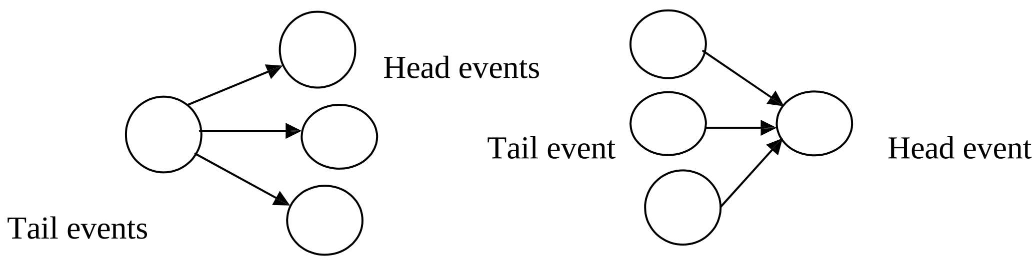
Figure 2



## 4.3 Rules for Drawing Networks

a) A complete network should have only one point of entry – a start event and only one point of exit – a finish event.

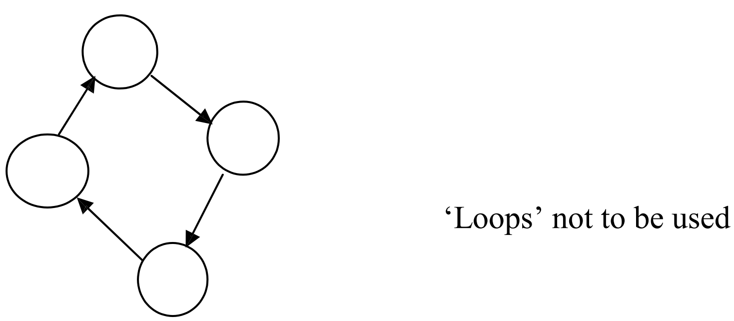
b) Every activity must have one preceding of 'tail' event and one succeeding or 'head' event. Note that many activities may use the same tail event and many may use the head event, e.g. Figure 3




However an activity must not share the same tail event and the same head event with any other activities.

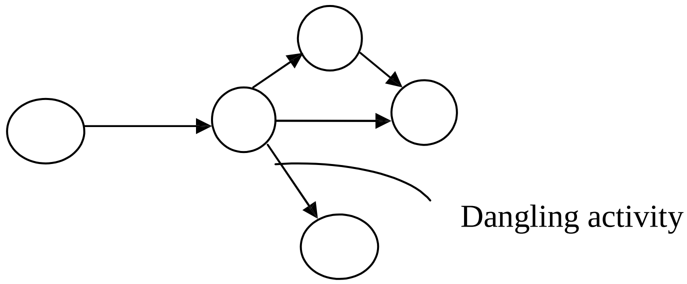
a) No activity can start until its tail event is reached.
b) An event is not complete until all activities leading in to it are complete.
c) A series of activities which lead back to the same event (loops) are not allowed since the essence of networks is a progression.

Figure 4




All activities must be tied into the network. Activities which do not link into the overall project are termed 'danglers'

Dangler not to be used                                  Figure5




## 4.4 Conventions for Drawing Networks

Networks proceed from left to right.

a) Networks are not drawn to scale.

b) Arrows need not be drawn in the horizontal plane but unless it is totally unavoidable they should proceed from left to right.

c) If there are not already numbered, events or nodes should be progressively numbered from left to right i.e. 0,1, 2, 3 or 0, 5, 10, 15 or 0, 10, 20, 30.

### 4.4.1 Methods of Identifying Activities

a) Shortened description of the job e.g. plaster wall, order timber.

b) Alphabetic or numeric code e.g. A, B, C etc or 100, 101, 102 etc.

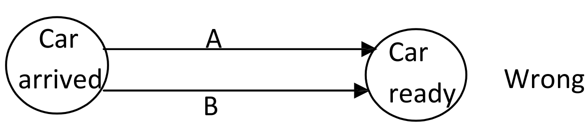
c) Identification by the tail and head event number e.g. 1-2, 2-3, 2-5 etc.

## 4.6 Dummy Activities

Example: Assume that part of the network involves a car arriving at a service station during which two independent activities take place; filling with petrol (A) and whipping the wind screen (B).

This could be shown incorrectly as follows:

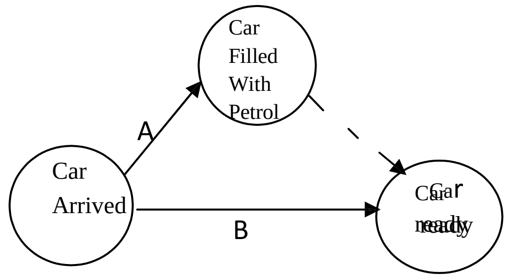
Figure 8




This is wrong because it contravenes rule b

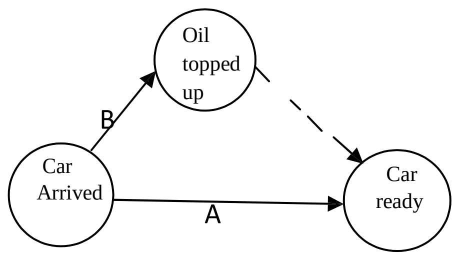
By the use of a dummy activity it could be shown correctly as follows:

Figure 6




Right

OR



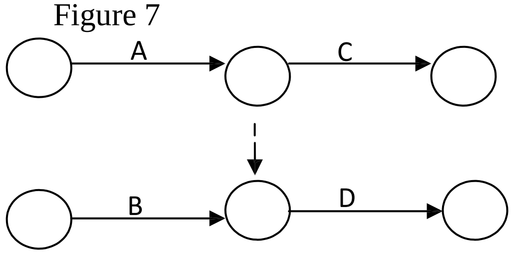

Right

Assume that part of a network involves a man lighting a cigarette. The activities involved, and their relationship are assumed to be as follows.

| Activity | Preceding activity |
|---|---|
| A-Remove cigarette from case | (Relates to earlier part of network) |
| C-Put cigarette case away | A |
| B-Strike match | (Relates to earlier part of network) |
| D-Light cigarette | A, B |

The network could be drawn thus

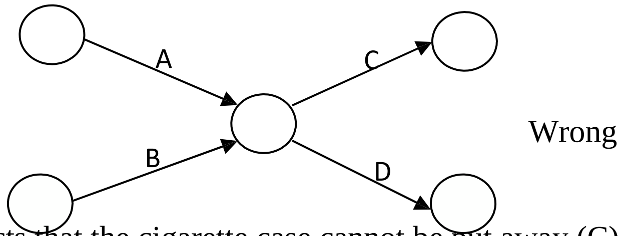
Figure 7




Right

If the network had been drawn as follows it would have been incorrect.

Figure 8



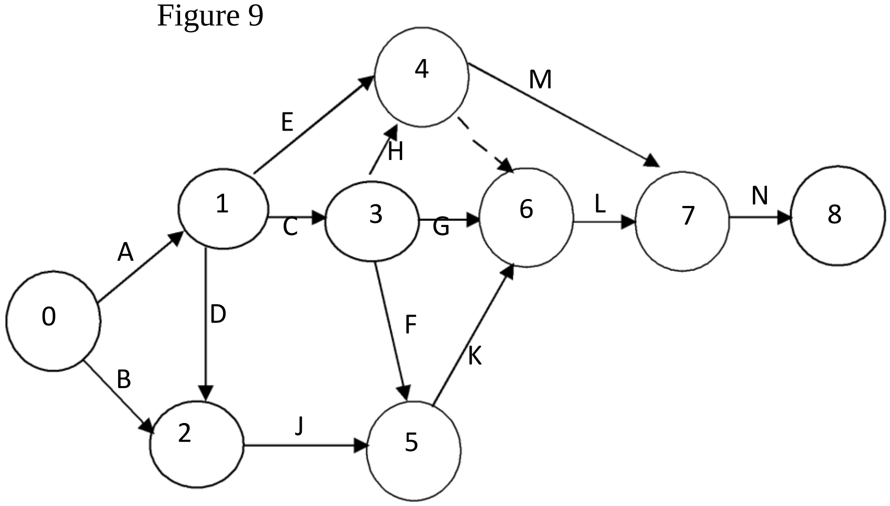

It is incorrect because it depicts that the cigarette case cannot be put away (C) until the match has been struck (B) which is incorrect according to the precedence rules given above.

Network example

Project One Constructing a house

| Activity | Preceding activity | Activity description |
|---|---|---|
| A | - | Draw a plan |
| B | - | Make a budget |
| C | A | Buy the materials |
| D | A | Transport the material to site |
| E | A | Dig up the foundation trenches |
| F | C | Lay the foundation |
| G | C | Build the First wall |
| H | C | Build the slab |
| J | B, D | Build the second wall |
| K | F, J | Construct the lido |
| L | E, H, G, K | Construct the roof |
| M | E, H | Fit the windows and doors |
| N | L,M | Do the plumbing, electrical work and finishing |

Figure 9




Project one constructing a house

Notes on solution given

a) The events have been numbered from 0 to 80.

b) A dummy (40-60) was necessary because of the preceding activity requirements of activity 10. If activities 5, 8 had not been specified as preceding activities 10, the dummy would not have been necessary.

c) The shape of the network is unimportant but the logic must be correct.

**4.7 Summary**

- Network analysis is used for the planning and control of large, complex projects.
- Networks comprise activities (represented as ⟶) and events (represented us O). Activities consume time and resources, events are points in time.
- Networks have one start event and one end event. An event is not complete until all activities leading into it are complete.
- The length of the arrows representing activities is not important because networks are not drawn to scale.
- Dummy activities (represented thus ----) are necessary to show logical relationships. They do not consume time or resources.

**Exercise**

1. ***Draw the network for the following project.***

| ***Activity*** | ***Preceding activity*** |
|---|---|
| ***A*** | **-** |
| ***B, C, D*** | ***A*** |
| ***E*** | ***B*** |
| ***F*** | ***C*** |
| ***G*** | ***F*** |
| ***H*** | ***G*** |
| ***I*** | ***H, I*** |
| ***J*** | ***C*** |
| ***K*** | ***D*** |
| ***L*** | ***I, J, K*** |

2. ***Draw the network in question 1 except that activity I is preceded by G and B.***

**References**:


- T. Lucey (2000) *Quantitative techniques, 5th edition. London*: Ashford colour press (Pg 322-329)
- Bishen Singh (2003) *Fundamentals of Operation Research New Connaught* place Dehra Dun-India (Pg310-323)

**Network Analysis -Time Analysis**

## 4.11 Objectives

After studying this subtopic the student will be able to:

- Know that there may be single or multiple time estimates for each activity
- Be able to define the critical path
- Understand how to find the critical path using the forward pass and the backward pass
- Know what is meant by float and how it is calculated
- Understand how basic time analysis can be extended using multiple time estimates
- Be able to estimate the standard deviation of the project duration
- Know how to use both continuous and discrete probabilities in networks

## 4.12 Assessing the Time

a) Time estimates. The analysis of project times can be achieved by using:

   i) Single time estimates for each activity. These are based on judgement of the individual responsible or by technical calculations using data from similar projects.

   ii) Multiple time estimates for each activity. The most usual multiple time estimates are three estimates for each activity. The most usual multiple time estimates are three estimates for each activity i.e. Optimistic (O), most likely (ML), and Pessimistic (P). These three estimates are combined to give an expected time and the accepted formula is:

$$\text{Expected time} = \frac{O + P + 4ML}{6}$$

b) Use of time estimates. On the completion of the basic time analysis, projects with multiple time estimates can be further analyzed to give an estimate of the probability of completing the project by a scheduled date.

c) Time units. Time estimates may be given in any unit i.e. minutes, hours, days, weeks, depending on the project. All time estimates within a project must be in the same units.

### 4.13 Basic Time Analysis – Critical Path

Critical path: The critical path of a network gives the shortest time in which the whole project can be completed. It is the chain of activities with the longest duration time.

There may be more than one critical path in a network and it is possible for the critical path to r-un through a dummy.

Earliest start time (EST). It is the earliest possible time at which a succeeding activity can start.

Figure10

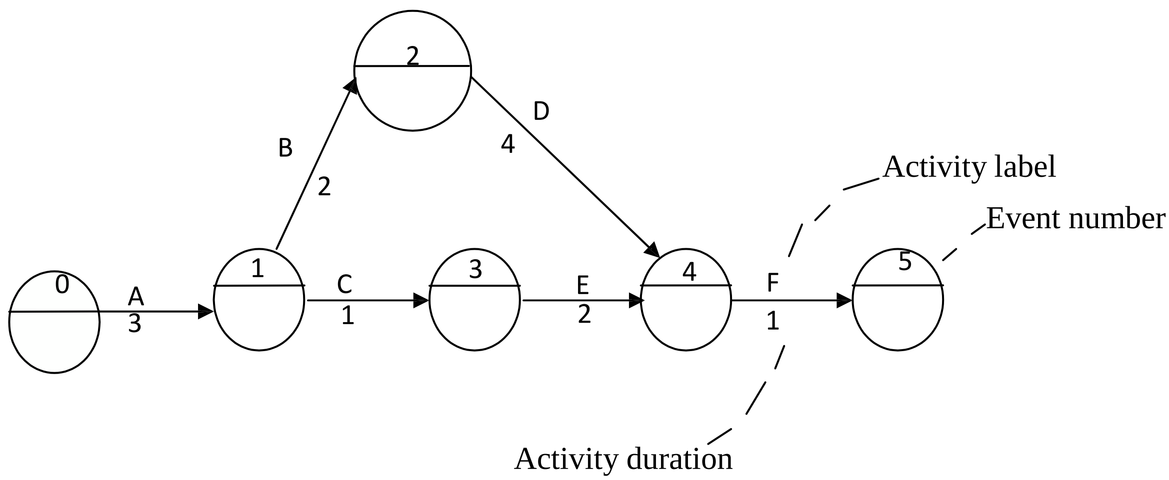
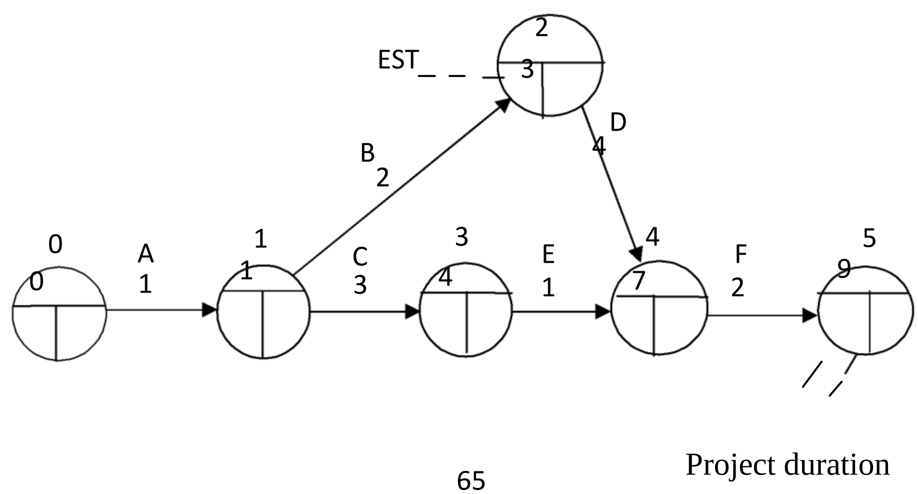



Figure 11

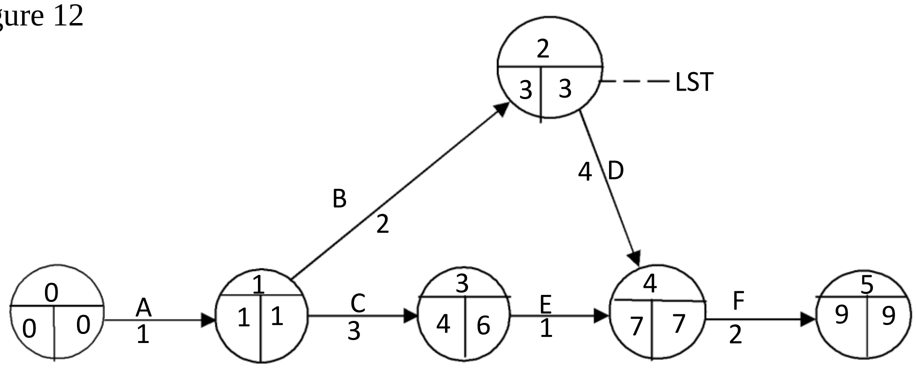


### 4.14 Calculation of EST (forward pass).

The EST of a head event is obtained by adding onto the EST of the tail event the linking activity duration starting from event 0, time 0 and working forward through the network. Where two or more routes arrive at an event the longest route time must be taken e.g. activity F depends on completion of D and E. E is completed by day 5 and D is not complete until day 7 thus F cannot start before day 7.The EST in the finish event no 5 is the project duration and is the shortest time in which the whole project can be completed.

Latest start times (LST). To enable the critical path to be isolated, the LST for each activity must be established. The LST is the latest possible time at which a preceding activity can finish without increasing the project duration. Using the example above the LSTs is as follows.

Figure 12



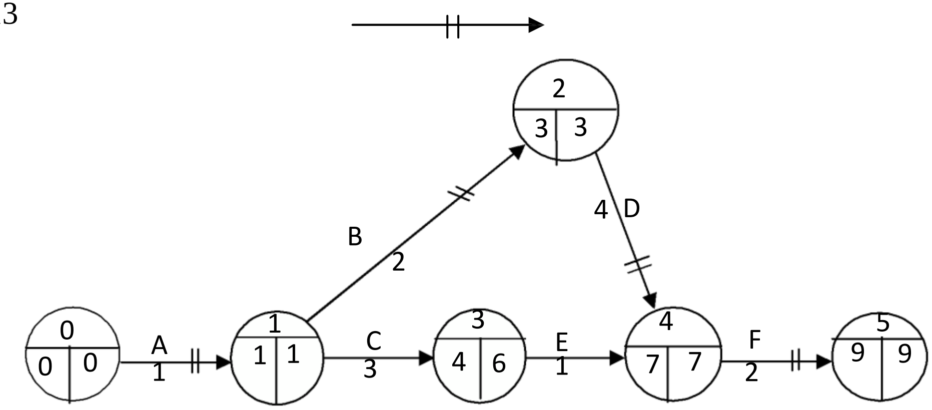

### 4.15 Calculating LST (termed the backward pass)

Starting at the finish event no 5, insert the LST (i.e. day 9) and work backwards through the network deducting each activity duration from the previously calculated LST. Where the tails of activities B and C join event no 1, the LST for C is day 3 and the LST for B is day 1. The lowest number is taken as the LST for event no 1 because if it occurred at day 3 then activities B and D could not be complete by day 7 as required and the project would be delayed.

### 4.16 Critical Path

Examination of the figure above shows that one path through the network (A, B, C, F) has EST's and LST's which are identical. This is the critical path which it should be noted is the chain of activities which has the longest duration. The critical path can be indicated on the network either by a different colour or by two small transverse lines across the arrows the path .

Figure 13

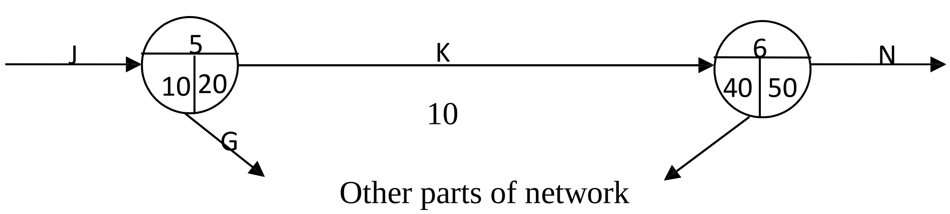



The activities along the critical path are vital activities which must be completed by their EST's/LST's otherwise the project will be delayed. The non critical activities i.e. C and E have spare time of float available thus could take up to an additional 2 days in total without delaying the project duration. If it is required to reduce the overall project duration then the time of one or more of the activities on the critical path must be reduced perhaps by using more labour, or more or better equipment or some other method of reducing job times.

**4.17 Float**

There are three types of float, total, free and independent float.

Using the diagram below, we will illustrate float.




i) Total float

This is the amount of time a path of activity could be delayed without affecting the overall project duration i.e. activity K.

Total float = Latest head time (Latest finishing time) – Earliest tail time – Activity Duration

Total float = 50 – 10 – 10

= 30 days

ii) Free float

This is the amount of time an activity can be delayed without affecting the commencement of subsequent activity at its earliest start time, but may affect float of a previous activity.

Free float = Earliest head time (Earliest finishing time) – Earliest tail time – Activity Duration

Free float = 40 – 10 – 10

= 20 days

iii) Independent float

This is the amount of time an activity can be delayed when all preceding activities are complete as late as possible and all succeeding activities complete as early as possible. Independent float therefore does not affect the float of either preceding or subsequent activities.

Independent float = Earliest head time – Latest tail time – Activity duration

Independent float = 40 – 20 – 10

= 10 days

The most important type of float is total float because it is involved with the overall project duration. When float is used without qualification assumes that total float is required.

The total float can be calculated separately for each activity but it is often useful to find the total float over chains of non-critical activities between critical events. For example in the example under critical path the non-critical chain activities is C, E for which the following calculation can be made:

| Non-critical chain | Time required | Time available | Total float over chain |
|---|---|---|---|
| C, E | 3+1 = 4 days | 7 – 1 = 6 days | = 2 days |

If some of the chain float is used up on one of the activities in a chain it reduces the leeway available to other activities in the chain.

Example: The construction of a house example is reproduced with the addition of activity durations.

Project One Constructing a house

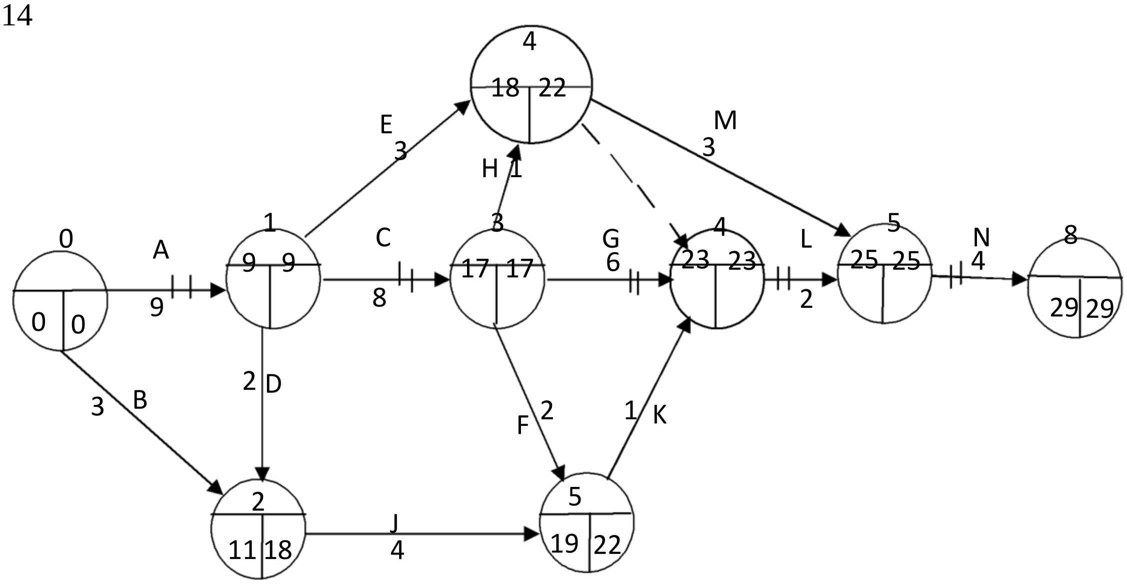
| Activity | Preceding Activity | Activity description | Activity duration (days) |
|---|---|---|---|
| A | - | Draw a plan | 9 |
| B | - | Make a budget | 3 |
| C | A | Buy the materials | 8 |
| D | A | Transport the material to site | 2 |
| E | A | Dig up the foundation trenches | 3 |
| F | C | Lay the foundation | 2 |
| G | C | Build the First wall | 6 |
| H | C | Build the slab | 1 |
| J | B, D | Build the second wall | 4 |
| K | F, J | Construct the lido | 1 |
| L | E,H,G,K | Construct the roof | 2 |
| M | E,H | Fit the windows and doors | 3 |
| N | L, M | Do the plumbing, electrical work and finishing | 4 |

Solution

The network is shown below from which it will be seen that the critical path is:

Activities 1, 3, 7, 12, 14 with duration of 29 days

Figure14




The float calculations are shown in the table below.

| Activity | EST | LST | EFT | LFT | D | (LFT-EST-D)Total float | (EFT-EST-D) free float | (EFT-LST-D) Independent float |
|---|---|---|---|---|---|---|---|---|
| A | *0 | 0 | 9 | 9 | 9 | - | - | - |
| B | 0 | 0 | 11 | 18 | 3 | 15 | 8 | 8 |
| C | * 9 | 9 | 17 | 17 | 8 | - | - | - |
| D | 9 | 9 | 11 | 18 | 2 | 7 | - | - |
| E | 9 | 9 | 18 | 22 | 3 | 10 | 6 | 6 |
| F | 17 | 17 | 19 | 22 | 2 | 3 | - | - |
| G | * 17 | 17 | 23 | 23 | 6 | - | - | - |
| H | 17 | 17 | 18 | 22 | 1 | 4 | - | - |
| J | 11 | 18 | 19 | 22 | 4 | 7 | 4 | - |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| K | 19 | 22 | 23 | 23 | 1 | 3 | 1 | - |
| L | *23 | 23 | 25 | 25 | 2 | - | - | - |
| M | 18 | 22 | 25 | 25 | 3 | 4 | 4 | - |
| N | 25 | 25 | 29 | 29 | 4 | - | - | - |
| *Critical activities | | | | | | | Float calculation | |

The total float on the non critical chains can also be calculated:

| Non-critical chain | Time float required chain | Time available | Total over |
|---|---|---|---|
| B, J, K | 8 | 23 | 15 |
| D, J, K | 7 | 14 | 7 |
| F, K | 3 | 6 | 3 |
| E, M | 6 | 16 | 10 |
| H, M | 4 | 8 | 4 |
| E, dummy | 3 | 14 | 9 |
| H, dummy | 1 | 6 | 5 |

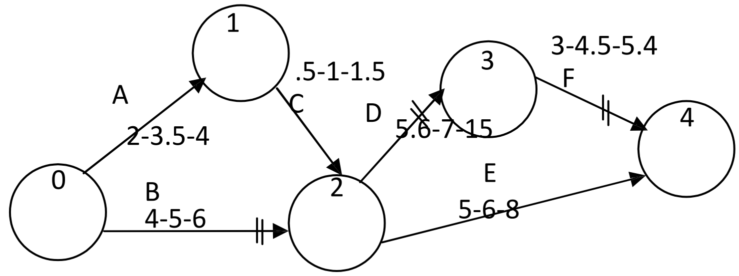
Slack is the difference between the EST and LST for each event. It does not apply to activities. Events on the critical path have zero slack.

### 4.19 Further Projects Time Analysis

Probability example

Assume that a simple project has the following network shown below. The activity times are in weeks and three estimates have been given for each activity.

Figure15




Scheduled date for completion week 19

Critical path (B, D, and F) expected duration

$$B = \frac{4+6+4(5)}{6} = 5 \text{ weeks}$$

$$D = \frac{5.6+15+4(7)}{6} = 8.1 \text{ weeks}$$

$$F = \frac{3+5.4+4(4.5)}{6} = \underline{4.4} \text{ weeks}$$

$$\underline{17.5}$$

If the critical activities were to occur at their optimistic times, event 4 would be reached in 12.6 weeks but if the critical activities occurred at their pessimistic times, event 4 would be reached in 26.4 weeks. As these durations span the scheduled date of week 19 some estimate of the probability of achieving the scheduled date of week must be calculated, as follows.

Make an estimate of the standard deviation for each of the critical activities. If no additional information is available the following PERT (Programme Evaluation and Review Technique) formula can be used.

$$\frac{\textit{Pessimistic time} - \textit{Optimistic time}}{6}$$

i.e. standard deviate on Activity B = $\frac{6-4}{6}$ = 0.33

Activity D = $\frac{15-5.6}{6}$ = 1.57

Activity F = $\frac{5.4-3}{6}$ = 0.4

Find the standard deviation of event 4 by calculating the statistical sum (the square root of the sum of the squares) of the standard deviation of all activities on the critical path.

$$\text{Standard deviation of Event 4} = \sqrt{\left(0.33^2 + 1.57^2 + 0.4^2\right)}$$

$$= 1.65 \text{ weeks}$$

Find the number of event standard deviations that the scheduled date is away from the expected duration.

$$\frac{19-17.5}{1.65} = 0.91$$

If you look in the table of areas under the normal curve the probability of achieving the scheduled date of week 19 is 82 %. If management consider that the probability of 82 % is not high enough, efforts must be made to reduce the times or the spread of time activities on the critical path. It is an efficient use of resources to try to make the probability of reaching the scheduled date 100 %. It is considered by some experts that the standard deviation, as calculated above, underestimates the true standard deviation. When activity times have variations the critical path will often change as the variations occur.

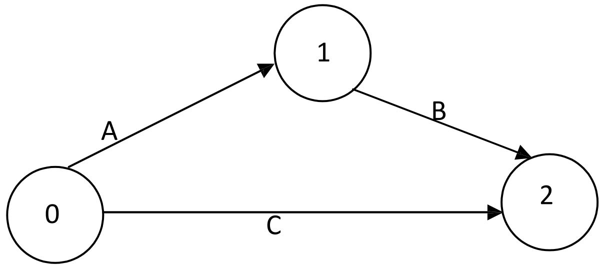
**4.19 Discrete Probabilities**

Probabilities are sometimes expressed in discrete terms. For example the time estimates for an activity could be given as follows:

| | Estimates | |
|---|---|---|
| A tivity | Time | Probability |
| A | 8weeks | 0.6 |
| | 11weeks | 0.4 |

The expected time for activity A would be (8 x 0.6) + (11 x 0.4) = 9.2 weeks.

Assume that time estimates have been made for the following network using discrete probabilities as follows:




Scheduled date week 14

Estimates

| Activities | Times (weeks) | Probability |
|---|---|---|
| A | 6 | 0.5 |
| | 10 | 0.5 |
| B | 3 | 0.4 |
| | 5 | 0.6 |
| C | 12 | 0.6 |
| | 14 | 0.3 |
| | 17 | 0.1 |

The expected times for the activities are:

A = (6 x 0.5) + (10 x 0.5) = 8

B = (3 x 0.4) + (5 x 0.6) = 4.2

C = (12 x 0.6) + (14 x 0.3) + (17 x 0.1) = 13.1

On the basis of the expected times the critical path is C with a duration of 13.1 weeks. However, numerous other possibilities exist and the probabilities of the various completion times and thus of achieving the schedule date of week 14 can be evaluated as follows:

The A, B route can have four durations, each with an associated probability as follows:

| A, B route | | | | | |
|---|---|---|---|---|---|
| Durations | 9 | 11 | 13 | 15 | Weeks |
| Probability | 0.2 | 0.3 | 0.2 | 0.3 | |

These values are found by combining the durations and probabilities of activities A and B. For example activity A duration of 6 weeks, probability 0.5, can be combined with activity B duration of 3 weeks, probability 0.4, to give 9 weeks duration and probability of 0.2(i.e. 0.5 x 0.4)

| C route | | | | |
|---|---|---|---|---|
| Durations | 12 | 14 | 17 | Weeks |
| Probability | 0.6 | 0.3 | 0.1 | |

The A, B route and the C route alternate as the critical path with varying probabilities as shown in the following table.

Table 17

| A, B route | | | | |
|---|---|---|---|---|
| 9 | 11 | 13 | 15 | Durations |
| 0.2 | 0.3 | 0.2 | 0.3 | Probability |

| | | | |
|---|---|---|---|
| 12<br>0.12 | 12<br>0.18 | 13*<br>0.12 | 15<br>0.18 |
| 14<br>0.06 | 14<br>0.09 | 14<br>0.06 | 15<br>0.09 |
| 17 | 17 | 17 | 17 |

| | | | |
|---|---|---|---|
| 0.02 | 0.03 | 0.02 | 0.03 |
| | | | |

| | Durations | Probability |
|---|---|---|
| | 12 | 0.6 |
| C route | 14 | 0.3 |
| | 17 | 0.1 |

*This means that if the A, B route with 13 weeks duration, probability 0.2, occurs at the same time as the C route duration of 12 weeks, probability 0.6, the critical path would be 13 weeks i.e. the longer duration, with the probability of 0.12 (0.6 x 0.2).

Summary of possible durations

| | |
|---|---|
| 12 weeks probability (0.12 + 0.18) | = 0.30 |
| 13 weeks probability (0.12) | = 0.12 |
| 14 weeks probability (0.06 + 0.09 + 0.06) | = 0.21 |
| 15 weeks probability (0.18 + 0.09) | = 0.27 |
| 17 weeks probability (0.2 + 0.3 + 0.2 + 0.3) | = 0.10 |
| | 1.00 |

Thus the probability of achieving 14 weeks or less is 0.63 (0.30 + 0.12 + 0.21) and the probability of exceeding the scheduled date is 0.37.

**4.20 Summary**

- Basic time analysis of a network involves calculating the critical path i.e. the shortest time in which the project can be completed.
- The critical path is established by calculating the EST (Earliest Start Times) and LST (Latest Start Time) for each event and comparing them. The critical path is the chain of activities where the EST's and LST's are the same.
- Float is the spare time available on non-critical activities. There are three types of float; total

float; free float and independent float.

- To calculate the probability of achieving scheduled dated it is necessary to establish what variability is likely to exist for each activity.
- Given activity time estimates the expected value of the project duration is calculated and an estimate made of the activity standard deviations.
- The activity standard deviations are combined to form the standard deviation of the overall project so that probability estimates can be made for various project duration possibilities.

**Exercise**

1. Find the critical path of the following network using the EST/LSTs.

| Activity | Preceding activity | Duration (days) |
|---|---|---|
| A | - | D |
| B | A | H |
| C | A | F |
| D | A | G |
| E | B | B |
| F | C | C |
| G | E | E |
| H | B, F | K |
| I | G, H | H |
| J | C | D |
| K | D | C |
| L | I, J, K | D |

2. Calculate the floats of the network in question 1.
3. The standard deviations of the activities on the critical path in question 1 are;1, 2, 1.5, 3, 2.5 and 3 respectively. Based on these values calculate the probability of achieving a scheduled time of 40 days for the project duration.

### 4.20 Network Analysis - Cost Scheduling

### 4.21 Objectives

After studying this subtopic the student will be able to:

- Understand the principles of least cost scheduling or crashing the network
- Know the meaning of normal and crash costs, normal and crash times, and cost slopes
- Be able to use the rules of least cost scheduling
- Know how to crash a network

### 4.22 Costs and Networks

This is sometimes known as PERT/COST.

The normal duration of a project incurs a given cost and by more labour, working overtime, more equipment etc the duration could be reduced but at the expense of higher costs. Some ways of reducing the project duration will be cheaper than others and network cost analysis seeks to find the cheapest way of reducing the overall duration.

A common feature of many projects is a penalty clause for delayed completion and / or a bonus for earlier completion. In examination questions, network costs analysis is often combined with a penalty and / or bonus situation with the general aim of calculating whether it is worthwhile paying extra to reduce the project time so as to save a penalty.

Basic definitions

a) Normal cost. The costs associated with a normal time estimate for an activity. Often the normal time estimate is set at the point where resources (men, machines etc) are used in the most efficient manner.

b) Crash cost. The costs associated with the minimum possible time for an activity. Crash costs, because of extra wages, overtime premiums, and extra facility costs are always higher than normal costs.

c) Crash time. The minimum possible time that an activity is planned to take, the minimum time is invariably brought about by the application of extra resources, e.g. more labour or machinery.

d) Cost slope. This is the average cost of shortening an activity by one time unit (day, week, month as appropriate). The cost slope is generally assumed to be linear and is calculated as follows;

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

| e.g. Activity A data: | <u>Normal</u> | <u>Crash</u> |
|---|---|---|
| | Time \| Cost | Time \| Cost |
| | 12 days at Ksh 4800 | 8 days at Ksh 6400 |

$$\text{Cost slope} = \frac{6400-4800}{12-8}$$

$$= \text{Ksh } 400 \text{ / day}$$

**4.23 Least Cost Scheduling Rules**

The basic rule of least cost scheduling is to reduce the time of the activity on the critical path with the lowest cost slope and progressively repeat this process until the desired reduction in time is achieved. Complications occur when time reductions cause several paths to become critical simultaneously thus necessitating several activities to be reduced at the same time. These complications are explained by example below.

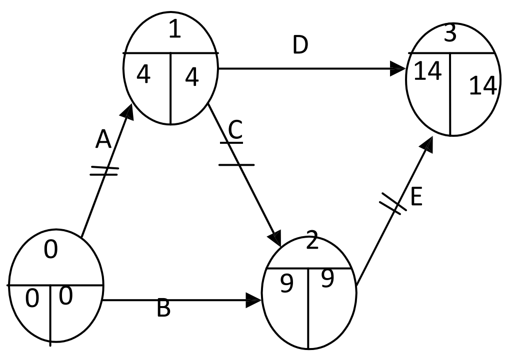
**4.24 Least Cost Scheduling Example**

A project has five activities and it is required to prepare the least cost schedules for all possible durations from 'normal time' - 'normal cost' to crash time'-crash cost'.

Table 18

| Project data Activity | Preceding activity | Time (days) Normal | Crash | Costs Normal (ksh) | Crash | Cost Slope (ksh) |
|---|---|---|---|---|---|---|
| A | - | 4 | 3 | 360 | 420 | 60 |
| B | - | 8 | 5 | 300 | 510 | 70 |
| C | A | 5 | 3 | 170 | 270 | 50 |
| D | A | 9 | 7 | 220 | 300 | 40 |
| E | B,C | 5 | 3 | 200 | 360 | 80 |

Figure 16




Project durations and costs

a) Normal duration 14 days

   Critical path A, C, E

   Project cost (i.e. cost of all activities at normal time) = Ksh1, 250

   (i.e., Ksh360 + 170 +220 +200)

b) Reduce by 1 day the activity on the critical path with the lowest cost slope.

   Reduce activity C at extra cost of Ksh 50

   Project duration 13 days

   Project cost Ksh1, 300

Note: all activities are now critical.

d) Several alternative ways are possible to reduce the project time by a further 1 day but note 2 or 3 activities need to be shortened because there are several critical paths.

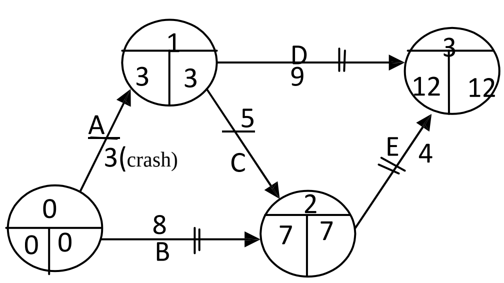
Possibilities available

Table 19

| Reduced by 1 day | Extra costs | Activities critical |
|---|---|---|
| A and B | Ksh60+70=Ksh130 | All |
| D and E | Ksh40+80=Ksh120 | All |
| B,C and D | Ksh70+50+40=Ksh160 | All |
| A and E | Ksh60+80=Ksh140 | A,D,B,E |

An indication of the total extra costs apparently indicates that the second alternative (i.e D and E reduced) is the cheapest. However, closer examination of the last alternative (i.e. A and E reduced) reveals that activity C is non-critical and with 1 day floats. It will be recalled that activity C was reduced by 1 day previously at an extra cost of ksh50. If in conjunction with the A and E reduction, activity C is increased by 1 day, the ksh50 is saved and all activities become critical. The net cost therefore for the 12 day duration is ksh1, 300 + (140-50) = ksh1,390. The network is now as follows:

Figure17



Duration 12 days

Cost 1,390

All activities critical

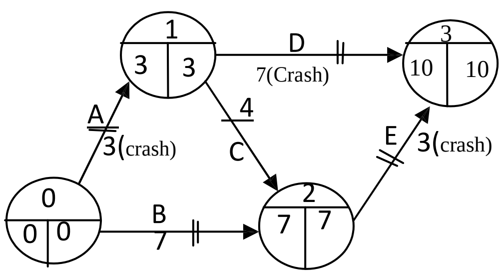
d) The next reduction would be achieved by reducing D and E at an increase of Ksh120 with once again all activities being critical.

Project duration 11days

Project cost Ksh1, 510

e) The final reduction possible is made by reducing B, C and D at an increased cost of Ksh160. The final network becomes:

Figure 18




Duration 10 days

Cost 1,670

All activities critical

Crashing networks in examinations

- All the principles necessary to crash networks have already been covered and the following points may save time in an examination.
- Only critical activities affect the project duration so take care not to crash non-critical activities.
- The minimum possible project duration is not necessarily the most profitable option it may be cost effective to pay some penalties to avoid higher crash costs.
- If there are several independent critical paths then several activities will need to be crashed simultaneously. If there are several critical paths which are not separate i.e. they share an activity or activities then it may be cost effective to crash the shared activities even though they may not have the lowest cost slopes.

- Always look for the possibility of increasing the duration of a previously crashed activity when subsequent crashing renders it non-critical, i.e. it has float.

**4.25 Summary**

- Cost analysis of network seeks the cheapest ways of reducing project times
- The crash cost is the cost associated with the minimum possible time for an activity which is known as the crash time.
- The average cost of shortening an activity by one time period (day, weeks etc) is known as the cost slope.
- Least cost scheduling finds the cheapest method of reducing the overall project time by reducing the time of the activity on the critical path with the lowest cost slope.
- The total project cost includes all activity costs not just those on the critical path.
- The usual assumption is that the cost slope is linear. This need not be so and care should be taken not to make the linearity assumption when circumstances point to some other conclusion.
- The example used in this chapter includes increasing the time of a subcritical activity
- Which has already been crashed, so saving the extra costs incurred? Always look for such possibilities.
- Dummy activities have zero slopes and cannot be crashed.

**4.26 Exercise**

1. Calculate the cost slopes and the critical path of the following network:

Table20

| Activity | Preceding activity | Time-normal | Time- crash | Cost-normal Ksh | Cost-crash Ksh |
|---|---|---|---|---|---|
| 1 | - | 6 | 4 | 500 | 620 |
| 2 | - | 4 | 2 | 300 | 390 |
| 3 | 1 | 7 | 6 | 650 | 680 |
| 4 | 1 | 3 | 2 | 400 | 450 |

| | | | | | |
|---|---|---|---|---|---|
| 5 | 2,3 | 5 | 3 | 850 | 1000 |

1. Construct a least cost schedule for the network in question 11 showing all durations from normal time- normal cost to crash time –crash cost.

### 4.30 Network Analysis - Resource Scheduling

### 4.31 Objectives

After studying this subtopic the student will be able to:

- Understand the principles of resource scheduling
- Known how to draw a Gantt chart
- Be able to prepare a resource aggregation profile
- Know how to use resource leveling
- Be able to prepare a resource allocation profile
- Resource and networks

### 4.32 Resource and Networks

The usefulness of networks is not confined only to the time and cost factors which have been discussed so far. Considerable assistance in planning and controlling the use of resources can be given to management by appropriate development of the basic network techniques.

Project resources: The resources (men of varying skills, machines of all types, the required materials, finance and space) used in a project are subject to varying demands and loadings as the project proceeds. Management need to know what activities and what resources are critical to the project duration and if resource limitations (e.g. shortage of materials, limited number of skilled craftsmen) might delay the project. They also wish to ensure, as far as possible, constant work rates to avoid paying overtime at one stage of a project and having short time working at another stage.

### 4.33 Resource Scheduling Requirements

a) To be able to schedule the resource requirements for a project the following details are required.
b) The customary activity times, descriptions and sequences as previously described

c) The resource requirements for each activity showing the classification of the resources and the quantity required

d) The resources in each classification that is available to the project. If variations in availability are likely during the project life, these must be specified.

e) Any management restrictions that need to be considered e.g. which activities may or may not be split or any limitations on labour mobility.

### 4.34 Resource Scheduling Example, Using a Gantt chart

A simple project has the following time and resource data (for simplicity, only the one resource of labour is considered but similar principles would apply to other inter- changeable resources.)

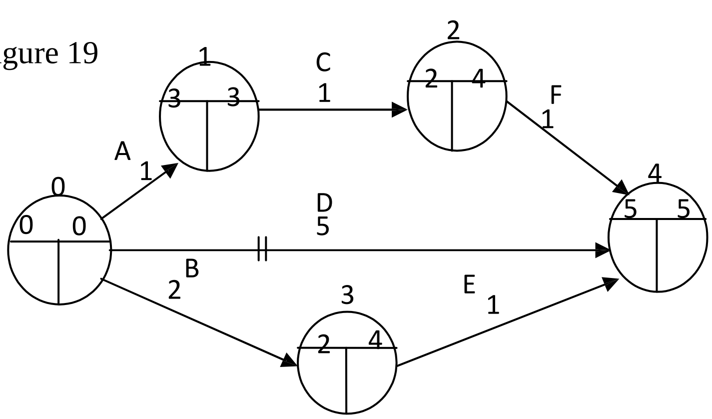
Table 21

Project data

| Activity | Preceding activity | Duration(days) | Labour requirements |
|---|---|---|---|
| A | - | 1 | 2 men |
| B | - | 2 | 1 man |
| C | A | 1 | 1 man |
| D | - | 5 | 1 man |
| E | B | 1 | 1 man |
| F | C | 1 | 1 man |

Resource constraint, 2 men only available

Figure 19

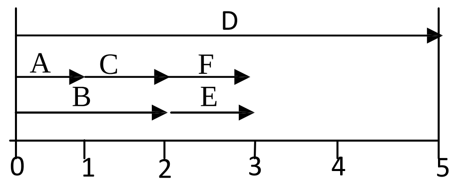


Critical path-activity D –durations 5 days (Without taking account of the resource constraint)

### 4.35 Resource Scheduling Steps

Draw the activity times on a Gantt or bar chart based on their ESTs

Figure 20

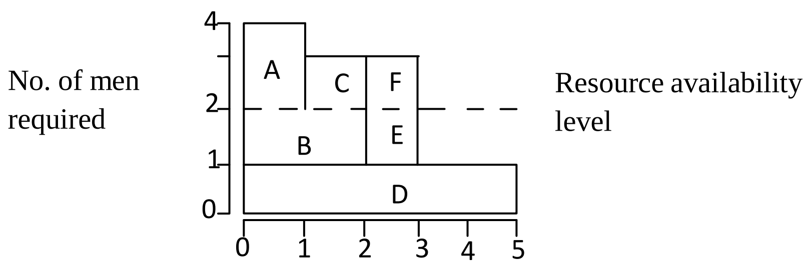



Time scale in days

Time scaled network

Based on the time bar chart prepares a resource aggregation profile i.e total resource requirements in each time period.

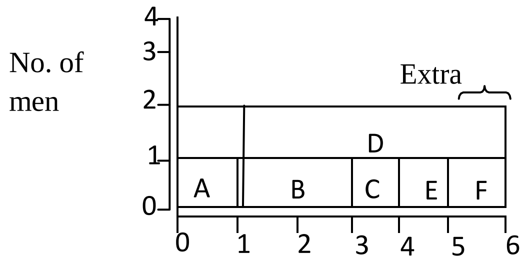
Figure 21




### 4.36 Resource Aggregation Profile Based on EST's

Examination of the above profile shows that at times more resources are required than are available if activities commence at their EST's/LSTs on the network show that float is available for activities A, C, F, B and E. having regard to these floats it is necessary to 'smooth out' the resource requirements so that the resources required do not exceed the resource constraint, i.e. delay the commencement of activities (within their float) and if this procedure is still not sufficient then delay the project as a whole. Carrying out this procedure results in the following resource profile

Figure 22



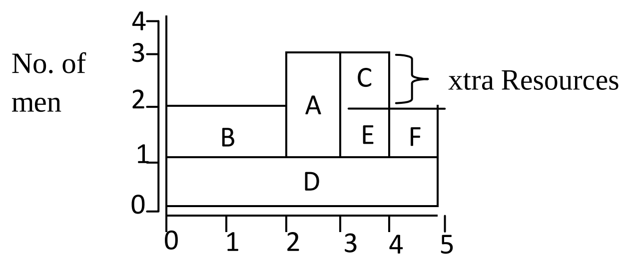

Time scale in days

Resource allocation with 2 man constraints

Note: this procedure is sometimes termed resource leveling.

Because of the resource constraint of 2 men it has been necessary to extend the project duration by 1 day. Assume that management state that the original project duration (5 days) must not be extended and they require this to be achieved with the minimum extra resources. In such cases a similar process of varying activity start times within their float is carried out, resulting in the following resource profile.

Figure 23




Time scale in days

Resources allocation profile- with 5 day constraints

The above profile shows that to achieve the 5 days duration it is necessary to have men available from day 2 to day 4.

**4.37 Summary**

- To enable resource scheduling to be carried out the resource requirement for each activity must be specified.
- In addition the various resources involved (men, machinery etc) must be classified and the

availability and constraints specified.

- After calculating the critical path in the usual manner a resource aggregation profile (s) is prepared i.e. the amount of the resource (s) required in each time periods of the project based on the EST's of each activity.
- If the resource aggregation indicates that a constraint is being exceeded, and float available the resource usage is 'smoothed' i.e. the start of activities is delayed.
- The smoothing of resource profiles is largely a matter of experimentation but if the time for the project is fixed concentrate attention on those activities with free float.

**4.38 Exercises**

1. A project has the following activity duration and resource requirements

Table 22

| Activity | Preceding activity | Duration (days) | Resource requirements (units) |
|---|---|---|---|
| A | - | 6 | 3 |
| B | - | 3 | 2 |
| C | - | 2 | 2 |
| D | C | 2 | 1 |
| E | B | 1 | 2 |
| F | D | 1 | 1 |

Assuming no restrictions show the network critical path and resource requirements on a day by day basis, assuming that starts are made on the EST of each activity.

2. Assume that there are only 6 units of resources what would be the plan?

### 4.40 Network Analysis – Activity on Nodes

### 4.41 Objectives

After studying this subtopic the students will be able to:

- Understand the characteristics of activity on node networks ( or precedence diagrams)
- Be able to draw activity on node networks
- Know how to identify the critical path in such networks
- Understand how computers can assist in network analysis.

### 4.42 Activity on Node Networks

This type of network, sometimes known as precedence diagram is an alternative form of presentation based on similar estimates to those previously described.

Activity on node network characteristics

a) The activities are shown as boxes, not as arrow

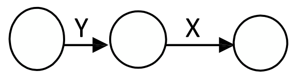
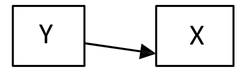
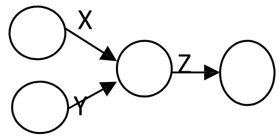
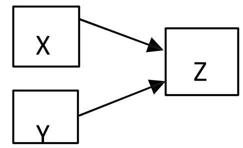
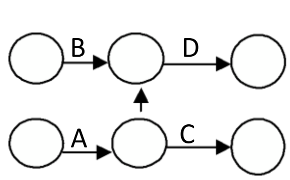
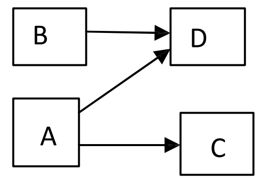
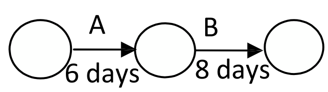
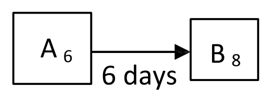
b) No events appear

c) Boxes are linked by lines to indicate their sequence or precedence

d) No dummies are necessary

e) Comparison with convectional networks

The activity on node approach is best illustrated by comparison with the procedures for drawing conventional networks

| Example | Conventional network | Activity on node |
| --- | --- | --- |
| A. Activity X depends upon |   |   |
| B Activity Z depends upon Activity x and y |   |   |
| C Activity C depends upon Activity A. Activity D depends upon Activities A and B |   |   |
| D Activity B can start immediately Activity A is complete. Durations are 8 days and 6 days respectively |   |   |

Notes:

a) Events as such do not appear in activity on node (A/N) networks

b) The lines shown in A/N networks indicate precedence and are not activities. The activities are represented by the square boxes.

c) From example C it will be seen that dummies are not necessary in A/N networks

d) The time (6 days) shown on the A/N network in example D is known as the dependency time.

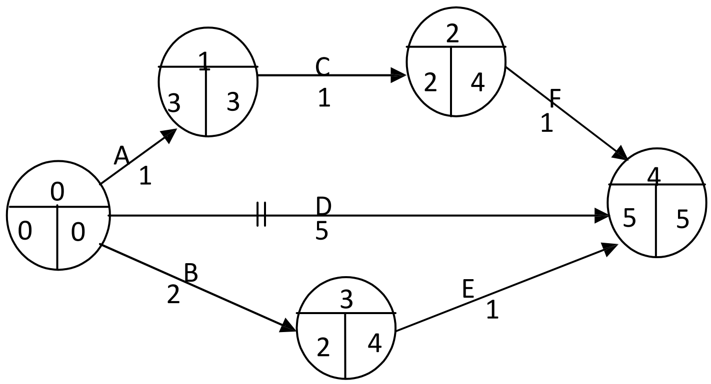
Note:

- Events as such do not appear in activity on node (A/N) networks.
- The lines shown in A/N networks indicate precedence and are not activities. The activities are represented by square boxes.
- From example C it will be seen that dummies are not necessary in A/N networks.

- The time (6 days) shown on the A/N network in example D is known as the dependency time.

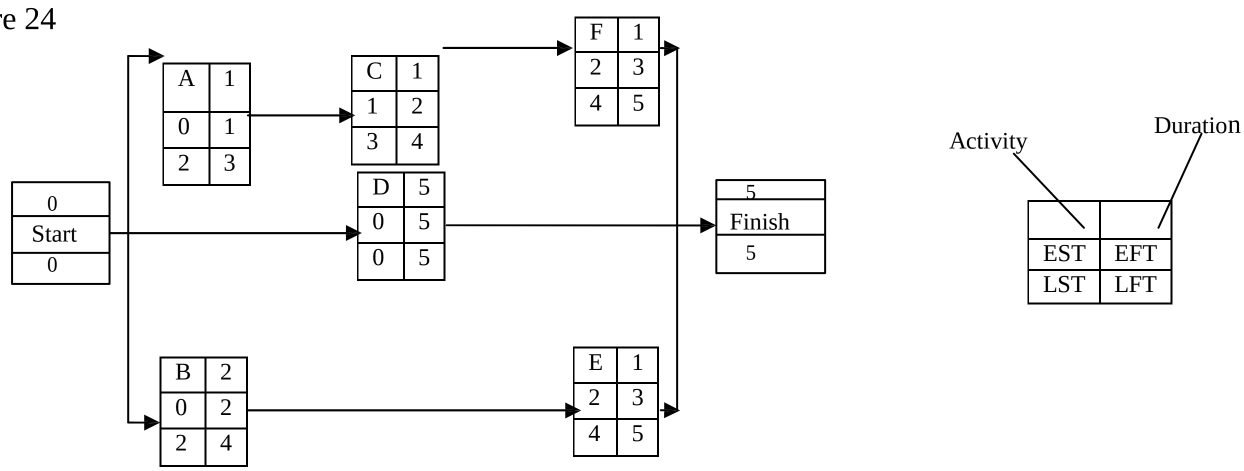
Activity on node example Based on the conventions described above a full (A/N) network is shown below. This uses the same activities, sequences and durations already used.

Figure 23




Convectional network

Figure 24




Notes on activity on node network

- The lines joining the boxes are not activities; they are precedence's as originally specifically specified.

- The EST/LSTs in the boxes are calculated by the same process as previously described but note that they are not the same values as shown in the events of the conventional network.
- The EFT (earliest finish time) and LFT (latest finish time) are calculated by adding the activity time on to the EST and LST respectively.
- The critical path is found by the usual method of comparing the EST and LST (or EFT and LFT)

**4.43 Networks and computers**

So far only the manual handling of networks has been considered but in practice most network analysis is handled by computers. Most installations have standard packages to deal with this technique and there are a number of advantages in utilizing these facilities.

a) By virtue of their speed computers can handle large networks with ease. Beyond a certain number of activities (say 50-100) networks become very difficult to handle manually and errors are almost certain to occur.
b) Full activity and event descriptions can be handled and printed out.
c) Input data has thorough validation checks and the network logic (loop danglers etc) is tested.
d) Once the data has been processed output can be presented in various ways, e.g. in activity or event sequence, scheduled data sequence activities by floats sequence, EST, LST, EFT or LFT sequence.
e) When changes in activity time's resource or scheduled dates occur the program can be quickly rerun to produce the new results.
f) More comprehensive time analysis and resource allocation can take place.
g) At one time the networks had to be drawn manually and the data fed into the computer. Packages are now available which allow the user to build up a network using the graphics facilities of the computer. The packages will then draw out the network diagram and calculate the project duration and isolate the critical path. More comprehensive packages will undertake cost scheduling and resource scheduling.

**4.44 Summary**

- Activity on node networks is an alternative method of presentation to

Conventional networks

- A/N networks do not show events but show the links (precedence) between activities.
- Forward and backward passes are made as usual. The forward pass gives the same EST as in conventional networks, but the backward pass gives the LST of the activity in the node rather than the LST of the succeeding activity as in the convectional networks.
- The critical path is established in the normal manner by comparing EST/LST or EFT/LFT.
- Computers are widely used for practical problems because of their greater speed, calculating ability and the existence of many standard packages.

**4.45 Exercise**

1. ***Draw an activity on node diagram for the following project.***

***Table 23***

| ***Activity*** | ***Preceding activity*** | ***Duration (days)*** |
|---|---|---|
| ***1*** | ***-*** | ***4*** |
| ***2*** | ***1*** | ***7*** |
| ***3*** | ***1*** | ***5*** |
| ***4*** | ***1*** | ***6*** |
| ***5*** | ***2*** | ***2*** |
| ***6*** | ***3*** | ***3*** |
| ***7*** | ***5*** | ***5*** |
| ***8*** | ***2,6*** | ***11*** |
| ***9*** | ***7,8*** | ***7*** |
| ***10*** | ***3*** | ***4*** |
| ***11*** | ***4*** | ***3*** |
| ***12*** | ***9,10,11*** | ***4*** |

2. ***Calculate the EST/LST and LFT values for each box.***

**References:**


Lucey T. (2000).*Quantitative techniques, 5th edition London:* Ashford colour press (pg 353-357).

## TOPIC FIVE: QUEUING THEORY

***5.0 Purpose:***

***Determining the cheapest and easiest ways of offering services***

***5.1 Objectives***

***After studying this chapter the students will be able to:***

- ***Identify situations where queuing is experienced.***
- ***Determine the number of items in the queuing system.***
- ***Determine the number of items actually waiting in the queues.***
- ***Determine amount of time an item spends in the queue.***
- ***Determine the probability of the number in the queue and the time in the queue.***

### 5.2 Life situations

Queues or waiting lines are experienced almost daily in real – life situations. Consider the following situations:

i) University students line up at the banks waiting to collect their 'booms' or loans, or line up to be registered at their respective campuses.

ii) Cars line up to be filled with petrol at petrol stations.

iii) Vehicles line up at toll stations for release after payment.

iv) Shoppers line up behind cash registers to pay.

v) Cinema goers wait to buy tickets.

vi) Letters wait in the secretary's tray to be typed.

vii) Aero planes circle around the airports waiting clearance to land.

viii) Patients line up in the doctor's clinic to be treated.

ix) Broken down machines await repair.

x) Cars wait for lights to turn green for them to move.

In all these situations there are arrivals, waiting and service. New arrivals wait because the existing demand for service exceeds the existing capacity of the service facility making it not possible for arriving customers to receive immediate service. It is also important to note that waiting occurs when there is variability in the arrival pattern of customers and also variability in the time required to provide required service.

Because there is no control to determine when the next customer should be arriving or how long it should take to serve a customer waiting lines are bound to occur. In those occasions where customers arrive too frequently, they must either wait for the service or do without it. But in cases where they arrive to infrequently, the service facility will have to remain idle while awaiting customers' arrival. Hence, in both cases, waiting customers and /or idle facilities form some waiting line or queue.

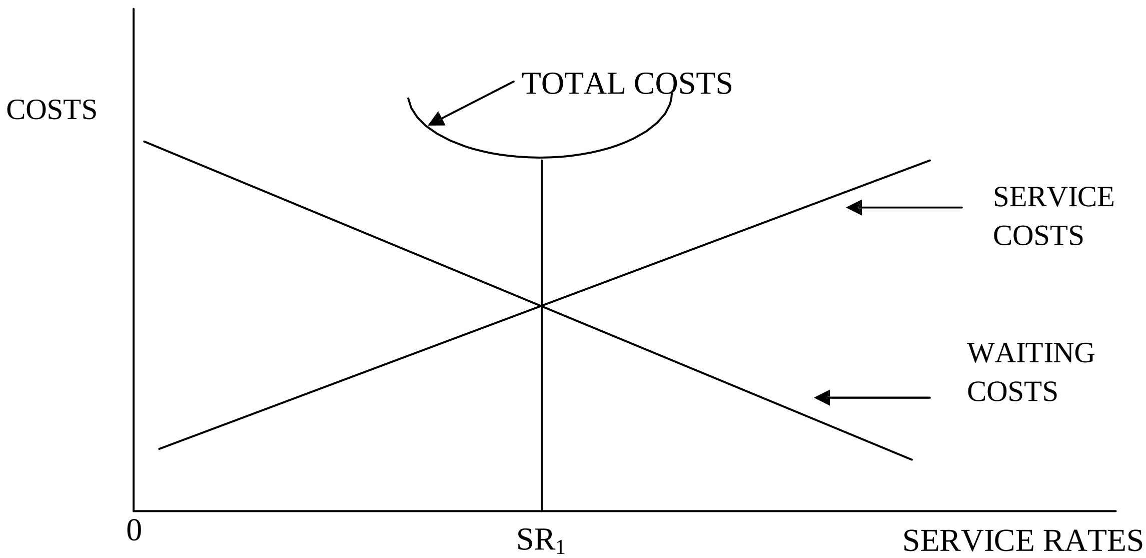
It is important to note that there need not be physical waiting line for a queuing situation to exist. Waiting lines or queues have some associated costs, which may either be monetary or non monetary.

Two primary costs associated with waiting lines are waiting costs, which are costs incurred because a customer is waiting for a service to be performed, and service costs, which are costs incurred for providing the service or having an idle facility.

An organization can reduce the waiting costs when it increases the rate of service, hence either increases the number of service facilities or increases the rate of providing the service.

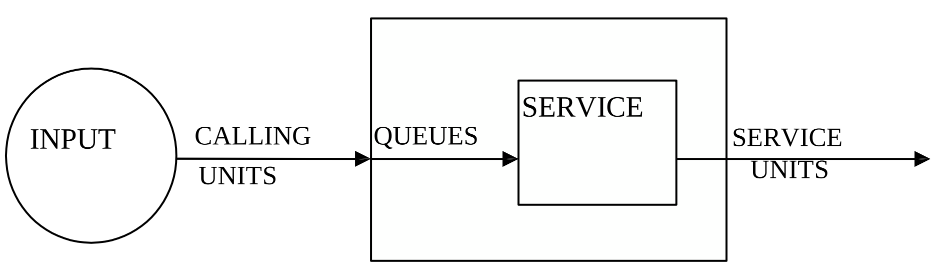
In either case, the service cost will be increased. But service costs can also be reduced by increasing the waiting costs. It is in the interest of management to reduce the inconvenience of waiting by finding that service rate that will minimize total costs, where total costs are composed of the sum of waiting and service costs.

Figure 25: General representation-costs and service rate




Service rate is the rate at which customers can be served. Note that the service rate SR1 which is directly below the lowest point on the total cost curve, is the required optimal service rate that yields to the organization the minimum total costs. Both service costs and waiting costs depend on the service rate. To the optimum service rate, the management requires to link these costs to the service rate.

Figure 26: General schematic representation of queuing process



These are the components of a queuing model: input source, calling units, Queues, service facility and served units.

Input source

This is the population or collection of people or things that generate people or things found in the queue, waiting for service. The people or things that are generated and waiting for service are called calling units. The input source may consist of finite or infinite number of calling units. If an organization has a fleet of 50 vehicles which are serviced by the organization's own mechanics, then this fleet forms a finite source because of the known number of vehicles available for serving. If on the other hand you sank a bore hole and then you pump water from it that water originates from an infinite source. Or a petrol station along a highway is fed by an infinite source. As more or less vehicles or a continuous stream of vehicles using the highway, might call at the station for service. The type of input source will affect the construction of queuing models as it influences the conclusions reached from such models.

**5.3 Input Source Type will affect**

a) The length of the queue. With 50 vehicles, the length of the queue cannot go beyond fifty, while with an infinite source; there is no agreed limit to the length of the queue.

b) It also affects the arrival of the calling units. In the finite source, the recent past history is capable of influencing the generation of calling units arriving at the present queues. For example, assume the mechanic had fixed new batteries in the 50 vehicles last week; there is little chance that a vehicle will require another battery this week. However, for an infinite source the experience of the immediate past is not likely to influence the current situation. The fact that some 50 vehicles called in last week at the highway petrol station for new batteries will have no effect on the number of vehicles calling in for batteries this week.

**5.4 Queuing System**

This consists of those who are actually in the queue and those being served at the service facility.

Served units are calling units who have already been served and now departing from the queue. Arrival process of calling units are considered as the mode of arrival and the timing of arrival of calling units waiting for service.

Modes arrivals include;

a) Bulk arrivals

Some calling units arrive at the point of service as a group. For example assume one takes stage coach bus to western Kenya. On arrival in Nakuru, the buses stop at shell B.P petrol for passengers to alight and have some refreshments from the restaurants at that station. Passengers, therefore, arrive for refreshments from these restaurants as a group (bulk arrivals).

b) Single arrivals

Some calling units arrive at service points individually, for example a customer may go to the post office alone to buy postage stamps.

c) Independent or conditional arrivals

The mode of arrival may be independent or conditional. If the state of the system or the sequence preceding arrivals does not affect subsequent arrivals, then the arrivals are said to be independent, otherwise conditional.

**5.5 Timing of Arrivals**

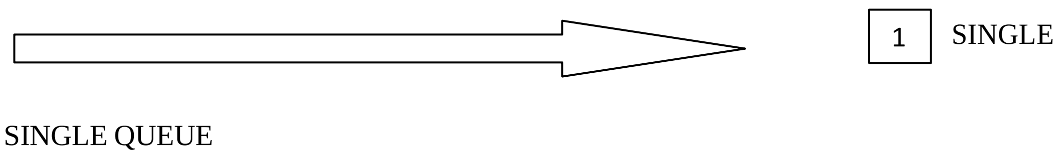
Calling units may arrive uniformly. In such a case, the amount of time separating each arrival is the same. For example, the arrival of a component to be assembled in an automated assembly line is totally controlled, hence the arrival is said to be uniform. If total control is exercised, then the arrival is deterministic. In some cases, the timing of arrival of calling units may be partially controlled. For example, aero planes arriving at an airport are partially controlled. Some calling units may arrive random; hence the inter-arrival times will vary randomly with unpredictable periods of time separating the arrivals. In such cases, the timing of arrival of calling units is not controlled, and the generating process of arrival is probabilistic. Most queuing problems are represented by this process. But probabilistic arrival process can either be described as empirical or a theoretical probability distribution. The widely used theoretical distribution that describes the arrival process is the poison model.

After specifying the details of arrivals, the average rate of arrival should be specified. The average rate of arrival determines the frequency of arrival. This average rate of arrival is denoted by the Greek letter lambda, $\lambda$

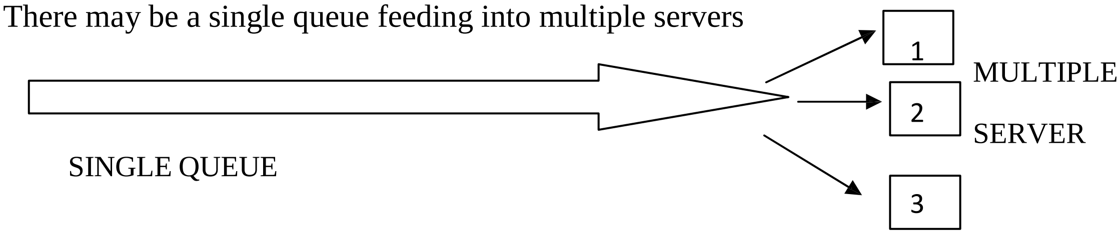
Hence, $\lambda$ = Mean arrival rate

The number and arrangement of queues

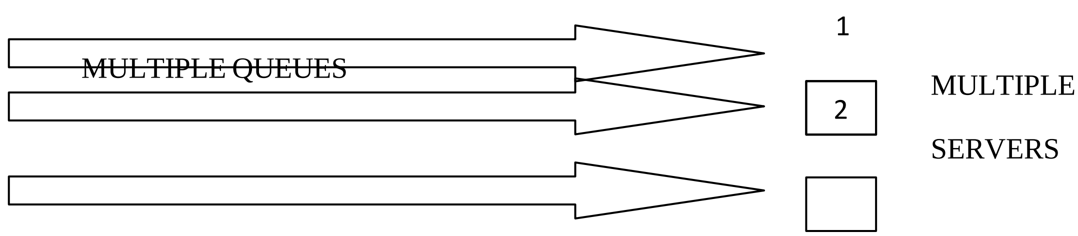
There may be a single queue feeding into a single server




There may be a single queue feeding into multiple servers




There may be multiple queues with equal numbers of servers




Queues may not exist at all, as when calls to a busy number are rejected. Or there may be a restriction as to the number of units which are allowed to form a queue, like the number of students allowed to form a class because of the existing facilities they will use for learning.

### 5.6 Behaviour Factors in Queues

When constructing queues, calling units behavior might influence the queuing system because such behavior may help in selecting or rejecting a queue, or determine how the calling unit will behave in the queue.

a) A human server may speed up rate of service when the waiting line is building up.

b) Rejection. If the system is filled to capacity, then the calling unit is rejected. For example when cinema theaters are full, newly arriving cinema goers may be rejected.

c) Jockey: in multiple - queue systems, calling units may jump from one line to another with the hope of reducing waiting time.

d) Balk: a human customer may balk or refuse joining a queue if he anticipates that there might be a long delay.

f) Renege: if a calling unit was already in the queue but thinks that additional waiting may not be worth it, he may renege or fall off and leave the queue altogether.

g) Collusion: if calling units feel that the explicit processing of one unit represents the implicit processing of more than one unit, then they may collude to have one of them perform for the others e.g. allowing one person to buy movie tickets for others.

Formulae – Simple queues

i) The probability of having no queue on arriving i.e. the probability that the service point is busy at any moment chosen at random = $P$ .This is $\frac{\lambda}{\mu}$ as there is only one channel.

ii) From (a) it follows that the probability that a customer is served immediately = $1-P$ .This is the same as the probability that there is no one in the system.

iii) The average number of people in system is $\frac{p}{1-P}$ or $\frac{\lambda}{\mu-\lambda}$

iv) The average number of people in the queue = $\frac{P^2}{1-P}$

v) The average time spent in the queue = $\frac{P}{\mu(1-P)}$

$$= \frac{\lambda}{\mu(\mu-\lambda)}$$

vi) From v it follows that the average time spent in the system is

$$\frac{P}{\mu(1-P)} + \frac{1}{\mu} = \frac{1}{\mu(1-P)} \text{ Or } \frac{1}{\mu-\lambda}$$

Multi- channel queuing formulae

The following formula can be derived using $\rho, \rho_0$ and $\eta$, the average rate of service for each channel:

$\rho_0$ = The probability that there are no customers in the system, and is given by the formula:

$$\rho_0 = \frac{c!(1-\rho)}{(\rho c)^c + c!(1-\rho)\left[\sum_{n=2}^{c-1} \frac{1}{n}(\rho c)^n\right]}$$

Average number of customers in the system = $\frac{\rho(\rho c)^c}{c!(1-\rho)^2}\rho_0 + \rho c$

Average number of customers in the queue = $\frac{\rho(\rho c)^c}{c!(1-\rho)^2 c\mu}\rho_0 + \frac{1}{\mu}$

Average time a customer spends in the queue = $\frac{(\rho c)^c}{c!(1-\rho)^2 c\mu}\rho_0$

This formula would also apply to a simple queue with only one service channel.

**5.7 Example**

In the newly created Sukuba district, there is Obongo health center.

The centre has a single medical doctor to take care of patients arriving from the rural community. The government is convinced that the situation faced here is a single- serve queuing situation with Poisson arrivals and Poisson service. After all, the calling units are the members of the community and the service mechanism is the doctor attending them.

From the minister's records, it has been established that patients arrive randomly at a rate of 0.20 patients per hour. Each patient requires different amount of time for treatment. The doctor reckons, however, that he treats his patients at an average rate of 0.21 patients per hour.

Calculate:

a) The average number of patients in the queuing system

b) The number of patients actually waiting in the queue

c) The average amount of time a patient spends in the queuing system

d) The average amount of time a patient spends in the queue

e) The probability of having exactly zero patients in the queuing system

f) The probability that there will be 10 or more patients in the queuing system

g) The utilization ratio with the doctor functioning as the service mechanism.

Solutions

a) The average number of patients in the queuing system:

$$L = \frac{\lambda}{\mu - \lambda} = \frac{0.2}{0.21 - 0.2} = 20 \text{ Patients}$$

b) The average number of patients waiting in the queue.

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{0.20^2}{0.21(0.21 - 0.20)} = 19 \text{ Patients}$$

c) The average amount of patient spends in the queuing system

$$W = \frac{1}{\mu - \lambda} = \frac{1}{0.21 - 0.20} = 100 \text{ Hours}$$

d) The average amount of time a patient spends in the queue.

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{0.20}{0.21(0.21 - 0.20)} = 95 \text{ Hours}$$

e) The probability of there being exactly zero patient in the queue.

$$P(n) = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right) = \left(\frac{0.20}{0.21}\right)^0 \left(1 - \frac{0.20}{0.21}\right) = 0.05$$

Hence, there is a probability of 0.05 that no patient is waiting to see a doctor or is actually being treated.

f) The probability that there will be 10 or more patients in the queuing system:

$$P(n \geq k) = \left(\frac{\lambda}{\mu}\right)^k = \left(\frac{0.20}{0.21}\right)^{10}$$

There is a probability of 0.60 that there will be 10 or more patients in the queuing system.

g) The utilization ratio with the doctor acting as the service mechanism

$$\mu = \frac{\lambda}{\mu} = \frac{0.20}{0.21} = 0.95$$

The doctor will be busy 95% of the time he is on duty.

**5.8 Classification of Queuing Models**

There are certain features which help in determining the number of distinct queuing models. But to completely specify any queuing model its respective assumptions must be stated. A system of notation that is used in classifying parallel server queuing model is as follows:

W/X/Y

Where,

W = this describes the particular distribution of inter-arrival tunes

X = this describes the specific distribution of service times

Y = this determines the number of parallel services.

Descriptive symbols for arrival and service distribution

M = Exponential inter-arrival or service time distribution or Poisson arrivals or departures

D = Deterministic inter-arrival or service time distribution.

$E_k$ = Erlangian or gamma distribution of inter-arrivals or service times of order k.

G = General distribution of inter-arrival times that are independent

G = General distribution of service times or departures.

### 5.9 Summary

a) The single - server queue structure

Customers arrive, wait for the server to be free, take a further period to be served, and leave the system. The server occasionally catches up with the arrivals. In effect we have two separate processes producing sequence of lives:

i) The arrival process, which is usually seen as being switched on exogenously (but can also be modeled as depend on the state of the serve)

ii) The service process, which is usually seen as being switched on by an arrival and running until it has run out of customers, then going idle until another customer arrives.

b) The multi - server queue structure

Customers are dealt with by one of several alternative servers, and then leave the system. There are now r service processes and l arrival process, impinging on each other.

c) Arrival processes

Generally distributed independent inter –arrival intervals, of which exponential interval gammas intervals and constant intervals are special cases. Generally distributed independent variations from an underlying schedule of arrivals, of which departure from fixed interval arrivals is a special case. In both cases, it is possible to link arrivals with the waste of the queue for example: deterrence by long queues getting tired of waiting limited pool of potential customer’s limited waiting room.

d) Service processes

Generally distributed independent service times, of which we have exponential service times enlarge service times and constant service times. It is possible to specify many variations of services; these include increased speed when the queue is long, fifo/lifo/random as the queue discipline, priority groups. (Pre-emptive and non-pre-emptive) A fairly widely used notation to describe simple queues takes the format: arrival process/service process/ no. of service; for example, GI/G/r, M/G/I, $E_k$ /$E_h$ /4. There are extensions to deal with things like priorities and queue discipline.

e) General results for stationary systems

i) Stationary and non-stationary systems

Basically the servers can cope if:

$$\frac{\text{E (service time)}}{\text{E (interval between arrivals)}} < r$$

In this case there will be a tendency for queues to build - up and clear again if service times Or inter-arrival show stochastic variation.

The system will, in the long run, be expected to spend:

- A proportion $\pi_0$ of total time with 0 customers
- A proportion $\pi_1$ of total time with 1 customer
- A proportion $\pi_2$ of total time with 2 customers.

And so on. If

$$\frac{\text{E (service time)}}{\text{E (interval between arrivals)}} > r$$

Then we expect queues to build-up without limit.

ii) General stationary system

In the long run, the expected number of customers in the system is:

$$\bar{n} = 0.\pi_0 + 1.\pi_1 + ... + r.\pi_r + (r+1)\pi_{r+1} + ...$$

And the expected number of customers queuing to be served is:

$$\bar{n} = 1.\pi_{r+1} + 2.\pi_{r+2} + ...$$

Evidently,

$\bar{n} = \bar{n}_q + E$ (Number of servers busy)

$$= \bar{n}_q + \frac{\text{E (Service time)}}{\text{E (Interval between arrivals)}}$$

We can also argue that:

$$\bar{n} = \frac{\text{E (time customer is in system)}}{\text{E (Interval between arrivals)}}$$

And $$\bar{n}_q = \frac{\text{E (time customer queues for service)}}{\text{E (Interval between arrivals)}}$$

That is:

$\bar{W}$ = E (time in system) = $\bar{n}$. E (intervals between arrivals)

$\bar{W}_q$ = E (time in system) = $\bar{n}_q$.E (intervals between arrivals)

We can also note that $\overline{W} = \overline{W}_q + E$ (service time)

iii) GI/G/r: unlimited waiting room

Although usable general solutions cannot be found, the conclusion of work by E. Page (Queuing theory in O.R, Butterworth's, 1972) and G.P Cosmetatos (A principle for approx Solutions of Q. systems, PhD London 1975) is that reasonable approximation can be found. Writing [ ] for $\bar{W}_q$ the approximation is:

$$[GI/G/r] \equiv V_a V_s [M/M/r] + (1-V_a)(1-V_s)[D/D/r] +$$
$$V_a(1-V_s)[M/D/r] + (1-V_a).V_s[D/M/r]$$

Where

$V_a$ = V (interval between arrivals) / (E (interval between arrivals)) $^2$

$V_s$ = V (service time) / (E (service time)) $^2$

Page has produced tables of [M/M/r], D/M/r] in units of E (service time) for traffic Intensities of 0.10 (0.05) 0.40 (0.02) (0.74) (0.01) 0.95 and for r =1(1) 20 (2) 40.

He has also tabulated Pr (having to queue) for these systems. For [M/M/r] and [M/D/r] the Results are almost identical; for [D/M/r] the probability is noticeable less.

iv) M/G/r: waiting room limited to r

In this system, customers who cannot be served immediately cannot join the system and cannot form a queue. It can be shown that:

$$\pi_0 = \left( \frac{E(servicetime)}{E(\text{int}\,erval\ between\ arrivals)} \right)^n \frac{\pi_0}{n^1}$$

This is clearly of the form of a Poisson distribution truncated at r

Probabilities can therefore be derived from tables of the Poisson distribution by dividing by: Pr (Poisson variables $\leq$ r)

In particular, because of the Markovian arrival process, we can argue that:

$\pi_n$ = Pr (customer is turned away)

Applications: telephone exchanges, maternity beds.

v) M/G/l: k Non-Pre-emptive priority groups

In these system customers, in group l jump ahead of customers of lower priority, so the waiting queue has the structure:

..4 4 3 2 2 2 1 $\boxed{3}$

Server

No one is displaced from service once started. If we write $P_j$ for:

$$\sum_{i=1}^{j} \frac{E(\text{service time for type } i)}{E(\text{interval between arrivals of type} i)}$$

And $p_0 = 0$

Then the general results:

E (time in system for a customer of type j)

$$\frac{\sum_{i=1}^{j} \left( \frac{E(\text{service time for type } i)}{E(\text{interval between arrivals of type} i)} \right)^2}{2(1-p_j-1)(1-p_j)}$$

Particular results for Markovian queues

The general Markovian queue is as follows

If every short interval kt, while there are n customer has

$\lambda_n \delta_t$ As the probability of an arrival

$\mu_n \delta_t$ As the probability of a service completion

Then the time - to- the next - event is exponential, mean $1/(\lambda_n + \mu_n)$ and we have a simple state- transition model with:

$\lambda_{n,n+1} = \lambda_n$ And $r_{n+1,n} = \mu_n$

Hence

$$\pi_n = \frac{\lambda_0 \ldots \lambda_{n-1} \pi_0}{\mu_1 \ldots \mu_n}$$

## 5.10 Particular Models

M/M/l (unlimited waiting room) has $\lambda_n = \lambda$ and $\mu_n = \mu$ (for all n > 0); it is a model of Poisson arrivals and exponential service times. From downstream its output appears to be a Poisson process, rate $\lambda$ and we can therefore consider problems which have networks of M/M/l (unlimited) queues

Machine breakdowns have $\lambda_n$= (m-n) $\lambda$ to describe the rate of breakdowns when for the total of m machines there are already n being repaired or waiting for repair.

**Exercise**

***The Operational Research group gongo goldmine constructed the following scenario for part of a new gold smelting works which was being designed. Trolleys will arrive at regular 15 minute interval. There will be four anodes on each trolley. Two identical processing machines will be available but, since trolles are very heavy, all four nodes will be processed by the same machine even if the other machine is idle. Processing will normally take exactly 15 minutes for each anode. However, 40 % of anodes will need an additional period of processing, with mean 24 minutes. The operational research group first calculated the means and standard deviations of:--***

- ***The total time to process an anode for the 40 % needing additional work.***
- ***The total time to process an anode chosen randomly from the 60:40 mixes,***
- ***The total time to process a randomly selected group of 4 anodes.***

***a. confirm their finding that it would, on average, take 24 minutes to process a trolley load of anodes, with a standard deviation of 4 minutes. After that, they disagreed about how to use queuing theory to estimate the mean time a trolley could expect to wait for processing machine to become free.***

***b. Kuojak found a formula for M/M/l queues and used it on the basis that each machine could be separately regarded as a server receiving 2 customers an hour. What would his estimate be?***

***c. Ajuoga found a more advanced book with formulae for M/M/r queues, and did a calculation for r = 2. What is his estimate?***

***d. Ragot discovered a reference book that had tablets for D/M/ queues. What would his estimate be?***

***e. Otuga used estimate and approximate interpolation method for GI/G/r queues. What would his estimate be, and what conclusions could be drawn about the calculations made by the other three?***

***f. since trolleys only need to stay while the first three anodes are processed, otuga also calculated how long on average, a trolley would take between arriving and leaving. What is the average? Having done that, he then suddenly began to doubt whether he had done the right calculations. Ought he to do all the calculations again with a mean service time of 18 minutes and a standard deviation of 3.464 minutes? Why?***

***2. Annie has found that inter-arrival times are almost exponential apart from a relative scarcity of values near zero, with mean 8 minutes and standard deviation 7.07 minutes.***

***a) She conjectures that arrivals might plausibly be modeled as consisting of a short exponential phased (with highish $\lambda_1$) followed by a longer exponential phase (with lowish $\lambda_2$). In order to estimate $\lambda_1$ and $\lambda_2$ she set up two equations.***

***8 = mean inter-arrival time as a function of λ1 and $\lambda_2$***

***7.07 = standard deviation of inter-arrival time as a function of $\lambda_1$ and $\lambda_2$.***

***Write down the equations and show they are satisfied by:***

$$\lambda_1 = 1, \lambda_2 = \frac{1}{7}$$

***Show that in general, writing m and s for the mean and standard deviation, the equations are satisfied by:***

$$\frac{1}{\lambda_1} = \frac{1}{2}(m - \sqrt{25^2 - m^2})\ \textbf{\textit{And}}$$

$$\frac{1}{\lambda_2} = \frac{1}{2}(m - \sqrt{25^2 - m^2})$$

***b)*** ***There are two servers each capable of dealing with a customer in a time averaging 12 minutes, with a standard deviation 4 minutes. The servers work separately. How long can a customer expect to wait before entering service? How many customers are on average waiting to enter service?***

***What difference does it make if we assume service is constant? Those inter-arrivals are exponential? That service is constant and arrivals are random?***

***[6.95 mins, 0.87mins, 6.15mins, 8.72mins, 7.88 mins]***

**References**:

Kumwar D.S and kongere T.O (2003) *Fundamentals of operations Research* India: Bishen Mahendra Pal Singh. (Pg 263-278)

## TOPIC SIX: INVENTORY CONTROL

***6.0 Purpose***

***Examination in depth of inventory control and planning and control techniques***

***6.1 Objectives***

***After studying this topic students will be able to***

- ***Have been introduced to the key principles of inventory control which are developed in the following three subtopics.***
- ***Know the reasons why stocks are held.***
- ***Understand the four costs associated with inventory.***
- ***Be able to define common inventory, control terms such as Lead time, Physical and Free***
  ***Stock, Economic Ordering Quantity***
- ***Know what is meant by Pareto or ABC analysis***

### 6.2 Types of Inventory

A convenient classification of the types of inventory is as follows:

a) Raw materials - the materials, components, fuels etc used in the manufacture of products.
b) Work-in-Progress-(W-I-P) – partly finished goods and materials, sub-assemblies etc. held between manufacturing stages.
c) Finished goods- completed products ready for sale or distribution.

The particular items included in each classification depend on the particular firm. What would be classified as a finished product for one company might be classified as raw materials for another. For example, steel bars would be classified as a finished product for a steel mill and as raw material for a nut and bolt manufacturer.

## 6.3 Reasons for Holding Stocks

- The main reasons for holding stocks can be summarized as follows:
- To ensure that sufficient goods are available to meet anticipated demand;
- To absorb variations in demand and production;
- To provide a buffer between production processes, this is applicable to work-in-progress stocks which effectively de-couple operations;
- To take advantage of bulk purchasing discounts;
- To meet possible shortages in the future;
- To absorb seasonal fluctuations in usage or demand;
- To enable production processes to flow smoothly and efficiently;
- As a necessary part of the production process e.g. the maturing of whisky;
- As deliberate investment policy particularly in times of inflation or possible shortage

## 6.4 Alternative Reasons for the Existence of Stocks

Reasons given in Para 3 Above are the logical ones based on deliberate decisions. However, stocks accumulate for other, less praiseworthy reasons, typical of which are the following:

- Obsolete items are retained in stock.
- Poor or nonexistent inventory control resulting in over-large orders, replenishment orders being out of Phase with production. Etc.
- Inadequate or non-existent stock records.
- Poor liaison between the production Control, Purchasing and Marketing departments.
- Sub-optimal decision making, e.g. the production Department might increase W-I-P stocks unduly so as to ensure long production runs.

## 6.5 Stock Costs

Where as a result of deliberate policy or not, stock represents an investment by the organization. As with any other investment, the costs of holding stock must be related to the benefits to be gained. To do this effectively, the costs must be identified. There are four categories; costs of holding stock, costs of obtaining stock, stock out costs, and the costs of the stock itself.

## 6.6 Costs of Holding Stock

These costs, also known as carrying costs, include the following:

Interest on capital invested in the stock, Storage charges (rent, lighting, heating, refrigeration, air conditioning, etc), Stores staffing, equipment maintenance and running costs, handling costs. Audit, stocktaking or perpetual inventory costs. Insurance, security deterioration and obsolescence Pilferage, vermin damage

**6.7 Costs of Obtaining Stock**

These costs, sometimes known as ordering or procurement costs, include the following:

a) The clerical and administrative costs associated with the purchasing, Accounting, and Goods Received departments.
b) Transport costs.
c) Where goods are manufactured internally the set up and tooling costs associated with each production run.

Note: Some students consider ordering costs to include only those costs associated with ordering external to the firm. However, internal ordering (i.e. own manufacture) may involve high costs for production planning, set – up.

**6.8 Stock out Costs**

These are the costs associated with running out of stock. The avoidance of these costs is the basic reason why stocks are held in first instance. These costs include the following:

i) Lost contribution through the lost sale caused by the stock out.
ii) Loss of future sale because customers go elsewhere.
iii) Loss of customer goodwill.
iv) Cost of production stoppages caused by stock outs of W-I-P or raw materials.
v) Labour frustration over stoppages.
vi) Extra costs associated with urgent, often small quantity, replenishment purchases. Clearly many of these costs are difficult to quantify, but they are often significant.

**6.9 Cost of Stock**

These costs are the buying in prices or the direct costs of production. These costs need to be considered when:

- Discounts are available for bulk purchase.
- Savings in production costs are possible with longer batch runs.

## 6.10 Objective of Inventory Control

The overall objective of inventory control is to maintain stock levels so that the combined costs, detailed in Para's. 6, 7 and 8 above are at a minimum. This is done by establishing two factors, when to order and how many to order. These decisions are the subject of the following subtopics.

## 6.11 Inventory Control Terminology

Brief definitions of common inventory control terms are given below and are illustrated below:

a) Lead or procurement time. The period of time, expressed in days, weeks, months, etc between ordering (either externally or internally) and replenishment, i.e. when the goods are available for use.

b) Demand. The amount required by sales, production, etc. Usually expressed as a rate of demand per week, month, etc Estimates of the rate of demand during the lead time are critical factors in inventory control systems.

c) Economic Ordering Quantity (EOQ) or Economic Batch Quantity (EBQ).This is a calculated ordering quantity which minimizes the balance of costs between inventory holding costs and re-orders costs.

d) Physical stock. The number of items physically in stock at a given time

e) Free stock. Physical stock plus outstanding replenishment orders minus unfulfilled requirements

f) Buffer stock or minimum stock or safety stock. A stock allowance to cover errors in forecasting the lead time or the demand during the lead time

g) Maximum Stock. A stock level selected as the maximum desirable which is used as an indicator to show when stocks have risen too high.

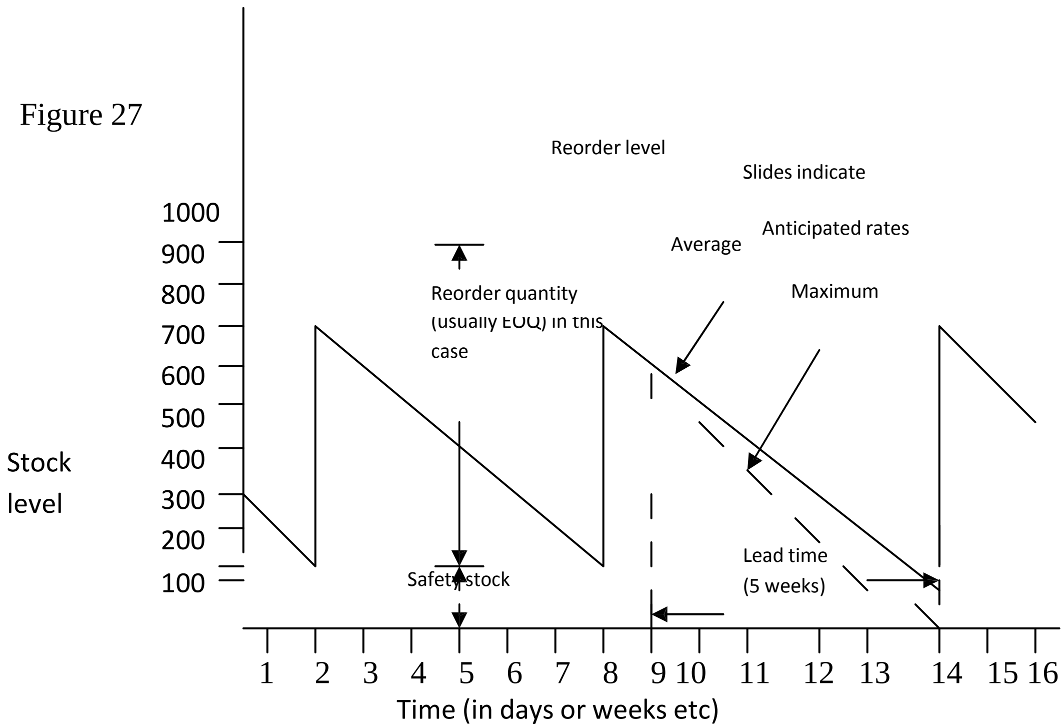
h) Re-order level. The level of stock at which a further replenishment order should be placed, The re-order level is dependent upon the lead time and the demand during the lead time.

i) Re-order Quantity. The quantity of the replenishment order, in some types of inventory control systems this is the EOQ but in some other systems a different value is used.

## 6.12 A Simple Stock Situation Illustration

The following diagram shows a stock situation simplified by the following assumptions: regular rates of demand, a fixed lead time, and replenishment in one batch

Figure 27




Stock terminology illustrated.

Notes:

a) It will be seen from figure 11/1 that the safety stock in this illustration is needed to cope with periods of maximum demand during the lead time.

b) The lead time as shown is 5 weeks, the safety stock 200 units, and the re-order quantity 600 units.

c) With constant rates of demand ,as shown, the average stock is the safety stock plus ½ Re- order quantity; for example, in Figure 11/1 the average stock is $200+\frac{1}{2}(600) = 500$ units.

### 6.13 Pareto Analysis

Detailed stock control uses time and resources and can cost a considerable amount of money. Because of this it is important that the effort is directed where it can be most cost effective – there is little point in elaborate and costly recording and control procedures for an item of insignificant value. Because of this it is worthwhile carrying out a so called Pareto or ABC analysis. It is often found that a few items account for a large proportion of the value and accordingly should have the closest monitoring. A typical analysis of stock items could be as follows:

Class A items- 80% of value in 20% of items – Close day-to-day control.

Class B items- 15% of value in30% of items –Regular review

Class C items- 5% of value in 50% of items –Infrequent review.

Such a review can help to ensure that resources are used to maximum advantage. Detailed, selective control will be more effective than a generalized approach which treats all items identically.

### 6.14 Deterministic and Stochastic Models

Like most other quantitative techniques, two types of models can be used for inventory control- deterministic and stochastic (or probabilistic) models.

- A deterministic model is one which assumes complete certainty. The values of all factors (e.g. demand, usage, lead time, carrying cost, etc.) are known exactly and there is no element of risk and uncertainty.
- A stochastic model exists where some or all of the factors are not known with certainty and can only be expressed in probabilistic or statistical terms. For example, if the usage rate or lead time was specified as a probability distribution, this would be a stochastic or probabilistic model.

### 6.15 Summary

- Stock may conveniently be classified into Raw materials, Work – in- progress, and finished Goods.
- The classification of an item depends upon the nature of the firm
- Stock are held to satisfy demands quickly, to allow unimpeded production, to take advantage of bulk purchasing, as a necessary part of the production process, and to absorb seasonal and other fluctuations.

- Stocks accumulate unnecessarily through poor control methods, obsolescence, poor liaison and sub-optimal decision making.

- The costs associated with stock are: holding costs, costs of obtaining stock, and stock outs cost

- The overall objective of inventory control is to maintain stock at a level which minimizes total stock costs.

- Inventory control has its own terminology, the basic contents of which are given in this chapter. The various definitions should be thoroughly learned.

### 6.16 Exercise

1. How may inventories be classified?
2. Why are stocks held?
3. What are the major categories of cost associated with stacks?
4. What is the overall objective of stock control systems?
5. Define at least six common terms used in Inventory Control.
6. What is ABC analysis?

### 6.20 Inventory control - Types of Control Systems

### 6.21 Objectives

After studying this subtopic students will be able to:

- Have been introduced to the two main inventory control systems.
- Know the principles of the re-order level or two-bin system.
- Be able to calculate the key control levels; re-order level, minimum level and maximum level
- Understand the characteristics of the periodic review system.

### 6.22 Re-order Level System

This system is also known as the two-bin system. Its characteristics are as follows:

a) A predetermined re-order level is set for each item.

b) When the stock level falls to the re-order level, a replenishment order is issued.

c) The replenishment order quantity is invariably the EOQ.

d) The name "two –bin system" comes from the simplest method of operating the system whereby the stock is segregated into two bins. Stock is initially drawn from the first bin and a replenishment order issued when it becomes empty.

e) Most organizations operating the re-order level system maintain stock records with calculated re-order levels which trigger off the required replenishment order.

**6.23 Illustrations of a Simple Manual Re-order Level System**

The following data relate to a particular stock item.

| | |
|---|---|
| Normal usage | 110 per day |
| Minimum usage | 50 per day |
| Maximum usage | 140 per day |
| Lead time | 25-30 days |
| EOQ (Previously calculated) | 5,000 |

Using this data the various control levels can be calculated.

Re-order Level = Maximum Usage x Maximum Lead Time

= 140x 30

= 4,200 units

Minimum Level = Re-order Level –Average Usage for Average Lead Time

= 4,200 – (110x27.5)

= 1,175 units

Maximum Level = Re-order level + EOQ – minimum Anticipated usage in Lead Time

=4,200+5,000-(50x25)

= 7,950 units

Notes:

a) The three levels would be entered on a Stock card comparisons made between the actual stock level and the control levels each time an entry was made on the card.
b) The re-order level is a definite action level, the minimum and maximum points are levels at which management would be warned that a potential danger may occur.

c) The re-order level is calculated so that if the worst anticipated position occurs, stock would be replenished in time.

d) The minimum level is calculated so that management will be warned when demand is above average and accordingly buffer stock is being used. There may be no danger, but the situation needs watching.

e) Maximum level is calculated so that management will be warned when demand is the minimum anticipated and consequently the stock level is likely to rise above the maximum intended.

f) In a manual system if warnings about maximum or minimum level violations were received, then it is likely that the re-order level and /or EOQ would be recalculated and adjusted. In a computer based system such adjustments would take place automatically to reflect current and forecast future conditions.

g) Critical factors in establishing re-order levels and for calculating EOQs is the forecast of expected demand. Forecasting has been covered earlier in the stock

**6.21 Periodic Review System**

This system is sometimes called the constant cycle system. The system has the following characteristics:

a) Stock levels for all parts are reviewed at fixed intervals e.g. every fortnight. Where necessary a replenishment order is issued.

b) The quantity of the replenishment order is not a previously calculated EOQ, but is based upon; the likely demand until the next review, the present stock level and the lead time.

c) The replenishment order quantity seeks to bring stocks up to a predetermined level. With

effect of the system is to order variable quantities at fixed intervals as compared with the re-order level system, described n Para.2, where fixed quantities are ordered at variable intervals.

### 6.22 Illustration of a Simple Manual Periodic Review System

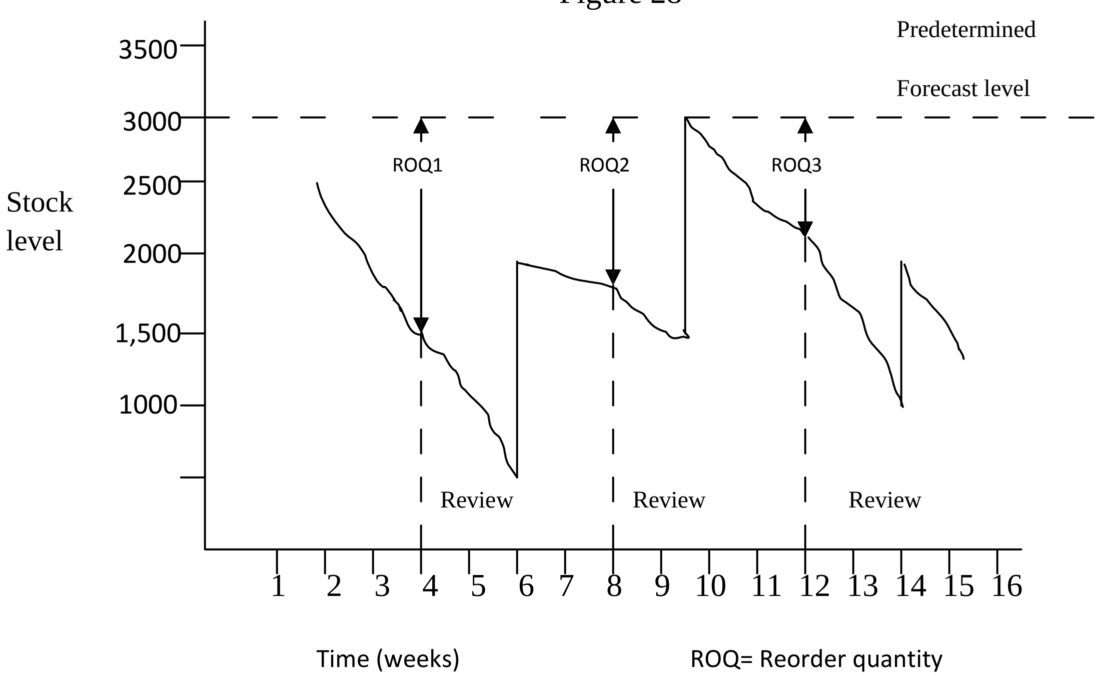
A production control department maintains control of the 500 piece parts used in the assembly of the finished products by the periodic review system.

The stock levels of all 500 parts are reviewed every 4 weeks and a replenishment order issued to bring the stock of each part up to a previously calculated level. This level is calculated at six-monthly intervals and is based on the anticipated demand for each part.

Based on the above system, the following graph shows the situation for one of the piece parts, part No.1101x, over a period of 16 weeks.

The following diagram shows a stock situation simplified by the following assumptions: regular rates of demand, a fixed lead time, and replenishment in one batch.

Figure 28




Stock Levels of Part No.1101

**Notes:**

The re-order quantities, based on the agreed system are 1,500 units, 1,200 units, and 1,000 units It will be that the rates of usage are assumed to be variable and the lead time has been assumed to be 2 weeks.

The above illustration is merely one of operating a periodic review system and many variants exist, particularly relating to the method of calculating the re-order quantities.

### 6.23 Periodic Review System

**Advantages:**

- All stock items are reviewed periodically so that there is more chance of obsolete items being eliminated.
- Economies in place orders may be gained by spreading the purchasing office load more evenly
- Large quantity discounts may be obtained when a range of stock items are ordered at the same time from a supplier.
- Because orders will always be in the same sequence, there may be production economies due to more efficient production planning being possible and lower set up costs. This is often a major advantage and results in the frequent use of some form of periodic review system in production control systems in firms where there is a preferred sequence of manufacture, so that full advantage can be gained from the predetermined sequence implied by the periodic review system.

**Disadvantages:**

- In general larger stocks are required, as re-order quantities must take account of the period between reviews as well as lead times
- Re-order quantities are not at the optimum level of a correctly calculated EOQ
- Less responsive to changes in consumption. If the rate of usage change shortly after a review, a stock out may well occur before the next review.
- Unless demands are reasonably consistent, it is difficult to set appropriate periods for review.

### 6.24 Re-order Level System

Advantages:

- Lower stocks on average.
- Items ordered in economic quantities via the EOQ calculation somewhat more responsive to fluctuations in demand.
- Automatic generation of a replenishment order at the appropriate time by comparison of stock level against re-order level.
- Appropriate for widely differing types of inventory within the sane firm.

**Disadvantages:**

- Many items may reach re-order level at the same time, thus overloading the reordering system.
- Items come up for re- ordering in a random fashion so that there is no set sequence.
- In certain circumstances (e.g. variable demand, ordering costs etc), the EOQ calculation may not be accurate.

### 6.25 Hybrid Systems

The two basic inventory control systems have been explained above but many variations exist in practice. A firm may develop a system to suit their organization which contains elements of both systems. In stable conditions of constant demand, lead times, and costs, both basic approaches are likely to be equally effective.

### 6.26 Summary

- There are two basic inventory control systems, the re-order level or two-bin system and the periodic review system
- The re-order level system usually has three control levels: re-order level, maximum level and minimum level.
- In the re-order level system the usual replenishment order quantity is the EOQ.
- The re-order level system results in fixed quantities being ordered at variable intervals dependent upon demand.
- The periodic review system means that all stocks are reviewed at fixed intervals and replenishment orders issued to bring stock back to predetermined level.

- The replenishment order quantity is based upon estimates of the likely demand until the next review period.
- The periodic review system results in variable quantities being ordered at fixed intervals.

### 6.27 Exercise

1) The following data relate to a given stock item.

| | |
|---|---|
| Normal usage | 1,300 per day |
| Minimum usage | 900 per day |
| Maximum usage | 2, 000 per day |
| Lead time | 15-20 days |
| EOQ | 30,000 |

Calculate the various control levels.

2) What are the advantages and disadvantages of the Periodic review system of Inventory Control?

## 6.30 INVENTORY CONTROL- ECONOMIC ORDER QUANTITY

### 6.31 Objectives

After studying this subtopic the student will be able to:

- Understand the assumptions necessary to calculate the basic Economic Order Quantity (EOQ).
- Be able to find the EOQ graphically and by formula.
- Know how calculate the EOQ with gradual replenishment and where stock outs are permitted.
- Understand how to find the best order quantity when price discounts are possible
- Know the importance of marginal costs in EOQ calculations.

EOQ assumptions

The EOQ has been previously defined as the ordering quantities which minimizes the balance of cost between inventory holding costs and re-order costs. To be able to calculate a basic EOQ certain assumptions are necessary:

- That there is a known, constant stockholding cost,
- That there is a known, constant ordering costs,
- That rate of demand is known,
- That there is a known, constant price per unit,
- That replenishment is made instantaneously, i.e. the whole batch is delivered at once,
- No stock outs allowed.

It will be apparent that the above assumptions are somewhat sweeping and they are a good reason for treating any EOQ calculation with caution. The rationale of EOQ ignores buffer stocks which are maintained to cater for variations in lead time and demand.

**6.32 A Graphical EOQ Example**

The following data will be used to develop a graphical solution to the EOQ problem.

Example1

A company uses 50,000 widgets per annum which are ksh10 each to purchase. The ordering and handling costs are Ksh 150 per order and carrying costs are 15% per annum, i.e. it costs ksh1.50 p.a. to carry a widget in stock (ksh10x15%).

To graph the various costs involved the following calculations are necessary:

Total costs p.a. = Ordering Costs p.a. + Carrying Cost p.a.

Where Ordering Costs p.a. = No. of orders x ksh150 and

The No. of orders = $\frac{Annual\ Demand}{order\ Quantity}$

[For example, if the order quantity was 5,000 widgets,

The no. of orders = $\frac{50,000}{5,000}$ =10 and the ordering cost p.a.

= 10 x ksh150 = ksh1, 500]

And carrying cost p.a = average stock level x ksh1.5

And the average stock = $\frac{order\ quantity}{2}$

[For example if the order quantity is 5,000, carrying costs p.a are $\frac{5,000}{2}$ x Ksh 1.5 = ksh3, 750]

Based on the above principles, the following table gives the costs for various order quantities.

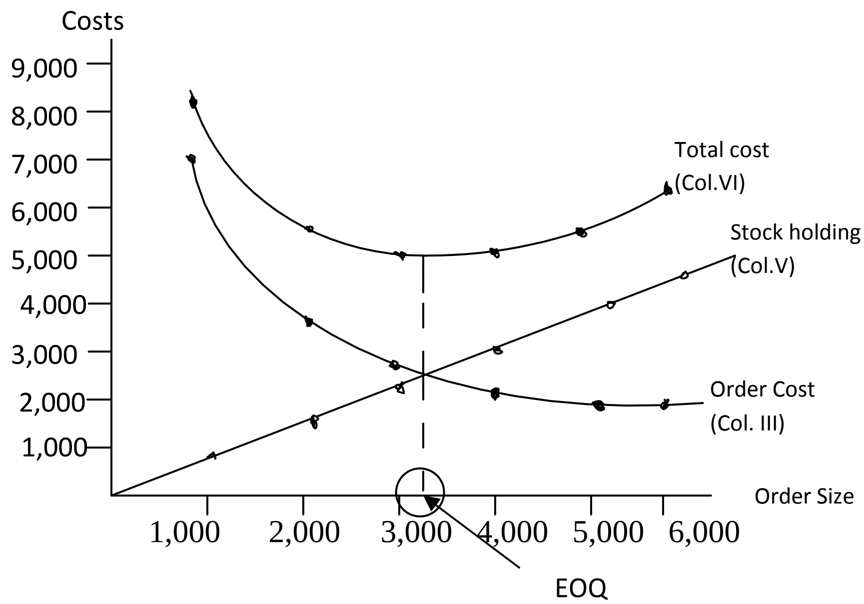
Table 24

| Column I | II | III | IV | V | VI |
|---|---|---|---|---|---|
| Order quantity | Average no. of orders p.a | Annual ordering cost | Average stock | Stock holding cost p.a | Total cost |
| | 50,000/Col.I | Col. II x ksh150 | Col.I/2 | Col. IV x ksh1.5 | Col. III+ col. v |
| | | Ksh | Ksh | Ksh | |
| 1,000 | 50 | 7,500 | 500 | 750 | 8,250 |
| 2,000 | 25 | 3,750 | 1,000 | 1,500 | 5,250 |
| 3,000 | 16 2/3 | 2,500 | 1,500 | 2,250 | 4,750 |
| 4,000 | 12 1/2 | 1,875 | 2,000 | 3,000 | 4,875 |
| 5,000 | 10 | 1,500 | 2,500 | 3,750 | 5,250 |
| 6,000 | 8 1/3 | 1,250 | 3,000 | 4,500 | 5,750 |

Ordering and stock holding costs for various order quantities.

The costs in Table 24 can be plotted in a graph and the approximate EOQ ascertained.

Graph of Data in Figure 29



From the graph it will be seen that the EOQ is approximately 3,200 widgets, which means that an average of slightly under 16 orders will have to be placed a year.

From a graph closer accuracy is not possible and is unnecessary anyway.

It will be seen from the graph that the bottom of the total cost curve is relatively fat, indicating that the exact value of the EOQ is not too critical. This is typical of most EOQ problems.

**6.32 The EOQ Formula**

It is possible, and more usual, to calculate the EOQ using a formula. The formula method gives an exact answer, but do not be misled into placing undue reliance upon the precise figure. The calculation is based on estimates of costs, demands etc which are, of course, subject to error. The EOQ formula is given below and should be learned. The mathematical derivation is given in Appendix 1 of this chapter.

Basic EOQ formula

$$EOQ = \sqrt{\frac{2.C_o.D}{C_c}}$$

Where Co = ordering cost per order

D = Demand per annum

Cc = Carrying cost per annum

Using the data from Example 1, the EOQ can be calculated:

Co = ksh150

D = 50,000 widgets

Cc = ksh10 x 15% or ksh1.5 per widget.

$$EOQ = \sqrt{\frac{2 \times 150 \times 50{,}000}{1.5}}$$

$$= \sqrt{10{,}000{,}000} = \underline{\mathbf{3162\ widgets}}$$

The closest value obtainable from the graph was approximately 3,200 which are very close to the exact figure.

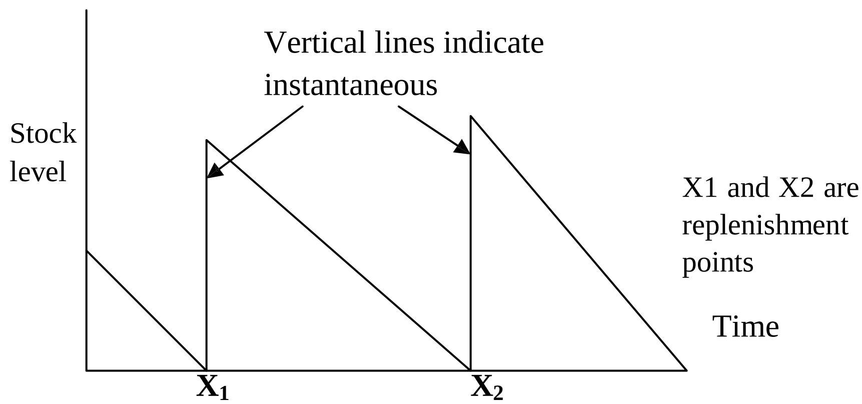
Always take care that demand and carrying costs are expressed for the same time period. A year is the usual period used.

In some problems the carrying cost is expressed as a percentage of the value whereas in others it is expressed directly as a cost per item. Both ways have been used in this example to provide a comparison.

**6.33 EOQ with Gradual Replenishment**

In example 1 above the assumption was that the widgets were ordered externally and that the order quantity was received as one batch, i.e. instantaneous replenishment as shown in the following diagram.

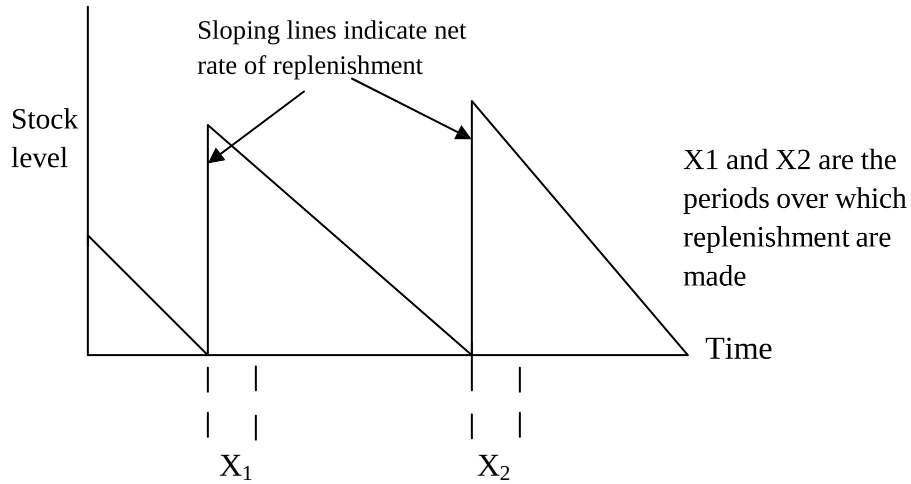
Figure 30




Stock Levels showing Instantaneous Replenishment

If however, the widgets were manufactured internally, they would probably be placed into stock over a period of time resulting in the following pattern:

Figure 31




Stock Levels showing Non-instantaneous Replenishment, The net rate of replenishment is determined by the rate of replenishment and the rate of usage during the replenishment period. To cope with such situations, the basic EOQ formula needs modification thus:

EOQ with gradual replenishment = $\sqrt{\dfrac{2.Co.D}{Cc(1-\dfrac{D}{R})}}$ .

Where R= Production rate per annum, i.e. the quantity that would be produced if production of the item was carried on the whole year.

All other elements in the formula have meanings as previously defined.

Example of EOQ with gradual replenishment

Example 2

Assume that the firm described in example 1 has decided to make the widgets in its own factory. The necessary machinery has been purchased which has a capacity of 250,000 widgets per annum. All other data are assumed to be the same.

EOQ with gradual replenishments $= \sqrt{\dfrac{2\,x\,150\,x\,50,000}{1.5(1-\dfrac{50,000}{250,000})}}$

$$= \sqrt{\frac{15,000,000}{1.5(.8)}}$$

= **<u>3,535 widgets</u>**

The value obtained above is larger than the basic EOQ because the usage during the replenishment period has the effect of lowering the average stock holding cost.

The ordering costs for internal ordering usually include set up and tooling costs as well as paper work and administration costs.

### 6.34 EOQ where Stock outs are Permitted

It will be recalled that the overall objective of stock control is to minimize the balance of the three main areas of costs i.e. holding costs, ordering costs and stock out costs.

Stock out costs are difficult to quantify but nevertheless may be significant and the avoidance of these costs is the main reason why stocks are held in the first place. Where stock out costs are known then they can be incorporated into the EOQ formula which thus becomes EOQ (where stock outs are permitted and stock out costs is known)

$$= \sqrt{\frac{2.Co}{Cc}}\;x\sqrt{\frac{Cc+C_s}{C_s}}$$

Where $C_s$ = stock costs per item per annum and the other symbols have the meanings previously given.

It will be seen that the formula is the basic EOQ formula multiplied by a new expression containing the stock out cost.

**Example of EOQ where Stock outs are Permitted**

Assume the same data as in 6.3.2 except that stock outs are now permitted. When a stock out occurs and an order is received for widgets the firm has agreed to retain the order and, when replenishments are received, to use express courier service for the delivery at a cost of ksh 0.75 per widget. Other administrative costs associated with stock outs are estimated at ksh 0.25 per unit. What is the EOQ?

$C_o$ = Ksh 150

D = 50,000

$C_c$= ksh1.5

$C_s$= ksh0.75 + ksh0.25 = ksh1

Thus EOQ (with stock outs) = $\sqrt{\frac{2\,x150\,x50{,}000}{1.5}}\ \sqrt{\frac{1.5+1}{1}}$

= 3162x 1.58

= 4,996

**6.35 EOQ with Discounts**

A particularly unrealistic assumption with the basic EOQ calculation is that the price per item remains constant. Usually some form of discount can be obtained by ordering increased quantities. Such price discounts can be incorporated into the EOQ formula, but it becomes much more complicated. A simpler approach is to consider the costs associated with the normal EOQ and compare these costs with the costs at each succeeding discount point and so find the best quantity to order.

**6.36 Financial Consequences of Discounts**

Price discounts for quantity purchases have three financial effects, two of which are beneficial and one adverse.

Beneficial Effects:

Savings come from **l**ower price per item**.** The larger order quantity means that fewer orders need to be placed so that total ordering costs are reduced.

Adverse Effects:

Increased costs arise from the extra stockholding costs caused by the average stock level being higher due to the large order quantity.

**Example 3 - EOQ with Discounts**

A company uses a special bracket in the manufacture of its products which it orders from outside suppliers. The appropriate data are

Demand = 2,000 per annum Order

cost = ksh20 per order Carrying

cost = 20% of item price Basic

item price =Ksh 10 per bracket

The company is offered the following discounts on the basic price:

| | | |
|---|---|---|
| For order quantities | 400-799 | less2 % |
| | 800-1,599 | less4% |
| | 1,600 and over | less 5% |

It is required to establish the most economic quantity to order.

This problem can be answered using the following procedure:

Calculate the EOQ using the basic price.

Compare the savings from the lower price and ordering costs and the extra stockholding costs at each discount point (i.e. 400, 800 and 1,600) with the costs associated with the basic EOQ, thus

Basic EOQ = $\sqrt{\dfrac{2\,x\,2{,}000\,x\,20}{10\,x\,2}}$

= 200 brackets

Based on this EOQ the various costs and savings comparisons are given in the following

Table: 25

| Order Quantity | 200(EOQ) | 400 | 800 | 1,600 | Line No. |
|---|---|---|---|---|---|
| Discount | - | 2% | 4% | 5% | 1 |
| Average N. of Orders p.a. | 10 | 5 | 2 ½ | 1 ¼ | 2 |
| Average N. of Orders saved p.a. | - | 5 | 7 ½ | 8 ¾ | 3 |
| Ordering cost savings p.a | - | (5x20)= ksh100 | (7 ½x20)<br>ksh150 | (8 ¾ x 20)= ksh175 | 4 |
| Price caving per item per annum | -<br>- | 20p<br>(2,000x20p)=<br>400 | 40p<br>(2,000x40p)=<br>800 | 50p<br>(2,000x50p)=1,000 | 5 |
| Total gains | | ksh500 | ksh950 | Ksh 1,175 | 6 |
| Stockholding cost P.a. | (100x10x2)= ksh200 | (200x9.8x2)= Ksh 392 | (400x 9.6x2)<br>=ksh768 | (800x9.5x2)= ksh1,520 | 7 |
| Additional costs incurred by increased order | - | (ksh392-ksh200)= ksh192 | (ksh768-ksh200)= ksh568 | (ksh1,520-ksh200)<br>= ksh1,320 | 8 |
| Net gain/ $\boxed{\text{loss}}$ | - | ksh308 | ksh382 | $\boxed{145}$ | 9 |

Cost/savings comparisons EOQ to discount points

From the above table it will be seen that the most economical order quantity is 800 brackets, thereby gaining the 4% discount.

Line 2 is Demand of 2,000/ Order quantity

Line 7 is the cost of carrying the average stock, i.e. $\frac{Order\ quantity}{2}$ x Cost per item x carrying cost percentage.

Line 9 is line 6 minus line 8

**6.37 Marginal Costs and EOQ Calculations**

It cannot be emphasized too strongly that the costs to be used in EOQ calculations must be true marginal costs, i.e. the costs that alter as a result of a further order or carrying another item in stock. It follows therefore that fixed costs should not be used in the calculations. In the examples used in this chapter the costs have been clearly and simply stated. In examination questions this is not always the case and considerable care is necessary to ensure that the appropriate costs are used.

**6.38 Summary**

- The EOQ is the order quantity which minimizes the total costs involved which include holding costs and order costs.
- The basic EOQ calculation is based on constant ordering and holding costs, constant demand and instantaneous replenishment.
- The basic EOQ formula is

$$\sqrt{\frac{2.Co.D}{Cc}}$$

- Where replenishment is not instantaneous, e.g. in own manufacture, the formula

$$\sqrt{\frac{2.Co.D}{Cc(1-\frac{D}{R})}}$$

- Where replenishment is not instantaneous, the EOQ calculated is larger than the basic EOQ.
- Where stock outs are permitted and stock-out costs are known, the formula becomes

$$\sqrt{\frac{2.Co.D}{C_c}} \, x \sqrt{\frac{C_c + C_s}{C_s}}$$

- Where larger quantities are ordered to take advantage of price discounts stockholding costs increase, but savings are made in the price reductions and reduced ordering costs.

- The costs to be used in EOQ calculations must be marginal costs. Fixed costs should not be included.

**6.39 Exercise**

***1. Calculate the various control levels given the following information:***

| | |
|---|---|
| ***Normal usage*** | ***560 per day*** |
| ***Minimum usage*** | ***240 per day*** |
| ***Maximum usage*** | ***710 per day*** |
| ***Lead Time*** | ***15-20 days*** |
| ***EOQ*** | ***10,000*** |

***2. a) A company uses 100,000 units per year which cost ksh3 each. Carrying costs are 1% per month and ordering costs are Ksh 250 per order. What is the EOQ?***

***b) What would be the EOQ if the company made the items themselves on a machine with a potential capacity of 600,000 units per year?***

***3. Demand is 5,000 units per year. Ordering costs are Ksh 100 per order and the basic unit price is ksh5. Carrying costs are 20% p.a.***

***Discounts are available thus:***

| | |
|---|---|
| ***1,200-1,399*** | ***less 10%*** |
| ***1,400- 1,499*** | ***less 15%*** |
| ***1,500 and over*** | ***less 20%*** |

***What is the most economical quantity to order?***

## 6.40 INVENTOR CONTROL-SAFETY STOCKS AND RE-ORDERS LEVELS

### 6.41 Objectives

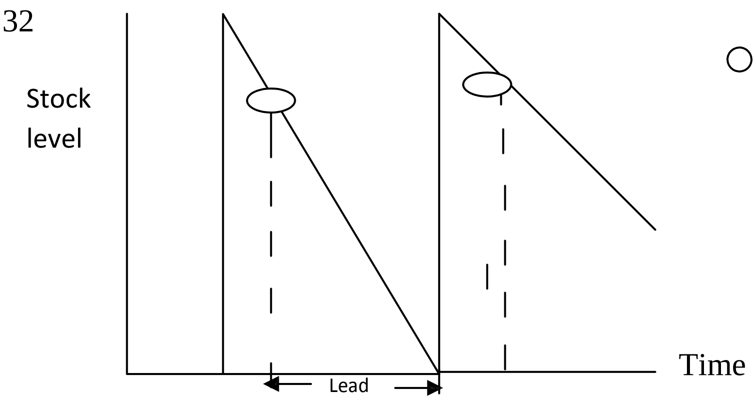
After studying this subtopic the students will be able to:

- Understand that safety or buffer stocks are necessary to cope with variations in demand and / or lead time.
- Know how to calculate the safety stock by tabulating costs.
- Be able to calculate the safety stock by statistical methods for various risk levels.
- Understand how sensitivity analysis can be applied to inventory control.

### 6.42 Setting Re-orders Levels in Condition of Certainty

Where the rate of demand and the lead time is known with certainty, the re-order level is the rate of demand times the lead time. This means that, regardless of the length of the lead time or of the rate of demand, no buffer stock is necessary. This results in a situation as follows:

Figure 32




Re-order level in conditions of certainty.

Re-order level and safety stock relationship

It will be seen from Figure 31 that, in conditions of certainty, the re-order level can be set so that stock just reaches zero and is then replenished. When demand and /or lead time vary, the re-order level must be set so that, on average, some safety stock is available to absorb variations in demand and/or lead time. In such circumstances the re-order level calculation can be conveniently considered in two parts.

a) The normal or average rate of usage times the normal or average lead time(i.e. as the re-order level calculation in conditions of certainty) *plus*
b) The safety stock

### 6.43 Safety Stock Calculation by Cost Tabulation

The amount of safety stock is the level where the total costs associated with safety stock are at a minimum. That is, where the safety stock holding cost plus the stock out cost is lowest. The appropriate calculations are given below based on the following illustration.

**Example 1**

An electrical company uses a particular type of thermostat which costs Ksh 5. The demand averages 800 p.a. and the EOQ has been calculated at 200. Holding costs are 20% p.a. and stock out costs have been estimated at Ksh 2 per item that is unavailable. Demand and lead times vary, but fortunately the company has kept records of usage over 50 lead times as follows:

Table 26

| (a) | (b) | (c) |
|---|---|---|
| Usage in lead time | Number of times recorded | Probability b/50 |
| 25-29 units | 1 | 0.02 |
| 30-34 units | 8 | 0.16 |
| 35-39 units | 10 | 0.20 |
| 40-44 units | 12 | 0.24 |
| 45-49 units | 9 | 0.18 |
| 50-54 units | 5 | 0.10 |
| 55-59 units | 5 | 0.10 |
| Total | 50 | 1.00 |

From the above re-order level and safety stock should be calculated.

Solution

Step1- Using the midpoint of each group calculate the average usage in the lead time.

Table 27

| x | t | xt |
|---|---|---|
| 27 | 1 | 27 |
| 328 | 2 | 56 |
| 37 | 10 | 370 |
| 42 | 12 | 504 |
| 47 | 9 | 423 |
| 52 | 5 | 260 |
| 57 | 5 | 285 |
| | 50 | 2,125 |

Average usage = 2,125/50 = 42.5

Step2- Find the holding and stock out costs for various re-order levels

Table 28

| A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| Re-order level | Safety stock (A-42.5) | Holding cost(B x ksh1) | Possible shortages (mid points) (Table 2-A) | Probability (from Table 1) | No of orders p.a. ( $\frac{800}{200}$ ) | Shortage cost (DxExFxksh2) | Total cost (C+G) |
| 45 | 2.5 | ksh<br>2.5 | 2<br>7<br>12 | .18<br>.10<br>.10 | 4<br>4<br>4 | Ksh<br>2.88<br>5.6<br>9.6 | ksh<br>20.58 |
| 50 | 7.5 | 7.5 | 2<br>7 | .10<br>.10 | 4<br>4 | 1.6<br>5.6 | 14.7 |
| 55 | 12.5 | 12.5 | 2 | .10 | 4 | 1.6 | 14.1 |
| 60 | 17.5 | 17.5 | | | | | 17.5 |

From Table 3 it will be seen that the most economical re-order level is 55 units. This re-order level, with the average demand in the lead time of 42.5, gives a safety stock of 12.5, say 13 units.

**6.44 Safety Stock Calculation by Statistical Methods**

The previous method of calculating Safety Stock was based on relative holding and stock out costs, but on occasions these costs, particularly the effects of stock outs, are not known. In such circumstances management may decided upon a particular risk level they are prepared to accept and the safety stock and re-order level are based upon this risk level. For example, management may have decided that they are prepared to accept a 5% possibility of a stock out.

Illustration of a safety stock calculation by statistical method

Example2.

Using the data from Example1, it was found that the average demand during the lead time was 42 ½ units. The company has carried out further analysis and has found that this average lead time demand is made up of an average demand (D) of 3.162 units per day over an average lead time (L) of 13.44 days. Both demand and lead time may vary and the company has estimated that the standard deviation of demand ($ó_D$) is 0.4 units and the standard deviation of lead time ($ó_L$) is 0.75 days. The companies are prepared to accept a 5% risk of a stock out and wish to know the safety stock required in the following three circumstances:

-Where demand varies and leads time is constant,

-Where the lead time varies and demand is constant,

-Where both demand and lead time vary.

- In each of these cases the average usage is D x L i.e.3.162 x 13.44 = 42 ½ units.
- Safety stock calculations based on risk levels are commonly used.
- Where lead time and demand vary it can be expensive to maintain sufficient safety stocks for low stock out risks.
- The above example uses the properties of the Normal Distribution and values obtained from Normal Area tables. It is but one further application of the statistical principles covered earlier in the book using continuous probability distributions. Discrete probabilities and expected values can also be used to incorporate variability.

**6.46 Inventory Control and Sensitivity Analysis**

When an inventory control value has been calculated; for example, the EOQ, the re-order level, the total stockholding cost etc. management may wish to know how sensitive the value is to changes in the factors used to calculate it. For example, is the EOQ greatly affected when ordering cost changes? The process by which this is done is known as sensitivity analysis. The following example shows a typical analysis.

Example 3:

Henderson Ltd made the following estimates for a component they use:

Annual usage 1.125

Ordering costs ksh 50 per order

Carrying costs per year ksh 5 per component

Based on these estimates, an EOQ of 150 and expected total stock costs of ksh 750 p.a. were calculated, as follows:

$$\text{EOQ} = \sqrt{\frac{2x50x1{,}125}{5}} = \underline{\mathbf{150}}$$

Expected total stock costs = Ordering Cost p.a. + Holding Cost p.a.

$$= \left(\frac{1{,}125}{150}\right)x\,50 + \frac{5\,x150}{2} = \underline{\mathbf{750}}$$

During a year the EOQ of 150 was used for re-ordering, but the actual usage of components turned out to be 20 % higher at 1,350.

a) Calculate the actual total stock costs.
b) Calculate what the total stock costs would have been if the correct EOQ had been used.
c) Find out how sensitive total costs are to errors in the usage estimates.

Solution

a) Actual total stock costs $= \left(\frac{1{,}350}{150}\right)x\,50 + \frac{5\,x150}{2} = \underline{\mathbf{825}}$

b) Correct EOQ $= \sqrt{\frac{2x50x1{,}350}{5}} = \underline{\mathbf{164}}$

Total costs if an EOQ of 164 was used $= \left(\frac{1{,}350}{164}\right)x\,50 + \frac{5\,x164}{2} = \underline{\mathbf{822}}$

c) Thus ksh3 (i.e.825-822) extra costs were incurred by using the 150 EOQ based on the incorrect usage. This means that a 20 % increase in demand has caused only approximately 0.5% change in total costs. We conclude that total costs are insensitive to errors in demand estimates. From management's viewpoint, this is good news. It means that even if errors are made in the estimates, which are all too likely, it doesn't make much difference to the final result.

**6.47 Summary**

- Safety stocks are necessary because of demand and/or lead time variations.
- Re-order level is the average demand over the average lead time plus safety stock.

- The safety stock level can be established by comparing the safety stock holding cost and the stock out cost at various re-order levels.

- Where it is necessary to calculate the safety stock level to cater for a given risk of stock out, statistical methods based on areas under the normal curve, can be used.

- The means and standard deviations of demand and lead time must be calculated or estimated and the number of standard deviations appropriate to the risk level added to the average demand and/or lead time before multiplying together to establish the re-order level and thus the safety stock

***Exercise***

***1. A company uses 2,000 components per annum and the cost is ksh6 per component. Holding costs are ksh2 per component p.a and stock out costs are ksh3 per component per item unavailable. The EOQ is 500 and demand is variable as follows:***

| ***Usage in lead time*** | ***Probability*** |
|---|---|
| ***80*** | ***0.2*** |
| ***90*** | ***0.5*** |
| ***100*** | ***0.3*** |

***What is the most economical re-order level: 90, 95 or 100?***

***2. The demand for an item is normally distributed with a mean of 50 per day and a standard deviation of 5 units. Given a lead time of 20 days what is the re-order level and safety stock to meet 90% of demands?***

**References**:

Lucey T. (2000). *Quantitative techniques*, 5th edn. London: Ashford colour press (Pg 217-223).

# TOPIC SEVEN: SEQUENCING MODELS/ASSIGNMENT

***7.0 Purpose***

***Determine the best supply and demand mix option involving one person with the least cost or***

## 7.2 Assignment Model is a Special type of Transportation Model

This is where:

i) The number of sources equals the number of destinations

ii) The assignment table is square (n = m)

iii)The model employs specific techniques in which each job is assigned to and is performed by only one man, and each man has the responsibility of only one job. Hence the supply at each source is one and the demand at each destination is one.

Three conditions of assignment problem

i) The value in each cell $x_{ij}$ has only two-aspects

ii) It receives a '1' if the facility i (man or capital) is assigned to the requirement j (job or Demand)

iii) It receives a '0' if the man i is not selected for the job j.

Each cell receives specific cost or profit or merit or scores or rates denoted by $Q_{ij}$, which represents the cost or return of the assignment of the facility i to the requirement j

The objective function is a minimization in case of cost or maximization in case of a profit. The assignment table of an optimum solution will have n filled cells.

Illustration

The following rating table indicates the scores obtained by three clerks A, B and C when the manager has analyzed each man for each job. The manager wants to assign each man to each job in order to get the maximum benefit.

Table 29: scores for assigning three people to 3 jobs

| | JOBS | | |
|---|---|---|---|
| Men | I | II | III |
| A | 62 | 31 | 82 |
| B | 30 | 53 | 29 |
| C | 72 | 33 | 23 |

The objective of this problem is of maximizing the scores when the jobs are assigned.

In the optimum solution each job will be assigned to only one man and each man will be responsible to only one job. Hence there will be only filled cells.

Optimum solution for assigning one job to each of the three men

| MEN | JOBS | | | |
|---|---|---|---|---|
| | I | II | III | MEN |
| A | 0 | 0 | 1 | 1 |
| B | 0 | 1 | 0 | 1 |
| C | 1 | 0 | 0 | 1 |
| JOB ASSIGNMENT | 1 | 2 | 2 | |

Each cell receives either '1' if the facility i is assigned to the requirement j or A '0' if the man i is not selected for the job j. In this case, the assignment problem is normally considered under certainty.

## 7.3 Assignment Method

Several appropriate techniques may be used in solving assignment problems. To efficiently solve such a problem the following steps are to be followed:

Steps 1: subtract the minimum value in each column from every entry in the same column

Step2: using the matrix resulting from step 1, subtract the minimum value in each row from every entry in the same row.

Step 3: draw a few vertical and /or horizontal lines as possible through the rows and columns to delete all the zero entries.

Step 4 : if the minimum number of lines drawn in step 3 to delete all the zero entries, equals the number of columns or rows, then make an optimal assignment using only the cells with zero entries. If the minimum number of lines drawn is less than the number of columns or rows, go to the next step.

Step 5: find the smallest entry which is not crossed by one of the lines of step 3, and subtract this entry from every entry in the matrix that is not crossed and add it to every entry which comes at an intersection of two of the lines of step 3.

To illustrate the assignment method, let us consider the following problem with 3 jobs and 3 machines. The initial solution (using the northwest corner rule) is obviously degenerate. This will always be the case in the assignment model regardless of the method used to obtain the starting basis. In fact, the solution will continue to be degenerate at every iteration

The special structure of the assignment model allows the development of an efficient method of solution. This method will be illustrated with the example:

Table 30

| | | MACHINE | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | |
| JOB | 1 | 5<br>1 | 7<br>0 | 9 | 1 |
| | 2 | 14 | 10<br>1 | 12<br>0 | 1 |
| | 3 | 15 | 13 | 16 | 1 |
| | | 1 | 1 | 1 | |

| | | |
|---|---|---|
| | | 1 |

The optimal solution of the assignment model remains the same if a constant is added or subtracted to any row or column of the cost matrix. This is proved as follows: If $P_i$ and $q_j$ are subtracted from the $i^{th}$ row and $j^{th}$ column, the new cost elements becomes: $C'_{ij} = C_{ij} - P_i - q_j$.
This yields the new objective function:

$$Z' = \sum_i \sum_j C'_{ij} = \sum_i \sum_j \left(C_{ij} - P_i - q\right) X_{ij} =$$

$$Z' = \sum_i \sum_j C'_{ij}\, X_{ij} - \sum_i P_i \sum_j X_{ij} - \sum_j q_j \sum_i X_j$$

Since $\sum_i X_{ij} = 1, \sum_j X_{ij} = 1$ We get: Z’= Z – constant

This shows that the minimization of the original objective function yields the same solution as the minimization of Z. This idea indicates that if one can create a new $C_{ij}$ matrix with zero entries, and if these zero elements or a subset thereof constitute a feasible solution, this feasible solution is optimal, because the cost cannot be negative.
In the table the zero elements are created by subtracting the smallest element in each row (column). If one considers the row first, the new $C'_{ij}$ matrix is:

Table 31

$//C_{ij}//$ =

| | 1 | 2 | 3 | |
|---|---|---|---|---|
| 1 | | | | $P_1$= 5 |
| 2 | 0 | 2 | 4 | $P_2$ =10 |
| 3 | | | | $P_3$ =13 |
| | 4 | 0 | 2 | |
| | | | | |

| | | |
|---|---|---|
| 2 | 0 | 3 |

The last matrix can be made to include more zeros by subtracting $q_3=2$ from the third column. This yields the following table:

Table 32

$$
//C_{ij}// \quad
\begin{array}{c|c|c|c|}
 & 1 & 2 & 3 \\
\hline
1 & 0 & 2 & 2 \\
\hline
2 & 4 & 0 & 0 \\
\hline
3 & 2 & 0 & 1 \\
\hline
\end{array}
$$

$q_3 = 2$

The squares give the feasible (and hence optimal) assignment: (1,1), (2,3) and (3,2); costing $4 + 13 + 12 = 30$. Notice that this cost is equal to $P_1 + P_2 + P_3 + q_3$. Unfortunately, it is not always possible to obtain a feasible assignment as in the example above. Further rules are thus required to find the optimal solution. These rules are illustrated with the following example:

Table 33

(1)

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |

| | | | |
|---|---|---|---|
| 1 | 4 | 6 | 3 |
| 9 | 7 | 10 | 9 |
| 4 | 5 | 11 | 7 |
| 8 | 7 | 8 | 5 |

(2)

| | | | |
|---|---|---|---|
| 0 | 3 | 5 | 2 |
| 2 | 0 | 3 | 2 |
| 0 | 1 | 7 | 3 |
| 3 | 2 | 3 | 0 |

(3)

| | | | |
|---|---|---|---|
| 0 | 3 | 2 | 2 |
| 2 | 0 | 0 | 2 |
| | | | |

| | | | |
|---|---|---|---|
| 0 | 1 | 4 | 3 |
| 3 | 2 | 0 | 0 |

(4)

| | | | |
|---|---|---|---|
| 0 | 3 | 2 | 2 |
| 2 | 0 | 0 | 2 |
| 0 | 1 | 4 | 3 |
| 3 | 2 | 0 | 0 |

After carrying out the same initial steps as in the previous example, one gets the table (3). A feasible assignment to the zero elements is not possible in this case. The procedure then is to draw a minimum number of lines through some of the rows and columns such that all zeros are crossed out (as shown in table (3)).

The next step is to select the smallest uncrossed-out element and added to every element at the intercession of two lines. This yields table (4), which gives the optimal assignment (1,1), (2,3), (3,2) and (4,4). The corresponding total cost is: 1 + 10 + 5 + 5 = 21. It should be noted that if the optimal solution was not obtained in the step above, the given procedure of drawing lines should be repeated until a feasible assignment is achieved.

## 7. 4 Integer Programming

It is a technique for solving a linear programming model with an added constraint that the decision variables must only be non-negative integers.

In the case of linear programming there is no single, method that can be used for solving all types of integer linear programming problems.

Two widely used techniques are:

i) The method of integer forms,

ii) Branch and bound techniques.

Simple method (or graphical method) is still valid if the required answer gives integer values.

### 7.5 Dynamic Programming

It is an extension which finds solutions to problems involving a number of decisions which have to be made sequentially. For example, the amount of a product to be made next month may depend on the amount sold this month and so on.

Thus dynamic programming is a quantitative technique which divides a given problem into stages (or sub-problems which are interrelated). Here we attempt to find a combination of decisions which will maximize overall effectiveness.

Usually, we work backwards from the natural end of the problem until the initial problem is finally solved (as in the decision trees).

The decision made at each stage influences the next stage. This method is also termed as recursive approach.

Some applications of dynamic programming

i) Production and distribution problems.

ii) Scheduling inventory control.

iii) Resource allocation.

iv) Replacement and maintenance problems.

Advantages of dynamic programming

a) In certain types of problems such as inventory control management, chemical engineering design, dynamic programming may be the only technique that can solve the problems.
b) Most problems requiring multistage, multi period or sequential decision process are solved using this type of programming.

c) Because of its wide range, it is applicable to linear or non-linear problems, discrete or continuous variables, deterministic or stochastic problems.
d) The mathematical techniques used can be adapted to the computer.

Limitations of dynamic programming

a) Each problem has to be modeled according to its own constraints and requirements. This requires great experience and ingenuity.
b) The number of state variables has to be kept low to prevent complicated calculation.
c) Where applicable, methods such as simplex are more efficient than general programming approach.

## 7.6 Summary of the Method for Determining an Optimum Solution in an Assignment Problem

The technique used for obtaining an optimal solution in an assignment problem may be summarized as follows:

Step 1

Prepare a rating table in which each cell receives an evaluation score or costs for performing each job by each man i.

Step 2

From the rating table, subtract the smallest value in each column from every entry in the same column.

Step 3

From the table obtained from step 2, subtract the smallest cell-value in each row from every entry in the same row.

Step 4

Find the smallest number of line s (row and /or columns) which delete all the zeros present in the table. If the number of lines equals the number of rows or columns (n=m), then the problem is optimal and solved. Secure the best solution. If however this number of lines is not equal to the number of rows or column, go to the next step.

Step 5

Checking the table, find the smallest value which is not crossed by the lines. When this values is found, add it to each value which is located at the intersection of a row and column. The same value must be subtracted from every entry in the table which has not been crossed by the lines go back to step 4.

Step 6

The solution is obtained by assigning n zeros in such a way that only one zero is present in each row and column.

### *7.7 Exercise*

***The dean of the faculty of commerce wants to assign some five responsibly to five academic members of staff. To assign each member a responsibility, the dean conducted an objective survey to evaluate the pros and cons of the assignments. After the evaluation of staff he prepared a rating table, where the scores represented the cost of assigning responsibility to each staff. The rating was as follows:***

***Scores of ability of the five staff members in relation to each responsibility are tabled below***

***Table 33***

| | ***Responsibility*** | | | | |
|---|---|---|---|---|---|
| ***Academic staff*** | ***Library*** | ***Time-table*** | ***Exams*** | ***research*** | ***Computing*** |
| ***Prof. Okelo*** | ***40*** | ***15*** | ***25*** | ***30*** | ***35*** |
| ***Prof. Gupta*** | ***30*** | ***10*** | ***30*** | ***40*** | ***35*** |
| ***Dr. Barton*** | ***35*** | ***25*** | ***40*** | ***10*** | ***45*** |
| ***Pro. Shalpiro*** | ***10*** | ***10*** | ***55*** | ***10*** | ***50*** |
| ***Dr. Ngeny*** | ***35*** | ***55*** | ***50*** | ***15*** | ***45*** |

***Assign each academic staff to each responsibility.***

### 7.8 References:

Kumwar D.s and Kongere,T.o. (2003). *Fundamentals of operations Research*: India: bishen Singh mahendra pal Singh (pg 209-220).

# TOPIC EIGHT: SIMULATION

***8.0 Purpose***

***Generate results of complex experimental processes.***

***8.1 Objectives***

***After studying this topic students will be able to:***

- ***Define simulation and know why it is used.***
- ***Understand what is meant by a model and how models are development.***
- ***Know what is meant by Monte Carlo simulation.***
- ***Be able to describe the typical variables used in simulation.***
- ***Know the advantages and disadvantages of simulation.***

## 8.2 Simulation Definition

Simulation is the process of experimenting or using a model and noting the results which occur. In a business context the process of experimenting with a model usually consists of inserting different input values and observing the resulting output values. For example, in a stimulation of a queuing situation the input values might be the number of arrivals and / or service points and the output values might be the numbers and / Or times in the queue.

## 8.3 Why is Simulation Used?

Simulation is used where analytical techniques are not available or would be overly complex. Typical business examples are: most queuing systems other than the very simple, inventory control problems, production planning problems; corporate planning etc. Simulation often provides an insight into a problem which would be unobtainable by other means. By means of simulation the behavior of a system can be observed over time and because only a model of the system is used the actual time span can be compressed.

Thus for example, the simulation of an inventory systems performance over 30 months could be carried out using a computer in as many minutes. In addition, the analyst can manipulate or experiment with the system at will. He could for example, alter the frequencies of receipts and issues, the decision rules governing re-order levels and reorder quantities and so on to observe the effects of these changes on the system being simulated or imitated. Fundamental to simulation is the concept of a model.

### 8.4 Business Model

A model is any representation (physical or abstract) of a real thing, event or circumstances. Physical models such as aircraft models for wind tunnel testing, models of space capsule controls, architected layout models are well known but for business planning and decision making purposes are rarely appropriate. Instead, symbolic models are generally used. These are models which represent reality in numeric, algebraic or graphical form. The most useful models for business simulated to be observed or sampled at selected time intervals. Thus an inventory simulation might show stock levels on a daily basis. Alternatively a queuing simulation for supermarket checkouts might use a five minute interval to asses' performance. The interval chosen should be that appropriate to the operational reality on the system being simulated.

### 8.5 Model Construction

The success of a simulation exercise is related to the predictive quality of the underlying model, so that considerable care should be taken with model construction. Important factors in model development are:

a) Objective oriented. The model should be constructed with some definite purpose and the model results must be directly related to this purpose.

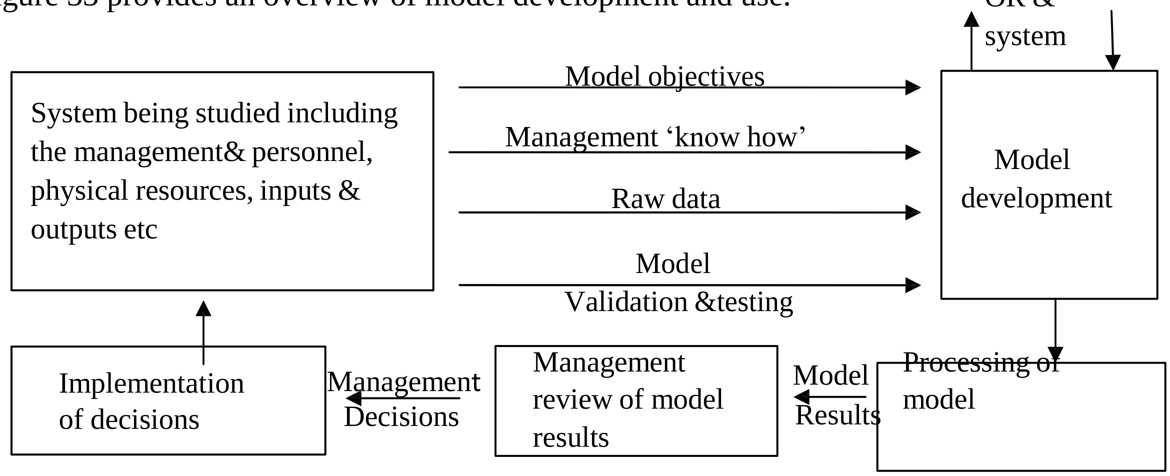
b) Critical variables and relationships. A model building is an alternative, creative process with the aim of identifying those variables and relationships which must be included in the model. It is not essential or indeed desirable to include all variables in the model

c) Simplicity. The best model is the simplest that has adequate predictive qualities.

d) Management involvement. To construct good models there must be a thorough understanding of actual operations. Only management has this knowledge, so they must be involved. If model building is left entirely to OR computer specialists, over-elaborate and sophisticated models may result which do not accurately represent reality.

e) Model development. If a model is to be used more than once, e.g. a corporate planning model used each quarter, care must be taken to modify and refine the model characteristics so that it continues to represent reality.

Figure 33 provides an overview of model development and use.




Model development and use

**8.6 Assessing a Model's Suitability**

Models are intelligent simplifications of reality and there is no way of providing that a model gives adequate prediction in unknown circumstances. However many models are developed to help forecast what might happen when an existing operation is enlarged or has extra demands made upon it. In such circumstances, when the model is constructed, values are input that correspond to current operational levels and the output values calculated. If these values correspond to the present position, then there are reasonable grounds for believing that the model is a fair representation of reality.

## 8.7 Monte Carlo Simulation

Any realistic business problem contains probabilistic or random features e.g. arrivals at a store may average 80 per day, but the actual arrival pattern is likely to be highly variable in a corporate planning exercise forecast of the likely sales will obviously vary according to different circumstances etc. because models must behave like the system under observation, the model must contain these probabilistic elements. Simulations involving such random elements are sometimes termed Monte Carlo simulations.

## 8.8 Random Selection

To carry out a realistic simulation involving probabilistic elements, it is necessary to avoid bias in the selection of the values which vary. This is done by selecting randomly (using the term in its statistical sense) using one of the following methods:

a) Random number generation by computer.
b) Random number tables. These consist of a table of randomly selected number which bias does not exist.
c) Lottery selection; e.g. put all the numbers in a bag, shake well, draw out a number.
d) Roulette wheel. The wheel is spun and a ball dropped in to select a number ((hence the name 'Monte Carlo simulation')
e) Dice or cards. These can also be used, although with cards the card drawn should be replaced and the pack reshuffled before another card is selected.

The repeated random selection of input values and the logging of the resultant output is the very essence of simulation. In this way an understanding is gained of the likely pattern of results so that a more informed decision can be taken.

Note: As all operational simulation uses computers, method (a) above is by far the most common.

## 8.9 Variables in a Simulation Model

A business model usually consists of linked series of equations and formulae arranged so that they 'behave' in a similar manner to the real system being investigated. The formulae and equation use a number of factors or variables which can be classified into 4 groups.

Input or exogenous variables

a) Parameters
b) Status variables
c) Output or endogenous variables

These are described below.

**Input variables**

These variables are of two types-controlled and non-controlled.

**Controlled variables**: these are the variables that can be controlled by management. Changing the input values of the controlled values, and noting the change in the output results, is the prime activity of simulation. For example, typical controlled variables in an inventory simulation might be the re-order level and re-order quantity. These could be altered and the effect on the system outputs noted.

**Non-controlled variables:** these are input variables which are not under management control. Typically these are probabilistic or stochastic variables i.e. they vary but in some uncontrollable, probabilistic fashion. For example, in a production simulation the number of breakdowns would be deemed to vary in accordance with a probability distribution derived from records of past breakdown frequencies in an inventory simulation, demand and lead time would also be generally classified as non-controlled, probabilistic variables.

**Parameters**

These are also input variables which for a given simulation have a constant value. Parameters are factors which help to satisfy the relationships between other types of variables. For example in a production simulation a parameter (or constant) might be the time taken for routine maintenance; in an inventory simulation a parameter might be the cost of a stock-out.

**Status variables**

In some types of simulation the behavior of the system (rates, usages, speeds, demand and so on) varies not only according to individual characteristics but also according to the general state of the system at various times or seasons. As an example in a simulation of supermarket demand and checkout queuing, demand will be probabilistic and variable on any given day but the general level of demand will be greatly influenced by the day of the week and the season of the year. Status variables would be required to specify the day(s) and season(s) to be used in a simulation.

Note: on occasions, status variables and parameters would both be termed just parameters, although strictly there is a difference between the two concepts.

**Output variables**

These are the results of the simulation. They arise from the calculations and tests performed in the model, the input values of the controlled values, the values derived for the probabilistic elements and the specified parameters and status values. The output variables must be carefully chosen to reflect the factors which are critical to the real system being stimulated and they must relate to the objectives of the real system. For the example, output variables for inventory simulation would typically include:

- Cost of stock holding,
- Number of stock outs,
- Number of unsatisfied orders,
- Numbers of replenishment orders,
- Cost of the re-ordering and so on.

**8.10 Constructing a Simulation Model**

Some broad guidelines for constructing a simulation model are given below.

Step1: identify the objective(s) of the simulation detailed listing of the results expected from the simulation will help to clarify step 5- the output variables.

Step2: identify the input variables. Distinguish between controlled and non-controlled variables.

Step3: where necessary determine the probability distribution for the non-controlled variables.

Step4: identify any parameters and status variables.

Step5: identify the output variables.

Step6: determine the logic of the model.

This is the heart of the simulation construction. The key questions are: how are the input variables changed into output results? What formulae/decision rules are required? How will the probabilistic elements be dealt with? How should the results be presented?

To illustrate these steps a simple problem follows together with a solution using the six step approach given above.

A simple inventory simulation

Example 1

A wholesaler stocks an item for which demand is uncertain. He wishes to assess two re-ordering policies i.e. order 10 units at a reorder level of 10, or order 15 units at a reorder level of 15units , to see which is most economical over a 10 day period.

The following information is available:

| Demand per day(units) | probability |
|---|---|
| 4 | 0.10 |
| 5 | 0.15 |
| 6 | 0.25 |
| 7 | 0.30 |
| 8 | 0.20 |

Carrying costs at ksh 15 per unit per day. Ordering costs ksh 50 per order. Loss of good- will for each Unit out of stock ksh 30. Lead time 3 days. Opening stock 17 units.

The probability distribution is to be based on the following random numbers

41 92 05 44 66 07 00 00 14 62

20 07 95 05 79 95 64 26 06 48

Note: the reorder level is physical stock plus any replenishment orders outstanding.

Solution

Step 1: objectives of simulation.

To simulate the behavior of two ordering policies-order 15 at reorder level of 15 and order 10 at reorder level of 10- to establish the cheaper policy.

Step 2: identify the input variables.

Controlled variables:

Order quantity

Reorder level

Non-controlled variables:

Probabilistic demand

Step 3: Determine probability distribution.

The random numbers are allocated to the demand as followers

| Demand | probability | cumulative probability | random numbers |
|---|---|---|---|
| 4 | 10% | 10% | 00-09 |
| 5 | 15% | 25% | 10-24 |
| 6 | 25% | 50% | 25-49 |
| 7 | 30% | 80% | 50-79 |
| 8 | 20% | 100 | 80-99 |

The two random number sequences supplied can then be used for the two runs of the simulation with each pair of digits used to select a demand. For example, for the 15 order quantity simulation, the first two digits, 41, give a day 1 demand of 6 units. The next two digits, 92, give a day 2 demand of 8 units and so on.

Step 4: identify parameters and status variables

| Parameters | |
|---|---|
| Opening stock | 17units |
| Carrying costs | ksh15 per unit per day |
| Ordering costs | ksh50 per order |
| Loss of goodwill | ksh30 |
| Lead time | 3 days |

There are no status variables in this simple example.

Step 5: identify the output variables

The main output variable is total cost. However ancillary output variables which arise from the simulation include:

N umber of orders placed

Number of stock outs

Cost of stock outs

Total carrying cost

Total order cost

Step 6: determine model logic

In this simple example the logic and rules required nothing more than the basics of inventory control thus:

Reorder level = physical stock + replenishment orders outstanding

Closing stock = opening stock - demand

Carrying cost per day = stock x ksh15 per unit

Goodwill cost = stock shortfall x ksh30 per unit

Total cost = goodwill costs + carrying costs + ordering costs.

The information, values and rules are then used to simulate the two ordering policies. The results of these simulations are shown in the schedules I figure 33 and figure 34 from which it will be seen that the simulation shows that the 'order 15' policy is more economical over the 10 days simulated.

It must be emphasized that in practice the simulations would be carried out over many more cycles than 10 days in order to obtain a truly representative picture.

Table 33

| Day | Opening stock | Demand | Closing stock | Order costs | Carrying costs | Stock costs | Total costs |
|---|---|---|---|---|---|---|---|
| 1 | 17 | 6 | 11 | ksh50 | ksh165 | | ksh215 |
| 2 | 11 | 8 | 3 | | ksh45 | | 45 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 3 | 4 | - | | - | ksh30 | 30 |
| 4 | +15 | 6 | 9 | ksh50 | ksh135 | | 185 |
| 5 | 9 | 7 | 2 | | ksh30 | | 30 |
| 6 | 2 | 4 | - | | - | ksh60 | 60 |
| 7 | +15 | 4 | 11 | ksh50 | ksh165 | | 215 |
| 8 | 11 | 4 | 7 | | ksh105 | | 105 |
| 9 | 7 | 5 | 2 | ksh50 | ksh30 | | 30 |
| 10 | +15+2 | 7 | 10 | | ksh150 | | 200 |
| | | | | ksh200 | ksh825 | ksh90 | ksh1,115 |

Results of simulation using order quantity and re-order level of 15 unit's.

The daily demand is found by using the random numbers in example 1 (i.e. 41 95 05 etc.) and determining the demand from the probability allocation in step 3 above. Thus 41 is in the 25-49 group and correspond to a demand of 6, 92 is in the 80-99 group corresponding to a demand of 8 and so on. An order for 15 is placed whenever physical stock and replenishment orders outstanding $\leq$ 15. This occurs on days 1, 4, 7 and 10.Table 33

| Day | Opening stock | Demand | Closing stock | Order costs | Carrying costs | Stock costs | Total costs |
|---|---|---|---|---|---|---|---|
| 1 | 17 | 5 | 12 | | ksh180 | | ksh180 |
| 2 | 12 | 4 | 8 | ksh50 | ksh120 | | 170 |
| 3 | 8 | 8 | - | | - | - | - |
| 4 | - | 5 | - | | - | ksh150 | 150 |
| 5 | +10 | 7 | 3 | 50 | ksh45 | | 95 |
| 6 | 3 | 8 | - | | - | 150 | 150 |
| 7 | - | 7 | - | | - | 210 | 210 |
| 8 | +10 | 6 | 4 | 50 | 60 | | 110 |
| 9 | 4 | 4 | - | | - | | - |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 10 | - | 6 | - | | - | 180 | 180 |
| | | | | ksh150 | ksh405 | ksh690 | ksh1,245 |

Results of simulation using order quantity and re-order level of 10 units'.

Further example of simulation

A common application of simulation is to examine the behavior of queues in circumstances where the use of queuing formulae is not possible. The following example is typical.

**Example 2**

A filling station is being planned and it is required to know how many attendants will be needed to maximize earnings. From traffic studies it has been forecast that customers will arrive in accordance with the following table:

Probability of 0 customer arriving in any minute 0.72

Probability of 1 customer arriving in any minute 0.24

Probability of 2 customer arriving in any minute 0.03

Probability of 3 customer arriving in any minute 0.01

From past experience it has been estimated that service times vary according to the following table

| Service time in Minutes | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Probability | 0.16 | 0.13 | 0.12 | 0.10 | 0.09 | 0.08 | 0.07 | 0.06 | 0.05 | 0.05 | 0.05 | 0.04 |

If there are more than two customers waiting, in addition to those being serviced, new arrivals drive on and the sale is lost.

A petrol pump attendant is paid ksh20 per 8 hour day, and the average contribution per customer is estimated to be ksh2.

How many attendants are needed?

**Solution**

Step 1: objectives of simulation

To find the number of attendants to maximize earnings

Step2: input variables

| | |
|---|---|
| Controlled | Number of attendants |
| Non- controlled | Customer arrival rate |
| | Service time |

Step3: probability distribution

As previously a random number table is used and an extract is given below.

| | | | | |
|---|---|---|---|---|
| 5053225496 | 9565241457 | 7354776952 | 2149630416 | 5579018342 |
| 7245174840 | 2275698645 | 8416549348 | 4676463101 | 2229367983 |
| 6749420382 | 4832530032 | 5670984959 | 5432114610 | 2966095680 |
| 7164238934 | 7666237259 | 5263097712 | 9999089966 | 7544056852 |
| 4192054466 | 0700014629 | 5169439659 | 8408705169 | 1074373131 |
| 9697426117 | 6488888550 | 4031652526 | 8123543276 | 0927534537 |
| 2007950579 | 9564268448 | 3457415998 | 1531027886 | 7016633739 |
| 4584768758 | 2389278610 | 3859431781 | 3643768456 | 4141314518 |
| 3840145867 | 9120831830 | 7228567652 | 1267173884 | 4020651657 |

The arrival pattern estimated is reproduced below with random numbers assigned.

| | | Random no.s assigned. |
|---|---|---|
| Probability of 0 customers arriving in any minute | = 0.72 | 01-72 |
| Probability of 1 customer arriving in any minute | = 0.24 | 73-96 |
| Probability of 2customer arriving in any minute | = 0.03 | 97-99 |
| Probability of 3 customer arriving in any minute | = 0.01 | 00 |

Similarly for the service pattern we assign the random numbers thus:

| Service time minutes | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Likelihood | 0.16 | 0.13 | 0.12 | 0.10 | 0.09 | 0.08 | 0.07 | 0.06 | 0.05 | 0.05 | 0.05 | 0.04 |
| Random numbers assigned | 01-16 | 17-29 | 30-41 | 42-51 | 52-60 | 61-68 | 69-75 | 76-81 | 82-86 | 87-91 | 92-96 | 97-100 |

The random number table is read in any direction in groups of two digits and, according to the digits, the appropriate arrival pattern or service time is selected.

For example, assume that for the arrival pattern the table is read from left to right starting from the first row.

| Minute No. | Random digits from table | no of arrivals |
|---|---|---|
| 1 | 50 | 0 |
| 2 | 53 | 0 |
| 3 | 22 | 0 |
| 4 | 54 | 0 |
| 5 | 96 | 1 |
| 6 | 95 | 1 |
| 7 | 65 | 0 |
| 8 | 24 | 0 |
| 9 | 14 | 0 |
| 10 | 57 | 0 |
| 11 | 73 | 1 |
| 12 | 54 | 0 |
| 13 | 77 | 1 |
| 14 | 69 | 0 |
| 15 | 52 | 0 |

| | | |
|---|---|---|
| 17 | 49 | 0 |
| Etc | | 0 |

Table 33

Selecting on this basis over say, a week's operations, will result in a random selection reflecting the estimated probabilities.

Step 4: parameters

Attendant cost ksh20 per day

Average contribution per customer ksh2

Step 5: output variables

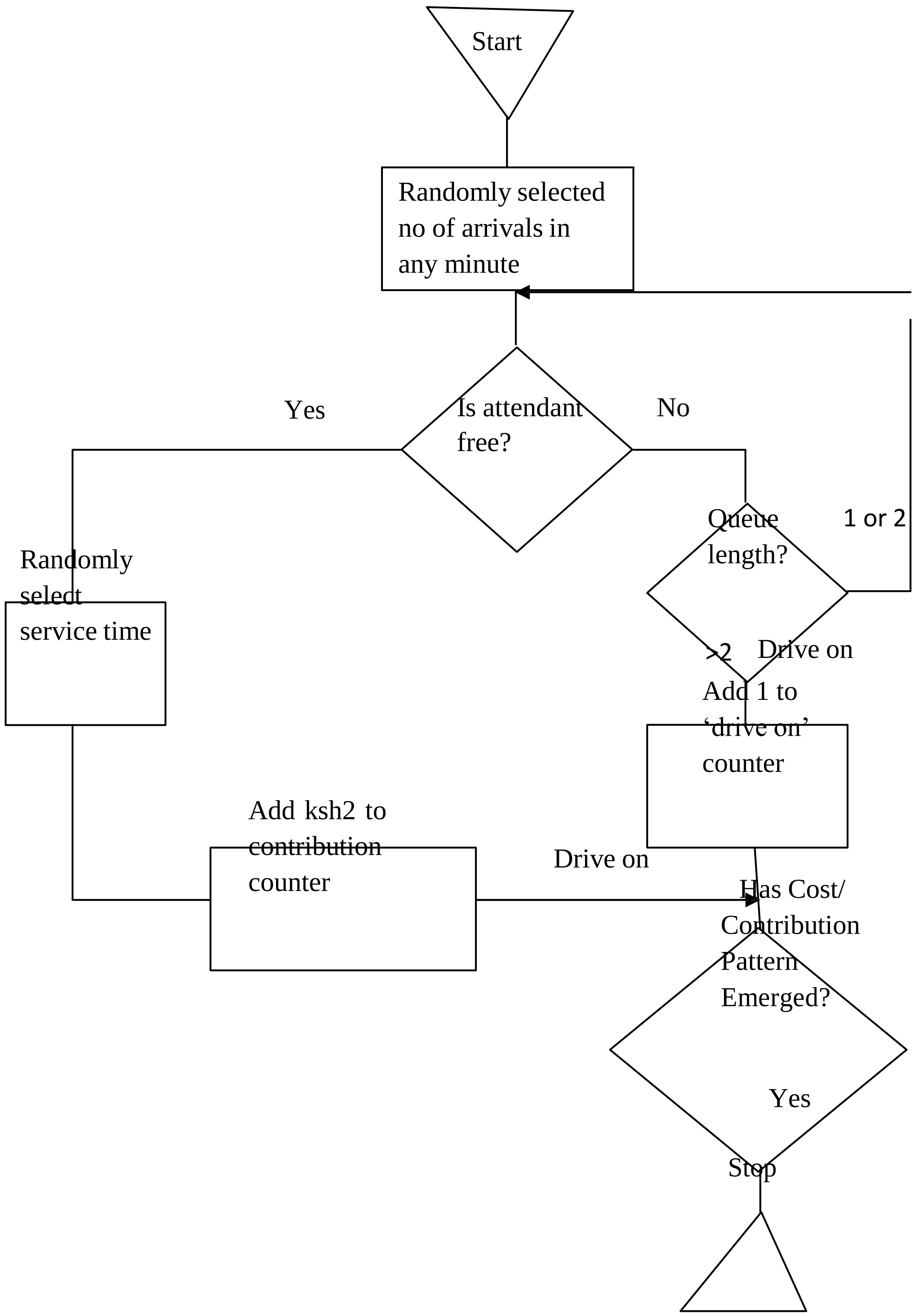
Average contributions per day

Attendant costs per day

No. of unsatisfied customers

Step 6: logic of simulation

In this case the logic is shown using a flowchart and the results of the simulation would be entered on a simple worksheet, Simulation logic




The results of such a simulation could be entered on a simple worksheet as follows

Figure33

| Minute | No. of arrivals | Queue length | Attendant | Attendant | Contribution earned | No. of unsatisfied customers |
|---|---|---|---|---|---|---|
| 1<br>2<br>3<br>4<br>5<br>6<br>7<br>8<br>Etc | | | | | | |

Simulation worksheet- for example 2. Figure 34

| Minute | No. Of arrivals | Queue length | Attendant A | Contribution earned | No. Of unsatisfied customers |
|---|---|---|---|---|---|
| 1 | 0 | 0 | | | |
| 2 | 0 | 0 | | | |
| 3 | 0 | 0 | | | |
| 4 | 0 | 0 | | | |
| 5 | 1 | 0 | Engaged | | |
| 6 | 1 | 1 | Engaged | | |
| 7 | 0 | 1 | Engaged | | |
| 8 | 0 | 1 | Engaged | | |
| 9 | 0 | 1 | Engaged | | |
| 10 | 0 | 1 | Engaged | | |
| 11 | 1 | 2 | Engaged | | |
| 12 | 0 | 2 | Engaged | | |
| 13 | 1 | 2 | Engaged | | |
| 14 | 0 | 2 | Engaged | | 1 |
| 15 | 0 | 1 | Engaged(and another random time selection made) | ksh2 | |

Simulation worksheet with one attendant- Figure 35

Carrying out the simulation

Now that the objectives, variables, logic and so on have been established the simulation is carried out for a number of iterations each representing 1 minutes operations; first with 1 attendant, then 2 attendants, then 3 and so on until a cost/contribution pattern emerges.

As the flowchart, figure 33, is worked through every minute, the number of arrivals would be randomly selected as in table 33. If an attendant was free, a random selection of service time would be made and the appropriate number of minutes logged against the attendant. For example, in table 2 an arrival occurred in minute 5. If the random selection of service time was, say 10 minutes, and the simulation was dealing with one attendant then that attendant would be engaged from minute 5 to minute 14.

In such circumstances the worksheet would be as figure 35.

It will be apparent that such a simulation is simple to set up and use but becomes very tedious indeed to repeat for hundreds of iterations.

**8.11 Results of Simulation**

The above simulation has worked through for several days' operation with 1, 2, 3 and 4 attendants and the results obtained are tabulated below.

Table 3

| No. of attendants | Average Contribution Per day (ksh) | Attendant(s) cost per day (ksh) | Average no.of vehicles/day driving on |
|---|---|---|---|
| 1 | 156 | 20 | 81 |
| 2 | 260 | 40 | 29 |
| 3 | 288 | 60 | 16 |
| 4 | 300 | 80 | 2 |

Results of simulation

From the table it will be seen that there is little difference is net profit per day between 2, 3 and 4 attendants, although there is of course a substantial difference in the average number of vehicles driving on. The results of a simulation do not necessarily indicate an optimal solution but provide more information upon which a reasoned decision can be taken.

**8.12 Computers and Simulation**

To carrying out any sort of realistic simulation, the use of a computer becomes a necessity. This is not because of any great complexity, but merely because of the large number of times that one has to work through the model. In the above example a computer was used to establish the table of results, but it did nothing more than could have been achieved using the model flowchart and the worksheets and a lot of patience. Because of the importance of computers in simulation, special simulation languages have been developed to facilitate the program writing and the use of the model. The GPSS (general purpose systems simulator) exists for most computer systems and the increasing availability of micro-computers and terminal access to larger machines renders manual simulation virtually obsolete in practice although it does occur in examination questions.

**8.13 Advantages of Simulation**

- Can be applied in areas where analytical techniques are not available or would be too complex.
- Constructing the model inevitably must involve management and this may enable a deeper insight to be obtained into a problem.
- A well constructed model does enable the results of various policies and decisions to be examined without any irreversible commitments being made.
- Simulation is cheaper and less risky than altering the real system.

**8.14 Disadvantages of Simulation**

- Although all models are simplifications of reality, they may still be complex and require a substantial amount of managerial and technical time.
- Practical simulation inevitably involves the use of computers which may be handicap to firms without computer facilities or ready access to a computer.
- Simulations do not produce optimal results. The manager makes the decision after testing a number of alternative policies. There is always the possibility that the optimum policy is not selected.

### 8.15 Summary

- Simulation is the process by which a model is experimented upon and the results of various policies examined
- At the heart of simulation is the concept of a model. A model is any representation of reality and business models can take the forms of flowcharts, formulae, equations etc.
- A model must reflect reality and reality inevitably involves variable or probabilistic elements.
- Practical simulations must include these variations and this is sometimes known as Monte Carlo simulation.
- Because of the iterative nature of simulation the use of a computer is a necessity for all practical problems.

***Exercise***

***i) Why is the concept of a model basic to the technique of simulation?***

***ii) What are the major factors to be considered in constructing a model?***

***iii) What is the essential feature of Monte Carlo simulation?***

***iv) What types of variables are found in simulation models?***

***v) What are the steps in contracting a simulation model?***

***vi) What is the role of computers in carrying out simulation?***

***vii) What are the advantages and disadvantages of simulation?***

**References**:

Lucey T. (2000). *Quantitative techniques*, *5th edition. London*: Ashford colour press (Pg 224-237).

**CAT**

**Answer all questions**

Question 1

i. An operations research model is as good as
   a) It provides some logical and systematic approach to the problem
   b) It incorporates useful tools which help in eliminating duplication of methods applied to solve specific problems
   c) It helps in finding avenues for new research and improvements in a system
   d) It indicates the nature of measurable quantities in a problem
   e) All of the above
   f) None of the above
ii. In business and management decision-making. The operation research study helps to have
   a) Better control
   b) Better system
   c) Better decisions
   d) All of the above
iii. The process of altering the inputs to operations research model in order to observe the effect upon its outputs is called:
   a) Model variation
   b) Input selection
   c) Sensitivity testing
   d) Cost benefit analysts
iv. Operations research analysts do not:
   a) Gather essential data
   b) Build one or more models
   c) Predict what future operations will be
   d) Recommend decisions and accept responsibility for the decision.

v. When applying the EOQ model to the production process, you must determine a set-up cost. In determining set-up costs, which one of the following should not be included:

   a) Paperwork cost of processing the work order and the production authorization
   b) Ordering cost to provide materials for the batch or order
   c) Cost of procuring new machinery to produce the order
   d) Engineering costs of setting up the production lines or machine

vi. Inventory theory is of no value when:

   a) Demand is not known
   b) Carrying costs are not known
   c) Ordering costs are not known
   d) None of the above

vii. When a transaction is due to arrive in a simulated system, it is not moved through the system until:

   a) The next transaction following the one currently arriving is assigned to a scheduled arrival time
   b) A check is made to see if there is room in the waiting line
   c) The transaction just ahead of the one currently arriving has completed service
   d) None of the above

viii. Unlike linear programming branch and bound and dynamic programming, simulation is not an analytical model. This means we view the results of simulation as:

   a) Unrealistic
   b) Exact
   c) Approximately
   d) Simplified

ix. Simulation often involves replication, where a system is simulated a number of time using different streams of random numbers. This procedure may help establish the validity of the simulation if:

   a) The output is consistent among the various replications
   b) The behavior is consistent among the various replications
   c) The simulated system itself is not changed in any of the replications

d) All of the above

x. Every path in a PERT network would be critical if all activities:

a) Were to start at their earliest finish times

b) Took as much time to accomplish as their pessimistic times

c) Have zero standard deviations

d) Were to start at their earliest start times.

Question 2

a) Explain the disadvantage of Queuing Theory as a tool for solving managerial problems and give suggestions on how these can be overcome.

b) The tuition-fee payment window at one of the professional for programme. Service times are exponentially distributed, averaging 6 minutes. Student arrivals at this window are approximately Poisson with an average of 8 per hour.

Required to determine

I. The mean time a student waits before service starts

II. The mean interval between the time a student arrives and departs

III. The mean number of students awaiting service

IV. The mean number of students either in line or at the window

V. The proportion of the time the clerk is idle.

MAIN EXAM

**EXAMINATION**

**PAPER 1**

**MOUNT KENYA UNIVERSITY**

**EXAMINATION 2010/2011**

**SCHOOL OF BUSINESS AND PUBLIC MANAGEMENT**

**DISTANCE LEARNING**

**UNIT CODE: BMA3201 TITLE: OPERATIONS RESEARCH**

**DATE: MAIN EXAMS TIME:2HRS**

**Instructions: Answer All Questions in Section A and any Two in Section B**

SECTION A (30 MKS)

QUESTION ONE

Out of the following multiple choice questions, indicate the appropriate answer:

i) A model

a) Forces a manager to identify explicitly the steps of decisions that influence objectives
b) Forces a manager to identity interactions and trade-offs between decisions involving different entities such as how much to produce of two different products that employ common resources.
c) Forces a manager to record explicitly constraints placed on the values that variables can assume.

ii) The scientific method in Operations Research study generally involves:

a) Judgement Phase

b) Research Phase

c) Action Phase

d) All of the above

e) None of the above

iii) The drawback to using the transportation problem method in solving an assignment problem is that:

a) A Degeneracy results from every improvement in the total cost

b) The assignment problem can't be formulated as a transportation problem

c) The assignment problem is an unbalanced transportation problem

d) Too many alternative optimal results.

iv) In the EOQ model

a) Orders arrive in a batch

b) Demand is known and occurs at a constant rate

c) All demand must be satisfied

d) All of the above

v) In the production lot size model, increasing the rate of production

a) Increases the optimal number of orders to place each year

b) Does not influence the optimal number of orders

c) Decreases the optimal number of orders to place each year

d) None of the above

vi) In the EOQ model with back jogging the optimal number of orders to backlog is

a) Directly proportional to

b) Directly proportional to the square root of

c) Not dependent on

d) Directly proportional to the reciprocal of

vii) Estimating expected activity times in a PERT network

a) Makes use of three estimates

b) Puts the greatest weight on the most likely time estimate

c) Is motivated by beta distribution

d) All of the above

viii) In the CPM time-cost trade-off function

a) The cost at normal time is 0

b) Within the range of feasible times, the activity increases linearly as time increases

c) Cost decreases linearly as time increases

d) None of the above.

ix) The PERT/cost model assumes that

a) Each activity achieves its optimistic time

b) Costs are uniformly distributed over the life of the activity

c) Activity times are statistically independent

d) None of the above

x) Simulation is widely accepted technique of Operations Research due to the following reasons:

a) Relatively straightforward and flexible and can be used to solve real-world problems

b) Once model has been constructed it may be used over and over to analyze all kinds of different situations

c) Each situation model is unique and its solutions and inferences are not usually transferable to other problems

d) Valuable and convenient method of breaking down a complicated system into sub-systems

e) All but not (c)

f) All of the above

xi) One of the disadvantages of simulation is that

a) It is very expensive and requires to develop large repetitions of data

b) Simulation is applicable in cases where there is an element of randomness in a system

c) Simulation solution can be zero per cent accurate

d) All of the above

xii) A random number refers to

a) An observation from a set of numbers (say integers 00-99) each of each is equally likely
b) In observation selected at random from a normal distribution
c) An observation selected at random from any distribution provided by the analyst
d) None of the above

xiii) In performing a simulation it is advisable to

a) Use the results of earlier decisions to suggest the next decision to try
b) Use the same number of trials for each decision
c) Simulate all possible decisions
d) None of the above

xiv) Out of the following multiple choice questions, indicate the appropriate answers:

In business and complex management decision-making, the study of Operations Research helps to have:-

a) Better control
b) Better decisions
c) Better system
d) All of the above

xv) Management science/operations research analyst do not:-

a) Gather essential data
b) Build one or more models
c) Predict what future operations will be
d) Recommend decisions and accept responsibility for the decisions

xvi) Which of the following is not an application of operations research?

a) Minimizing the number of airline stewards required at each base
b) Planning inventory at a super-market
c) Determining a bank’s investment plan
d) Determining the pollution generated by a new car engine.

xvii) Model formulation is important because.

a) It enables us to use algebraic techniques

b) In a business context, most managers prefer to work with formal models
c) It forces management to address a clear defined problem
d) It allows the manager to better communicate with the management scientists and therefore to be more discriminating in hiring policies.

xviii) Suppose you had an assignment problem where 5 jobs are to be assigned to 5 people, but there are in fact 7 people available to do the 5 jobs. You could solve the problem in the same manner as an unbalanced transportation problem is solved by:-

a) Creating two dummy jobs with zero cross and solve a 7 x 7 problem.
b) Arbitrarily eliminating two of the people and solve a 5 x 5 problem
c) Solving the assignment problem as a 5 x 7 problem
d) All of the above

xix) PERT assumes that the span of time between the optimistic and pessimistic time estimates of an activity are:-

a) 3 standard deviations
b) 6 standard deviations
c) 8 standard deviations
d) 12 standard deviations

xx) Which of the following represents a method of readjusting a PERT network to achieve better project results?

a) Shifting resources from the critical Path to non-critical paths.
b) Reassessing the optimistic, pessimistic, and most likely times for all activities on the critical path.
c) Modifying the precedence relationships so that series-connected activities can be performed at the same time
d) All of the above

xxi) If activity X is an immediate predecessor to activity Y:

a) Both X and Y are members of the critical path
b) Activity X's earliest finish time must equal activity Y's earliest start time.
c) Both X and Y have the same slack

d) None of the above

xxii) All of the following may be used to finish EOQ except:

a) Optimal number of orders per year.
b) Optimal number of days supply to order
c) Number of orders which minimize ordering costs
d) Optimal amount of shillings per order.

xxiii) In the application of EOQ to minimize inventory level without increasing purchasing workload, average inventory is used as a surrogate for:-

a) Purchase cost
b) Annual demand
c) Price per unit of the inventory
d) Carrying cost

xxiv) Ordering costs do not include:

a) Obsolescence
b) Issuing purchase orders
c) Placing goods into inventory
d) None of the above

xxv) Set up costs do not include:-

a) Labor costs for setting up machines
b) Ordering costs of raw materials
c) Maintenance cost of the machines
d) Paperwork cost of preparing the work order

xxvi) Inventories perform the following functions:

a) Provide operating flexibility
b) Smooth out irregularities in demand
c) Smooth out variations in labor requirements
d) All of the above

xxvii) With the absence of any random components, all simulations of a given process would produce:-

a) Invalid results
b) The same answers
c) Long waiting lines
d) None of the above

xxviii) In order to ensure consistency when comparing different sets of system parameters changed, we can ensure consistency in our comparisons by:

a) Using the same random number sequence in each of the runs
b) Using the same length of sample size
c) Both (a) and (b)

xxix Unlike linear programming, simulation is not an analytic model. This means we must view the results of the simulation as:-

a) Unrealistic
b) Exact
c) Approximations
d) Simplified

xxx The Operations Research Models can be classified according to :

a) Degree of abstraction
b) Structure
c) Purpose
d) Nature of environment
e) Behaviour characteristics
f) Procedure of solution
g) All of the above
h) None of the above

## SECTION B

## QUESTION TWO

a) Simulation techniques have been used to analyze problems of two distinct types: Practical real life problems and theoretical problems related to basic sciences. Illustrate the statement giving examples of each type. (5 marks)"

b) Eastland's supermarket has only one clerk who performs the task of receiving payments and handing over goods to customers whose arrival pattern at the counter is random phenomenon. Time between arrival at service varies from 1 minute to 10 minutes, the frequency distribution of which is as follows:- (15 marks)

| Time between Arrivals (minutes) | Frequency% | Service time (minutes) | Frequency % |
|---|---|---|---|
| 1-2 | 5 | 1-2 | 10 |
| 2-3 | 10 | 2-3 | 10 |
| 3-4 | 25 | 3-4 | 20 |
| 4-5 | 30 | 4-5 | 25 |
| 5-6 | 10 | 5-6 | 10 |
| 6-7 | 5 | 6-7 | 10 |
| 7-8 | 5 | 7-8 | 5 |
| 8-9 | 5 | 8-9 | 5 |
| 9-10 | 5 | 9-10 | 5 |

The owner of Eastland's supermarket knows that the mean arrival rate of customers is 8 minutes and mean service time is 4 minutes. The store opens at 10.00am and closes at 12 pm for prayers at the nearby mosque. The owner pays the clerk at the rate of sh.10 per hour and if the customer waits, it costs the owner Ksh 20 per hour

REQUIRED

Using the following random numbers for arrival and service, simulate for 15 runs and determine:-

i) Average waiting time before service

ii) Average service time

iii) Time a customer has to spend in the supermarket

iv) Whether to engage another clerk or not

ARRIVAL RANDOM NUMBERS

29,47,24,67,24,45,76,39,81,76,87,05,18,35 AND 36

SERVICERANDOM NUMBERS

90,24,32,04,72,90,90,68,14,32,47,64,38,96,24 AND 33

QUESTION THREE

a) How would PERT and CPM techniques be used in decision making? (5 marks)

b) State the major similarities between PERT and CPM (5 marks)

c) Under what circumstances is CPM a better technique than PERT? (5 marks)

Activities A, B and E may be started at the beginning of the project when activity B is completed D and G may start.

When A is completed F and D are completed and is a final activity, H can start after C is completed and is a final activity.

K must follow the finish of E. L cannot start until both G and K are completed J start when L is completed and is a final activity.

<u>ACTIVITY DURATION TIMES</u>

| ACTIVITIES | A | M | B |
|---|---|---|---|
| A | 3 | 4 | 5 |
| B | 6 | 8 | 10 |
| C | 2 | 3 | 4 |
| D | 4 | 5 | 12 |
| E | 5 | 7 | 9 |
| F | 9 | 16 | 17 |
| G | 8 | 12 | 10 |
| H | 8 | 10 | 12 |
| J | 4 | 5 | 6 |
| K | 7 | 8 | 15 |
| L | 8 | 11 | 14 |

Determine the critical path, critical activities and project duration, EST, LST, EFT and LFT for each of the activities (5 marks)

QUESTION FOUR

a) Describe the advantages and disadvantages of using simulation to investigate business problems compared with the use of mathematical formulae. (10marks)

b) A wholesale company has three warehouses from which supplies are drawn to four retails which supplies are drawn to four retail customers. The company deals in a single product, the supplies at each warehouse are.

| WAREHOUSE | SUPPLY(TONS) |
|---|---|
| 1 | 20 |
| 2 | 28 |
| 3 | 17 |

| CUSTOMER | DEMAND(TONS) |
|---|---|
| 1 | 15 |
| 2 | 19 |
| 3 | 13 |
| 4 | 18 |

The following table gives the transportation costs per ton shipment from each warehouse to each customer.

CUSTOMER

| WAREHOUSE | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | Ksh 210 | Ksh 420 | Ksh 560 | Ksh350 |
| 2 | 420 | 70 | 140 | 350 |
| 3 | 490 | 560 | 270 | 630 |

Using the stepping stone method, determine what supplies to dispatch from each of the warehouses to each customer so as to minimize overall transportation cost.

QUESTION FIVE

Mradi Company is in the business of training people to be computer literate. It trains people in four areas: Programming, Computer Applications, Networking and Graphics. Trainees are charged on an hourly basis. Profit matrix: Days expected profit in Kshs.

| | Computer areas | | | |
|---|---|---|---|---|
| Trainees | programming | applications | Networking | Graphics |
| Kirit | 1 | 8 | 4 | 1 |
| Talesu | 5 | 7 | 6 | 5 |
| Sokaru | 3 | 5 | 4 | 2 |
| Simba | 4 | 1 | 6 | 3 |

Assign the four trainees to the four computer areas so as to maximize the day's expected profit.

(20 marks)

**EXAMINATION**

**PAPER 2**

**MOUNT KENYA UNIVERSITY**

**EXAMINATION 2010/2011**

**SCHOOL OF BUSINESS AND PUBLIC MANAGEMENT**

**DISTANCE LEARNING**

**UNIT CODE: BMA3201 TITLE: OPERATIONS RESEARCH**

**DATE:** **MAIN EXAMS** **TIME: 2HRS**

**Instructions: Answer All Questions in Section A and any Two in Section B**

SECTION A (30 MKS)

OUESTION ONE

The total revenue (TR) and total cost (TC) functions for a business (for output between 1,000 and 1,300 units per period) are as follows:

$$TR = \frac{-x^2}{2} + 1{,}500x$$

$$TC = 330x + 415{,}500$$

(Where x is the level of output in units and TR and TC are in (Ksh)

Required:

a. Give an expression for total profit (5marks)

b. Using differential calculus:

i .Determine the expressions for marginal revenue and marginal profit (5marks)

ii. Find the profit maximizing output and confirm that it is a maximum value. (5marks)

Draw a graph showing marginal revenue and marginal cost for output between 1,000 and 1,300 units per period, clearly indicating the profit maximizing output. (15 marks)

SECTION B

QUESTION TWO

Production line turns out about 50 trucks per day. Fluctuations in production occur for many reasons. The production can be described by a probability distribution as follows:

| PRODUCTION/DAY | PROBABILITY |
|---|---|
| 45 | 0.03 |
| 46 | 0.05 |
| 47 | 0.07 |
| 48 | 0.10 |
| 49 | 0.15 |
| 50 | 0.20 |
| 51 | 0.15 |
| 52 | 0.10 |
| 53 | 0.07 |
| 54 | 0.05 |
| 55 | 0.03 |

Finished trucks are transported by train at the end of the day. The train capacity is only 51 trucks.

REQUIRED

a. Using the following random numbers, simulate the production for the next 8 days. (10 marks)

b. Determine the number of empty spaces (10 marks)

QUESTION THREE

The university is having its workshop at Thika campus where most of the maintenance job is done. The storekeeper in charge of the vital spare parts is concerned about the stock level of these parts. He feels that the level is too low. Currently the workshop orders these parts at a cost of ksh 40 a unit in batches of 200 units. The workshop operates for 250 days in a year, and for each day there is a demand for 100 units of these parts. Ordering costs per batch is ksh 25 while the inventory carrying cost is 12%.

a). Calculate the economic order quantity. (5 marks)

b) .Determine the annual savings which would be made over the current policy of ordering if the economic order quantity were adopted. (5 marks)

c). Of late the economy has been experiencing lots of difficulties, resulting into the demand and lead time for these parts becoming quite unstable. The university invited the members of the department of management science to study the situation. After thorough scrutiny of the situation after some time, the department established that average demand per day is 150 units with a standard deviation of 10 units; and average lead time is 8 days with a standard deviation of 2 days. Determine the reorder level for the workshop if they want to maintain a buffer stock with 99 % safety factor. (10 marks)

QUESTION FOUR

Ndege Airline Company has drawn up a new flight schedule to eight destinations: Kisumu, Eldoret, Nakuru, Nairobi, Garissa, Mandera, Malindi, and Mombasa. To assist in assigning eight pilots to these flight destinations, the management has asked them to state their present scores by giving each destination a number out of 10.The higher the number the greater the preference.

| | FLIGHT DESTINATIONS | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Pilot | KSM | ELD | NKR | NRB | GARSA | MADRA | MALDI | MBSA |
| Chepkwony | 8 | 2 | 0 | 5 | 4 | 9 | 3 | 10 |
| Purado | 10 | 9 | 2 | 8 | 4 | 6 | 1 | 7 |
| Nzuve | 5 | 4 | 9 | 6 | 0 | 10 | 3 | 7 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Heikh | 3 | 6 | 2 | 8 | 7 | 1 | 10 | 9 |
| Raziella | 5 | 6 | 10 | 4 | 3 | 0 | 8 | 7 |
| Nyamo | 10 | 1 | 7 | 2 | 9 | 8 | 4 | 4 |
| Buya | 8 | 2 | 9 | 1 | 3 | 10 | 5 | 6 |
| Ndieki | 7 | 3 | 9 | 10 | 3 | 4 | 2 | 6 |

Assign the pilots to flight destination in order to meet as many preferences as possible.

(20 marks)

OUESTION FIVE

a.Define the following terms

i.Game theory (2marks)

ii. Dynamic programming (2marks)

iii. Integer programming (2 marks)

b. Differentiate between integer programming and linear programming (2marks)

c. What contributes a game (5 marks)

d. List some applications of dynamic programming. (4marks)

e. Give widely used techniques of integer programming. (3marks)