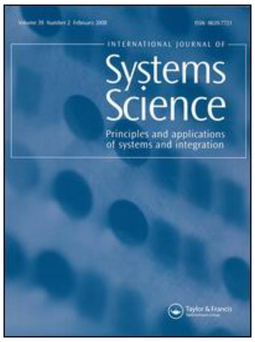

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Predefined-time integral sliding mode control of second-order systems

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ABSTRACT
This manuscript introduces the design of a controller that ensures predefined-time convergence for a class of second-order systems. In contrast to finite- and fixed-time controllers, predefined-time schemes allow to prescribe a bound for the convergence time as a control parameter. First, a predefined-time integral sliding mode controller allows rejecting unknown but bounded matched disturbances. Then, the system dynamics evolve free of the effect of disturbances during the integral sliding motion. Finally, an ideal controller enforces convergence also in predefined-time. A Lyapunov-like characterisation for predefined-time stability is conducted, and numerical results are provided to illustrate the validity of the proposed technique.

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Predefined-time stability; integral sliding mode control; Lyapunov stability

1. Introduction

For control, observation and/or estimation, the performance constraints are usually related to fast responses, while being robust to uncertainties, such as external disturbances and parameter variations. For those cases, sliding mode algorithms are ones of the most promising methods (Chu et al., 2019; Drakunov & Utkin, 1992; J. Fei & Chen, 2020; Y. Fei et al., 2020; Galván-Guerra et al., 2018; Rahmani et al., 2020; Sun & Zheng, 2017; Utkin, 1992).

A primary feature of sliding mode control is the finite-time stability (Bhat & Bernstein, 2000; J. Fei & Feng, 2020; Moulay & Perruquetti, 2006; Roxin, 1966; Yu & Ji, 2018; Zhao et al., 2018). However, the stabilisation time is often an unbounded function of the system initial conditions. To overcome this drawback, making the settling time-bounded for any initial condition, a stronger form of stability, called fixed-time stability, was introduced by Andrieu et al. (2008) for homogeneous systems, and by Cruz-Zavala et al. (2010), Polyakov (2012), Polyakov and Fridman (2014) for systems with sliding modes. The settling time of fixed-time stable systems presents uniformity with respect to initial conditions. Successful applications of fixed-time controllers are available in literature (Li & Cai, 2017; Ning et al., 2017; Zeng et al., 2019; Zhang et al., 2019; Zuo, 2015). The references Sánchez-Torres et al. (2014), Jiménez-Rodríguez et al. (2017a), Sánchez-Torres, Gómez-Gutiérrez, et al. (2018) and Sánchez-Torres et al. (2020) analyse a class of systems where an upper bound of the stabilisation time is a tunable parameter. This notion is related to the Posicast controller given in Smith (1957), Tallman and Smith (1958) and the fixed terminal point regulator problem presented in Rekasius (1964). The exposed structural advantage allows coping with problems related to the estimation of the convergence time.

Predefined-time stability assures that the state of a dynamical system converges to zero before time T_c, which appears as a tunable parameter during the control design. This feature provides a high degree of certainty on the system performance, and it is potentially applicable to a large variety of engineering systems, in which, time constraints need to be fulfilled; for instance, robotic manipulators, tracking systems, flexible manufacturing processes, satellite systems and/or camera-based systems, to name a few.

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Recent efforts to design predefined-time controllers for high-order systems are exposed in Jiménez-Rodríguez et al. (2017b, 2017c). However, the presence of singularities in the closed-loop dynamics makes the use of these controllers restricted to particular cases.

The contribution of this paper is the design of a novel controller with predefined-time convergence based on an integral sliding mode. In contrast to the mentioned methods, the current proposal allows designing a non-singular controller without the use of any switching between regimes. Besides, the integral action makes the closed-loop system to be insensitive to a wide class of perturbations (Matthews & DeCarlo, 1988; Utkin & Shi, 1996). With this aim, a generalised Lyapunov condition for predefined-time stability is presented. Then, a second-order sliding mode controller is proposed. Compared to existing works, the proposed scheme has the following advantages: (i) the upper bound for the convergence time can be easily tuned, (ii) an accurate estimation of the upper bound for the convergence time is obtained since no time is neglected. Finally, through the paper, some rigorous proofs are provided the demonstrate the validity of the proposed method.

The outline of the paper is as follows. In Section 2, some basics on predefined-time stability and on gamma function are recalled. The main results concerning the Lyapunov characterisation of predefined-time stability are given in Section 3. Based on this characterisation, a robust controller is derived for second-order systems with bounded matched perturbations. Numerical simulations are presented in Section 4 to illustrate the effectiveness of the proposed controller.

2. Preliminaries

2.1. On predefined-time stability

Consider the system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}; \boldsymbol{\rho}), \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the system state. The vector $\boldsymbol{\rho} \in \mathbb{R}^b$ stands for the parameters of system (1), which are assumed to be constant, i.e. $\dot{\boldsymbol{\rho}} = 0$, where b is the number of system parameters. The function $\boldsymbol{f} : \mathbb{R}^n \to \mathbb{R}^n$ is nonlinear, and the origin is assumed to be an equilibrium point of system (1), so $\boldsymbol{f}(\boldsymbol{0}; \boldsymbol{\rho}) = \boldsymbol{0}$. The initial conditions of this system are $\boldsymbol{x}_0 = \boldsymbol{x}(0) \in \mathbb{R}^n$.

Definition 2.1 (Finite-time stability Bhat and Bernstein (2000)): The origin of (1) is *globally finite-time stable* if it is globally asymptotically stable and any solution $\boldsymbol{x}(t, \boldsymbol{x}_0)$ of (1) reaches the equilibrium point at some finite time moment, i.e. $\forall t \geq T(\boldsymbol{x}_0) : \boldsymbol{x}(t, \boldsymbol{x}_0) = \boldsymbol{0}$, where $T : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ is called the *settling-time function*.

Definition 2.2 (Fixed-time stability Polyakov (2012)): The origin of (1) is *fixed-time stable* if it is globally finite-time stable and the settling-time function is bounded, i.e. $\exists T_{\max} > 0 : \forall \boldsymbol{x}_0 \in \mathbb{R}^n,\ T(\boldsymbol{x}_0) \leq T_{\max}$.

Definition 2.3 (Bound set Sánchez-Torres et al. (2014)): Let the origin of system (1) be fixed-time-stable. The set of all the bounds of the settling-time function is defined as

$$\mathcal{T} = \{T_{\max} \in \mathbb{R}_+ : T(\boldsymbol{x}_0) \leq T_{\max},\ \forall\, \boldsymbol{x}_0 \in \mathbb{R}^n\}.$$

Remark 2.1: For some engineering applications such as state estimation, dynamic optimisation, fault detection, among others, it would be convenient that the trajectories of (1) reach the origin within a time $T_c \in \mathcal{T}$, which can be defined in advance during the system design.

Definition 2.4 (Predefined-time stability Sánchez–Torres, Gómez-Gutiérrez, et al. (2018)): For the system parameters $\boldsymbol{\rho}$ and a constant $T_c := T_c(\boldsymbol{\rho}) > 0$, the origin of (1) is said to be *predefined-time-stable* for system (1) if it is fixed-time-stable and the settling-time function $T : \mathbb{R}^n \to \mathbb{R}$ is such that

$$T(\boldsymbol{x}_0) \leq T_c \quad \forall \boldsymbol{x}_0 \in \mathbb{R}^n.$$

If this is the case, T_c is called a *predefined-time*.

Lemma 2.5 (Finite-time stability characterisation for scalar systems (Haimo, 1986, Fact 1)): *Let $n = 1$ in system (1) (scalar system). The origin is globally finite-time stable if and only if for all $x \in \mathbb{R} \setminus \{0\}$*

(i) $xf(x; \rho) < 0$, *and*
(ii) $\int_x^0 \frac{\mathrm{d}z}{f(z;\rho)} < +\infty$.

Remark 2.2: A proof can be found in Moulay and Perruquetti (2008, Lemma 3.1). Nevertheless, intuitively, condition (i) implies Lyapunov stability. Let us note the settling time function as $T(x_0) = \int_0^{T(x_0)} \mathrm{d}t$. Since first-order systems do not oscillate, the solution $x(\cdot, x_0) : [0, T(x_0)) \to [x_0, 0)$ of system (1) as a function of t defines a bijection. Using it as a change of variable, the following equality holds:

$$T(x_0) = \int_0^{T(x_0)} \mathrm{d}t = \int_{x_0}^{0} \frac{\mathrm{d}x}{f(x;\rho)}. \tag{2}$$

Note that $\frac{1}{f(x;\rho)}$ is defined for all $x \in \mathbb{R}^n \setminus \{0\}$ from condition (i). Thus, condition (ii) of Lemma 2.5 implies that the settling-time function is finite.

2.2. On the incomplete gamma function inverse and the upper right-hand derivative

Let us recall the definition of the *gamma* function.

Definition 2.6 (Gamma function Bateman and Erdélyi (1955) and Abromowitz and Stegun (1965)): Let $a > 0$. The *gamma* function is defined as

$$\Gamma(a) = \int_0^{\infty} t^{a-1} \exp(-t)\,\mathrm{d}t. \tag{3}$$

Splitting the integral (3) at a point $x \geq 0$, two incomplete gamma functions are obtained. This motivates the following definitions.

Definition 2.7 (Incomplete gamma function Bateman and Erdélyi (1955) and Abromowitz and Stegun (1965)): Let $a > 0$ and $x \geq 0$. The *incomplete gamma function* is defined as

$$\Gamma(a, x) = \int_x^{\infty} t^{a-1} \exp(-t)\,\mathrm{d}t.$$

Definition 2.8 (Regularised incomplete gamma function Bateman and Erdélyi (1955) and Abromowitz and Stegun (1965)): Let $a > 0$ and $x \geq 0$. The *regularised incomplete gamma function* is defined as

$$Q(a, x) = \frac{\Gamma(a, x)}{\Gamma(a)}.$$

Definition 2.9 (Regularised incomplete gamma function inverse Bateman and Erdélyi (1955) and Abromowitz and Stegun (1965)): Let $a > 0$ and $x \geq 0$. The *regularised incomplete gamma function inverse* $Q^{-1}(a, \cdot) : (0, 1] \to [0, \infty)$, is defined as the unique function satisfying $Q^{-1}(a, Q(a, x)) = x$.

Remark 2.3: Note that $Q(1, x) = \exp(-x)$, $Q(a, 0) = 1$, and $Q(a, x) \to 0$ as $x \to \infty$. Consequently, from Definition 2.9, $Q^{-1}(a, 1) = 0$.

Definition 2.10 (Upper right-hand derivative Kannan and Krueger (2012)): Let ϕ a continuous function on $\mathbb{R}$, then $D^+ f(x)$ denotes the Dini upper right-hand derivative of $\phi(x)$, defined as the following one-sided limit:

$$D^+\phi(x) = \limsup_{h \to 0^+} \frac{\phi(x+h) - \phi(x)}{h}.$$

3. Main results

Consider the system

$$\dot{x} = -\exp(\alpha\,|x|^{mp})\,\lfloor x \rceil^{m(\beta q - 1)+1}, \tag{4}$$

where $x \in \mathbb{R}$, $\lfloor x \rceil = |x|\,\mathrm{sign}(x)$, with the parameters $m > 1, \alpha > 0, \beta > 0, p > 0, q > 0$ such that $\beta q < 1$.

The following theorem provides the least upper bound of the settling-time function of system (4):

Theorem 3.1: *The origin $x = 0$ of system* (4) *is fixed-time stable and the settling time function satisfies* $\sup_{x_0 \in \mathbb{R}^n} T(x_0) = \frac{\alpha^{(\beta q - 1)/p}\Gamma((1-\beta q)/p)}{mp}$.

Proof: Note that for system (4), the field is $f(x;\rho) = -\exp(\alpha\,|x|^{mp})\,\lfloor x \rceil^{m(\beta q - 1)+1}$, where the parameter vector is $\rho = [\alpha \;\; \beta \;\; p \;\; q \;\; m]^{\mathrm{T}} \in \mathbb{R}^5$. Furthermore, the product $xf(x;\rho) = -\exp(\alpha\,|x|^{mp})\,|x|^{m(\beta q - 1)+1} < 0$ for all $x \in \mathbb{R} \setminus \{0\}$. Thus, $V(x) = \frac{1}{2}x^2$ is a Lyapunov function for system (4), so its origin $x = 0$ is Lyapunov stable (Khalil Grizzle, 2002, Theorem 4.2).

Now, let $x_0 \in \mathbb{R} \setminus \{0\}$ (if $x_0 = 0$, then $x(t, 0) = 0$ is the unique solution of (4) and $T(0) = 0$). From (2), the settling time function is

$$\begin{aligned} T(x_0) &= \int_{x_0}^{0} \frac{\mathrm{d}x}{f(x;\rho)} \\ &= \int_0^{x_0} \frac{\mathrm{d}x}{\exp(\alpha\,|x|^{mp})\,\lfloor x \rceil^{m(\beta q - 1)+1}}, \qquad z = |x| \\ &= \int_0^{|x_0|} \frac{\mathrm{d}z}{\exp(\alpha z^{mp}) z^{m(\beta q - 1)+1}}. \end{aligned}$$

Since $\frac{\mathrm{d}z}{\exp(\alpha z^{mp}) z^{m(\beta q - 1)+1}}$ is positive for $z \in (0, |x_0|)$, the settling time function is increasing with respect to $|x_0|$.

Hence, the least upper bound of $T(x_0)$ is obtained as

$$\sup_{x_0\in\mathbb{R}} T(x_0) = \lim_{|x_0|\to+\infty} T(x_0) = \int_0^{+\infty} \frac{\mathrm{d}z}{\exp(\alpha z^{mp}) z^{m(\beta q-1)+1}} = \frac{\alpha^{(\beta q-1)/p}\Gamma\left(\frac{1-\beta q}{p}\right)}{mp}.$$

Then, using Lemma 2.5 and by the definitions of fixed-time stability, the origin $x = 0$ of system (4) is fixed-time stable and $T_{\max} = \frac{\alpha^{(\beta q-1)/p}\Gamma((1-\beta q)/p)}{mp}$. ■

Theorem 3.2: *The system*

$$\dot{x} = -\frac{\alpha^{(\beta q-1)/p}\Gamma\left(\frac{1-\beta q}{p}\right)}{mpT_c} \exp(\alpha\, |x|^{mp})\, \lfloor x\rceil^{m(\beta q-1)+1} \tag{5}$$

with $T_c > 0$ is predefined-time stable, with a predefined-time T_c.

Proof: The proof follows directly from Theorem 3.1. ■

Consider now the candidate Lyapunov function for system (5) as $V(x) = |x|^m$. Then,

$$D^+V(x) = -\frac{\alpha^{(\beta q-1)/p}\Gamma(\frac{1-\beta q}{p})}{pT_c} \exp(\alpha\, |x|^{mp})\, |x|^{m\beta q} = -\frac{\alpha^{(\beta q-1)/p}\Gamma(\frac{1-\beta q}{p})}{pT_c} \exp(\alpha V(x)^p) V(x)^{\beta q}.$$

This motivates the following subsection which presents a characterisation of predefined-time stability based on the Lyapunov approach.

3.1. A Lyapunov characterisation of predefined-time stability

Motivated by Sánchez-Torres, Defoort, et al. (2018), one can present the following theorem which introduces a Lyapunov characterisation of predefined-time stable systems.

Theorem 3.3: *If there exists a continuous radially unbounded function $V : \mathbb{R}^n \to \mathbb{R}$ such that*

(i) *$V(\boldsymbol{x}) = 0$ if and only if $\boldsymbol{x} = \boldsymbol{0}$,*
(ii) *$V(\boldsymbol{x}) \geq 0$, and*
(iii) *any solution $\boldsymbol{x}(t)$ of (1) satisfies*

$$D^+V(\boldsymbol{x}) \leq -\frac{\alpha^{(\beta q-1)/p}\Gamma\left(\frac{1-\beta q}{p}\right)}{pT_c} \times \exp\left(\alpha V(\boldsymbol{x})^p\right) V(\boldsymbol{x})^{\beta q} \tag{6}$$

for $\boldsymbol{x} \in \mathbb{R}^n \setminus \{\boldsymbol{0}\}$ and constants $T_c := T_c(\boldsymbol{\rho}) > 0$, $\alpha > 0$, $\beta > 0$, $p > 0$, $q > 0$ such that $\beta q < 1$.

Then, the origin of system (1) is predefined-time stable with predefined time equal to T_c.

Proof: From the differential inequality (6), the Lyapunov function satisfies $V(\boldsymbol{x}(t)) \leq [\frac{1}{\alpha}Q^{-1}(\frac{1-\beta q}{p}, \frac{t-t_0}{T_c} + Q(\frac{1-\beta q}{p}, -V(\boldsymbol{x}_0)^p))]^{1/p}$. Thus, from Remark 2.3, the settling-time function for system (1) complies with $T(\boldsymbol{x}_0) \leq T_c[1 - Q(\frac{1-\beta q}{p}, -V(\boldsymbol{x}_0)^p)] \leq T_c, \forall \boldsymbol{x}_0 \in \mathbb{R}^n$. Hence, the origin of system (1) is predefined-time-stable, with a predefined-time T_c. ■

The following corollary exposes how Theorem 3.3 generalises the results proposed in Sánchez-Torres et al. (2014) and Sánchez-Torres, Gómez-Gutiérrez, et al. (2018).

Corollary 3.4 (See Sánchez-Torres, Gómez-Gutiérrez, et al. (2018)): *If there exists a continuous radially unbounded function $V : \mathbb{R}^n \to \mathbb{R}$ such that*

(i) *$V(\boldsymbol{x}) = 0$ if and only if $\boldsymbol{x} = \boldsymbol{0}$,*
(ii) *$V(\boldsymbol{x}) \geq 0$, and*
(iii) *any solution $\boldsymbol{x}(t)$ of (1) satisfies*

$$D^+V(\boldsymbol{x}) \leq -\frac{1}{pT_c} \exp(V(\boldsymbol{x})^p) V(\boldsymbol{x})^{1-p} \tag{7}$$

for $\boldsymbol{x} \in \mathbb{R}^n \setminus \{\boldsymbol{0}\}$ and, constants $T_c := T_c(\boldsymbol{\rho}) > 0$ and $0 < p \leq 1$.

Then, the origin of system (1) is predefined-time-stable, with a predefined time T_c.

Proof: (See Sánchez-Torres, Gómez-Gutiérrez, et al. (2018)) From the differential inequality (7), $V(\boldsymbol{x}(t))$ satisfies $V(\boldsymbol{x}(t)) \leq [\ln(\frac{1}{(t-t_0)/T_c+\exp(-V(\boldsymbol{x}_0)^p)})]^{1/p}$. Thus, the settling-time function for system (1) complies with $T(\boldsymbol{x}_0) \leq T_c[1 - \exp(-V(\boldsymbol{x}_0)^p)] \leq T_c, \forall \boldsymbol{x}_0 \in$

$\mathbb{R}^n$. Hence, the origin of system (1) is predefined-time-stable, with a predefined-time T_c.

Similarly, the proof of this corollary easily follows from Theorem 3.3 with $\alpha = \beta = 1$ and $p = 1 - q$ and considering Remark 2.3. ■

Remark 3.1: With a similar approach to the previous one, it follows the predefined-time characterisation presented in Jiménez-Rodríguez et al. (2017b).

Remark 3.2: Consider the example $\dot{x} = -\exp(|x|) \lfloor x \rceil^{1/2}$ proposed in Lopez-Ramirez et al. (2018). In this paper, it is shown that this system is fixed-time stable with a fixed time equal to $\sqrt{\pi}$. First, note that the same results is obtained applying Theorem 3.3 with $V(x) = |x|$, $\alpha = p = q = 1$, $\beta = 1/2$ which results in $T_c = \sqrt{\pi}$ since $\Gamma(1/2) = \sqrt{\pi}$. Secondly, the system in its current form does not provides a straightforward approach to select in advance the convergence time. Namely, the time $\sqrt{\pi}$ cannot be easily change to another value. For this reason, the given system cannot be considered yet as predefined-time stable. However, this constraint is easily removed applying again Theorem 3.3 with $V(x) = |x|$, $\alpha = p = q = 1$, $\beta = 1/2$ and $T_c > 0$. This procedure leads to the modified system $\dot{x} = -\frac{\sqrt{\pi}}{T_c} \exp(|x|) \lfloor x \rceil^{1/2}$, which is predefined-time stable, with a predefined time T_c. Finally, the modified system is a particular case of the system (5) presented in Example 3.2 with $\alpha = m = p = q = 1$, $\beta = 1/2$ and $T_c > 0$.

3.2. Predefined time stabilisation of uncertain second-order systems

Consider a second-order system in regular form with a matched disturbance

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= u + \Delta, \end{aligned} \tag{8}$$

where $x = [x_1, x_2]^{\mathrm{T}} \in \mathbb{R}^2$ is the state, $u \in \mathbb{R}$ is the control input and $\Delta \in \mathbb{R}$ is the perturbation which is assumed to be bounded by a known constant, i.e. $|\Delta| < \delta$.

The first step is to design a controller which rejects the effect of Δ for all instant $t \geq T_{c_0}$, where $T_{c_0} < 0$ is a predefined time. Thus, consider u_1 as the robust control term which compensates for Δ, and u_0 the controller that enforces the sliding motion (x_1, x_2) for system (8) with $\Delta = 0$. In this sense, as in Utkin and Shi (1996), consider the integral sliding variable

$$\begin{aligned} s &= x_2 + z, \\ \dot{z} &= -u_0. \end{aligned} \tag{9}$$

It yields

$$\begin{aligned} \dot{s} &= \dot{x}_2 - u_0, \\ &= u + \Delta - u_0. \end{aligned} \tag{10}$$

The controller is the sum of the ideal controller u_0 and the robust controller u_1, i.e.

$$u = u_0 + u_1. \tag{11}$$

Hence, one has

$$\dot{s} = u_1 + \Delta. \tag{12}$$

In the following lemma, the design of the robust controller u_1 which ensures the establishment of the sliding motion before the predefined-time T_{c_0}, is introduced. Therefore, the equivalent term $u_{1,eq}$ can be seen as a sliding mode disturbance observer since in sliding mode $u_{1,eq} = -\Delta$ for all $t \geq T_{c_0}$. Therefore, in sliding mode, the system behaves free of the effect of the disturbance Δ.

Lemma 3.5: *Let the following robust controller*:

$$u_1 = -\gamma_0 \exp\left(\frac{1}{2}|s|^{p_0}\right) \lfloor s \rceil^{q_0/2} - \kappa \operatorname{sign}(s), \tag{13}$$

with $\gamma_0 = 2^{(1-q_0/2)/p_0} \Gamma(\frac{1-q_0/2}{p_0})/p_0 T_{c_0}$ *and* $\kappa > \delta$, *depending on* $T_{c_0} > 0$, $p_0 > 0$ *and* $1 \leq q_0 < 2$. *Therefore, system* (12) *closed by* (13) *is predefined-time stable, with a predefined time* T_{c_0} *despite the effect of* Δ.

Proof: The dynamics of the integral sliding variable s is

$$\dot{s} = -\gamma_0 \exp\left(\tfrac{1}{2}|s|^{p_0}\right) \lfloor s \rceil^{q_0/2} - \kappa \operatorname{sign}(s) + \Delta.$$

To analyse the stability of $s = 0$, let the candidate Lyapunov function be $V_0 = |s|$. Then, for

$s \neq 0$,

$$\begin{aligned} D^+V_0 &= \operatorname{sign}(s)\dot{s} \\ &= -\gamma_0 \exp\left(\frac{1}{2}|s|^{p_0}\right)|s|^{q_0/2} - \kappa + \Delta \operatorname{sign}(s) \\ &\leq -\frac{2^{(1-q_0/2)/p_0}\Gamma\left(\frac{1-q_0/2}{p_0}\right)}{p_0 T_{c0}} \\ &\quad \times \exp\left(\frac{1}{2}V_0^{p_0}\right)V_0^{q_0/2} \end{aligned} \tag{14}$$

which implies the sliding mode $s = 0$ for $t \geq T_{c_0}$. ■

The following lemma presents a controller that stabilises system (8) in predefined-time for $\Delta = 0$, that is $(x_1, x_2) = (x_1, \dot{x}_1) = (0, 0)$. Therefore, the proposed scheme is a second-order sliding mode controller with predefined-time convergence. The basis of the controller is the sliding variable proposed in Polyakov (2012). The main advantage of this proposal is that despite the singularity presented in the dynamics of this variable, it is possible to design a non-singular controller to stabilise it.

Lemma 3.6: *Let the following control input for system* (8) *with* $u_1 = \Delta = 0$

$$\begin{aligned} u_0 = &-\gamma_1^2(q_1 + p_1|x_1|^{p_1})|x_1|^{q_1-1} \\ &\times \exp(|x_1|^{p_1})\operatorname{sign}(\sigma) \\ &- \gamma_2 \exp(\alpha_2|\sigma|^{p_2})\lfloor\sigma\rceil^{\beta_2 q_2}, \end{aligned} \tag{15}$$

where $\sigma = x_2 + \left\lfloor \lfloor x_2\rceil^2 + 2\gamma_1^2 \exp(|x_1|^{p_1})\lfloor x_1\rceil^{q_1}\right\rceil^{1/2}$, *with parameters* $\gamma_1 = 2^{(1-q_1/2)/p_1}\Gamma(\frac{1-q_1/2}{p_1})/(p_1 T_{c_1})$, *and* $\gamma_2 = \alpha_2^{(\beta_2 q_2-1)/p_2}\Gamma(\frac{1-\beta_2 q_2}{p_2})/(p_2 T_{c_2})$, *depending on* $T_{c_1} > 0, p_1 > 0, 1 \leq q_1 < 2, T_{c_2} > 0, \alpha_2 > 0, \beta_2 > 0, p_2 > 0, q_2 > 0$ *such that* $\beta_2 q_2 < 1$.

Therefore, system (8) *closed by* (15) *is predefined-time stable, with a predefined time* $T_{c_1} + T_{c_2}$.

Proof: The dynamics of the variable σ is given by

$$\begin{aligned} \dot{\sigma} &= u + \frac{|x_2|\,u + \gamma_1^2(q_1 + p_1|x_1|^{p_1})|x_1|^{q_1-1}\exp(|x_1|^{p_1})x_2}{\left|\lfloor x_2\rceil^2 + 2\gamma_1^2\exp(|x_1|^{p_1})\lfloor x_1\rceil^{q_1}\right|^{1/2}} \\ &= -\gamma_1^2(q_1 + p_1|x_1|^{p_1})|x_1|^{q_1-1} \\ &\quad \times \exp(|x_1|^{p_1})\operatorname{sign}(\sigma) - \gamma_2\exp(\alpha_2|\sigma|^{p_2})\lfloor\sigma\rceil^{\beta_2 q_2} \\ &\quad - \frac{|x_2|\,\gamma_2\exp(\alpha_2|\sigma|^{p_2})\lfloor\sigma\rceil^{\beta_2 q_2}}{\left|\lfloor x_2\rceil^2 + 2\gamma_1^2\exp(|x_1|^{p_1})\lfloor x_1\rceil^{q_1}\right|^{1/2}} \\ &\quad - \frac{\gamma_1^2(q_1 + p_1|x_1|^{p_1})|x_1|^{q_1-1}\exp(|x_1|^{p_1})(|x_2|\operatorname{sign}(\sigma) - x_2)}{\left|\lfloor x_2\rceil^2 + 2\gamma_1^2\exp(|x_1|^{p_1})\lfloor x_1\rceil^{q_1}\right|^{1/2}}. \end{aligned}$$

Thus, to analyse the stability of the variable σ, let the candidate Lyapunov function $V_2 = |\sigma|$. Then, for $\sigma \neq 0$,

$$\begin{aligned} D^+V_2 &= \dot{\sigma}\operatorname{sign}(\sigma) \\ &= -\gamma_1^2(q_1 + p_1|x_1|^{p_1})|x_1|^{q_1-1}\exp(|x_1|^{p_1}) \\ &\quad - \gamma_2\exp(\alpha_2|\sigma|^{p_2})|\sigma|^{\beta_2 q_2} \\ &\quad - \frac{|x_2|\,\gamma_2\exp(\alpha_2|\sigma|^{p_2})|\sigma|^{\beta_2 q_2}}{\left|\lfloor x_2\rceil^2 + 2\gamma_1^2\exp(|x_1|^{p_1})\lfloor x_1\rceil^{q_1}\right|^{1/2}} \\ &\quad - \frac{\gamma_1^2(q_1 + p_1|x_1|^{p_1})|x_1|^{q_1-1}\exp(|x_1|^{p_1})(|x_2| - x_2\operatorname{sign}(\sigma))}{\left|\lfloor x_2\rceil^2 + 2\gamma_1^2\exp(|x_1|^{p_1})\lfloor x_1\rceil^{q_1}\right|^{1/2}}. \end{aligned}$$

Hence

$$\begin{aligned} D^+V_2 &\leq -\gamma_2\exp(\alpha_2|\sigma|^{p_2})|\sigma|^{\beta_2 q_2} \\ &\leq -\frac{\alpha_2^{(\beta_2 q_2-1)/p_2}\Gamma\left(\frac{1-\beta_2 q_2}{p_2}\right)}{p_2 T_{c_2}}\exp(\alpha_2 V_2^{p_2})V_2^{\beta_2 q_2} \end{aligned}$$

which implies the sliding mode $\sigma = 0$ for $t \geq T_{c_2}$.

During the sliding motion $\sigma = 0$, system (8) reduces to

$$\dot{x}_1 = -\gamma_1\exp\left(\frac{1}{2}|x_1|^{p_1}\right)\lfloor x_1\rceil^{q_1/2}. \tag{16}$$

To analyse the stability of the sliding dynamics (16), let the candidate Lyapunov function be $V_1 = |x_1|$. Then, for $x_1 \neq 0$,

$$\begin{aligned} D^+V_1 &= \mathrm{sign}(x_1)\dot{x}_1 \\ &= -\gamma_1 \exp\left(\frac{1}{2}|x_1|^{p_1}\right)|x_1|^{q_1/2} \\ &= -\frac{2^{(1-q_1/2)/p_1}\Gamma\left(\frac{1-q_1/2}{p_1}\right)}{p_1 T_{c_1}} \\ &\quad \times \exp\left(\frac{1}{2}V_1^{p_1}\right)V_1^{q_1/2} \end{aligned} \tag{17}$$

which implies the sliding mode $x_1 = 0$. Besides, since $\sigma = 0$, $x_2 = 0$ is maintained for any $t \geq T_{c_1} + T_{c_2}$. ■

Remark 3.3: Theorem 3.3 is very important since it allows the construction of the second-order sliding mode controller presented in Lemma 3.6. Since $2 > q_1 \geq 1$, no singularity occurs in the control input (15), as x_1 reaches zero. In this form, the generalisation allowed by Theorem 3.3 extends the applicability of Corollary 3.4, since the latter does not provide a straightforward method to the predefined time stabilisation of second-order systems.

The following theorem presents a controller which stabilises system (8) in predefined-time, even in the presence of unknown but bounded disturbances, that is, $(x_1, x_2) = (x_1, \dot{x}_1) = (0, 0)$ for a given $T_c > 0$, which is based on an integral sliding mode-like disturbance observer.

Theorem 3.7: *Let the following control input for system* (8) *with* $\Delta \neq 0$

$$u = u_0 + u_1$$

with the ideal controller u_0 *as in* (15), *and the robust controller* u_1 *as in* (13).

Therefore, system (8) *closed by* (15) *is predefined-time stable, with a predefined time* $T_c = T_{c_0} + T_{c_1} + T_{c_2}$.

Proof: The action of u_1 enforces the integral sliding mode $s = 0$, guaranteeing, in an equivalent sense, that $\dot{x}_2 = u_0$ after the predefined time T_{c_0}. Besides, the nominal controller u_0 assures $(x_1, x_2) = (0, 0)$, at instant $T_{c_1} + T_{c_2}$ after the integral sliding mode $s = 0$ is enforced. Thus, the second-order sliding mode $(x_1, x_2) = (0, 0)$ is enforced and maintained for all $t \geq T_c = T_{c_0} + T_{c_1} + T_{c_2}$. ■

A block diagram showing the flow signals of a second-order dynamical system closed by the proposed scheme is shown in Figure 1.

4. Simulation example

The simulation is performed in Matlab/Simulink, based on the Euler solver with 0.01ms as sampling time.

Consider the inverted pendulum problem whose dynamics is given by

$$ml^2\ddot{\theta} + mgl\sin(\theta) = \tau + d,$$

where $m = 0.15$ kg and $l = 0.2$ m are the mass and length of the pendulum, respectively, $g = 9.81$ m/s^2 is gravitational constant, τ is the control torque, and $d = 0.1\sin(t)$ is an unknown but bounded disturbance.

The control objective is to stabilise the angle θ to $\theta_d = \pi$ rad. Let us define the regulation error as $x_1 = \theta - \pi$. Besides, the torque controller is proposed as

$$\tau = ml^2 u + mgl\sin(\theta),$$

which leads to

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= u + \Delta \end{aligned} \tag{18}$$

for $\Delta = d/(ml^2)$, and u a control term defined as in (15), with $q_0 = q_1 = 1.1$, $q_2 = 0.25$, $p_0 = p_1 = p_2 = 1$, $T_{c_1} = 0.25$, $T_{c_1} = 1.5$, $T_{c_2} = 0.25$, $\alpha_2 = 0.01$ and $\beta_2 = 1$. It is worth noticing that the proposed controller provides the convergence of x_1 and x_2 toward 0 before the predefined-time $T_c = T_{c_0} + T_{c_1} + T_{c_2} = 2$ s. The system initial conditions is set at $\theta(0) = 0$, $\dot{\theta}(0) = 1$.

From the simulation results depicted in Figure 2, one can appreciate the convergence of the regulation error and its derivative within the predefined time T_c. This is provided by virtue of the integral sliding mode which is established in a predefined-time T_{c_0}. When sliding mode occurs, the system evolves invariant to the effect of matched disturbance. Then, the second-order sliding motion is established by the action of the ideal controller.

5. Conclusion

This paper introduced an integral sliding mode-like controller, which guarantees the robust stabilisation of

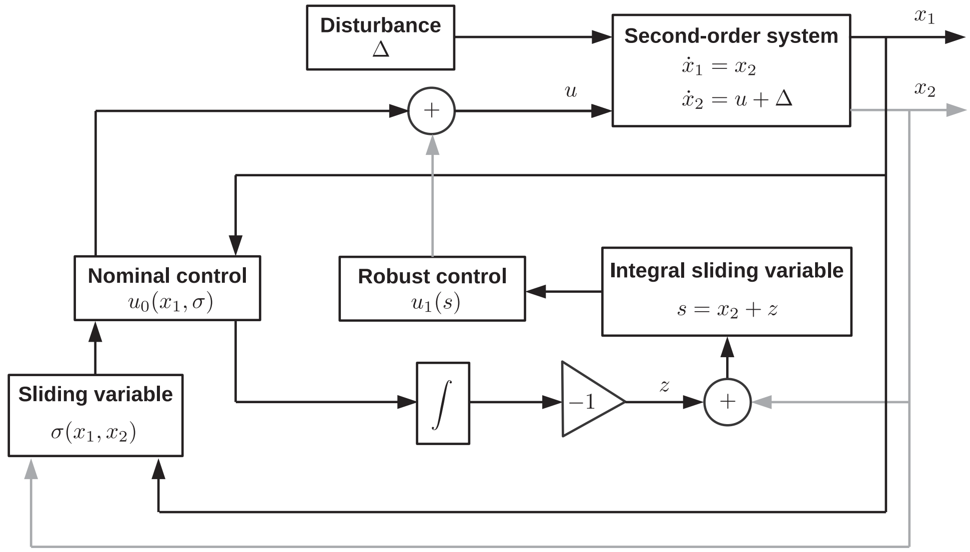

Figure 1. Block diagram of the second-order system under the proposed controller.

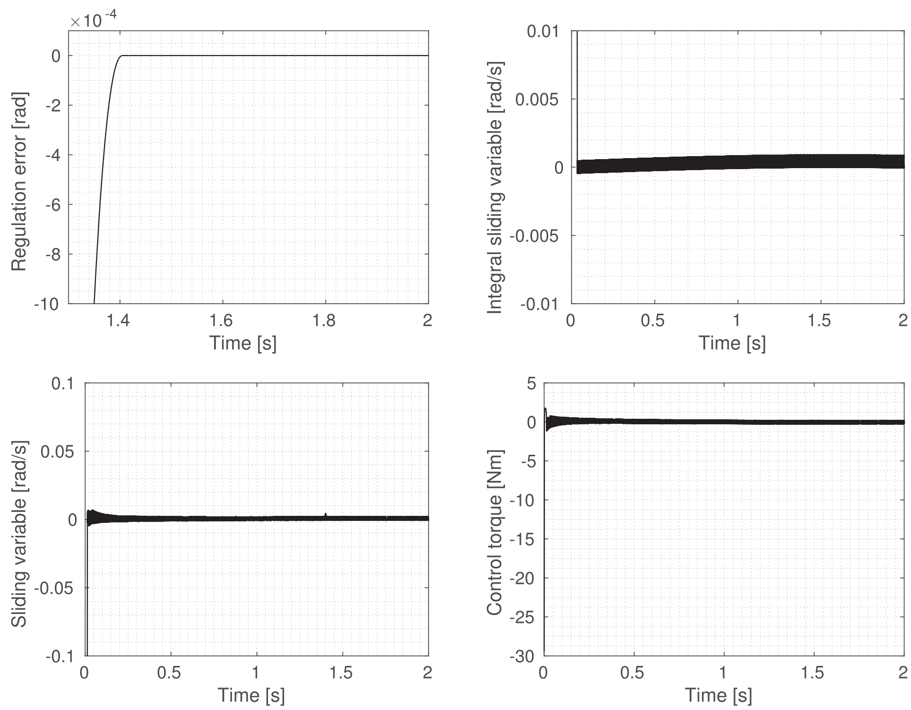

Figure 2. Simulation results.

second-order systems in predefined-time. First, a Lyapunov analysis that allows for the characterisation and design of this controller was presented. Then, based on the theoretical basis provided by the proposed stability analysis, a predefined-time robust integral sliding mode was proposed, and an ideal predefined-time second-order sliding mode controller was designed. The closed-loop system presents the practical advantage that the least upper bound for this settling time is known through an explicit and straightforward relationship with the controller gains. In future, we will investigate the use of norm-normalised sign function to simplify the sliding-mode control design for multi-input systems.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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References

Abromowitz, M., & Stegun, I. A. (1965). *Handbook of mathematical functions: With formulas, graphs, and mathematical tables* (9th ed.). D. Publications.

Andrieu, V., Praly, L., & Astolfi, A. (2008). Homogeneous approximation, recursive observer design, and output feedback. *SIAM Journal on Control and Optimization*, *47*(4), 1814–1850. https://doi.org/10.1137/060675861

Bateman, H., & Erdélyi, A. (1955). *Higher transcendental functions*. McGraw-Hill.

Bhat, S. P., & Bernstein, D. S. (2000). Finite-time stability of continuous autonomous systems. *SIAM Journal on Control and Optimization*, *38*(3), 751–766. https://doi.org/10.1137/S0363012997321358

Chu, Y., Fei, J., & Hou, S. (2019). Adaptive global sliding-mode control for dynamic systems using double hidden layer recurrent neural network structure. *IEEE Transactions on Neural Networks and Learning Systems*, *31*(4), 1297–1309. https://doi.org/10.1109/TNNLS.5962385

Cruz-Zavala, E., Moreno, J., & Fridman, L. (2010, June 26–28). *Uniform second-order sliding mode observer for mechanical systems*. 11th international workshop on Variable Structure Systems (VSS), Mexico City, Mexico (pp. 14–19). IEEE. https://doi.org/10.1109/VSS.2010.5544656

Drakunov, S. V., & Utkin, V. I. (1992). Sliding mode control in dynamic systems. *International Journal of Control*, *55*(4), 1029–1037. https://doi.org/10.1080/00207179208934270

Fei, J., & Chen, Y. (2020). Dynamic terminal sliding-mode control for single-phase active power filter using new feedback recurrent neural network. *IEEE Transactions on Power Electronics*, *35*(9), 9906–9924. https://doi.org/10.1109/TPEL.63

Fei, J., & Feng, Z. (2020). Fractional-order finite-time super-twisting sliding mode control of micro gyroscope based on double-loop fuzzy neural network. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*. https://doi.org/10.1109/TSMC.2020.2979979

Fei, Y., Shi, P., & Lim, C.-C. (2020). Neural network adaptive dynamic sliding mode formation control of multi-agent systems. *International Journal of Systems Science*, *51*(11), 2025–2040. https://doi.org/10.1080/00207721.2020.1783385

Galván-Guerra, R., Fridman, L., Iriarte, R., Velázquez-Velázquez, J., & Steinberger, M. (2018). Integral sliding mode observation and control for switched uncertain linear time invariant systems: A robustifying strategy. *Asian Journal of Control*, *20*(4), 1551–1565. https://doi.org/10.1002/asjc.v20.4

Haimo, V. T. (1986). Finite time controllers. *SIAM Journal on Control and Optimization*, *24*(4), 760–770. https://doi.org/10.1137/0324047

Jiménez-Rodríguez, E., Sánchez-Torres, J. D., & Loukianov, A. G. (2017a). On optimal predefined-time stabilization. *International Journal of Robust and Nonlinear Control*, *27*(17), 3620–3642. https://doi.org/10.1002/rnc.3757

Jiménez-Rodríguez, E., Sanchez-Torres, J. D., & Loukianov, A. G. (2017b, July 9–14). *Predefined-time backstepping control for tracking a class of mechanical systems*. Proceedings of the 20th IFAC World Congress, Toulouse, France.

Jiménez-Rodríguez, E., J. D. Sánchez-Torres, & Loukianov, A. G. (2017c, October, 20–22). *A second order predefined-time control algorithm*. 14th International conference on Electrical Engineering, Computing Science and Automatic Control (CCE), Mexico City, Mexico (pp. 1–6). IEEE. https://doi.org/10.1109/ICEEE.2017.8108833.

Kannan, R., & Krueger, C. (2012). *Advanced analysis: On the real line*. Springer.

Khalil, H. K., & Grizzle, J. (2002). *Nonlinear systems* (Vol. 3). Prentice Hall.

Li, H., & Cai, Y. (2017). On SFTSM control with fixed-time convergence. *IET Control Theory & Applications*, *11*(6), 766–773. https://doi.org/10.1049/iet-cta.2016.1457

Lopez-Ramirez, F., Efimov, D., Polyakov, A., & Perruquett, W. (2018, June, 12–15). *On necessary and sufficient conditions for fixed-time stability of continuous autonomous system*. Proceedings of the 17th European Control Conference (ECC), Limassol, Cyprus. IEEE. https://doi.org/10.23919/ECC.2018.8550571

Matthews, G. P., & DeCarlo, R. A. (1988). Decentralized tracking for a class of interconnected nonlinear systems using variable structure control. *Automatica*, *24*(2), 187–193. https://doi.org/10.1016/0005-1098(88)90027-1

Moulay, E., & Perruquetti, W. (2006). Finite-time stability and stabilization: State of the art. In C. Edwards, E. Fossas Colet, & L. Fridman (Eds.), *Advances in variable structure and sliding mode control* (Vol. 334, pp. 23–41). Springer.

Moulay, E., & Perruquetti, W. (2008). Finite time stability conditions for non-autonomous continuous systems. *International Journal of Control*, *81*(5), 797–803. https://doi.org/10.1080/00207170701650303

Ning, B., Jin, J., & Zheng, J. (2017). Fixed-time consensus for multi-agent systems with discontinuous inherent dynamics over switching topology. *International Journal of Systems Science*, *48*(10), 2023–2032. https://doi.org/10.1080/00207721.2017.1308579

Polyakov, A. (2012). Nonlinear feedback design for fixed-time stabilization of linear control systems. *IEEE Transactions on Automatic Control*, *57*(8), 2106–2110. https://doi.org/10.1109/TAC.2011.2179869

Polyakov, A., & Fridman, L. (2014). Stability notions and Lyapunov functions for sliding mode control systems. *Journal of the Franklin Institute*, *351*(4), 1831–1865. https://doi.org/10.1016/j.jfranklin.2014.01.002 Special Issue on 2010–2012 Advances in Variable Structure Systems and Sliding Mode Algorithms.

Rahmani, R., Toshani, H., & Mobayen, S. (2020). Consensus tracking of multi-agent systems using constrained neural-optimiser-based sliding mode control. *International Journal of Systems Science*, 1–22. https://doi.org/10.1080/00207721.2020.1799257

Rekasius, Z. (1964, July). An alternate approach to the fixed terminal point regulator problem. *IEEE Transactions on Automatic Control*, *9*(3), 290–292. https://doi.org/10.1109/TAC.1964.1105700

Roxin, E. (1966). On finite stability in control systems. *Rendiconti del Circolo Matematico di Palermo*, *15*(3), 273–282. https://doi.org/10.1007/BF02844106

Sánchez-Torres, J. D., Defoort, M., & Muñoz-Vázquez, A. J. (2018, September, 5–7). *A second order sliding mode controller with predefined-time convergence*. 15th international conference on Electrical Engineering, Computing Science and Automatic Control (CCE), Mexico City, Mexico. IEEE. https://doi.org/10.1109/ICEEE.2018.8533952.

Sánchez-Torres, J. D., Gómez-Gutiérrez, D., López, E., & Loukianov, A. G. (2018). A class of predefined-time stable dynamical systems. *IMA Journal of Mathematical Control and Information*, *35*(Suppl 1), i1–i29. https://doi.org/10.1093/imamci/dnx004

Sánchez-Torres, J. D., Muñoz-Vázquez, A. J., Defoort, M., Jiménez-Rodríguez, E., & Loukianov, A. G. (2020). A class of predefined-time controllers for uncertain second-order systems. *European Journal of Control*, *53*, 52–58. https://doi.org/10.1016/j.ejcon.2019.10.003

Sánchez-Torres, J. D., Sánchez, E. N., & Loukianov, A. G. (2014, December 9–12). *A discontinuous recurrent neural network with predefined time convergence for solution of linear programming*. IEEE Symposium on Swarm Intelligence (SIS), Orlando, FL (pp. 9–12). IEEE. https://doi.org/10.1109/SIS.2014.7011799

Smith, O. J. M. (1957, September). Posicast control of damped oscillatory systems. *Proceedings of the IRE*, *45*(9), 1249–1255. https://doi.org/10.1109/JRPROC.1957.278530

Sun, L., & Zheng, Z. (2017). Finite-time sliding mode trajectory tracking control of uncertain mechanical systems. *Asian Journal of Control*, *19*(1), 399–404. https://doi.org/10.1002/asjc.v19.1

Tallman, G., & Smith, O. J. M. (1958, March). Analog study of dead-beat Posicast control. *IRE Transactions on Automatic Control*, *4*(1), 14–21. https://doi.org/10.1109/TAC.1958.1104844

Utkin, V. I. (1992). *Sliding modes in control and optimization*. Springer Verlag.

Utkin, V. I., & Shi, J (1996, December 13). *Integral sliding mode in systems operating under uncertainty conditions*. Proceedings of the 35th IEEE conference on Decision and Control, Kobe, Japan (Vol. 4, pp. 4591–4596). IEEE. https://doi.org/10.1109/CDC.1996.577594.

Yu, D., & Ji, X. Y. (2018). Finite-time containment control of perturbed multi-agent systems based on sliding-modecontrol.

International Journal of Systems Science, *49*(2), 299–311. https://doi.org/10.1080/00207721.2017.1406553

Zeng, T., Ren, X., & Zhang, Y. (2019). Fixed-time sliding mode control based plant/controller co-design of dual-motor driving system. *International Journal of Systems Science*, *50*(9), 1847–1859. https://doi.org/10.1080/00207721.2019.1645232

Zhang, Y., Hua, C., & Li, K. (2019). Disturbance observer-based fixed-time prescribed performance tracking control for robotic manipulator. *International Journal of Systems Science*, *50*(13), 2437–2448. https://doi.org/10.1080/00207721.2019.1622818

Zhao, H., Niu, Y., & Song, J. (2018). Finite-time output feedback control of uncertain switched systems via sliding mode design. *International Journal of Systems Science*, *49*(5), 984–996. https://doi.org/10.1080/00207721.2018.1435838

Zuo, Z. (2015). Nonsingular fixed-time consensus tracking for second-order multi-agent networks. *Automatica*, *54*, 305–309. https://doi.org/10.1016/j.automatica.2015.01.021