“Actividad 2 módulo 1: variable aleatoria continua”

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Grupo 101

Problema 1:

1) Sea $f(x)$ una función definida por:

$$f(x) = \begin{cases} cx^2 & 0 \leq x \leq 2 \\ 0 & \text{en otro caso} \end{cases}$$

* Encuentra el valor de c que hace que $f(x)$ sea una función de densidad de la variable aleatoria X y calcula $P(0 < x < 1)$

$$\int_0^2 cx^2\,dx = 1$$

$$c\int_0^2 x^2\,dx = 1$$

$$c\left(\frac{x^3}{3}\Big|_0^2\right) = 1$$

$$c\left(\frac{2^3}{3} - \frac{0^3}{3}\right) = 1$$

$$c\left(\frac{8}{3} - \frac{0}{3}\right) = 1$$

$$c\left(\frac{8}{3} - 0\right) = 1$$

$$\frac{8}{3}c = 1$$

$$c = (1)\left(\frac{3}{8}\right)$$

$$\boxed{c = \frac{3}{8}}$$

$$f(x) = \begin{cases} \frac{3}{8}x^2 & 0 \leq x \leq 2 \\ 0 & \text{en otro caso} \end{cases}$$

$$P(0 < x < 1) = \int_0^1 \frac{3}{8}x^2\,dx$$

$$P(0 < x < 1) = \frac{3}{8}\int_0^1 x^2\,dx$$

$$P(0 < x < 1) = \frac{3}{8}\left(\frac{x^3}{3}\Big|_0^1\right)$$

$$P(0 < x < 1) = \frac{3}{8}\left(\frac{1^3}{3} - \frac{0^3}{3}\right)$$

$$P(0 < x < 1) = \frac{3}{8}\left(\frac{1}{3} - \frac{0}{3}\right)$$

$$P(0 < x < 1) = \frac{3}{8}\left(\frac{1}{3} - 0\right)$$

$$P(0 < x < 1) = \frac{3}{8}\left(\frac{1}{3}\right)$$

$$P(0 < x < 1) = \frac{3}{24}$$

$$\boxed{P(0 < x < 1) = \frac{1}{8}}$$

Problema 2:

② Flujo vehicular

a) $K = ?$ para el cual $f(x)$ es una fdp (función de densidad de probabilidad)

X: segundos entre 2 autos que pasan.

$$f(x) = \begin{cases} \frac{K}{x^4} & si \quad x > 1 \\ 0 & si \quad x \leq 1 \end{cases}$$

$$\int_1^{\infty} \frac{K}{x^4}\, dx = 1$$

$$K \int_1^{\infty} \frac{1}{x^4}\, dx = 1$$

$$K \int_1^{\infty} x^{-4}\, dx = 1$$

$$K \left(\frac{x^{-3}}{-3} \Big|_1^{\infty} \right) = 1$$

$$K \left(-\frac{1}{3x^3} \Big|_1^{\infty} \right) = 1$$

$$K \left[-\frac{1}{3 \cdot \infty^3} - \left(-\frac{1}{3(1)^3} \right) \right] = 1$$

$$K \left[-\frac{1}{3 \cdot \infty} + \frac{1}{3(1)} \right] = 1$$

$$K \left[-\frac{1}{\infty} + \frac{1}{3} \right] = 1$$

$$K \left[0 + \frac{1}{3} \right] = 1$$

$$K \left[\frac{1}{3} \right] = 1$$

$$\frac{K}{3} = 1$$

$$K = (1)(3)$$

$$\boxed{K = 3}$$

b) $E[X] = ?$ y $V[X] = ?$

$$f(x) = \begin{cases} \frac{3}{x^4} & si \quad x > 1 \\ 0 & si \quad x \leq 1 \end{cases}$$

$$E[X] = \int_1^{\infty} \cancel{x} \cdot \frac{3}{x^{\cancel{4}\,3}}\, dx$$

$$E[X] = \int_1^{\infty} \frac{3}{x^3}\, dx$$

$$E[X] = 3 \int_1^{\infty} \frac{1}{x^3}\, dx$$

$$E[X] = 3 \int_1^{\infty} x^{-3}\, dx$$

$$E[X] = 3 \left(-\frac{x^{-2}}{2} \Big|_1^{\infty} \right)$$

$$E[X] = 3\left(-\frac{1}{2x^2}\Big|_1^\infty\right)$$

$$E[X] = 3\left(-\frac{1}{2\cdot\infty^2} + \frac{1}{2(1)^2}\right)$$

$$E[X] = 3\left(\cancelto{0}{-\frac{1}{\infty}} + \frac{1}{2(1)}\right) = 3\left(\frac{1}{2}\right)$$

$$\boxed{E[X] = \frac{3}{2}}$$

$$V[X] = E\left[\left(X-\frac{3}{2}\right)^2\right] = \int_1^\infty \left(x-\frac{3}{2}\right)^2 \cdot \frac{3}{x^4}\,dx$$

$$V[X] = 3\int_1^\infty \frac{1}{x^4}\left(x-\frac{3}{2}\right)^2 dx$$

$$\left(x-\frac{3}{2}\right)^2 = x^2 + 2(x)(-3/2) + (-3/2)^2 = x^2 - 3x + \frac{9}{4}$$

$$V[X] = 3\int_1^\infty \frac{1}{x^4}\left(x^2-3x+\frac{9}{4}\right) dx = 3\int_1^\infty \left(\frac{1}{x^2} - \frac{3}{x^3} + \frac{9}{4x^4}\right) dx$$

$$V[x] = 3\int_1^\infty \left(x^{-2} - 3x^{-3} + \frac{9}{4}x^{-4}\right) dx = 3\left[\int_1^\infty x^{-2}\,dx - \int_1^\infty 3x^{-3}\,dx + \int_1^\infty \frac{9}{4}x^{-4}\,dx\right]$$

$$V[x] = 3\left[\left(-\frac{1}{x}\Big|_1^\infty\right) - 3\left(-\frac{1}{2x^2}\Big|_1^\infty\right) + \frac{9}{4}\left(-\frac{1}{3x^3}\Big|_1^\infty\right)\right]$$

$$V[x] = 3\left[\left(\cancelto{0}{-\frac{1}{\infty}} + \frac{1}{1}\right) - 3\left(\cancelto{0}{-\frac{1}{2(\infty)^2}} + \frac{1}{2(1)^2}\right) + \frac{9}{4}\left(\cancelto{0}{-\frac{1}{3(\infty)^3}} + \frac{1}{3(1)^3}\right)\right]$$

$$V[x] = 3\left[1 - 3\left(\frac{1}{2}\right) + \frac{9}{4}\left(\frac{1}{3}\right)\right] = 3\left[1 - \frac{3}{2} + \frac{3}{4}\right] = 3\left(\frac{1}{4}\right)$$

$$\boxed{V[x] = \frac{3}{4}}$$

c) $P(X>2) = ?$; $P(X\leq 2) = ?$; $P(X\leq x) = ?$

$$P(X\leq 2) = 1 - P(X>2)$$

$$P(X>2) = \int_2^\infty \frac{3}{x^4}\,dx = 3\int_2^\infty x^{-4}\,dx = 3\left(-\frac{1}{3x^3}\Big|_2^\infty\right) = 3\left(-\frac{1}{3(\infty)^3} + \frac{1}{3(2)^3}\right)$$

$$P(X>2) = 3\left(\cancelto{0}{-\frac{1}{\infty}} + \frac{1}{3(8)}\right) = 3\left(\frac{1}{24}\right) = \frac{3}{24} \longrightarrow \boxed{P(X>2) = \frac{1}{8}}$$

$$P(X\leq 2) = 1 - \frac{1}{8} = \frac{7}{8} \longrightarrow \boxed{P(X\leq 2) = \frac{7}{8}}$$

$$P(X \leq x) = 1 - P(X > x)$$

$$P(X > x) = \int_x^{\infty} \frac{3}{x^4}\, dx$$

$$P(X > x) = 3 \int_x^{\infty} \frac{1}{x^4}\, dx$$

$$P(X > x) = 3 \int_x^{\infty} x^{-4}\, dx$$

$$P(X > x) = 3\left(-\frac{x^{-3}}{3}\Big|_x^{\infty}\right)$$

$$P(X > x) = 3\left(-\frac{1}{3x^3}\Big|_x^{\infty}\right)$$

$$P(X > x) = 3\left(\cancel{-\frac{1}{3(\infty)^3}}^{\,0} + \frac{1}{3x^3}\right)$$

$$P(X > x) = \cancel{3}\left(\frac{1}{\cancel{3}x^3}\right)$$

$$\boxed{P(X > x) = \frac{1}{x^3}}\ //$$

$$\boxed{P(X \leq x) = 1 - \frac{1}{x^3}}\ //$$

***Nota:** Integrales resueltas a mano.