U.S. NAVAL RESEARCH LABORATORY

Quantum Computational Complexity in Curved Spacetime

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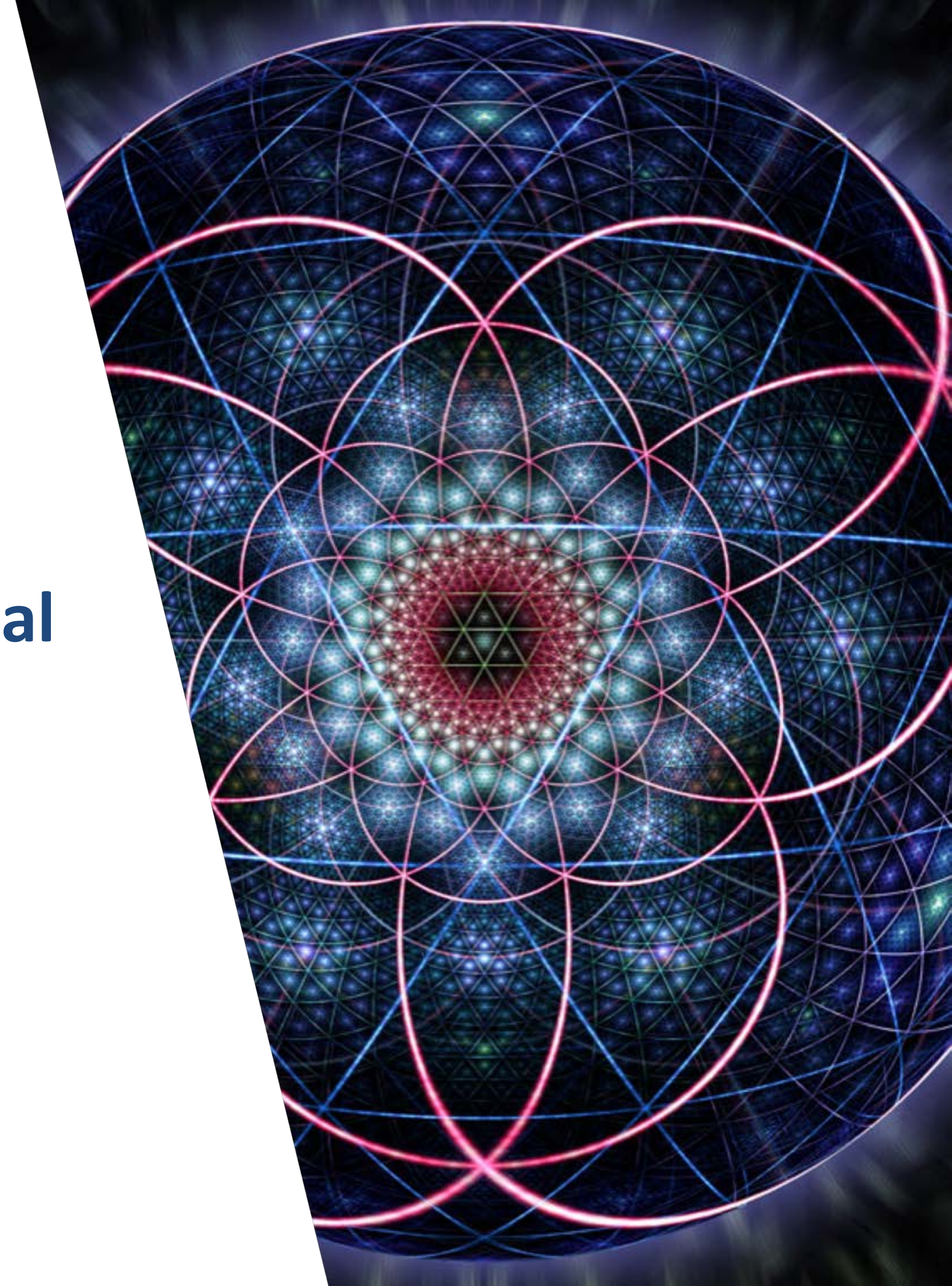
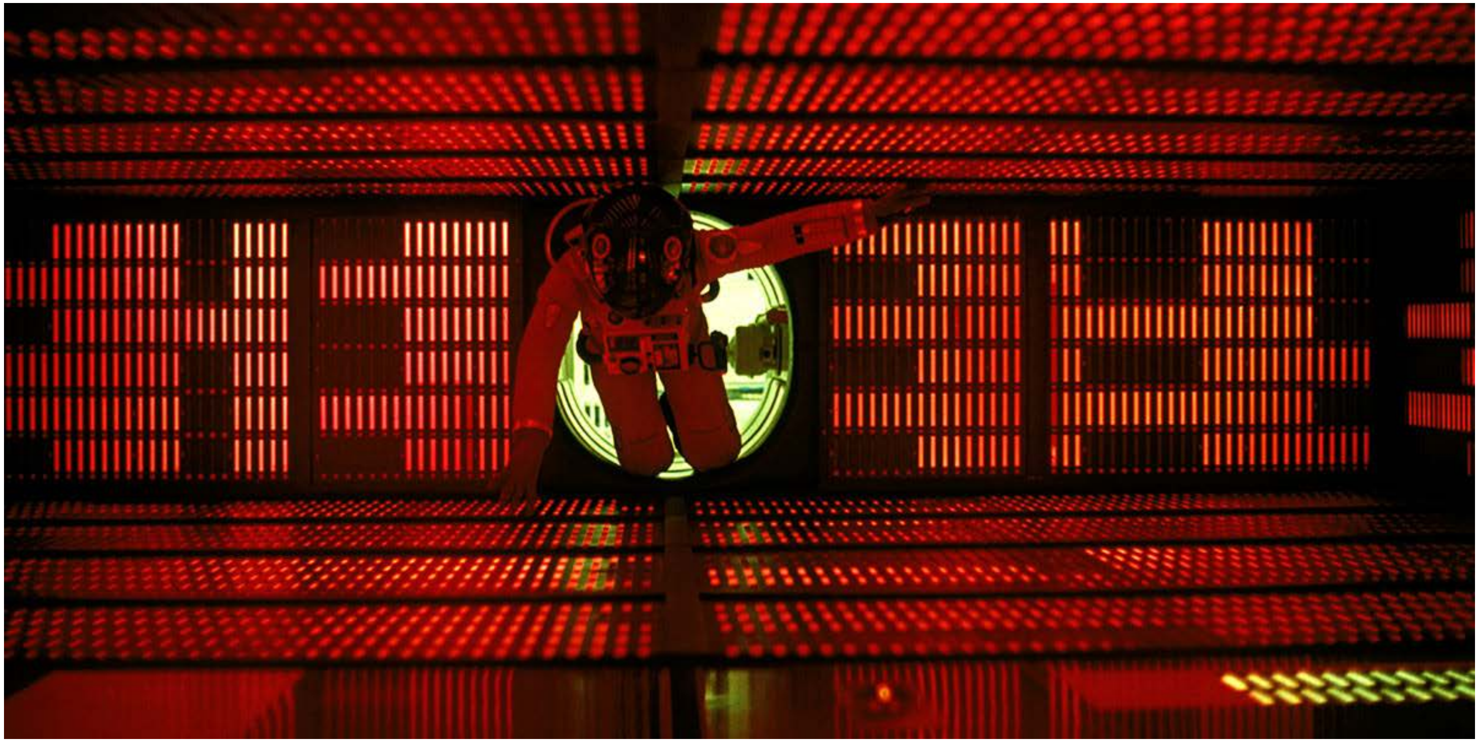


Clarke's Second Law

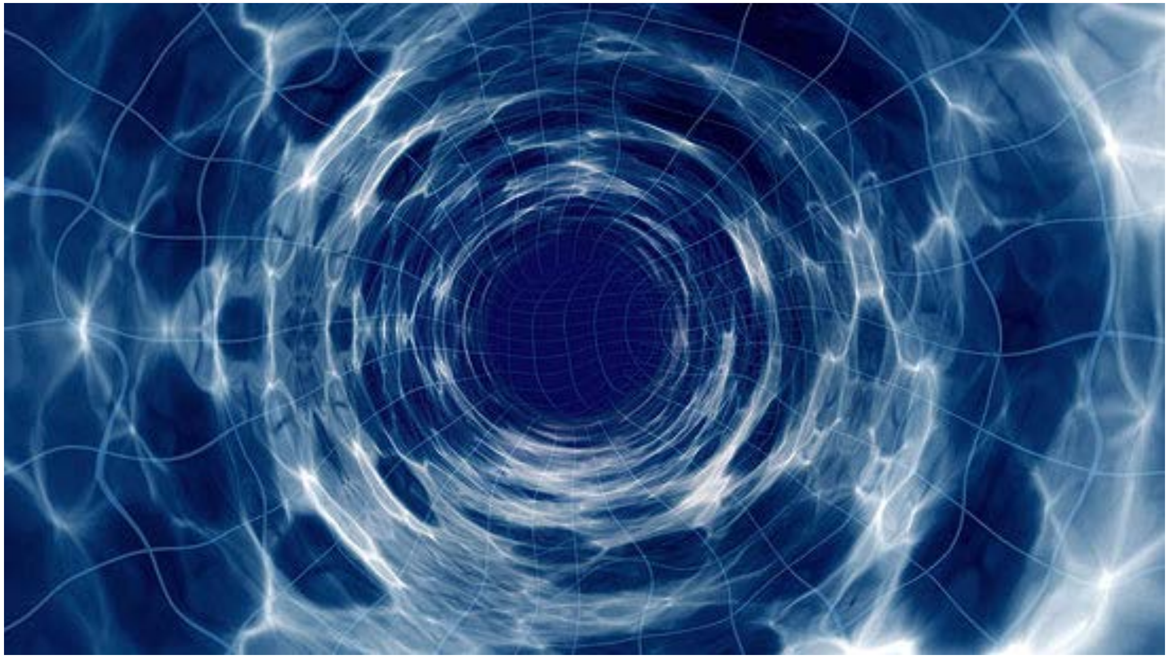


The only way of discovering the limits of the possible is to venture a little way past them into the impossible

Clarke’s Third Law

Any sufficiently advanced technology is indistinguishable from magic

What Lies Ahead?



- We want to explore what lies ahead, beyond the horizon… what are the limits of the possible with reasonable R&D assumptions and without breaking the laws of physics?
- Quantum computation is affected by gravity!

Outline

- Qubits in Gravity
- Quantum Computing in Gravity
- Quantum Gravimetry
- Conclusions

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- **Qubits in Gravity**
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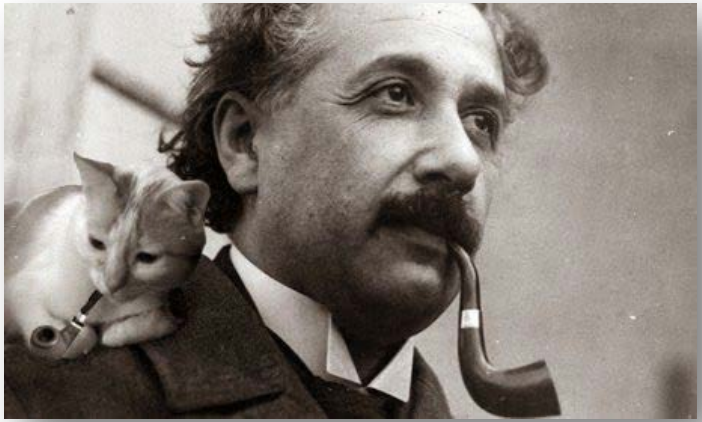
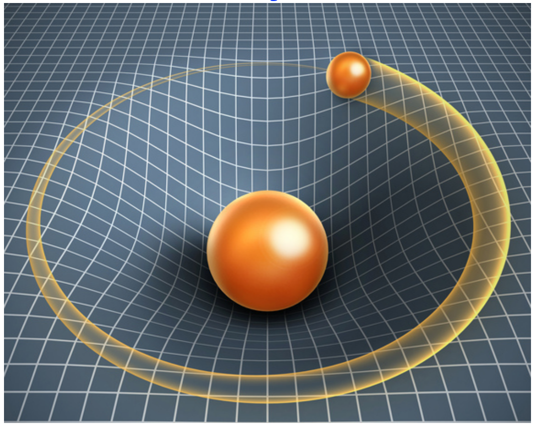


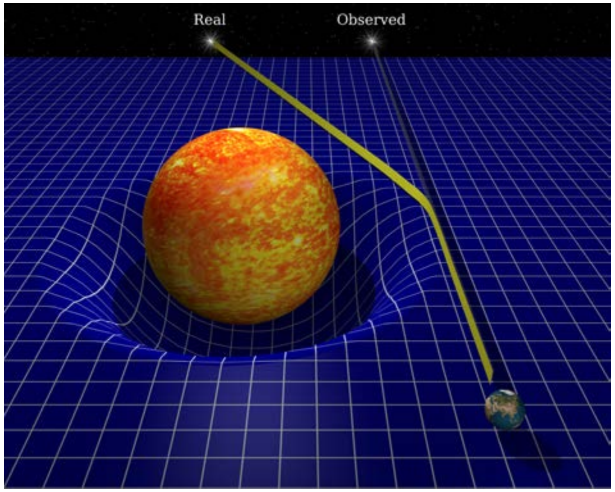
Principle of Relativity

- The laws of nature are identical for all physical observers.
- That is, the laws of nature are independent of the frame of reference of the observer.
- Mathematically, this implies that “the equations of physics are the same”, or better said, “the equations of physics have the same form”, in all frames of reference.
- Of course, the individual values of some physical properties may not be the same in different frames of reference.

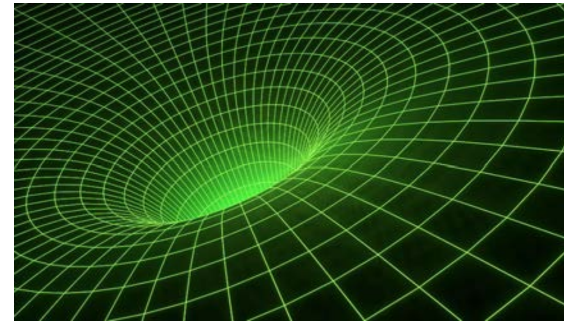
General Relativity

- Principle of Equivalence: A non-inertial reference frame is equivalent to some gravitational field.
- Invariance of the laws of physics in accelerated frames implies that the gravitational interaction is described as the effect of spacetime curvature.
- Therefore, an orbiting test particle is not being “attracted” by the mass of a star, but it merely follows a geodesic in the curved spacetime produced by the star.





Einstein Field Equations



- The Einstein Field Equations describe how matter and energy curve spacetime.

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{Spacetime Geometry}} = -8\pi G \underbrace{T_{\mu\nu}}_{\text{Energy-Momentum}}$$

Non-homogenous system of 16 coupled second order, nonlinear, hyperbolic-elliptic, partial differential equations (but can be reduced to 6 independent equations + 4 gauge fixing degrees of freedom)

- Exact solutions are rare and very difficult to find.
- Because of the nonlinearities, even the full self-gravitating two body problem is extremely difficult to solve.
- We will consider test particles immersed in external gravitational fields (no self-gravitation and smaller than the curvature scale).

Spin-Based Qubits in Gravity

- Intrinsic spin is the most common degree of freedom used to define qubits in Quantum Information Science
- Spin is a relativistic concept formally described by Quantum Field Theory
- Thus, spin-based implementations of quantum information are *intrinsically* relativistic and are *directly coupled* to classical gravitational fields described by Einstein's General Theory of Relativity
- Furthermore, Einstein's field equations couple all kinds of matter-energy with spacetime curvature, thus, if information is physical, then it will warp spacetime (i.e., it will gravitate):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}$$

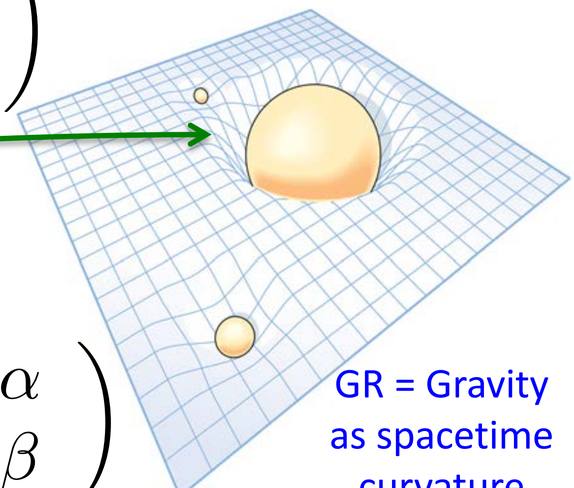
The Wigner Rotation

Consider 1 qubit encoded with massive spin states:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Gravity rotates massive spin states

+ gravity



$$|\psi'\rangle = \begin{pmatrix} \cos\Omega/2 & \sin\Omega/2 \\ -\sin\Omega/2 & \cos\Omega/2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Wigner rotation angle induced by gravity

This is a purely general relativistic effect.

In contrast to the much more abstract concept of a classical bit of information, *spin-based quantum information invariably gravitates*

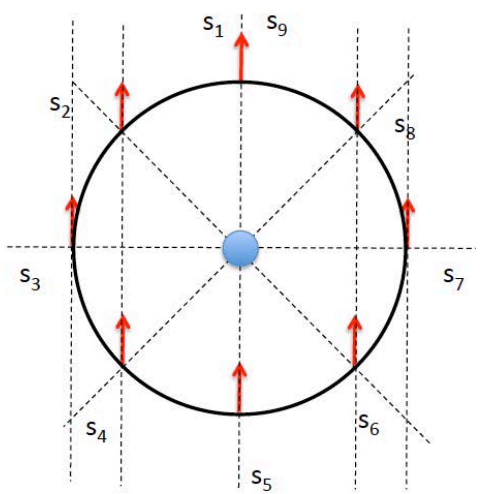
Single Qubits as Gyroscopes in Gravity

Spin-based qubits behave as gyroscopes in gravity

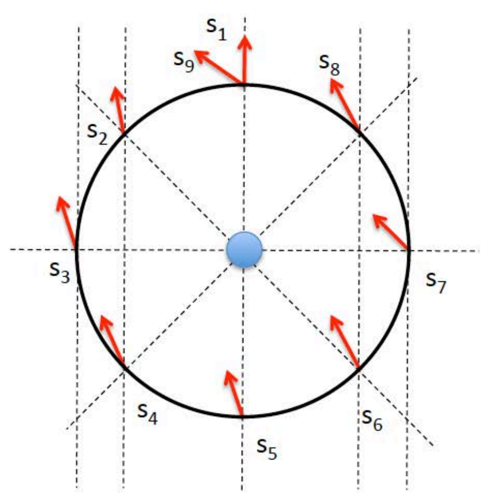
The Wigner rotation of an orbiting qubit is the quantum mechanical equivalent to the geodetic precession for classical orbiting gyroscopes

In general, we understand a "gyroscope" as an angular momentum *pseudo-vector* that gets reversed under improper rotations

Example: consider a gyroscope or a spin-based qubit orbiting around a spherical body in Newtonian and in General Relativistic gravitation:



Newtonian Gravitation



General Relativity

S_i is a classical or quantum angular momentum pseudo-vector

For quantum particles:
the Wigner rotation leads to the angle between S_i and the vertical axis

For classical particles:
the geodetic precession rotation leads to the angle between S_i and the vertical axis

1 Qubit State in Gravity

We begin by considering a single qubit in the uniform superposition, i.e., it will be measured as "0" or "1" with equal probability:

$$|\psi_1\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \tag{1.4}$$

In the presence of gravity this state will "drift" into:

$$|\psi_1\rangle \quad \longrightarrow \quad \frac{\cos\frac{\Omega}{2} + \sin\frac{\Omega}{2}}{\sqrt{2}}|0\rangle + \frac{\cos\frac{\Omega}{2} - \sin\frac{\Omega}{2}}{\sqrt{2}}|1\rangle \tag{1.5}$$

$$|\phi\rangle_0 = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

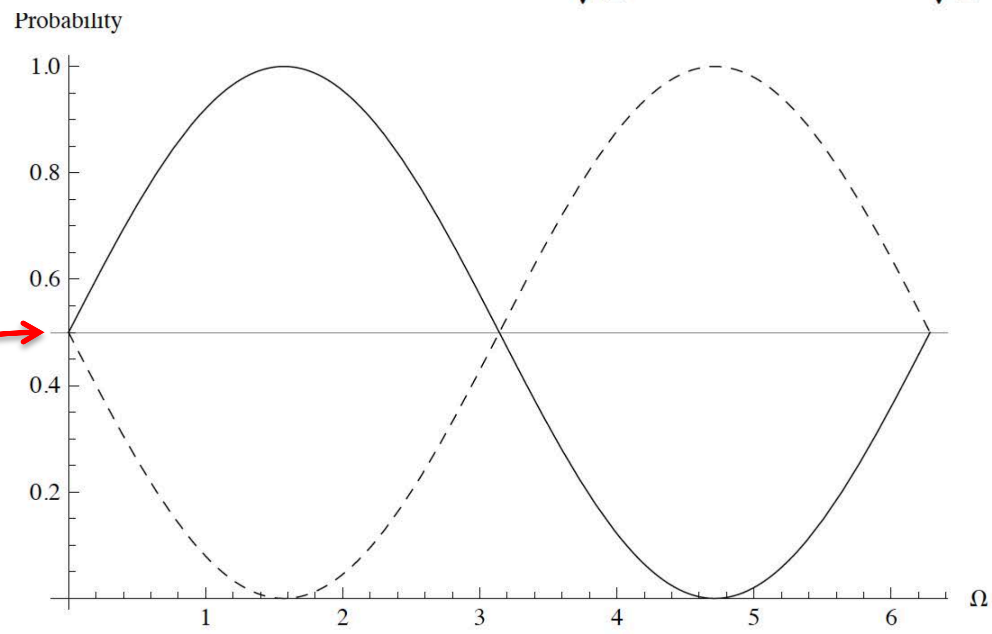
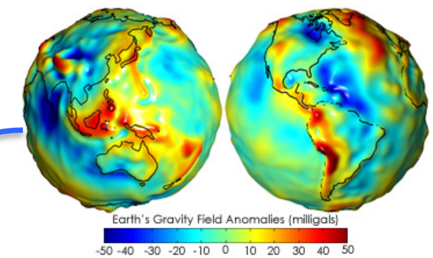


Fig. 1.2 Drifting of the probability of measuring "1" (solid line) and "0" (dashed line) for a 1 qubit state in the uniform superposition in the presence of a gravitational field described by the Wigner rotation angle $\Omega(t)$.

Computation of the Wigner Angle for Electrons and Photons



Gravitational Models

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}$$

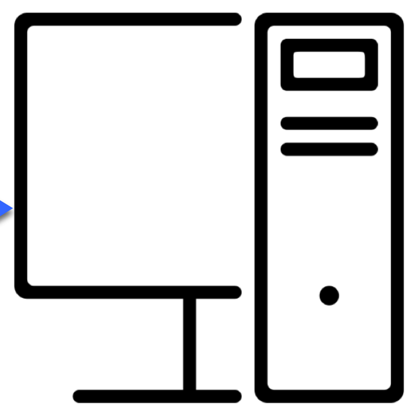
Einstein Field Equations

$$g^{\mu\nu}(x) = e_a{}^{\mu}(x)\ e_b{}^{\nu}(x)\ \eta^{ab}$$

Tetrad Field

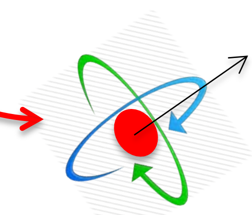
$$g^{\mu\nu} \mathcal{D}_\mu \mathcal{D}_\nu A^\alpha + A^\beta R_\beta{}^\alpha = 0$$

Maxwell Equations in Curved Spacetime



$$\left(i e_a{}^{\mu} \gamma^a \partial_\mu - i e_a{}^{\mu} \gamma^a \Gamma_\mu - m\right) \psi = 0$$

Dirac Equation in Curved Spacetime



Ω

Wigner Angle

Wigner Angle for a Qubit in an Equatorial Circular Orbit Around a Spherically Symmetric Rotating Mass

All the complexity of the gravitational interaction is "hidden" in the Wigner angle: $\Omega = 2\pi \frac{\vartheta^1{}_3}{u^\phi}$

$$\vartheta^1{}_3(x) = \frac{a r_s^2 \sqrt{f}}{2r^4 \Delta (K + \sqrt{f})} \left(J - \frac{K}{\omega} + aK + \frac{fJr}{r_s} \right) \times \left(-\frac{aK}{f} + aK + J \right)$$

$$+ \frac{a^2 r_s^2}{2\Delta r^4 \sqrt{f}} \left(\frac{K}{\omega} - J \right) - \frac{a r_s}{\Delta r} \left(K + \frac{fJr}{a r_s} \right) \left(\frac{a^2 r_s}{2r^3 \sqrt{f}} - \sqrt{f} \right)$$

$$J = \mp \frac{1 + \frac{a^2}{r^2} \pm 2a\sqrt{\frac{r_s}{2r^3}}}{\sqrt{1 - \frac{3r_s}{2r} \mp 2a\sqrt{\frac{r_s}{2r^3}}}} \sqrt{\frac{r r_s}{2}}$$

$$\Delta = r^2 - r r_s + a^2$$

$$f = 1 - \frac{r_s}{r}$$

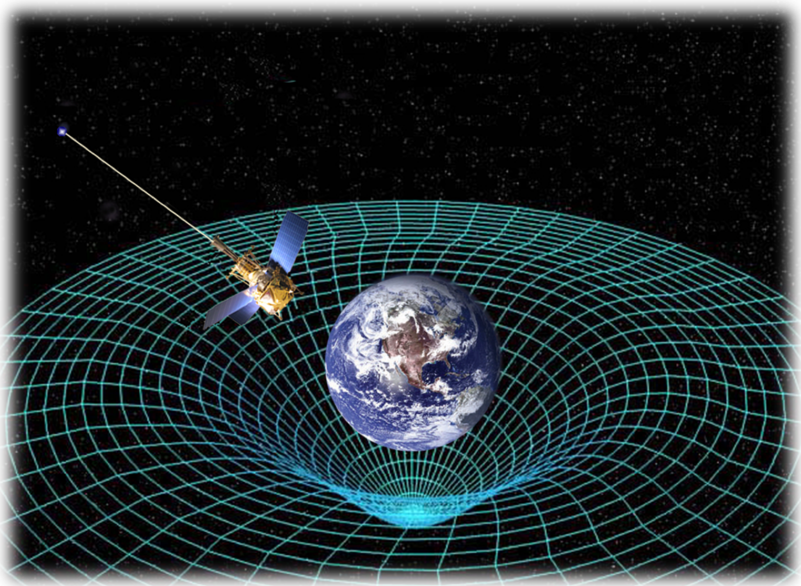
$$\omega = \frac{r_s r a}{(r^2 + a^2)^2}$$

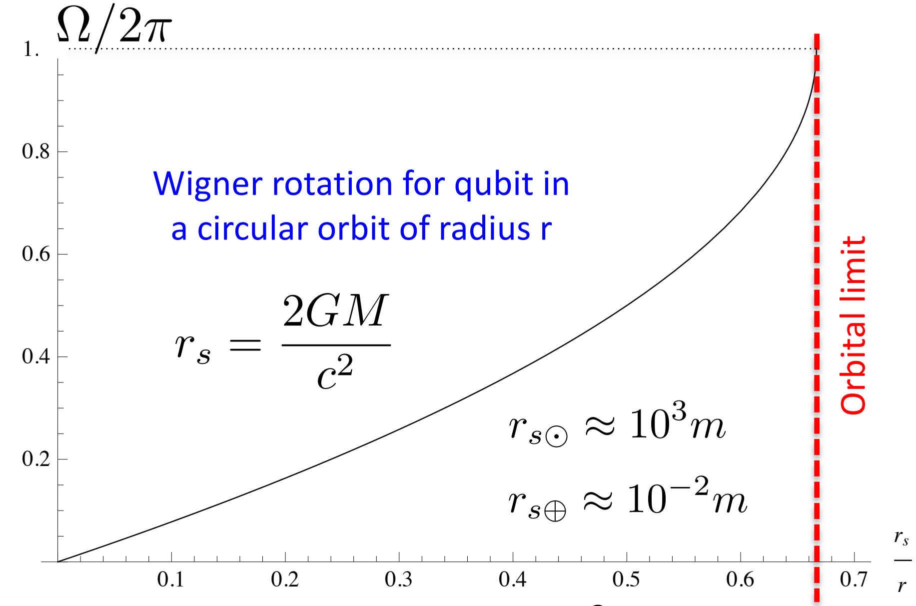
$$u^\phi(x) = \frac{1}{\Delta} \left(\frac{aKr_s}{r} + fJ \right)$$

$$K = \frac{1 - \frac{r_s}{r} \mp a\sqrt{\frac{r_s}{2r^3}}}{\sqrt{1 - \frac{3r_s}{2r} \mp 2a\sqrt{\frac{r_s}{2r^3}}}}$$

Only three free parameters: orbital distance (r), star's angular momentum (a) and star's mass (r_s).

Qubits Onboard a Satellite in a Circular Orbit Around Earth

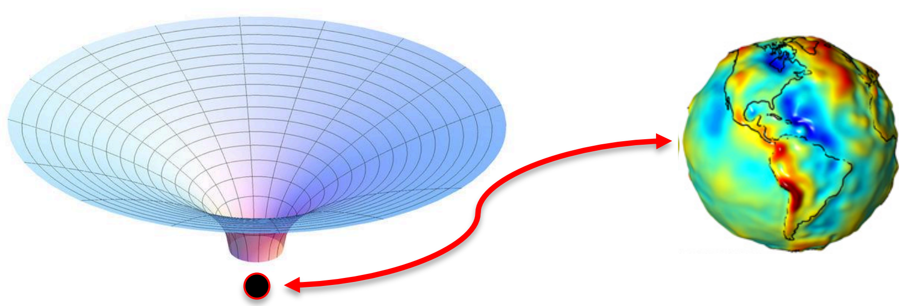




Approximation to Earth's field:
Schwarzschild curved spacetime produced by a static, isotropic, spherically symmetric massive body (black hole).

For Earth: $\Omega \approx 10^{-9}$ per day

Small, but cumulative effect
(similar to relativistic corrections to GPS)

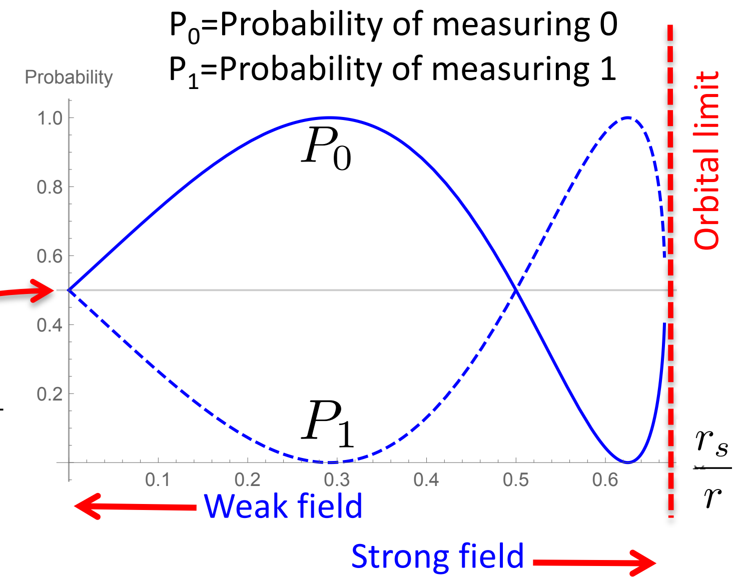


Gravitational Drifting

The state of a spin-based qubit "drifts" as it moves in a gravitational field

Example: consider a qubit in circular orbit in a spherically symmetric gravitational field (Schwarzschild spacetime)

$$|\phi\rangle_0 = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



Traditional classical information devices do not showcase gravitational drifting

Traditional classical information processing devices that use charge as a logical state variable could be affected by gravity (e.g., time dilation in a communication protocol)

Because these physical implementations of classical bits do not depend on angular momentum variables, they do not exhibit geodetic precession / gravitational drifting

However, classical information processing based on spintronic logic devices could showcase geodetic gravitational drifting if spin is used as a logical state variable

2-Qubit State in Gravity

Consider now a 2-qubit state uniform superposition:

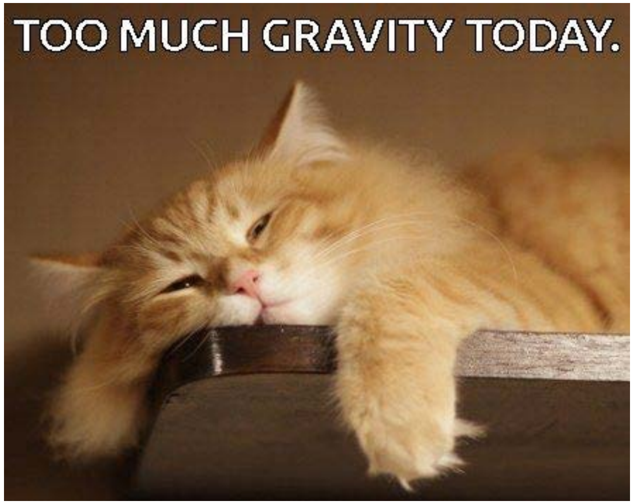
$$|\psi_2\rangle = \frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2} \tag{1.14}$$

which under the presence of gravity is transformed into:

$$|\psi_2\rangle \to \frac{\left(\cos\frac{\Omega}{2} - \sin\frac{\Omega}{2}\right)^2}{2}|00\rangle + \frac{\cos^2\frac{\Omega}{2} - \sin^2\frac{\Omega}{2}}{2}|01\rangle \tag{1.15}$$

$$+\frac{\cos^2\frac{\Omega}{2} - \sin^2\frac{\Omega}{2}}{2}|10\rangle + \frac{\left(\cos\frac{\Omega}{2} + \sin\frac{\Omega}{2}\right)^2}{2}|11\rangle$$

where we have assumed that the two qubits have negligible spatial separation so that their Wigner angles are identical.



Probability Drifting for a 2-Qubit State

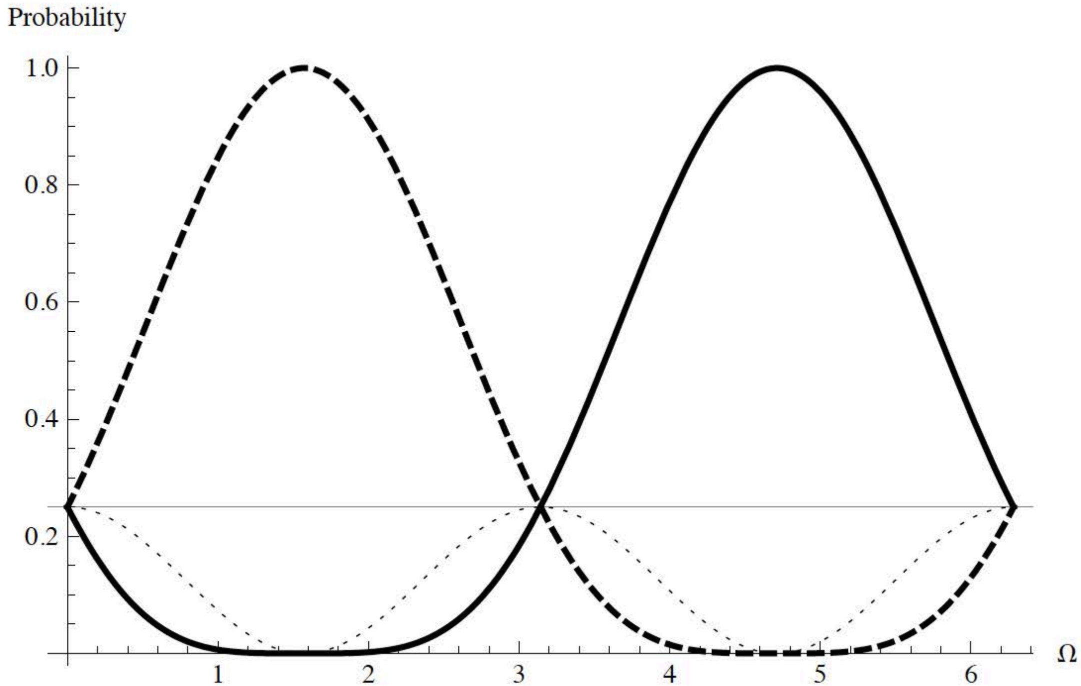


Fig. 1.5 Drifting of the probability of measuring "00" (solid line), "01" and "10" (dotted line), and "11" (dashed line) for a 2 qubit state in the uniform superposition in the presence of a gravitational field described by the Wigner rotation angle $\Omega(t)$.

Drifting in a Speherically Symmetric Gravitational Field

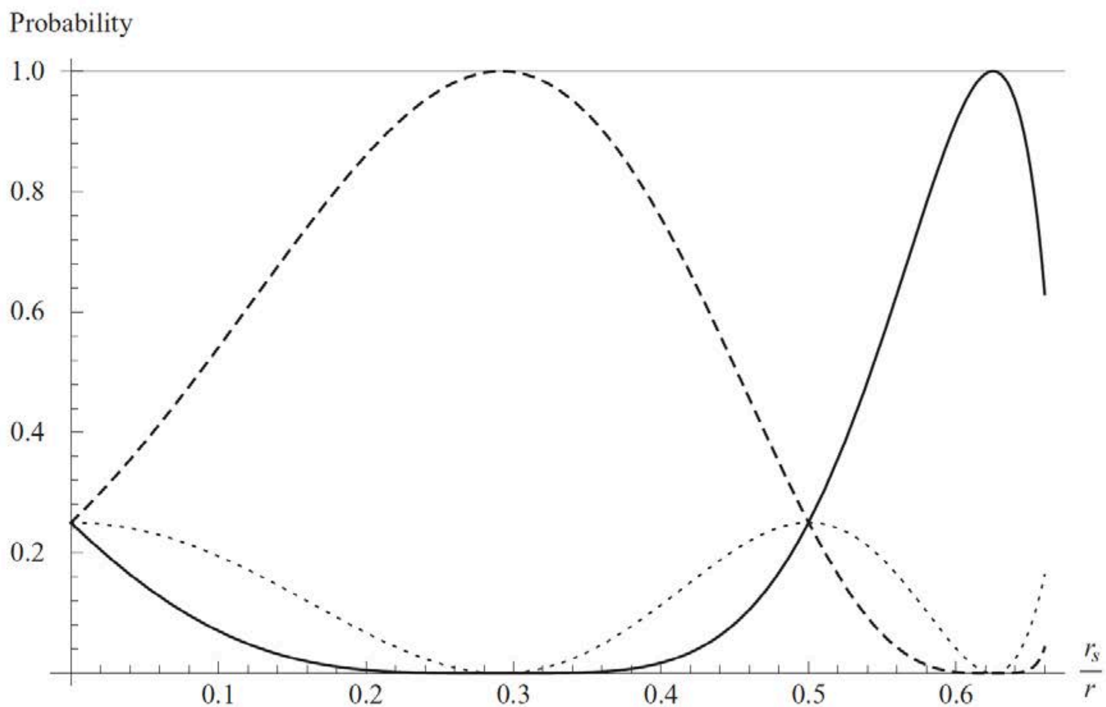


Figure 9.4. Drifting of the probability of measuring '00' (solid line), '01' and '10' (dotted line) and '11' (dashed line) for a 2-qubit state after completion of a circular orbit of radius r in the Schwarzschild spacetime produced by an object of mass $r_s/2$.

Relativistic Effects on a Momentum Superposition

On the other hand, let us assume a particle with a non-definite momentum state, but definite helicity state, described by:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|p\rangle + |q\rangle) \otimes |+\rangle \tag{4.83}$$

which clearly is a separable state in the spin and momentum degrees of freedom. Then, the Lorentz transformation of this state leads to:

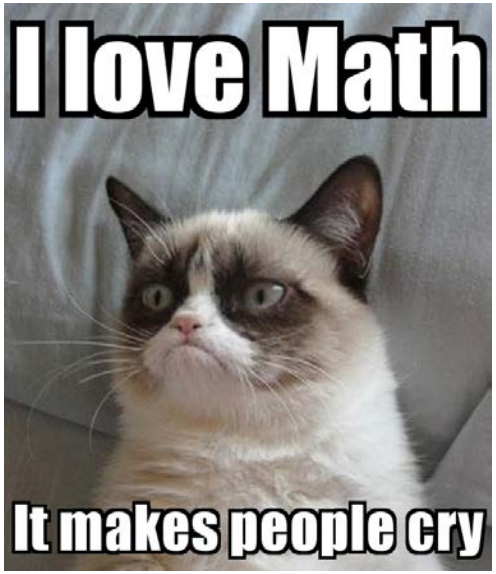
$$\begin{aligned}
|\psi\rangle \to |\tilde{\psi}\rangle &= \hat{U}(\Lambda)|\psi\rangle \\
&= \frac{1}{\sqrt{2}}\hat{U}(\Lambda)|p\rangle \otimes |+\rangle + \frac{1}{\sqrt{2}}\hat{U}(\Lambda)|q\rangle \otimes |+\rangle \\
&= \frac{1}{\sqrt{2}}|\Lambda p\rangle \otimes \left(\cos\left(\frac{\Omega_p}{2}\right)|+\rangle + \sin\left(\frac{\Omega_p}{2}\right)|-\rangle\right) \\
&\quad + \frac{1}{\sqrt{2}}|\Lambda q\rangle \otimes \left(\cos\left(\frac{\Omega_q}{2}\right)|+\rangle + \sin\left(\frac{\Omega_q}{2}\right)|-\rangle\right) \\
&= \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\Omega_p}{2}\right)|\Lambda p\rangle + \cos\left(\frac{\Omega_q}{2}\right)|\Lambda q\rangle\right) \otimes |+\rangle \\
&\quad + \frac{1}{\sqrt{2}}\left(\sin\left(\frac{\Omega_p}{2}\right)|\Lambda p\rangle + \sin\left(\frac{\Omega_q}{2}\right)|\Lambda q\rangle\right) \otimes |-\rangle
\end{aligned} \tag{4.84}$$

which in general is a non-separable state. The two Schmidt coefficients are given by:

Schmidt Coefficients

which in general is a non-separable state. The two Schmidt coefficients are given by:

$$\begin{aligned}\lambda_{\pm}^2 &= \frac{1}{2}\left(1 \pm \sqrt{1 - \left(\cos\frac{\Omega_p}{2}\sin\frac{\Omega_q}{2} - \cos\frac{\Omega_q}{2}\sin\frac{\Omega_p}{2}\right)^2}\right)\\ &= \frac{1}{2}\left(1 \pm \sqrt{1 - \sin^2\left(\frac{\Omega_q - \Omega_p}{2}\right)}\right)\\ &= \frac{1}{2}\left(1 \pm \cos\left(\frac{\Omega_q - \Omega_p}{2}\right)\right)\end{aligned} \tag{4.85}$$

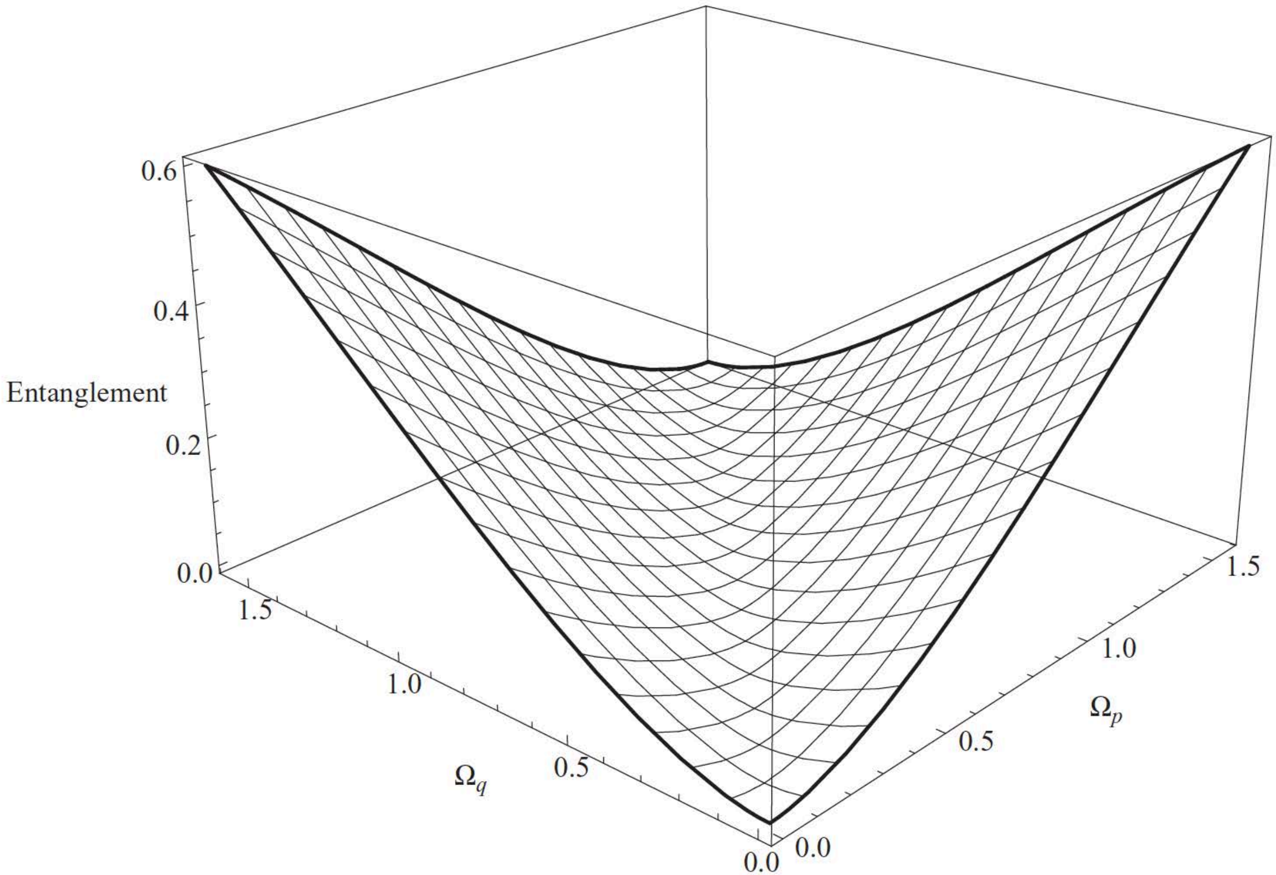


and therefore the two Schmidt coefficients can be simply expressed as:

$$\begin{aligned}\lambda_{+}^2 &= \cos^2\left(\frac{\Omega_q - \Omega_p}{4}\right)\\ \lambda_{-}^2 &= \sin^2\left(\frac{\Omega_q - \Omega_p}{4}\right)\end{aligned} \tag{4.86}$$

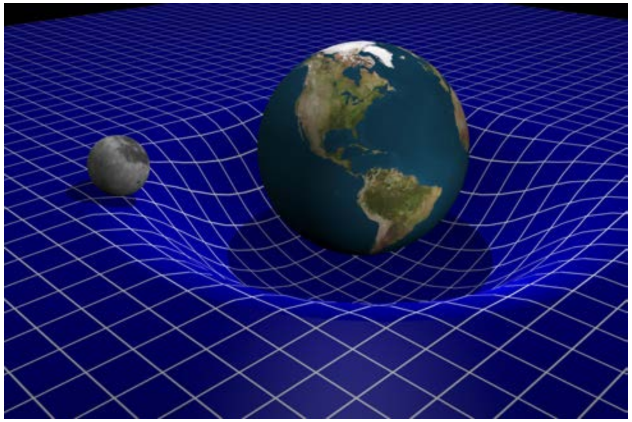
which leads to a level of entanglement given by equation (4.58). Thus, for non-definite momentum states, a Lorentz transformation could generate entangled states out of non-entangled states.

Gravitational Entanglement

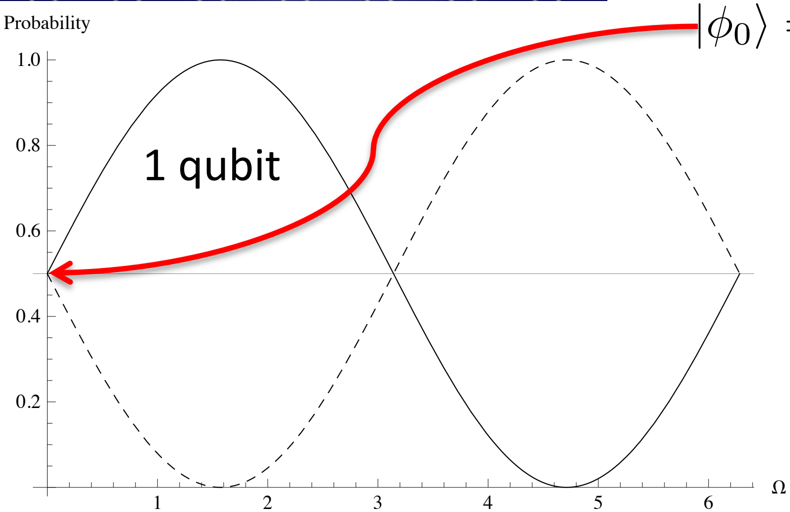


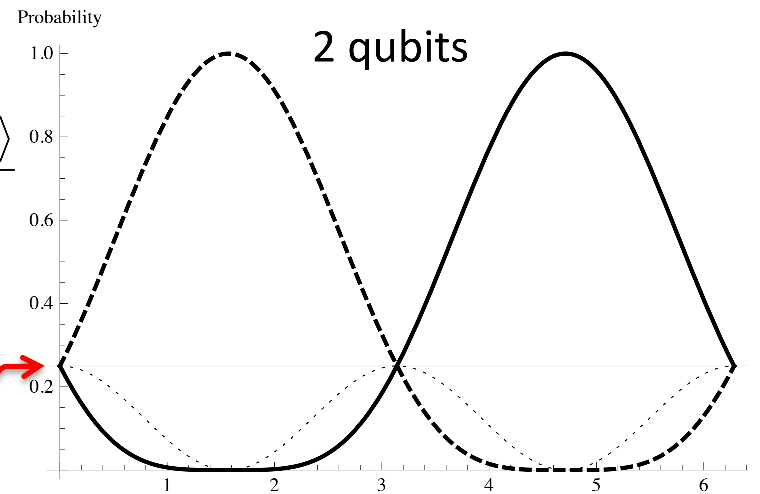
Depending on the initial state, gravity may produce entanglement!

Gravity Introduces Noise in Communication Systems



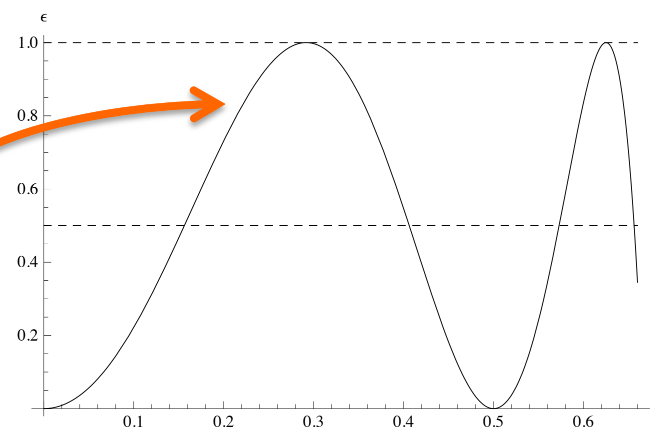
$$|\phi_0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$





$$|\psi_0\rangle = \frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{4}$$

The state of a qubit will drift due to its coupling to gravity

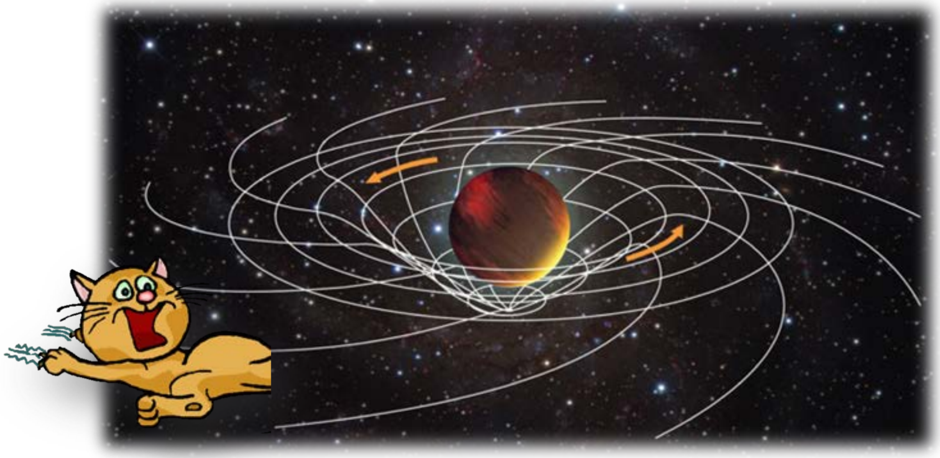


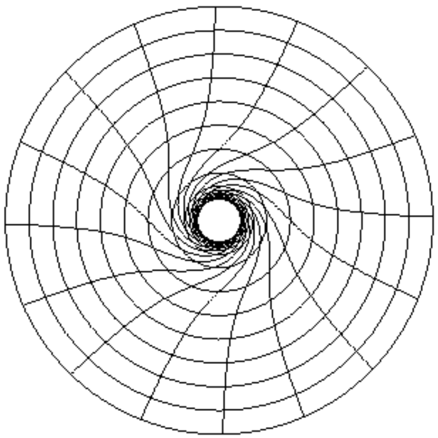
Gravity degrades the performance of communication devices

Earth's gravity is too weak to break standard send/receive QKD

$$\epsilon \approx 10^{-18}$$

Frame Dragging





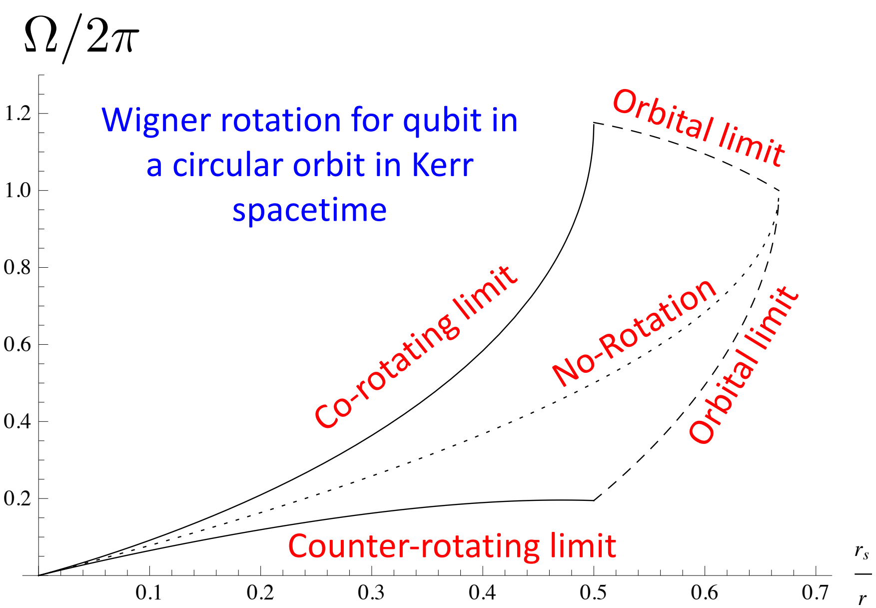
Kerr spacetime produced by a stationary, axisymmetric, rotating spherically symmetric massive body.

Rotating bodies *drag* the spacetime as they rotate.

Predicted by Einstein's General Relativity, but not yet satisfactorily observed.

How can we use quantum information to detect frame dragging?

Would this approach be better than current unconvincing data (e.g. Gravity Probe B's unreliable 19% error)?



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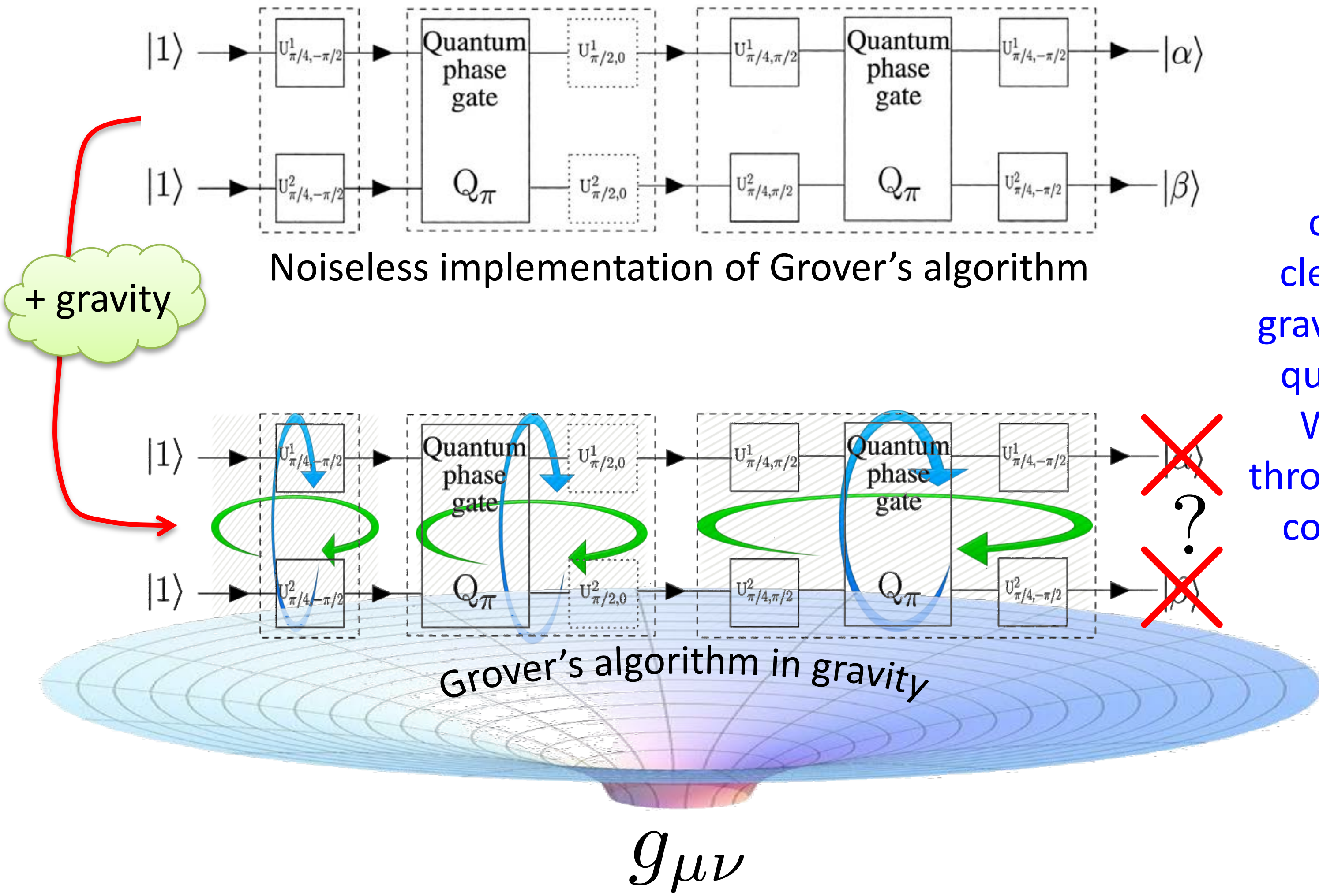
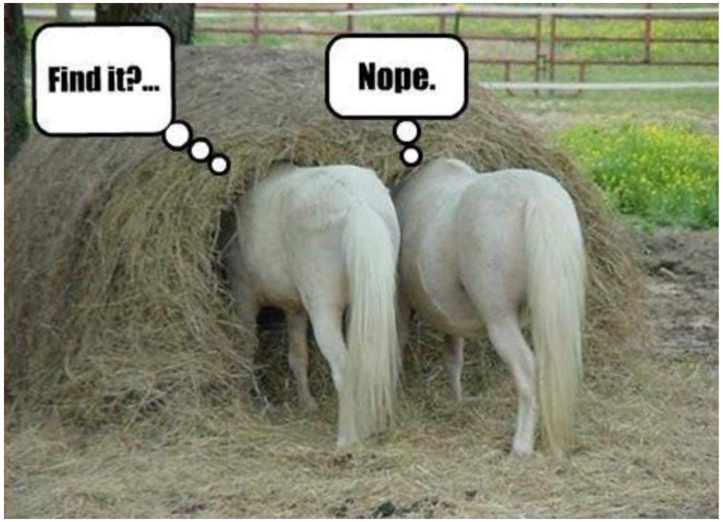
Quantum Computation in Gravity



Quantum computation is clearly affected by gravity because each qubit undergoes a Wigner rotation throughout the entire computation time



Grover's Algorithm

- Amplitude amplification quantum algorithm developed by Lov Grover to perform a search of an item from an unsorted and unstructured list of *N* records.
  - Performs optimal search in $O(\sqrt{N})$
  - Instead of the *O(N)* required by brute force methods in classical computing.
  - Example: reverse phone pages problem with 10,000 entries. Classically, we may require up to 10,000 time steps. But Grover's algorithm gives the right answer with high probability in about 100 time steps.

Grover's Algorithm (1)

1. Initialize the qubit sensors to the zero state:

$$|\Psi\rangle = |000...00\rangle$$

2. Transform to the uniform superposition:

$$|\Psi\rangle \longrightarrow \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |i\rangle$$

3. Apply the quantum oracle:

$$|\Psi\rangle \longrightarrow \hat{O}|\Psi\rangle$$

4. Apply the Inverse Around the Mean operator:

$$\hat{O}|\Psi\rangle \longrightarrow \hat{D}\hat{O}|\Psi\rangle$$

Grover's Algorithm (2)

5. Apply the Grover operator $O(N^{1/2})$ times:

$$\left(\hat{D}\hat{O}\right)^{\sqrt{N}}|\Psi\rangle$$

6. The resulting state will give the searched item with high probability

Grover's Algorithm in Gravity

5. Apply the Grover operator $O(N^{1/2})$ times:

$$\left(\hat{D}\hat{R}\hat{O}\hat{R}\right)^{\sqrt{N}}|\Psi\rangle$$

Wigner Rotations

6. The resulting state **may not** give the searched item with high probability

2 Qubit Grover's Probabilities

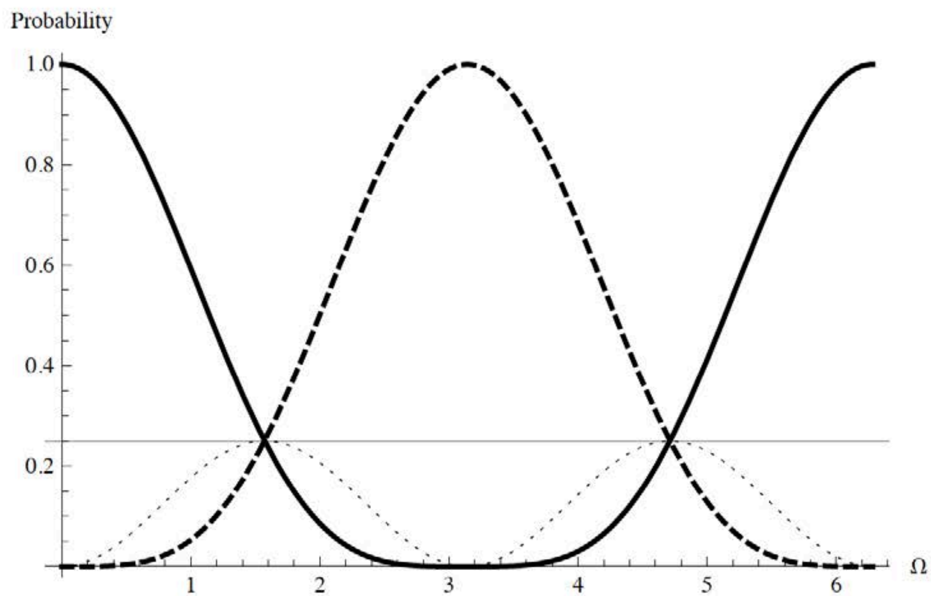


Fig. 1.6 Probability of measuring "00" (solid line), "01" and "10" (dotted line), and "11" (dashed line) for a 2 qubit state after a single iteration of the Grover operator in the presence of a gravitational field described by the Wigner rotation angle $\Omega(t)$.

3 Qubit Grover Probabilities

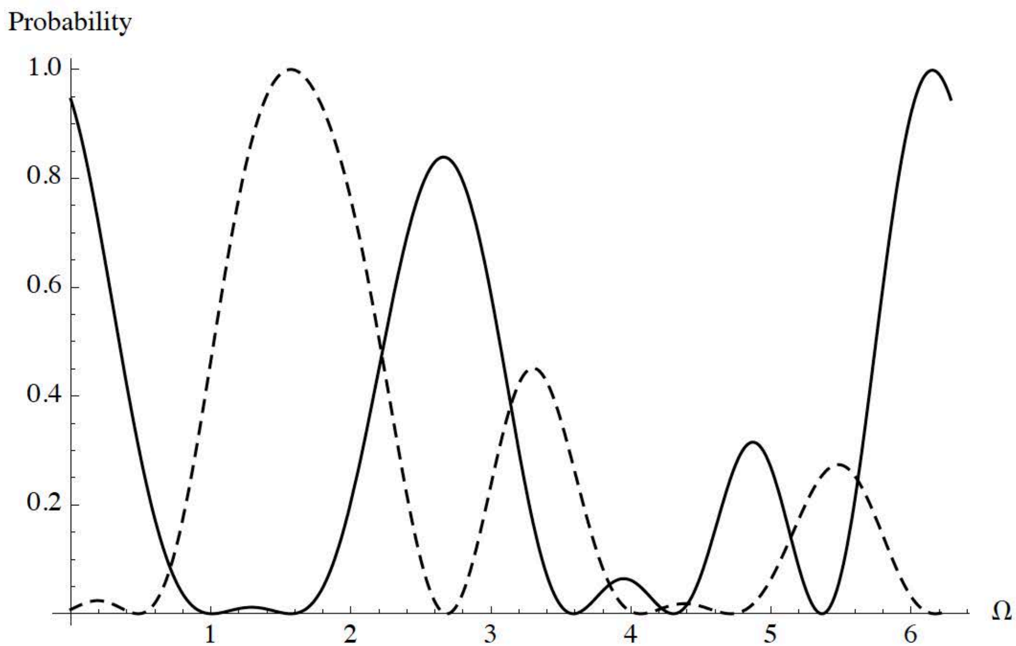


Fig. 1.7 Probability of measuring "000" (solid line) and "111" (dashed line) for a 3 qubit state after two iterations of the Grover operator in the presence of a gravitational field described by the Wigner rotation angle $\Omega(t)$.

General Iterative Quantum Algorithm

Consider now a quantum computer running an algorithm that iteratively applies a quantum operation to an initial n-qubit quantum state represented by the density matrix $\rho^{(0)}$. We can examine the result of this computation in the noiseless (i.e., flat spacetime) and weak-field regimes.

Suppose the quantum algorithm performs m sequential applications of a single multi-qubit gate $\hat{U}$. The system has n qubits that represent $N = 2^n$ possible states. In the absence of gravity, and ignoring all possible sources of noise and error, the state's density matrix after the first computational step is:

$$\rho^{(1)} = \hat{U}\rho^{(0)}\hat{U}^{\dagger} \tag{1.27}$$

and the state after m iterations is given by:

$$\rho^{(m)} = \hat{U}^{m}\rho^{(0)}\hat{U}^{\dagger m} \tag{1.28}$$

The Effect of Gravity

By contrast, the state after one iteration in the presence of a weak gravitational field is:

$$\rho^{(1)} = \hat{D}\hat{U}\rho^{(0)}\hat{U}^\dagger\hat{D}^\dagger \tag{1.29}$$

where $\hat{D}$ is the unitary operation that represents the n-qubit Wigner rotation between computational steps. After two iterations the state will be:

$$\rho^{(2)} = \hat{D}\hat{U}\hat{D}\hat{U}\rho^{(0)}\hat{U}^\dagger\hat{D}^\dagger\hat{U}^\dagger\hat{D}^\dagger \tag{1.30}$$

and so on. In this case the Wigner rotation can be written as:

$$\hat{D} = \bigotimes_{j=1}^{n} e^{i\hat{\mathbf{\Sigma}}_j\cdot\mathbf{\Omega}_j/2} \tag{1.31}$$

where $\mathbf{\Omega}_j$ is the vector of Wigner rotation angles for the j^{th} qubit and $\hat{\mathbf{\Sigma}}_j$ is the vector of spin rotation generators that acts on the j^{th} qubit. This expression reflects the fact that, in general, qubits will occupy different positions in spacetime, their spins may be aligned in different directions, and the quantization axes may be different for each of them. To simplify the analysis we assume not only that the gravitational field is weak but also that all qubits undergo the exact same Wigner rotation Ω such that:

$$|\Omega_j - \Omega| \ll 1 \quad \forall j \tag{1.32}$$

where in the weak field limit $\Omega \ll 1$.

Weak Field Limit

In the weak field limit, the qubit rotation operator $\hat{D}$ can be expressed as:

$$\hat{D} \approx \mathbb{I} + \frac{i\Omega}{2}\sum_{j=1}^{n}\hat{\Sigma}_j + \mathcal{O}(\Omega^2) \tag{1.33}$$

where $\hat{\Sigma}_j$ is a unitary operator that acts in a non-trivial manner only on the j^{th}-qubit:

$$\hat{\Sigma}_j = \underbrace{\mathbb{I}\otimes\mathbb{I}\otimes\ldots\otimes\mathbb{I}}_{j-1}\otimes\sigma\otimes\underbrace{\mathbb{I}\otimes\mathbb{I}\otimes\ldots\otimes\mathbb{I}}_{n-j} \tag{1.34}$$

where the under-brace denotes the number of single-qubit identity operators in the tensor product and σ is the spin rotation generator over a given axis.

The State After One Iteration

As a consequence, at order $\mathcal{O}(\Omega)$ the state after one iteration can approximately be written as:

$$\rho^{(1)} \approx \left(1 + \frac{i\Omega}{2}\sum_{j=1}^{n}\hat{\Sigma}_j\right)\hat{U}\rho^{(0)}\hat{U}^\dagger\left(1 - \frac{i\Omega}{2}\sum_{k=1}^{n}\hat{\Sigma}_k^\dagger\right) + \mathcal{O}(\Omega^2) \quad (1.35)$$

$$\approx \hat{U}\rho^{(0)}\hat{U}^\dagger + \frac{i\Omega}{2}\sum_{j=1}^{n}\left(\hat{\Sigma}_j\hat{U}\rho^{(0)}\hat{U}^\dagger - \hat{U}\rho^{(0)}\hat{U}^\dagger\hat{\Sigma}_j^\dagger\right) + \mathcal{O}(\Omega^2)$$

Defining Ξ as:

$$\Xi(\rho) \equiv \frac{i}{2}\sum_{j=1}^{n}\left(\hat{\Sigma}_j\hat{U}\rho\hat{U}^\dagger - \hat{U}\rho\hat{U}^\dagger\hat{\Sigma}_j^\dagger\right) \quad (1.36)$$

$$= \frac{i}{2}\sum_{j=1}^{n}\left[\hat{\Sigma}_j, U\rho\hat{U}^\dagger\right]$$

allows $\rho^{(1)}$ at first order in Ω to be expressed as:

$$\rho^{(1)} \approx \hat{U}\rho^{(0)}\hat{U}^\dagger + \Omega\ \Xi(\rho^{(0)}) \quad (1.37)$$

The State After m Iterations

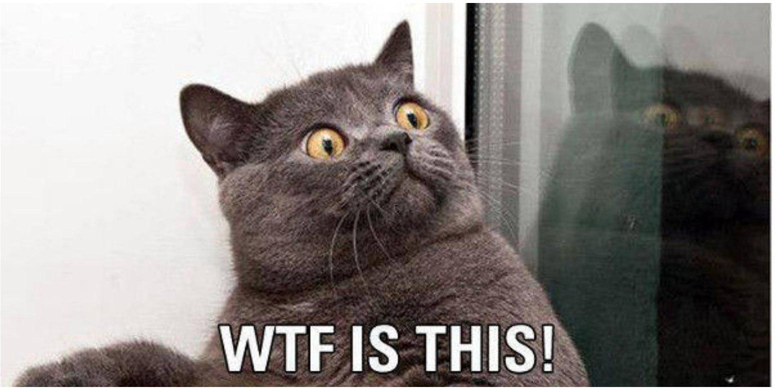
Thus, the second iteration is:

$$\begin{aligned}\rho^{(2)} &\approx \hat{U}\rho^{(1)}\hat{U}^\dagger + \Omega\ \Xi(\rho^{(1)}) \\ &\approx \hat{U}\hat{U}\rho^{(0)}\hat{U}^\dagger\hat{U}^\dagger + \Omega\hat{U}\ \Xi(\rho^{(0)})\hat{U}^\dagger + \Omega\ \Xi(\hat{U}\rho^{(0)}\hat{U}^\dagger) + \mathcal{O}\left(\Omega^2\right) \\ &\approx \hat{U}\hat{U}\rho^{(0)}\hat{U}^\dagger\hat{U}^\dagger + \Omega\left(\hat{U}\ \Xi(\rho^{(0)})\hat{U}^\dagger + \Xi(\hat{U}\rho^{(0)}\hat{U}^\dagger)\right) + \mathcal{O}\left(\Omega^2\right)\end{aligned} \tag{1.38}$$

and the state after m iterations will be of the form:

$$\rho^{(m)} \approx \left(\hat{U}\right)^m \rho^{(0)} \left(\hat{U}^\dagger\right)^m + \Omega\,\underbrace{(\ldots)}_{m} + \mathcal{O}(\Omega^2) \tag{1.39}$$

where the underbrace denotes how many n-sum terms Ξ are contained inside the parentheses.



The Effect on the Number of Iterations

If it is assumed/conjectured that the gravitational field of the operating environment can only be estimated with bounded accuracy then the probability of measuring the correct result after m iterations can be expressed as:

$$p \gtrsim (1 - n\epsilon)^m \approx 1 - nm\epsilon + \mathcal{O}(\epsilon^2) \tag{1.40}$$

where $\epsilon = \mathcal{O}(\Omega)$ and we have used a small limit approximation $n\epsilon \lesssim 1$. Under these conditions the error probability for the quantum algorithm is approximately bounded by:

$$e \lesssim 1 - (1 - n\epsilon)^m \tag{1.41}$$

In principle it would seem that the algorithm could be iterated to increase the probability of success to satisfy any desired threshold. For example, after k runs of the entire algorithm, the bound for the probability of error could be reduced to:

$$e^k \lesssim (1 - (1 - n\epsilon)^m)^k \tag{1.42}$$

and k is chosen such that:

$$e^k \approx \delta \tag{1.43}$$

where δ is the maximum error probability desired for the computational process. That is, k is given by:

$$k \lesssim \frac{\log \delta}{\log\left(1 - (1 - n\epsilon)^m\right)} \tag{1.44}$$

The True Complexity of the Algorithm

In the asymptotic limit for large m we have:

$$k \lesssim -\log \delta \times \left(\frac{1}{1-n\epsilon}\right)^m = \mathcal{O}\left(\left(\frac{1}{1-n\epsilon}\right)^m\right) \tag{1.45}$$

and the *true complexity* of the algorithm (i.e. the total number of iterations necessary to complete the computational task with error probability δ) is:

$$\mathcal{O}\left(m \times k\right) \approx \mathcal{O}\left(m \times \left(\frac{1}{1-n\epsilon}\right)^m\right). \tag{1.46}$$

Thus, as the number of qubits n grows - approaching the value $1/\epsilon$ - the algorithmic complexity becomes exponential in the number of iterations m. In other words, the presence of gravity increases the computational dependence on m from linear to exponential. This motivates an examination of the specific consequences of this for practical n-quibit applications of Grover's algorithm and Shor's algorithm.

Grover's Algorithm Computational Complexity in Gravity

In light of the analysis of the previous section, uncorrected gravitation-induced errors increase the complexity of Grover's algorithm to search an N-element database from $\mathcal{O}(\sqrt{N})$ to:

$$\mathcal{O}\left(\sqrt{N} \times \left(\frac{1}{1 - n\epsilon}\right)^{\sqrt{N}}\right) \tag{1.47}$$

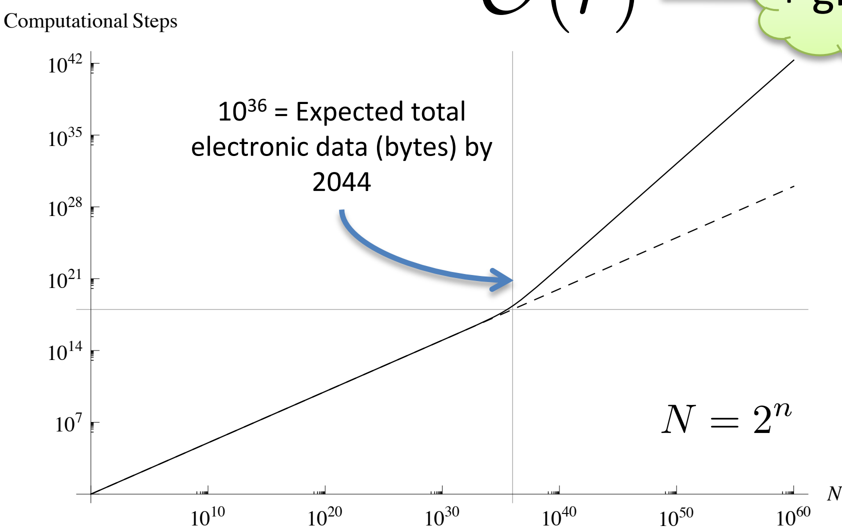
which is exponential in N for $n = \log N$ qubits.

QC in Earth's Gravity

Therefore, if the quantum processor operates at 1 Hz on the surface of Earth, then $\epsilon \approx \Omega \approx 10^{-14}$, which implies that Grover's algorithm could outperform classical brute force for datasets of size up to $N \approx 2^{80}$. This breakeven analysis is of course very crude, but it does suggest the possibility of a practical performance advantage of Grover's algorithm over classical despite what is implied by our asymptotic analysis. On the other hand, a slightly more complete and/or detailed analysis may yield an opposite conclusion.

Notice that even current laptops may become useless in the presence of Earth's gravitational field…

Gravity's Effect on Computational Complexity



$$\mathcal{O}(r) \xrightarrow{+\text{ gravity}} \mathcal{O}\left(r \times \left(\frac{1}{1-n\epsilon}\right)^{r}\right)$$

This is a formal complexity theory result (i.e. in the large asymptotic limit)

Assumes a simple error model in an iterative algorithm

$r \longrightarrow$ Number of iterations w/o gravity

$n \longrightarrow$ Number of qubits $\quad n\epsilon \lesssim 1$

$\epsilon \longrightarrow$ Error due to gravity $\approx \Omega$

$\Rightarrow \Omega \approx 10^{-14}/s$ on Earth's surface

Grover's Algorithm: $r = \sqrt{N}$

Assume 1 MHz QC, all running constants = 1 (note: this is **not** a formal result, as such example is outside the scope of complexity theory!)

True scaling due to gravitation

Gravity free scaling

What about other types of quantum algorithms?

Can we design fault tolerant quantum computers in gravity?

Shor's Algorithm

For completeness we now consider the other major quantum algorithm, Shor's algorithm, which can factorize an n-bit co-prime number using $\mathcal{O}(\log N)$ quantum computational steps with probability close to one [Nielsen and Chuang (2000)]. In the presence of gravity, however, the complexity becomes:

$$\mathcal{O}\left(\log N \times \left(\frac{1}{1-\epsilon \log N}\right)^{\log N}\right) \tag{1.52}$$

or, equivalently, in terms of number of qubits:

$$\mathcal{O}\left(n \times \left(\frac{1}{1-n\epsilon}\right)^{n}\right) \tag{1.53}$$

which is exponential in n.

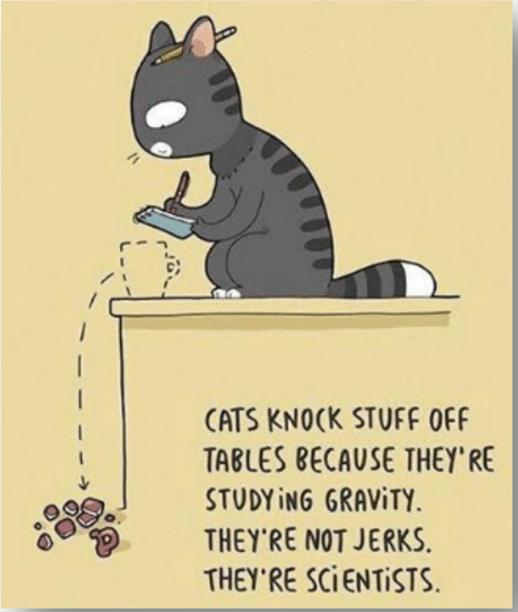
Clearly, Shor's algorithm is more resilient to the effects of gravitation than Grover's algorithm. The reason is that Shor's algorithm has linear dependency on the number of qubits while the dependency of Grover's algorithm is exponential.

The Effect of Gravity

At this point it is clear that the presence of a gravitational field affects the probability of measuring the correct final state of the system. The exact deviation from the expected performance of the quantum algorithm will depend in a non-trivial manner on the type and number of quantum gates, data qubits, ancillary qubits, and error correction encoding used to implement the computation. While it is notationally straightforward to represent all gravitational effects in the form of a single unitary transformation, the actual identification of that transformation at each computational step may not be feasible.

Gravitational noise produces correlated errors which cannot be removed by quantum error correction codes and fault tolerance is not guaranteed.

Quantum error correction and fault tolerant quantum computation may not be possible in the presence of gravity.



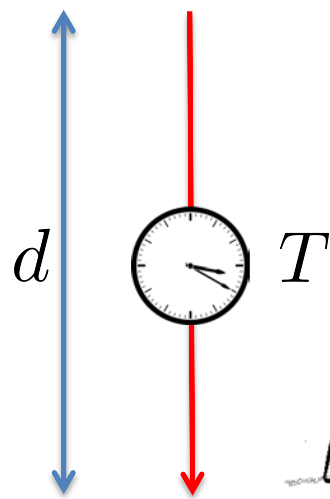
Outline

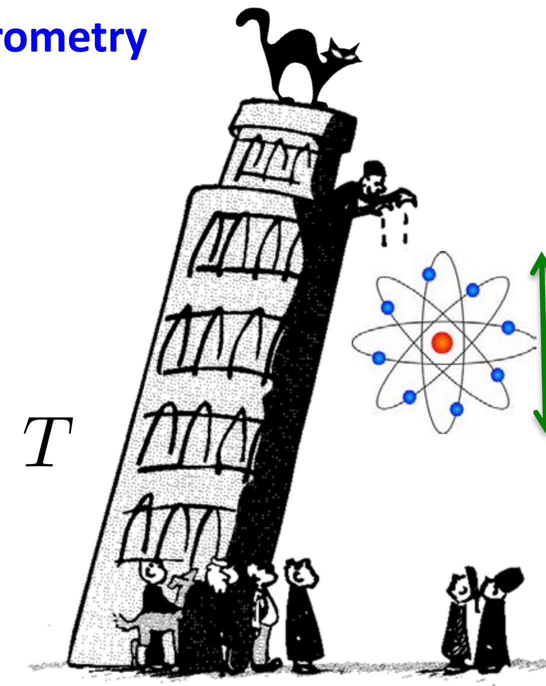
- Qubits in Gravity
- Quantum Computing in Gravity
- **Quantum Gravimetry**
- Conclusions

Current Approach to Gravimetry

Best known approach: **atom interferometry**

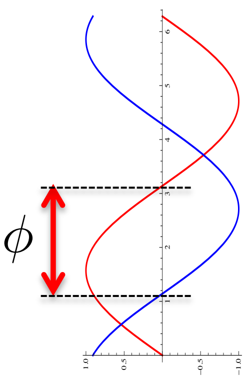
Circa 1589





$$\lambda = \frac{h}{mv}$$

$\Delta\phi$



Circa 2005

$$d = \frac{gT^2}{2}$$

Same idea, new tools

$$\Delta\phi \approx \frac{gT^2}{\lambda}$$

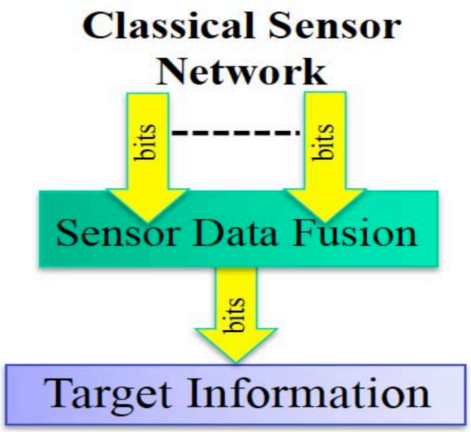
$$\frac{\delta g}{g} \approx 10^{-9}$$

But performance is restricted by the standard quantum limit and the de Broglie's wavelength of the atom

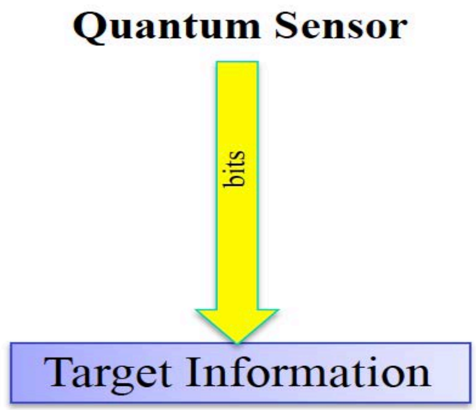
This technique does **not** use the Wigner angle

State-of-the-Art

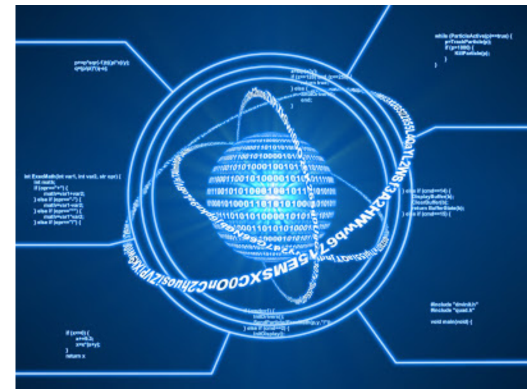
Data Fusion



Quantum Sensing



Quantum Computing



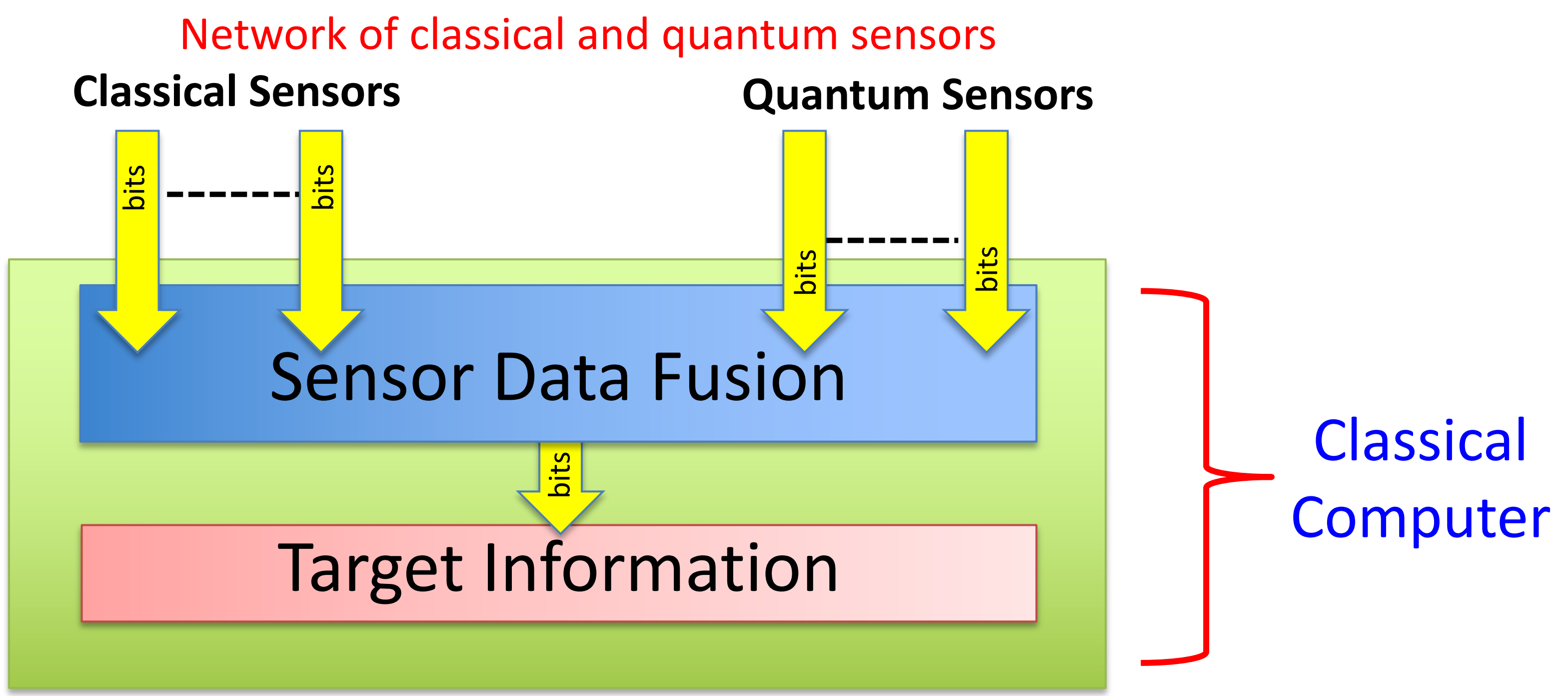
Active areas of research, but not much interaction between them

Sensor Data Fusion: how to combine the sensor data from each node in the network to provide the most accurate, complete, timely, and dependable target information available

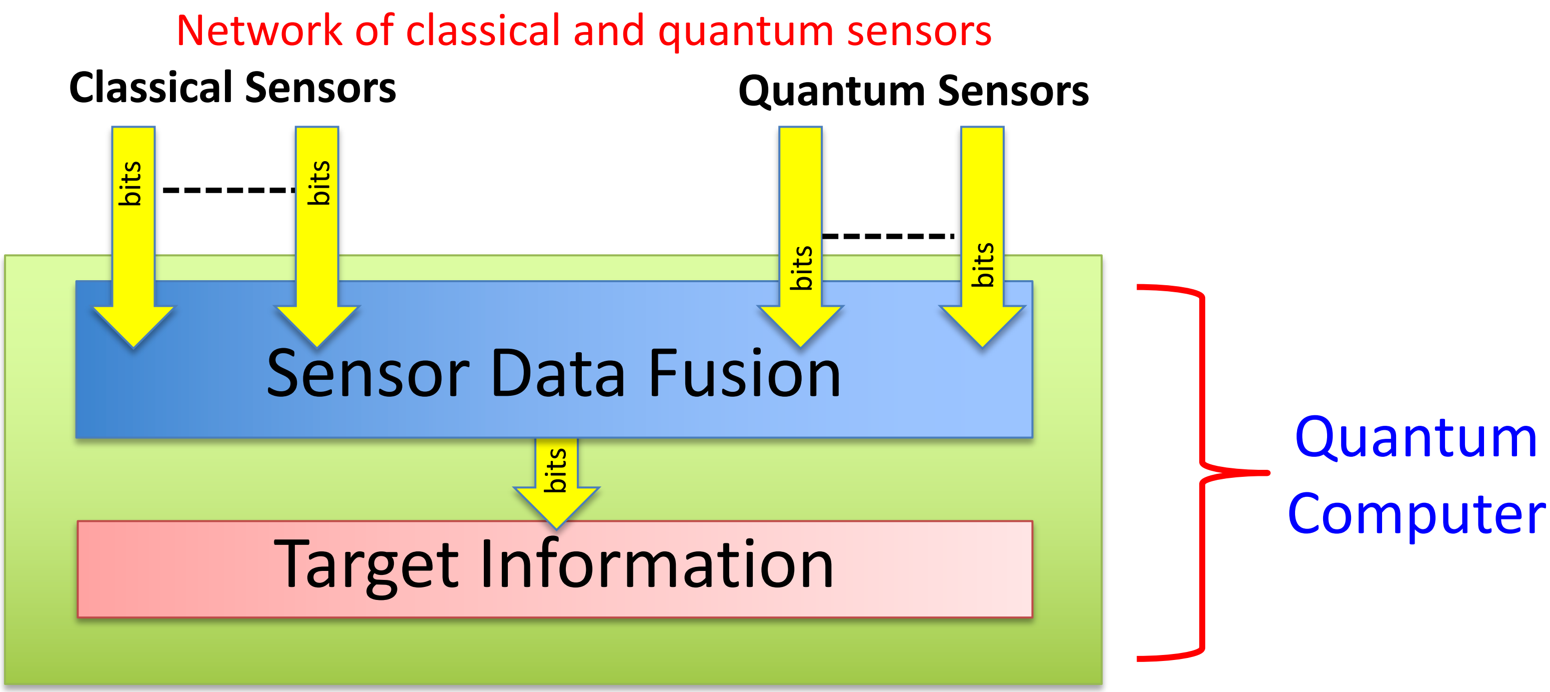
Quantum Sensing: how to harness quantum phenomena to improve the performance of sensing devices

Quantum Computing: how to harness quantum phenomena to improve the performance of information processing systems

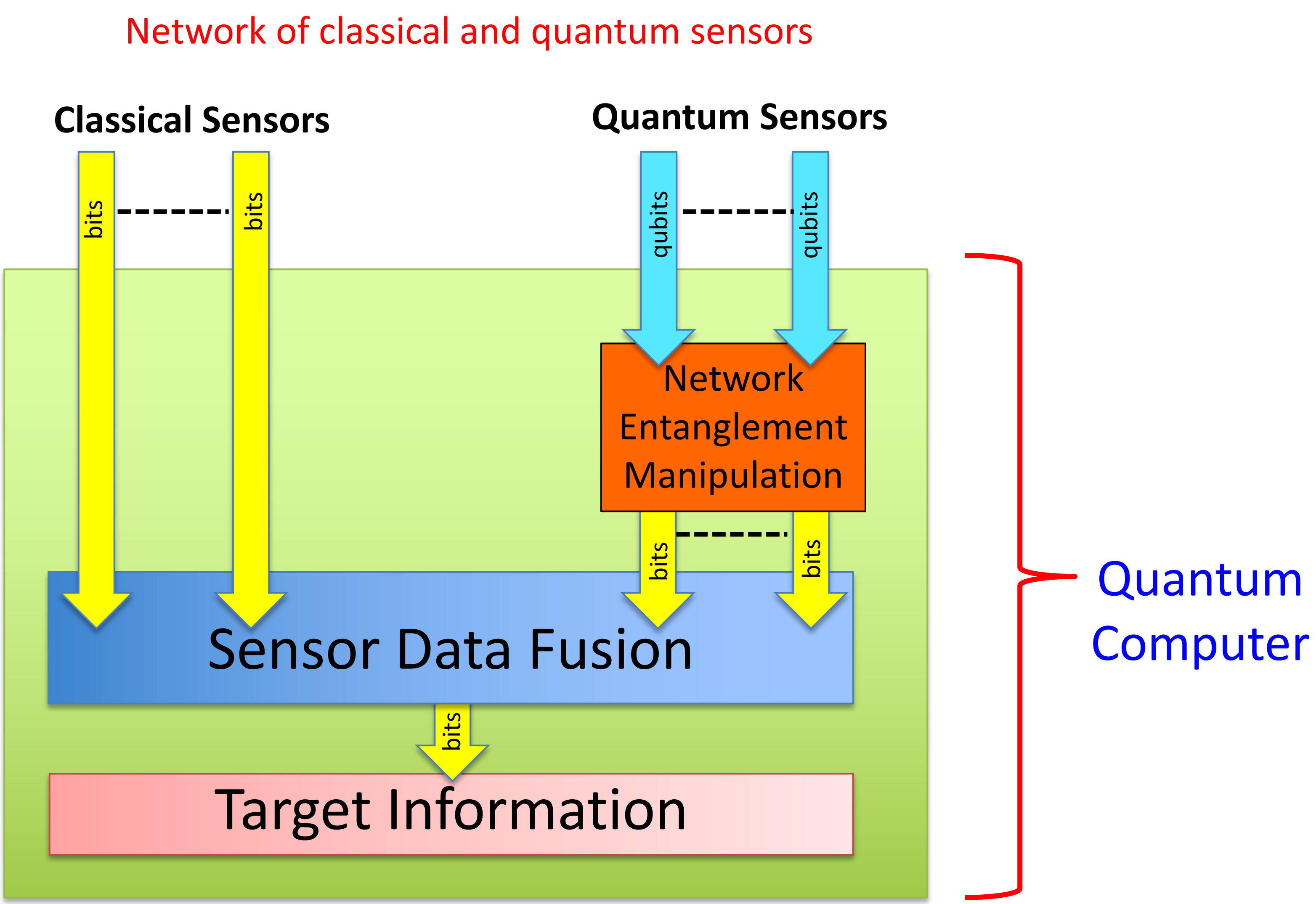
Classical Data Fusion



Quantum (Data Fusion)

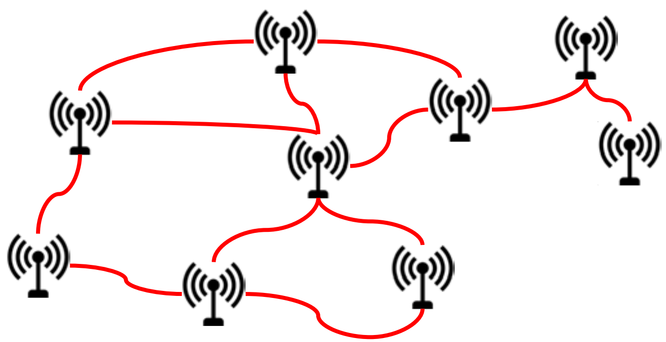


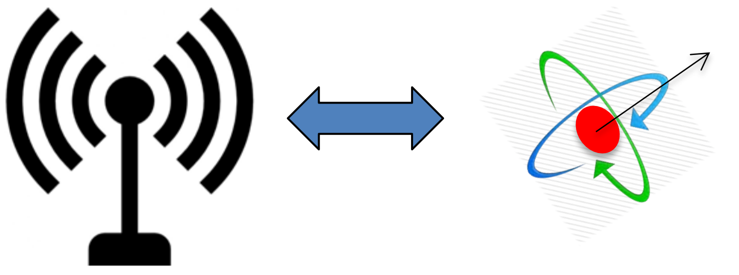
(Quantum Data) Fusion



(Quantum Data) Fusion

Assume a network of quantum sensors, where the physical response of each sensor is represented by a qubit state (e.g., gravity and EM fields).





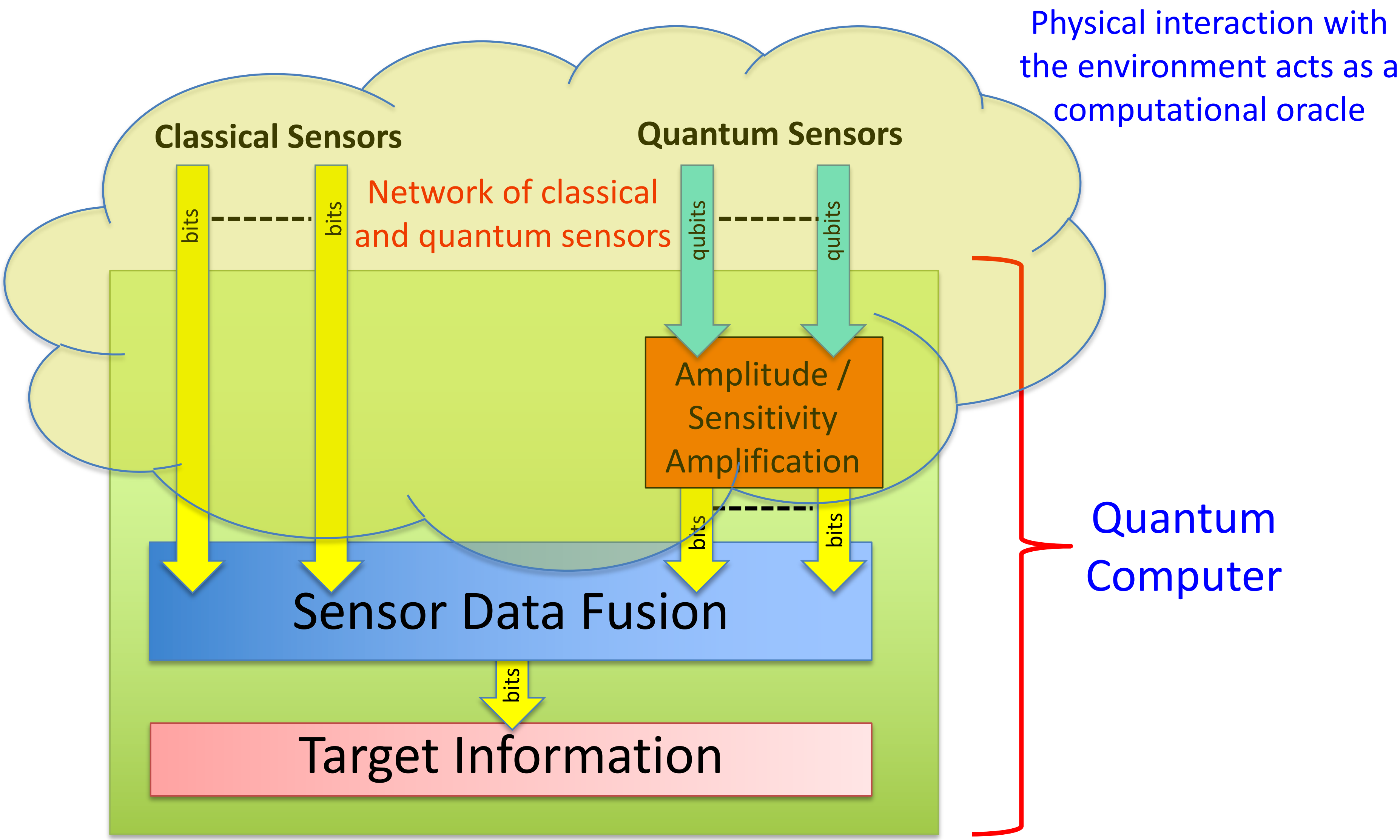
If each sensor performs a measurement and reports the result to a central node, then the simplest data fusion algorithm is the average of all measurements.

From a sensor network perspective, there is no quantum advantage whatsoever, even though each sensor may be better than its classical counterpart.

Radical idea: treat the quantum sensor state as an evolving qubit in a quantum computer, and perform amplitude amplification algorithms before measurements.

Non-local operations manipulate entanglement in the sensor network to amplify the response of the overall network: The network becomes a quantum computer where the physical interaction acts as a computational oracle.

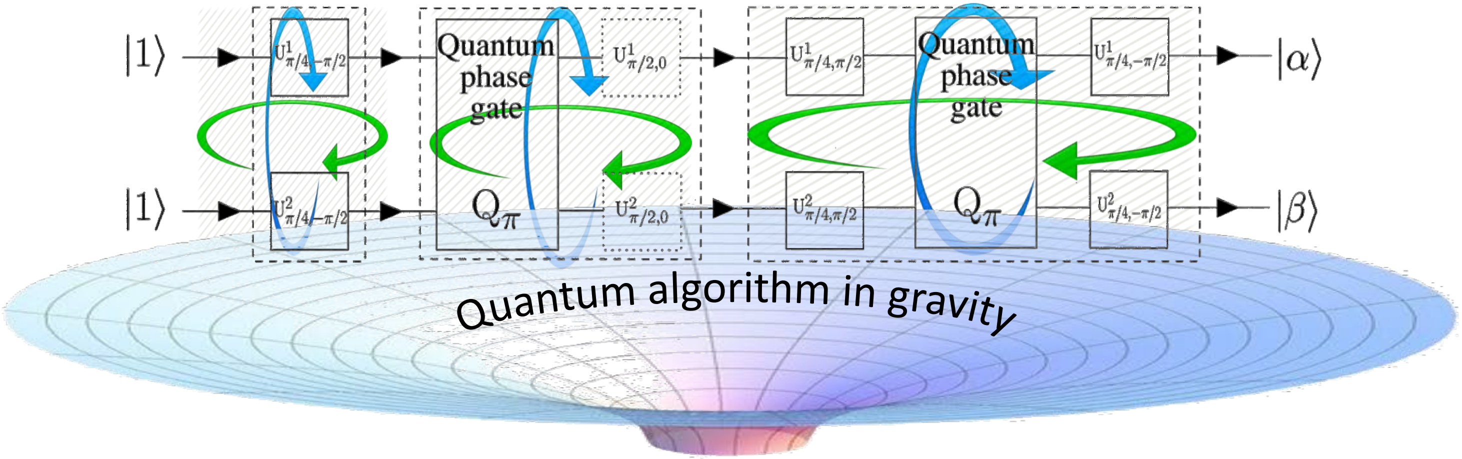
(Quantum Data) Fusion



Wigner Gravimetry

A quantum computer is an array of quantum gyroscopes affected by gravity which can be individually manipulated

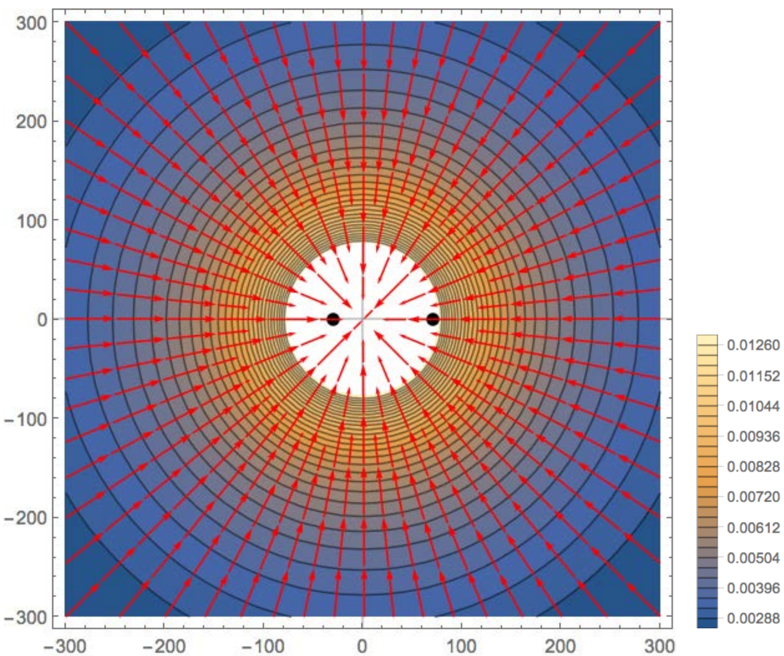
Thus, we use a quantum computer that performs a quantum amplitude amplification algorithm on a quantum state affected by the gravitational field. In a sense, the gravitational field acts as an oracle.

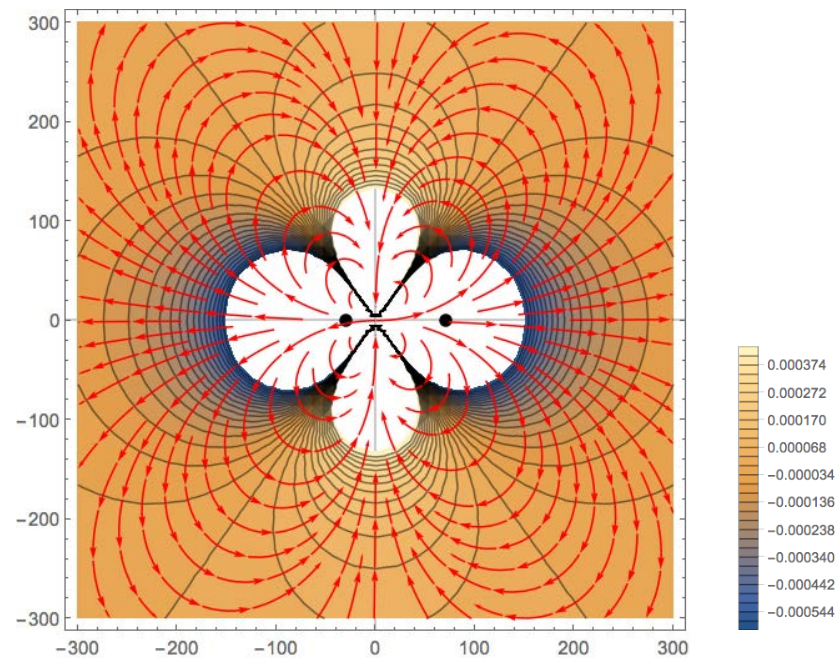


$g_{\mu\nu}$

Wigner Gravimetry: use a universal quantum computer to measure the gravitational field.

Monopole and Quadrupole Fields

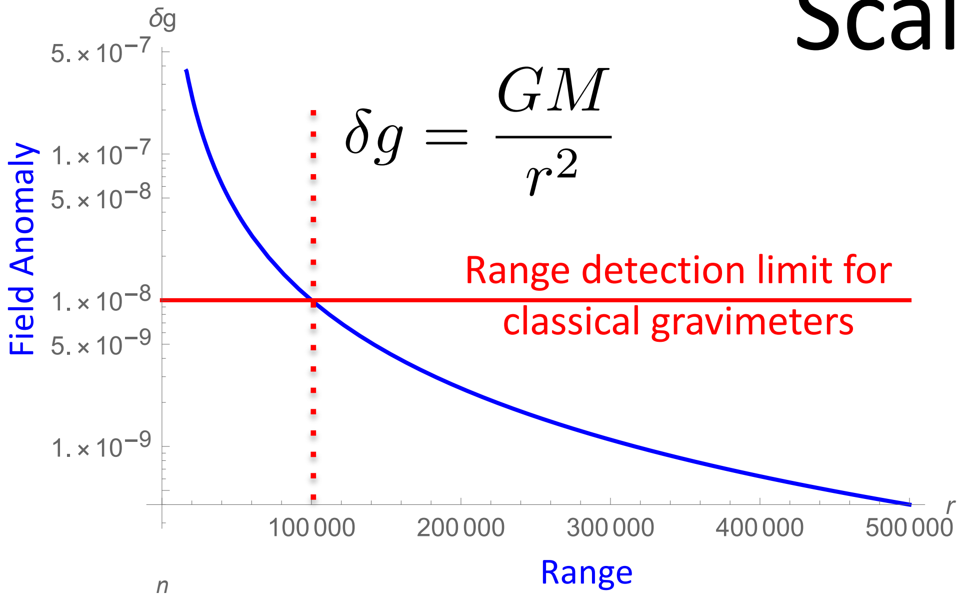




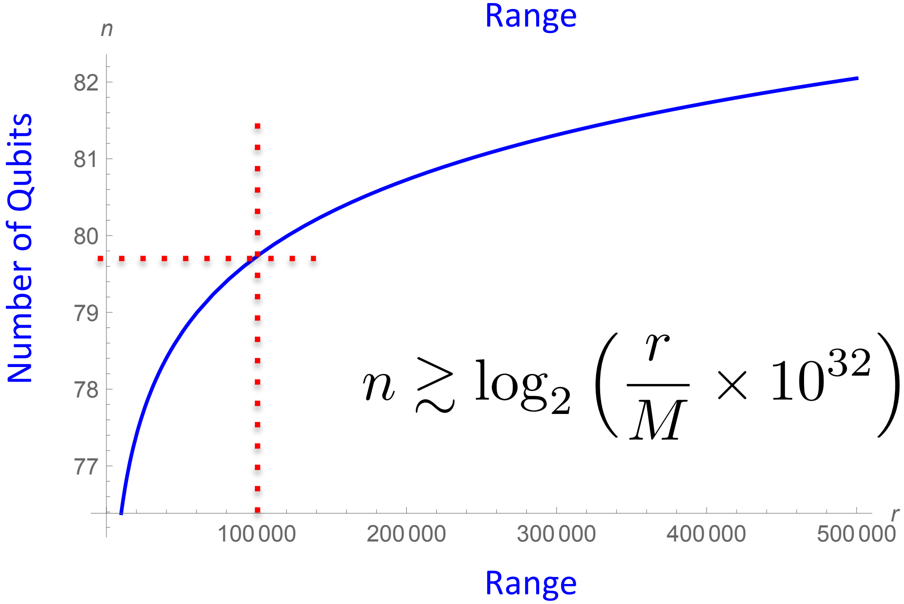
$$\Phi_M(r) = G\ \frac{m_1 + m_2}{r}$$

$$\Phi_Q(r,\theta) = G\ \frac{L^2}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)\frac{1 - 3\cos^2\theta}{r^3}$$

Quantum Monopole Gravimetry Scaling



$$\delta g = \frac{GM}{r^2}$$



$$n \gtrsim \log_2\left(\frac{r}{M} \times 10^{32}\right)$$

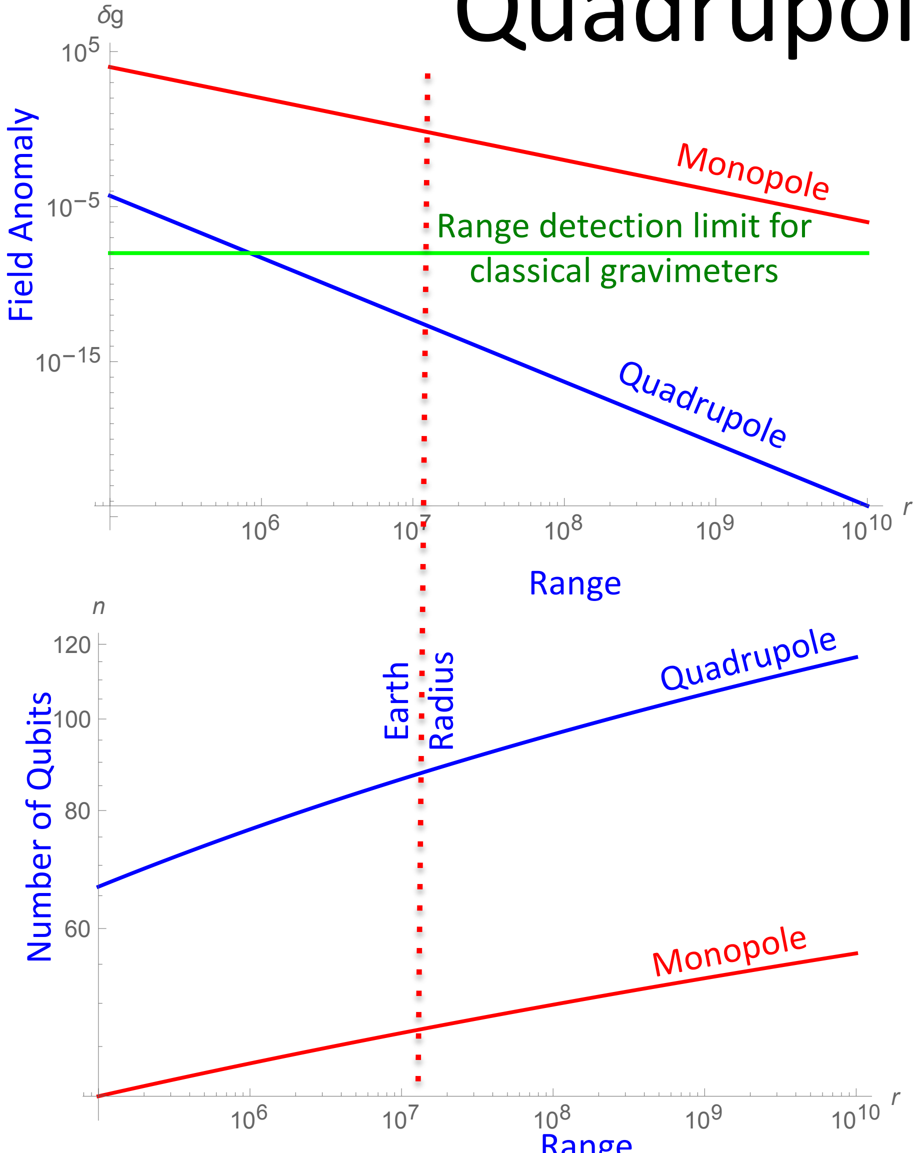
Example: Mount Everest

$$M \approx 10^{13}\ kg$$

At the most, the traditional techniques could measure Mount Everest at about 100 km

With 80 qubits we get similar performance as with the traditional classical gravimeter

Earth-Everest Gravitational Quadrupole Detection



$\alpha \approx 100$

Traditional gravimetric techniques appear to be unable to detect the quadrupole moment outside the Earth surface, but this could be done with about at least 90 qubits

Outline

- Qubits in Gravity
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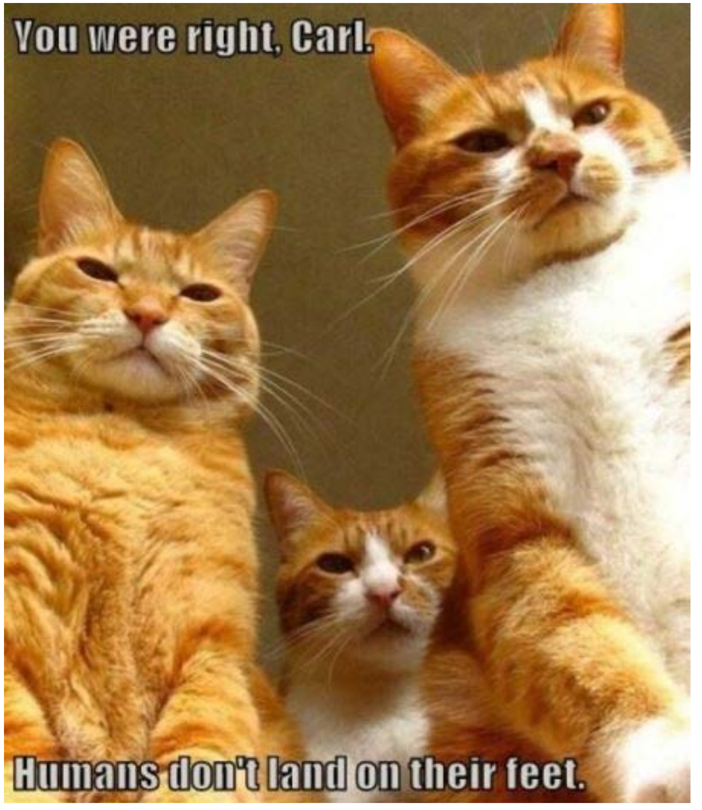
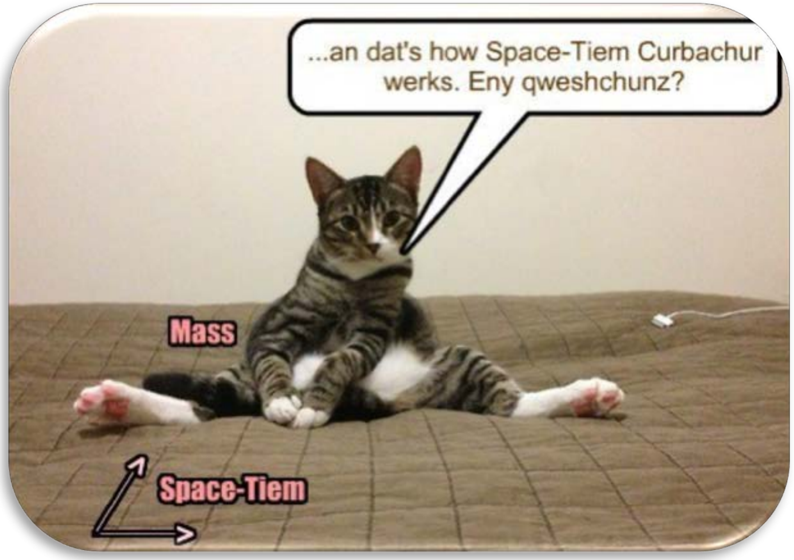
- **Conclusions**



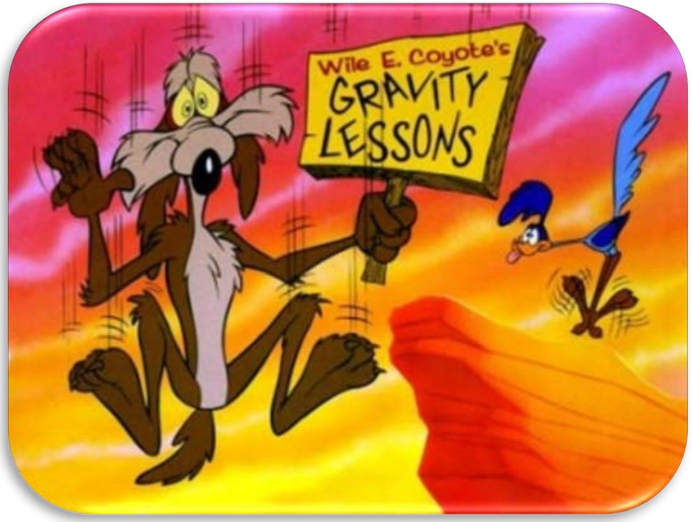
Conclusions

- Gravity affects quantum information systems.
- Formal result: computational complexity of quantum algorithms in a spin-based gate model is exponential because of gravity.
- The effect may be small in our planet's gravitational field, but it is cumulative and grows with the number of qubits...



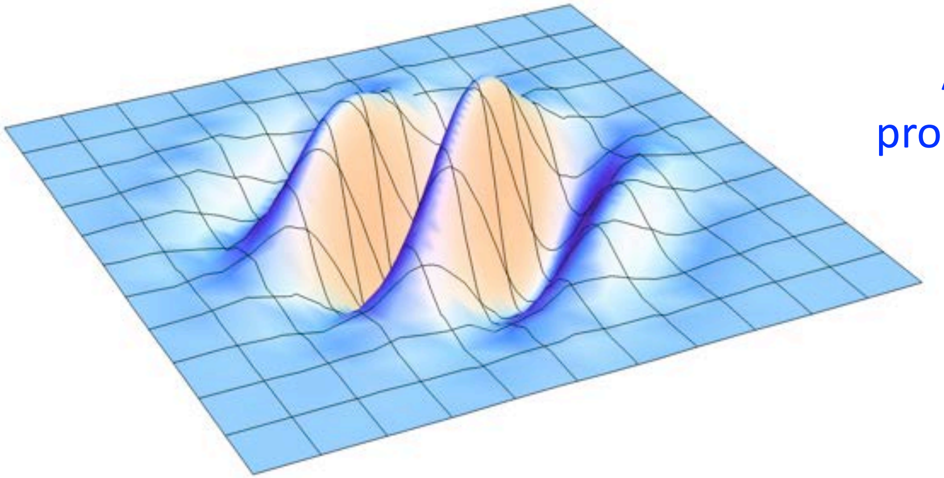
Current Challenges

- Our previous research presumed simple classical gravitational fields and oversimplified architectures for the quantum information processing systems (communication, computation and sensing).

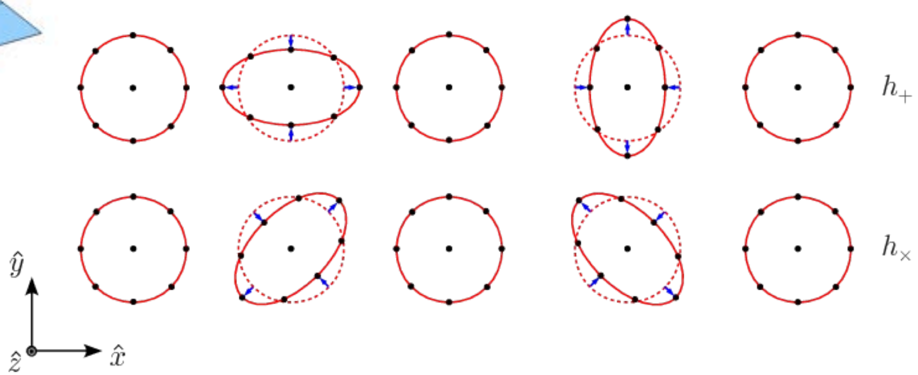


We need to improve our understanding of the effects of gravity on quantum information devices

Gravitational Communications



A gravitational wave is a propagating disturbance in the spacetime manifold.



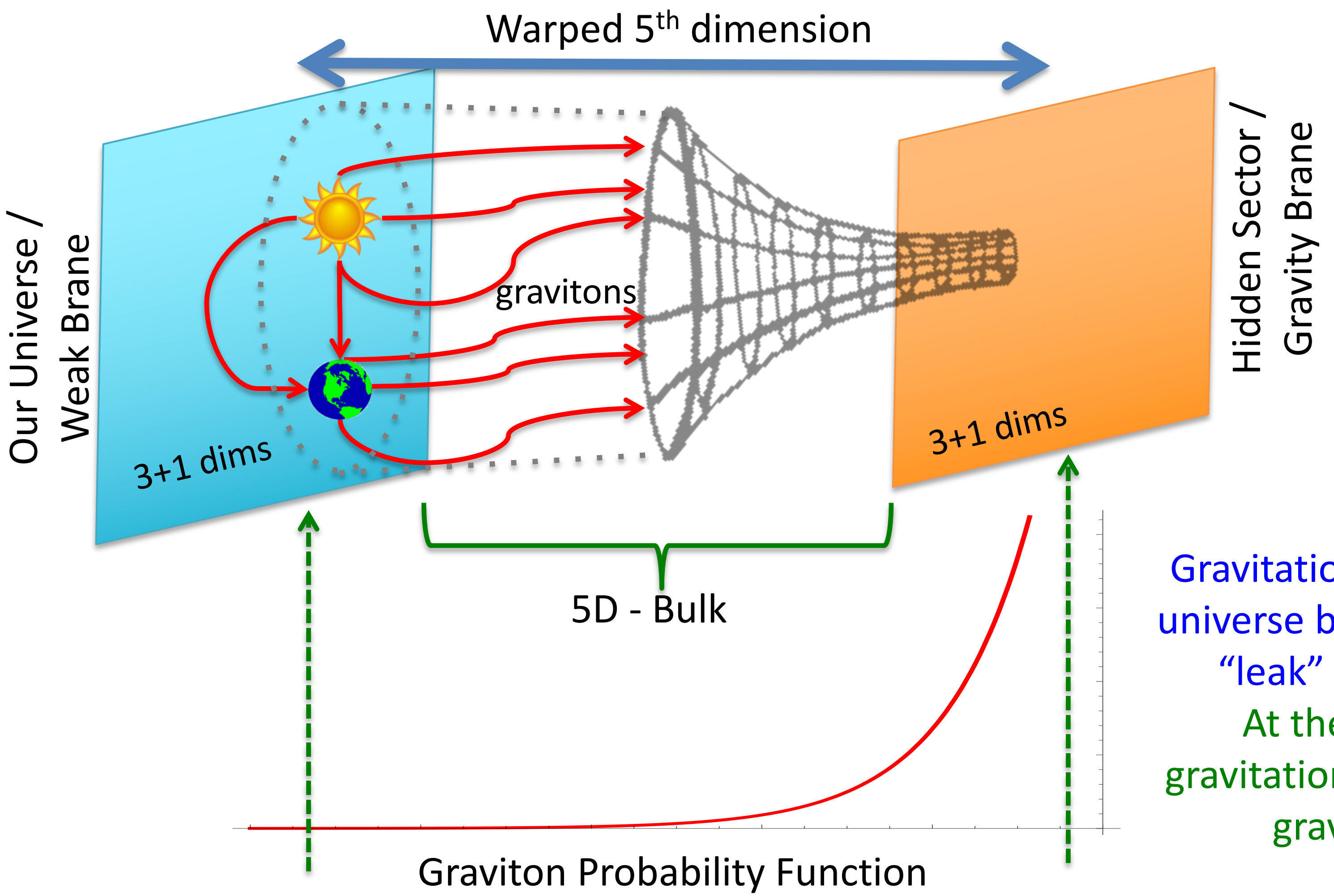
Gravitational waves affect the relative (Lorentzian) distance between test particles, even if they are firmly anchored in spacetime.

A cross-polarized (h_x) gravitational wave may induce Wigner rotations on qubits: we are currently studying the possibility of using quantum Wigner gravimeters to detect gravitational waves.

We are also studying the intriguing theoretical possibility of classical gravitational waves detected with compact quantum Wigner gravimeters for long range stealth communications…

Quantum Information in Quantum Gravitational Fields

If we subscribe to the Randall – Sundrum superstring quantum theory of gravity…

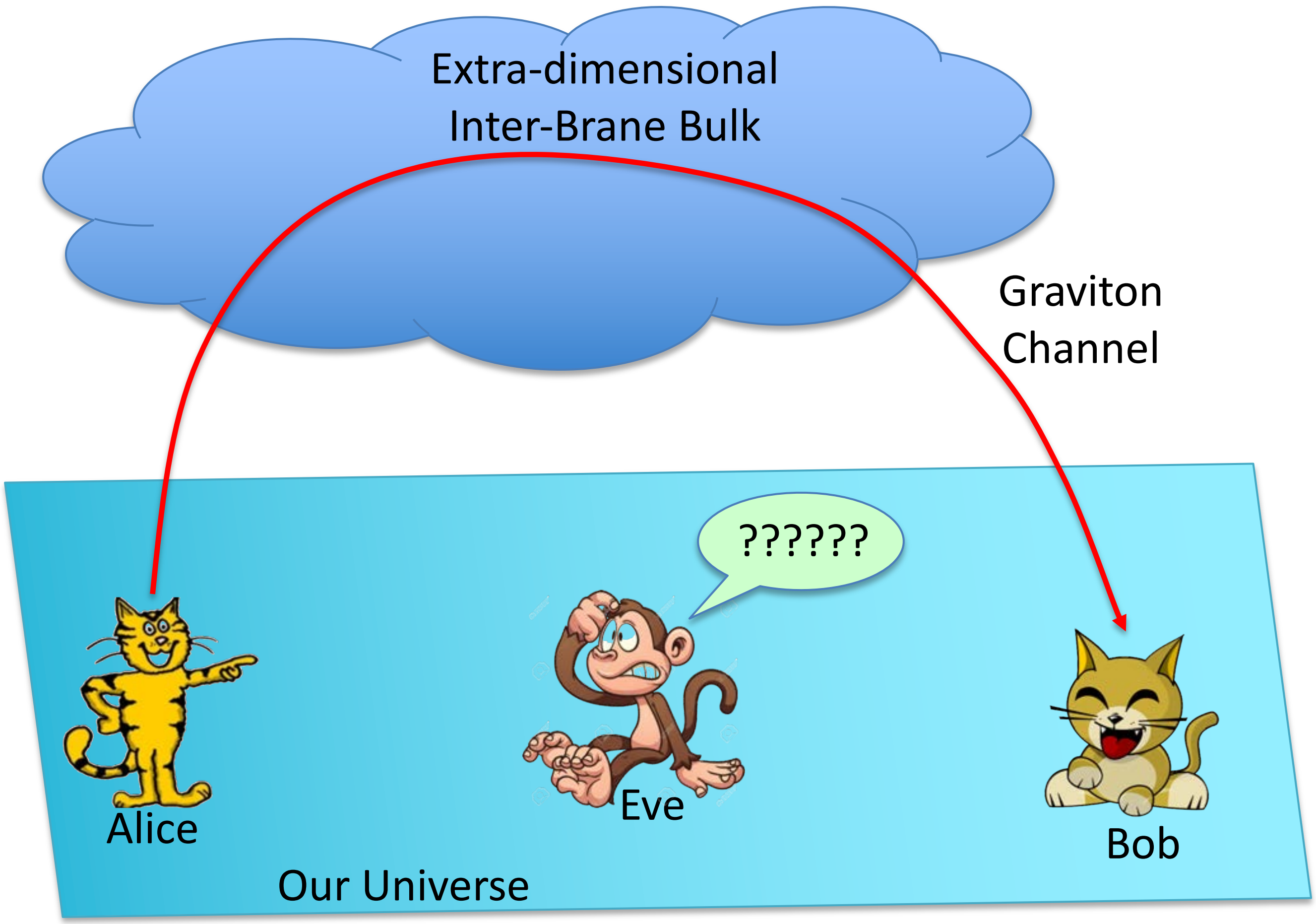


The graviton is the only particle not confined to our brane

Gravitation is weak in our universe because gravitons "leak" into the bulk.

At the other end, gravitation is strong in the gravity brane.

Graviton Channel In Extra Dimensions



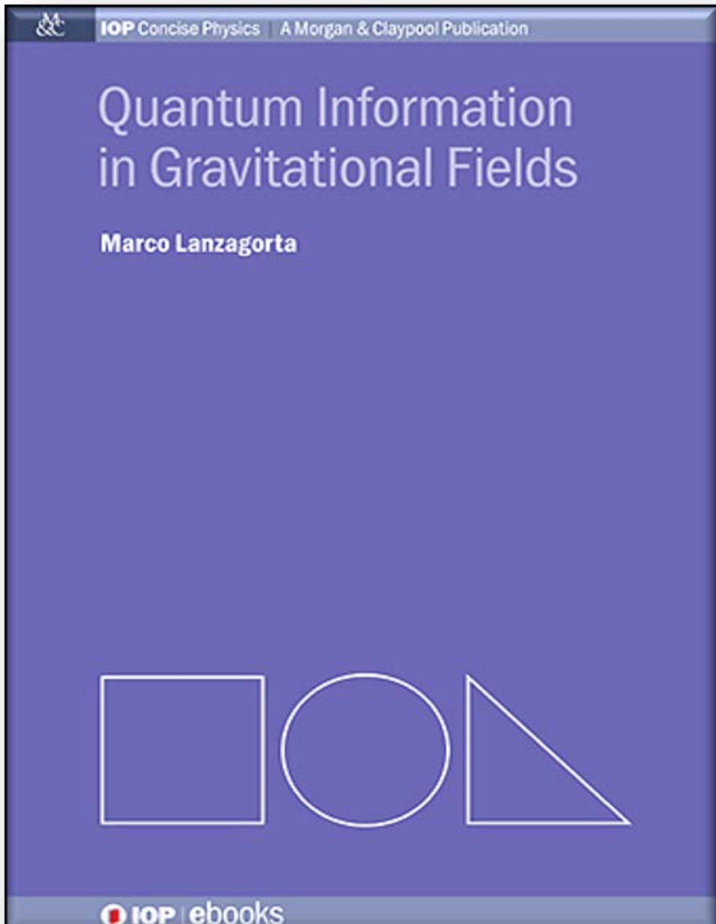
A secure communications concept for the 25th Century Navy

Quantum information is transmitted through the graviton channel across the bulk.

No information is passed through our 3+1 spacetime!

There is nothing for Eve to intercept!

Further Reading



And references therein…

I am done, just another silly cat meme that I like but didn't find a good place where to put it…

Thank You