BST Balance

Learning Objectives

- Think about the runtime of basic binary tree operations.
- Understand the motivation behind binary search tree balance.
- Implement a rotation.

Outline

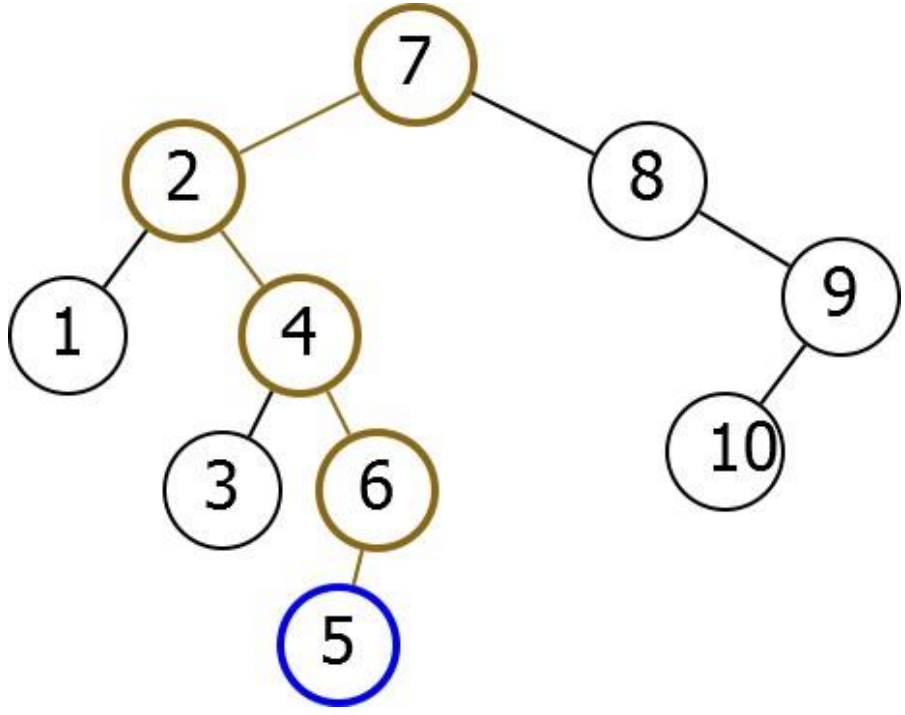
1. Runtime
2. Balanced Trees
3. Rotations

Runtime

How long do Binary Search Tree operations take?

Find

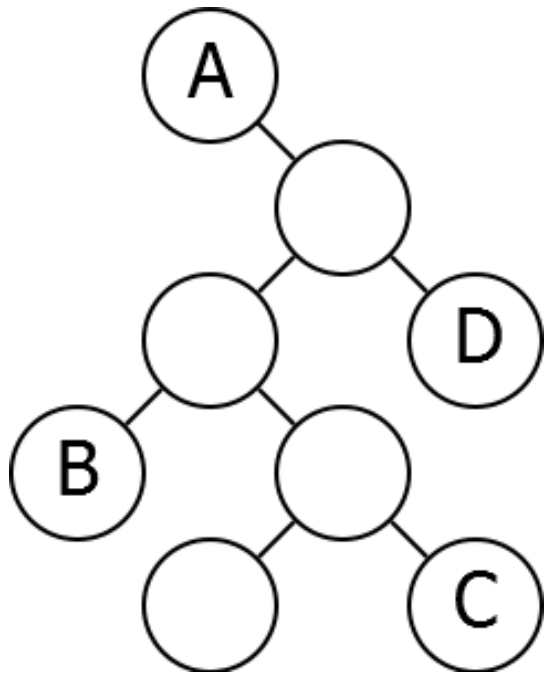
Find(5)



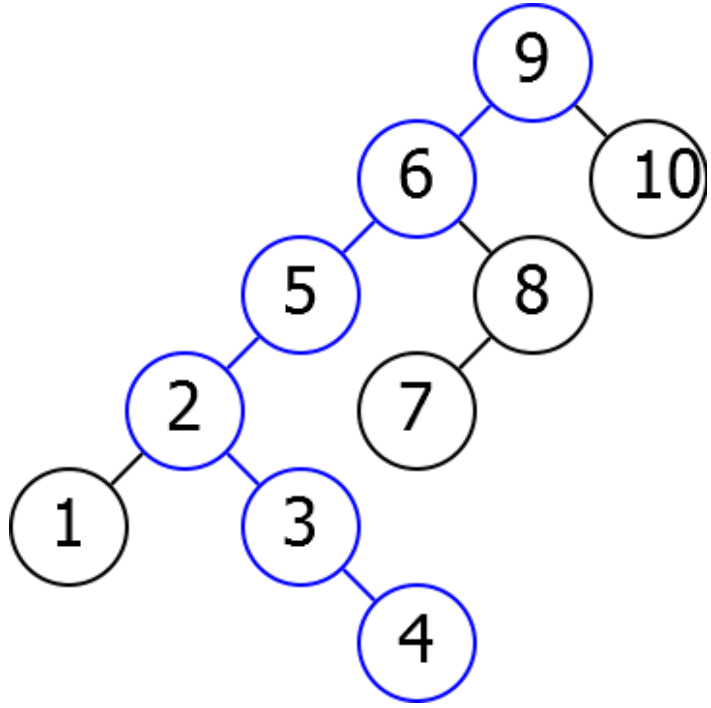
Number of operations = O(Depth)

Problem

Which nodes will be faster to search for in the following tree?



Example I



Depth can be as bad as n.

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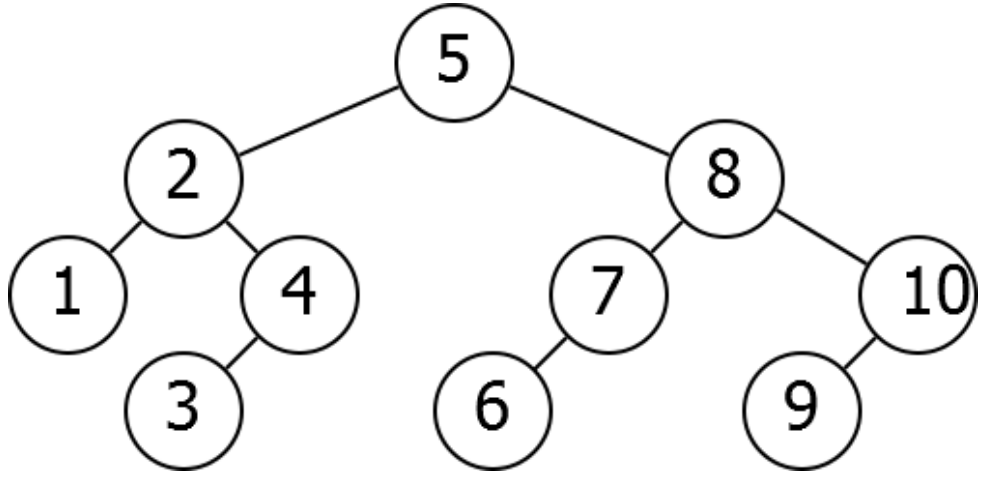
Example II



Depth can be much smaller.

Balance

- Want left and right subtrees to have approximately the same size.

Balance

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- Suppose perfectly balanced:

Balance

- Want left and right subtrees to have approximately the same size.
- Suppose perfectly balanced:
  - Each subtree half the size of its parent.
  - After $\log_2(n)$ levels, subtree of size 1.
  - Operations run in $O(\log(n))$ time.

Problem

Insertions and deletions can destroy balance!

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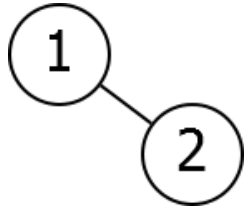
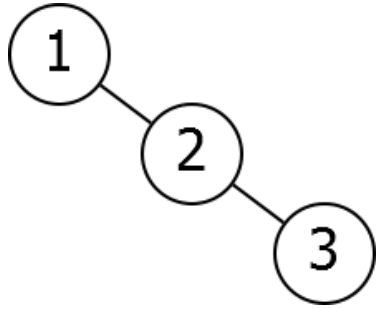


Problem

Insertions and deletions can destroy balance!



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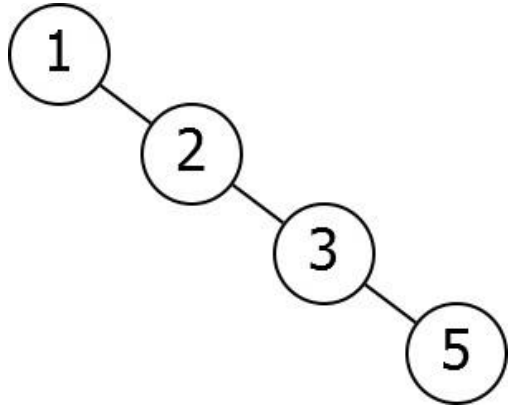
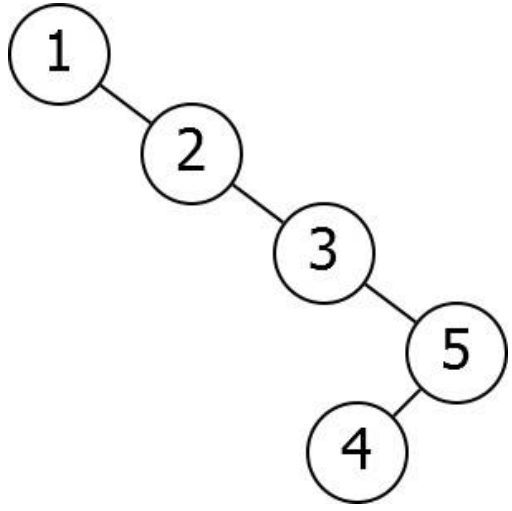


Problem

Insertions and deletions can destroy balance!



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Rebalancing

Idea: Rearrange tree to maintain balance.

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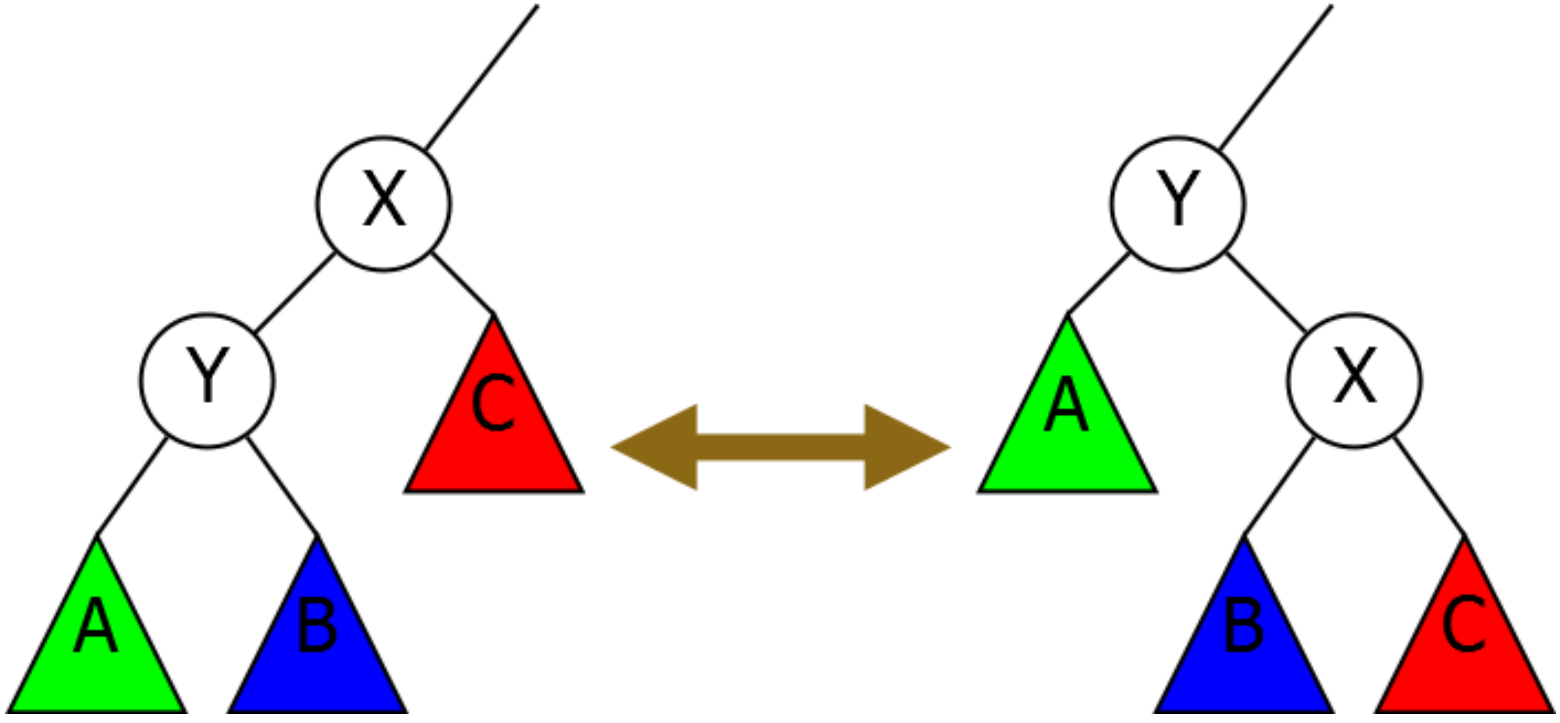
Problem: How do we rearrange tree while maintaining order?

Rotations



$$A < Y < B < X < C$$

Implementation

RotateRight(*X*)

$P \leftarrow X.\text{Parent}$

$Y \leftarrow X.\text{Left}$

$B \leftarrow Y.\text{Right}$

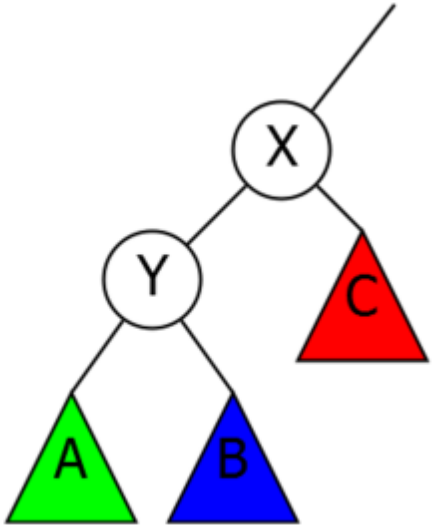
$Y.\text{Parent} \leftarrow P$

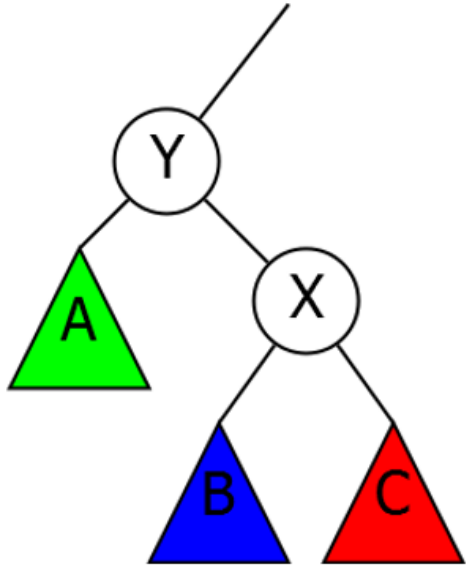
$P.\text{AppropriateChild} \leftarrow Y$

$X.\text{Parent} \leftarrow Y, Y.\text{Right} \leftarrow X$

$B.\text{Parent} \leftarrow X, X.\text{Left} \leftarrow B$

Rotate the tree by yourself





RotateRight(*X*)

P ← *X*.Parent
Y ← *X*.Left
B ← *Y* .Right
Y .Parent ← *P*
P.AppropriateChild ← *Y*
X.Parent ← *Y* , *Y* .Right ← *X*
B.Parent ← *X*, *X*.Left ← *B*

Next Time

How to keep a tree balanced. AVL trees.