

# BINARY SEARCH TREES

Basic operations

# Learning Objectives

- Implement basic operations on Binary Search Trees.
- Understand some of the difficulties with making updates.

# Outline

- 1 Find
- 2 Next Element
- 3 Search
- 4 Insert
- 5 Delete

# Find

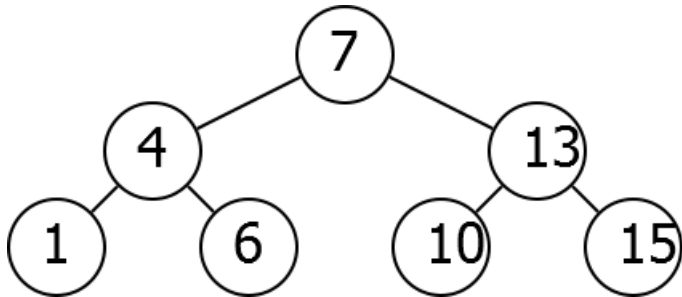
## Find

**Input:** Key  $k$ , Root  $R$

**Output:** The node in the tree of  $R$  with key  $k$

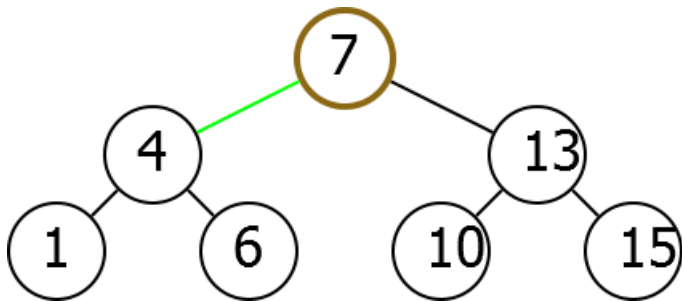
# Idea

Find(6)



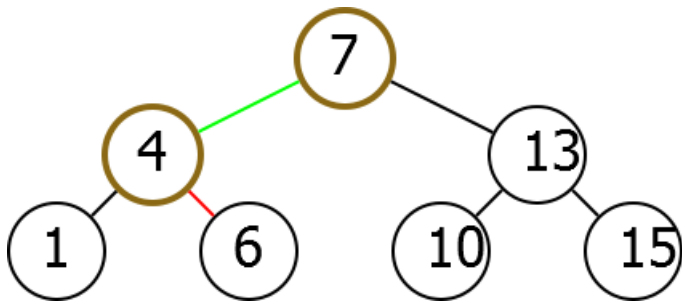
# Idea

Find(6)



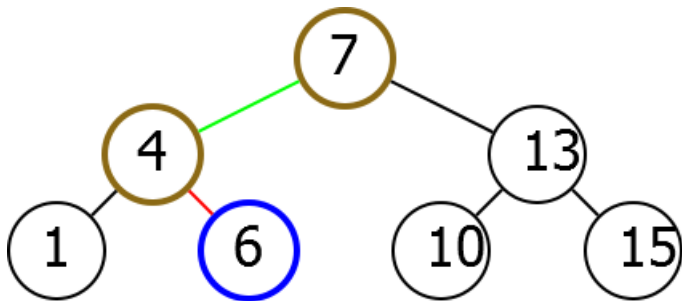
# Idea

Find(6)



# Idea

Find(6)





# Algorithm

Find( $k, R$ )

if  $R.\text{Key} = k$ :

    return  $R$

else if  $R.\text{Key} > k$ :

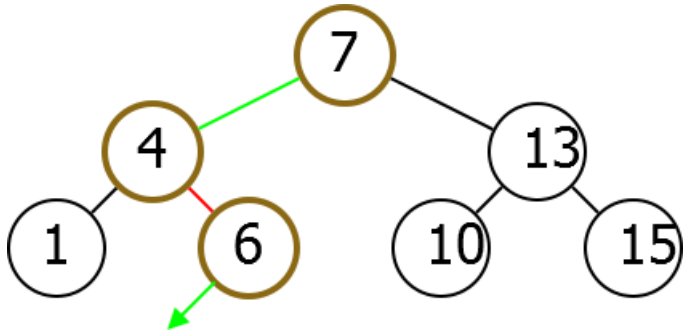
    return Find( $k, R.\text{Left}$ )

else if  $R.\text{Key} < k$ :

    return Find( $k, R.\text{Right}$ )

# Missing Key

Run Find(5).



Key not in tree. Did find point where it should be.

# Missing Key

If you stop before reaching a null pointer, you find the place in the tree where  $k$  would fit.

# Modification

Find (modified)

```
else if  $R.\text{Key} > k$  :  
    if  $R.\text{Left} \neq \text{null}$ :  
        return Find( $k, R.\text{Left}$ )  
    return  $R$ 
```

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# Adjacent Elements

Given a node  $N$  in a Binary Search Tree, would like to find adjacent elements.

# Next

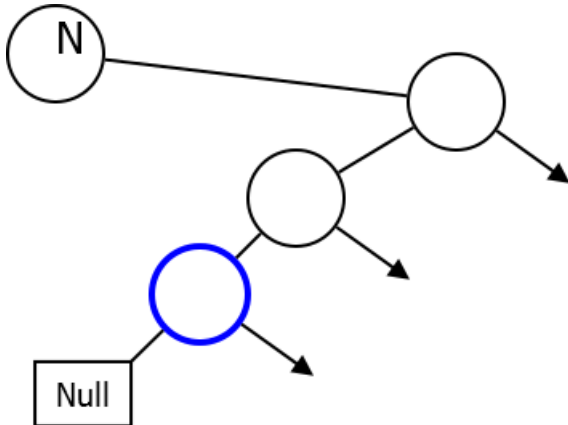
## Next

**Input:** Node  $N$

**Output:** The node in the tree with the next largest key.

# Case I

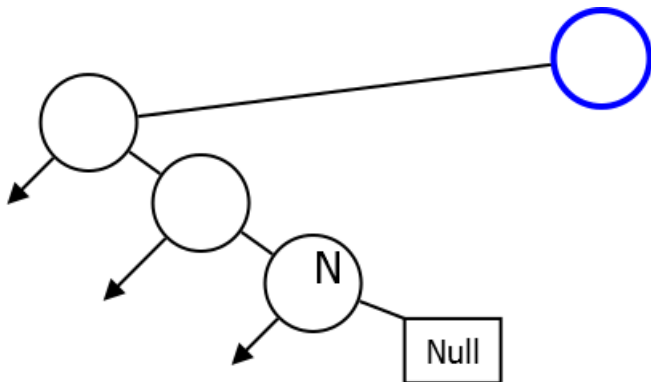
If you have right child.





## Case II

No right child.



# Next

**Next(*N*)**

```
if N.Right ≠ null:  
    return LeftDescendant(N.Right)  
else:  
    return RightAncestor(N)
```

# Left Descendant

LeftDescendant(*N*)

```
if N.Left = null
```

```
    return N
```

```
else:
```

```
    return LeftDescendant(N.Left)
```

# Right Ancestor

RightAncestor( $N$ )

```
if  $N.Key < N.Parent.Key$   
    return  $N.Parent$   
else:  
    return RightAncestor( $N.Parent$ )
```

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# Range Search

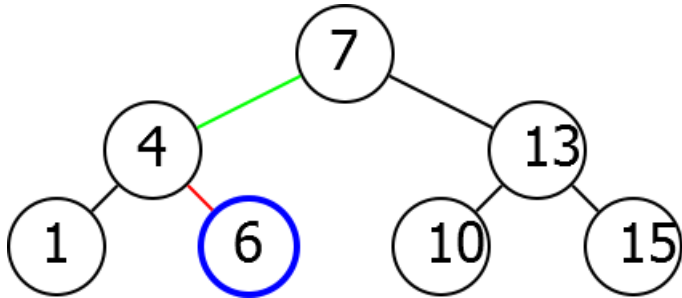
## Range Search

**Input:** Numbers  $x$ ,  $y$ , root  $R$

**Output:** A list of nodes with key between  $x$  and  $y$

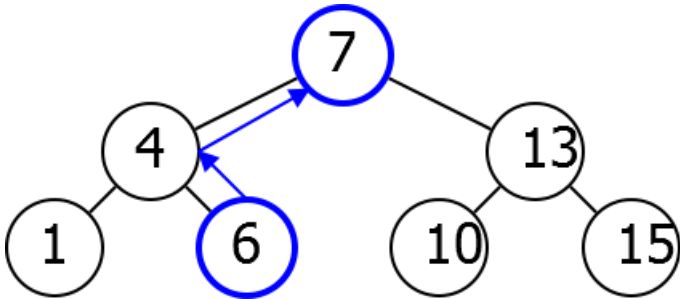
# Idea

RangeSearch(5, 12).



# Idea

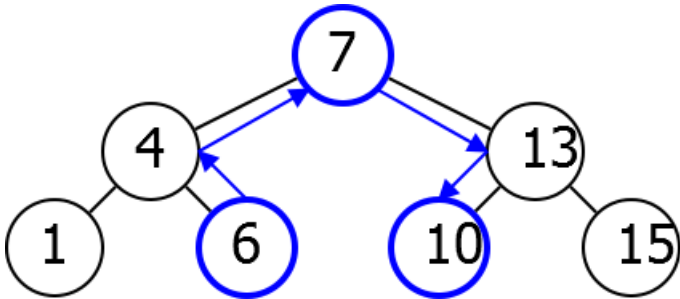
RangeSearch(5, 12).





# Idea

RangeSearch(5, 12).



# Implementation

RangeSearch( $x, y, R$ )

$L \leftarrow \emptyset$

$N \leftarrow \text{Find}(x, R)$

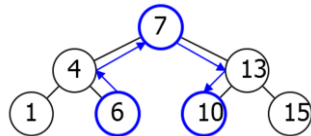
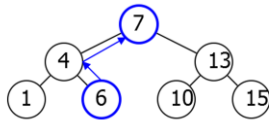
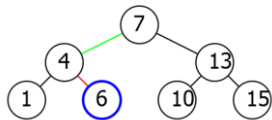
while  $N.\text{Key} \leq y$

    if  $N.\text{Key} \geq x$ :

$L \leftarrow L.\text{Append}(N)$

$N \leftarrow \text{Next}(N)$

return  $L$



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# Insert

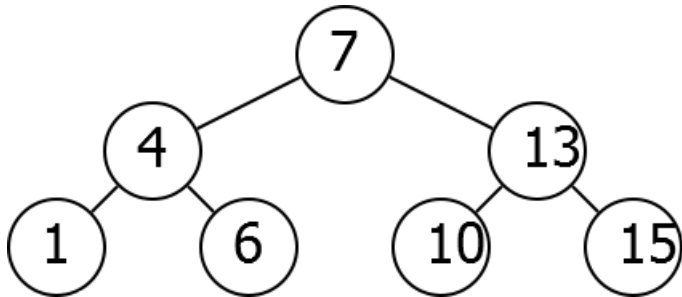
## Insert

**Input:** Key  $k$  and root  $R$

**Output:** Adds node with key  $k$  to the tree

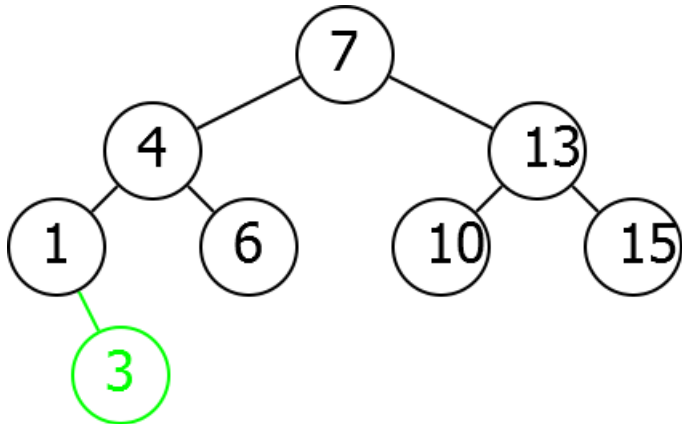
# Insert Idea

Insert(3)



# Insert Idea

Insert(3)



# Implementation

Insert( $k, R$ )

$P \leftarrow \text{Find}(k, R)$

Add new node with key  $k$  as child of  $P$

# Outline

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# Delete

## Delete

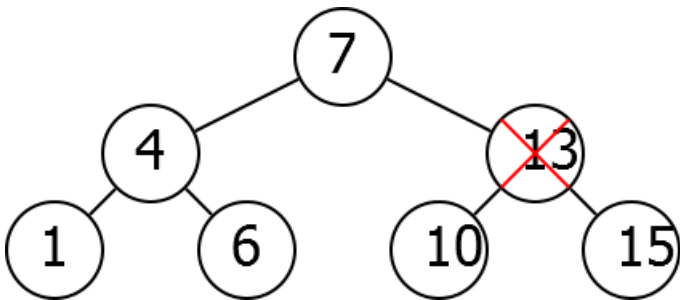
Input: Node  $N$

Output: Removes node  $N$  from the tree

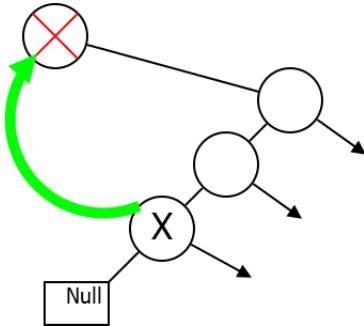
# Difficulty

Cannot simply remove.

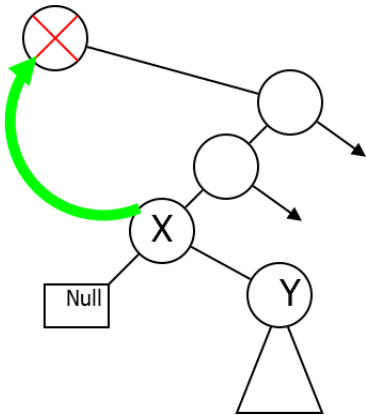
Delete(13)



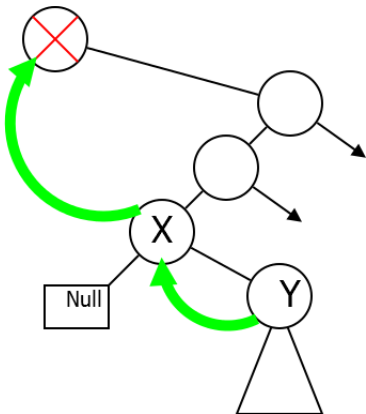
# Idea



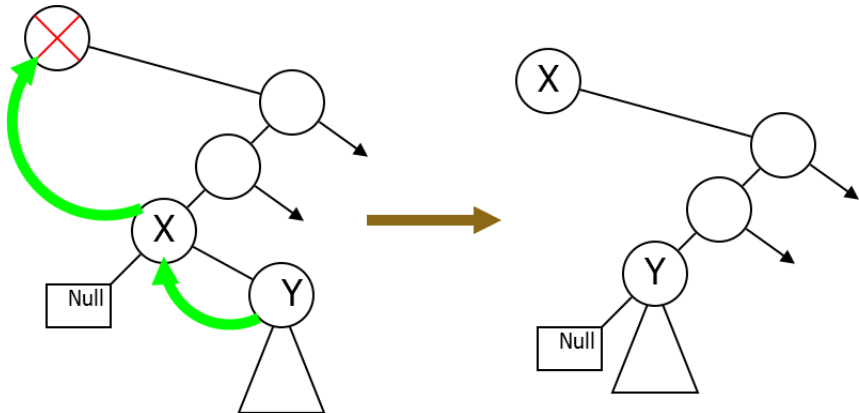
# Idea



# Idea



# Idea



# Implementation

## Delete( $N$ )

if  $N.Right = \text{null}$ :

    Remove  $N$ , promote  $N.Left$

else:

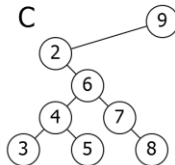
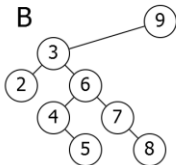
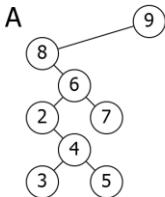
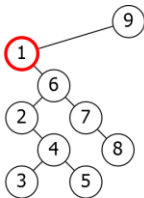
$X \leftarrow \text{Next}(N)$

$X.Left = \text{null}$

    Replace  $N$  by  $X$ , promote  $X.Right$

# Problem

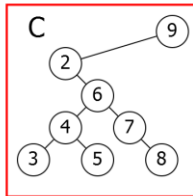
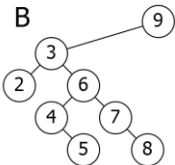
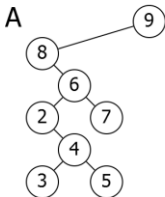
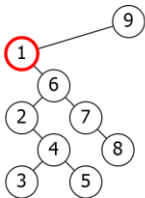
Which of the following trees is obtained when the selected node is deleted?





# Problem

Which of the following trees is obtained when the selected node is deleted?



# Next Time

Runtime and balance.