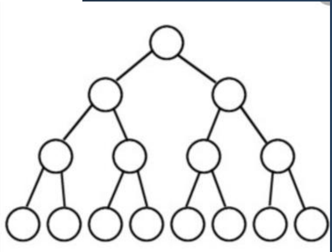
BST BALANCE AND HEIGHT

Advantages of keeping a BST balance

Search= O(height)



Learning Objectives

- Understand what the height of a node is.
- State the AVL property.
- Show that trees satisfying the AVL property have low depth.

Outline

1. Basic Idea
2. Analysis

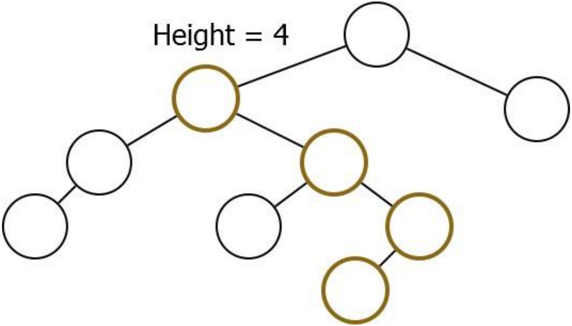
Balance

- Want to maintain balance.
- Need a way to measure balance.

Height

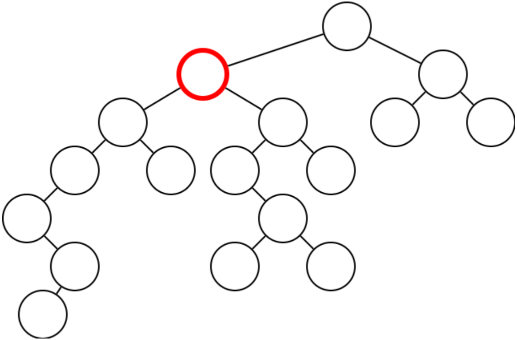
Definition

The height of a node is the maximum depth



Problem

What is the height of the selected node?



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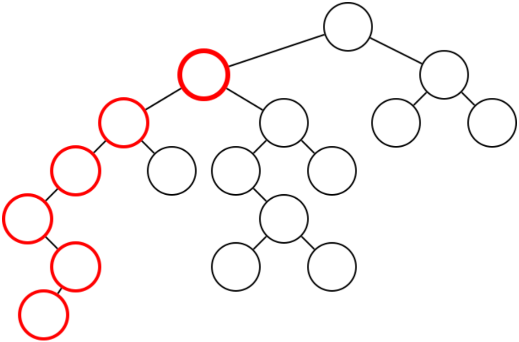


Recursive Definition

$N.\mathtt{Height}$ equals

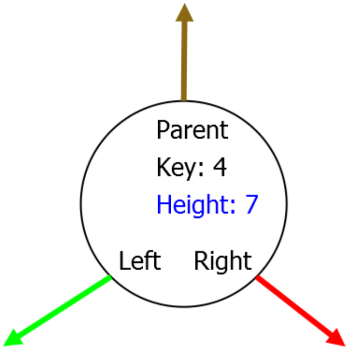
1 if N is a leaf,

$1 + \max(N.\mathtt{Left.Height}, N.\mathtt{Right.Height})$

otherwise.

Field

Add height field to nodes.



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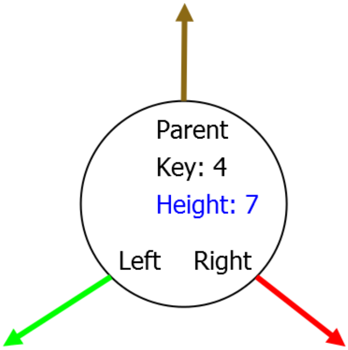


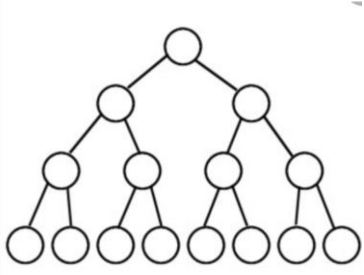
(Note: We'll have to work to ensure that this is kept up to date)

Balance

- Height is a rough measure of subtree size.
- Want size of subtrees roughly the same.
- Force heights to be roughly the same.

Why?

What is this telling me about Time of operations?



AVL Property

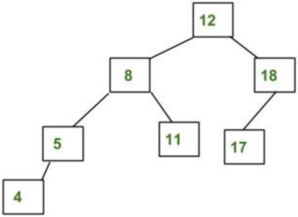
AVL trees maintain the following property:

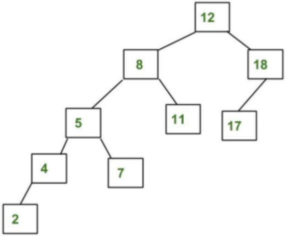
For all nodes *N*,

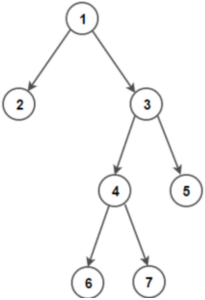
$$|N.\mathtt{Left}.\mathtt{Height} - N.\mathtt{Right}.\mathtt{Height}| \leq 1$$

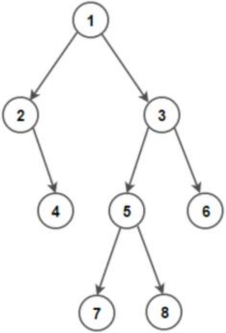
We claim that this ensures balance.

Check here









Conclusion

AVL Property

If you can maintain the AVL property, you can perform operations in $O(\log(n))$ time.