Smart Grid Fundamentals

MEEC

Assignment IX: Power Demand Management

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Contents

1 Objectives

For this assignment we had to work with the *GAMS Studio 41* program to optimize a given problem.

The problem to be optimized is the total cost given the power consumption of a household with 5 loads and a set of flexibilities during a 24 hours period.

2 Problems

In this section we answer to the three problems presented in the assignment. To do so we need to take in account the tables for the power demand, the demand flexibility and the cost in Kwh, presented in figure 1,2 and 3, respectively:

```
** Node demand in m^3/h
Table Pd(i,t) "Power demand (kW)"
        1      2      3      4      5      6      7      8      9      10     11     12     13     14     15     16     17     18     19     20     21     22     23     24
   1  144.0  145.8  147.6  140.4  133.2  117.0  100.8  91.8   82.8   75.6   69.6   63.6   57.6   55.8   54.0   61.2   68.4   80.4   92.4   104.4  115.2  126.0  136.8  147.6
   2  219.6  223.2  226.8  217.8  208.8  180.0  151.2  136.8  122.4  111.6  104.4  97.2   90.0   84.6   79.2   90.0   100.8  118.8  136.8  154.8  174.6  194.4  214.2  234.0
   3  219.6  223.2  226.8  217.8  208.8  180.0  151.2  136.8  122.4  111.6  104.4  97.2   90.0   84.6   79.2   90.0   100.8  118.8  136.8  154.8  174.6  194.4  214.2  234.0
   4  194.4  198.0  201.6  190.8  180.0  158.4  136.8  122.4  108.0  100.8  93.6   86.4   79.2   75.6   72.0   81.0   90.0   106.8  123.6  140.4  154.8  169.2  183.6  198.0
   5  64.8   64.8   64.8   61.2   57.6   50.4   43.2   37.8   32.4   28.8   27.6   26.4   25.2   25.2   25.2   27.0   28.8   32.4   36.0   39.6   47.7   55.8   63.9   72.0
```

Figure 1: Power demand for each load during 24h period.

```
Table DeltaP(i,t) "Demand Flexibility (%)"
        1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19    20    21    22    23    24
   1  0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.05  0.1   0.1   0.1   0.1   0.1
   2  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05
   3  0.25  0.25  0.25  0.25  0.25  0.25  0.25  0.25  0.25  0.25  0.25  0.25  0.25  0.25  0.25  0.25  0     0     0     0     0     0     0     0
   4  0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0.25  0.25  0.25  0.25  0.25  0.25  0.25  0.25
   5  0.2   0.2   0.2   0.2   0.2   0.2   0.2   0.1   0.1   0.2   0.2   0.2   0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.05  0.1   0.1   0.1   0.1
;
```

Figure 2: Demand flexibility for each load during 24h period.

```
** cost in � KWH per hour
Parameter
 cost(t)
/ 1 0.016, 2 0.016, 3 0.018, 4 0.018, 5 0.021, 6 0.021, 7 0.022, 8 0.022,
  9 0.023, 10 0.023, 11 0.024, 12 0.024, 13 0.025, 14 0.025, 15 0.029, 16 0.029,
 17 0.031, 18 0.031, 19 0.035, 20 0.035, 21 0.0325, 22 0.0325, 23 0.03, 24 0.03/;
```

Figure 3: Cost per Kwh during 24h period.

To create the optimization problem, a couple of constraints had to be defined. The general for all three problems where as the following **items**:

1. The objective cost-related function
2. Real power demand, taking demand flexibility into account
3. The installation's maximum accepted power consumption
4. When demand flexibility is applied or not, the power usage must remain the same.

For the **first item**, the equation created was the same for the two first problems and partially for the third. The cost function to be minimized was the sum of the real power demand of all loads (as described on the item 2) in the 24h period, multiplied by the cost per Kwh, as demonstrated by equation (1):

$$\sum_i \sum_t Pf(i,t) * cost(t) \tag{1}$$

Which translates into the equation on *GAMS Studio 41* language:

$$costs..\;\; z = e = sum((i,t), Pf(i,t) * cost(t)); \tag{2}$$

For the **second item**, part of the constraint was also the same for all of the problems. The real power demand of each load in each hour is equal to the actual power demand as represented in figure 1, plus a variable that evolves from the demand flexibility which we called *Flex(i, t)*:

$$Flexeq(i,t)..\;\; Pf(i,t) = e = Pd(i,t) + Flex(i,t); \tag{3}$$

As we will see next, the *Flex(i, t)* variable will take different values according with the problem. This variable will also take an effect on the **third item**, which is way it will be discussed later on.

The **fourth item** is used, to make sure that the total power consumption is equal if we use or not the demand flexibility. This restriction is necessary because, absent it, the software will always utilize demand flexibility to reduce power consumption in all circumstances in order to minimize the cost function. This translates into the following equation:

$$Flexeq4..\;\; sum((i,t), Pf(i,t)) = e = sum((i,t), Pd(i,t)); \tag{4}$$

2.1 Problem 1

On this first problem, we complement the **second item** by adding the following definitions for *Flex(i, t)*:

$$Flexeq3(i,t)..\;\; Flex(i,t) = g = -Pd(i,t) * DeltaP(i,t); \tag{5}$$

$$Flexeq2(i,t)..\;\; Flex(i,t) = l = Pd(i,t) * DeltaP(i,t); \tag{6}$$

This equations for *Flex(i, t)*, represent the fact that each of its value at each hour needs to be confined to the interval:

$$Flex(i,t) \in [-Pd(i,t) * DeltaP(i,t);\; Pd(i,t) * DeltaP(i,t)] \tag{7}$$

This part of the **second item**, influences the power demand, so that the final constraint represented by the **third item** can be met. This tells us that at each hour the power utilized by the 5 loads needs to be below a maximum value. This third constraint, in this case is represented by the following equation:

$$ConsoMax(t)..\;\; sum(i, Pf(i,t)) = l = Pmax; \tag{8}$$

Where $Pmax = 850\ kW$.

With all the constraints defined, we could finally run the code and retrieve the following information to respond to the questions of the assignment:

```
104 VARIABLE z.L                          =       345.369  total costs during the day
```

Figure 4: Total cost during the day

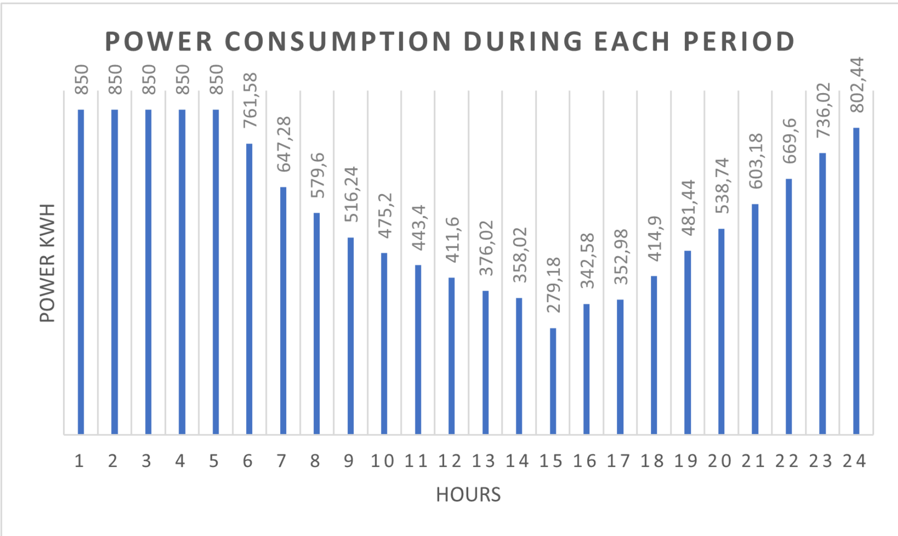

Figure 5: Power consumption of the installation during each period

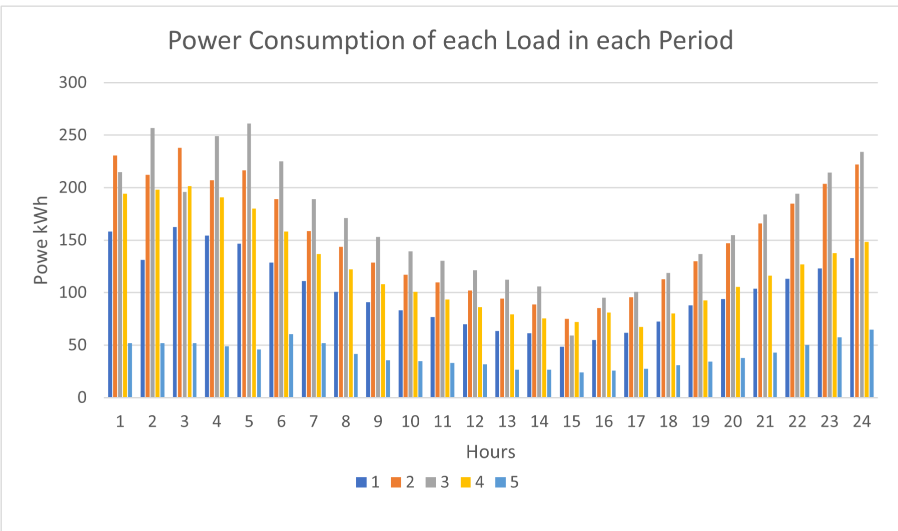

Figure 6: Power consumption of each load in each period

2.1.1 Preliminary Conclusions

From the analysis of Figures 5 and 6, as well as the tables from **Appendix B**, we can make some initial conclusions.

It's possible to analyse that all constraints are met. For all hours the maximum power of all loads is less or equal than the set Pmax, which was in this case 850kW and the final power is equal to the initial, which is 14040kW. Also, with low constraints for the demand flexibility variable, all of them that where different of zero, where used in order to achieve the optimum value for the cost.

Another reflection that we had, was that the amount of variation for the power consumption of the loads at each hour, relative to its initial values was overall pretty similar. Initially we assumed that hours that where close to the maximum power value where all going to have smaller variations, than the others that could be moved more freely. On **Table 2**, for the 1h to the 4h we can say that our belief was correct, but then when we look more closely to the 23h and 24h, we can see that the variation aligns with the rest. Therefore we can assume that our belief can't be yet proved.

2.2 Problem 2

On this second problem we also had to substitute the **second item**, but reused the constraint for the **third item**, from *Problem 1*.

According to the assignment guidelines, there where two different situations. One referred to the loads 1 and 2 which was the same as before, while the other pertained the rest, imposing that the demand for this loads had to be either entirely activated (0 or 1). This way we inputted the following constraints:

$$Flexeq2(i,t)\$(ord(i) <= 2)..\ \ Flex(i,t) = l = Pd(i,t) * DeltaP(i,t); \tag{9}$$

$$Flexeq5(i,t)\$(ord(i) <= 2)..\ \ Flex(i,t) = g = -Pd(i,t) * DeltaP(i,t); \tag{10}$$

$$Flexeq3(i,t)\$(ord(i) > 2)..\ \ Flex(i,t) = e = -Pd(i,t) * DeltaP(i,t) * X(i,t); \tag{11}$$

The equations (9) and (10) created the same environment as before for loads 1 and 2. The equation (11), introduces a **Binary Variable (X(i,t))** that only allows the *Flex(i, t)* variable assume the full demand flexibility or not. There is a negative sign given to the *Flex(i, t)*, to overcome the infeasible solution. This happens because otherwise the values for the the loads could only be either the same, or higher and seen that on the 24h, the total value of the power is above the Pmax, we need to be able to reduce it.

With all the constraints defined, we could finally run the code and retrieve the following information to respond to the questions of the assignment:

```
108 VARIABLE z.L                    =       348.106  total costs during the day
```

Figure 7: Total cost during the day

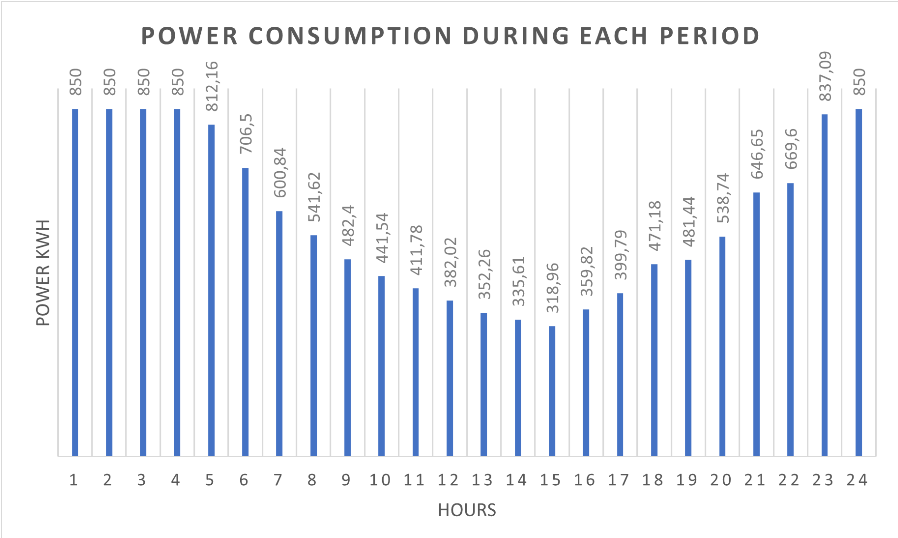

Figure 8: Power consumption of the installation during each period

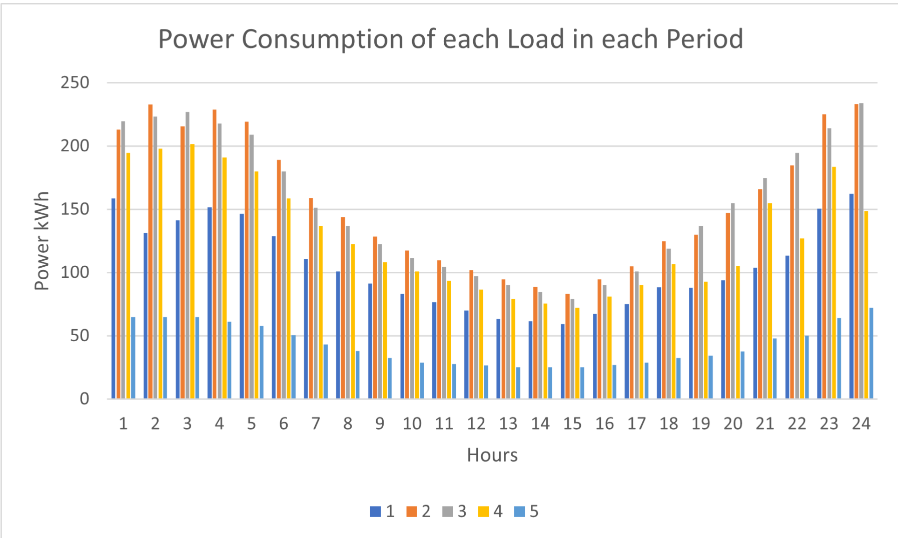

Figure 9: Power consumption of each load in each period

2.2.1 Preliminary Conclusions

From the analysis of Figures 8 and 9, as well as the tables from **Appendix C**, we can make some initial conclusions.

Again it's possible to analyse that all constraints are met. For all hours the maximum power of all loads is less or equal than the set Pmax, which was in this case 850kW and the final power is equal to the initial, which is 14040kW. On **Table 6**, from 1h to 4h, it is visible that the same variation for the power is the same as in **Problem 1**, even if the demand flexibilities used are different. Therefore we can assume that there is more than one way to get to the same results and in addition to the optimal solution.

The cost function of *Problem 2* was higher than *Problem 1.* This fact leads us to believe that the extra constraints thrown in this problem caused the result to be less than optimal. In fact it happens, because since the problem as less flexibility to get to the optimum value and the constraints used influenced negatively, the solution achieved was worse.

Lastly the variation of the power for each hour again refutes our initial belief that it had something to do with the closeness to the maximum allowed power.

2.3 Problem 3

On this problem, there was a new constraint, which was that from 19h to 24h the maximum allowed power to be consumed at each hour by all loads was now *Pmax = 700 kW.*

In order to comply with this demand, we where faced with two different situations, again like in *Problem 2*, which lead us to make the following changes to the **first and third items**:

$$costs..\ \ z = e = sum((i,t), Pf(i,t) * cost(t)) - sum((i,t), Psheding(i,t) * cost(t) * 5); \quad (12)$$

$$ConsoMax2(t)\$(ord(t) > 18)..\ \ sum(i, Psheding(i,t)) + sum(i, Pf(i,t)) = l = 700; \quad (13)$$

$$ConsoMax3(t)\$(ord(t) <= 18)..\ \ sum(i, Pf(i,t)) = l = 850; \quad (14)$$

There was a big issue with this new situation that was making it infeasible. At some point, even when using the demand flexibility it was impossible for the system to reduce the maximum power to lower or equal to the Pmax.

In order to pinpoint the error, we used a facultative variable named *Pshedding(i, t).* This variable was set to be negative and the purposed was for it to only be used when the power was above the Pmax and needed to be lowered.

To do so, we changed the cost function (**first item**), so that the cost for this new variable was higher than the cost of our power demand, as it is demonstrated on equation (12). The negative value comes from the fact that the sum of the cost of *Pshedding(i, t)* is negative, because its power is negative.

Relative to the **third item**, for the period of time below or equal to the 18h, the equation for the maximum consumption stays the same, as it is visible on equation (14) and for the rest of the time we add the sum(i, Pshedding(i, t)), in order for it to be taken in account.

Next, we decided to test this new constraints, for the two previous problems, in order to compare the results.

2.3.1 With constraints from Problem 1

Taking in account the new constraints for the **first and third items** created specifically for the *Problem 3*, as well as the constraints for the **second and fourth items** from *Problem 1*, we could finally get some results.

First by analysing the *Pshedding(i, t)* variable we can see in which hours the consumer can provide or not the service described in the assignment. If it's equal to zero, then it can provide the service, otherwise it can't and as shown in Figure 10:

```
----    134 VARIABLE Pshading.L  power to

            23          24

5     -36.020    -102.440
```

Figure 10: Power consumption of Pshedding(i, t) in each hour and in each load

We can see that *Pshedding(i, t)* it is only used on the **23h and 24h** by the load 5. This shows that at this time the **consumer can't provide this service.**

As it is seen in equation (12) we decided to give a weight of 5 to the cost provided by the *Pshedding(i, t)* power or in other words a **cost for service participation**. Which results in the final value for this cost to be:

```
134 VARIABLE CostAux.L              =        20.769  cost for service participation
```

Figure 11: Cost for service participation

This extra cost that we add because the costumer wasn't able to comply with the imposed constraints is the difference between the cost that we imposed on *Problem 1* subtracted to the one obtained on this Problem. In a optimal scenario, the cost values should be the same for both scenarios, because overall the total power consumed by the loads is the same, as imposed by the constraint from **item 4**, which didn't occurred, seen that the demand flexibility didn't allowed.

The answers for the rest of the questions for this problem are presented in the following figures:

```
134 VARIABLE z.L                    =       366.138  total costs during the day
```

Figure 12: Total cost during the day

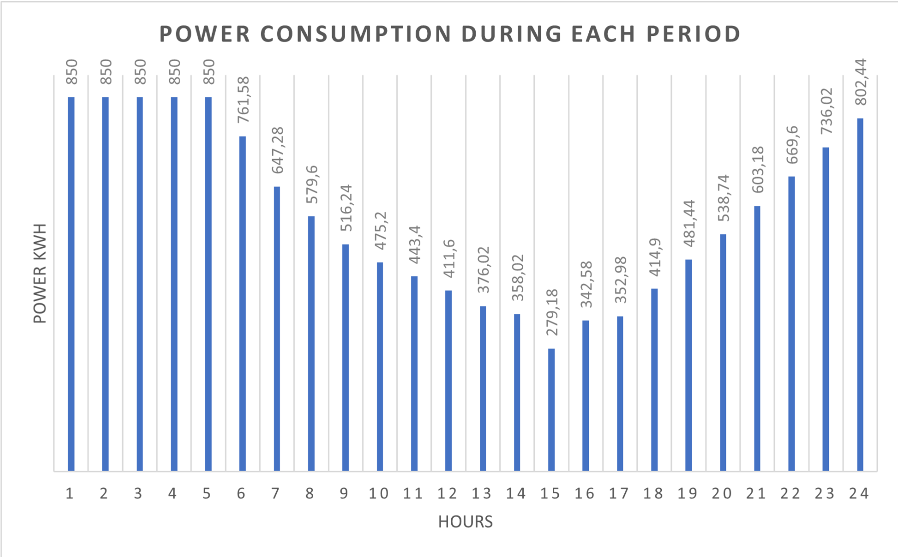

Figure 13: Power consumption of the installation during each period

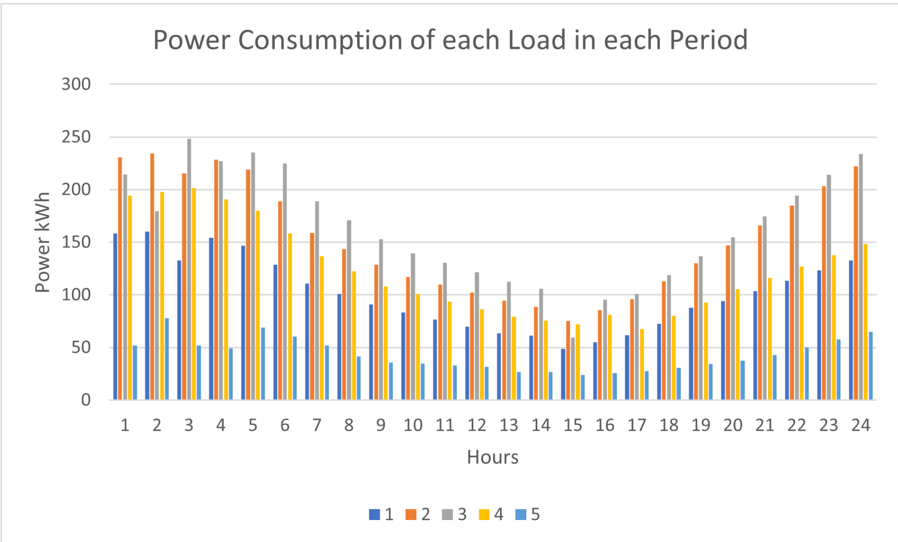

Figure 14: Power consumption of each load in each period

2.3.2 Preliminary Conclusions with constraints from Problem 1

From the analysis of Figures 13 and 14, as well as the tables from **Appendix D.1**, we can make some initial conclusions.

The issue with the constraints utilized on this Problem, was the fact that it made it infeasible. As it is visible on **Table 8**, for the 23h and 24h, the maximum power is above the maximum limit. This fact made it necessary to use the *Pshedding(i,t)* variable to lower the values to the maximum allowed.

Even so, something important to notice is the fact that the solution achieved for the consumption of the installation during each period, and therefore its total cost was exactly the same as the one achieved for *Problem 1.* Also the same assumptions for the demand flexibility and variation of the power in each hour can be made.

2.3.3 With constraints from Problem 2

Taking in account the new constraints for the **first and third items** created specifically for the *Problem 3*, as well as the constraints for the **second and fourth items** from *Problem 2*, we could finally get some results.

Again by analysing the *Pshedding(i, t)* variable as shown in Figure 10:

```
----    134 VARIABLE Pshading.L  power to counter infisability

            23          24

3     -36.020
5                 -102.440
```

Figure 15: Power consumption of Pshedding(i, t) in each hour and in each load

We can see that *Pshedding(i, t)* it is only used on the **23h and 24h** by the load 3 and 5. Again this shows that at this time the **consumer can't provide this service.**

As it is seen in equation (12) we kept the weight of 5 to the cost provided by the **service participation**, in order to have comparable results. This leads to the final value for this cost:

```
134 VARIABLE CostAux.L           =        20.769  cost for service participation
```

Figure 16: Cost for service participation

The answers for the rest of the questions for this problem are presented in the following figures:

```
134 VARIABLE z.L                 =       369.438  total costs during the day
```

Figure 17: Total cost during the day

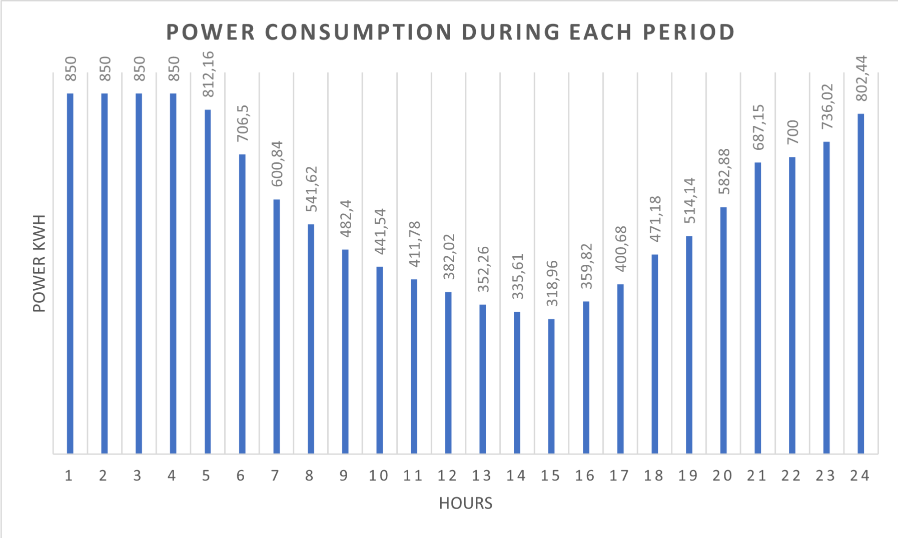

Figure 18: Power consumption of the installation during each period

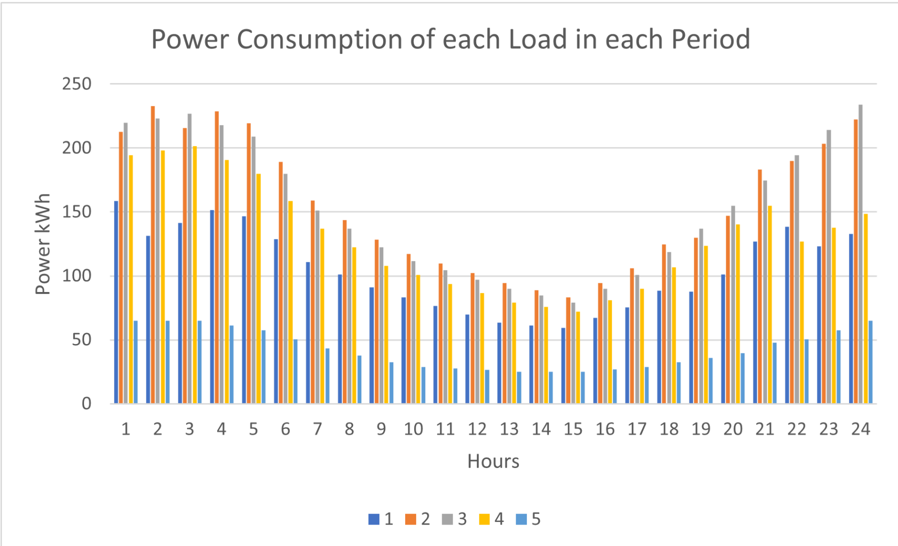

Figure 19: Power consumption of each load in each period

2.3.4 Preliminary Conclusions with constraints from Problem 2

From the analysis of Figures 18 and 19, as well as the tables from **Appendix D.2**, we can make some initial conclusions.

The infeasible solution happens again in the 23h and 24h, same as with the constraints from the *Problem 1*. This leads us to conclude that the problem with the maximum power demand is independent of the constraints defined and is impossible to overcome in this scenario.

In first instance the solution derived for the cost of the consumption of the installation during each period seems that it differs from the solution achieved on *Problem 2*, partially because on **Table 11**, the values differ. This would ultimately make the cost between them different. But when a closer look is taken we see that the price of the **Cost for service participation** is exactly the same as for the constraints used with *Problem 1*. With this we can finally realize that the cost is exactly the same as on *Problem 2*.

Lastly the same assumptions for the demand flexibility and variation of the power in each hour can be made.

3 Conclusions

By the end of this assignment, we believe we where able to retrieve crucial information about the constraints used, as well as with the overall problem definition.

First something that's very straightforward is that the less constraints we use to optimize a problem, the better the solution will be or at least the more probable we are to find a more optimal solution, as it is visible when comparing the costs from *Problem 1* and *Problem 2*. Even when adding more constraints, like we saw on *Problem 3*, the costs change proportionally and again both of the costs where higher than from previous Problems due to the added constraint.

Another important finding was that the optimal solution can be achieved from multiple different settings and the variation of the power demand, by the use of a flexibility term, is independent of its actual value.

Lastly, *Problem 3* was essential to show that the optimal solution for the cost of the power demand was the same for the ones achieved in both *Problems 1 and 2* (the added value for the cost, came with the penalization given to the cost for participation service). This way is possible to say that for a given maximization or minimization problem, the optimal solution of a given variable, is always the same and independent of others that might be used to calculate the cost function, if the variables themselves don't influence each other.

References

1)SGF lecture notes
2)Provided code examples

Appendix

A Initial condition for power demand

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**TOTAL**
1	144	145,8	147,6	140,4	133,2	117	100,8	91,8	82,8	75,6	69,6	63,6	57,6	55,8	54	61,2	68,4	80,4	92,4	104,4	115,2	126	136,8	147,6	2412
2	219,6	223,2	226,8	217,8	208,8	180	151,2	136,8	122,4	111,6	104,4	97,2	90	84,6	79,2	90	100,8	118,8	136,8	154,8	174,6	194,4	214,2	234	3672
3	219,6	223,2	226,8	217,8	208,8	180	151,2	136,8	122,4	111,6	104,4	97,2	90	84,6	79,2	90	100,8	118,8	136,8	154,8	174,6	194,4	214,2	234	3672
4	194,4	198	201,6	190,8	180	158,4	136,8	122,4	108	100,8	93,6	86,4	79,2	75,6	72	81	90	106,8	123,6	140,4	154,8	169,2	183,6	198	3245,4
5	64,8	64,8	64,8	61,2	57,6	50,4	43,2	37,8	32,4	28,8	27,6	26,4	25,2	25,2	25,2	27	28,8	32,4	36	39,6	47,7	55,8	63,9	72	1038,6
TOTAL	842,4	855	867,6	828	788,4	685,8	583,2	525,6	468	428,4	399,6	370,8	342	325,8	309,6	349,2	388,8	457,2	525,6	594	666,9	739,8	812,7	885,6	

Table 1: Power consumption of the installation without using demand flexibility

B Problem 1

B.1 Power consumption of the installation

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	TOTAL
1	158,4	131,22	162,36	154,44	146,52	128,7	110,88	100,98	91,08	83,16	76,56	69,96	63,36	61,38	48,6	55,08	61,56	72,36	87,78	93,96	103,68	113,4	123,12	132,84	2460,54
2	230,58	212,04	238,14	206,91	216,4	189	158,76	143,64	128,52	117,18	109,62	102,06	94,5	88,83	75,24	85,5	95,76	112,86	129,96	147,06	165,87	184,68	203,49	222,3	3705,84
3	214,78	256,9	196,06	248,89	261	225	189	171	153	139,5	130,5	121,5	112,5	105,75	59,4	95,35	100,8	118,8	136,8	154,8	174,6	194,4	214,2	234	3932,43
4	194,4	198	201,6	190,8	180	158,4	136,8	122,4	108	100,8	93,6	86,4	79,2	75,6	72	81	67,5	80,1	92,7	105,3	116,1	126,9	137,7	148,5	2953,8
5	51,84	51,84	51,84	48,96	46,08	60,48	51,84	41,58	35,64	34,56	33,12	31,68	26,46	26,46	23,94	25,65	27,36	30,78	34,2	37,62	42,93	50,22	57,51	64,8	987,39
TOTAL	850	850	850	850	850	761,58	647,28	579,6	516,24	475,2	443,4	411,6	376,02	358,02	279,18	342,58	352,98	414,9	481,44	538,74	603,18	669,6	736,02	802,44	

Table 2: Power consumption of the installation

B.2 Error of the power consumption of the installation

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	TOTAL
1	0,1	0,023876	0,029468	0,081088	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,036896	0,04432	0,1	0,1	0,05	0,1	0,1	0,1	0,1	0,1	0,020124
2	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,012138	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,009216
3	0,021949	0,00126	0,051281	0,054937	0,229288	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,054143	0	0	0	0	0	0	0	0	0,070923
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,08985
5	0,2	0,2	0,2	0,2	0,174224	0,2	0,2	0,1	0,1	0,2	0,2	0,2	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,1	0,1	0,1	0,1	0,049307
TOTAL	0,009022	0,005848	0,020286	0,02657	0,078133	0,110499	0,109877	0,10274	0,103077	0,109244	0,10961	0,110032	0,099474	0,098895	0,098256	0,018958	0,09213	0,09252	0,084018	0,09303	0,095547	0,094891	0,094352	0,093902	

Table 3: Error in percentage (%), between the initial power demand (Pd) and the final(Pf)

B.3 Table for *Flex(i,t)* variable

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	14,4	14,58	14,76	14,04	13,32	11,7	10,08	9,18	8,28	7,56	6,96	6,36	5,76	5,58	-5,4	-6,12	-6,84	-8,04	-4,62	-10,44	-11,52	-12,6	-13,68	-14,76
2	10,98	11,16	11,34	10,89	10,44	9	7,56	6,84	6,12	5,58	5,22	4,86	4,5	4,23	-3,96	-4,5	-5,04	-5,94	-6,84	-7,74	-8,73	-9,72	-10,71	-11,7
3	-4,82	-17,78	-30,74	9,31	49,36	45	37,8	34,2	30,6	27,9	26,1	24,3	22,5	21,15	-19,8	5,35	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-22,5	-26,7	-30,9	-35,1	-38,7	-42,3	-45,9	-49,5
5	-12,96	-12,96	-12,96	-12,24	-11,52	10,08	8,64	3,78	3,24	5,76	5,52	5,28	1,26	1,26	-1,26	-1,35	-1,44	-1,62	-1,8	-1,98	-4,77	-5,58	-6,39	-7,2

Table 4: *Flex(i,t)* for each load in each period

C Problem 2

C.1 Power consumption of the installation

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	TOTAL
1	158,4	131,22	141,34	151,51	146,52	128,7	110,88	100,98	91,08	83,16	76,56	69,96	63,36	61,38	59,4	67,32	75,24	88,44	87,78	93,96	103,68	113,4	150,48	162,36	2517,11
2	212,8	232,78	215,46	228,69	219,24	189	158,76	143,64	128,52	117,18	109,62	102,06	94,5	88,83	83,16	94,5	104,95	124,74	129,96	147,06	165,87	184,68	224,91	233,14	3734,05
3	219,6	223,2	226,8	217,8	208,8	180	151,2	136,8	122,4	111,6	104,4	97,2	90	84,6	79,2	90	100,8	118,8	136,8	154,8	174,6	194,4	214,2	234	3672
4	194,4	198	201,6	190,8	180	158,4	136,8	122,4	108	100,8	93,6	86,4	79,2	75,6	72	81	90	106,8	92,7	105,3	154,8	126,9	183,6	148,5	3087,6
5	64,8	64,8	64,8	61,2	57,6	50,4	43,2	37,8	32,4	28,8	27,6	26,4	25,2	25,2	25,2	27	28,8	32,4	34,2	37,62	47,7	50,22	63,9	72	1029,24
TOTAL	850	850	850	850	812,16	706,5	600,84	541,62	482,4	441,54	411,78	382,02	352,26	335,61	318,96	359,82	399,79	471,18	481,44	538,74	646,65	669,6	837,09	850	

Table 5: Power consumption of the installation

C.2 Error of the power consumption of the installation

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	TOTAL
1	0,1	0,1	0,042412	0,079131	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,05	0,1	0,1	0,1	0,1	0,1	**0,043578**
2	0,030965	0,042921	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,041171	0,05	0,05	0,05	0,05	0,05	0,05	0,003675	**0,016898**
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,25	0,25	0	0,25	0	0,25	**0,048623**
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,05	0,05	0	0,1	0	0	**0,009012**
TOTAL	**0,009022**	**0,005848**	**0,020286**	**0,02657**	**0,030137**	**0,030184**	**0,030247**	**0,030479**	**0,030769**	**0,030672**	**0,03048**	**0,030259**	**0,03**	**0,03011**	**0,030233**	**0,030412**	**0,028266**	**0,030577**	**0,084018**	**0,09303**	**0,030364**	**0,094891**	**0,030011**	**0,040199**	

Table 6: Error in percentage (%), between the initial power demand (Pd) and the final(Pf)

C.3 Table for *Flex(i,t)* variable

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	14,4	-14,58	-6,26	11,11	13,32	11,7	10,08	9,18	8,28	7,56	6,96	6,36	5,76	5,58	5,4	6,12	6,84	8,04	-4,62	-10,44	-11,52	-12,6	13,68	14,76
2	-6,8	9,58	-11,34	10,89	10,44	9	7,56	6,84	6,12	5,58	5,22	4,86	4,5	4,23	3,96	4,5	4,15	5,94	-6,84	-7,74	-8,73	-9,72	10,71	-0,86
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-30,9	-35,1	0	-42,3	0	-49,5
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1,8	-1,98	0	-5,58	0	0

Table 7: *Flex(i,t)* for each load in each period

D Problem 3

D.1 With constraints from problem 1

D.1.1 Power consumption of the installation

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	TOTAL
1	158,4	160,38	132,84	154,44	146,52	128,7	110,88	100,98	91,08	83,16	76,56	69,96	63,36	61,38	48,6	55,08	61,56	72,36	87,78	93,96	103,68	113,4	123,12	132,84	2431,02
2	230,58	234,36	215,46	228,69	219,24	189	158,76	143,64	128,52	117,18	109,62	102,06	94,5	88,83	75,24	85,5	95,76	112,86	129,96	147,06	165,87	184,68	203,49	222,3	3683,16
3	214,78	179,5	248,26	227,11	235,12	225	189	171	153	139,5	130,5	121,5	112,5	105,75	59,4	95,35	100,8	118,8	136,8	154,8	174,6	194,4	214,2	234	3935,67
4	194,4	198	201,6	190,8	180	158,4	136,8	122,4	108	100,8	93,6	86,4	79,2	75,6	72	81	67,5	80,1	92,7	105,3	116,1	126,9	137,7	148,5	2953,8
5	51,84	77,76	51,84	48,96	69,12	60,48	51,84	41,58	35,64	34,56	33,12	31,68	26,46	26,46	23,94	25,65	27,36	30,78	34,2	37,62	42,93	50,22	57,51	64,8	1036,35
TOTAL	850	850	850	850	850	761,58	647,28	579,6	516,24	475,2	443,4	411,6	376,02	358,02	279,18	342,58	352,98	414,9	481,44	538,74	603,18	669,6	736,02	802,44	

Table 8: Power consumption of the installation

D.1.2 Error of the power consumption of the installation

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	TOTAL
1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,05	0,1	0,1	0,1	0,1	0,1	0,007886
2	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,003039
3	0,021949	0,195789	0,094621	0,042746	0,126054	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,059444	0	0	0	0	0	0	0	0	0,071806
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,25	0,08985
5	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,1	0,1	0,2	0,2	0,2	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,1	0,1	0,1	0,1	0,002166
TOTAL	0,009022	0,005848	0,020286	0,02657	0,078133	0,110499	0,109877	0,10274	0,103077	0,109244	0,10961	0,110032	0,099474	0,098895	0,098256	0,018958	0,09213	0,09252	0,084018	0,09303	0,095547	0,094891	0,094352	0,093902	

Table 9: Error in percentage (%), between the initial power demand (Pd) and the final(Pf)

D.1.3 Table for *Flex(i,t)* variable

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	14,4	14,58	-14,76	14,04	13,32	11,7	10,08	9,18	8,28	7,56	6,96	6,36	5,76	5,58	-5,4	-6,12	-6,84	-8,04	-4,62	-10,44	-11,52	-12,6	-13,68	-14,76
2	10,98	11,16	-11,34	10,89	10,44	9	7,56	6,84	6,12	5,58	5,22	4,86	4,5	4,23	-3,96	-4,5	-5,04	-5,94	-6,84	-7,74	-8,73	-9,72	-10,71	-11,7
3	-4,82	-43,7	21,46	9,31	26,32	45	37,8	34,2	30,6	27,9	26,1	24,3	22,5	21,15	-19,8	5,35	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-22,5	-26,7	-30,9	-35,1	-38,7	-42,3	-45,9	-49,5
5	-12,96	12,96	-12,96	-12,24	11,52	10,08	8,64	3,78	3,24	5,76	5,52	5,28	1,26	1,26	-1,26	-1,35	-1,44	-1,62	-1,8	-1,98	-4,77	-5,58	-6,39	-7,2

Table 10: *Flex(i,t)* for each load in each period

D.2 With constraints from problem 2

D.2.1 Power consumption of the installation

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	TOTAL	Total
1	158,4	131,22	141,34	151,51	146,52	128,7	110,88	100,98	91,08	83,16	76,56	69,96	63,36	61,38	59,4	67,32	75,24	88,44	87,78	101,02	126,72	138,6	123,12	132,84	**2515,53**
2	212,8	232,78	215,46	228,69	219,24	189	158,76	143,64	128,52	117,18	109,62	102,06	94,5	88,83	83,16	94,5	105,84	124,74	129,96	147,06	183,33	189,88	203,49	222,3	**3725,34**
3	219,6	223,2	226,8	217,8	208,8	180	151,2	136,8	122,4	111,6	104,4	97,2	90	84,6	79,2	90	100,8	118,8	136,8	154,8	174,6	194,4	214,2	234	**3672**
4	194,4	198	201,6	190,8	180	158,4	136,8	122,4	108	100,8	93,6	86,4	79,2	75,6	72	81	90	106,8	123,6	140,4	154,8	126,9	137,7	148,5	**3107,7**
5	64,8	64,8	64,8	61,2	57,6	50,4	43,2	37,8	32,4	28,8	27,6	26,4	25,2	25,2	25,2	27	28,8	32,4	36	39,6	47,7	50,22	57,51	64,8	**1019,43**
TOTAL	**850**	**850**	**850**	**850**	**812,16**	**706,5**	**600,84**	**541,62**	**482,4**	**441,54**	**411,78**	**382,02**	**352,26**	**335,61**	**318,96**	**359,82**	**400,68**	**471,18**	**514,14**	**582,88**	**687,15**	**700**	**736,02**	**802,44**	

Table 11: Power consumption of the installation

D.2.2 Error of the power consumption of the installation

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total
1	0,1	0,1	0,042412	0,079131	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,05	0,032375	0,1	0,1	0,1	0,1	**0,042923**
2	0,030965	0,042921	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,023251	0,05	0,05	**0,014526**
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1,24E-16**
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,25	0,25	0,25	**0,042429**
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,1	0,1	0,1	**0,018458**
Total	**0,009022**	**0,005848**	**0,020286**	**0,02657**	**0,030137**	**0,030184**	**0,030247**	**0,030479**	**0,030769**	**0,030672**	**0,03048**	**0,030259**	**0,03**	**0,03011**	**0,030233**	**0,030412**	**0,030556**	**0,030577**	**0,021804**	**0,018721**	**0,030364**	**0,053798**	**0,094352**	**0,093902**	

Table 12: Error in percentage (%), between the initial power demand (Pd) and the final(Pf)

D.2.3 Table for ***Flex(i,t)*** variable

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	14,4	-14,58	-6,26	11,11	13,32	11,7	10,08	9,18	8,28	7,56	6,96	6,36	5,76	5,58	5,4	6,12	6,84	8,04	-4,62	-3,38	11,52	12,6	-13,68	-14,76
2	-6,8	9,58	-11,34	10,89	10,44	9	7,56	6,84	6,12	5,58	5,22	4,86	4,5	4,23	3,96	4,5	5,04	5,94	-6,84	-7,74	8,73	-4,52	-10,71	-11,7
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-42,3	-45,9	-49,5
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-5,58	-6,39	-7,2

Table 13: *Flex(i,t)* for each load in each period