UTBM
K. Deschinkel

Ariadne's thread

1 Context

Your friend, a big fan of video games, asks you to help him. He noticed that many video games consist of moving a character within a particular environment. This character moves from places to places, and may revisit a place when you cross a game level (elevators, secret passages, etc.). After playing a game for hours, your friend says that he's sometimes lost with all these levels. His objective is to establish a small map of the game. For that, he notes all the places encountered, and all the passages from one place to another. This includes only the different levels of play and opportunities to move from one level to another. Each level corresponds to a set of accessible places from other places by any path.
He provides his observations as a binary matrix A of size $n \times n$ (composed of only 0 and 1), where the row and column indices correspond to the different places encountered. Any "1" at the position (i,j) in the matrix means that there is a direct passage from the place i to the place j. If there is no direct passage from i to j, then $A(i,j) = 0$. Finally, he asks you to help him with:

- The list of the different game levels with the places inside each levels
- The map of the game levels, presented as a square matrix N. $N(i,j) = k$, if there are k direct passages from level i to level j (possibly $k = 0$)
- A longest path linking the level including place 1 (input of the game), to the level including place n (output of the game)

2 Questions

1. Which classical problem of graph theory do you recognize in order to address the first point of your friend? Write a program to implement this algorithm. Data may be read in a .txt or .csv file. An example of matrix A is given below:

0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

2. Your program should also provide the different game levels and the places included within each level (for example, here there are five levels: {1,2,3}, {4,5,6,7}, {8,9,10,11,12,13}, {14,15}, {16}).

3. Your program should give the reduced matrix N with the different game levels and the number of direct passages k.

4. What is the longest path that your friend asks you to find? Implement an algorithm to identify such a path.