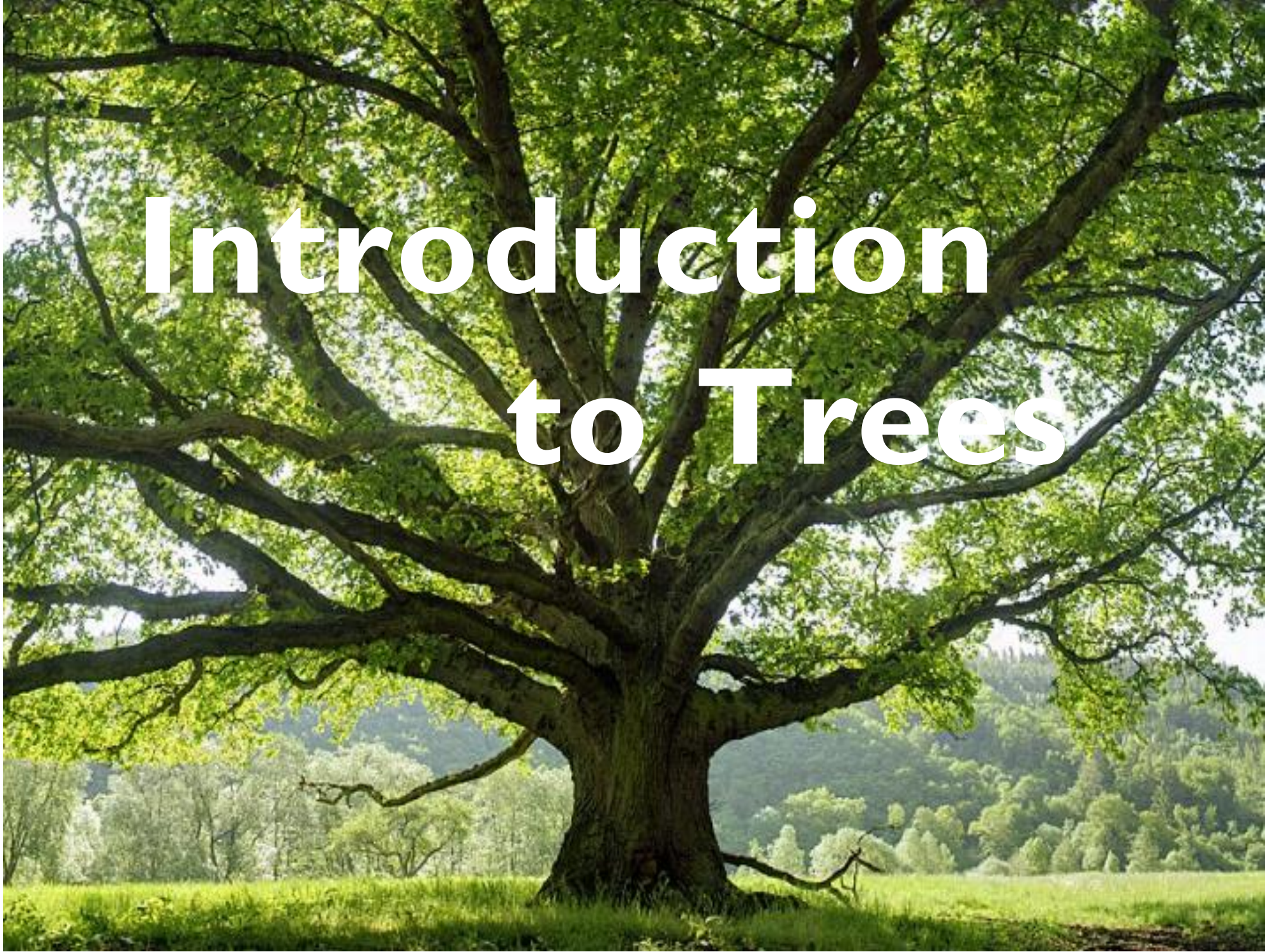
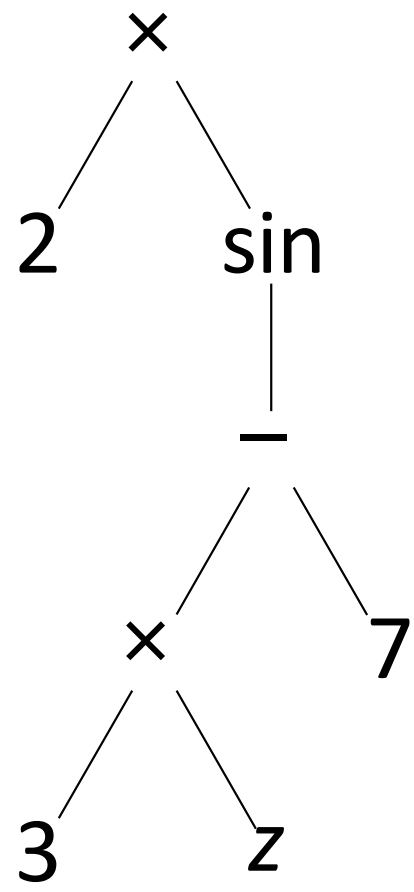
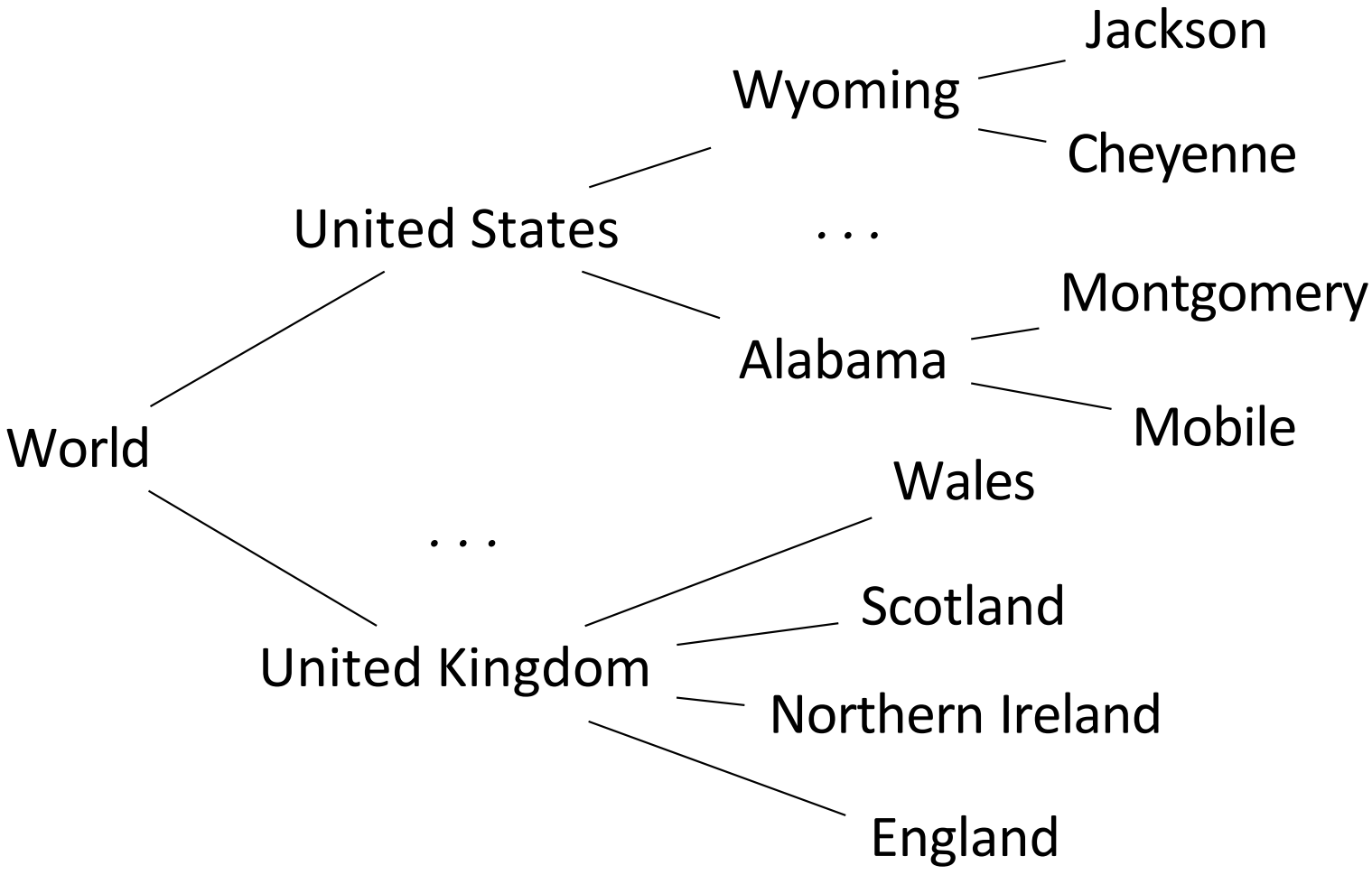

Introduction
to Trees

Syntax tree for an Expression

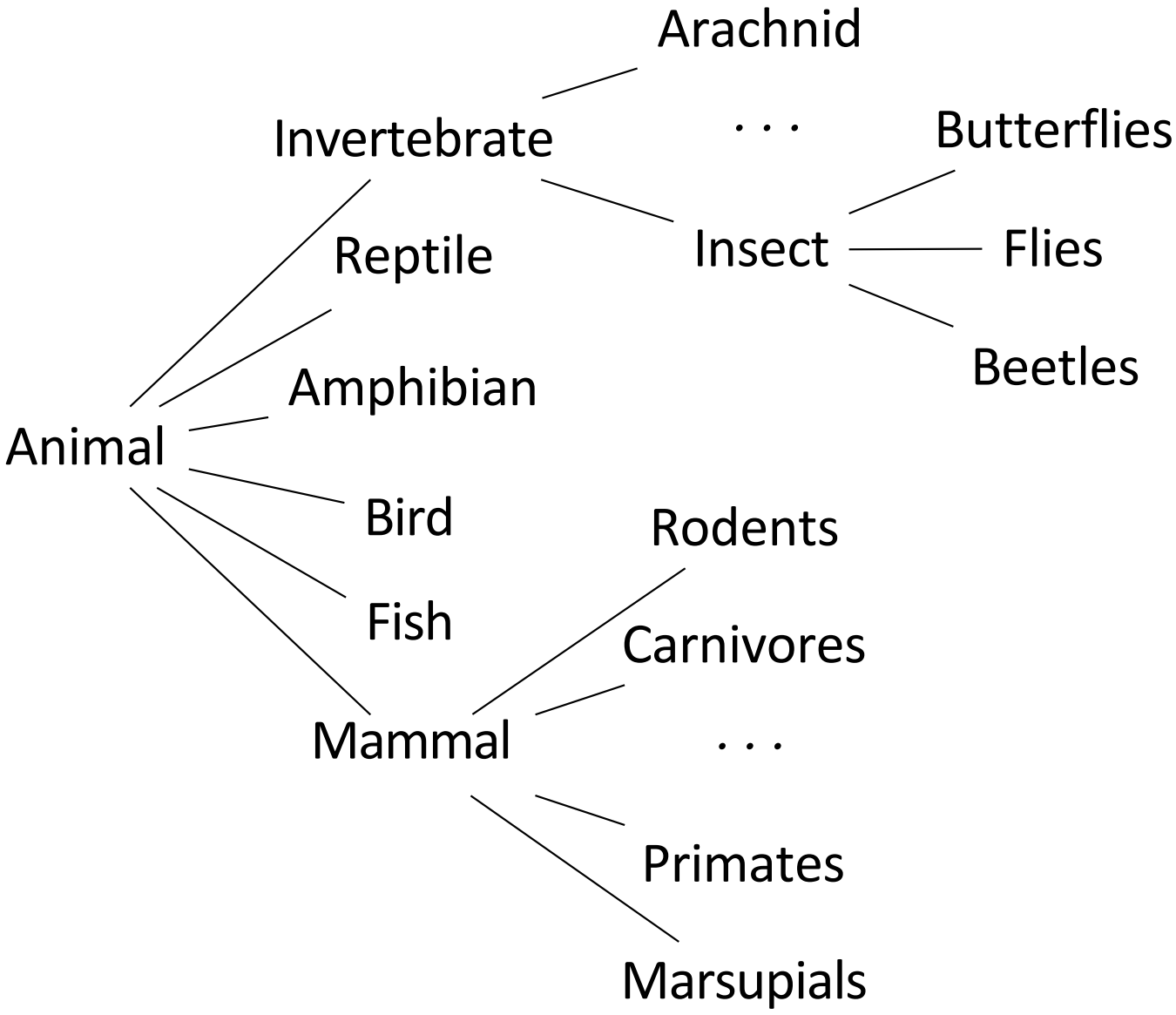
$$2\sin(3z - 7)$$



Geography Hierarchy

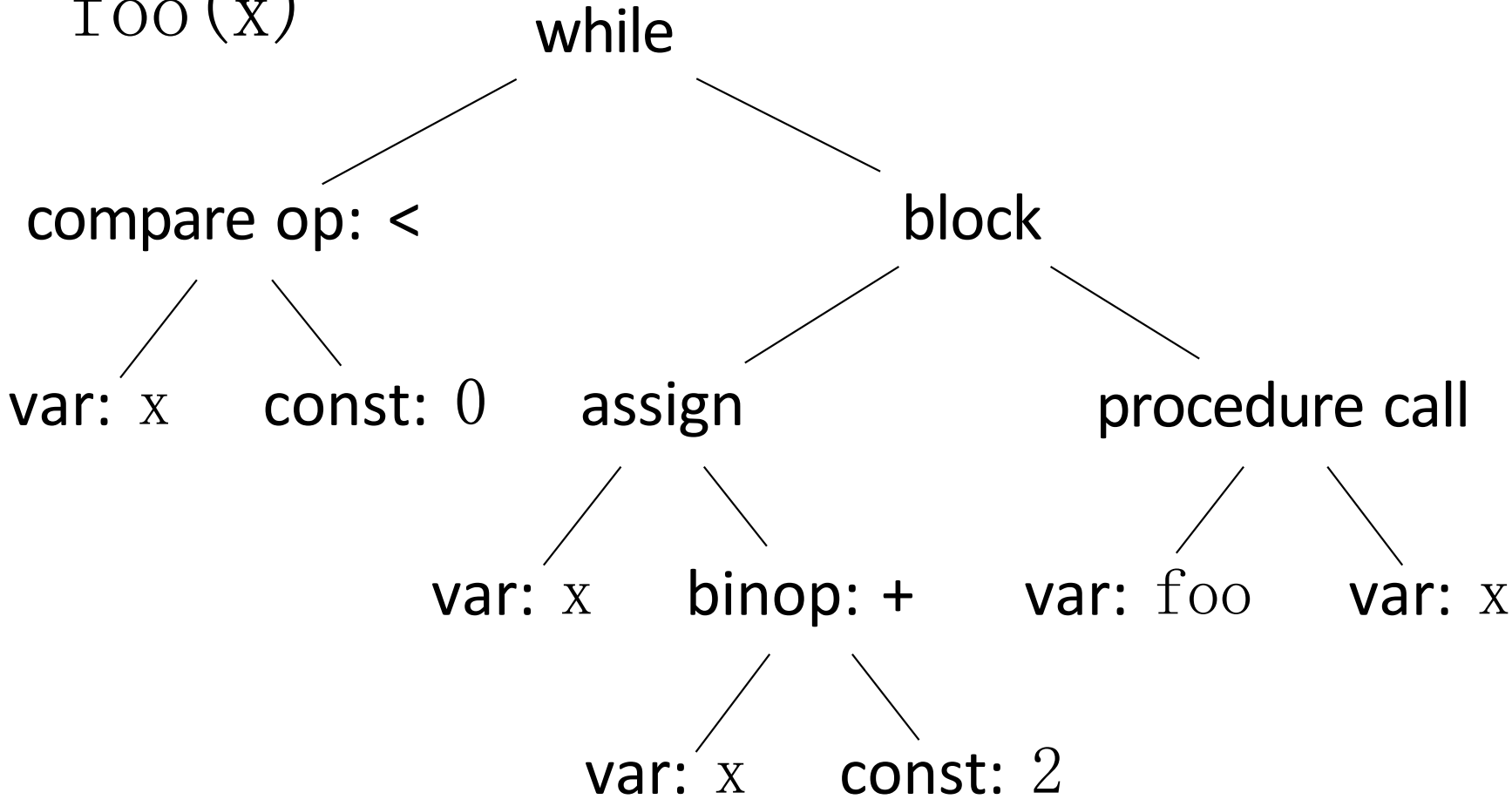


Animal Kingdom (partial)

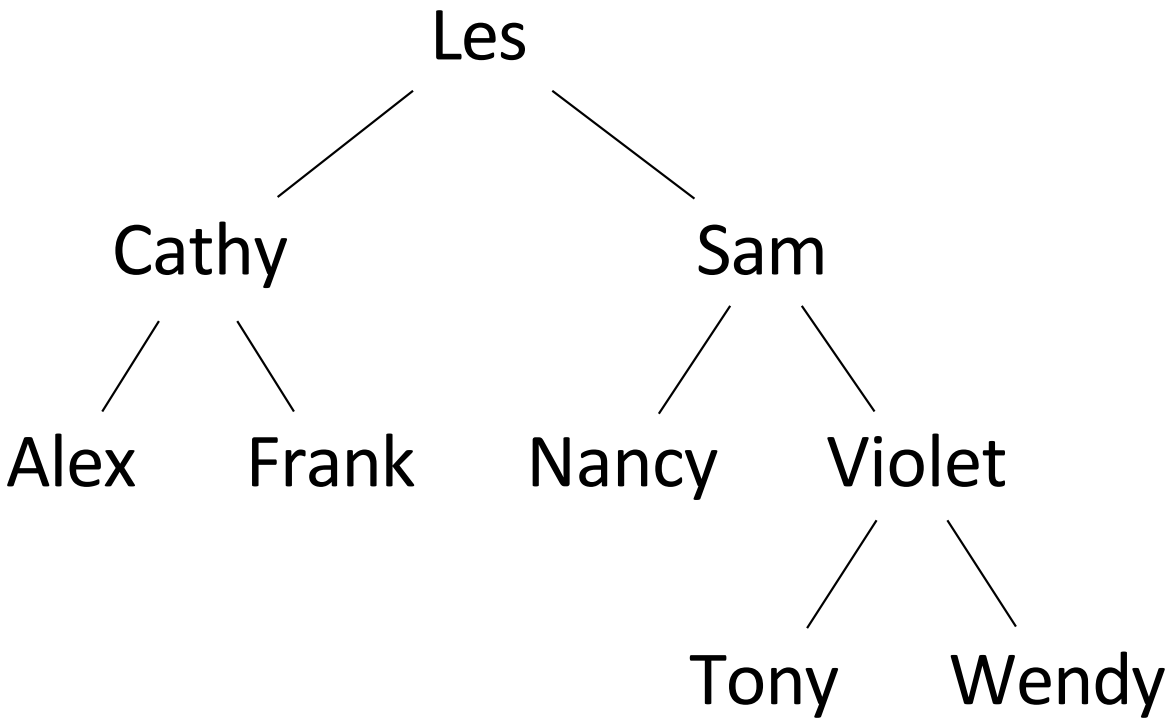


Abstract Syntax Tree for Code

```
while x < 0:
  x = x + 2
  foo(x)
```



Binary Search Tree



Definition

A Tree is:

- empty, or
- a node with:
  - a key, and
  - a list of child trees.

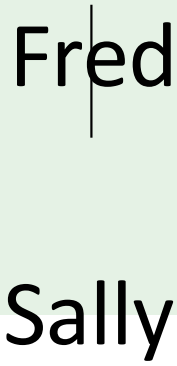
Simple

Empty tree:

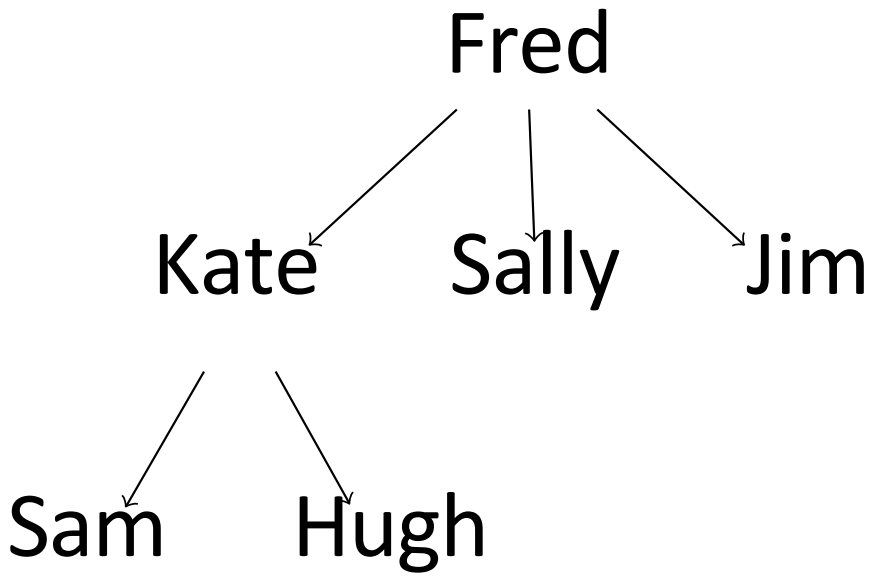
Tree with one node:

Fred

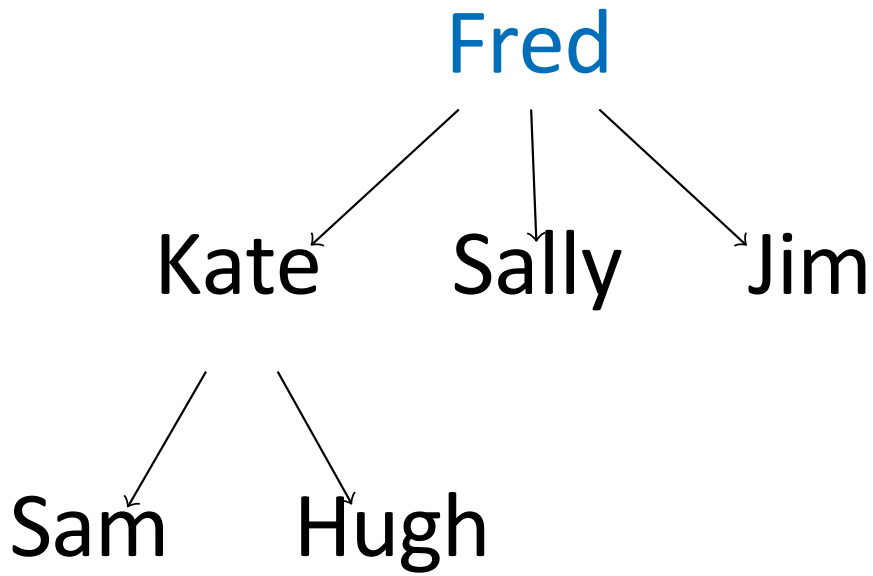
Tree with two nodes:



Terminology



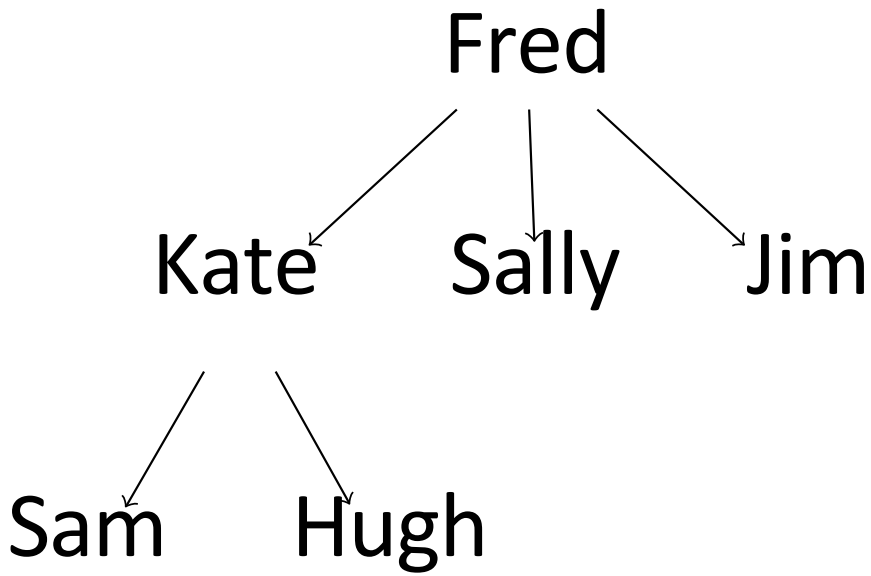
Terminology



***Root*:**

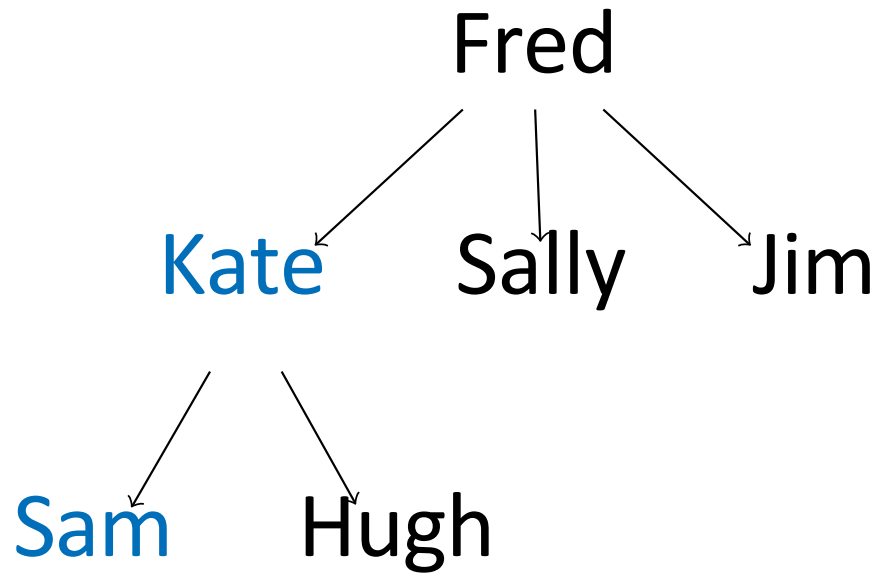
top node in the tree

Terminology



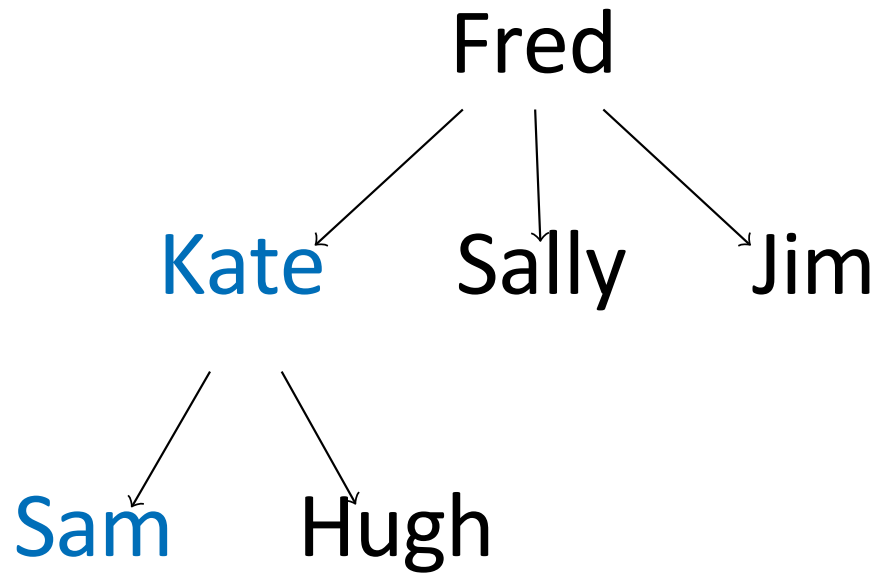
A *child* has a line down directly from a *parent*

Terminology



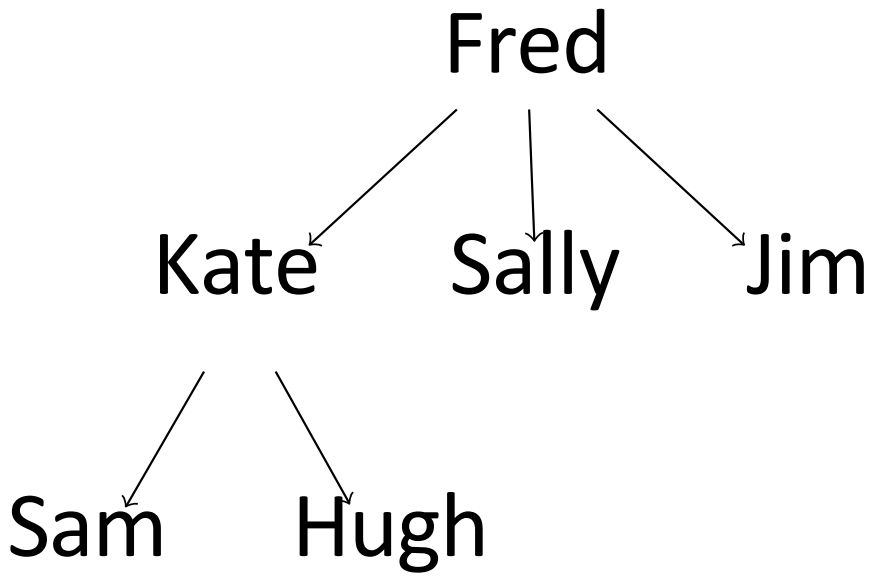
Kate is a *parent* of Sam

Terminology



Sam is a *child* of Kate

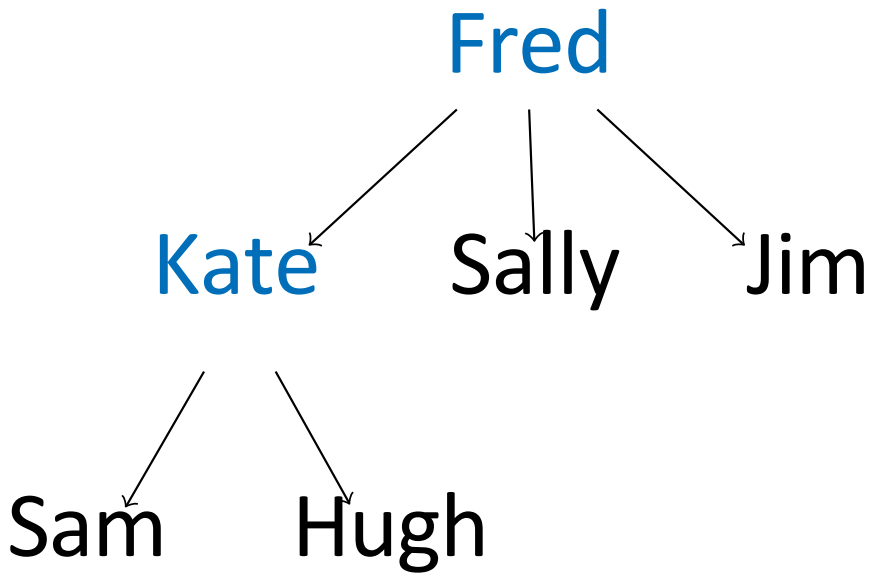
Terminology



***Ancestor* :**

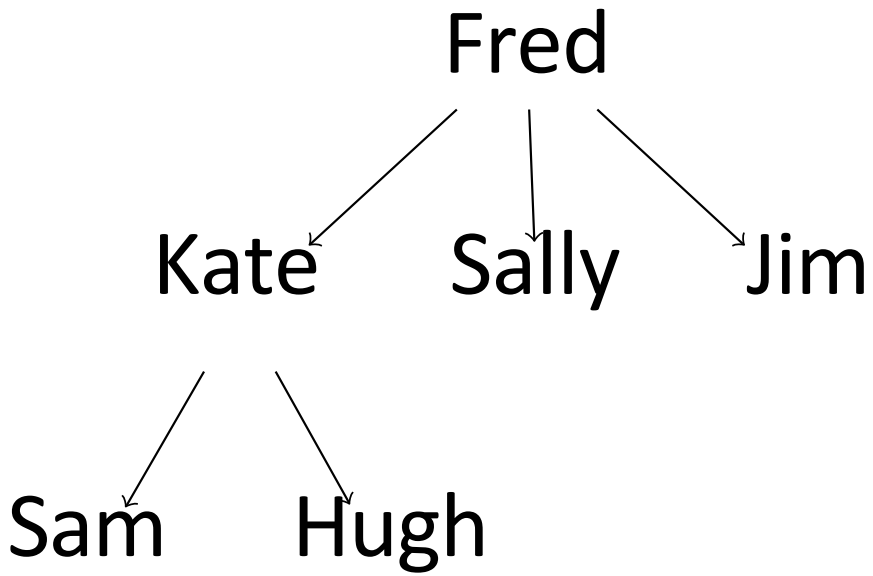
parent, or parent of parent, etc.

Terminology



***Ancestors* of Sam**

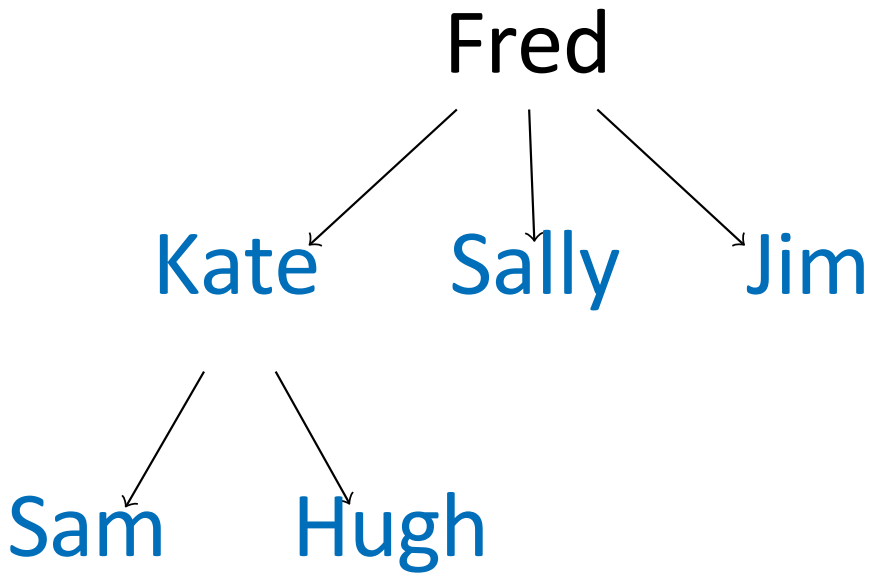
Terminology



***Descendant*:**

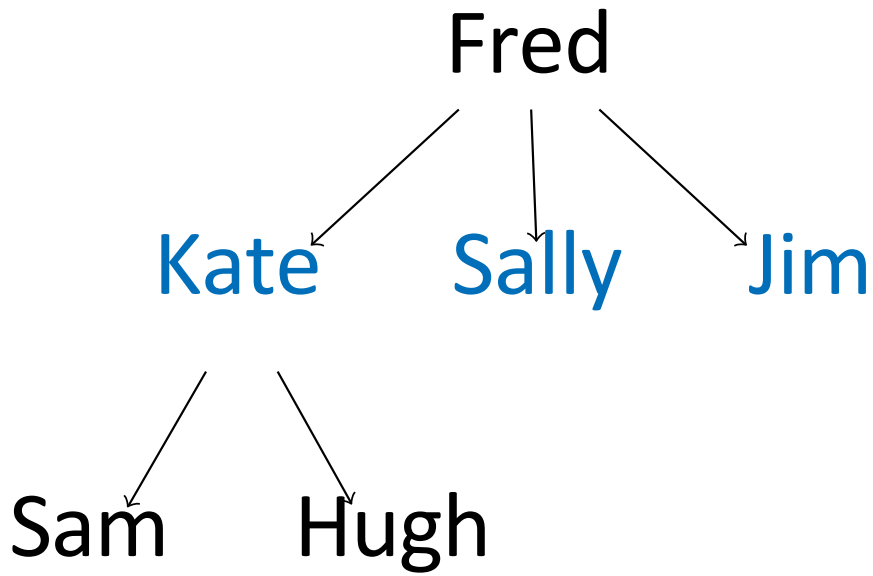
child, or child of child, etc.

Terminology



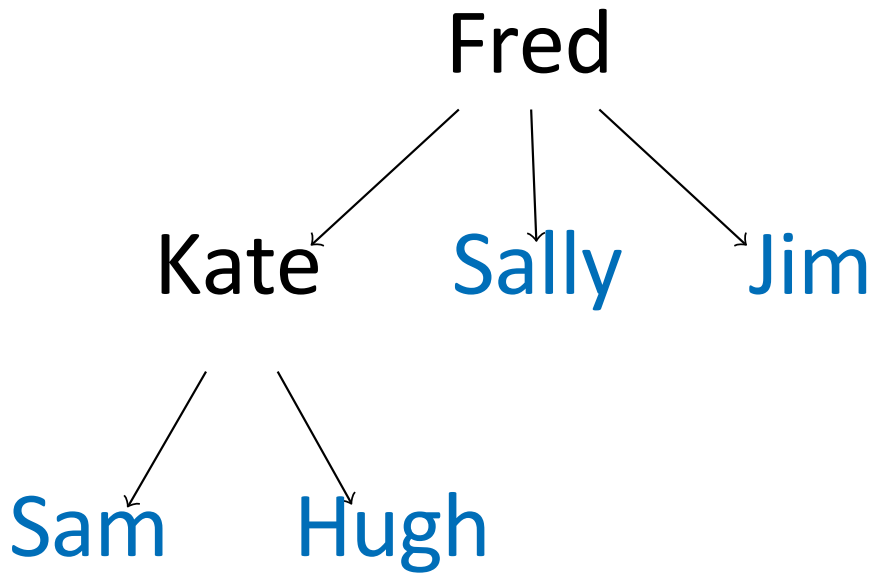
***Descendants* of Fred**

Terminology



Sibling **: sharing the same parent**

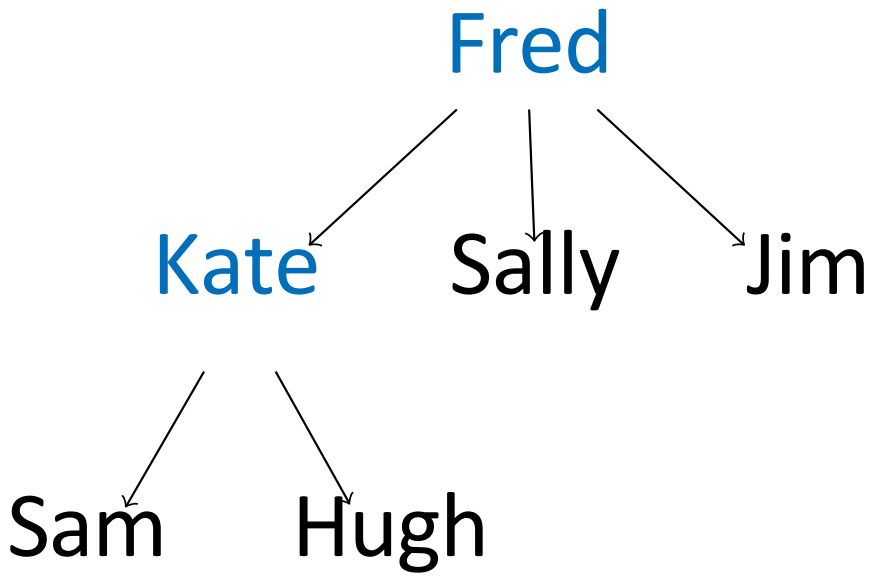
Terminology



Leaf **:**

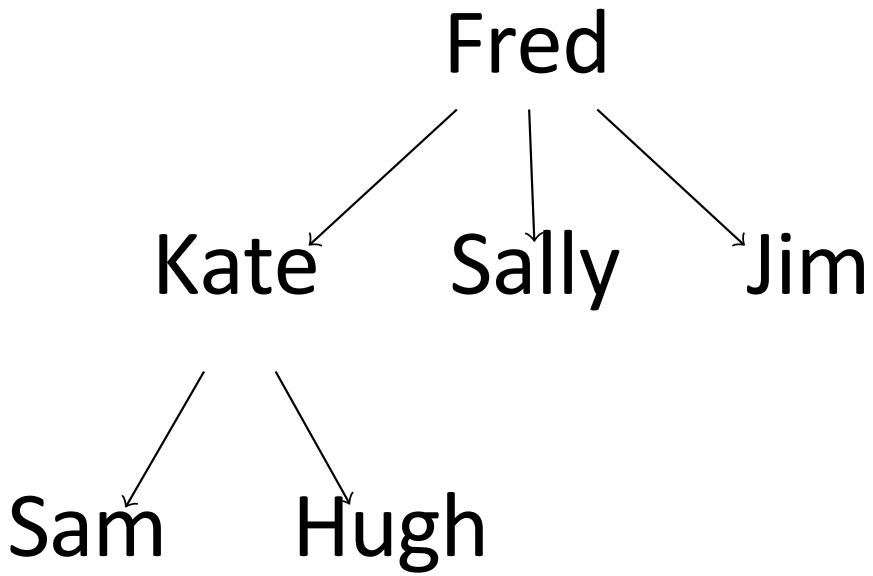
node with no children

Terminology



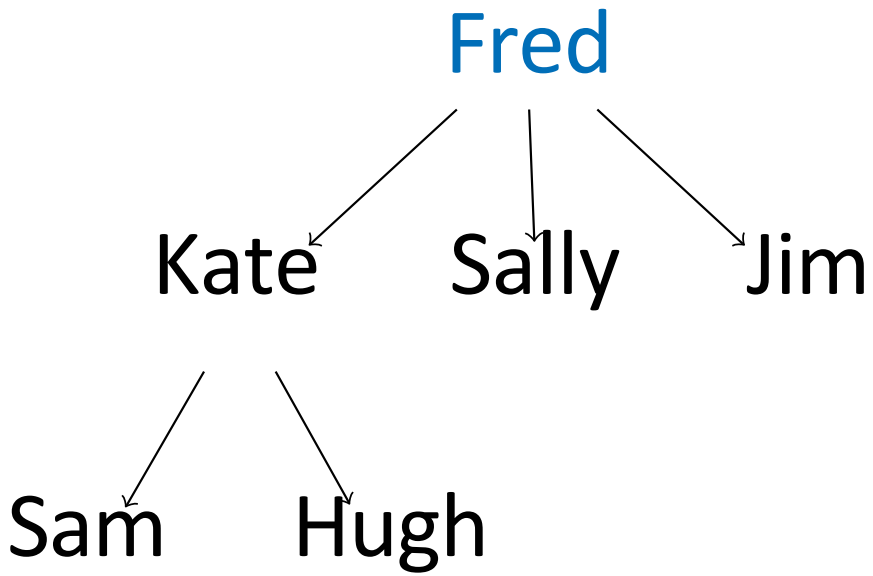
Interior node (non-leaf)

Terminology



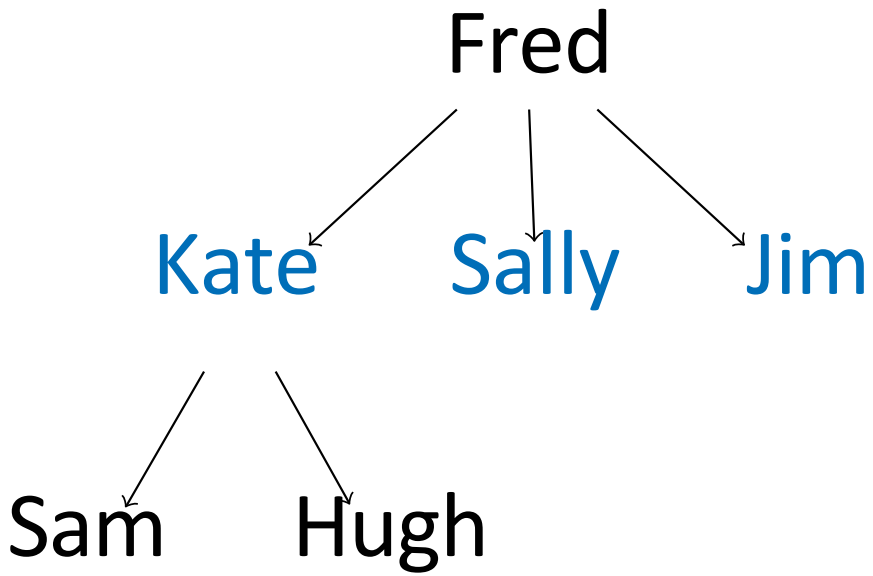
***Level*: 1 + num edges between root and node**

Terminology



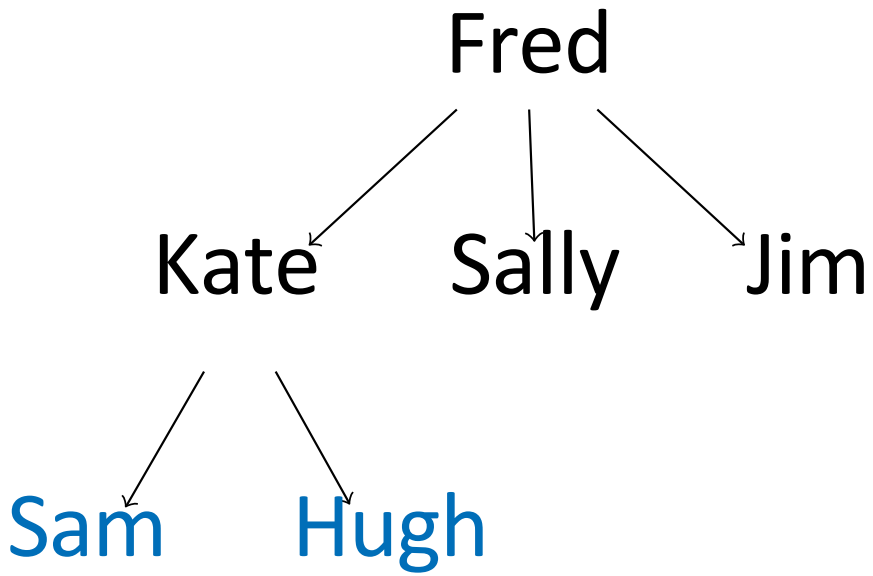
Level 1

Terminology



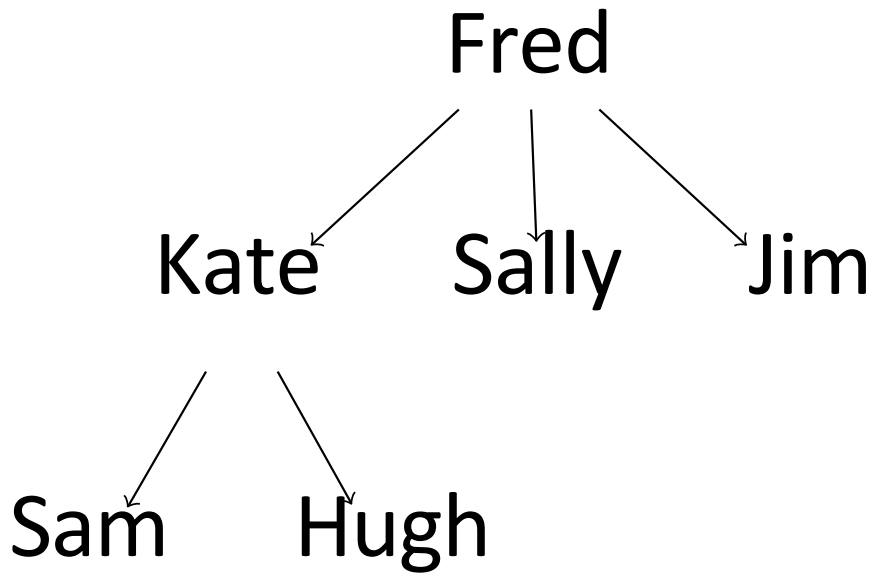
Level 2

Terminology



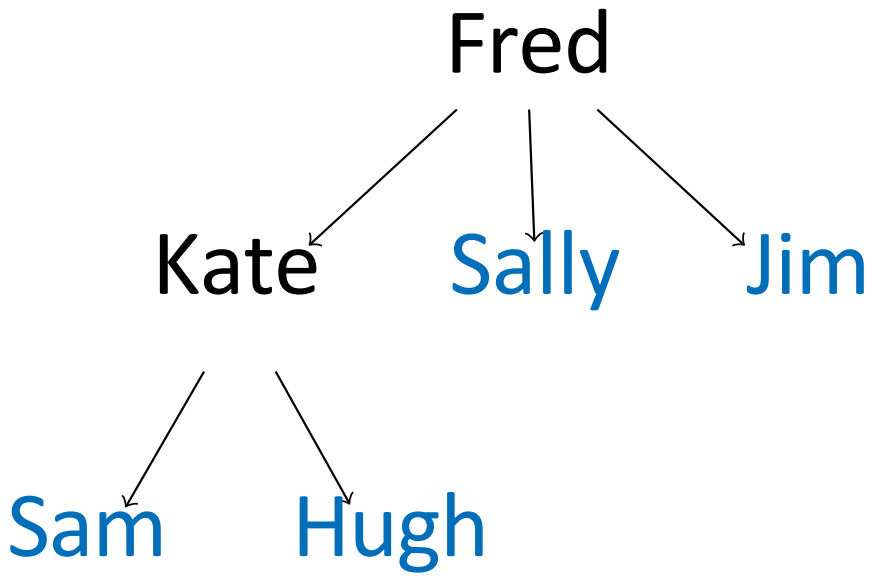
Level 3

Terminology



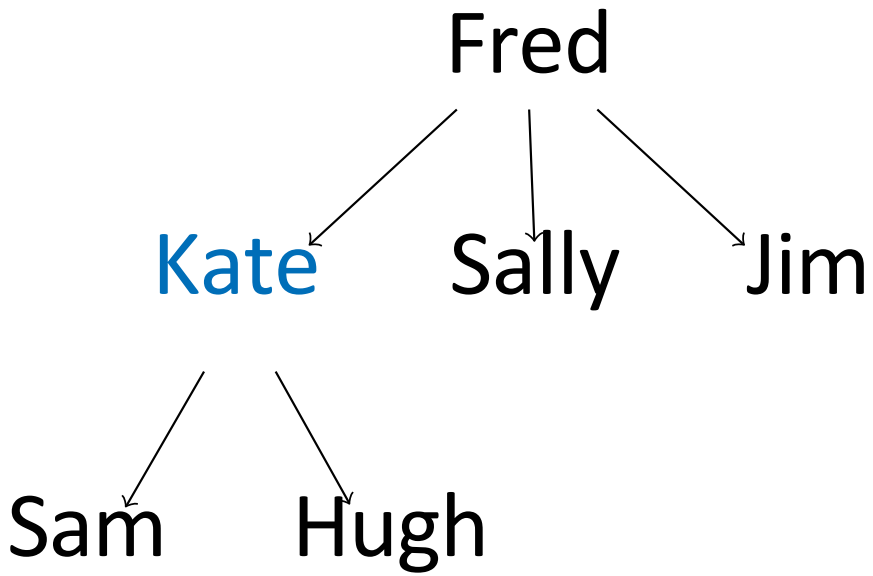
***Height*: maximum depth of subtree node and farthest leaf**

Terminology



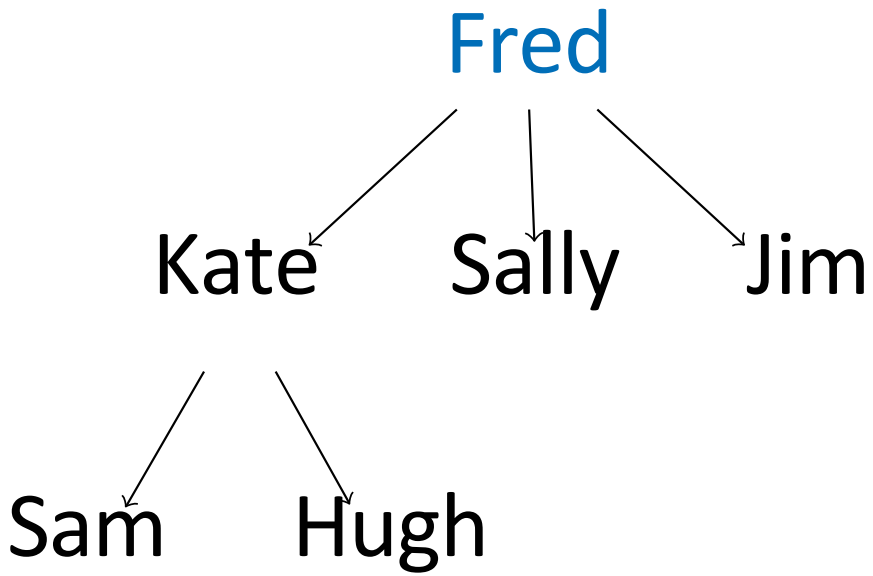
Height 1

Terminology



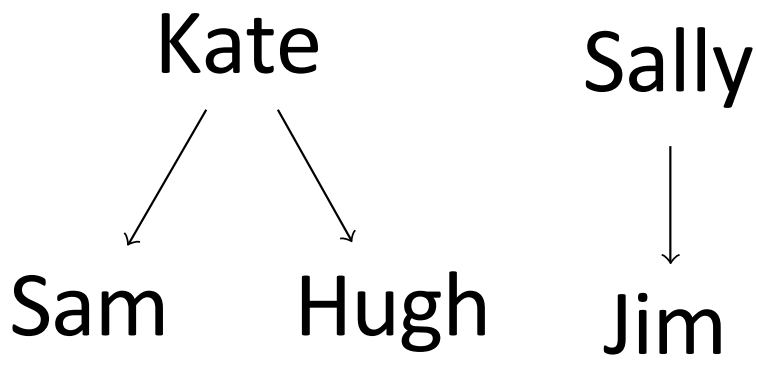
Height 2

Terminology



Height 3

Terminology



***Forest*:**

collection of trees

Node contains:

- `key`
- `children`: list of children nodes
- (optional) `parent`

For binary tree, node contains:

- `key`
- `left`
- `right`
- (optional) `parent`

Height(*tree*)

```
if tree = nil:
   return 0
return 1 + Max(Height(tree.left),
                Height(tree.right))
```

Size(*tree*)

```
if tree = nil
   return 0
return 1 + Size(tree.left)
+
                Size(tree.right)
```

Walking a Tree

Often we want to visit the nodes of a tree in a particular order.

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For example, print the nodes of the tree.

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- Depth-first: We completely traverse one sub-tree before exploring a sibling sub-tree.

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Often we want to visit the nodes of a tree in a particular order.

For example, print the nodes of the tree.

- Depth-first: We completely traverse one sub-tree before exploring a sibling sub-tree.
- Breadth-first: We traverse all nodes at one level before progressing to the next level.

Walking a Tree

Depth-first:

- Pre-order (Visit, Left, Right)
- In-order (Left, Visit, Right)
- Post-order (Left, Right, Visit)

Breadth-first:

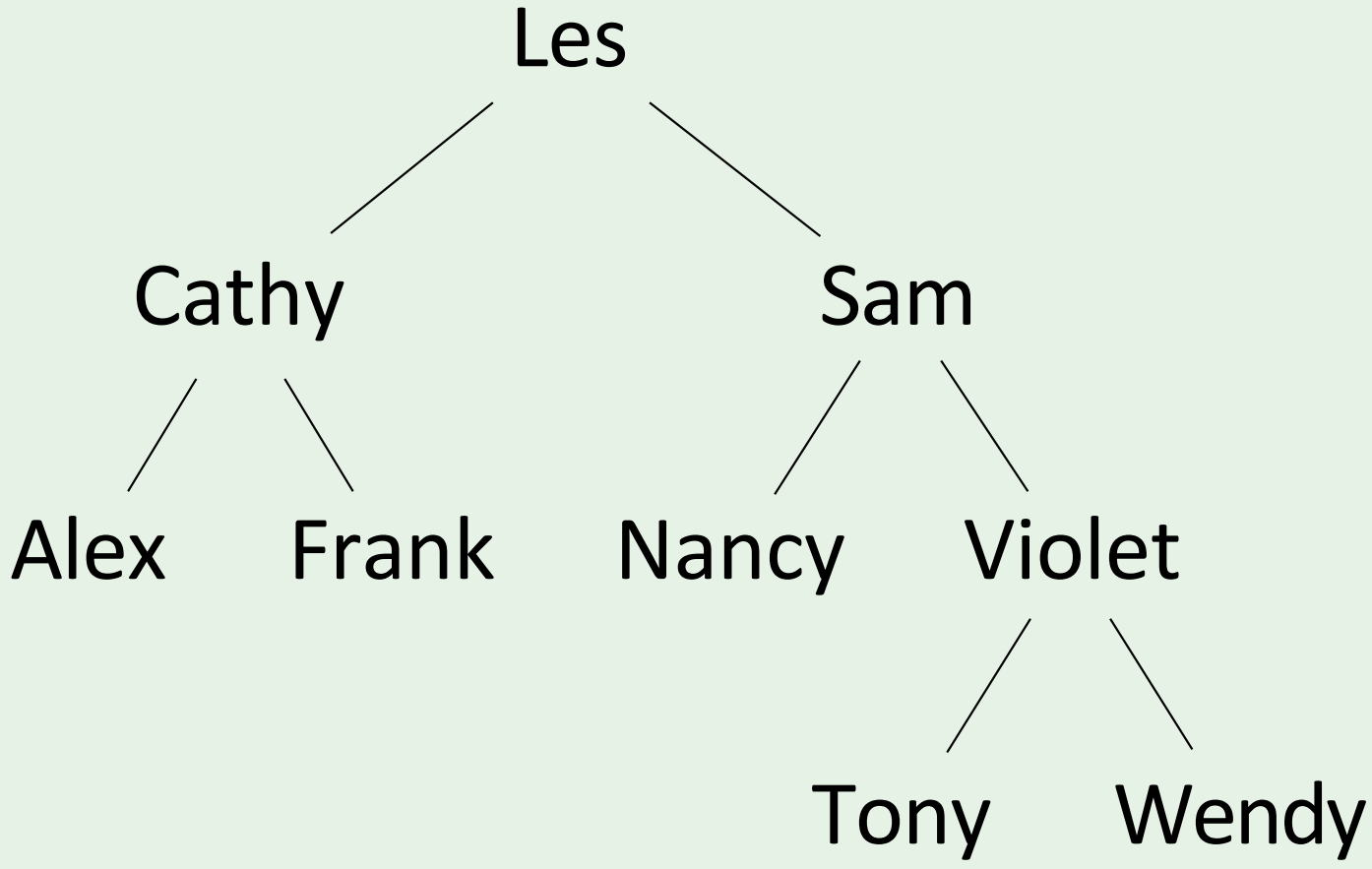
- We traverse all nodes at one level before progressing to the next level.

  1^{st} level (left to right), 2^{nd} level (left to right), ..

Depth-first

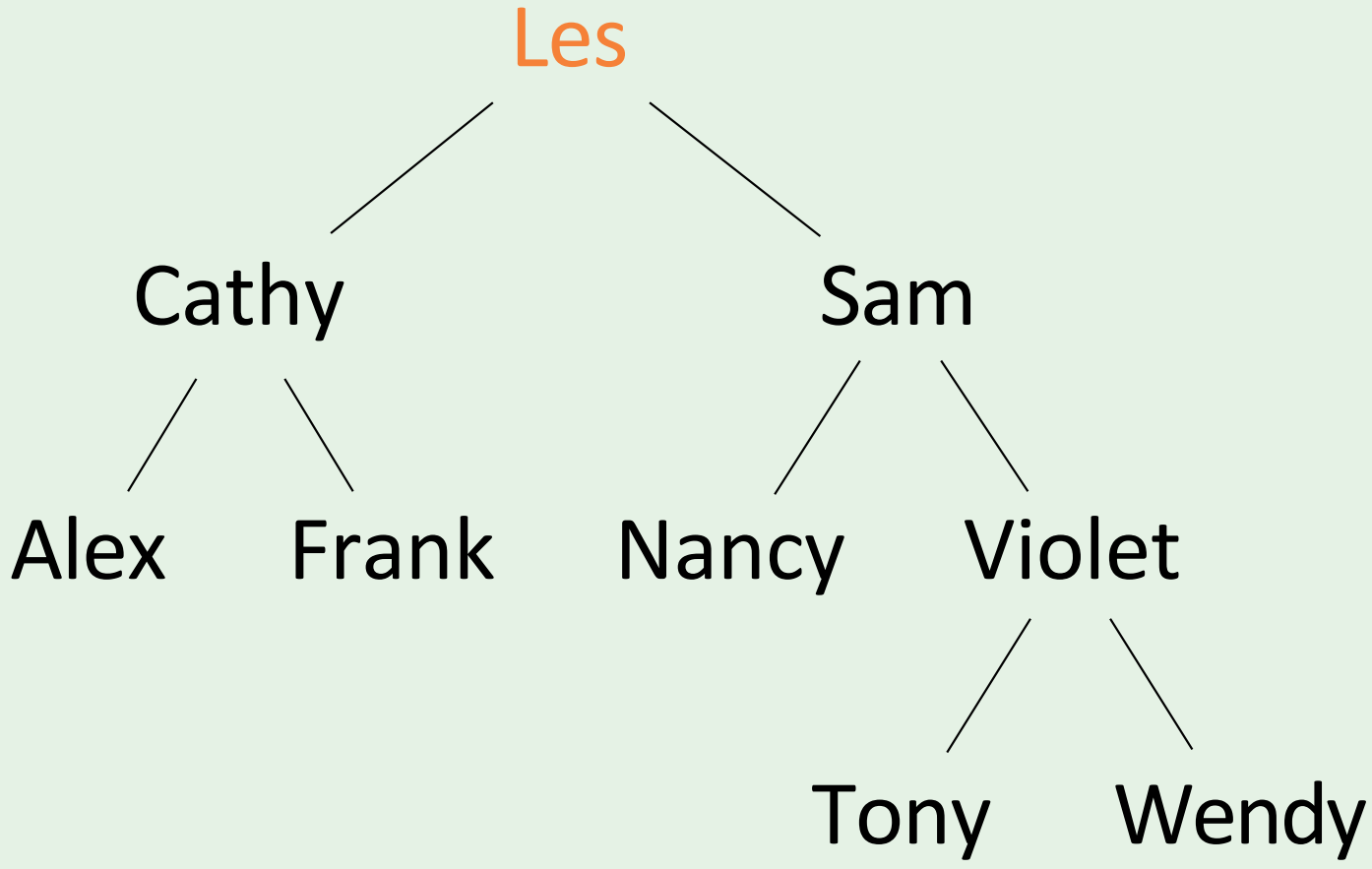
```
InOrderTraversal(tree)
if tree = nil :
   return
InOrderTraversal(tree.left)
Print(tree.key )
InOrderTraversal(tree.right)
```

InOrderTraversal



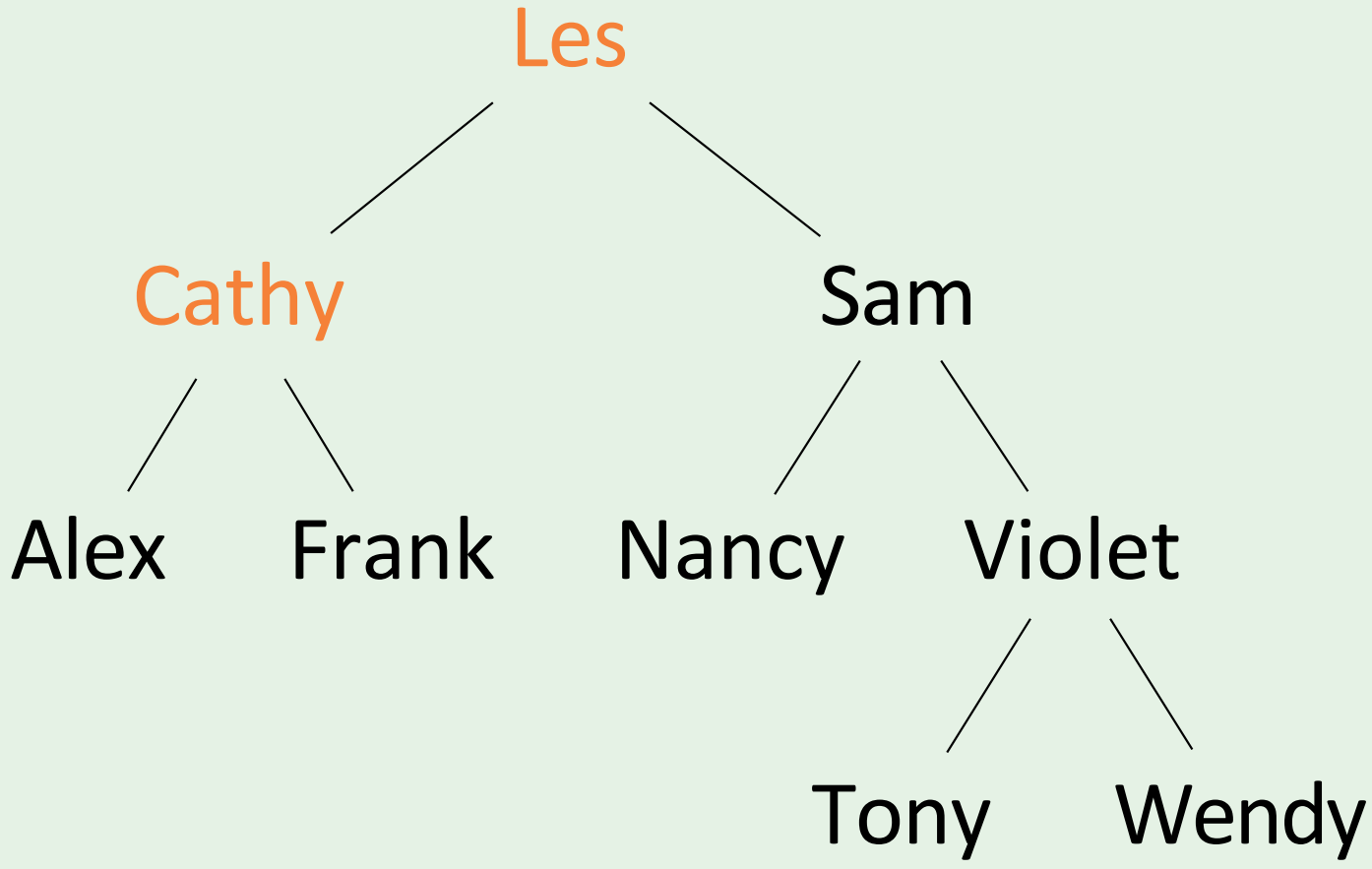
Output:

InOrderTraversal



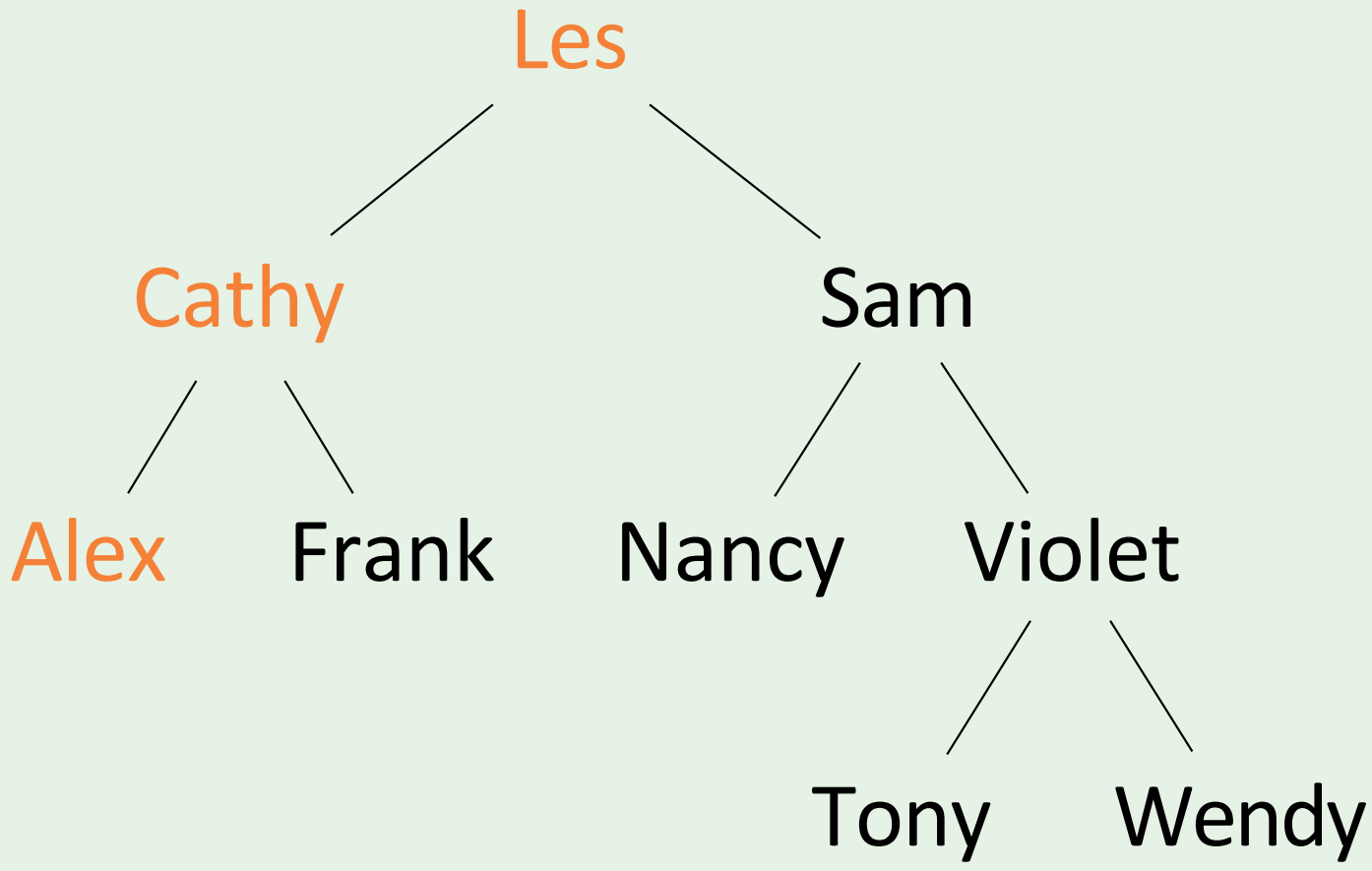
Output:

InOrderTraversal



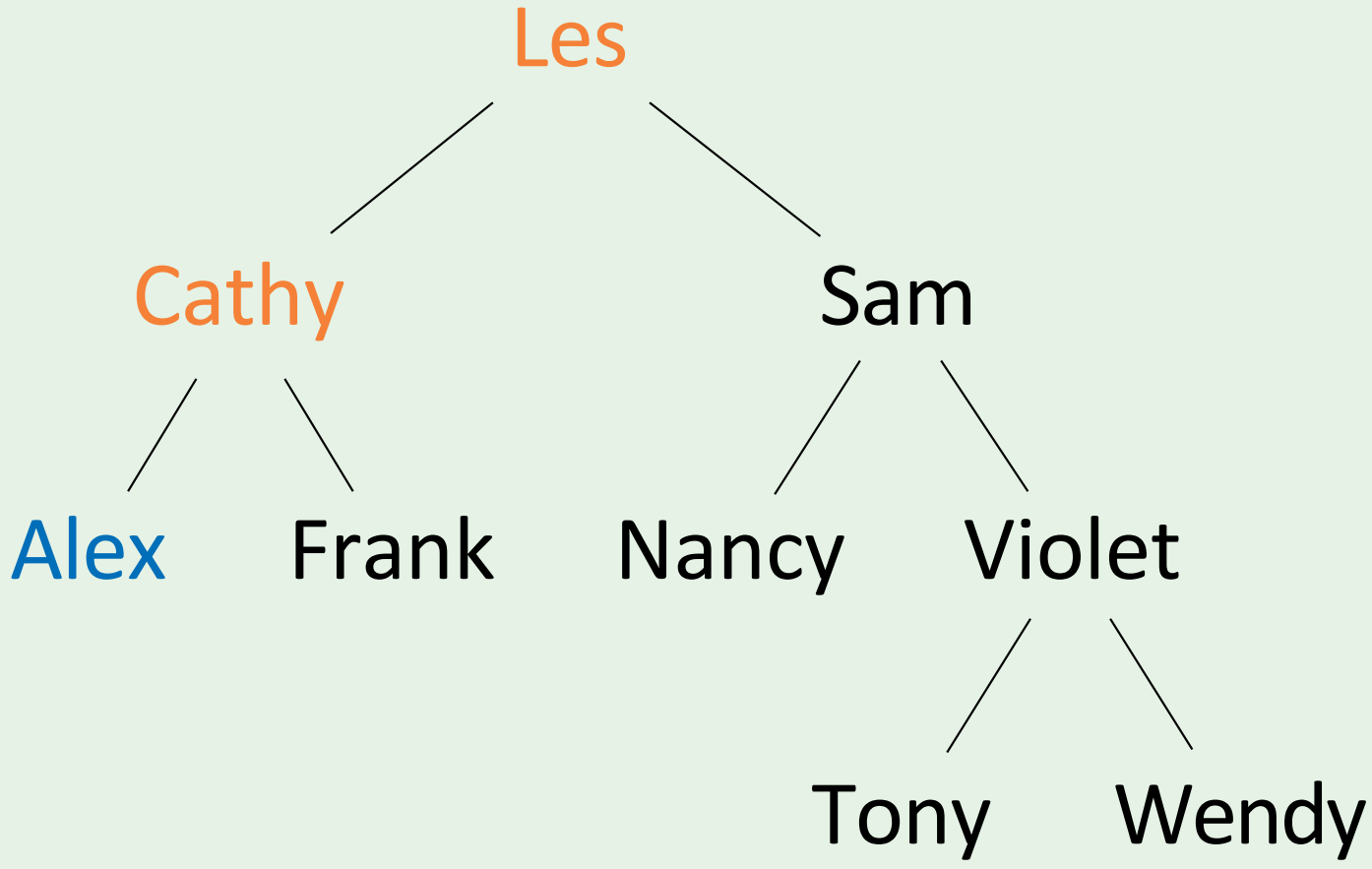
Output:

InOrderTraversal



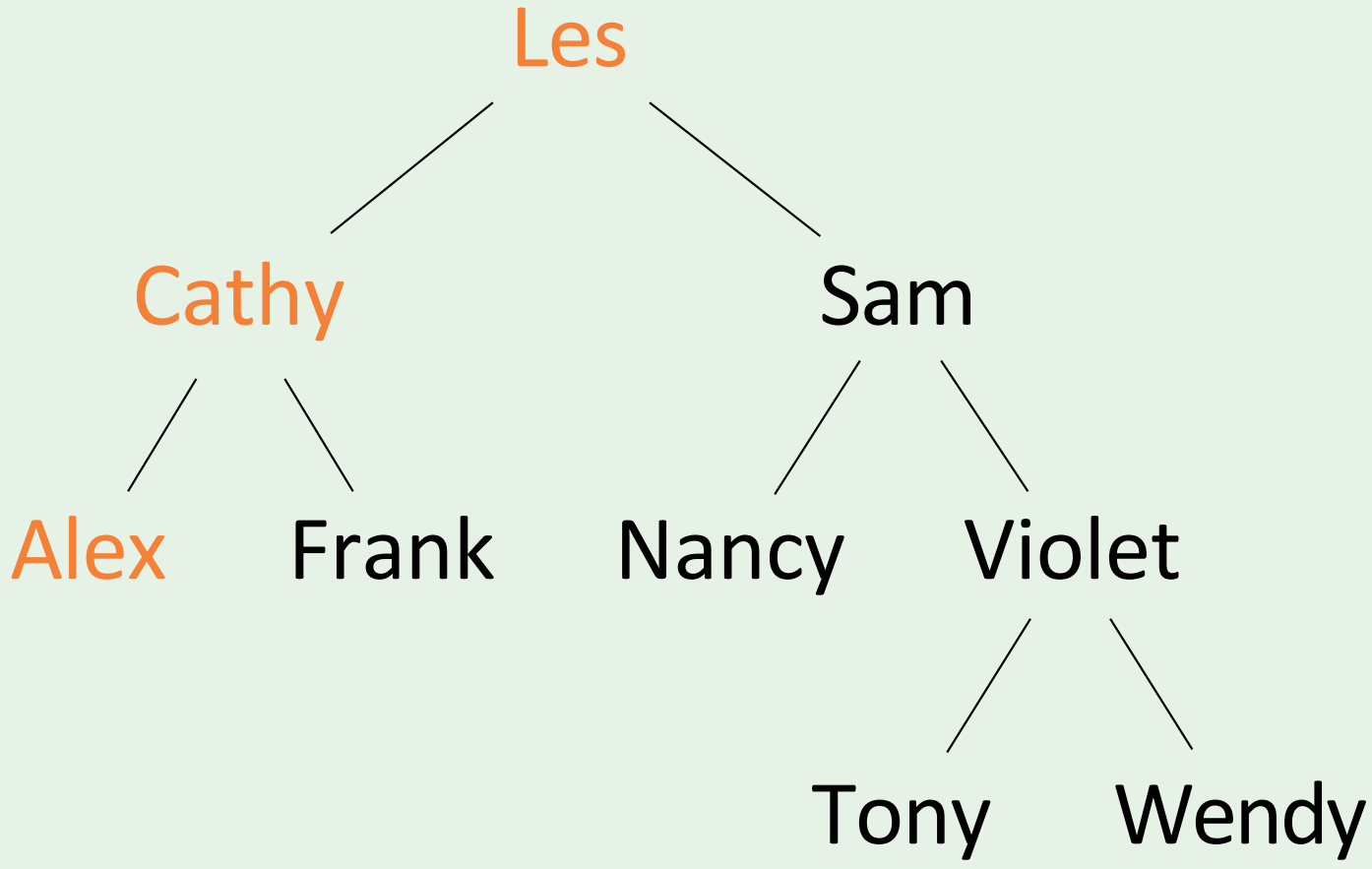
Output:

InOrderTraversal



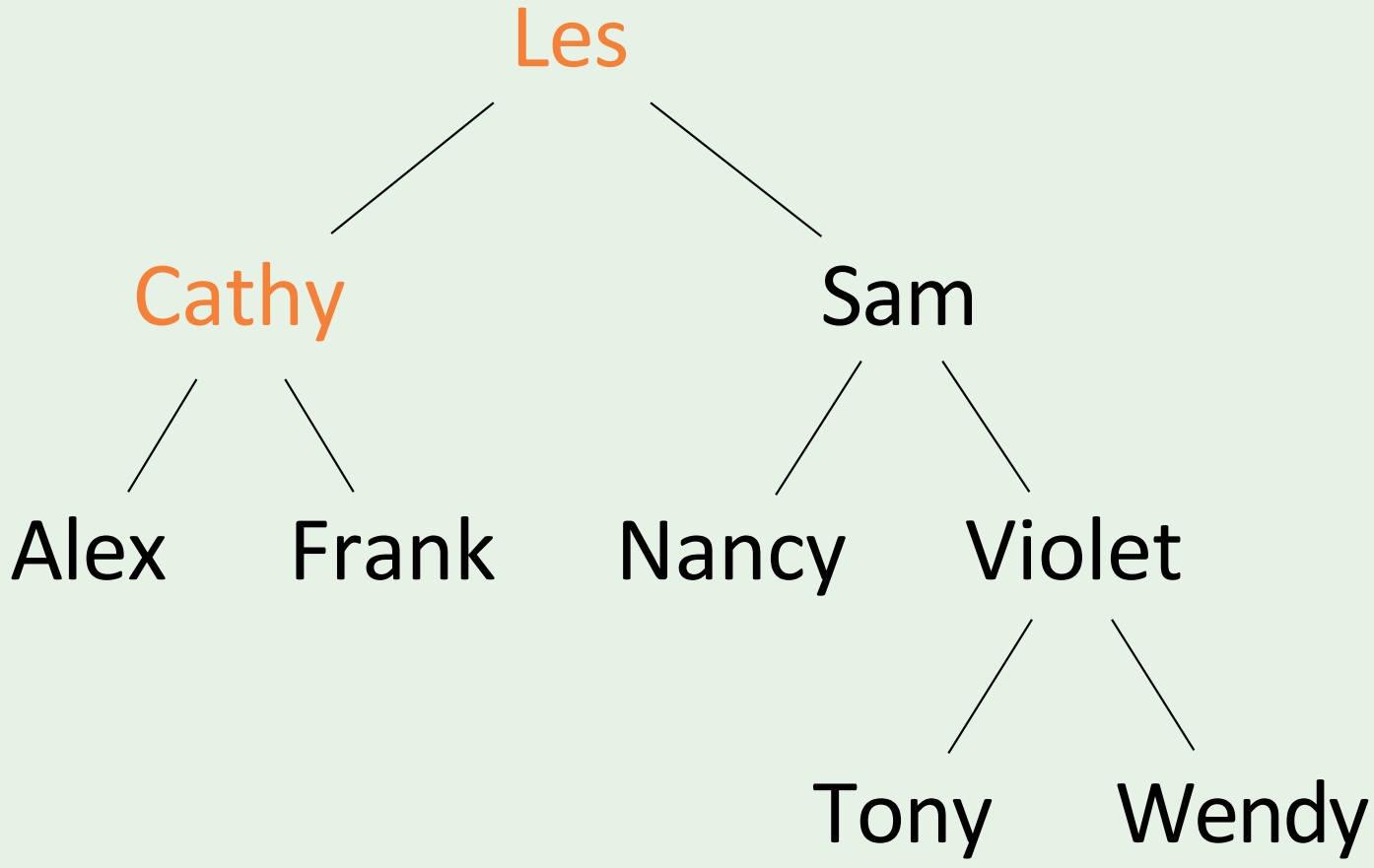
Output: Alex

InOrderTraversal



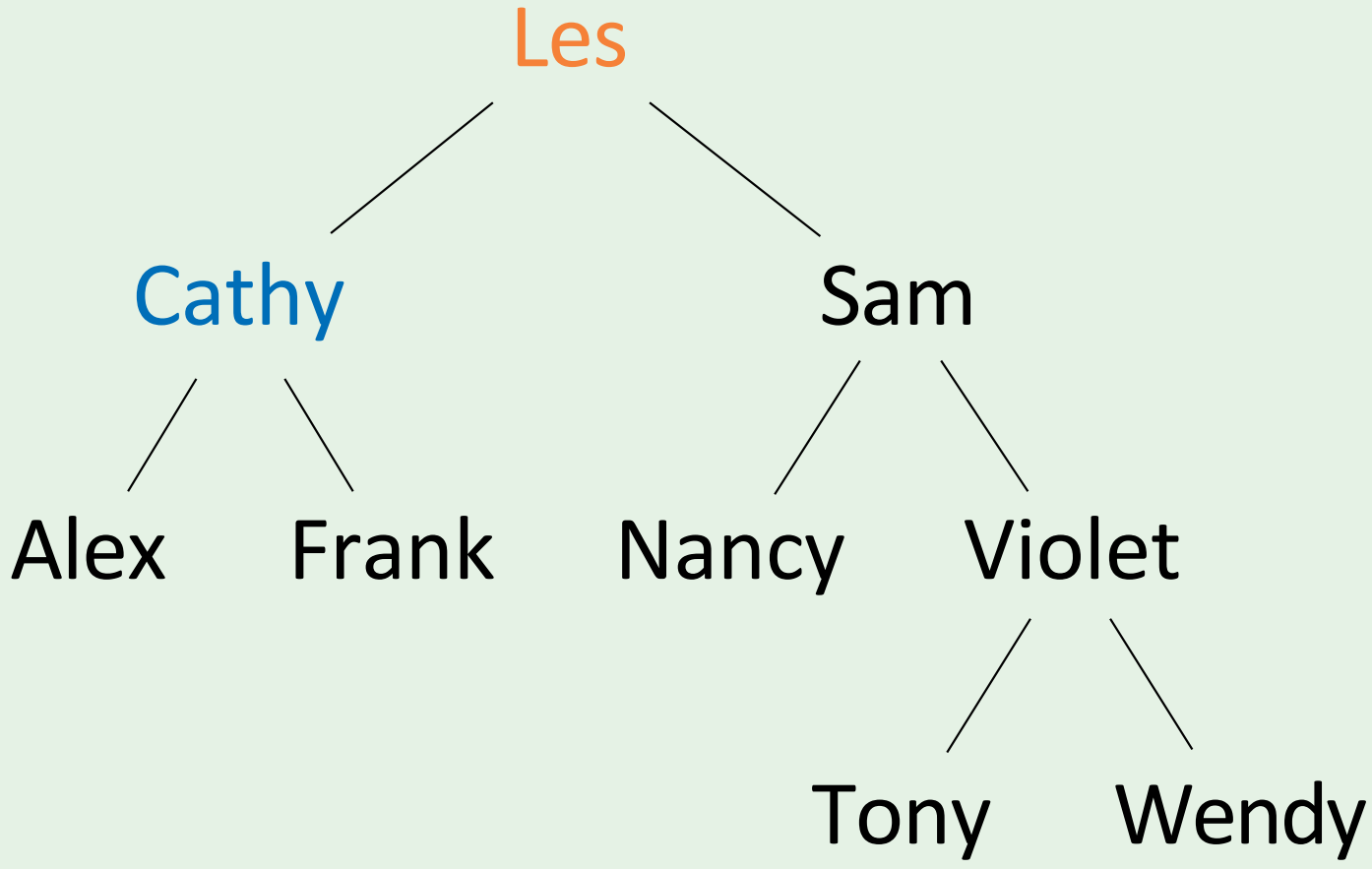
Output: Alex

InOrderTraversal



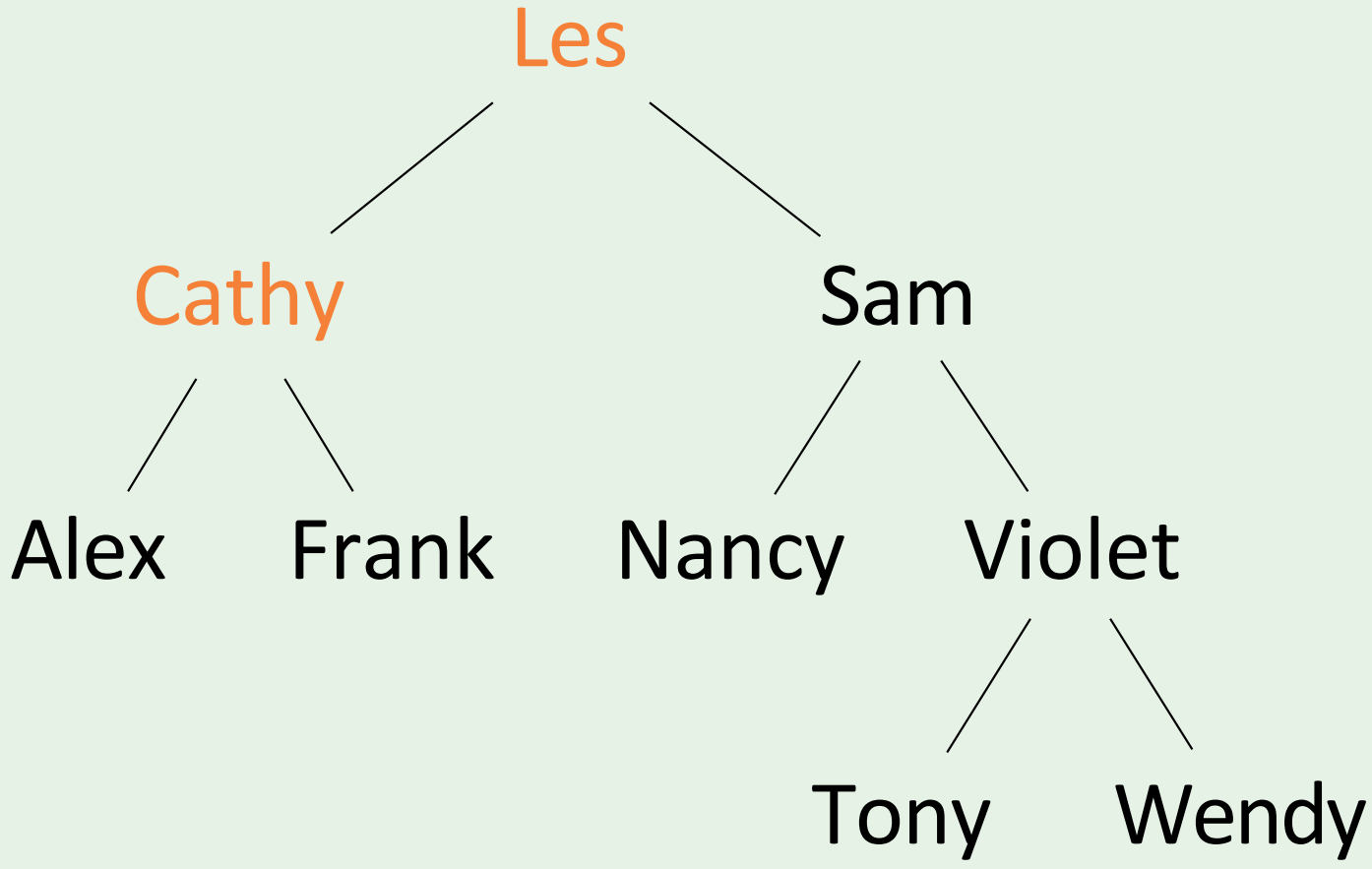
Output: Alex

InOrderTraversal



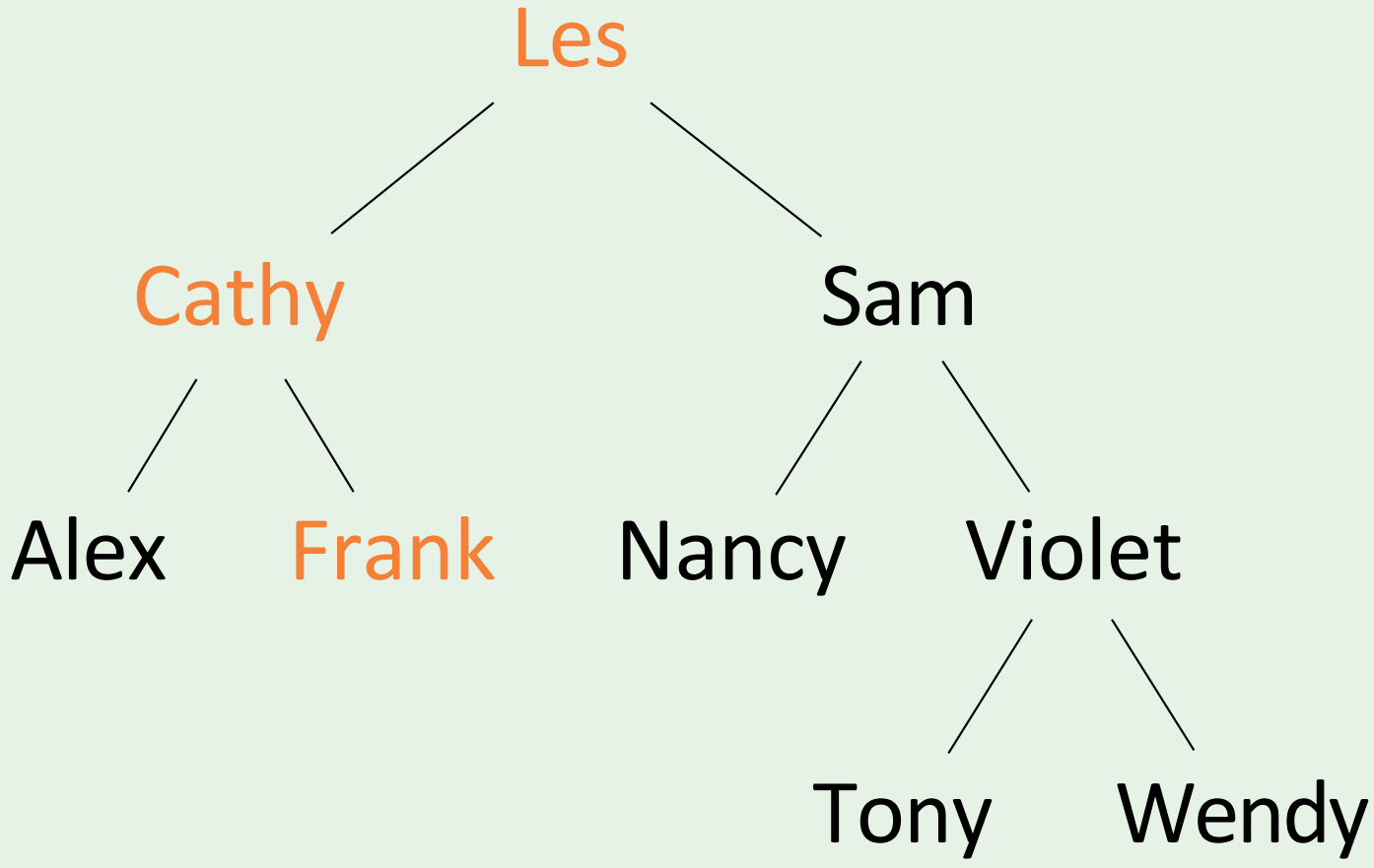
Output: Alex Cathy

InOrderTraversal



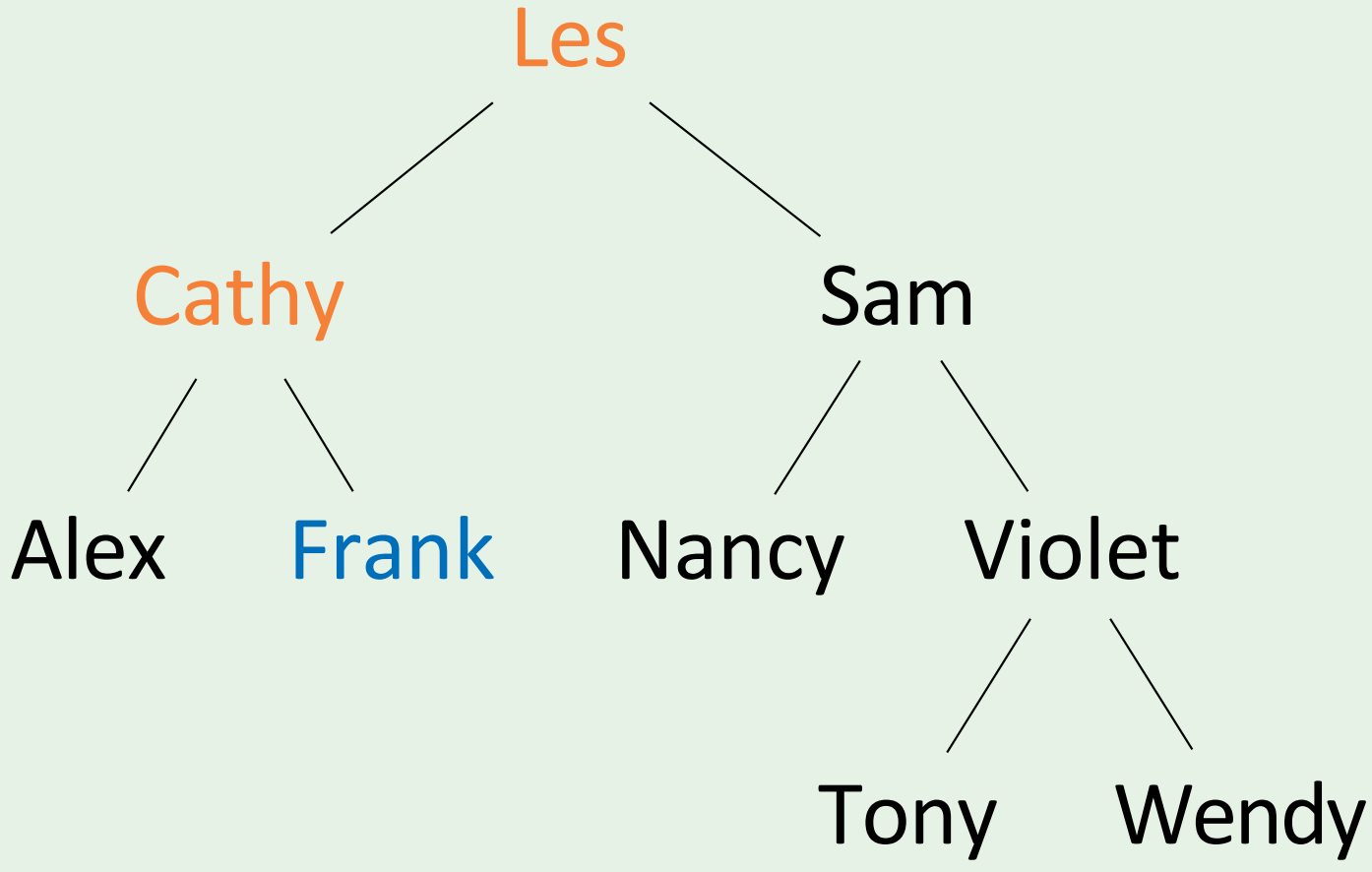
Output: Alex Cathy

InOrderTraversal



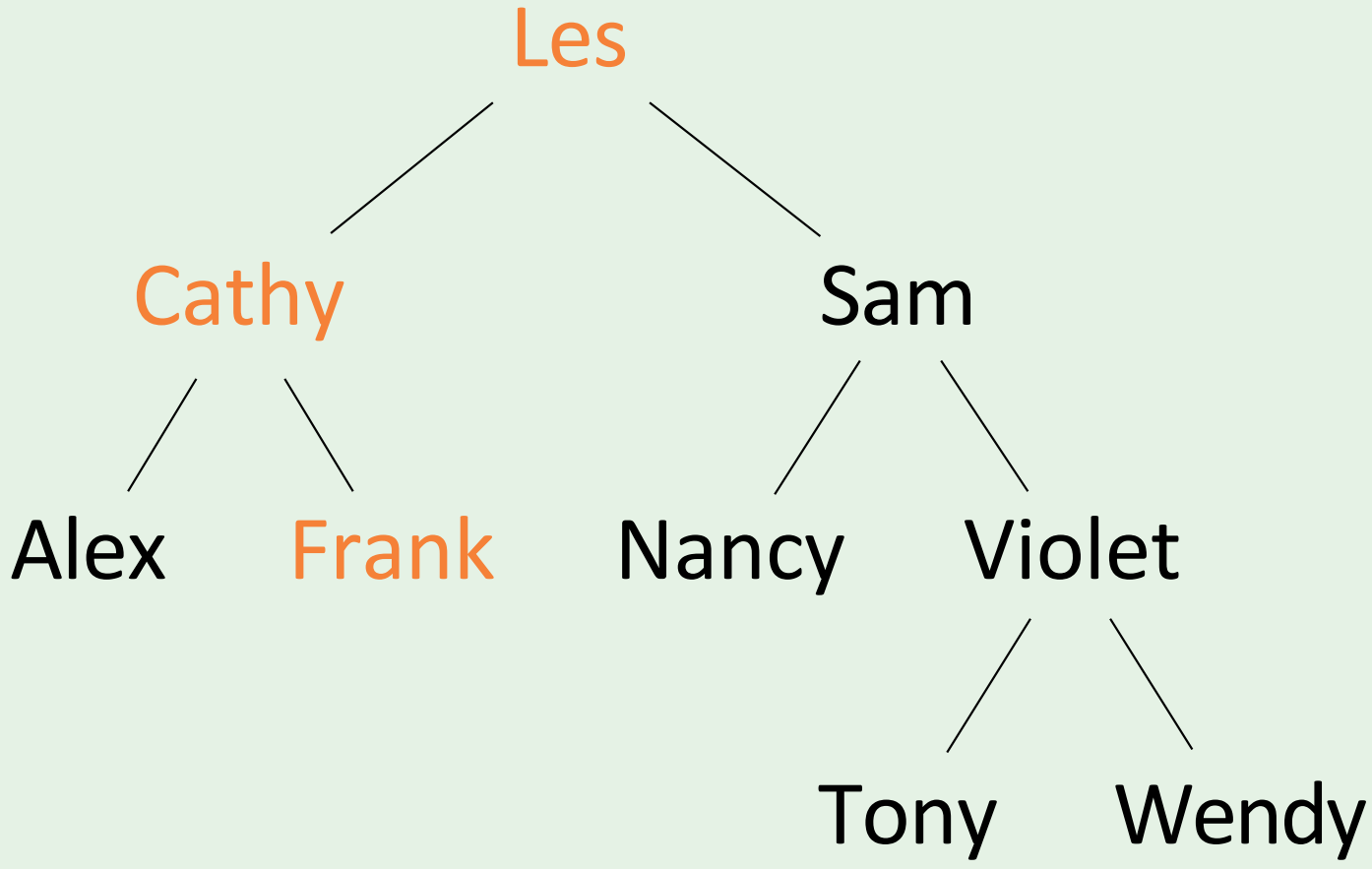
Output: Alex Cathy

InOrderTraversal



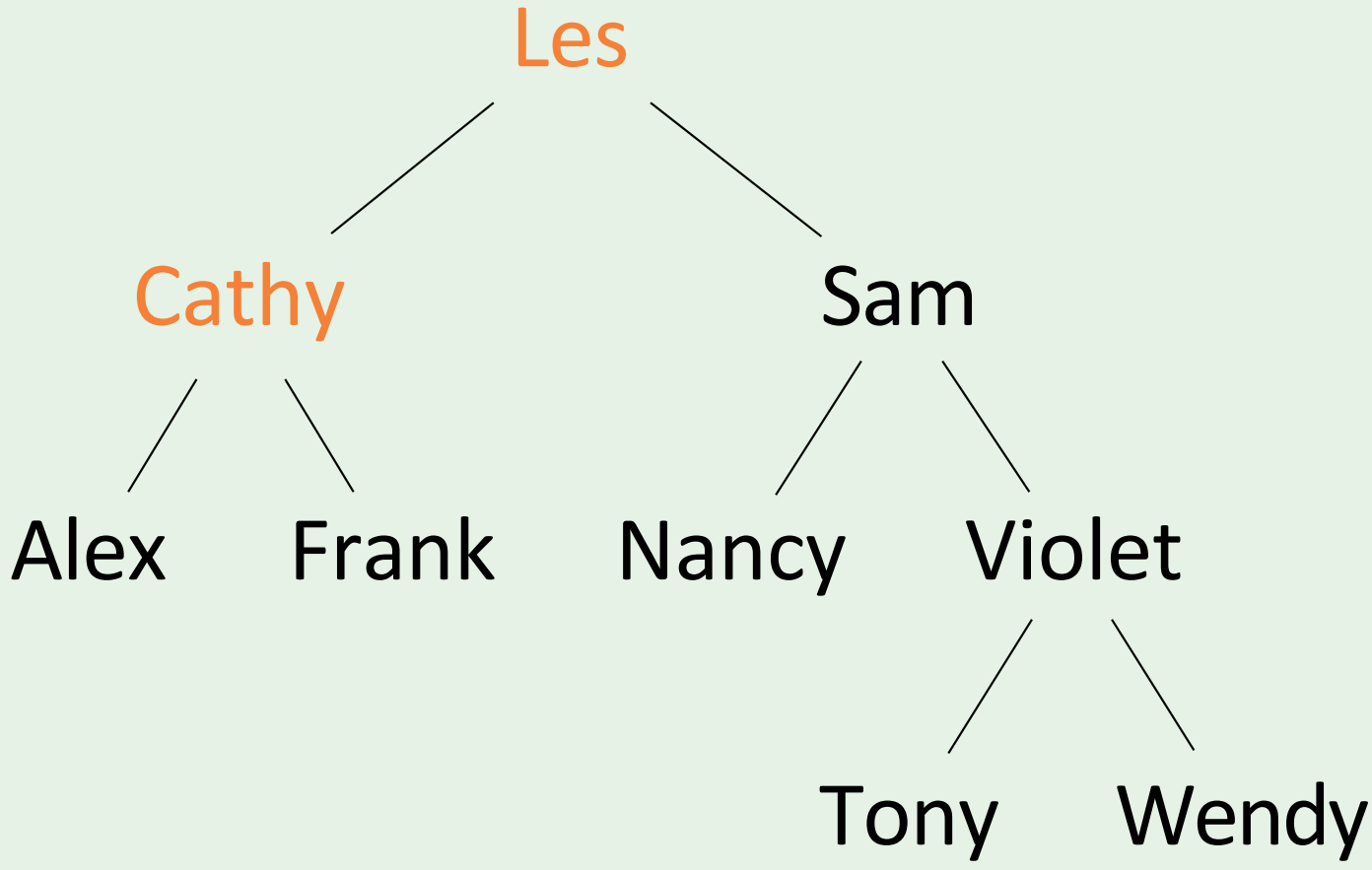
Output: Alex Cathy Frank

InOrderTraversal



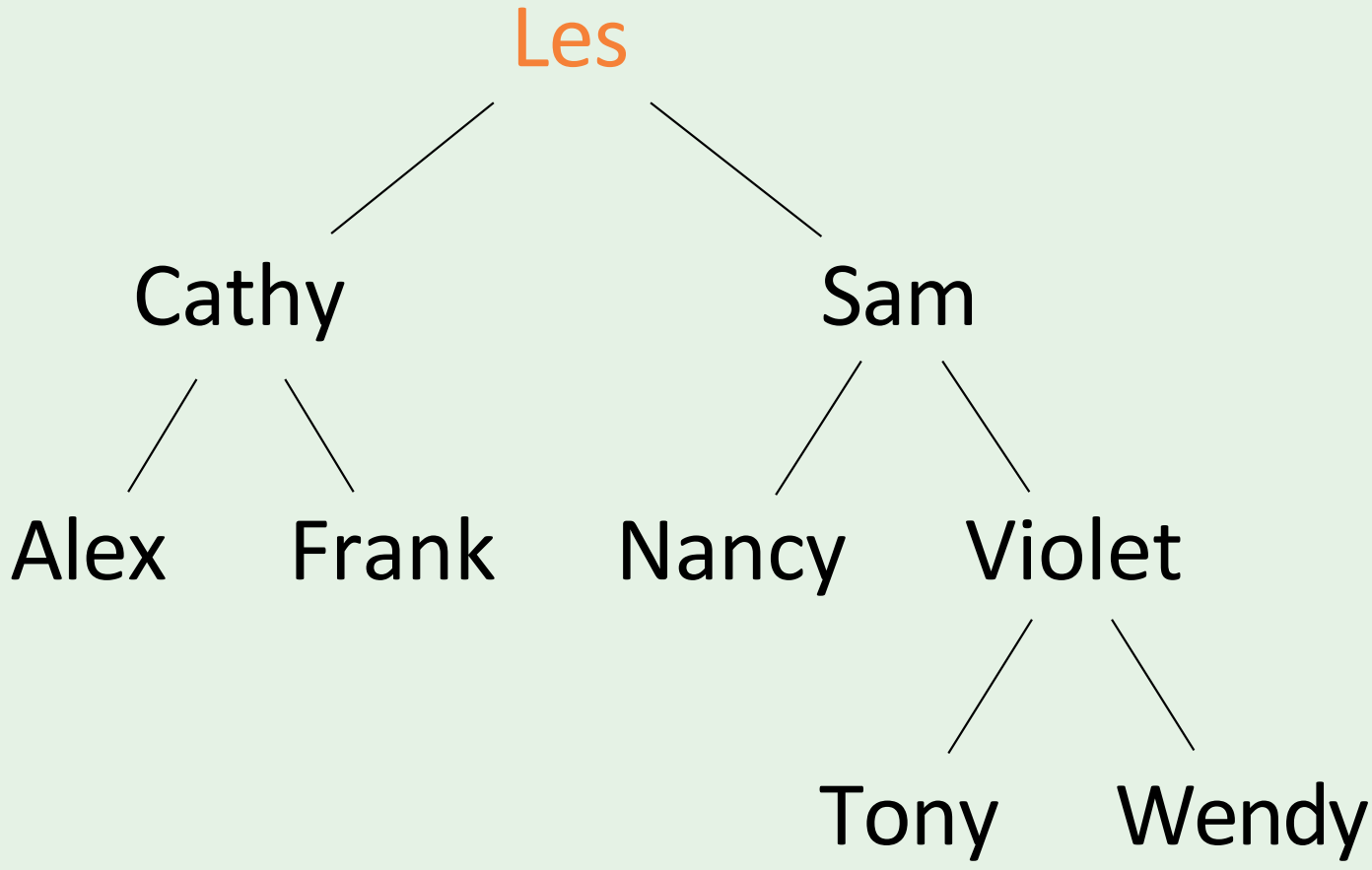
Output: Alex Cathy Frank

InOrderTraversal



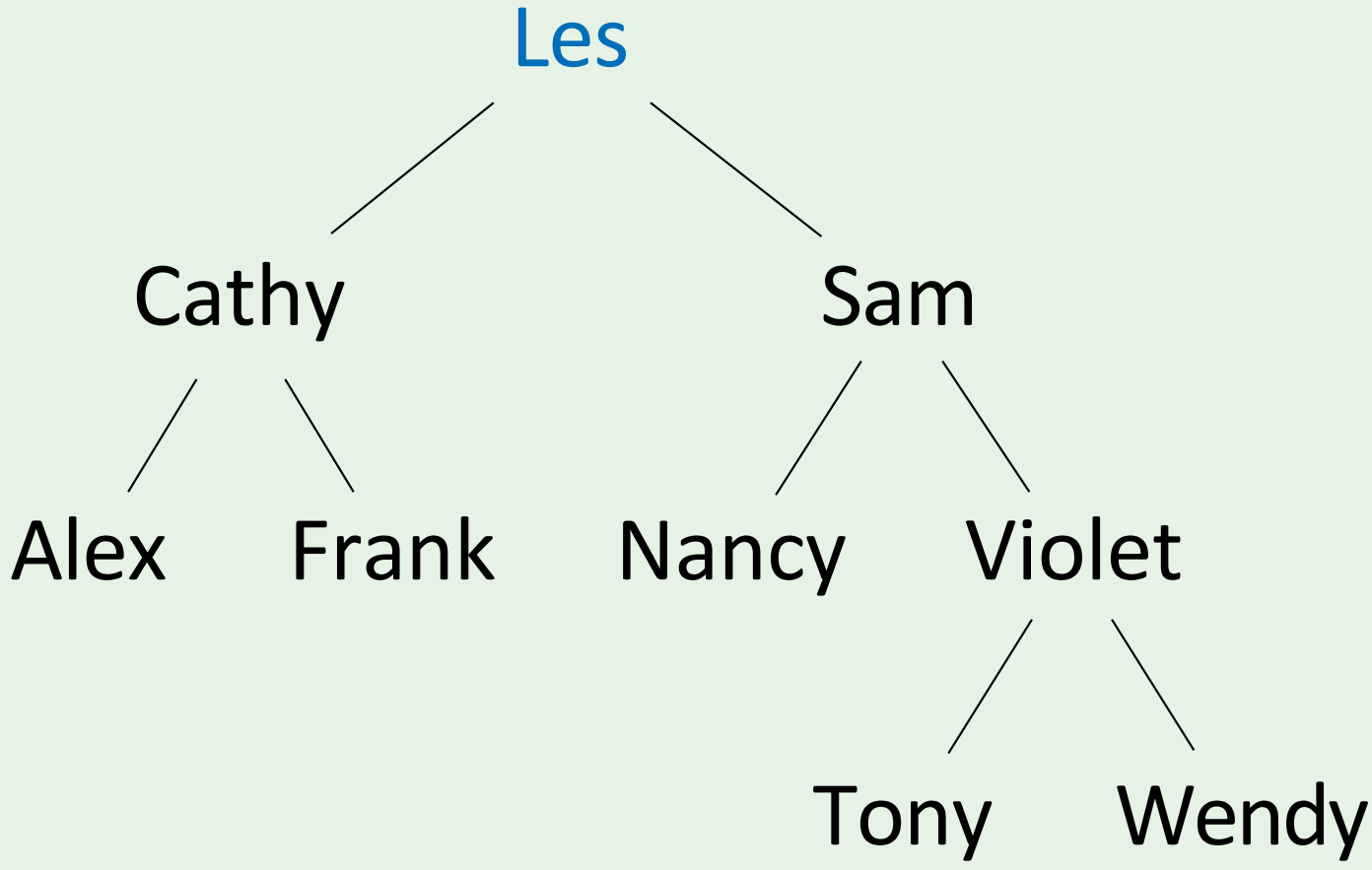
Output: Alex Cathy Frank

InOrderTraversal



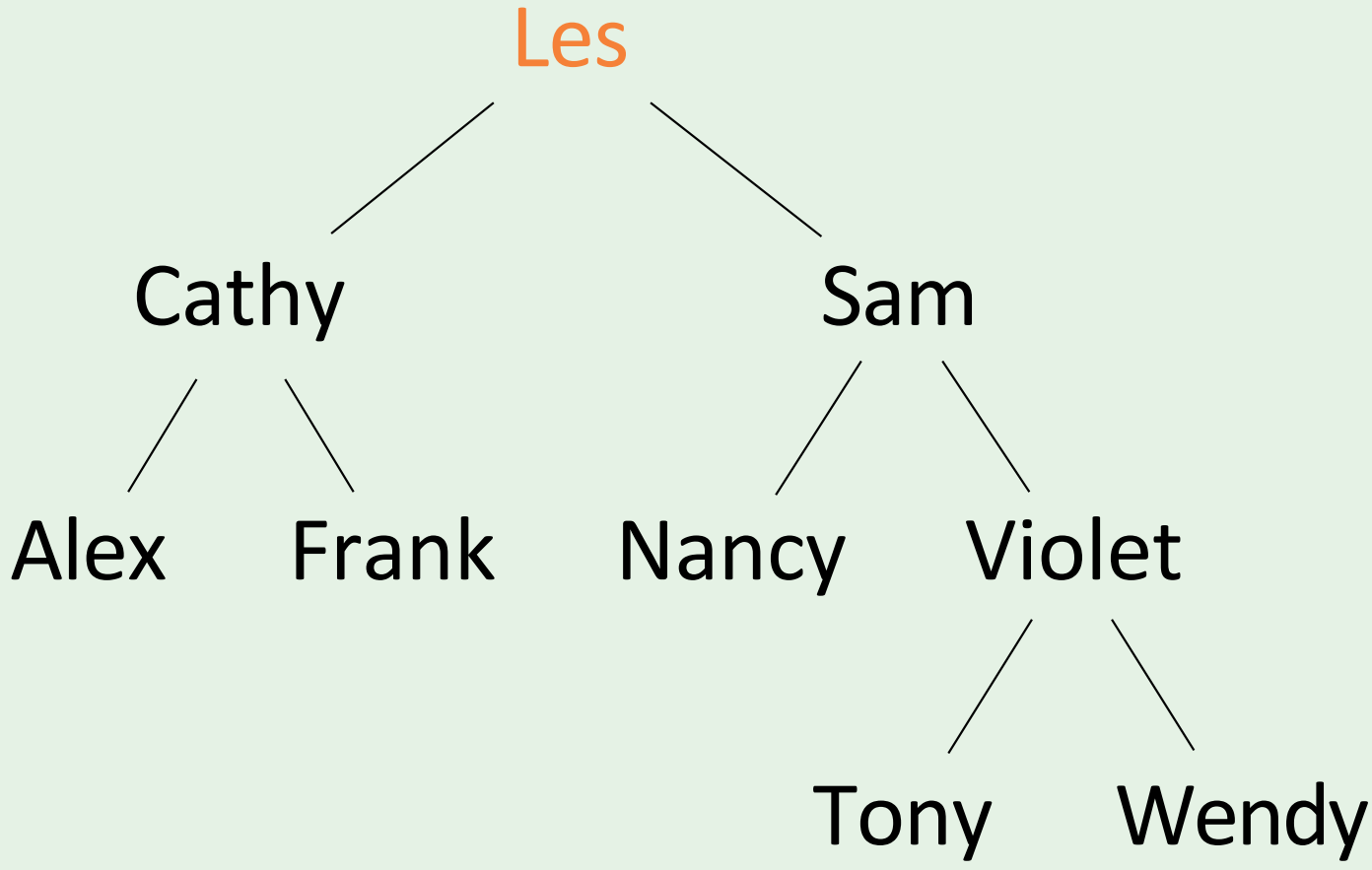
Output: Alex Cathy Frank

InOrderTraversal



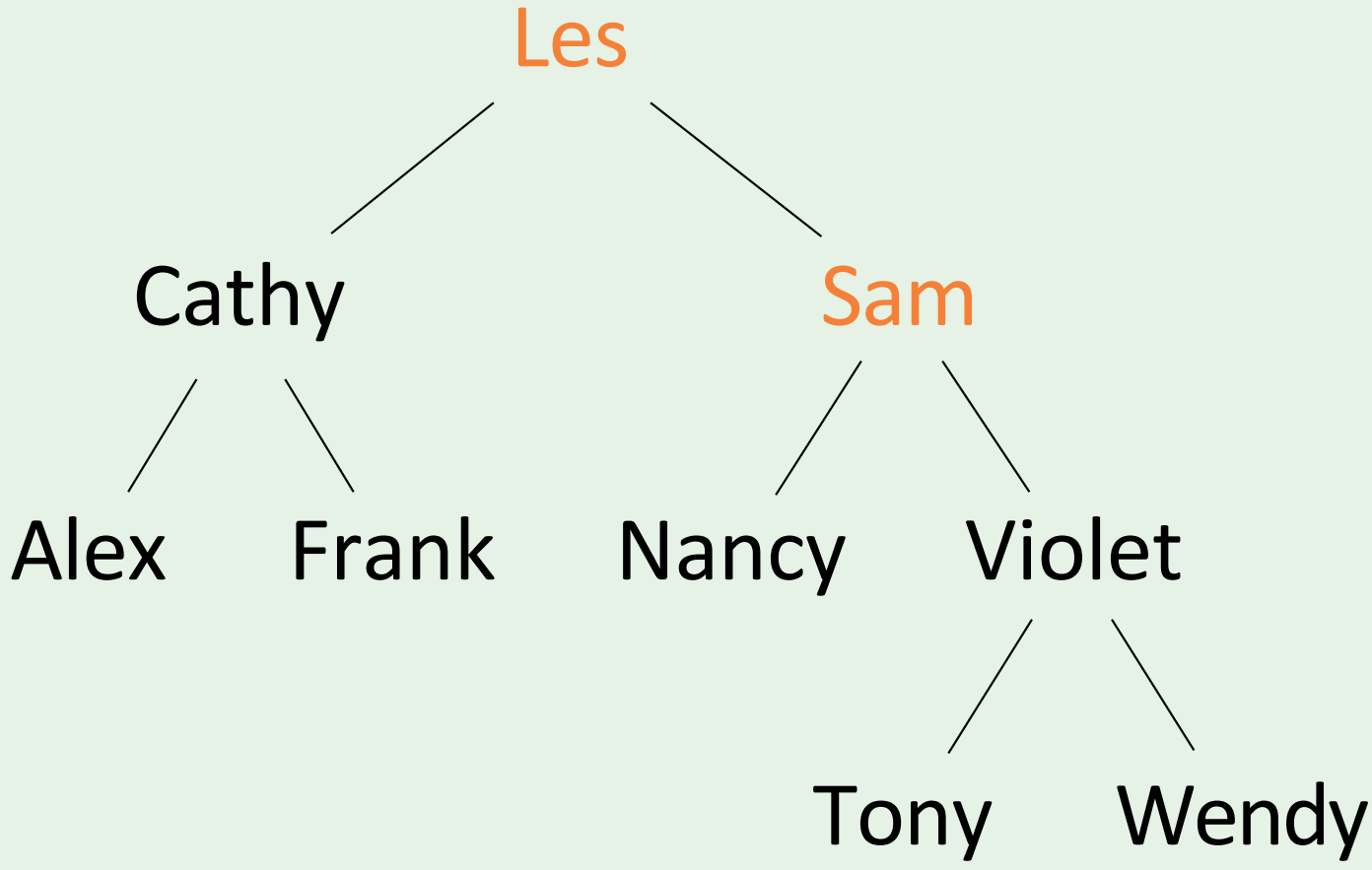
Output: Alex Cathy Frank Les

InOrderTraversal



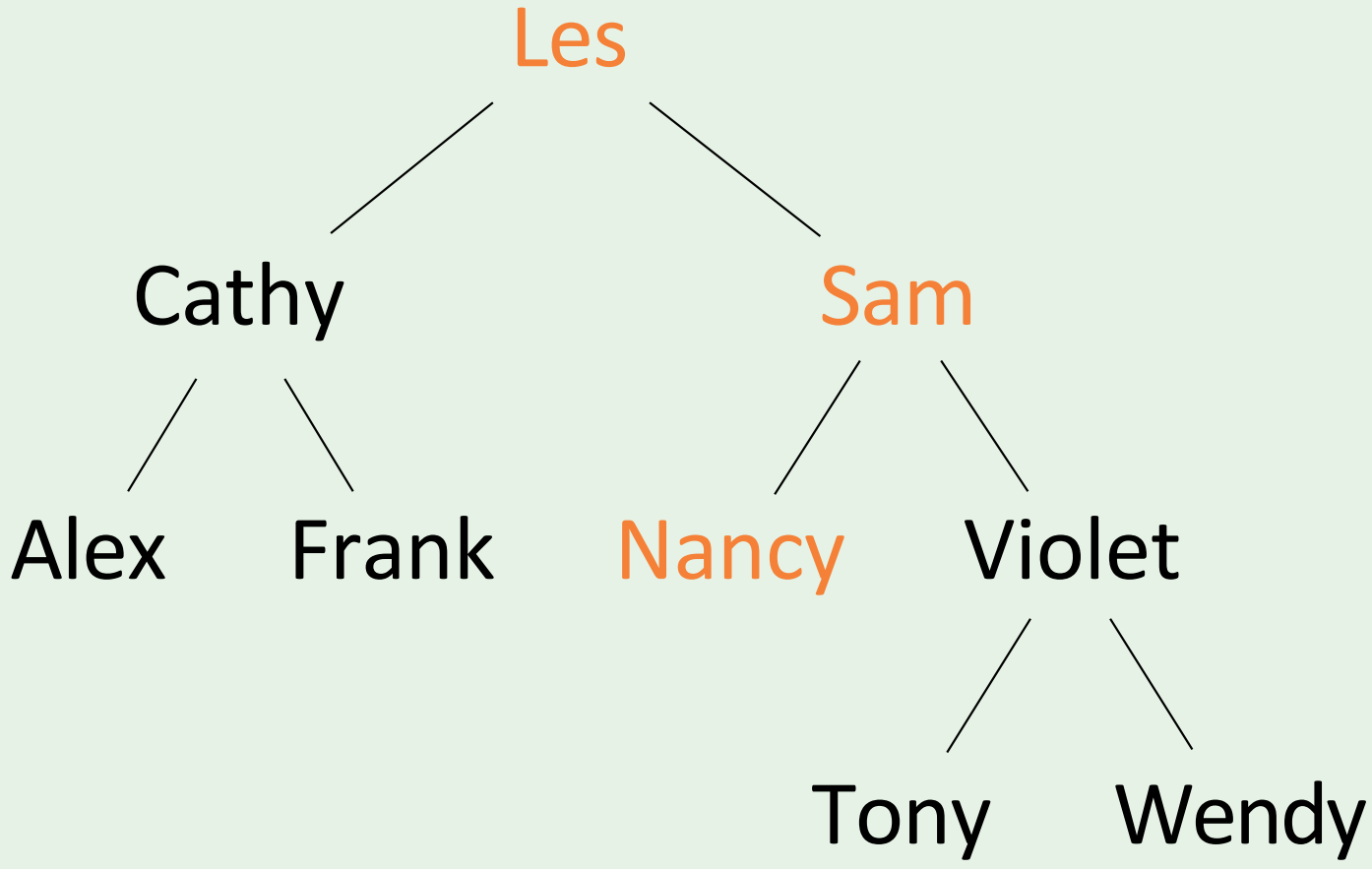
Output: Alex Cathy Frank Les

InOrderTraversal



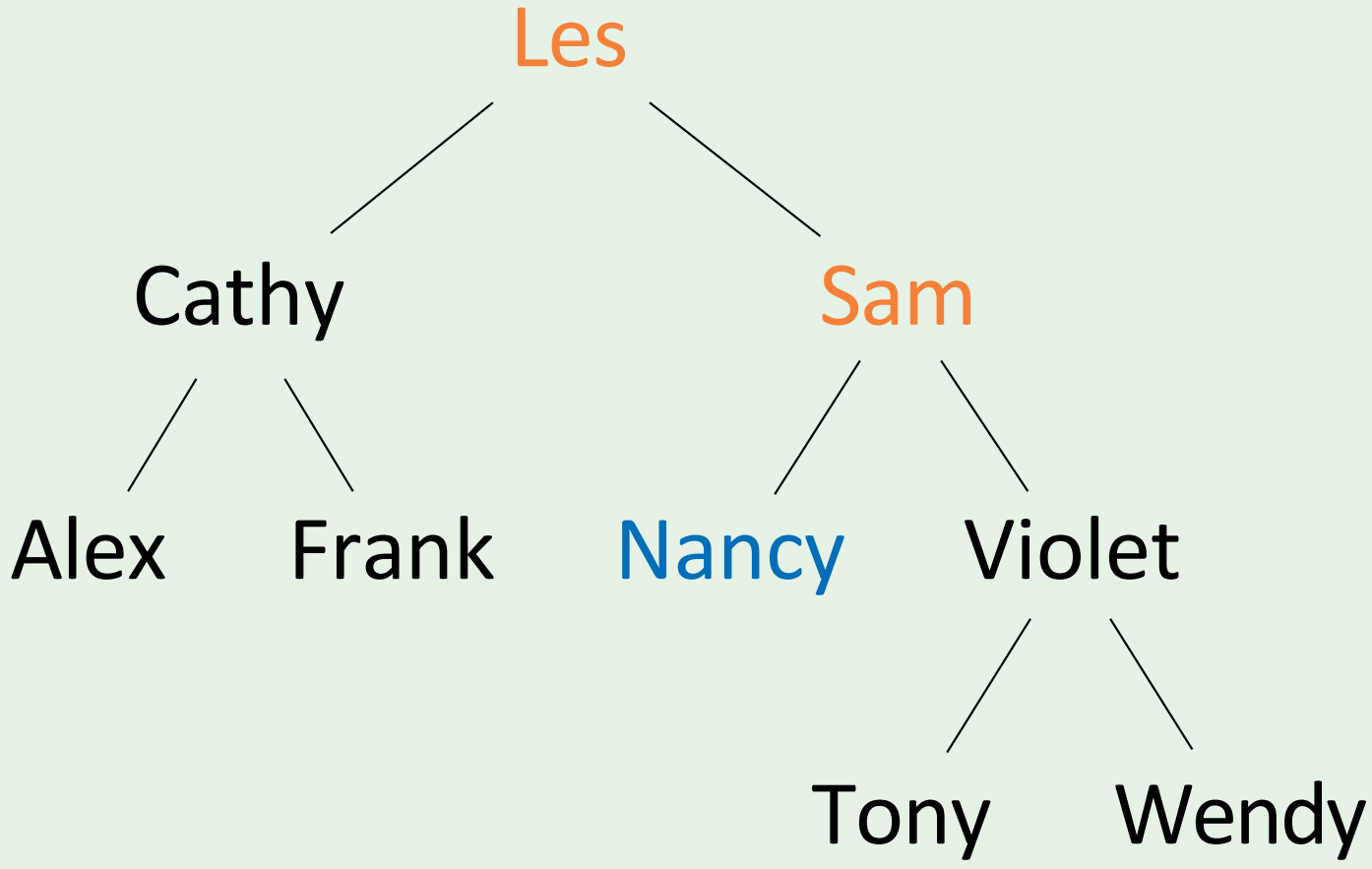
Output: Alex Cathy Frank Les

InOrderTraversal



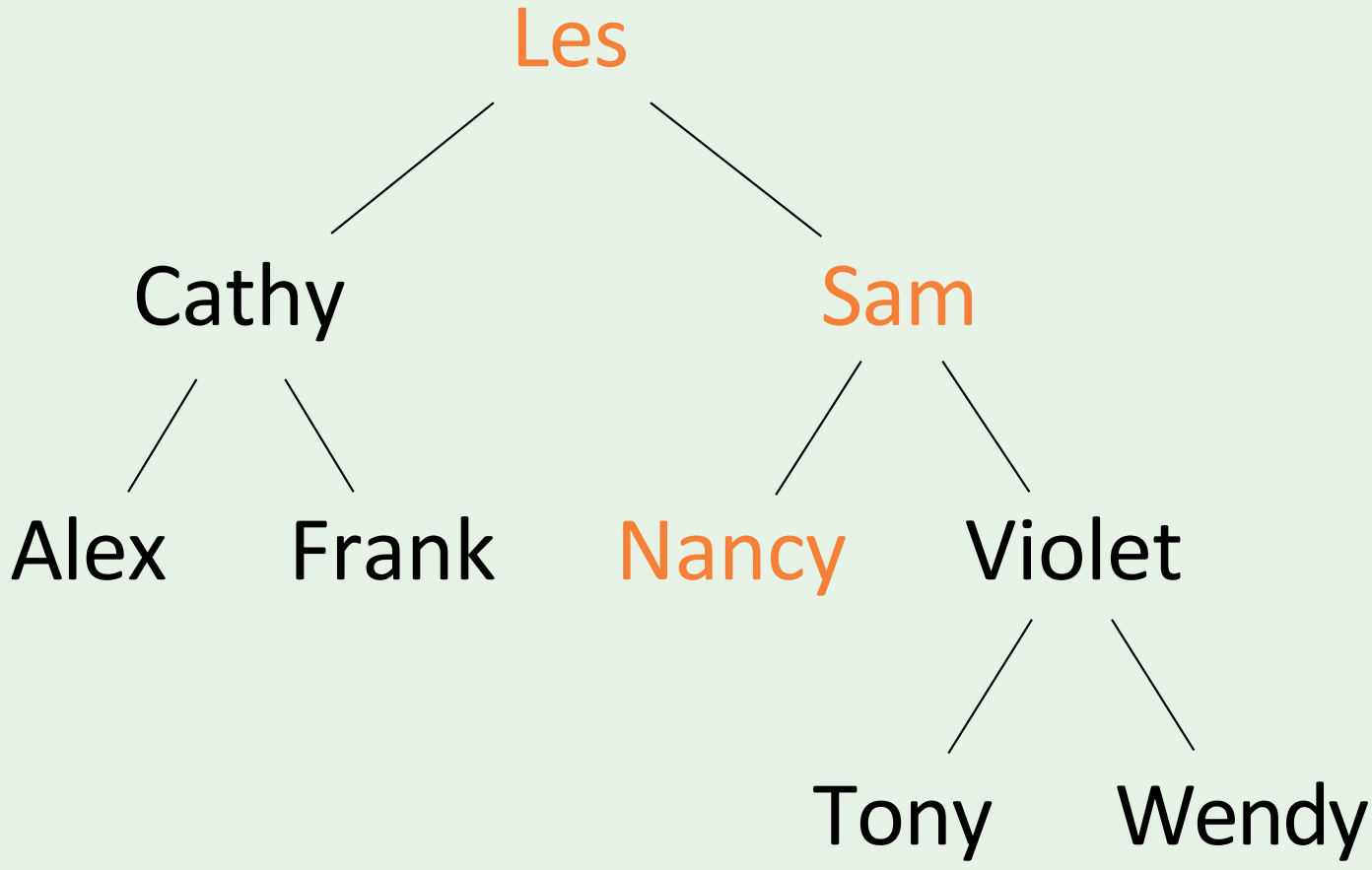
Output: Alex Cathy Frank Les

InOrderTraversal



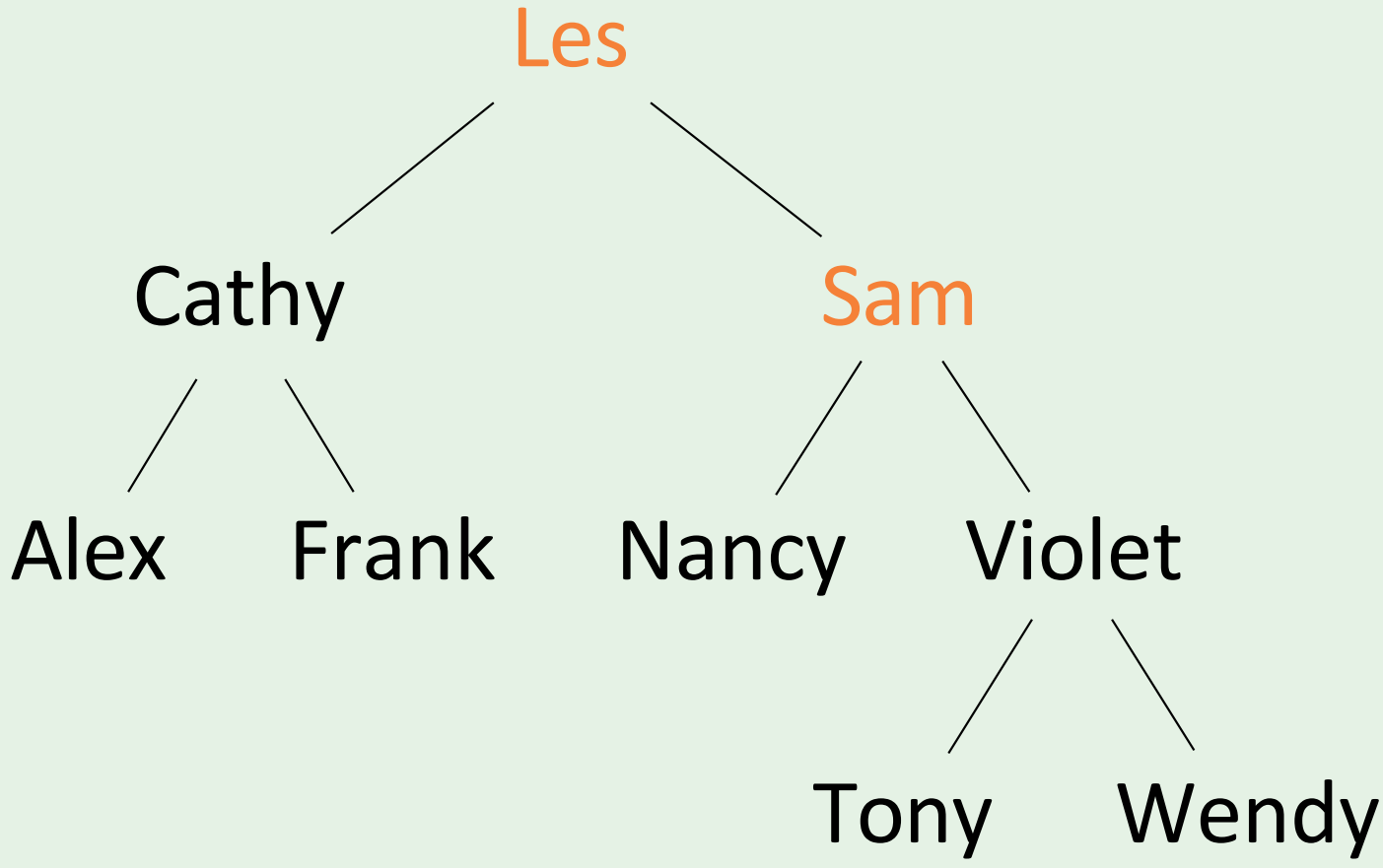
Output: Alex Cathy Frank Les Nancy

InOrderTraversal



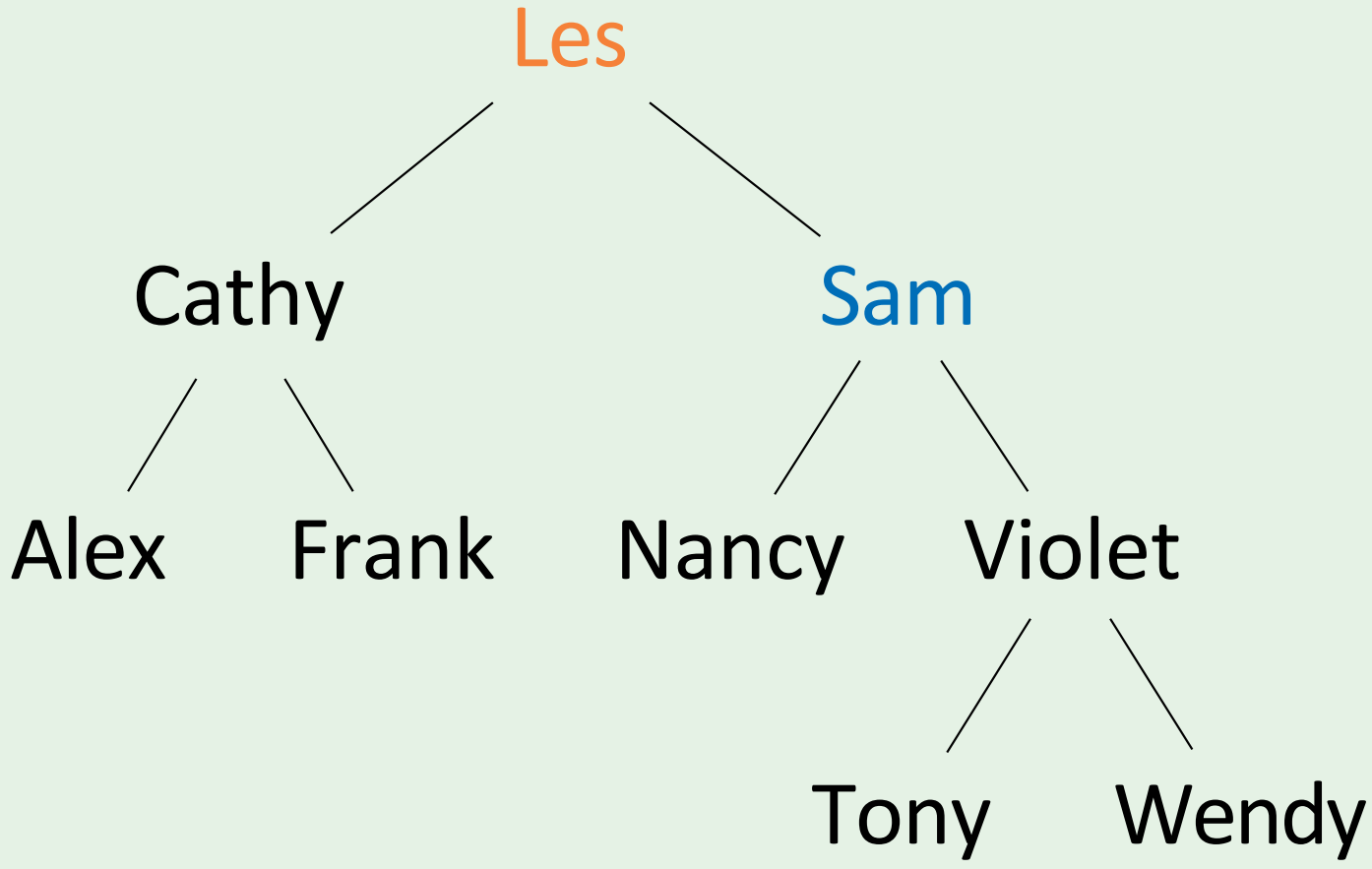
Output: Alex Cathy Frank Les Nancy

InOrderTraversal



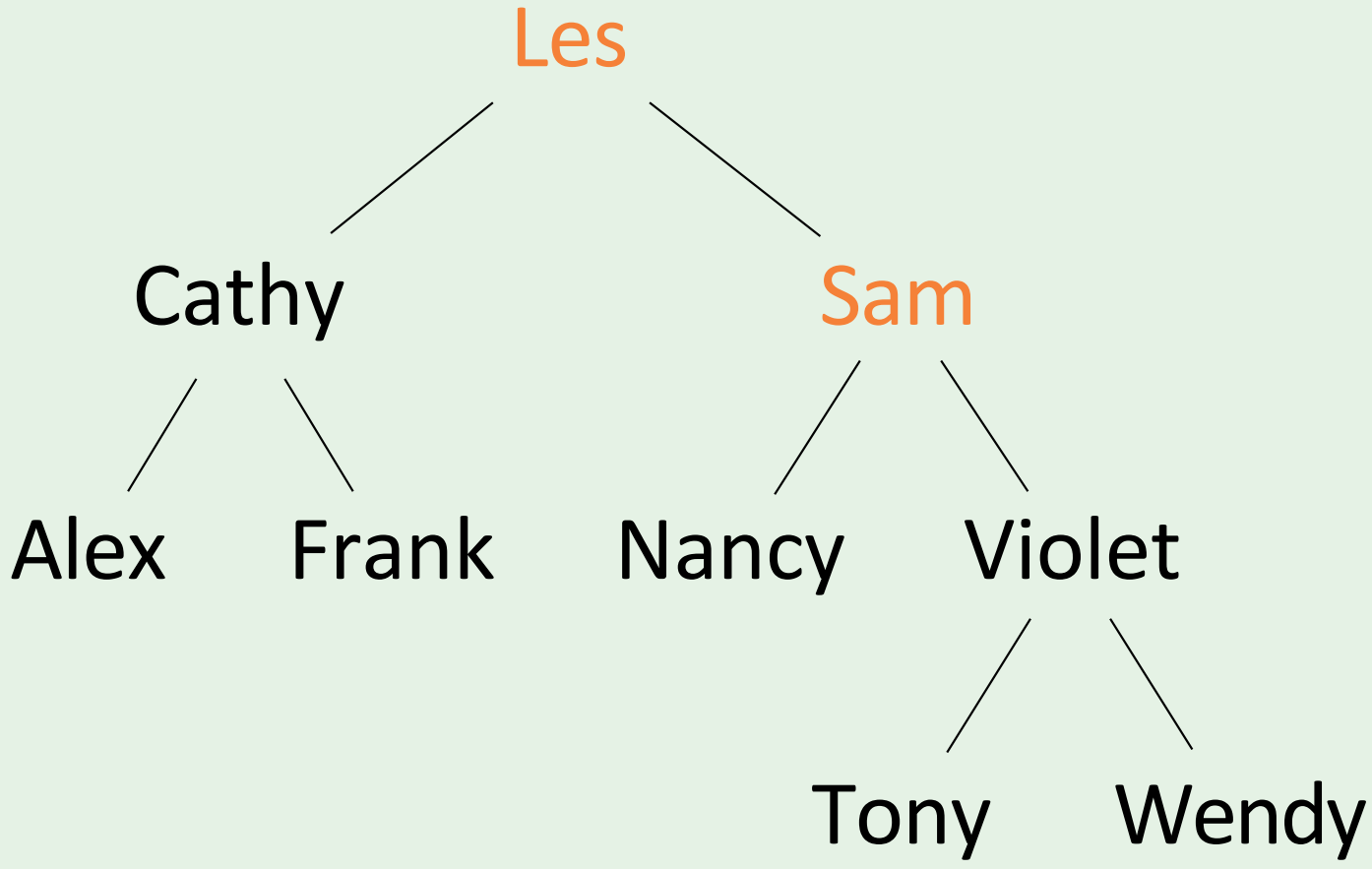
Output: Alex Cathy Frank Les Nancy

InOrderTraversal



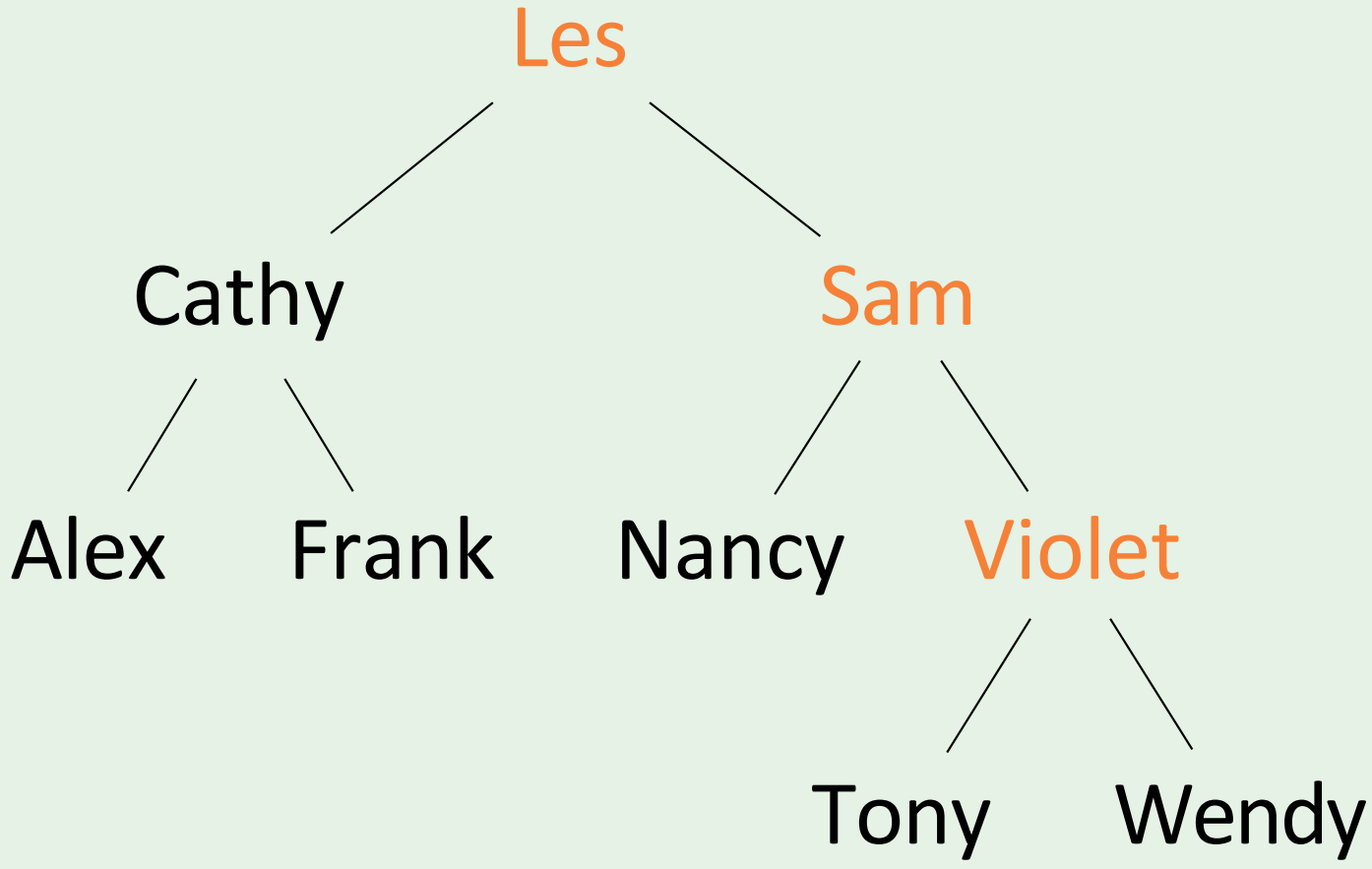
Output: Alex Cathy Frank Les Nancy Sam

InOrderTraversal



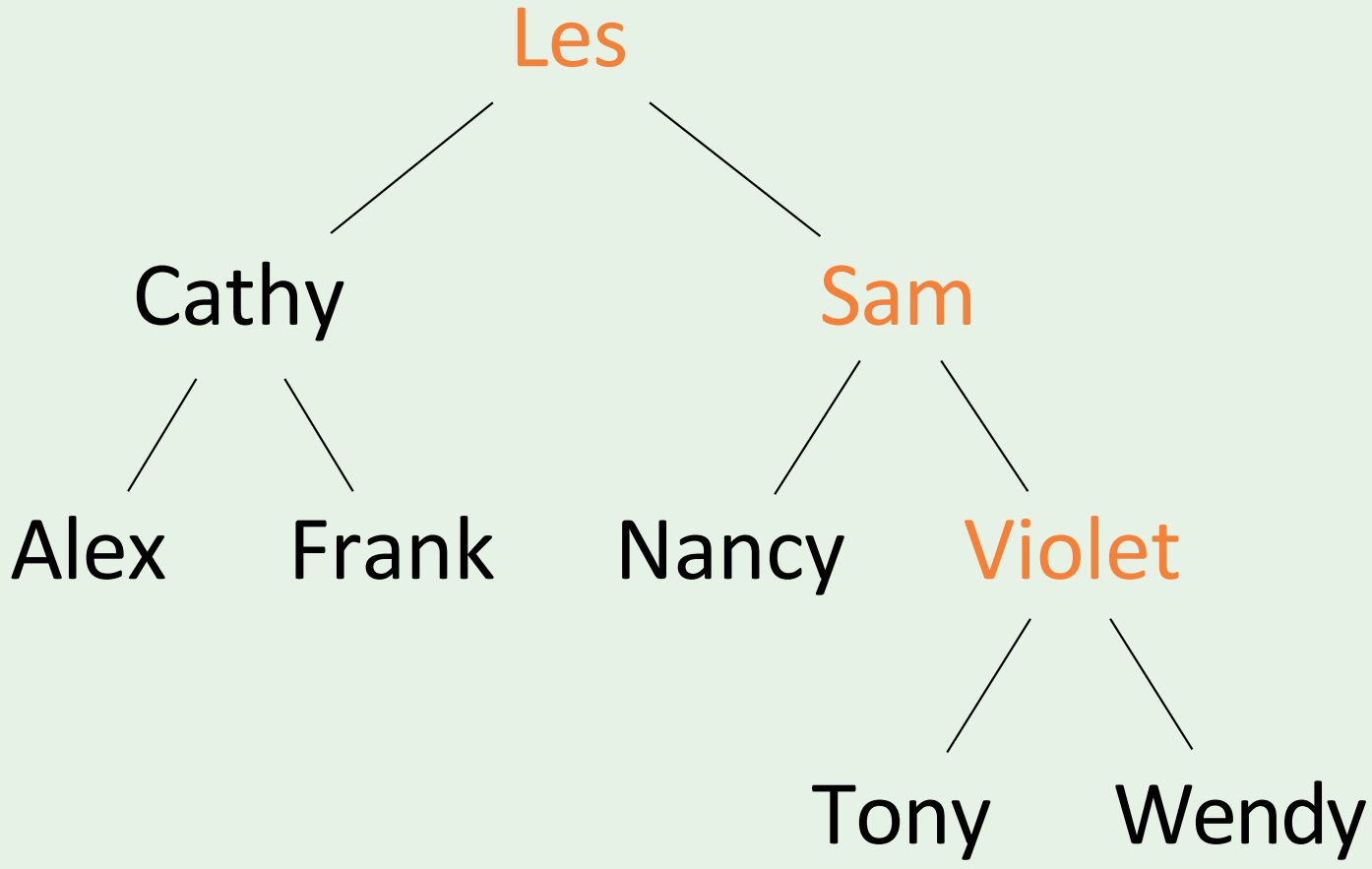
Output: Alex Cathy Frank Les Nancy Sam

InOrderTraversal



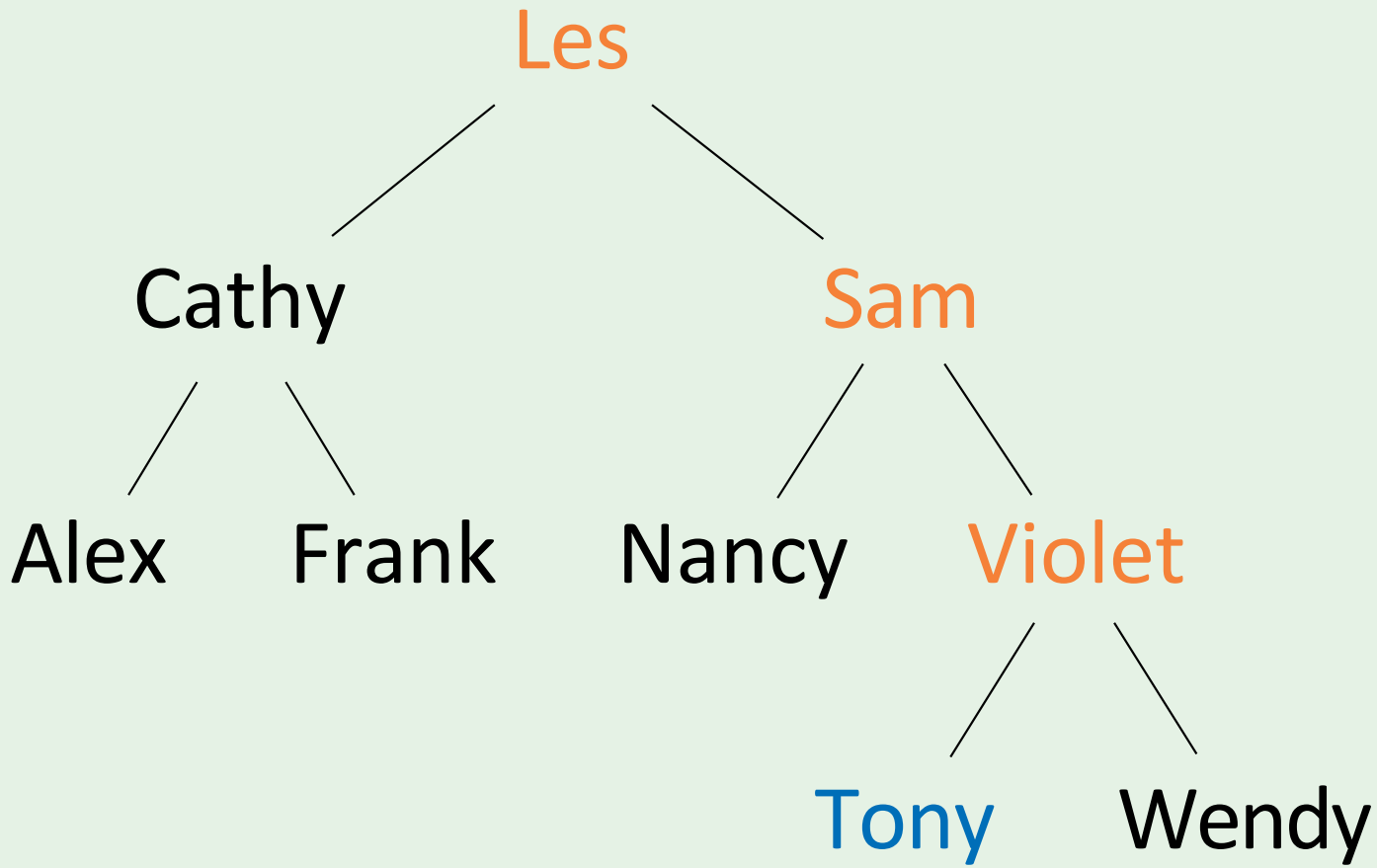
Output: Alex Cathy Frank Les Nancy Sam

InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam

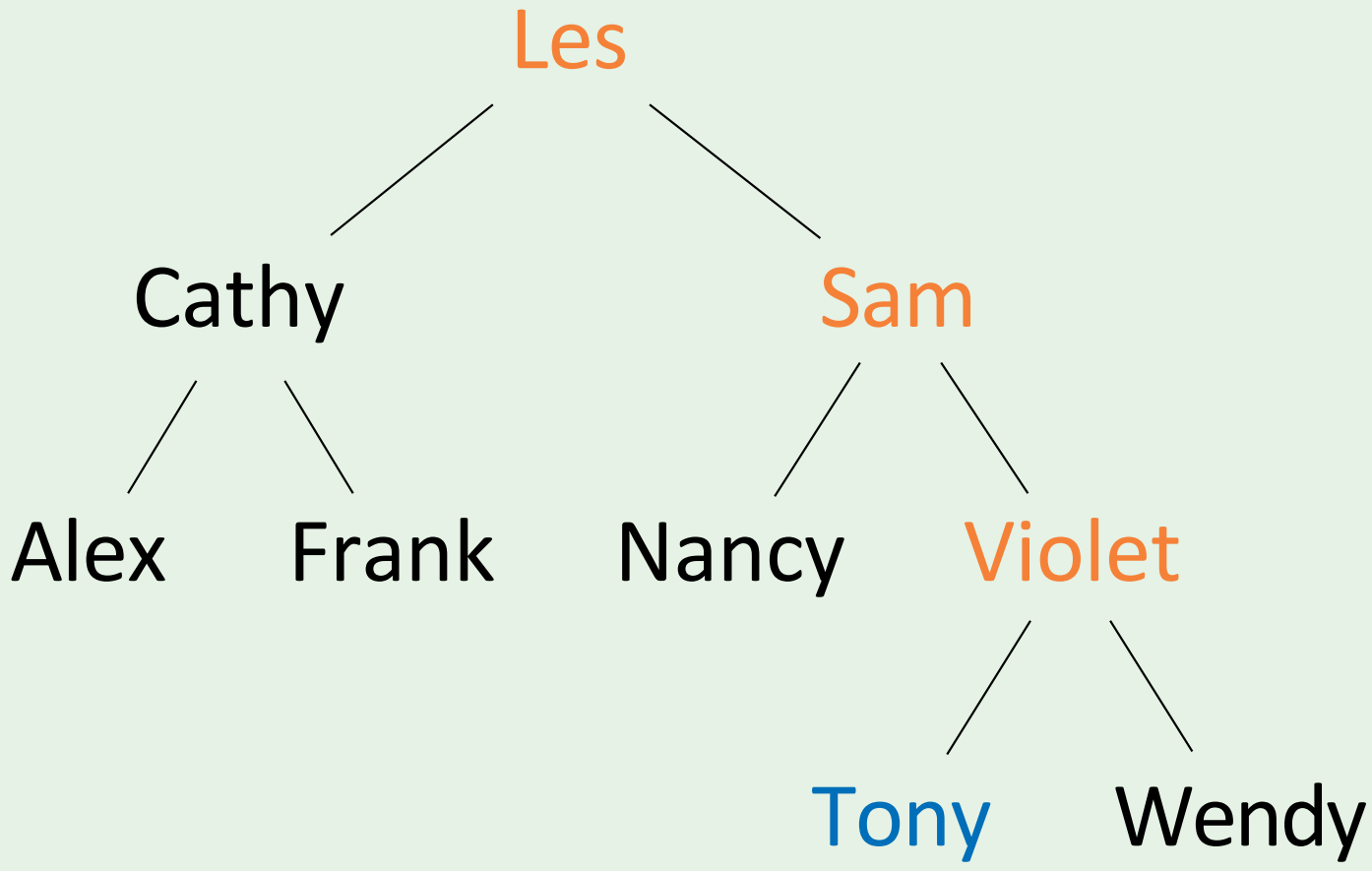
InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam

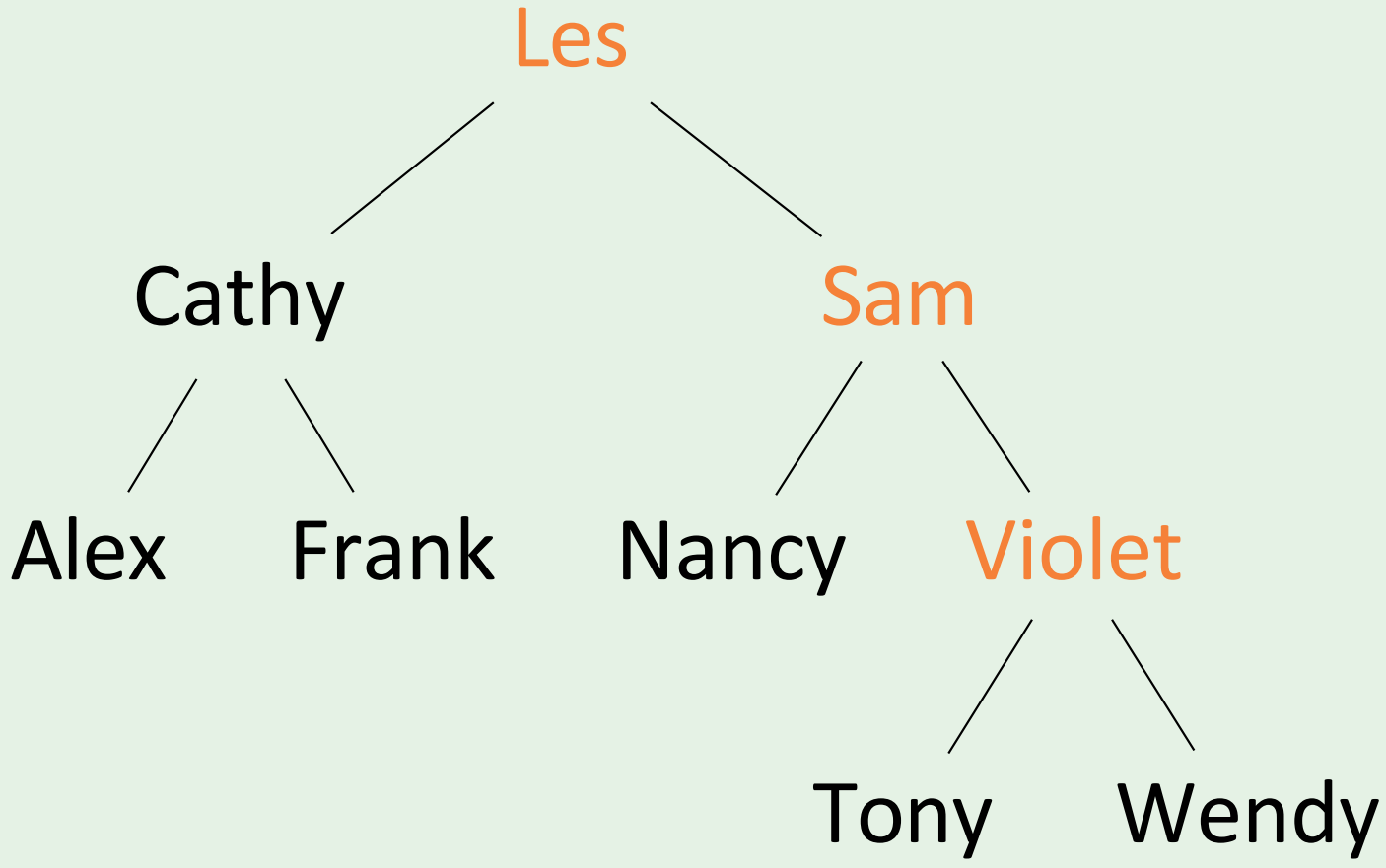
Tony

InOrderTraversal



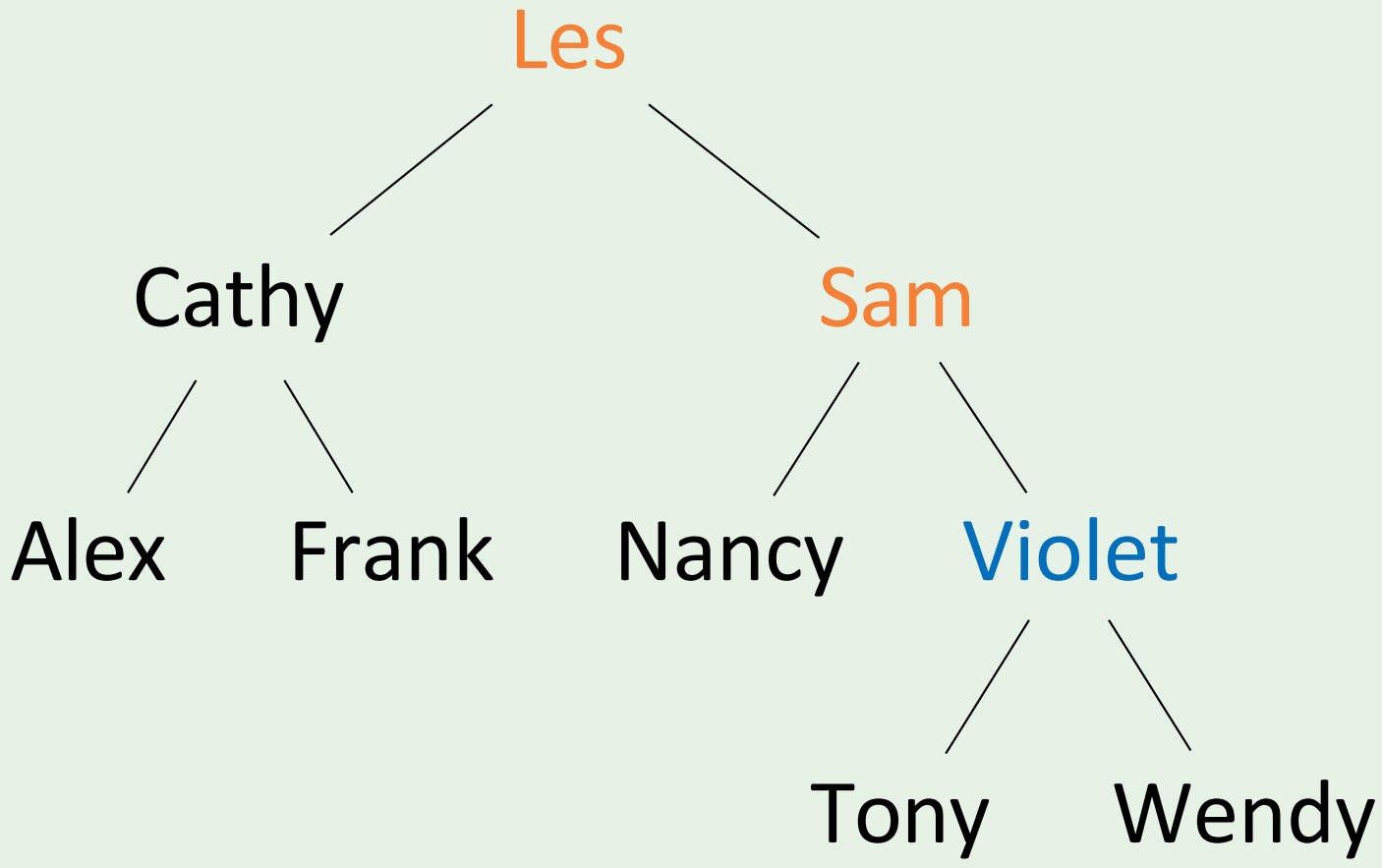
Output: Alex Cathy Frank Les Nancy Sam Tony

InOrderTraversal



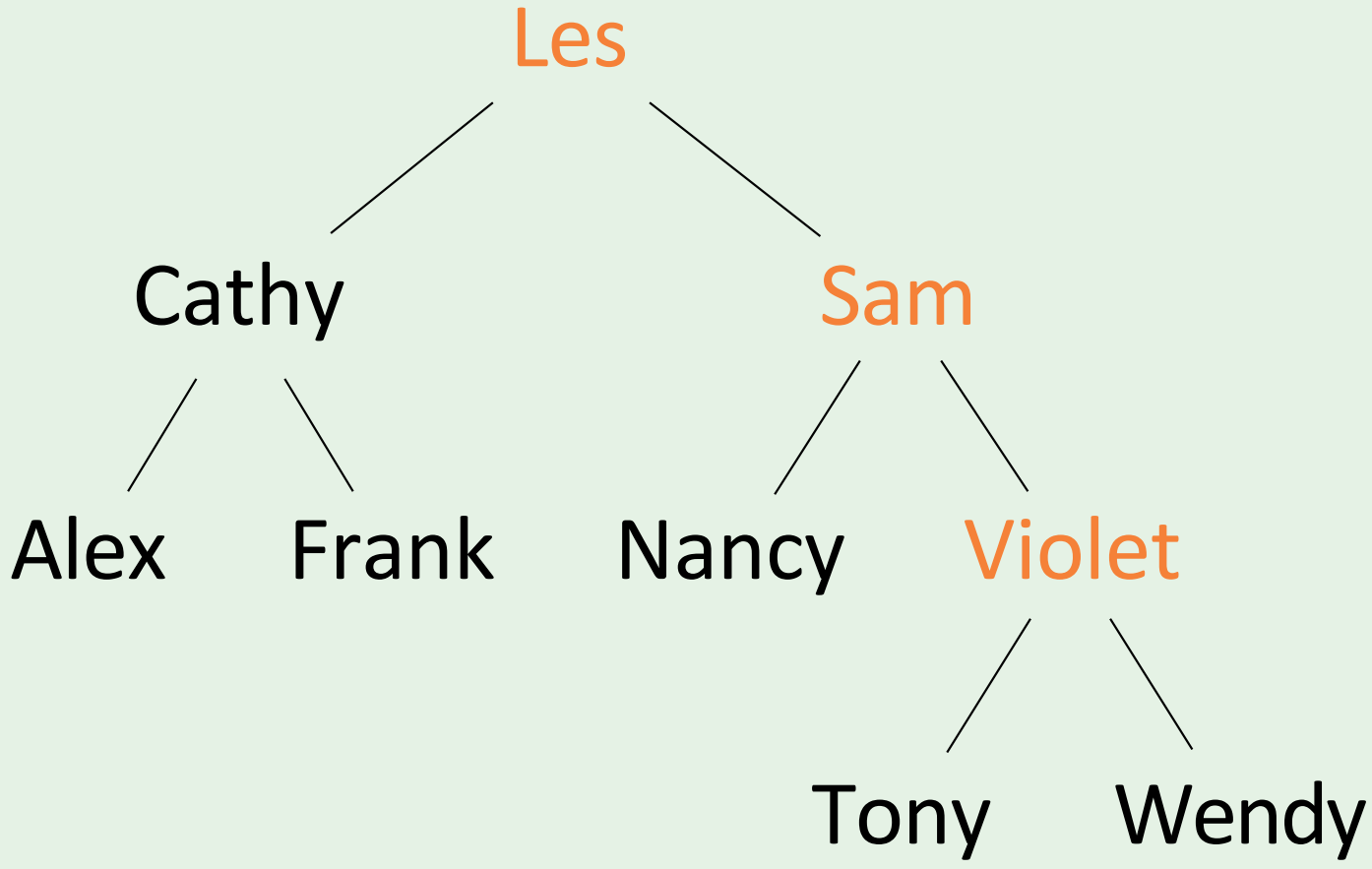
Output: Alex Cathy Frank Les Nancy Tony Sam

InOrderTraversal



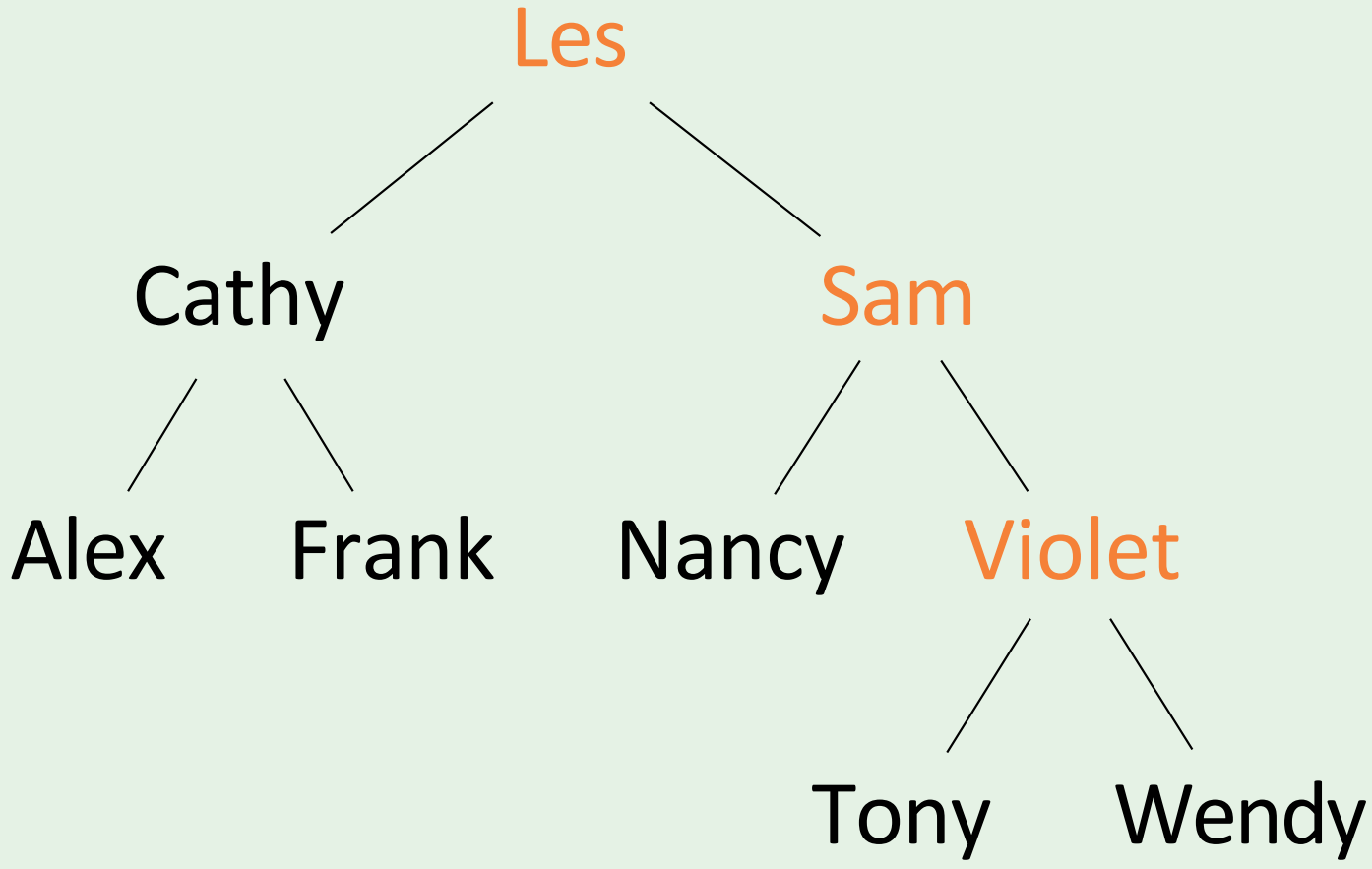
Output: Alex Cathy Frank Les Nancy Sam Tony Violet

InOrderTraversal



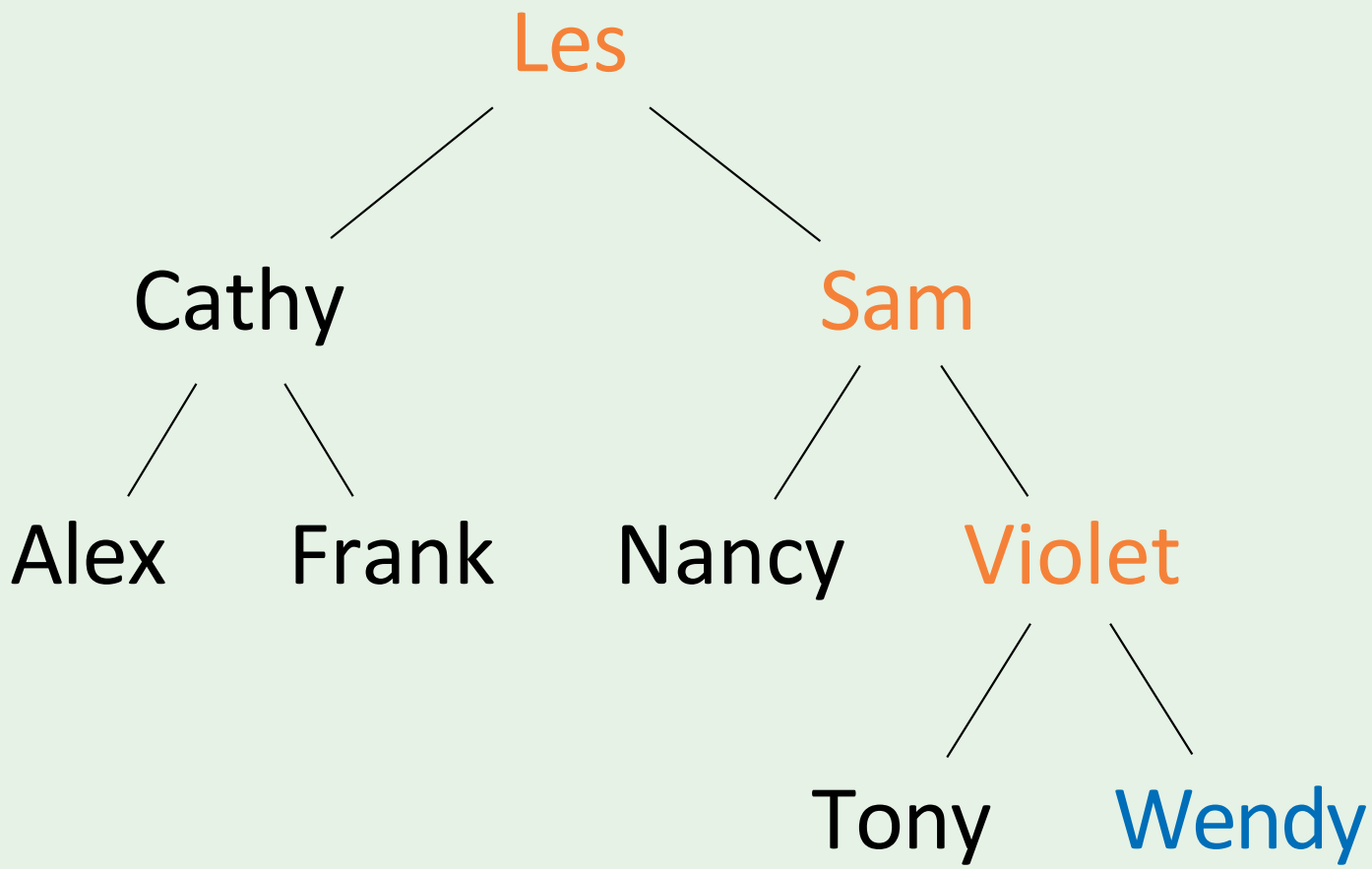
Output: Alex Cathy Frank Les Nancy Sam Tony Violet

InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam Tony Violet

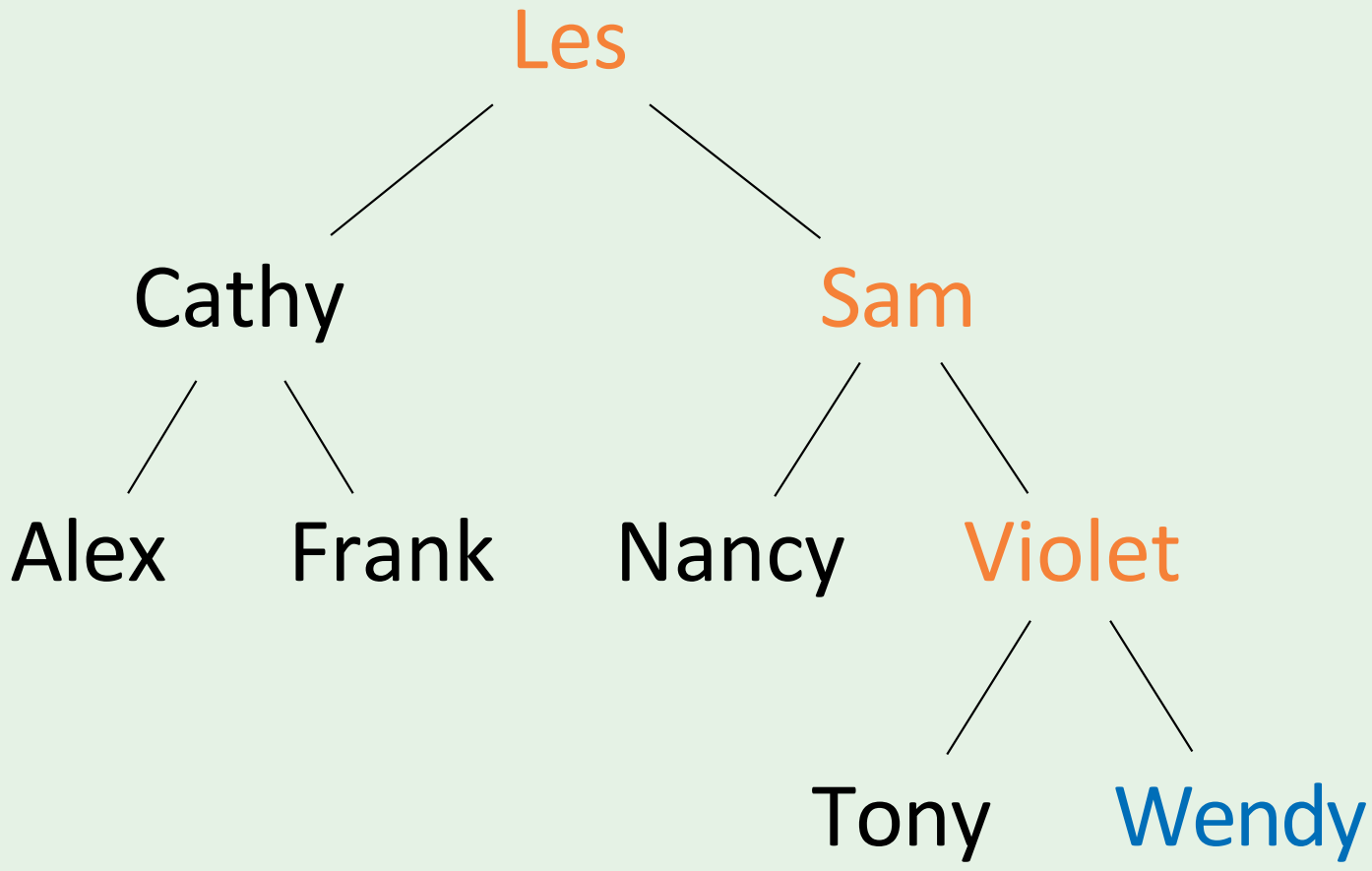
InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam

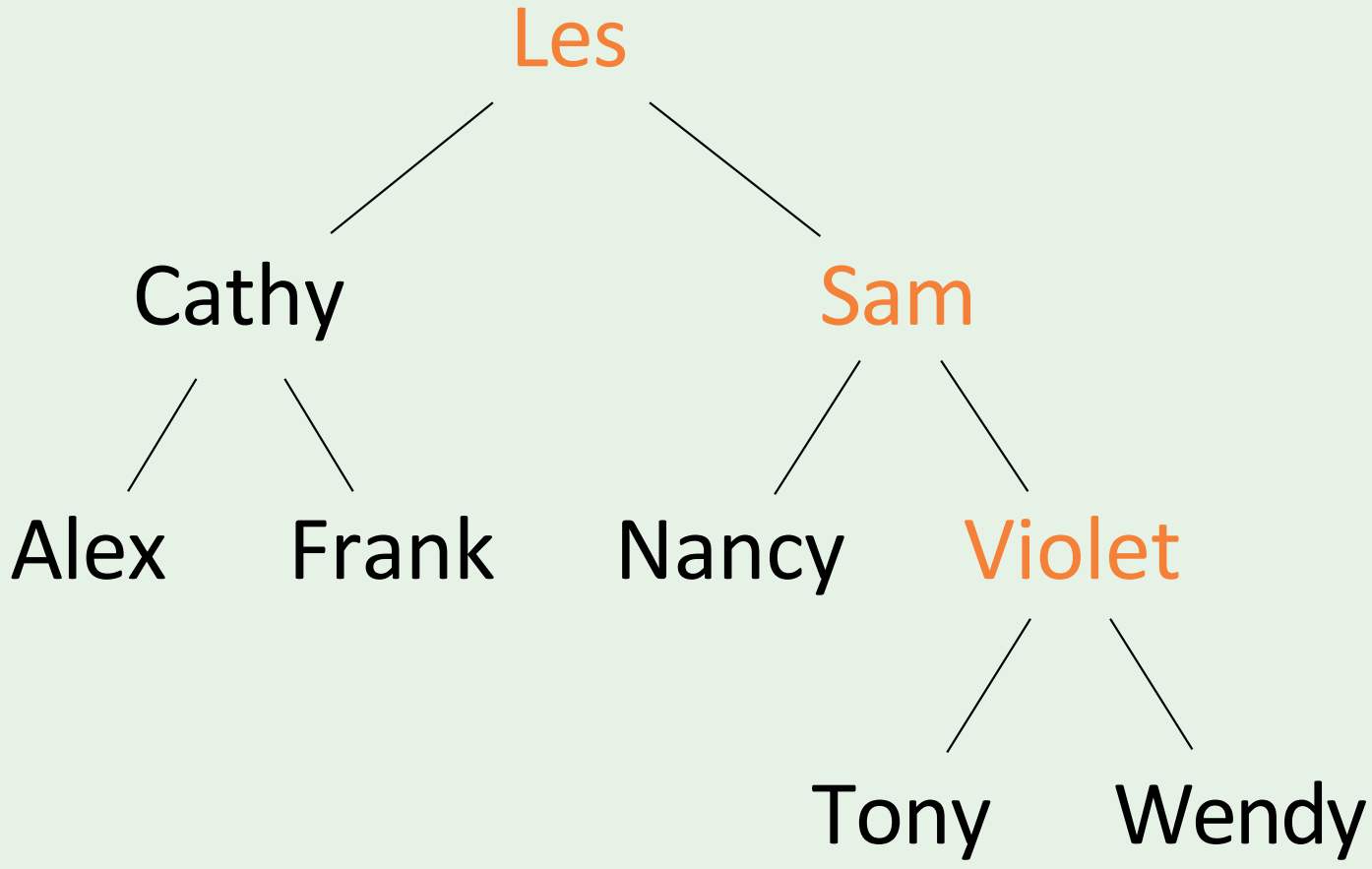
Tony Violet Wendy

InOrderTraversal



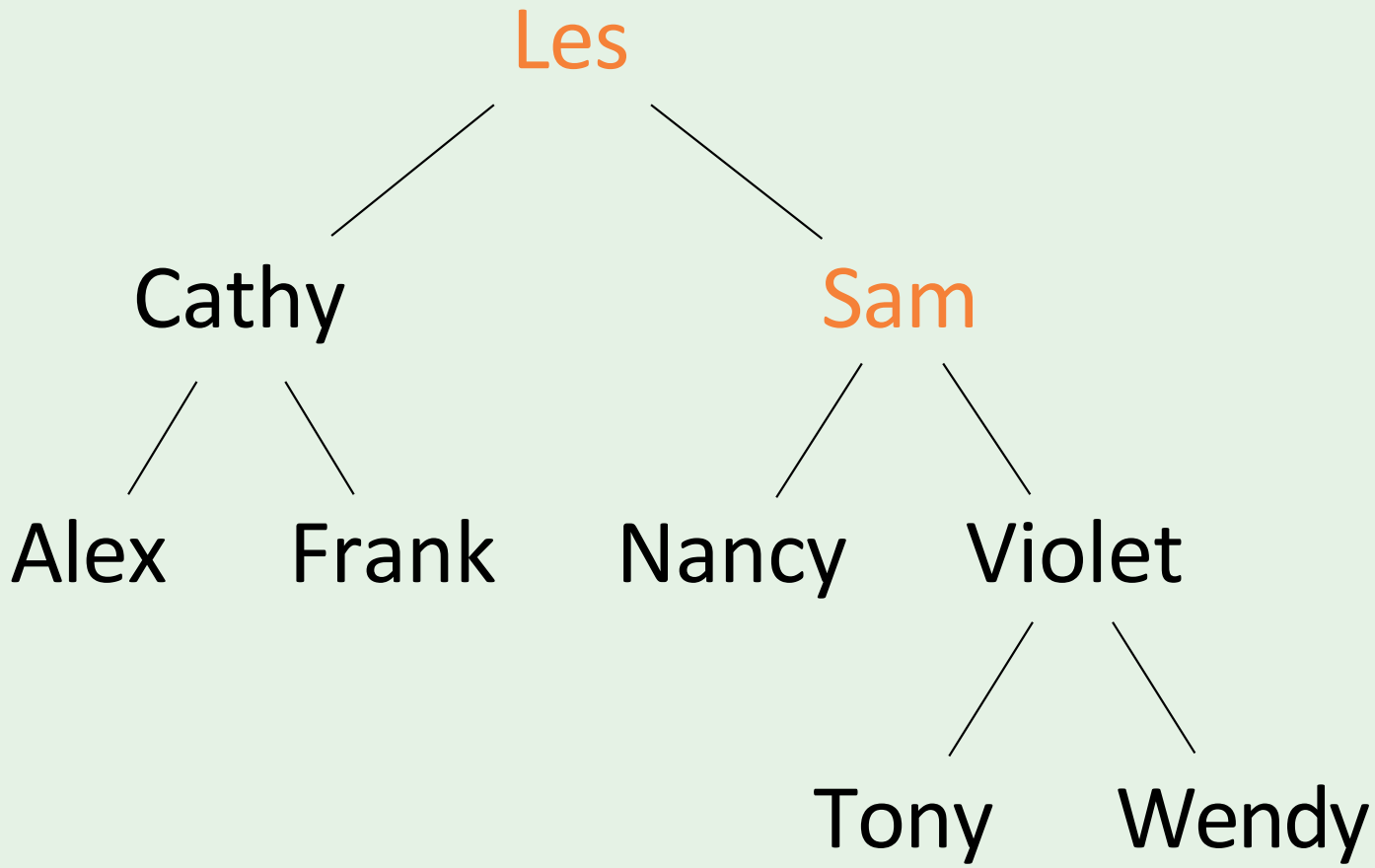
Output: Alex Cathy Frank Les Nancy Sam Tony Violet Wendy

InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam Tony Violet Wendy

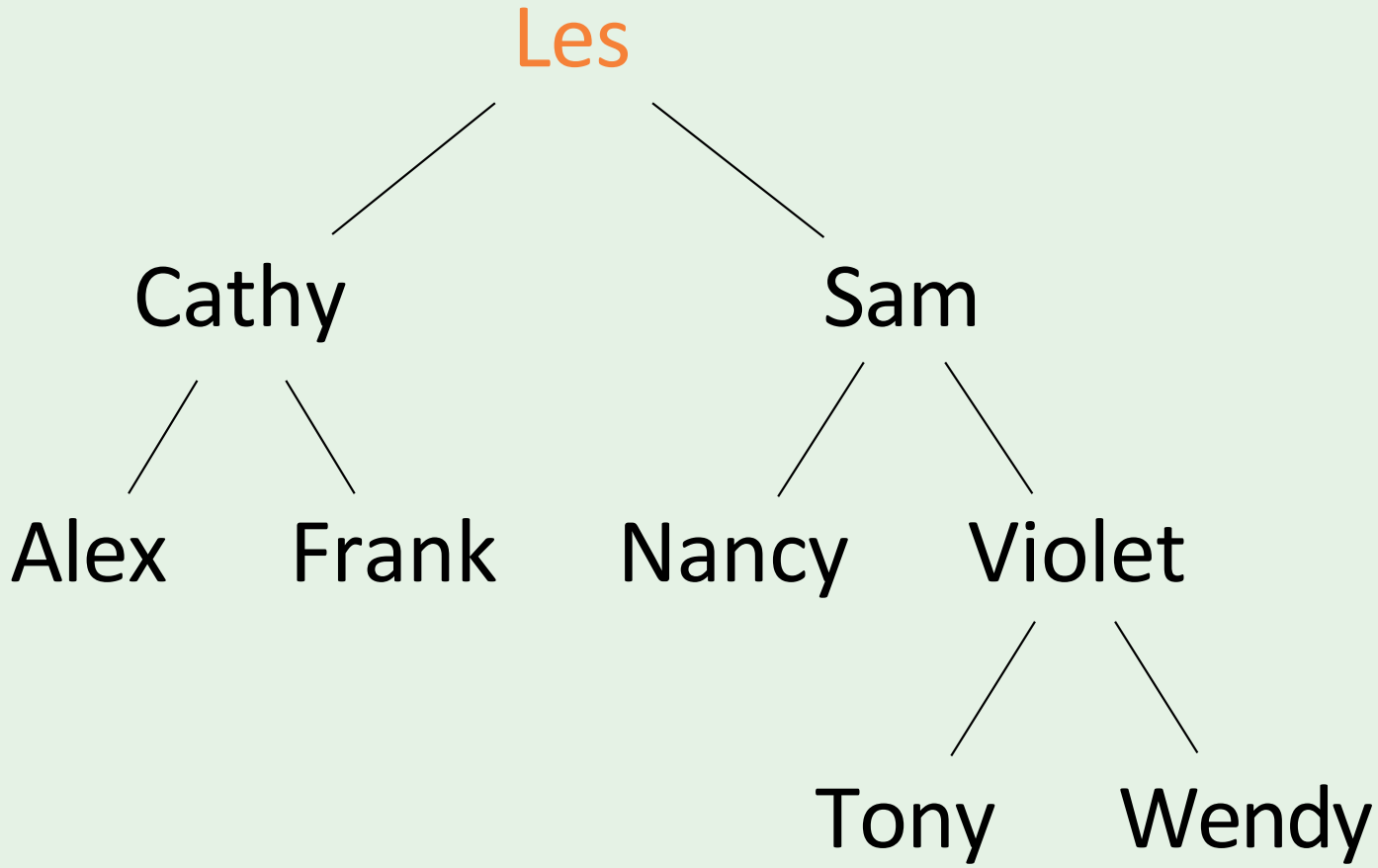
InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam

Tony Violet Wendy

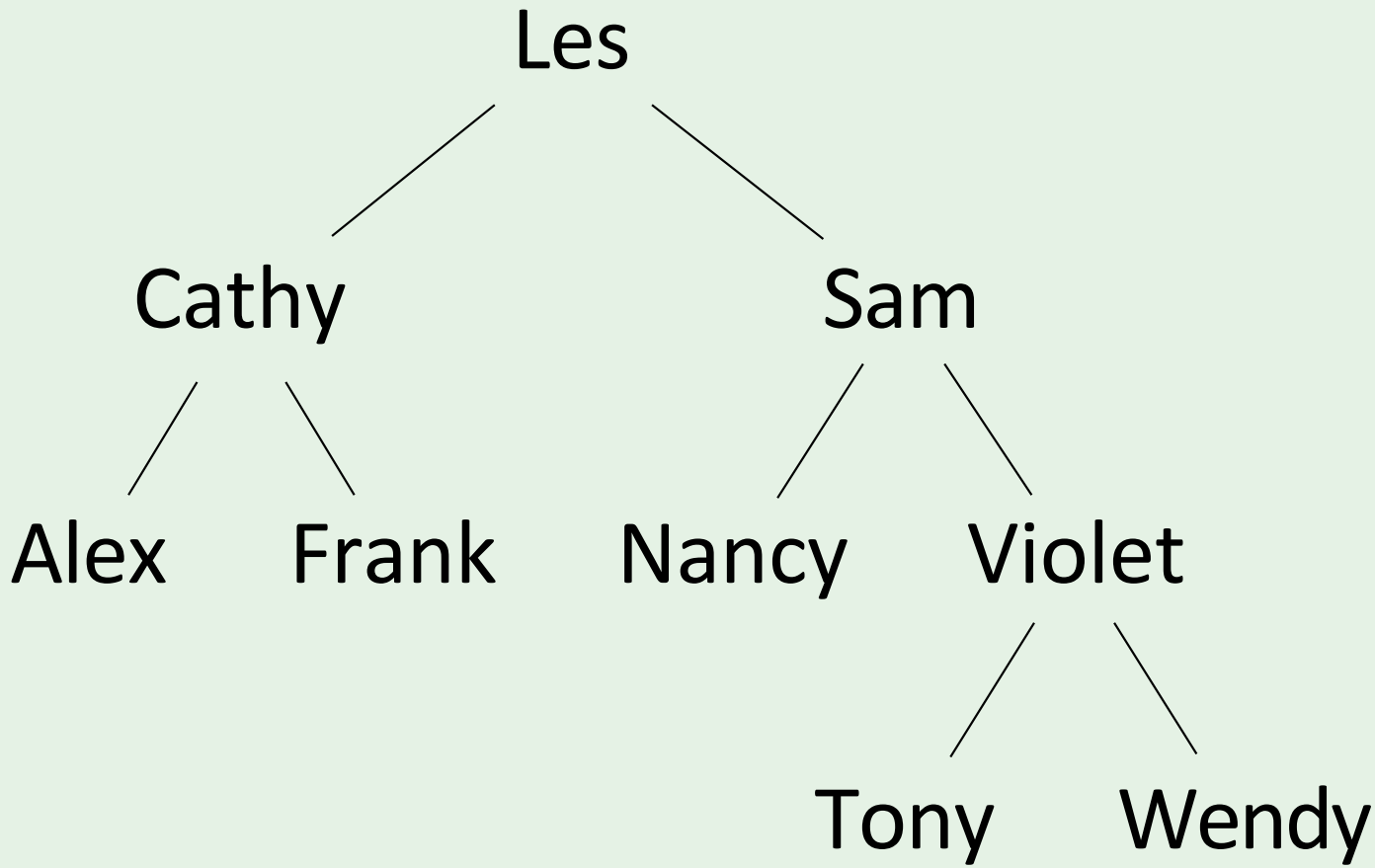
InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam

Tony Violet Wendy

InOrderTraversal



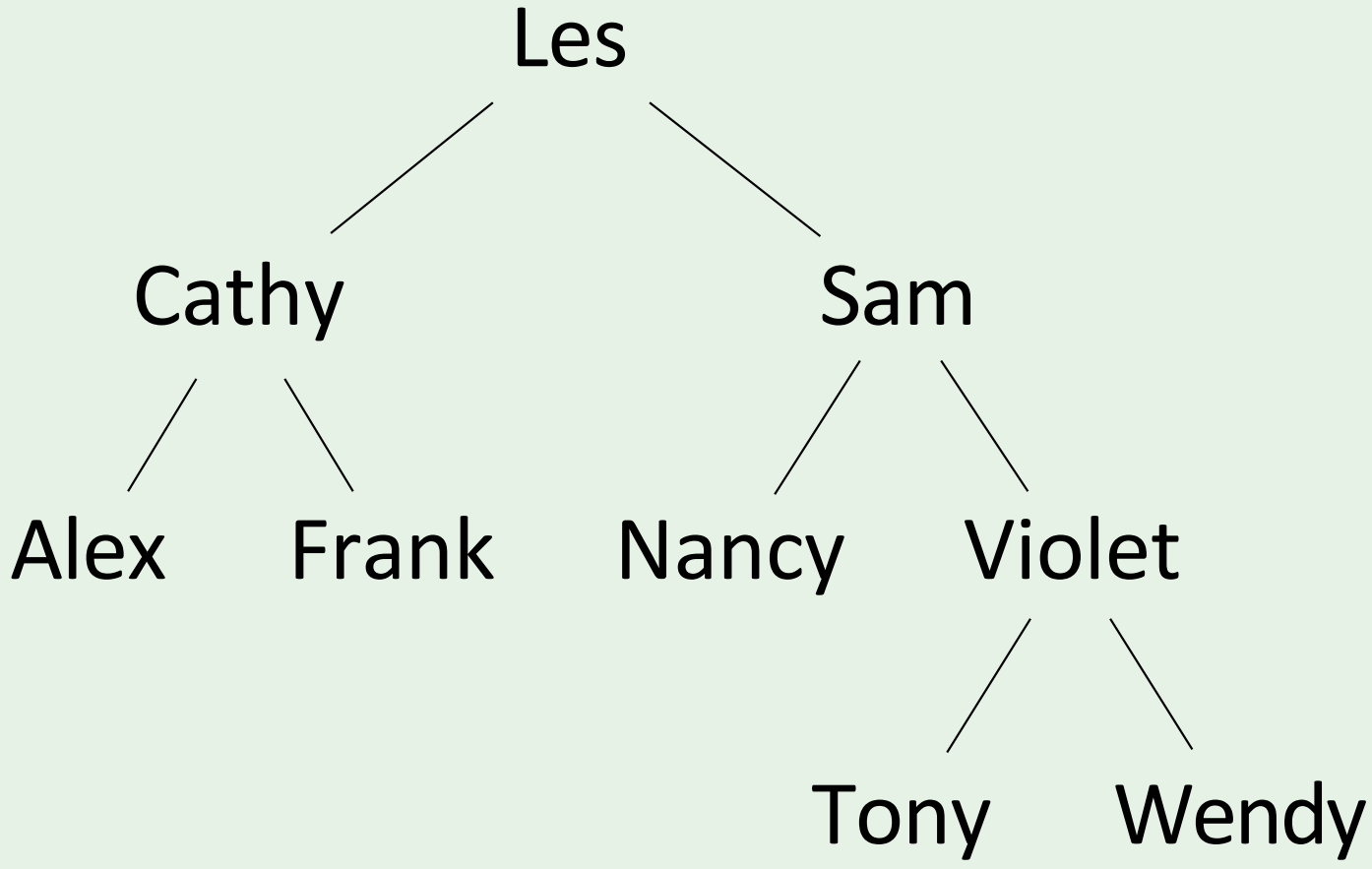
Output: Alex Cathy Frank Les Nancy Sam

Tony Violet Wendy

Depth-first

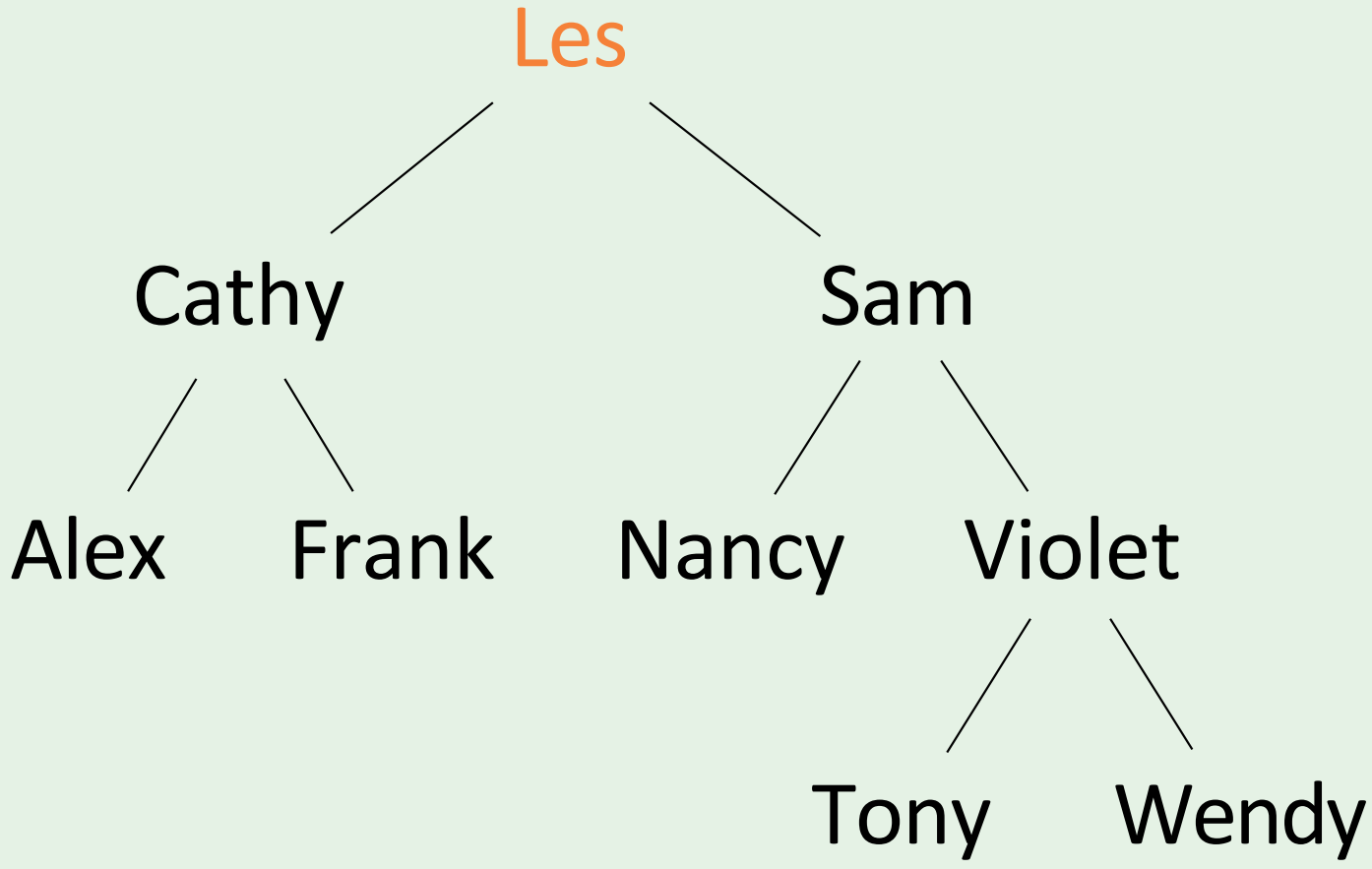
```
PreOrderTraversal(tree)
if tree = nil :
   return
Print(tree.key )
PreOrderTraversal(tree.left)
PreOrderTraversal(tree.right)
```

PreOrderTraversal



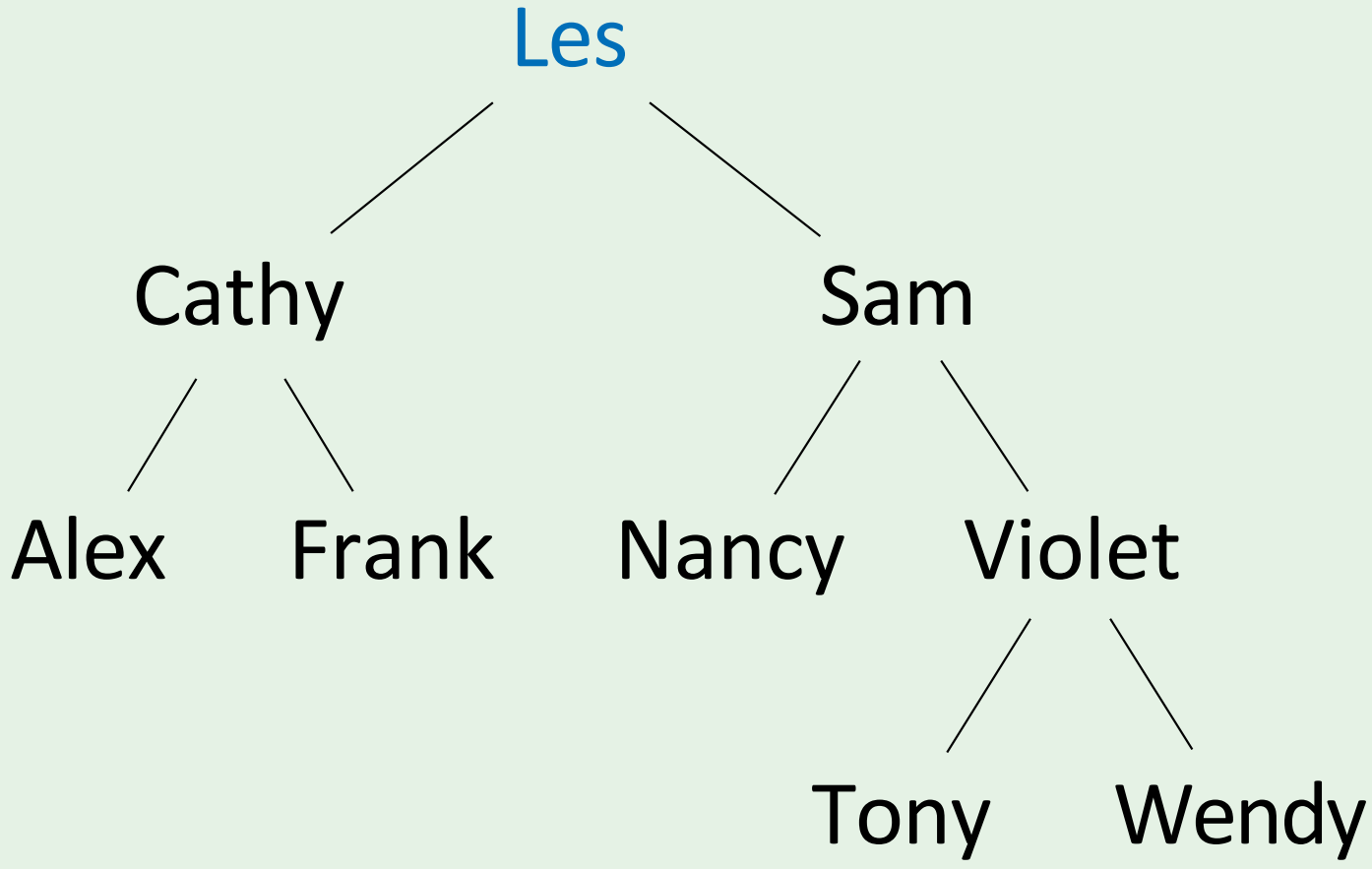
Output:

PreOrderTraversal



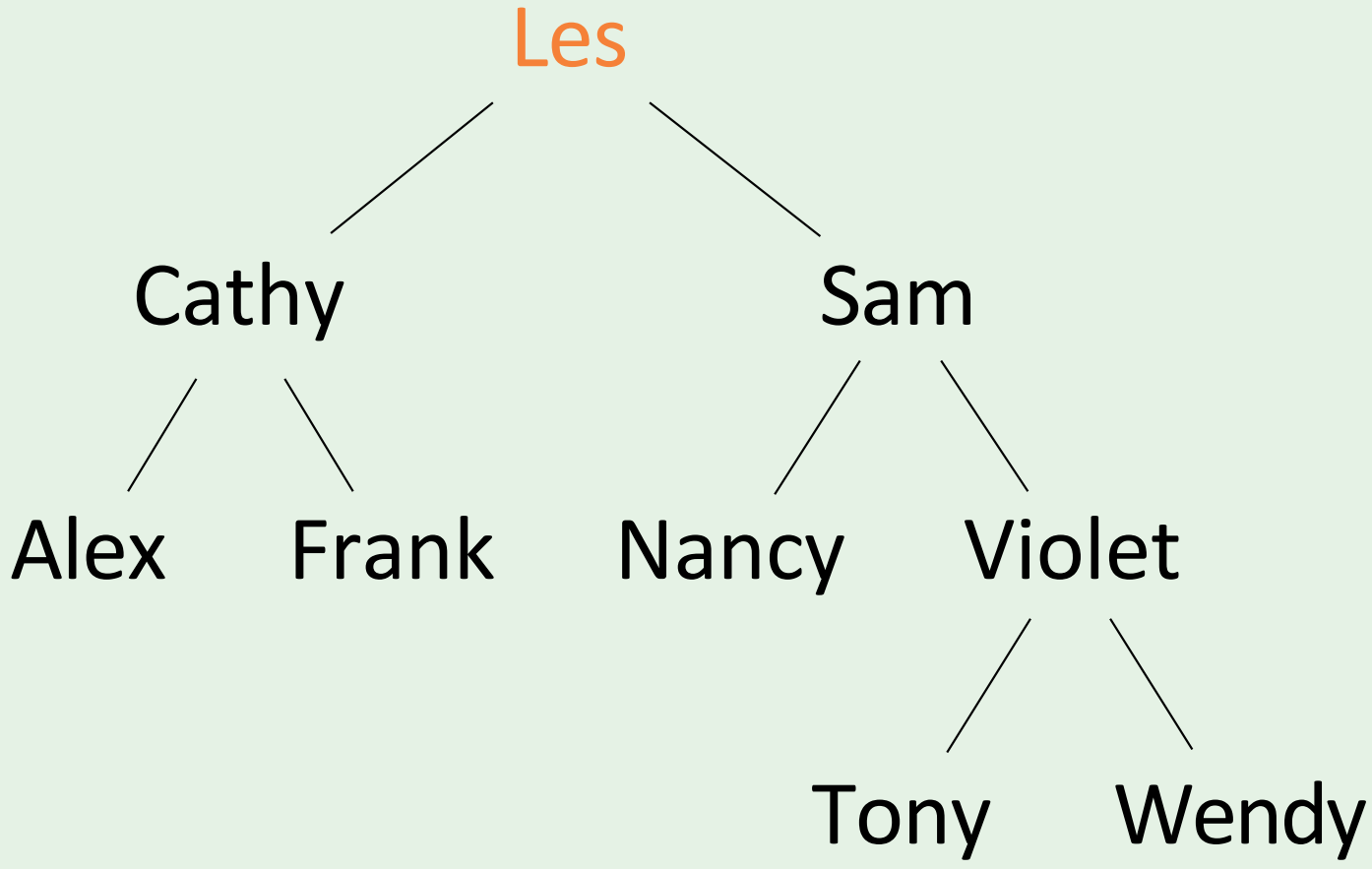
Output:

PreOrderTraversal



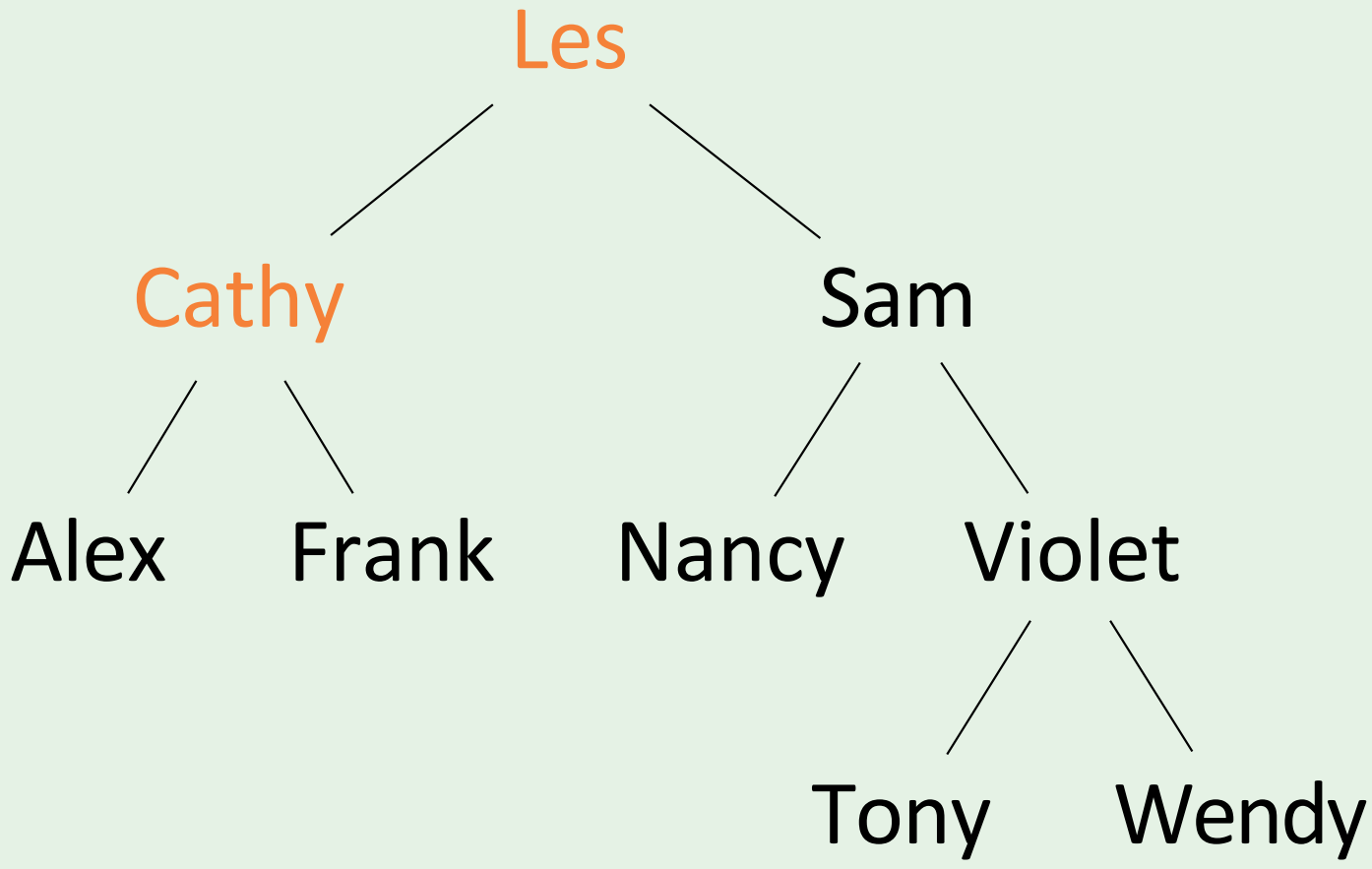
Output: Les

PreOrderTraversal



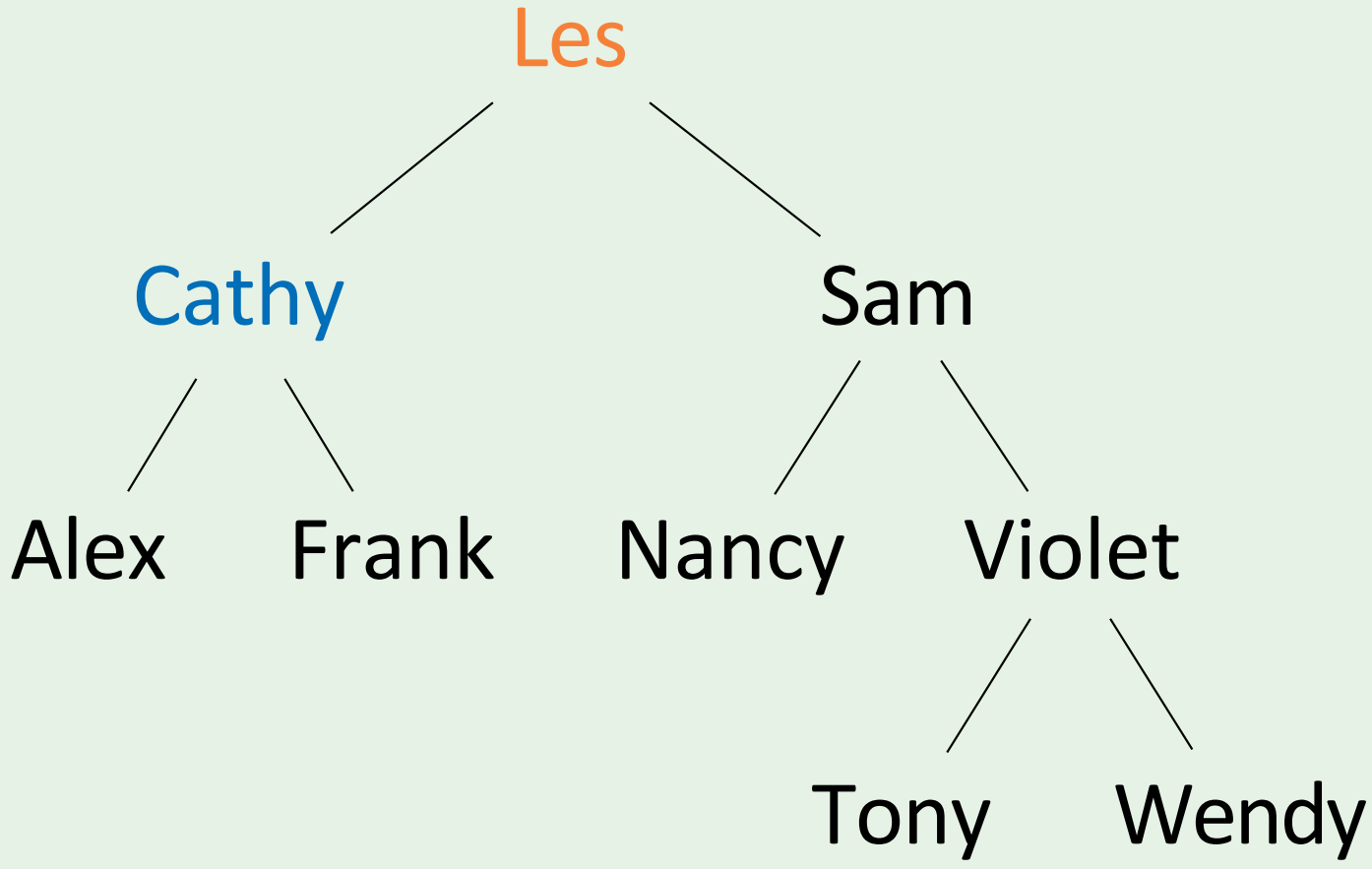
Output: Les

PreOrderTraversal



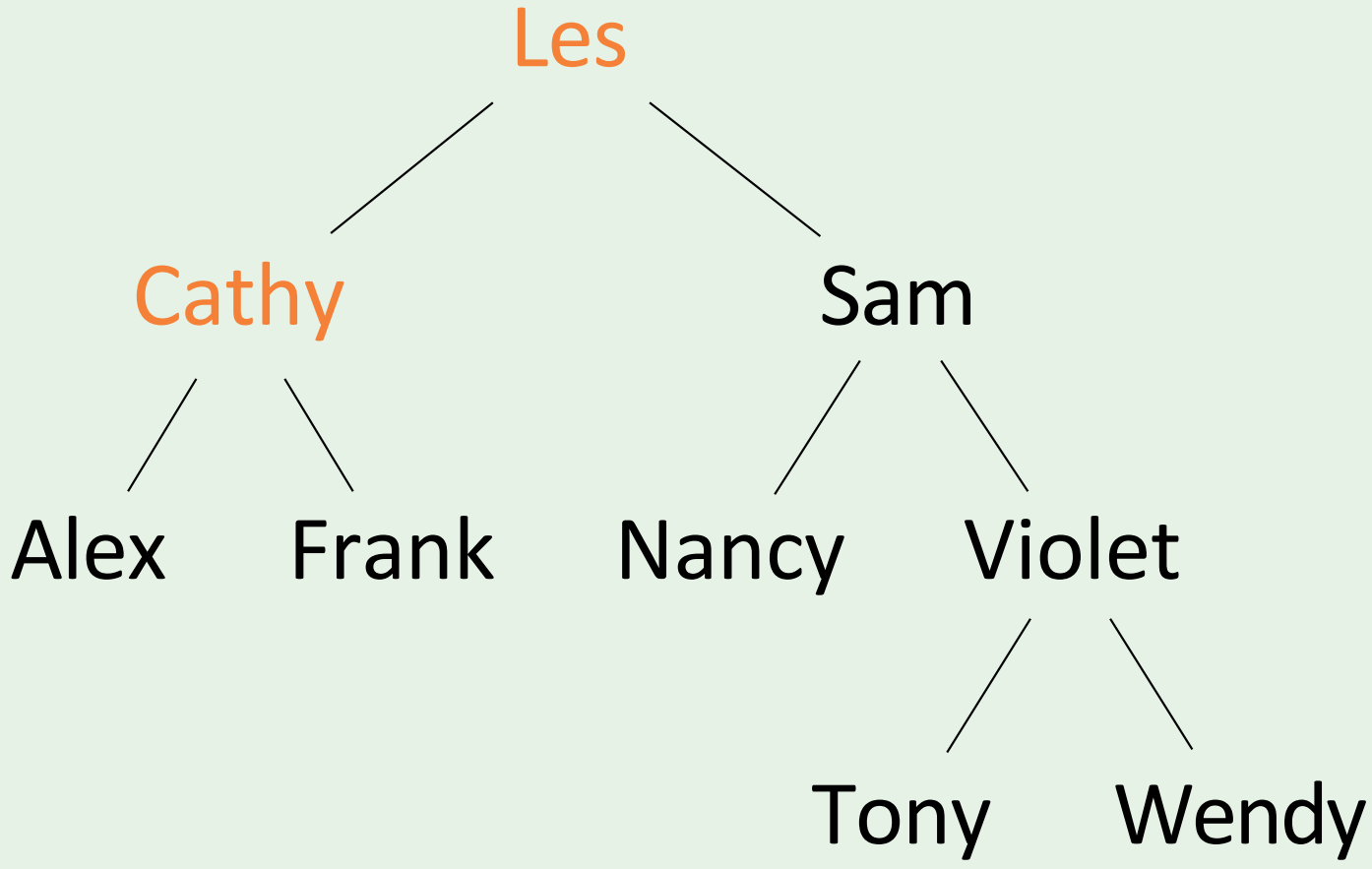
Output: Les

PreOrderTraversal



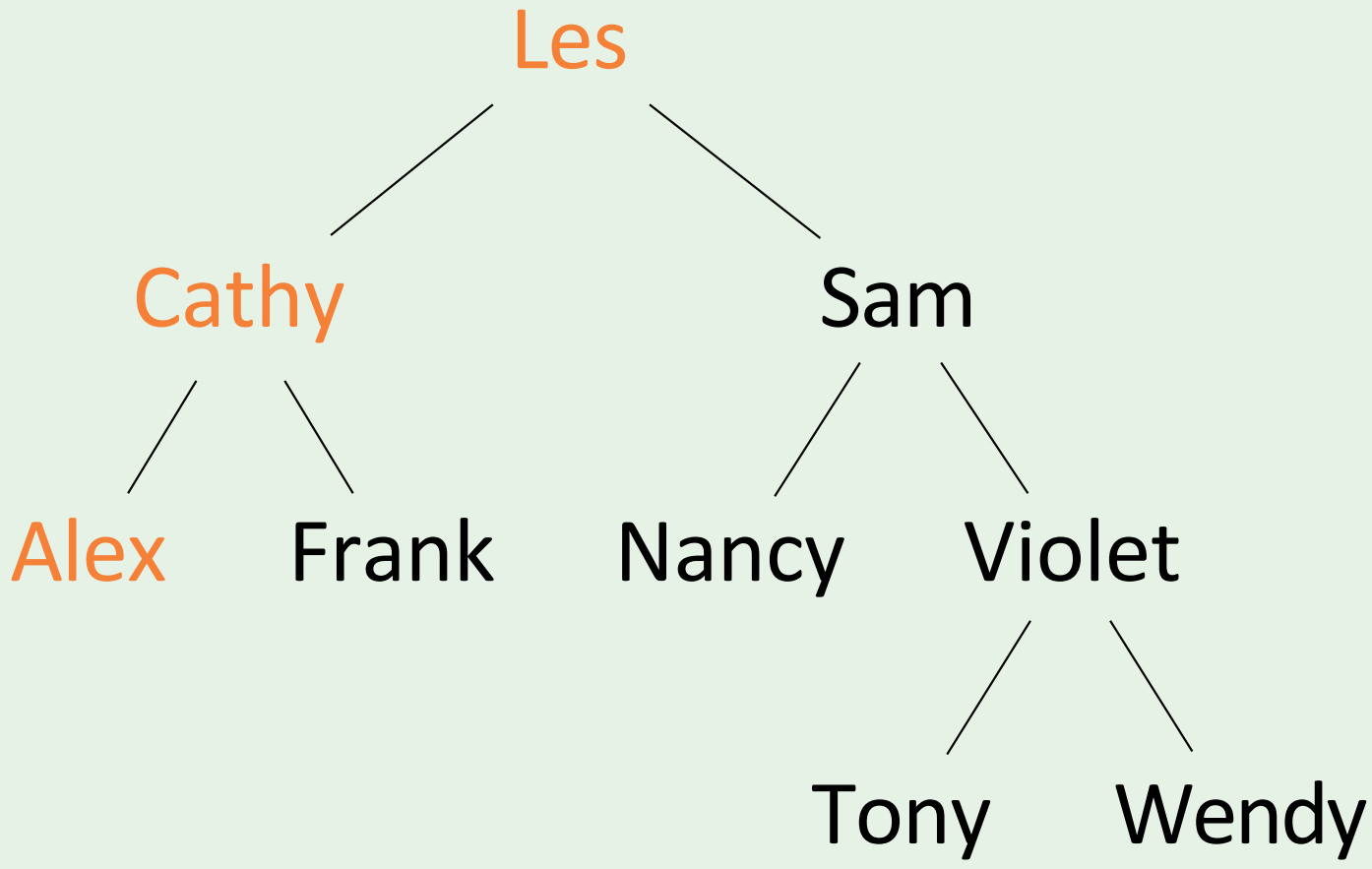
Output: Les
Cathy

PreOrderTraversal



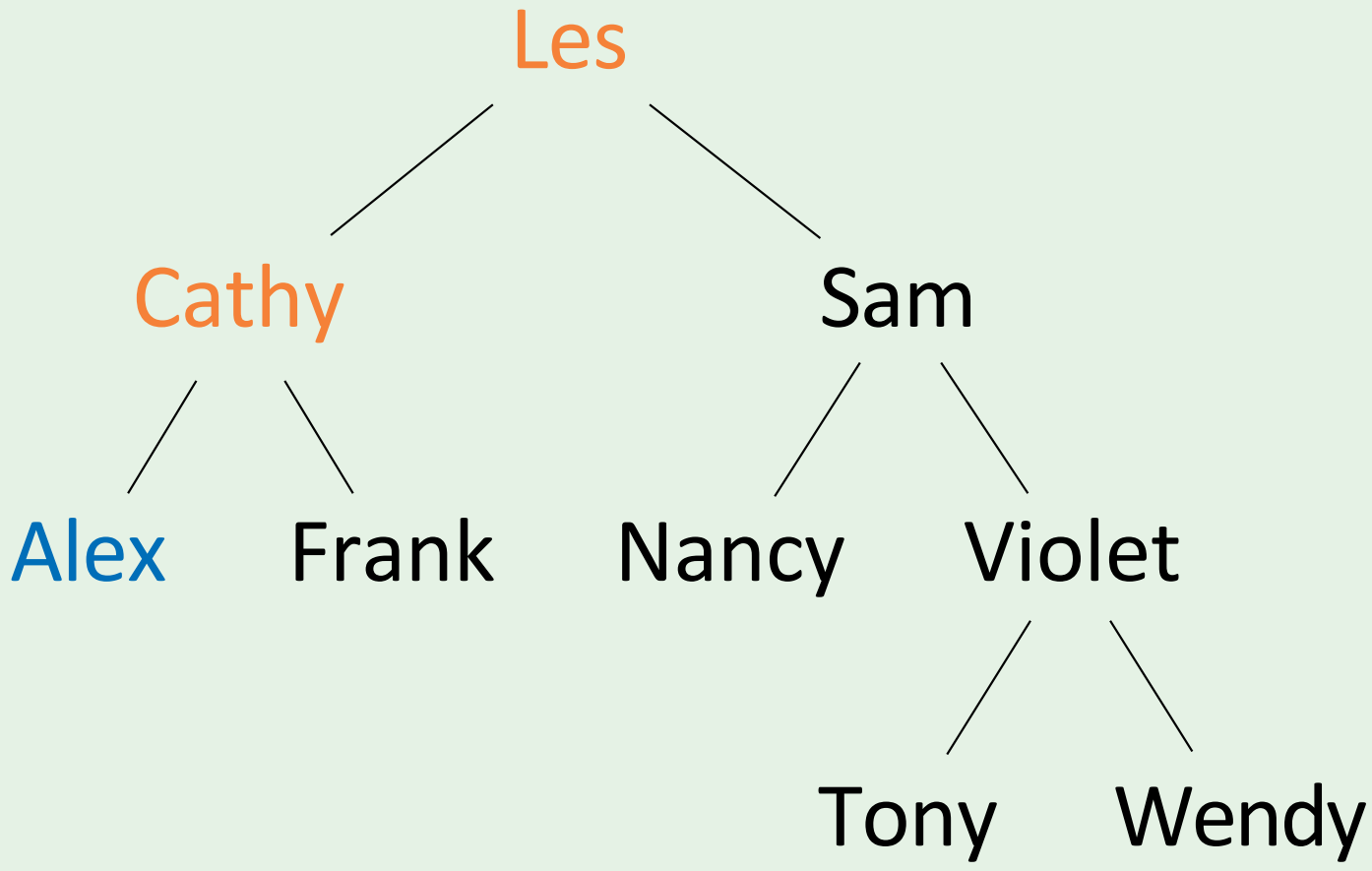
Output: Les
Cathy

PreOrderTraversal



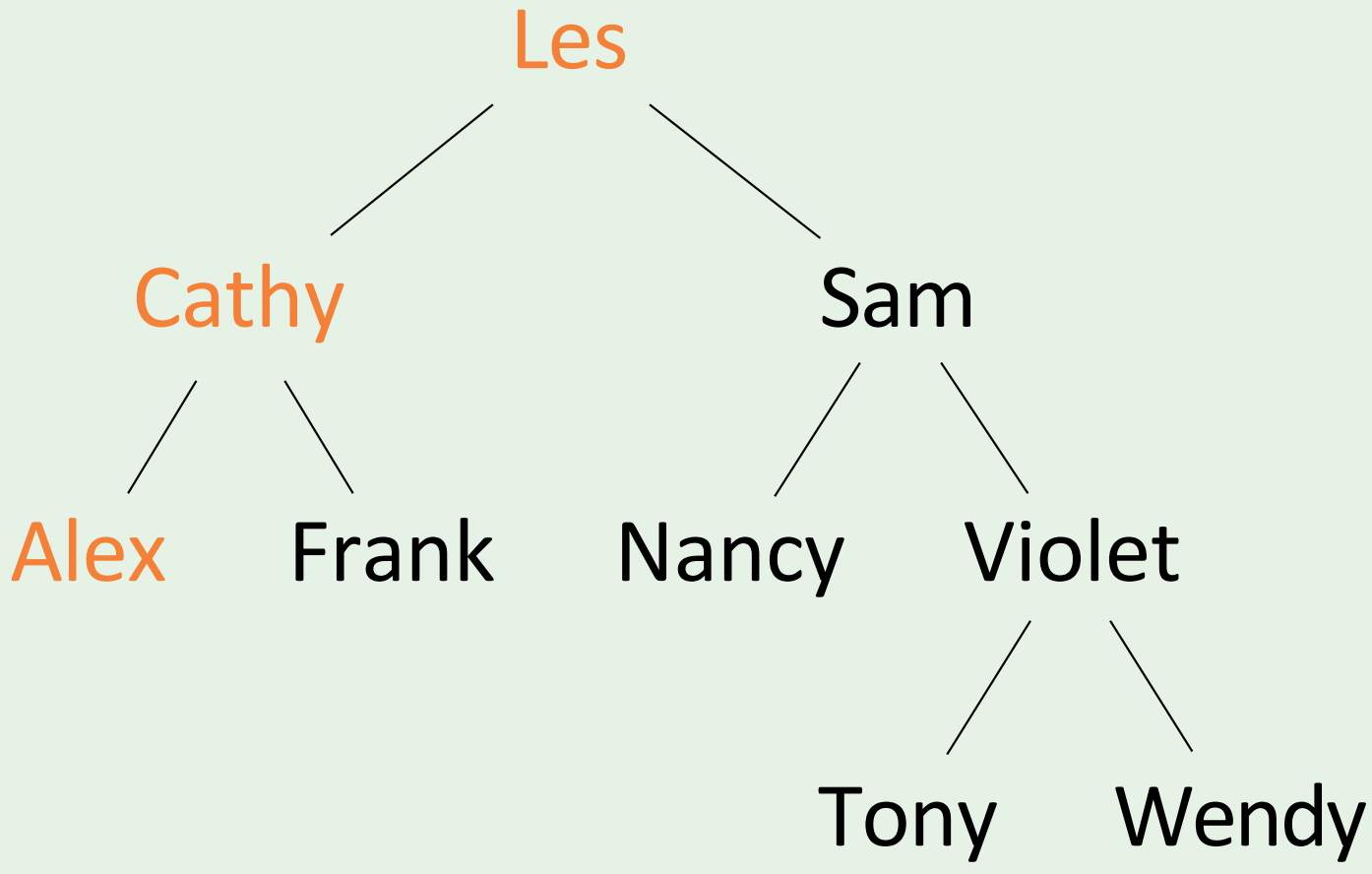
Output: Les Cathy

PreOrderTraversal



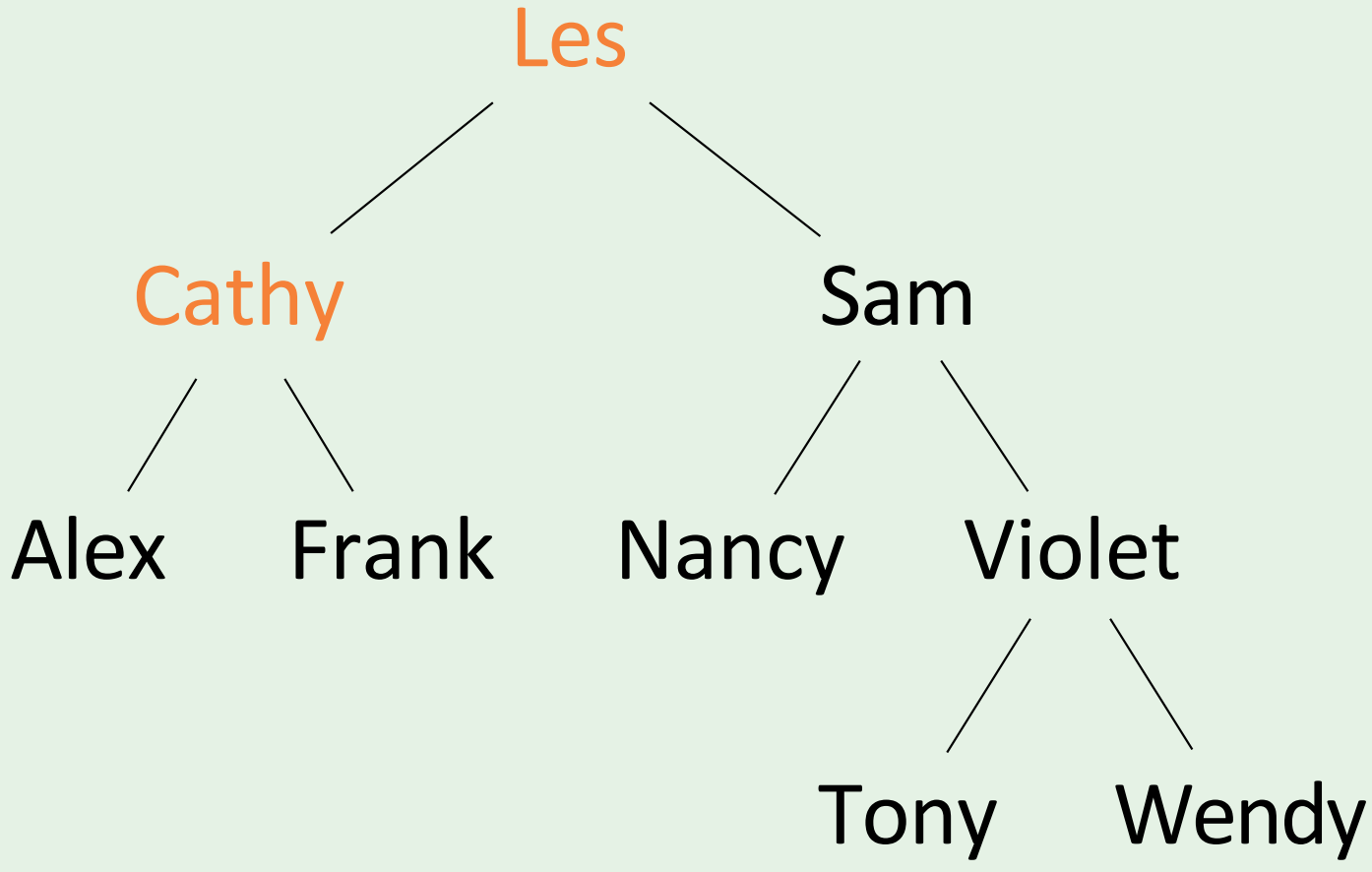
Output: Les Cathy Alex

PreOrderTraversal



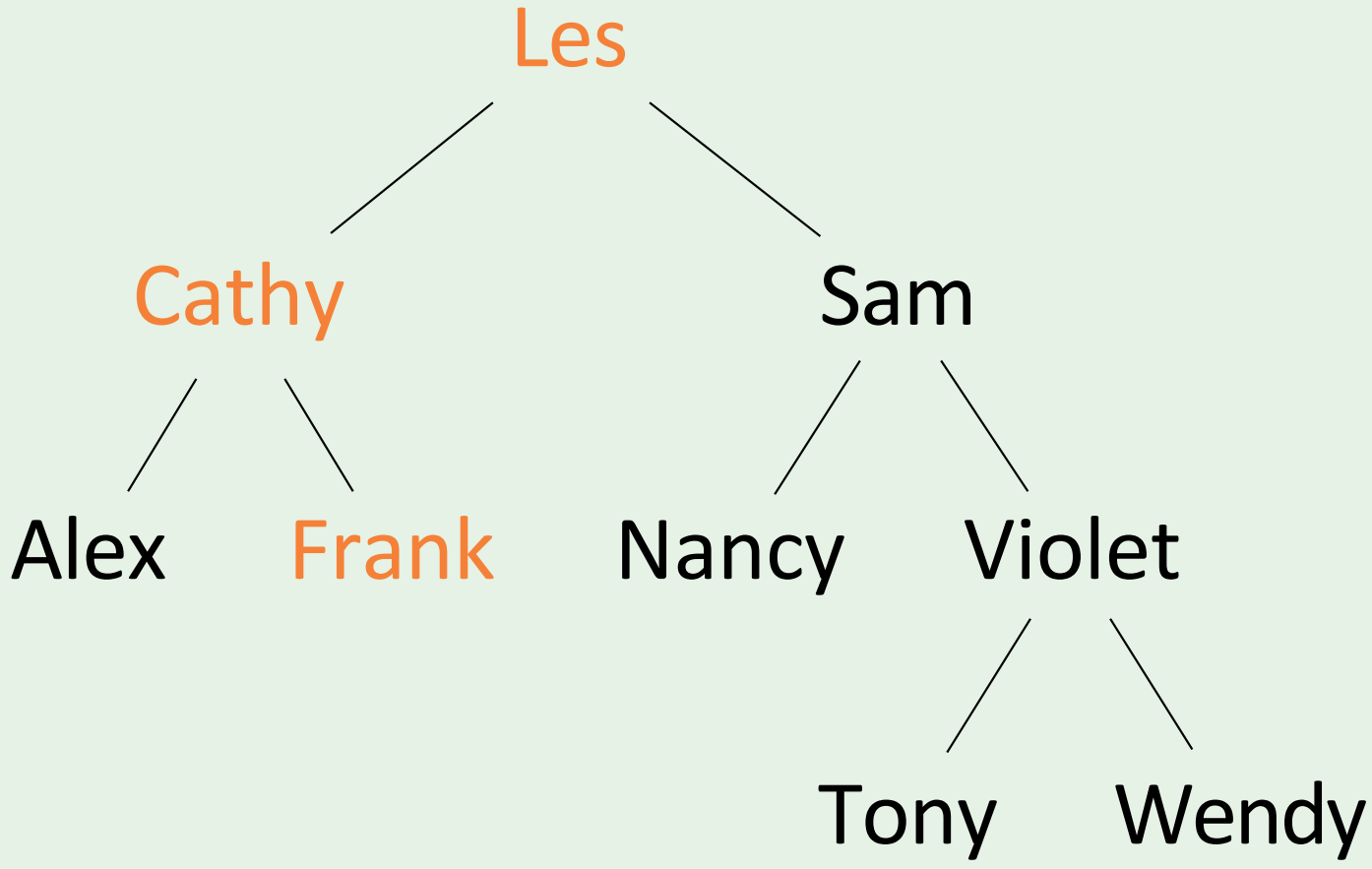
Output: Les Cathy Alex

PreOrderTraversal



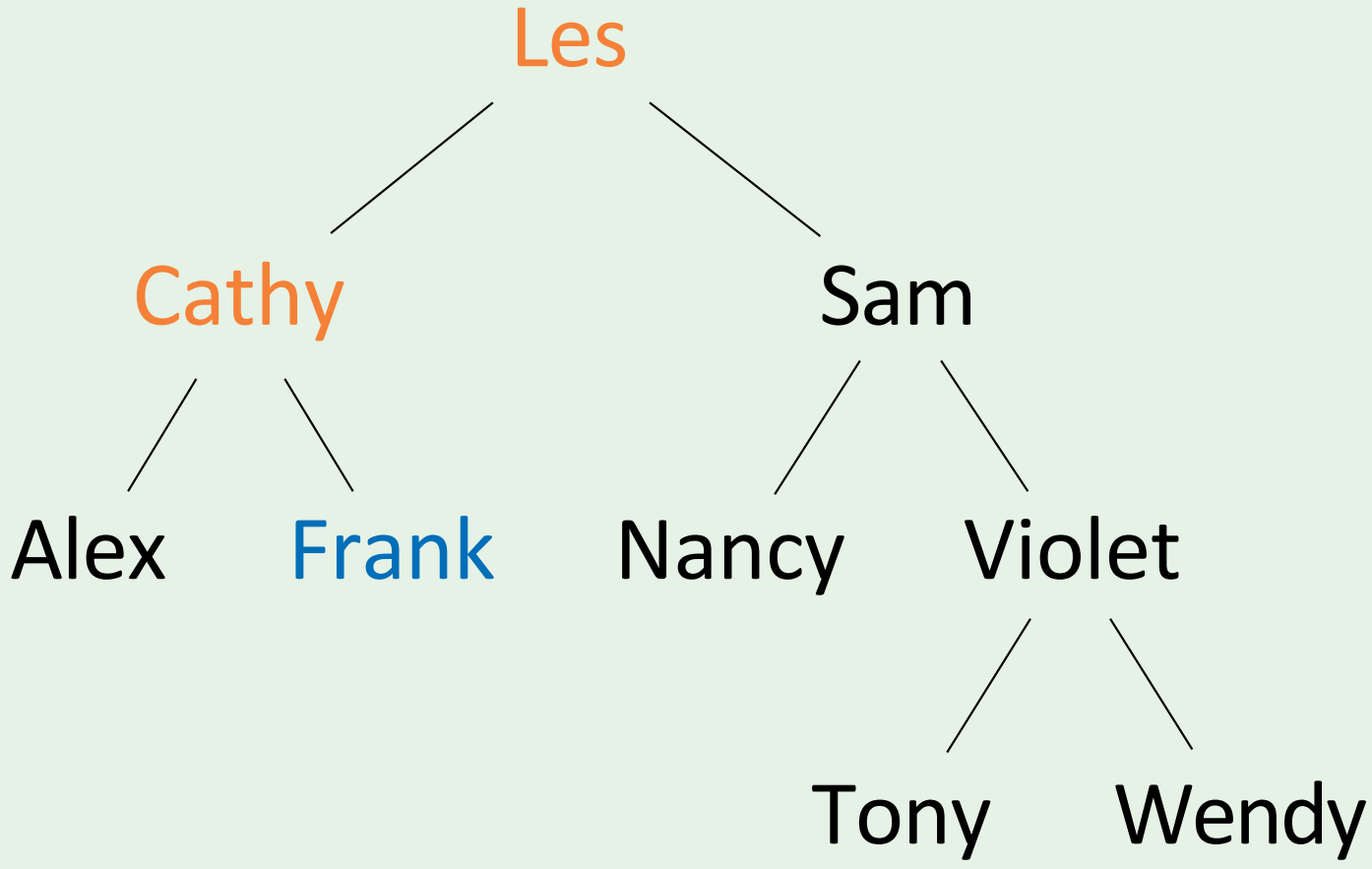
Output: Les Cathy Alex

PreOrderTraversal



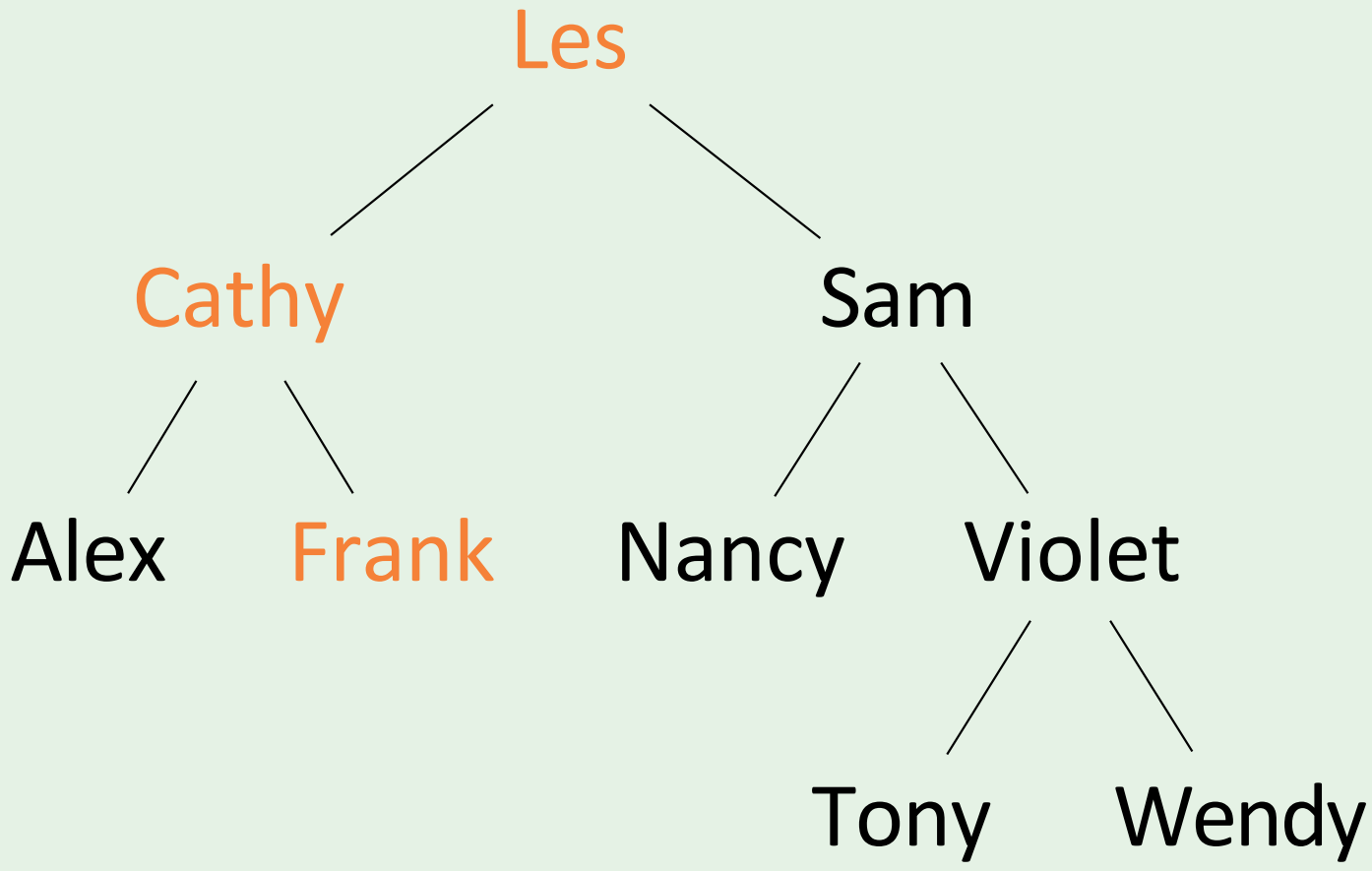
Output: Les Cathy Alex

PreOrderTraversal



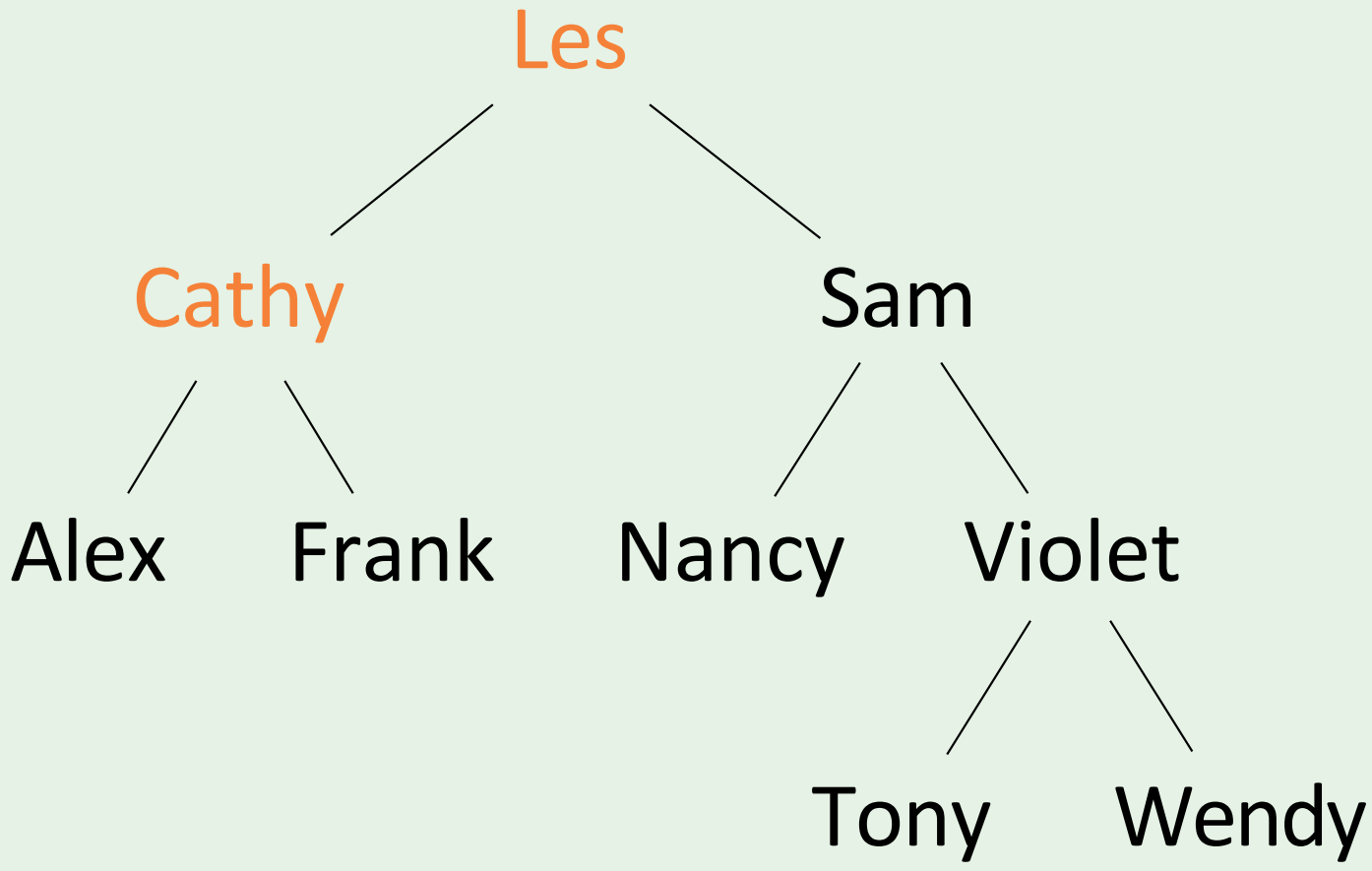
Output: Les Cathy Alex Frank

PreOrderTraversal



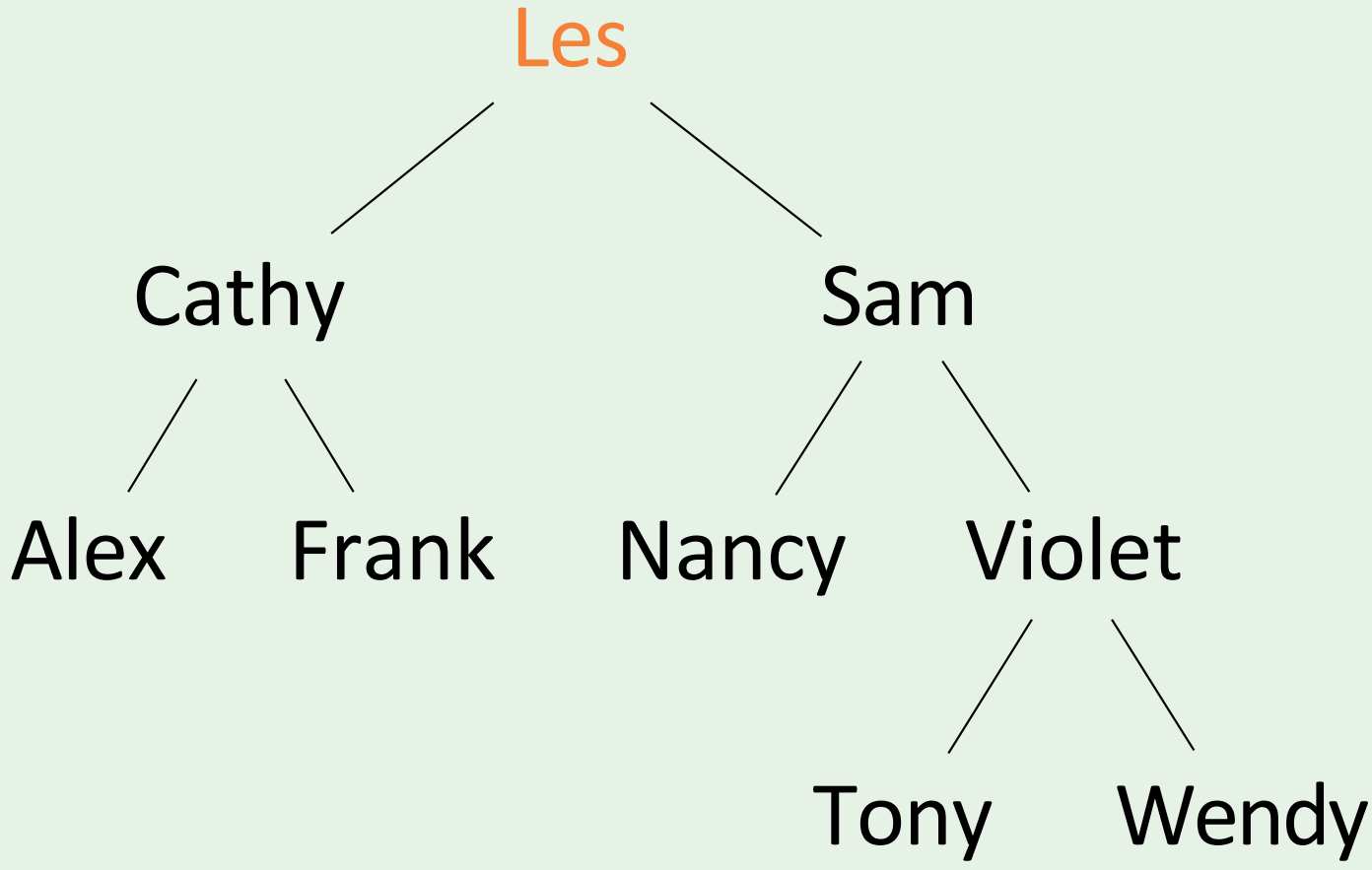
Output: Les Cathy Alex Frank

PreOrderTraversal



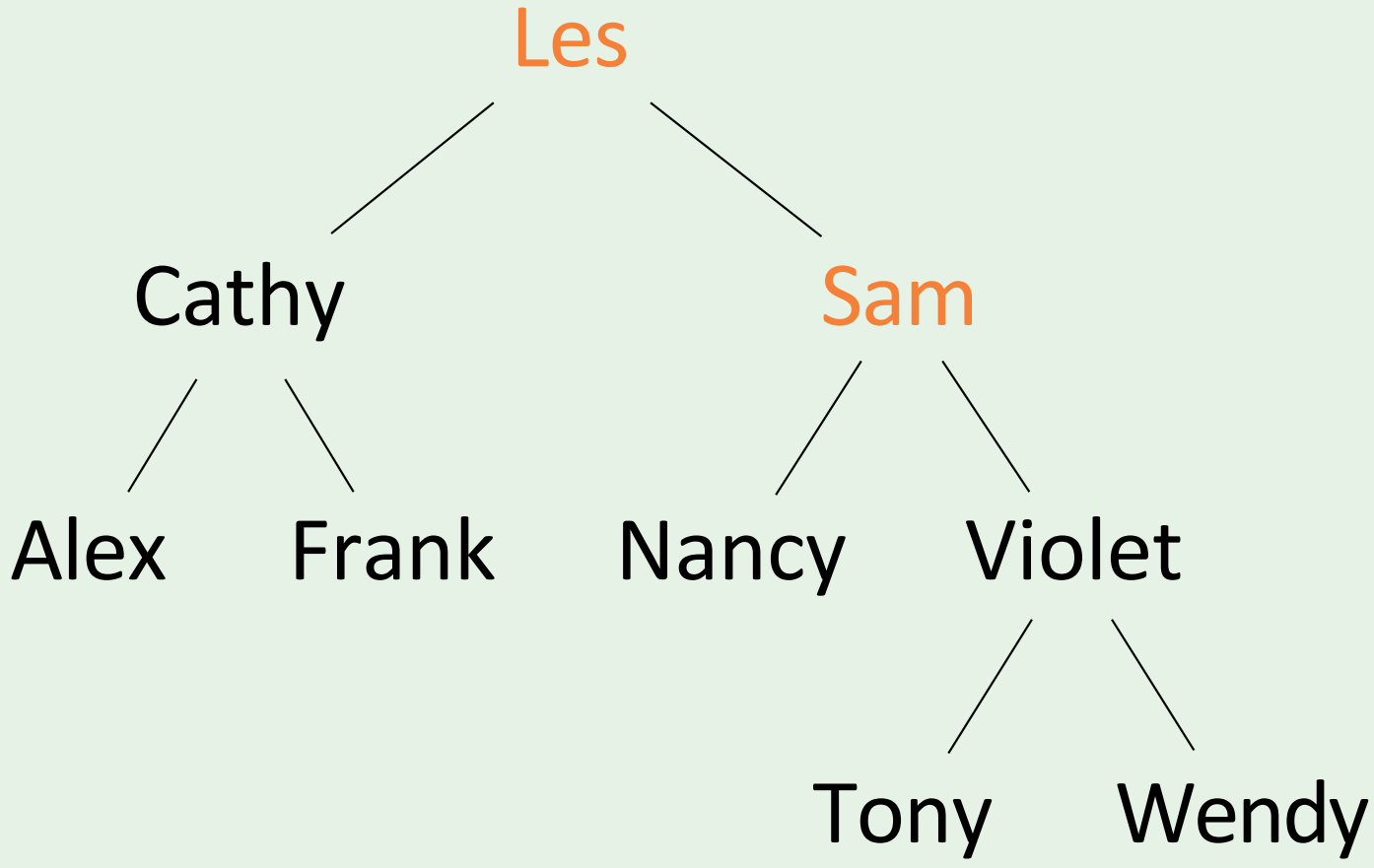
Output: Les Cathy Alex Frank

PreOrderTraversal



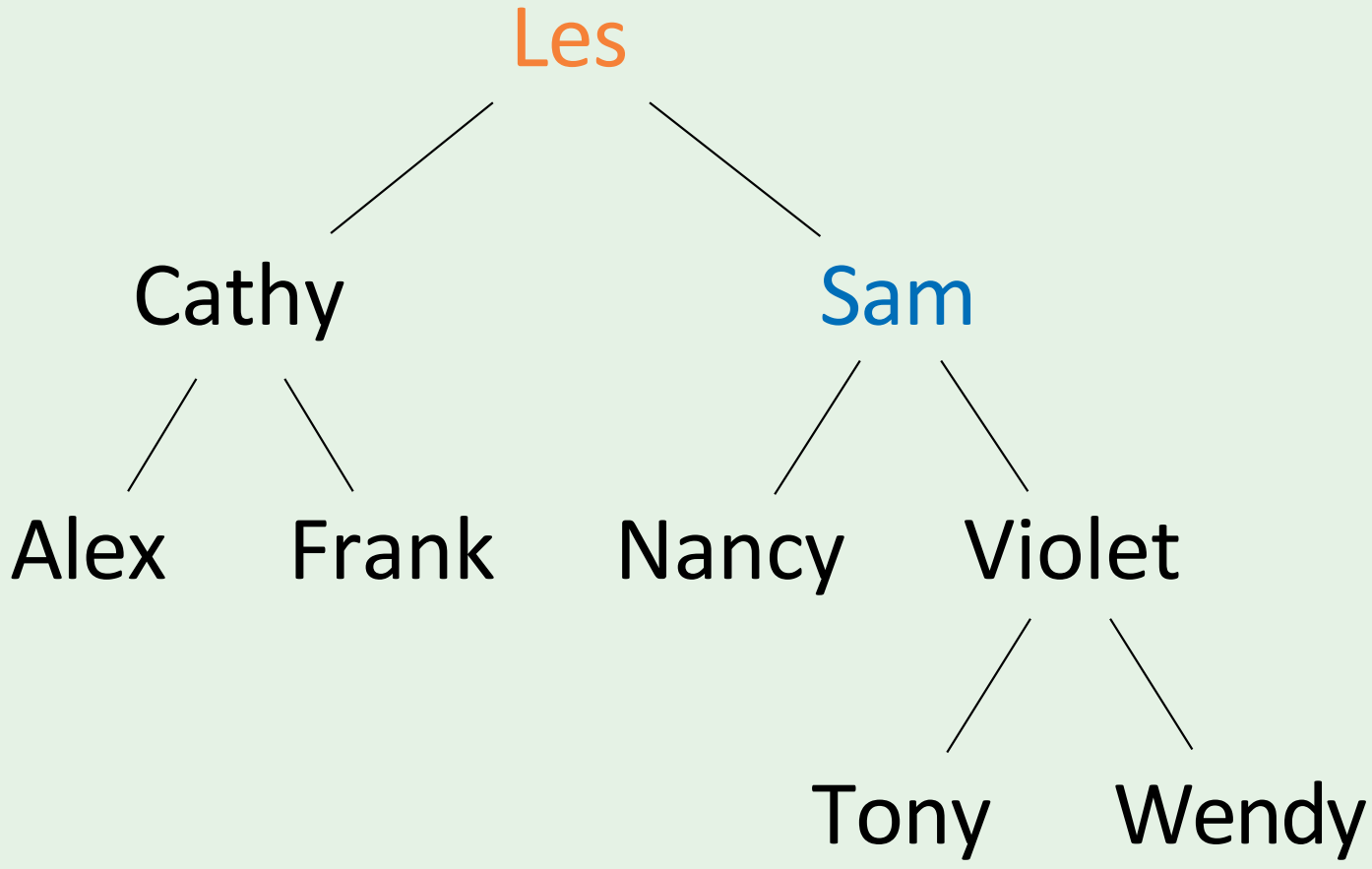
Output: Les Cathy Alex Frank

PreOrderTraversal



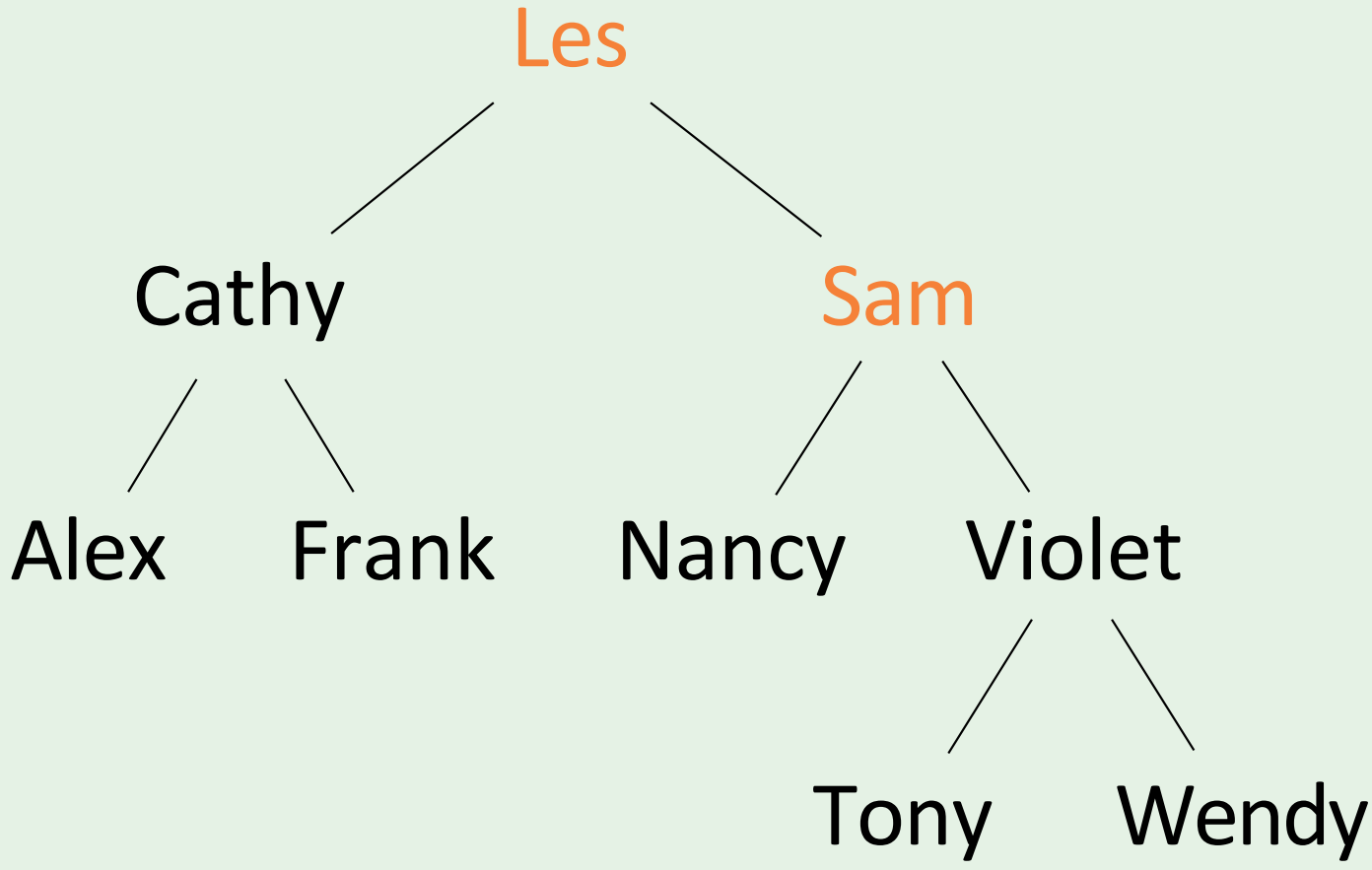
Output: Les Cathy Alex Frank

PreOrderTraversal



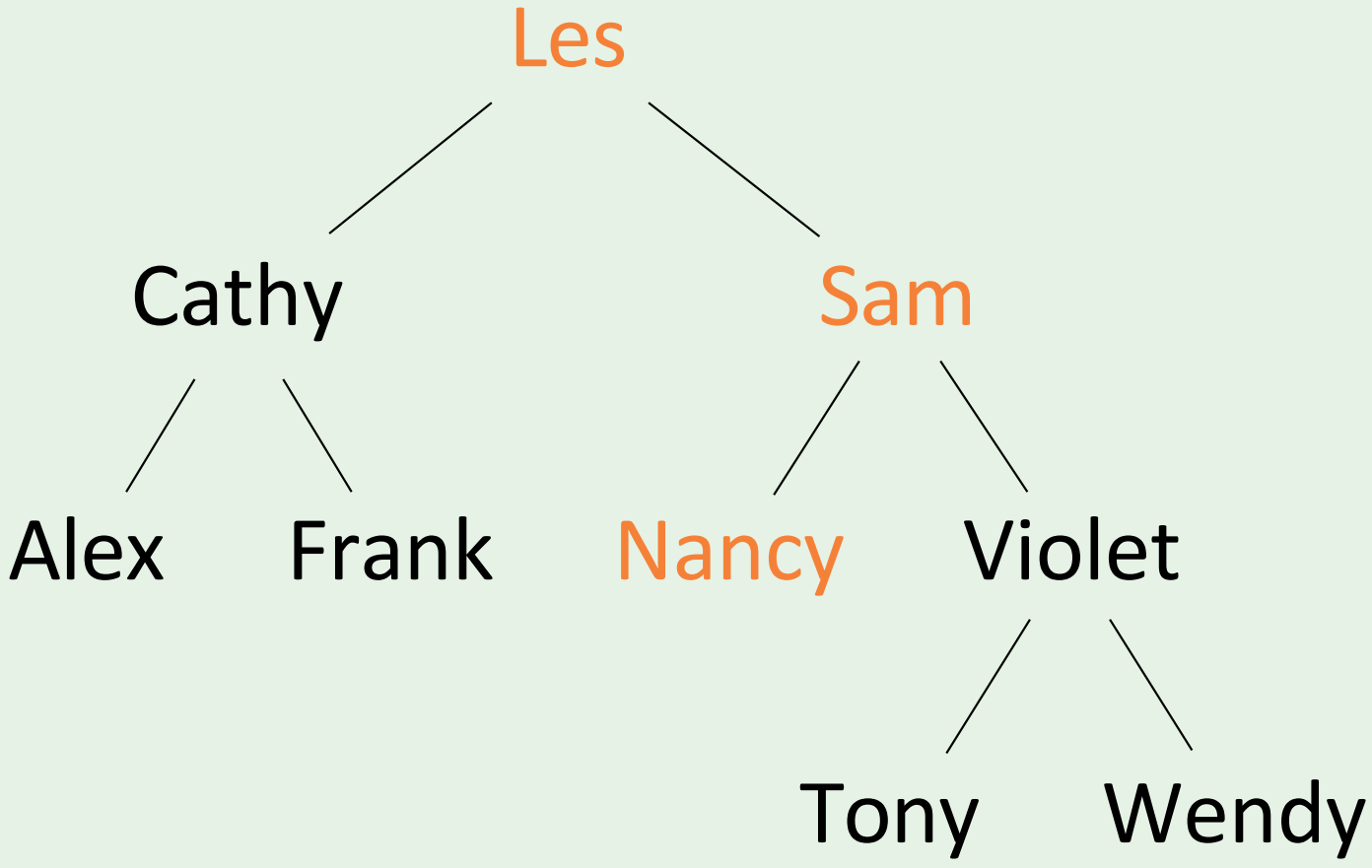
Output: Les Cathy Alex Frank Sam

PreOrderTraversal



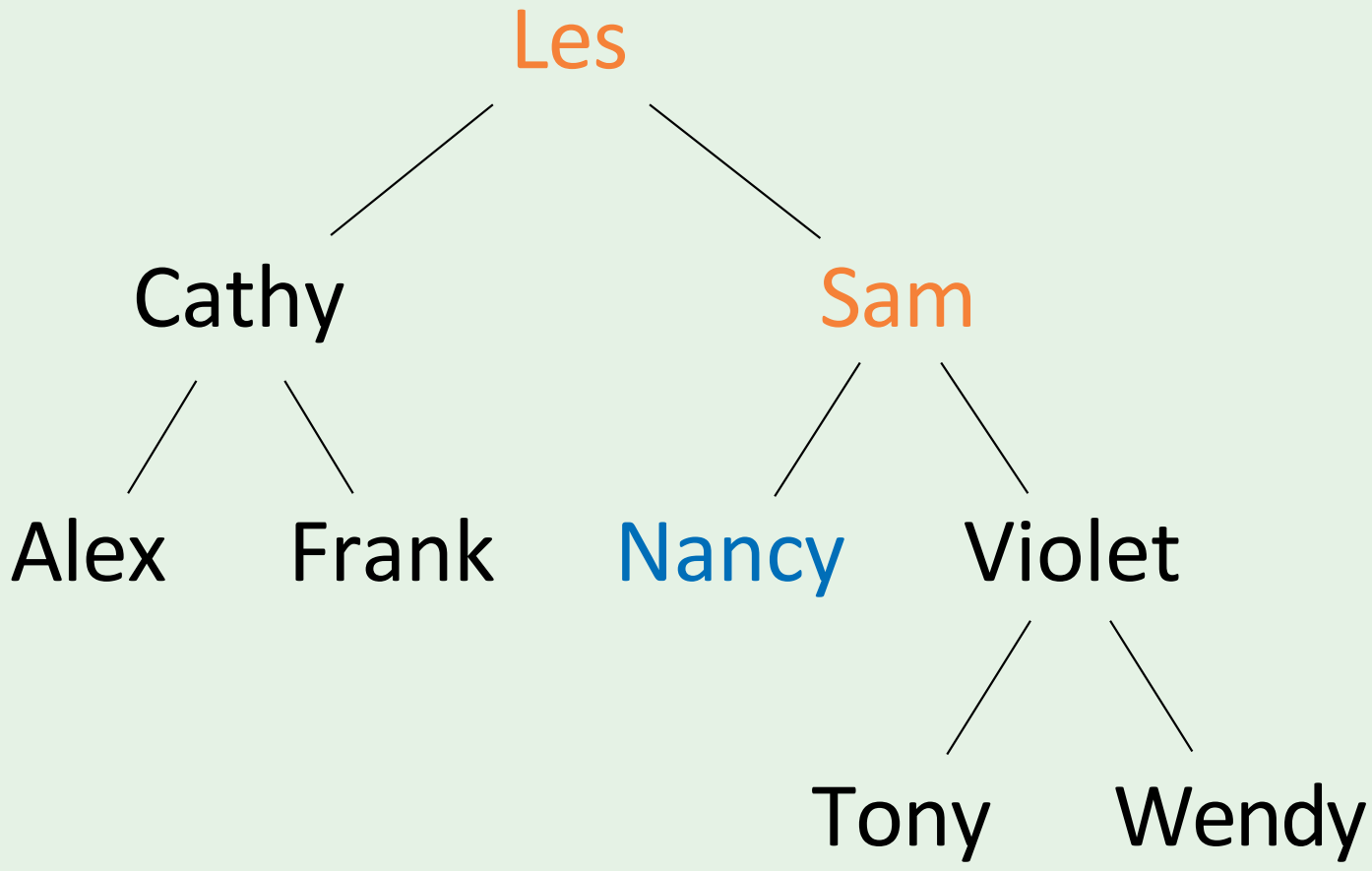
Output: Les Cathy Alex Frank Sam

PreOrderTraversal



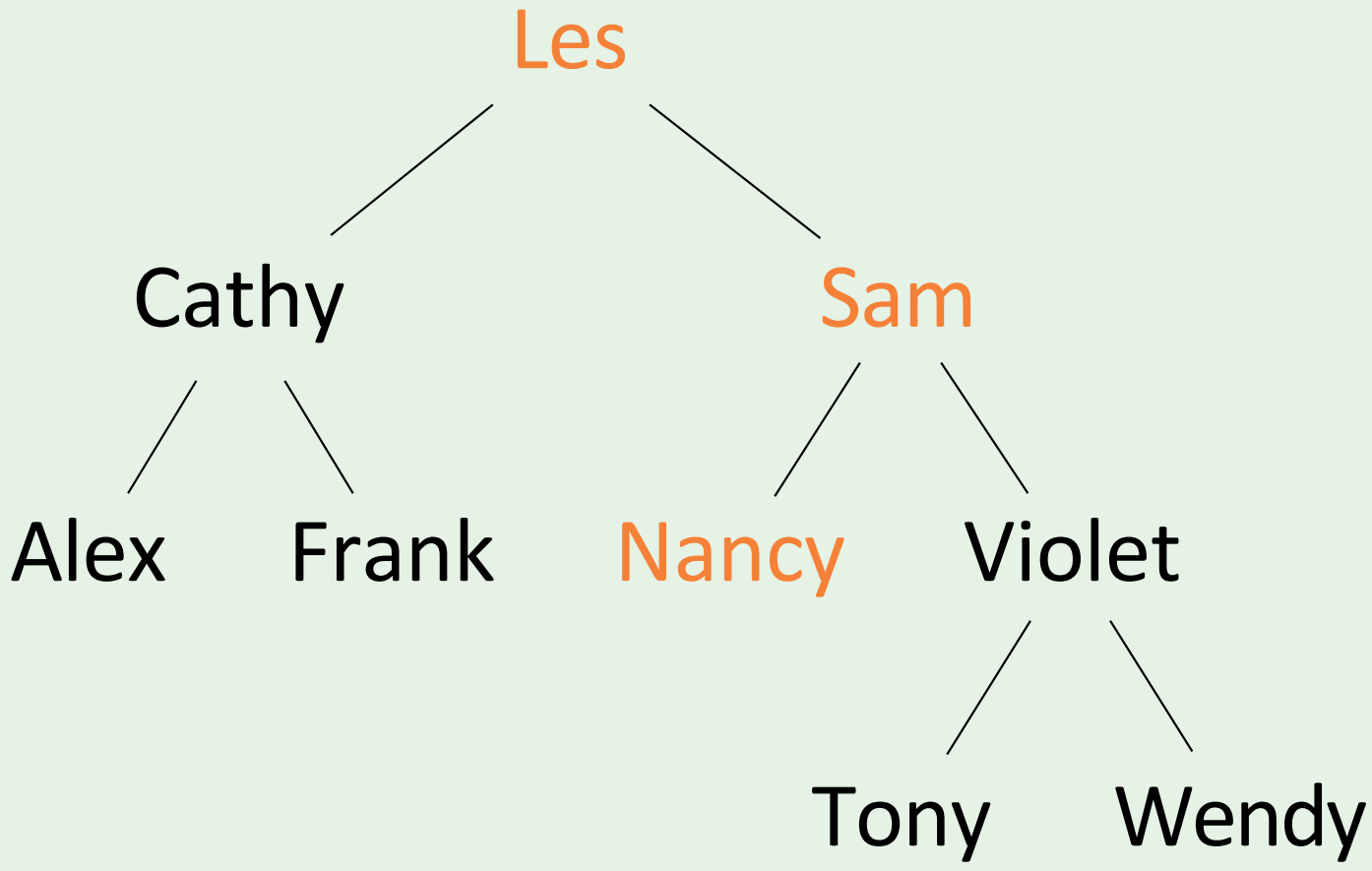
Output: Les Cathy Alex Frank Sam

PreOrderTraversal



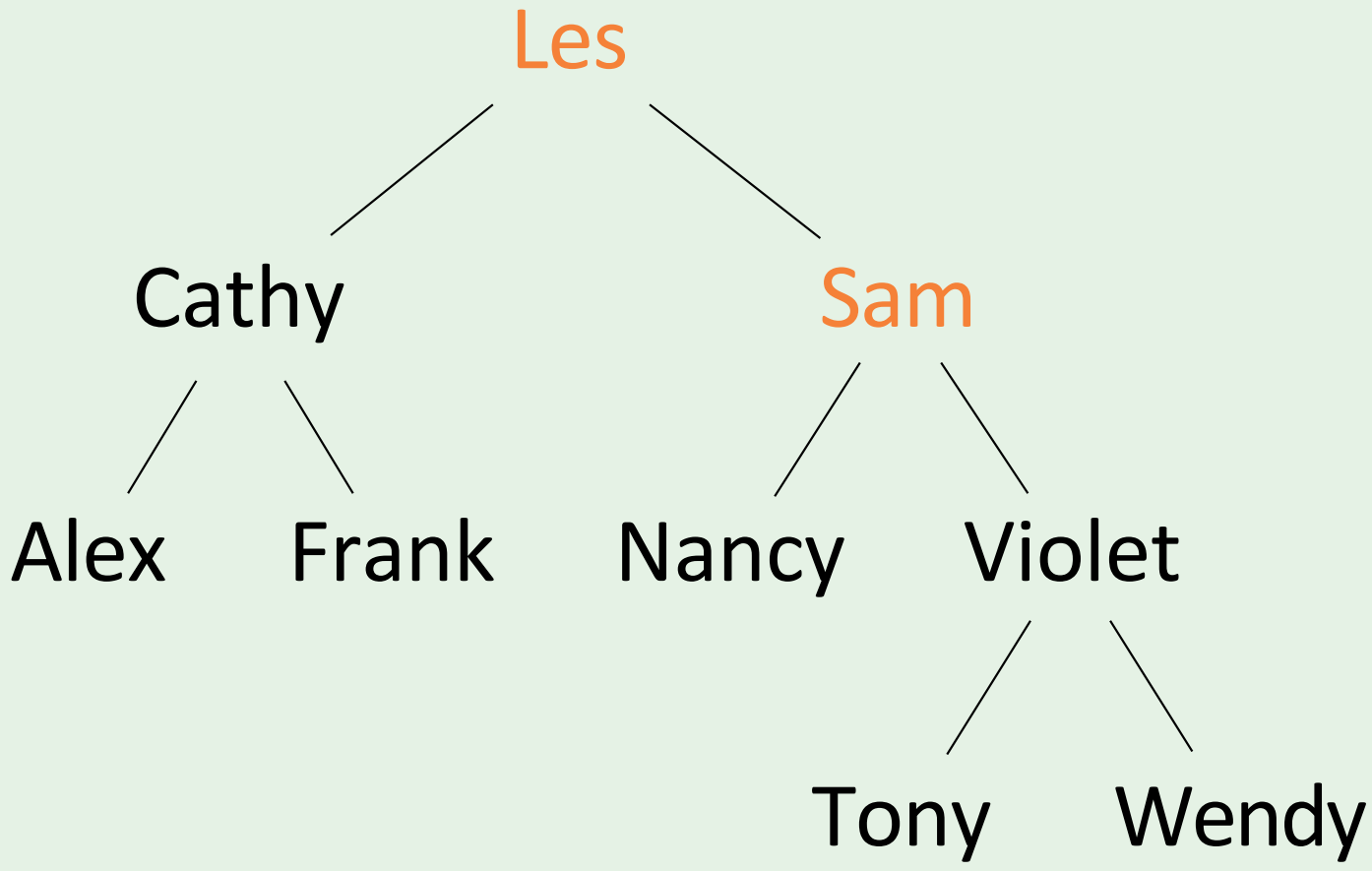
Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



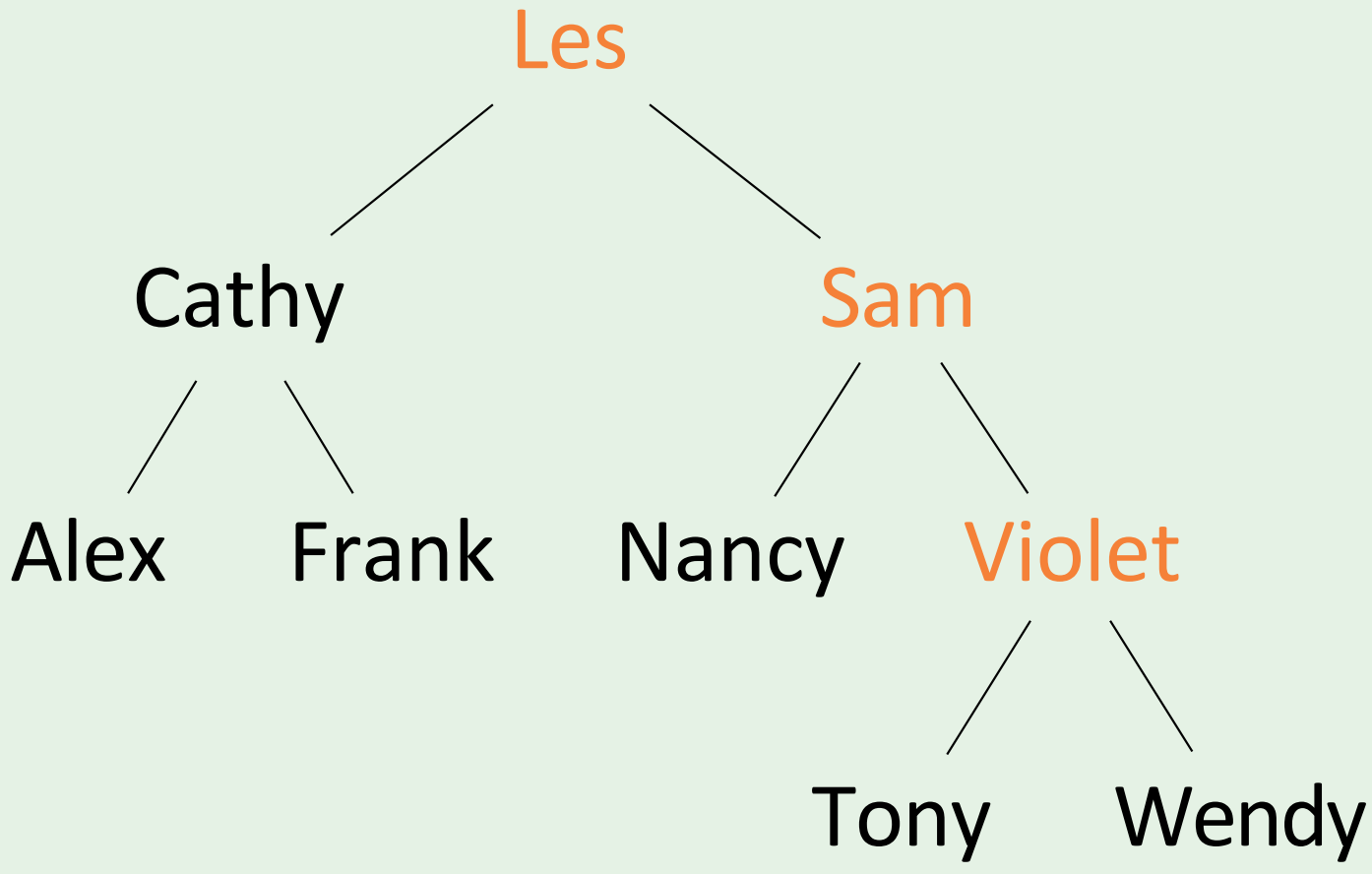
Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



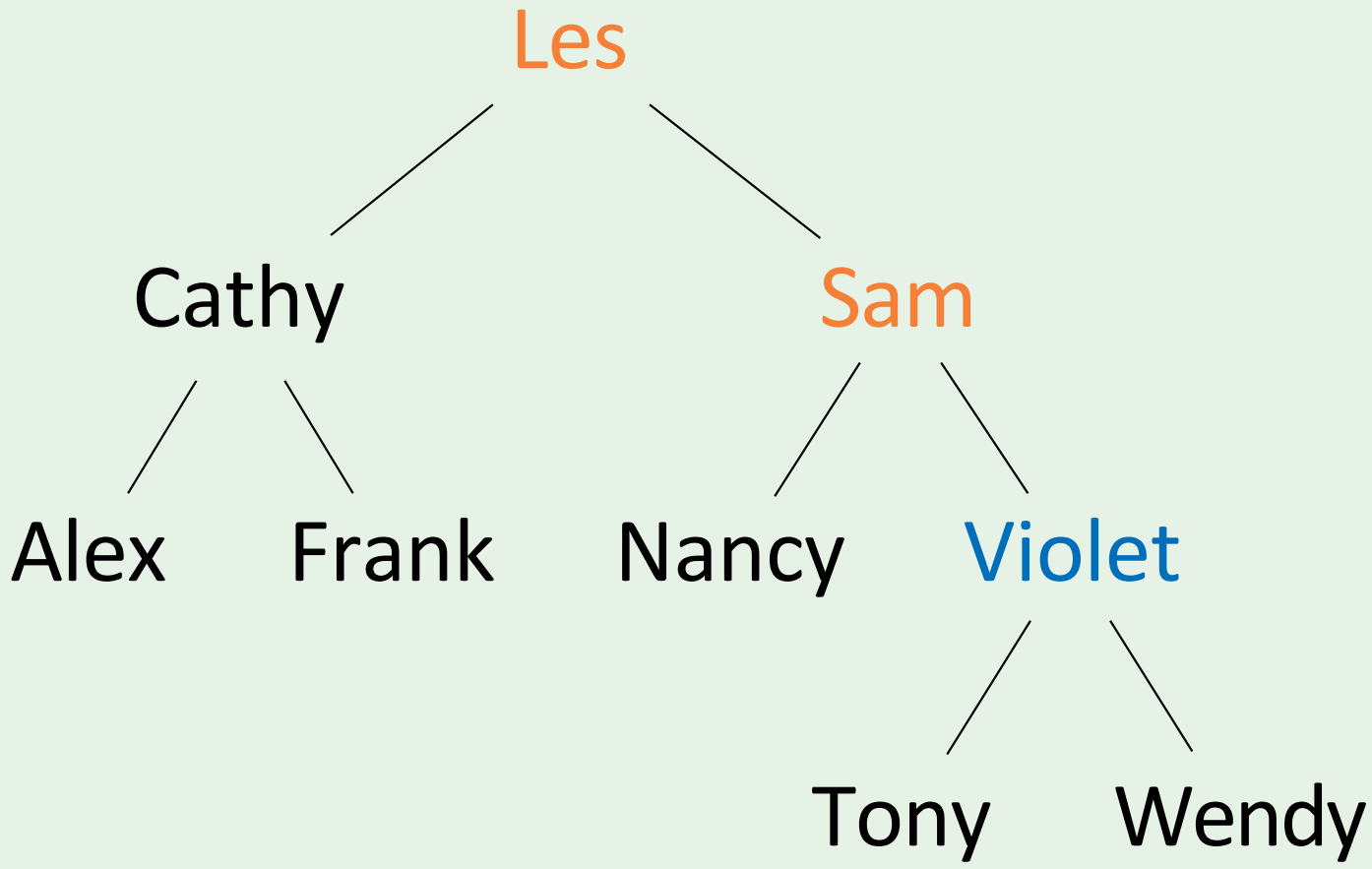
Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



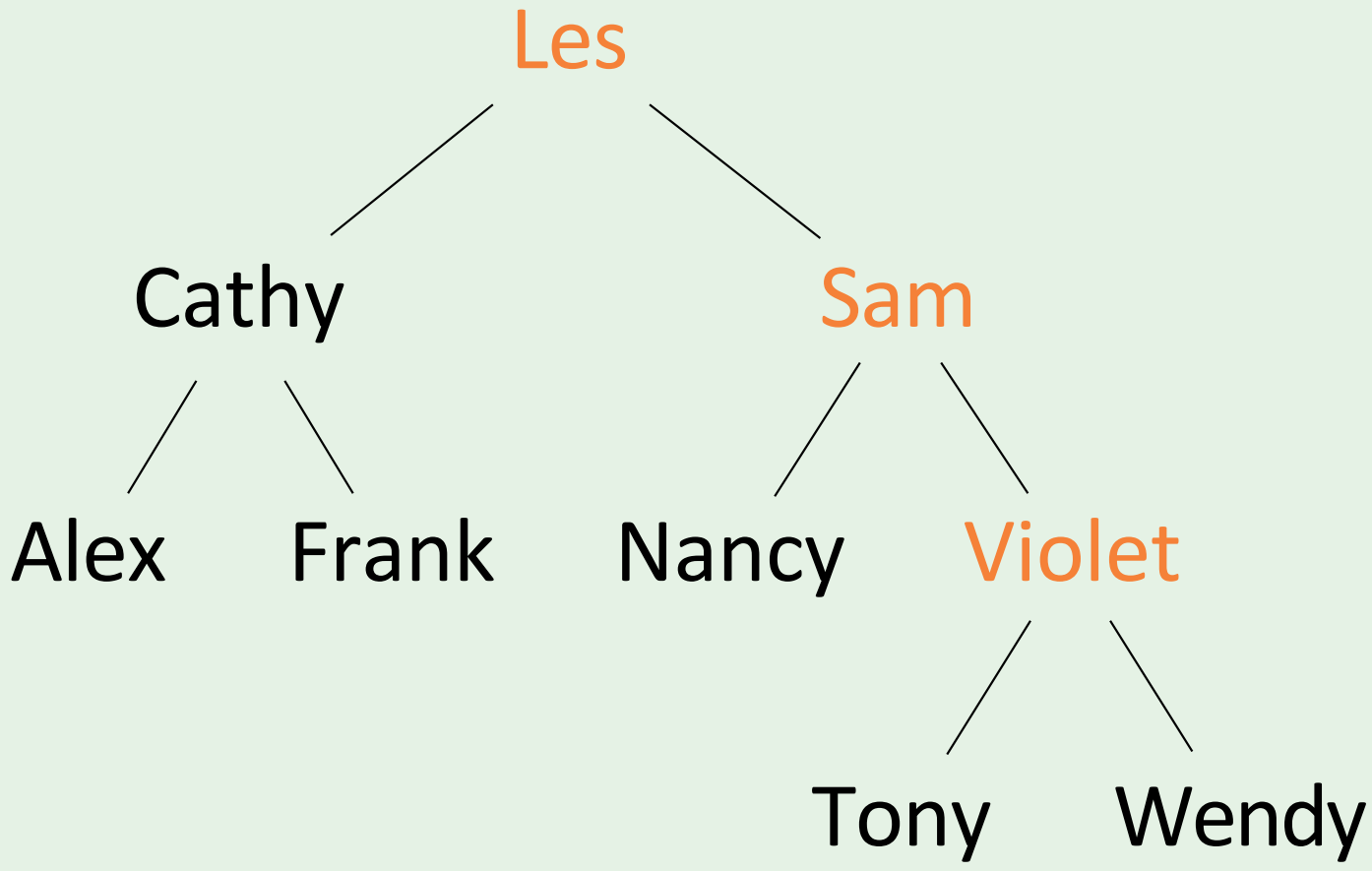
Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



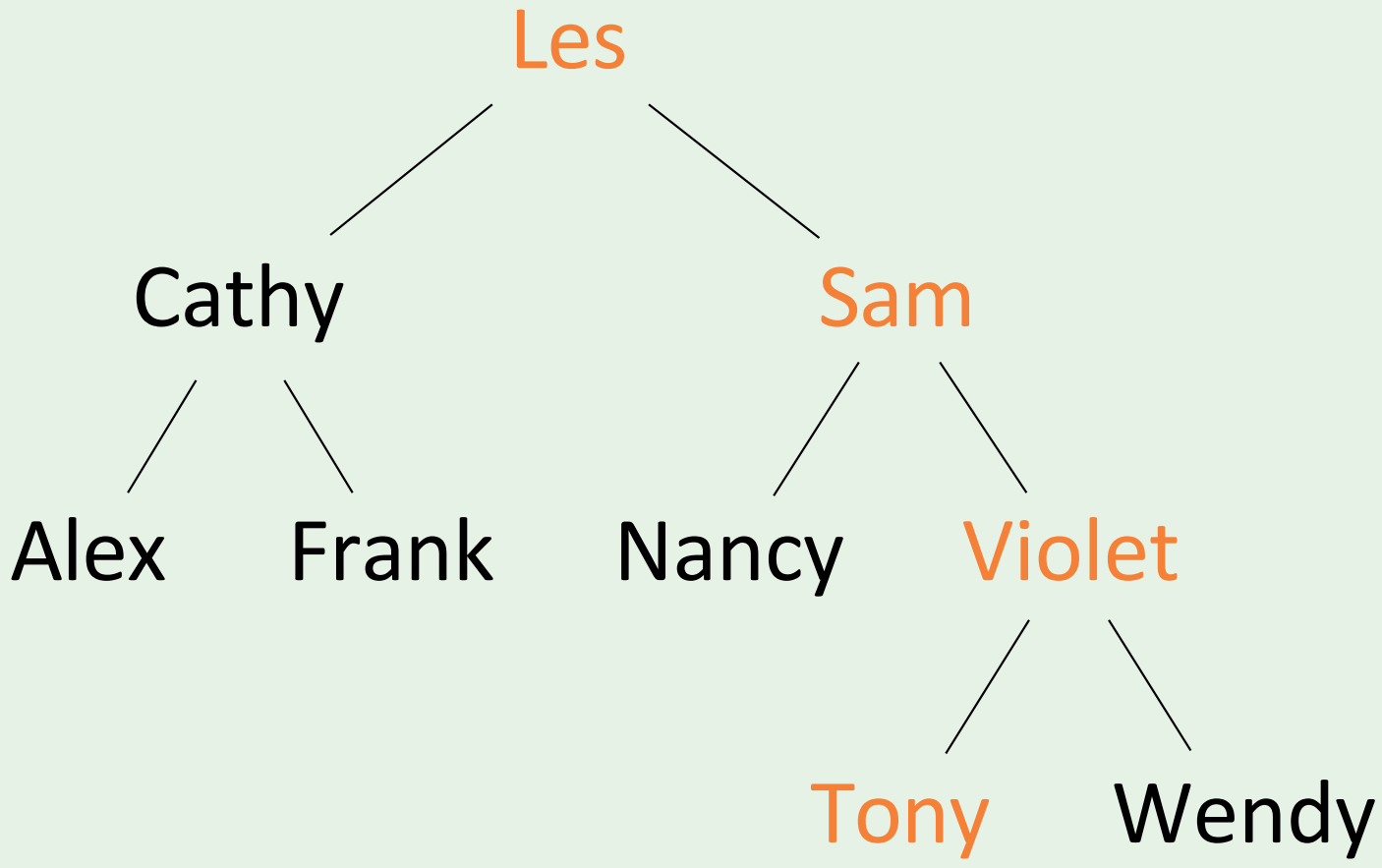
Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

PreOrderTraversal



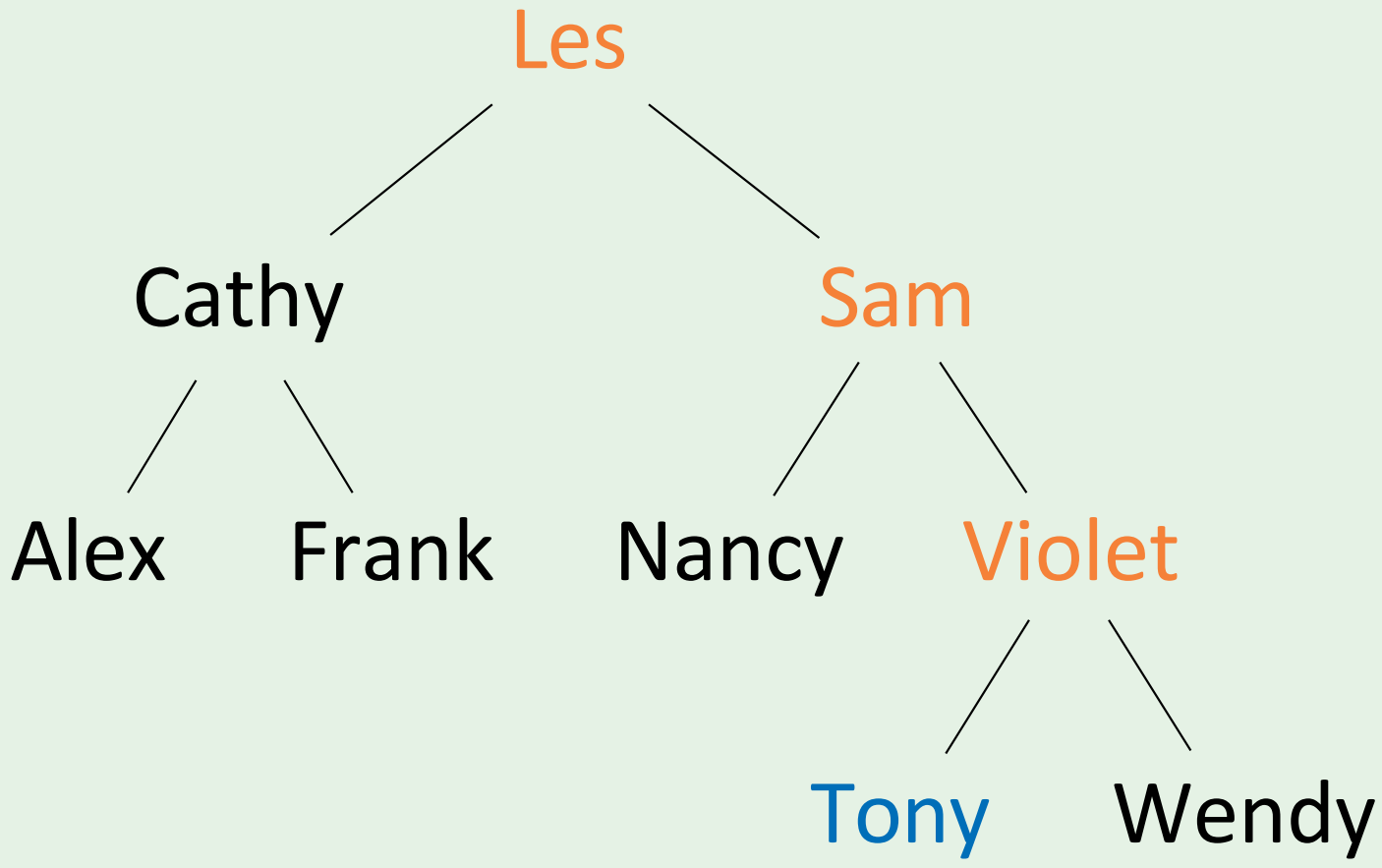
Output: Les Cathy Alex Frank Sam Nancy Violet

PreOrderTraversal



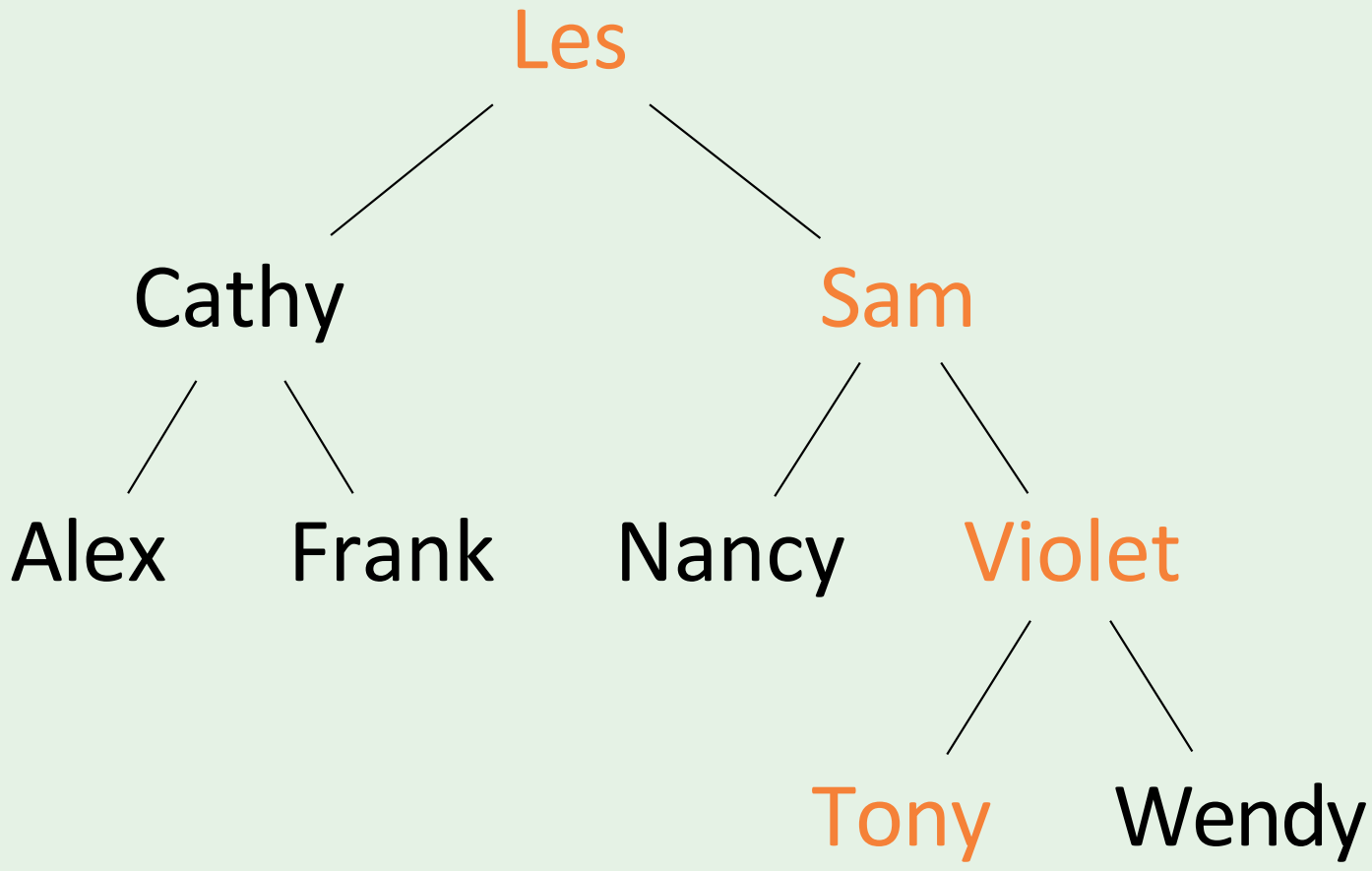
Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

PreOrderTraversal



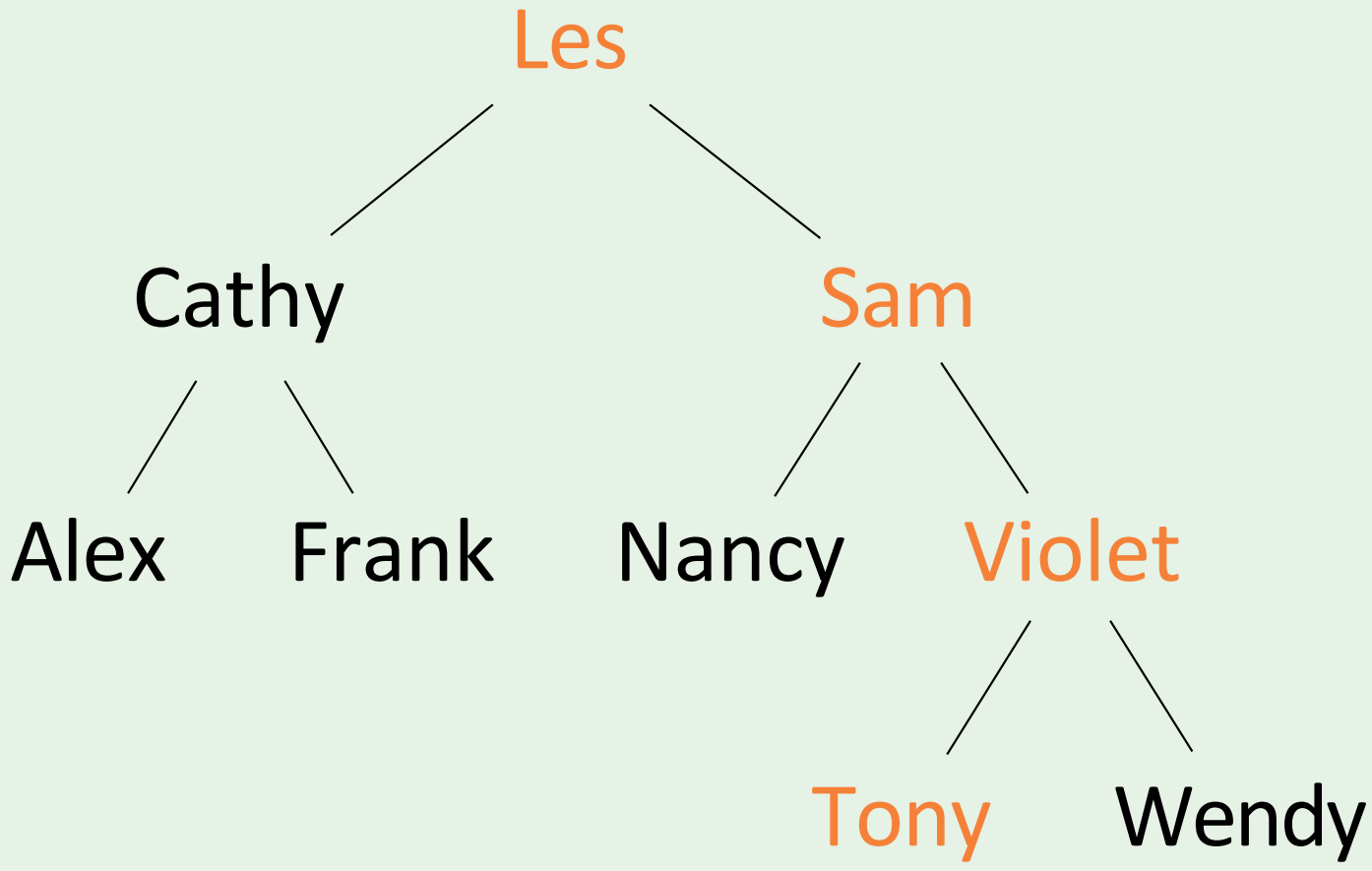
Output: Les Cathy Alex Frank Sam Nancy Violet Tony

PreOrderTraversal



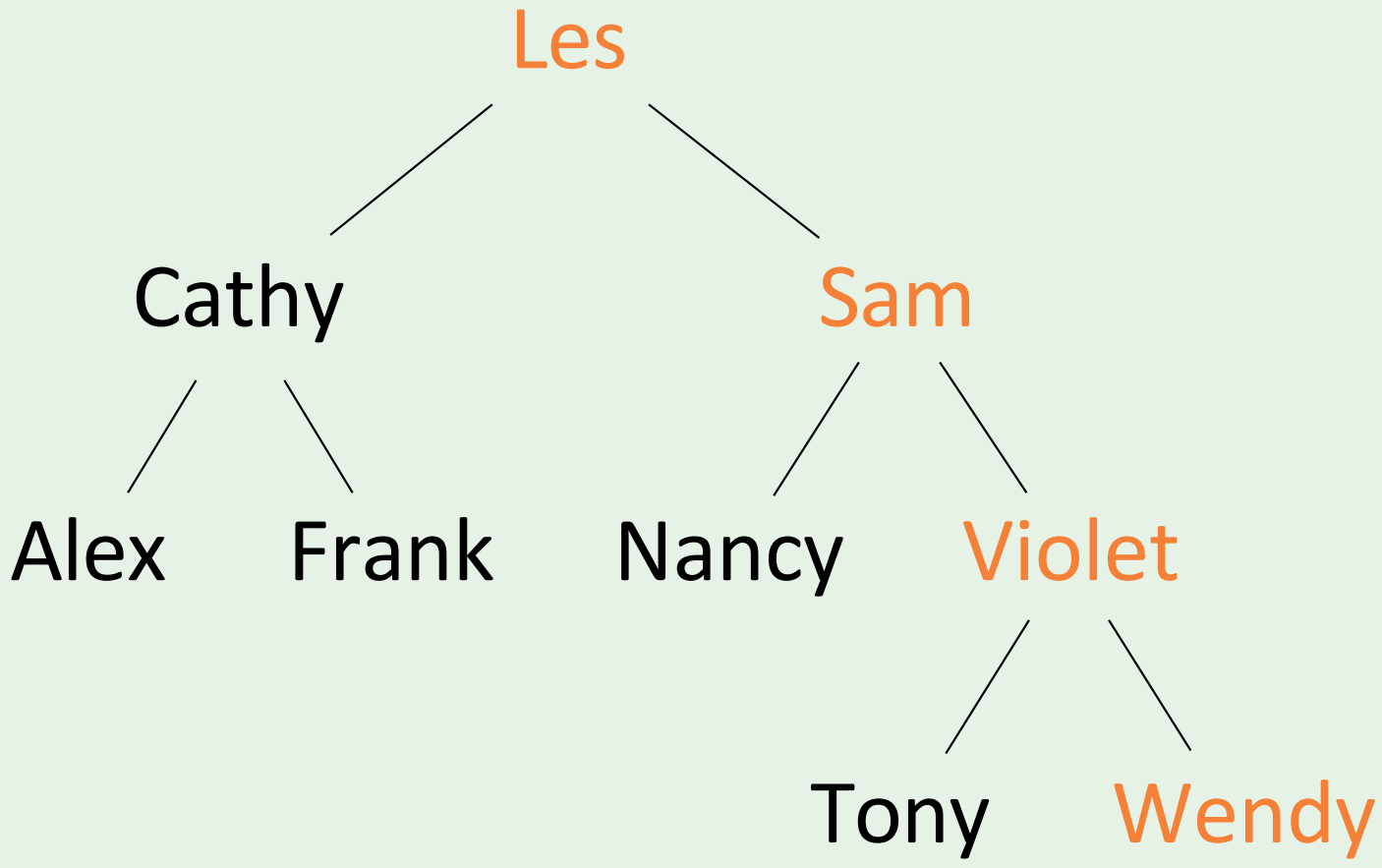
Output: Les Cathy Alex Frank Sam Nancy Violet Tony

PreOrderTraversal



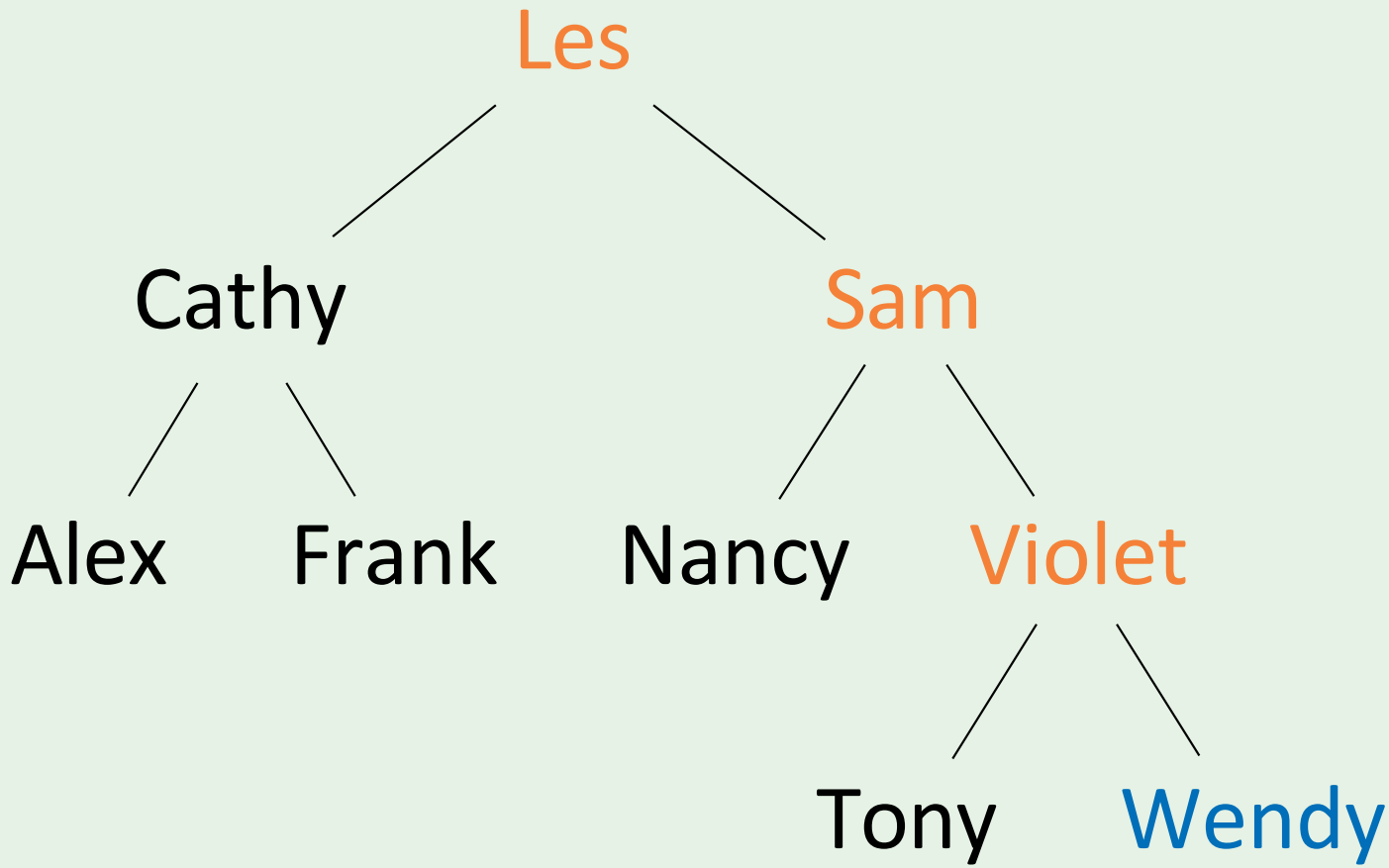
Output: Les Cathy Alex Frank Sam Nancy Violet Tony

PreOrderTraversal



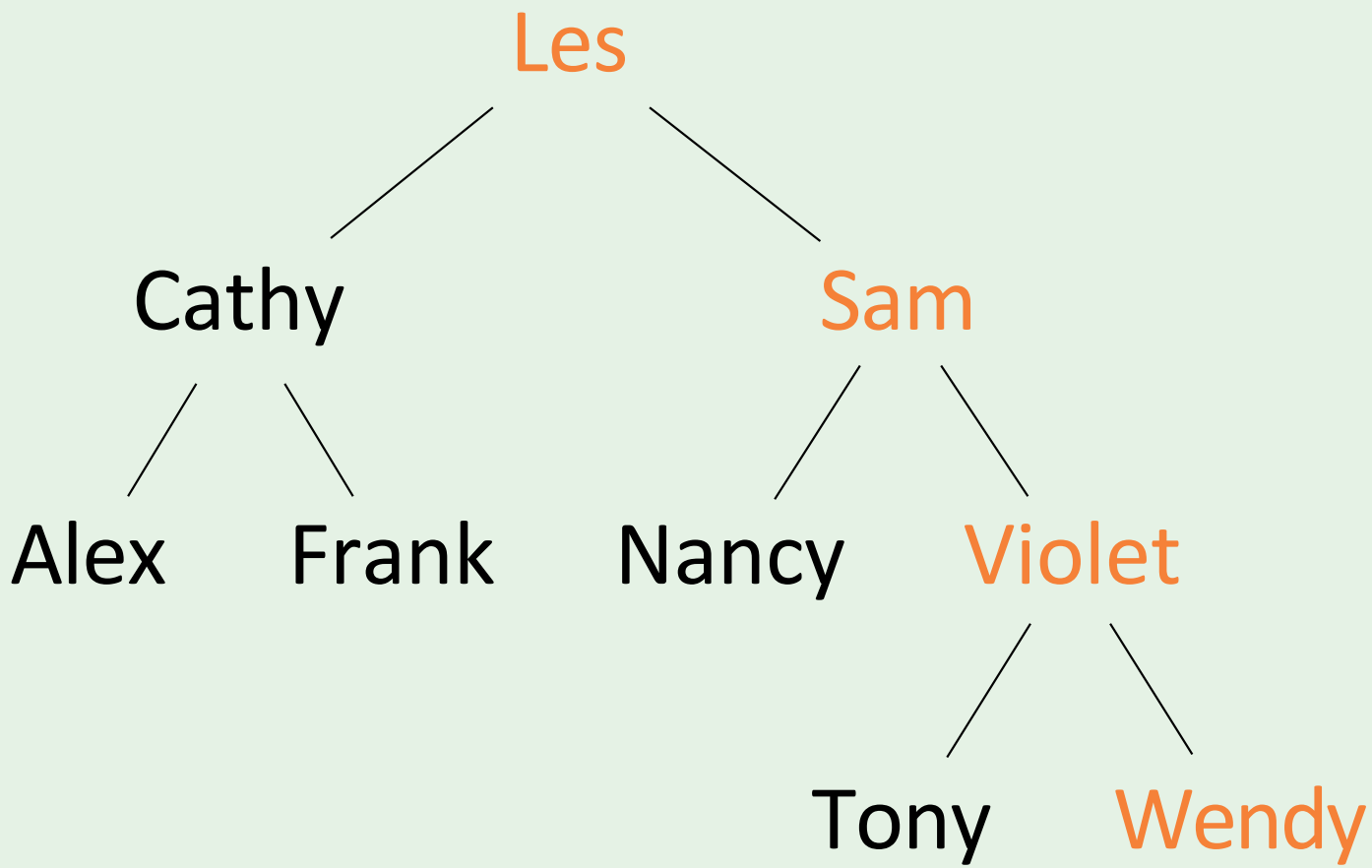
Output: Les Cathy Alex Frank Sam Nancy Violet Tony

PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

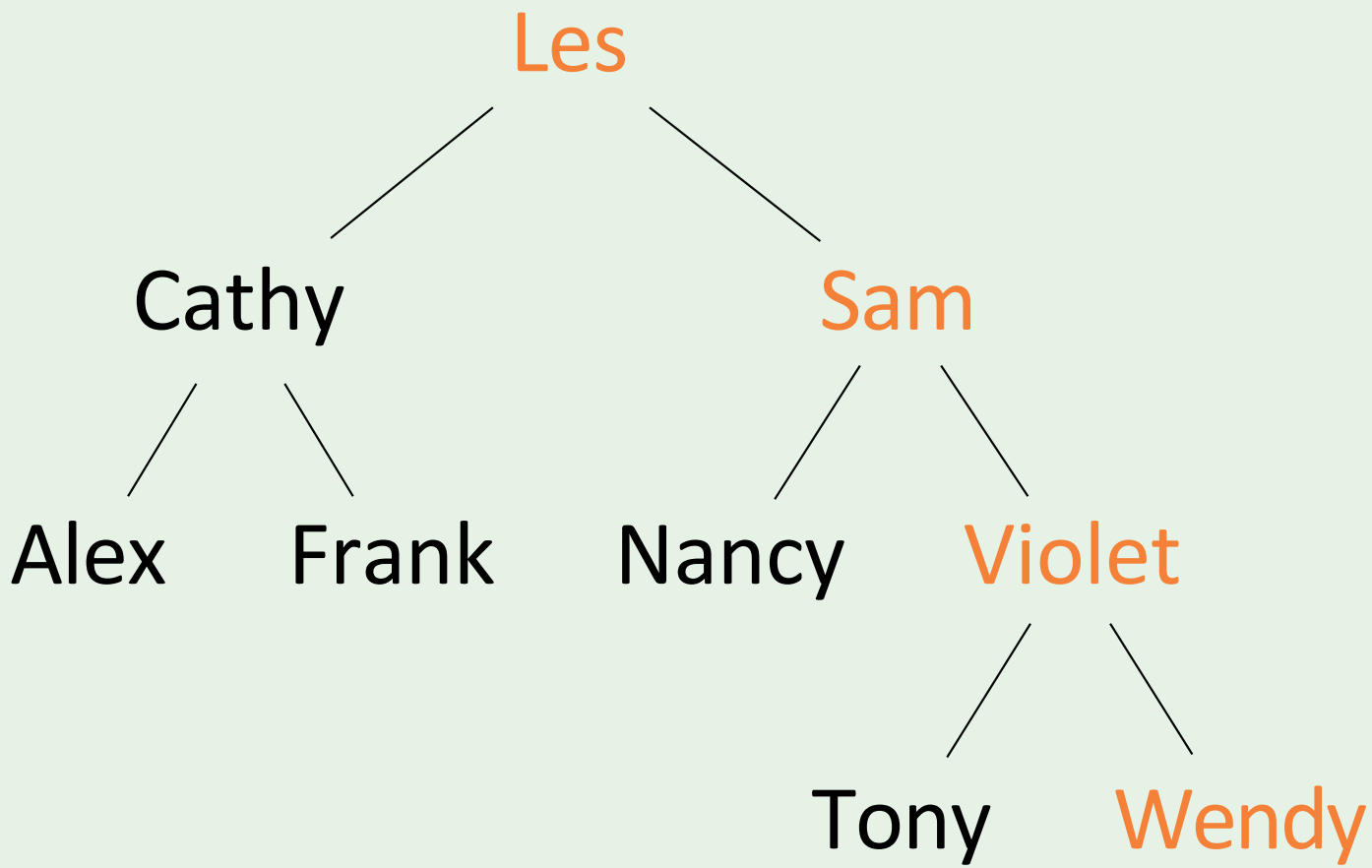
PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy

Violet Tony Wendy

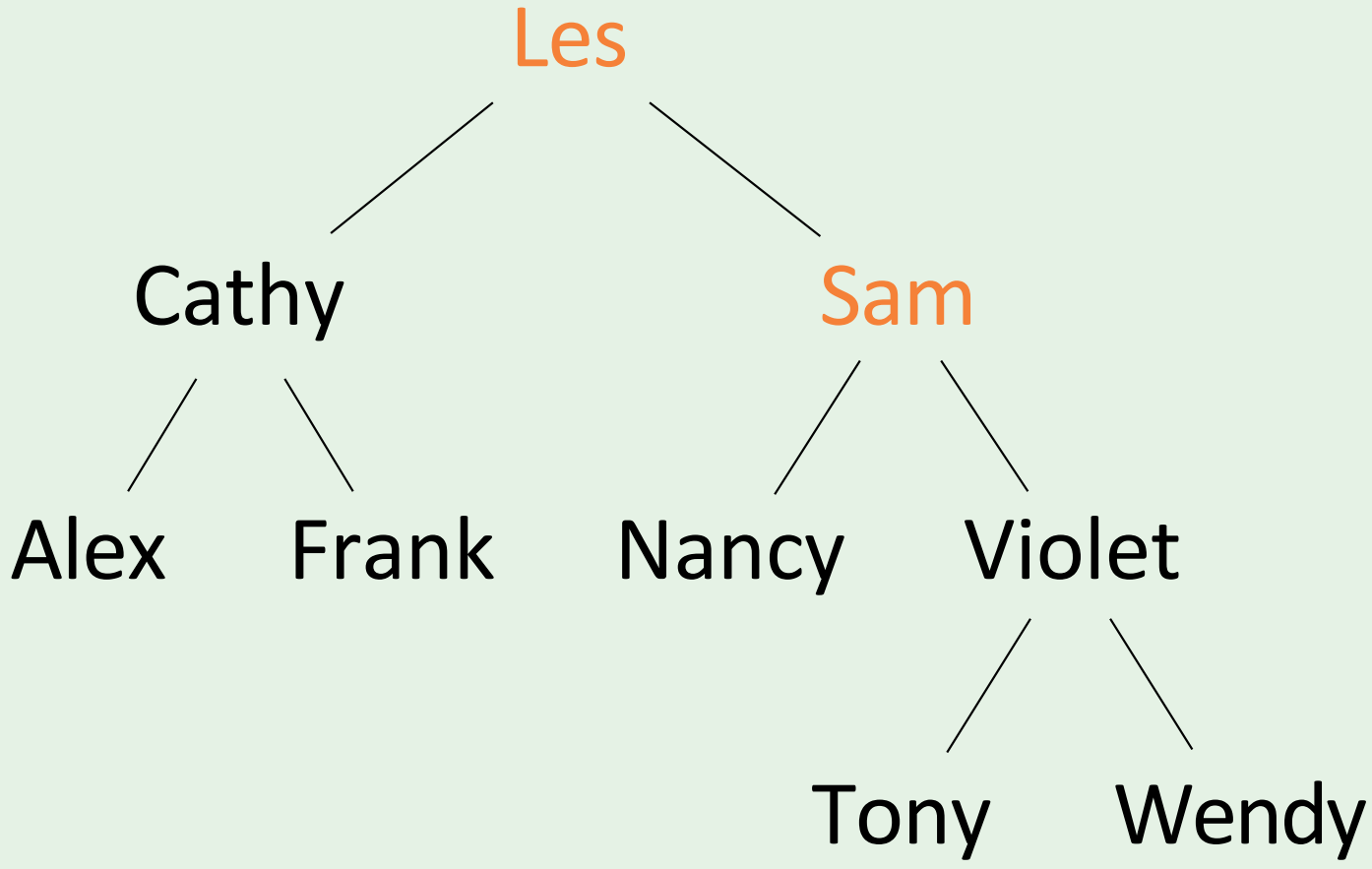
PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy

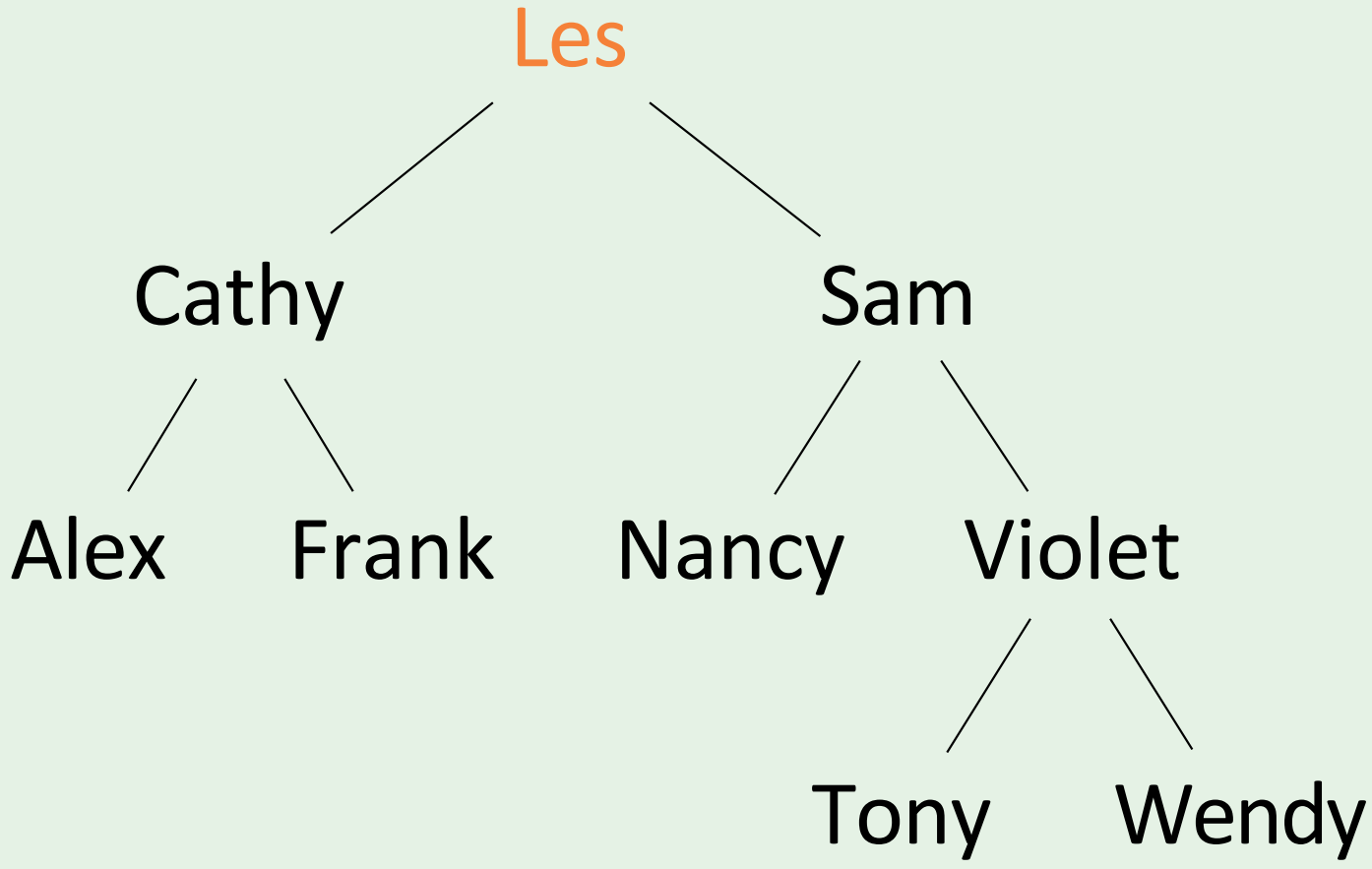
Violet Tony Wendy

PreOrderTraversal



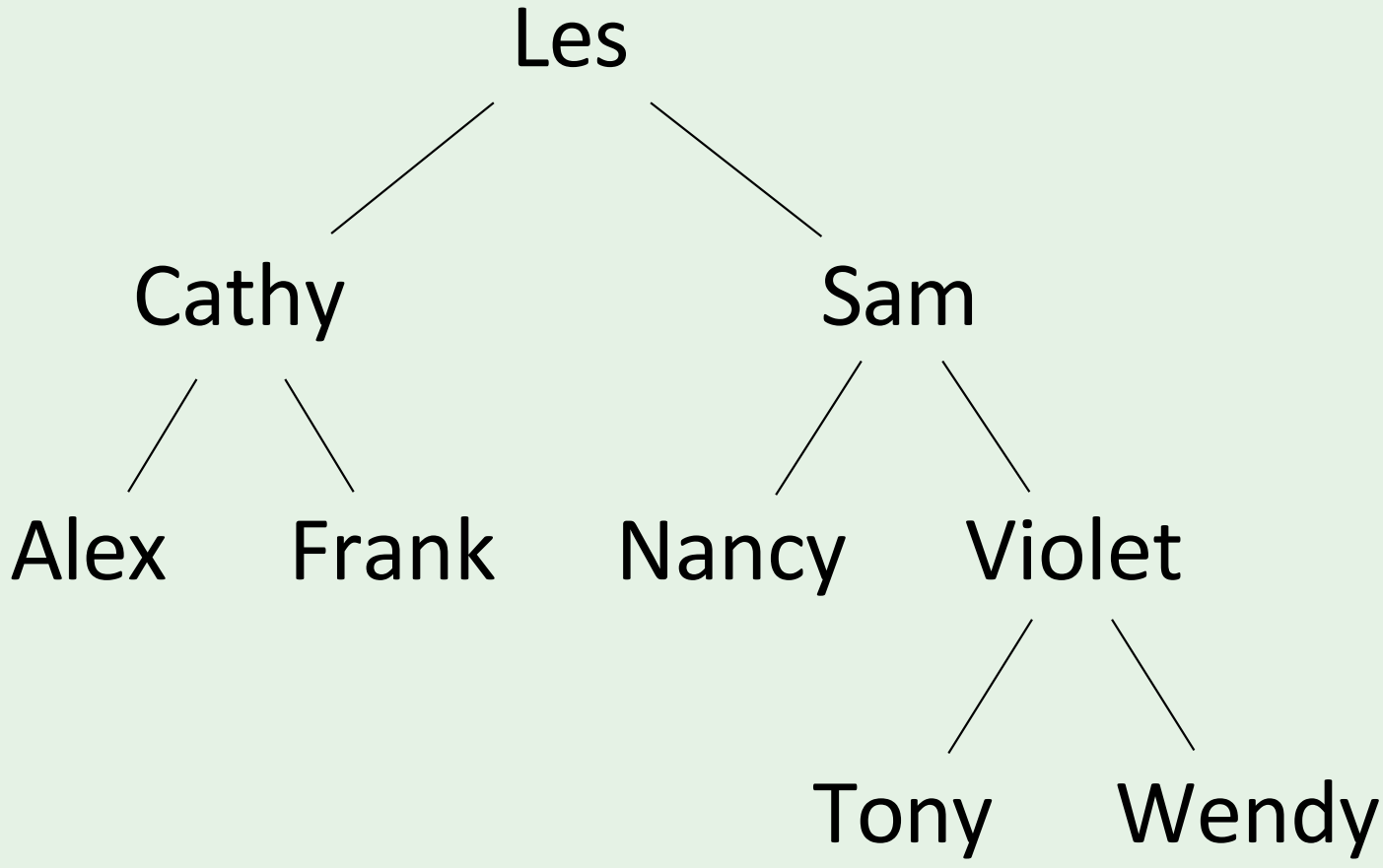
Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

PreOrderTraversal

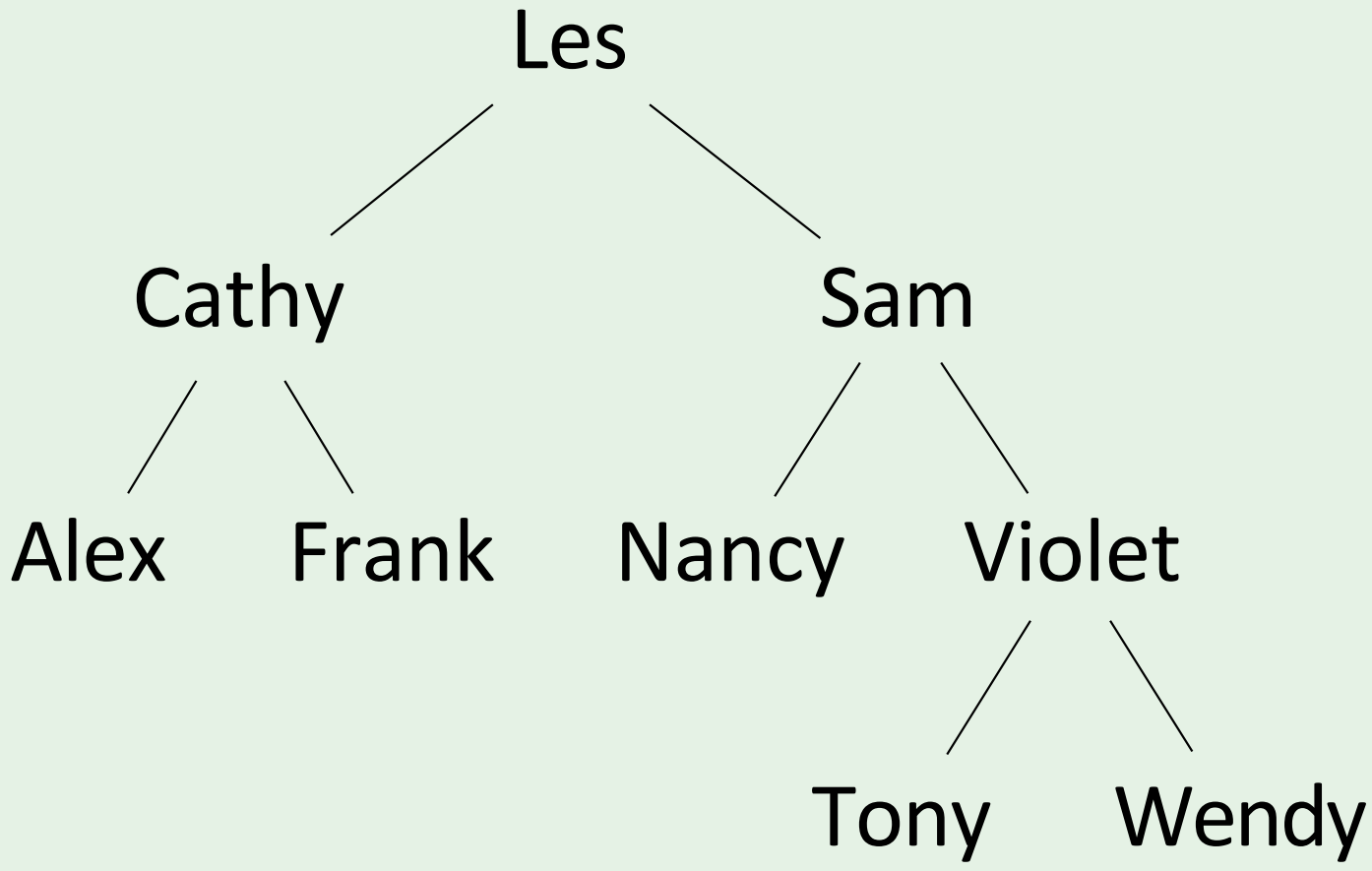


Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

Depth-first

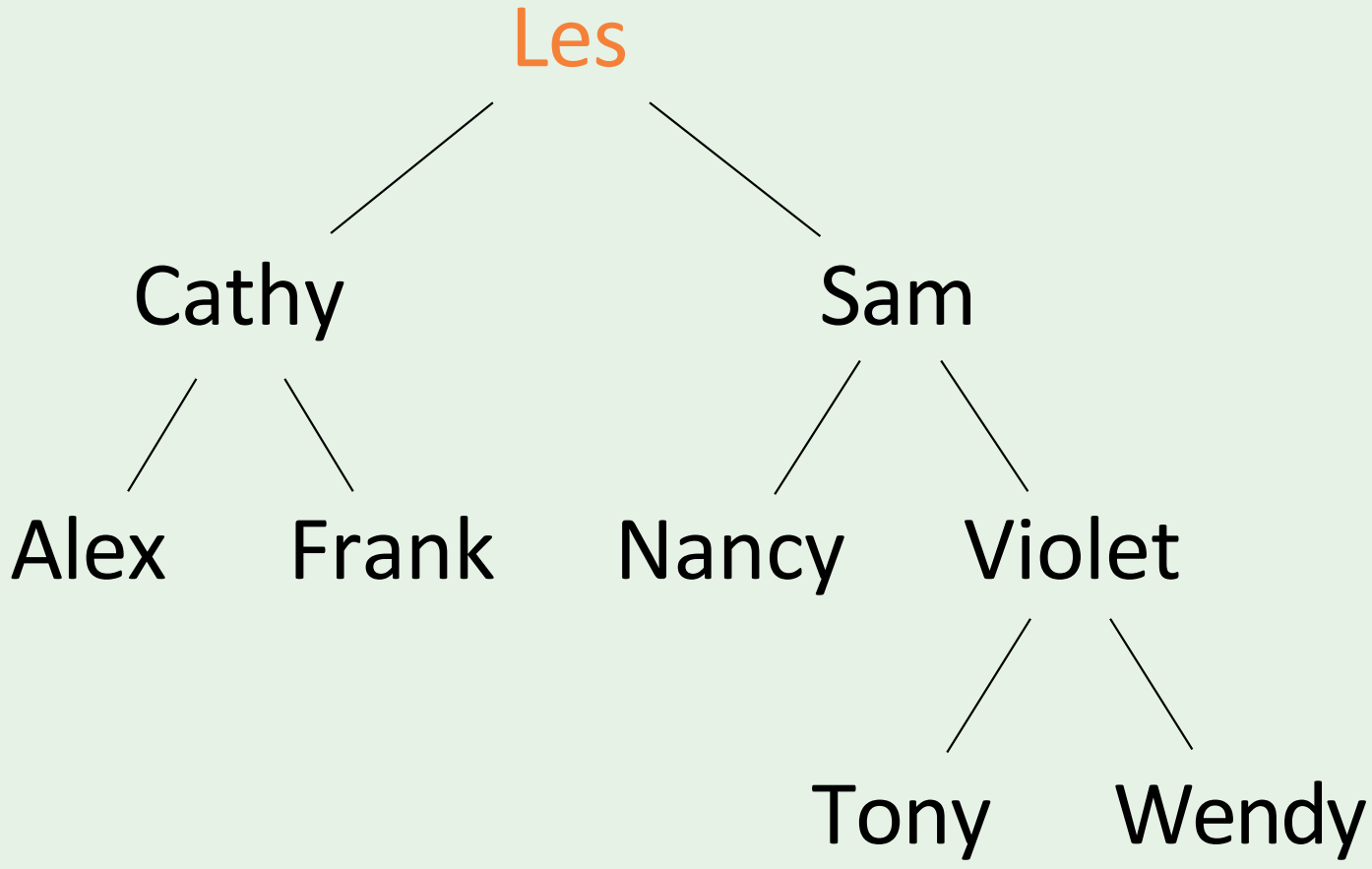
```
PostOrderTraversal(tree)
if tree = nil:
   return
PostOrderTraversal(tree.left)
PostOrderTraversal(tree.right)
Print(tree.key )
```

PostOrderTraversal



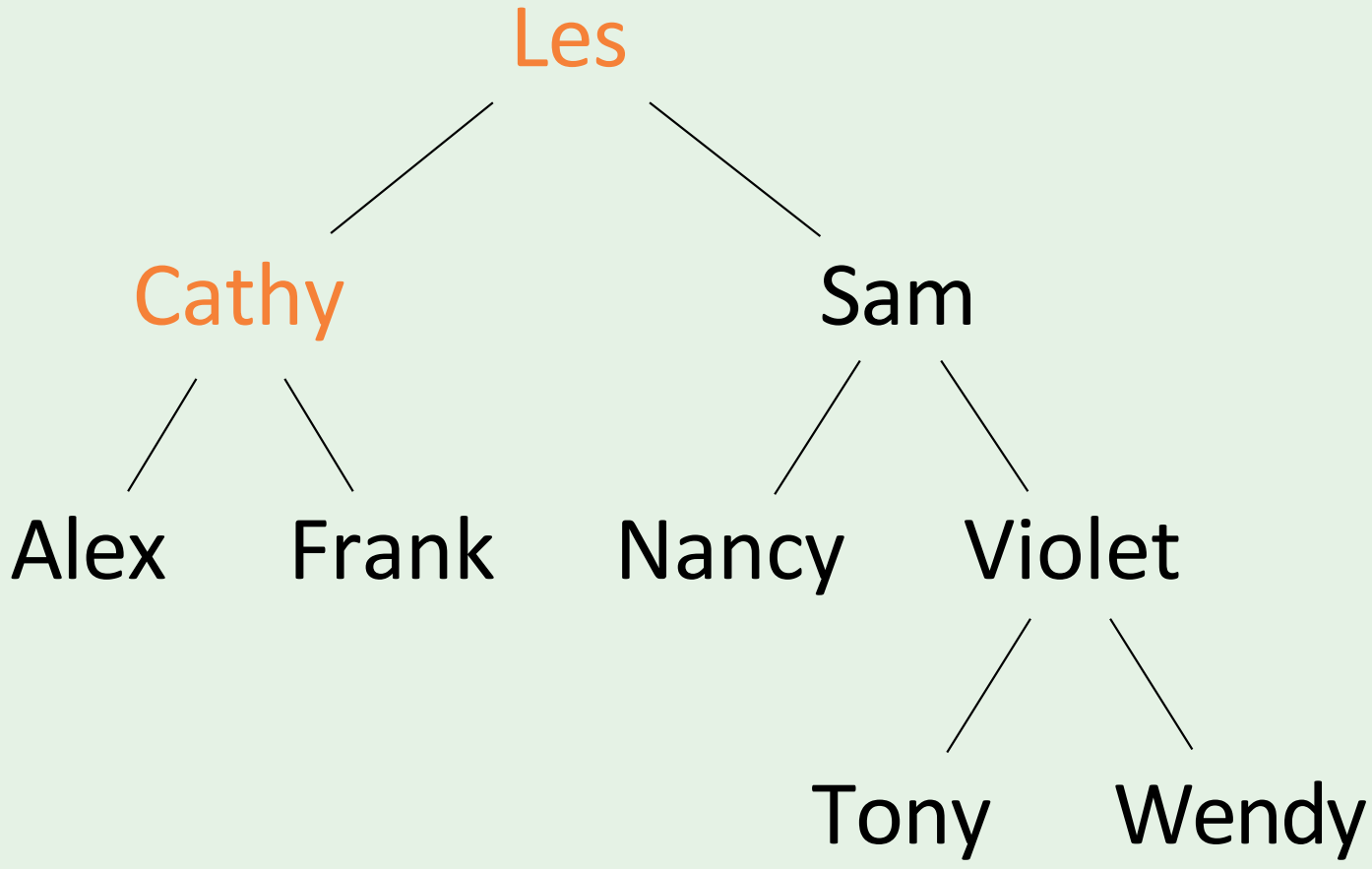
Output:

PostOrderTraversal



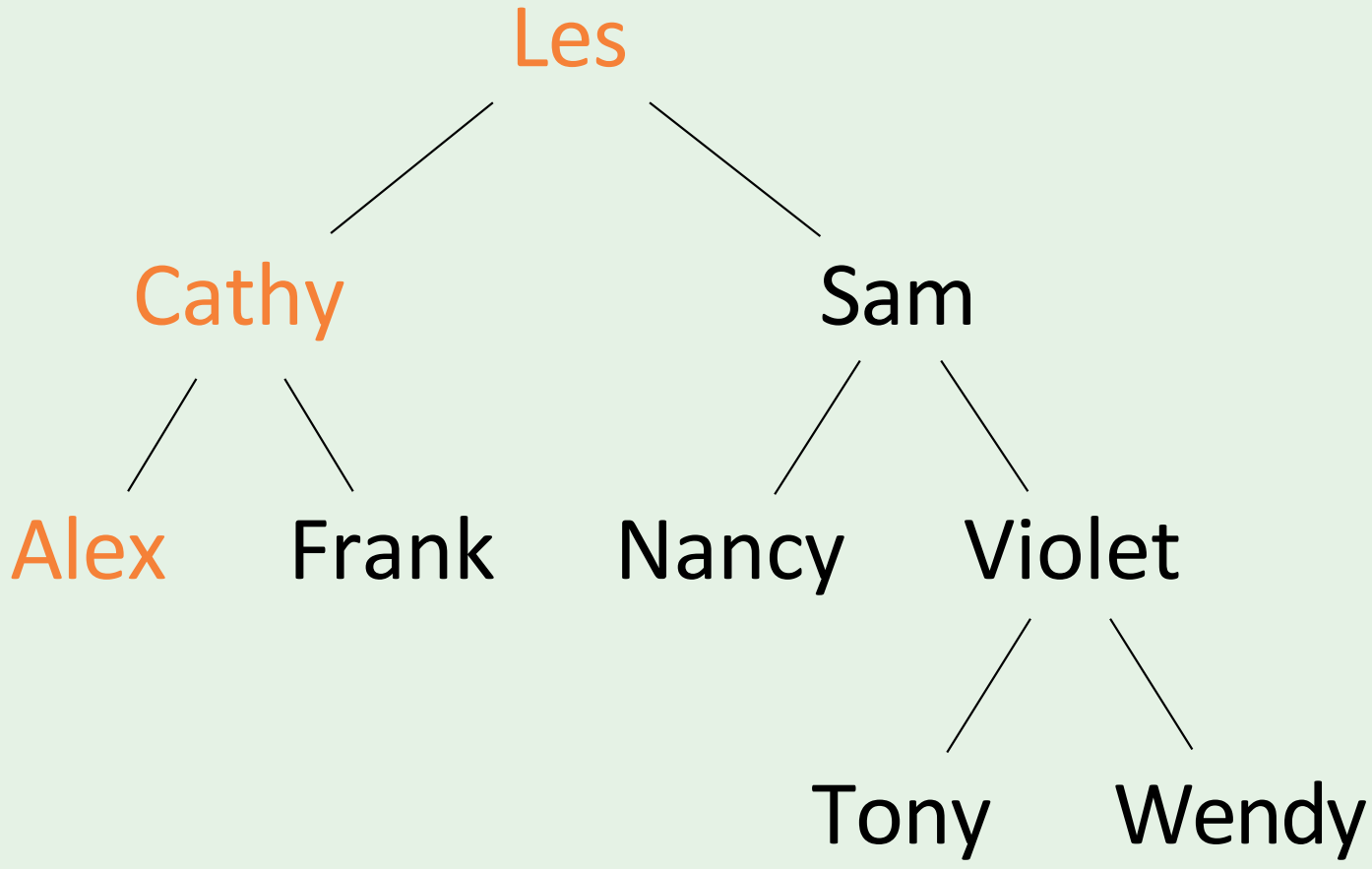
Output:

PostOrderTraversal



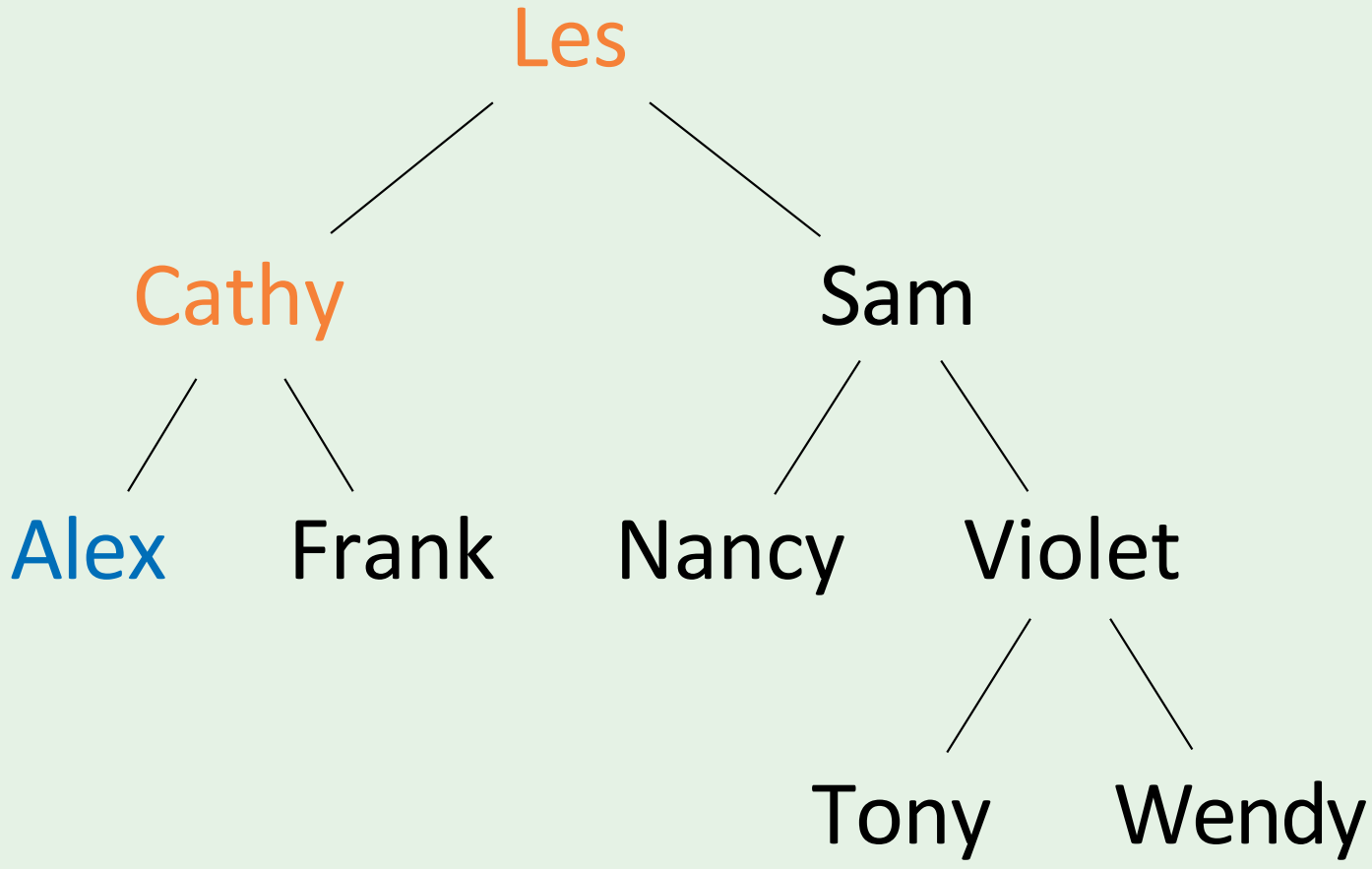
Output:

PostOrderTraversal



Output:

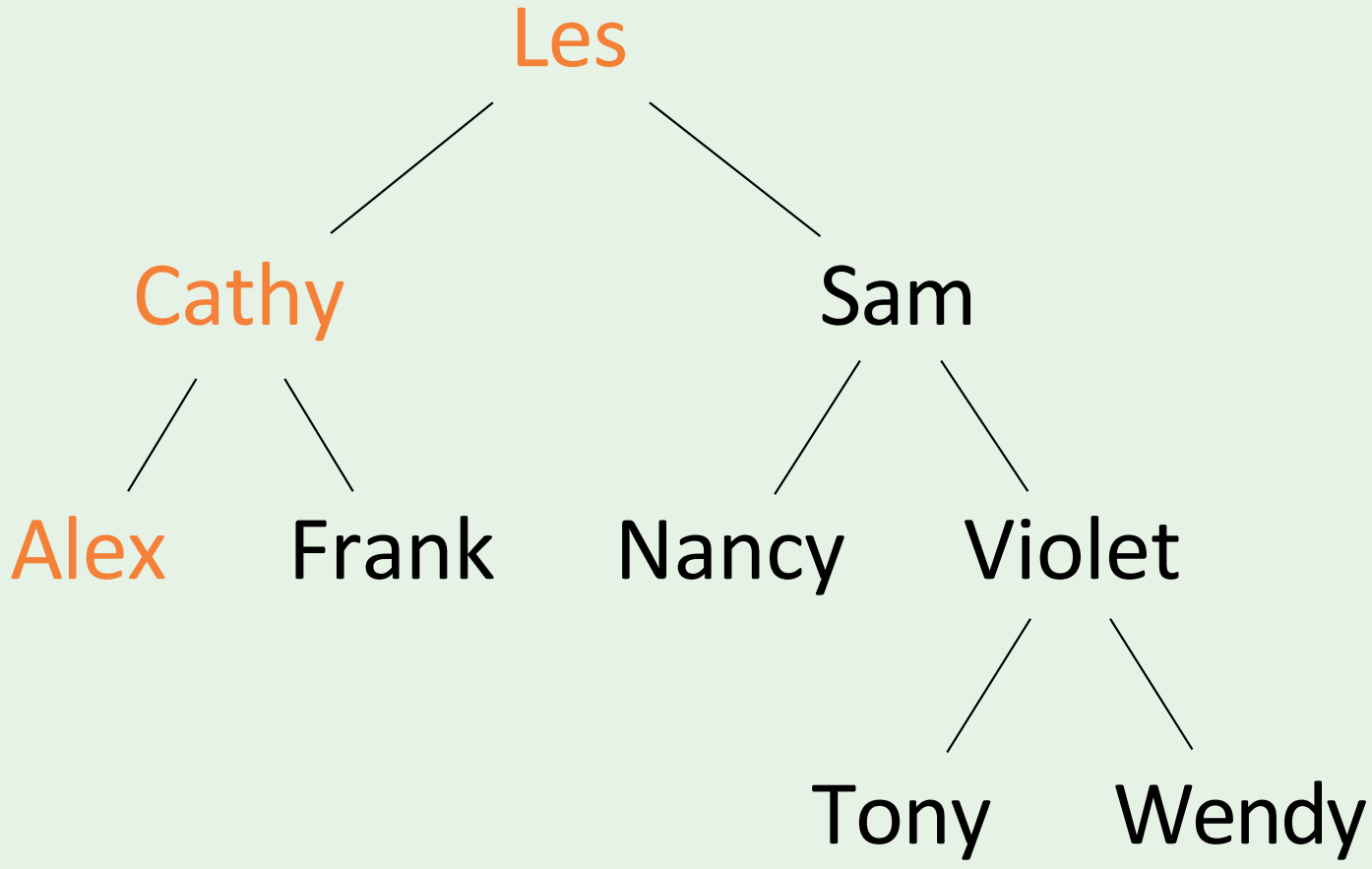
PostOrderTraversal



Output:

Alex

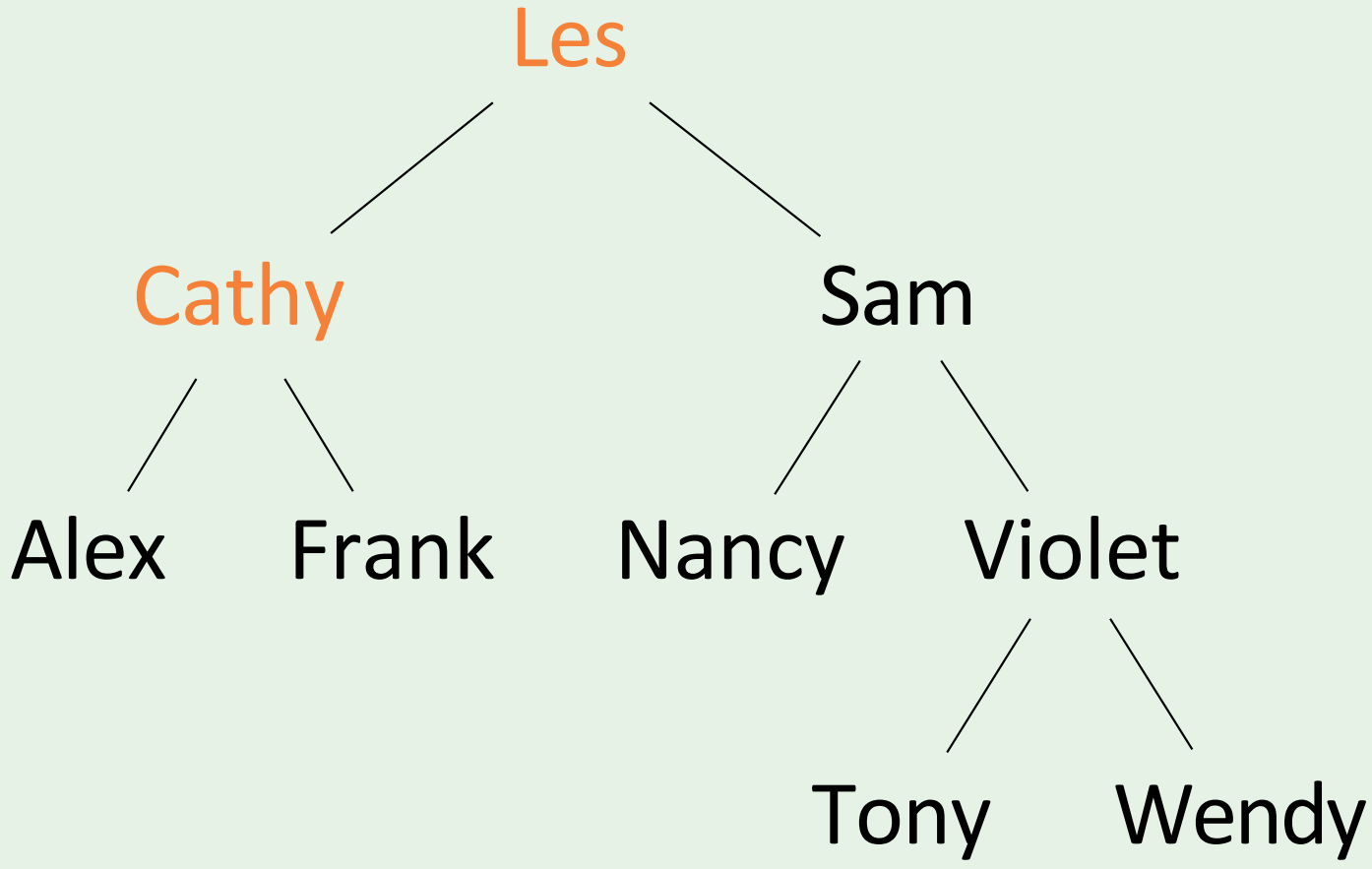
PostOrderTraversal



Output:

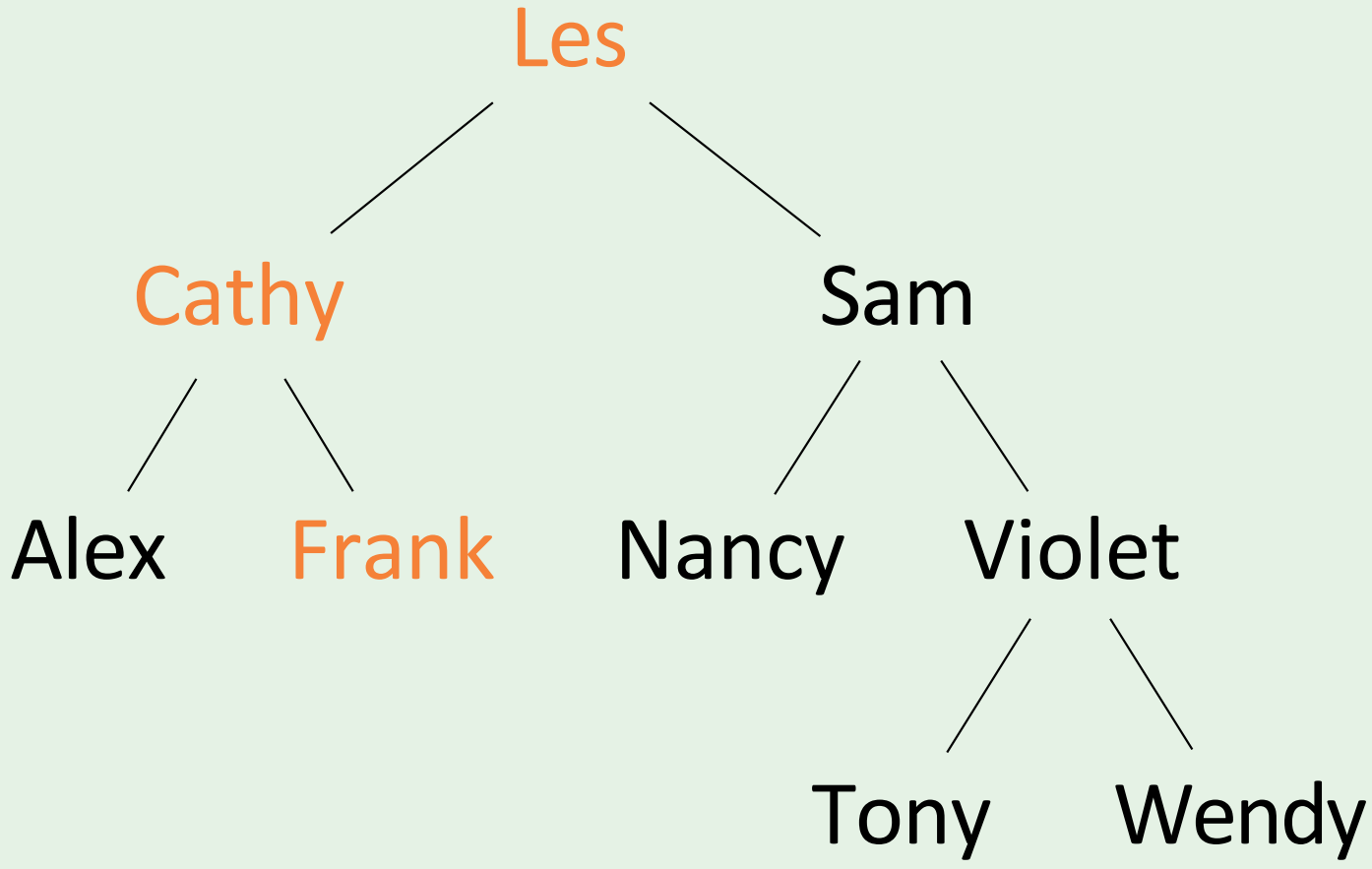
Alex

PostOrderTraversal



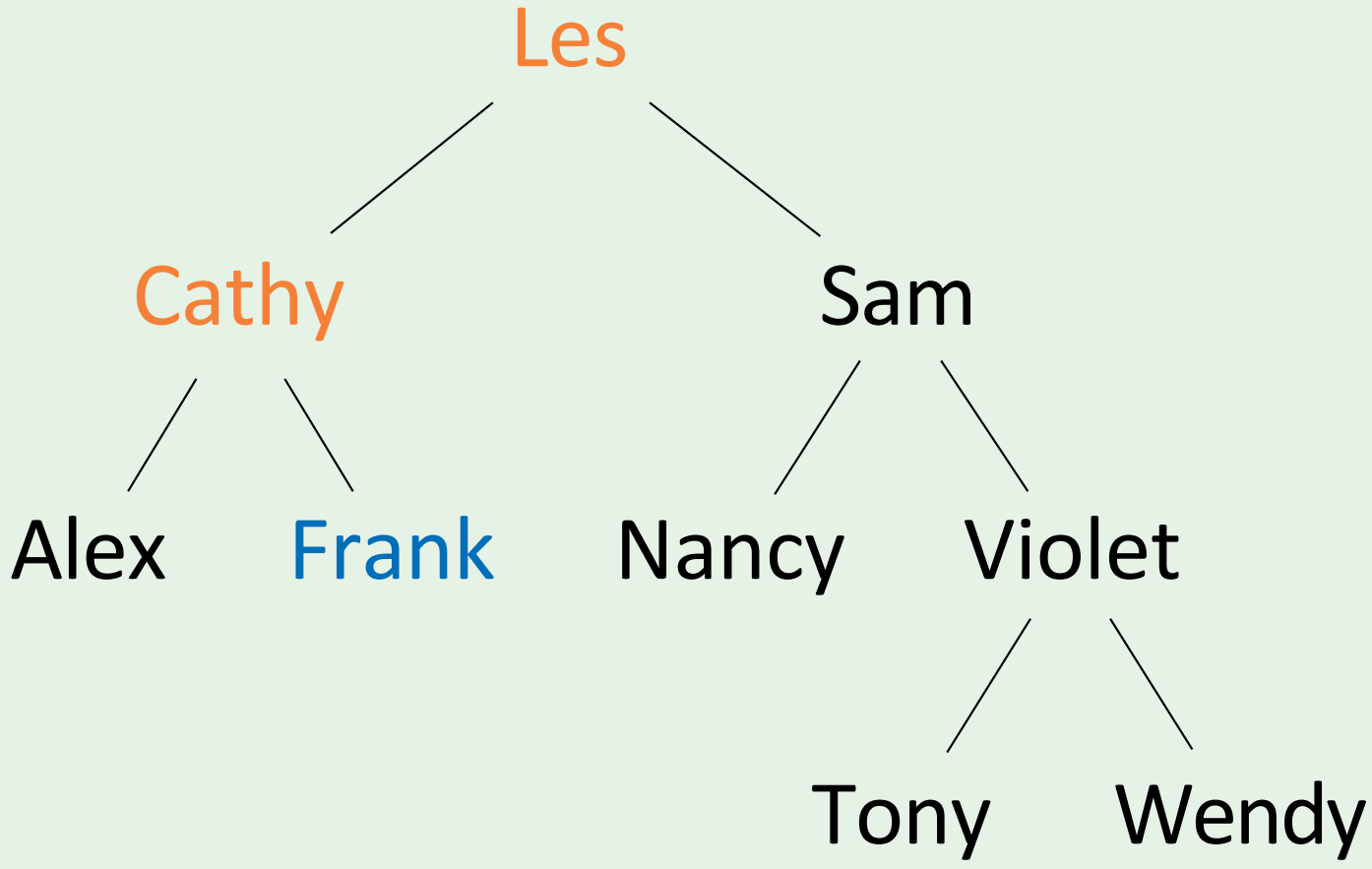
Output: Alex

PostOrderTraversal



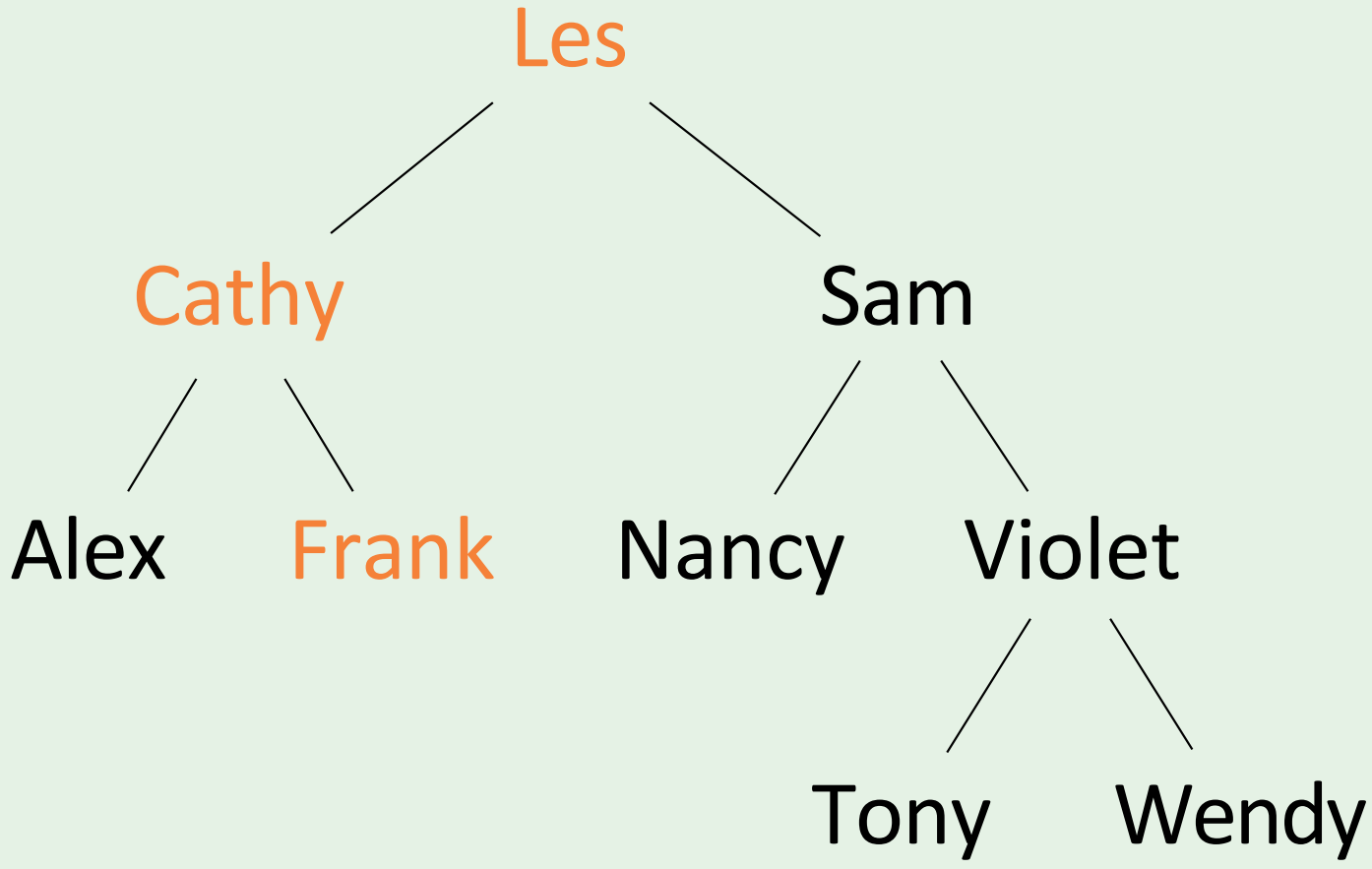
Output: Alex

PostOrderTraversal



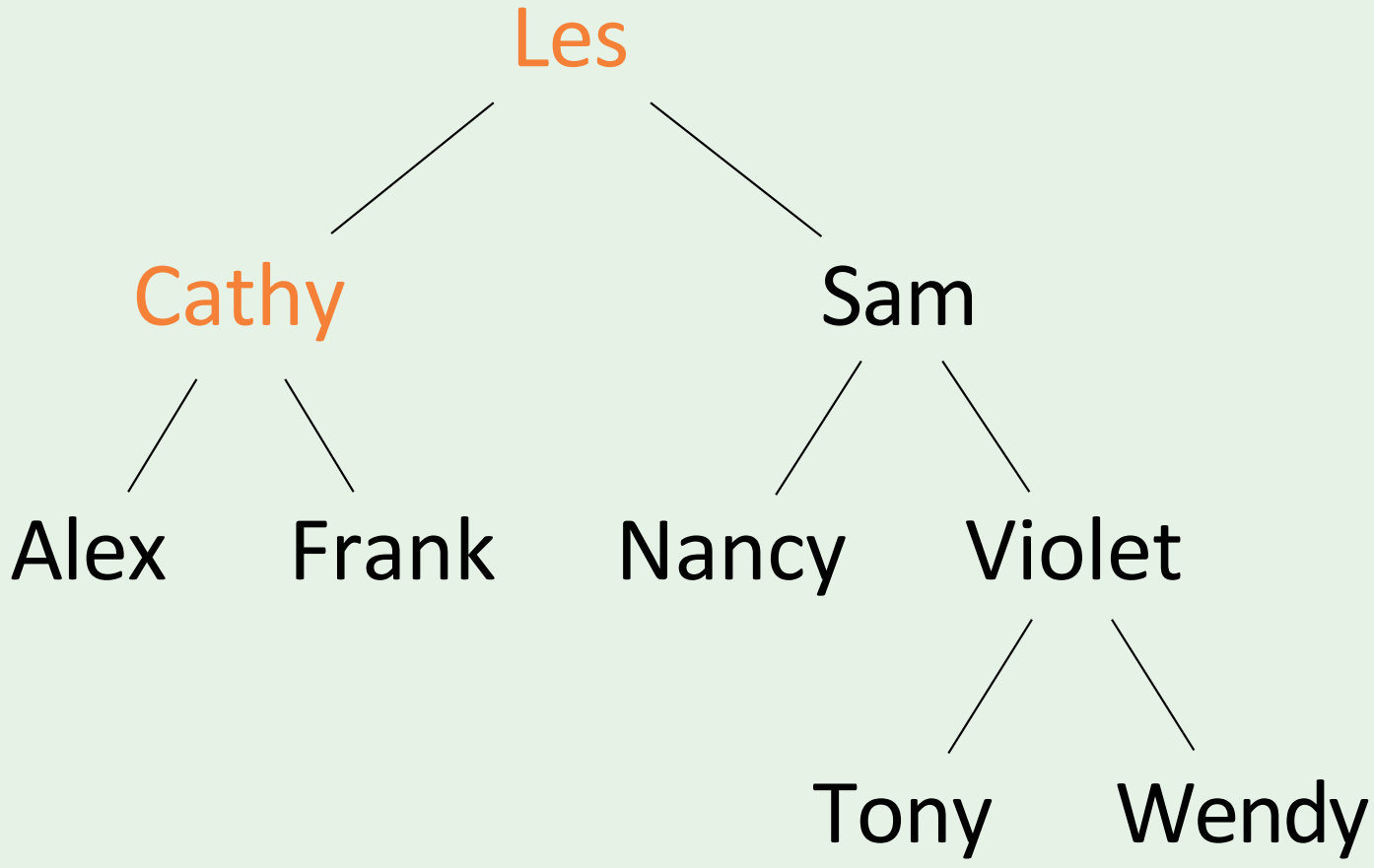
Output: Alex
Frank

PostOrderTraversal



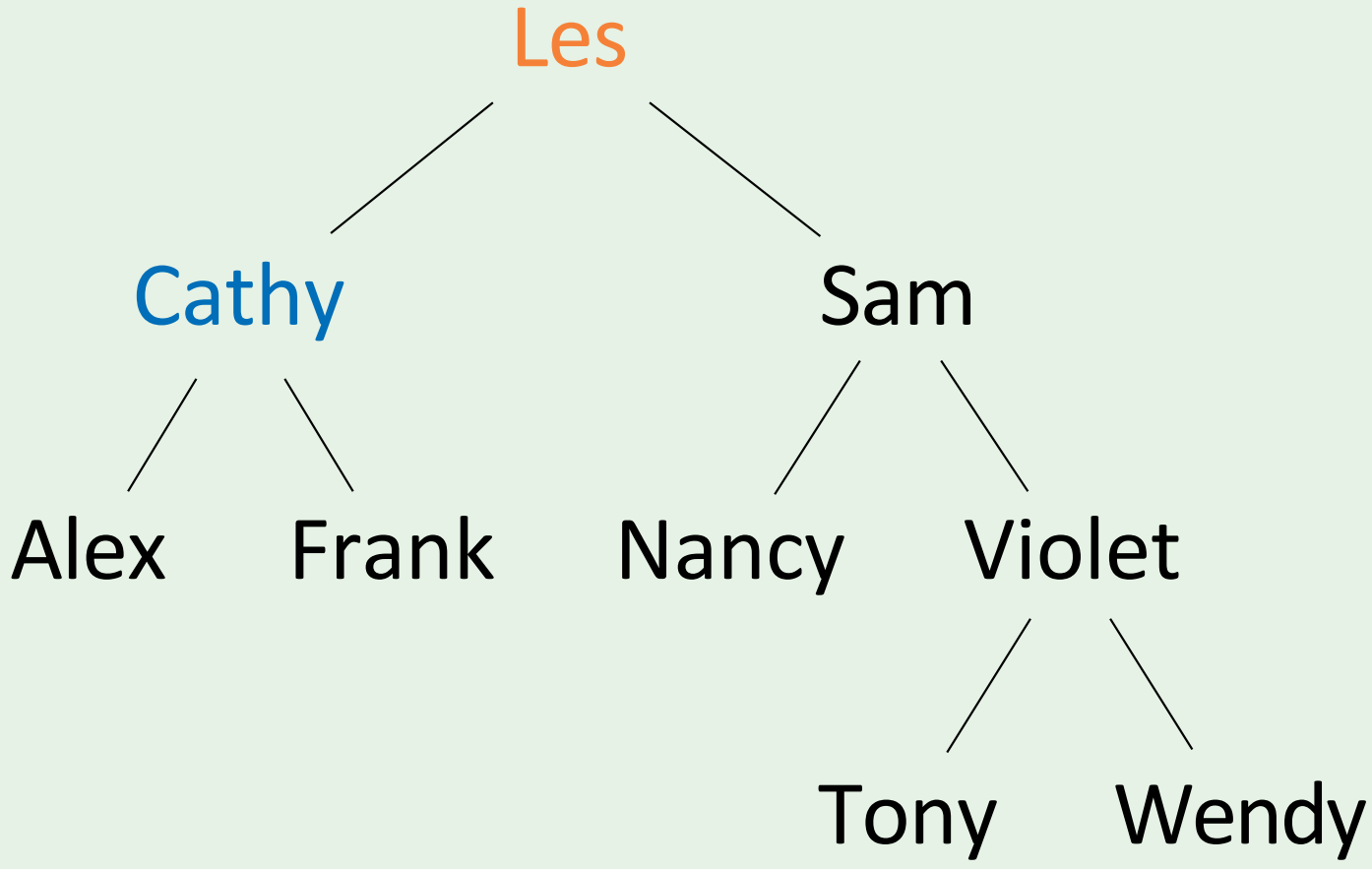
Output: Alex Frank

PostOrderTraversal



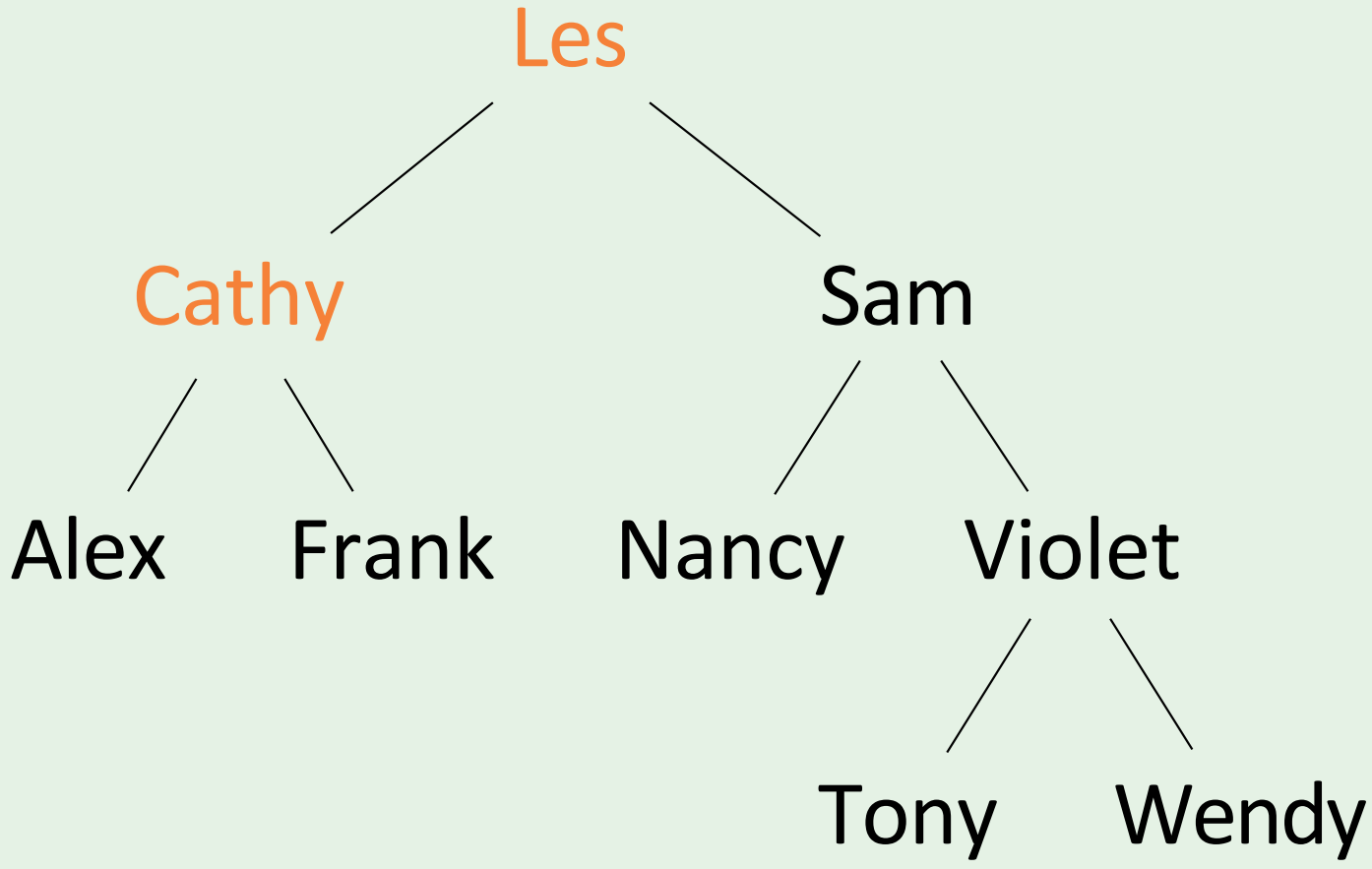
Output: Alex Frank

PostOrderTraversal



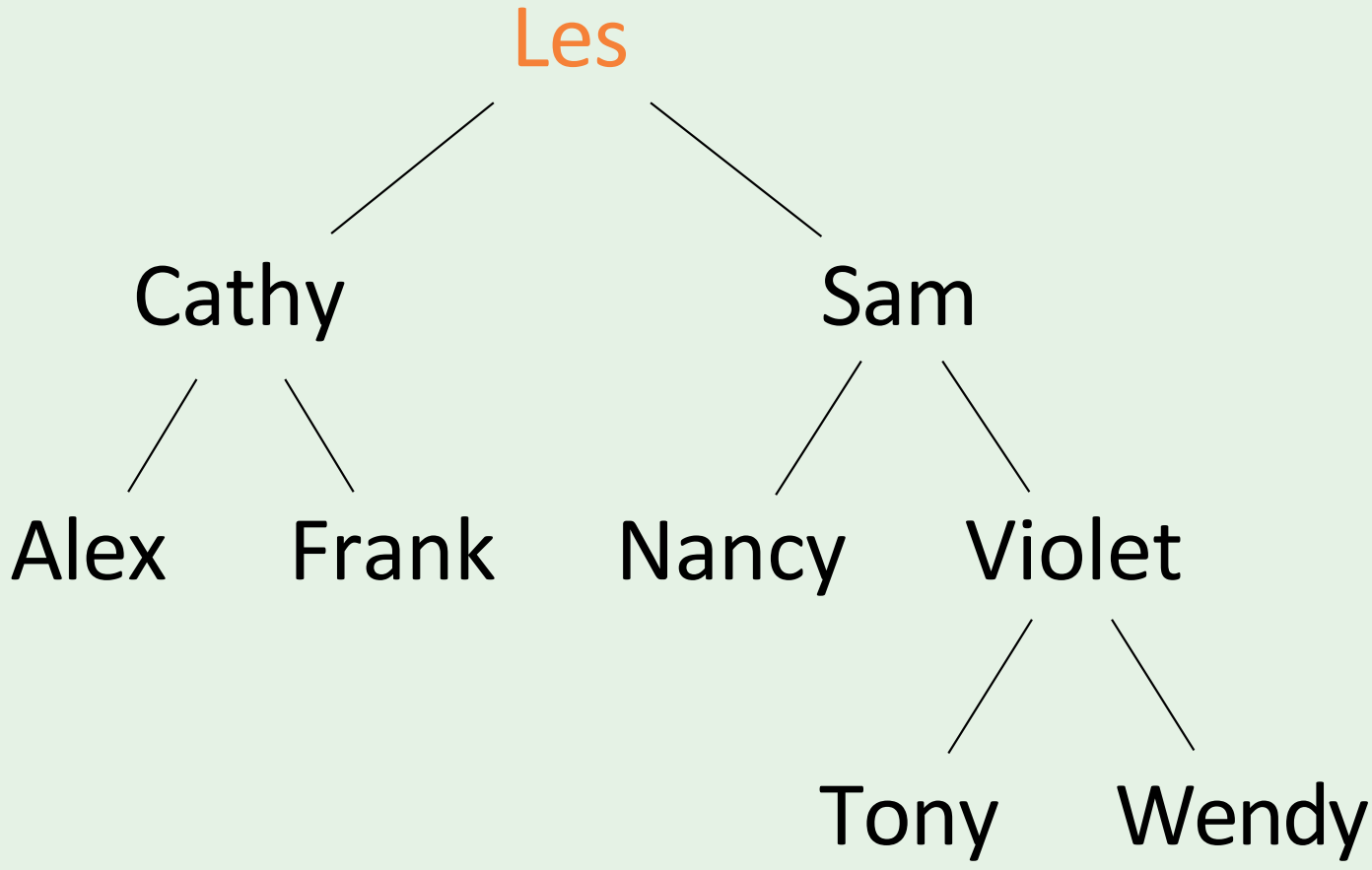
Output: Alex Frank Cathy

PostOrderTraversal



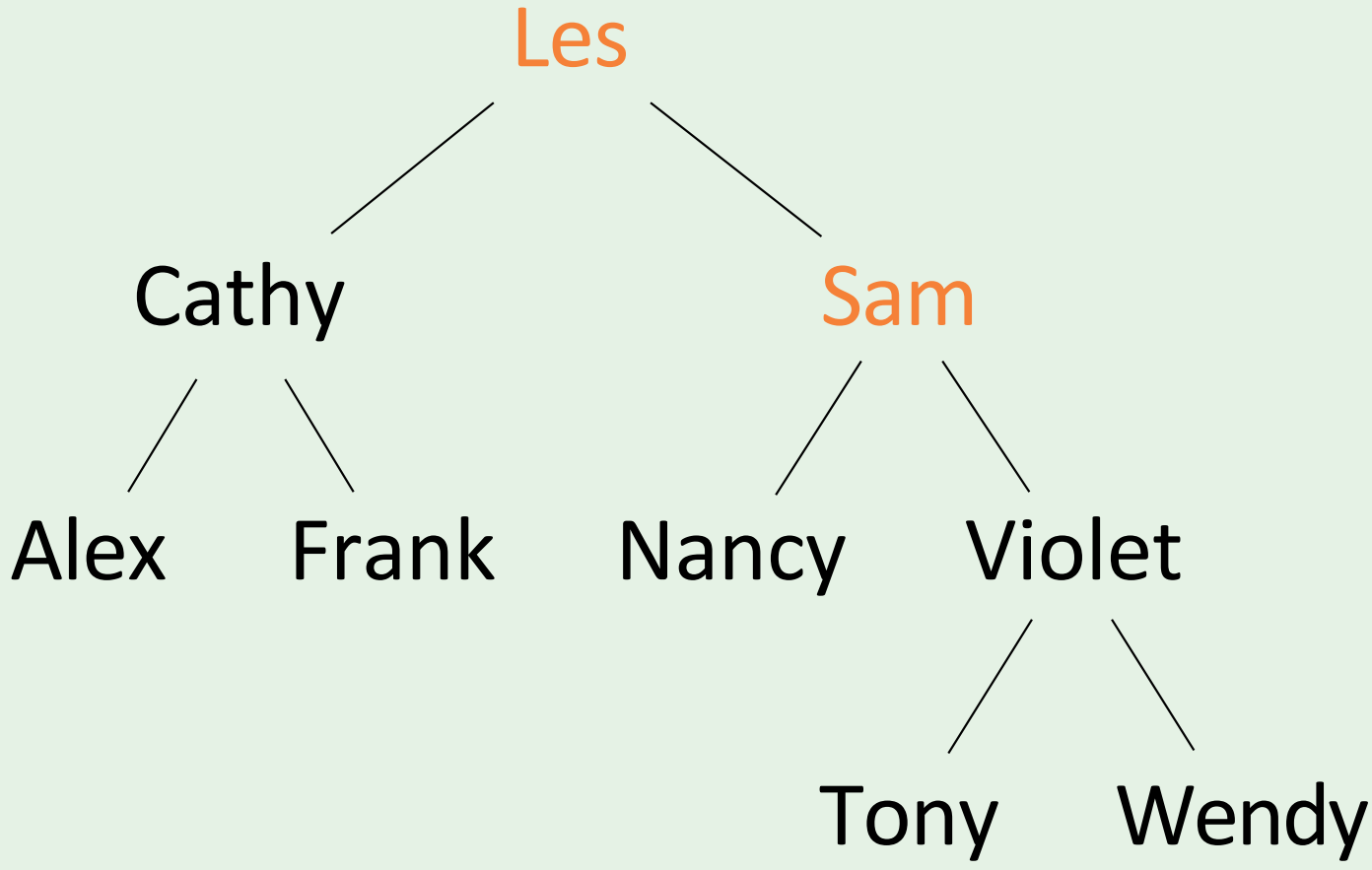
Output: Alex Frank Cathy

PostOrderTraversal



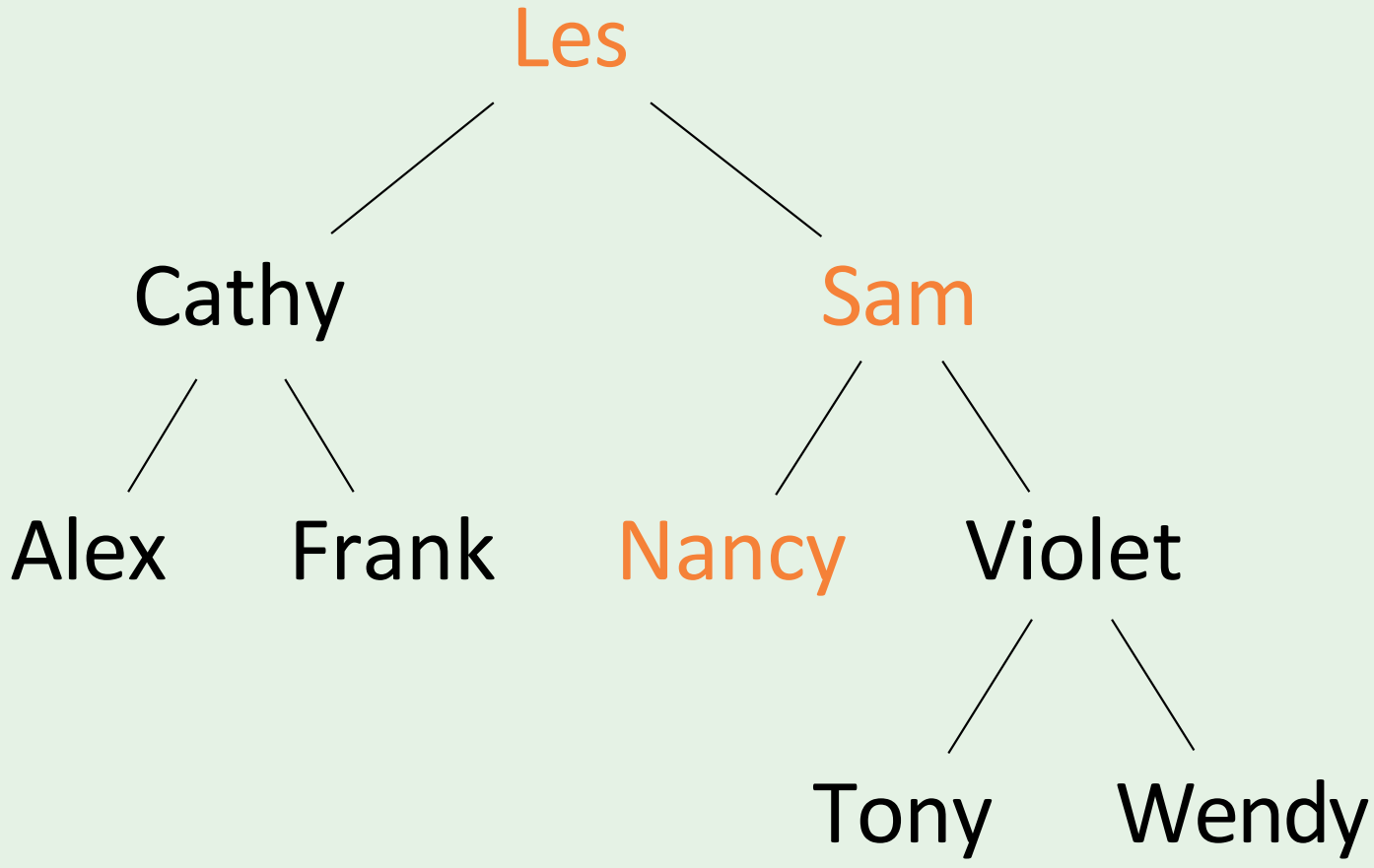
Output: Alex Frank Cathy

PostOrderTraversal



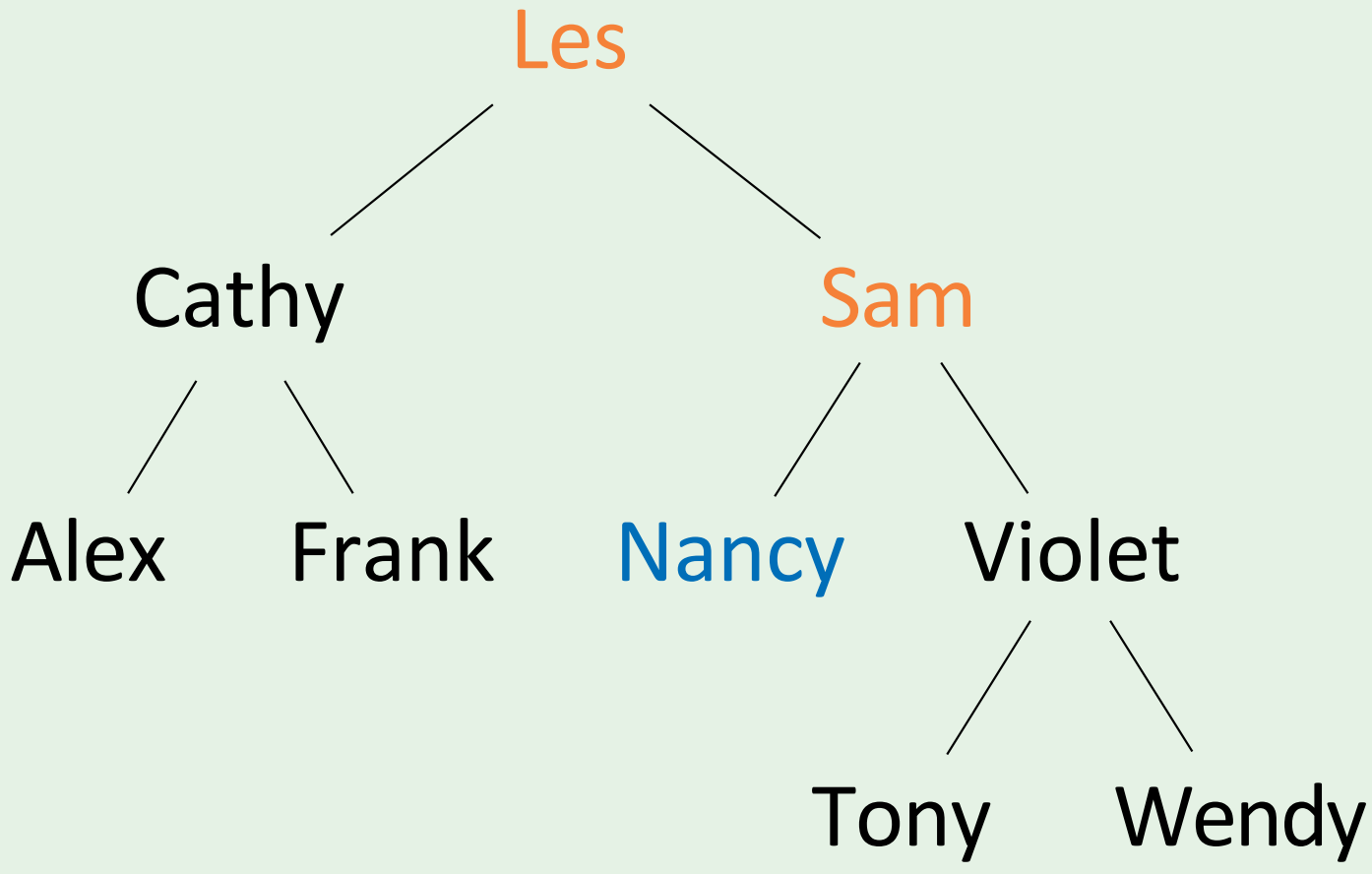
Output: Alex Frank
Cathy

PostOrderTraversal



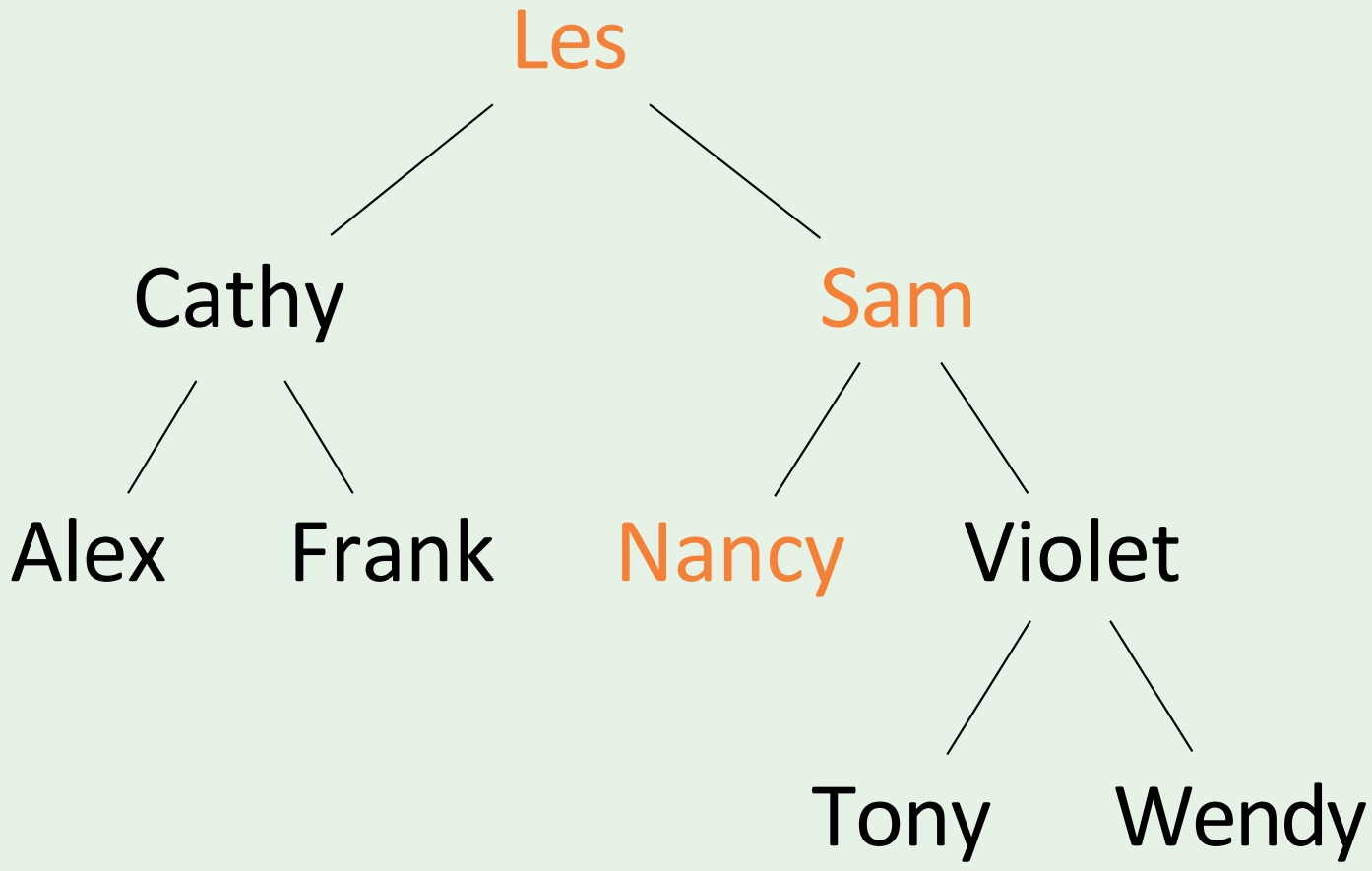
Output: Alex Frank Cathy

PostOrderTraversal



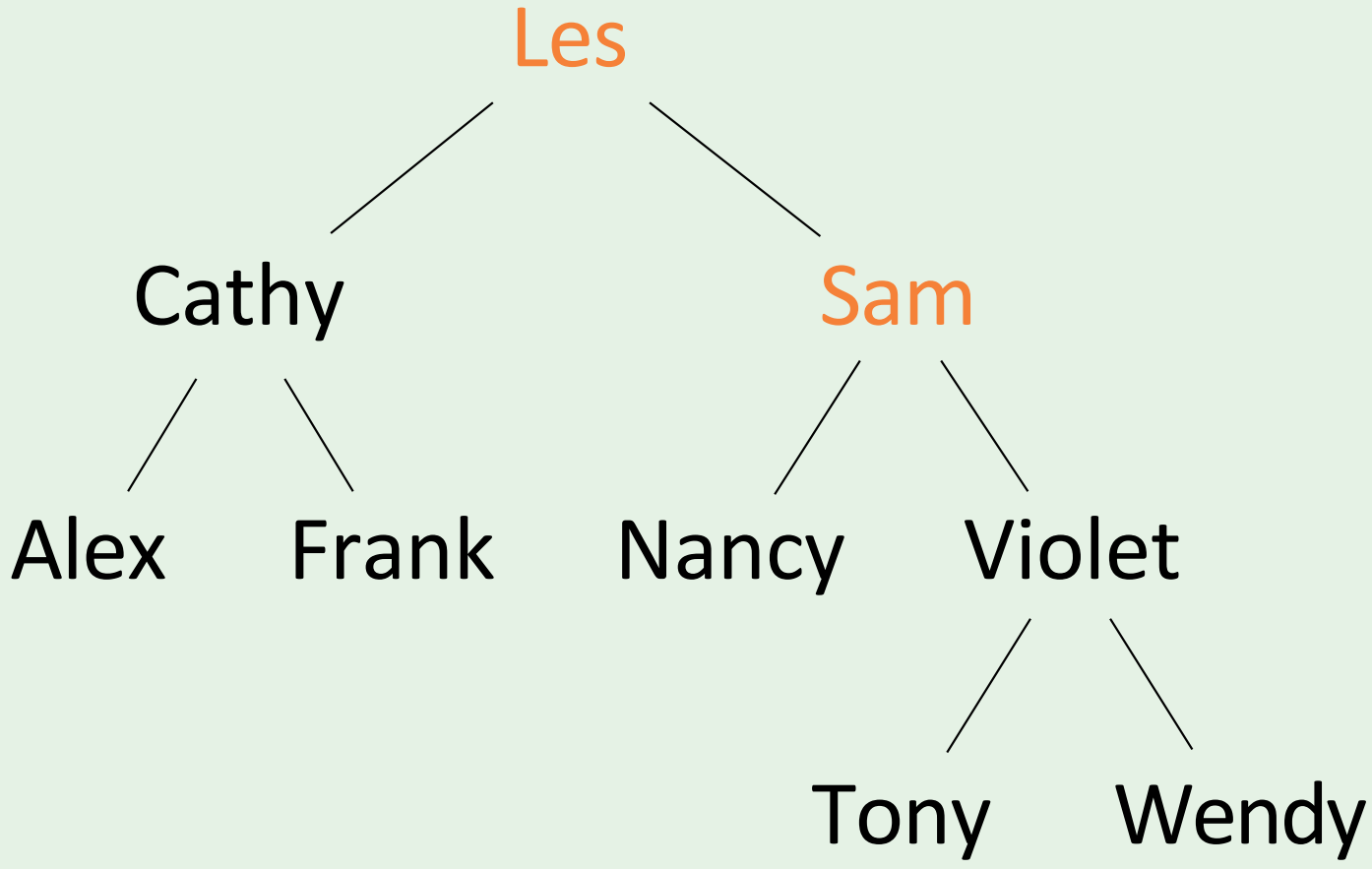
Output: Alex Frank Cathy Nancy

PostOrderTraversal



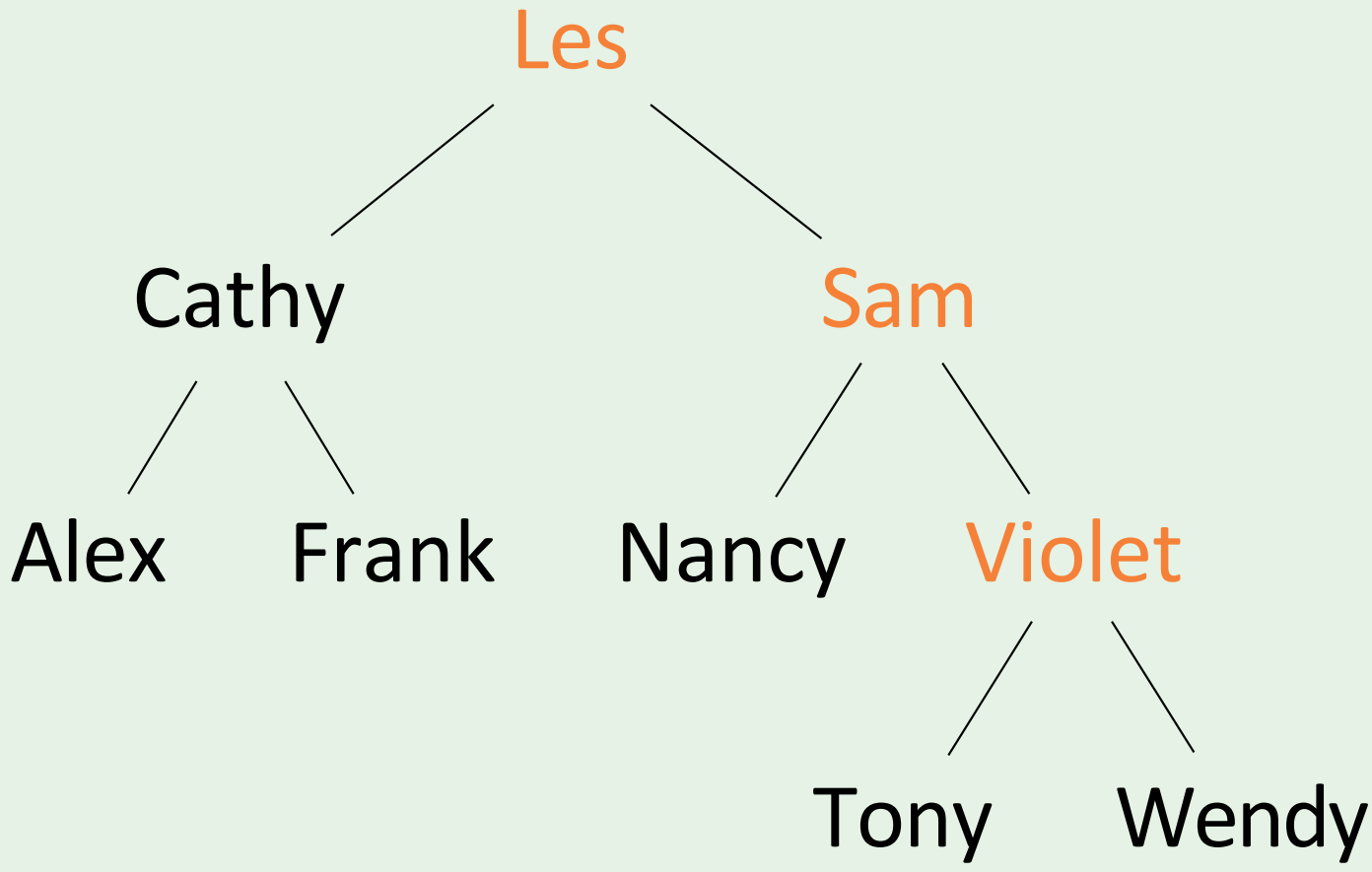
Output: Alex Frank Cathy Nancy

PostOrderTraversal



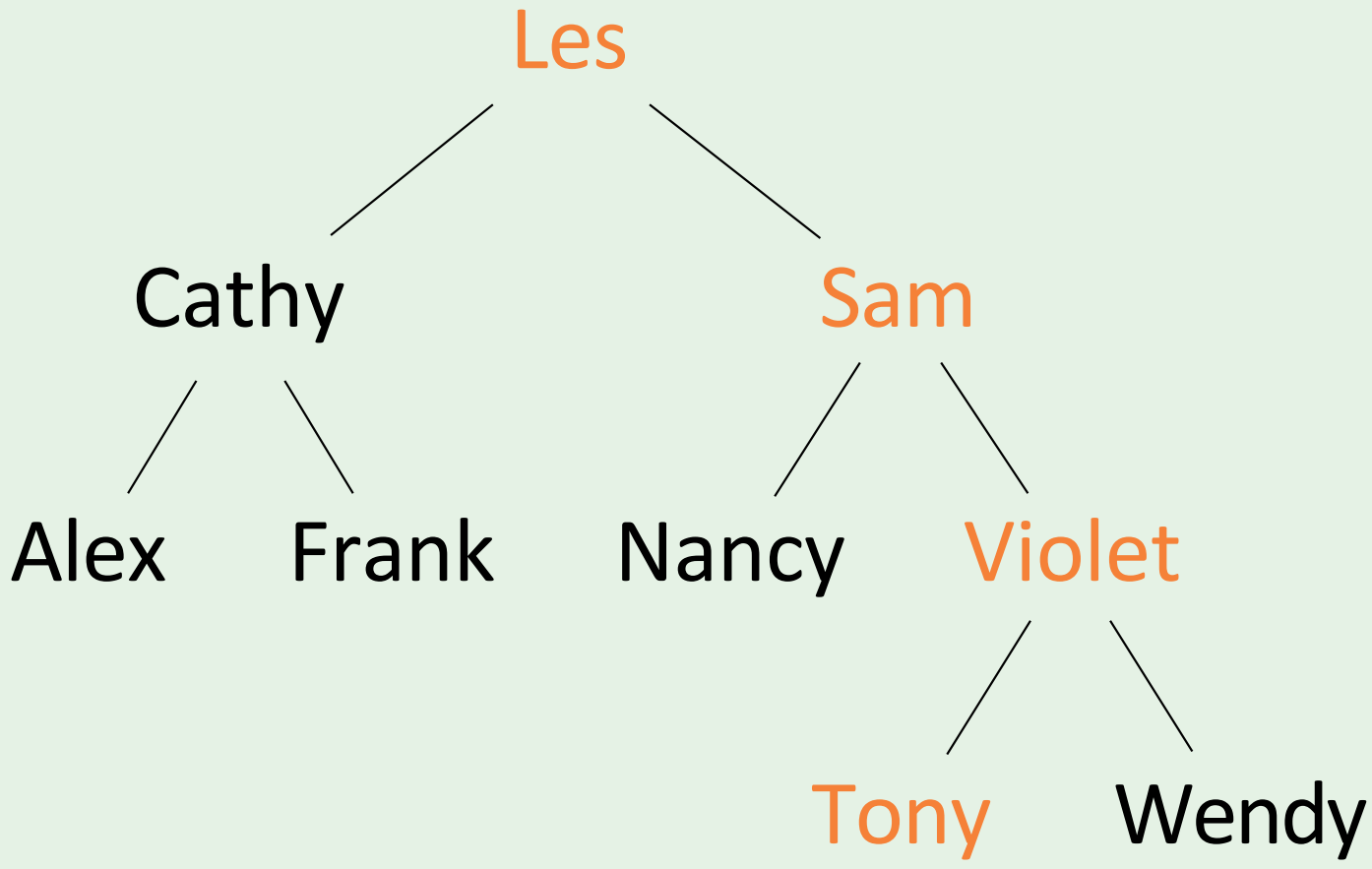
Output: Alex Frank Cathy Nancy

PostOrderTraversal



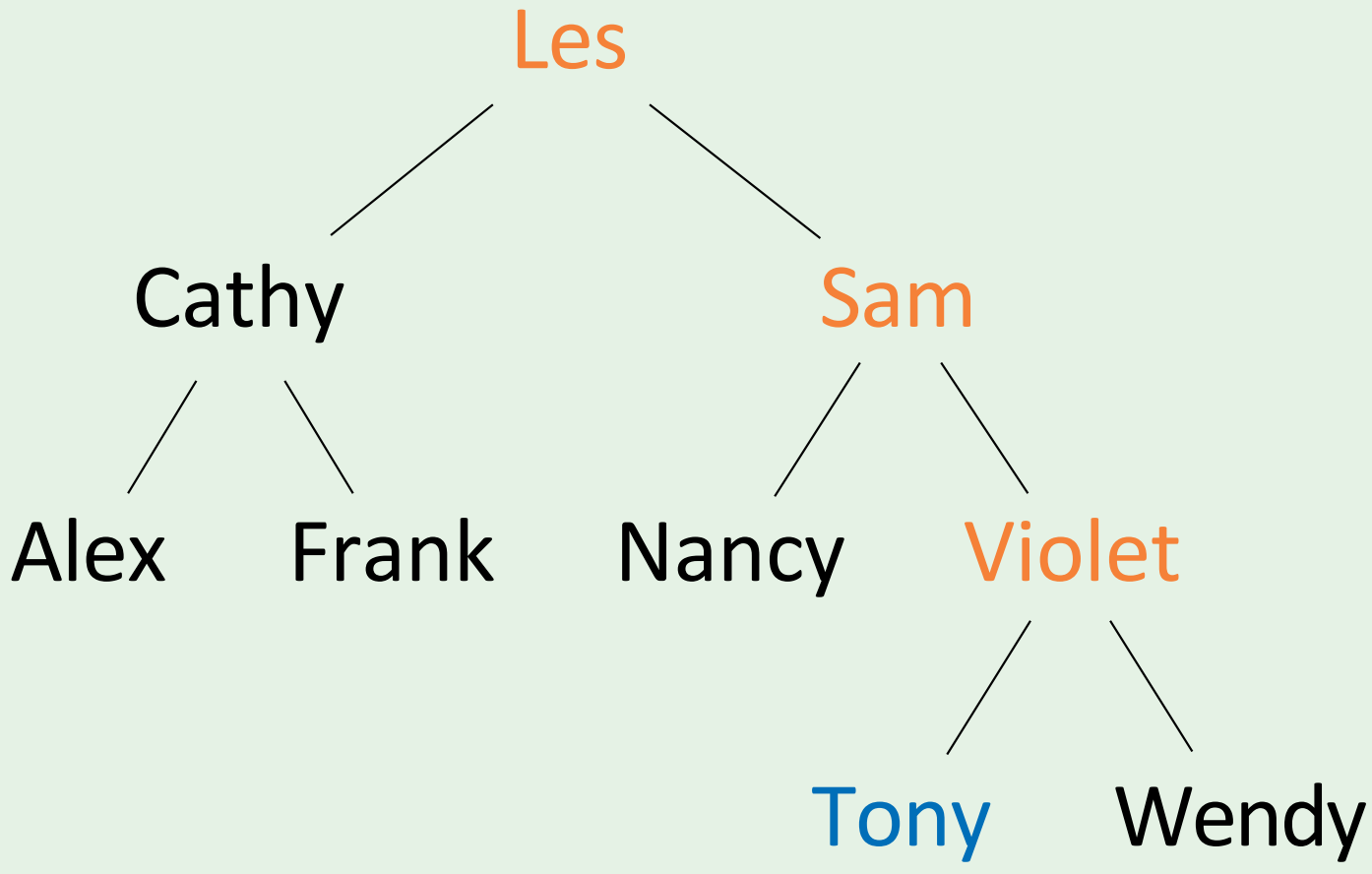
Output: Alex Frank Cathy Nancy

PostOrderTraversal



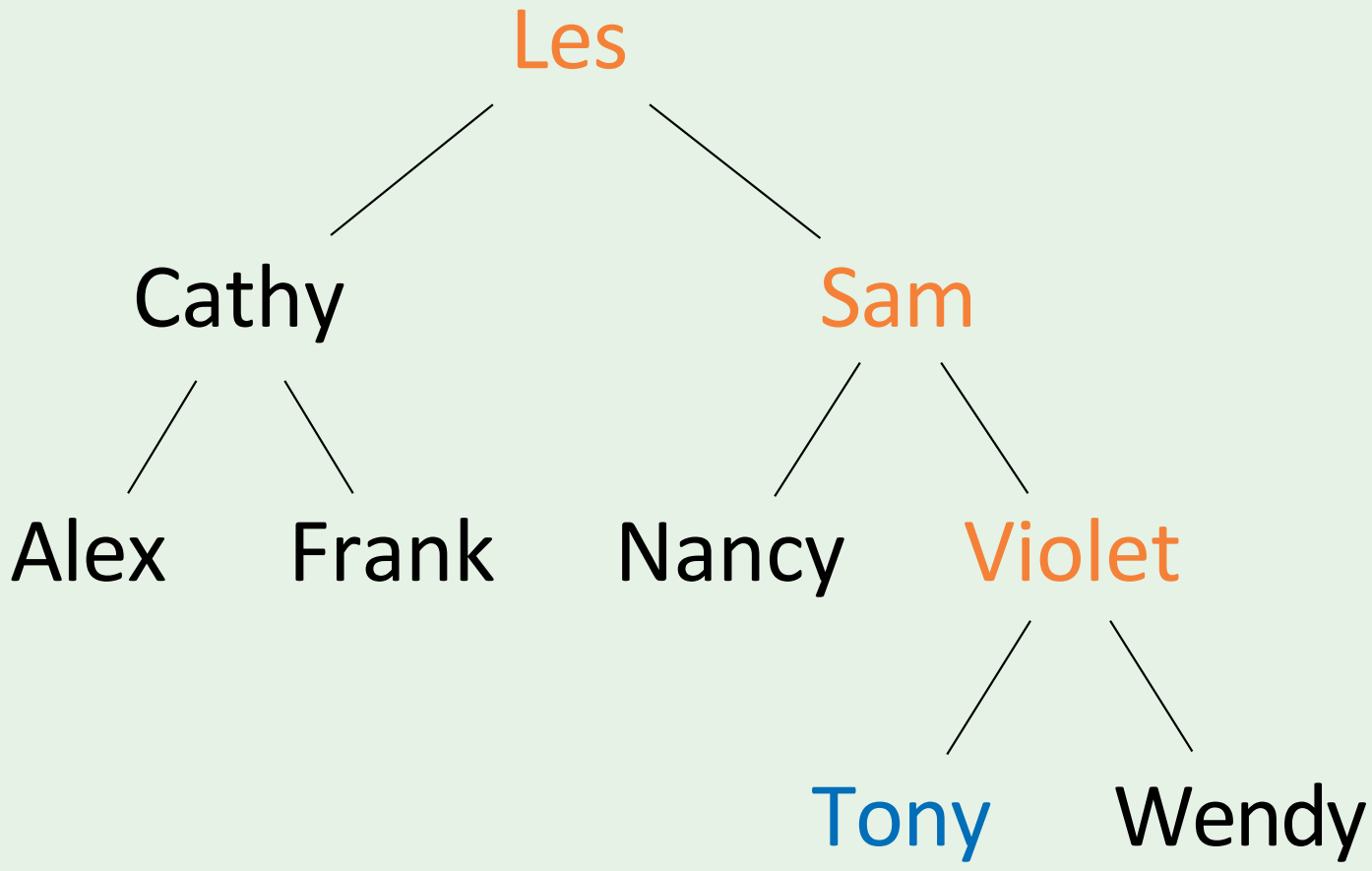
Output: Alex Frank Cathy Nancy

PostOrderTraversal



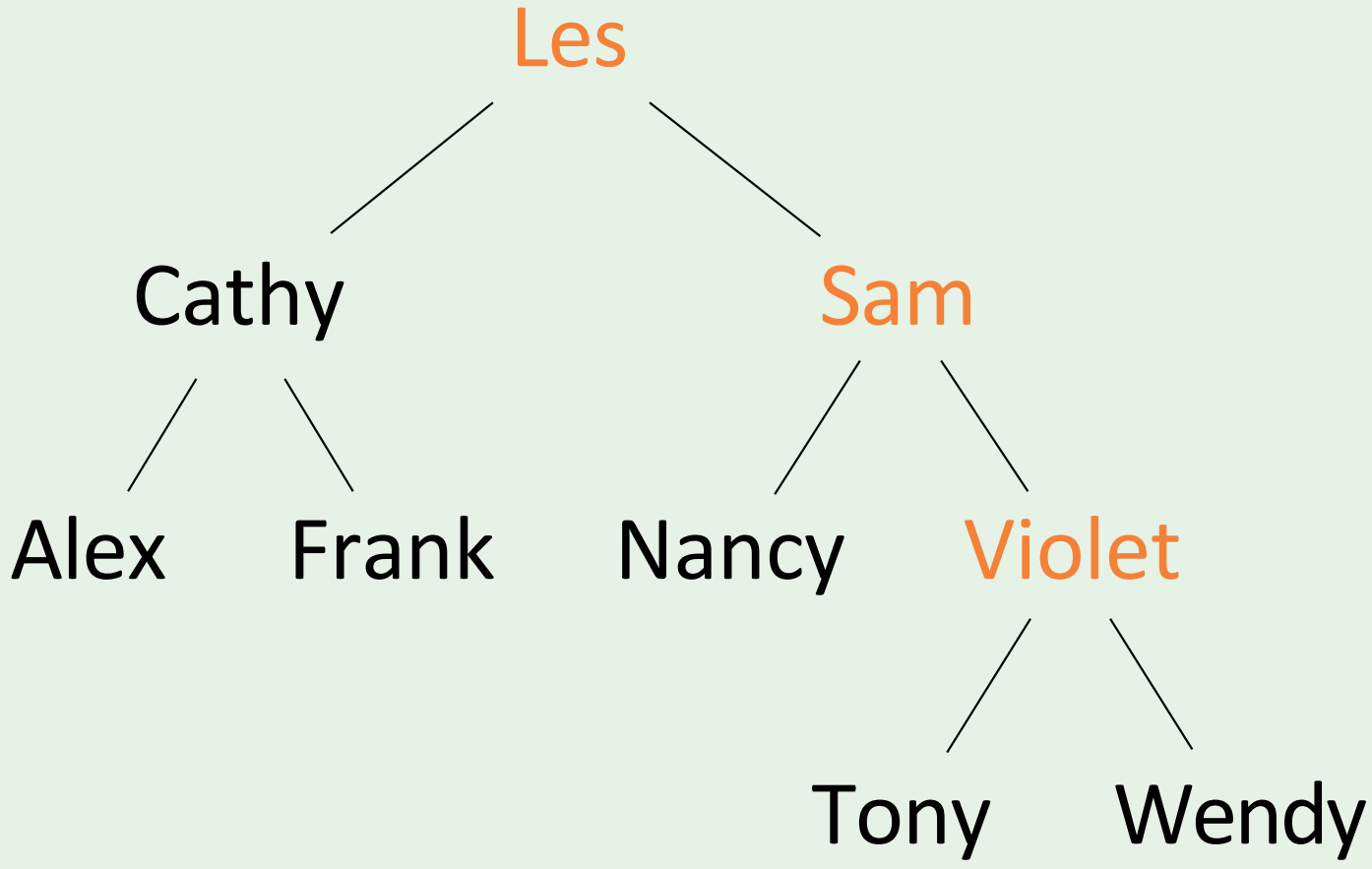
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



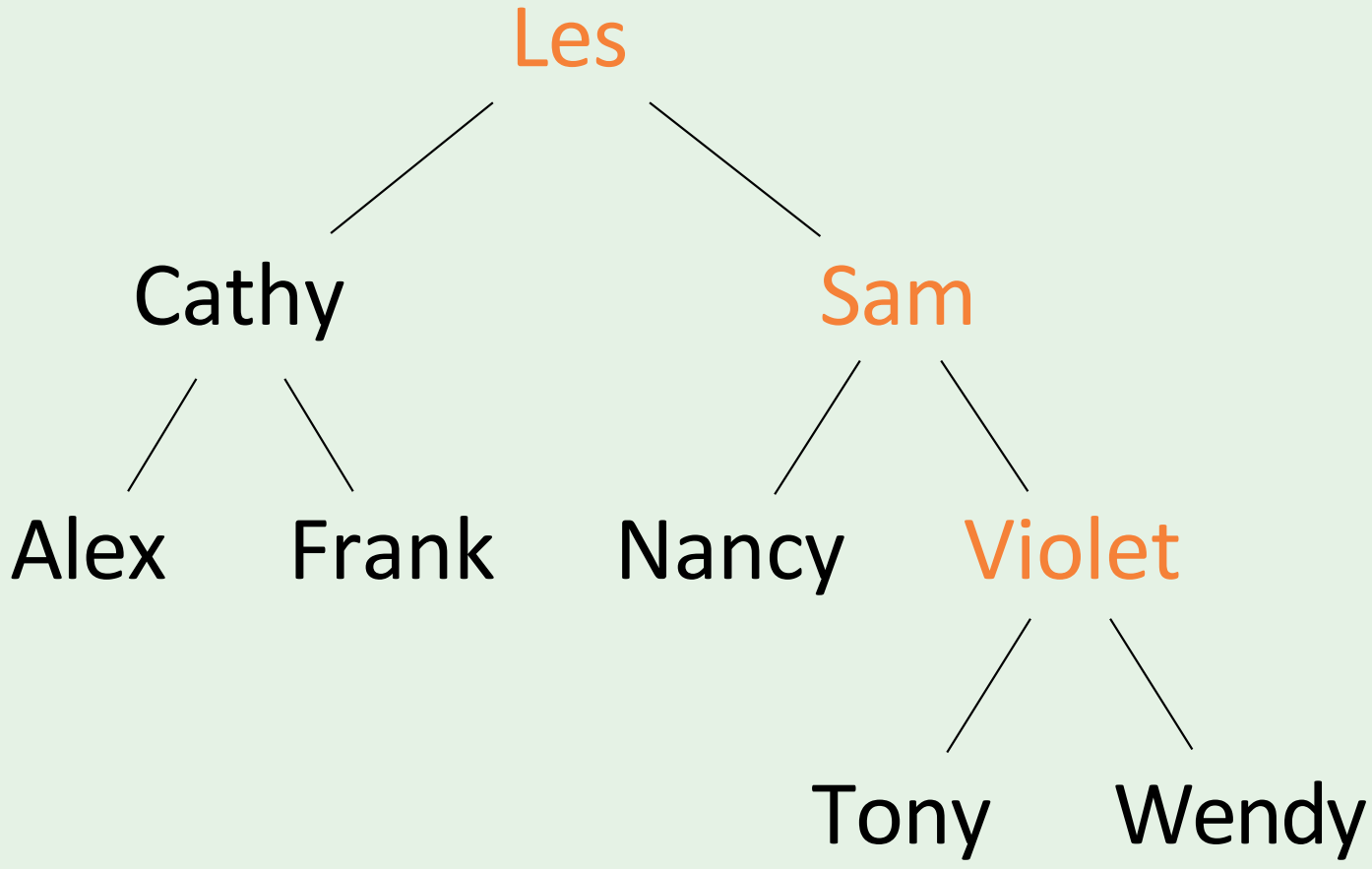
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



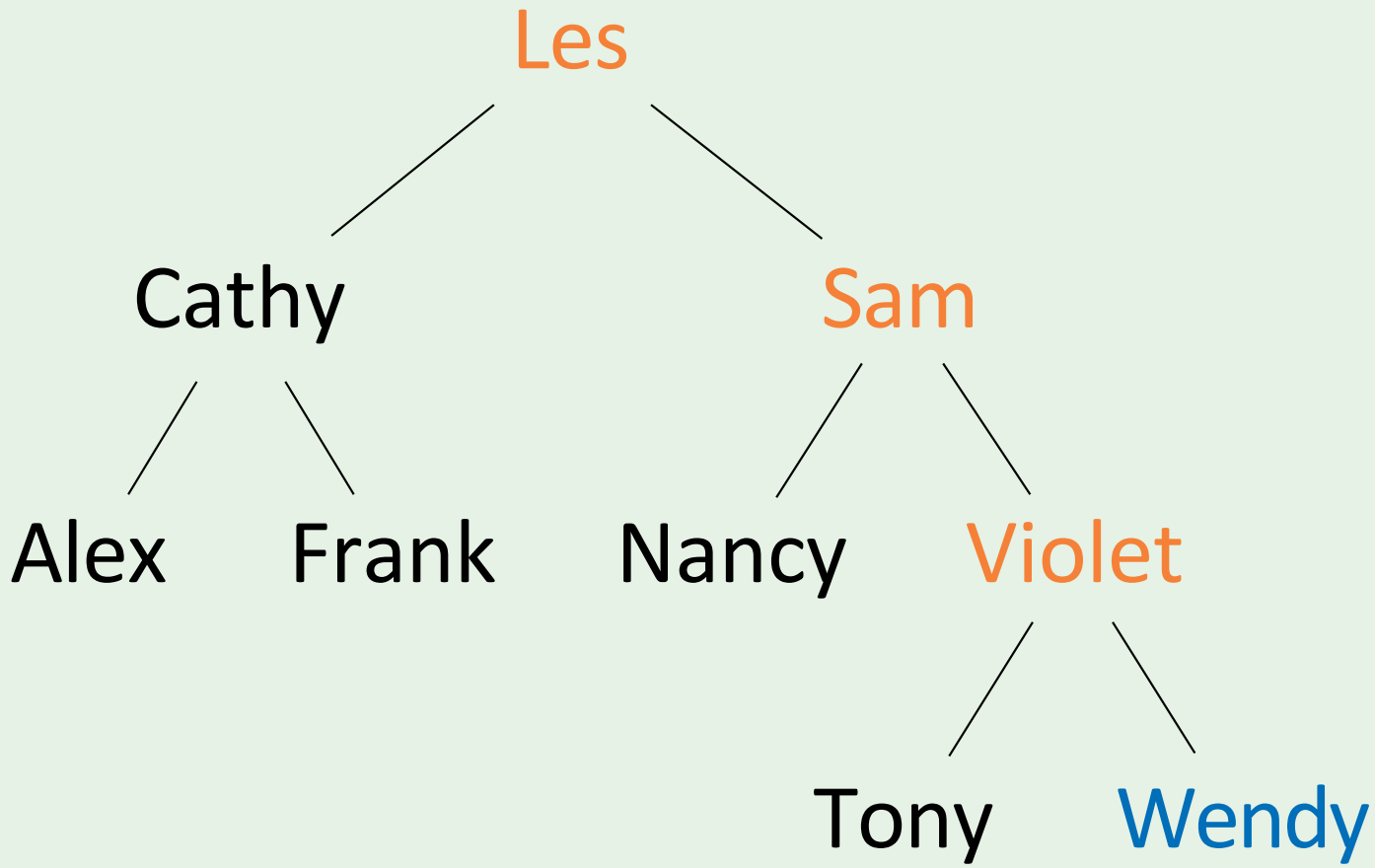
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



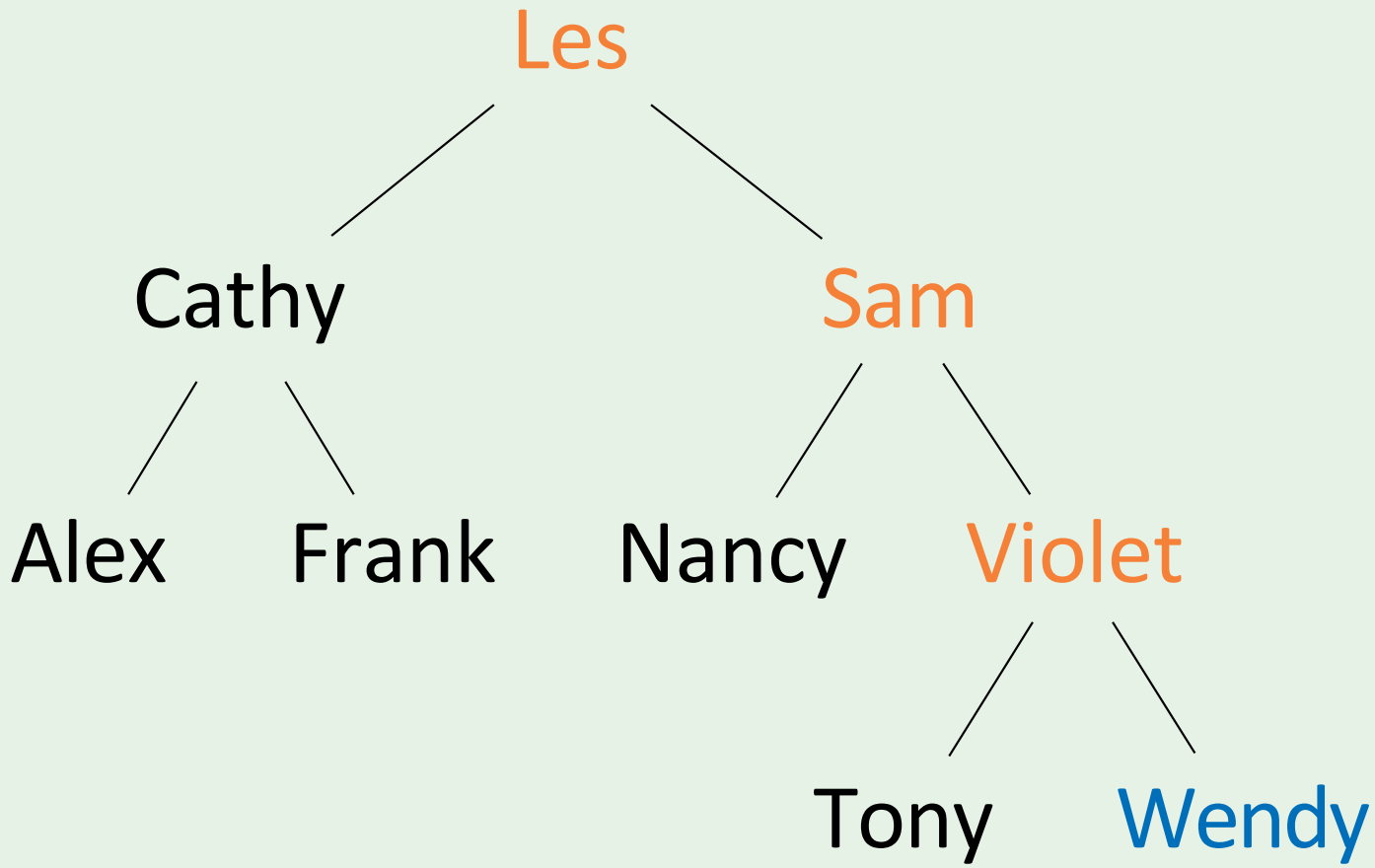
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



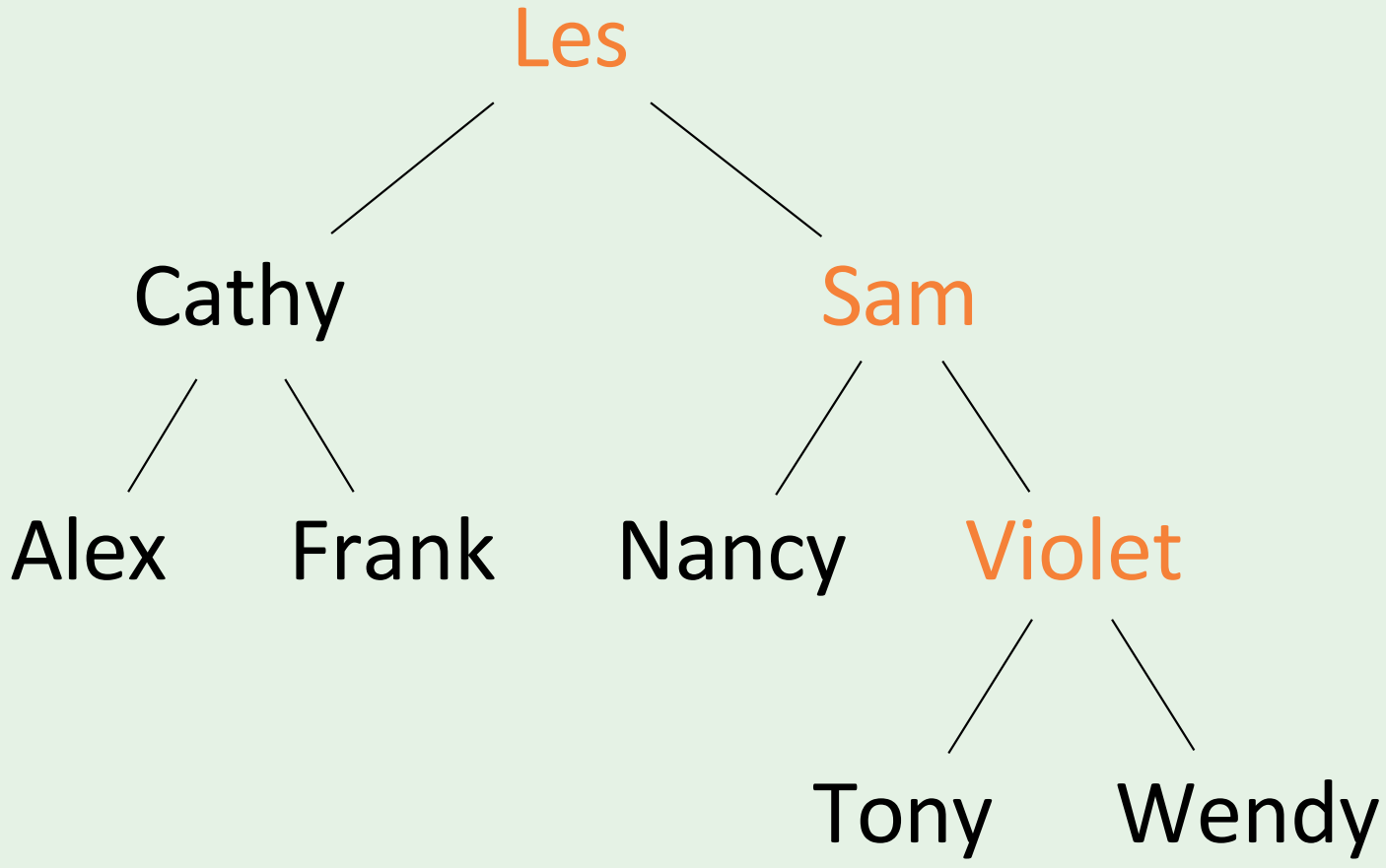
Output: Alex Frank Cathy Nancy Tony Wendy

PostOrderTraversal



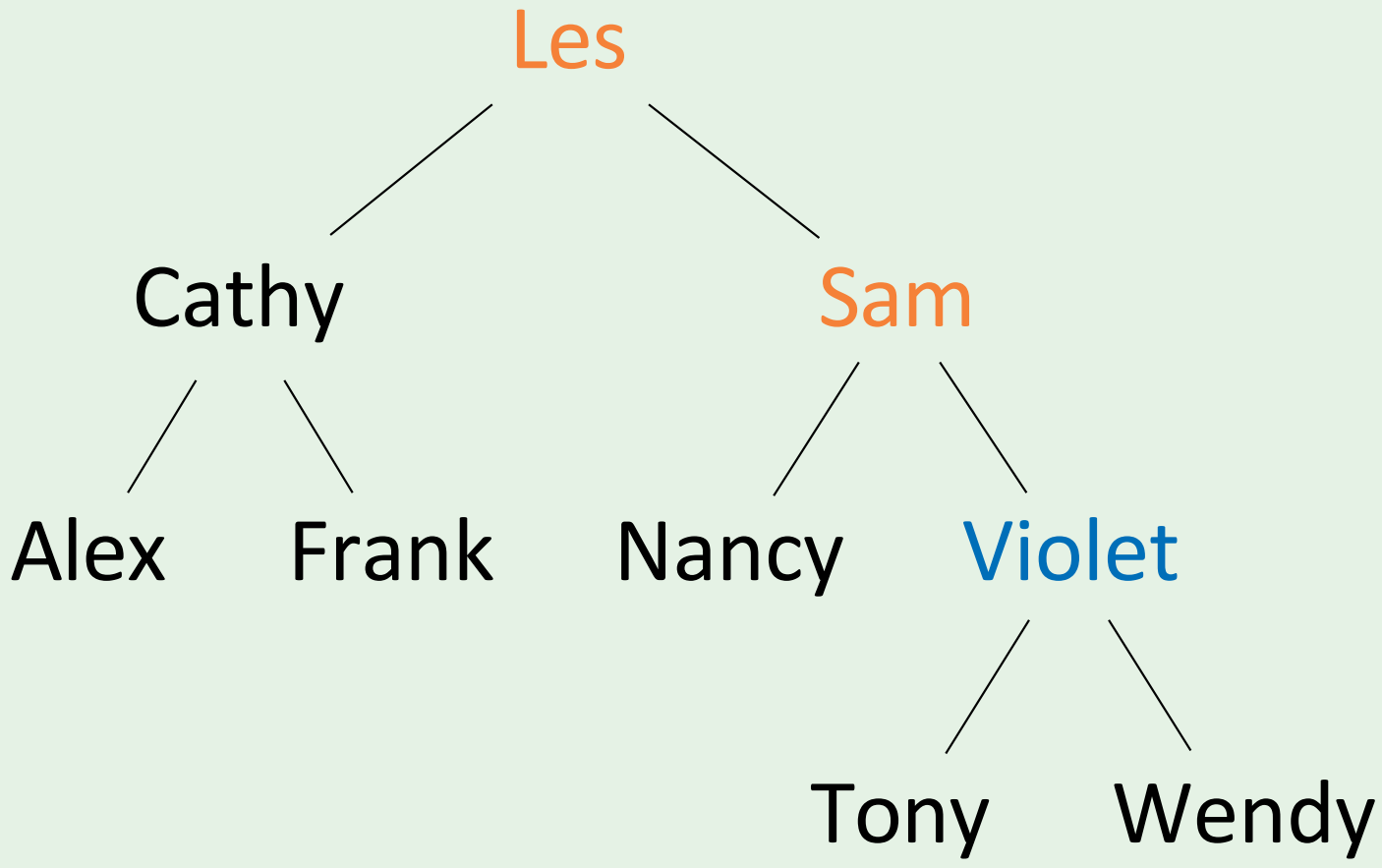
Output: Alex Frank Cathy Nancy Tony Wendy

PostOrderTraversal



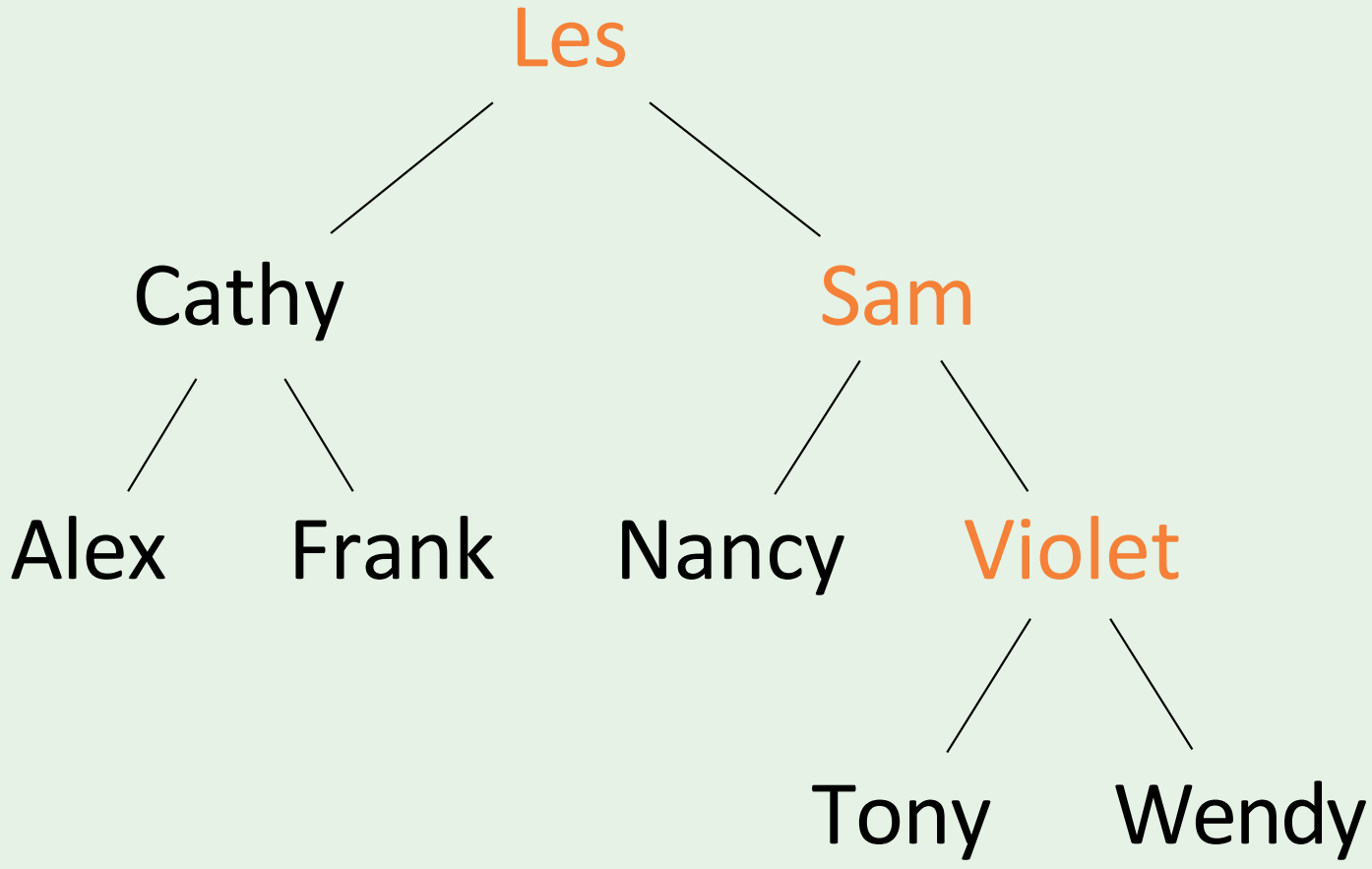
Output: Alex Frank Cathy Nancy Tony Wendy

PostOrderTraversal



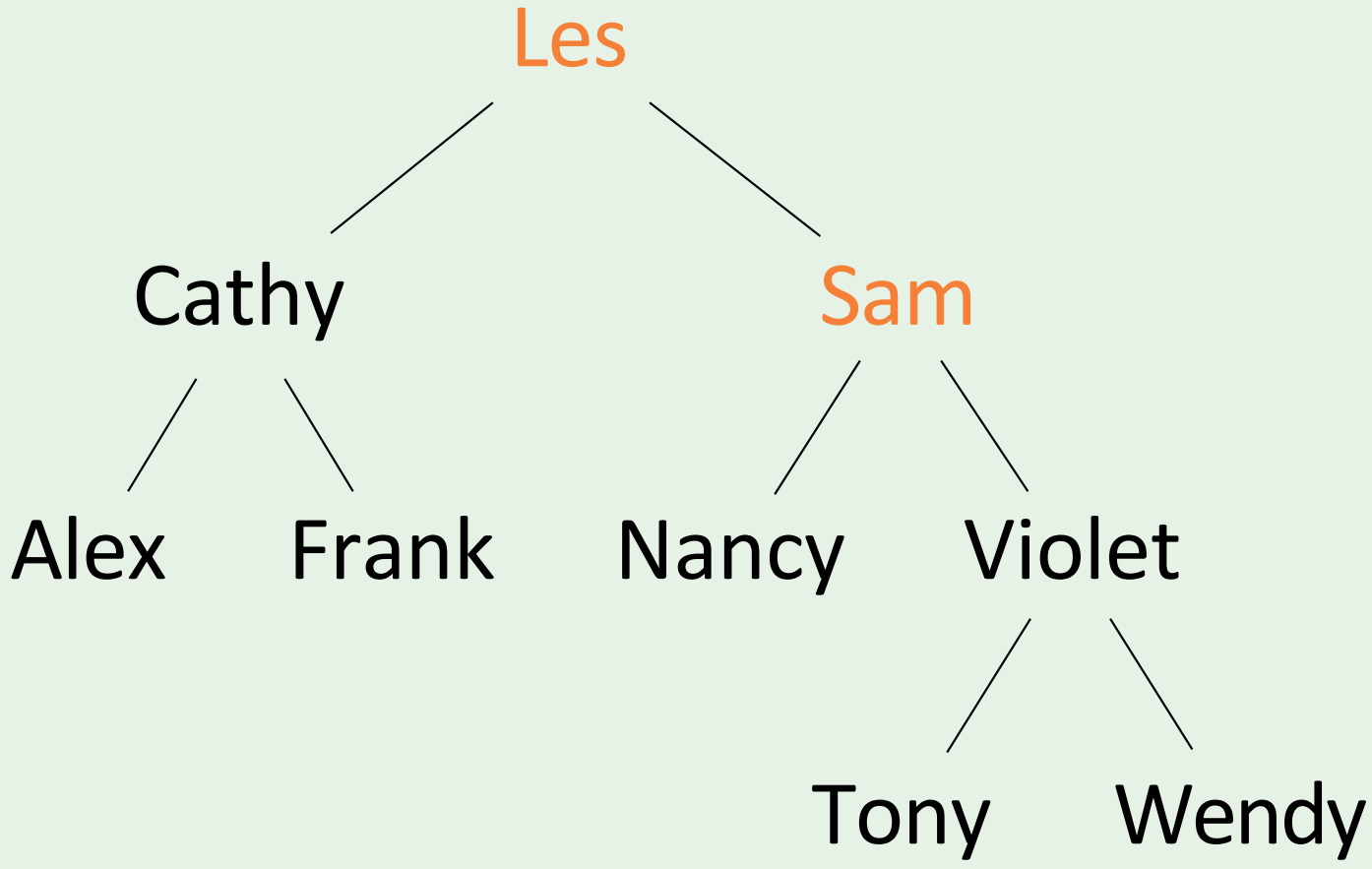
Output: Alex Frank Cathy Nancy Tony Wendy Violet

PostOrderTraversal



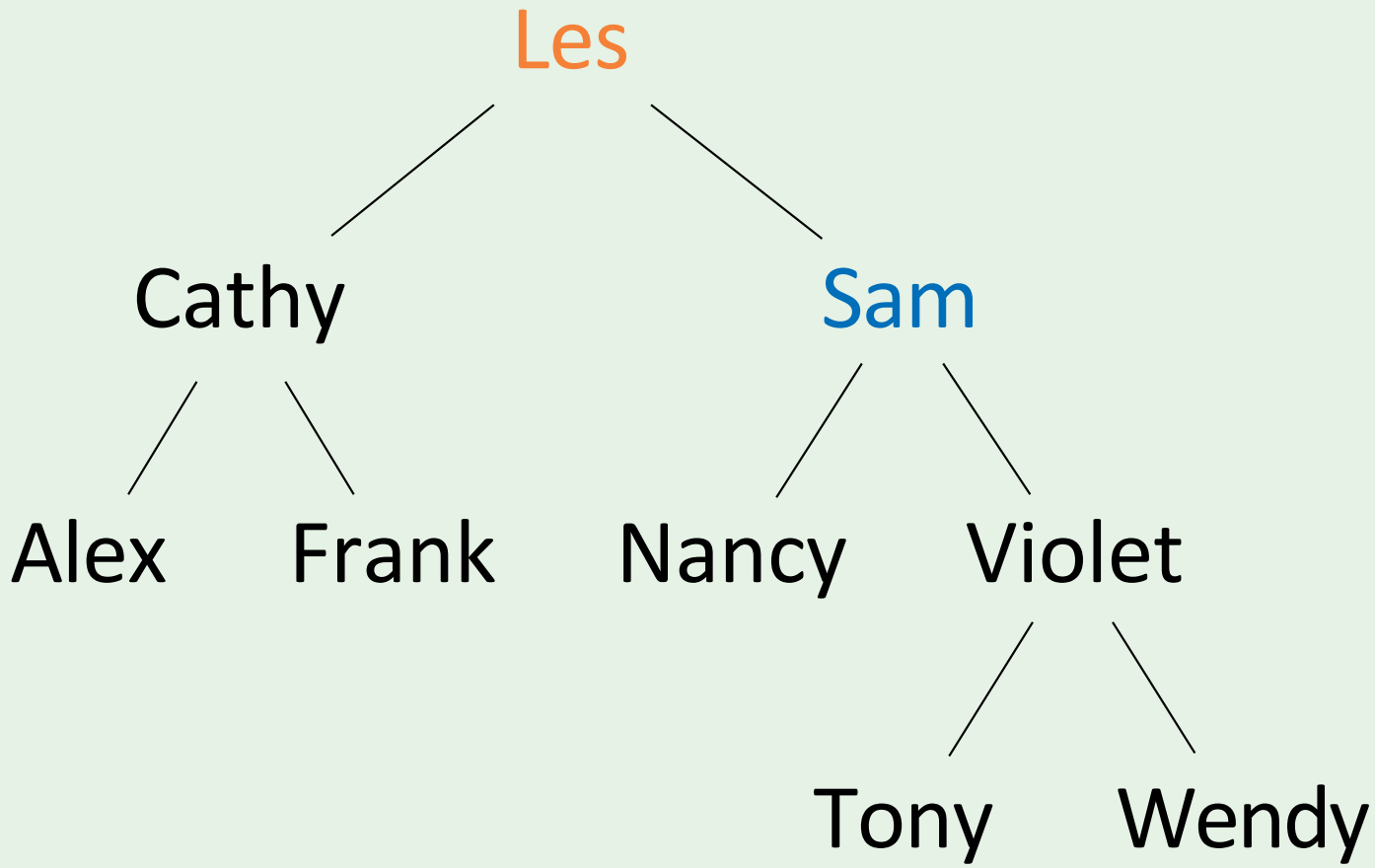
Output: Alex Frank Cathy Nancy Tony Wendy Violet

PostOrderTraversal



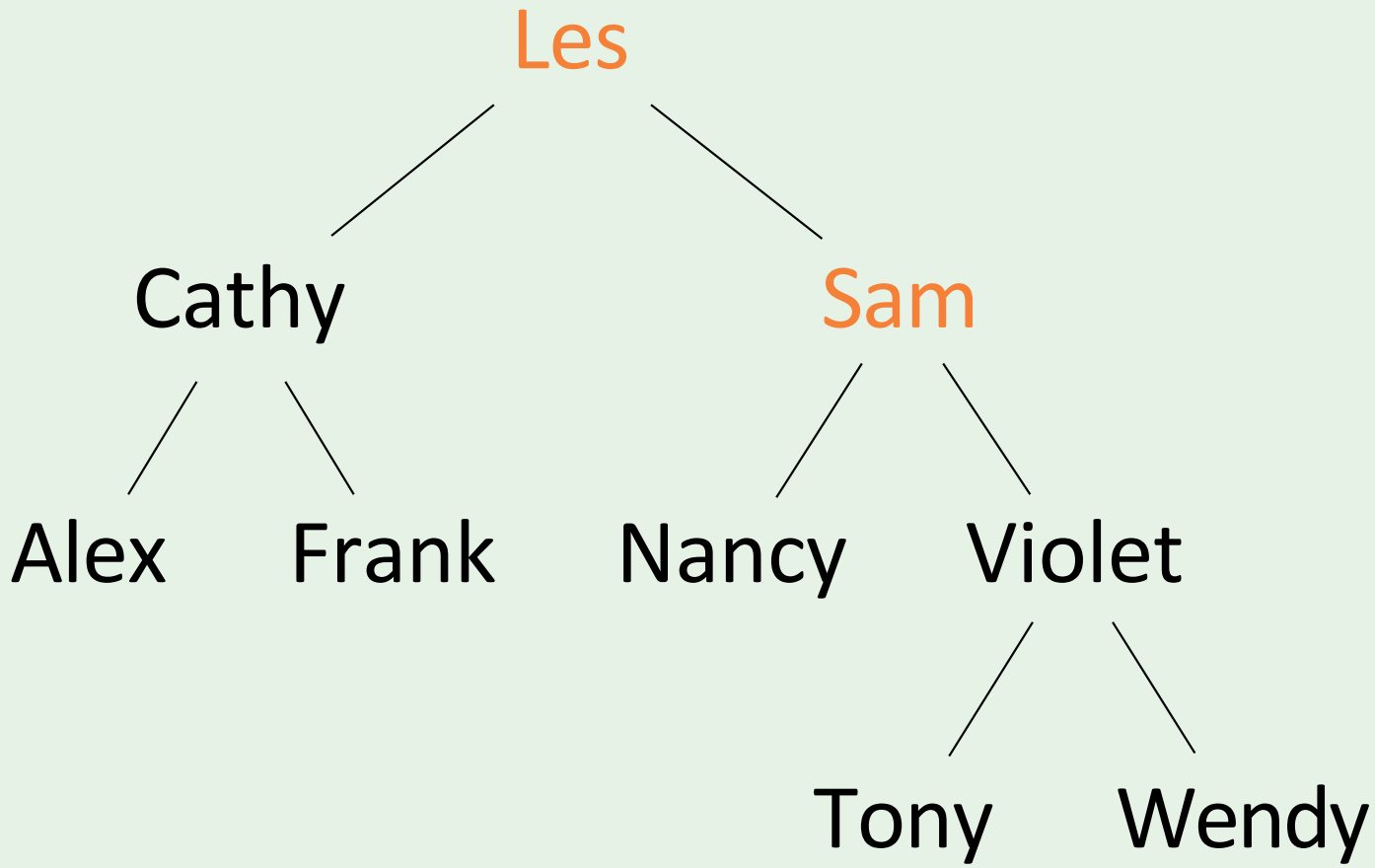
Output: Alex Frank Cathy Nancy Tony Wendy Violet

PostOrderTraversal



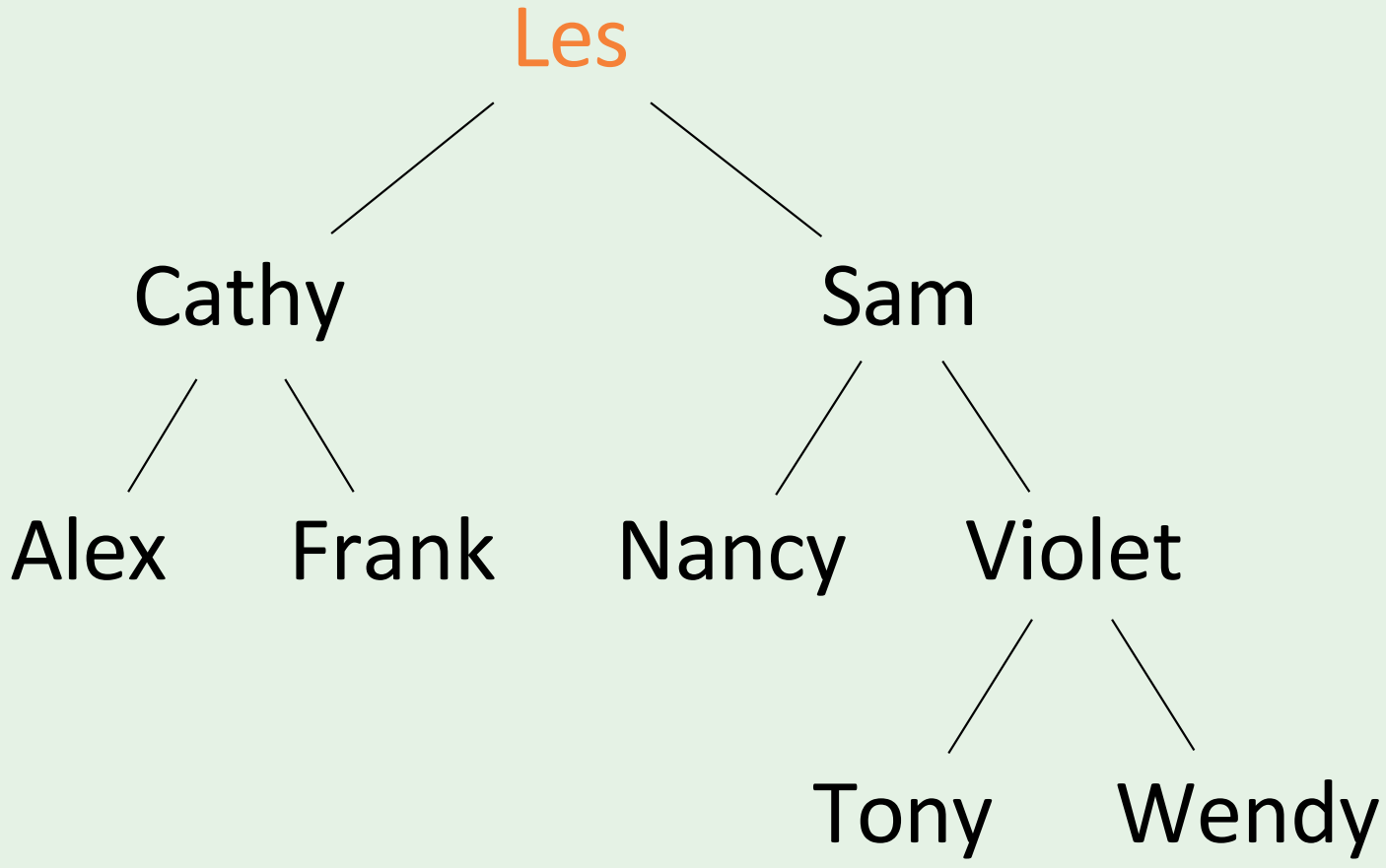
Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam Les

PostOrderTraversal



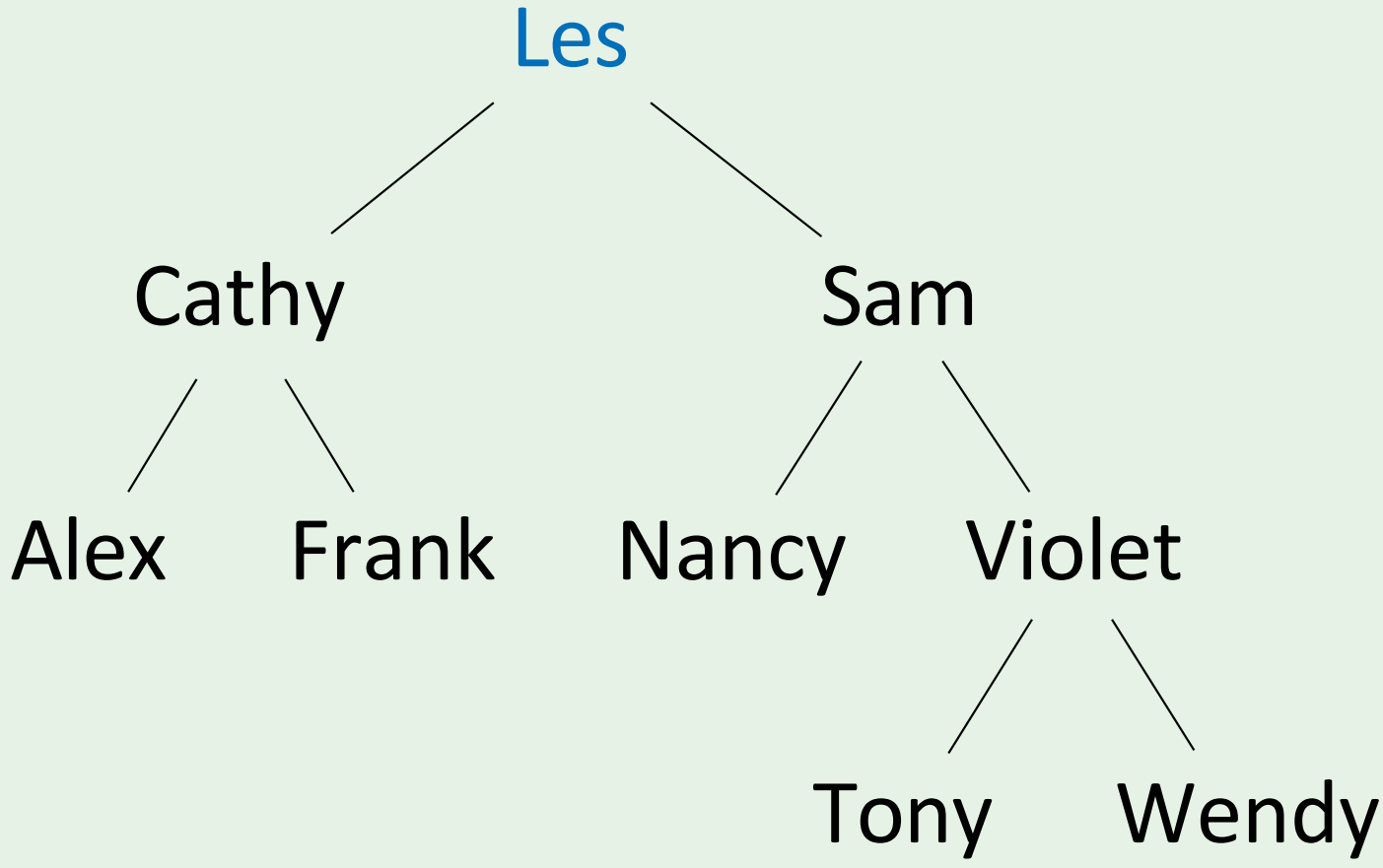
Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam Les

PostOrderTraversal



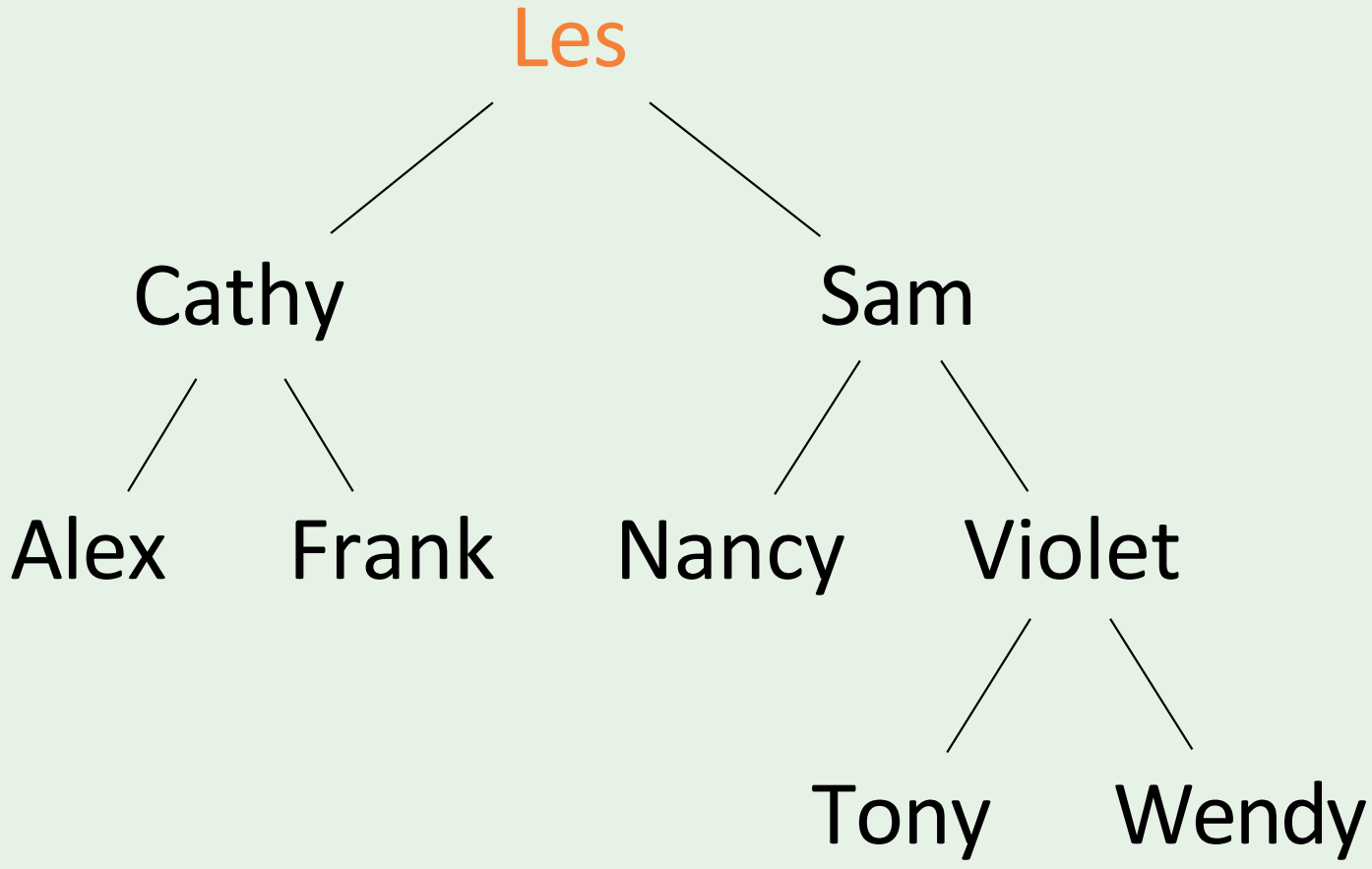
Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam

PostOrderTraversal



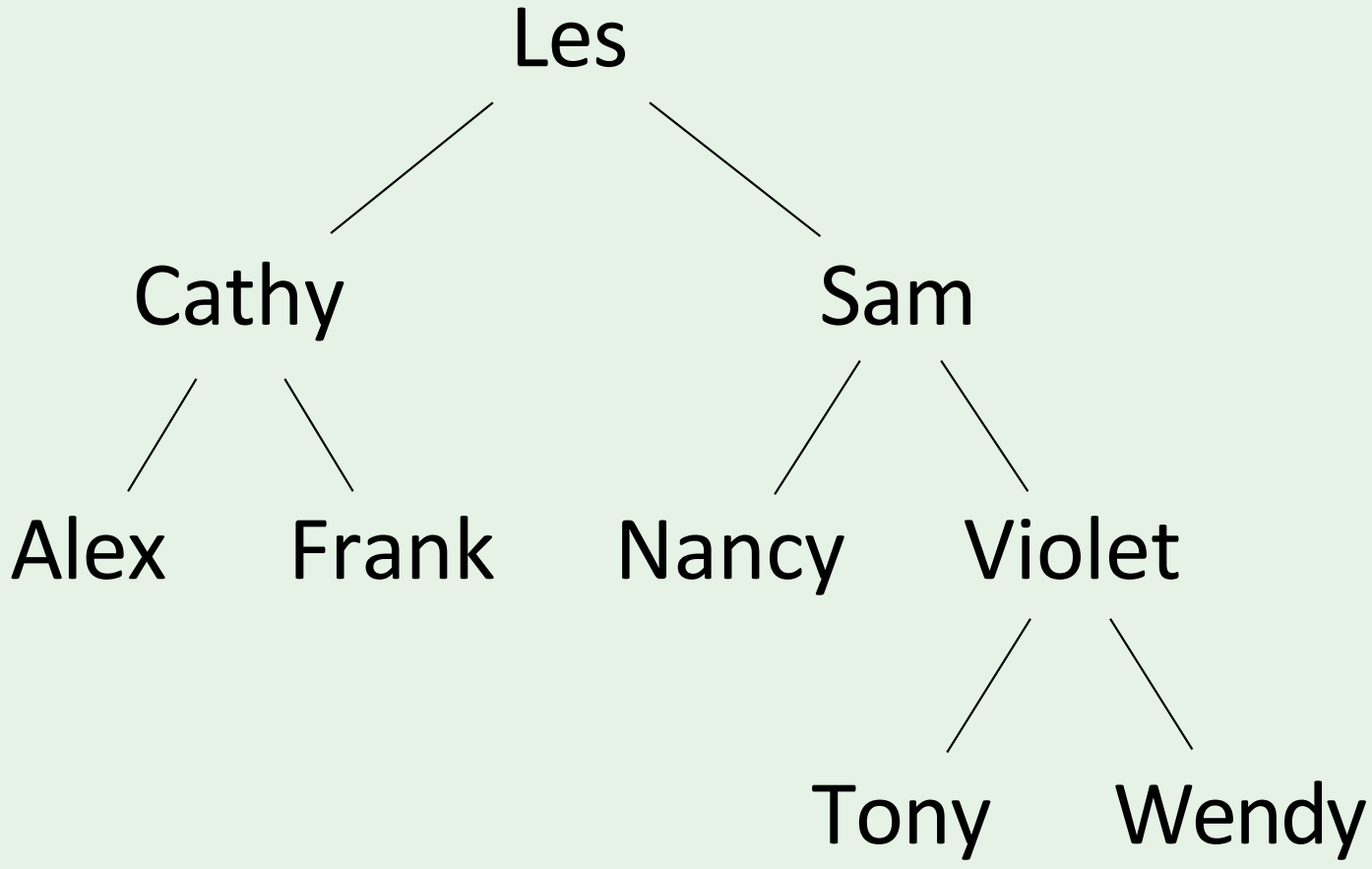
Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam Les

PostOrderTraversal



Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam Les

PostOrderTraversal



Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam Les

Breadth-first

```
LevelTraversal(tree)
if tree = nil:  return

Queue q
q.Enqueue(tree)
```

Breadth-first

```
LevelTraversal(tree)
if tree = nil :  return

Queue q
q.Enqueue(tree)
while not q.Empty() :
    node ← q.Dequeue()
```

Breadth-first

```
LevelTraversal(tree)
if tree = nil:  return

Queue q
q.Enqueue(tree)
while not q.Empty():
    node ← q.Dequeue()
    Print(node)
```

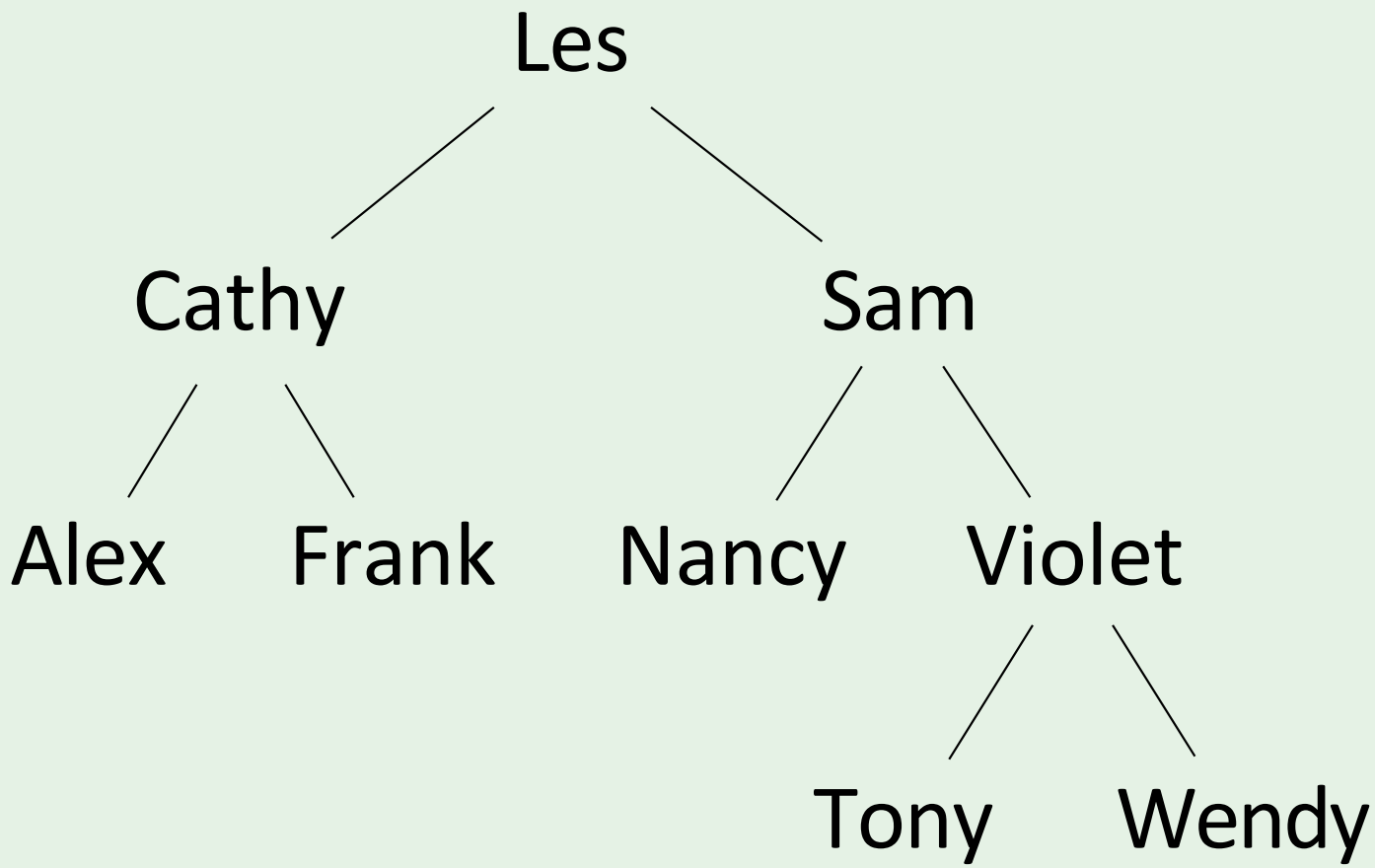
Breadth-first

```
LevelTraversal(tree)
if tree = nil :  return

Queue q
q.Enqueue(tree)
while not q.Empty() :
    node ← q.Dequeue()
    Print(node)
    if node.left ≠ nil :
        q.Enqueue(node.left)
```

Breadth-first

```
LevelTraversal(tree)
if tree = nil :  return

Queue q
q.Enqueue(tree)
while not q.Empty() :
    node ← q.Dequeue()
    Print(node)
    if node.left ≠ nil :
        q.Enqueue(node.left)
    if node.right ≠ nil :
        q.Enqueue(node.right)
```

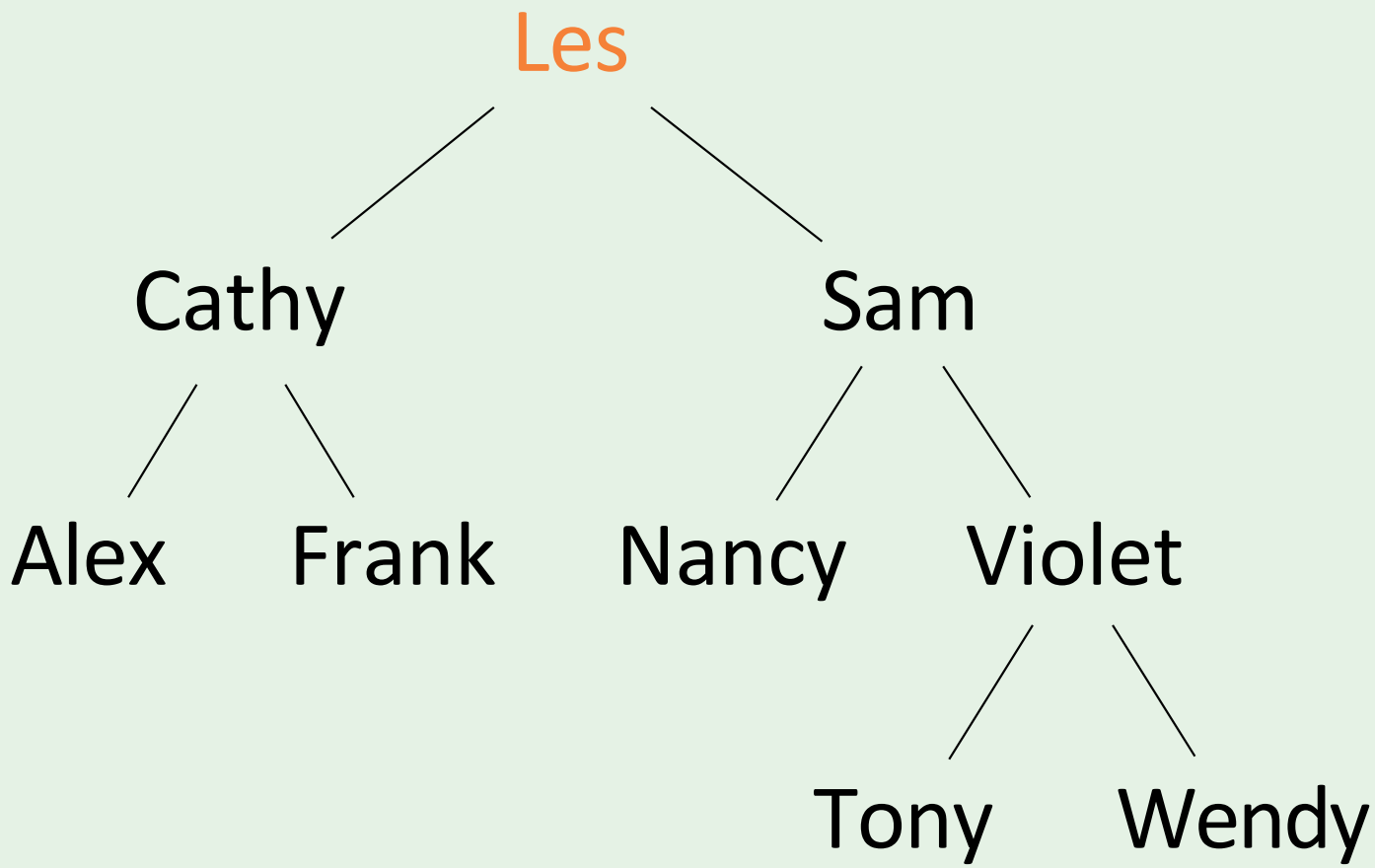
LevelTraversal



Output:

Queue: Les

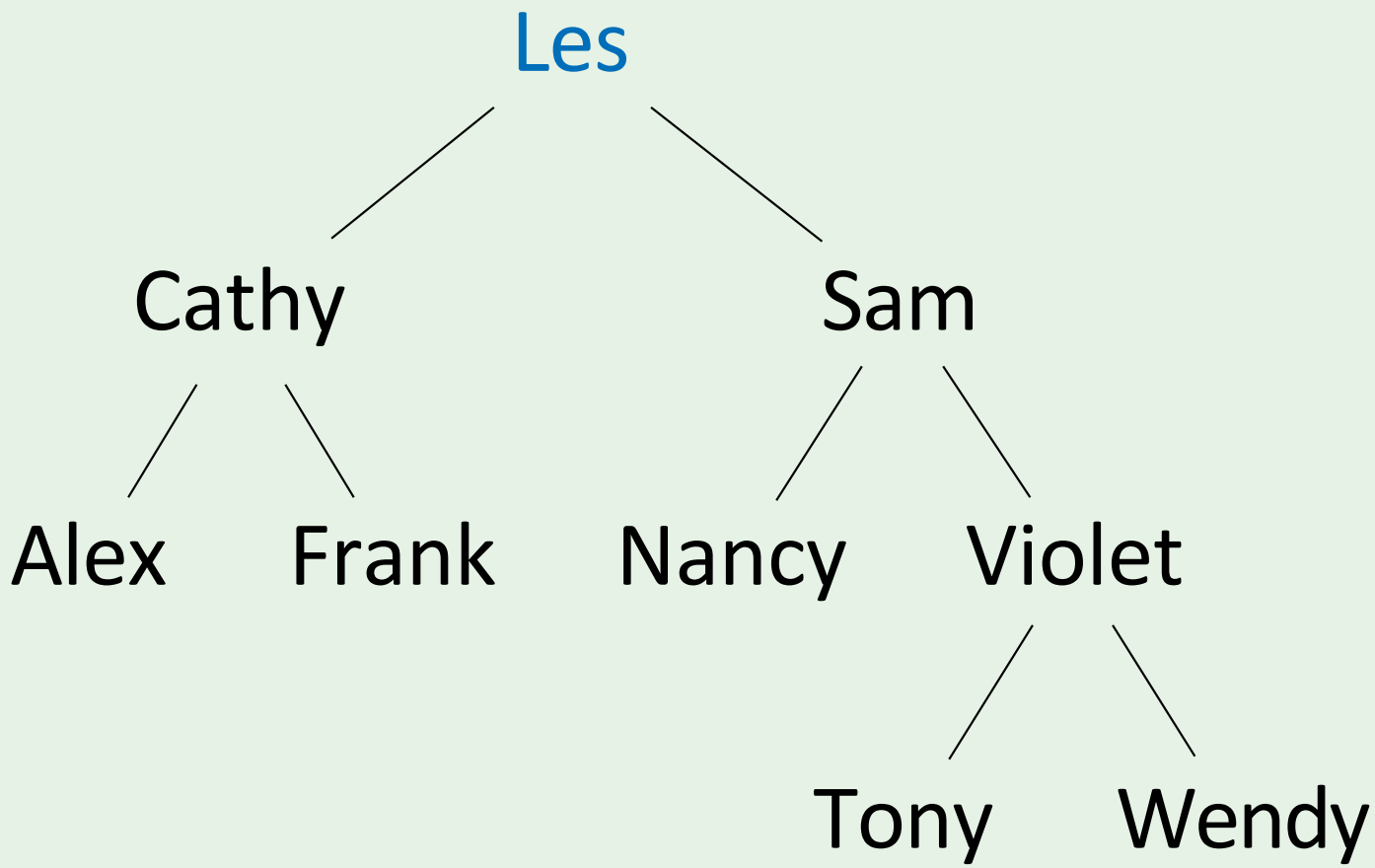
LevelTraversal



Output:

Queue:

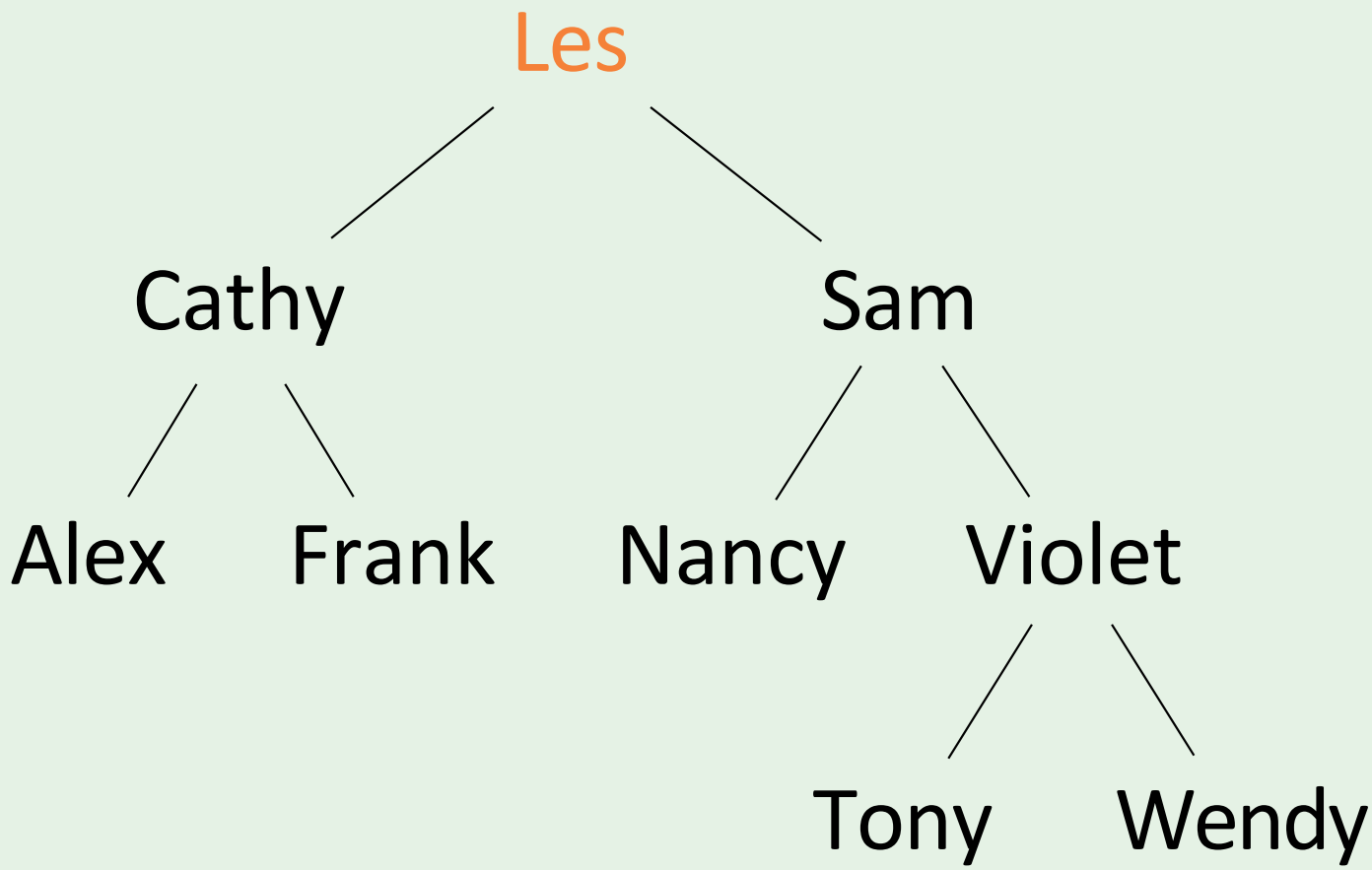
LevelTraversal



Output: Les

Queue:

LevelTraversal

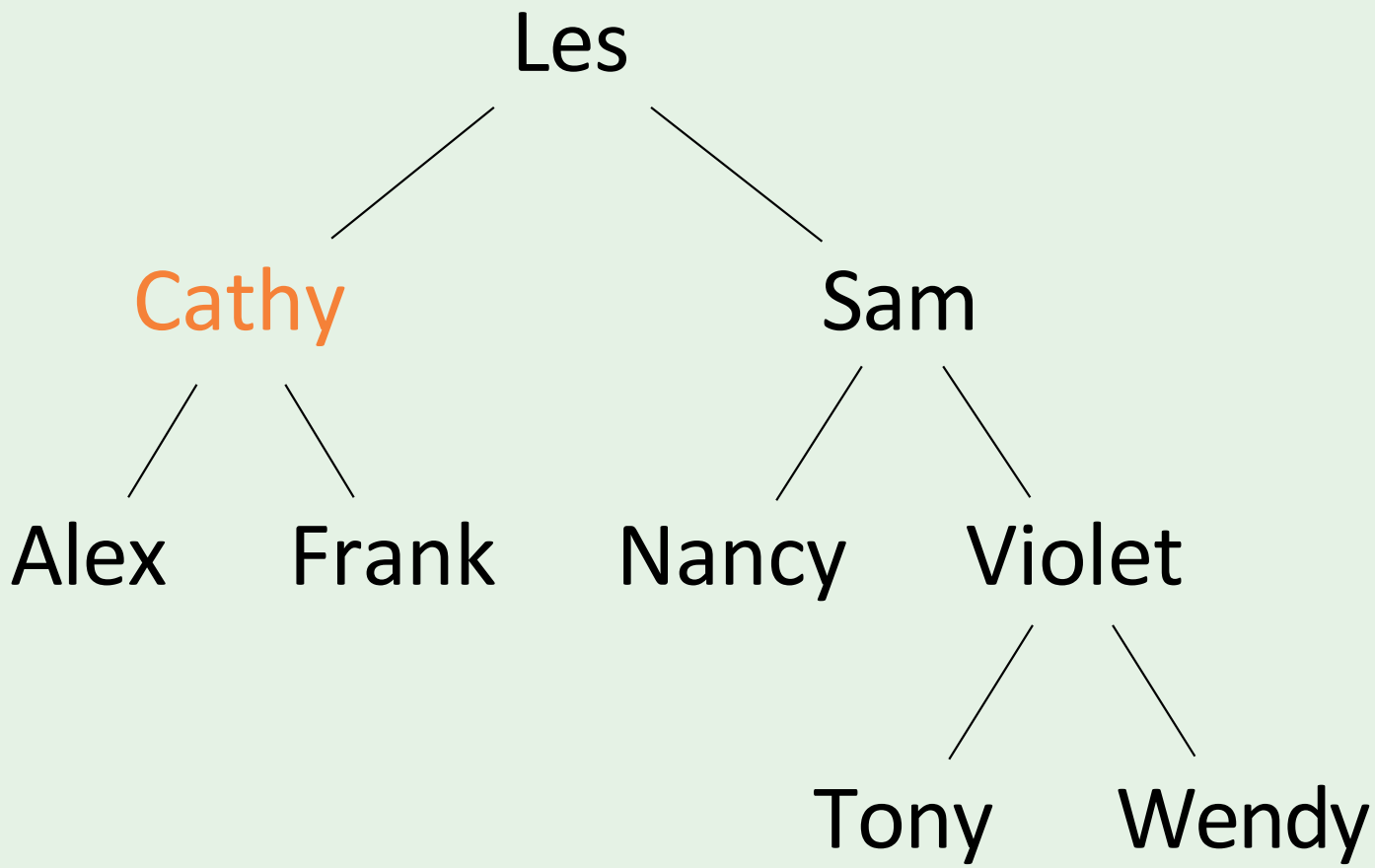


Output:

Les

Queue: Cathy,

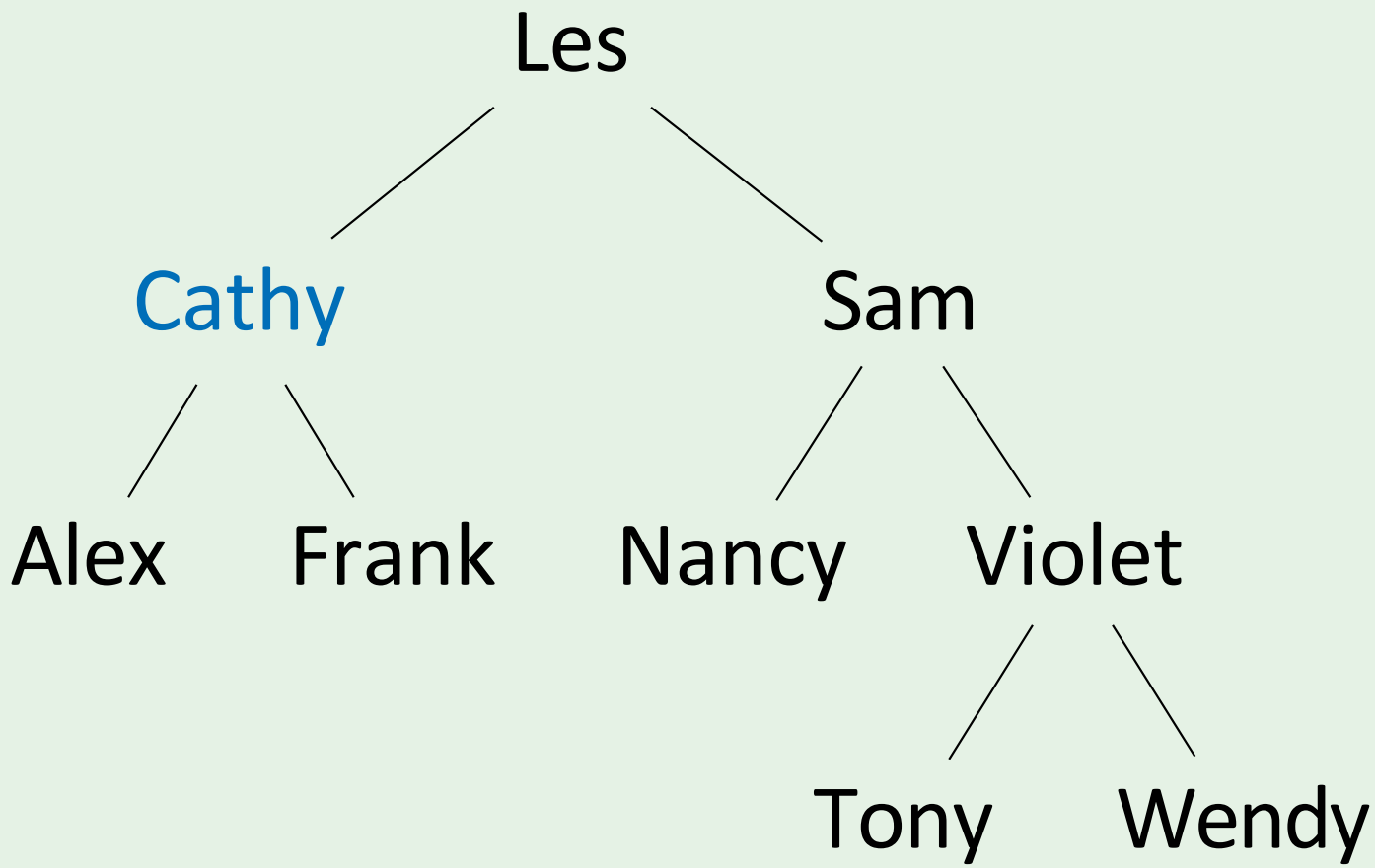
LevelTraversal



Output: Les

Queue: Sam

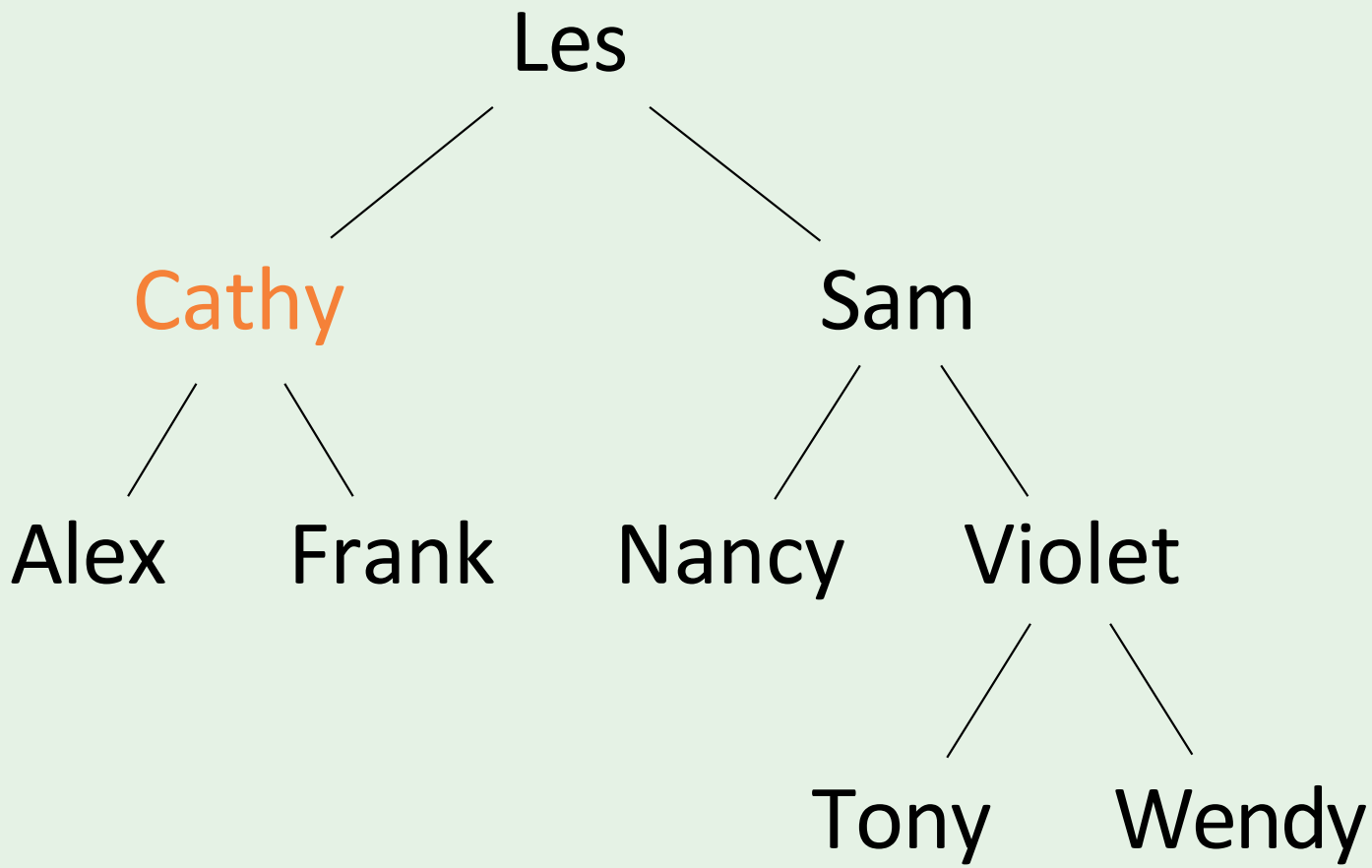
LevelTraversal



Output: Les Cathy

Queue: Sam

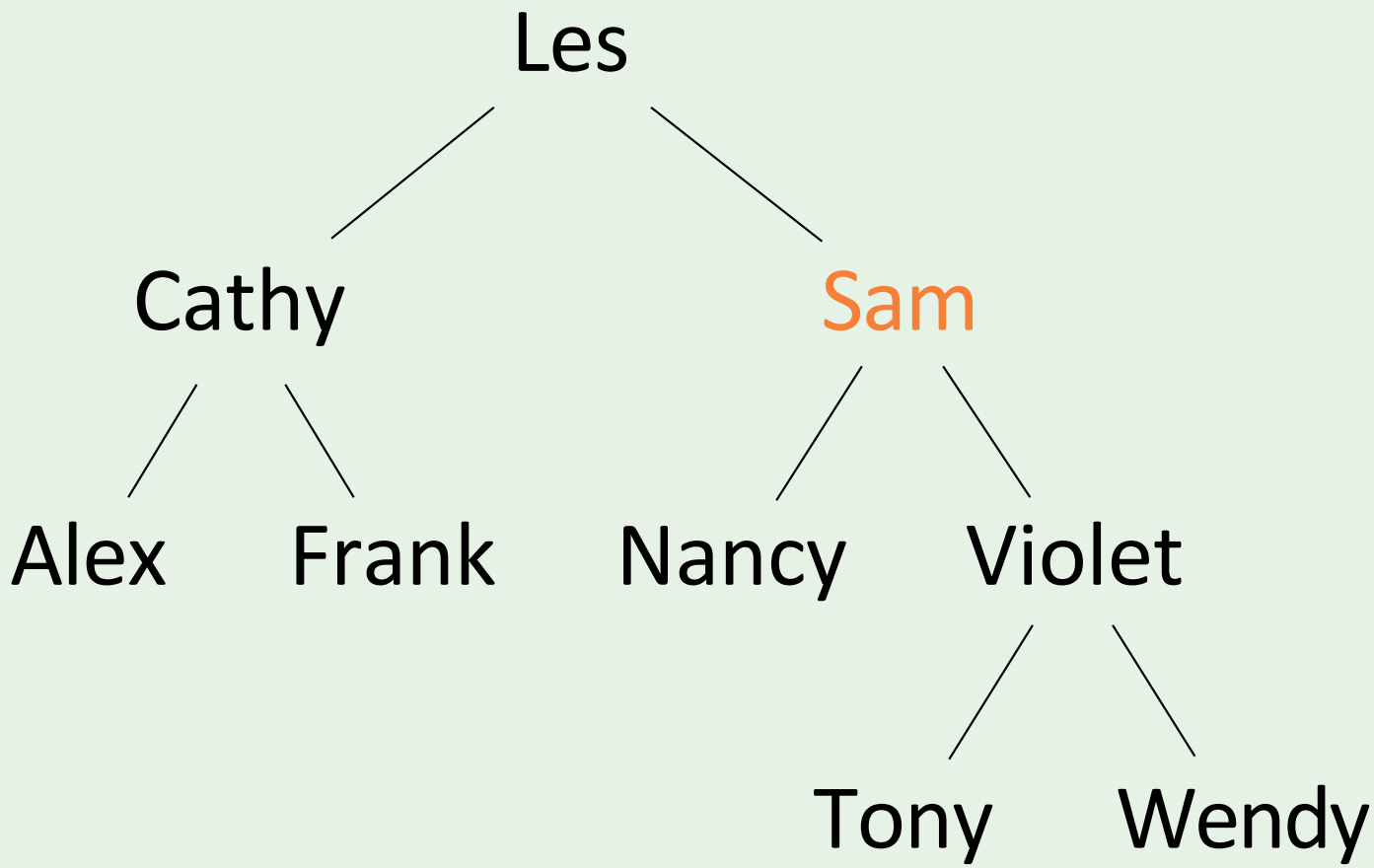
LevelTraversal



Output: Les Cathy

Queue: Sam,Alex, Frank

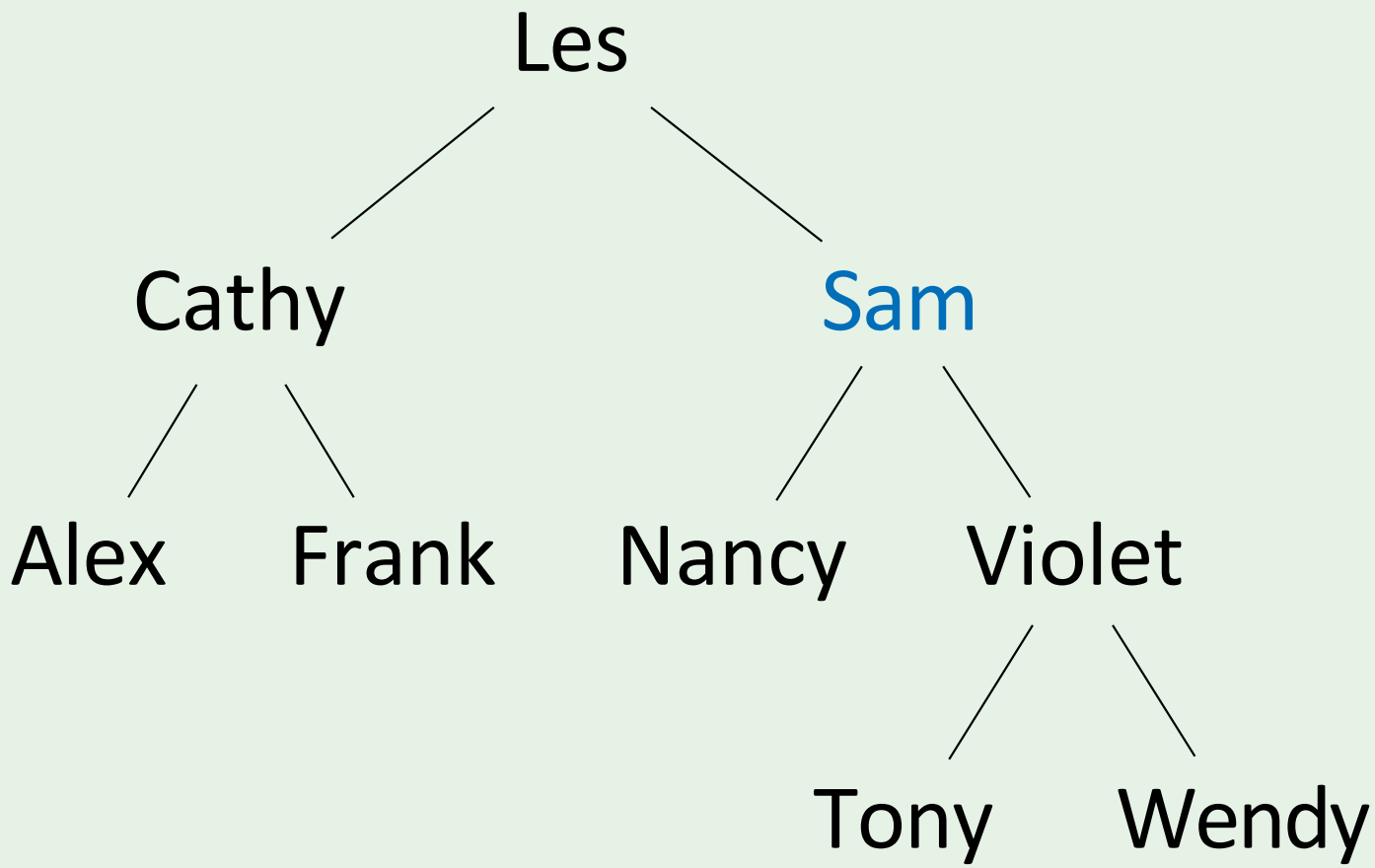
LevelTraversal



Output: Les Cathy

Queue: Alex,

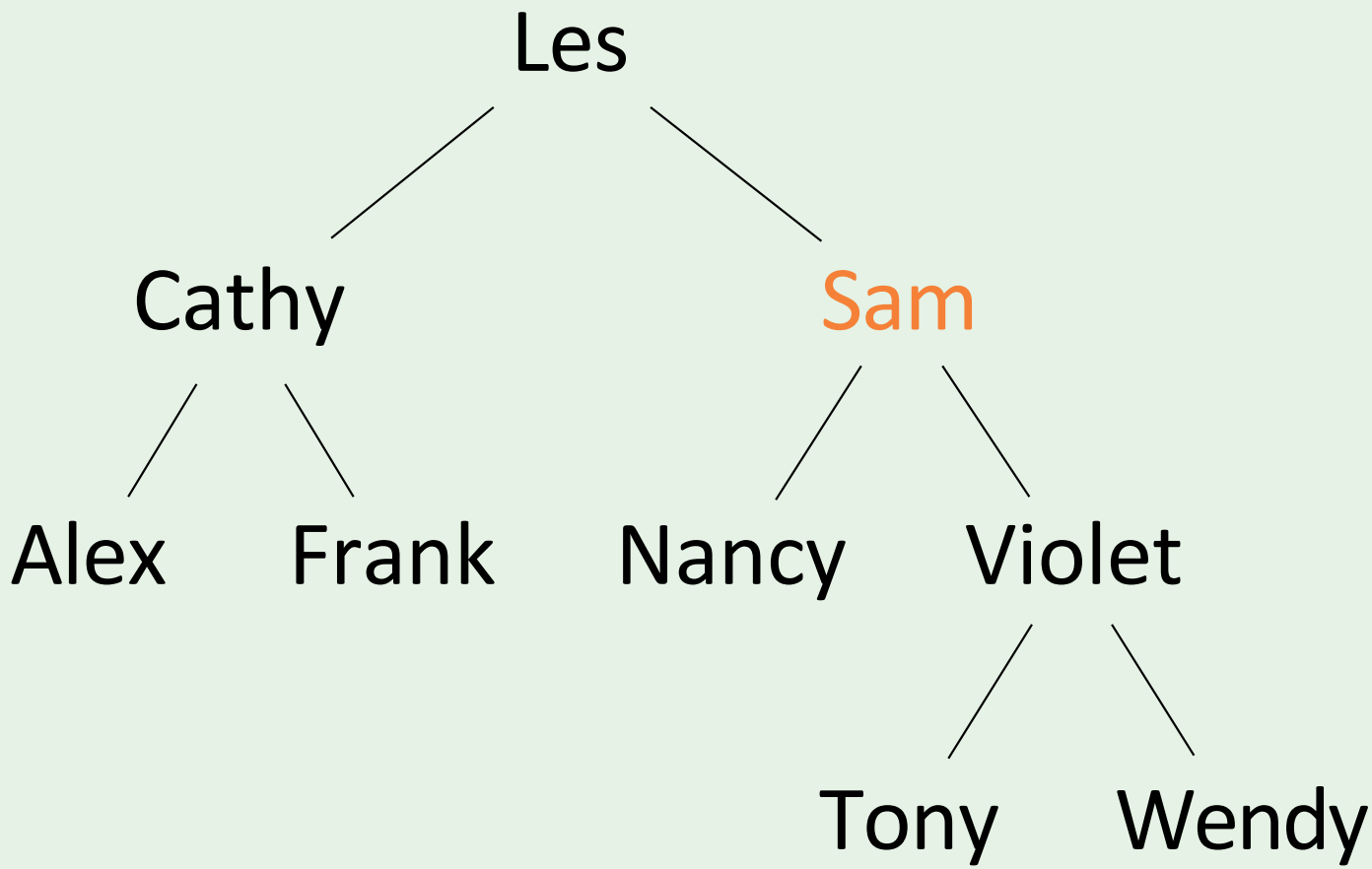
LevelTraversal



Output: Les Cathy Sam

Queue: Alex,

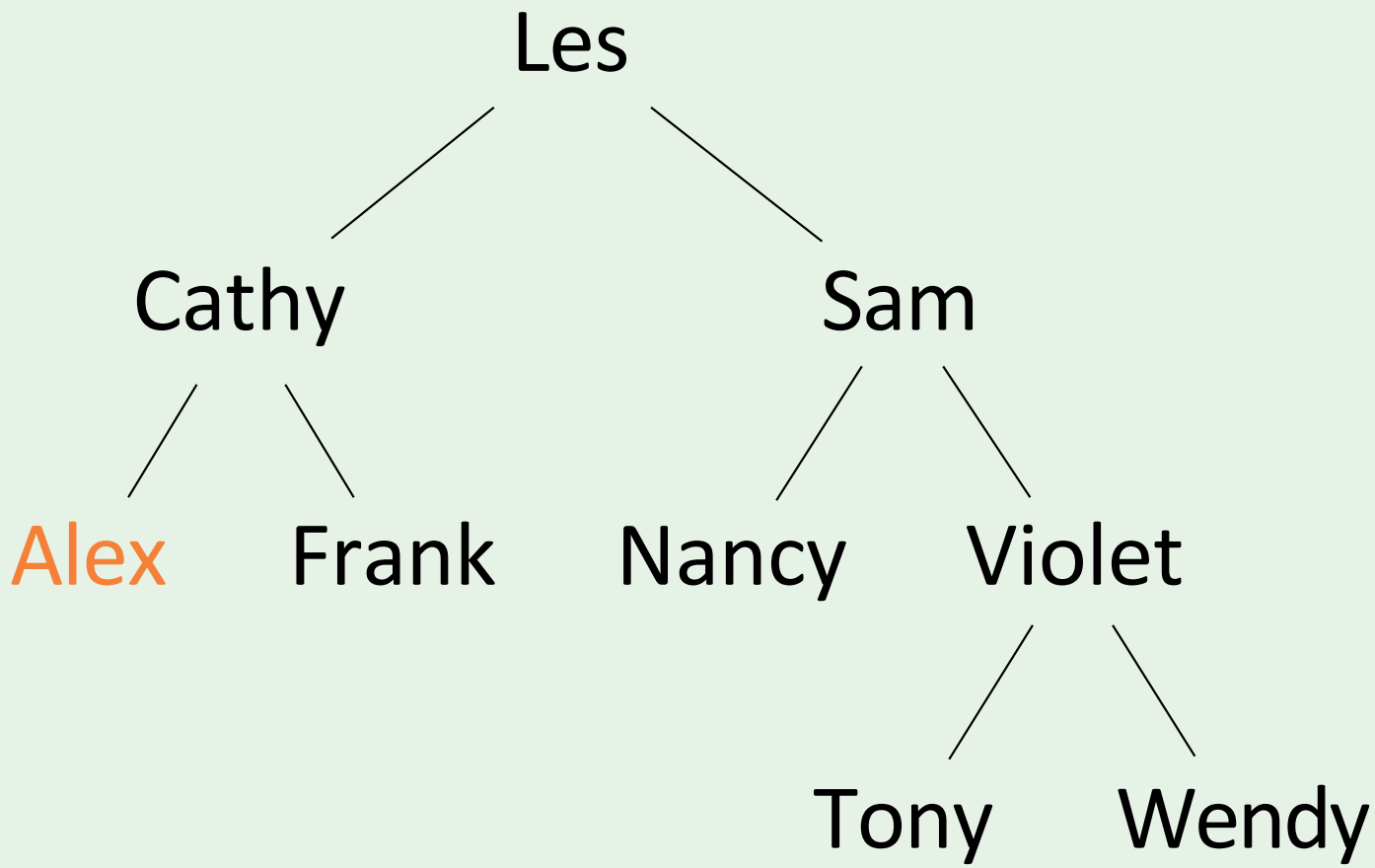
LevelTraversal



Output: Les Cathy Sam

Queue: Alex, Frank, Nancy,

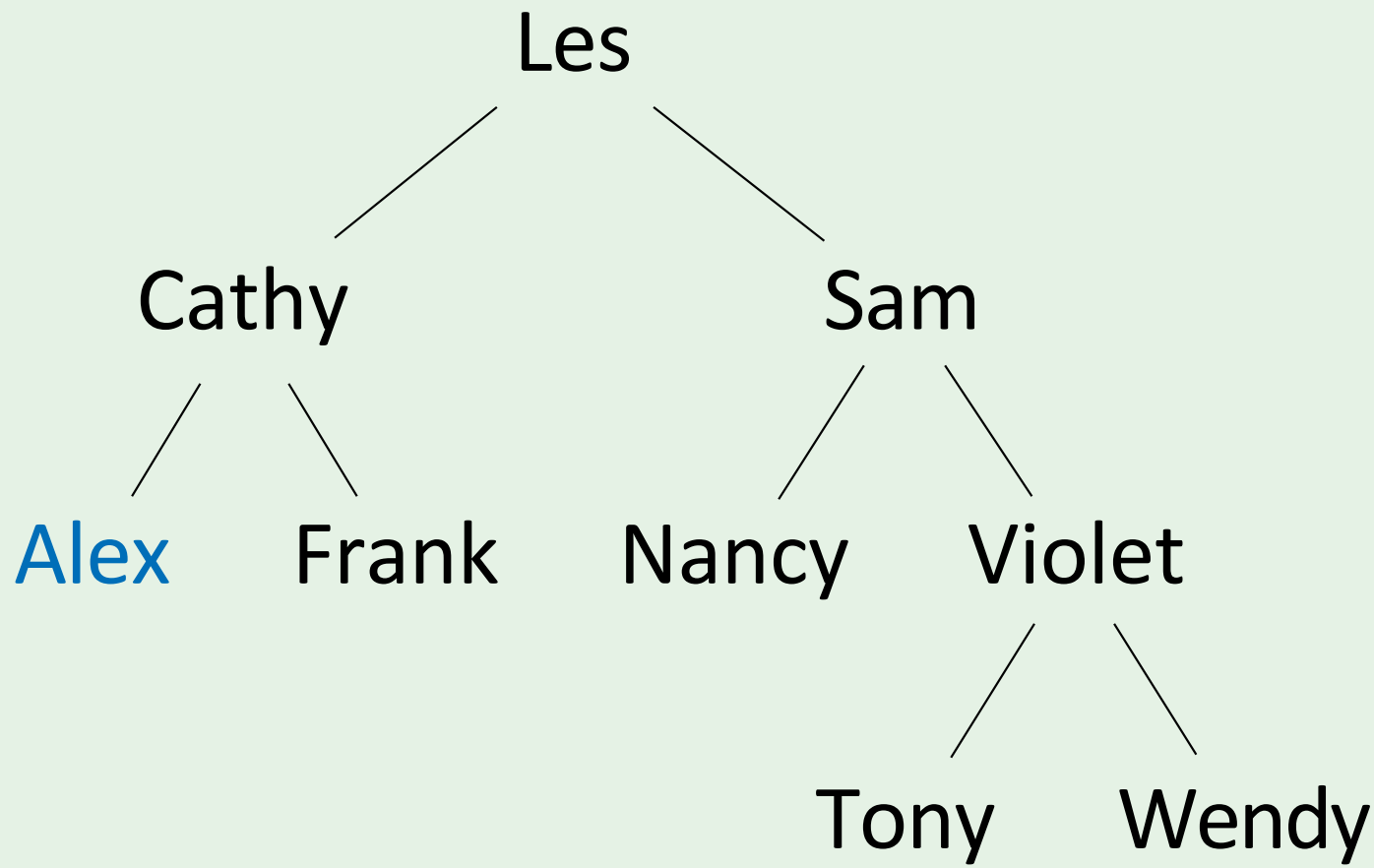
LevelTraversal



Output: Les Cathy Sam

Queue: Frank, Nancy,

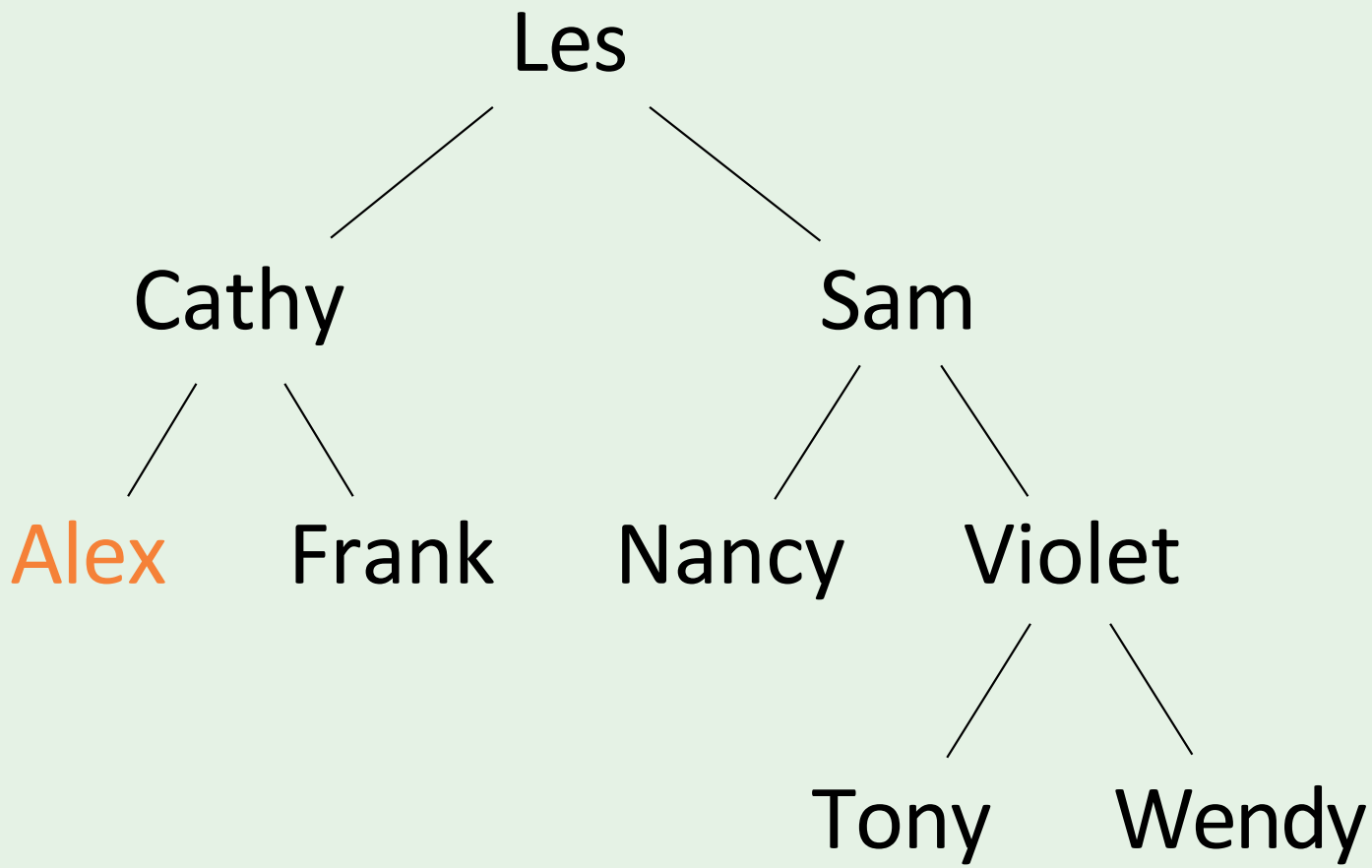
LevelTraversal



Output: Les Cathy Sam Alex

Queue: Frank, Nancy,

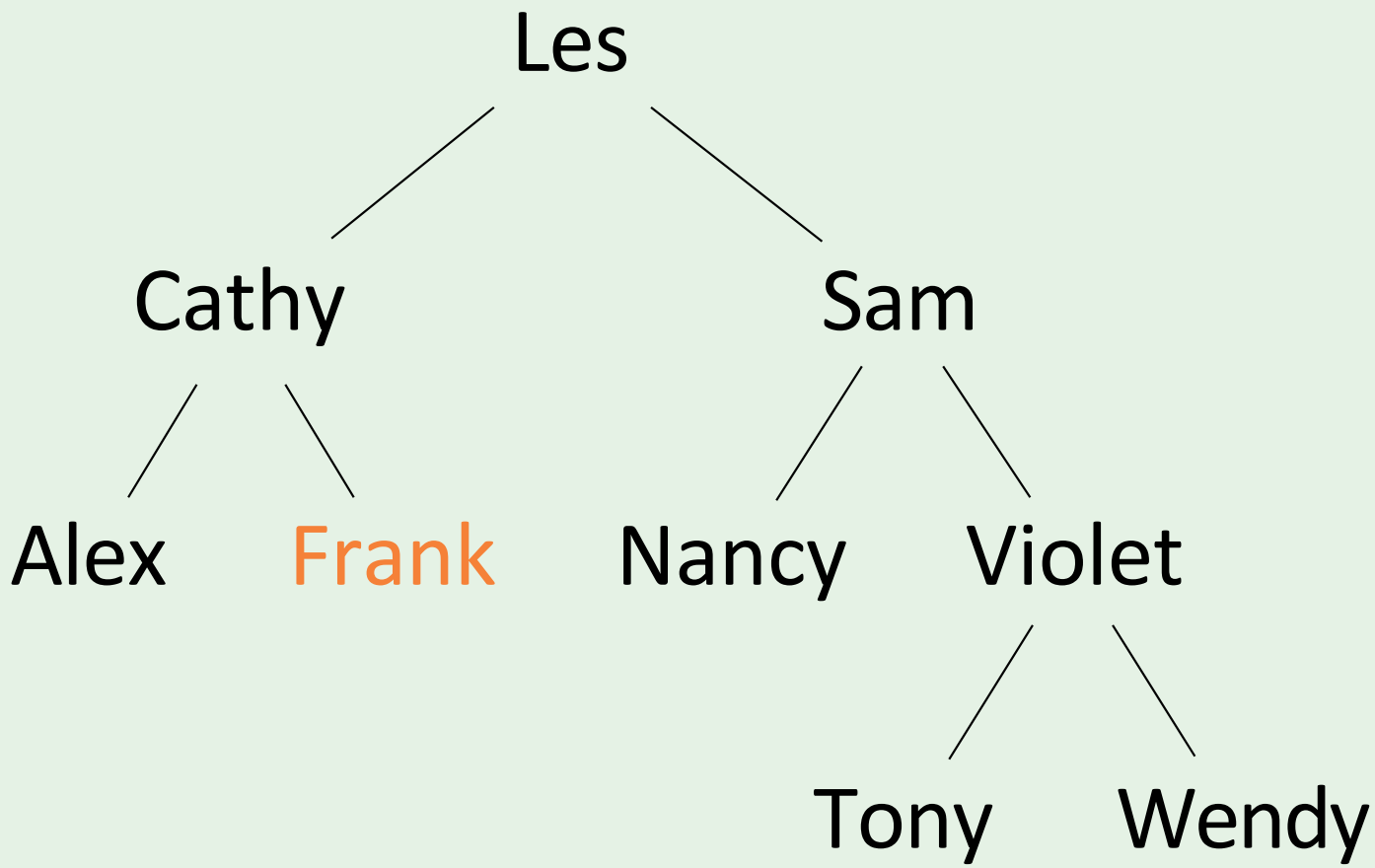
LevelTraversal



Output: Les Cathy Sam Alex

Queue: Frank, Nancy,

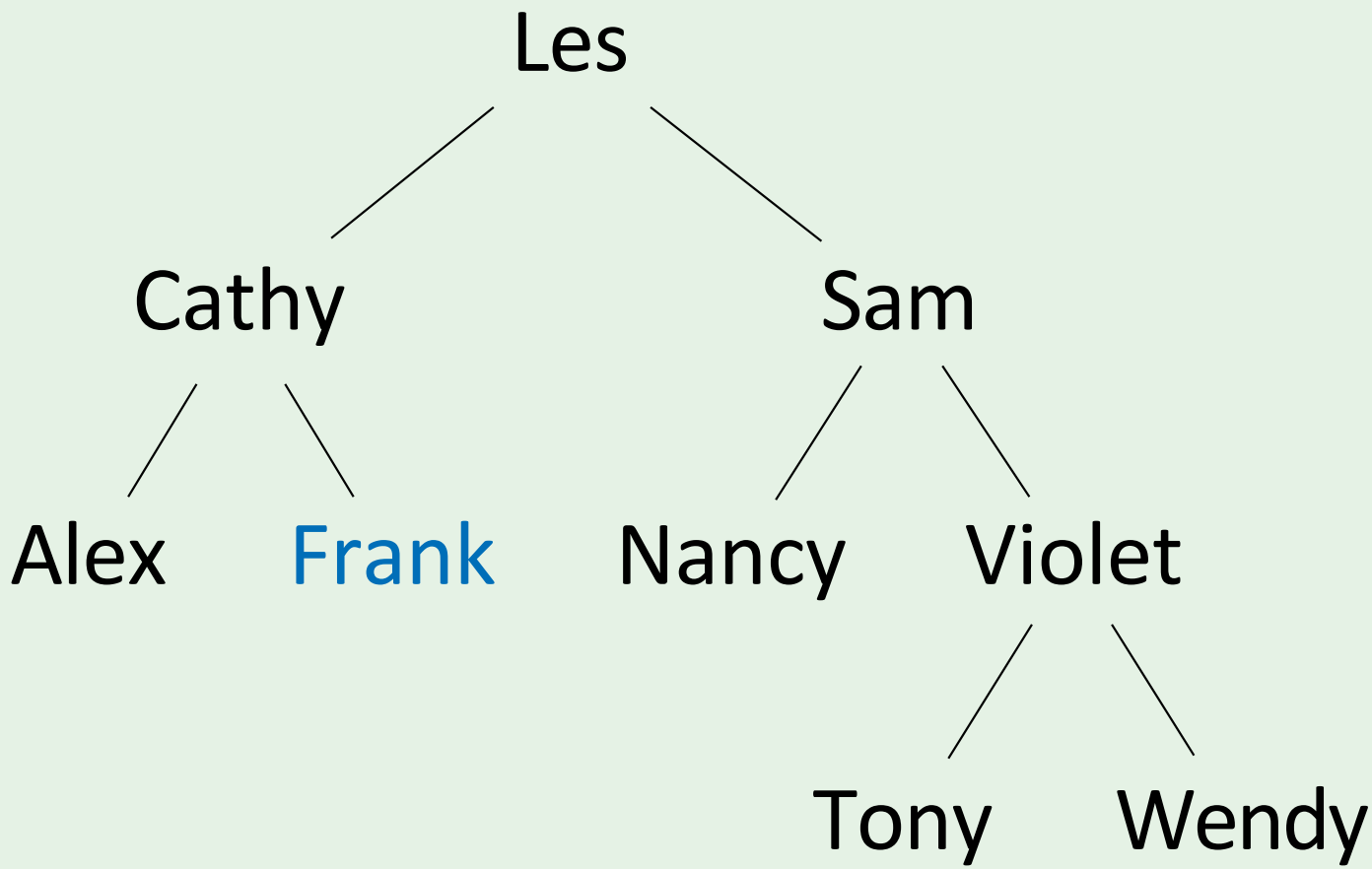
LevelTraversal



Output: Les Cathy Sam Alex

Queue: Nancy, Violet

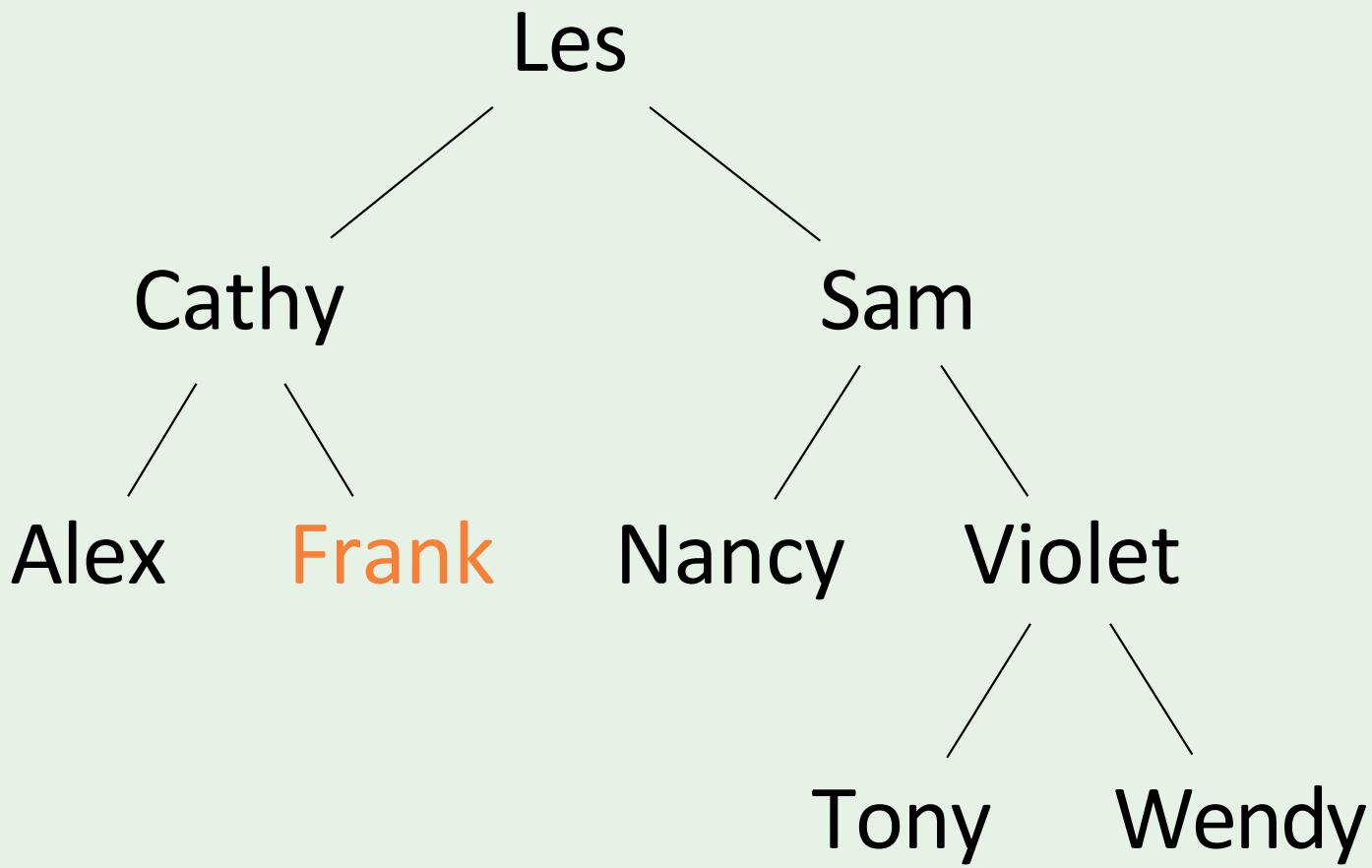
LevelTraversal



Output: Les Cathy Sam Alex Frank

Queue: Nancy, Violet

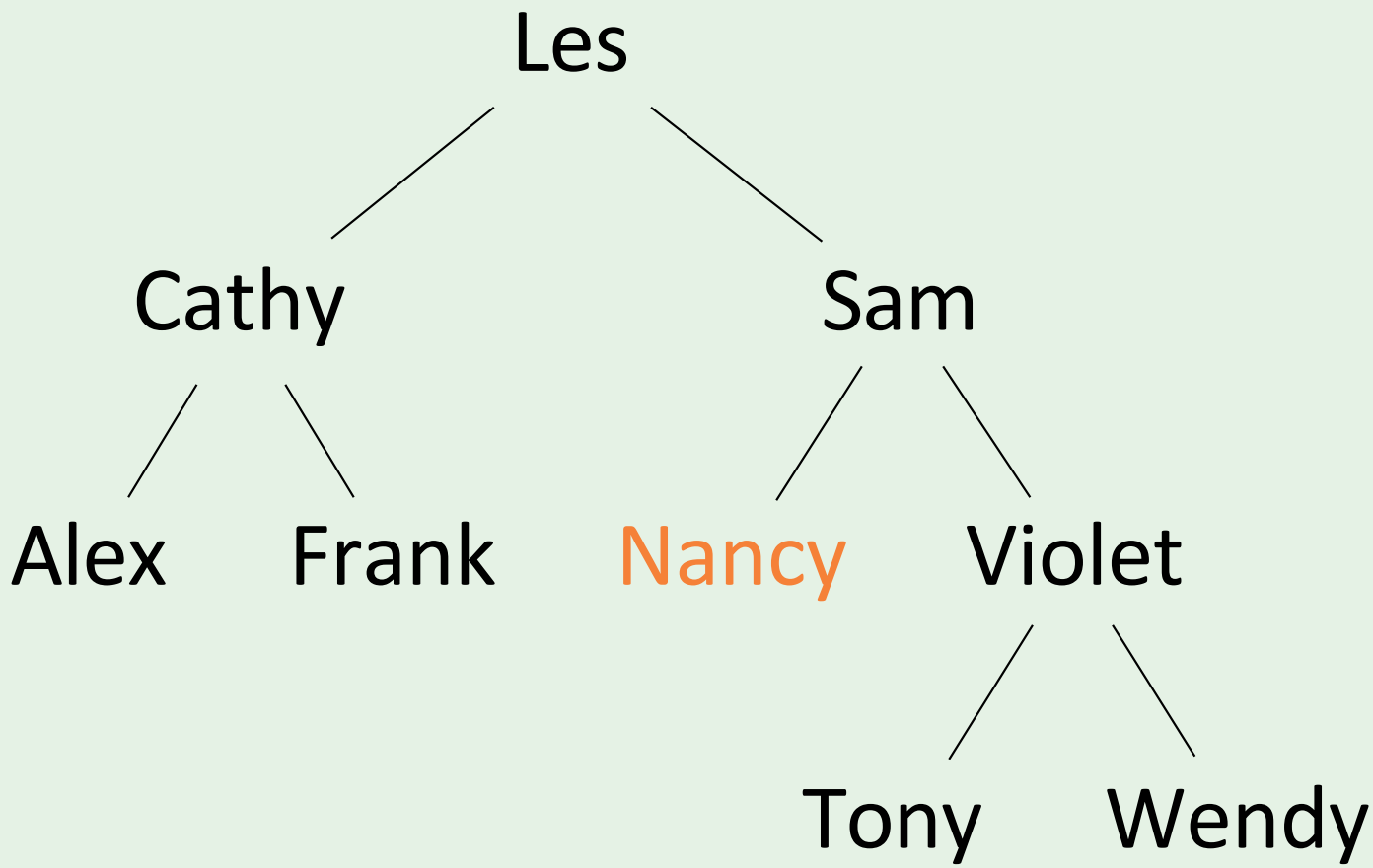
LevelTraversal



Output: Les Cathy Sam Alex Frank

Queue: Nancy, Violet

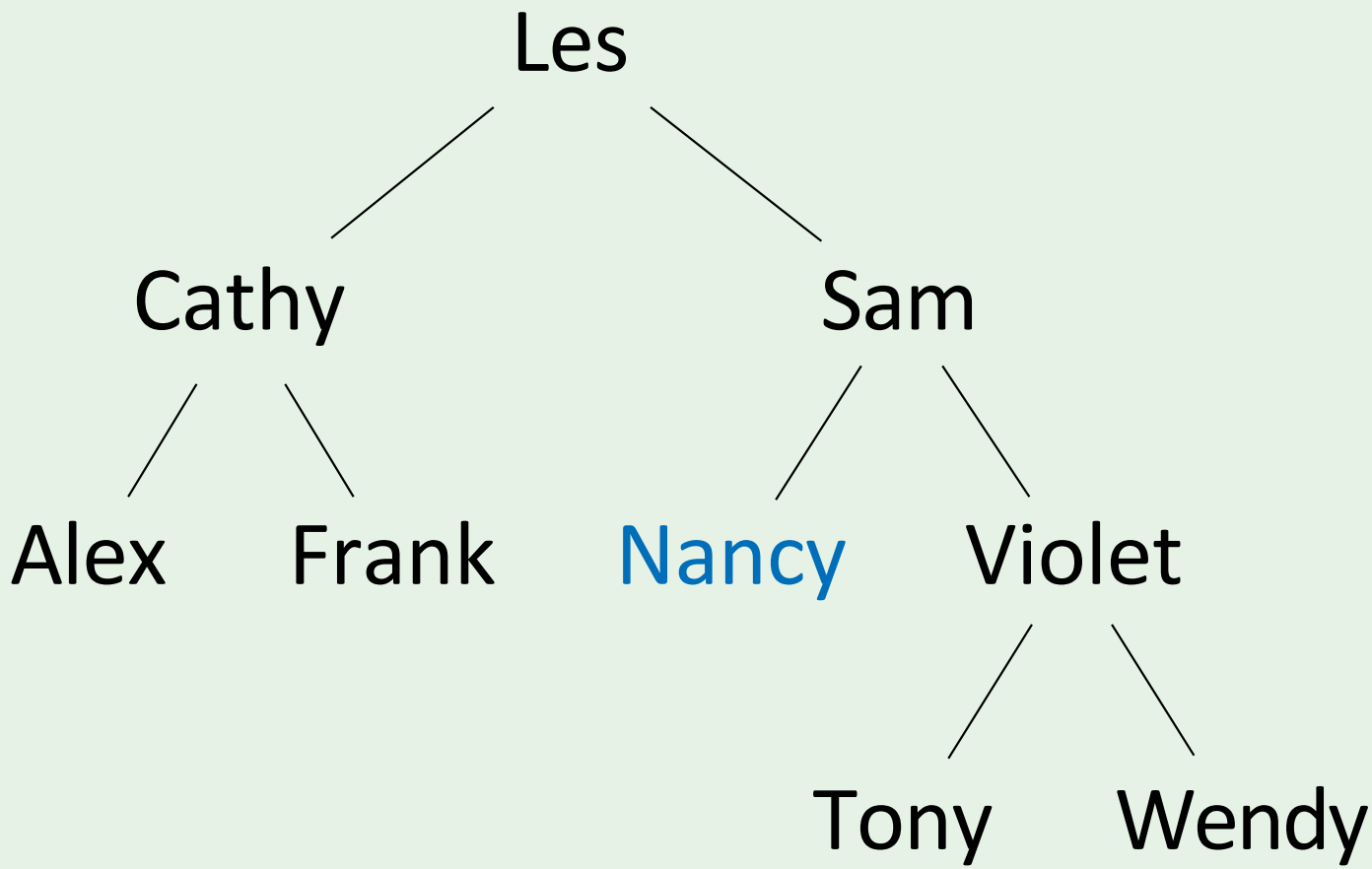
LevelTraversal



Output: Les Cathy Sam Alex Frank

Queue:

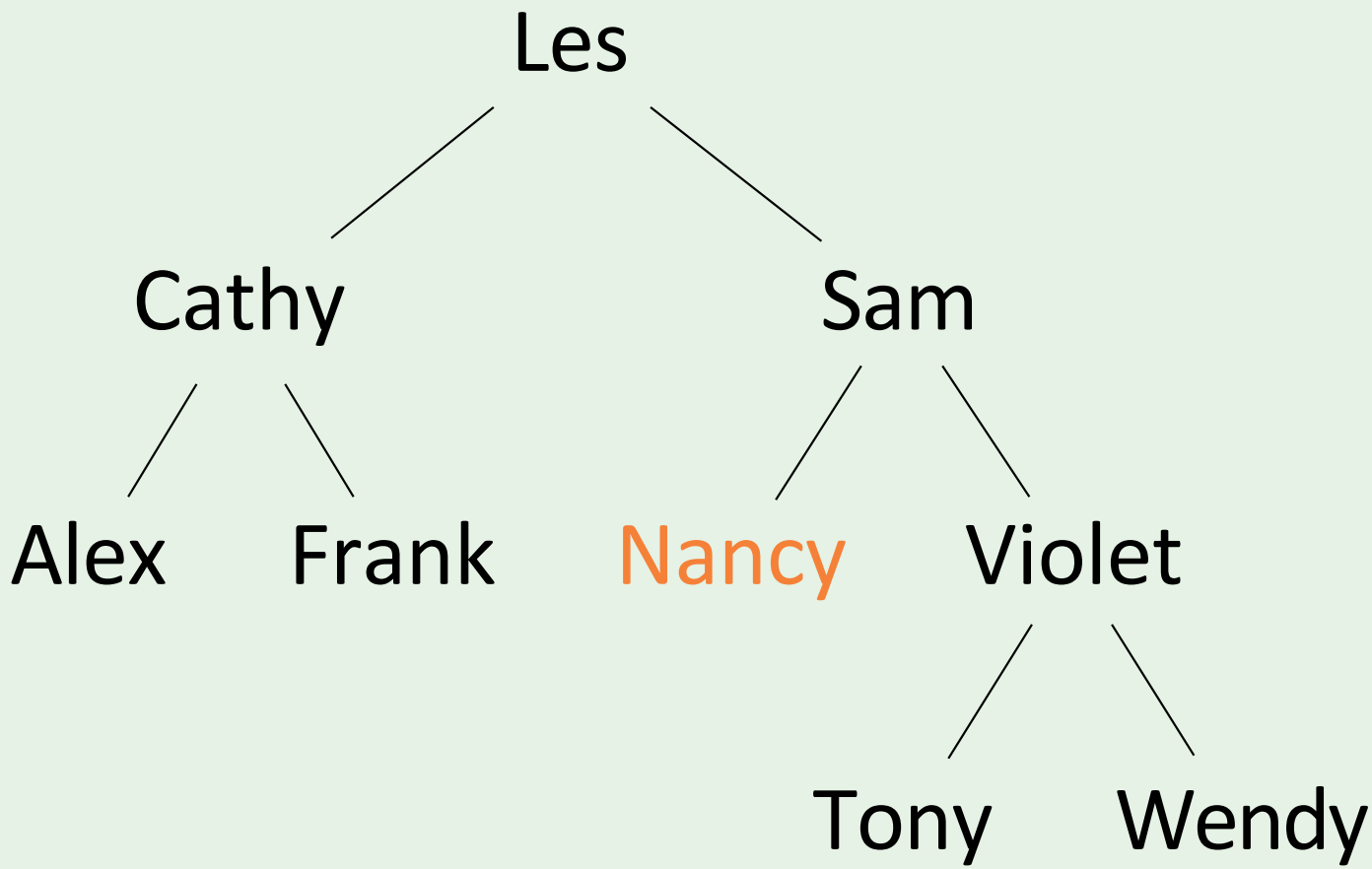
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy

Queue:

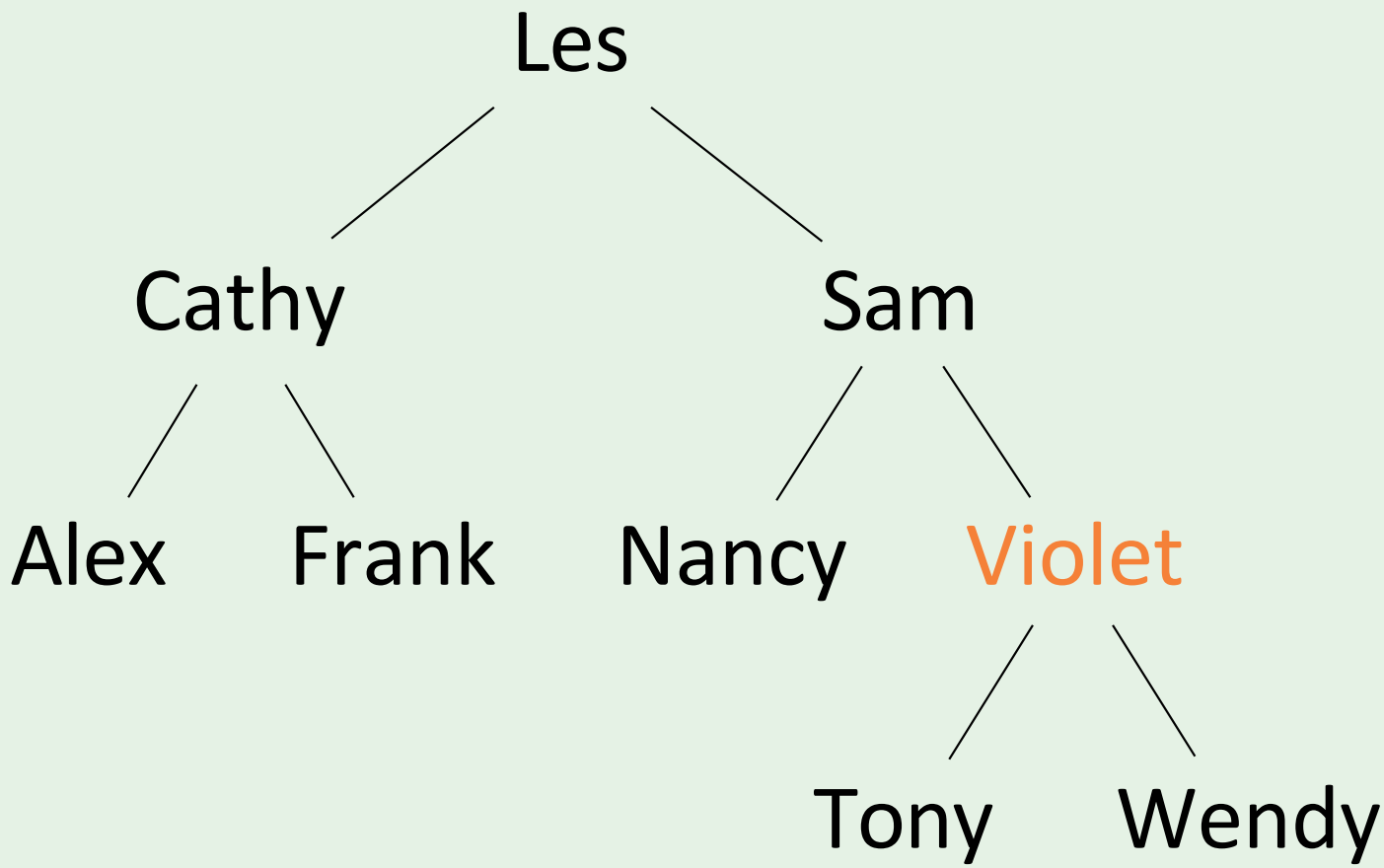
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy

Queue:

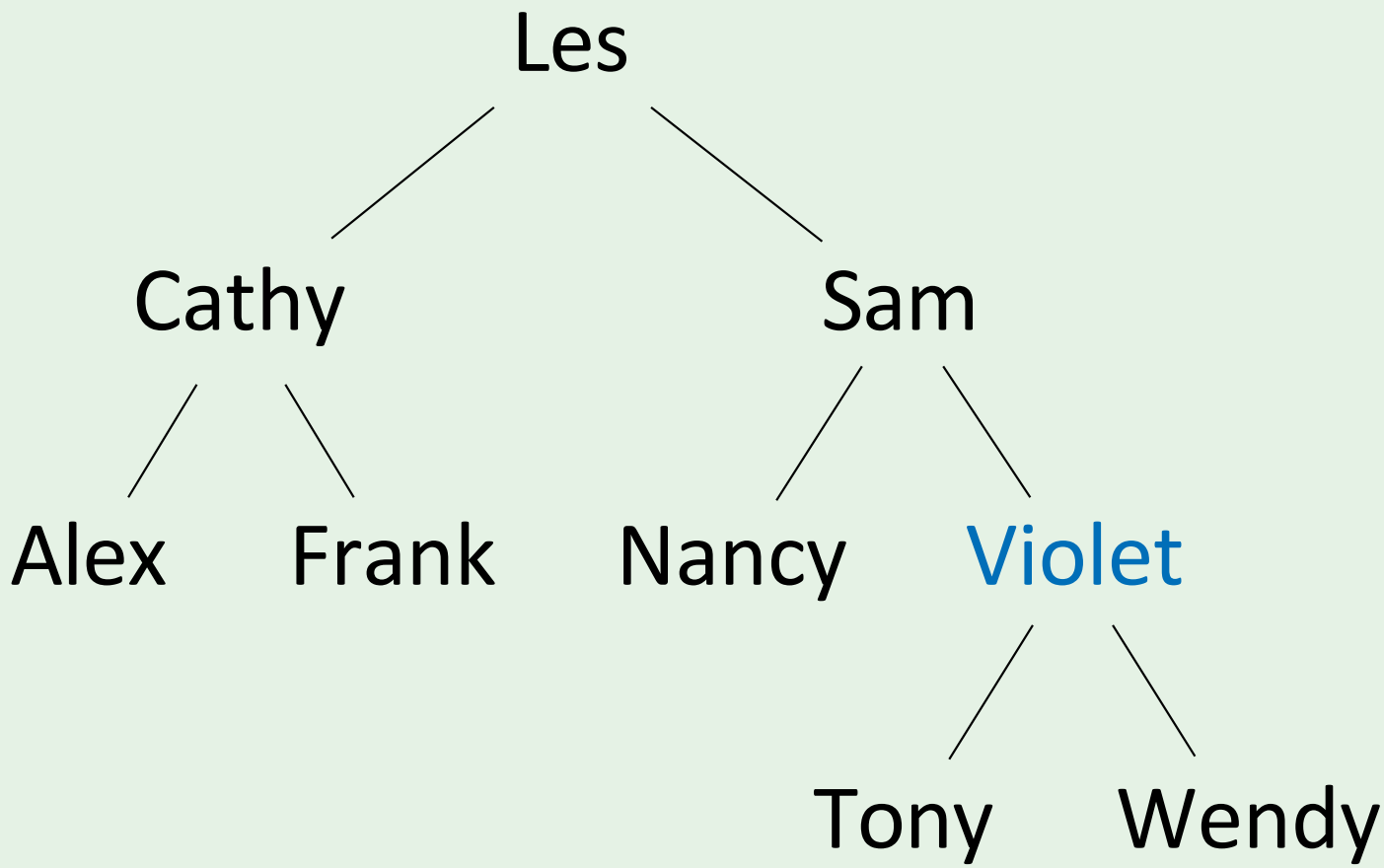
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy

Queue:

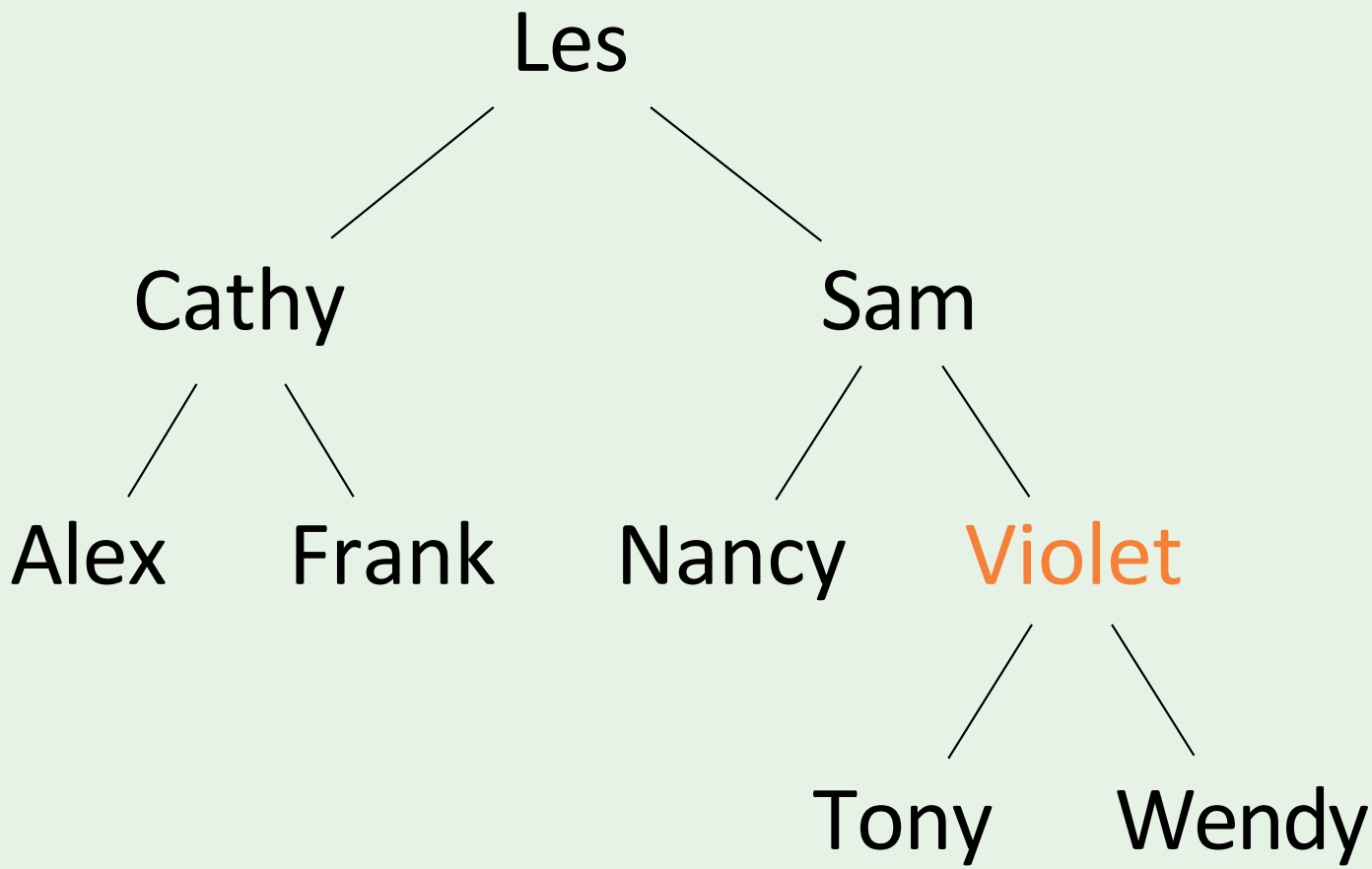
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet

Queue:

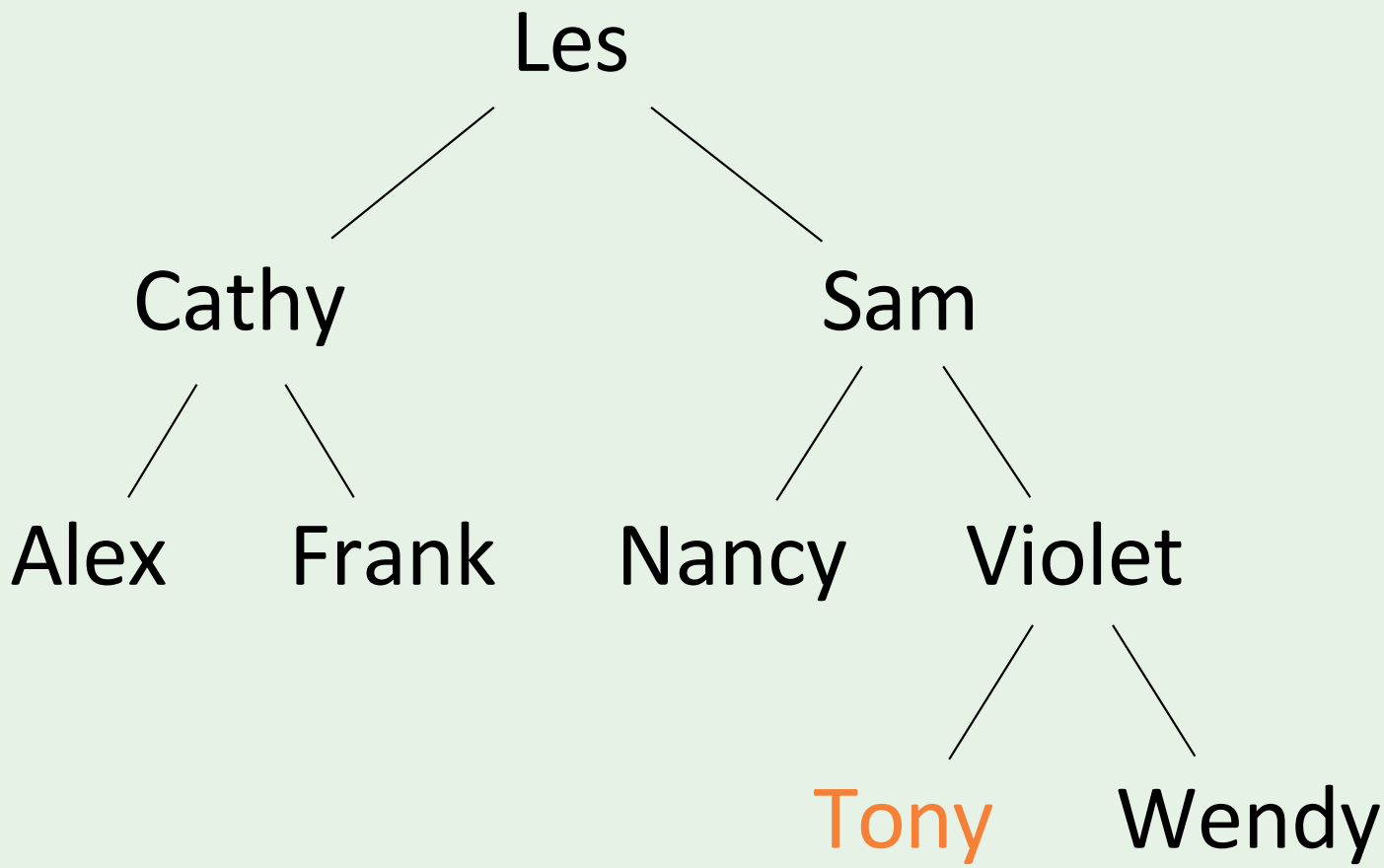
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet

Queue: Tony Wendy

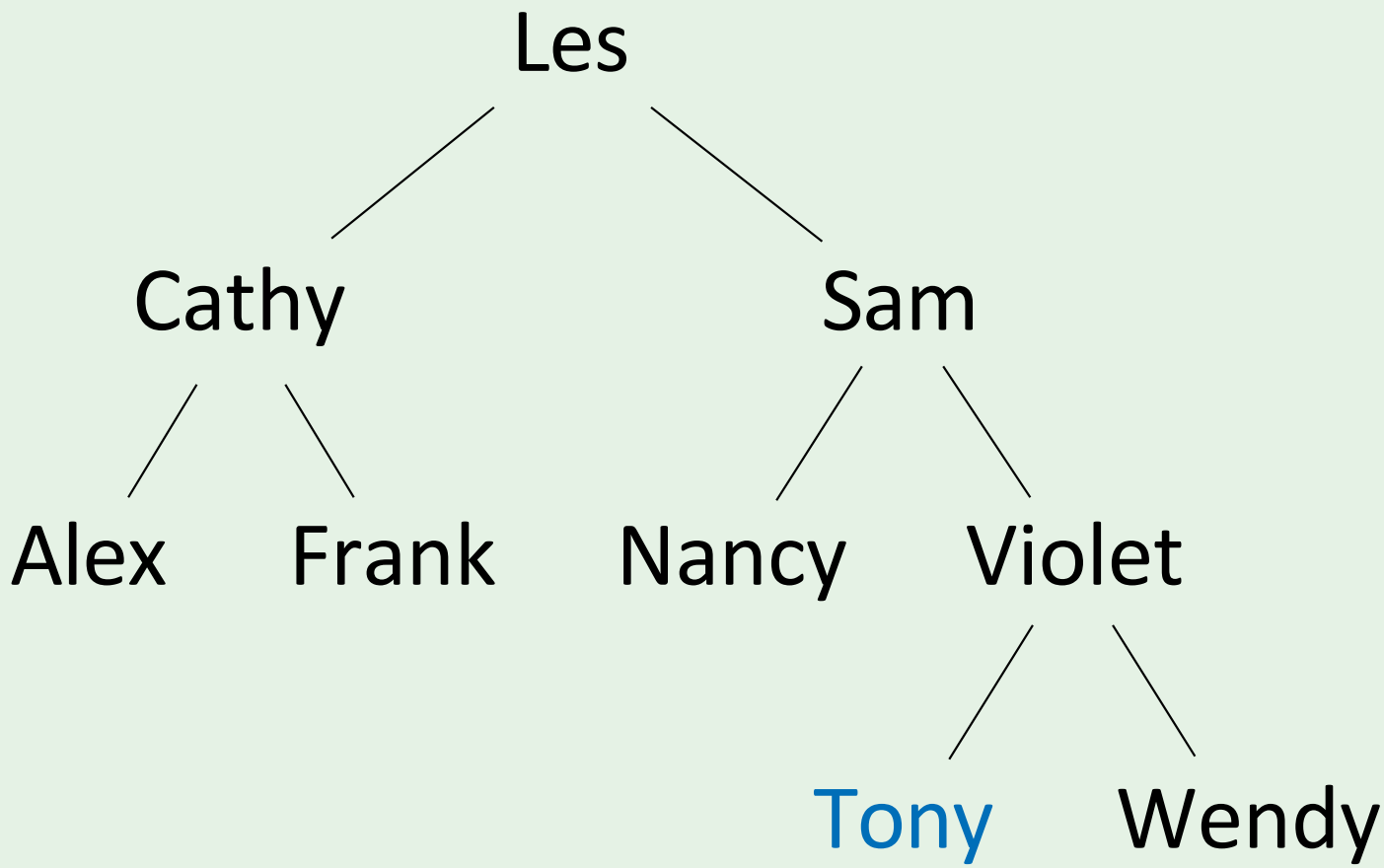
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet

Queue:

LevelTraversal



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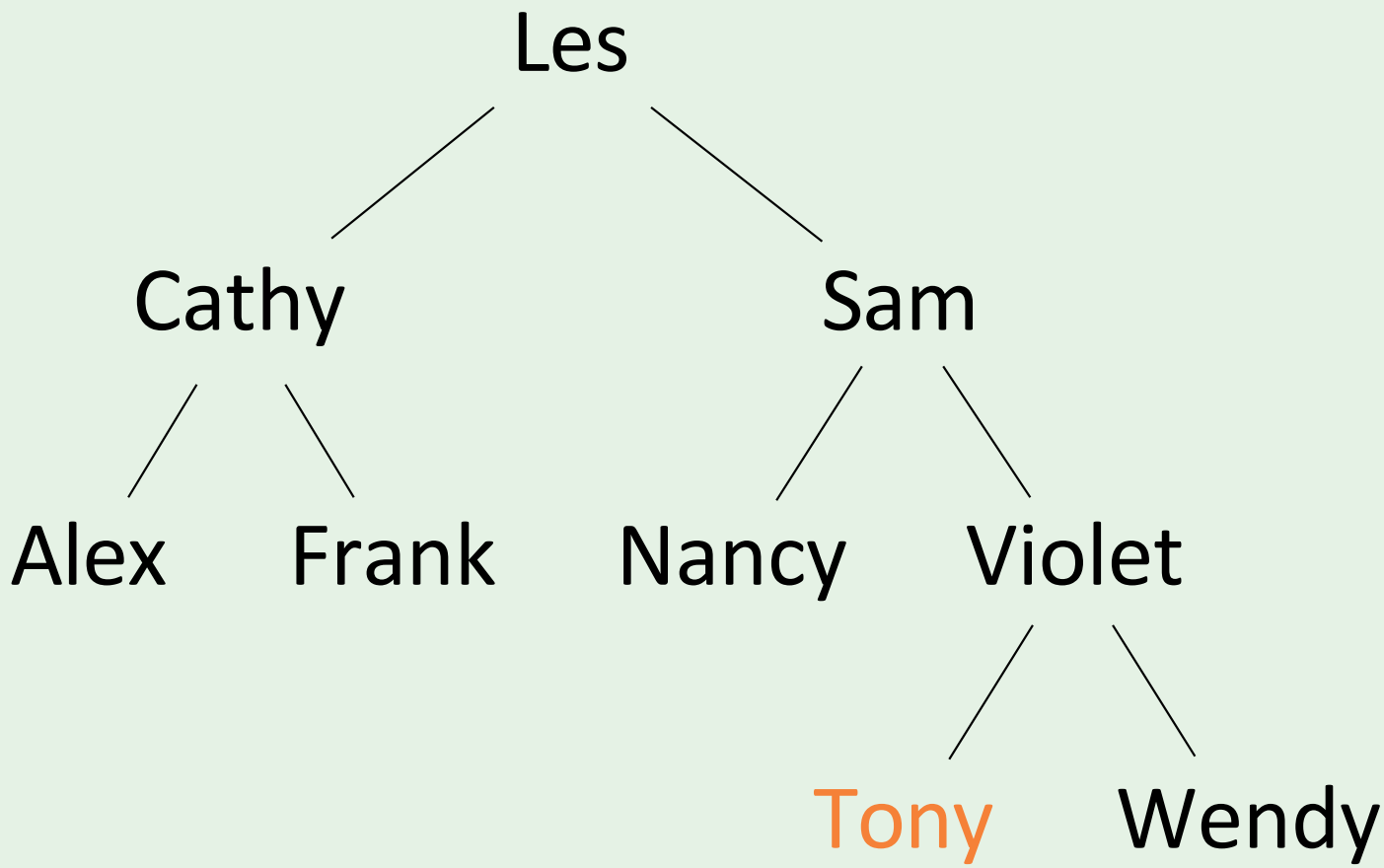
Queue:

LevelTraversal



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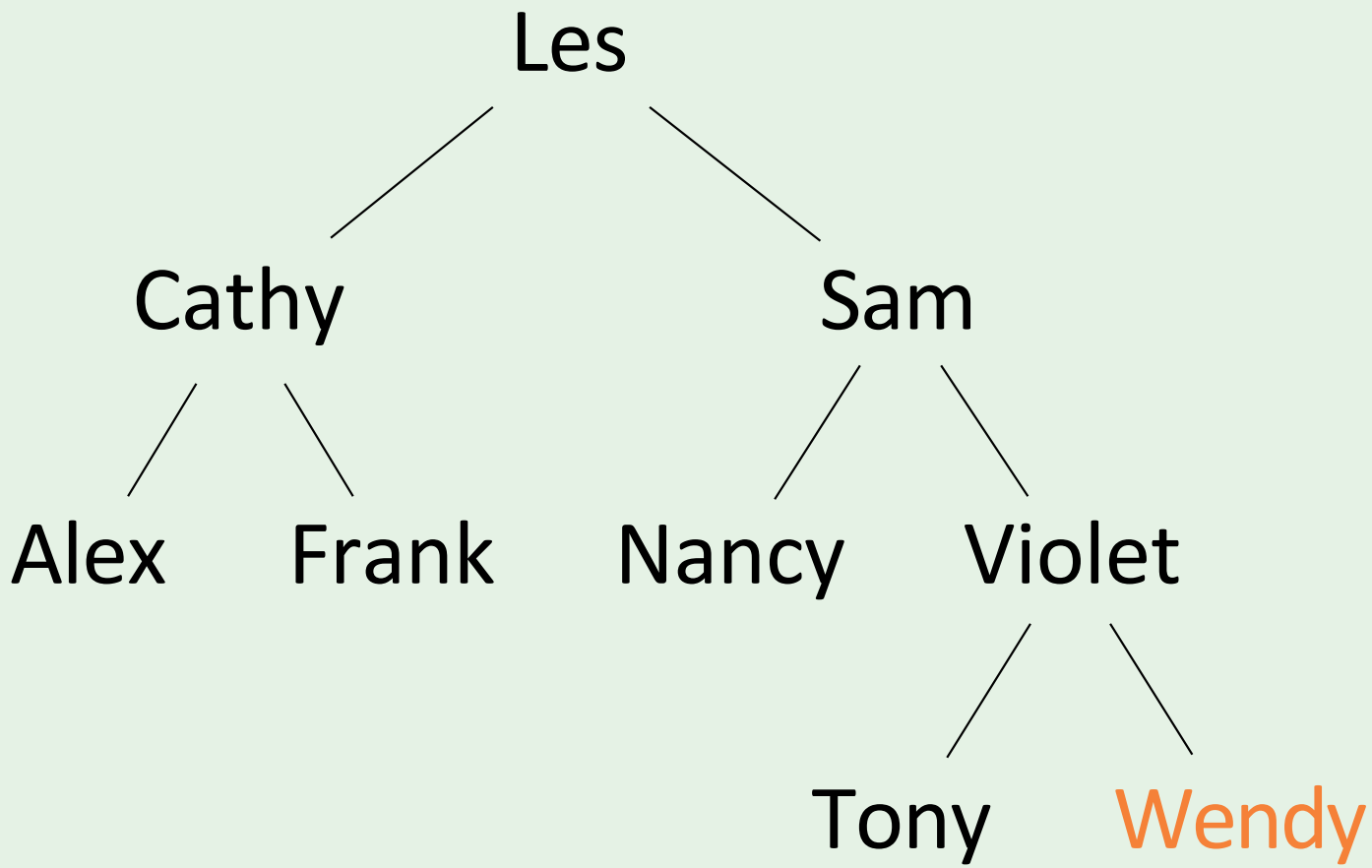

Output: Les Cathy Sam Alex Frank Nancy

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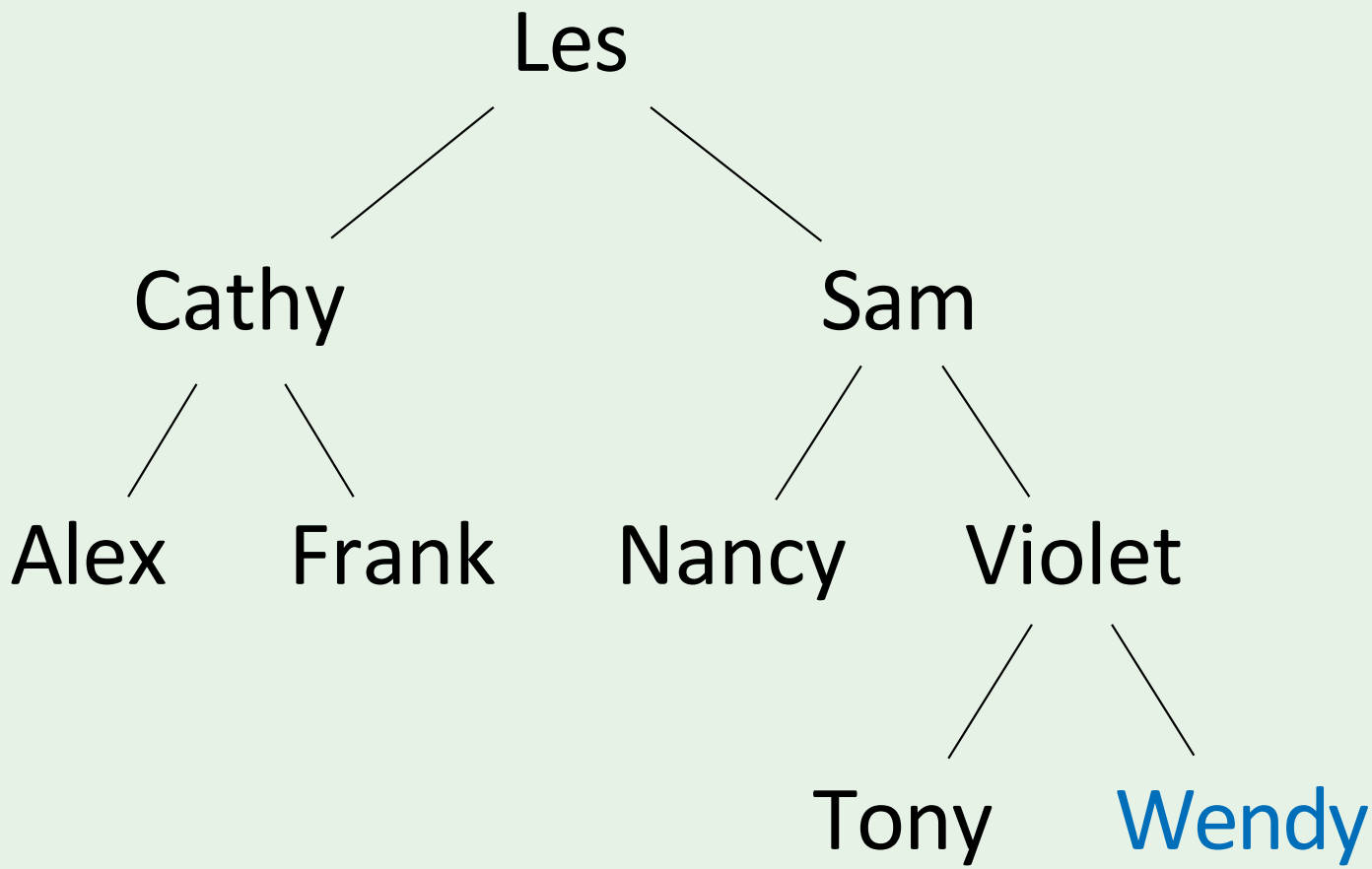

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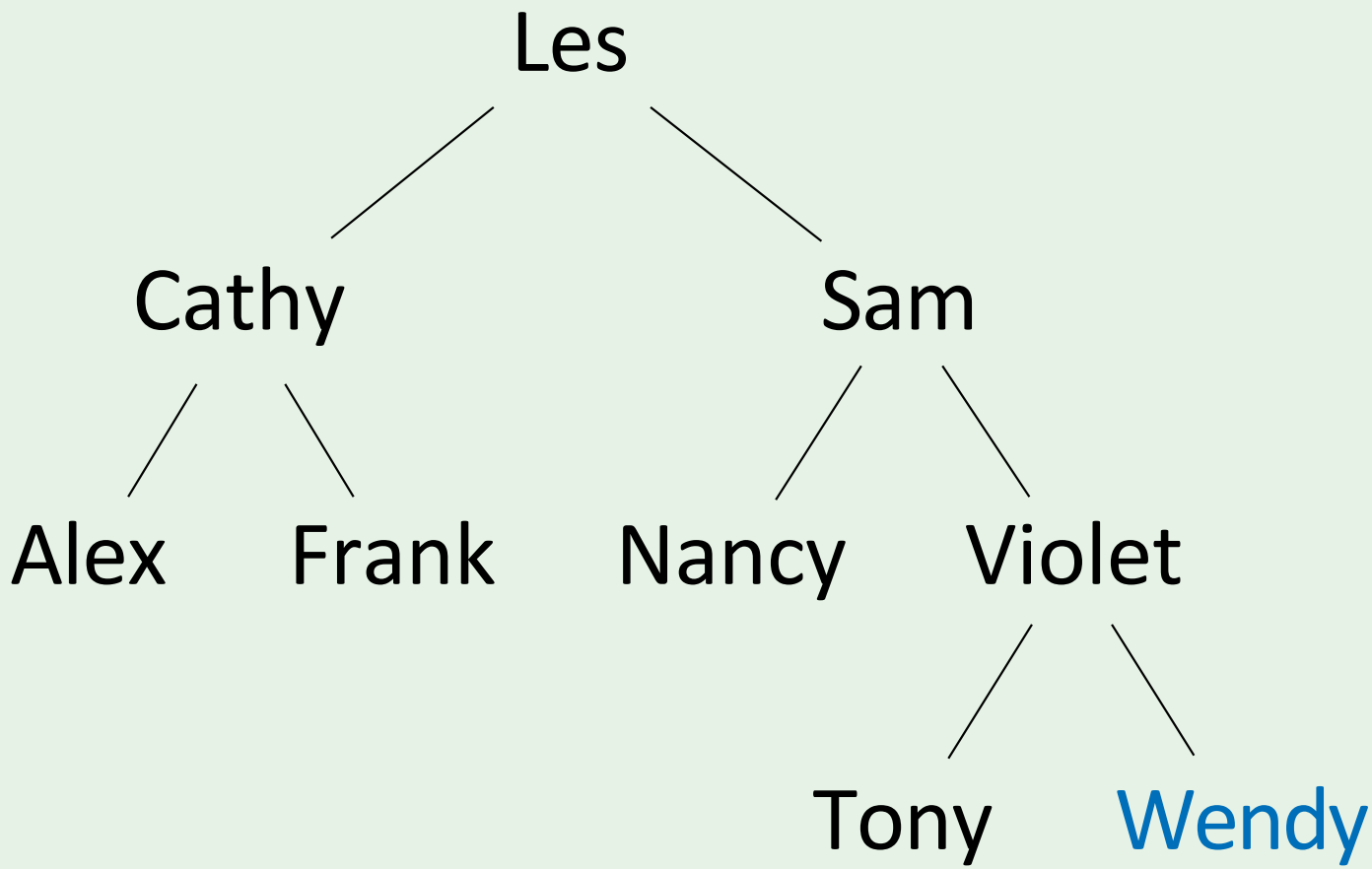

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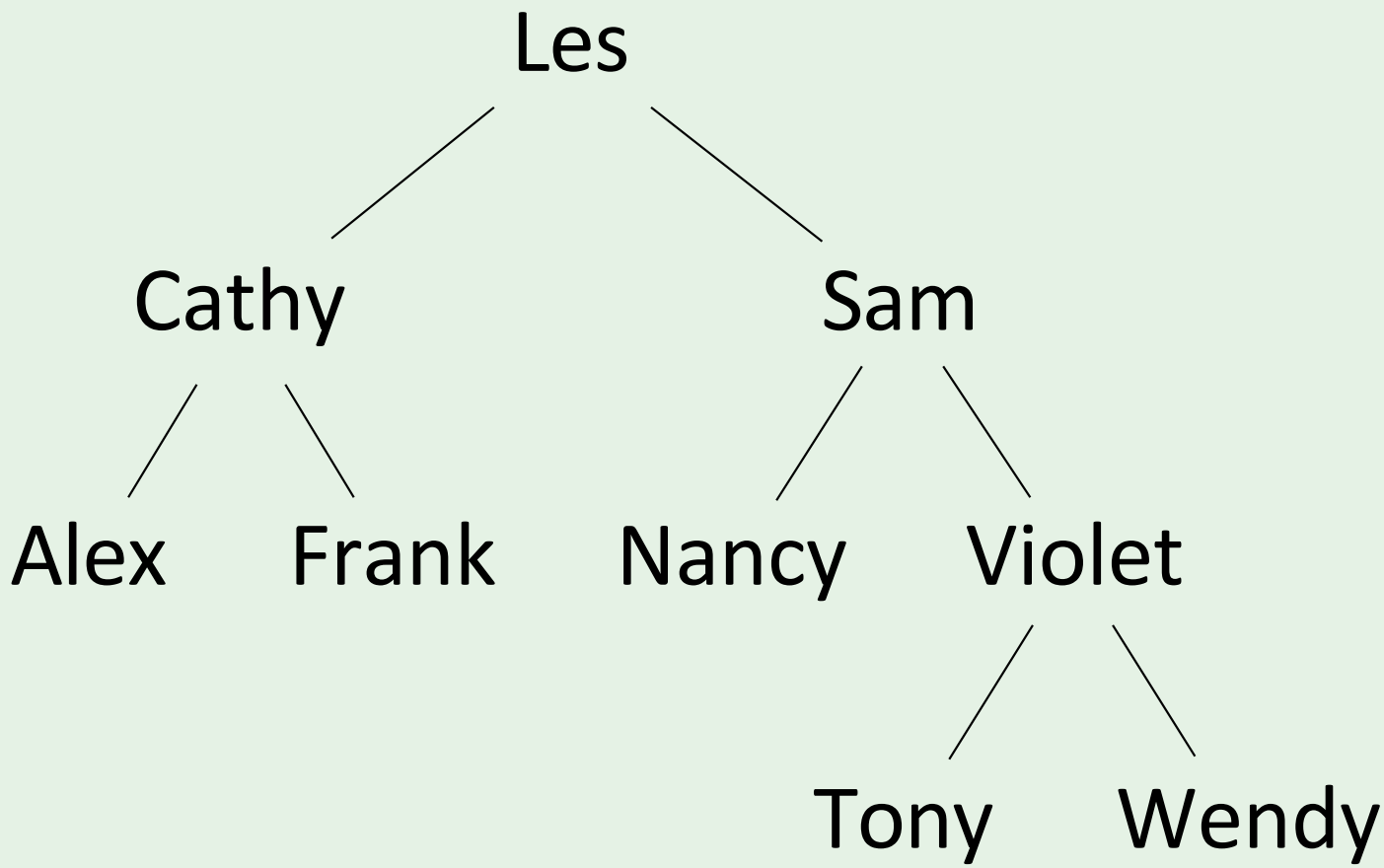


Output: Les Cathy Sam Alex Frank Nancy Violet Tony Wendy

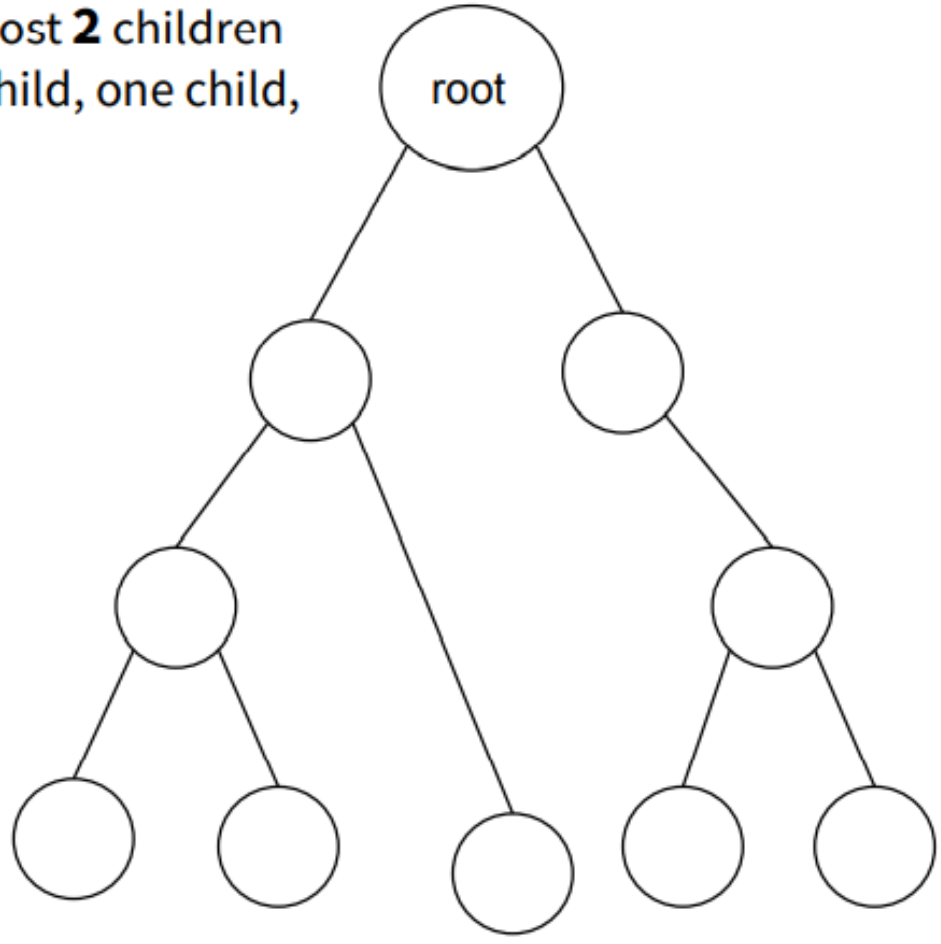
Summary

- Trees are used for lots of different things.

Other properties

Binary Trees

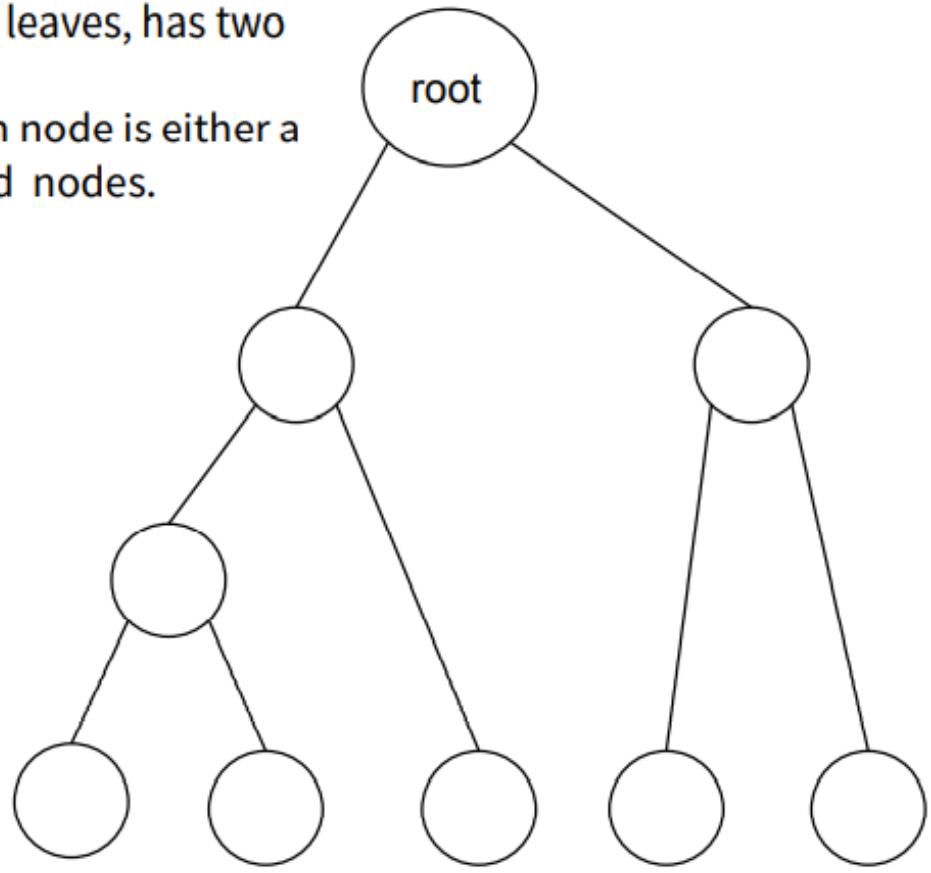
Each node can have at most **2** children
Each node can have no child, one child, or two children



Other properties

Full Binary Trees

Every node, other than the leaves, has two children
A binary tree T is full if each node is either a leaf or has exactly two child nodes.



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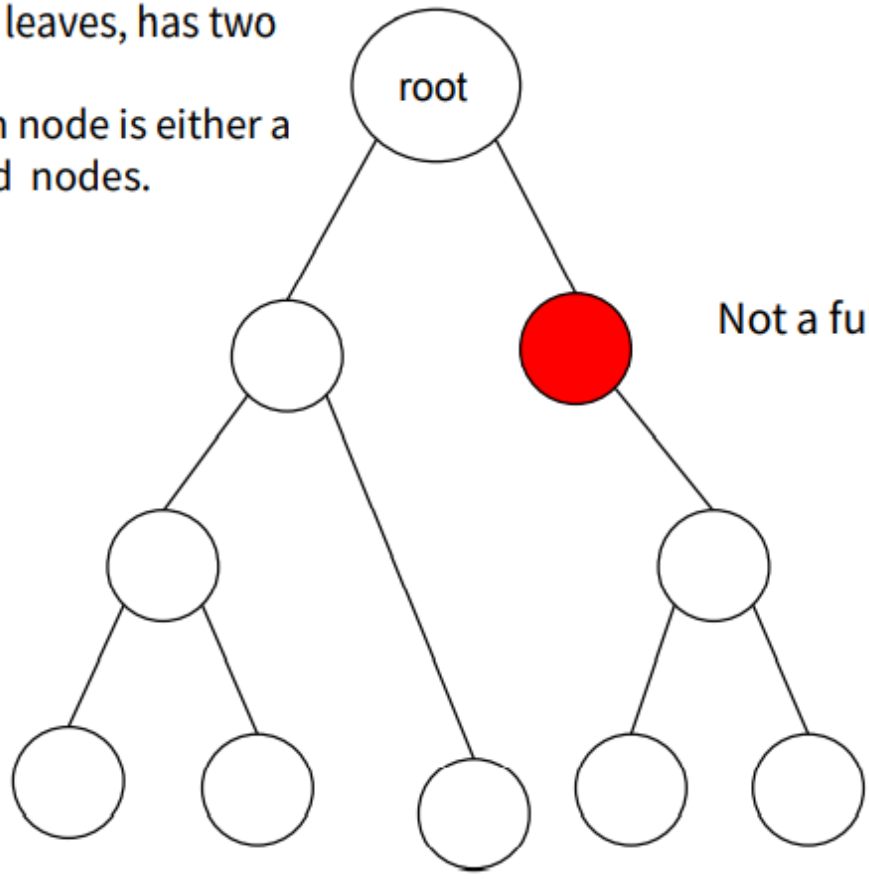

Not a full binary tree!

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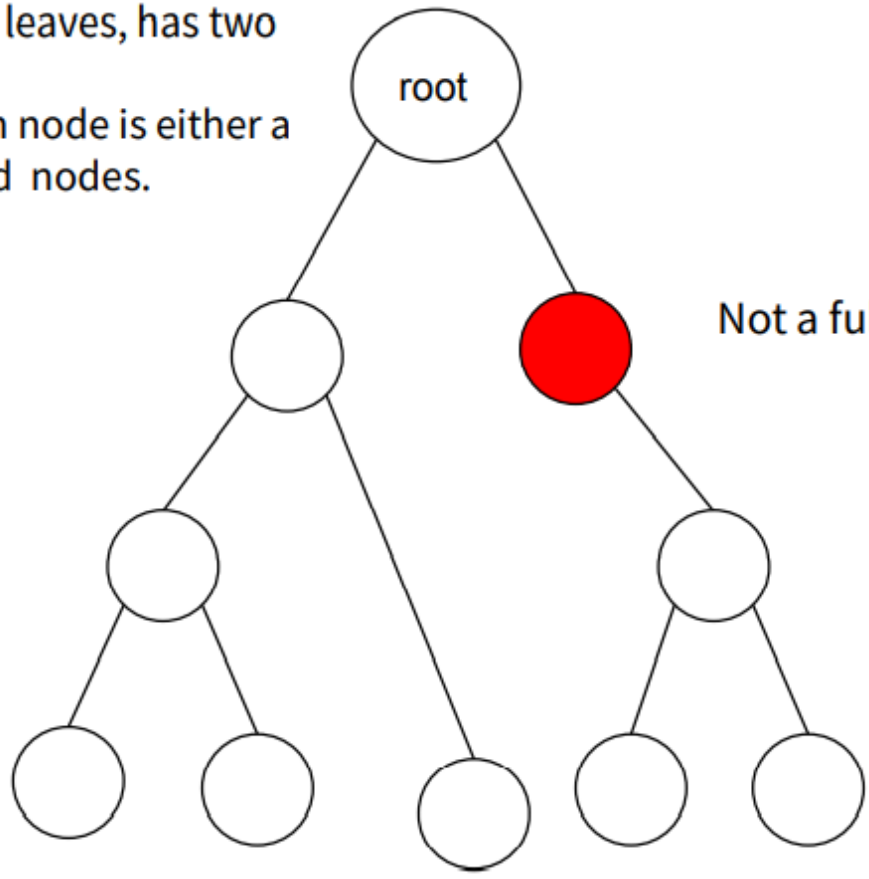
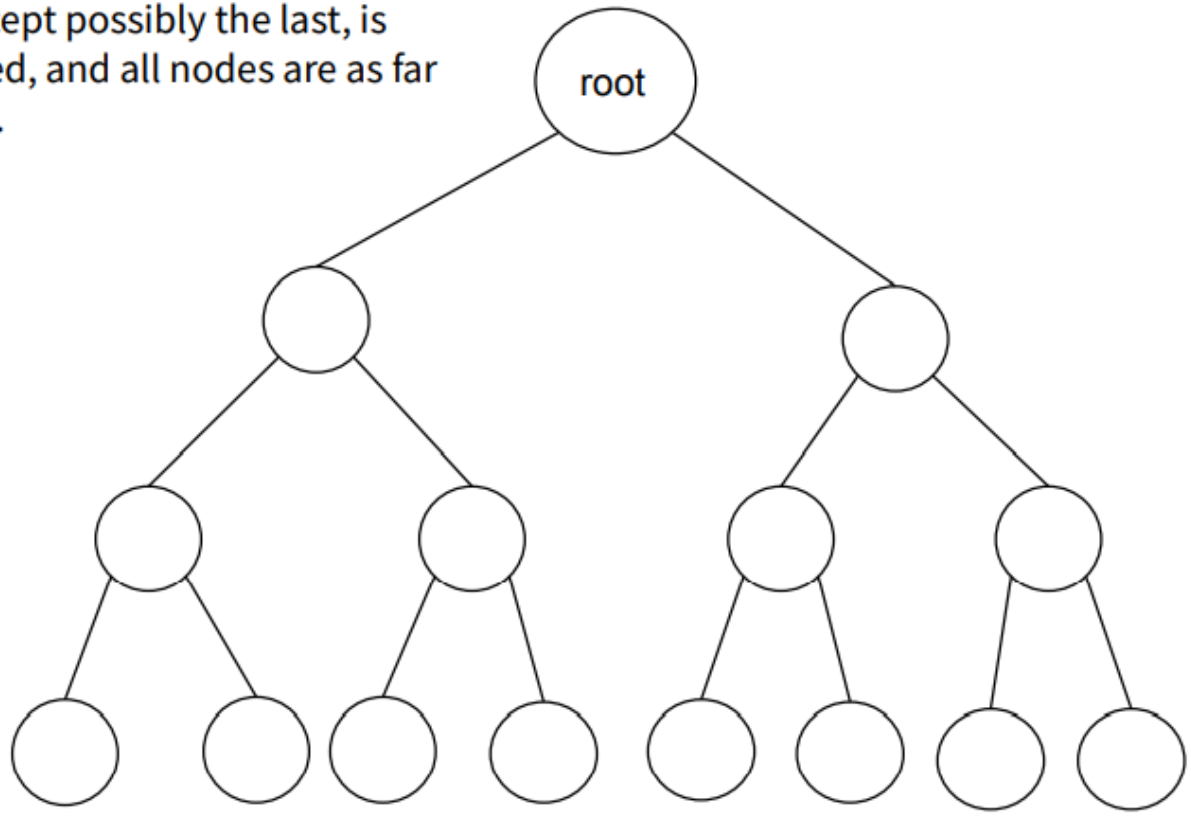


Not a full binary tree!

Other properties

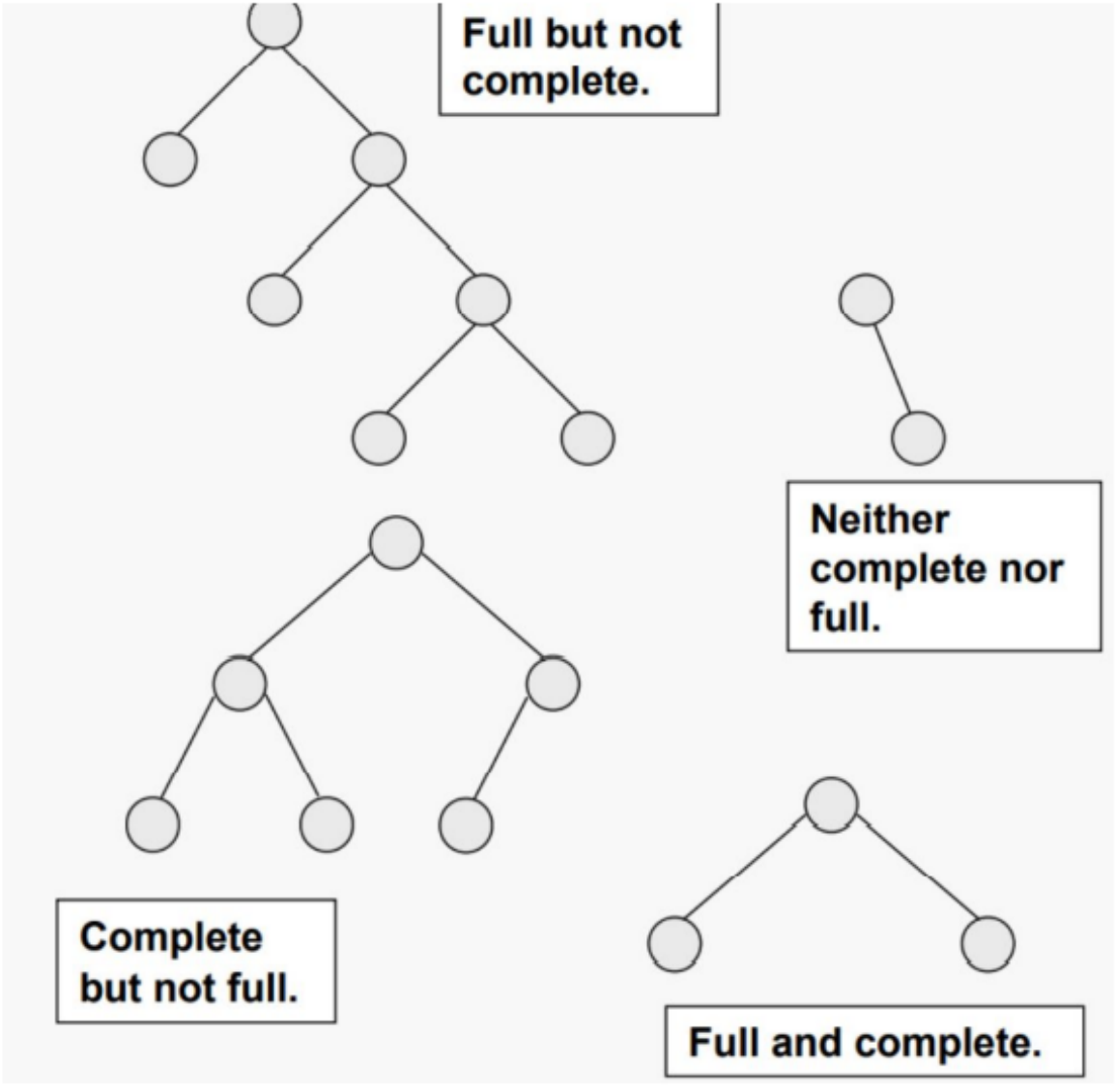
Complete Binary Trees

Every level, except possibly the last, is completely filled, and all nodes are as far left as possible.



Other properties

Complete Vs Full Binary Tree



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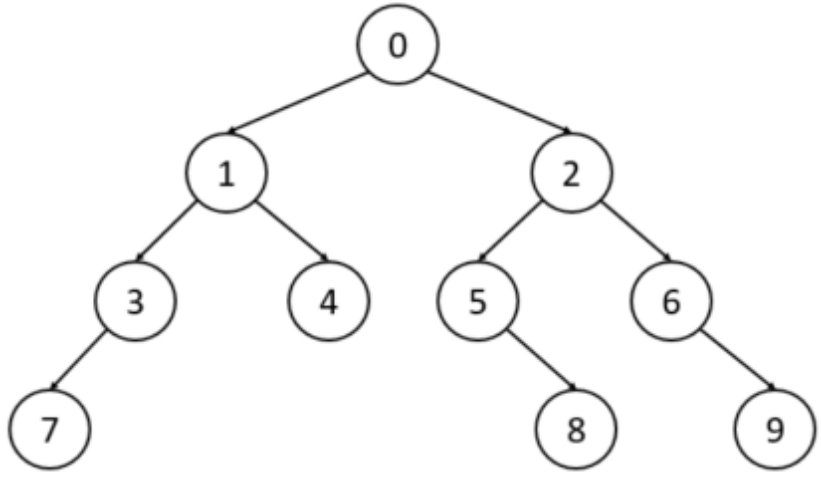
- When working with a tree, recursive algorithms are common.

Summary

- Trees are used for lots of different things.
- Trees have a key and children.
- Tree walks: DFS (pre-order, in-order, post-order) and BFS.
- When working with a tree, recursive algorithms are common.
- In Computer Science, trees grow down!

For Tree-traversal quiz



Depth First Search (BFS)

pre-order VLR (Visit-Left-Right)=

in-order LVR (Left-Visit-Right)=

post-order LRV (Left-Right-Visit)=

Breadth First Search (BFS)

level-order =