Land Use Transitions Optimization Framework for SISEPUEDE

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Parameters and Indicies

Let:

- $n \in \mathbb{N}^+$ be the number of land use classes
- $t \in \mathbb{N}$ be the time period
- $x_t \in \mathbb{R}^n$ be the prevalence vector at time t. For convenience, this is sometimes shown as simply x. Furthermore, since x and q are combined in the objective function, prevalence vectors should be expressed as a stochastic vector, i.e., so that $\sum_i x_i = 1$ and $x_i \geq 0$. Since the area of a region is generally fixed–sea level rise can be represented through transitions to flooded states–expressing land use prevalence as a fraction is relatively straight-forward.
- $\hat{x}$ be the target prevalence vector. Depending on the costs r_i (see below), this vector may only value include legitimate target prevalence values for some classes.
- $Q(t) \in \mathbb{R}^{n \times n}$ be the exogenously specified row-stochastic transition matrix at time t.
- s_{ij} be the negative cost applied to transition probability deviations from i to j (in general, $s_{ij} \leq 0$)
- r_i be the negative cost applied to prevalence deviations for class i (in general, $r_i \leq 0$)

Variables

Let:

- $q_{ij}(t)$ be the adjusted transition matrix at time t. Since $q \in \mathbb{R}^{n \times n}$ a matrix, we use $q^{(j)}$ to represent column j and q_i to represent row i.
- $d(x, x)$ be a distance metric on $\mathbb{R}^n$

Problem

$$
\begin{gathered}
\textbf{maximize} \sum_{i,j} d(q_{ij}, Q_{ij}) s_{ij} + \sum_{j} d(x_t q^{(j)} - \hat{x}_j) r_j \\
\textbf{maximize} \sum_{i,j} d(q_{ij}, Q_{ij}) s_{ij} + \sum_{j} d(x_t q^{(j)} - \hat{x}_j) r_j
\end{gathered}
\tag{1}
$$