Empirical methods - Final project

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Abstract

This brief paper presents the results of simulating the data and some of the results presented in the paper "A Structural Model of the Retail Market for Illicit Drugs" by Manolis Galenianos and Alessandro Gavazza (AER, 2017). Additionally, it presents a counterfactual scenario about what would happen in the market after an exogenous increase in the matching efficiency of the function that connects buyers and sellers. Under the parameters estimated by the authors, this new scenario implies more low quality sellers and more matches, but less consumers active in the market.

1 The model

The model resembles a market of illegal drugs where buyers and sellers search for each other and where quality is unobservable and non-contractible, implying the existence of moral hazard. When a buyer meets a seller for the first time, the buyer makes the purchase and realize about drug quality after consuming. The authors estimate the model for the market of crack cocaine in the US, a drug that closely resembles the feature of having unobservable quality (purity) at the time of the purchase.

In the model, buyers B and sellers S meet randomly according to an aggregated homogeneous of degree one matching function $m(B,S)$ that controls the flow of matches in the market. Market tightness $\theta = \frac{B}{S}$ influences how often buyers and sellers meet: A buyer meets a seller at rate $\frac{m(B,S)}{B} = \alpha_B(\theta)$, while a seller meets a buyer at rate $\frac{m(B,S)}{S} = \alpha_S(\theta)$.

1.1 Buyers

Buyers are characterized by their heterogeneous taste for the drug z_i and when consuming have instantaneous utility given by $u_i = z_i q - p$, with p the price and q the quality of the drug. They are distributed according to a continuous log-normal CDF $\bar{M}(z)$ and can be on either one of two possible states: 1) matched with a seller providing them drugs regularly at frequency γ, or 2) unmatched waiting for a random meeting to occur at rate α_B. Matches are exogenously destroyed at rate δ.

Each time buyers meet a new seller, they purchase the drug no matter in which state they are. When they are unmatched, if the drug is above their reservation quality $R_i = \frac{p}{z_i}$[1], they form a long term relationship. Alternatively, if quality q is below R_i, buyers continue searching for a new seller. When they are matched, consumers compare the quality provided by the new seller with the one they were receiving before. If the new quality is higher, they will switch to the new provider, while if not, they will keep buying from the previous one[2].

[1] The authors fixed $p = 100$ and used the quality q that a consumer receives per \$100 as one of the variables of the model.

[2] The authors assume that sellers commit to offering the same quality for ever.

Consumption decisions of buyers depend on market conditions and on how strong their addiction is (how high z_i is). In equilibrium exist an agent type $z = z^*$ indifferent between paying the entry cost K_B to consume, and staying out of the market. The indifferent buyer solves the following equation[3].

$$\begin{aligned}&\alpha_B\left(\frac{\bar{B}(1-\bar{M}(z^*))}{S}\right)\\&\times\left(z^*\int_0^{\bar{q}} x dF(x) + z^*\int_{p/z^*}^{\bar{q}} \frac{\gamma(1-F(x))}{r+\delta+\alpha_B\left(\frac{\bar{B}(1-\bar{M}(z^*))}{S}\right)(1-F(x))}dx - p\right) = K_B\end{aligned} \tag{1}$$

where the CDF $F(q)$ represents the quality distribution faced by first time buyers [4]. Then, a fraction $\bar{M}(z^*)$ of consumers stay out of the market, while $1 - \bar{M}(z^*)$ participate.

1.2 Sellers

Sellers are heterogeneous in their constant marginal cost c of supplying the drug, which comes from a draw from a Pareto CDF $\bar{D}(c) = \left(\frac{c}{\bar{c}}\right)^{\xi}$. Sellers decide optimally the quality q they want to offer in order to maximize profits given their cost c and the expected number of purchases from regular $t_R(q)$ and non-regular t_N consumers.

$$\pi_c = (t_N + t_R(q))(p - cq) \tag{2}$$

The authors assume that sellers have to pay a fixed costs K_s to enter the market, a helpful device to identify those that participate in equilibrium. Assuming free entry, the marginal seller c^* will be identified, after substituting for the expressions of $t_R(q)$ and t_N, by the zero profits condition.

$$\underline{\pi}_c(q) = \alpha_S(\theta)(p - cq)\left(1 + \frac{\gamma\delta H(q)}{(\delta + \alpha_B(\theta)(1 - F(0)))^2}\right) = K_s \tag{3}$$

Additionally, sellers offering positive quality with types $c < c^*$ solve the following differential equation, with initial condition $q^*(c^*) = \underline{q}(c^*)$, to find the optimal level of q.

$$\begin{aligned}&q^{*\prime}(c)\\&= -\frac{2\gamma\delta\left(\frac{p}{c}-q^*(c)\right)H(q^*(c))\alpha_B(\theta)(1-F(0))D'(c)}{(\delta+\alpha_B(\theta)(1-F(0))D(c))\left[(\delta+\alpha_B(\theta)(1-F(0))D(c))^2+\gamma\delta H(q^*(c))-\gamma\delta\left(\frac{p}{c}-q^*(c)\right)H'(q^*(c))\right]}\end{aligned} \tag{4}$$

[3]This is one of the key equations of the estimation procedure.

[4]See the appendix.

2 Estimation

I used the set of parameters estimated in the paper[5]. However, I had to complement those estimates by finding threshold values that come out of the equilibrium of the model. More precisely, the equilibrium values for $\theta = \{\underline{\text{q}}, c^*, F_0, \bar{\text{c}}, z^*\}$ are included in the paper (except for $\bar{c}$), but they are not part of the set of parameters directly estimated (except for c^*)[6].

Parameter	Description
$\underline{\text{q}}$	Lowest quality offered, given $q > 0$
$c*$	Marginal cost of marginal seller with $q > 0$
F_0	Fraction of quality $q = 0$ in the market (rip-offs)
$\bar{c}$	Upper bound of sellers' marginal costs distribution
z^*	Marginal utility of the marginal consumer

To find those values, I used the equilibrium conditions presented above, plus an extra unknown and an extra constraint that follows from the model, to identify $\bar{c}$. I used the "Fminsearch" algorithm because it provides more sensible results. The results are displayed in the following table.

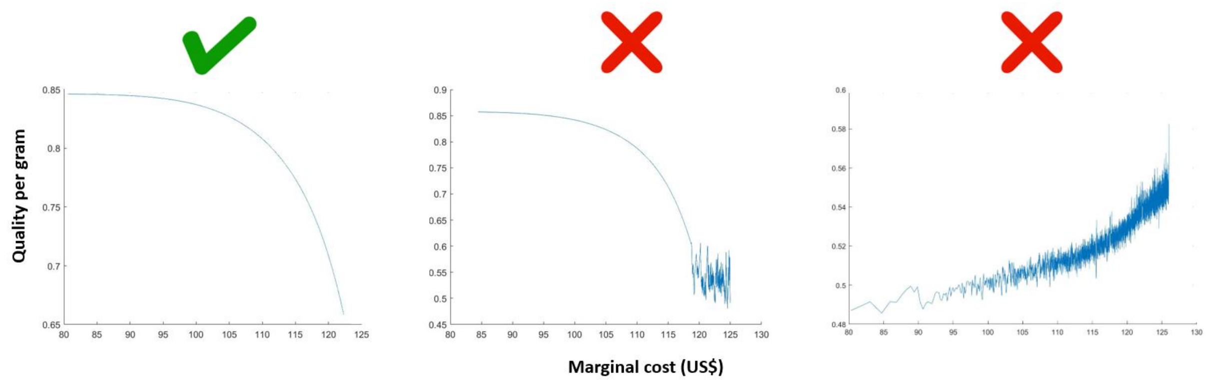
Parameter	$\underline{\text{q}}$	c^*	$\bar{c}$	F_0	z^*
Paper	0.63	124.37	NaN	0.158	150.71
Estimation	0.64	122.22	125.83	0.174	149.82

With these parameters it is possible to separately identify the types z and c of buyers and sellers that enter the market in equilibrium (from z^* and c^*), and those who don't. Also, it is possible to know the proportion of sellers F_0 that rip-offs buyers offering $q = 0$, the lowest positive quality offered $\underline{\text{q}}$ and the maximum potential marginal cost of a seller $\bar{c}$.

[5] See Table 2 of the paper in the appendix.

[6] In the case of $\bar{c}$, this is most probably my fault, but as I wasn't getting an appropriate equilibrium, I included an extra constraint that follows from the model in order to estimate $\bar{c}$. As the results make sense, I kept working that way in the rest of the project.

One of the most challenging parts of finding the equilibrium was to solve numerically the differential equation that sellers use to decide the optimal quality they offer in the market. The authors proved that there is a one to one mapping between c and q and that there are no pair of sellers offering the same q, meaning that the solution to the differential equation should be strictly monotone. However, this is not always the case. The following graph displays the right solution on the left, and a couple of wrong attempts on the right.



3 Simulation

To run the simulation, I take a slightly different approach from the one followed in the paper. Instead of taking draws from steady state distributions of matched and unmatched buyers, I assume that every buyer starts in the same state of the world (unmatched). Then, I let the randoms events of the model operate and I take a snapshot of the market situation after a burn-in period.

The good thing is that both approaches deliver very similar results. These are presented in the following table[7]. In order of appearance, the columns contain frequencies extracted from the data, the authors' simulation and the ones from this replication exercise.

[7] Table 3 of the paper

Simulation results	Data	Paper	Simul
Fraction of rip-offs (percent)	15.338	15.862	17.434
Average pure grams per \$100, $\hat{q} > 0$	0.735	0.732	0.672
Standard deviation pure grams per \$100, $\hat{q} > 0$	0.505	0.416	0.406
Median pure grams per \$100, $\hat{q} > 0$	0.591	0.635	0.697
Skewness pure grams per \$100, $\hat{q} > 0$	1.952	1.870	2.384
Kurtosis pure grams per \$100, $\hat{q} > 0$	8.516	9.521	11.406
Fraction obtained drug in last 30 days (percent)	16.900	16.899	16.822
Fraction last purchased from regular dealer (percent)	52.481	52.960	60.557
Average number of purchases, matched buyer	16.331	16.771	17.207
Average number of purchases, unmatched buyer	11.548	10.756	9.894
Standard deviation number of purchases, matched buyer	11.124	11.477	12.132
Standard deviation number of purchases, unmatched buyer	10.419	10.337	9.622
Median number of purchases, matched buyer	15	14	14
Median number of purchases, unmatched buyer	7	7	7
Fraction consuming drug in NSDUH (percent)	0.800	0.800	0.794
Arrest rate (percent)	3.776	3.775	3.768

Note: Table 3 of the paper plus a new column for simulation results.

I think that the results are broadly in line with the ones presented in the paper, except for some statistics related to the shape of the quality distribution, as the fraction of rip-offs, for example. As I mentioned in the estimation section, the solution to the differential equation relating q and c is key for understanding the quality distribution observed in the market. Therefore, the differences are probably the consequence of the different values I'm using for the set $\{\underline{\mathrm{q}}, c^{*}, F_0, \bar{\mathrm{c}}, z^{*}\}$.

4 Improvements in matching technology

4.1 Changes to the model

The original paper runs counter factual scenarios for 1) changing to observable the unobservable feature of the quality of the drug (learning q before paying for it), 2) increasing and decreasing entry costs K_S and K_B, and 3) raising or reducing matching destruction and frequent consumers rates δ and γ[8].

What the paper didn't do is to estimate what would happen in the market if the matching efficiency summarized in the parameter ω exogenously increases. The model was estimated with data for the period 2001-2003, so maybe technology improvements like cellphones and the internet now make it easier for buyers and sellers to find each other in the drugs market. Therefore, I will estimate what would happen if the matching efficiency increases by 25% and 40%, and compare those two scenarios with the baseline model.

In this case, I will stick to the Cobb-Douglas matching function with Constant Returns to Scale they used in the main body of the paper, where the flow of matches $m(B,S)$ in the market over a 30 days period is given by

$$m(B,S) = \omega B^{\nu} S^{1-\nu} \text{ with } \nu = 0.5 \tag{5}$$

The first step was to find the matching efficiency of the baseline scenario $\omega^0 = 4.3646$. Then, I assumed that efficiency increases in the amounts specified earlier. As a result, the rate $\alpha_B(\theta) = \frac{m(B,S)}{B}$ at which a buyer meets a seller and the rate $\alpha_S(\theta) = \frac{m(B,S)}{S}$ at which a seller meets a buyer should be expected to change. However, the free entry condition imposed on sellers makes the relationship $K_S \leq p\alpha_S(\theta)$ still hold as an equality in equilibrium. This implies that the efficiency improvement is going to affect meeting rates only through a higher α_B. The new rate α_B at which a buyer meets a seller will then be the solution to an equation that uses $K_s = p\alpha_S(\theta)$ and the functional form of the matching function.

$$\alpha_B^1(\theta) = (\omega^0 \Delta)^2 p / K_s \tag{6}$$

[8] I originally wanted to run those counter factual scenarios until I realized in a footnote of the paper that they were already done and included in the appendix

In this case, the values of ω and of the associated $\alpha_B(\theta)$ are displayed in the following table

	Baseline (paper)	$\Delta\omega$ = 25%	$\Delta\omega$ = 40%
ω	4.3646	5.4558	6.1105
$\alpha_B(\theta)$	1.27	1.984	2.489

I choose productivity increases of 25% and 40% because, while in the paper buyers meet a new seller on average every 24 days ($24 = \frac{30}{\alpha_B}$), with this coefficients they will meet a new one on average either every 15 or every 12 days, respectively. Finally, using these new values for $\alpha_B(\theta)$ and the primitives of the paper, I estimated the new equilibrium of the model in each case.

4.2 Results

A raise in $\alpha_B(\theta)$ with α_S unchanged, necessarily implies an decrease in market tightness B/S. However, it is unknown what will happen with each component individually.

	Baseline (Paper)	Baseline (Simulation)	$\Delta\omega$ = 25%	$\Delta\omega$ = 40%
Fraction of rip-offs (percent)	15.862	17.434	16.938	17.299
Average pure grams per \$100	0.616	0.554	0.594	0.599
Standard deviation pure grams per \$100	0.271	0.224	0.194	0.202
Active buyers, in millions	3.431	3.404	3.248	2.909
Active sellers, in millions	0.290	0.288	0.430	0.483
Fraction of matched buyers (percent)	54.04	52.20	63.20	67.30
Average number of purchases per month	12.726	12.358	14.74	16.47
Average pure grams consumed per month	9.464	9.983	11.052	12.04
Number of trans. with $q = 0$ (percent)	1.57	2.313	2.436	2.62

According to the results, the effect is a massive entry of less efficient sellers and a decrease in the number of consumers. The increase in the flow of meetings allows high cost sellers to enter, but offering lower quality (notice how c^* is now higher and $\underline{q}$ lower than before). Also, more competition pushes sellers already in the market to increase the quality they offer ($\bar{q}$ increases from 0.84 to 0.86 in the last scenario). Remember that buyers purchase every time they meet a new seller, so now sellers have to offer higher quality to sustain long term relationships. Matched consumers are still more frequent buyers than unmatched ones as they consume, on average, every 1.6 days $1.57 = \frac{30}{\gamma}$. The number of transactions increases mainly because of the increase in the fraction of matched consumers.

For buyers, there are two competing forces at play. On one side, the quality increase that matched consumers receive and on the other, the increase in low quality suppliers they meet now more frequently than before. This implies that if they are lucky and find a low cost seller, they receive high quality drugs and form a long term relationship. However, if they find a high cost seller (and now there are much more of them so the probability of finding one of these increased), they will be either rip-offs or will receive a quality below their reservation R. In fact, in the new equilibrium the number of transactions with $q = 0$ increases and the mean quality that unmatched consumers receive slightly decreases.

Overall, there are less active buyers (z^* is now higher) as the lower quality effect for lower valuation consumers dominates the higher quality for the matched effect. I find this a reasonable result considering the log-normal distribution of consumers preferences. Then, the increase in the matching efficiency increases the number of low quality sellers in the market and reduces the number of active buyers, improving market conditions for higher valuation (more addictive consumers)[9].

[9]However, I must acknowledge that the final effect on buyers is still a bit unconvincing to me, as the mean quality in the market increases. This might be either because the consumers that remained now get a better deal (selection), or because I have an error in the solution that I'm not able to see.

5 Appendix

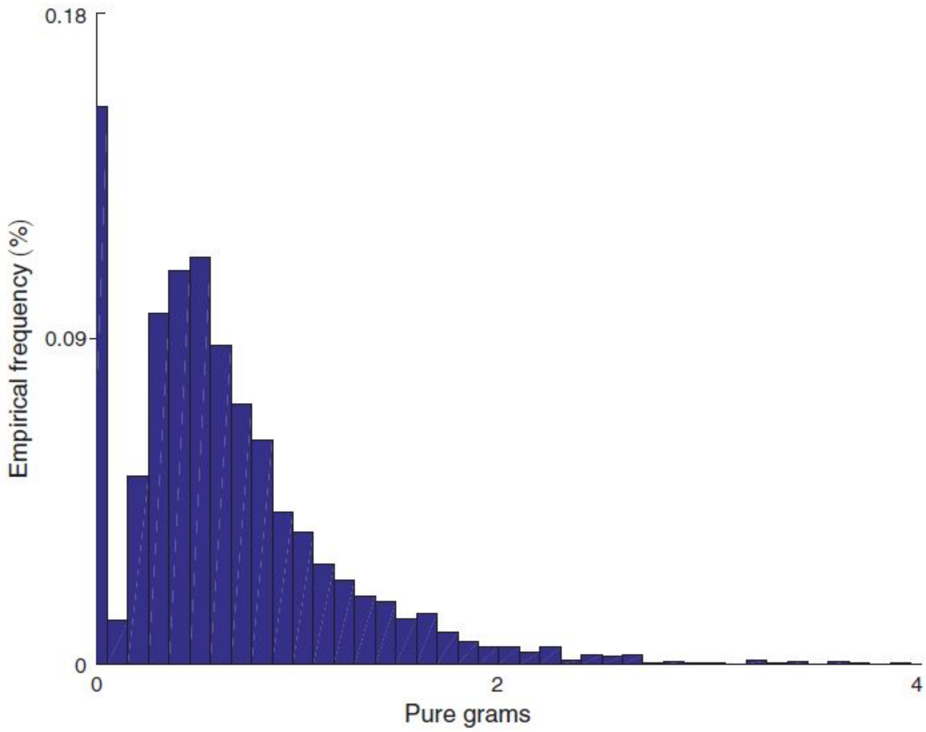


FIGURE 1. HISTOGRAM OF PURE GRAMS OF CRACK COCAINE PER $100

TABLE 2—ESTIMATES

α_B	1.267 [1.217, 1.267]	λ	0.982 [0.980, 0.983]
γ	19.399 [18.868, 20.658]	μ_z	5.118 [5.097, 5.137]
δ	0.731 [0.700, 0.734]	σ_z	0.114 [0.103, 0.125]
K_B	152.639 [112.710, 166.035]	c^*	124.368 [123.597, 128.681]
σ_ϵ	0.526 [0.457, 0.572]	ξ	20.443 [20.443, 31.284]
σ_η	2.803 [2.723, 2.852]	μ_η	−5.237 [−5.329, − 5.103]
σ_ν	0.522 [0.479, 0.548]		

TABLE 3—MODEL FIT

	Data	Model
Fraction of rip-offs (percent)	15.338	15.862
Average pure grams per \$100, $\hat{q} > 0$	0.735	0.732
Standard deviation pure grams per \$100, $\hat{q} > 0$	0.505	0.416
Median pure grams per \$100, $\hat{q} > 0$	0.591	0.635
Skewness pure grams per \$100, $\hat{q} > 0$	1.952	1.870
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Fraction obtained drug in last 30 days (percent)	16.900	16.899
Fraction last purchased from regular dealer (percent)	52.481	52.960
Average number of purchases, matched buyer	16.331	16.771
Average number of purchases, unmatched buyer	11.548	10.756
Standard deviation number of purchases, matched buyer	11.124	11.477
Standard deviation number of purchases, unmatched buyer	10.419	10.337
Median number of purchases, matched buyer	15.000	14.000
Median number of purchases, unmatched buyer	7.000	7.000
Fraction consuming drug in NSDUH (percent)	0.800	0.800
Arrest rate (percent)	3.776	3.775

Note: This table reports the values of the empirical moments and of the simulated moments calculated at the estimated parameters reported in Table 2.