Physical and computational properties of quantum walks

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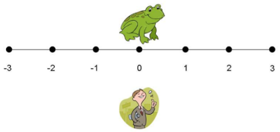
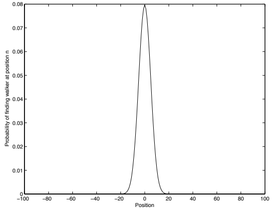
Classical random walks as comp. resource (1/8)

Froggy jumps either forward or backwards, depending on coin tosses.



$$P_{ok}^{(n)} = \binom{n}{\frac{1}{2}(k+n)} p^{\frac{1}{2}(k+n)} q^{\frac{1}{2}(n-k)}$$

for $\frac{1}{2}(k+n) \in \{0, \ldots, n\}$, 0 OTRW.



Classical random walks as comp. resource (2/8)

Random Walks are used to develop sophisticated stochastic algorithms for solving complex problems. For example:

Classical random walks as comp. resource (3/8)

Earthquake Simulation by Restricted Random Walks

Steven N. Ward
Institute of Geophysics and Planetary Physics
University of California, Santa Cruz

Abstract. This article simulates earthquake slip distributions as restricted random walks. Random walks offer several unifying insights into earthquake behaviors that physically-based simulations do not. With properly tailored variables, random walks generate observed power law rates of earthquake number versus earthquake magnitude (the Gutenberg-Richter relation). Curiously b-value, the slope of this distribution, not only fixes the ratio of small to law events but it also dictates diverse earthquake scaling laws such as mean slip versus fault length and moment versus mean slip. Moreover, b-value determines the overall shape and roughness of earthquake ruptures. For example, mean random walk quakes with b=-1/2 have elliptical slip distributions characteristic of a uniform stress drop on a crack. Random walk earthquake simulators, tuned by comparison with field data, provide improved bases for statistical inference of earthquake behavior and hazard.

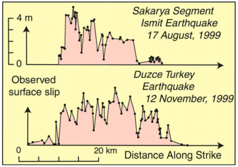


Figure 1. Measured dextral displacement along the surface fault trace of the Sakarya Segment, M7.4 Ismit, Turkey earthquake of August 17, 1999 (*Barka et al.*, 2002) and the M7.1 Duzce, Turkey earthquake of November 12, 1999 (*Akyuz et al.*, 2002).

Here, Δu is some small slip value and the $\chi(\nu,\sigma)$ are unitless random variables with

Classical random walks as comp. resource (4/8)

IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, VOL. 28, NO. 5, JUNE 2010 677

Random-Walk Based Approach to Detect Clone Attacks in Wireless Sensor Networks

Yingpei Zeng, Jiannong Cao, *Senior Member, IEEE,* Shigeng Zhang, Shanqing Guo and Li Xie

***Abstract*—Wireless sensor networks (WSNs) deployed in hostile environments are vulnerable to clone attacks. In such attack, an adversary compromises a few nodes, replicates them, and inserts arbitrary number of replicas into the network. Consequently, the adversary can carry out many internal attacks. Previous solutions on detecting clone attacks have several drawbacks. First, some of them require a central control, which introduces several inherent limits. Second, some of them are deterministic and vulnerable to simple witness compromising attacks. Third, in some solutions the adversary can easily learn the critical witness nodes to start *smart attacks* and protect replicas from being detected. In this paper, we first show that in order to avoid existing drawbacks, replica-detection protocols must be non-deterministic and fully distributed (NDFD), and fulfill three security requirements on witness selection. To our knowledge,**

information from the WSNs or disable the functions of the WSNs. For example, on the battlefield, the enemies would hope to learn the private locations of soldiers from, or inject wrong commands into the sensor network. So it is critical to ensure the security of sensor networks in such scenarios.

Clone attack [2] (also called node replication attack) is a severe attack in WSNs. In this attack, an adversary captures only a few of nodes, replicates them and then deploys arbitrary number of replicas throughout the network. The capture of nodes is plausible [3], [4] because sensor nodes are usually unprotected by physical shielding due to cost considerations [2], and are often left unattended after deployment. If we do not detect these replicas, the network will be vulnerable to a

Classical random walks as comp. resource (5/8)

1965–1974

Random Walks in Stock Market Prices

Eugene F. Fama

For many years economists, statisticians, and teachers of finance have been interested in developing and testing models of stock price behavior. One important model that has evolved from this research is the theory of random walks. This theory casts serious doubt on many other methods for describing and predicting stock price behavior—methods that have considerable popularity outside the academic world. For example, we shall see later that if the random walk theory is an accurate description of reality, then the various "technical" or "chartist" procedures for predicting stock prices are completely without value.

In general the theory of random walks raises challenging questions for anyone who has more than a passing interest in understanding the behavior of stock prices. Unfortunately, however, most discussions of the theory have appeared in itself, i.e., past patterns of price behavior in individual securities will tend to recur in the future. Thus the way to predict stock prices (and, of course, increase one's potential gains) is to develop a familiarity with past patterns of price behavior in order to recognize situations of likely recurrence.

Essentially, then, chartist techniques attempt to use knowledge of the past behavior of a price series to predict the probable future behavior of the series. A statistician would characterize such techniques as assuming that successive price changes in individual securities are dependent. That is, the various chartist theories assume that the *sequence* of price changes prior to any given day is important in predicting the price change for that day.[1]

The techniques of the chartist have always

Classical random walks as comp. resource (6/8)

K-SAT, a Fundamental NP-Complete Problem

Let $B = \{x_1, x_2, \ldots, x_n, \bar{x_1}, \bar{x_2}, \ldots \bar{x_n}\}$ be a set of Boolean variables. Also, let C_i be a disjunction of k elements of B and F be a conjunction of m clauses C_i.
Question: Is there an assignment of Boolean variables in F that satisfies all clauses simultaneously, i.e. F=1?
For example:

Classical random walks as comp. resource (7/8)

Suppose $B = \{x_1, x_2, x_3, x_4, x_5, x_6, \bar{x_1}, \bar{x_2}, \bar{x_3}, \bar{x_4}, \bar{x_5}, \bar{x_6}\}$ and

$$
\begin{aligned}
P \quad = \quad & (\bar{x_1} \vee \bar{x_4} \vee \bar{x_5}) \wedge (\bar{x_2} \vee \bar{x_3} \vee \bar{x_4}) \wedge (x_1 \vee x_2 \vee \bar{x_5}) \wedge (x_3 \vee x_4 \vee x_5) \wedge \\
& (x_4 \vee x_5 \vee \bar{x_6}) \wedge (\bar{x_1} \vee \bar{x_3} \vee \bar{x_5}) \wedge (x_1 \vee \bar{x_2} \vee \bar{x_5}) \wedge (x_2 \vee \bar{x_3} \vee \bar{x_6}) \wedge \\
& (\bar{x_1} \vee \bar{x_2} \vee \bar{x_6}) \wedge (x_3 \vee \bar{x_5} \vee \bar{x_6}) \wedge (\bar{x_1} \vee \bar{x_2} \vee \bar{x_4}) \wedge (x_2 \vee x_3 \vee \bar{x_4}) \wedge \\
& (x_2 \vee x_5 \vee \bar{x_6}) \wedge (x_2 \vee \bar{x_3} \vee \bar{x_5}) \wedge (\bar{x_2} \vee \bar{x_3} \vee \bar{x_4}) \wedge (x_2 \vee x_3 \vee x_6) \wedge \\
& (\bar{x_1} \vee \bar{x_2} \vee \bar{x_3}) \wedge (\bar{x_1} \vee \bar{x_4} \vee \bar{x_5}) \wedge (\bar{x_3} \vee \bar{x_4} \vee \bar{x_6}) \wedge (\bar{x_4} \vee \bar{x_5} \vee x_6) \wedge \\
& (\bar{x_2} \vee x_3 \vee \bar{x_6}) \wedge (x_2 \vee x_5 \vee x_6) \wedge (x_3 \vee x_5 \vee \bar{x_6}) \wedge (\bar{x_1} \vee x_3 \vee \bar{x_6}) \wedge \\
& (x_3 \vee \bar{x_5} \vee x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_1 \vee x_2 \vee \bar{x_3})
\end{aligned}
$$

Finding the solutions (if any) of even a modest SAT instance can become difficult quite easily. In fact, only $\{x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 1, x_5 = 0, x_6 = 0\}$ satisfies P.

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Classical random walks as comp. resource (8/8)

So far, one of the best algorithms for solving the K-SAT problem is based on a classical random walk (U. Schöning. IEEE Proc. Ann. Symp. Found. Comp. Sci., 410-14 (1999)):

- Given an instance of the K-SAT problem, a brute force algorithm would take $O(2^n)$ steps, where n is the number of binary variables.

- Instead, Schöning proposed a stochastic algorithm that, starting on random initial conditions, executes a random walk on the set of binary variables. The complexity of this algorithm is $O(p(n)(2(1-\frac{1}{k}))^n$, where $p(n)$ is a polynomial.

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Quantum walks and their role in quantum computing

Quantum walks, the quantum mechanical counterpart of classical random walks, is an advanced tool for building quantum algorithms that has been recently shown to constitute a universal model of quantum computation.

Let us now provide a concise introduction to quantum walks.

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Basic structure of a quantum walk (1/16)

A quantum walk is a generalisation of classical random walks in the quantum world. There are two kinds of quantum walks:

Basic structure of a quantum walk (2/16)

Discrete quantum walks, characterized by having walker(s) and coin(s), discrete time steps and evolution via Unitary operators:

$$|\psi\rangle_n = \hat{U}^n |\psi\rangle_0$$

Basic structure of a quantum walk (3/16)

Continuous quantum walks, a coinless model characterized by no timing restrictions at all, having only walker(s) and evolution via the Schrödinger equation:

$$i\hbar\frac{\mathrm{d}|\psi\rangle}{\mathrm{d}\beta} = \hat{H}|\psi\rangle$$

where the Hamiltonian $\hat{H}$ is built based on the structure of the graph the quantum walk is expected to run on.

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Basic structure of a quantum walk (4/16)

The simplest and possibly most studied model of quantum walk is the discrete quantum walk on an unlimited line (DQWL). The main components of a coined DQWL are a walker, a coin, evolution operators for both walker and coin, and a set of observables:

Basic structure of a quantum walk (5/16)

Walker. The walker is a quantum system living in a Hilbert space $\mathcal{H}_p$ with $\#(\mathcal{H}_p) = \aleph_0$. It is customary to use the computational basis $\{|i\rangle, i \in \mathbb{N}\}$ of $\mathcal{H}_p$ as "position sites" for the walker.

We denote the walker as $|\text{position}\rangle \in \mathcal{H}_p$ and affirm that the canonical basis states $|i\rangle_p$ that span $\mathcal{H}_p$, as well as any superposition of the form $\sum_i \alpha_i |i\rangle_p$ subject to $\sum_i |\alpha_i|^2 = 1$, are valid states for $|\text{position}\rangle$

The walker is usually initialized at the 'origin', i.e. $|\text{position}\rangle_{\text{initial}} = |0\rangle_p$

Basic structure of a quantum walk (6/16)

Coin. The coin is a quantum system living in a 2-dimensional Hilbert space $\mathcal{H}_c$.

The coin may take the canonical basis states $|0\rangle$ and $|1\rangle$ as well as any superposition of these basis states.

Therefore $|\text{coin}\rangle \in \mathcal{H}_c$ and a general normalized state of the coin may be written as $|\text{coin}\rangle = a|0\rangle_c + b|1\rangle_c$, where $|a|^2 + |b|^2 = 1$.

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Basic structure of a quantum walk (7/16)

Total state. The total state of the quantum walk resides in

$$\mathcal{H}_t = \mathcal{H}_c \otimes \mathcal{H}_p$$

It is customary to use product states of $\mathcal{H}_t$ as initial states, that is,

$$|\psi\rangle_{\text{initial}} = |\text{coin}\rangle_{\text{initial}} \otimes |\text{position}\rangle_{\text{initial}}$$

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Basic structure of a quantum walk (8/16)

Evolution Operators: The evolution of a DQWL is divided into two parts that closely resemble the behavior of a classical random walk:

Apply, to the total quantum system, an evolution operator to the coin state followed by a conditional shift operator

$$|\psi\rangle_{t_1} = \hat{S}(\hat{C} \otimes \hat{I})(|\text{coin}\rangle_{t_0} \otimes |\text{position}\rangle_{t_0})$$

The coin operator renders the coin state in a superposition, the shift operator spreads the walker state over $\mathbb{N}$, and randomness is introduced by performing a measurement on the system after both evolution operators have been applied several times.

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Basic structure of a quantum walk (9/16)

In general, an n-step discrete quantum walk can be written as

$$|\psi\rangle_{t_n} = \hat{U}^n |\psi\rangle_{t_0}$$

or, equivalently, as

$$|\psi\rangle_{t_n} = \sum_k [a_k|0\rangle_c + b_k|1\rangle_c]|k\rangle_p$$

where $|0\rangle_c$, $|1\rangle_c$ are the coin state components and $|k\rangle_p$ are the walker state components.

Basic structure of a quantum walk (10/16)

The Hadamard operator has been extensively employed as coin operator:

$$\hat{H} = \frac{1}{\sqrt{2}}(|0\rangle_c\langle 0| + |0\rangle_c\langle 1| + |1\rangle_c\langle 0| - |1\rangle_c\langle 1|) \tag{1}$$

Basic structure of a quantum walk (11/16)

For the conditional shift operator use is made of a unitary operator that allows the walker to go one step forward if the accompanying coin state is one of the two basis states (e.g. $|0\rangle$), or one step backwards if the accompanying coin state is the other basis state (e.g. $|1\rangle$). A suitable conditional shift operator has the form

$$\hat{S} = |0\rangle_c\langle 0| \otimes \sum_i |i+1\rangle_p\langle i| + |1\rangle_c\langle 1| \otimes \sum_i |i-1\rangle_p\langle i| \tag{2}$$

Basic structure of a quantum walk (12/16)

For example, let us suppose we have

$|\psi\rangle_0 = |0\rangle_c \otimes |0\rangle_p$ as the quantum walk initial state

$\hat{H} = \frac{1}{\sqrt{2}}(|0\rangle_c\langle 0| + |0\rangle_c\langle 1| + |1\rangle_c\langle 0| - |1\rangle_c\langle 1|)$ as coin operator, and

$\hat{S} = |0\rangle_c\langle 0| \otimes \sum_i |i+1\rangle_p\langle i| + |1\rangle_c\langle 1| \otimes \sum_i |i-1\rangle_p\langle i|$ as shift operator.

Then, the first three steps of this quantum walk can be written as:

Basic structure of a quantum walk (13/16)

$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}|0\rangle_c|1\rangle_p + \frac{1}{\sqrt{2}}|1\rangle_c|-1\rangle_p$$

$$|\psi\rangle_2 = (\frac{1}{2}|0\rangle_c + 0|1\rangle_c)|2\rangle_p + (\frac{1}{2}|0\rangle_c + \frac{1}{2}|1\rangle_c)|0\rangle_p + (0|0\rangle_c - \frac{1}{2}|1\rangle_c)|-2\rangle_p$$

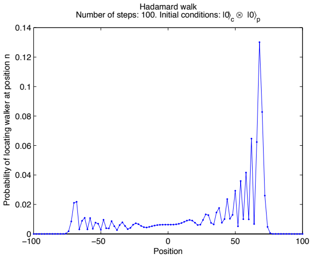
$$|\psi\rangle_3 = (\frac{1}{2\sqrt{2}}|0\rangle_c + 0|1\rangle_c)|3\rangle_p + (\frac{1}{\sqrt{2}}|0\rangle_c + \frac{1}{2\sqrt{2}}|1\rangle_c)|1\rangle_p + \\ (\frac{-1}{2\sqrt{2}}|0\rangle_c + 0|1\rangle_c)|-1\rangle_p + (0|0\rangle_c + \frac{1}{2\sqrt{2}}|1\rangle_c)|-3\rangle_p$$

Basic structure of a quantum walk (14/16)

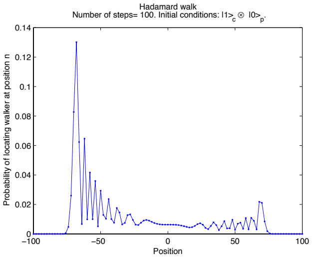
Quantum walks produce position probability distributions that are very different from those probability distributions obtained by the computation of classical random walks. We expect to use those new probability distributions produced by quantum walks for the development of new and faster algorithms.

Basic structure of a quantum walk (15/16)

Probability distributions of 100 steps DQWLs using coin and shift operators given by $\hat{H}$ and $\hat{S}$, respectively. Note the skewness of probability distributions.



$$|\psi\rangle_{t_0} = |0\rangle_c \otimes |0\rangle_p$$



$$|\psi\rangle_{t_0} = |1\rangle_c \otimes |0\rangle_p$$

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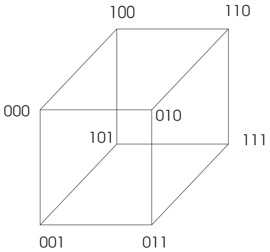
Quantum walk-based algorithms

Quantum walk-based algorithms have been developed to solve different instances of an abstract search problem: given a structure (e.g. a a list, an unordered set or a graph G), translate it into a graph and, by running a[n] [oracle-based] quantum walk on it, find a (possibly marked) state or a property of the structure.

Quantum random-walk search algorithm (1/3)

Quantum random-walk search algorithm. N. Shenvi, J. Kempe, K.B. Whaley. Phys. Rev. A 67, 052307 (2003)



Objective: to find a marked vertex v contained in a hypercube with 2^n nodes.

Quantum random-walk search algorithm (2/3)

Solution: an oracle-based quantum walk is run over a hypercube using the following coin and shift operators:

$$\hat{C} = \hat{C}_0 \otimes \hat{\mathbb{I}} = (-\hat{\mathbb{I}} + 2|s^c\rangle\langle s^c|) \otimes \hat{\mathbb{I}}$$

where $|s^c\rangle$ is the equal superposition over all n directions, i.e. $|s^c\rangle = \frac{1}{\sqrt{n}} \sum_{d=1}^{n} |d\rangle$, and

$$\hat{S} = \sum_{d=0}^{n-1} \sum_{\vec{x}} |d, \vec{x} \otimes \vec{e}_d\rangle\langle d, \vec{x}|$$

where $|\vec{e}_d\rangle$ is the d^{th} basis vector of the hypercube.

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Quantum random-walk search algorithm (3/3)

Use the eigenvalues and eigenvectors of the evolution operator

$$\hat{U} = \hat{S}\hat{C}$$

to build a slightly modified coin operator $\hat{C}'$ and an evolution operator $\hat{U}'$.

After applying $\hat{U}'$ a number of $t_f = \frac{\pi}{2}\sqrt{2^n} = O(\sqrt{N})$ times, the outcome of this algorithm is x_{target} with probability $\frac{1}{2} - O(\frac{1}{n})$.

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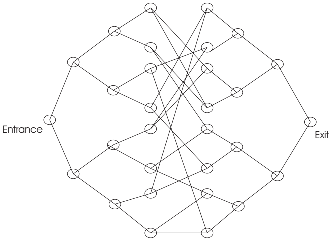
Continuous quantum walk-based algorithm (1/4)

Exponential Algorithmic Speedup by a Quantum Walk.
A.M. Childs et al. Proc. ACM-STOC 03, pp. 59-68 (2003)



Let G be a graph composed of two binary trees whose leaves are randomly connected pairwise.

Continuous quantum walk-based algorithm (2/4)

We now define the following Hamiltonian

$$\langle a|\hat{H}|b\rangle = \begin{cases} \gamma, & (a,b) \in E(G) \\ 0, & \text{otherwise} \end{cases}$$

where $E(G)$ is the set of edges of graph G and γ is a constant probability per unit time to jump between adjacent nodes.

Continuous quantum walk-based algorithm (3/4)

Then, after long calculations on the eigenvalues of H and e^{-iHt}, the authors conclude the following:

Theorem. For n sufficiently large, running the quantum walk for a time chosen uniformly in $[0, \frac{n^4}{2\epsilon}]$ and then measuring in the computational basis yields a probability of finding the exit that is greater than $\frac{1}{2n}(1-\epsilon)$

Continuous quantum walk-based algorithm (4/4)

Moreover, it is also found that, for any possible classical (stochastic) algorithm:

Theorem. Any classical algorithm that makes at most $2^{\frac{n}{6}}$ queries to the oracle finds the EXIT with probability at most $4 \times 2^{\frac{-n}{6}}$

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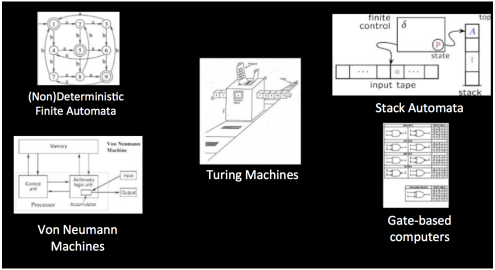
Universal Computation in Classical Computer Science (1/3)

Are these models of computation equally powerful, i.e. are their sets of computable functions identical?



No! Different models of computation have different computability power.

Universal Computation in Classical Computer Science (2/3)

Universality in Classical Computer Science is based on the following notion:

Two computer machines are equivalent if they are capable of simulating one another (i.e., of recognizing the same set of languages)

For example, Finite Deterministic Automata and Finite Nondeterministic Automata are **equivalent**, while Finite Deterministic Automata and Turing Machines are **not equivalent.**

Universal Computation in Classical Computer Science (3/3)

Universality in Classical Computer Science has two meanings:

- The computing (computability) capacity of the Universal Turing Machine.
- Suppose we have a computer machine A. We say that A is Universal if and only if its set of computable functions is identical to the set of computable functions by the Universal Turing Machine.

Quantum Computational Universality (1/2)

In 1995, Deutsch, Barenco and Ekert proposed the following notion of Universal computer:

A universal set of components is one that is adequate for the building of computers to perform *any physically possible computation*.

A **Universal Computer** is a single machine that can perform any physically possible computation.

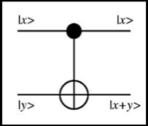
D. Deutsch, A. Barenco, A. Ekert. Universality in Quantum Computation. Proc. R. Soc. Lond. 449, 669-677 (1995)

Quantum Computational Universality (2/2)

In this sense, the computational models known as Quantum Circuit/gate model, Quantum Adiabatic Computation and Quantum Walks constitute Universal Models of Computation.

Quantum Gate Model

So, Universal Quantum Computation can be achieved by any experiment/physical platform/theoretical proposal that succeeds at implementing the following gates:



Controlled-not gate

$$\hat{H} = \frac{1}{\sqrt{2}}(|0\rangle_c\langle 0| + |0\rangle_c\langle 1| + |1\rangle_c\langle 0| - |1\rangle_c\langle 1|)$$

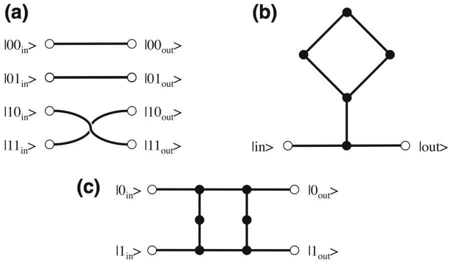
Hadamard gate

$$R(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{2\pi i\theta} \end{bmatrix}$$

A phase-rotation gate

D. DiVincenzo. Phys. Rev. A. 51(2), 1015-1021 (1995)

Universal Computation by Quantum Walks (1/2)



a Widget for C_{not} gate, **b** widget for phase gate, and **c** widget for the basis-changing gate

A.M. Childs. Universal computation by quantum walk. Phys. Rev. Lett., 102:180501 (2009)

Universal Computation by Quantum Walks (2/2)

Two fundamental contributions:

- First to prove that the continuous quantum walk model is universal for quantum computation
- Scattering algorithms

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Research Horizons in Quantum Walks

- Simulation of complex phenomena via quantum walks
- Direct manipulation of physical properties for computational speed-up on quantum walk-based algorithms
- Quantum walk-based algorithms for $\mathrm{NP}-\text{complete}$ and $\mathrm{NP}-\text{hard}$ problems
- Development of quantum algorithms based on the Quantum Walk Universal Model of Computation

Moral: **We need a killer app!**

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My group's research (1/2)

- **Quantum walks**
  - Quantum entanglement and decoherence as controlled computational resources for quantum walk-based algorithms
  - Quantum algorithm development for $\mathrm{NP-complete}$ and $\mathrm{NP-hard}$, based on the Quantum Walk Universal Model of Computation
  - Quantum walks as scattering algorithms
  - Automatic translation of quantum walk-based algorithms to other universal models of quantum computation (quantum circuit and quantum adiabatic models)

My group's research (2/2)

- **More on quantum walks**
  - High-level quantum programming language based on quantum walks (currently, quantum programming resembles using assembly or machine language in a digital computer system)
  - Simulation of quantum walk-based algorithms via digital platforms
- **Other topics**
  - Quantum Image Processing
  - Quantum annealing algorithms running on a D-Wave computer (NASA Ames Research Centre, US Navy, UAEM)
  - Quantum Pattern Recognition

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Names to be remembered

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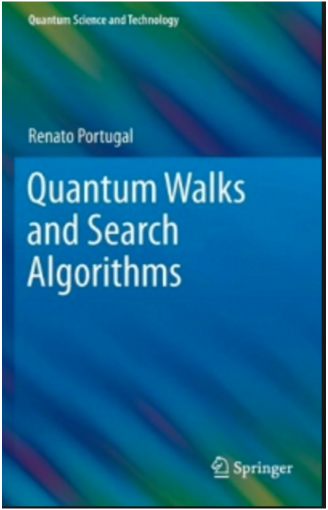

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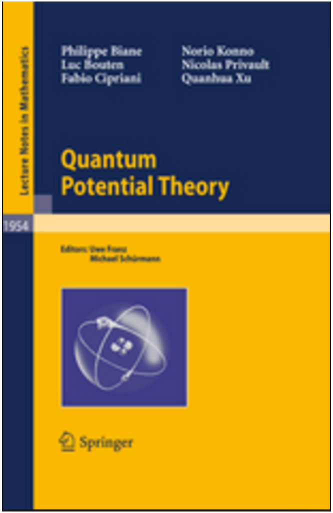

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Quantum walks: a comprehensive review

Salvador Elías Venegas-Andraca

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Abstract Quantum walks, the quantum mechanical counterpart of classical random walks, is an advanced tool for building quantum algorithms that has been recently shown to constitute a universal model of quantum computation. Quantum walks is now a solid field of research of quantum computation full of exciting open problems for physicists, computer scientists and engineers. In this paper we review theoretical advances on the foundations of both discrete- and continuous-time quantum walks, together with the role that randomness plays in quantum walks, the connections between the mathematical models of coined discrete quantum walks and continuous quantum walks, the quantumness of quantum walks, a summary of papers published on discrete quantum walks and entanglement as well as a succinct review of experimental proposals and realizations of discrete-time quantum walks. Furthermore, we have reviewed several algorithms based on both discrete- and continuous-time quantum walks as well as a most important result: the computational universality of both continuous- and discrete-time quantum walks.

Keywords Quantum walks · Quantum algorithms · Quantum computing · Quantum and classical simulation of quantum systems

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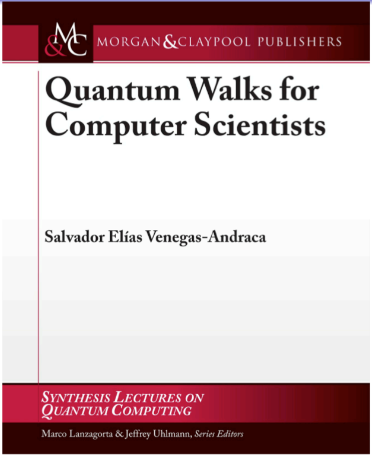

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Thanks again!

Thank you very much!
Questions?