

Depth First Search

About this lecture

- Depth First Search
 - DFS Tree and DFS Forest
- Parenthesis theorem


Depth First Search (DFS)

- An alternative algorithm to find all vertices reachable from a particular source vertex s
- Idea:
 - Explore a branch as far as possible before exploring another branch
- Easily done by recursion or stack

The DFS Algorithm

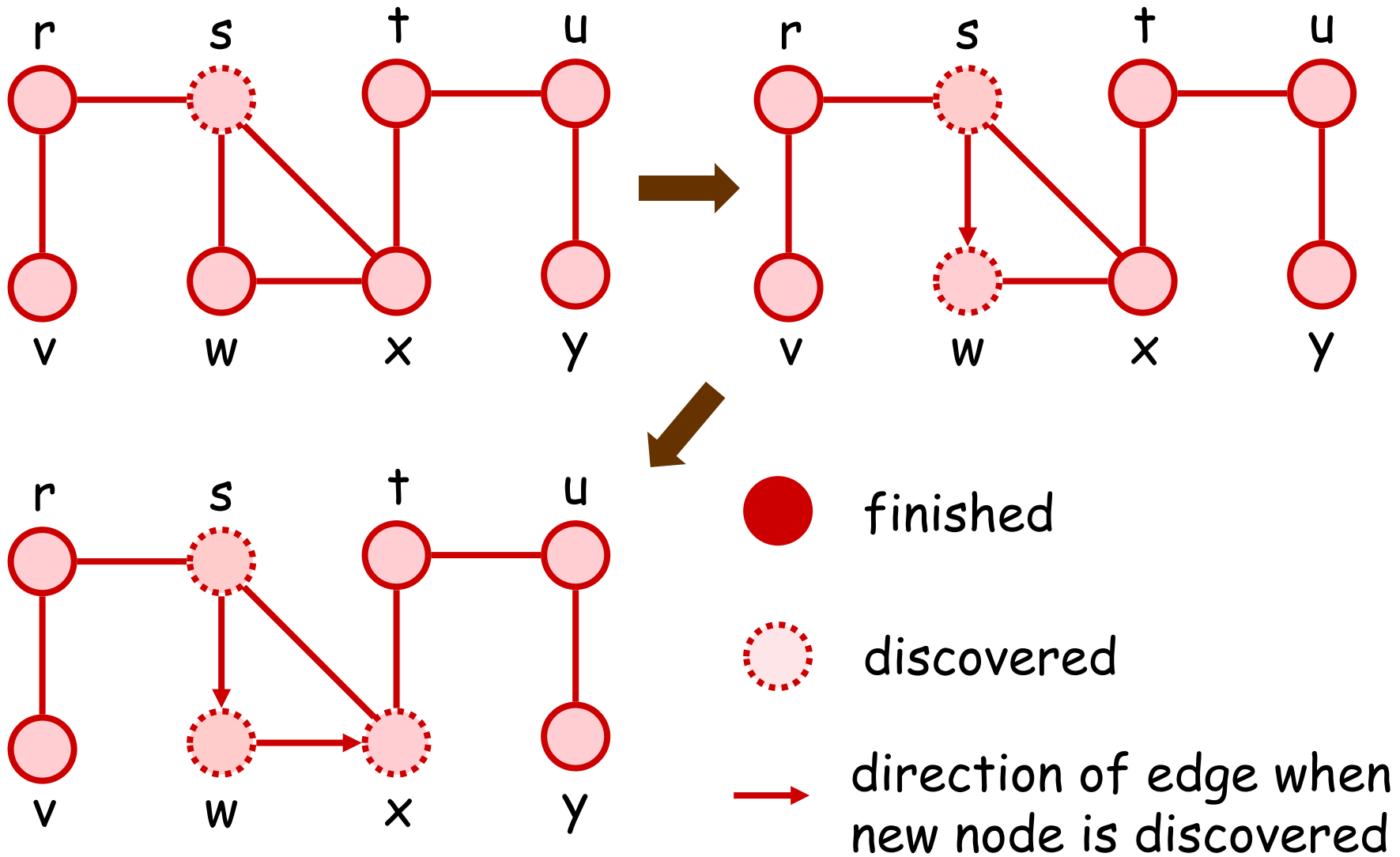
DFS(*u*)

```
{  Mark u as discovered ;  
    while (u has unvisited neighbor v)  
        DFS(v);  
    Mark u as finished ;  
}
```

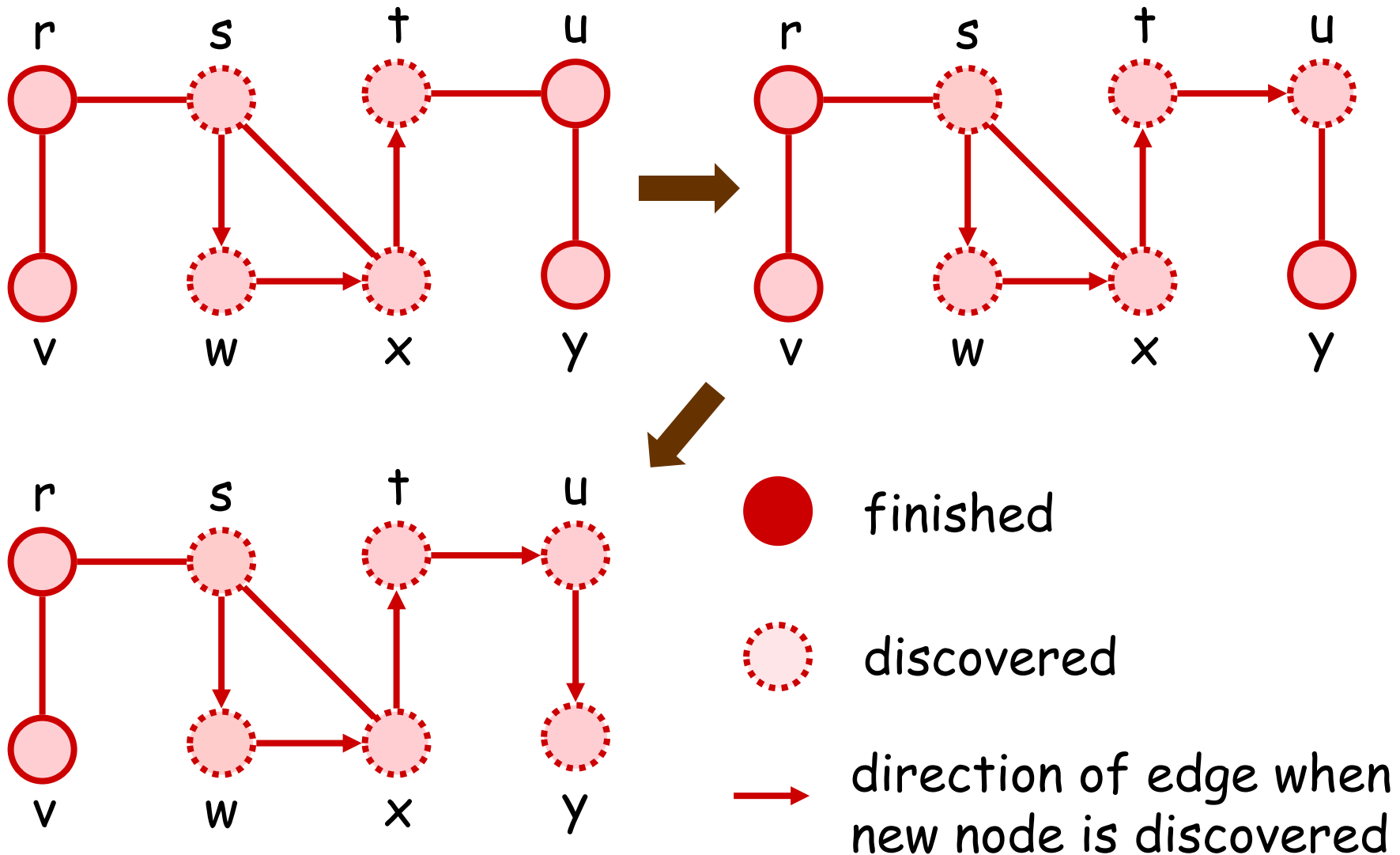


The while-loop explores a branch as far as possible before the next branch

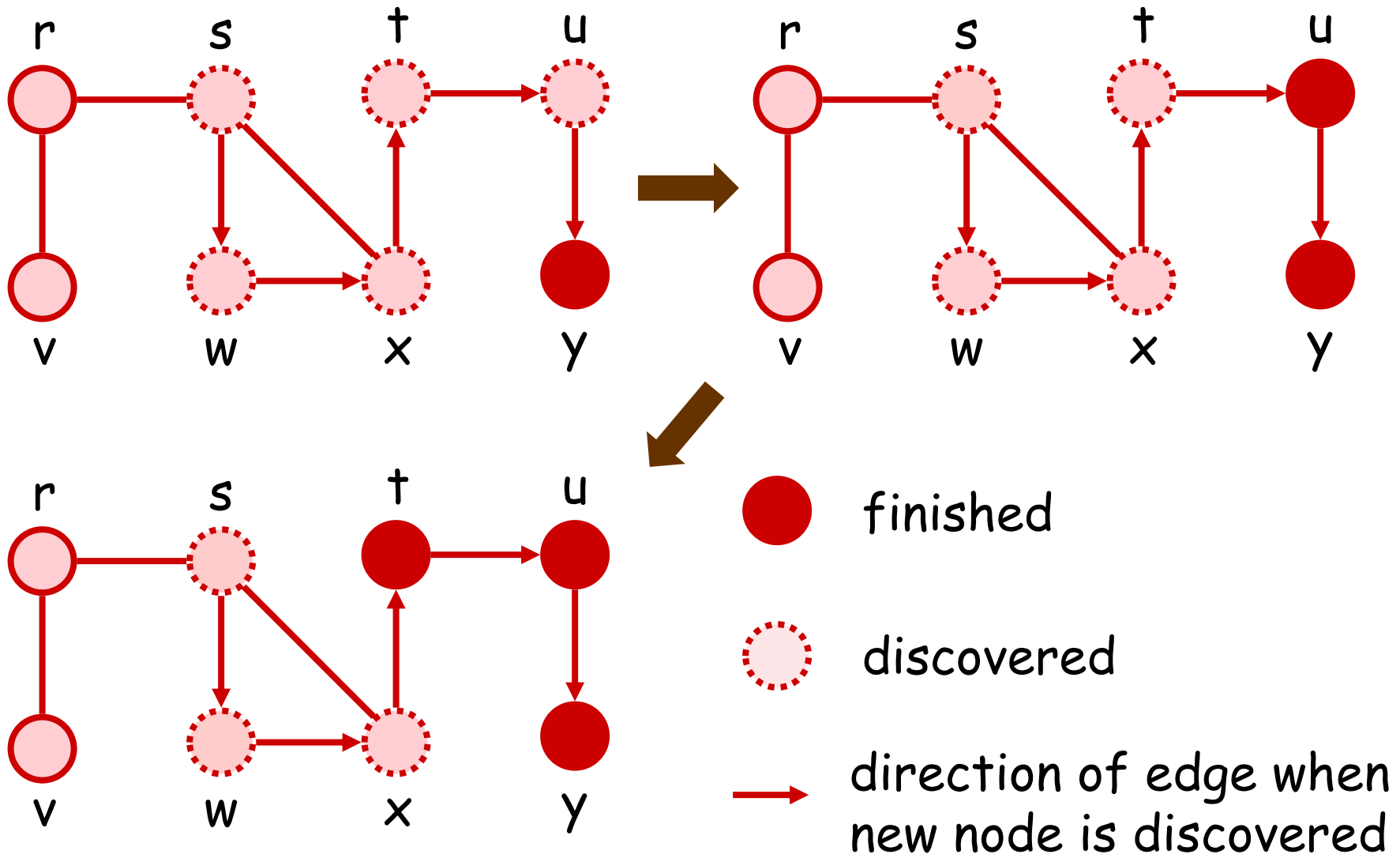
Example (s = source)



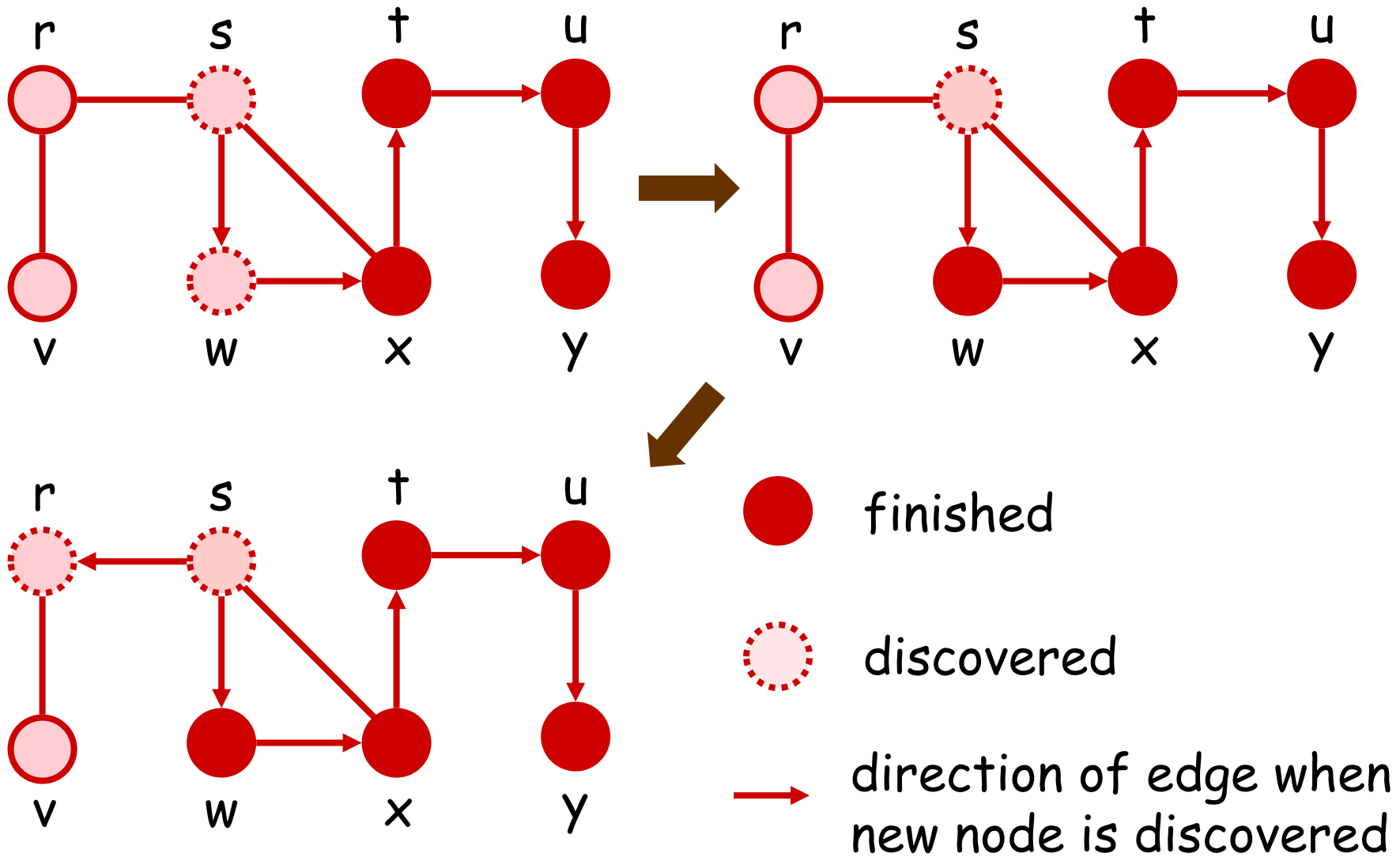
Example ($s = \text{source}$)



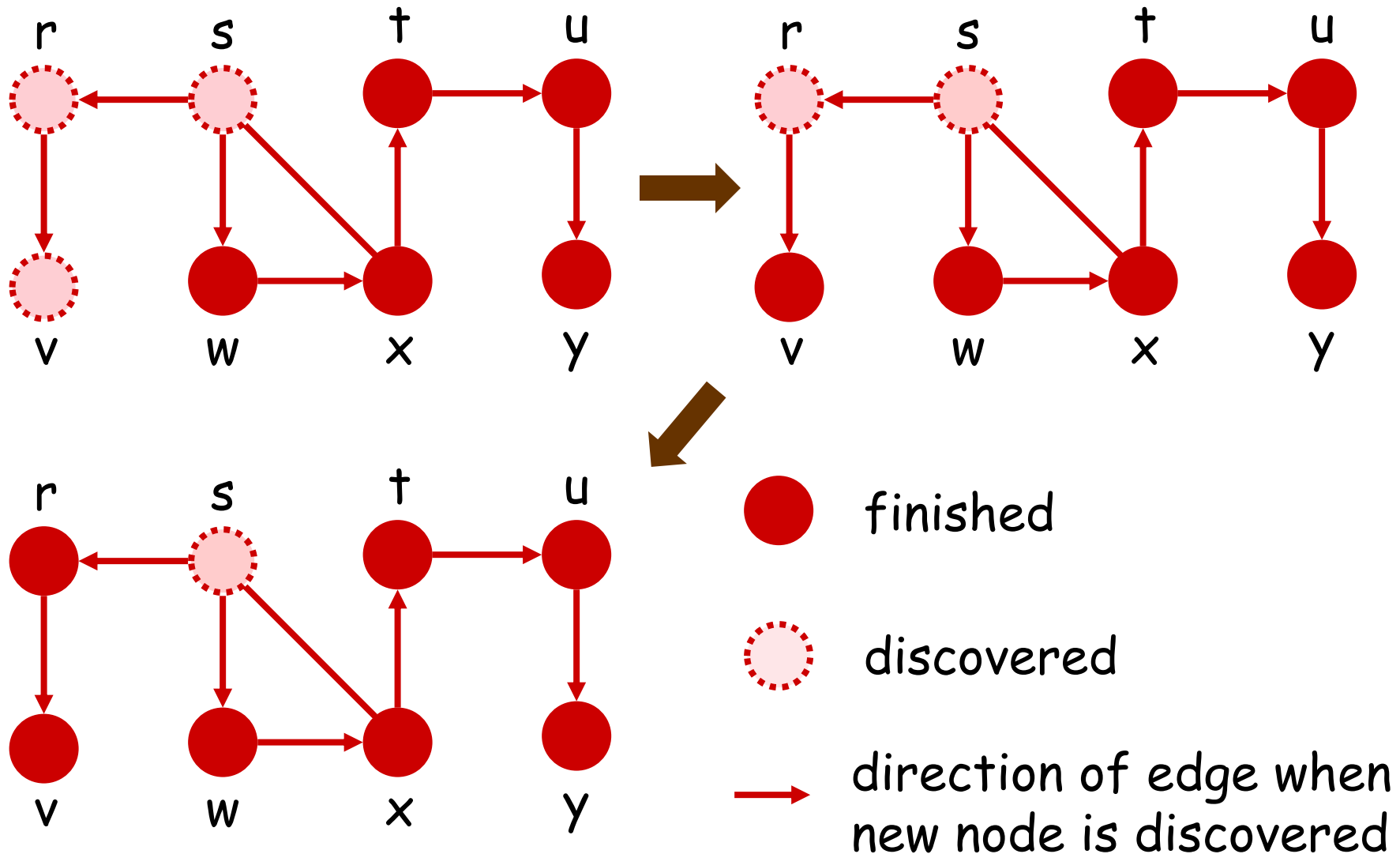
Example ($s = \text{source}$)



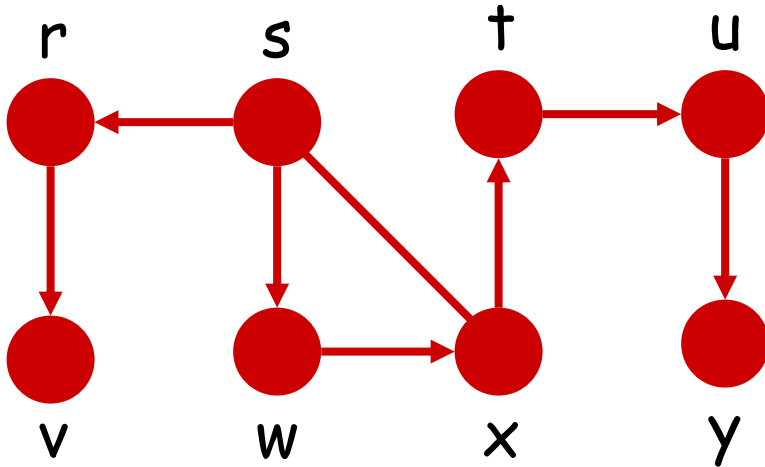
Example (s = source)



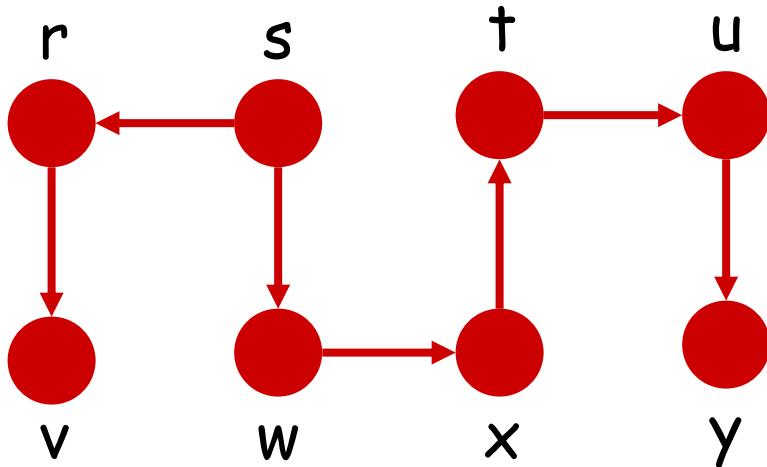
Example (s = source)



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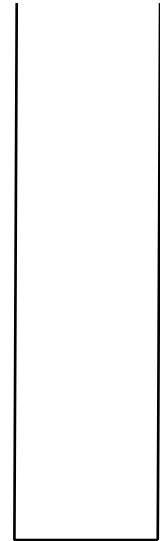
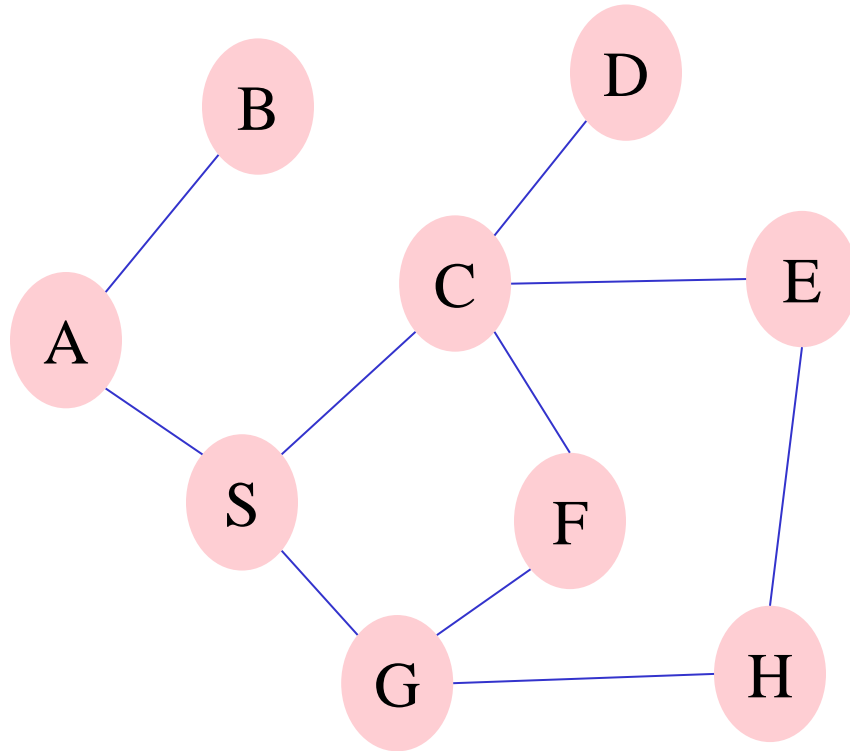
Done when s is discovered



The directed edges form a tree that contains all nodes *reachable* from s

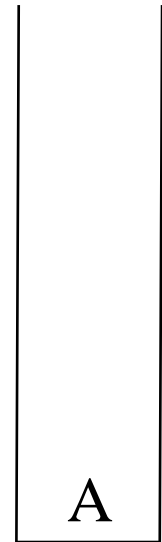
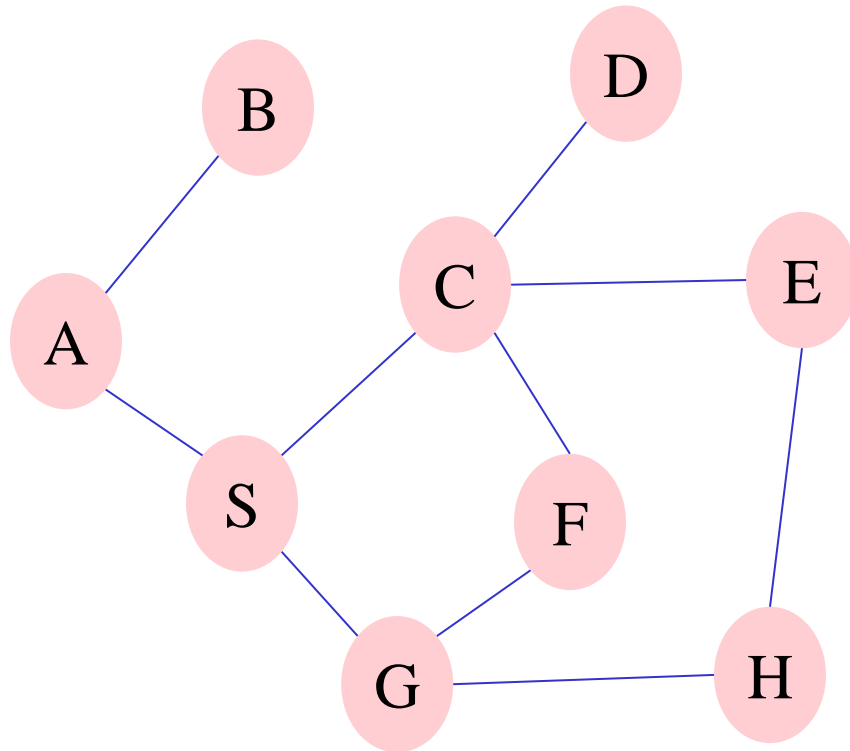
Called **DFS tree** of s

DFS using Stacks



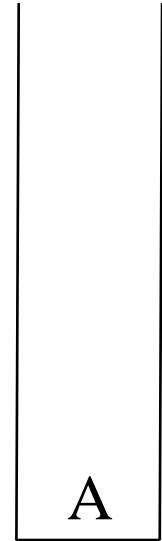
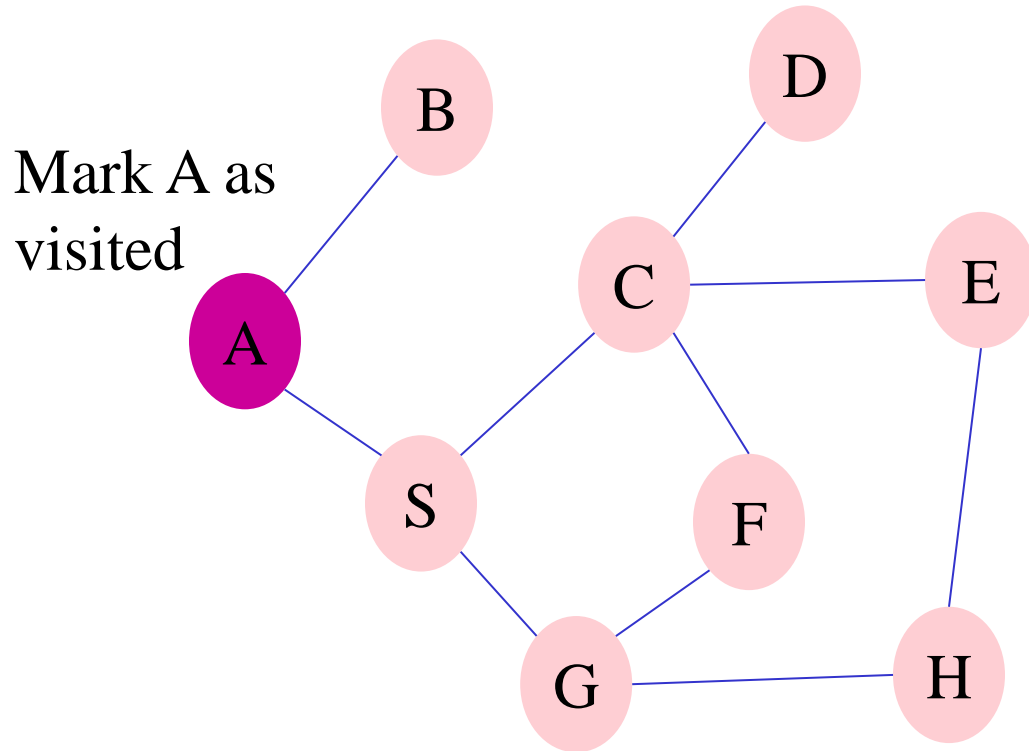
Output:

DFS using Stacks



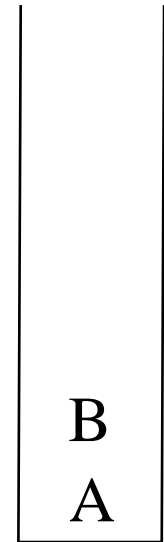
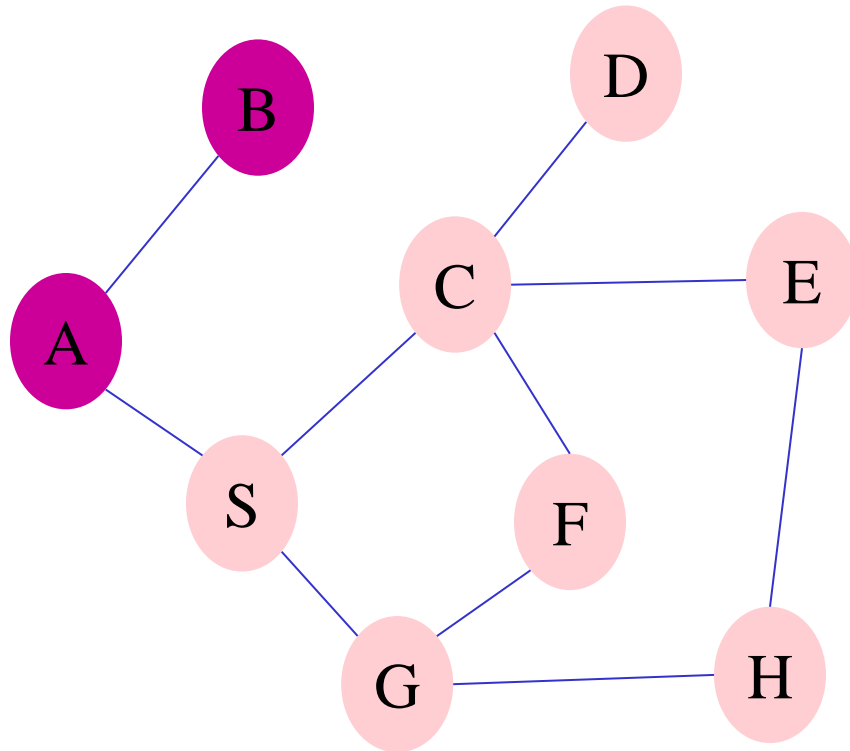
Output: A

DFS using Stacks



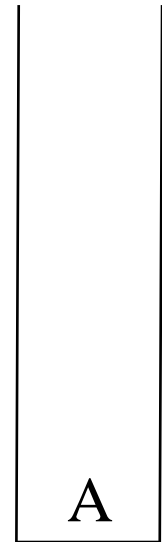
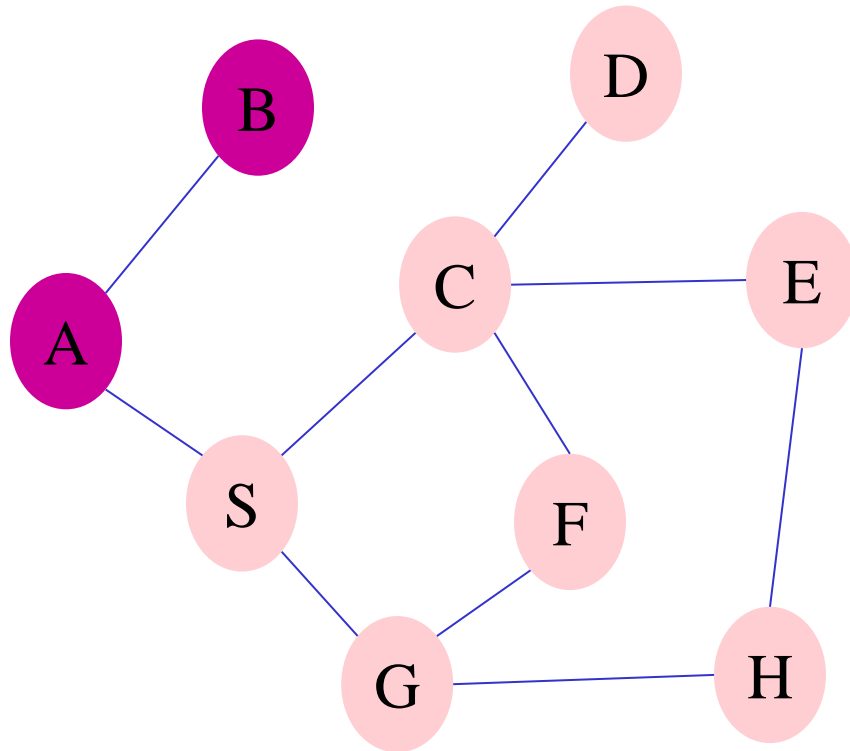
Output: A

DFS using Stacks



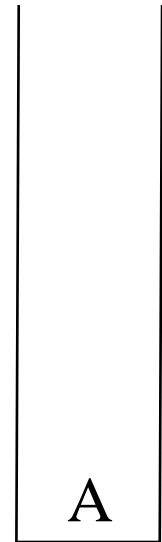
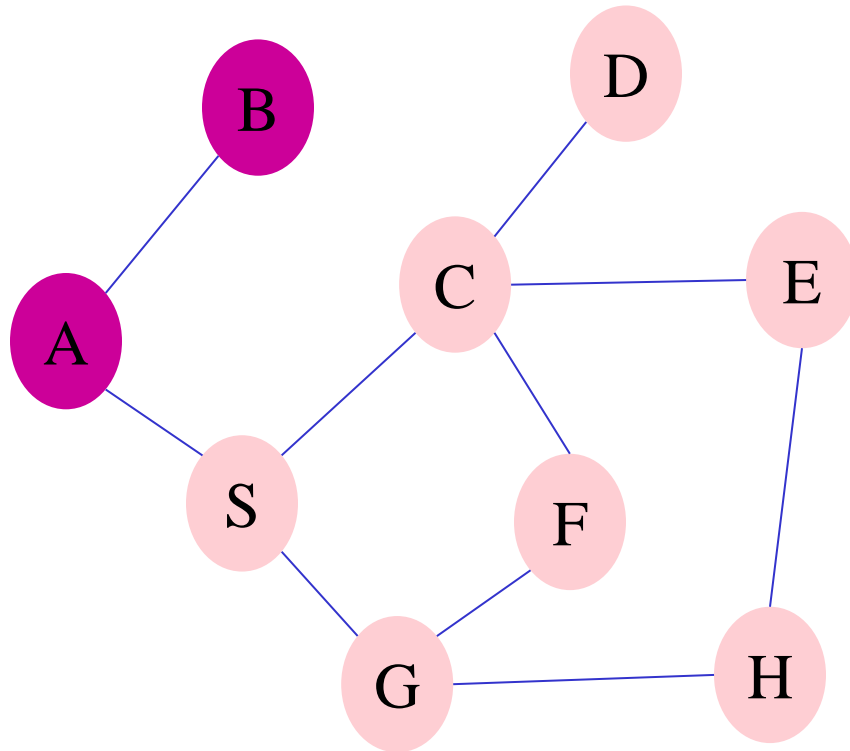
Output: A B

DFS using Stacks



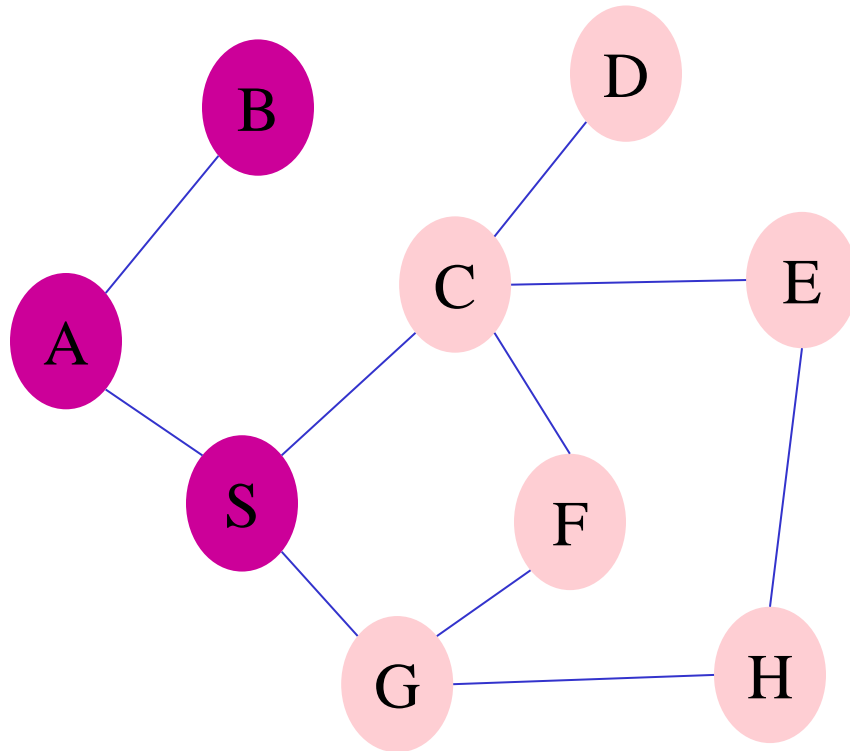
Output: A B S

DFS using Stacks



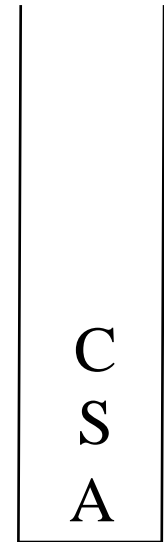
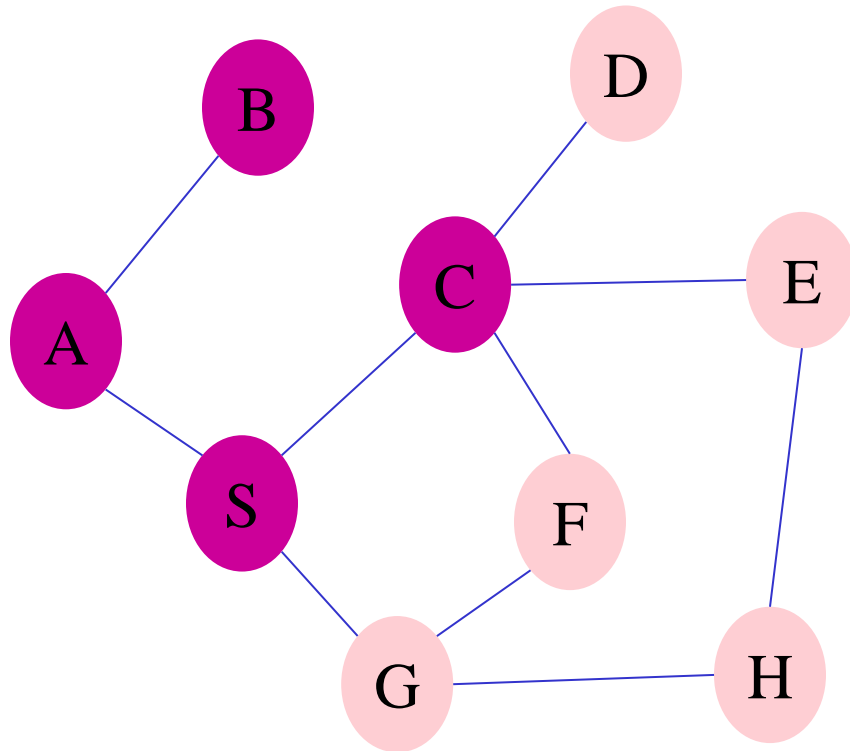
Output: A B S

DFS using Stacks



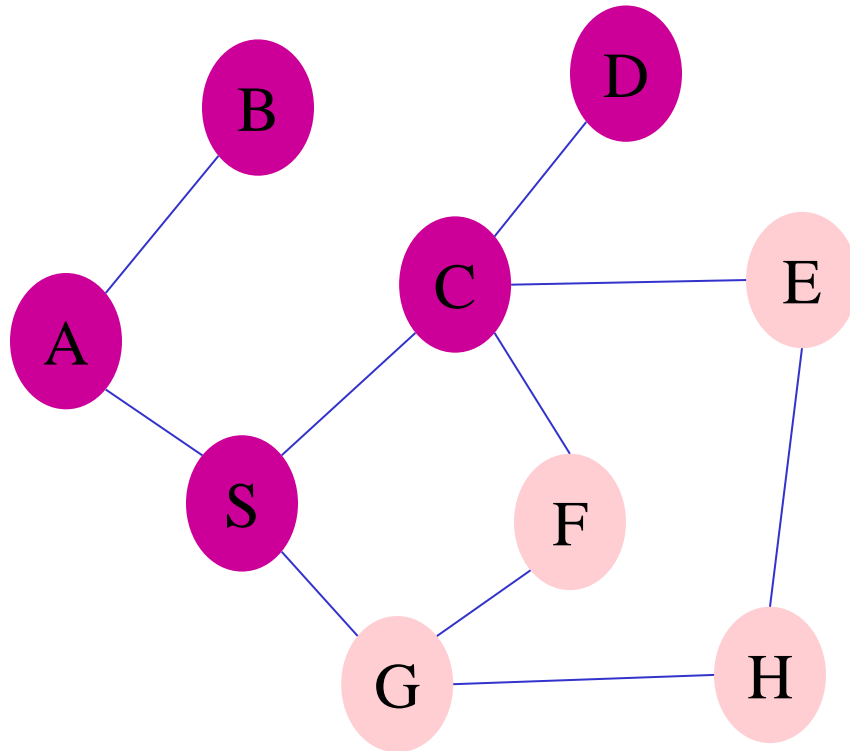
Output: A B S

DFS using Stacks



Output: A B S C

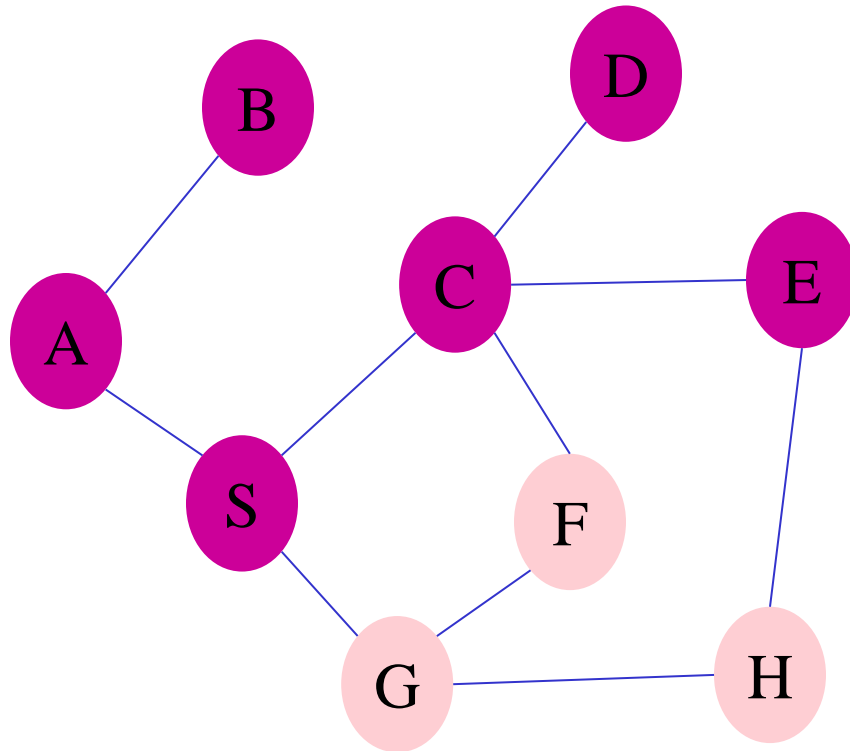
DFS using Stacks



D
C
S
A

Output: A B S C D

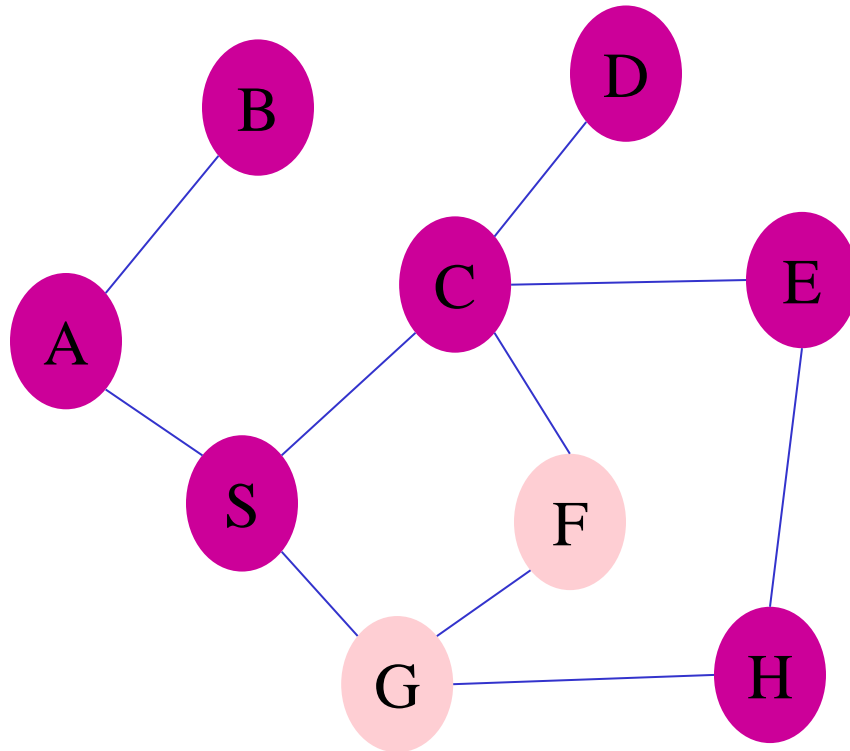
DFS using Stacks



E
C
S
A

Output: A B S C D E

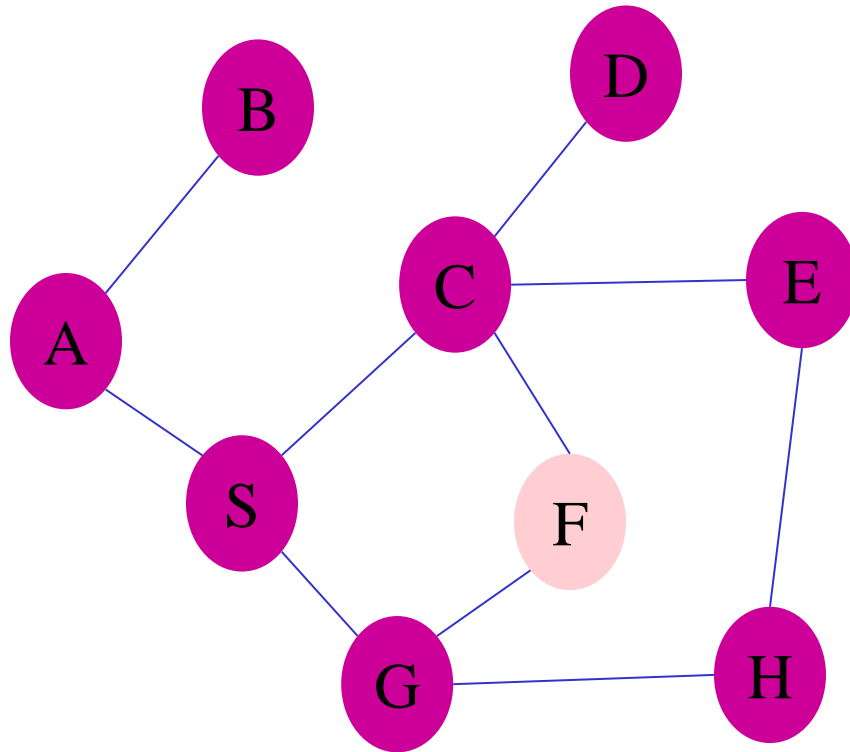
DFS using Stacks



H
E
C
S
A

Output: A B S C D E H

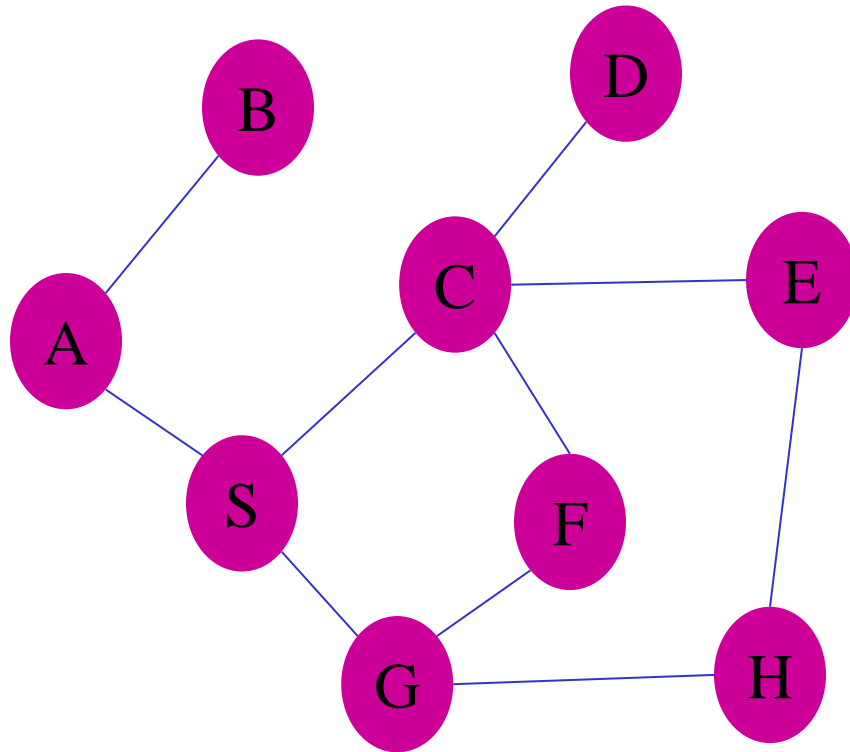
DFS using Stacks



G
H
E
C
S
A

Output: A B S C D E H G

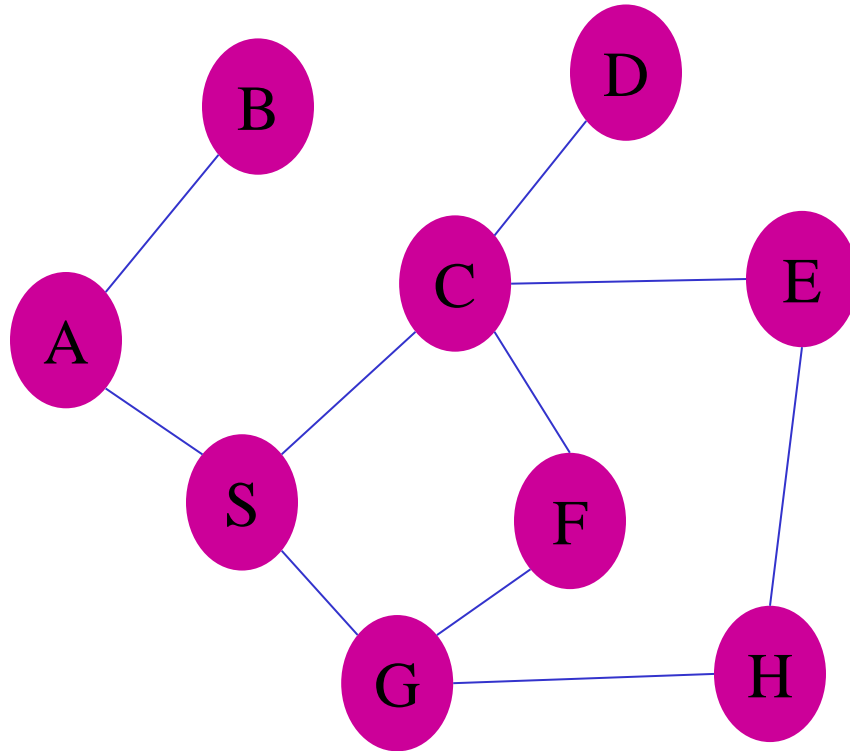
DFS using Stacks



F
G
H
E
C
S
A

Output: A B S C D E H G F

DFS using Stacks

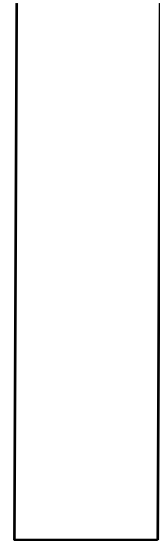
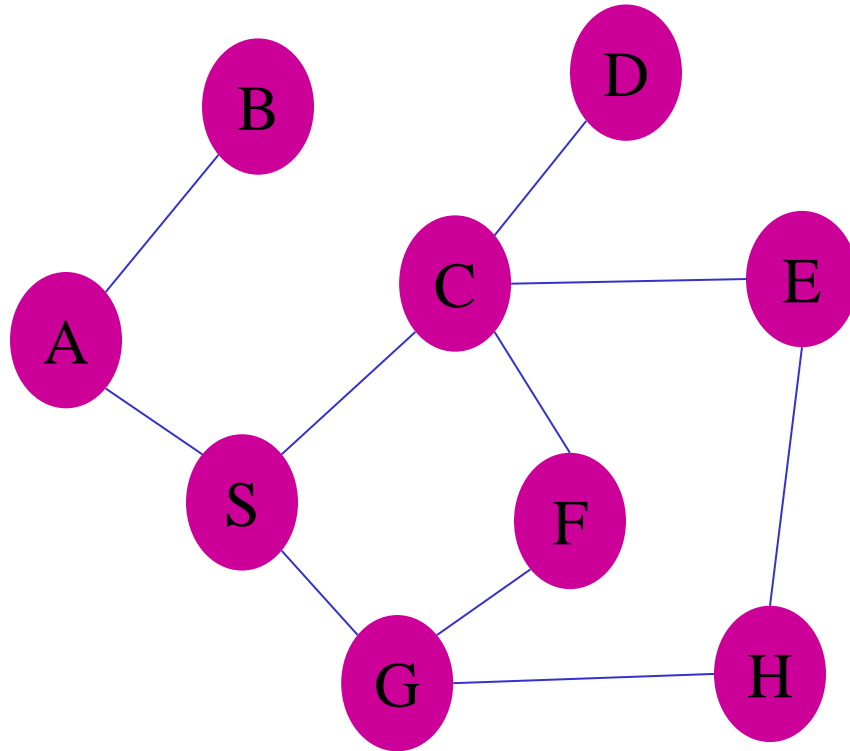


Pop every node with no unvisited child

F
G
H
E
C
S
A

Output: A B S C D E H G F

DFS using Stacks



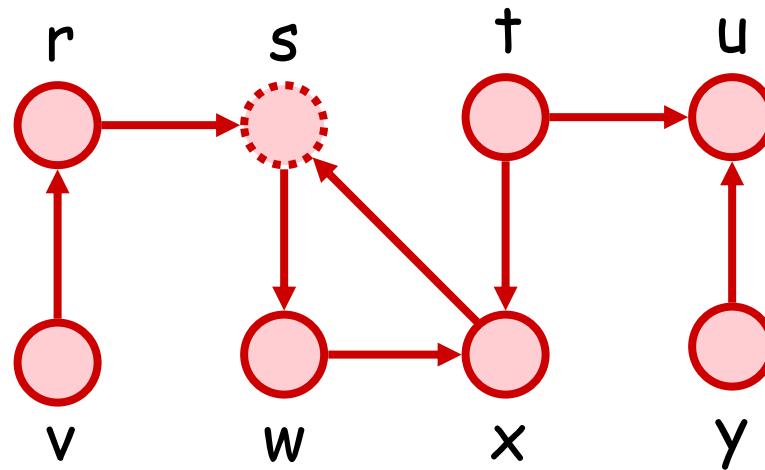
Output: A B S C D E H G F

Generalization

- Just like BFS, DFS may not visit all the vertices of the input graph G , because :
 - G may be disconnected
 - G may be directed, and there is no directed path from s to some vertex
- In most application of DFS (as a subroutine) , once DFS tree of s is obtained, we will continue to apply DFS algorithm on any unvisited vertices ...

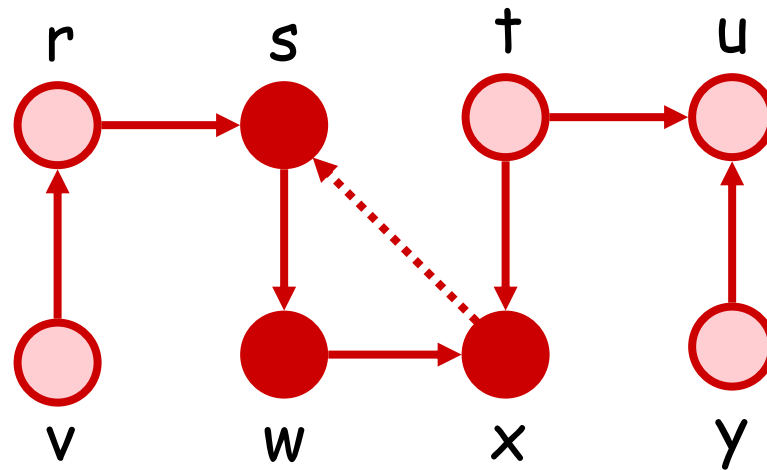
Generalization (Example)

Suppose the input graph is directed



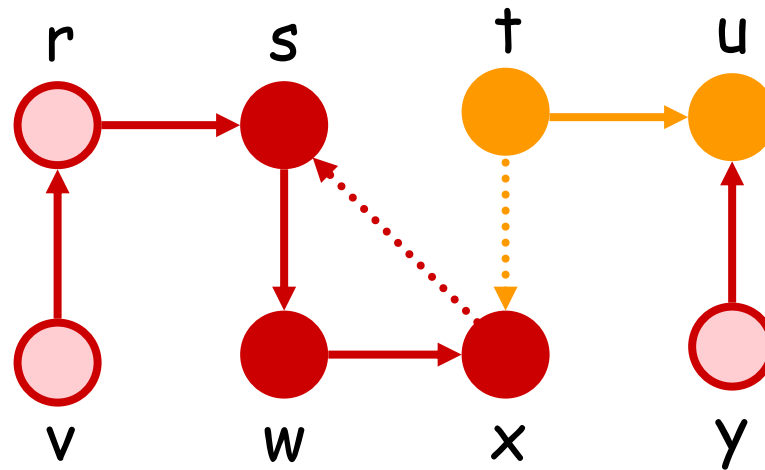
Generalization (Example)

1. After applying DFS on s



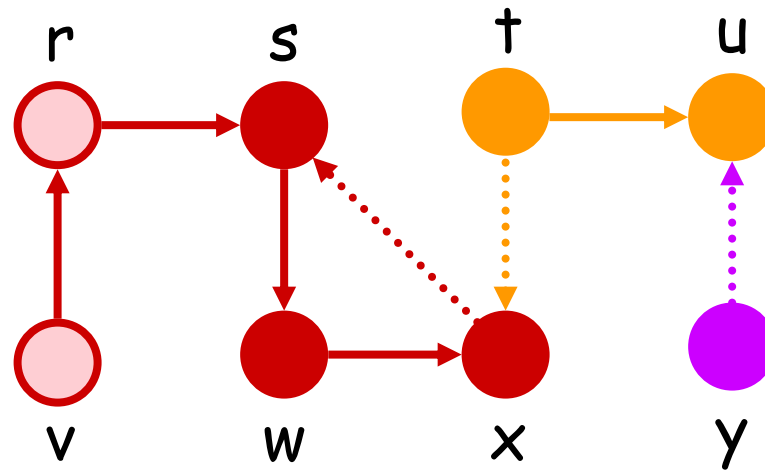
Generalization (Example)

2. Then, after applying DFS on t



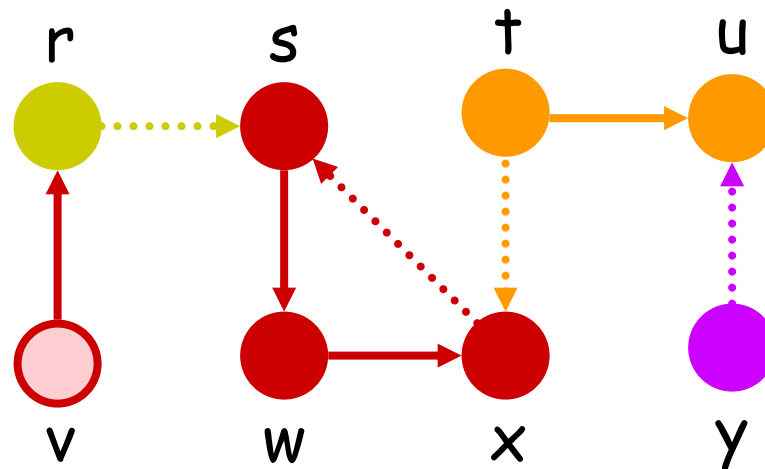
Generalization (Example)

3. Then, after applying DFS on y



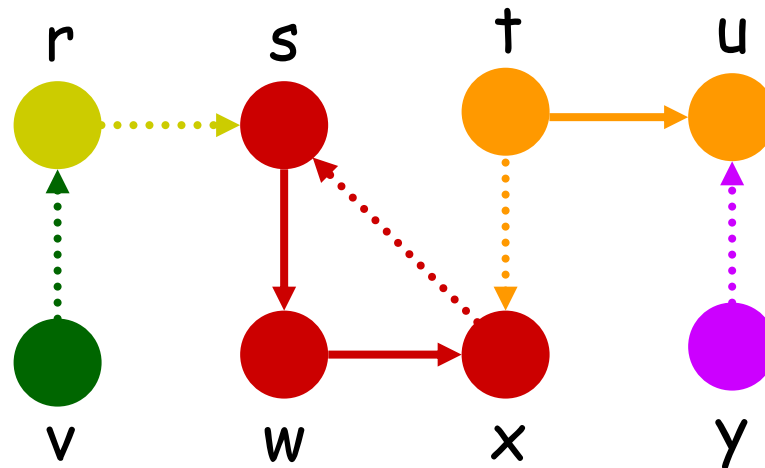
Generalization (Example)

4. Then, after applying DFS on r



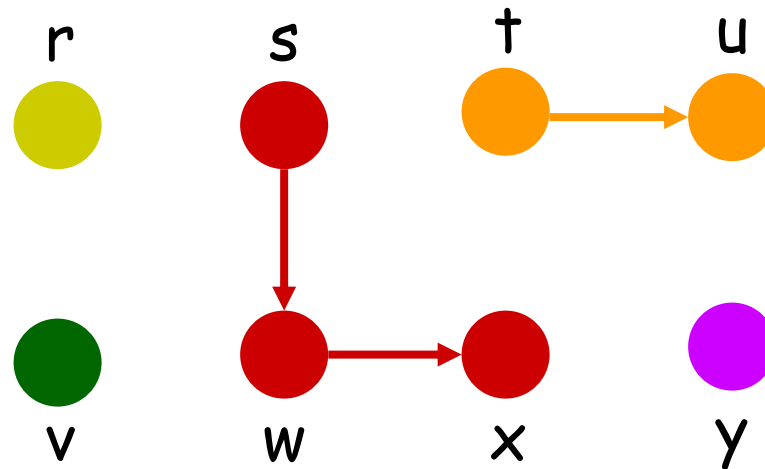
Generalization (Example)

5. Then, after applying DFS on v



Generalization (Example)

Result : a collection of rooted trees
called **DFS forest**



Performance

- Since no vertex is discovered twice, and each edge is visited at most twice (why?)
→ Total time: $O(|V|+|E|)$
- As mentioned, apart from recursion, we can also perform DFS using a LIFO stack (Do you know how?)