Ambiguity, Trading Volume and Liquidity

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Abstract

In this paper, I build a trading volume model that explicitly considers the impact of ambiguity surrounding public information announcements. I derive a closed form expression that illustrates how ambiguity influences trading volume through two channels: expectations and volatility. The empirical results highlight the significant role played by the expectations channel of ambiguity in driving trading activity. Specifically, on a monthly basis, I observe that a one-standard-deviation increase in ambiguity results in approximately a 20%-standard-deviation increase in trading volume. Additionally, by employing this model, I demonstrate that a substantial portion of the positive returns of a standard US Turnover sorted portfolio, approximately 70% of the returns, traditionally attributed to Liquidity, are actually driven by information ambiguity.

Keywords: ambiguity; knightian uncertainty; trading volume; turnover; liquidity; information; beliefs; heterogeneity; learning.

JEL Classification: G12, G4, D8

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1 Introduction

What generates the large trading volume activity observed in markets is a long standing question in the finance literature. While rational expectations models have been widely employed, they encounter limitations when attempting to explain trading activity solely based on public information (Banerjee & Kremer, 2010; Milgrom & Stokey, 1982). These models require the inclusion of an external process, such as private information, liquidity shocks, or the consideration of heterogeneous prior information to generate trading activity. However, these mechanisms often come with restrictions or rely on external processes, making it challenging to explain the large trading volumes observed in markets(Glaser & Weber, 2007; Hirshleifer, 2001). One modeling approach that potentially can generate larger differences and swings in expectations to account for trading volume is ambiguity (Knightian Uncertainty).

In this paper, I study the effect of ambiguity on trading volume. I propose a trading volume model that explicitly incorporates ambiguity about public information announcements. To develop this model, I introduce agents with ambiguous preferences into the difference in prior beliefs model of Kandel and Pearson (1995). These ambiguous preferences induce heterogeneous interpretations of public information by affecting expectations, and through this heterogeneity the expectations channel of ambiguity generates trading activity. The volatility channel of ambiguity plays a role in amplifying or smoothing the main price channel of trading volume and the differences in expectations. I derive three testable hypotheses. My first hypothesis H1 postulates that on average ambiguity positively influences trading volume. My second hypothesis H2 is that ambiguity affects the conventional positive relationship between price volatility and trading volume by reducing the elasticity of trading volume to price volatility. Lastly, my third hypothesis H3 proposes that trading activity linked to ambiguity significantly contributes to the returns of turnover sorted portfolios, returns typically attributed to liquidity in the literature.

I bring my model to equity data and show that on average a one-standard-deviation increase in daily Ambiguity translates approximately in a 13%-standard-deviation increase in trading volume after controlling for price movements and differences in prior beliefs. On a monthly basis, I find that a one-standard-deviation in Ambiguity translates approximately in a 20%-standard-deviation increase in trading volume. I also find that Ambiguity distorts the on average positive elasticity between trading volume and price volatility, weakening this channel. A one-standard-deviation increase in price change volatility times daily Ambiguity translates in a -9%-standard-deviation decrease in trading volume, rather than an increase. I also empirically find that since 1990 approximately 70% of the positive returns of a US long-short Turnover sorted portfolio are associated to trading volume driven by Ambiguity. The literature traditionally associates these returns to Liquidity.

Trading volume is a relevant market measure that is widely used for purposes such as measuring liquidity (Chordia et al., 2001), understanding information transmission or the structure of markets (Karpoff, 1986) among other uses. Despite its variety of practical uses, it is still an open puzzle what are the drivers behind the large trading volume observed in most markets and its relation to market liquidity. Cochrane (Cochrane, 2016) noted that "volume is the great unsolved problem of financial economics", Shleifer (2000) ranked the volume puzzle among the top 20 issues of behavioural finance, and De Bondt and Thaler (1995) mentioned that volume "is perhaps the single most embarrassing fact to the standard finance paradigms".

There are several empirical studies that have documented the on average high levels of trading volume in markets. Barber and Odean (2000) report that between 1991 to 1996 the average US household had a portfolio turnover of around 75%, while the top active quantile of retail investors showed annual turnover rates of 250%. Dorn and Huberman (2005) report average turnover rates of around 100% among German retail investors, Barber et al. (2009) report turnover rates of up to 300% in Taiwan, and Gao (2002) reports turnover rates of 500% in China. According to the literature, such levels of trading activity can not be explained by rational expectations models (Dorn & Sengmueller, 2009). Using a rational expectations model calibrated on Nasdaq data, Sen (2002) obtained an average holding period of 98 months for stocks, a much longer period than the real Nasdaq average holding period of 5.1 months.

In relation to the practical use of trading volume as a proxy for liquidity (Abdi & Ranaldo, 2017; Avramov & Chordia, 2006; Becker-Blease & Paul, 2006; Eckbo & Norli, 2005; Illeditsch, 2011; Lee, 1993; Rouwenhorst, 1999), there are several empirical studies that have found results inconsistent with the finance literature (Bekaert et al., 2007; Chordia et al., 2001). One of the most famous is the study of Chordia et al. (2001), who found a puzzling negative correlation between stock returns and variability of trading volume. According to the asset pricing literature, for assets with higher liquidity risk one would have expected higher returns. Bekaert et al. (2007) found that across 19 emerging stock markets a return based measures of liquidity was priced, while turnover was not priced. These inconsistencies have cast doubts on the suitability of volume based measures of liquidity (Barinov, 2014; Gabrielsen et al., 2011; Johnson, 2008; Le & Gregoriou, 2020; Lee & Swaminathan, 2000), and have open the question of whether trading volume might be capturing some other factor(s)?.

With this model I show that the expectations channel of ambiguity plays an important role in the generation of trading activity and that a large portion of a standard US Turnover portfolio returns usually associated to liquidity are actually driven by information ambiguity. This research will contribute to the growing literature on the effects of Ambiguity on markets and information transmission through prices (Condie & Ganguli, 2017; Easley & O'HARA, 2010; Epstein & Schneider, 2010; Mele & Sangiorgi, 2015; Ozsoylev & Werner, 2011). In relation to previous trading volume models or closed form equations that explicitly incorporate Ambiguity about public information, to the best of my knowledge there are two previously existing works. The model of Caskey (2009) about the ambiguous perception of public information and the partially related model of Hsiao (2019) about the ambiguous perception of others' beliefs. Both models exploit the volatility contribution of Ambiguity to generate trading activity, while my model uses the expectations channel of Ambiguity. The introduction of this expectations channel in a closed form equation for trading volume and the empirical results showing its statistical significance are the main contributions of this paper.

The rest of the paper is organized as follows. Section-2 describes the theoretical model. Section-3 summarizes the data used in the empirical estimations. Section-4 presents the main empirical results regarding the proposed trading volume, Ambiguity and price change relation. Section-5 presents the empirical results regarding the effects of Ambiguity on the trading volume to price volatility elasticity. Section-**??** presents an empirical application of the model that dissects the returns of a standard US Turnover sorted Portfolio. Finally, section 7 presents my conclusions.

2 Model

The model is based on a setup similar to Kandel and Pearson (1995). The setup is composed of two competitive markets, each with two investor types selecting optimal portfolios based on their own prior beliefs. The key feature of the model is that type-A investors interpret public information ambiguously. The proportion of these ambiguity-averse (or ambiguity-loving) type-A investors operating in both risk-less and risky asset markets is denoted by π, while the proportion of type-B ambiguity-neutral investors is represented by $1 - \pi$.

Both investor types receive a public signal denoted as S, which contains information about the unknown payoff $\widetilde{X}$. The type-A investors prone to ambiguity-averse (or ambiguity loving) behavior, interprets this signal S in an uncertain manner, considering various mental representations or models. After observing the public signal S, both investor types update their beliefs and adjust their portfolios accordingly.

The model dynamics consist of three time periods. In period 1, both investor types construct their initial portfolios based on their individual prior beliefs. In period 2, both investor types observe the public signal S, which provides information about the unknown payoff $\widetilde{X}$ of the risky asset. Armed with this new information, all investor types update their beliefs and adjust their portfolios accordingly. This portfolio adjustment leads to trading volume from period (1) to period (2). Finally, in the last period (3), the risky-asset payoff is realized. For the sake of simplicity in this setup (Kandel & Pearson, 1995), investors, when building their portfolios at time (1), do not expect a second opportunity or reason to trade in the future.

2.1 Ambiguity Neutral Investors (Type-B)

The type-B investors utility function is denoted by $U^B\left(\theta_t^B\right)$, where θ_t^B represents this investor type allocation in the risky asset during period (t). $W\left(\theta_t^B\right)$ represents the final wealth level for type-B investors at period 3. This final wealth level $W\left(\theta_t^B\right)$ depends on its initial wealth w_t^B at period (t) and the risky asset allocation θ_t^B determined during the same period. In period 1, this investor type maximizes its expected final utility based on his prior beliefs. Moving to period 2, they update their beliefs about the distribution of the risky-asset payoff and adjusts its portfolio accordingly. Finally, in period 3, the risky asset pays off.

This investor type has a standard CARA utility function with absolute risk aversion given by γ and maximizes the following expected utility.

$$\begin{cases} E^B\left[\ -e^{-\gamma\ *\ W\left(\theta_t^B\right)}\ \right] & \text{if } t = t1 \\ E^B\left[-e^{\ -\gamma\ *\ W\left(\theta_t^B\right)}\ \middle| S\right] & \text{if } t = t2 \end{cases} \tag{2.1}$$

The final wealth $W\left(\theta_t^B\right)$ of this investor type at period 3 is given by the expression below.

$$W\left(\theta_t^B\right) = w_t^B + \theta_t^B * \left(\widetilde{X} - P_t\right) \tag{2.2}$$

In period 1, this investor type initially believes that the risky-asset payoff $\widetilde{X}$ follows a normal distribution with parameters $N\left(\mu_X^B,\ \sigma_X^{2\,B}\right)$ and precision $\rho_X^B = 1/\sigma_X^{2\,B}$. However, upon observing the unexpected signal S, this investor type updates its beliefs. They perceive the signal as biased, containing information about both the risky-payoff amount $\widetilde{X}$ and a measurement bias or error ϵ. This investor type believes that the measurement error distributes $N\left(\mu_\epsilon^B,\ \sigma_\epsilon^{2\,B}\right)$ with precision $\rho_\epsilon^B = 1/\sigma_\epsilon^{2\,B}$. The total error term of the signal is $\widetilde{\mathcal{E}}$ which according to this investor type is composed of just a measurement error and has precision $\rho_{\mathcal{E}}^B$.

$$S = \widetilde{X} + \widetilde{\epsilon} \tag{2.3}$$

A type-B investor with bullish prior beliefs or a positive interpretation of public information would assume that the measurement bias mean μ_ϵ^B is negative. Conversely, a type-B investor with bearish prior beliefs would believe that the signal has a positive bias μ_ϵ^B, misleadingly indicating an expectation above the true expected value of $\widetilde{X}$.

2.2 Ambiguity Averse/Loving Investors (Type-A)

The ambiguity-averse/loving A investor type is characterized by a utility function denoted as $U^A\left(\theta_t^A\right)$, with θ_t^A representing their risky asset allocation decided during period (t). $W_t\left(\theta_t^{I_A}\right)$ represents this investor type final wealth level at period 3 based on his allocation at (t). Initially, in the first period, this investor type expected utility looks similar to that of a CARA agent (ambiguity-neutral), as the source of ambiguity arises in period 2 after observing the signal S. In this second period 2, after incorporating the information from signal S, this investor type maximizes their expected utility by considering the ambiguity surrounding signal S according to their Smooth Ambiguity Utility function.

$$\begin{cases} E^A\left[\ -e^{-\gamma\ *\ W\left(\theta_t^A\right)}\ \right] & \text{if } t = t1 \\ E^A\left[-\left(\ -E^A\left[-e^{\ -\gamma\ *\ W\left(\theta_t^A\right)}\ \middle|S, M\right]\ \right)^{\gamma_a}\middle|S\right] & \text{if } t = t2 \end{cases} \tag{2.4}$$

The parameters γ and γ_a are the risk aversion and the ambiguity aversion coefficients of the utility function, and $W\left(\theta_t^A\right)$ represents the final wealth of this investor type given his risky asset allocation θ_t^A at (t).

$$W\left(\theta_t^A\right) = w_t^A + \theta_t^A * \left(\widetilde{X} - P_t\right) \tag{2.5}$$

The investor type-A maximizes at time 2 his expected utilities obtained after evaluating his terminal wealth under n-different models M^n by selecting the optimal asset mixture. In this function, the first expectation operator addresses the traditional risk concept associated with the payoff $\widetilde{X}$, while the second expectation handles the ambiguity surrounding S, by evaluating the terminal wealth under different M models. In this particular setup, each model M represents a specific way of interpreting the public information.

Regarding this investor type beliefs about the risky-asset payoff $\widetilde{X}$, they initially assume a normal distribution with parameters $N\left(\mu_X^A,\ \sigma_X^{2A}\right)$ and precision $\rho_X^A = 1/\sigma_X^{2\,A}$. Additionally, they believe that the signal S is subject to a measurement bias or error $\widetilde{\epsilon}$ that distributes $N\left(\mu_\epsilon^A,\ \sigma_\epsilon^{2\,A}\right)$ and has precision $\rho_\epsilon^A = 1/\sigma_\epsilon^{2\,A}$.

Despite these prior beliefs, this investor type is not completely certain about the appropriate model for interpreting the signal S. This ambiguity is represented by different models $M \in M^n$, each characterized by a model-dependent signal component $\widetilde{\delta}$. This $\widetilde{\delta}$ distributes across M^n according to the normal distribution $N\left(\mu_\delta^A,\ \sigma_\delta^{2\,A}\right)$. Similarly to the previous investor type, a bullish investor who tends to interpret information positively would perceive the mean bias $\mu_\delta^A + \mu_\epsilon^A$ of the signal as negative, causing the signal S to misleadingly appear below the true expected value of $\widetilde{X}$.

In summary, the investor type-A believes that the signal S consists of three components: the risky payoff $\widetilde{X}$ information, an ambiguous model-dependent component $\widetilde{\delta}$, and a measurement error $\widetilde{\epsilon}$. The total error term of the signal is denoted as $\widetilde{\mathcal{E}}$ and has precision $\rho_{\mathcal{E}}^A$ according to investor type-A.

$$S = \widetilde{X} + \widetilde{\delta} + \widetilde{\epsilon} = S + \widetilde{\mathcal{E}} \tag{2.6}$$
$$\widetilde{X} \sim N\big(\mu_X^A,\ \sigma_X^{2\,A}\ \big)$$
$$\widetilde{\delta} \sim N\big(\mu_\delta^A,\ \sigma_\delta^{2\,A}\ \big)$$
$$\widetilde{\epsilon} \sim N\big(\mu_\epsilon^A,\ \sigma_\epsilon^{2\,A}\ \big)$$

2.3 Market Equilibrium in Period 1

In the initial period 1, each investor type seeks to maximize their expected final utility at period 3 based on their own prior beliefs.

The type-B ambiguity neutral investors maximize the following expected utility at period 1.

$$\max_{\theta_{t1}^B} E^B\big[U^B\big(\theta_{t1}^B\big)\big] = \max_{\theta_{t1}^B} -e^{-\gamma*\left(w_{t1}^B\ +\ \theta_{t1}^B*\ \left(E^B\left[\widetilde{X}\right]-P_{t1}\right)\right)\ +\ \frac{1}{2}*\gamma^2*\theta_{t1}^{2B}*\mathrm{VAR}^B[\widetilde{X}]} \tag{2.7}$$

The type-B investors optimal allocation in the risky asset at period 1 is given by the expression θ_{t1}^B below.

$$\theta_{t1}^B = \frac{\big(\mu_X^B - P_{t1}\big) * \rho_X^B}{\gamma}$$

The ambiguity-averse/loving type-A investors maximize at period 1 the following expected utility.

$$\max_{\theta_{t1}^A} E^A\big[U^A\big(\theta_{t1}^A\big)\big] = \max_{\theta_{t1}^A} -e^{-\gamma*\left(w_{t1}^A\ +\ \theta_{t1}^A*\ \left(E^A\left[\widetilde{X}\right]-P_{t1}\right)\right)\ +\ \frac{1}{2}*\gamma^2*\theta_{t1}^{2A}*\mathrm{VAR}^A[\widetilde{X}]} \tag{2.8}$$

The type-A investors optimal investment in the risky asset at period 1 is determined by the expression $\theta_{t1}^{I_A}$ below.

$$\theta_{t1}^A = \frac{\big(\mu_X^A - P_{t1}\big) * \rho_X^A}{\gamma}$$

The aggregate demands of both investor types will converge in the risky asset market, establishing a market clearing price in equilibrium.The type-B ambiguity neutral investors aggregate demand is $(1-\pi)*\theta_{t1}^B$, while the type-A investors aggregate demand is $\pi*\theta_{t1}^A$. In equilibrium the

0-net-supply risky-asset market clears according to the following equation.

$$(1-\pi) * \theta_{t1}^{B} + \pi * \theta_{t1}^{A} = 0 \tag{2.9}$$

The price P_{t1} represents the equilibrium price at which the risky asset market clears in period 1 and is given by the following expression.

$$P_{t1} = \frac{\bar{\mu}_{X_{t1}}}{\bar{\rho}_{X_{t1}}} \tag{2.10}$$

$$\begin{aligned} &where \\ &\bar{\mu}_{X_{t1}} = \pi * \mu_X^A * \rho_X^A + (1-\pi) * \mu_X^B * \rho_X^B \\ &\bar{\rho}_{X_{t1}} = \pi * \rho_X^A + (1-\pi) * \rho_X^B \end{aligned}$$

The resulting equilibrium allocation at period 1 for both ambiguity-averse/loving type-A and ambiguity neutral type-B investors is given by the expressions θ_{t1}^A and θ_{t1}^B below.

$$\begin{aligned} \theta_{t1}^B &= \frac{\pi * \rho_X^A * \rho_X^B * \left(\mu_X^B - \mu_X^A\right)}{\gamma * \bar{\rho}_X} \\ \theta_{t1}^A &= \frac{(1-\pi) * \rho_X^A * \rho_X^B * \left(\mu_X^A - \mu_X^B\right)}{\gamma * \bar{\rho}_X} \end{aligned} \tag{2.11}$$

2.4 Arrival of Public Information and Update of Beliefs at Period 2

At the beginning of period 2, public information arrives through the signal S, which is visible to both investor types. Although both types of investors receive the same raw information, their interpretations differ due to their prior heterogeneous beliefs and the ambiguity faced by type-A investors.

The type-B ambiguity neutral investors believe that the signal S they are receiving follows this functional form $S = \widetilde{X} + \widetilde{\epsilon}$. Based on this, they update their beliefs about the mean and volatility of the future risky asset payoff $\widetilde{X}$, constructing the posterior beliefs $E^B\left[\widetilde{X}\middle|S\right]$ and $\mathrm{VAR}^B\left[\widetilde{X}\middle|S\right]$. These posterior beliefs are derived from normal conditional distributions.

$$E^B\left[\widetilde{X}\middle|S\right] = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * \left(S - \mu_\epsilon^B\right)}{\rho_X^B + \rho_{\mathcal{E}}^B} \tag{2.12}$$

$$\text{VAR}^B\left[\widetilde{X}\middle|S\right] = \left(\rho_X^B + \rho_{\mathcal{E}}^B\right)^{-1}$$

The type-A ambiguity averse/lover investors believe that the signal S comprises the risky-payoff $\widetilde{X}$ information, an ambiguous model-dependent component $\widetilde{\delta}$ and a measurement bias or error $\widetilde{\epsilon}$. $S = \widetilde{X} + \widetilde{\delta} + \widetilde{\epsilon}$. Based on this assumption, this investor type consistently updates his beliefs about the mean and volatility of the future risky-asset payoff $\widetilde{X}$, constructing the conditional M model-dependent posterior beliefs $E^A\left[\widetilde{X}\middle|S, M\right]$ and $\text{VAR}^A\left[\widetilde{X}\middle|S, M\right]$, and the unconditional posterior beliefs $E^A\left[\widetilde{X}\middle|S\right]$ and $\text{VAR}^A\left[\widetilde{X}\middle|S\right]$. These posterior beliefs are also based on normal conditional distributions.

$$\begin{aligned}
E^A\left[\widetilde{X}\middle|S\right] &= \frac{\rho_X^A * \mu_X^A + \rho_{\mathcal{E}}^A * \left(S - \mu_\delta^A - \mu_\epsilon^A\right)}{\rho_X^A + \rho_{\mathcal{E}}^A} \qquad (2.13)\\
Var^A[\widetilde{X}|S] &= \left[\rho_X^A + \rho_{\mathcal{E}}^A\right]^{-1}\\
E^A\left[\widetilde{X}\middle|S, M\right] &= \frac{\rho_X^A * \mu_X^A + \rho_\epsilon^A * \left((S - \delta) - \mu_\epsilon^A\right)}{\rho_X^A + \rho_\epsilon^A}\\
Var^A[\widetilde{X}|S, M] &= \frac{1}{\rho_X^A} - \frac{\rho_S^A * \frac{\rho_\delta^{2\,A}}{\rho_X^{2\,A}}}{\rho_\delta^{2\,A} - \rho_S^{2\,A}}
\end{aligned}$$

2.5 Market Equilibrium in Period 2

After integrating the information from signal S into their beliefs, both investor types update their portfolio compositions. The ambiguity neutral type-B investors update their risky asset allocations θ_{t2}^B following the expected utility maximization below.

$$\max_{\theta_{t2}^B} -e^{-\gamma * \left(w_{t2}^B \,+\, \theta_{t2}^B * \,\left(E^B\left[\widetilde{X}\middle|S\right] - P_{t2}\right)\right) \,+\, \frac{1}{2} * \gamma^2 * \theta_{t2}^{2B} * \text{VAR}^B[\widetilde{X}|S]} \qquad (2.14)$$

The resulting optimal risky asset allocation of the type-B investors is given by the following expression.

$$\theta_{t2}^B = \frac{E^B[\widetilde{X}|S] - P_{t2}}{\gamma * \text{VAR}^B[\widetilde{X}|S]}$$

$$\theta_{t2}^B = \frac{\rho_X^B * \mu_X^B + \rho_e^B * \,\left(S - \mu_{e_{ME}}^B\right) - P_{t2} * \left(\rho_X^B + \rho_e^B\right)}{\gamma}$$

The ambiguity averse/loving type-A investors update their portfolios according to their Smooth

Ambiguity utility functions, employing various M models to assess the information from signal S. This investor type optimizes their portfolios according to the maximization of the following expected utility.

$$\underset{\theta_{t2}^{A}}{max}\ w_{t2}^{A} \ + \ \theta_{t2}^{A} * \ \left(E^{A}[\widetilde{X}|S] - P_{t2}\right) - \tfrac{1}{2} * \gamma * \theta_{t2}^{2A} * \mathrm{VAR}^{A}[\widetilde{X}|S] * \left[1 + (\gamma_a - 1) * \left(\tfrac{\mathrm{VAR}^{A}[\widetilde{X}|S] - \mathrm{VAR}^{A}[\widetilde{X}|S,M]}{\mathrm{VAR}^{A}[\widetilde{X}|S]}\right)\right] \tag{2.15}$$

The optimal risky asset allocation $\theta_{t2}^{I_A}$ of the type-A investors is given by the following expression.

$$\theta_{t2}^{A} = \frac{E^{A}[\widetilde{X}|S] - P_{t2}}{\gamma * Var^{A}[\widetilde{X}|S] * \nu^{A}}$$

$$\theta_{t2}^{A} = \frac{\rho_X^A * \mu_X^A + \rho_{\mathcal{E}}^A * (S - \mu_\epsilon^A - \mu_\delta^A) - P_{t2} * (\rho_X^A + \rho_{\mathcal{E}}^A)}{\gamma * \nu^A}$$

$$where$$

$$\nu^A = \left[1 + (\gamma_a - 1) * \left(\frac{\mathrm{VAR}^{A}[\widetilde{X}|S] - \mathrm{VAR}^{A}[\widetilde{X}|S,M]}{\mathrm{VAR}^{A}[\widetilde{X}|S]}\right)\right]$$

$$\frac{Var^{A}[\widetilde{X}|S] - Var^{A}[\widetilde{X}|S,M]}{Var^{A}[\widetilde{X}|S]} = \frac{\rho_X^A * \rho_\epsilon^{A\,3}}{(\rho_\delta^A * \rho_\epsilon^A + 2 * \rho_X^A * \rho_\epsilon^A + \rho_X^A * \rho_\delta^A) * (\rho_X^A + \rho_\epsilon^A) * (\rho_\epsilon^A + \rho_\delta^A)}$$

At the end of period 2, after both investor types have optimized their portfolios based on the new information from S, their risky asset demands will come together in the market, establishing a market clearing price in equilibrium. The aggregate demand of the ambiguity neutral type-B investors is denoted by $(1 - \alpha) * \theta_{t2}^{B}$, and the demand of the ambiguity-averse/loving type-A investors is denoted by $\alpha * \theta_{t2}^{I_A}$. The 0-net-supply risky-asset market clears at time 2 according to the following equation.

$$(1 - \pi_A) * \theta_{t2}^{B} + \pi_A * \theta_{t2}^{I_A} = 0 \tag{2.16}$$

The market clearing price at time 2, price P_{t2}, is given by the expression below.

$$\mathrm{P}_{t2} = \frac{\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * \ \mathrm{S} - \bar{\mu}_{\epsilon} - \dfrac{\pi}{\nu^A} * \rho^A_{\mathcal{E}} * \mu^A_{\delta}}{\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}} \tag{2.17}$$

$$\begin{aligned}
&\textit{where}\\
\bar{\mu}_X &= \frac{\pi}{\nu^A} * \rho^A_X * \mu^A_X + (1-\pi) * \rho^B_X * \mu^B_X\\
\bar{\mu}_{\epsilon} &= \frac{\pi}{\nu^A} * \rho^A_{\mathcal{E}} * \mu^A_{\epsilon} + (1-\pi) * \rho^B_{\mathcal{E}} * \mu^B_{\epsilon}\\
\bar{\rho}_X &= \frac{\pi}{\nu^A} * \rho^A_X + (1-\pi) * \rho^B_X\\
\bar{\rho}_{\mathcal{E}} &= \frac{\pi}{\nu^A} * \rho^A_{\mathcal{E}} + (1-\pi) * \rho^B_{\mathcal{E}}
\end{aligned}$$

The expressions $\bar{\mu}_X$ and $\bar{\mu}_{\epsilon}$ represent population and precision-weighted averages of both investor types prior beliefs about the risky asset payoff $\widetilde{X}$, and the signal measurement errors, respectively. μ^A_{δ} in the numerator represents the mean bias effect of the ambiguity expectations channel on the price P_{t2}. On the other hand, $\bar{\rho}_X$ and $\bar{\rho}_{\mathcal{E}}$ denote population-weighted averages of the two investor types prior belief precisions regarding the risky asset payoff $\widetilde{X}$ and the signal S total error.

2.6 Trading Volume Expression

In this section, I present the main results of this work: the relationship between trading volume, ambiguity and price changes. I will begin by introducing a simplified model version 1, which omits differences in prior beliefs and the influence of the price channel on trading volume. Following that, I will delve into a model version 2 that incorporates the price channel of trading volume. Finally, I will conclude with a full model version 3, which encompasses ambiguity, differences in prior beliefs and the impact of the price channel on trading volume.

Regarding the ambiguity mechanism driving trading volume in this model, put plainly, when signal S conveys fresh information, it spurs investors to trade the risky asset in order to align their portfolios with their updated beliefs. Despite both investor types receiving identical signals, ambiguity introduces heterogeneity, giving rise to distinct posterior beliefs about the risky asset's payoff (X). This heterogeneity, induced by ambiguity, is the driving force behind this trading activity.

To derive the closed form trading volume model presented below, I utilize the change in allocation of the risky asset for one investor type, multiplied by the proportion of these investors in the economy. In this market with a net supply of zero, the buying activity of one investor type corresponds to the selling activity of the other, and vice versa.

$$\left|(1-\pi) * \Delta\theta_{t2}^{I_B}\right| = \left|\pi * \Delta\theta_{t2}^{I_A}\right| \tag{2.18}$$

$$where$$

$$\Delta\theta^{I_A} = \theta_{t2}^{I_A} \; - \; \theta_{t1}^{I_A}$$

$$\Delta\theta^{I_{NA}} = \theta_{t2}^{I_{NA}} \; - \; \theta_{t1}^{I_{NA}}$$

Based on these symmetrical trading volume expressions, I arrive at the model below. Additional details provided in the Appendix-B.

Version 1: Only ambiguity channel

Market Trading Volume V_{21} from period 1 to period 2, only ambiguity channel present while price channel and heterogeneous prior beliefs switched off.

$$V_{21} = |\alpha_V| \tag{2.19}$$

$$where$$

$$\alpha_V = \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \mu_\delta^A$$

In this simplified model version, trading volume is directly linked to the bias effect of μ_δ^A, originating from the influence of the expectations channel of ambiguity on the posterior beliefs held by type-A investors. This effect is further modulated by the factor F_1. It is worth highlighting that both the numerator and denominator of F_1 encompass the inverse of the coefficient ν^A and the precision term $\rho_{\mathcal{E}}^A$ from type-A investors. The value of ν^A is determined by the level of ambiguity aversion γ_a, and similarly, both the precision term $\rho_{\mathcal{E}}^A$ and ν^A depend on the ambiguity volatility $\sigma_\delta^{2\,A}$.

As illustrated in the top panel of figure-1 below, the factor F_1 exhibits a decreasing trend as ambiguity aversion γ_a increases. It reaches its minimum value of zero for type-A investors characterized by extreme ambiguity aversion γ_a approaching infinity, and conversely, it attains a maximum positive value for those investors who favor ambiguity with γ_a values close to 0. The intuition here is that as ambiguity aversion grows, type-A investors tend to avoid risky assets, regardless of signal-related expectations, including those associated with ambiguity. In this simplified scenario, since both types of investors shared the same prior beliefs at time 1, they did not take any positions in the risky asset. Consequently, at time 2, an increased level of ambiguity aversion leads type-A investors to stay out of the market, resulting in a lack of trading activity.

The lower panel of figure-1 shows that the factor F_1 diminishes as the ambiguity volatility $\sigma_\delta^{2\,A}$ increases. F_1 reaches its peak when the ambiguity volatility $\sigma_\delta^{2\,A}$ approaches zero, and it approaches zero as the ambiguity volatility tends towards infinity. Here the intuition is that as ambiguity volatility rises, the signal-to-noise ratio of S decreases. This leads to a disregard of all information contained in signal S because of its low quality, including the expectations associated to ambiguity.

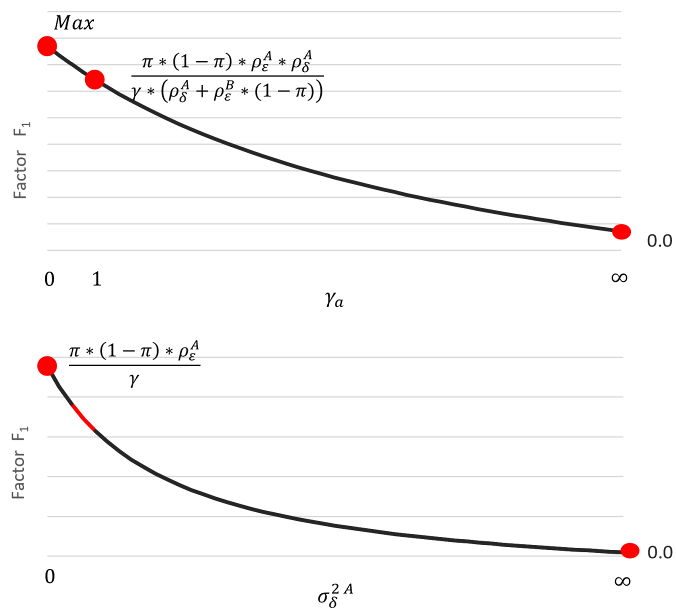

Figure 1. Effect of γ_a and $\sigma_\delta^{2\,A}$ on factor F_1 . This graph shows how the the values of γ_a and $\sigma_\delta^{2\,A}$ affect the factor F_1 multiplying μ_δ^A. The top panel achieves a maximum of $\frac{\rho_\delta^A*\rho_\epsilon^A*\pi*(1-\pi)*\left(\rho_\epsilon^A+\rho_X^A\right)*\left(2\rho_\epsilon^A*\rho_X^A+\rho_\delta^A*(\rho_\epsilon^A+\rho_X^A)\right)}{\gamma*\left(\rho_\delta^{A2}*\left(\rho_\epsilon^A+\rho_X^A\right)+\rho_\epsilon^{A2}*(1-\pi)*\rho_X^A*\left(\rho_\epsilon^A+2\rho_X^A\right)+\rho_\delta^A*\rho_\epsilon^A*\left(\rho_\epsilon^A+\rho_X^A\right)*\left(\rho_\epsilon^A*(1-\pi)+\rho_X^A*(3-\pi)\right)\right)}$ when γ_a goes to 0, a value of $\frac{\pi*(1-\pi)*\rho_\epsilon^A*\rho_\delta^A}{\gamma*\left(\rho_\delta^A+\rho_\epsilon^B*(1-\pi)\right)}$ when γ_a is 1 and a value of 0 when γ_a tends to infinity.

Version 2: Ambiguity channel and price channel

Market trading volume V_{21} from period 1 to period 2, when ambiguity channel and price channel are present while heterogeneity in prior beliefs is switched off.

$$V_{21} = |\alpha_V + \beta_V * \Delta P_{21}| \tag{2.20}$$

where

$$\alpha_V = \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \mu_\delta^A$$

$$\beta_V = \left[\frac{\frac{\pi}{\nu^A} * \rho_X^A * \left(\rho_{\mathcal{E}}^B - \rho_{\mathcal{E}}^A\right)}{\bar{\rho}_{\mathcal{E}}}\right]_{F_2}$$

In this model version, trading volume is determined by the combination of two drivers: the bias effect μ_δ^A resulting from the expectations channel of ambiguity, and the trading volume linked to price changes. The impact of price changes is then scaled by β_V, which incorporates both the inverse of the type-A investors coefficient ν^A and the precision term $\rho_{\mathcal{E}}^A$ in its numerator and denominator. The value of the coefficient ν^A hinges on both the degree of ambiguity aversion γ_a and the ambiguity volatility d $\sigma^{2\,A}$. Similarly, the precision term $\rho_{\mathcal{E}}^A$ also depends on the ambiguity volatility $\sigma^{2\,A}$. This last interdependence creates a distinction between $\rho_{\mathcal{E}}^B$ and $\rho_{\mathcal{E}}^A$, even when the prior beliefs ex ambiguity about the signal S and the measurement error are homogeneous.

As shown in the upper panel of figure-2 above, this last introduced factor F_2 is positive and declines as ambiguity aversion γ_a increases. It reaches its lowest point at zero for type-A investors characterized by extreme ambiguity aversion with γ_a tending toward infinity. Conversely, for investors who embrace ambiguity and possess γ_a values close to 0, this factor F_3 attains its maximum value. he intuition here echoes the previous version. As ambiguity aversion increases, type-A investors choose to steer clear of the risky asset, preserving their initial allocation risk-free asset allocation established at time 1 under homogeneous prior beliefs.

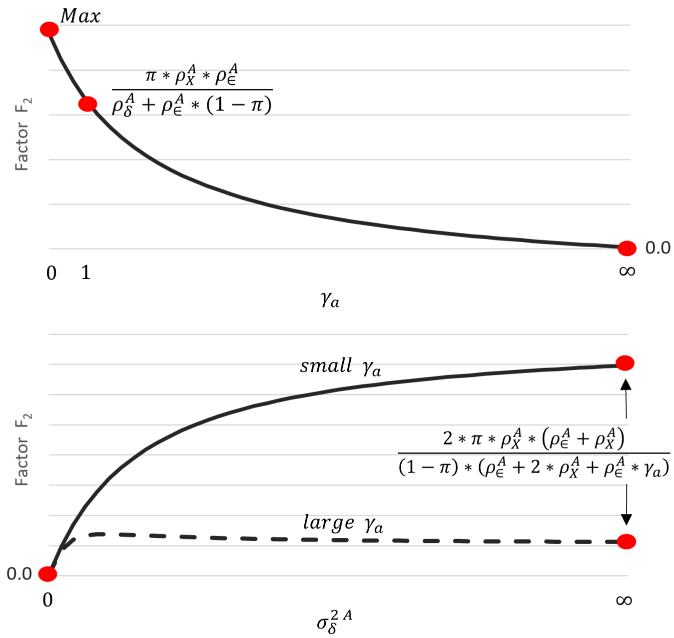

Figure 2. Effect of γ_a and $\sigma_\delta^{2\,A}$ on factor F_2 . This graph shows how the the values of γ_a and $\sigma_\delta^{2\,A}$ affect the factor F_2 multiplying. The top panel achieves a maximum of $\frac{\pi*\rho_\epsilon^A*\rho_X^A*\left(\rho_\epsilon^A+\rho_X^A\right)*\left(2*\rho_\epsilon^A*\rho_X^A+\rho_\delta^A*(\rho_\epsilon^A+\rho_X^A)\right)}{\gamma*\left(\rho_\delta^{A\,2}*\left(\rho_\epsilon^A+\rho_X^A\right)^2+\rho_\epsilon^{A\,2}*(1-\pi)*\rho_X^A*\left(\rho_\epsilon^A+2*\rho_X^A\right)+\rho_\delta^A*\rho_\epsilon^A*\left(\rho_\epsilon^A+\rho_X^A\right)*\left(\rho_\epsilon^A*(1-\pi)+\rho_X^A*(3-\pi)\right)\right)}$ when γ_a goes to 0, and a value of $\frac{\pi*\rho_X^A*\rho_\epsilon^A}{\rho_\delta^A+\rho_\epsilon^A*(1-\pi)}$ when γ_a is 1.

The lower panel of figure-2, shows that the behavior of the factor F_2 varies for different levels of ambiguity volatility and is influenced by the degree of ambiguity aversion γ_a. For small values of γ_a, the factor F_2 increases with increasing ambiguity volatility, eventually reaching the maximum value as depicted in the figure. Conversely, for larger values of γ_a, the factor F_2 exhibits a more complex behavior, characterized by two distinct regions. Initially, in the first region, this factor increases with ambiguity volatility, but it then reverses its trend and decreases until it reaches a limit when the ambiguity volatility tends to infinity, as illustrated on the right side of the figure. Here, the intuition is that as ambiguity volatility rises, type-A investors who have a preference for ambiguity (γ_a¡1) become more inclined to trade the risky asset in response to changes in market conditions as reflected by prices. Conversely, for ambiguity-averse investors with larger ambiguity aversion (γ_a¿1), an increase in ambiguity volatility motivates type-A investors to maintain their risk-free asset holdings from time 1 established under homogeneous prior beliefs, reducing their inclination to trade as ambiguity volatility intensifies.

Version 3: Ambiguity channel, price channel and heterogeneous prior beliefs

Full model market trading volume V_{21} from period 1 to period 2, when ambiguity channel, price channel and heterogeneous initial beliefs are present.

$$V_{21} = |\alpha_V + \beta_V * \Delta P_{21}| \tag{2.21}$$

where

$$\begin{aligned}\alpha_V = & \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \mu_\delta^A \\ & + \left[\pi * (1-\pi)^2 * \left(\frac{1}{\nu^A} - 1\right) * \left(\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}} * \gamma}\right) * \left(\frac{\rho_X^A * \rho_X^B}{\bar{\rho}_{\underset{t1}{X}}}\right)\right]_{F_3} * \left(\mu_X^B - \mu_X^A\right) \\ & + \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \left(\mu_\epsilon^A - \mu_\epsilon^B\right)\end{aligned}$$

$$\beta_V = \left[\frac{\frac{\pi}{\nu^A} * \rho_X^A * \left(\rho_{\mathcal{E}}^B - \rho_{\mathcal{E}}^A\right)}{\bar{\rho}_{\mathcal{E}}}\right]_{F_2}$$

In this full version of the model, the trading volume is proportional to the bias effect μ_δ^A created by the expectations channel of ambiguity, the difference in prior beliefs about the risky asset payoff $\left(\mu_X^B - \mu_X^A\right)$, the difference in prior beliefs about the measurement error $\left(\mu_\epsilon^A - \mu_\epsilon^B\right)$, plus the trading volume associated to the change in price. The difference in prior beliefs about the risky asset $\left(\mu_X^B - \mu_X^A\right)$ gets multiplied by the factor F_3 which has on the denominator the contribution of the inverse of the coefficient ν^A and the type-A investor precision $\rho_{\mathcal{E}}^A$ that's averaged inside the term $\bar{\rho}_{\mathcal{E}}$. The difference in prior beliefs about the measurement error $\left(\mu_\epsilon^A - \mu_\epsilon^B\right)$ gets multiplied by the factor F_1 previously analyzed, factor that decreases when either ambiguity aversion γ_a or ambiguity volatility $\sigma_\delta^{2\,A}$ increases.

As illustrated in the upper panel of figure-3 below, the newly introduced factor F_3, which multiplies the difference in prior beliefs regarding the risky asset payoff $\left(\mu_X^B - \mu_X^A\right)$, exhibits a decreasing pattern with respect to ambiguity aversion γ_a. Specifically, it starts at a positive maximum when γ_a is 0, then crosses through zero when γ_a equals 1, and ultimately approaches a negative limit when γ_a tends to infinity. Due to the absolute value operator in the final trading volume expression, this behavior gives rise to two distinct regions. The first region corresponds to γ_a values within the interval $[0, 1]$, where the factor F_3 decreases gradually, reaching zero as ambiguity aversion increases toward 1. In the second region, for γ_a values in the range $[1, \infty]$, the factor F_3 increases steadily, ultimately reaching the absolute value of the limit depicted in the figure. The underlying intuition here is that as ambiguity aversion decreases from 1 to 0 in the initial region, type-A investors will increase the size of their positions in the risky asset. The direction of this adjustment is driven by the difference in net expectations between type-A and type-B investors, which incorporates their difference in beliefs regarding the risky asset's payoff $(\mu_X^B - \mu_X^A)$. In the subsequent region, characterized by an increase in ambiguity aversion from 1 to infinity, type-A investors tend to exit the risky asset market. They liquidate their positions in the risky asset, initially established in period 1, resulting in higher trading activity. In instances of extreme ambiguity aversion, type-A investors divest their entire position established in period 1, regardless of the information contained in the signal S.

The lower panel of Figure 3 illustrates how the factor F_3 behaves concerning ambiguity volatility $\sigma_\delta^{2\,A}$ and ambiguity aversion γ_a. When γ_a is positive, an increase in ambiguity translates in an increase of F_3, whereas negative γ_a values lead to a decrease in F_3 with rising ambiguity volatility. When γ_a equals one, F_3 remains at zero. Furthermore, due to the absolute value operator in the final trading volume formula, an overall surge in ambiguity volatility results in an elevated F_3 as long as γ_a is not zero. The intuition here is that for investors who embrace ambiguity with γ_a below 1, as ambiguity volatility increases, they will seek to increase the size of their positions in the risky asset. Conversely, for averse ambiguity investors with γ_a greater than one, rising ambiguity volatility encourages them to liquidate their initial non-zero position in the risky asset. In terms of trading volume, both scenarios lead to increased trading activity. However, for investors who are ambiguity-neutral with γ_a equal to 1, this ambiguity volatility mechanism has no impact.

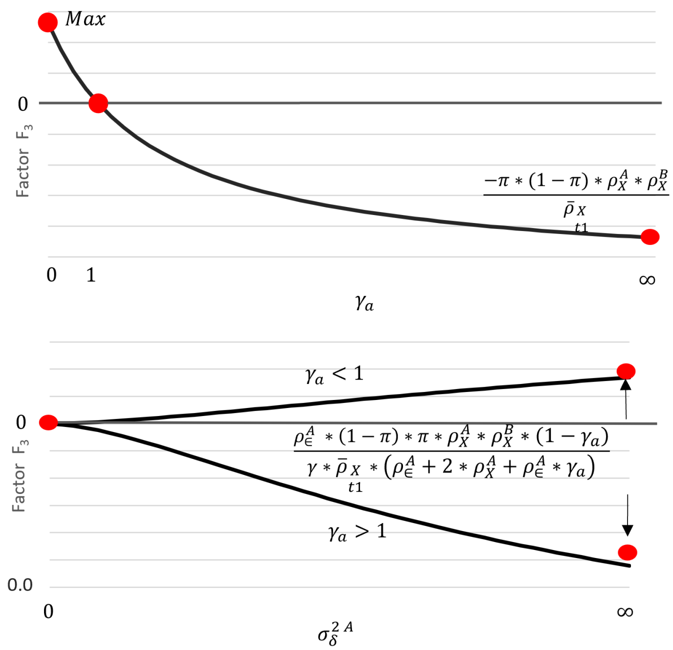

Figure 3. Effect of γ_a and $\sigma_\delta^{2\,A}$ on factor F_3 . This graph shows how the the values of γ_a and $\sigma_\delta^{2\,A}$ affect the factor F_3 multiplying. The top panel achieves a maximum of $\frac{\pi*(1-\pi)^2*\rho_\epsilon^{A\,3}*\rho_\epsilon^B*\rho_X^{A\,2}*\rho_X^B}{\gamma*\left(\rho_\delta^{A\,2}*\bar{\rho}_{\underset{t1}{X}}*\left(\rho_\epsilon^A+\rho_X^A\right)^2+\rho_\epsilon^{A\,2}*\rho_\epsilon^B*(1-\pi)*\rho_X^A*\left(\rho_\epsilon^A+2*\rho_X^A\right)+\rho_\delta^A*\rho_\epsilon^A*\left(\rho_\epsilon^A+\rho_X^A\right)*\left(3*(1-\pi)*\rho_\epsilon^B*\rho_X^A+\rho_\epsilon^A*(\pi*\rho_X^A+\bar{\rho}_{\underset{t1}{X}})\right)\right)*\bar{\rho}_{\underset{t1}{X}}}$ when γ_a goes to 0, a value of 0 when γ_a is 1 and a value of $\frac{-\pi*(1-\pi)*\rho_X^A*\rho_X^B}{\bar{\rho}_{\underset{t1}{X}}}$ when γ_a tends to infinity. The bottom panel shows the effect of the ambiguity volatility on the factor F_3 for different levels of ambiguity aversion.

2.7 Ambiguity and the Trading Volume to Price Volatility Elasticity

In this section, I examine the influence of ambiguity on the elasticity relationship between trading volume and price volatility. To derive this elasticity relationship, I make the assumption that, on average, changes in price ΔP_{21} follow a normal distribution, akin to the approach taken by Bollerslev et al. (2018) for differences in prior beliefs. I begin by presenting a simplified version 1 of the elasticity relationship, which includes both ambiguity and the price channel of trading volume. In the complete version 2 of the elasticity relationship, I introduce heterogeneity in prior beliefs. Further details can be found in Appendix C.

Version 1: Price and ambiguity channel

Elasticity ξ between trading volume and price volatility when the price and ambiguity channels are present, while heterogeneous prior beliefs are switched off.

$$\xi = \frac{\partial E[V_{21}]/E[V_{21}]}{\partial \sigma_p/\sigma_p} = \frac{1}{1 + \Psi\left(\dfrac{|\alpha_v|}{|\beta_v| * \sigma_p}\right)} \tag{2.22}$$

$$\begin{aligned}
&where \\
\Psi(x) &= \frac{x * (\Phi(x) - 1/2)}{\phi(x)} \\
\alpha_V &= \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B}{\gamma * \bar{\rho}_{\mathcal{E}}}\right] * \mu_\delta^A \\
\beta_V &= \left[\frac{\frac{\pi}{\nu^A} * \rho_X^A * \left(\rho_{\mathcal{E}}^B - \rho_{\mathcal{E}}^A\right)}{\bar{\rho}_{\mathcal{E}}}\right] \\
&and \\
\Phi(x) &= \text{Normal CDF} \\
\phi(x) &= \text{Normal Density}
\end{aligned}$$

In this simplified version, the elasticity is influenced by a combination of two factors: the impact of the bias effect, denoted as μ_δ^A, within the term α_V from the expectations channel of ambiguity, and the influence of price changes on trading volume, associated to the term β_V. Both α_V and β_V include the inverse of the coefficient ν^A, as well as the precision term $\rho_{\mathcal{E}}^A$ in their numerators and denominators. Ambiguity aversion, denoted as γ_a, affects this elasticity through the ν^A coefficient, while ambiguity volatility also plays a role by affecting both the ν^A coefficient and the precision term $\rho_{\mathcal{E}}^A$.

In the top panel of Figure 4 shows that when the magnitude of the expectation channel bias generated by ambiguity, represented as μ_δ^A, is large in either positive or negative terms, the elasticity decreases. The intuition here is that when the expectation channel of ambiguity becomes the predominant driver of trading activity, trading volume appears increasingly disconnected from price fluctuations. In the most extreme scenarios, the elasticity approaches zero.

The middle panel of the figure indicates that ambiguity aversion γ_a has no impact on elasticity. This is because ambiguity aversion affects both α_V and β_V in equal proportions. As a result, any alterations in ambiguity aversion do not change the relative importance between the expectations channel of ambiguity and price changes as drivers of trading volume.

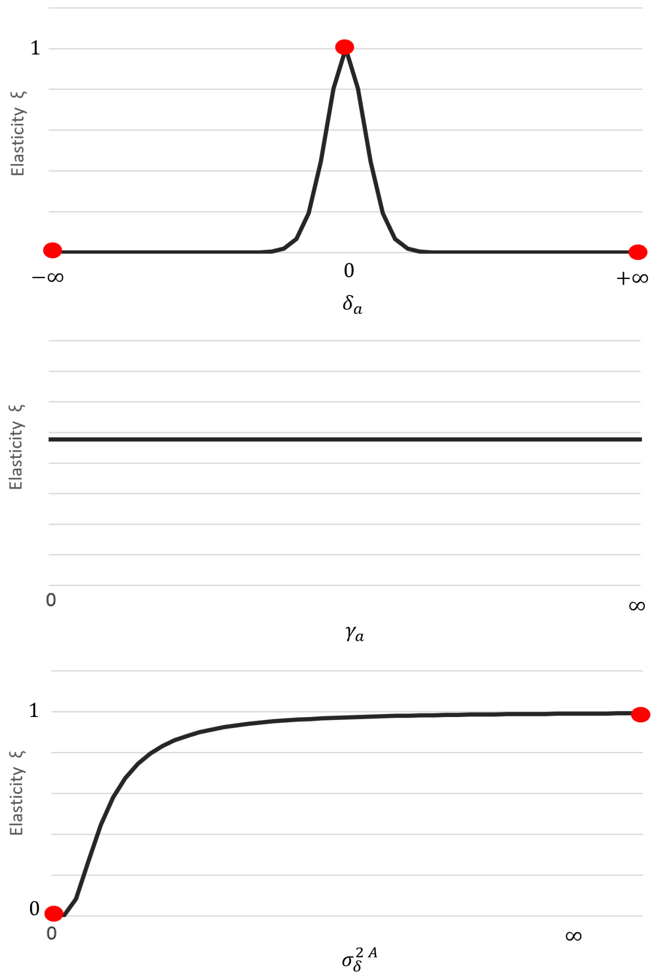

Figure 4. Effect of γ_a and $\sigma_\delta^{2\,A}$ on Elasticity. This graph shows how the values of γ_a and $\sigma_\delta^{2\,A}$ affect the elasticity ξ between trading volume and price volatility.

The bottom panel of Figure 4 shows that as ambiguity's volatility increases, the elasticity $\Psi(x)$ tends to its maximum value of 1. The underlying intuition here is that heightened ambiguity volatility diminishes the quality of the signal S, which includes the effects stemming from the expectations channel of ambiguity μ_δ^A. As ambiguity volatility approaches infinity, it dampens the expectations channel of ambiguity as a driver of trading volume. Consequently the primary force influencing trading volume becomes price changes alone. This is why, in extreme volatility, elasticity converges to a one-to-one relationship between trading volume and price volatility.

Version 2: Price channel, ambiguity channel and heterogeneous beliefs

Full elasticity expression ξ between trading volume and price volatility when the price channel, ambiguity channel and heterogeneous prior beliefs are present.

$$\xi = \frac{\partial E[V_{21}]/E[V_{21}]}{\partial \sigma_p/\sigma_p} = \frac{1}{1+\Psi\left(\dfrac{|\alpha_v|}{|\beta_v| * \sigma_p}\right)} \tag{2.23}$$

$$\begin{aligned}
&\textit{where}\\
\Psi(x) &= \frac{x * (\Phi(x) - 1/2)}{\phi(x)}\\
\alpha_V &= \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho^A_{\mathcal{E}} * \rho^B_{\mathcal{E}}}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \mu^A_\delta\\
&+ \left[\pi * (1-\pi)^2 * \left(\frac{1}{\nu^A} - 1\right) * \left(\frac{\rho^B_{\mathcal{E}}}{\bar{\rho}_{\mathcal{E}} * \gamma}\right) * \left(\frac{\rho^A_X * \rho^B_X}{\bar{\rho}_{\underset{t1}{X}}}\right)\right]_{F_3} * \left(\mu^B_X - \mu^A_X\right)\\
&+ \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho^A_{\mathcal{E}} * \rho^B_{\mathcal{E}}}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \left(\mu^A_\epsilon - \mu^B_\epsilon\right)\\
\beta_V &= \left[\frac{\frac{\pi}{\nu^A} * \rho^A_X * \left(\rho^B_{\mathcal{E}} - \rho^A_{\mathcal{E}}\right)}{\bar{\rho}_{\mathcal{E}}}\right]_{F_2}\\
&\textit{and}\\
\Phi(x) &= \text{Normal CDF}\\
\phi(x) &= \text{Normal Density}
\end{aligned}$$

In this full version of the elasticity expression, the introduction of heterogeneous prior beliefs results in two additional terms associated to $\left(\mu^B_X - \mu^A_X\right)$ and $\left(\mu^A_\epsilon - \mu^B_\epsilon\right)$. Here, the elasticity relating trading volume to price volatility is influenced by the bias effect μ^A_δ arising from the expectations channel of ambiguity, as well as the differences in prior beliefs regarding the risky asset payoff $\left(\mu^B_X - \mu^A_X\right)$ and the difference in prior beliefs concerning the measurement error $\left(\mu^A_\epsilon - \mu^B_\epsilon\right)$.

In terms of ambiguity-related coefficients, the expectations channel of ambiguityδ_A exclusively influences the first component of the α_V term. This effect can lead to a maximum elasticity of 1, which occurs when the bias μ^A_ϵ neutralized the impact of differences in prior beliefs regarding the risky asset payoff $\left(\mu^B_X - \mu^A_X\right)$ and the measurement error $\left(\mu^A_\epsilon - \mu^B_\epsilon\right)$. This is illustrated in the top panel of figure-5 below. The underlying intuition is that when the expectations channel of ambiguity becomes the dominant factor driving trading volume in any direction, the sensitivity of trading volume to price changes diminishes.

The ambiguity aversion coefficient γ_a since it appears on the portions of $alpha_V$ associated to the expectations channel of ambiguity μ_ϵ^A, the differences in prior beliefs about the measurement error $\left(\mu_\epsilon^A - \mu_\epsilon^B\right)$ and the $beta_V$ coefficient that multiplies changes in prices, does mainly affect the elasticity through the differences in prior beliefs about the risky asset payoff $\left(\mu_X^B - \mu_X^A\right)$. The mechanism is that, as the ambiguity aversion increases, the effect of $\left(\mu_X^B - \mu_X^A\right)$ starts increasing and this contributes to decrease the heterogeneity if the sign of $\left(\mu_X^B - \mu_X^A\right)$ points in the same direction as the effect of $\left(\mu_\epsilon^A + \mu_\epsilon^A - \mu_\epsilon^B\right)$, or initially increase the net heterogeneity if the they point in the opposite directions. The intuition here is that as ambiguity aversion increases, type-A investors will tend to sell their assets regardless of market price conditions, thereby reducing elasticity. In extreme cases, they liquidate their entire t1 position, which are proportional to $\left(\mu_X^B - \mu_X^A\right)$, bringing the elasticity down to 0. Conversely, when ambiguity aversion decreases below 1, type-A investors aim to increase the magnitude of their positions in the risky asset independently of market price conditions, also resulting in decreased elasticity to price changes.

Changes in ambiguity volatility influence the impact of the ambiguity expectations channel δ_A, the impact of the difference in prior beliefs about the measurement error $\left(\mu_\epsilon^A - \mu_\epsilon^B\right)$, and the impact of the difference in the in prior beliefs about the risky asset payoff $\left(\mu_X^B - \mu_X^A\right)$. As ambiguity volatility increases, the net effects caused by δ_A and $\left(\mu_\epsilon^A - \mu_\epsilon^B\right)$ start to diminish. In extreme cases, the primary driving force of trading volume becomes price changes, leading to an elasticity that tends to approach 1. However, ambiguity volatility also impacts the influence of the difference in prior beliefs about the risky asset payoff $\left(\mu_X^B - \mu_X^A\right)$, causing the elasticity to deviate from its maximum as ambiguity volatility increases on the right-hand side of the figure. The intuition here is that as ambiguity volatility increases, the impact of the signal S on type-A investors weakens, rendering the trading activity of type-A investors more responsive to market price conditions alone. This initially results in an upward trend in elasticity as ambiguity volatility increases. Additionally, there is a secondary effect associated with the sizing of type-A investors' positions, which depends on their ambiguity aversion. When ambiguity volatility rises, ambiguity-averse type-A investors will seek to reduce the magnitude of their positions, while ambiguity-loving type-A investors will seek to increase them, regardless of market conditions like prices. This second source of trading activity, not directly linked to price conditions, causes the elasticity to decrease. In cases of extreme ambiguity volatility, this latter effect takes precedence, reducing the elasticity towards 0.

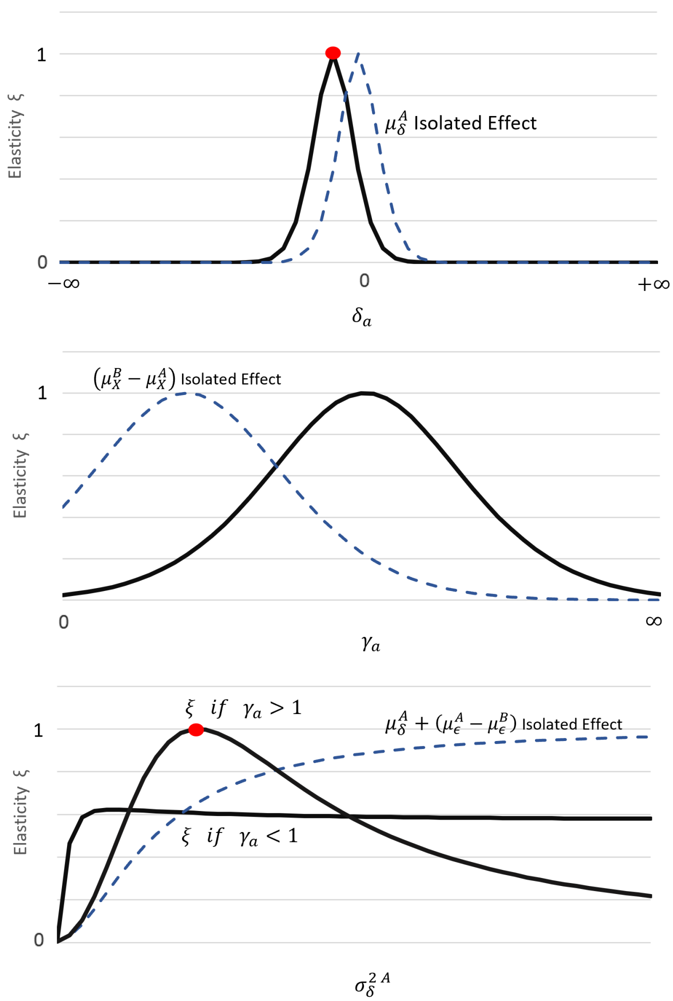

Figure 5. Effect of μ_δ^A, γ_a and $\sigma_\delta^{2\,A}$ on Elasticity. This graph shows how the values of μ_δ^A, γ_a and $\sigma_\delta^{2\,A}$ affect the elasticity ξ between trading volume and price volatility.

3 Empirical Evidence: Data

The empirical validation of the model proposed in this work relies on data comprising price and trading volume series of the SPY ETF, along with measures of Ambiguity and differences in prior beliefs.

Prices and Trading Volume

The data for SPY prices and trading volumes in daily and monthly frequencies is sourced from Bloomberg. Daily regressions utilize data spanning from 2013 to 2018, while monthly regressions cover the period from 2000 to 2020. Monthly regressions relying on daily frequency data cover the period 2013 to 2018.

Intraday calculations utilize 15-minute sampled TAQ data from NASDAQ to construct daily frequency α_V and β_V coefficients.

Ambiguity

This research employs two Ambiguity measures: the news-based Economic Policy Uncertainty Index (EPU)(Baker et al., 2016) and the market price based measure of Izhakian (2020).

I utilize both the daily (Baker et al., 2021) and monthly (Baker et al., 2016) versions of the Economic Policy Uncertainty Index. The daily measure (TEU-SCA) represents an ambiguity index extracted daily from Twitter text messages up to 4 pm U.S. EDT Time, while the monthly measure corresponds to the original EPU measure extracted from U.S. newspapers. The daily measure spans from 2013 to 2018, and the monthly measure covers the period from 2000 to 2020. In the rest of the text I identify these series by AMB_{EPUD} and AMB_{EPUM} respectively.

The Izhakian (2020) ambiguity measure represents a market price-based indicator designed to approximate the implicit volatility of investors' beliefs. Unlike the previous type of measures, this index is directly extracted from prices rather than from text and correlates negatively with the EPU measure. To maintain consistency in the economic interpretation of both measures' results, I inverted the sign of the Izhakian (2020) measure. The series used in this research is of monthly frequency and covers the period from 2000 to 2020. In the rest of the text I identify this serie by AMB_{IZHM}.

Differences in Prior Beliefs

As proxies for differences in prior beliefs, I employ two types of a daily disagreement measures extracted from the social media investing platform StockTwits (Cookson & Niessner, 2020) and a monthly analysts' forecast dispersion measure extracted from the IBES database.

The two types of daily investor disagreement are based on data collected from the social media investing platform StockTwits by Cookson and Niessner (2020) on a daily basis up to 4pm

U.S. EDT Time from 2013 to 2018. Both series types represent the standard deviation of bullishness/bearishness beliefs across users of the StockTwits platform. The users bullishness/bearishness information is extracted from messages posted on the platform. For messages not explicitly tagged as bullish or bearish by the users, their text content is used to infer their sentiment. This text-based classification of unlabeled messages is performed by a maximum-entropy machine learning algorithm (Cookson & Niessner, 2020). One disagreement series type is calculated within the same type of users' investment approach, and it is more closely related to differences in information sets (Cookson & Niessner, 2020). The other type is extracted across users with different investment approaches and is more likely caused by different ways or models of interpreting information (Cookson & Niessner, 2020). In the following sections I refer to these two types of measures by $PBEL_{WI}$ and $PBEL_{AC}$ respectively. For both types of measures, I use a direct SPY extracted reading and a S&P 500 proxy, constructed as an equally weighted mean of the S&P 500 stocks' readings. In the rest of the text I identify these series by $\{PBEL_{WI,ETF}, PBEL_{AC,ETF}\}$ and $\{PBEL_{WI,IND}, PBEL_{AC,IND}\}$ respectively. Monthly regressions are performed using the monthly means of these series.

The IBES proxy for differences in prior beliefs is a monthly equally weighted average of the individual S&P500 companies' forecast dispersion measures. Each company's forecast dispersion measure is calculated as the standard deviation across the most recent earnings forecasts, scaled by the corresponding company's monthly average price. This series covers the period from 2000 to 2020. In the rest of the text I identify this serie by $PBEL_{IBES}$.

4 Empirical Evidence: Trading Volume and Price Relation

In this section, I empirically test the trading volume model presented in section 2.6 using SPY data at both daily and monthly frequencies.

To validate the hypothesis that ambiguity affects trading volume, I examine the statistical significance of the ambiguity time series in explaining the α_V term of the volume-price relation. This mechanism represents the expectations channel of ambiguity. I also assess the statistical significance of ambiguity volatility in explaining the β_V term of the volume-price relation. Furthermore, I investigate whether ambiguity has, on average, a positive effect on trading volume.

Methodology

I validate the theoretical model of section 2.6 through daily and monthly frequency regressions. These regressions involve the SPY ETF trading volume series as the dependent variable and include as explanatory variables changes in price, proxies of ambiguity, proxies of differences in prior beliefs, proxies of ambiguity volatility, price volatility, and time controls. To test the hypothesis that ambiguity affects the volume-price relation even after accounting for price changes, I examine the statistical significance of the coefficients associated with mean ambiguity $\{\alpha_{amb}\}$ and ambiguity volatility $\{\beta_{\sigma_{amb}}, \beta_{\Delta\sigma_{amb}}\}$. Additionally, I expect that mean ambiguity U would positively impact trading volume $\{\alpha_{amb}\}$ activity.

$$\begin{aligned} V &= \alpha_V + \beta_V * \Delta_P + \epsilon \\ \alpha_V &= 1 + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \epsilon \\ \beta_V &= 1 + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_p * \sigma_P + \epsilon \end{aligned} \tag{4.1}$$

$$\begin{aligned} V = &\big(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\big)_{\alpha} \;+\; \big(\beta_p + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_{\sigma_p} * \sigma_P\big)_{\beta} * \\ &\Delta_P \;+ V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \end{aligned} \tag{4.2}$$

$$\begin{aligned} \Delta V = &\; c \;+\; \big(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\big)_{\Delta\alpha} \;+\; \big(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{AMB} + \beta_{\Delta\sigma_p} * \\ &\Delta\sigma_P\big)_{\Delta\beta} * \Delta_P \;+\; \big(\beta_P + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_{\sigma_p * \Delta_P^2} * \sigma_P\big)_{\beta} * \Delta_P^2 \;+ \Delta V_{t-1} + \\ &\textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \end{aligned} \tag{4.3}$$

In (1), I begin by regressing intraday trading volume on price changes to obtain daily time series of α_V and β_V. Following the theoretical model, I then conduct regressions of the daily α_V time series on the difference in beliefs and ambiguity proxy series. These ambiguity series act as proxies for the mean ambiguity term μ_δ^A within the model. I also regress the β_V coefficient on the SPY price volatility and the ambiguity volatility series. I use the SPY price volatility series as a proxy for the prior beliefs precisions $\rho_X^{I_B}$ and ρ_X^A, and the ambiguity volatility series as a proxy for the ambiguity volatility $\sigma_\delta^{2\,A}$ within $\rho_{\mathcal{E}}^A$. The daily time series of α_V and β_V were obtained using the 15-minutes SPY TAQ data. The monthly time series of α_V and β_V were obtained using monthly SPY trading volume and prices from Bloomberg.

In the second regression (2), I directly regress the trading volume measure on the explanatory series that make up the α_V and β_V coefficients of the model. I do not perform a previous regression step to obtain time series of α_V and β_V.

The last regression (3) is a differenced version of regression (2), where I regress changes in trading volume on the changes of the explanatory variables that, according to the model, explain the volume.

In all regressions, I include a lagged series of the dependent variable as an explanatory variable to account for trading volume persistence and also use yearly time controls. Additionally, all series were detrended beforehand.

These regressions are conducted using all combinations of the difference in beliefs and ambiguity series described in Section-3. For daily frequency, I have 4 different measures of differences in beliefs obtained from the Cookson and Niessner (2020) data and one measure of ambiguity (EPU) (Baker et al., 2016, 2021). Both sources of information are extracted from text rather than market prices or trading volumes. This results in four distinct explanatory daily datasets for each regression. For the monthly regressions, I have five different series of differences in beliefs and two series of ambiguity, resulting in a total of 10 distinct explanatory datasets for each regression. Among the differences in beliefs series, four are obtained by converting the daily measures from Cookson and Niessner (2020) into monthly frequency through averaging, and a fifth one is extracted from the IBES database. The two ambiguity series are the monthly version of the EPU index (Baker et al., 2016) and the monthly market-based measure from Izhakian (2020).

Results

The regression results demonstrate that the ambiguity series (regression coefficient α_{amb}, μ_δ^A in the model) is statistically significant in explaining the α_V component of the volume-price relation across all regression types, data sets combinations and frequencies (daily, monthly). However, no clear supporting evidence of the relevance of ambiguity volatility was found. Overall, these results provide positive validation for the hypotheses (H1) that ambiguity influences trading volume activity, even after controlling for price changes, and (H2) that higher levels of ambiguity are associated with larger trading volumes. Moreover, these findings suggest that, in the context of the model, the expectations channel of ambiguity plays a more significant role in explaining trading volume compared to ambiguity volatility.

In concrete, effects of the ambiguity series that proxy for mean ambiguity (regression coefficient α_{amb}, μ_δ^A in the model), the daily frequency regressions (1) and (2) for the period 2013-2018 yielded the most statistically significant results. Table-?? below shows that the EPU ambiguity measure exhibited t-stats ranging from 3.6 to 4.10 in regressions (1) and (2). For the regression (3), the EPU ambiguity measure t-stats were around 3.6 .

Between 2013 and 2018, regression (1) in Table-1 shows that a one-standard-deviation increase in the EPU daily ambiguity measure is linked to an approximately 11%-standard-deviation increase in the coefficient α_V. In regression (2), where the impact of explanatory variables on trading volume is directly measured, a one-standard-deviation increase in ambiguity is associated with a +10%-standard-deviation increase in trading volume. Regression (3), which employs variables in differences, indicates a +12%-standard-deviation increase in the delta of trading volume per one-standard-deviation increase in the delta of EPU daily ambiguity.

The monthly regressions in Table-2 indicate that a one-standard-deviation increase in the EPU monthly ambiguity measure is associated with approximately a +23%-standard-deviation increase in the monthly coefficient α_V. Regression (2), directly measuring the effect of ambiguity on trading volume, demonstrates an approximately +20%-standard-deviation increase in trading volume per one-standard-deviation increase in monthly EPU ambiguity. The first differences regression (3) suggests an effect of around +23%-standard-deviation increase in the delta of trading volume per one-standard-deviation increase in the delta of EPU ambiguity. The effect using the Izhakian (2020) and the IBES measures is also statistically significant. Across regressions (1), (2), and (3), the results suggest that a one-standard-deviation increase in these measures leads to an effect ranging between a +25%-standard-deviation increase to a +58%-standard-deviation increase in trading volume. To maintain consistency in the economic interpretation, I previously inverted the sign of the Izhakian (2020) series due to its negative correlation with the EPU and IBES measures.

In relation to the proxy measures of differences in prior beliefs (regression coefficient α_{pbel}, $(\mu_e^A - \mu_e^B)$ and $(\mu_X^B - \mu_X^A)$ in the model), I obtain for the daily frequency regressions between 2013 to 2018 that both measures sourced from Stocktwits (Cookson & Niessner, 2020) are statistically significant, with t-stats above 3. Here I obtain that on average a one-standard-deviation increase in any of these measures leads to a change between a -15%-standard-deviation to +13%-standard-deviation in trading volume. For almost all versions of these series, the direction of the impact on trading volume is positive with the exception of the ETF based measure obtained from users with different investment approaches ($PBEL_{AC,ETF}$), here the direction of the effect is negative.

In the monthly frequency regressions, the results for the difference in prior beliefs coefficient are not as clear as in the daily frequency regressions in terms of sign and statistical significance (Appendix-F). For regressions (1) and (2), the Cookson and Niessner (2020) measures looses its statistical significance and switches sign to negative in most cases, while in regression (3), the sign is mostly positive across the different datasets. The IBES measure demonstrates strong statistical significance with t-stats above 3 in all regressions (1) and (2), albeit with a negative sign. Specifically, a one-standard-deviation increase in the IBES measure leads to a -4%-standard-deviation to -14%-standard-deviation decrease in trading volume. In the monthly first differenced regression (3), the IBES measure does not show statistical significance but maintains the negative sign.

Regarding the influence of ambiguity volatility on the β_V component of the volume-price relation (regression coefficients $\beta_{\sigma_{amb}}$ and $\beta_{\Delta\sigma_{amb}}$), I did not find clear supporting evidence. In the daily frequency regression (1) (Table-1 below), the results show statistical significance in support of the daily EPU ambiguity volatility interaction with the change in price ΔP. However, for the daily regressions (2) and (3), the coefficient did not exhibit clear statistical significance. In the monthly Table-3 below, the regression (3) provides strong statistical evidence in favor of the Izhakian (2020) ambiguity volatility series when combined with the (Cookson & Niessner, 2020) difference in prior beliefs measures. However, for the same regression (3) using the monthly IBES difference in beliefs measure, I only found support in favor of the EPU monthly ambiguity volatility serie. As for the monthly regressions (1) and (2), I did not find clear statistical evidence supporting any of the ambiguity volatility series. For detailed regression results refer to Appendix-F.

Table 1. Daily Alpha and Beta Regressions.

This table summarizes the statistical significance of the Ambiguity mean term AMB (α_{amb}) belonging to α_V (Panel A) and the ambiguity volatility term σ_{amb} ($\beta_{\sigma_{amb}}$) belonging to β_V (Panel B) across the different regression setups (1), (2) and (3). The first column on the left indicates the dataset. $\text{PBEL}_{\text{WI, IND}}$, $\text{PBEL}_{\text{AC, IND}}$, $\text{PBEL}_{\text{WI, ETF}}$ and $\text{PBEL}_{\text{AC, ETF}}$ are the Cookson and Niessner (2020) difference in previous beliefs measures extracted from Stocktwits. AMB_{EPUD} refers to the daily EPU ambiguity measure extracted from Twitter. The regressions were performed for the period 2013 to 2018. T-values in round brackets are Newey-West autocorrelation robust values. γ_p represent time fixed effects. Full regression results in Appendix-D.

$$\alpha_V =^1 c + \alpha_{amb} * AMB + \alpha_{pbel} * PBEL + \alpha_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$V =^2 \left(c+\alpha_{pbel}*PBEL+\alpha_{amb}*AMB\right)_\alpha + \left(\beta_p+\beta_{\sigma_{amb}}*\sigma_{amb}+\beta_{\sigma_p}*\sigma_P\right)_\beta*\Delta_P + V_{t-1}+\textstyle\sum_{p=t+1}^{T}\gamma_p+\epsilon$$

$$\Delta V =^3 c + \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\right)_{\Delta\alpha} + \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p * \Delta_P^2} * \sigma_P\right)_\beta * \Delta_P^2 + \Delta V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon$$

Dataset	R(1)	R(2)	R(3)	
Panel A	α_{amb}	α_{amb}	α_{amb}	
$\text{PBEL}_{\text{WI, IND}}$ & AMB_{EPUD}	0.11***	0.10***	0.12***	
	(3.62)	(3.78)	(3.59)	
$\text{PBEL}_{\text{AC, IND}}$ & AMB_{EPUD}	0.11***	0.10***	0.12***	
	(3.80)	(3.94)	(3.62)	
$\text{PBEL}_{\text{WI, ETF}}$ & AMB_{EPUD}	0.11***	0.10***	0.12***	
	(3.71)	(3.92)	(3.53)	
$\text{PBEL}_{\text{AC, ETF}}$ & AMB_{EPUD}	0.11***	0.10***	0.12***	
	(3.84)	(4.10)	(3.62)	
Panel B	$\beta_{\sigma_{\text{amb}}}$	$\beta_{\sigma_{\text{amb}}}$	$\beta_{\triangle\sigma_{\text{amb}}}$	$\beta_{\sigma_{\text{amb}}}$
$\text{PBEL}_{\text{WI, IND}}$ & AMB_{EPUD}	-0.09*	-0.05	-0.02	-0.05
	(-1.74)	(-1.30)	(-0.26)	(-1.17)
$\text{PBEL}_{\text{AC, IND}}$ & AMB_{EPUD}	-0.09*	-0.05	-0.02	-0.05
	(-1.74)	(-1.37)	(-0.25)	(-1.21)
$\text{PBEL}_{\text{WI, ETF}}$ & AMB_{EPUD}	-0.09*	-0.05	-0.02	-0.05
	(-1.74)	(-1.40)	(-0.26)	(-1.24)
$\text{PBEL}_{\text{AC, ETF}}$ & AMB_{EPUD}	-0.09*	-0.05	-0.02	-0.05
	(-1.74)	(-1.44)	(-0.26)	(-1.24)

Table 2. Monthly Alpha Regression.

This table summarizes the statistical significance of the Ambiguity mean term AMB (α_{amb}) belonging to α_V across the different regression setups (1), (2) and (3). The first column on the left indicates the dataset. $\text{PBEL}_{\text{WI, IND}}$, $\text{PBEL}_{\text{AC, IND}}$, $\text{PBEL}_{\text{WI, ETF}}$ and $\text{PBEL}_{\text{AC, ETF}}$ are the Cookson and Niessner (2020) difference in previous beliefs measures extracted from Stocktwits and $\text{PBEL}_{\text{IBES}}$ is the difference in previous beliefs measure extracted from IBES. Regarding the Ambiguity measures, AMB_{EPUM} refers to the monthly EPU ambiguity measure extracted from newspapers and AMB_{IZHM} refers to the monthly Ambiguity market based measure of Izhakian (2020). The regressions involving the IBES measure cover the period 2000 to 2020 and the rest of the regressions cover the period 2013 to 2018. γ_p represent time fixed effects. T-values in round brackets are Newey-West autocorrelation robust values. Full regression results in Appendix-F.

$$\alpha_V =^1 c + \alpha_{amb} * AMB + \alpha_{pbel} * PBEL + \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$V =^2 \left(c + \alpha_{pbel} * PBEL + \ddot{\alpha}_{amb} * AMB\right)_{\alpha} + \left(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\right)_{\beta} * \Delta_P + V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\Delta V =^3 c + \left(\alpha_{pbel} * \Delta PBEL + \ddot{\alpha}_{amb} * \Delta AMB\right)_{\Delta\alpha} + \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p * \Delta_P^2} * \sigma_P\right)_{\beta} * \Delta_P^2 + \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

Dataset	R(1)	R(2)	R(3)
	α_{amb}	α_{amb}	α_{amb}
$\text{PBEL}_{\text{WI, IND}}$ & AMB_{EPUM}	0.23*	0.22***	0.24***
	(1.99)	(3.87)	(4.08)
$\text{PBEL}_{\text{AC, IND}}$ & AMB_{EPUM}	0.23**	0.21***	0.25***
	(2.07)	(3.75)	(4.03)
$\text{PBEL}_{\text{WI, ETF}}$ & AMB_{EPUM}	0.20*	0.19***	0.19***
	(1.99)	(5.48)	(4.17)
$\text{PBEL}_{\text{AC, ETF}}$ & AMB_{EPUM}	0.23*	0.20***	0.23***
	(1.95)	(3.14)	(3.43)
$\text{PBEL}_{\text{WI, IND}}$ & AMB_{IZHM}	0.46***	0.35***	0.38***
	(4.61)	(3.18)	(4.05)
$\text{PBEL}_{\text{AC, IND}}$ & AMB_{IZHM}	0.46***	0.36***	0.38***
	(4.68)	(3.27)	(4.03)
$\text{PBEL}_{\text{WI, ETF}}$ & AMB_{IZHM}	0.47***	0.36***	0.33***
	(4.98)	(3.65)	(4.58)
$\text{PBEL}_{\text{AC, ETF}}$ & AMB_{IZHM}	0.46***	0.36***	0.38***
	(4.75)	(3.09)	(3.94)
$\text{PBEL}_{\text{IBES}}$ & AMB_{EPUM}	0.37***	0.36***	0.25***
	(3.03)	(3.26)	(2.80)
$\text{PBEL}_{\text{IBES}}$ & AMB_{IZHM}	0.58***	0.48***	0.53***
	(5.19)	(4.15)	(5.30)

Table 3. Monthly Betas Regression.

This table summarizes the statistical significance of the Ambiguity volatility term σ_{amb} ($\beta_{\sigma_{amb}}$) belonging to β_V across the different regression setups (1), (2) and (3). The first column on the left indicates the dataset. $\text{PBEL}_{\text{WI, IND}}$, $\text{PBEL}_{\text{AC, IND}}$, $\text{PBEL}_{\text{WI, ETF}}$ and $\text{PBEL}_{\text{AC, ETF}}$ are the Cookson and Niessner (2020) difference in previous beliefs measures extracted from Stocktwits and $\text{PBEL}_{\text{IBES}}$ is the difference in previous beliefs measure extracted from IBES. Regarding the Ambiguity measures, AMB_{EPUM} refers to the monthly EPU ambiguity measure extracted from newspapers and AMB_{IZHM} refers to the monthly Ambiguity market based measure of Izhakian (2020). The regressions involving the IBES measure cover the period 2000 to 2020 and the rest of the regressions cover the period 2013 to 2018. γ_p represent time fixed effects. T-values in round brackets are Newey-West autocorrelation robust values. Full regression results in Appendix-F.

$$\beta_V =^1 c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P \; + \; \beta_{t-1} + \; \sum_{p=t+1}^{T} \gamma_p + \; \epsilon$$

$$V =^2 \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\right)_\alpha + \left(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\right)_\beta * \Delta_P + V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\Delta V =^3 c \; + \; \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\right)_{\Delta\alpha} \; + \; \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P \; + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p * \Delta_P^2} * \sigma_P\right)_\beta * \Delta_P^2 \; + \; \Delta V_{t-1} \; + \; \sum_{p=t+1}^{T} \gamma_p \; + \; \epsilon$$

Dataset	R(1)	R(2)	R(3)	
	$\beta_{\sigma_{\text{amb}}}$	$\beta_{\sigma_{\text{amb}}}$	$\beta_{\triangle\sigma_{\text{amb}}}$	$\beta_{\sigma_{\text{amb}}}$
$\text{PBEL}_{\text{WI, IND}}$ & AMB_{EPUM}	-0.15	0.05	-0.51	0.31
	(-0.43)	(0.39)	(-1.13)	(1.01)
$\text{PBEL}_{\text{AC, IND}}$ & AMB_{EPUM}	-0.15	0.07	-0.45	0.27
	(-0.43)	(0.53)	(-1.00)	(0.92)
$\text{PBEL}_{\text{WI, ETF}}$ & AMB_{EPUM}	-0.15	0.05	-0.43	0.29
	(-0.43)	(0.38)	(-1.15)	(1.32)
$\text{PBEL}_{\text{AC, ETF}}$ & AMB_{EPUM}	-0.15	0.12	-0.42	0.28
	(-0.43)	(1.00)	(-0.99)	(1.20)
$\text{PBEL}_{\text{WI, IND}}$ & AMB_{IZHM}	0.17	-0.03	0.12***	-0.09*
	(0.93)	(-0.63)	(4.08)	(-1.83)
$\text{PBEL}_{\text{AC, IND}}$ & AMB_{IZHM}	0.17	-0.04	0.12***	-0.09*
	(0.93)	(-0.66)	(4.13)	(-1.90)
$\text{PBEL}_{\text{WI, ETF}}$ & AMB_{IZHM}	0.17	-0.03	0.15***	-0.10**
	(0.93)	(-0.61)	(5.99)	(-2.05)
$\text{PBEL}_{\text{AC, ETF}}$ & AMB_{IZHM}	0.17	-0.05	0.12***	-0.10**
	(0.93)	(-0.88)	(5.55)	(-2.09)
$\text{PBEL}_{\text{IBES}}$ & AMB_{EPUM}	0.20	-0.10	-0.35***	0.19**
	(0.99)	(-1.39)	(-7.09)	(2.54)
$\text{PBEL}_{\text{IBES}}$ & AMB_{IZHM}	0.31	0.03	-0.09	0.02
	(1.54)	(0.39)	(-1.14)	(0.35)

5 Empirical Evidence: Trading Volume and Price Volatility Elasticity

In this section, I empirically test the trading volume elasticity model presented in section 2.7 using SPY data at daily frequency.

To validate the hypothesis that Ambiguity affects the elasticity relation between trading volume and price volatility, I test whether the series proxying for mean Ambiguity and Ambiguity volatility are statistically significant in explaining this elasticity. Additionally, I expect high levels of Ambiguity to weaken the trading volume to price volatility elasticity, as it reflects that trading volume responds not only to price changes but also to Ambiguity.

Methodology

I test the theoretical elasticity model presented in section 2.7 through a daily frequency regression, where I regress the changes in log SPY trading volume against changes in log delta price-volatility $\sigma_{\Delta p}$ multiplied by the elasticity ξ. To validate the hypothesis regarding the impact of Ambiguity on this elasticity, I check the statistical significance of the coefficients ξ_{amb}, ξ_{amb^2} associated with mean Ambiguity, as well as the coefficients $\xi_{\sigma_{amb}}, \xi_{\sigma_{amb^2}}$ associated with Ambiguity volatility in the regression. Additionally, I expect that mean Ambiguity (ξ_{amb}) will negatively impact the overall elasticity, thereby weakening the link between price volatility and trading volume.

To conduct the linear regression, I use an expression that accounts for the non-linear normal density and cumulative distribution function (CDF) inside the elasticity model. This expression involves linear and quadratic terms of the elasticity factors: mean delta price change ($\mu_{\Delta p}$, ξ_{μ}), mean Ambiguity (μ_{δ}^{A}, ξ_{amb}), difference in previous beliefs (($\mu_e^A - \mu_e^B$) and ($\mu_X^B - \mu_X^A$), ξ_{pbel}), and ambiguity volatility ($\sigma_{\delta}^{2\,A}$, $\xi_{\sigma_{amb}}$).

$$
\begin{aligned}
&\Delta log(V) = \xi * \Delta log(\sigma_{\Delta p}) \qquad (5.1)\\
&\qquad \xi = F(\mu_{\Delta p}\ ,\ \mu_{\delta}^{A}\ ,\ \sigma_{\delta}^{A}\ ,\ \mu_e^A - \mu_e^B\ ,\ \mu_X^B - \mu_X^A,\ \mu_{\Delta p}^2\ ,\ \mu_{\delta}^{2\,A}\ , \sigma_{\delta}^{2\,A},\ (\mu_e^A - \mu_e^B)^2\ ,\ (\mu_X^B - \mu_X^A)^2\ )
\end{aligned}
$$

I employ the regression below,

$$\Delta log(V) = c + \Big(\xi_1 + \xi_\mu * \mu_{\Delta p} + \xi_{pbel} * PBEL + \xi_{amb} * AMB + \xi_{\mu^2} * \mu^2_{\Delta p} + \xi_{pbel^2} * PBEL^2 + \xi_{amb^2} * AMB^2 + \xi_{\sigma_{amb}} * \sigma_{amb} + \xi_{\sigma_{amb^2}} * \sigma^2_{amb} \Big)_\xi * \Delta log(\sigma_{\Delta p}) + \Delta log(V)_{t-1} + \sum_{p=t+1}^{T} \gamma_p \tag{5.2}$$

that also include a constant, a lagged series of the dependent variable to control for persistence and yearly time controls.

I perform the regressions using daily data. The proxy series for differences in previous beliefs (PBEL) are based on the daily measures from Cookson and Niessner (2020). The Ambiguity measure used (AMB) is the daily version of the EPU index extracted from Twitter messages (Baker et al., 2021). For the mean price change ($\mu_{\Delta p}$), I use a lagged rolling moving average based on the last 200 days of information. To address potential collinearity issues between the explanatory variables and their squared terms, I orthogonalize them by regressing each term on its squared counterpart and retain the residuals as the explanatory variable. I implement a similar procedure for the mean price change ($\mu_{\Delta p}$), as well as for the interactions of ($\sigma_{\Delta p}$) and the single regressor ($\sigma_{\Delta p}$). The regression is performed using daily data for the period 2013 to 2018.

Results

The regression results in Table-4 below highlight the statistical significance of the Ambiguity measure (AMB_{EPUD}), represented by the coefficient ξ_{amb}, across all four daily dataset combinations. This coefficient shows an average value of -0.09, t-values exceeding 2.90 and p-values close to 1%.

According to these results, I obtain that a one-standard-deviation rise in the interaction between the daily Ambiguity measure and the delta of log price volatility corresponds to an approximate -9% standard-deviation reduction in the log trading volume delta from 2013 to 2018. This validates the hypothesis that (H1) Ambiguity distorts the conventional volume-to-price volatility relationship and (H2) weakens the trading volume's responsiveness to price volatility.

Across all datasets, other relevant explanatory variables are the delta in log price volatility (coefficient ξ_1) and the squared mean price change (coefficient ξ_{μ^2}). The log price volatility series (coefficient ξ_1), displaying t-values comfortably above 3 and a mean coefficient of +0.23, represents the well known principal channel between price volatility and trading volume. Based on these results, a one-standard-deviation rise in the log price volatility delta corresponds to an approximate

+23% standard-deviation increase in the log trading volume delta. The squared mean price change (coefficient ξ_{μ^2}), featuring t-values above 2 and an average coefficient around -0.10, is linked to the normal distribution assumption of price changes.

Regarding the difference in prior beliefs measures (PBEL), none of the measures demonstrate statistical significance. However, the coefficients (ξ_{pbel}, ξ_{pbel^2}) linked to the ETF measure across various investor styles ($PBEL_{AC,ETF}$) exhibit comparatively greater significance than the others. I also find that these elasticity results reveal a difference in the impact direction of the measure associated with the same investment approach ($PBEL_{WI,ETF}$) versus the one associated to investors with varied investment approaches ($PBEL_{AC,ETF}$). Similar as for the trading volume relationship discussed in Section 4.

Overall, the results obtained here from different datasets are consistent with the prior empirical findings presented by Bollerslev et al. (2018) concerning the role and impact of Ambiguity (EPU) on the elasticity between trading volume and price volatility. Moreover, these findings underscore the significance of the Ambiguity expectations channel in the volume-price volatility relationship. However, there are distinctions: (1) the results here differentiate between the prior belief channel and the Ambiguity expectations channel, and (2) they reveal divergent behavior of the prior belief measures depending on whether they originate from investors with similar or distinct investment approaches. Building upon the differences in construction and nuances between these two beliefs measures (Cookson & Niessner, 2020), the observed divergence in outcomes indicates diverse elasticity effects, contingent on whether differing beliefs stem from distinct information sets or distinct methods of interpreting information. Detailed regression results in Appendix-D Table-15.

Table 4. Daily Elasticity Regression.

This table summarizes the main results of the trading-volume to price volatility elasticity regression (5.2). The regressions were performed for the period 2013 to 2018. The row on top indicates the dataset. The dataset D(1): $\text{PBEL}_{\text{WI, IND}}$ & AMB_{EPUD}, D(2): $\text{PBEL}_{\text{AC, IND}}$ & AMB_{EPUD}, D(3): $\text{PBEL}_{\text{WI, ETF}}$ & AMB_{EPUD} and D(4): $\text{PBEL}_{\text{AC, ETF}}$ & AMB_{EPUD}. Where $\text{PBEL}_{\text{WI, IND}}$, $\text{PBEL}_{\text{AC, IND}}$, $\text{PBEL}_{\text{WI, ETF}}$ and $\text{PBEL}_{\text{AC, ETF}}$ are the Cookson and Niessner (2020) previous beliefs measures extracted from Stocktwits and AMB_{EPUD} refers to the daily EPU ambiguity measure extracted from Twitter. T-values in round brackets are Newey-West autocorrelation robust values. Full regression results in Appendix-D.

$$\Delta log(V) = c + \big(\, \xi_1 + \xi_\mu * \mu_{\Delta p} + \xi_{pbel} * PBEL + \xi_{amb} * AMB + \xi_{\mu^2} * \mu_{\Delta p}^2 + \xi_{pbel^2} * PBEL^2 + \xi_{amb^2} * AMB^2 + \xi_{\sigma_{amb}} * \sigma_{amb} + \xi_{\sigma_{amb^2}} * \sigma_{amb}^2 \,\big)_\xi \; * \; \Delta log(\sigma_{\Delta p}) + \; \Delta log(V)_{t-1} + \; \sum_{p=t+1}^{T} \gamma_p$$

	D(1)	D(2)	D(3)	D(4)
c	-0.02	-0.02	-0.02	-0.02
	(-0.48)	(-0.52)	(-0.46)	(-0.61)
ξ_1	0.23***	0.23***	0.23***	0.23***
	(9.08)	(8.87)	(8.59)	(9.14)
ξ_μ	-0.01	-0.01	-0.02	-0.00
	(-0.30)	(-0.37)	(-0.61)	(-0.00)
ξ_{pbel}	-0.01	-0.02	-0.04	0.04
	(-0.30)	(-0.78)	(-1.45)	(0.98)
ξ_{amb}	-0.09***	-0.09***	-0.09***	-0.08***
	(-3.34)	(-2.98)	(-3.41)	(-2.95)
ξ_{p_1}	-0.02	-0.01	-0.01	-0.00
	(-0.53)	(-0.35)	(-0.12)	(-0.06)
ξ_{μ^2}	-0.10***	-0.10***	-0.11***	-0.08**
	(-3.22)	(-2.63)	(-3.74)	(-2.18)
ξ_{pbel^2}	0.02	-0.01	0.03	0.05
	(0.45)	(-0.15)	(0.80)	(1.43)
ξ_{amb^2}	0.01	0.01	0.01	0.01
	(0.39)	(0.37)	(0.37)	(0.58)
$\xi_{p_1^2}$	0.02	0.02	0.02	0.00
	(0.87)	(0.75)	(1.10)	(0.18)
$\xi_{\sigma_{amb}}$	-0.04	-0.04	-0.03	-0.04
	(-1.55)	(-1.36)	(-1.01)	(-1.51)
$\xi_{\sigma^2_{amb}}$	-0.02	-0.02	-0.01	-0.03
	(-0.56)	(-0.52)	(-0.25)	(-0.70)
$\xi_{\Delta log(V)_{t-1}}$	-0.38***	-0.38***	-0.38***	-0.38***
	(-18.71)	(-18.58)	(-18.62)	(-18.46)
N	1510	1510	1510	1510
R_a^2	0.204	0.204	0.206	0.206

6 Application: Turnover Sorted Portfolios

In this section, I employ the model outlined in section-2.6 to identify the primary drivers of returns in Turnover sorted portfolios, which are commonly associated to Liquidity in the literature. Furthermore, based on the trading volume components of the trading volume model of section-2.6, I introduce an improved method for constructing Turnover sorted portfolios with better risk/return profiles. I begin by introducing the current state of the literature on Turnover sorted portfolios and liquidity. Subsequently, I present my key findings, followed by a detailed explanation of the methodology and numerical outcomes.

Several empirical studies have shown that stocks with low turnover tend to exhibit higher future returns (Amihud, 2002, 2018; Chou et al., 2013; Datar et al., 1998; Haugen & Baker, 1996; Lee & Swaminathan, 2000). The asset pricing literature typically regards Turnover as a gauge of liquidity and liquidity risk. According to this perspective, theoretically, a greater exposure to low turnover, implying heightened liquidity risk, should be associated to higher returns (Acharya & Pedersen, 2005; Pástor & Stambaugh, 2003). However, second-moment volume-based measures of liquidity risk point in the opposite direction (Chordia et al., 2001). This contradiction poses a puzzle within the established asset pricing literature. So, what do Turnover sorted portfolios truly measure?.

Utilizing the model outlined in section 2.6, I provide empirical evidence showing that a significant portion of the returns from a standard Turnover sorted portfolio can be attributed to Ambiguity. Summary in figure-6 below. In detail, the results indicate that around 70% of the positive returns within a US long-short Turnover sorted portfolio, starting in 1990, can be linked to trading volume driven by Ambiguity. These findings align with the Microstructure perspective (Harris & Raviv, 1993), which views turnover not only as an indicator of liquidity risk but also as a manifestation of Ambiguity and divergence in beliefs.

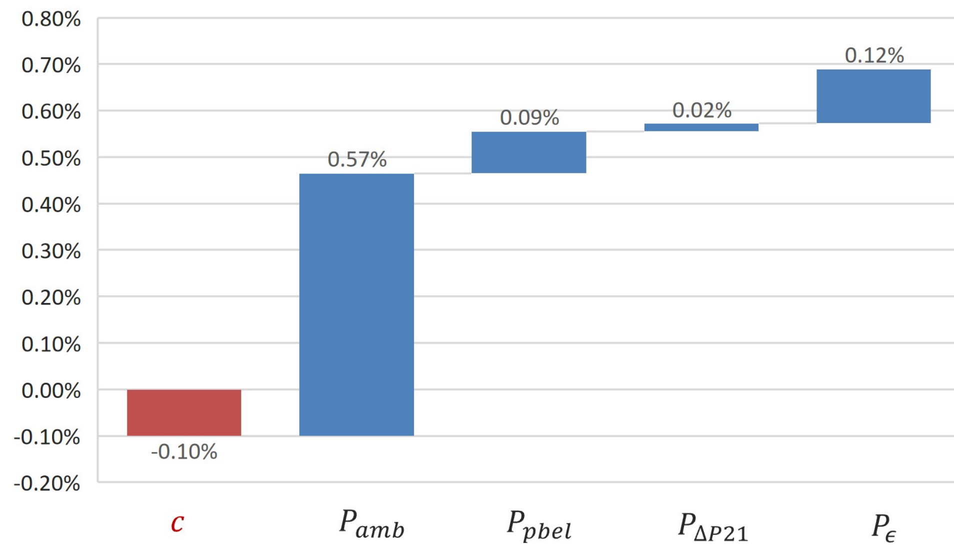

Figure 6. LMH Turnover Portfolio Returns Attribution. Attribution of the monthly returns of the LMH Turnover sorted portfolio described in section *Data and Portfolio Construction* below between 1990 to 2020. The returns attribution is obtained in regression (4.2) below by regressing the returns on a constant and the LMH portfolios obtained from sorting on the Turnover components $\{\alpha_{V_{pbel}}, \alpha_{V_{amb}}, \beta_{V_{\Delta P_{21}}}, \epsilon\}$. The sorting variables of these last portfolios are: Turnover driven by difference in prior beliefs ($\alpha_{V_{pbel}}$), Turnover driven by Ambiguity ($\alpha_{V_{amb}}$), Turnover driven by price fluctuations ($\beta_{V_{\Delta P_{21}}}$) and an unexplained Turnover portion or error term (ϵ).

Furthermore, I empirically demonstrate that constructing a bisorted portfolio utilizing turnover driven by both Ambiguity and price changes results in a 2x to 3x higher Sharpe Ratio compared to a counterfactual portfolio using solely the Turnover measure. Refer to figure-7 below.

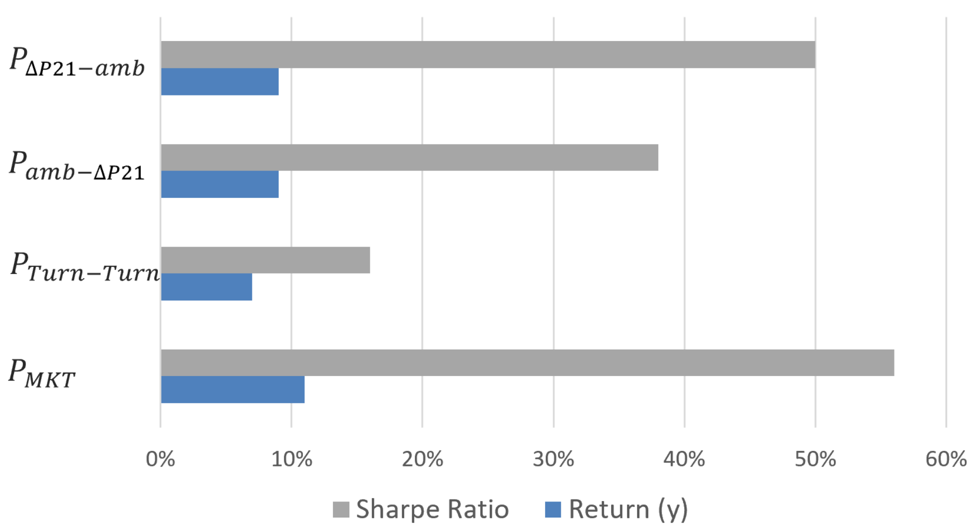

Figure 7. Bisorted Portfolios Returns and Sharpe-Ratios. Yearly returns and Sharpe-Ratios of bisorted portfolios between 1990 to 2020. P_{MKT} is the Fama-French market factor.

Methodology

I apply the theoretical framework presented in section 2.6 to empirically deconstruct the Turnover measure of each stock analyzed in this chapter. This process results in four distinct Turnover components per stock, which in turn serve as the basis for constructing four low-minus-high (LMH) portfolios. Subsequently, I project the initial Turnover LMH portfolio onto these four distinct LMH portfolios, with the goal of identifying the individual contribution of each Turnover component to the overall returns of the original Turnover portfolio.

Regarding these four components of Turnover, according to the model in section 2.6 they are: Ambiguity related Turnover, Turnover stemming from variations in prior beliefs, Turnover associated to price fluctuations, and an unexplained residual or error term. Sorting LMH on these measures originates the aforementioned four portfolios.

$$Turnover = \alpha_{V_{pbel}} + \alpha_{V_{amb}} + \beta_{V_{\Delta P_{21}}} + \epsilon \tag{6.1}$$
$$LMH\ Portfolios = \{P_{Turn}, P_{amb}, P_{pbel}, P_{\Delta P_{21}}, P_{\epsilon}\}$$

In a subsequent analysis, taking into account the higher Sharpe ratios of portfolios P_{amb} and $P_{\Delta P_{21}}$ from the prior step, I utilize the respective Turnover components associated with Ambiguity and changes in price $\{\alpha_{V_{amb}}, \beta_{V_{\Delta P_{21}}}\}$ to form bisorted LMH portfolios. I then compare these portfolios against a similar counterfactual bisorted portfolio based solely on the original Turnover metric.

To characterize and compare the above mentioned portfolios, I calculate their average annual excess returns, standard deviations, and Sharpe ratios. Additionally, I conduct regressions (1), (2), and (3) on the monthly returns of the LMH P_{Turn} portfolio resulting from sorting on Turnover in order to analyze and attribute its returns.

$$R_{P_{Turn}} = c + \epsilon \tag{6.2}$$

$$R_{P_{Turn}} = c + R_{free} + \beta_{MKTRF} * R_{MKTRF} + \beta_{HML} * R_{HML} + \beta_{SMB} * R_{SMB} + \epsilon \tag{6.3}$$

$$R_{P_{Turn}} = c + \beta_{P_{amb}} * R_{P_{amb}} + \beta_{P_{pbel}} * R_{P_{pbel}} + \beta_{P_{\Delta P_{21}}} * R_{P_{\Delta P_{21}}} + \beta_{P_{\epsilon}} * R_{P_{\epsilon}} + \epsilon \tag{6.4}$$

In regression (4.1), I analyze the monthly portfolio returns by regressing them against a constant term to determine the average monthly returns and their statistical significance. Regression (4.2) examines the returns using the Fama-French three-factor model $\{MKT - R_{Free}, HML, SMB\}$ to gauge the extent to which conventional factors account for the LMH Turnover portfolio's behavior. In regression (4.3), I regress the initial LMH Turnover portfolio against the LMH portfolios obtained from sorting on $\{\alpha_{V_{pbel}}, \alpha_{V_{amb}}, \beta_{V_{\Delta P_{21}}}, \epsilon\}$, aiming to identify the main Turnover components that explains the original Turnover portfolio returns. The results of regressions (2), (3) and (4) are displayed in tables (5) and (6) below. Utilizing the betas obtained from regression (3) alongside the mean average returns of each explanatory portfolio, I build the LMH Turnover portfolio P_{Turn} returns attribution displayed in figure-6 above. In Table 21 within Appendix G, I validate the regression outcomes from (3) by introducing alternative liquidity measures.

Data and Portfolio Construction

I create the portfolios analyzed here by using all CRSP stocks traded on the NYSE and NASDAQ. The monthly portfolio returns span from 1990 to 2020. To capture Ambiguity, I utilize the monthly series of the Economic Policy Uncertainty Index (Baker et al., 2016). To account for differing prior beliefs, I employ an analysts' forecast dispersion measure retrieved from the IBES database. The Fama-French factors for the regression analysis are sourced from the Kenneth French Online Data Library.

I create unisorted portfolios by averaging the Turnover measure and its four components: $\{\alpha_{V_{amb}}, \alpha_{V_{pbel}}, \beta_{V_{\Delta P_{21}}}, \epsilon\}$ over a 3-month rolling window for each stock. These five averages represent Turnover itself, Turnover explained by Ambiguity ($\alpha_{V_{amb}}$), Turnover explained by divergence in prior beliefs ($\alpha_{V_{pbel}}$), and the unexplained portion of Turnover (ϵ). On a 3-month basis, using 1-month lagged averages, I segment the stock universe into 10 quantiles (10x), generating ten equally weighted portfolios corresponding to each quantile. This monthly process relies solely on past data. The final unisorted portfolios $\{P_{Turn}, P_{amb}, P_{pbel}, P_{\Delta P_{21}}, P_{\epsilon}\}$ representing one of the five series above, emerge as the difference between the lowest and highest quantile portfolios (LMH).

For the second analysis, I construct bisorted portfolios based on $\{\alpha_{V_{amb}}, \beta_{V_{\Delta P_{21}}}\}$ following the same methodology. Using a lagged 3-month rolling window average of the initially selected measure, I segment the stock universe into 5 quantiles on a 3-month basis. Within these five quantiles, utilizing the lagged 3-month rolling average of the second selected measure, I further segment them into two smaller quantiles. This process solely relies on past data, resulting in a set of 5x2 portfolios. The final portfolio emerges by taking a long position in the portfolio with the lowest $\alpha_{V_{amb}}$ and highest $\beta_{V_{\Delta P_{21}}}$, while shorting the portfolio with the highest $\alpha_{V_{amb}}$ and lowest $\beta_{V_{\Delta P_{21}}}$. I create two versions of this bisorted portfolio, one version employs $\alpha_{V_{amb}}$ as the first sorting dimension (5x) and the other one uses $\beta_{V_{\Delta P_{21}}}$. I refer to these portfolios as $\{P_{amb-\triangle P_{21}}, P_{\triangle P_{21}-amb}\}$. For comparison and benchmarking purposes I also construct a counterfactal Turnover-Turnover bisorted portfolio $P_{Turn-Turn}$ following the same methodology. This portfolio employs on both dimensions (5x2) the Turnover measure.

Results

The regression results (4.1) presented in Table-5 below reveal that the LMH Turnover portfolio P_{Turn} yielded a positive and statistically significant average monthly return of +0.60% during the 1990-2020 period. A similar outcome emerged from the LMH portfolio P_{amb} exclusively driven by the Turnover component $\alpha_{V_{amb}}$ associated to Ambiguity. During this same period, arranging the portfolio from low to high (LMH according to the Turnover component associated with price changes ($\triangle P_{21}$) would have led to statistically significant average negative returns of -0.50% per month in portfolio $P_{\triangle P_{21}}$. These two portfolios { P_{amb}, $P_{\triangle P_{21}}$} linked to the Turnover components $\alpha_{V_{amb}}$ and $\triangle P_{21}$ respectively exhibit the highest Sharpe ratios, albeit in inverse LMH and HML directions.

The results of regressions (4.2) presented in Table-6 highlight that the traditional Fama-French three-factor model fails to account for the positive returns of the LMH Turnover portfolio P_{Turn}. The constant term captures an average monthly return of +1.9% (22% annually) with a robust t-statistic exceeding 4 that can not be explained by the Fama-French factors.

Regression (4.3) results in Table-3, utilizing LMH Portfolios $\{P_{Turn}, P_{amb}, P_{pbel}, P_{\Delta P_{21}}, P_{\epsilon}\}$ as explanatory variables, reveal that a significant proportion of the LMH Turnover portfolio P_{Turn} positive returns can be attributed to these explanatory portfolios. In this regression, the constant term captures a residual -0.1% monthly return (-1.2% annually) that is not statistically significant. The findings from this Table-6 and the average monthly returns detailed in Table-5 above, indicate that the explanatory portfolio P_{amb} with a beta of 0.942 contributes with an approximate monthly return of +0.57% to the P_{turn} portfolio. In the second, third, and fourth positions, the explanatory portfolios $\{P_{\epsilon}, P_{pbel}, P_{\Delta P_{21}}\}$ contribute with average monthly returns of {+0.12%, +0.09%, +0.02%} respectively to the P_{turn} portfolio. The robustness tests in Table-21 within

Appendix-G show that the regression coefficients, t-stats, and p-values of regression (4.3) remain largely unaffected after the incorporation of alternative liquidity measures.

In relation to the construction of portfolios with better statistical properties using the information contained in the Turnover measure, in Table-7 below I show that a bisorted portfolio utilizing solely the components $\{\alpha_{V_{amb}}, \beta_{V_{\Delta P_{21}}}\}$ of Turnover for sorting purposes achieves a superior risk-return profile. The bisorted portfolio of dimensions 5x2 that first sorts LMH on $\alpha_{V_{amb}}$ and then HML on $\beta_{V_{\Delta P_{21}}}$ achieves a sharpe ratio of +0.48. This outperforms the Sharpe ratio of +0.16 achieved by a comparable counterfactual Turnover-Turnover bisorted portfolio. Similarly, the 5x2 bisorted portfolio that first sorts HML on $\beta_{V_{\Delta P_{21}}}$ and then LMH on $\alpha_{V_{amb}}$ achieves a sharpe ratio of +0.50 that exceeds the counterfactual Turnover-Turnover bisorted portfolio sharp ratio and comes close to Fama-French Market factor sharpe ratio.

These findings highlight the important role that Ambiguity plays in the dynamics of Turnover sorted portfolios. Furthermore, they contribute to the longstanding debate on whether Turnover primarily captures liquidity or something else. Additionally, these results offer an alternative explanation on the puzzling relationship between volume-based measures of liquidity risk and returns (Chordia et al., 2001).

Table 5. Portfolios Monthly Returns and Sharpe Ratios.

This table summarizes in Panel A the monthly returns of the Market Factor Portfolio, the LMH Turnover Portfolio (P_{Turn}), the LMH Uncertainty driven Turnover Portfolio (P_{unc}), the LMH Disagreement driven Turnover Portfolio (ΔP_{21}), the LMH Price Change driven Turnover Portfolio (ΔP_{21}) and the LMH Non-Explained Turnover driven Portfolio (P_{ϵ}). Panel B shows the yearly Sharpe-Ratios for the same portfolios.

	P_{MKT}	P_{Turn}	P_{amb}	P_{pbel}	$P_{\triangle P_{21}}$	P_{ϵ}
Panel A: Monthly Portfolio Returns (1990 - 2020)						
c	0.009***	0.006*	0.006*	0.002	-0.005***	0.003*
	(3.96)	(1.68)	(1.70)	(0.76)	(-3.27)	(1.75)
N	372	372	372	372	372	372
R_a^2	0	0	0	0	0	0
Panel B: Sharpe Ratios (1990 - 2020, yearly)						
Excess Return	0.09	0.05	0.05	-0.01	-0.03	0.01
Std. Deviation	0.14	0.25	0.21	0.16	0.15	0.11
Sharpe Ratio	0.56	0.20	0.22	-0.05	-0.22	0.04

Table 6. Portfolio Monthly Returns Regressions.

This table summarizes the attribution of the LMH Turnover Sorted Portfolio (P_{Turn}) using the Fama-French Factors as well as the Ambiguity (P_{unc}), Disagreement (P_{pbel}), Price Change ($P_{\Delta P_{21}}$) and Non-Explained (P_{ϵ}) Turnover components LMH portfolios.

	P_{Turn}	P_{Turn}	P_{Turn}
c	0.006*	0.019***	-0.001
	(1.68)	(4.81)	(-1.27)
β_{R_f}		-2.380*	
		(-1.77)	
β_{MKT-R_f}		-0.953***	
		(-12.19)	
β_{HML}		0.854***	
		(6.82)	
β_{SMB}		-0.789***	
		(-7.96)	
$\beta_{P_{amb}}$			0.942***
			(41.18)
$\beta_{P_{pbel}}$			0.450***
			(5.93)
$\beta_{P_{\triangle P_{21}}}$			-0.034
			(-0.52)
$\beta_{P_{\epsilon}}$			0.390***
			(7.21)
N	372	372	372
R_a^2	0.000	0.728	0.944

Table 7. Bisorted Portfolios Monthly Returns and Sharpe Ratios.

This tables summarize the statistics of the market portfolio plus three differential bisorted portfolios portfolios $P_{Turn-Turn}$, $P_{unc-\Delta P_{21}}$ and $P_{\Delta P_{21}-unc}$. Each bisorted portfolio is obtained as the spread between the extreme quantiles of (bisorted) portfolios sorted on two measures. Each portfolio that compose the long and short leg of the differential portfolio is obtained by double sorting on a 5x2 grid. Panel A shows the monthly returns of the Market Factor, the bisorted Turnover-Turnover Portfolio, the bisorted Price Change - Ambiguity driven (Turnover) Portfolio, and the bisorted Ambiguity - Price Change (Turnover) Portfolio. Panel B shows the yearly Sharpe-Ratios for the same portfolios.

	P_{MKT}	$\mathrm{P}_{Turn-Turn}$	$\mathrm{P}_{amb-\triangle P_{21}}$	$\mathrm{P}_{\triangle P_{21}-amb}$
Panel A: Monthly Portfolio Returnds (1990 - 2020)				
c	0.009***	0.006	0.008***	0.008***
	(3.96)	(1.39)	(2.79)	(3.53)
N	372	372	372	372
R_a^2	0	0	0	0
Panel B: Sharpe Ratios (1990 - 2020, yearly)				
Excess Return	0.09	0.04	0.07	0.07
Std. Deviation	0.14	0.25	0.16	0.13
Sharpe Ratio	0.56	0.16	0.38	0.50

7 Conclusions

In this work, I explore the influence of Ambiguity, also referred to as Knightian Uncertainty, on the connection between trading volume and prices within financial markets. I created a model that sheds light on how the ambiguous interpretation of public information by a subset of market participants affects trading activity. Through both theoretical analysis and empirical evidence, I reveal how the expectations channel of Ambiguity can trigger trading activity, even amidst stable scenarios lacking visible price changes.

The empirical sections of this study validate the theoretical model, demonstrating that, on average, a one-standard-deviation rise in daily Ambiguity translate in an approximately +11%-standard-deviation surge in trading volume, even after accounting for price shifts. On a monthly frequency, I observe that a one-standard-deviation rise in Ambiguity is associated with an approximate +20%-standard-deviation to +58%-standard-deviation surge in trading volume.

By relaxing the assumptions of my initial framework, I derive a trading volume to price volatility elasticity relation that incorporates the influence of Ambiguity within market agents. This connection illustrates the impact of Ambiguity on the well-established positive correlation between price volatility and trading volume. My empirical estimates demonstrate that a one-standard-deviation rise in the product of price volatility and daily Ambiguity leads to a decrease of approximately 9% in trading volume, rather than an increase. This outcome underscores how Ambiguity weakens the typically positive link between trading activity and price volatility.

In terms of applications, the model's decomposition of trading volume suggests that roughly 70% of the positive returns of a standard US Turnover sorted portfolio from 1990 to 2020 can be attributed to Turnover driven by Ambiguity.

The model and empirical findings presented here shed light on how Ambiguity can help to explain the puzzling large average trading volume observed in markets (Cochrane, 2016), even in situations marked by minor price shifts and subdued volatility. Furthermore, these results bolster the notion that Turnover is more a manifestation of Ambiguity and divergent viewpoints rather than mere liquidity, thereby contributing to the understanding of the puzzling relationship between volume-based liquidity risk measures and returns (Chordia et al., 2001).

Appendix A Volume and Price Change Relation for Max Min Utility

Market Equilibrium at Time 1

The price P_{t1} represents the equilibrium price at which the risky asset market clears in period 1 and is determined by the following formula.

$$
\begin{aligned}
& P_{t1} = \bar{\mu}_X / \bar{\rho}_X \\
where & \\
& \bar{\mu}_X = \pi * \mu_X^A * \rho_X^A + (1-\pi) * \mu_X^B * \rho_X^B \\
& \bar{\rho}_X = \pi * \rho_X^A + (1-\pi) * \rho_X^B
\end{aligned}
$$

The resulting equilibrium allocation at period 1 for both ambiguous and non-ambiguous investors is given by the formulas θ_{t1}^A and θ_{t1}^B below.

$$
\begin{aligned}
\theta_{t1}^B &= \frac{\pi * \rho_X^A * \rho_X^B * \left(\mu_X^B - \mu_X^A\right)}{\gamma * \bar{\rho}_X} \\
\theta_{t1}^A &= \frac{(1-\pi) * \rho_X^A * \rho_X^B * \left(\mu_X^A - \mu_X^B\right)}{\gamma * \bar{\rho}_X}
\end{aligned}
$$

Ambiguity Neutral Investor Type (Type-B) At Time 2

The ambiguity-neutral type-B investor takes the following optimal decision at time (2)

$$
\begin{aligned}
\max_{\theta_{t2}^B} \; E^B\left[U^B\left(\theta_{t2}^B\right)\middle|S\right] &= \max_{\theta_{t2}^B} E^B\left[-e^{-\gamma \, * \, \left(w_{t2}^B \, + \, \theta_{t2}^B * \, \left(\widetilde{X} - P_{t1}\right)\right)} \middle| S\right] \\
\max_{\theta_{t2}^B} \; E^B\left[U^B\left(\theta_{t2}^B\right)\middle|S\right] &= \max_{\theta_{t2}^B} -e^{-\gamma * \left(w_{t2}^B \, + \, \theta_{t2}^B * \, \left(E^B\left[\widetilde{X}\middle|S\right] - P_{t2}\right)\right) \, + \, \frac{1}{2} * \gamma^2 * \theta_{t2}^{2B} * \mathrm{VAR}^B[\widetilde{X}|S]}
\end{aligned}
$$

Given his prior beliefs and the information received through the signal S, his optimal allocation at time 2 is the following

$$
\theta_{t2}^B = \frac{E^B[\widetilde{X}|S] - P_{t2}}{\gamma * \mathrm{VAR}^B[\widetilde{X}|S]}
$$

$$\theta_{t2}^{B} = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * \ \left(S - \mu_\epsilon^B\right) - P_{t2} * \left(\rho_X^B + \rho_{\mathcal{E}}^B\right)}{\gamma}$$

$$where$$

$$\rho_X^B = \frac{1}{\sigma_X^{2\ B}}$$

$$\rho_{\mathcal{E}}^B = \frac{1}{\sigma_{\mathcal{E}}^{2\ B}} = \frac{1}{\sigma_\epsilon^{2\ B}}$$

$$E^B\left[\widetilde{X}\middle|S\right] = \mu_X^B + \frac{\sigma_X^{2\ B}}{\sigma_X^{2\ B} + \sigma_e^{2\ B}} * \left(S - \mu_X^B - \mu_e^B\right)$$

$$E^B\left[\widetilde{X}\middle|S\right] = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * \left(S - \mu_e^B\right)}{\rho_X^B + \rho_{\mathcal{E}}^B}$$

$$\mathrm{VAR}^B\left[\widetilde{X}\middle|S\right] = \sigma_X^{2\ B} - \frac{\sigma_X^{4\ B}}{\sigma_X^{2\ B} + \sigma_e^{2\ B}}$$

$$\mathrm{VAR}^B\left[\widetilde{X}\middle|S\right] = \left(\rho_X^B + \rho_{\mathcal{E}}^B\right)^{-1}$$

The parameter ρ_X^B corresponds to the type-B investor prior belief precision about the payoff X and $\rho_{\mathcal{E}}^B$ corresponds to the type-B investor belief about the precision of the signal S total error $\widetilde{\mathcal{E}}$.

Ambiguity Averse Investor (Type-A) At Time 2

The investor type maximizes at time 2 his expected utilitity by selecting the optimal asset mixture in accordance with its Max Min Utility function. In this function, the expectation operator addresses the traditional risk concept associated with the payoff $\widetilde{X}$, while the minimization operator handles the ambiguity surrounding $\widetilde{X}$ by selecting the most pessimistic M model. In this particular setup, each model M represents a specific way of interpreting the public information. Details below.

Regarding this investor type beliefs about the risky-asset payoff $\widetilde{X}$, he initially assumes a normal distribution with parameters $N\left(\mu_X^A,\ \sigma_X^{2\ A}\right)$. Additionally, he believes that the signal S is subject to a measurement bias or error $\widetilde{\epsilon}$ that distributes $N\left(\mu_\epsilon^A,\ \sigma_\epsilon^{2\ A}\right)$.

Despite these prior beliefs, this investor type is not completely certain about the appropriate model for interpreting the signal S. This ambiguity is represented by different models $M \in M^n$, each characterized by a model-dependent signal component $\widetilde{\delta}$ following the normal distribution $N\left(\mu_\delta^A,\ \sigma_\delta^{2\ A}\right)$. The mean and variance of $\widetilde{\delta}$ are specific to each M-model. The mean of $\widetilde{\delta}$ across all models falls within the range $\left[\underline{\mu}_\delta, \overline{\mu}_\delta\right]$ and its variance falls within the range $[\underline{\sigma}_\delta^2, \overline{\sigma}_\delta^2]$.

In summary, the investor type-A believes that the signal S consists of three components: the risky payoff $\widetilde{X}$ information, an ambiguous model-dependent component $\widetilde{\delta}$, and a measurement error $\widetilde{\epsilon}$. The total error term of the signal is denoted as $\widetilde{\mathcal{E}}$.

$$
\begin{aligned}
S &= \widetilde{X} + \widetilde{\delta} + \widetilde{\epsilon} = S + \widetilde{\mathcal{E}} \\
\widetilde{\delta} &\sim N\left(\mu_\delta^A,\ \sigma_\delta^{2\ A}\right) \\
\mu_\delta^A &\in \left[\underline{\mu}_\delta, \overline{\mu}_\delta\right] \\
\sigma_\delta^{2\ A} &\in \left[\underline{\sigma}_\delta^2, \overline{\sigma}_A^2\right]
\end{aligned}
$$

The ambiguity averse type-A investor takes the following optimal decision at time 2.

$$
\begin{aligned}
&\max_{\theta_{t2}^A} \min_M E^A\left[-e^{\,-\gamma\,*\,\left(w_{t2}^A\ +\ \theta_{t2}^{I_A}*\ \left(\widetilde{X}\ -\ P_{t2}\right)\right)}\middle|S, M\right] \\
&\max_{\theta_{t2}^A} \min_M -e^{-\gamma*\left(w_{t2}^A\ +\ \theta_{t2}^A*\left(E^A\left[\widetilde{X}\middle|S,M\right]-P_{t2}\right)\right)\ +\ 1/2*\gamma^2*\theta_{t2}^{2\ A}*\mathrm{Var}^A\left[\widetilde{X}\middle|S,M\right]} \\
&\max_{\theta_{t2}^A}\ -e^{-\gamma*\left(w_{t2}^A\ +\ \theta_{t2}^A*\left(E^A\left[\widetilde{X}\middle|S,M^*\right]-P_{t2}\right)\right)\ +\ 1/2*\gamma^2*\theta_{t2}^{2\ A}*\mathrm{Var}^A\left[\widetilde{X}\middle|S,M^*\right]}
\end{aligned}
$$

Given his prior beliefs, the information received through signal S, and his personal interpretation of information (model M) his optimal allocation at time 2 is the following.

$$
\begin{aligned}
\theta_{t2}^A &= \frac{E^A\left[\widetilde{X}\middle|\ S, M^*\right] - P_{t2}}{\gamma * \mathrm{VAR}^A\left[\widetilde{X}\middle|S, M^*\right]} \\
\theta_{t2}^A &= \frac{\rho_X^A * \mu_X^A + \rho_{\mathcal{E}}^{A|M^*} *\ \left(S - \mu_\epsilon^A - \mu_\delta^{A|M^*}\right) - P_{t2} * \left(\rho_X^A + \rho_{\mathcal{E}}^{A|M^*}\right)}{\gamma} \\
&\mathit{where} \\
\rho_X^A &= \frac{1}{\sigma_X^{2\ A}} \\
\rho_{\mathcal{E}}^{A|M^*} &= \frac{1}{\sigma_{\mathcal{E}}^{2\ A|M^*}} = \frac{1}{\sigma_\epsilon^{2\ A} + \sigma_\delta^{2\ A|M^*}}
\end{aligned}
$$

Ambiguity Averse Investor (Type-A) At Time 2: Optimal Paremeters

In this section, I elaborate on the optimal model M parameters, which the Ambiguous agents will employ to interpret the signal S according to their Max Min utility.

To ensure tractability, I adopt a simpler parametrization wherein ambiguity is represented by a range of models with different means u_{AM}, while sharing a fixed ambiguity volatility $\sigma_{A_F}^{2\ A}$.

$\widetilde{A} \sim N\Big(\mu_A^A \in \left[\ \underline{\mu}_A^A, \overline{\mu}_A^A\right],\ \sigma_A^{2\ A} \in \left[\sigma_{A_F}^{2\ A}\right]\Big).$

In this parametrization, the Min component of the ambiguous agent's utility gives rise to a function with one kink, located at point $\theta_{t2}^A = 0$. This results in a piecewise function with three regions where the agent maximizes its utility (Condie & Ganguli, 2017). In the first region, the agent selects a model with a mean ambiguity $\underline{u}_A^A$. Moving to the next region, the agent opts for a model with a mean ambiguity $\underline{\bar{\mu}}^A$ inside the range $[\underline{u}_A^A, \overline{u}_A^A]$, ensuring that the expected return $E^A\Big[\widetilde{X}\Big|S, M\Big] - P_{t2}$ equals 0. Lastly, in the final region, the agent goes for the mean ambiguity $\overline{u}_A$. The following equations summarize the piecewise utility of the ambiguous agent across these different regions.

$$\max_{\theta_{t2}^A} E^{I_A}\left[U^A\big(\theta_{t2}^A\big)\big|S\right] = \begin{cases} \max\limits_{\theta_{t2}^A}\ E^A\Big[-e^{\ -\gamma\ *\ \left(w_{t2}+\theta_{t2}^B*\ \left(\tilde{X}\ -\ P_{t2}\right)\right)}\ \Big|\ S, M = \{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] & \text{if } \theta_{t2}^A < 0 \\ \max\limits_{\theta_{t2}^A}\ E^A\Big[-e^{\ -\gamma\ *\ \left(w_{t2}+\theta_{t2}^B*\ \left(\tilde{X}\ -\ P_{t2}\right)\right)}\ \Big|\ S, M = \{\underline{\bar{\mu}}_A^A, \sigma_{A_F}^{2\ A}\}\Big] & \text{if } \theta_{t2}^A = 0 \\ \max\limits_{\theta_{t2}^A}\ E^A\Big[-e^{\ -\gamma\ *\ \left(w_{t2}+\theta_{t2}^B*\ \left(\tilde{X}\ -\ P_{t2}\right)\right)}\ \Big|\ S, M = \{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] & \text{if } \theta_{t2}^A > 0 \end{cases}$$

One interesting aspect of this utility function is the abrupt changes it exhibits when market conditions and signals shift, causing the utility to transition between different regions. In such instances, adjustments in the optimal mean ambiguity parameter μ_A^A leads to alterations in the utility function's shape.

Another noteworthy aspect of this utility function is the agent's portfolio inertia, wherein $\theta_{t2}^A = 0$. Within this particular region, the concrete allocation quantity remains entirely insensitive to the agent's current ambiguity.

This piecewise function leads to variations in the optimal allocation θ_{t2}^A depending on the region of operation. In one region, the allocation corresponds to the optimal mean variance portfolio resulting from a mean ambiguity $\overline{\mu}_A^A$. In the next region, the agent withdraws completely from the market. Finally, in the last region, the allocation represents the optimal mean variance portfolio resulting from a mean ambiguity $\underline{\mu}_A^A$.

$$\theta_{t2}^{A*} = \begin{cases} \dfrac{E^A\Big[\widetilde{X}\Big|S, M=\{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] - P_{t2}}{\gamma * \mathrm{VAR}^A\Big[\widetilde{X}\Big|S, M=\{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big]} & \text{if } P_{t2} < E^A\Big[\widetilde{X}\Big|S, M=\{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] \\ \dfrac{E^A\Big[\widetilde{X}\Big|S, M=\{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] - P_{t2}}{\gamma * \mathrm{VAR}^A\Big[\widetilde{X}\Big|S, M=\{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big]} & \text{if } E^A\Big[\widetilde{X}\Big|S, M=\{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] \leq P_{t2} \\ & \text{and } P_{t2} \leq E^A\Big[\widetilde{X}\Big|S, M=\{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] \\ \dfrac{E^A\Big[\widetilde{X}\Big|S, M=\{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] - P_{t2}}{\gamma * \mathrm{VAR}^A\Big[\widetilde{X}\Big|S, M=\{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big]} & \text{if } P_{t2} > E^A\Big[\widetilde{X}\Big|S, M=\{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] \end{cases}$$

Equilibrium At Time 2

In equilibrium at time 2 the following equation has to hold.

$$(1-\pi) * \theta_{t2}^B + \pi * \theta_{t2}^A = 0$$

Replacing the investor types A and B optimal allocations θ_{t2}^B and θ_{t2}^A in the market clearing condition.

$\pi * [\rho_X^A * \ \mu_X^A \ + \rho_{\mathcal{E}}^{A|M^*} * (S - \mu_\epsilon^A - \mu_\delta^{A|M^*}) \ - \ P_{t2} * \ (\rho_X^A + \rho_{\mathcal{E}}^{A|M^*}) \ ] \ + \ (1-\pi) \ * \ [\rho_X^B * \mu_X^B \ + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) \ - \ P_{t2} * (\rho_X^B + \rho_{\mathcal{E}}^B) \ ] = 0.$

Grouping together terms associated to the investors previous beliefs about the risky asset expected payment, the signal S, the signal S error and price P_{t2}

$[\pi * \rho_X^A * \mu_X^A + (1-\pi) * \rho_X^B * \mu_X^B]_{\bar{\mu}_X} + [\pi * \rho_{\mathcal{E}}^{A|M^*} + (1-\pi) * \rho_{\mathcal{E}}^B]_{\bar{\rho}_{\mathcal{E}}} * \mathrm{S} - \Big[\pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\epsilon^A + (1-\pi) * \rho_{\mathcal{E}}^B * \mu_\epsilon^B\Big]_{\bar{\mu}_\epsilon} - [\pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}] - P_{t2} * [\pi * \rho_X^A + (1-\pi) * \rho_X^B]_{\bar{\rho}_X} - P_{t2} * [\pi * \rho_{\mathcal{E}}^{A|M^*} + (1-\pi) * \rho_{\mathcal{E}}^B]_{\bar{\rho}_{\mathcal{E}}} = 0$

Replacing the expression in brackets by the terms $\bar{\mu}_X, \bar{\rho}_{\mathcal{E}}, \bar{\mu}_\epsilon$ and $\bar{\rho}_X$ we can rewrite the market clearing condition as

$$\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * \ \mathrm{S} - \bar{\mu}_\epsilon - \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*} - P_{t2} * (\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}) = 0.$$

Signal S At Time 2

From here we can rewrite the price at time 2.

$$P_{t2} = \frac{\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * \ \mathrm{S} - \bar{\mu}_\epsilon - \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}}{\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}}.$$

We can use this last expression for P_{t2} to rewrite the signal S as a function of the change in price ΔP

$$\begin{aligned}
\Delta\mathrm{P} = P_{t2} - P_{t1} &= \frac{\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * S - \bar{\mu}_\epsilon - \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}}{\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}} - P_{t1} \\
\Delta\mathrm{P} * [\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] &= \left[\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * S - \bar{\mu}_\epsilon - \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}\right] - P_1 * [\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] \\
\mathrm{S} &= \frac{\Delta\mathrm{P} * [\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] + \bar{\mu}_\epsilon + \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}}{\bar{\rho}_{\mathcal{E}}} + P_{t1}
\end{aligned}$$

In this last equation we replaced the price P_{t1} by the formula obtained for it as a result of the market equilibrium at time 1. $P_{t1} = \dfrac{\bar{\mu}_X}{\bar{\rho}_X}$.

Trading Volume from Time 1 to Time 2

We measure the trading volume from time 1 to 2 as the change in the risky-asset allocation of the ambiguity-neutral investor type (B) multiplied by the quantity of this type of investor. By symmetry of this market equilibrium, the volume of risky asset this investor type-B buys/sells is equivalent to the volume the ambiguous investor type-A sells/buys.

$$\begin{aligned}
V &= (1-\pi) * (\theta_{t2}^B - \theta_{t1}^B) \\
&\textit{where} \\
\theta_{t1}^B &= (\mu_X^B - P_{t1}) * \rho_X^B/\gamma \\
\theta_{t2}^B &= \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) - P_{t2} * (\rho_X^B + \rho_{\mathcal{E}}^B)}{\gamma}
\end{aligned}$$

Replacing the allocations of the ambiguity-neutral investor type-B in the expression $(\theta_{t2}^B - \theta_{t1}^B)$ we obtain

$$\theta_{t2}^B - \theta_{t1}^B = \left[\frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) - P_{t2} * (\rho_X^B + \rho_{\mathcal{E}}^B)}{\gamma}\right] - \left[\frac{(\mu_X^B - P_{t1}) * \rho_X^B}{\gamma}\right]$$

$$\theta_{t2}^B - \theta_{t1}^B = \frac{\rho_{\mathcal{E}}^B * \left[\ \mathrm{S} - \mu_\epsilon^B \right]}{\gamma} - \frac{P_{t2} * \rho_{\mathcal{E}}^B}{\gamma} - \Delta\mathrm{P} * \frac{\rho_X^B}{\gamma}$$

At this stage we can rewrite the signal S expression $(S - \mu_\epsilon^B)$ as a function of the change in prices between period (1) and (2) using the formula we found in the previous section.

$$\theta_{t2}^B - \theta_{t1}^B = \rho_{\mathcal{E}}^B * \frac{\left[\left(\frac{\Delta\mathrm{P}*[\bar{\rho}_X+\bar{\rho}_{\mathcal{E}}]+\bar{\mu}_\epsilon+\pi*\rho_{\mathcal{E}}^{A|M^*}*\mu_\delta^{A|M^*}}{\bar{\rho}_{\mathcal{E}}} + P_{t1}\right)_{\mathrm{S}} - \mu_\epsilon^B\right]}{\gamma} - \frac{P_{t2} * \rho_{\mathcal{E}}^B}{\gamma} - \Delta\mathrm{P} * \frac{\rho_X^B}{\gamma}$$

$$\theta_{t2}^B - \theta_{t1}^B = \frac{\Delta\mathrm{P}}{\gamma} * \left[\rho_{\mathcal{E}}^B * \frac{(\bar{\rho}_X+\bar{\rho}_{\mathcal{E}})}{\bar{\rho}_{\mathcal{E}}} - \rho_{\mathcal{E}}^B - \rho_X^B\right] + \left[\frac{\rho_{\mathcal{E}}^B*\bar{\mu}_\epsilon}{\gamma*\bar{\rho}_{\mathcal{E}}} - \frac{\rho_{\mathcal{E}}^B*\mu_\epsilon^B}{\gamma} + \frac{\pi*\rho_{\mathcal{E}}^B*\rho_{\mathcal{E}}^{A|M^*}*\mu_\delta^{A|M^*}}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]$$

$$\theta_{t2}^B - \theta_{t1}^B = \frac{\Delta\mathrm{P}}{\gamma} * \left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 + \left[\frac{\rho_{\mathcal{E}}^B*\bar{\mu}_\epsilon}{\gamma*\bar{\rho}_{\mathcal{E}}} - \frac{\rho_{\mathcal{E}}^B*\mu_\epsilon^B}{\gamma} + \frac{\pi*\rho_{\mathcal{E}}^B*\rho_{\mathcal{E}}^{A|M^*}*\mu_\delta^{A|M^*}}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]_1$$

We simplify the expressions inside bracket (2)

$$\left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 = \left[\frac{\rho_{\mathcal{E}}^B * \pi * \rho_X^A - \rho_X^B * \pi * \rho_{\mathcal{E}}^{A|M^*}}{\bar{B}}\right]$$

$$\left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 = \pi * \left[\frac{\rho_{\mathcal{E}}^B * \rho_X^A - \rho_X^B * \rho_{\mathcal{E}}^{A|M^*}}{\bar{\rho}_{\mathcal{E}}}\right]$$

We also rewrite the expression inside bracket (1)

$$[...]_1 = \left[\frac{\left(\rho_{\mathcal{E}}^B*\pi*\rho_{\mathcal{E}}^{A|M^*}*\mu_\epsilon^A+\rho_{\mathcal{E}}^B*(1-\pi)*\rho_{\mathcal{E}}^B*\mu_\epsilon^B\right)-\left(\rho_{\mathcal{E}}^B*\mu_\epsilon^B*\pi*\rho_{\mathcal{E}}^{A|M^*}+\rho_{\mathcal{E}}^B*\mu_\epsilon^B*(1-\pi)*\rho_{\mathcal{E}}^B\right)}{\gamma*\bar{\rho}_{\mathcal{E}}} + \frac{\pi*\rho_{\mathcal{E}}^B*\rho_{\mathcal{E}}^{A|M^*}*\mu_\delta^{A|M^*}}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]$$

$$[...]_1 = \left[\frac{\rho_{\mathcal{E}}^B * \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\epsilon^A - \rho_{\mathcal{E}}^B * \mu_\epsilon^B * \pi * \rho_{\mathcal{E}}^{A|M^*}}{\gamma * \bar{\rho}_{\mathcal{E}}} + \frac{\pi * \rho_{\mathcal{E}}^B * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]$$

$$[...]_1 = \frac{\pi * \rho_{\mathcal{E}}^{A|M^*} * \rho_{\mathcal{E}}^B * \left[\mu_\epsilon^A - \mu_\epsilon^B + \mu_\delta^{A|M^*}\right]}{\gamma * \bar{\rho}_{\mathcal{E}}}$$

Then we can rewrite the expression for the trade-volume originated by the ambiguity neutral investor type-B as

$$\theta_{t2}^B - \theta_{t1}^B = \left[\frac{\pi*\rho_{\mathcal{E}}^{A|M^*}*\rho_{\mathcal{E}}^B*\left[\mu_\epsilon^A-\mu_\epsilon^B+\mu_\delta^{A|M^*}\right]}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]_1 + \pi * \left[\frac{\rho_{\mathcal{E}}^B*\rho_X^A-\rho_X^B*\rho_{\mathcal{E}}^{A|M^*}}{\bar{\rho}_{\mathcal{E}}}\right]_2 * \frac{\Delta\mathrm{P}_{21}}{\gamma}$$

The total trading volume in the economy is given by the expression below. The ambiguity about the signal term δ manifests within this expression through the model dependent mean $\mu_\delta^{A|M^*}$

and the precissions $\rho_{\mathcal{E}}^{A|M^*}$ and $\bar{\rho}_{\mathcal{E}}$, whose specific values depend on the model M^* chosen by the ambiguous investor type-B on the minimization stage of his utility function.

$$V = |(1-\pi) * (\theta_{t2}^B - \theta_{t1}^B)|$$

$$V = \left| \left[\frac{\pi * (1-\pi) * \rho_{\mathcal{E}}^{A|M^*} * \rho_{\mathcal{E}}^B * \left[\mu_\epsilon^A - \mu_\epsilon^B + \mu_\delta^{A|M^*} \right]}{\gamma * \bar{\rho}_{\mathcal{E}}} \right]_1 + \left[\frac{\pi * (1-\pi) * (\rho_{\mathcal{E}}^B * \rho_X^A - \rho_X^B * \rho_{\mathcal{E}}^{A|M^*})}{\gamma * \bar{\rho}_{\mathcal{E}}} \right]_2 * \Delta \mathrm{P}_{21} \right|$$

$$V_{21} = |\alpha_V + \beta_V * \Delta P_{21}|$$

Appendix B Volume and Price Change Relation for Smooth Ambiguity Utility

Ambiguity Neutral Investor (Type-B) At Time 2

The ambiguity-neutral type-B investor takes the following optimal decision at time 2.

$$\max_{\theta_{t2}^B} \; E^B\left[U^B\left(\theta_{t2}^B\right)\middle|S\right] = \max_{\theta_{t2}^B} E^B\left[-e^{-\gamma \,*\, \left(w_{t2}^B \,+\, \theta_{t2}^B * \,\left(\widetilde{X} - P_{t1}\right)\right)}\,\middle|S\right]$$

$$\max_{\theta_{t2}^B} \; E^B\left[U^B\left(\theta_{t2}^B\right)\middle|S\right] = \max_{\theta_{t2}^B} -e^{-\gamma * \left(w_{t2}^B \,+\, \theta_{t2}^B * \,\left(E^B\left[\widetilde{X}\middle|S\right] - P_{t2}\right)\right) \,+\, \frac{1}{2} * \gamma^2 * \theta_{t2}^{2B} * \mathrm{VAR}^B[\widetilde{X}|S]}$$

Given his prior beliefs and the information received through the signal S, his optimal allocation at time 2 is the following.

$$\theta_{t2}^B = \frac{E^B[\widetilde{X}|S] - P_{t2}}{\gamma * \mathrm{VAR}^B[\widetilde{X}|S]}$$

$$\theta_{t2}^B = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * \,\left(S - \mu_\epsilon^B\right) - P_{t2} * \left(\rho_X^B + \rho_{\mathcal{E}}^B\right)}{\gamma}$$

where

$$\rho_X^B = \frac{1}{\sigma_X^{2\;B}}$$

$$\rho_{\mathcal{E}}^B = \frac{1}{\sigma_e^{2\;B}} = \frac{1}{\sigma_{\mathcal{E}}^{2\;B}}$$

$$E^B\left[\widetilde{X}\middle|S\right] = \mu_X^B + \frac{\sigma_X^{2\;B}}{\sigma_X^{2\;B} + \sigma_\epsilon^{2\;B}} * \left(S - \mu_X^B - \mu_\epsilon^B\right)$$

$$E^B\left[\widetilde{X}\middle|S\right] = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * \left(S - \mu_\epsilon^B\right)}{\rho_X^B + \rho_{\mathcal{E}}^B}$$

$$\mathrm{VAR}^B\left[\widetilde{X}\middle|S\right] = \sigma_X^{2\;B} - \frac{\sigma_X^{4\;B}}{\sigma_X^{2\;B} + \sigma_e^{2\;B}}$$

$$\mathrm{VAR}^B\left[\widetilde{X}\middle|S\right] = \left(\rho_X^B + \rho_{\mathcal{E}}^B\right)^{-1}$$

The parameter ρ_X^B corresponds to the type-B investor prior belief precision about the payoff X and $\rho_{\mathcal{E}}^B$ corresponds to the agent's belief about the precision of the signal S total error $\mathcal{E}$.

Ambiguity Averse/Loving Investor (Type-A) At Time 2

The ambiguous agent-1 takes interprets the signal in an ambiguous way, according to his Smooth Ambiguity Utility Function (Klibanoff et al., 2005).

The agent's preferences for risk are given by a CARA utility function $U_{1R}(\theta_{t2}^A) = -e^{-\gamma * W(\theta_{t2}^A)}$, where $W(\theta_{t2}^A)$ represents the final wealth of the agent at time 3. The agents preferences for ambiguity are given by the function $U_{1a}(E[U_{1R}]) = -(-E[U_{1R}])^{\gamma_a}$, where γ_a represents the ambiguity attitude of the agent.

The ambiguous agent beliefs that the signal S can be interpreted as $S_M = \widetilde{X} + \widetilde{\delta} + \widetilde{\epsilon}$, where $\widetilde{\delta}$ represents the agent's ambiguity about the correct model M under which he should interpret the signal. The prior belief of the agents is that the ambiguous component $\widetilde{\delta}$ distributes $N(\mu_{\delta}^A, \sigma_{\delta}^{2\ A})$.

The type-A investor problem maximizes the following expected utility.

$$\max_{\theta_{t2}^A}\ E^A\Big[-\Big(\ -E^A\Big[-e^{\ -\gamma\ *\ \left(w_{t2}^A\ +\ \theta_{t2}^A *\ \left(\widetilde{X}\ -\ P_{t2}\right)\right)}\ \Big|S, M\Big]\ \Big)^{\gamma_a}\Big|S\Big] =$$

$$\mathop{max}_{\theta_{t2}^A}\ E^A\Big[-\Big(e^{\ -\gamma\ *\ \left(w_{t2}^A\ +\ \theta_{t2}^A *\ \left(E^A[\widetilde{X}|S,M]-P_{t2}\right)+\frac{1}{2}*\gamma^2*\theta_{t2}^{2A}*\mathrm{VAR}^A[\widetilde{X}|S,M]\right)}\ \Big)^{\gamma_a}\Big|S\Big] =$$

$$\mathop{max}_{\theta_{t2}^A}\ -e^{\ -\gamma*\gamma_a*\ \left(w_{t2}^A\ +\ \theta_{t2}^A *\ \left(E^A[\widetilde{X}|S]-P_{t2}\right)\right)\ +\ \frac{1}{2}*\gamma^2*\gamma_a*\theta_{t2}^{2A}*\mathrm{E}^A[\ \mathrm{VAR}^A[\widetilde{X}|S,M]\ |S]+\ \frac{1}{2}*\gamma^2*\gamma_a^2*\theta_{t2}^{2A}*\mathrm{VAR}^A[\ \mathrm{E}^A[\widetilde{X}|S,M]\ |S]}$$

Considering that the variance $\mathrm{VAR}^A[\widetilde{X}|S, M]$ is already known before the last expectation operator conditional on the signal S, and that the variance $\mathrm{VAR}^A[\ \mathrm{E}^A[\widetilde{X}|S, M]\ |S]$ can be rewritten using the Law of Total Variance as $(\mathrm{VAR}^A[\widetilde{X}|S] - \mathrm{VAR}^A[\widetilde{X}|S, M])$ (Caskey, 2009), the optimization problem reduces to the maximization of the following certainty equivalence.

$$\mathop{max}_{\theta_{t2}^A}\ w_{t2}^A\ +\ \theta_{t2}^A *\ \left(E^A[\widetilde{X}|S] - P_{t2}\right) - \tfrac{1}{2} * \gamma * \theta_{t2}^{2A} * \mathrm{VAR}^A[\widetilde{X}|S] * \left[1 + (\gamma_a - 1) * \left(\tfrac{\mathrm{VAR}^A[\widetilde{X}|S]-\mathrm{VAR}^A[\widetilde{X}|S,M]}{\mathrm{VAR}^A[\widetilde{X}|S]}\right)\right]$$

Given his prior beliefs, the information received through signal S, and his personal interpretation of information the information contained in S, the optimal allocation of the type-A investor at time 2 is the following bellow. In our parametrization of the signal S, the signal and the payoff $\widetilde{X}$ have a covariance equal to the variance of the payoff $\widetilde{X}$, and we do not consider correlation between the payoff $\widetilde{X}$ and the ambiguity term $\widetilde{\delta}$.

$$\theta_{t2}^A = \frac{E^A[\widetilde{X}|S] - P_{t2}}{\gamma * Var^A[\widetilde{X}|S] * \nu^A}$$

$$\theta_{t2}^{A} = \frac{\rho_X^A * \mu_X^A + \rho_{\mathcal{E}}^A * (S - \mu_\epsilon^A - \mu_\delta^A) - P_{t2} * (\rho_X^A + \rho_{\mathcal{E}}^A)}{\gamma * \nu^A}$$

$$\begin{aligned}
&where \\
\rho_X^A &= \frac{1}{\sigma_X^{2\ A}} \\
\rho_{\mathcal{E}}^A &= \frac{1}{\sigma_\delta^{2\ A} + \sigma_\epsilon^{2\ A}} \\
\nu^A &= \left[1 + (\gamma_a - 1) * \left(\frac{\mathrm{VAR}^A[\widetilde{X}|S] - \mathrm{VAR}^A[\widetilde{X}|S, M]}{\mathrm{VAR}^A[\widetilde{X}|S]}\right)\right]
\end{aligned}$$

$$\begin{aligned}
E^A\left[\widetilde{X}\middle|S\right] &= \mu_X^A + \frac{\sigma_X^{2\ A}}{\sigma_X^{2\ A} + \sigma_\delta^{2\ A} + \sigma_\epsilon^{2\ A}} * \left(S - \mu_X^A - \mu_\delta^A - \mu_\epsilon^A\right) \\
&= \frac{\rho_X^A * \mu_X^A + \rho_{\mathcal{E}}^A * \left(S - \mu_\delta^A - \mu_\epsilon^A\right)}{\rho_X^A + \rho_{\mathcal{E}}^A} \\
Var^A[\widetilde{X}|S] &= \left[\rho_X^A + \rho_{\mathcal{E}}^A\right]^{-1}
\end{aligned}$$

$$\begin{aligned}
E^A\left[\widetilde{X}\middle|S, M\right] &= \mu_X^A + \begin{bmatrix} \sigma_{X,S}^A & \sigma_{X,\delta}^A \end{bmatrix} * \begin{bmatrix} \sigma_S^{2\ A} & \sigma_{S,\delta}^A \\ \sigma_{S,\delta}^A & \sigma_\delta^{2\ A} \end{bmatrix}^{-1} * \begin{bmatrix} S - \mu_S^A \\ \delta - \mu_\delta^A \end{bmatrix} \\
&= \mu_X^A + \begin{bmatrix} \sigma_X^{2\ A} & 0 \end{bmatrix} * \begin{bmatrix} \sigma_S^{2\ A} & \sigma_\delta^{2\ A} \\ \sigma_\delta^{2\ A} & \sigma_\delta^{2\ A} \end{bmatrix}^{-1} * \begin{bmatrix} S - \mu_S^A \\ \delta - \mu_\delta^A \end{bmatrix} \\
&= \mu_X^A + \frac{\sigma_X^{2\ A} * (S - \delta) - \sigma_X^{2\ A} * \left(\mu_X^A + \mu_\epsilon^A\right)}{\sigma_S^{2\ A} - \sigma_\delta^{2\ A}} \\
&= \frac{\rho_X^A * \mu_X^A + \rho_\epsilon^A * \left((S - \delta) - \mu_\epsilon^A\right)}{\rho_X^A + \rho_\epsilon^A} \\
Var^A[\widetilde{X}|S, M] &= \sigma_X^{2\ A} - \begin{bmatrix} \sigma_{X,S}^A & \sigma_{X,\delta}^A \end{bmatrix} * \begin{bmatrix} \sigma_S^{2\ A} & \sigma_{S,\delta}^A \\ \sigma_{S,\delta}^A & \sigma_S^{2\ A} \end{bmatrix}^{-1} * \begin{bmatrix} \sigma_{X,S}^A \\ \sigma_{X,\delta}^A \end{bmatrix} \\
&= \sigma_X^{2\ A} - \begin{bmatrix} \sigma_X^{2\ A} & 0 \end{bmatrix} * \begin{bmatrix} \sigma_S^{2\ A} & \sigma_\delta^{2\ A} \\ \sigma_\delta^{2\ A} & \sigma_S^{2\ A} \end{bmatrix}^{-1} * \begin{bmatrix} \sigma_X^{2\ A} \\ 0 \end{bmatrix} \\
&= \sigma_X^{2\ A} - \frac{\sigma_X^{4\ A} * (\sigma_X^{2\ A} + \sigma_\delta^{2\ A} + \sigma_\epsilon^{2\ A})}{(\sigma_X^{2\ A} + \sigma_\delta^{2\ A} + \sigma_\epsilon^{2\ A})^2 - \sigma_\delta^{4\ A}} = \frac{1}{\rho_X^A} - \frac{\rho_S^A * \frac{\rho_\delta^{2\ A}}{\rho_X^{2\ A}}}{\rho_\delta^{2\ A} - \rho_S^{2\ A}} \\
\frac{Var^A[\widetilde{X}|S] - Var^A[\widetilde{X}|S, M]}{Var^A[\widetilde{X}|S]} &= \frac{\rho_X^A * \rho_\epsilon^{A\ 3}}{(\rho_\delta^A * \rho_\epsilon^A + 2 * \rho_X^A * \rho_\epsilon^A + \rho_X^A * \rho_\delta^A) * (\rho_X^A + \rho_\epsilon^A) * (\rho_\epsilon^A + \rho_\delta^A)}
\end{aligned}$$

The term ν^A weights by the ambiguity aversion coefficient a the ratio of $\widetilde{X}$'s volatility caused by the ambiguity of the agent's interpretation of the signal. This ratio increases with the ambiguity volatility $\sigma_\delta^{2\ A}$, increasing the volatility divisor $\gamma * Var^A[\widetilde{X}|S] * \nu^A$ that goes inside the type-B investor's optimal allocation.

Equilibrium At Time 2

In equilibrium at time 2 the following equation has to hold.

$$(1-\pi) * \theta_{t2}^B + \pi * \theta_{t2}^A = 0.$$

Replacing the individual agents optimal allocations θ_{t2}^B and θ_{t2}^A in the market clearing condition.

$$\frac{\pi}{\nu^A} * \left[\rho_X^A * \ \mu_X^A \ + \ \rho_{\mathcal{E}}^A \ (S - \mu_\epsilon^A - \mu_\delta^A) \ - \ P_{t2} * \ (\rho_X^A + \rho_{\mathcal{E}}^A) \ \right] + (1-\pi) *$$
$$\left[\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) \ - \ P_2 * (\rho_X^B + \rho_{\mathcal{E}}^B) \ \right] = 0$$

Grouping together terms associated to the type-A investor previous beliefs about the risky asset expected payment, the signal S, the signal S error and price P_2

$$\left[\frac{\pi}{\nu^A} * \rho_X^A * \mu_X^A + (1-\pi) * \rho_X^B * \mu_X^B\right]_{\bar{\mu}_X} + \left[\frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A + (1-\pi) * \rho_{\mathcal{E}}^B\right]_{\bar{\rho}_{\mathcal{E}}} * \ \mathrm{S} \ -$$
$$\left[\frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\epsilon^A + (1-\pi) * \rho_{\mathcal{E}}^B * \mu_\epsilon^B\right]_{\bar{\mu}_\epsilon} - \ \left[\frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A\right] - \mathrm{P}_{t2} * \left[\frac{\pi}{\nu^A} * \rho_X^A + (1-\pi) * \rho_X^B\right]_{\bar{\rho}_X} \ -$$
$$\mathrm{P}_{t2} * \left[\frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A + (1-\pi) * \rho_{\mathcal{E}}^B\right]_{\bar{\rho}_{\mathcal{E}}} = 0$$

Replacing the expression in brackets by the terms $\bar{\mu}_X, \bar{\rho}_{\mathcal{E}}, \bar{\mu}_\epsilon$ and $\bar{\rho}_X$ we can rewrite the market clearing condition as the expression below.

$$\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * \ \mathrm{S} - \bar{\mu}_\epsilon - \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A - \mathrm{P}_{t2} * (\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}) = 0$$

From here we can rewrite the price at time 2.

$$\mathrm{P}_{t2} = \frac{\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * \ \mathrm{S} - \bar{\mu}_\epsilon - \dfrac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A}{\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}}$$

Signal S At Time 2

We can use this last expression for P_{t2} to rewrite the signal S as a function of the change in price ΔP.

$$\Delta \mathrm{P} = P_{t2} - P_{t1} = \frac{\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * \ \mathrm{S} - \bar{\mu}_\epsilon - \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A}{\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}} - P_{t1}$$

$$\Delta \mathrm{P} * \Big[\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}\Big] = \Big[\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * S - \bar{\mu}_\epsilon - \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A\Big] - P_1 * \Big[\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}\Big]$$

$$\mathrm{S} = \frac{\Delta \mathrm{P} * [\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] + \bar{\mu}_\epsilon + \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A - [\bar{\mu}_X - P_{t1} * \bar{\rho}_X]}{\bar{\rho}_{\mathcal{E}}} + P_{t1}$$

In comparison to the Max Min Utility, in this last expression we have the additional term $[\bar{\mu}_X - P_{t1} * \bar{\rho}_X]$ that is different from 0. In the next lines we show that this term is proportional to the investor type-A versus type-B difference in prior beliefs about the mean of the payoff X. From the optimization at time 1 we know the following.

$$\mathrm{P}_{t1} = \frac{\bar{\mu}_{\underset{t1}{X}}}{\bar{\rho}_{\underset{t1}{X}}}$$

$$where$$

$$\bar{\mu}_{\underset{t1}{X}} = \pi * \rho_X^A * \mu_X^A + (1-\pi) * \rho_X^B * \mu_X^B$$

$$\bar{\rho}_{\underset{t1}{X}} = \pi * \rho_X^A + (1-\pi) * \rho_X^B$$

These last terms from time (1) are related to the average terms $\bar{\mu}_X$ and $\bar{\rho}_X$ that appear at time 2 through the following equations.

$$\bar{\mu}_X = \bar{\mu}_{\underset{t1}{X}} + \Big(\bar{\mu}_X - \bar{\mu}_{\underset{t1}{X}}\Big) = \bar{\mu}_{\underset{t1}{X}} + \pi * \left(\frac{1}{\nu^A} - 1\right) * \mu_X^A * \rho_X^A$$

$$\bar{\rho}_X = \bar{\rho}_{\underset{t1}{X}} + \Big(\bar{\rho}_X - \bar{\rho}_{\underset{t1}{X}}\Big) = \bar{\rho}_{\underset{t1}{X}} + \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A$$

Using these last relation, we can rewrite the term $-[\bar{\mu}_X - P_{t1} * \bar{\rho}_X]$ that appears above in the equation for S.

$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = -\left[\bar{\mu}_{\underset{t1}{X}} + \pi * \left(\frac{1}{\nu^A} - 1\right) * \mu_X^A * \rho_X^A\right] + \left[\mathrm{P}_{t1} * \bar{\rho}_{\underset{t1}{X}} + \mathrm{P}_{t1} * \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A\right]$$

$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = -\left[\bar{\mu}_{\underset{t1}{X}} + \pi * \left(\frac{1}{\nu^A} - 1\right) * \mu_X^A * \rho_X^A\right] + \left[\bar{\mu}_{\underset{t1}{X}} + \mathrm{P}_{t1} * \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A\right]$$
$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A * \left(P_{t1} - \mu_X^A\right)$$
$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A * \left(\frac{\pi * \rho_X^A * \mu_X^A + (1-\pi) * \rho_X^B * \mu_X^B}{\pi * \rho_X^A + (1-\pi) * \rho_X^B} - \mu_X^A\right)$$
$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A * \left(\frac{(1-\pi) * \rho_X^B * (\mu_X^B - \mu_X^A)}{\bar{\rho}_{\underset{t1}{X}}}\right)$$
$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = \pi * (1-\pi) * \left(\frac{1}{\nu^A} - 1\right) * \left(\frac{\rho_X^A * \rho_X^B}{\bar{\rho}_{\underset{t1}{X}}}\right) * \left(\mu_X^B - \mu_X^A\right)$$
$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = \eta * \left(\mu_X^B - \mu_X^A\right)$$

Replacing the term $[\bar{\mu}_X - P_{t1} * \bar{\rho}_X]$ by this last expression in the equation for signal S, we obtain the following term.

$$\mathrm{S} = \frac{\Delta \mathrm{P} * [\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] + \bar{\mu}_\epsilon + \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A + \eta * \left(\mu_X^B - \mu_X^A\right)}{\bar{\rho}_{\mathcal{E}}} + P_{t1}$$

Trading Volume At Time 2

We measure the trading volume from time 1 to 2 as the change in the risky-asset allocation of the type-B agent multiplied by the proportion of this agent type in the economy. By symmetry of this market equilibrium, the volume of risky asset this type-B investors buys/sells is equivalent to the volume the type-A investors sells/buys.

$$V = (1-\pi) * (\theta_{t2}^B - \theta_{t1}^B)$$
$$where$$
$$\theta_{t1}^B = \frac{(\mu_X^B - P_{t1}) * \rho_X^B}{\gamma}$$
$$\theta_{t2}^B = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) - P_{t2} * (\rho_X^B + \rho_{\mathcal{E}}^B)}{\gamma}$$

Replacing the allocations of the ambiguity-neutral type-B investor in the expression $(\theta_{t2}^B - \theta_{t1}^B)$ we obtain the following expression.

$$\theta_{t2}^B - \theta_{t1}^B = \left[\frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) - P_{t2} * (\rho_X^B + \rho_{\mathcal{E}}^B)}{\gamma}\right] - \left[\frac{(\mu_X^B - P_{t1}) * \rho_X^B}{\gamma}\right]$$

$$\theta_{t2}^B - \theta_{t1}^B = \frac{\rho_{\mathcal{E}}^B * \left[\, \mathrm{S} - \mu_\epsilon^B \right]}{\gamma} - \frac{P_{t2} * \rho_{\mathcal{E}}^B}{\gamma} - \Delta\mathrm{P} * \frac{\rho_X^B}{\gamma}$$

At this stage we rewrite the signal S expression $(S - U_2)$ as a function of the change in prices between period (1) and (2) using the relation of the previous section.

$$\theta_{t2}^B - \theta_{t1}^B = \rho_{\mathcal{E}}^B * \frac{\left[\left(\frac{\Delta\mathrm{P}*[\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] + \bar{\mu}_\epsilon + \frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^A*\mu_\delta^A + \eta*\left(\mu_X^B - \mu_X^A\right)}{\bar{\rho}_{\mathcal{E}}} + P_{t1}\right)_{\mathrm{S}} - \mu_\epsilon^B\right]}{\gamma} - \frac{P_{t2} * \rho_{\mathcal{E}}^B}{\gamma} - \Delta\mathrm{P} * \frac{\rho_X^B}{\gamma}$$

$$\theta_{t2}^B - \theta_{t1}^B = \frac{\Delta\mathrm{P}}{\gamma} * \left[\rho_{\mathcal{E}}^B * \frac{(\bar{\rho}_X + \bar{\rho}_{\mathcal{E}})}{\bar{\rho}_{\mathcal{E}}} - \rho_{\mathcal{E}}^B - \rho_X^B\right] + \left[\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}}*\gamma} * \eta * \left(\mu_X^B - \mu_X^A\right)\right] + \left[\frac{\rho_{\mathcal{E}}^B*\bar{\mu}_\epsilon}{\gamma*\bar{\rho}_{\mathcal{E}}} - \frac{\rho_{\mathcal{E}}^B*\mu_\epsilon^B}{\gamma} + \frac{\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^B*\rho_{\mathcal{E}}^A*\mu_\delta^A}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]$$

$$\theta_{t2}^B - \theta_{t1}^B = \frac{\Delta\mathrm{P}}{\gamma} * \left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 + \left[\frac{\rho_{\mathcal{E}}^B*\bar{\mu}_\epsilon}{\gamma*\bar{\rho}_{\mathcal{E}}} - \frac{\rho_{\mathcal{E}}^B*\mu_\epsilon^B}{\gamma} + \frac{\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^B*\rho_{\mathcal{E}}^A*\mu_\delta^A}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]_1 + \left[\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}}*\gamma} * \eta * \left(\mu_X^B - \mu_X^A\right)\right]_0$$

We simplify the expressions inside bracket (2)

$$\left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 = \left[\frac{\rho_{\mathcal{E}}^B * \frac{\pi}{\nu^A} * \rho_X^A - \rho_X^B * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A}{\bar{B}}\right]$$

$$\left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 = \frac{\pi}{\nu^A} * \left[\frac{\rho_{\mathcal{E}}^B * \rho_X^A - \rho_X^B * \rho_{\mathcal{E}}^A}{\bar{\rho}_{\mathcal{E}}}\right]$$

We simplify the expression inside bracket (1).

$$[...]_1 = \left[\frac{\left(\rho_{\mathcal{E}}^B*\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^A*\mu_\epsilon^A + \rho_{\mathcal{E}}^B*(1-\pi)*\rho_{\mathcal{E}}^B*\mu_\epsilon^B\right) - \left(\rho_{\mathcal{E}}^B*\mu_\epsilon^B*\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^A + \rho_{\mathcal{E}}^B*\mu_\epsilon^B*(1-\pi)*\rho_{\mathcal{E}}^B\right)}{\gamma*\bar{\rho}_{\mathcal{E}}} + \frac{\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^B*\rho_{\mathcal{E}}^A*\mu_\delta^A}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]$$

$$[...]_1 = \left[\frac{\rho_{\mathcal{E}}^B * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\epsilon^A - \rho_{\mathcal{E}}^B * \mu_\epsilon^B * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A}{\gamma * \bar{\rho}_{\mathcal{E}}} + \frac{\frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^B * \rho_{\mathcal{E}}^A * \mu_\delta^A}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]$$

$$[...]_1 = \frac{\frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B * \left[\mu_\epsilon^A - \mu_\epsilon^B + \mu_\delta^A\right]}{\gamma * \bar{\rho}_{\mathcal{E}}}$$

Then we can rewrite the expression for the trading volume associated to the type-B investor.

$$\theta_{t2}^B - \theta_{t1}^B = \left[\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}} * \gamma} * \eta * \left(\mu_X^B - \mu_X^A\right)\right]_0 + \left[\frac{\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^A*\rho_{\mathcal{E}}^B*\left(\mu_\epsilon^A - \mu_\epsilon^B + \mu_\delta^A\right)}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]_1 + \left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 * \frac{\Delta\mathrm{P}}{\gamma}$$

$$\theta_{t2}^B - \theta_{t1}^B = \left[\pi * (1-\pi) * \left(\frac{1}{\nu^A} - 1\right) * \left(\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}} * \gamma}\right) * \left(\frac{\rho_X^A * \rho_X^B}{\bar{\rho}_{\underset{t1}{X}}}\right) * \left(\mu_X^B - \mu_X^A\right)\right]_0$$
$$+ \left[\frac{\frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B * \left(\mu_\epsilon^A - \mu_\epsilon^B + \mu_\delta^A\right)}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_1 + \left[\frac{\frac{\pi}{\nu^A} * \left(\rho_{\mathcal{E}}^B * \rho_X^A - \rho_X^B * \rho_{\mathcal{E}}^A\right)}{\bar{\rho}_{\mathcal{E}}}\right]_2 * \frac{\Delta\mathrm{P}}{\gamma}$$

The total trading-volume in the economy is given by the expression below. Here, α_V accounts for the difference in prior beliefs $(\mu_X^B - \mu_X^A)$ between investor types A-B and the expectations channel of ambiguity represented by μ_δ^A. The coefficient β_V is a the term that amplifies or smooths the traditiona trading volume channel associated to changes in prices.

$$V = \left|(1-\pi) * (\theta_{t2}^B - \theta_{t1}^B)\right|$$

$$\begin{aligned}
V = \Bigg| \Bigg\{ & \left[\pi * (1-\pi)^2 * \left(\frac{1}{\nu^A} - 1\right) * \left(\frac{\rho_\mathcal{E}^B}{\bar{\rho}_\mathcal{E} * \gamma}\right) * \left(\frac{\rho_X^A * \rho_X^B}{\bar{\rho}_{\substack{X \\ t1}}}\right)\right] * \left(\mu_X^B - \mu_X^A\right) \\
& + \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_\mathcal{E}^A * \rho_\mathcal{E}^B}{\gamma * \bar{\rho}_\mathcal{E}}\right] * \left(\mu_\epsilon^A - \mu_\epsilon^B + \mu_\delta^A\right) \Bigg\}_\alpha \\
& + \left\{ \left[\frac{\frac{\pi}{\nu^A} * \left(\rho_\mathcal{E}^B * \rho_X^A - \rho_X^B * \rho_\mathcal{E}^A\right)}{\bar{\rho}_\mathcal{E}}\right] \right\}_\beta * \frac{\Delta \mathrm{P}}{\gamma} \Bigg|
\end{aligned}$$

$$V = |\alpha_V + \beta_V * \Delta P_{21}|$$

Appendix C Volume and Price Volatility Elasticity

This section derives the expected market's trading volume one period ahead, as well as the trading volume to price volatility elasticity. These formulas expand the work of Bollerslev et al. (2018) by introducing (1) Ambiguity (Knightian Uncertainty) and (2) a non standard normal distribution.

C.1 Expected Trading Volume

This part shows the derivation of the market's expected trading volume one period ahead, $E[V_{21}]$ in the presence of an Ambiguous Agent. The formula uses the theoretical expression for trading volume V_{21} presented in Section 2.7 and Appendix B. We calculate a closed form expression for $E[V_{21}]$ using the volume formula $V_{21} = |\alpha_V + \beta_V * \Delta P_{21}|$ and assuming a normal distribution $\Delta P_{21} \sim\ N(\mu_{\Delta P}, \sigma_{\Delta P})$.

Using the standard normal distribution $Z = \frac{P_{21} - \mu_{\Delta P}}{\sigma_{\Delta P}}$ we rewrite the expression for trading volume as $V_{21} = |\alpha_V + \beta_V * \sigma_{\Delta P} * Z|$ The expected volume calculation can be split in two parts conditional of the sign that the expression inside the absolute value takes

$$E[V_{21}] = \begin{aligned} &E[V_{21} \mid \alpha_V + \beta_V * \sigma_{\Delta P} * Z > 0] * P(\alpha_V + \beta_V * \sigma_{\Delta P} * Z > 0) \\ &+ E[V_{21} \mid \alpha_V + \beta_V * \sigma_{\Delta P} * Z < 0] * P(\alpha_V + \beta_V * \sigma_{\Delta P} * Z < 0) \end{aligned}$$

Taking into account the conditional density $f_{z|condition}$ of the standard normal distribution

$$f_{z|\alpha_V + \beta_V * \sigma_{\Delta P} * Z > 0} = \frac{f_z * I_[[\alpha_V + \beta_V * \sigma_{\Delta P} * Z > 0]}{P(\alpha_V + \beta_V * \sigma_{\Delta P} * Z > 0)}$$

$$f_{z|\alpha_V + \beta_V * \sigma_{\Delta P} * Z < 0} = \frac{f_Z * I_[[\alpha_V + \beta_V * \sigma_{\Delta P} * Z < 0]}{P(\alpha_V + \beta_V * \sigma_{\Delta P} * Z < 0)}.$$

the expected volume formula simplifies to

$$E[V_{21}] = \int_{\alpha_V + \beta_V * \sigma_{\Delta P} * Z > 0} (\alpha_V + \beta_V * \sigma_{\Delta P} * Z) * f_z dZ + \int_{\alpha_V + \beta_V * \sigma_{\Delta P} * Z < 0} -(\alpha_V + \beta_V * \sigma_{\Delta P} * Z) * f_z dZ$$

Integral of β_V Section

First we calculate the integrals of the expressions $(\beta_V * \sigma_{\Delta P} * Z)$ and $(-\beta_V * \sigma_{\Delta P} * Z)$. For both integrals, there are two sub-cases depending on the sign of the constant β_V, because the inequality that defines the integrals' regions flips direction.

For $\beta_V \geq 0$:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\beta_V * \sigma_{\Delta P} * Z) * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[-\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}}^{+\infty}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\beta_V * \sigma_{\Delta P} * Z) * f_z dZ = \frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} = \frac{|\beta_V| * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\beta_V * \sigma_{\Delta P} * Z) * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\infty}^{-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\beta_V * \sigma_{\Delta P} * Z) * f_z dZ = \frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} = \frac{|\beta_V| * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}$$

For $\beta_V < 0$:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\beta_V * \sigma_{\Delta P} * Z) * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\infty}^{-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\beta_V * \sigma_{\Delta P} * Z) * f_z dZ = -\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} = \frac{|\beta_V| * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\beta_V * \sigma_{\Delta P} * Z) * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}}^{+\infty}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\beta_V * \sigma_{\Delta P} * Z) * f_z dZ = -\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}*} e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} = \frac{|\beta_V| * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}$$

We get that independent of the sign of phi1, the sum of both integrals of the expressions $(\beta_V * \sigma_{\Delta P} * Z)$ and $(-\beta_V * \sigma_{\Delta P} * Z)$ is equal to

$$|\beta_V| * \sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}.$$

Integral of α_V Section

Second, we calculate the integrals of the expressions (α_V) and $(-\alpha_V)$. For both integrals, there are four sub-cases depending on the sign of the constants $\{\alpha_V, \beta_V\}$, because the inequality that defines the integrals' regions flips direction with β_V and the sign of both constants affect the standard normal density function.

For α_V^+ and β_V^+:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}}^{+\infty} = \alpha_V * \left(1 - \Phi\left(-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)\right)$$
$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = |\alpha_V| * \left[1 - \Phi\left(-\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)\right] = |\alpha_V| * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[-\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\infty}^{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}} = -\alpha_V * \Phi\left(-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)$$
$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = -\alpha_V * \left[1 - \Phi\left(\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)\right] = |\alpha_V| * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]$$

For α_V^+ and β_V^-:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P*}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\alpha_V * \frac{1}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\infty}^{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}} = \alpha_V * \Phi\left(-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)$$
$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = |\alpha_V| * \Phi\left(\frac{\alpha_V}{-\beta_V * \sigma_{\Delta P}}\right) = |\alpha_V| * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[-\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}}^{+\infty} = -\alpha_V * \left[1 - \Phi\left(\frac{\alpha_V}{-\beta_V * \sigma_{\Delta P}}\right)\right]$$
$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \alpha_V * \left[\Phi\left(\frac{\alpha_V}{-\beta_V * \sigma_{\Delta P}}\right) - 1\right] = |\alpha_V| * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]$$

For α_V^- and β_V^+:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\frac{\alpha_V}{\beta_V*\sigma}}^{+\infty} = \alpha_V * \left(1 - \Phi\left(-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)\right)$$
$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = (-\alpha_V) * \left[\Phi\left(\frac{-\alpha_V}{\beta_V * \sigma_{\Delta P}}\right) - 1\right] = |\alpha_V| * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[-\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\infty}^{-\frac{\alpha_V}{\beta_V^* \sigma_{\Delta P}}} = -\alpha_V * \Phi\left(\frac{\alpha_V}{-\beta_V * \sigma_{\Delta P}}\right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = |\alpha_V| * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)$$

For α_V^- and β_V^-:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\infty}^{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}} = \alpha_V * \Phi\left(-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \alpha_V * \left[1 - \Phi\left(\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)\right] = (-\alpha_V) * \left[\Phi\left(\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right) - 1\right]$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \alpha_V * \left[1 - \Phi\left(\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)\right] = |\alpha_V| * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[-\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}}^{+\infty} = -\alpha_V * \left[1 - \Phi\left(\frac{-\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)\right]$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = -\alpha_V * \Phi\left(\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right) = |\alpha_V| * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)$$

We get that independent of the constants $\{\alpha_V, \beta_V\}$ signs, the sum the sum of both integrals of the expressions (α_V) and $(-\alpha_V)$ is equal to

$$|\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]$$

Final Result

Finally, we get that independent of the constants $\{\alpha_V, \beta_V\}$ signs, the expression for the one period ahead expected trading volume is

$$E[V_{21}] = \int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\alpha_V + \beta_V * \sigma_{\Delta P} * Z) * f_z dZ + \int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\alpha_V + \beta_V * \sigma_{\Delta P} * Z) * f_z dZ$$

$$E[V_{21}] = |\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right] + |\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}\right]$$

C.2 Trading Volume to Price Volatility Elasticity

This part makes use of the expected trading volume derived above, to calculate the expected trading volume to price volatility elasticity $\xi = \frac{\partial E[V_{21}]/E[V_{21}]}{\partial\sigma_{\Delta P}/\sigma_{\Delta P}}$ presented in Section-2.7.

The derivation starts by calculating the derivative of the expected trading volume

$$\xi = \frac{\partial E[V_{21}]/E[V_{21}]}{\partial\sigma_{\Delta P}/\sigma_{\Delta P}} = \frac{\partial E[V_{21}]}{\partial\sigma_{\Delta P}} * \frac{\sigma_{\Delta P}}{E[V_{21}]}$$

We can divide the expected trading volume in two parts

$$E[V_{21}] = P1 + P2 = |\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right] + |\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}\right].$$

Then, we proceed to derive the first part of the expected volume P_1 in relation to price volatility $\sigma_{\Delta P}$

$$\frac{\partial \mathrm{P}_1}{\partial\sigma_{\Delta P}} = \frac{\partial\left\{|\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V|*\sigma_{\Delta P}}\right) - 1\right]\right\}}{\partial\sigma_{\Delta P}} = |\alpha_V| * 2 * \frac{\partial\left\{\int_{-\infty}^{\frac{|\alpha_V|}{|\beta_V|*\sigma_{\Delta P}}} \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ\right\}}{\partial\sigma_{\Delta P}}$$

$$\frac{\partial \mathrm{P}_1}{\partial\sigma_{\Delta P}} = |\alpha_V|^* 2^* \frac{-|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}^2} * \frac{1}{\sqrt{2*\pi}} e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} = \frac{\sqrt{2}}{\sqrt{\pi}} * \frac{-|\alpha_V|^2}{|\beta_V| * \sigma_{\Delta P}^2} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}.$$

Now, we continue by deriving the second part P_2 of the expected trading volume in relation to price volatility $\sigma_{\Delta P}$

$$\frac{\partial \mathrm{P}_2}{\partial\sigma_{\Delta P}} = \frac{\partial\left\{|\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V{}^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}\right]\right\}}{\partial\sigma_{\Delta P}} = |\beta_V|^* \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} + \frac{\sqrt{2}}{\sqrt{\pi}} * \frac{|\alpha_V|^2}{|\beta_V| * \sigma_{\Delta P}^2} * e^{\frac{-\alpha_V{}^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}$$

$$\frac{\partial E[V_{21}]}{\partial\sigma_{\Delta P}} = \frac{\partial(\mathrm{P}_1 + \mathrm{P}_2)}{\partial\sigma_{\Delta P}} = \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}*} |\beta_V| = \frac{\mathrm{P}_2}{\sigma_{\Delta P}}$$

With the previous results, we can proceed to calculate the elasticity ξ

$$\xi = \frac{\partial E[V_{21}]/E[V_{21}]}{\partial\sigma_{\Delta P}/\sigma_{\Delta P}} = \frac{\partial E[V_{21}]}{\partial\sigma_{\Delta P}} * \frac{\sigma_{\Delta P}}{E[V_{21}]} = \left(\frac{\partial E[V_{21}]}{\partial\sigma_{\Delta P}} * \sigma_{\Delta P}\right) * \frac{1}{E[V_{21}]}$$

$$\xi = \mathrm{P}_2 * \frac{1}{\mathrm{P}_1 + \mathrm{P}_2} = \frac{1}{1 + \mathrm{P}_1/\mathrm{P}_2}.$$

Replacing for the terms P_1 and P_2 we get the final expected trading volume to price volatility elasticity formula ξ used in section Section-2.7. In the formula below ϕ refers to the standard normal density function and Φ refers to the standard normal CDF.

$$\xi = \frac{1}{1 + \frac{|\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]}{|\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2 * \beta_V^2 * \sigma_{\Delta P}^2}}\right]}} = \frac{1}{1 + \frac{|\alpha_V| / (|\beta_V|^* \sigma_{\Delta P}) * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1/2\right]}{\frac{1}{\sqrt{2 * \pi}} * e^{\frac{-\alpha_V^2}{2 * \beta_V^2 * \sigma_{\Delta P}^2}}}}$$

$$\xi = \frac{1}{1 + \frac{|\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]}{|\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2 * \beta_V^2 * \sigma_{\Delta P}^2}}\right]}} = \frac{1}{1 + \frac{|\alpha_V| / (|\beta_V|^* \sigma_{\Delta P}) * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1/2\right]}{\phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)}}$$

$$\xi = \frac{1}{1 + \frac{|\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]}{|\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2 * \beta_V^2 * \sigma_{\Delta P}^2}}\right]}} = \frac{1}{1 + \frac{|\alpha_V| / (|\beta_V|^* \sigma_{\Delta P}) * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1/2\right]}{\phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)}}$$

Appendix D Model Daily Regressions Results

In this appendix, I present the daily frequency regressions for the trading volume model outlined in Section-2.6, along with the elasticity model detailed in Section-2.7.

Tables 8, 13, and 14 below present the regressions (1), (2), and (3) results of the trading volume model as discussed in Section-4.

$$V = \alpha_V + \beta_V * \Delta_P + \epsilon \tag{4.1}$$
$$\alpha_V = 1 + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \epsilon$$
$$\beta_V = 1 + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_p * \sigma_P + \epsilon$$

$$V = \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\right)_{\alpha} + \left(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\right)_{\beta} * \Delta_P + V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon \tag{4.2}$$

$$\Delta V = c + \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\right)_{\Delta\alpha} + \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p * \Delta_P^2} * \sigma_P\right)_{\beta} * \Delta_P^2 + \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon \tag{4.3}$$

The outcomes of regression (1) in Section-4 for the elasticity model are illustrated in table-15 below.

$$\Delta log(V) = c + \Big(\xi_1 + \xi_\mu * \mu_{\Delta p} + \xi_{pbel} * PBEL + \xi_{amb} * AMB + \xi_{p1} * P_1 + \xi_{\mu^2} * \mu_{\Delta p}^2 + \xi_{pbel^2} * PBEL^2 + \xi_{amb^2} * AMB^2 + \xi_{p1^2} * P_1^2 + \xi_{\sigma_{amb}} * \sigma_{amb} + \xi_{\sigma_{amb^2}} * \sigma_{amb}^2 \Big)_{\xi} * \Delta log(\sigma_{\Delta p}) + \Delta log(V)_{t-1} + \sum_{p=t+1}^{T} \gamma_p \tag{5.2}$$

The regressions cover the daily period from 2013 to 2018 for the SPY and were conducted on four distinct datasets: D(1), D(2), D(3) and D(4). Each dataset employs the daily EPU measure extracted from Twitter as a proxy for Ambiguity, alongside a distinct proxy for differences in prior beliefs from Stocktwits (Cookson & Niessner, 2020).

Daily Datasets Description	
D(1)	Prior beliefs differences proxied by $\text{PBEL}_{\text{WI, IND}}$ and Ambiguity proxied by AMB_{EPUD}
D(2)	Prior beliefs differences proxied by $\text{PBEL}_{\text{AC, IND}}$ and Ambiguity proxied by AMB_{EPUD}
D(3)	Prior beliefs differences proxied by $\text{PBEL}_{\text{WI, ETF}}$ and Ambiguity proxied by AMB_{EPUD}
D(4)	Prior beliefs differences proxied by $\text{PBEL}_{\text{AC, ETF}}$ and Ambiguity proxied by AMB_{EPUD}

Table 8. Daily Regressions α_V and β_V

This table summarizes the daily regression results for the coefficients α_V and β_V of the trading volume formula, as detailed in regression (1) of Section-4. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\alpha_V = c + \alpha_{amb} * AMB + \alpha_{pbel} * PBEL + \alpha_{V_{t-1}} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_{\sigma_p} * \sigma_P \ + \ \beta_{V_{t-1}} + \ \sum_{p=t+1}^{T} \gamma_p + \ \epsilon$$

	D(1)	D(2)	D(3)	D(4)
Panel A: Regression on α_V				
c	0.15	0.16	0.19	0.16
	(3.31) (3.32) (2.80)	(3.46) (3.47) (3.32)	(4.06) (4.07) (4.34)	(3.40) (3.41) (3.43)
	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{pbel}	0.12	0.08	0.10	-0.12
	(6.64) (6.66) (5.39)	(4.21) (4.22) (3.46)	(5.16) (5.18) (6.03)	(-6.62) (-6.64) (-7.24)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.11	0.11	0.11	0.11
	(5.10) (5.11) (3.62)	(5.30) (5.31) (3.80)	(5.13) (5.14) (3.71)	(5.37) (5.38) (3.84)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha_{V_{\text{t-1}}}$	0.58	0.58	0.58	0.57
	(28.32) (28.41) (17.50)	(28.32) (28.41) (17.50)	(28.30) (28.38) (16.89)	(27.93) (28.01) (16.75)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1509	1509	1509	1509
R_a^2	0.489	0.481	0.484	0.489
Panel B: Regression on β_V(same for 1, 2, 3, 4)				
c	0.02	0.02	0.02	0.02
	(0.24) (0.24) (0.17)	(0.24) (0.24) (0.17)	(0.24) (0.24) (0.17)	(0.24) (0.24) (0.17)
	[0.81] [0.81] [0.86]	[0.81] [0.81] [0.86]	[0.81] [0.81] [0.86]	[0.81] [0.81] [0.86]
σ_{amb}	-0.09	-0.09	-0.09	-0.09
	(-1.75) (-1.75) (-1.74)	(-1.75) (-1.75) (-1.74)	(-1.75) (-1.75) (-1.74)	(-1.75) (-1.75) (-1.74)
	[0.08] [0.08] [0.08]	[0.08] [0.08] [0.08]	[0.08] [0.08] [0.08]	[0.08] [0.08] [0.08]
$\triangle\sigma_{\text{p}}$	-0.06	-0.06	-0.06	-0.06
	(-1.44) (-1.44) (-1.77)	(-1.44) (-1.44) (-1.77)	(-1.44) (-1.44) (-1.77)	(-1.44) (-1.44) (-1.77)
	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]
$\beta_{V_{\text{t-1}}}$	0.02	0.02	0.02	0.02
	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)
	[0.48] [0.48] [0.62]	[0.48] [0.48] [0.62]	[0.48] [0.48] [0.62]	[0.48] [0.48] [0.62]
N	1509	1509	1509	1509
R_a^2	0.000	0.000	0.000	0.000

Table 9. Daily Regressions for Trading Volume

This table summarizes the daily regression results of the trading volume formula, as detailed in regression (2) of Section-4. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$V = \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\right)_{\alpha} + \left(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\right)_{\beta} * \Delta_P + V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
c	-0.11	-0.09	-0.08	-0.10
	(-2.46) (-2.47) (-2.15)	(-2.16) (-2.17) (-2.06)	(-1.84) (-1.85) (-2.02)	(-2.40) (-2.41) (-2.54)
	[0.01] [0.01] [0.03]	[0.03] [0.03] [0.04]	[0.07] [0.07] [0.04]	[0.02] [0.02] [0.01]
α_{pbel}	0.11	0.08	0.07	-0.13
	(6.21) (6.23) (5.48)	(4.13) (4.15) (3.64)	(3.50) (3.51) (3.85)	(-7.39) (-7.42) (-7.70)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.10	0.10	0.10	0.10
	(5.02) (5.04) (3.78)	(5.17) (5.19) (3.94)	(5.11) (5.13) (3.92)	(5.30) (5.32) (4.10)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
β_{p}	-0.33	-0.33	-0.33	-0.33
	(-14.87) (-14.93) (-8.32)	(-15.04) (-15.10) (-8.48)	(-14.68) (-14.74) (-8.16)	(-15.32) (-15.38) (-8.71)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\sigma_{\text{amb}}}$	-0.05	-0.05	-0.05	-0.05
	(-1.96) (-1.96) (-1.30)	(-2.05) (-2.06) (-1.37)	(-2.11) (-2.12) (-1.40)	(-2.15) (-2.16) (-1.44)
	[0.05] [0.05] [0.20]	[0.04] [0.04] [0.17]	[0.03] [0.03] [0.16]	[0.03] [0.03] [0.15]
$\beta_{\sigma_{\text{p}}}$	0.10	0.10	0.09	0.10
	(4.18) (4.19) (2.37)	(4.08) (4.10) (2.29)	(3.94) (3.96) (2.18)	(4.25) (4.27) (2.41)
	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.03]	[0.00] [0.00] [0.02]
$\beta_{V_{\text{t-1}}}$	0.61	0.61	0.61	0.60
	(32.09) (32.22) (17.38)	(32.01) (32.14) (17.61)	(32.00) (32.13) (16.97)	(31.68) (31.81) (16.91)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1510	1510	1510	1510
R_a^2	0.538	0.532	0.53	0.543

Table 10. Daily Regressions for ΔTrading Volume

This table summarizes the daily regression results of the first difference trading volume formula, as detailed in regression (3) of Section-4. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\Delta V = c \ + \ \big(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\big)_{\Delta\alpha} \ + \ \big(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\big)_{\Delta\beta} * \Delta_P \ + \big(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p * \Delta_P^2} * \sigma_P\big)_{\beta} * \Delta_P^2 \ + \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
c	-0.01	-0.01	-0.01	-0.01
	(-0.15) (-0.15) (-0.27)	(-0.15) (-0.15) (-0.28)	(-0.16) (-0.16) (-0.29)	(-0.16) (-0.16) (-0.29)
	[0.88] [0.88] [0.78]	[0.88] [0.88] [0.78]	[0.88] [0.88] [0.77]	[0.87] [0.87] [0.77]
α_{pbel}	0.13	0.08	0.08	-0.15
	(5.46) (5.49) (4.59)	(3.53) (3.55) (2.93)	(3.18) (3.20) (3.43)	(-6.27) (-6.29) (-6.65)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.12	0.12	0.12	0.12
	(5.11) (5.14) (3.59)	(5.21) (5.24) (3.62)	(5.10) (5.12) (3.53)	(5.23) (5.25) (3.62)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
β_{p}	-0.23	-0.23	-0.22	-0.23
	(-7.70) (-7.74) (-4.89)	(-7.56) (-7.59) (-4.81)	(-7.28) (-7.32) (-4.64)	(-7.78) (-7.81) (-4.95)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\sigma_{\text{amb}}}$	-0.05	-0.05	-0.05	-0.05
	(-1.52) (-1.53) (-1.17)	(-1.59) (-1.60) (-1.21)	(-1.65) (-1.65) (-1.24)	(-1.65) (-1.65) (-1.24)
	[0.13] [0.13] [0.24]	[0.11] [0.11] [0.23]	[0.10] [0.10] [0.21]	[0.10] [0.10] [0.21]
$\beta_{\triangle\sigma_{\text{amb}}}$	-0.02	-0.02	-0.02	-0.02
	(-0.75) (-0.75) (-0.26)	(-0.73) (-0.74) (-0.25)	(-0.75) (-0.76) (-0.26)	(-0.76) (-0.76) (-0.26)
	[0.45] [0.45] [0.79]	[0.46] [0.46] [0.80]	[0.45] [0.45] [0.79]	[0.45] [0.45] [0.80]
$\beta_{\sigma_{\text{p}}*\triangle^2\text{p}}$	0.11	0.11	0.11	0.11
	(3.68) (3.69) (2.63)	(3.49) (3.51) (2.47)	(3.40) (3.42) (2.37)	(3.55) (3.56) (2.44)
	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.01]
$\beta_{\triangle\sigma_{\text{p}}}$	-0.04	-0.04	-0.03	-0.04
	(-1.57) (-1.58) (-0.48)	(-1.47) (-1.48) (-0.45)	(-1.40) (-1.40) (-0.42)	(-1.43) (-1.44) (-0.42)
	[0.12] [0.12] [0.63]	[0.14] [0.14] [0.65]	[0.16] [0.16] [0.67]	[0.15] [0.15] [0.67]
$\beta_{V_{\text{t-1}}}$	-0.29	-0.30	-0.30	-0.30
	(-11.84) (-11.90) (-8.61)	(-12.13) (-12.19) (-8.93)	(-12.07) (-12.12) (-9.17)	(-12.10) (-12.15) (-9.06)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1509	1509	1509	1509
R_a^2	0.176	0.167	0.166	0.181

Table 11. Daily Regressions for Trading Volume Elasticity ξ

This table summarizes the daily regression of the elasticity model, as detailed in regression (1) of Section-5. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}.

$$\Delta log(V) = c + \Big(\xi_1 + \xi_\mu * \mu_{\Delta p} + \xi_{pbel} * PBEL + \xi_{amb} * AMB + \xi_{p_1} * P_1 + \xi_{\mu^2} * \mu^2_{\Delta p} + \xi_{pbel^2} * PBEL^2 + \xi_{AMB^2} * AMB^2 + \xi_{p_1{}^2} * P_1^2 + \xi_{\sigma_{amb}} * \sigma_{amb} + \xi_{\sigma_{AMB^2}} * \sigma^2_{amb} \Big)_\xi * \Delta log(\sigma_{\Delta p}) + \Delta log(V)_{t-1} + \sum_{p=t+1}^{T} \gamma_p$$

	D(1)	D(2)	D(3)	D(4)
c	-0.01	-0.01	-0.01	-0.01
	(-0.37) (-0.37) (-0.62)	(-0.39) (-0.39) (-0.64)	(-0.37) (-0.38) (-0.62)	(-0.44) (-0.44) (-0.73)
	[0.71] [0.71] [0.54]	[0.70] [0.70] [0.52]	[0.71] [0.71] [0.53]	[0.66] [0.66] [0.46]
ξ_1	13.59	13.86	13.49	14.27
	(9.89) (9.95) (8.13)	(9.93) (9.99) (8.09)	(9.96) (10.02) (8.22)	(10.12) (10.18) (8.78)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
ξ_μ	-0.01	-0.01	-0.01	-0.00
	(-0.37) (-0.37) (-0.30)	(-0.46) (-0.46) (-0.36)	(-0.76) (-0.77) (-0.61)	(-0.00) (-0.00) (-0.00)
	[0.71] [0.71] [0.76]	[0.65] [0.64] [0.72]	[0.44] [0.44] [0.54]	[1.00] [1.00] [1.00]
ξ_{pbel}	-0.02	-0.04	-0.38	0.05
	(-0.36) (-0.36) (-0.31)	(-0.87) (-0.87) (-0.78)	(-1.73) (-1.74) (-1.45)	(1.37) (1.38) (0.99)
	[0.72] [0.72] [0.76]	[0.39] [0.38] [0.43]	[0.08] [0.08] [0.15]	[0.17] [0.17] [0.32]
ξ_{amb}	-0.08	-0.08	-0.08	-0.07
	(-3.34) (-3.36) (-3.34)	(-3.23) (-3.25) (-2.98)	(-3.32) (-3.34) (-3.41)	(-2.88) (-2.90) (-2.95)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
ξ_{p_1}	0.02	0.03	0.04	0.02
	(1.24) (1.25) (0.96)	(1.34) (1.34) (0.95)	(1.79) (1.80) (1.30)	(1.28) (1.29) (0.86)
	[0.21] [0.21] [0.34]	[0.18] [0.18] [0.34]	[0.07] [0.07] [0.19]	[0.20] [0.20] [0.39]
ξ_{μ^2}	-0.04	-0.03	-0.04	-0.03
	(-3.02) (-3.04) (-2.68)	(-2.97) (-2.99) (-2.24)	(-3.34) (-3.36) (-2.97)	(-2.29) (-2.31) (-1.94)
	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.03]	[0.00] [0.00] [0.00]	[0.02] [0.02] [0.05]
ξ_{pbel^2}	0.03	0.04	0.38	-0.04
	(0.46) (0.46) (0.42)	(0.80) (0.81) (0.76)	(1.76) (1.77) (1.47)	(-0.99) (-0.99) (-0.75)
	[0.65] [0.64] [0.67]	[0.42] [0.42] [0.45]	[0.08] [0.08] [0.14]	[0.32] [0.32] [0.45]
ξ_{amb^2}	0.07	0.07	0.07	0.06
	(3.46) (3.48) (3.67)	(3.39) (3.41) (3.30)	(3.44) (3.47) (3.79)	(3.07) (3.09) (3.23)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\xi_{p_1^2}$	0.04	0.04	0.04	0.03
	(1.92) (1.93) (1.65)	(1.86) (1.87) (1.59)	(2.31) (2.32) (2.14)	(1.70) (1.71) (1.49)
	[0.06] [0.05] [0.10]	[0.06] [0.06] [0.11]	[0.02] [0.02] [0.03]	[0.09] [0.09] [0.14]
$\xi_{\sigma_{amb}}$	-0.11	-0.10	-0.08	-0.11
	(-1.75) (-1.76) (-1.56)	(-1.54) (-1.55) (-1.37)	(-1.22) (-1.22) (-1.02)	(-1.77) (-1.78) (-1.52)
	[0.08] [0.08] [0.12]	[0.12] [0.12] [0.17]	[0.22] [0.22] [0.31]	[0.08] [0.08] [0.13]
$\xi_{\sigma^2_{amb}}$	0.11	0.09	0.08	0.10
	(1.65) (1.66) (1.53)	(1.43) (1.44) (1.34)	(1.16) (1.17) (1.04)	(1.61) (1.62) (1.48)
	[0.10] [0.10] [0.13]	[0.15] [0.15] [0.18]	[0.25] [0.24] [0.30]	[0.11] [0.10] [0.14]
N	1510	1510	1510	1510
R^2_a	0.204	0.204	0.206	0.206

Appendix E Model Daily Regressions Results with Volatility Controls

In this appendix, I present the daily frequency regressions for the trading volume model outlined in Section-2.6, along with the elasticity model detailed in Section-2.7.

Tables 8, 13, and 14 below present the regressions (1), (2), and (3) results of the trading volume model as discussed in Section-4.

$$V = \alpha_V + \beta_V * \Delta_P + \epsilon \tag{1}$$

$$\alpha_V = c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb} + \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P + \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$V = \big(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb}\big)_{\alpha} + \big(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P\big)_{\beta} * \Delta_P + \varsigma_{t-1} * V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon \tag{4.2}$$

$$\Delta V = c + \big(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB + \alpha_{amb\sigma} * \Delta AMB * \sigma_{amb}\big)_{\Delta\alpha} + \big(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{h\Delta\sigma_{amb}} * \Delta\sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\big)_{\Delta\beta} * \Delta_P + \big(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\big)_{\beta} * \Delta_P^2 + \varsigma_{t-1} * \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon \tag{4.3}$$

The outcomes of regression (1) in Section-4 for the elasticity model are illustrated in table-15 below.

$$\Delta log(V) = c + \big(\, \xi_1 + \xi_\mu * \mu_{\Delta p} + \xi_{pbel} * PBEL + \xi_{amb} * AMB + \xi_{\mu^2} * \mu_{\Delta p}^2 + \xi_{pbel^2} * PBEL^2 + \xi_{amb^2} * AMB^2 + \xi_{\sigma_{amb}} * \sigma_{amb} + \xi_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \xi_{\sigma_{amb}^2} * \sigma_{amb}^2 + \xi_{h\sigma_{amb}^2} * \sigma_{amb}^2 * I_{h\sigma_{amb}} \,\big)_{\xi} * \Delta log(\sigma_{\Delta p}) + \varsigma_{t-1} * \Delta log(V)_{t-1} + \sum_{p=t+1}^{T} \gamma_p \tag{5.2}$$

The regressions cover the daily period from 2013 to 2018 for the SPY and were conducted on four distinct datasets: D(1), D(2), D(3) and D(4) used in the previous appendix. The dummy $I_{h\sigma_{amb}}$ marks days with high levels of ambiguity above the 50% quantile of the sample.

Table 12. Daily Regressions α_V and β_V

This table summarizes the daily regression results for the coefficients α_V and β_V of the trading volume formula, as detailed in regression (1) of Section-4. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\alpha_V = c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb} + \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P + \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
Panel A: Regression on α_V				
c	0.13	0.14	0.17	0.13
	(2.79) (2.79) (2.56)	(2.97) (2.98) (3.07)	(3.53) (3.54) (4.00)	(2.91) (2.92) (3.09)
	[0.01] [0.01] [0.01]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{pbel}	0.13	0.08	0.10	-0.12
	(6.80) (6.82) (5.55)	(4.36) (4.37) (3.63)	(4.96) (4.97) (5.73)	(-6.31) (-6.33) (-6.94)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.16	0.16	0.15	0.15
	(6.77) (6.79) (4.12)	(6.90) (6.93) (4.21)	(6.58) (6.60) (4.03)	(6.69) (6.72) (4.08)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.10	-0.10	-0.09	-0.09
	(-4.81) (-4.83) (-3.83)	(-4.72) (-4.73) (-3.80)	(-4.35) (-4.36) (-3.53)	(-4.13) (-4.14) (-3.44)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{t-1}	0.56	0.56	0.56	0.56
	(27.18) (27.27) (17.60)	(27.18) (27.27) (17.49)	(27.26) (27.35) (16.75)	(26.97) (27.06) (16.68)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1509	1509	1509	1509
R_a^2	0.497	0.488	0.49	0.495
Panel B: Regression on β_V (same for 1, 2, 3, 4)				
c	0.05	0.05	0.05	0.05
	(0.54) (0.55) (0.41)	(0.54) (0.55) (0.41)	(0.54) (0.55) (0.41)	(0.54) (0.55) (0.41)
	[0.59] [0.59] [0.68]	[0.59] [0.59] [0.68]	[0.59] [0.59] [0.68]	[0.59] [0.59] [0.68]
$\beta_{\sigma_{amb}}$	-0.02	-0.02	-0.02	-0.02
	(-0.16) (-0.16) (-0.18)	(-0.16) (-0.16) (-0.18)	(-0.16) (-0.16) (-0.18)	(-0.16) (-0.16) (-0.18)
	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]
$\beta_{h\sigma_{amb}}$	-0.10	-0.10	-0.10	-0.10
	(-0.75) (-0.75) (-0.75)	(-0.75) (-0.75) (-0.75)	(-0.75) (-0.75) (-0.75)	(-0.75) (-0.75) (-0.75)
	[0.46] [0.45] [0.45]	[0.46] [0.45] [0.45]	[0.46] [0.45] [0.45]	[0.46] [0.45] [0.45]
β_{σ_p}	-0.06	-0.06	-0.06	-0.06
	(-1.44) (-1.44) (-1.78)	(-1.44) (-1.44) (-1.78)	(-1.44) (-1.44) (-1.78)	(-1.44) (-1.44) (-1.78)
	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]
$\beta_{\text{t-1}}$	0.02	0.02	0.02	0.02
	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)
	[0.49] [0.48] [0.62]	[0.49] [0.48] [0.62]	[0.49] [0.48] [0.62]	[0.49] [0.48] [0.62]
N	1509	1509	1509	1509
R_a^2	0.000	0.000	0.000	0.000

Table 13. Daily Regressions for Trading Volume

This table summarizes the daily regression results of the trading volume formula, as detailed in regression (2) of Section-4. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$V = \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb}\right)_{\alpha} + \\ \left(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P\right)_{\beta} * \Delta_P + \varsigma_{t-1} * V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
c	-0.13	-0.12	-0.10	-0.12
	(-2.96) (-2.97) (-2.77)	(-2.64) (-2.65) (-2.65)	(-2.31) (-2.32) (-2.62)	(-2.78) (-2.79) (-3.06)
	[0.00] [0.00] [0.01]	[0.01] [0.01] [0.01]	[0.02] [0.02] [0.01]	[0.01] [0.01] [0.00]
α_{pbel}	0.11	0.08	0.06	-0.13
	(6.29) (6.32) (5.58)	(4.22) (4.24) (3.78)	(3.33) (3.34) (3.69)	(-7.13) (-7.17) (-7.54)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.13	0.14	0.13	0.13
	(6.03) (6.06) (4.23)	(6.14) (6.17) (4.31)	(5.95) (5.98) (4.15)	(5.92) (5.95) (4.18)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.07	-0.07	-0.06	-0.06
	(-3.45) (-3.47) (-2.68)	(-3.40) (-3.41) (-2.65)	(-3.11) (-3.12) (-2.44)	(-2.70) (-2.71) (-2.20)
	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.01] [0.01] [0.03]
β_{p}	-0.34	-0.35	-0.34	-0.35
	(-10.42) (-10.47) (-5.95)	(-10.56) (-10.61) (-6.07)	(-10.32) (-10.37) (-5.78)	(-10.70) (-10.75) (-6.12)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\sigma_{\text{amb}}}$	-0.07	-0.07	-0.08	-0.07
	(-1.91) (-1.92) (-1.34)	(-2.00) (-2.01) (-1.41)	(-2.01) (-2.02) (-1.38)	(-1.99) (-2.00) (-1.40)
	[0.06] [0.05] [0.18]	[0.05] [0.04] [0.16]	[0.04] [0.04] [0.17]	[0.05] [0.05] [0.16]
$\beta_{h\sigma_{amb}}$	0.07	0.07	0.07	0.06
	(0.92) (0.92) (0.55)	(0.95) (0.95) (0.58)	(0.91) (0.92) (0.54)	(0.85) (0.85) (0.52)
	[0.36] [0.36] [0.58]	[0.34] [0.34] [0.56]	[0.36] [0.36] [0.59]	[0.40] [0.40] [0.61]
$\beta_{\sigma_{\text{p}}}$	0.10	0.10	0.09	0.10
	(4.16) (4.18) (2.43)	(4.06) (4.08) (2.34)	(3.92) (3.94) (2.22)	(4.20) (4.22) (2.45)
	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.03]	[0.00] [0.00] [0.01]
$\varsigma t-1$	0.60	0.60	0.60	0.59
	(31.13) (31.27) (17.41)	(31.05) (31.20) (17.58)	(31.12) (31.27) (16.86)	(30.91) (31.05) (16.83)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1510	1510	1510	1510
R_a^2	0.542	0.535	0.533	0.545

Table 14. Daily Regressions for ΔTrading Volume

This table summarizes the daily regression results of the first difference trading volume formula, as detailed in regression (3) of Section-4. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\Delta V = c \ + \ \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB + \alpha_{amb\sigma} * \Delta AMB * \sigma_{amb}\right)_{\Delta\alpha} + \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{h\Delta\sigma_{amb}} * \Delta\sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P \ + \ \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p \Delta_P^2} * \sigma_P\right)_{\beta} * \Delta_P^2 \ + \varsigma_{t-1} * \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
c	0.01	0.01	0.01	0.01
	(0.09) (0.09) (0.17)	(0.10) (0.10) (0.18)	(0.09) (0.09) (0.16)	(0.09) (0.09) (0.16)
	[0.93] [0.92] [0.86]	[0.92] [0.92] [0.86]	[0.93] [0.93] [0.87]	[0.93] [0.93] [0.87]
α_{pbel}	0.12	0.08	0.07	-0.15
	(5.28) (5.31) (4.63)	(3.37) (3.39) (2.89)	(3.06) (3.07) (3.28)	(-6.29) (-6.32) (-6.73)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.13	0.13	0.13	0.13
	(5.17) (5.20) (3.77)	(5.23) (5.26) (3.78)	(5.15) (5.18) (3.74)	(5.28) (5.31) (3.86)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.00	-0.00	-0.00	-0.00
	(-0.16) (-0.16) (-0.10)	(-0.04) (-0.04) (-0.03)	(-0.14) (-0.14) (-0.09)	(-0.14) (-0.14) (-0.09)
	[0.87] [0.87] [0.92]	[0.96] [0.96] [0.98]	[0.89] [0.89] [0.93]	[0.89] [0.89] [0.93]
β_{p}	-0.22	-0.23	-0.22	-0.24
	(-4.96) (-4.99) (-3.32)	(-5.02) (-5.05) (-3.36)	(-4.83) (-4.86) (-3.26)	(-5.28) (-5.31) (-3.54)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\sigma_{\text{amb}}}$	-0.04	-0.05	-0.05	-0.06
	(-0.77) (-0.78) (-0.69)	(-0.97) (-0.97) (-0.85)	(-1.00) (-1.00) (-0.86)	(-1.12) (-1.12) (-0.99)
	[0.44] [0.44] [0.49]	[0.33] [0.33] [0.40]	[0.32] [0.32] [0.39]	[0.26] [0.26] [0.32]
$\beta_{h\sigma_{amb}}$	0.02	0.04	0.04	0.05
	(0.20) (0.20) (0.15)	(0.41) (0.41) (0.31)	(0.40) (0.40) (0.30)	(0.56) (0.56) (0.42)
	[0.84] [0.84] [0.88]	[0.68] [0.68] [0.76]	[0.69] [0.69] [0.76]	[0.58] [0.57] [0.67]
$\beta_{\triangle\sigma_{\text{amb}}}$	0.11	0.11	0.11	0.11
	(3.25) (3.27) (2.72)	(3.29) (3.30) (2.73)	(3.30) (3.32) (2.72)	(3.36) (3.37) (2.64)
	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]
$\beta_{h\triangle\sigma_{amb}}$	-0.27	-0.28	-0.28	-0.28
	(-5.69) (-5.73) (-4.24)	(-5.74) (-5.77) (-4.28)	(-5.78) (-5.81) (-4.21)	(-5.86) (-5.89) (-4.19)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\sigma_{\text{p}}\triangle^2{\text{p}}}$	0.13	0.12	0.12	0.12
	(4.02) (4.04) (2.99)	(3.81) (3.84) (2.81)	(3.73) (3.76) (2.70)	(3.86) (3.89) (2.77)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]
$\beta_{\triangle\sigma_{\text{p}}}$	-0.06	-0.06	-0.06	-0.06
	(-2.41) (-2.43) (-0.98)	(-2.34) (-2.35) (-0.96)	(-2.26) (-2.27) (-0.91)	(-2.30) (-2.32) (-0.91)
	[0.02] [0.02] [0.33]	[0.02] [0.02] [0.34]	[0.02] [0.02] [0.36]	[0.02] [0.02] [0.36]
$\varsigma t-1$	-0.31	-0.31	-0.31	-0.31
	(-12.51) (-12.58) (-9.66)	(-12.79) (-12.87) (-9.94)	(-12.74) (-12.81) (-10.20)	(-12.78) (-12.85) (-10.00)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1509	1509	1509	1509
R_a^2	0.192	0.184	0.182	0.199

Table 15. Daily Regressions for Trading Volume Elasticity ξ

This table summarizes the daily regression of the elasticity model, as detailed in regression (1) of Section-5. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}.

$$\Delta log(V) = c \ + \ \Big(\ \xi_1 \ + \ \xi_\mu * \mu_{\Delta p} \ + \ \xi_{pbel} * PBEL \ + \ \xi_{amb} * AMB \ + \ \xi_{\mu^2} * \mu_{\Delta p}^2 \ + \ \xi_{pbel^2} * PBEL^2 \ + \ \xi_{amb^2} * AMB^2 \ + \ \xi_{\sigma_{amb}} * \sigma_{amb} \ + \ \xi_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} \ + \ \xi_{\sigma_{amb}^2} * \sigma_{amb}^2 \ + \ \xi_{h\sigma_{amb}^2} * \sigma_{amb}^2 * I_{h\sigma_{amb}} \ \Big)_\xi \ * \ \Delta log(\sigma_{\Delta p}) + \ \varsigma_{t-1} * \Delta log(V)_{t-1} + \ \sum_{p=t+1}^{T} \gamma_p$$

	D(1)	D(2)	D(3)	D(4)
c	-0.02	-0.02	-0.02	-0.02
	(-1.04) (-1.04) (-1.56)	(-1.07) (-1.08) (-1.60)	(-1.02) (-1.03) (-1.52)	(-1.08) (-1.09) (-1.64)
	[0.30] [0.30] [0.12]	[0.28] [0.28] [0.11]	[0.31] [0.30] [0.13]	[0.28] [0.28] [0.10]
ξ_1	30.65	31.47	30.22	30.50
	(5.43) (5.47) (5.26)	(5.39) (5.42) (5.27)	(5.47) (5.51) (4.60)	(5.61) (5.64) (5.29)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
ξ_μ	-0.00	-0.00	-0.00	0.01
	(-0.06) (-0.06) (-0.07)	(-0.13) (-0.13) (-0.13)	(-0.27) (-0.27) (-0.27)	(0.73) (0.74) (0.76)
	[0.95] [0.95] [0.95]	[0.90] [0.89] [0.89]	[0.79] [0.79] [0.79]	[0.46] [0.46] [0.45]
ξ_{pbel}	-0.02	-0.02	-0.20	0.06
	(-0.37) (-0.37) (-0.36)	(-0.50) (-0.50) (-0.49)	(-0.95) (-0.95) (-0.73)	(1.53) (1.54) (1.38)
	[0.71] [0.71] [0.72]	[0.62] [0.62] [0.62]	[0.34] [0.34] [0.46]	[0.13] [0.12] [0.17]
ξ_{amb}	-0.08	-0.08	-0.08	-0.07
	(-3.31) (-3.33) (-3.39)	(-3.20) (-3.22) (-3.19)	(-3.27) (-3.29) (-3.44)	(-2.92) (-2.94) (-2.94)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
ξ_{μ^2}	-0.02	-0.02	-0.02	-0.01
	(-1.73) (-1.74) (-1.84)	(-1.84) (-1.85) (-1.62)	(-1.89) (-1.91) (-1.78)	(-1.14) (-1.15) (-1.12)
	[0.08] [0.08] [0.07]	[0.07] [0.06] [0.11]	[0.06] [0.06] [0.08]	[0.25] [0.25] [0.26]
ξ_{pbel^2}	0.02	0.02	0.20	-0.05
	(0.36) (0.37) (0.36)	(0.37) (0.37) (0.36)	(0.95) (0.96) (0.74)	(-1.18) (-1.18) (-1.07)
	[0.72] [0.71] [0.72]	[0.71] [0.71] [0.72]	[0.34] [0.34] [0.46]	[0.24] [0.24] [0.29]
ξ_{amb^2}	0.07	0.07	0.07	0.07
	(3.72) (3.75) (3.74)	(3.69) (3.71) (3.58)	(3.71) (3.73) (3.89)	(3.35) (3.37) (3.26)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\xi_{\sigma_{amb}}$	-0.73	-0.74	-0.71	-0.71
	(-3.56) (-3.58) (-3.48)	(-3.55) (-3.57) (-3.49)	(-3.45) (-3.47) (-2.95)	(-3.56) (-3.59) (-3.50)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\xi_{h\sigma_{amb}}$	-0.24	-0.24	-0.24	-0.23
	(-2.12) (-2.14) (-2.12)	(-2.16) (-2.17) (-2.22)	(-2.12) (-2.13) (-1.97)	(-2.10) (-2.12) (-2.22)
	[0.03] [0.03] [0.03]	[0.03] [0.03] [0.03]	[0.03] [0.03] [0.05]	[0.04] [0.03] [0.03]
$\xi_{\sigma^2_{amb}}$	0.74	0.76	0.72	0.72
	(3.54) (3.56) (3.44)	(3.52) (3.54) (3.45)	(3.41) (3.43) (2.91)	(3.51) (3.54) (3.37)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\xi_{h\sigma^2_{amb}}$	0.04	0.04	0.05	0.04
	(0.55) (0.55) (0.50)	(0.55) (0.56) (0.52)	(0.58) (0.59) (0.54)	(0.55) (0.56) (0.53)
	[0.58] [0.58] [0.62]	[0.58] [0.58] [0.60]	[0.56] [0.56] [0.59]	[0.58] [0.58] [0.60]
$\varsigma t-1$	-0.38	-0.38	-0.39	-0.38
	(-16.73) (-16.83) (-18.74)	(-16.71) (-16.82) (-18.76)	(-16.77) (-16.87) (-18.74)	(-16.62) (-16.72) (-18.51)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1510	1510	1510	1510
R^2_a	0.208	0.208	0.208	0.21

Appendix F Model Monthly Regressions Results

In this appendix, I present the monthly frequency regressions for the trading volume model outlined in Section-2.6. These regressions cover the monthly periods 2013-2018 and 2000-2020.

Tables 16, 17, and 18 below present the regressions (1), (2) and (3) results of the trading volume model as discussed in Section-4.

$$\begin{aligned} V &= \alpha_V + \beta_V * \Delta_P + \epsilon \\ \alpha_V &= 1 + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \epsilon \\ \beta_V &= 1 + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_p * \sigma_P + \epsilon \end{aligned} \tag{4.1}$$

$$V = \big(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\big)_\alpha \; + \; \big(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\big)_\beta * \Delta_P \; + V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon \tag{4.2}$$

$$\Delta V = c \; + \; \big(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\big)_{\Delta\alpha} \; + \; \big(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\big)_{\Delta\beta} * \Delta_P \; + \; \big(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p * \Delta_P^2} * \sigma_P\big)_\beta * \Delta_P^2 \; + \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon \tag{4.3}$$

The regressions using datasets D(1) to D(8) cover the monthly period from 2013 to 2018 for the SPY. The regressions using the datasets D(9) to D(10) cover the monthly period from 2000 to 2020. Datasets (1) ,(2), (3), (4) and (9) employ the monthly EPU measure extracted from newspapers as a proxy for Ambiguity, alongside a distinct proxy for differences in prior beliefs extracted from Stocktwits (Cookson & Niessner, 2020) and the IBES database. Datasets (4), (5), (6), (7) and (10) employ the monthly Ambiguity measure of Izhakian (2020).

Monthly Datasets Description	
D(1)	Prior beliefs differences proxied by $\text{PBEL}_{\text{WI, IND}}$ and Ambiguity proxied by AMB_{EPUM}
D(2)	Prior beliefs differences proxied by $\text{PBEL}_{\text{AC, IND}}$ and Ambiguity proxied by AMB_{EPUM}
D(3)	Prior beliefs differences proxied by $\text{PBEL}_{\text{WI, ETF}}$ and Ambiguity proxied by AMB_{EPUM}
D(4)	Prior beliefs differences proxied by $\text{PBEL}_{\text{AC, ETF}}$ and Ambiguity proxied by AMB_{EPUM}
D(5)	Prior beliefs differences proxied by $\text{PBEL}_{\text{WI, IND}}$ and Ambiguity proxied by AMB_{IZHM}
D(6)	Prior beliefs differences proxied by $\text{PBEL}_{\text{AC, IND}}$ and Ambiguity proxied by AMB_{IZHM}
D(7)	Prior beliefs differences proxied by $\text{PBEL}_{\text{WI, ETF}}$ and Ambiguity proxied by AMB_{IZHM}
D(8)	Prior beliefs differences proxied by $\text{PBEL}_{\text{AC, ETF}}$ and Ambiguity proxied by AMB_{IZHM}
D(9)	Prior beliefs differences proxied by $\text{PBEL}_{\text{IBES}}$ and Ambiguity proxied by AMB_{EPUM}
D(10)	Prior beliefs differences proxied by $\text{PBEL}_{\text{IBES}}$ and Ambiguity proxied by AMB_{IZHM}

Table 16. Monthly Regressions α_V and β_V

This table summarizes the monthly regression results for the coefficients α_V and β_V of the trading volume formula, as detailed in regression (1) of Section-4. The regressions were performed for the SPY ETF using ten different monthly data sets {D(1), D(2), D(3), D(4), D(6), D(7), D(8), D(9), D(10)}. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\alpha_V = c + \alpha_{amb} * AMB + \alpha_{pbel} * PBEL + \alpha_{V_{t-1}} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_{\sigma_p} * \sigma_P \ + \ \beta_{V_{t-1}} + \ \sum_{p=t+1}^{T} \gamma_p + \ \epsilon$$

	D(1)	D(2)	D(3)	D(4)	D(5)
Panel A: Regression on α_V					
c	-0.33	-0.38	-0.19	-0.34	-0.10
	(-2.57) (-2.74) (-5.21)	(-2.85) (-3.05) (-5.43)	(-1.29) (-1.38) (-1.68)	(-2.62) (-2.80) (-6.39)	(-1.04) (-1.11) (-1.20)
	[0.01] [0.01] [0.00]	[0.01] [0.00] [0.00]	[0.20] [0.17] [0.10]	[0.01] [0.01] [0.00]	[0.30] [0.27] [0.24]
α_{pbel}	-0.04	-0.08	0.11	-0.04	-0.06
	(-0.64) (-0.68) (-0.67)	(-1.35) (-1.45) (-1.64)	(1.52) (1.63) (1.30)	(-0.81) (-0.87) (-0.89)	(-1.42) (-1.52) (-1.61)
	[0.53] [0.50] [0.50]	[0.18] [0.15] [0.11]	[0.13] [0.11] [0.20]	[0.42] [0.39] [0.38]	[0.16] [0.13] [0.11]
α_{amb}	0.23	0.23	0.20	0.23	0.46
	(2.40) (2.57) (1.99)	(2.38) (2.54) (2.07)	(1.99) (2.13) (1.99)	(2.37) (2.53) (1.95)	(8.82) (9.43) (4.61)
	[0.02] [0.01] [0.05]	[0.02] [0.01] [0.04]	[0.05] [0.04] [0.05]	[0.02] [0.01] [0.06]	[0.00] [0.00] [0.00]
$\alpha V_{\text{t-1}}$	0.16	0.17	0.11	0.16	-0.20
	(1.25) (1.34) (1.31)	(1.31) (1.40) (1.31)	(0.84) (0.90) (0.78)	(1.25) (1.34) (1.28)	(-1.97) (-2.11) (-2.60)
	[0.22] [0.19] [0.19]	[0.20] [0.17] [0.19]	[0.40] [0.37] [0.44]	[0.21] [0.18] [0.21]	[0.05] [0.04] [0.01]
N	72	72	72	72	72
R_a^2	0.255	0.271	0.277	0.258	0.636
Panel B: Regression on β_V(same for 1, 2, 3, 4)					
c	0.05	0.05	0.05	0.05	0.14
	(0.16) (0.17) (0.17)	(0.16) (0.17) (0.17)	(0.16) (0.17) (0.17)	(0.16) (0.17) (0.17)	(0.38) (0.40) (0.45)
	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]	[0.71] [0.69] [0.65]
σ_{amb}	-0.15	-0.15	-0.15	-0.15	0.17
	(-0.60) (-0.64) (-0.43)	(-0.60) (-0.64) (-0.43)	(-0.60) (-0.64) (-0.43)	(-0.60) (-0.64) (-0.43)	(0.65) (0.69) (0.93)
	[0.55] [0.53] [0.67]	[0.55] [0.53] [0.67]	[0.55] [0.53] [0.67]	[0.55] [0.53] [0.67]	[0.52] [0.49] [0.35]
$\triangle\sigma_{\text{p}}$	0.09	0.09	0.09	0.09	0.17
	(0.30) (0.32) (0.42)	(0.30) (0.32) (0.42)	(0.30) (0.32) (0.42)	(0.30) (0.32) (0.42)	(0.52) (0.55) (0.90)
	[0.77] [0.75] [0.68]	[0.77] [0.75] [0.68]	[0.77] [0.75] [0.68]	[0.77] [0.75] [0.68]	[0.61] [0.58] [0.37]
$\beta V_{\text{t-1}}$	0.01	0.01	0.01	0.01	0.01
	(0.07) (0.08) (0.10)	(0.07) (0.08) (0.10)	(0.07) (0.08) (0.10)	(0.07) (0.08) (0.10)	(0.09) (0.10) (0.11)
	[0.94] [0.94] [0.92]	[0.94] [0.94] [0.92]	[0.94] [0.94] [0.92]	[0.94] [0.94] [0.92]	[0.93] [0.92] [0.91]
N	72	72	72	72	72
R_a^2	-0.033	-0.033	-0.033	-0.033	-0.032

Table 16. Monthly Regressions α_V and β_V (continuation)

This table summarizes the monthly regression results for the coefficients α_V and β_V of the trading volume formula, as detailed in regression (1) of Section-4. The regressions were performed for the SPY ETF using ten different monthly data sets {D(1), D(2), D(3), D(4), D(6), D(7), D(8), D(9), D(10)}. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\alpha_V = c + \alpha_{amb} * AMB + \alpha_{pbel} * PBEL + \alpha_{V_{t-1}} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_{\sigma_p} * \sigma_P \; + \; \beta_{V_{t-1}} + \; \sum_{p=t+1}^{T} \gamma_p + \; \epsilon$$

	D(6)	D(7)	D(8)	D(9)	D(10)
Panel A: Regression on α_V					
c	-0.13	-0.09	-0.11	-0.10	-1.08
	(-1.39) (-1.48) (-1.43)	(-0.84) (-0.89) (-0.88)	(-1.18) (-1.27) (-1.53)	(-0.57) (-0.69) (-0.79)	(-6.05) (-7.28) (-6.96)
	[0.17] [0.14] [0.16]	[0.41] [0.37] [0.38]	[0.24] [0.21] [0.13]	[0.57] [0.49] [0.43]	[0.00] [0.00] [0.00]
α_{pbel}	-0.08	-0.03	-0.06	-0.14	-0.10
	(-1.91) (-2.04) (-1.99)	(-0.54) (-0.58) (-0.42)	(-1.63) (-1.74) (-2.10)	(-3.38) (-3.30) (-3.34)	(-2.54) (-2.43) (-2.32)
	[0.06] [0.05] [0.05]	[0.59] [0.57] [0.68]	[0.11] [0.09] [0.04]	[0.00] [0.00] [0.00]	[0.01] [0.02] [0.02]
α_{amb}	0.46	0.47	0.46	0.37	0.58
	(8.82) (9.43) (4.68)	(8.16) (8.72) (4.98)	(8.85) (9.46) (4.75)	(6.04) (5.64) (3.03)	(9.33) (10.15) (5.19)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha V_{\text{t-1}}$	-0.19	-0.21	-0.20	0.39	0.26
	(-1.95) (-2.09) (-2.40)	(-2.02) (-2.16) (-2.70)	(-1.99) (-2.13) (-2.52)	(7.19) (5.65) (4.97)	(4.93) (3.57) (2.65)
	[0.06] [0.04] [0.02]	[0.05] [0.03] [0.01]	[0.05] [0.04] [0.01]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.01]
N	72	72	72	247	247
R_a^2	0.645	0.626	0.639	0.697	0.746
Panel B: Regression on β_V(same for 1, 2, 3, 4)					
c	0.14	0.14	0.14	0.41	0.68
	(0.38) (0.40) (0.45)	(0.38) (0.40) (0.45)	(0.38) (0.40) (0.45)	(1.30) (1.33) (2.54)	(1.73) (1.78) (2.61)
	[0.71] [0.69] [0.65]	[0.71] [0.69] [0.65]	[0.71] [0.69] [0.65]	[0.20] [0.18] [0.01]	[0.09] [0.08] [0.01]
σ_{amb}	0.17	0.17	0.17	0.20	0.31
	(0.65) (0.69) (0.93)	(0.65) (0.69) (0.93)	(0.65) (0.69) (0.93)	(1.51) (1.57) (0.99)	(1.63) (1.71) (1.54)
	[0.52] [0.49] [0.35]	[0.52] [0.49] [0.35]	[0.52] [0.49] [0.35]	[0.13] [0.12] [0.32]	[0.10] [0.09] [0.12]
$\triangle\sigma_{\text{p}}$	0.17	0.17	0.17	-0.22	-0.16
	(0.52) (0.55) (0.90)	(0.52) (0.55) (0.90)	(0.52) (0.55) (0.90)	(-1.49) (-1.59) (-0.56)	(-1.14) (-1.22) (-0.46)
	[0.61] [0.58] [0.37]	[0.61] [0.58] [0.37]	[0.61] [0.58] [0.37]	[0.14] [0.11] [0.58]	[0.26] [0.22] [0.65]
$\beta V_{\text{t-1}}$	0.01	0.01	0.01	0.00	0.01
	(0.09) (0.10) (0.11)	(0.09) (0.10) (0.11)	(0.09) (0.10) (0.11)	(0.07) (0.07) (0.10)	(0.16) (0.16) (0.22)
	[0.93] [0.92] [0.91]	[0.93] [0.92] [0.91]	[0.93] [0.92] [0.91]	[0.94] [0.94] [0.92]	[0.88] [0.87] [0.83]
N	72	72	72	247	247
R_a^2	-0.032	-0.032	-0.032	0.111	0.113

Table 17. Monthly Regressions for Trading Volume

This table summarizes the monthly regression results of the trading volume formula, as detailed in regression (2) of Section-4. The regressions were performed for the SPY ETF using ten different data sets {D(1), D(2), D(3), D(4), D(6), D(7), D(8), D(9), D(10)}.

$$V = \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\right)_{\alpha} + \left(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\right)_{\beta} * \Delta_P + V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)	D(5)
c	-0.23	-0.25	-0.16	-0.29	-0.12
	(-2.13) (-2.33) (-3.37)	(-2.20) (-2.41) (-3.37)	(-1.32) (-1.44) (-1.16)	(-2.65) (-2.90) (-4.47)	(-1.27) (-1.39) (-1.33)
	[0.04] [0.02] [0.00]	[0.03] [0.02] [0.00]	[0.19] [0.15] [0.25]	[0.01] [0.01] [0.00]	[0.21] [0.17] [0.19]
α_{pbel}	0.02	-0.00	0.08	-0.05	-0.01
	(0.51) (0.56) (0.55)	(-0.10) (-0.11) (-0.13)	(1.27) (1.39) (0.69)	(-1.13) (-1.24) (-0.86)	(-0.32) (-0.35) (-0.36)
	[0.61] [0.58] [0.58]	[0.92] [0.92] [0.90]	[0.21] [0.17] [0.49]	[0.26] [0.22] [0.40]	[0.75] [0.73] [0.72]
α_{amb}	0.22	0.21	0.19	0.20	0.35
	(2.64) (2.89) (3.87)	(2.57) (2.82) (3.75)	(2.31) (2.53) (5.48)	(2.41) (2.64) (3.14)	(5.71) (6.26) (3.18)
	[0.01] [0.01] [0.00]	[0.01] [0.01] [0.00]	[0.02] [0.01] [0.00]	[0.02] [0.01] [0.00]	[0.00] [0.00] [0.00]
β_{p}	-0.27	-0.26	-0.26	-0.25	-0.09
	(-4.15) (-4.55) (-6.05)	(-3.93) (-4.31) (-5.86)	(-4.16) (-4.56) (-6.54)	(-3.86) (-4.23) (-6.16)	(-1.31) (-1.43) (-1.20)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.20] [0.16] [0.23]
$\beta_{\sigma_{\text{amb}}}$	0.05	0.07	0.05	0.12	-0.03
	(0.24) (0.26) (0.39)	(0.31) (0.34) (0.53)	(0.23) (0.25) (0.38)	(0.58) (0.64) (1.00)	(-0.50) (-0.55) (-0.63)
	[0.81] [0.79] [0.70]	[0.75] [0.73] [0.59]	[0.82] [0.80] [0.71]	[0.56] [0.53] [0.32]	[0.62] [0.59] [0.53]
$\beta_{\sigma_{\text{p}}}$	0.09	0.08	0.11	0.08	-0.01
	(0.62) (0.68) (1.04)	(0.60) (0.65) (1.05)	(0.78) (0.86) (1.31)	(0.57) (0.62) (1.00)	(-0.12) (-0.13) (-0.22)
	[0.54] [0.50] [0.30]	[0.55] [0.52] [0.30]	[0.44] [0.40] [0.19]	[0.57] [0.54] [0.32]	[0.91] [0.90] [0.83]
$\beta_{V_{\text{t-1}}}$	0.15	0.16	0.13	0.16	-0.16
	(1.43) (1.57) (1.17)	(1.49) (1.63) (1.23)	(1.23) (1.34) (0.96)	(1.55) (1.70) (1.25)	(-1.48) (-1.62) (-1.41)
	[0.16] [0.12] [0.25]	[0.14] [0.11] [0.22]	[0.23] [0.18] [0.34]	[0.13] [0.09] [0.22]	[0.14] [0.11] [0.16]
N	72	72	72	72	72
R_a^2	0.493	0.49	0.504	0.501	0.629
	D(6)	D(7)	D(8)	D(9)	D(10)
c	-0.15	-0.12	-0.18	-0.13	-0.97
	(-1.50) (-1.64) (-1.42)	(-1.18) (-1.29) (-1.05)	(-1.88) (-2.06) (-2.03)	(-0.87) (-1.02) (-1.06)	(-5.53) (-6.55) (-5.56)
	[0.14] [0.11] [0.16]	[0.24] [0.20] [0.30]	[0.07] [0.04] [0.05]	[0.39] [0.31] [0.29]	[0.00] [0.00] [0.00]
α_{pbel}	-0.04	-0.01	-0.07	-0.09	-0.08
	(-0.92) (-1.00) (-0.98)	(-0.21) (-0.23) (-0.17)	(-2.03) (-2.22) (-2.89)	(-2.38) (-2.38) (-3.64)	(-2.07) (-2.03) (-3.24)
	[0.36] [0.32] [0.33]	[0.83] [0.82] [0.86]	[0.05] [0.03] [0.01]	[0.02] [0.02] [0.00]	[0.04] [0.04] [0.00]
α_{amb}	0.36	0.36	0.36	0.36	0.48
	(5.82) (6.37) (3.27)	(5.24) (5.74) (3.65)	(6.04) (6.61) (3.09)	(6.30) (6.07) (3.26)	(7.25) (8.09) (4.15)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
β_{p}	-0.08	-0.09	-0.08	-0.30	-0.19
	(-1.17) (-1.28) (-1.07)	(-1.37) (-1.50) (-1.23)	(-1.16) (-1.27) (-0.99)	(-5.42) (-5.85) (-4.22)	(-3.14) (-3.16) (-2.10)
	[0.25] [0.20] [0.29]	[0.18] [0.14] [0.22]	[0.25] [0.21] [0.33]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.04]
$\beta_{\sigma_{\text{amb}}}$	-0.04	-0.03	-0.05	-0.10	0.03
	(-0.53) (-0.58) (-0.66)	(-0.45) (-0.49) (-0.61)	(-0.71) (-0.78) (-0.88)	(-1.69) (-1.63) (-1.39)	(0.61) (0.54) (0.39)
	[0.60] [0.56] [0.51]	[0.66] [0.63] [0.54]	[0.48] [0.44] [0.38]	[0.09] [0.10] [0.17]	[0.54] [0.59] [0.70]
$\beta_{\sigma_{\text{p}}}$	-0.01	-0.02	-0.00	0.10	-0.04
	(-0.10) (-0.10) (-0.17)	(-0.17) (-0.19) (-0.29)	(-0.02) (-0.02) (-0.03)	(1.37) (1.50) (0.88)	(-0.66) (-0.70) (-0.57)
	[0.92] [0.92] [0.86]	[0.86] [0.85] [0.77]	[0.98] [0.98] [0.97]	[0.17] [0.13] [0.38]	[0.51] [0.48] [0.57]
$\beta_{V_{\text{t-1}}}$	-0.16	-0.16	-0.17	0.38	0.27
	(-1.52) (-1.66) (-1.45)	(-1.49) (-1.64) (-1.44)	(-1.60) (-1.75) (-1.35)	(7.52) (6.27) (5.31)	(4.90) (3.81) (3.10)
	[0.13] [0.10] [0.15]	[0.14] [0.11] [0.15]	[0.12] [0.09] [0.18]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	72	72	72	248	248
R_a^2	0.634	0.629	0.653	0.732	0.743

Table 18. Monthly Regressions for ΔTrading Volume

This table summarizes the monthly regression results of the first difference trading volume formula, as detailed in regression (3) of Section-4.

$$\Delta V = c \ + \ \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\right)_{\Delta\alpha} \ + \ \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P \ + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p * \Delta_P^2} * \sigma_P\right)_\beta * \Delta_P^2 \ + \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)	D(5)
c	0.08	0.06	0.05	0.04	0.01
	(0.46) (0.51) (0.82)	(0.36) (0.41) (0.68)	(0.35) (0.39) (0.75)	(0.23) (0.26) (0.50)	(0.09) (0.10) (0.27)
	[0.65] [0.61] [0.42]	[0.72] [0.69] [0.50]	[0.73] [0.70] [0.46]	[0.82] [0.80] [0.62]	[0.93] [0.92] [0.79]
α_{pbel}	0.14	0.10	0.25	-0.02	0.03
	(2.00) (2.24) (1.61)	(1.39) (1.55) (1.09)	(3.89) (4.35) (2.23)	(-0.30) (-0.34) (-0.25)	(0.53) (0.59) (0.53)
	[0.05] [0.03] [0.11]	[0.17] [0.13] [0.28]	[0.00] [0.00] [0.03]	[0.76] [0.74] [0.80]	[0.60] [0.56] [0.60]
α_{amb}	0.24	0.25	0.19	0.23	0.38
	(3.33) (3.72) (4.08)	(3.31) (3.70) (4.03)	(2.86) (3.20) (4.17)	(3.07) (3.43) (3.43)	(7.97) (8.90) (4.05)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.01] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
β_{p}	-0.28	-0.30	-0.26	-0.29	-0.03
	(-2.39) (-2.66) (-2.28)	(-2.47) (-2.75) (-2.52)	(-2.37) (-2.65) (-2.73)	(-2.39) (-2.67) (-3.11)	(-0.25) (-0.28) (-0.31)
	[0.02] [0.01] [0.03]	[0.02] [0.01] [0.01]	[0.02] [0.01] [0.01]	[0.02] [0.01] [0.00]	[0.80] [0.78] [0.76]
$\beta_{\sigma_{\text{amb}}}$	0.31	0.27	0.29	0.28	-0.09
	(0.91) (1.01) (1.01)	(0.77) (0.85) (0.92)	(0.94) (1.04) (1.32)	(0.78) (0.88) (1.20)	(-0.81) (-0.90) (-1.83)
	[0.37] [0.32] [0.32]	[0.45] [0.40] [0.36]	[0.35] [0.30] [0.19]	[0.44] [0.38] [0.23]	[0.42] [0.37] [0.07]
$\beta_{\triangle\sigma_{\text{amb}}}$	-0.51	-0.45	-0.43	-0.42	0.12
	(-1.53) (-1.71) (-1.13)	(-1.34) (-1.50) (-1.00)	(-1.41) (-1.57) (-1.15)	(-1.25) (-1.39) (-0.99)	(1.67) (1.86) (4.08)
	[0.13] [0.09] [0.26]	[0.18] [0.14] [0.32]	[0.16] [0.12] [0.25]	[0.22] [0.17] [0.32]	[0.10] [0.07] [0.00]
$\beta_{\sigma_{\text{p}}*\triangle^2{\text{p}}}$	0.21	0.17	0.32	0.10	-0.04
	(0.68) (0.76) (0.83)	(0.54) (0.61) (0.64)	(1.10) (1.23) (1.83)	(0.31) (0.34) (0.36)	(-0.14) (-0.15) (-0.19)
	[0.50] [0.45] [0.41]	[0.59] [0.55] [0.53]	[0.28] [0.23] [0.07]	[0.76] [0.73] [0.72]	[0.89] [0.88] [0.85]
$\beta_{\triangle\sigma_{\text{p}}}$	0.19	0.20	0.20	0.23	0.10
	(0.75) (0.84) (0.77)	(0.78) (0.87) (0.76)	(0.87) (0.98) (1.04)	(0.89) (0.99) (0.81)	(0.51) (0.57) (0.77)
	[0.45] [0.40] [0.44]	[0.44] [0.39] [0.45]	[0.39] [0.33] [0.30]	[0.38] [0.33] [0.42]	[0.61] [0.57] [0.45]
N	71	71	71	71	71
R_a^2	0.404	0.383	0.496	0.363	0.641
	D(6)	D(7)	D(8)	D(9)	D(10)
c	0.01	0.01	0.01	-0.05	-0.05
	(0.07) (0.08) (0.21)	(0.10) (0.11) (0.33)	(0.05) (0.06) (0.17)	(-0.22) (-0.28) (-0.93)	(-0.20) (-0.26) (-1.72)
	[0.94] [0.94] [0.83]	[0.92] [0.91] [0.74]	[0.96] [0.95] [0.87]	[0.83] [0.78] [0.35]	[0.84] [0.80] [0.09]
α_{pbel}	0.02	0.17	0.00	-0.07	-0.04
	(0.28) (0.32) (0.28)	(3.27) (3.65) (3.28)	(0.02) (0.02) (0.02)	(-1.16) (-1.12) (-2.04)	(-0.70) (-0.67) (-0.93)
	[0.78] [0.75] [0.78]	[0.00] [0.00] [0.00]	[0.99] [0.99] [0.98]	[0.25] [0.26] [0.04]	[0.48] [0.51] [0.35]
α_{amb}	0.38	0.33	0.38	0.25	0.53
	(8.08) (9.02) (4.03)	(7.36) (8.21) (4.58)	(8.12) (9.07) (3.94)	(4.42) (4.55) (2.80)	(9.33) (11.13) (5.30)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.00]
β_{p}	-0.03	-0.04	-0.02	-0.29	-0.08
	(-0.23) (-0.26) (-0.29)	(-0.36) (-0.40) (-0.50)	(-0.20) (-0.22) (-0.24)	(-3.51) (-3.97) (-3.50)	(-0.89) (-0.97) (-1.17)
	[0.82] [0.80] [0.77]	[0.72] [0.69] [0.62]	[0.85] [0.83] [0.81]	[0.00] [0.00] [0.00]	[0.38] [0.33] [0.24]
$\beta_{\sigma_{\text{amb}}}$	-0.09	-0.10	-0.10	0.19	0.02
	(-0.83) (-0.93) (-1.90)	(-0.98) (-1.10) (-2.05)	(-0.87) (-0.97) (-2.09)	(1.93) (1.84) (2.54)	(0.26) (0.24) (0.35)
	[0.41] [0.36] [0.06]	[0.33] [0.28] [0.05]	[0.39] [0.34] [0.04]	[0.05] [0.07] [0.01]	[0.79] [0.81] [0.73]
$\beta_{\triangle\sigma_{\text{amb}}}$	0.12	0.15	0.12	-0.35	-0.09
	(1.70) (1.90) (4.13)	(2.26) (2.53) (5.99)	(1.62) (1.81) (5.55)	(-5.28) (-4.19) (-7.09)	(-1.40) (-1.16) (-1.14)
	[0.09] [0.06] [0.00]	[0.03] [0.01] [0.00]	[0.11] [0.08] [0.00]	[0.00] [0.00] [0.00]	[0.16] [0.25] [0.25]
$\beta_{\sigma_{\text{p}}*\triangle^2{\text{p}}}$	-0.04	0.12	-0.05	-0.04	-0.01
	(-0.16) (-0.18) (-0.23)	(0.47) (0.52) (0.62)	(-0.18) (-0.20) (-0.25)	(-0.32) (-0.37) (-0.34)	(-0.11) (-0.13) (-0.12)
	[0.87] [0.86] [0.82]	[0.64] [0.60] [0.54]	[0.86] [0.84] [0.80]	[0.75] [0.71] [0.73]	[0.91] [0.90] [0.91]
$\beta_{\triangle\sigma_{\text{p}}}$	0.11	0.14	0.11	0.14	-0.03
	(0.57) (0.64) (0.87)	(0.81) (0.91) (1.34)	(0.61) (0.68) (1.01)	(1.87) (2.16) (2.82)	(-0.48) (-0.53) (-0.36)
	[0.57] [0.52] [0.39]	[0.42] [0.37] [0.19]	[0.55] [0.50] [0.31]	[0.06] [0.03] [0.01]	[0.63] [0.60] [0.72]
N	71	71	71	247	247
R_a^2	0.639	0.696	0.639	0.317	0.400

Appendix G Single Sorted Portfolio Returns

This section shows the regression results for the single sorted portfolios described in Section-6.

Table-21 on this page shows the results of regressing the Fama-French market portfolio, the LMH Turnover Portfolio P_{Turn}, LMH Ambiguity related Turnover sorted portfolio known as P_{amb}, the LMH differences in prior beliefs related Turnover sorted portfolio called P_{pbel}, the LMH price fluctuations related Turnover sorted portfolio called $P_{\Delta P_{21}}$, and finally, the LMH unexplained residual related Turnover sorted portfolio named P_{ϵ}, on a constant as described in regression (2) of Section-6.

$$R_{P_{Turn}} = c + \epsilon \tag{2}$$

$$R_{P_{Turn}} = c + R_{free} + \beta_{MKTRF} * R_{MKTRF} + \beta_{HML} * R_{HML} + \beta_{SMB} * R_{SMB} + \epsilon \tag{3}$$

$$R_{P_{Turn}} = c + \beta_{P_{amb}} * R_{P_{amb}} + \beta_{P_{pbel}} * R_{P_{pbel}} + \beta_{P_{\Delta P_{21}}} * R_{P_{\Delta P_{21}}} + \beta_{P_{\epsilon}} * R_{P_{\epsilon}} + \epsilon \tag{4}$$

Table 19. Monthly Single Sorted Portfolio Returns

Single sorted portfolio returns statistics using monthly data between 1990 to 2020.

	P_{MKT}	P_{Turn}	P_{amb}	P_{pbel}	$P_{\Delta P_{21}}$	P_{ϵ}
Panel A: Monthly Portfolio Returnds (1990 - 2020)						
c	0.009	0.006	0.006	0.002	-0.005	0.003
	(4.10) (4.10) (3.96)	(1.66) (1.67) (1.68)	(1.88) (1.88) (1.70)	(0.65) (0.65) (0.76)	(-2.24) (-2.24) (-3.27)	(1.51) (1.52) (1.75)
	[0.00] [0.00] [0.00]	[0.10] [0.10] [0.09]	[0.06] [0.06] [0.09]	[0.51] [0.51] [0.45]	[0.03] [0.03] [0.00]	[0.13] [0.13] [0.08]
N	372	372	372	372	372	372
R_a^2	0	0	0	0	0	0
Panel B: Sharpe Ratios (1990 - 2020, yearly)						
Excess Return	0.09	0.05	0.05	-0.01	-0.03	0.01
Std. Deviation	0.14	0.25	0.21	0.16	0.15	0.11
Sharpe Ratio	0.56	0.20	0.22	-0.05	-0.22	0.04

Table-20 below shows the results of regressing the LMH Turnover sorted portfolio on a constant, on the Fama-French 3-Factors (MKT, SMB, HML) and on the LMH portfolios $\{P_{amb}, P_{pbel}, P_{\Delta P_{21}}, P_{\epsilon}\}$ derived from the four Turnover components, as described in regressions (2), (3) and (4) of Section-6.

Lastly, you find in Table-21 the outcomes of the regression of the LMH Turnover sorted portfolio P_{Turn} on the LMH portfolios $\{P_{amb}, P_{pbel}, P_{\Delta P_{21}}, P_{\epsilon}\}$ based on the four components of Turnover described in Section 6, plus an explanatory variable controlling for Liquidity. The

Liquidity proxies are the month-end SPY Bid-Ask, the monthly mean SPY Bid-Ask, the Liquidity measure proposed by Hu et al. (2013), and the Intermediary Capital Risk Factor and Ratio introduced by He et al. (2017). These variables are employed both in their level form and as first differences.

Table 20. Monthly Single Sorted Portfolio Returns Attribution

Single sorted portfolio returns statistics using monthly data between 1990 to 2020.

	P_{Turn}	P_{Turn}	P_{Turn}
c	0.006	0.019	-0.001
	(1.66) (1.67) (1.68)	(6.08) (6.07) (4.81)	(-1.53) (-1.54) (-1.27)
	[0.10] [0.10] [0.09]	[0.00] [0.00] [0.00]	[0.13] [0.12] [0.20]
β_{R_f}		-2.380	
		(-2.20) (-2.16) (-1.77)	
		[0.03] [0.03] [0.08]	
β_{MKT-R_f}		-0.953	
		(-19.98) (-17.55)	
		(-12.19)	
		[0.00] [0.00] [0.00]	
β_{HML}		0.854	
		(12.61) (11.43) (6.82)	
		[0.00] [0.00] [0.00]	
β_{SMB}		-0.789	
		(-11.68) (-11.83) (-7.96)	
		[0.00] [0.00] [0.00]	
$\beta_{P_{amb}}$			0.942
			(63.44) (62.43) (41.18)
			[0.00] [0.00] [0.00]
$\beta_{P_{pbel}}$			0.450
			(20.46) (20.70) (5.93)
			[0.00] [0.00] [0.00]
$\beta_{P_{\triangle P_{21}}}$			-0.034
			(-1.26) (-1.27) (-0.52)
			[0.21] [0.21] [0.60]
$\beta_{P_{\epsilon}}$			0.390
			(11.94) (11.94) (7.21)
			[0.00] [0.00] [0.00]
N	372	372	372
R_a^2	0.000	0.728	0.944

Table 21. Monthly Single Sorted Portfolio Returns Attribution with Liquidity Controls

Single sorted portfolio returns statistics using monthly data between 1990 to 2020. Last row indicates the liquidity control used.

	P_{Turn}	P_{Turn}	P_{Turn}	P_{Turn}	P_{Turn}
Panel A: LMH Turnover Portfolio Regressions with Liquidity Controls in Levels					
c	-0.001	-0.001	-0.000	-0.001	-0.002
	(-0.69) (-0.70) (-0.63)	(-0.60) (-0.60) (-0.61)	(-0.16) (-0.16) (-0.17)	(-1.48) (-1.49) (-1.25)	(-0.81) (-0.80) (-0.69)
	[0.49] [0.49] [0.53]	[0.55] [0.55] [0.55]	[0.87] [0.88] [0.87]	[0.14] [0.14] [0.21]	[0.42] [0.42] [0.49]
$\beta_{P_{amb}}$	0.944	0.941	0.942	0.939	0.942
	(61.72) (60.53)	(62.43) (61.13)	(63.43) (62.44)	(57.21) (56.50)	(63.32) (62.41)
	(38.43)	(37.79)	(41.46)	(37.45)	(41.11)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{P_{pbel}}$	0.457	0.457	0.449	0.451	0.450
	(20.31) (20.60) (6.27)	(20.30) (20.60) (6.25)	(20.31) (20.60) (5.84)	(20.44) (20.72) (5.90)	(20.44) (20.71) (5.93)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{P_{\triangle P_{21}}}$	-0.056	-0.056	-0.036	-0.034	-0.033
	(-2.05) (-2.08) (-0.89)	(-2.05) (-2.07) (-0.88)	(-1.32) (-1.34) (-0.54)	(-1.24) (-1.25) (-0.51)	(-1.22) (-1.23) (-0.49)
	[0.04] [0.04] [0.38]	[0.04] [0.04] [0.38]	[0.19] [0.18] [0.59]	[0.21] [0.21] [0.61]	[0.22] [0.22] [0.62]
$\beta_{P_{\epsilon}}$	0.440	0.443	0.392	0.392	0.389
	(12.85) (12.88) (8.85)	(12.89) (12.93) (8.90)	(11.97) (11.97) (7.32)	(11.91) (11.93) (7.37)	(11.85) (11.86) (7.07)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{Control}$	0.001	0.001	-0.000	-0.007	0.013
	(1.06) (1.01) (1.17)	(0.92) (0.89) (0.85)	(-0.92) (-0.85) (-0.86)	(-0.46) (-0.45) (-0.40)	(0.33) (0.33) (0.25)
	[0.29] [0.31] [0.24]	[0.36] [0.38] [0.39]	[0.36] [0.39] [0.39]	[0.65] [0.65] [0.69]	[0.74] [0.75] [0.80]
N	324	324	372	372	372
R_a^2	0.949	0.949	0.944	0.944	0.944
Panel B: LMH Turnover Portfolio Regressions with Liquidity Controls in Differences					
c	-0.001	-0.001	-0.001	-0.001	-0.001
	(-0.92) (-0.93) (-0.80)	(-0.93) (-0.93) (-0.79)	(-1.57) (-1.58) (-1.30)	(-1.55) (-1.56) (-1.28)	(-1.51) (-1.52) (-1.26)
	[0.36] [0.35] [0.43]	[0.35] [0.35] [0.43]	[0.12] [0.12] [0.19]	[0.12] [0.12] [0.20]	[0.13] [0.13] [0.21]
$\beta_{P_{amb}}$	0.942	0.939	0.942	0.946	0.939
	(61.98) (60.71)	(62.36) (61.29)	(63.43) (62.37)	(58.99) (58.44)	(58.02) (57.26)
	(37.18)	(36.47)	(41.23)	(38.07)	(38.48)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{P_{pbel}}$	0.456	0.455	0.450	0.450	0.451
	(20.21) (20.51) (6.23)	(20.27) (20.55) (6.35)	(20.44) (20.73) (5.90)	(20.45) (20.72) (5.97)	(20.44) (20.72) (5.90)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{P_{\triangle P_{21}}}$	-0.055	-0.056	-0.033	-0.033	-0.033
	(-2.02) (-2.04) (-0.86)	(-2.07) (-2.09) (-0.91)	(-1.24) (-1.25) (-0.51)	(-1.22) (-1.23) (-0.51)	(-1.22) (-1.24) (-0.51)
	[0.04] [0.04] [0.39]	[0.04] [0.04] [0.37]	[0.22] [0.21] [0.61]	[0.22] [0.22] [0.61]	[0.22] [0.22] [0.61]
$\beta_{P_{\epsilon}}$	0.440	0.447	0.389	0.389	0.391
	(12.78) (12.81) (8.77)	(13.03) (13.09) (9.55)	(11.90) (11.91) (7.20)	(11.85) (11.86) (7.26)	(11.94) (11.95) (7.29)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\triangle Control}$	0.000	-0.004	-0.000	0.007	-0.097
	(0.31) (0.30) (0.44)	(-2.02) (-2.02) (-3.25)	(-0.98) (-0.94) (-1.18)	(0.62) (0.62) (0.68)	(-0.47) (-0.48) (-0.51)
	[0.75] [0.76] [0.66]	[0.04] [0.04] [0.00]	[0.33] [0.35] [0.24]	[0.54] [0.54] [0.50]	[0.64] [0.63] [0.61]
N	323	323	372	372	372
R_a^2	0.949	0.950	0.944	0.944	0.944
Control	BidAsk_{last}	BidAsk_{mean}	Noise	ICapital_{factor}	ICapital_{ratio}

Appendix H Bisorted Portfolio Returns

In this appendix, I provide the monthly frequency return statistics of the bisorted portfolios outlined in Section 6.

Table-22 below displays the mean monthly returns, standard deviations and Sharpe-Ratios of the Fama-French Market Factor, the bisorted LMH Turnover-Turnover portfolio called P_{Turn}, the LMH Turnover related to Ambiguity and Turnover related to price fluctuations bisorted portfolio called $P_{amb-\Delta P_{21}}$ and the LMH Turnover related to price fluctuations and Turnover related to Ambiguity bisorted portfolio called $P_{\Delta P_{21}-amb}$.

Table 23 below presents the annualized average differential returns of the bisorted portfolios. In Panel A, the table illustrates the differential returns achieved by going long on the quantile bisorted portfolio indicated in the top row and simultaneously shorting the quantile bisorted portfolio described in the leftmost column. The first dimension in Panel A, pertaining to Ambiguity driven Turnover, is divided into 5 quantiles, while the second dimension, associated to price fluctuations driven Turnover, is distributed across two quantiles. This arrangement yields a total of 10 bisorted portfolios {00,01,10,11,20,21,30,31,40,41} . In Panel-B of the same table, you can observe the annualized differential returns stemming from bisorted portfolios created by employing Turnover driven by price fluctuations as the first sorting dimension, and Turnover driven by Ambiguity as the second sorting dimension.

Table 22. Bisorted Portfolios Monthly Returns Statistics

This tables summarize the returns of the HML bisorted portfolios of section 5. Each portfolio is obtained by double sorting on a 5x2 grid.

	P_{MKT}	$P_{Turn-Turn}$	$P_{amb-\triangle P_{21}}$	$P_{\triangle P_{21}-amb}$
Panel A: Monthly Portfolio Returns (1990 - 2020)				
c	0.009	0.006	0.008	0.008
	(4.10) (4.10) (3.96)	(1.44) (1.44) (1.39)	(2.96) (2.96) (2.79)	(3.90) (3.91) (3.53)
	[0.00] [0.00] [0.00]	[0.15] [0.15] [0.17]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.00]
N	372	372	372	372
R^2_a	0	0	0	0
Panel B: Sharpe Ratios (1990 - 2020, yearly)				
Excess Return	0.09	0.04	0.07	0.07
Std. Deviation	0.14	0.25	0.16	0.13
Sharpe Ratio	0.56	0.16	0.38	0.50

Table 23. Bisorted Portfolios Monthly Returns

This tables summarize the annualized differential returns of the bisorted portfolios of Section-6. Each portfolio is obtained by double sorting on a 5x2 grid. The portfolios dimensions are Ambiguity - Price Change driven Turnover in Panel-A, and Price Change - Ambiguity driven Turnover in panel-B. Dimensions are in decimal format, 0.09 means 9% pear year.

Panel A: Ambiguity and $\triangle P_{21}$

	00	01	10	11	20	21	30	31	40	41
00	0.00	-0.03	-0.01	-0.04	0.00	-0.03	0.02	-0.01	0.06	0.03
01	0.03	0.00	0.03	-0.00	0.04	0.01	0.05	0.03	**0.09**	0.07
10	0.01	-0.03	0.00	-0.03	0.01	-0.02	0.03	-0.00	0.07	0.04
11	0.04	0.00	0.03	0.00	0.04	0.01	0.05	0.03	0.10	0.07
20	-0.00	-0.04	-0.01	-0.04	0.00	-0.03	0.01	-0.01	0.06	0.03
21	0.03	-0.01	0.02	-0.01	0.03	0.00	0.05	0.02	0.09	0.06
30	-0.02	-0.05	-0.03	-0.05	-0.01	-0.05	0.00	-0.03	0.04	0.02
31	0.01	-0.03	0.00	-0.03	0.01	-0.02	0.03	0.00	0.07	0.04
40	-0.06	**-0.09**	-0.07	-0.10	-0.06	-0.09	-0.04	-0.07	0.00	-0.03
41	-0.03	-0.07	-0.04	-0.07	-0.03	-0.06	-0.02	-0.04	0.03	0.00

Panel B: $\triangle P_{21}$ and Ambiguity

	00	01	10	11	20	21	30	31	40	41
00	0.00	0.04	-0.02	0.01	-0.02	0.00	-0.05	0.00	-0.05	0.01
01	-0.04	0.00	-0.06	-0.03	-0.06	-0.04	-0.09	-0.04	**-0.09**	-0.03
10	0.02	0.06	0.00	0.03	-0.00	0.02	-0.03	0.02	-0.04	0.03
11	-0.01	0.03	-0.03	0.00	-0.03	-0.01	-0.06	-0.01	-0.07	-0.00
20	0.02	0.06	0.00	0.03	0.00	0.02	-0.03	0.02	-0.04	0.03
21	-0.00	0.04	-0.02	0.01	-0.02	0.00	-0.05	-0.00	-0.06	0.01
30	0.05	0.09	0.03	0.06	0.03	0.05	0.00	0.05	-0.01	0.06
31	-0.00	0.04	-0.02	0.01	-0.02	0.00	-0.05	0.00	-0.06	0.01
40	0.05	**0.09**	0.04	0.07	0.04	0.06	0.01	0.06	0.00	0.07
41	-0.01	0.03	-0.03	0.00	-0.03	-0.01	-0.06	-0.01	-0.07	0.00

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