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C02 - Quiz 2 of the course 2025-26

30214 - Teoría de la computación
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Context-Free Languages, Grammars and Pushdown Automata

Problem 2.1. Give Context-Free Grammars (CFGs) that generate the following languages. Each context-free language has infinitely many correct CFGs, but you only need to provide one.

1. $\{w \in \{0,1\} | w \text{ contains at least three } 1s\}$
2. $\{a^i b^j c^k | i, j, k \geq 0, \text{ and } i = j \text{ or } i = k\}$
3. $\{a^i b^j c^k | i, j, k \geq 0, \text{ and } i + j = k\}$
4. $\{a^i b^j c^k | i, j, k \geq 0, \text{ and } i + k = j\}$
5. $\{ab^n acab^n a | n \geq 0\}$
6. $\emptyset$ □

Problem 2.2. Let $T = \{0, 1, (,), +, ^*, \emptyset, e\}$ be an alphabet used by regular expressions describing languages over the alphabet $\{0, 1\}$; the only difference is that the e symbol is used for symbol ε, to avoid potential confusion.

1. Design a Context-Free Grammar (CFG), G, with set of terminals T that generates exactly the regular expressions with alphabet $\{0, 1\}$.
2. Using the previous CFG G, give a derivation and the corresponding parse tree for the string $(0{+}(10)^*1)^*$. □

Problem 2.3. Let A be a language to describe arithmetic expressions with natural numbers of only one digit, and the CFG G has been designed to generate the words of A: $G = (N, \Sigma, P, S)$, with $N = \{S\}$, $\Sigma = \{+, \text{-}, \text{x}, /, (,), 0, 1, 2, ..., 9\}$, S as the starting variable, and the following production rules in P.

$$S \longrightarrow S{+}S | S\text{-}S | S\text{x}S | S/S | (S) | \text{-}S | 0 | 1 | \ldots | 9$$

1. Consider the strings ---5 and 2+--4. Give derivations showing that each string belongs to $L(G)$, i.e. they are words generated by the grammar G.
2. Give another CFG that disallows the strings as the previous arithmetic expressions. More specifically, strings such as 2-3, 2+-3 and 2--3 are allowed, but not 2+--3 nor 2---3. □

Problem 2.4. For each of the following languages, state whether it is regular, context-free but not regular, or neither. Prove your answer. Make sure, if you say that a language is context free, that you show that it is not also regular.

1. $A = \{w \in \{0,1\}^* | \exists k \geq 0$ and w is a binary encoding of $2^k + 1$ (leading zeros allowed) $\}$
2. $B = L(a^*b^*c^*) \setminus \{a^n b^n c^n | n \geq 0\}$
3. $C = \{(ab)^n a^n b^n | n \geq 0\}$
4. $D = \{x \in \{a,b\}^* || x|$ is even and the first half of x has one more a than does the second half $\}$ □

Problem 2.5. Convert the following CFG, G, into an equivalent CFG in Chomsky normal form: $G = (N, \Sigma, P, S)$, where $N = \{S, B\}$, $\Sigma = \{0\}$ and the set P of production rules contains,

$$S \longrightarrow BSB|B|\varepsilon$$
$$B \longrightarrow 00|\varepsilon$$

□

Problem 2.6. Let G be a Context-Free Grammar (CFG), $G = (N, \Sigma, P, S)$, where $N = \{S\}$, $\Sigma = \{0,1\}$ and the set P of production rules contains,

$$S \longrightarrow 0S|S1|0|1|\varepsilon$$

Prove that no string in the language generated by this grammar has 10 as a factor. □

Problem 2.7. Give Pushdown Automata (PDA) that recognize the following languages. For each PDA, give both a state diagram and 6-tuple specification. Every context-free language has infinitely many (correct) PDAs, but you only need to give one.

1. $A = \{w \in \{0,1\}^* | w$ contains at least three $1s\}$
2. $B = \{a^i b^j c^k | i,j,k \geq 0$ and $i + j = k\}$
3. $C = \{a^{2n} b^{3n} | n \geq 0\}$
4. $D = \{a^i b^j c^k | i,j,k \geq 0$ and $i + k = j\}$
5. $E = \{ab^n acab^n a | n \geq 0\}$ □
6. $H = \emptyset$, with $\Sigma = \{0,1\}$.

Problem 2.8. For each of the following languages, demonstrate which class of languages it belongs to: Regular, Context-Free, Not Context-Free.

1. $A = \{0^{2i} 1^{3j} 0^k | i,j,k \geq 0$, and $i = j = k\}$
2. $B = \{0^{2i} 1^{3j} 0^k | i,j,k \geq 0\}$
3. $C = \{0^{2i} 1^{3j} 0^k | i,j,k \geq 0$, and $i = k\}$ □

Problem 2.9.
Prove that the language $L = \{a^n b^n | n \geq 0$ and n is not a multiple of $5\}$ is a context-free language. □

Problem 2.10. Define $H(L)$ as the set of even-length strings in L. That is,

$$H(L) = \{w | w \in L \text{ and } |w| = 2k \text{ for some } k \geq 0\}$$

If L is a Context-Free Language, is $H(L)$ a Context-Free Language? $\square$