ELEC 4700-ASSIGNMENT-2

FINITE DIFFERENCE METHOD

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Purpose:

The purpose of this experiment is to use Laplace equation to solve electrostatic potential problems to see how accurate our model is.

Introduction:

This assignment comes in 2 parts where we will be using Finite difference method to solve for the electrostatic potential rectangular region in part 1 and current flow in rectangular region in part 2. MATLAB will be using for the study.

Experiment

Question 1.

a. In part a of question 1, a simple case where V= V_0 at x = 0 and V=0 at x = L was solved, plotted as shown in figure 1 bellow. This was treated as 1-D case.
b. In part b, V= V_0 at x=0, x=1 and V = 0 at y=0 and y=W was studied and plotted as a 2-D case as shown in figure 2 below.
   In comparison, the meshing can be seen close to the analytical solution around 100 iterations, there's no change in the graph higher than 100. The advantage of numerical solution is that it's easier for partial differential equation since it's almost same to the actual value for a complicated analytical problem but will take longer since it's simple meshing. A disadvantage to the numerical is the difficulty for a non-rectangular geometric and non-uniform meshing.

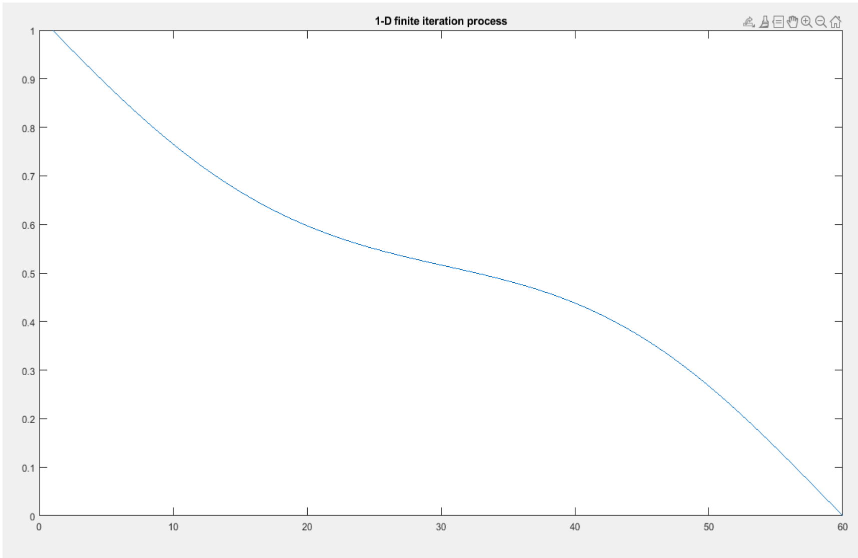

Figure 1: 1-D iteration process

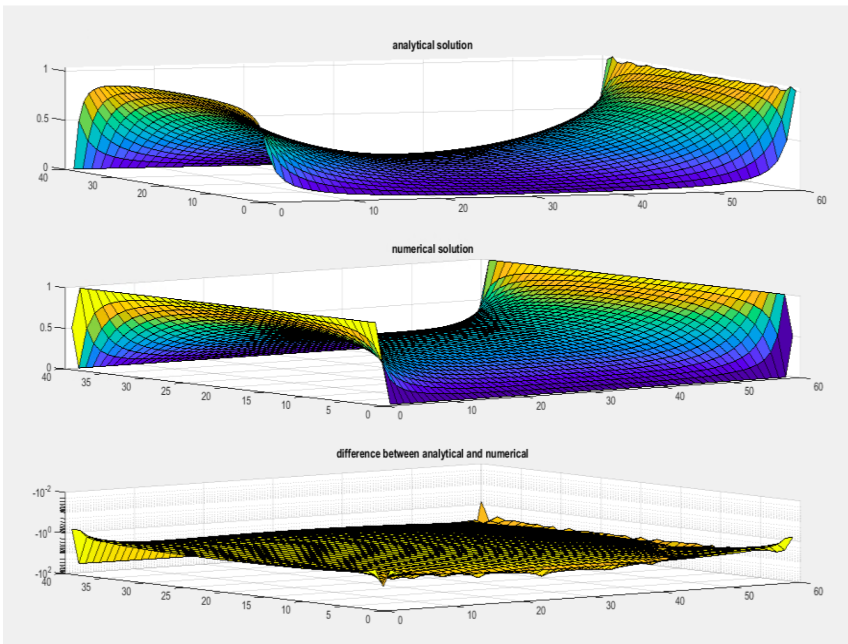

Figure 2: 2-D iteration process

Question 2.

a. In part a of question 2, the current flow at two contacts was calculated to generate the plots for the conductivity sig(x,y), V(x,y), E(x,y), and J(x,y) as shown in figure 3.

b. In part b, the mesh density was investigate by changing the mesh size to half, double, triple of the standard as shown in figure 4 to 6 below.

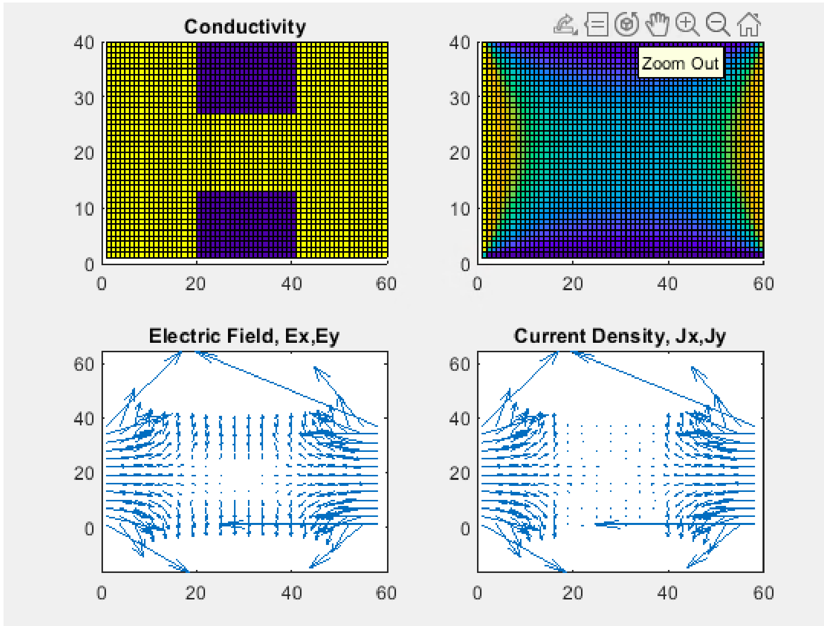

Figure 3: Plots for the conductivity sig(x,y), V(x,y), E(x,y), and J(x,y). (Standard)

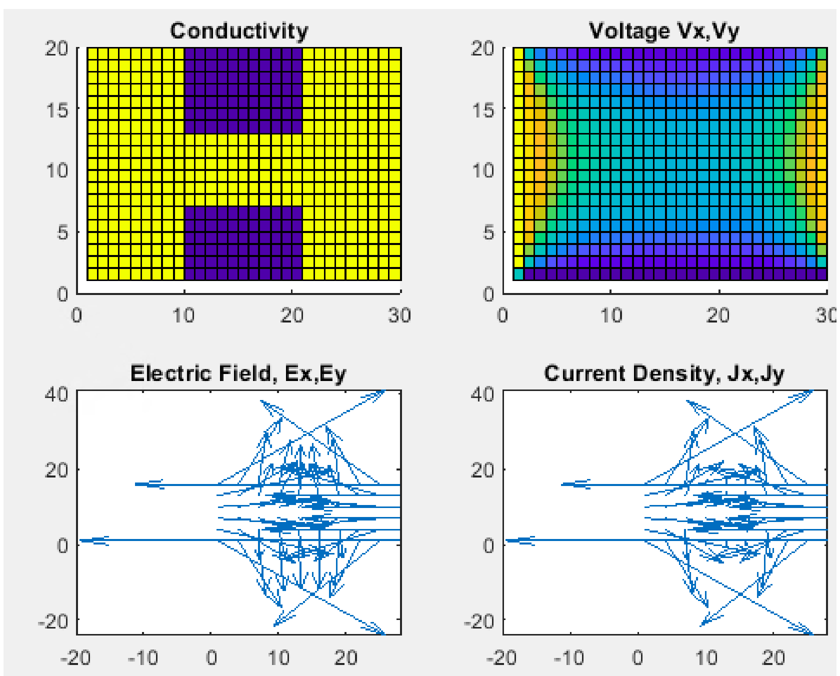

Figure 4: Plots for the conductivity sig(x,y), V(x,y), E(x,y), and J(x,y). (half Standard)

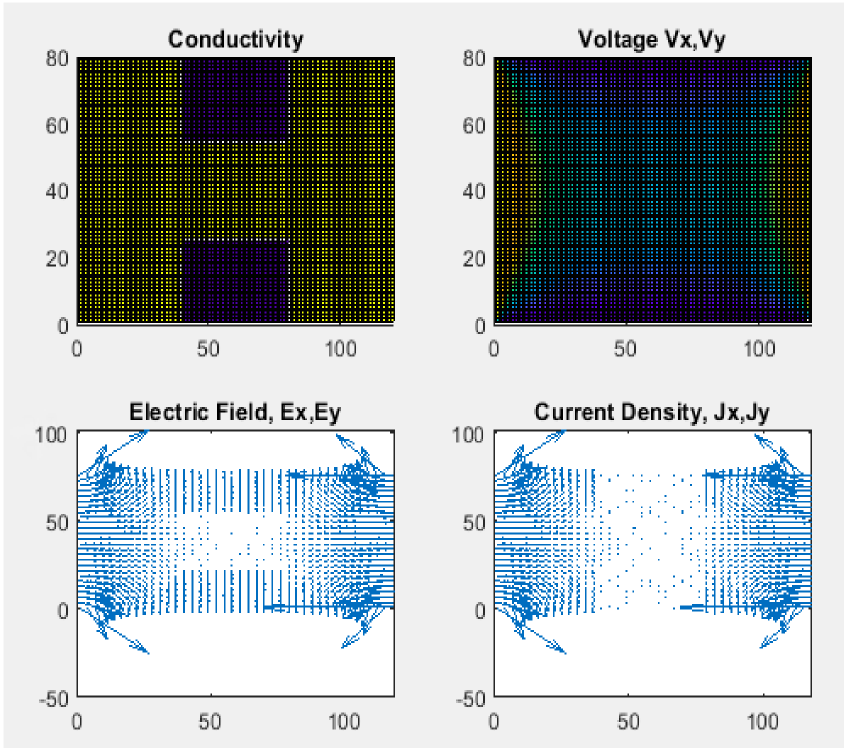

Figure 5: Plots for the conductivity sig(x,y), V(x,y), E(x,y), and J(x,y). (double Standard)

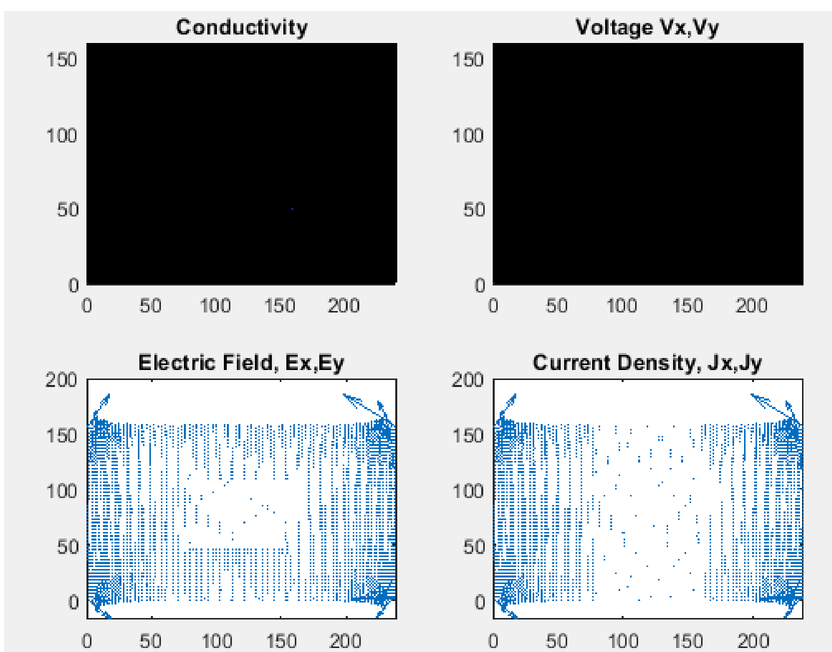

Figure 6: Plots for the conductivity sig(x,y), V(x,y), E(x,y), and J(x,y). (Triple Standard)

c. In part c, the narrowing of “bottle-neck” was investigated by plotting for different bottle-neck size. As can be seen in figure 7, 8 and 9, the conductivity and the meshing remain the same but the bottle neck size change.

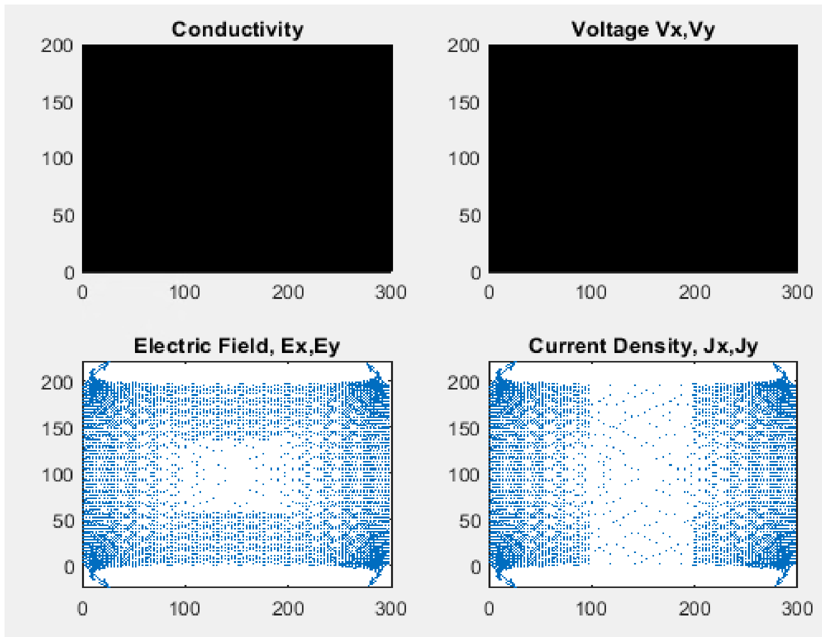

Figure 7: narrowing of “bottle-neck”

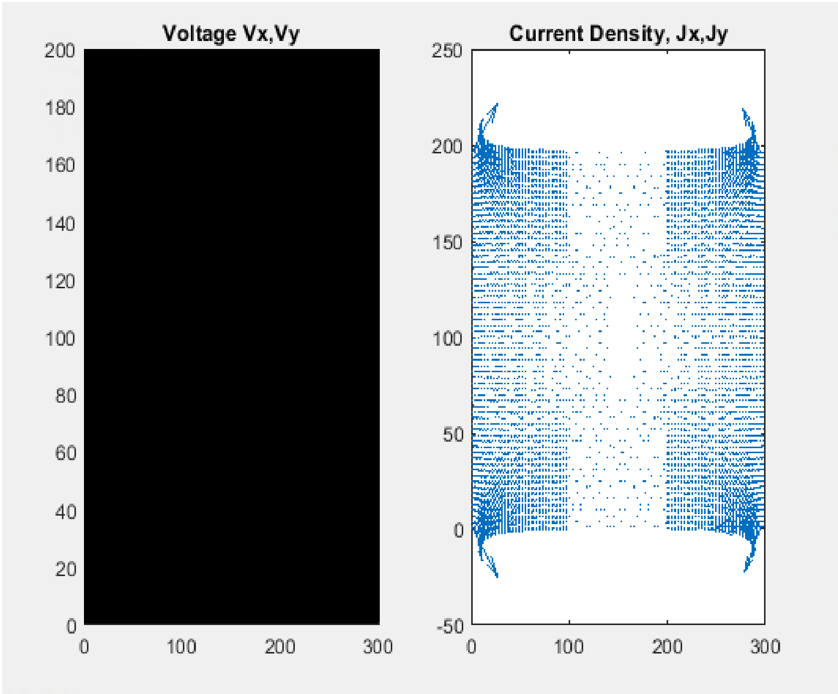

Figure 8: narrowing of "bottle-neck"

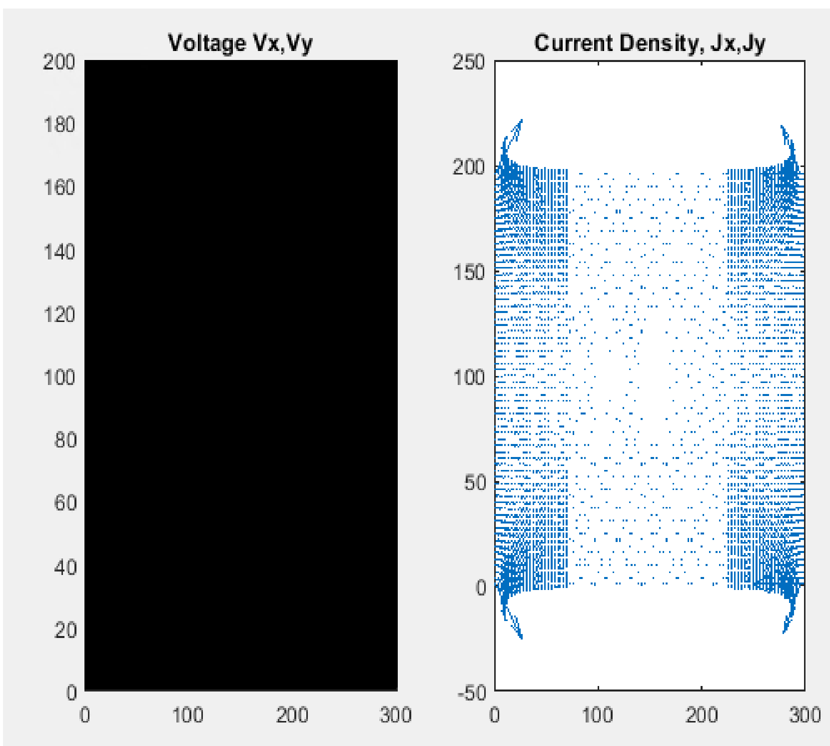

Figure 9: narrowing of "bottle-neck"

d. In part d, sigma was investigated by varying it and then plotted for higher resistivity and lower resistivity as shown in figure 10 and 11 below.

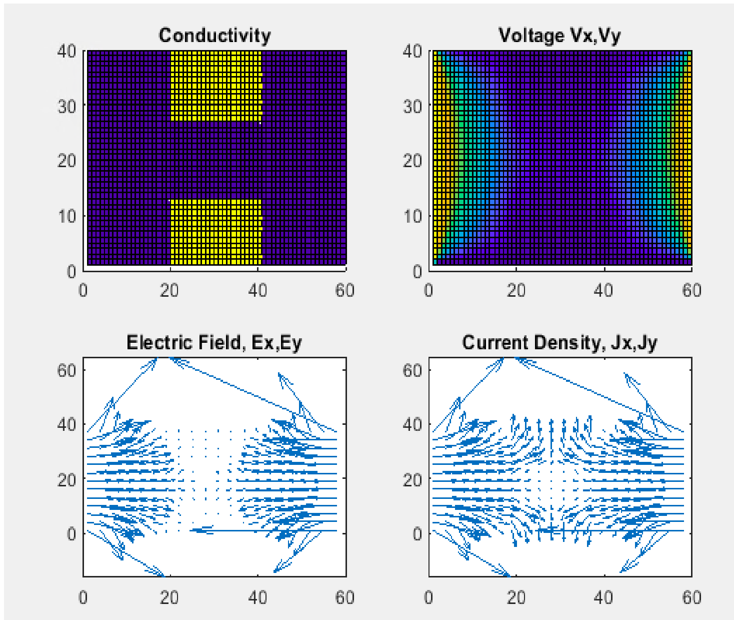

Figure 10: Higher resistivity

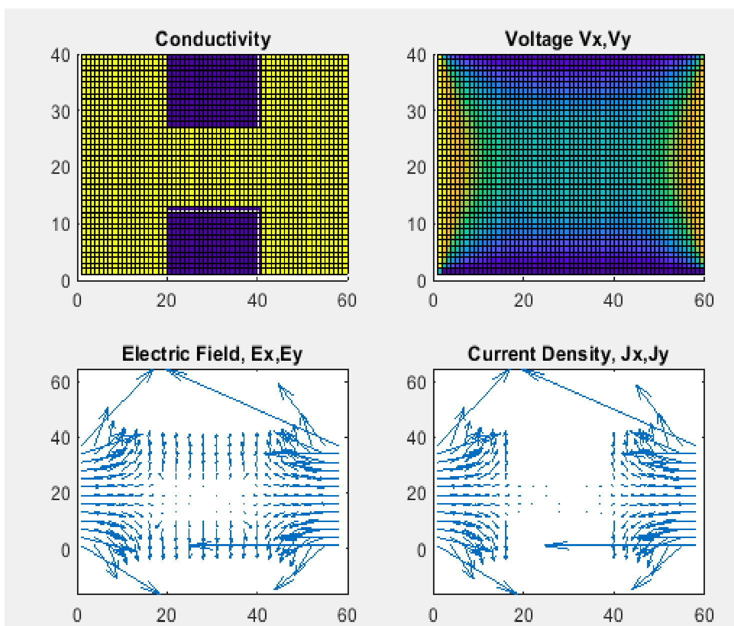

Figure 11: Lower resistivity

Conclusion

In this Assignment, the Finite Difference Method was used to solve for electrostatic potential and current flow in rectangular. A simple case was studied in part a of question 1 and then compared to part b of question 1. The solution of a bunch of mesh sizes to the analytical series solution was compared. The current flow was calculated in part 2, the mesh density, narrowing of “bottle-neck” and sigma of the box were investigated. The code used is attached in appendix A below.

Appendix A

```
% Rodolphe Rejouis-101089193
% Assignement 2

%clear all
clearvars
clearvars -GLOBAL
close all
format shortE

%% Parameters for question 1
size = 20;
W = 2*size;
L = 3*size;
Vo=1;
G1 = sparse(1,L); % Vector for question 1a
G2 = sparse(W,L); % Matrix for question 1b.
G3 = sparse(W,L); % Matrix to solve question 1 b analytical
BC1 = 0;
BC2 = Vo;
iterations =100;

%% Part a of question 1:
f1 = figure;
G1(L)=BC1; % Set the boundary condition for the Vector to be V=0 at x=L
G1(1)=BC2; % Set the boundary condition for the Vector to be V=Vo at x=0
G1(2:L-1)=deal(0.5*(BC1+BC2)); %populate the rest of the vector with the average of the Boundary Conditions
figure(f1)
for steps =1:iterations
    plot(G1);
    title('1-D finite iteration process')
    pause(0.01);
    for i=2:L-1
        G1(i)= 0.5*(G1(i-1)+G1(i+1));
    end
end

%% Part b of question 1
%Analytical solution for the equation :
% $$V(x,y)=\frac{4Vo}{\pi}\sum_{n=1,3,5...}^\infty\frac{1}{n}\frac{\cosh\left(\frac{n\pi x}{a}\right)}{\cosh\left(\frac{n\pi b}{a}\right)}\sin\left(\frac{n\pi y}{a}\right)$$

Nterms = iterations;
f2 = figure;
figure(f2)
for i = 1:L
    for j = 1:W
        G3(j,i) = analyticalsol2(i,j,Vo,L,W,Nterms);
    end
end
subplot(3,1,1);
surf(G3);
title('analytical solution');
pause(0.01);

% V=Vo at x=0,x=L and V=0 at y=0, y=W
[G2(1,1),G2(1,L),G2(W,1),G2(W,L),G2(2:W-1,2:L-1)]=deal( 0.5*(BC1+BC2) );
[G2(1,2:L-1),G2(W,2:L-1)]=deal(BC1);
[G2(2:W-1,1),G2(2:W-1,L)]=deal(BC2);

for step = 1:iterations
    subplot(3,1,2);
    surf(G2);
    title('numerical solution');
    subplot(3,1,3);
    surf(log(abs(G2-G3)))
    set(gca,'ZScale','log')
    title('difference between analytical and numerical')
    pause(0.01);
    for i=2:L-1
        for j=2:W-1
```

```
    for i=2:L-1
        for j=2:W-1
            G2(j,i) = 0.25*(G2(j+1,i)+G2(j-1,i)+G2(j,i+1)+G2(j,i-1));
        end
    end
end



%% Question 2 parameters:
% The length of the bottle neck equal to 1/3 of the total length of the rectangular
% region and the width is about 2/8 of the total width of the
% rectangular region. The conductivity inside the boxes is sig = 10e-2.

cMap2=ones(L,W); % Conductivity mapping
LGap = L/3; %For the length of the bottle neck
WGap =3*W/8; % Width of the boxes enclosing the bottle neck
sig = 10e-2;
[cMap2(floor((L-LGap)/2):floor((L+LGap)/2),1:floor((W-WGap)/2)),...
    cMap2(floor((L-LGap)/2):floor((L+LGap)/2),floor((W+WGap)/2):W)]= deal(sig);


G4 = sparse(L*W,L*W);
B3 = zeros(1,L*W);

[B3(2:W-1),B3((L-1)*W+1:L*W-1)]=deal(BC2);
[B3(1),B3(W),B3((L-1)*W),B3(L*W)]=deal(0.5*(BC1+BC2));
for i=1:L
    for j=1:W
        n= j + (i-1) * W;
        if i==1 || i == L || j == 1 || j == W
            G4(n,n) = 1;
        else
            nxp = j+i*W;
            nxm = j+(i-2)*W;
            nyp = n+1;
            nym = n-1;

            rxp = 0.5* (cMap2(i,j)+cMap2(i+1,j));
            rxm = 0.5* (cMap2(i,j)+cMap2(i-1,j));
            ryp = 0.5* (cMap2(i,j)+cMap2(i,j+1));
            rym = 0.5* (cMap2(i,j)+cMap2(i,j-1));

            G4(n,n) = -(rxp+rxm+ryp+rym);
            G4(n,nxp)=rxp;
            G4(n,nxm)=rxm;
            G4(n,nyp)=ryp;
            G4(n,nym)=rym;
        end
    end
end

V2Vec=G4\(B3'); % The matrix neccessary to obtained our V(x,y) from the conductivity map

V2Map= zeros(L,W);
for i=1:L
    for j=1:W
        V2Map(i,j)=V2Vec(j+(i-1)*W);
    end
end
%% Process to obtain the Electric Field values for the vector format of the Electric Field

eX= zeros(L,W);
eY= zeros(L,W);
for i=1:L
    for j=1:W
        if i == 1
            eX(i,j) = V2Map(i+1,j)-V2Map(i,j);
        elseif i == L
            eX(i,j) = V2Map(i,j)-V2Map(i-1,j);
```

```
            eX(i,j) = V2Map(i+1,j)-V2Map(i,j);
        elseif i == L
            eX(i,j) = V2Map(i,j)-V2Map(i-1,j);
        else
            eX(i,j) = (V2Map(i+1,j)-V2Map(i-1,j))*0.5;
        end
        if j == 1
            eY(i,j) = V2Map(i,j+1)-V2Map(i,j);
        elseif j == W
            eY(i,j) = V2Map(i,j)-V2Map(i,j-1);
        else
            eY(i,j) = (V2Map(i,j+1)-V2Map(i,j-1))*0.5;
        end
    end
end

eX=-eX;
eY=-eY;
% Code responsible for creating the Current Density Vectors
% The following code lines are responsible for making the Current Density
% Vector, by first assigning the value of the current Density to their
% matching position with the Electric Field (since J = sig*E).
Jx=cMap2.*eX;% This line of code create the X value of the Current Density vector
Jy=cMap2.*eY;% This line of code create the Y value of the Current Density vector

pointSampling = 3;
[x,y]= meshgrid(1:floor(W/pointSampling),1:floor(L/pointSampling));
x=pointSampling*x-pointSampling+1;
y=pointSampling*y-pointSampling+1;

eXSample = zeros(floor(L/pointSampling),floor(W/pointSampling));
eYSample = zeros(floor(L/pointSampling),floor(W/pointSampling));
JxSample = zeros(floor(L/pointSampling),floor(W/pointSampling));
JySample = zeros(floor(L/pointSampling),floor(W/pointSampling));

for i=1:floor(L/pointSampling)
    for j = 1:floor(W/pointSampling)
        eXSample(i,j)=eX(pointSampling*i,pointSampling*j);
        eYSample(i,j)=eY(pointSampling*i,pointSampling*j);
        JxSample(i,j)=Jx(pointSampling*i,pointSampling*j);
        JySample(i,j)=Jy(pointSampling*i,pointSampling*j);
    end
end
%% Plot of the question 2.a
% This is plots for the conductivity sig(x,y), V(x,y), E(x,y), and J(x,y).
% This will be standard plot that will compare to the following figures,
% where the setting are modified to investigate the changes.
f3=figure;
figure(f3)
subplot(2,2,1)
surface(cMap2');
title('Conductivity');
subplot(2,2,2)
surface(V2Map');
title('Voltage Vx,Vy');
subplot(2,2,3)
quiver(y,x,eXSample,eYSample,10);
title('Electric Field, Ex,Ey');
subplot(2,2,4)
quiver(y,x,JxSample,JySample,10);
title('Current Density, Jx,Jy');

%% Answer of Question 2.b

% This is with meshing size being double of the standard set:
Plotconductivity(40,10e-2,3);

% This is with meshing size being triple of the standard set:
Plotconductivity(80,10e-2,3);
```

```
Jx=cMap2.*eX;% This line of code create the X value of the Current Density vector
Jy=cMap2.*eY;% This line of code create the Y value of the Current Density vector

pointSampling = 3;
[x,y]= meshgrid(1:floor(W/pointSampling),1:floor(L/pointSampling));
x=pointSampling*x-pointSampling+1;
y=pointSampling*y-pointSampling+1;

eXSample = zeros(floor(L/pointSampling),floor(W/pointSampling));
eYSample = zeros(floor(L/pointSampling),floor(W/pointSampling));
JxSample = zeros(floor(L/pointSampling),floor(W/pointSampling));
JySample = zeros(floor(L/pointSampling),floor(W/pointSampling));

for i=1:floor(L/pointSampling)
    for j = 1:floor(W/pointSampling)
        eXSample(i,j)=eX(pointSampling*i,pointSampling*j);
        eYSample(i,j)=eY(pointSampling*i,pointSampling*j);
        JxSample(i,j)=Jx(pointSampling*i,pointSampling*j);
        JySample(i,j)=Jy(pointSampling*i,pointSampling*j);
    end
end
%% Plot of the question 2.a
% This is plots for the conductivity sig(x,y), V(x,y), E(x,y), and J(x,y).
% This will be standard plot that will compare to the following figures,
% where the setting are modified to investigate the changes.
f3=figure;
figure(f3)
subplot(2,2,1)
surface(cMap2');
title('Conductivity');
subplot(2,2,2)
surface(V2Map');
title('Voltage Vx,Vy');
subplot(2,2,3)
quiver(y,x,eXSample,eYSample,10);
title('Electric Field, Ex,Ey');
subplot(2,2,4)
quiver(y,x,JxSample,JySample,10);
title('Current Density, Jx,Jy');

%% Answer of Question 2.b

% This is with meshing size being double of the standard set:
Plotconductivity(40,10e-2,3);

% This is with meshing size being triple of the standard set:
Plotconductivity(80,10e-2,3);

% This is with meshing size being half of the standard set:
Plotconductivity(10,10e-2,3);

%% Answer of Question 2.c:
% Here are the different version of the a set with different bottle neck size
%
% Compariso:
Plotconductivity(100,10e-2,3);
% This is one with longer bottle neck:
Plotconductivity2(100,10e-2,3,0.5,0.375);
% This is one with narrower bottle neck:
Plotconductivity2(100,10e-2,3,0.3333,0.5);

%% Answer of Question 2.d:
% Comparing the results with different resistivity inside the box:

% Higher resistivity
Plotconductivity(20,10,3);

% Lower resistivity
Plotconductivity(20,10e-5,3);
```