Ambiguity, Trading Volume and Liquidity

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Abstract

In this paper, I construct a trading volume model that explicitly incorporates the impact of ambiguity surrounding public information announcements. I derive a closed form expression that illustrates how ambiguity influences trading volume through two channels: expectations and volatility. The empirical results highlight the significant role played by the expectations channel of ambiguity in driving trading activity. Specifically, on a monthly basis, I observe that a one-standard-deviation increase in ambiguity results in approximately a 19% to 58%-standard-deviation increase in trading volume. Additionally, by employing this model, I demonstrate that a substantial portion of the positive returns of a standard U.S. equity turnover sorted portfolio, approximately 80% of the returns, traditionally attributed to liquidity, are actually driven by information ambiguity.

Keywords: ambiguity; knightian uncertainty; trading volume; turnover; liquidity; information; beliefs; heterogeneity; learning.

JEL Classification: G12, G4, D8

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1 Introduction

What generates the large trading volume activity observed in markets is a long standing question in the finance literature. While rational expectations models have been widely employed, they encounter limitations when attempting to explain trading activity solely based on public information (Banerjee & Kremer, 2010; Milgrom & Stokey, 1982). These models require the inclusion of an external process, such as private information, liquidity shocks, or the consideration of heterogeneous prior information to generate trading activity. However, these mechanisms often come with restrictions or rely on external processes, making it challenging to explain the large trading volumes observed in markets (Glaser & Weber, 2007; Hirshleifer, 2001). One modeling approach that potentially can generate larger differences and swings in expectations to account for trading volume is ambiguity (Knightian Uncertainty).

In this paper, I study the effects of ambiguity on trading volume. I propose a trading volume model that explicitly incorporates ambiguity about public information announcements. To develop this model, I introduce agents with ambiguous preferences into the difference in prior beliefs model of Kandel and Pearson (1995). These ambiguous preferences induce heterogeneous interpretations of public information by affecting expectations, and through this heterogeneity the expectations channel of ambiguity generates trading activity. Additionally, the volatility channel of ambiguity plays a role in amplifying or smoothing the price channel of trading volume, the expectations channel of ambiguity and the differences in prior beliefs. I derive four testable hypotheses. My first hypothesis H1 postulates that on average the expectations level of ambiguity positively influences trading volume. My second hypothesis H2 posits that ambiguity volatility weakens the positive channel between the level of ambiguity and trading volume. My third hypothesis H3 is that the expectations channel of ambiguity affects the conventional positive relationship between price volatility and trading volume by reducing the elasticity between them. Lastly, my fourth hypothesis H4 proposes that trading activity driven by information ambiguity significantly contributes to the positive returns of turnover sorted portfolios, returns typically attributed to liquidity in the literature.

I bring my model to equity data and find that, on average, a one-standard-deviation increase in daily ambiguity translates approximately in a 11%-standard-deviation increase in trading volume after controlling for price movements and differences in prior beliefs. On a monthly basis, I find that a one-standard-deviation increase in ambiguity translates in a 19% to 58%-standard-deviation increase in trading volume. The daily frequency results indicate that ambiguity volatility weakens this positive relationship between the expectations channel of ambiguity and trading volume. Specifically, a one-standard-deviation increase in the interaction between the level of ambiguity and its volatility leads to a statistically significant -6% to -10%-standard-deviation decrease in trading volume. Furthermore, my findings reveal that the level of ambiguity distorts the on average positive elasticity between trading volume and price volatility, weakening this connection. A one-standard-deviation increase in the interaction between the level of ambiguity and price volatility translates in a -9% to -12%-standard-deviation decrease in trading volume, rather than an increase. Regarding the returns of turnover sorted portfolios, my empirical findings reveal that since 1990, approximately 80% of the positive returns of a standard US long-short equity turnover sorted portfolio can be attributed to trading volume driven by information ambiguity. Traditionally, the literature associates these returns with liquidity.

Trading volume stands as a crucial market metric, widely employed for purposes such as measuring liquidity (Chordia et al., 2001), understanding information transmission or the structure of markets (Karpoff, 1986) among other uses. Despite its variety of practical uses, it is still an open puzzle what are the drivers behind the large trading volume observed in most markets and its relation to market liquidity. Cochrane (Cochrane, 2016) noted that "volume is the great unsolved problem of financial economics", Shleifer (2000) ranked the volume puzzle among the top 20 issues of behavioural finance, and De Bondt and Thaler (1995) mentioned that volume "is perhaps the single most embarrassing fact to the standard finance paradigms".

There are several empirical studies that have documented the on average high levels of trading volume in markets. Barber and Odean (2000) report that between 1991 to 1996 the average US household had a portfolio turnover of around 75%, while the top active quantile of retail investors showed annual turnover rates of 250%. Dorn and Huberman (2005) report average turnover rates of around 100% among German retail investors, Barber et al. (2009) report turnover rates of up to 300% in Taiwan, and Gao (2002) reports turnover rates of 500% in China. According to the literature, such levels of trading activity can not be explained by rational expectations models (Dorn & Sengmueller, 2009). Using a rational expectations model calibrated on Nasdaq data, Sen (2002) obtained an average holding period of 98 months for stocks, a much longer period than the real Nasdaq average holding period of 5.1 months. What can explain such high levels of trading activity ?.

In relation to the practical use of trading volume as a proxy for liquidity (Abdi & Ranaldo, 2017; Avramov & Chordia, 2006; Becker-Blease & Paul, 2006; Eckbo & Norli, 2005; Illeditsch, 2011; Lee, 1993; Rouwenhorst, 1999), there are several empirical studies that have found results inconsistent with the finance literature (Bekaert et al., 2007; Chordia et al., 2001). One of the most famous is the study of Chordia et al. (2001), who found a puzzling negative correlation between stock returns and the dispersion of trading volume. According to the asset pricing literature, for assets with higher liquidity risk one would have expected higher returns. Bekaert et al. (2007) found that across 19 emerging stock markets a returns based measures of liquidity was priced, while turnover was not priced. These inconsistencies have cast doubts on the suitability of volume based measures of liquidity (Barinov, 2014; Gabrielsen et al., 2011; Johnson, 2008; Le & Gregoriou, 2020; Lee & Swaminathan, 2000), and have open the question of whether trading volume might be capturing some other factor(s)?.

With this model and empirical results, I demonstrate the pivotal role of the expectations channel of ambiguity in driving trading volume and show that a considerable portion of a standard U.S. turnover sorted portfolio returns, returns often attributed to liquidity, are actually driven by information ambiguity. This research will contribute to the growing literature on the effects of ambiguity on markets and information transmission through prices (Condie & Ganguli, 2017; Easley & O'HARA, 2010; Epstein & Schneider, 2010; Mele & Sangiorgi, 2015; Ozsoylev & Werner, 2011). In relation to previous trading volume models or closed form equations that explicitly incorporate ambiguity about public information, to the best of my knowledge there are two previously existing works. The model of Caskey (2009) about the ambiguous perception of public information and the partially related model of Hsiao (2019) about the ambiguous perception of others' beliefs. Both models exploit the volatility contribution of ambiguity to generate trading activity, while my model uses the expectations channel of ambiguity. The introduction of this expectations channel in a closed form equation for trading volume and the empirical results showing its statistical significance are the main contributions of this paper.

The rest of the paper is organized as follows. Section 2 describes the theoretical model. Section 4 summarizes the data used in the empirical estimations. Section 3 describes the four hypotheses I test. Section 5 presents the main empirical results regarding the effects of ambiguity on trading volume. Section 6 presents the empirical results regarding the effects of ambiguity on the trading volume to price volatility elasticity. Section 7 presents an empirical application of the model that dissects the returns of a standard U.S. equity turnover sorted portfolio. Finally, Section 8 presents my conclusions.

2 Model

The model is based on a setup similar to Kandel and Pearson (1995). The setup consists of two competitive markets: a riskless asset market and a risky asset market. Across both markets, two investor types choose optimal portfolios according to their individual prior beliefs. The key feature of the model is that type-A investors interpret public information ambiguously. The proportion of these ambiguity-averse (or ambiguity-loving) type-A investors operating in both risk-less and risky asset markets is denoted by π, while the proportion of type-B ambiguity-neutral investors is represented by $1-\pi$.

Both investor types receive a public signal denoted as S, which contains information about the unknown risky asset payoff $\widetilde{X}$. The type-A investors prone to ambiguity-averse (or ambiguity loving) behavior, interprets this signal S in an uncertain manner, considering various mental representations or models. After observing the public signal S, both investor types update their beliefs and adjust their portfolios accordingly.

The model dynamics consist of three time periods. In period 1, both investor types construct their initial portfolios based on their individual prior beliefs. In period 2, both investor types observe the public signal S, which provides information about the unknown payoff $\widetilde{X}$ of the risky asset. Armed with this new information, both investor types update their beliefs and adjust their portfolios accordingly. These portfolio adjustments lead to trading volume from period (1) to period (2). Finally, in the last period (3), the risky-asset payoff is realized. For the sake of simplicity in this setup (Kandel & Pearson, 1995), investors, when building their portfolios at time (1), do not expect a second opportunity or reason to trade in the future.

2.1 Ambiguity Neutral Investors (Type-B)

The type-B investors utility function is denoted by $U^B\left(\theta_t^B\right)$, where θ_t^B represents this investor type allocation in the risky asset during period (t). $W\left(\theta_t^B\right)$ represents the final wealth level for type-B investors at period 3. This final wealth level $W\left(\theta_t^B\right)$ depends on their initial wealth w_t^B

at period (t) and the risky asset allocation θ_t^B determined during the same period. In period 1, this investor type maximizes their expected final utility based on their prior beliefs. Moving to period 2, they update their beliefs about the distribution of the risky-asset payoff and adjusts their portfolio accordingly. Finally, in period 3, the risky asset pays off.

This investor type has a standard CARA utility function with absolute risk aversion given by γ and maximizes the following expected utility.

$$\begin{cases} E^B\left[\ -e^{-\gamma\ *\ W\left(\theta_t^B\right)}\ \right] & \text{if } t = t1 \\ E^B\left[-e^{\ -\gamma\ *\ W\left(\theta_t^B\right)}\ \middle|S\right] & \text{if } t = t2 \end{cases} \tag{2.1}$$

The final wealth $W\left(\theta_t^B\right)$ of this investor type at period 3 is given by the expression below.

$$W\left(\theta_t^B\right) = w_t^B + \theta_t^B * \left(\widetilde{X} - P_t\right) \tag{2.2}$$

In period 1, this investor type initially believes that the risky-asset payoff $\widetilde{X}$ follows a normal distribution with parameters $N\left(\mu_X^B,\ \sigma_X^{2\,B}\right)$ and precision $\rho_X^B = 1/\sigma_X^{2\,B}$. However, upon observing the unexpected signal S, this investor type updates their beliefs. They perceive the signal as biased, containing information about both the risky-payoff amount $\widetilde{X}$ and a measurement bias or error ϵ. This investor type believes that the measurement error distributes $N\left(\mu_\epsilon^B,\ \sigma_\epsilon^{2\,B}\right)$ with precision $\rho_\epsilon^B = 1/\sigma_\epsilon^{2\,B}$. The total error term of the signal is $\widetilde{\mathcal{E}}$ which according to this investor type is composed of just a measurement error and has precision $\rho_{\mathcal{E}}^B$.

$$S = \widetilde{X} + \widetilde{\epsilon} \tag{2.3}$$

A type-B investor with bullish prior beliefs or a positive interpretation of public information would assume that the measurement bias mean μ_ϵ^B is negative. Conversely, a type-B investor with bearish prior beliefs would believe that the signal has a positive bias μ_ϵ^B, misleadingly indicating an expectation above the true expected value of $\widetilde{X}$.

2.2 Ambiguity Averse/Loving Investors (Type-A)

The ambiguity-averse/loving A investor type is characterized by a utility function denoted as $U^A\left(\theta_t^A\right)$, with θ_t^A representing their risky asset allocation decided during period (t). $W_t\left(\theta_t^{I_A}\right)$

represents this investor type final wealth level at period 3 based on his allocation at (t). Initially, in the first period, this investor type expected utility looks similar to that of a CARA agent (ambiguity-neutral), as the source of ambiguity arises in period 2 after observing the signal S. In this second period 2, after incorporating the information from signal S, this investor type maximizes their expected utility by considering the ambiguity surrounding signal S according to their Smooth Ambiguity Utility function.

$$\begin{cases} E^A\Big[\ -e^{-\gamma\ *\ W\left(\theta_t^A\right)}\ \Big] & \text{if } t = t1 \\ E^A\Big[-\Big(\ -E^A\Big[-e^{\ -\gamma\ *\ W\left(\theta_t^A\right)}\ \Big|S, M\Big]\ \Big)^{\gamma_a}\Big|S\Big] & \text{if } t = t2 \end{cases} \tag{2.4}$$

The parameters γ and γ_a are the risk aversion and the ambiguity aversion coefficients of the utility function, and $W\left(\theta_t^A\right)$ represents the final wealth of this investor type given his risky asset allocation θ_t^A at (t).

$$W\left(\theta_t^A\right) = w_t^A + \theta_t^A * \left(\widetilde{X} - P_t\right) \tag{2.5}$$

The investor type-A maximizes at time 2 their expected utility obtained after evaluating their terminal wealth under n-different models M^n by selecting the optimal asset mixture. In this utility function, the first expectation operator addresses the traditional concept of risk associated with the payoff $\widetilde{X}$, while the second expectation handles the ambiguity surrounding S, by evaluating the terminal wealth under different M models. In this particular setup, each model M represents a specific way of interpreting the public information.

Regarding this investor type beliefs about the risky-asset payoff $\widetilde{X}$, they initially assume a normal distribution with parameters $N\left(\mu_X^A,\ \sigma_X^{2\,A}\right)$ and precision $\rho_X^A = 1/\sigma_X^{2\,A}$. Additionally, they believe that the signal S is subject to a measurement bias or error $\widetilde{\epsilon}$ that distributes $N\left(\mu_\epsilon^A,\ \sigma_\epsilon^{2\,A}\right)$ and has precision $\rho_\epsilon^A = 1/\sigma_\epsilon^{2\,A}$.

Despite these prior beliefs, this investor type is not completely certain about the appropriate model for interpreting the signal S. This ambiguity is represented by different models $M \in M^n$, each characterized by a model-dependent signal component $\widetilde{\delta}$. This $\widetilde{\delta}$ distributes across M^n according to the normal distribution $N\left(\mu_\delta^A,\ \sigma_\delta^{2\,A}\right)$. Similarly to the previous investor type, a bullish investor who tends to interpret information positively would perceive the mean bias $\mu_\delta^A + \mu_\epsilon^A$ of the signal as negative, causing the signal S to misleadingly appear below the true expected value of $\widetilde{X}$.

In summary, the investor type-A believes that the signal S consists of three components: the risky payoff $\widetilde{X}$ information, an ambiguous model-dependent component $\widetilde{\delta}$ and a measurement error $\widetilde{\epsilon}$. The total error term of the signal is denoted as $\widetilde{\mathcal{E}}$ and has precision $\rho_{\mathcal{E}}^{A}$ according to investor type-A.

$$\begin{aligned} S &= \widetilde{X} + \widetilde{\delta} + \widetilde{\epsilon} = S + \widetilde{\mathcal{E}} \\ \widetilde{X} &\sim N\big(\mu_X^A,\ \sigma_X^{2\,A}\ \big) \\ \widetilde{\delta} &\sim N\big(\mu_\delta^A,\ \sigma_\delta^{2\,A}\ \big) \\ \widetilde{\epsilon} &\sim N\big(\mu_\epsilon^A,\ \sigma_\epsilon^{2\,A}\ \big) \end{aligned} \tag{2.6}$$

2.3 Market Equilibrium in Period 1

In the initial period 1, each investor type seeks to maximize their expected final utility at period 3 based on their own prior beliefs.

The type-B ambiguity neutral investors maximize the following expected utility at period 1.

$$\max_{\theta_{t1}^B}\ E^B\big[U^B\big(\theta_{t1}^B\big)\big] = \max_{\theta_{t1}^B} -e^{-\gamma*\left(w_{t1}^B\ +\ \theta_{t1}^B*\ \left(E^B[\widetilde{X}]-P_{t1}\right)\right)\ +\ \frac{1}{2}*\gamma^2*\theta_{t1}^{2B}*\mathrm{VAR}^B[\widetilde{X}]} \tag{2.7}$$

The type-B investors optimal allocation in the risky asset at period 1 is given by the expression θ_{t1}^B below.

$$\theta_{t1}^B = \frac{\big(\mu_X^B - P_{t1}\big) * \rho_X^B}{\gamma}$$

The ambiguity-averse/loving type-A investors maximize at period 1 the following expected utility.

$$\max_{\theta_{t1}^A}\ E^A\big[U^A\big(\theta_{t1}^A\big)\big] = \max_{\theta_{t1}^A} -e^{-\gamma*\left(w_{t1}^A\ +\ \theta_{t1}^A*\ \left(E^A[\widetilde{X}]-P_{t1}\right)\right)\ +\ \frac{1}{2}*\gamma^2*\theta_{t1}^{2A}*\mathrm{VAR}^A[\widetilde{X}]} \tag{2.8}$$

The type-A investors optimal investment in the risky asset at period 1 is determined by the expression $\theta_{t1}^{I_A}$ below.

$$\theta_{t1}^{A} = \frac{\left(\mu_X^A - P_{t1}\right) * \rho_X^A}{\gamma}$$

The aggregate demands of both investor types will converge in the risky asset market, establishing a market clearing price in equilibrium. The type-B ambiguity neutral investors aggregate demand is $(1-\pi) * \theta_{t1}^B$, while the type-A investors aggregate demand is $\pi * \theta_{t1}^A$. In equilibrium the zero net supply risky-asset market clears according to the following equation.

$$(1-\pi) * \theta_{t1}^B + \pi * \theta_{t1}^A = 0 \tag{2.9}$$

The price P_{t1} represents the equilibrium price at which the risky asset market clears in period 1 and is given by the following expression.

$$P_{t1} = \frac{\bar{\mu}_{\underset{t1}{X}}}{\bar{\rho}_{\underset{t1}{X}}} \tag{2.10}$$

$$\begin{aligned} &where \\ &\bar{\mu}_{\underset{t1}{X}} = \pi * \mu_X^A * \rho_X^A + (1-\pi) * \mu_X^B * \rho_X^B \\ &\bar{\rho}_{\underset{t1}{X}} = \pi * \rho_X^A + (1-\pi) * \rho_X^B \end{aligned}$$

The resulting equilibrium allocation at period 1 for both ambiguity-averse/loving type-A and ambiguity neutral type-B investors is given by the expressions θ_{t1}^A and θ_{t1}^B below.

$$\begin{aligned} \theta_{t1}^B &= \frac{\pi * \rho_X^A * \rho_X^B * \left(\mu_X^B - \mu_X^A\right)}{\gamma * \bar{\rho}_X} \\ \theta_{t1}^A &= \frac{(1-\pi) * \rho_X^A * \rho_X^B * \left(\mu_X^A - \mu_X^B\right)}{\gamma * \bar{\rho}_X} \end{aligned} \tag{2.11}$$

2.4 Arrival of Public Information and Update of Beliefs at Period 2

At the beginning of period 2, public information arrives through the signal S, which is visible to both investor types. Although both types of investors receive the same raw information, their interpretations differ due to their prior heterogeneous beliefs and the ambiguity faced by type-A investors.

The type-B ambiguity neutral investors believe that the signal S they are receiving follows this functional form $S = \widetilde{X} + \widetilde{\epsilon}$. Based on this, they update their beliefs about the mean and volatility of the future risky asset payoff $\widetilde{X}$, constructing the posterior beliefs $E^B\left[\widetilde{X}\middle|S\right]$ and $\mathrm{VAR}^B\left[\widetilde{X}\middle|S\right]$. These posterior beliefs are derived from normal conditional distributions.

$$\begin{aligned} E^B\left[\widetilde{X}\middle|S\right] &= \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * \left(S - \mu_\epsilon^B\right)}{\rho_X^B + \rho_{\mathcal{E}}^B} \\ \mathrm{VAR}^B\left[\widetilde{X}\middle|S\right] &= \left(\rho_X^B + \rho_{\mathcal{E}}^B\right)^{-1} \end{aligned} \tag{2.12}$$

The type-A ambiguity averse/lover investors believe that the signal S comprises the risky-payoff $\widetilde{X}$ information, an ambiguous model-dependent component $\widetilde{\delta}$ and a measurement bias or error $\widetilde{\epsilon}$. $S = \widetilde{X} + \widetilde{\delta} + \widetilde{\epsilon}$. Based on this assumption, this investor type consistently updates their beliefs about the mean and volatility of the future risky-asset payoff $\widetilde{X}$, constructing the conditional M model-dependent posterior beliefs $E^A\left[\widetilde{X}\middle|S, M\right]$ and $\mathrm{VAR}^A\left[\widetilde{X}\middle|S, M\right]$, and the unconditional posterior beliefs $E^A\left[\widetilde{X}\middle|S\right]$ and $\mathrm{VAR}^A\left[\widetilde{X}\middle|S\right]$. These posterior beliefs are also based on normal conditional distributions.

$$\begin{aligned} E^A\left[\widetilde{X}\middle|S\right] &= \frac{\rho_X^A * \mu_X^A + \rho_{\mathcal{E}}^A * \left(S - \mu_\delta^A - \mu_\epsilon^A\right)}{\rho_X^A + \rho_{\mathcal{E}}^A} \\ Var^A[\widetilde{X}|S] &= \left[\rho_X^A + \rho_{\mathcal{E}}^A\right]^{-1} \end{aligned} \tag{2.13}$$

$$\begin{aligned} E^A\left[\widetilde{X}\middle|S, M\right] &= \frac{\rho_X^A * \mu_X^A + \rho_\epsilon^A * \left(S - \widetilde{\delta} - \mu_\epsilon^A\right)}{\rho_X^A + \rho_\epsilon^A} \\ Var^A[\widetilde{X}|S, M] &= \frac{1}{\rho_X^A} - \frac{\rho_S^A * \frac{\rho_\delta^{2\,A}}{\rho_X^{2\,A}}}{\rho_\delta^{2\,A} - \rho_S^{2\,A}} \end{aligned}$$

2.5 Market Equilibrium in Period 2

After integrating the information from signal S into their beliefs, both investor types update their portfolio compositions. The ambiguity neutral type-B investors update their risky asset allocations θ_{t2}^B following the expected utility maximization below.

$$\max_{\theta_{t2}^B} -e^{-\gamma*\left(w_{t2}^B \;+\; \theta_{t2}^B* \;\left(E^B\left[\widetilde{X}\middle|S\right]-P_{t2}\right)\right) \;+\; \frac{1}{2}*\gamma^2*\theta_{t2}^{2B}*\mathrm{VAR}^B[\widetilde{X}|S]} \tag{2.14}$$

The resulting optimal risky asset allocation of the type-B investors is given by the following expression.

$$\theta^B_{t2} = \frac{E^B[\widetilde{X}|S] - P_{t2}}{\gamma * \text{VAR}^B[\widetilde{X}|S]}$$

$$\theta^B_{t2} = \frac{\rho^B_X * \mu^B_X + \rho^B_{\mathcal{E}} * \ \left(S - \mu^B_\epsilon\right) - P_{t2} * \left(\rho^B_X + \rho^B_{\mathcal{E}}\right)}{\gamma}$$

The ambiguity averse/loving type-A investors update their portfolios according to their Smooth Ambiguity utility functions, employing various M models to assess the information from signal S. This investor type optimizes their portfolios according to the maximization of the following expected utility.

$$\max_{\theta^A_{t2}} w^A_{t2} \ + \ \theta^A_{t2} * \ \left(E^A[\widetilde{X}|S] - P_{t2}\right) - \tfrac{1}{2} * \gamma * \theta^{2A}_{t2} * \text{VAR}^A[\widetilde{X}|S] * \left[1 + (\gamma_a - 1) * \left(\tfrac{\text{VAR}^A[\widetilde{X}|S] - \text{VAR}^A[\widetilde{X}|S,M]}{\text{VAR}^A[\widetilde{X}|S]}\right)\right] \tag{2.15}$$

The optimal risky asset allocation $\theta^{I_A}_{t2}$ of the type-A investors is given by the following expression.

$$\theta^A_{t2} = \frac{E^A[\widetilde{X}|S] - P_{t2}}{\gamma * Var^A[\widetilde{X}|S] * \nu^A}$$

$$\theta^A_{t2} = \frac{\rho^A_X * \mu^A_X + \rho^A_{\mathcal{E}} * (S - \mu^A_\epsilon - \mu^A_\delta) - P_{t2} * (\rho^A_X + \rho^A_{\mathcal{E}})}{\gamma * \nu^A}$$

$$where$$

$$\nu^A = \left[1 + (\gamma_a - 1) * \left(\frac{\text{VAR}^A[\widetilde{X}|S] - \text{VAR}^A[\widetilde{X}|S, M]}{\text{VAR}^A[\widetilde{X}|S]}\right)\right]$$

$$\frac{Var^A[\widetilde{X}|S] - Var^A[\widetilde{X}|S, M]}{Var^A[\widetilde{X}|S]} = \frac{\rho^A_X * \rho^{A\,3}_\epsilon}{(\rho^A_\delta * \rho^A_\epsilon + 2 * \rho^A_X * \rho^A_\epsilon + \rho^A_X * \rho^A_\delta) * (\rho^A_X + \rho^A_\epsilon) * (\rho^A_\epsilon + \rho^A_\delta)}$$

At the end of period 2, after both investor types have optimized their portfolios based on the new information from S, their risky asset demands will come together in the market, establishing a market clearing price in equilibrium. The aggregate demand of the ambiguity neutral type-B investors is denoted by $(1 - \pi) * \theta^B_{t2}$, and the demand of the ambiguity-averse/loving type-A investors is denoted by $\pi * \theta^{I_A}_{t2}$. The zero net supply risky-asset market clears at time 2 according to the following equation.

$$(1 - \pi_A) * \theta^B_{t2} + \pi_A * \theta^{I_A}_{t2} = 0 \tag{2.16}$$

The market clearing price at time 2, price P_{t2}, is given by the expression below.

$$\mathrm{P}_{t2} = \frac{\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * \ \mathrm{S} - \bar{\mu}_{\epsilon} - \dfrac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_{\delta}^A}{\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}} \tag{2.17}$$

$$\begin{aligned} &where \\ \bar{\mu}_X &= \frac{\pi}{\nu^A} * \rho_X^A * \mu_X^A + (1-\pi) * \rho_X^B * \mu_X^B \\ \bar{\mu}_{\epsilon} &= \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_{\epsilon}^A + (1-\pi) * \rho_{\mathcal{E}}^B * \mu_{\epsilon}^B \\ \bar{\rho}_X &= \frac{\pi}{\nu^A} * \rho_X^A + (1-\pi) * \rho_X^B \\ \bar{\rho}_{\mathcal{E}} &= \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A + (1-\pi) * \rho_{\mathcal{E}}^B \end{aligned}$$

The expressions $\bar{\mu}_X$ and $\bar{\mu}_{\epsilon}$ represent population and precision-weighted averages of both investor types prior beliefs about the risky asset payoff $\widetilde{X}$, and the signal measurement errors, respectively. μ_{δ}^A in the numerator represents the mean bias effect of the ambiguity expectations channel on the price P_{t2}. On the other hand, $\bar{\rho}_X$ and $\bar{\rho}_{\mathcal{E}}$ denote population-weighted averages of the two investor types prior beliefs precisions regarding the risky asset payoff $\widetilde{X}$ and the signal S total error.

2.6 Trading Volume Expression

In this section, I present the main results of this work: the relationship between trading volume, ambiguity and price changes. I will begin by introducing a simplified model version 1, which omits differences in prior beliefs and the influence of the price channel on trading volume. Following that, I will delve into a model version 2 that incorporates the price channel of trading volume. Finally, I will conclude with a full model version 3, which encompasses ambiguity, differences in prior beliefs and the impact of the price channel on trading volume.

Regarding the ambiguity mechanism driving trading volume in this model, put plainly, when signal S conveys fresh information, it spurs investors to trade the risky asset in order to align their portfolios with their updated beliefs. Despite both investor types receiving identical signals, ambiguity introduces heterogeneity, giving rise to distinct posterior beliefs about the risky asset's payoff ($\widetilde{X}$). This heterogeneity, induced by ambiguity, is the driving force behind this trading activity.

To derive the closed form trading volume model presented below, I utilize the change in allocation of the risky asset for one investor type, multiplied by the proportion of these investors in the economy. In this market with zero net supply, the buying activity of one investor type corresponds to the selling activity of the other, and vice versa.

$$\left|(1-\pi) * \Delta\theta^{I_B}\right| = \left|\pi * \Delta\theta^{I_A}\right| \tag{2.18}$$

$$where$$

$$\Delta\theta^{I_A} = \theta_{t2}^{I_A} \ - \ \theta_{t1}^{I_A}$$

$$\Delta\theta^{I_B} = \theta_{t2}^{I_B} \ - \ \theta_{t1}^{I_B}$$

Based on these symmetrical trading volume expressions, I arrive at the model below. Additional details provided in Appendix A. Details of the derivation of a similar trading volume model using Max Min utility in Appendix B.

Version 1: Only Ambiguity Channel

Market Trading Volume V_{21} from period 1 to period 2, only ambiguity channel present while price channel and heterogeneous prior beliefs switched off.

$$V_{21} = |\alpha_V| \tag{2.19}$$

$$where$$

$$\alpha_V = \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \mu_\delta^A$$

In this simplified model version, trading volume is directly linked to the bias effect of μ_δ^A, originating from the influence of the expectations channel of ambiguity on the posterior beliefs held by type-A investors. This effect is further modulated by the factor F_1. It is worth highlighting that both the numerator and denominator of F_1 encompass the inverse of the coefficient ν^A and the precision term $\rho_{\mathcal{E}}^A$ from type-A investors. The value of ν^A is determined by the level of ambiguity aversion γ_a, and similarly, both the precision term $\rho_{\mathcal{E}}^A$ and ν^A depend on the ambiguity volatility $\sigma_\delta^{2\,A}$.

As illustrated in the top panel of Figure 1 below, the factor F_1 exhibits a decreasing trend as ambiguity aversion γ_a increases. It reaches its minimum value of zero for type-A investors characterized by extreme ambiguity aversion γ_a approaching infinity, and conversely, it attains a maximum positive value for those investors who favor ambiguity with γ_a values close to 0. The intuition here is that as ambiguity aversion grows, type-A investors tend to avoid risky assets, regardless of signal related expectations, including those associated with ambiguity. In this simplified scenario, since both types of investors shared the same prior beliefs at time 1, they did not take any positions in the risky asset. Consequently, at time 2, an increased level of ambiguity aversion leads type-A investors to stay out of the market, resulting in a lack of trading activity.

The lower panel of Figure 1 shows that the factor F_1 diminishes as the ambiguity volatility $\sigma_\delta^{2\,A}$ increases. F_1 reaches its peak when the ambiguity volatility $\sigma_\delta^{2\,A}$ approaches zero, and it approaches zero as the ambiguity volatility tends towards infinity. Here the intuition is that as ambiguity volatility rises, the signal-to-noise ratio of S decreases. This leads to a disregard of all information contained in signal S because of its low quality, including the expectations associated to ambiguity.

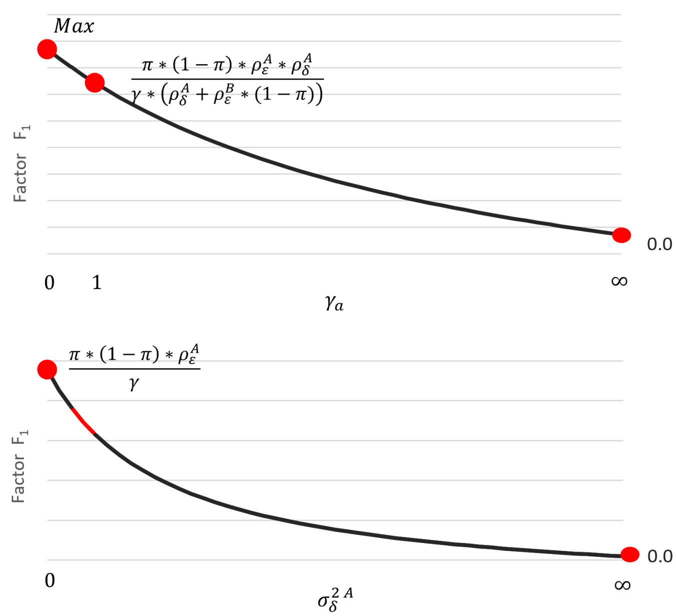

Figure 1. Effect of γ_a and $\sigma_\delta^{2\,A}$ on factor F_1 . This graph shows how the the values of γ_a and $\sigma_\delta^{2\,A}$ affect the factor F_1 multiplying μ_δ^A. The top panel achieves a maximum of $\frac{\rho_\delta^A*\rho_\epsilon^A*\pi*(1-\pi)*\left(\rho_\epsilon^A+\rho_X^A\right)*\left(2\rho_\epsilon^A*\rho_X^A+\rho_\delta^A*(\rho_\epsilon^A+\rho_X^A)\right)}{\gamma*\left(\rho_\delta^{A2}*\left(\rho_\epsilon^A+\rho_X^A\right)+\rho_\epsilon^{A2}*(1-\pi)*\rho_X^A*\left(\rho_\epsilon^A+2\rho_X^A\right)+\rho_\delta^A*\rho_\epsilon^A*\left(\rho_\epsilon^A+\rho_X^A\right)*\left(\rho_\epsilon^A*(1-\pi)+\rho_X^A*(3-\pi)\right)\right)}$ when γ_a goes to 0, a value of $\frac{\pi*(1-\pi)*\rho_\epsilon^A*\rho_\delta^A}{\gamma*\left(\rho_\delta^A+\rho_\epsilon^B*(1-\pi)\right)}$ when γ_a is 1 and a value of 0 when γ_a tends to infinity.

Version 2: Ambiguity Channel and Price Channel

Market trading volume V_{21} from period 1 to period 2, when the ambiguity channel and the price channel are present while heterogeneity in prior beliefs is switched off.

$$V_{21} = |\alpha_V + \beta_V * \Delta P_{21}| \tag{2.20}$$

where

$$\alpha_V = \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \mu_\delta^A$$

$$\beta_V = \left[\frac{\frac{\pi}{\nu^A} * \rho_X^A * \left(\rho_{\mathcal{E}}^B - \rho_{\mathcal{E}}^A\right)}{\bar{\rho}_{\mathcal{E}}}\right]_{F_2}$$

In this model version, trading volume is determined by the combination of two drivers: the bias effect μ_δ^A resulting from the expectations channel of ambiguity, and the trading volume linked to price changes. The impact of price changes is then scaled by β_V, which incorporates both the inverse of the type-A investors coefficient ν^A and the precision term $\rho_{\mathcal{E}}^A$ in its numerator and denominator. The value of the coefficient ν^A hinges on both the degree of ambiguity aversion γ_a and the ambiguity volatility d $\sigma^{2\,A}$. Similarly, the precision term $\rho_{\mathcal{E}}^A$ also depends on the ambiguity volatility $\sigma^{2\,A}$. This last interdependence creates a distinction between $\rho_{\mathcal{E}}^B$ and $\rho_{\mathcal{E}}^A$, even when the prior beliefs ex ambiguity about the signal S and the measurement error are homogeneous.

As shown in the upper panel of figure-2 above, this last introduced factor F_2 is positive and declines as ambiguity aversion γ_a increases. It reaches its lowest point at zero for type-A investors characterized by extreme ambiguity aversion with γ_a tending towards infinity. Conversely, for investors who embrace ambiguity and possess γ_a values close to 0, this factor F_3 attains its maximum value. The intuition here echoes the previous version. As ambiguity aversion increases, type-A investors choose to steer clear of the risky asset, preserving their initial risk-free asset allocation established at time 1 under homogeneous prior beliefs.

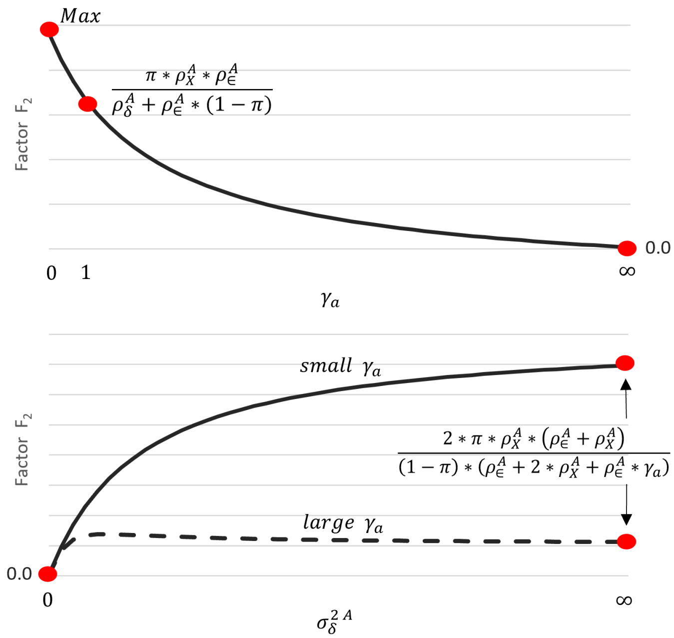

Figure 2. Effect of γ_a and $\sigma_\delta^{2\,A}$ on factor F_2 . This graph shows how the the values of γ_a and $\sigma_\delta^{2\,A}$ affect the factor F_2 multiplying ΔP_{21}. The top panel achieves a maximum of $\frac{\pi*\rho_\epsilon^A*\rho_X^A*\left(\rho_\epsilon^A+\rho_X^A\right)*\left(2*\rho_\epsilon^A*\rho_X^A+\rho_\delta^A*(\rho_\epsilon^A+\rho_X^A)\right)}{\gamma*\left(\rho_\delta^{A\,2}*\left(\rho_\epsilon^A+\rho_X^A\right)^2+\rho_\epsilon^{A\,2}*(1-\pi)*\rho_X^A*\left(\rho_\epsilon^A+2*\rho_X^A\right)+\rho_\delta^A*\rho_\epsilon^A*\left(\rho_\epsilon^A+\rho_X^A\right)*\left(\rho_\epsilon^A*(1-\pi)+\rho_X^A*(3-\pi)\right)\right)}$ when γ_a goes to 0, and a value of $\frac{\pi*\rho_X^A*\rho_\epsilon^A}{\rho_\delta^A+\rho_\epsilon^A*(1-\pi)}$ when γ_a is 1.

The lower panel of Figure 2, shows that the behavior of the factor F_2 varies for different levels of ambiguity volatility and is influenced by the degree of ambiguity aversion γ_a. For small values of γ_a, the factor F_2 increases with increasing ambiguity volatility, eventually reaching the maximum value as depicted in the figure. Conversely, for larger values of γ_a, the factor F_2 exhibits a more complex behavior, characterized by two distinct regions. Initially, in the first region, this factor increases with ambiguity volatility, but it then reverses its trend and decreases until it reaches a limit when the ambiguity volatility tends to infinity, as illustrated on the right side of the figure. Here, the intuition is that as ambiguity volatility rises, type-A investors who have a preference for ambiguity ($\gamma_a < 1$) become more inclined to trade the risky asset in response to changes in market conditions as reflected by prices. Conversely, for ambiguity-averse investors with larger ambiguity aversion ($\gamma_a > 1$), an increase in ambiguity volatility motivates type-A investors to maintain their risk-free asset holdings from time 1 established under homogeneous prior beliefs, reducing their inclination to trade as ambiguity volatility intensifies.

Version 3: Ambiguity channel, price channel and heterogeneous prior beliefs

Full model of market trading volume V_{21} from period 1 to period 2, when ambiguity channel, price channel and heterogeneous initial beliefs are present.

$$V_{21} = |\alpha_V + \beta_V * \Delta P_{21}| \tag{2.21}$$

$$\textit{where}$$

$$\begin{aligned}
\alpha_V = & \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \mu_\delta^A \\
& + \left[\pi * (1-\pi)^2 * \left(\frac{1}{\nu^A} - 1\right) * \left(\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}} * \gamma}\right) * \left(\frac{\rho_X^A * \rho_X^B}{\bar{\rho}_{\substack{X \\ t1}}}\right)\right]_{F_3} * \left(\mu_X^B - \mu_X^A\right) \\
& + \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \left(\mu_\epsilon^A - \mu_\epsilon^B\right)
\end{aligned}$$

$$\beta_V = \left[\frac{\frac{\pi}{\nu^A} * \rho_X^A * \left(\rho_{\mathcal{E}}^B - \rho_{\mathcal{E}}^A\right)}{\bar{\rho}_{\mathcal{E}}}\right]_{F_2}$$

In this full version of the model, the trading volume is proportional to the bias effect μ_δ^A created by the expectations channel of ambiguity, the difference in prior beliefs about the risky asset payoff $\left(\mu_X^B - \mu_X^A\right)$, the difference in prior beliefs about the measurement error $\left(\mu_\epsilon^A - \mu_\epsilon^B\right)$, plus the trading volume associated to the change in price. The difference in prior beliefs about the risky asset $\left(\mu_X^B - \mu_X^A\right)$ gets multiplied by the factor F_3 which has on the denominator the contribution of the inverse of the coefficient ν^A and the type-A investor precision $\rho_{\mathcal{E}}^A$ that's averaged inside the term $\bar{\rho}_{\mathcal{E}}$. The difference in prior beliefs about the measurement error $\left(\mu_\epsilon^A - \mu_\epsilon^B\right)$ gets multiplied by the factor F_1 previously analyzed, factor that decreases when either ambiguity aversion γ_a or ambiguity volatility $\sigma_\delta^{2\,A}$ increases.

As illustrated in the upper panel of Figure 3 below, the newly introduced factor F_3, which multiplies the difference in prior beliefs regarding the risky asset payoff $\left(\mu_X^B - \mu_X^A\right)$, exhibits a decreasing pattern with respect to ambiguity aversion γ_a. Specifically, it starts at a positive maximum when γ_a is 0, then crosses through zero when γ_a equals 1, and ultimately approaches a negative limit when γ_a tends to infinity. Due to the absolute value operator in the final trading volume expression, this behavior gives rise to two distinct regions. The first region corresponds to γ_a values within the interval $[0, 1]$, where the factor F_3 decreases gradually, reaching zero as ambiguity aversion increases toward 1. In the second region, for γ_a values in the range $[1, \infty]$, the factor F_3 increases steadily, ultimately reaching the absolute value of the limit depicted in the figure. The underlying intuition here is that as ambiguity aversion decreases from 1 to 0 in the initial region, type-A investors will increase the size of their positions in the risky asset. The direction of this adjustment is driven by the difference in net expectations between type-A and type-B investors, which incorporates their difference in beliefs regarding the risky asset's payoff $(\mu_X^B - \mu_X^A)$. In the subsequent region, characterized by an increase in ambiguity aversion from 1 to infinity, type-A investors tend to exit the risky asset market. They liquidate their positions in the risky asset, initially established in period 1, resulting in higher trading activity. In instances of extreme ambiguity aversion, type-A investors divest their entire position established in period 1, regardless of the information contained in the signal S.

The lower panel of Figure 3 illustrates how the factor F_3 behaves concerning ambiguity volatility $\sigma_\delta^{2\,A}$ and ambiguity aversion γ_a. When γ_a is bigger than one, an increase in ambiguity volatility translates in an increase of F_3, whereas values of γ_a below one lead to a decrease in F_3 with rising ambiguity volatility. When γ_a equals one, F_3 remains at zero. Furthermore, due to the absolute value operator in the final trading volume formula, an overall surge in ambiguity volatility results in an elevated F_3 as long as γ_a is not zero. The intuition here is that for investors who embrace ambiguity with γ_a below 1, as ambiguity volatility increases, they will seek to increase the size of their positions in the risky asset. Conversely, for averse ambiguity investors with γ_a greater than one, rising ambiguity volatility encourages them to liquidate their initial non zero position in the risky asset. In terms of trading volume, both scenarios lead to increased trading activity. However, for investors who are ambiguity-neutral with γ_a equal to 1, this ambiguity volatility mechanism has no impact.

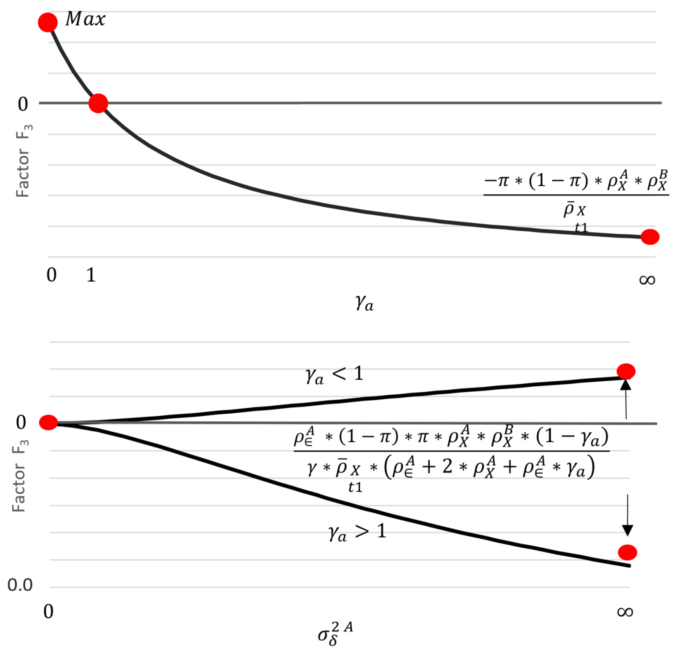

Figure 3. Effect of γ_a and $\sigma_\delta^{2\,A}$ on factor F_3 . This graph shows how the the values of γ_a and $\sigma_\delta^{2\,A}$ affect the factor F_3 multiplying $(\mu_X^B - \mu_X^A)$. The top panel achieves a maximum of $\frac{\pi*(1-\pi)^2*\rho_\epsilon^{A\,3}*\rho_\epsilon^B*\rho_X^{A\,2}*\rho_X^B}{\gamma*\left(\rho_\delta^{A\,2}*\bar{\rho}_{\underset{t1}{X}}*\left(\rho_\epsilon^A+\rho_X^A\right)^2+\rho_\epsilon^{A\,2}*\rho_\epsilon^B*(1-\pi)*\rho_X^A*\left(\rho_\epsilon^A+2*\rho_X^A\right)+\rho_\delta^A*\rho_\epsilon^A*\left(\rho_\epsilon^A+\rho_X^A\right)*\left(3*(1-\pi)*\rho_\epsilon^B*\rho_X^A+\rho_\epsilon^A*(\pi*\rho_X^A+\bar{\rho}_{\underset{t1}{X}})\right)\right)*\bar{\rho}_{\underset{t1}{X}}}$ when γ_a goes to 0, a value of 0 when γ_a is 1 and a value of $\frac{-\pi*(1-\pi)*\rho_X^A*\rho_X^B}{\bar{\rho}_{\underset{t1}{X}}}$ when γ_a tends to infinity. The bottom panel shows the effect of the ambiguity volatility on the factor F_3 for different levels of ambiguity aversion.

2.7 Ambiguity and the Trading Volume to Price Volatility Elasticity

In this section, I examine the influence of ambiguity on the elasticity relationship between trading volume and price volatility. To derive this elasticity relationship, I make the assumption that, on average, changes in price ΔP_{21} follow a normal distribution, akin to the approach taken by Bollerslev et al. (2018) for differences in prior beliefs. I begin by presenting a simplified version 1 of the elasticity relationship, which includes both ambiguity and the price channel of trading volume. In the complete version 2 of the elasticity relationship, I introduce heterogeneity in prior beliefs. Further details can be found in Appendix C.

Version 1: Price and Ambiguity Channel

Elasticity ξ between trading volume and price volatility when the price and ambiguity channels are present, while heterogeneous prior beliefs are switched off.

$$\xi = \frac{\partial E[V_{21}]/E[V_{21}]}{\partial \sigma_p/\sigma_p} = \frac{1}{1 + \Psi\left(\dfrac{|\alpha_v|}{|\beta_v| * \sigma_p}\right)} \tag{2.22}$$

$$\begin{aligned}
&where \\
&\Psi(x) = \frac{x * (\Phi(x) - 1/2)}{\phi(x)} \\
&\alpha_V = \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho^A_{\mathcal{E}} * \rho^B_{\mathcal{E}}}{\gamma * \bar{\rho}_{\mathcal{E}}}\right] * \mu^A_\delta \\
&\beta_V = \left[\frac{\frac{\pi}{\nu^A} * \rho^A_X * \left(\rho^B_{\mathcal{E}} - \rho^A_{\mathcal{E}}\right)}{\bar{\rho}_{\mathcal{E}}}\right] \\
&and \\
&\Phi(x) = \text{Normal CDF} \\
&\phi(x) = \text{Normal Density}
\end{aligned}$$

In this simplified version, the elasticity is influenced by a combination of two main drivers: the impact of the the expectations channel of ambiguity within the term α_V, channel represented by μ^A_δ in the model, and the influence of price changes on trading volume, associated to the term β_V. Both α_V and β_V include the inverse of the coefficient ν^A, as well as the precision term $\rho^A_{\mathcal{E}}$ in their numerators and denominators. Ambiguity aversion, denoted as γ_a, affects this elasticity through the ν^A coefficient, while ambiguity volatility also plays a role by affecting both the ν^A coefficient and the precision term $\rho^A_{\mathcal{E}}$.

The top panel of Figure 4 shows that when the magnitude of the expectation channel bias generated by ambiguity, represented as μ^A_δ, is large in either positive or negative terms, the elasticity decreases. The intuition here is that when the expectation channel of ambiguity becomes the predominant driver of trading activity, trading volume appears increasingly disconnected from price fluctuations. In the most extreme scenarios, the elasticity approaches zero.

The middle panel of the figure shows that ambiguity aversion γ_a has no impact on elasticity. This is because ambiguity aversion affects both α_V and β_V in equal proportions. As a result, any alterations in ambiguity aversion do not change the relative importance between the expectations channel of ambiguity and price changes as drivers of trading volume.

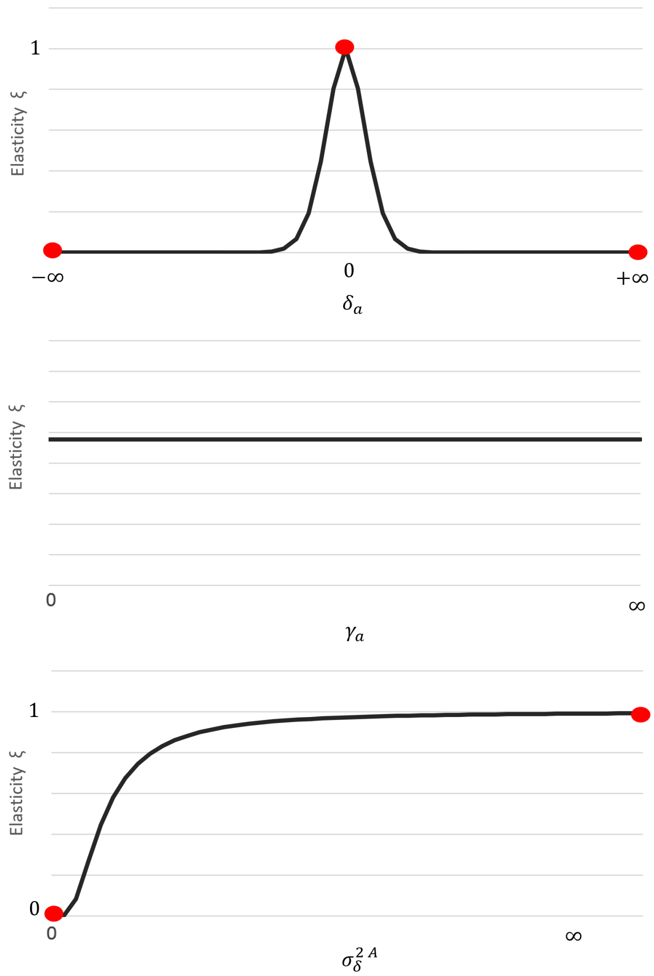

Figure 4. Effect of γ_a and $\sigma_\delta^{2\,A}$ on Elasticity. This graph shows how the values of γ_a and $\sigma_\delta^{2\,A}$ affect the elasticity ξ between trading volume and price volatility.

The bottom panel of Figure 4 shows that as ambiguity's volatility increases, the elasticity ξ tends to its maximum value of 1. The underlying intuition here is that heightened ambiguity volatility diminishes the quality of the signal S, which includes the effects stemming from the expectations channel of ambiguity μ_δ^A. As ambiguity volatility approaches infinity, it dampens the expectations channel of ambiguity as a driver of trading volume. Consequently the primary force influencing trading volume becomes price changes alone. This is why, in extreme volatility, elasticity converges to a one-to-one relationship between trading volume and price volatility.

Version 2: Price channel, Ambiguity Channel and Heterogeneous Prior Beliefs

Full elasticity expression ξ between trading volume and price volatility when the price channel, ambiguity channel and heterogeneous prior beliefs are present.

$$\xi = \frac{\partial E[V_{21}]/E[V_{21}]}{\partial\sigma_p/\sigma_p} = \frac{1}{1+\Psi\left(\dfrac{|\alpha_v|}{|\beta_v| * \sigma_p}\right)} \tag{2.23}$$

$$\begin{aligned}
&\textit{where}\\
\Psi(x) &= \frac{x * (\Phi(x) - 1/2)}{\phi(x)}\\
\alpha_V &= \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho^A_{\mathcal{E}} * \rho^B_{\mathcal{E}}}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \mu^A_\delta\\
&+ \left[\pi * (1-\pi)^2 * \left(\frac{1}{\nu^A} - 1\right) * \left(\frac{\rho^B_{\mathcal{E}}}{\bar{\rho}_{\mathcal{E}} * \gamma}\right) * \left(\frac{\rho^A_X * \rho^B_X}{\bar{\rho}_{\substack{X\\t1}}}\right)\right]_{F_3} * \left(\mu^B_X - \mu^A_X\right)\\
&+ \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho^A_{\mathcal{E}} * \rho^B_{\mathcal{E}}}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_{F_1} * \left(\mu^A_\epsilon - \mu^B_\epsilon\right)\\
\beta_V &= \left[\frac{\frac{\pi}{\nu^A} * \rho^A_X * \left(\rho^B_{\mathcal{E}} - \rho^A_{\mathcal{E}}\right)}{\bar{\rho}_{\mathcal{E}}}\right]_{F_2}\\
&\textit{and}\\
\Phi(x) &= \text{Normal CDF}\\
\phi(x) &= \text{Normal Density}
\end{aligned}$$

In this full version of the elasticity expression, the introduction of heterogeneous prior beliefs results in two additional terms associated to $\left(\mu^B_X - \mu^A_X\right)$ and $\left(\mu^A_\epsilon - \mu^B_\epsilon\right)$. Here, the elasticity relating trading volume to price volatility is influenced by the bias effect μ^A_δ arising from the expectations channel of ambiguity, as well as the differences in prior beliefs regarding the risky asset payoff $\left(\mu^B_X - \mu^A_X\right)$ and the difference in prior beliefs concerning the measurement error $\left(\mu^A_\epsilon - \mu^B_\epsilon\right)$.

In terms of ambiguity related coefficients, the expectations channel of ambiguity μ^A_δ exclusively influences the first component of the α_V term. This effect can lead to a maximum elasticity of 1, which occurs when the bias caused by μ^A_δ neutralizes the impact of differences in prior beliefs regarding the risky asset payoff $\left(\mu^B_X - \mu^A_X\right)$ and the measurement error $\left(\mu^A_\epsilon - \mu^B_\epsilon\right)$ or when the expectations channel of ambiguity as well as heterogeneous prior beliefs are not present. This is illustrated in the top panel of Figure 5 below. The underlying intuition is that when the expectations channel of ambiguity becomes the dominant factor driving trading volume in any direction, the sensitivity of trading volume to price changes diminishes.

While the ambiguity aversion coefficient γ_a appears on the portions of α_V associated to the expectations channel of ambiguity μ_ϵ^A, the differences in prior beliefs about the measurement error $\left(\mu_\epsilon^A - \mu_\epsilon^B\right)$ and the β_V coefficient that multiplies changes in prices, it does mainly affect the elasticity through the differences in prior beliefs about the risky asset payoff $\left(\mu_X^B - \mu_X^A\right)$. The mechanism is that, as the ambiguity aversion increases, the effect of $\left(\mu_X^B - \mu_X^A\right)$ starts increasing and this contributes to decrease the heterogeneity if the sign of $\left(\mu_X^B - \mu_X^A\right)$ points in the same direction as the effect of $\left(\mu_\epsilon^A + \mu_\epsilon^A - \mu_\epsilon^B\right)$, or initially increase the net heterogeneity if the they point in the opposite directions. The intuition here is that as ambiguity aversion increases, type-A investors will tend to sell their assets regardless of market price conditions, thereby reducing elasticity. In extreme cases, they liquidate their entire period 1 positions, which are proportional to $\left(\mu_X^B - \mu_X^A\right)$, bringing the elasticity down to 0. Conversely, when ambiguity aversion decreases below 1, type-A investors aim to increase the magnitude of their positions in the risky asset independently of market price conditions, also resulting in decreased elasticity to price changes.

Changes in ambiguity volatility influence the impact of the ambiguity expectations channel δ_A, the impact of the difference in prior beliefs about the measurement error $\left(\mu_\epsilon^A - \mu_\epsilon^B\right)$, and the impact of the difference in the in prior beliefs about the risky asset payoff $\left(\mu_X^B - \mu_X^A\right)$. As ambiguity volatility increases, the net effects caused by δ_A and $\left(\mu_\epsilon^A - \mu_\epsilon^B\right)$ start to diminish. In extreme cases, the primary driving force of trading volume becomes price changes, leading to an elasticity that tends to approach 1. However, ambiguity volatility also impacts the influence of the difference in prior beliefs about the risky asset payoff $\left(\mu_X^B - \mu_X^A\right)$, causing the elasticity to deviate from its maximum as ambiguity volatility increases on the right hand side of the figure. The intuition here is that as ambiguity volatility increases, the impact of the signal S on type-A investors weakens, rendering the trading activity of type-A investors more responsive to market price conditions alone. This initially results in an upward trend in elasticity as ambiguity volatility increases. Additionally, there is a secondary effect associated with the sizing of type-A investors' positions, which depends on their ambiguity aversion. When ambiguity volatility rises, ambiguity-averse type-A investors will seek to reduce the magnitude of their positions, while ambiguity-loving type-A investors will seek to increase them, regardless of market conditions like prices. This second source of trading activity, not directly linked to price conditions, causes the elasticity to decrease. In cases of extreme ambiguity volatility, this latter effect takes precedence, reducing the elasticity towards 0.

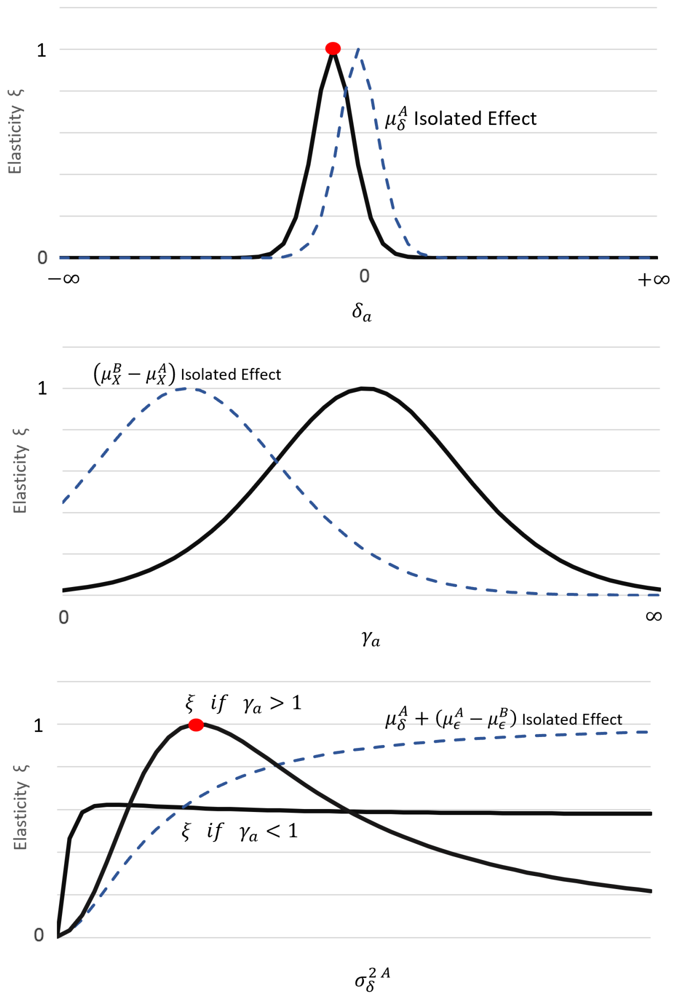

Figure 5. Effect of μ_δ^A, γ_a and $\sigma_\delta^{2\,A}$ on Elasticity. This graph shows how the values of μ_δ^A, γ_a and $\sigma_\delta^{2\,A}$ affect the elasticity ξ between trading volume and price volatility.

3 Hypothesis

I test the following four hypothesis regarding the role of the expectations channel of ambiguity in generating trading volume.

H1: I anticipate that trading volume increases at daily and monthly frequencies as the expectations channel of ambiguity strengthens. This is based on the notion that the expectations channel of ambiguity introduces heterogeneity in investors' posterior beliefs, stimulating trading activity.

H2: I anticipate that the impact of the expectations channel of ambiguity on trading volume diminishes as ambiguity volatility rises. See bottom panel of Figure 1. The rationale behind this is that heightened ambiguity volatility deteriorates the signal-to-noise ratio of public signals, thus dampening the expectations channel of ambiguity.

H3: I anticipate that, on average, the expectations channel of ambiguity reduces the elasticity between price volatility and trading volume. See the top panel of Figure 5. The intuition here is that as the expectations channel of ambiguity gains prominence in driving trading volume, the connection between trading volume dynamics and price changes weakens in relative terms, resulting in a decreased elasticity with respect to price volatility.

H4: In the context of portfolios constructed by sorting on stocks turnover and considering the impact of the expectations channel of ambiguity on trading activity, I anticipate that turnover influenced by information ambiguity significantly contributes to explain the positive returns observed in these portfolios.

In the following Section 6, I will assess hypotheses H1 and H2. In Section 5, I will examine hypothesis H3, and in Section 7 I will evaluate hypothesis H4.

4 Empirical Evidence: Data

The empirical validation of the model proposed in this work relies on data comprising price and trading volume series of the SPY ETF, along with measures of ambiguity and differences in prior beliefs.

Prices and Trading Volume

The SPY prices and trading volume data at daily and monthly frequencies is sourced from Bloomberg. Daily regressions utilize data spanning from 2013 to 2018, while monthly regressions cover the period from 2000 to 2020. Monthly regressions relying on daily frequency data cover the period 2013 to 2018.

Intraday calculations utilize 15-minute sampled SPY TAQ data from NASDAQ to construct daily frequency α_V and β_V coefficients.

Ambiguity

This research employs two ambiguity measures: the news based Economic Policy Uncertainty Index (EPU)(Baker et al., 2016) and the market price based measure of Izhakian (2020).

I utilize both the daily (Baker et al., 2021) and monthly (Baker et al., 2016) versions of the Economic Policy Uncertainty Index. The daily measure (TEU-SCA) represents an ambiguity index extracted daily from Twitter text messages up to 4 pm U.S. EDT time, while the monthly measure corresponds to the original EPU measure extracted from U.S. newspapers. The daily measure spans from 2013 to 2018, and the monthly measure covers the period from 2000 to 2020. In the rest of the text I identify these series by AMB_{EPUD} and AMB_{EPUM} respectively.

The Izhakian (2020) ambiguity measure represents a market price based indicator designed to approximate the implicit dispersion of investors' beliefs. Unlike the previous type of measures, this index is directly extracted from prices rather than from text and correlates negatively with the EPU measure. To maintain consistency in the economic interpretation of both measures' results, I inverted the sign of the Izhakian (2020) measure. The series used in this research is of monthly frequency and covers the period from 2000 to 2020. In the rest of the text I identify this serie by AMB_{IZHM}.

Differences in Prior Beliefs

As proxies for differences in prior beliefs, I employ two types of a daily disagreement measures extracted from the social media investing platform StockTwits (Cookson & Niessner, 2020) and a monthly analysts' forecast dispersion measure extracted from the IBES database.

The two types of daily investor disagreement are based on data collected from the social media investing platform StockTwits by Cookson and Niessner (2020) on a daily basis up to 4pm U.S. EDT time from 2013 to 2018. Both series types represent the standard deviation of bullishness/bearishness beliefs across users of the StockTwits platform. The users bullishness/bearishness information is extracted from messages posted on the platform. For messages not explicitly tagged as bullish or bearish by the users, their text content is used to infer their sentiment. This text-based classification of unlabeled messages is performed by a maximum entropy machine learning algorithm (Cookson & Niessner, 2020). One disagreement series type is calculated within the same type of users' investment approach, and it is more closely related to differences in information sets (Cookson & Niessner, 2020). The other type is extracted across users with different investment approaches and is more likely caused by different ways or models of interpreting information (Cookson & Niessner, 2020). In the following sections I refer to these two types of measures by $PBEL_{WI}$ and $PBEL_{AC}$ respectively. For both types of measures, I use a direct SPY extracted reading and a S&P 500 proxy, constructed as an equally weighted mean of the S&P 500 stocks' readings. In the rest of the text I identify these series by $\{PBEL_{WI,ETF}, PBEL_{AC,ETF}\}$ and $\{PBEL_{WI,IND}, PBEL_{AC,IND}\}$ respectively. Monthly regressions are performed using the monthly means of these series.

The IBES proxy for differences in prior beliefs is a monthly equally weighted average of the individual S&P500 companies' forecast dispersion measures. Each company's forecast dispersion measure is calculated as the standard deviation across the most recent earnings forecasts, scaled by the corresponding company's monthly average price. This series covers the period from 2000 to 2020. In the rest of the text I identify this serie by $PBEL_{IBES}$.

5 Empirical Evidence: Trading Volume and Price Relation

In this section, I empirically test the trading volume model presented in section 2.6 using SPY data at both daily and monthly frequencies.

To validate hypotheses H1 and H2 that ambiguity influences trading volume, I analyze the statistical significance of the ambiguity time series to explain the α_V term of the trading volume-price relation and how they interact with ambiguity volatility. These ambiguity series represent the expectations channel of ambiguity. Additionally, I assess the statistical significance of ambiguity volatility in explaining the β_V term of the trading volume-price relation.

Methodology

I validate the theoretical model presented in section 2.6 through daily and monthly frequency regressions. These regressions feature the SPY ETF trading volume as the dependent variable and include as explanatory variables the SPY price change, proxies for the level of ambiguity, proxies for differences in prior beliefs, proxies for ambiguity volatility, the SPY price volatility and time controls. To explore the initial hypothesis H1, asserting that the expectations channel of ambiguity positively impacts trading volume, I evaluate the statistical significance of the regression coefficient $\{\alpha_{amb}\}$ linked to the mean level of ambiguity. Additionally, to assess the impact of ambiguity volatility within the β_V segment of the relation, I analyze the results associated to the regression coefficients $\{\beta_{\sigma_{amb}}, \beta_{\Delta\sigma_{amb}}\}$.

$$
\begin{aligned}
V &= \alpha_V + \beta_V * \Delta_P + \epsilon \qquad (5.1)\\
\alpha_V &= c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon\\
\beta_V &= c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P + \varsigma_{t-1} * \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon
\end{aligned}
$$

$$
\begin{aligned}
V = &\left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\right)_{\alpha} \;+\; \left(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\right)_{\beta} * \\
&\Delta_P \;+ \varsigma_{t-1} * V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon
\end{aligned} \qquad (5.2)
$$

$$
\begin{aligned}
\Delta V = c \;+\; &\left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\right)_{\Delta\alpha} \;+\; \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \right.\\
&\left.\Delta\sigma_P\right)_{\Delta\beta} * \Delta_P \;+\; \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\right)_{\beta} * \Delta_P^2 \;+ \varsigma_{t-1} * \Delta V_{t-1} +\\
&\textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon
\end{aligned} \qquad (5.3)
$$

In regression (5.1), I begin the analysis by regressing the intraday trading volume on price changes, resulting in daily time series of α_V and β_V. Following the theoretical model, I then conduct regressions of the daily α_V time series on the series proxying for ambiguity and difference in prior beliefs. The ambiguity series accounts for the mean level of ambiguity μ_δ^A within the model. Simultaneously, I regress the β_V coefficient on the SPY price volatility and the ambiguity volatility series. The SPY price volatility series serves as a proxy for the prior beliefs precisions $\rho_X^{I_B}$ and ρ_X^A in the model, while the ambiguity volatility series acts as a proxy for the ambiguity volatility $\sigma^{2\,A}\delta$ within the model's coefficient $\rho_{\mathcal{E}}^A$. The daily time series of α_V and β_V are derived from 15-minute SPY TAQ data, while the monthly trading volumes and prices series are obtained from Bloomberg.

In the second regression (5.2), I directly regress the trading volume on the explanatory series that make up the α_V and β_V coefficients of the model. No prior regression is used to obtain time series for α_V and β_V.

The final regression (5.3) is a differenced version of regression (5.2), where I regress changes in trading volume on the changes of the explanatory variables that, according to the model, account for trading volume.

In all regressions, I incorporate a lagged dependent variable to address trading volume persistence and utilize yearly time controls γ_p. Furthermore, all series were detrended beforehand.

To validate hypothesis H2, which focuses on the smoothing effect of ambiguity volatility on the expectations channel of ambiguity as volatility increases, I include in Appendices E and G the coefficient $\{\alpha_{amb\sigma}\}$ in the regressions (5.1), (5.2) and (5.3). This coefficient captures the interaction between the expectation level of ambiguity and its volatility.

Furthermore, to address the varying impact of ambiguity volatility on the β_V coefficient at elevated levels of ambiguity volatility, Figure 2, I also include in Appendices E and G the regression coefficients $\{\beta_{h\sigma_{amb}}, \beta_{h\Delta\sigma_{amb}}\}$. These coefficients account for the effect of ambiguity volatility on β_V under states where ambiguity volatility exceeds its 50% quantile.

These regressions encompass all combinations of difference in beliefs and ambiguity series, as described in Section 4. For daily frequency, I utilize four distinct measures of differences in beliefs sourced from Cookson and Niessner (2020) and one measure of ambiguity (EPU) (Baker et al., 2016, 2021). Both information sources are derived from text rather than market prices, yielding four unique explanatory daily datasets for each regression. More details in Appendix D. In the context of monthly regressions, I use five different series of differences in prior beliefs and two series of ambiguity, resulting in a total of 10 distinct explanatory datasets for each regression. Among the differences in prior beliefs series, four are derived by converting the daily measures from Cookson and Niessner (2020) into monthly frequency through averaging, while the fifth serie is extracted from the IBES database. The two ambiguity series consist of the monthly version of the EPU index (Baker et al., 2016) and the monthly market based measure from Izhakian (2020).

Results Summary

The regression results reveal that the ambiguity level, denoted as μ_δ^A in the model and represented by the coefficients α_{amb} in the regressions, is statistically significant in explaining the α_V component of the volume-price relationship across all regression types, dataset combinations, and frequencies (daily and monthly). The findings suggest that as ambiguity levels increase, trading volume also increases. Furthermore, both daily and monthly results illustrate that this generally positive relation is negatively affected by ambiguity volatility. While the negative impact of ambiguity volatility is economically observable in all daily and monthly regressions, it achieves statistical significance only in the daily regressions (5.1) and (5.2), but not in daily regression

(5.3) and several monthly regressions. These results confirm hypothesis H1 and the average effect of hypothesis H2, though the data suggests that the latter effect may be relatively weaker. Analyzing the impact of ambiguity volatility on the β_V coefficient in the trading volume relation, daily frequency regressions (5.2) and (5.3) show an effect that amplifies the base magnitude of the β_V model coefficient, consistent with Figure 1 when ambiguity volatility levels are close to zero or ambiguity aversion is below one. However, the monthly regression results do not consistently exhibit this behavior. In summary, the statistical significance and magnitude of these findings indicate that, within the model's framework, the expectations channel of ambiguity plays a more substantial and consistent role in explaining trading volume when compared to ambiguity volatility. Moreover, while the influence of ambiguity volatility on trading volume is confined within a certain range, refer to Figures 1, 2, and 3, the expectations channel of ambiguity exerts a linear unbounded effect on trading volume.

H1 and H2 Results

In the context of hypothesis H1 concerning the impact of the ambiguity series representing mean ambiguity, specifically μ_δ^A in the model and coefficient $\{\alpha_{amb}\}$ in the regressions, daily frequency regressions (5.1), (5.2) and (5.3) for the period 2013-2018 reveal economically and statistically significant results. The EPU ambiguity measure shows t-stats ranging from 3.6 to 4.10 across regressions (5.1), (5.2) and (5.3). From 2013 to 2018, regression (1) in Table 1 indicates that a one-standard-deviation increase in the EPU daily ambiguity measure translates in an approximately 11%-standard-deviation increase in the coefficient α_V. For regression (5.2), directly measuring the impact of the explanatory variables on trading volume, a one-standard-deviation increase in ambiguity is linked to a 10%-standard-deviation increase in trading volume. Regression (5.3), utilizing variables in differences, suggests a 12%-standard-deviation increase in the delta of trading volume per one-standard-deviation increase in the delta of EPU daily ambiguity.

The monthly regressions also reveal both economically and statistically significant results. Regression (5.1) of Table 2 reveals that a one-standard-deviation rise in the EPU monthly ambiguity measure is associated with approximately a 23%-standard-deviation increase in the monthly coefficient α_V. Regression (5.2), directly measuring the impact of ambiguity on trading volume, shows an approximately 20%-standard-deviation increase in trading volume per one-standard-deviation rise in monthly EPU ambiguity. The first differences regression (5.3) suggests an effect ranging from a 19% to 23%-standard-deviation increase in the delta of trading volume per one-standard-deviation rise in the delta of EPU ambiguity. The effect using the Izhakian (2020) and the IBES series is also statistically significant. Across regressions (5.1), (5.2) and (5.3), the results indicate that an increase of a one-standard-deviation in these measures leads to an effect ranging between a 25%-standard-deviation increase to a 58%-standard-deviation increase in trading volume. To maintain consistency in the economic interpretation, I previously inverted the sign of the Izhakian (2020) series due to its negative correlation with the EPU and IBES measures.

Concerning hypothesis H2, the daily regression results in Appendix E, which incorporate the interaction between the expectations level of ambiguity and ambiguity volatility, reveal for regression (5.1) an average effect of a 15% to 16%-standard-deviation increase in trading volume per one-standard-deviation rise in EPU ambiguity. This effect is tempered by a negative impact of -9% to -10%-standard-deviation decrease in trading volume per one-standard-deviation increase in ambiguity volatility interacted with the ambiguity level, regression coefficient $\{\alpha_{amb\sigma}\}$, with t-stats exceeding 4. In regression (5.2) of the same appendix, which directly measures the impact of the explanatory variable on trading volume and includes the interaction between the level of ambiguity and its volatility, a statistically positive effect of 0.13% to 0.14%-standard-deviation increase in trading volume per one-standard-deviation increase in the level of ambiguity is observed. This effect is weakened by the interaction between the ambiguity level and ambiguity volatility, regression coefficient $\{\alpha_{amb\sigma}\}$, resulting in a counter effect of -6% to -7%-standard-deviation decrease in trading volume. Both effects are statistically significant, with t-stats exceeding four and 2.20, respectively. The version (5.3) in differences of the regression between trading volume and the explanatory variables, including the interaction term between ambiguity level and volatility in the appendix, demonstrates a similar effect, although the weakening effect caused by the interaction between ambiguity level and ambiguity volatility is smaller than a -1%-standard-deviation and not statistically significant in this regression.

The monthly regressions (5.1), (5.2), and (5.3) in Appendix G, incorporating the interaction term between ambiguity and ambiguity volatility, first show the statistically significant positive effect of the level of ambiguity on trading volume across all three regression versions. This effect ranges from a 0.20% to a 0.69%-standard-deviation increase in trading volume per one-standard-deviation rise in the level of ambiguity. These monthly regressions also demonstrate that this positive relationship is weakened by the interaction between ambiguity and its volatility, regression coefficient $\{\alpha_{amb\sigma}\}$, resulting in a counteracting -2% to -0.38%-standard-deviation decrease in trading volume per one-standard-deviation increase in this interaction. Across all regressions, the average effect of this interaction is negative, though it is statistically significant for only one to two out of ten monthly datasets across regression versions (5.1), (5.2) and (5.3).

Additional Results

Regarding the series acting as proxies for differences in prior beliefs, specifically $(\mu_e^A - \mu_e^B)$ and $(\mu_X^B - \mu_X^A)$ in the model, coefficient $\{\alpha_{pbel}\}$ in the regressions, the daily frequency regressions spanning from 2013 to 2018 unveil a statistically significant effect on trading volume for all series derived from Stocktwits (Cookson & Niessner, 2020), boasting t-stats exceeding 3. On average, a one-standard-deviation increase in any of these measures results in a change ranging from a -15%-standard-deviation to 13%-standard-deviation in trading volume. For three out of the four daily series, the impact on trading volume is positive, except for the ETF-based measure sourced from users with distinct investment approaches ($PBEL_{AC,ETF}$), where the effect is negative. In the context of monthly frequency regressions, the results for the difference in prior beliefs series are less consistent when compared to the daily frequency regressions, both in terms of sign and statistical significance. For regressions (5.1) and (5.2), the measures from Cookson and Niessner (2020) lose statistical significance and predominantly switch to a negative sign. Conversely, in regression (5.3), the sign is mostly positive across diverse datasets. The IBES measure consistently displays strong statistical significance with t-stats surpassing 3 in regressions (5.1) and (5.2), albeit with a negative sign. Specifically, a one-standard-deviation increase in the IBES measure corresponds to a -4% to -14%-standard-deviation decrease in trading volume. In the monthly first differenced regression version (5.3), the IBES measure lacks statistical significance but maintains the negative effect on trading volume. For further details refer to Appendix F.

Regarding the impact of ambiguity volatility on the β_V component of the trading volume-price relation, specifically regression coefficient $\{\beta_{\sigma_{amb}}\}$, the evidence is not entirely clear. In the daily frequency regression (5.1) of Table 1, the statistically significant results support the idea that daily EPU ambiguity volatility reduces the base magnitude of the β_V coefficient. This aligns with the expected effect of ambiguity volatility when it is distant from zero for ambiguity-averse investors, as illustrated in Figure 3. According to these findings, a one-standard-deviation increase in ambiguity volatility decreases the value of β_V by approximately -9%-standard-deviations from its base value of 0.02. However, for daily regressions (5.2) and (5.3), the direction of the effect is opposite: ambiguity volatility increases the magnitude of the β_V coefficient. This, according to Figure 3, would be consistent with ambiguity volatility levels close to 0 for both ambiguity-averse and ambiguity-loving investors. In the monthly results in Table 3, regressions (5.1), (5.2), and (5.3) using the monthly EPU ambiguity measure and the difference in prior beliefs series from Cookson and Niessner (2020) suggest that, on average, ambiguity volatility decreases the magnitude of the β_V coefficient, though lacking statistical significance. These estimates align with ambiguity volatility levels distant from 0 for ambiguity-averse investors. According to the results in regression (5.1) using the monthly EPU ambiguity measure, a one-standard-deviation increase in ambiguity volatility decreases the average magnitude of β_V by approximately -0.15%-standard-deviations from its average value of 0.05. Regression (5.2) shows a similar effect, with a decrease between 5% to 12%-standard-deviations from its average base value. Regression (5.3) also exhibits the same behavior, with a decrease of its base value by approximately 30%-standard-deviations per one-standard-deviation increase in ambiguity volatility. However, the same regressions using the ambiguity measure from Izhakian (2020) and the difference in prior beliefs series from Cookson and Niessner (2020) show a statistically significant effect in the opposite direction: ambiguity volatility increases the average magnitude of the β_V coefficient. This effect would be consistent with levels of ambiguity volatility close to zero. Regression (5.1) indicates an increase of around 17%-standard-deviations in the average base value of β_V per one-standard-deviation increase in ambiguity volatility. Regression (5.2) shows an increase between 3% to 5%-standard-deviations per one-standard-deviation increase in ambiguity volatility, and regression (5.3) exhibits an increase between 9% to 10%-standard-deviations per one-standard-deviation increase in ambiguity volatility.

Regarding regression versions (5.1), (5.2), and (5.3) in Appendices E and G, which include controls $\{\beta_{h\sigma_{amb}}, \beta_{h\delta\sigma_{amb}}\}$ to address changes in the sign of the marginal effect of ambiguity volatility on β_V as it transitions from levels close to 0 to high levels, as depicted in Figure 3, the majority of the regression versions show evidence of a change in the sign of the marginal effect of ambiguity volatility on β_V depending on whether the volatility is close or distant from zero. Some exceptions not showing this dynamic include daily regression version (5.1) using the EPU ambiguity measure and one monthly regression using the Izhakian (2020) ambiguity measure. The signs of this marginal effect change, however, are not consistent across datasets, and the majority lack statistical significance. In daily regressions (5.2) and (5.3) and monthly regression (5.2) using the EPU ambiguity measure, there is an indication that ambiguity volatility initially strengthens the average base value of β_V. Then, as ambiguity volatility crosses above the 50% quantile, the effect reverses or practically disappears, aligning with the behavior depicted in Figure 3. In concrete, according to daily regression (5.2) in Appendix E, a one-standard-deviation increase in ambiguity volatility increases the average magnitude of the β_V coefficient by around 7%-standard-deviations. Later, as ambiguity volatility crosses above its 50% quantile, this effect practically disappears, reaching 0. In monthly regression (5.2) using the EPU ambiguity measure, there is an initial incremental effect of ambiguity volatility on β_V, ranging between a 20% to 29%-standard-deviations increase on the base value of β_V per one-standard-deviation increase in ambiguity volatility. As ambiguity volatility crosses above the 50% quantile, this effect reverses into a net decrease of the base value of β_V by a 96%-standard-deviations to 107%-standard-deviations per one-standard-deviation increase in ambiguity volatility.

Table 1. Daily Alpha and Beta Regressions.

This table summarizes the daily trading volume regression results for the level of ambiguity, represented by μ_δ^A in the model and by the coefficient $\{\alpha_{amb}\}$ in the regressions, as well as for the ambiguity volatility, represented by σ_{amb} in the model and by the coefficient $\{\beta_{\sigma_{amb}}\}$ in the regressions. The table covers the results of regressions (5.1), (5.2) and (5.3) for the four daily datasets described in Section 4. The first column on the left indicates the dataset. $\mathrm{PBEL}_{\mathrm{WI,\ IND}}$, $\mathrm{PBEL}_{\mathrm{AC,\ IND}}$, $\mathrm{PBEL}_{\mathrm{WI,\ ETF}}$ and $\mathrm{PBEL}_{\mathrm{AC,\ ETF}}$ are the Cookson and Niessner (2020) difference in previous beliefs measures extracted from Stocktwits. $\mathrm{AMB}_{\mathrm{EPUD}}$ refers to the daily EPU ambiguity measure extracted from Twitter. The regressions were performed for the period 2013 to 2018. T-values in round brackets are Newey-West autocorrelation robust values. γ_p represent time fixed effects. Full regression results in Appendix-D.

$$\alpha_V =^1 c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \varsigma_{t-1} * \alpha_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V =^1 c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P + \varsigma_{t-1} * \beta_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$V =^2 \big(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\big)_\alpha \; + \; \big(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\big)_\beta * \Delta_P \; + \varsigma_{t-1} * V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\Delta V =^3 c \; + \; \big(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\big)_{\Delta\alpha} \; + \; \big(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\big)_{\Delta\beta} * \Delta_P \; + \big(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p \Delta_P^2} * \sigma_P\big)_\beta * \Delta_P^2 \; + \varsigma_{t-1} * \Delta V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon$$

Dataset	R(1)	R(2)	R(3)	
Panel A	α_{amb}	α_{amb}	α_{amb}	
$\mathrm{PBEL}_{\mathrm{WI,\ IND}}$ & $\mathrm{AMB}_{\mathrm{EPUD}}$	0.11***	0.10***	0.12***	
	(3.62)	(3.78)	(3.59)	
$\mathrm{PBEL}_{\mathrm{AC,\ IND}}$ & $\mathrm{AMB}_{\mathrm{EPUD}}$	0.11***	0.10***	0.12***	
	(3.80)	(3.94)	(3.62)	
$\mathrm{PBEL}_{\mathrm{WI,\ ETF}}$ & $\mathrm{AMB}_{\mathrm{EPUD}}$	0.11***	0.10***	0.12***	
	(3.71)	(3.92)	(3.53)	
$\mathrm{PBEL}_{\mathrm{AC,\ ETF}}$ & $\mathrm{AMB}_{\mathrm{EPUD}}$	0.11***	0.10***	0.12***	
	(3.84)	(4.10)	(3.62)	
Panel B	$\beta_{\sigma_{\mathrm{amb}}}$	$\beta_{\sigma_{\mathrm{amb}}}$	$\beta_{\Delta\sigma_{\mathrm{amb}}}$	$\beta_{\sigma_{\mathrm{amb}}}$
$\mathrm{PBEL}_{\mathrm{WI,\ IND}}$ & $\mathrm{AMB}_{\mathrm{EPUD}}$	-0.09*	-0.05	-0.02	-0.05
	(-1.74)	(-1.30)	(-0.26)	(-1.17)
$\mathrm{PBEL}_{\mathrm{AC,\ IND}}$ & $\mathrm{AMB}_{\mathrm{EPUD}}$	-0.09*	-0.05	-0.02	-0.05
	(-1.74)	(-1.37)	(-0.25)	(-1.21)
$\mathrm{PBEL}_{\mathrm{WI,\ ETF}}$ & $\mathrm{AMB}_{\mathrm{EPUD}}$	-0.09*	-0.05	-0.02	-0.05
	(-1.74)	(-1.40)	(-0.26)	(-1.24)
$\mathrm{PBEL}_{\mathrm{AC,\ ETF}}$ & $\mathrm{AMB}_{\mathrm{EPUD}}$	-0.09*	-0.05	-0.02	-0.05
	(-1.74)	(-1.44)	(-0.26)	(-1.24)

Table 2. Monthly Alpha Regression.

This table summarizes the monthly trading volume regression results for the level of ambiguity, represented by μ_δ^A in the model and by the coefficient $\{\alpha_{amb}\}$ in the regressions. The table covers the results of regressions (5.1), (5.2) and (5.3) for the ten monthly datasets described in Section 4. The first column on the left indicates the dataset. $\text{PBEL}_{\text{WI, IND}}$, $\text{PBEL}_{\text{AC, IND}}$, $\text{PBEL}_{\text{WI, ETF}}$ and $\text{PBEL}_{\text{AC, ETF}}$ are the Cookson and Niessner (2020) difference in previous beliefs measures extracted from Stocktwits and $\text{PBEL}_{\text{IBES}}$ is the difference in previous beliefs measure extracted from IBES. Regarding the Ambiguity measures, AMB_{EPUM} refers to the monthly EPU ambiguity measure extracted from newspapers and AMB_{IZHM} refers to the monthly Ambiguity market based measure of Izhakian (2020). The regressions involving the IBES measure cover the period 2000 to 2020 and the rest of the regressions cover the period 2013 to 2018. γ_p represent time fixed effects. T-values in round brackets are Newey-West autocorrelation robust values. Full regression results in Appendix-F.

$$\alpha_V =^1 c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \varsigma_{t-1} * \alpha_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$V =^2 \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\right)_\alpha + \left(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\right)_\beta * \Delta_P + \varsigma_{t-1} * V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\Delta V =^3 c + \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\right)_{\Delta\alpha} + \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\right)_\beta * \Delta_P^2 + \varsigma_{t-1} * \Delta V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon$$

Dataset	R(1)	R(2)	R(3)
	α_{amb}	α_{amb}	α_{amb}
$\text{PBEL}_{\text{WI, IND}}$ & AMB_{EPUM}	0.23*	0.22***	0.24***
	(1.99)	(3.87)	(4.08)
$\text{PBEL}_{\text{AC, IND}}$ & AMB_{EPUM}	0.23**	0.21***	0.25***
	(2.07)	(3.75)	(4.03)
$\text{PBEL}_{\text{WI, ETF}}$ & AMB_{EPUM}	0.20*	0.19***	0.19***
	(1.99)	(5.48)	(4.17)
$\text{PBEL}_{\text{AC, ETF}}$ & AMB_{EPUM}	0.23*	0.20***	0.23***
	(1.95)	(3.14)	(3.43)
$\text{PBEL}_{\text{WI, IND}}$ & AMB_{IZHM}	0.46***	0.35***	0.38***
	(4.61)	(3.18)	(4.05)
$\text{PBEL}_{\text{AC, IND}}$ & AMB_{IZHM}	0.46***	0.36***	0.38***
	(4.68)	(3.27)	(4.03)
$\text{PBEL}_{\text{WI, ETF}}$ & AMB_{IZHM}	0.47***	0.36***	0.33***
	(4.98)	(3.65)	(4.58)
$\text{PBEL}_{\text{AC, ETF}}$ & AMB_{IZHM}	0.46***	0.36***	0.38***
	(4.75)	(3.09)	(3.94)
$\text{PBEL}_{\text{IBES}}$ & AMB_{EPUM}	0.37***	0.36***	0.25***
	(3.03)	(3.26)	(2.80)
$\text{PBEL}_{\text{IBES}}$ & AMB_{IZHM}	0.58***	0.48***	0.53***
	(5.19)	(4.15)	(5.30)

Table 3. Monthly Betas Regression.

This table summarizes the statistical significance of the Ambiguity volatility term σ_{amb} ($\beta_{\sigma_{amb}}$) belonging to β_V across the different regression setups (1), (2) and (3). The first column on the left indicates the dataset. $\text{PBEL}_{\text{WI, IND}}$, $\text{PBEL}_{\text{AC, IND}}$, $\text{PBEL}_{\text{WI, ETF}}$ and $\text{PBEL}_{\text{AC, ETF}}$ are the Cookson and Niessner (2020) difference in previous beliefs measures extracted from Stocktwits and $\text{PBEL}_{\text{IBES}}$ is the difference in previous beliefs measure extracted from IBES. Regarding the Ambiguity measures, AMB_{EPUM} refers to the monthly EPU ambiguity measure extracted from newspapers and AMB_{IZHM} refers to the monthly Ambiguity market based measure of Izhakian (2020). The regressions involving the IBES measure cover the period 2000 to 2020 and the rest of the regressions cover the period 2013 to 2018. γ_p represent time fixed effects. T-values in round brackets are Newey-West autocorrelation robust values. Full regression results in Appendix-F.

$$\beta_V =^1 c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P + \varsigma_{t-1} * \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$V =^2 \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\right)_\alpha + \left(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\right)_\beta * \Delta_P + \varsigma_{t-1} * V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\Delta V =^3 c + \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\right)_{\Delta\alpha} + \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\right)_\beta * \Delta_P^2 + \varsigma_{t-1} * \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

Dataset	R(1)	R(2)	R(3)	
	$\beta_{\sigma_{\text{amb}}}$	$\beta_{\sigma_{\text{amb}}}$	$\beta_{\triangle\sigma_{\text{amb}}}$	$\beta_{\sigma_{\text{amb}}}$
$\text{PBEL}_{\text{WI, IND}}$ & AMB_{EPUM}	-0.15	0.05	-0.51	0.31
	(-0.43)	(0.39)	(-1.13)	(1.01)
$\text{PBEL}_{\text{AC, IND}}$ & AMB_{EPUM}	-0.15	0.07	-0.45	0.27
	(-0.43)	(0.53)	(-1.00)	(0.92)
$\text{PBEL}_{\text{WI, ETF}}$ & AMB_{EPUM}	-0.15	0.05	-0.43	0.29
	(-0.43)	(0.38)	(-1.15)	(1.32)
$\text{PBEL}_{\text{AC, ETF}}$ & AMB_{EPUM}	-0.15	0.12	-0.42	0.28
	(-0.43)	(1.00)	(-0.99)	(1.20)
$\text{PBEL}_{\text{WI, IND}}$ & AMB_{IZHM}	0.17	-0.03	0.12***	-0.09*
	(0.93)	(-0.63)	(4.08)	(-1.83)
$\text{PBEL}_{\text{AC, IND}}$ & AMB_{IZHM}	0.17	-0.04	0.12***	-0.09*
	(0.93)	(-0.66)	(4.13)	(-1.90)
$\text{PBEL}_{\text{WI, ETF}}$ & AMB_{IZHM}	0.17	-0.03	0.15***	-0.10**
	(0.93)	(-0.61)	(5.99)	(-2.05)
$\text{PBEL}_{\text{AC, ETF}}$ & AMB_{IZHM}	0.17	-0.05	0.12***	-0.10**
	(0.93)	(-0.88)	(5.55)	(-2.09)
$\text{PBEL}_{\text{IBES}}$ & AMB_{EPUM}	0.20	-0.10	-0.35***	0.19**
	(0.99)	(-1.39)	(-7.09)	(2.54)
$\text{PBEL}_{\text{IBES}}$ & AMB_{IZHM}	0.31	0.03	-0.09	0.02
	(1.54)	(0.39)	(-1.14)	(0.35)

6 Empirical Evidence: Trading Volume and Price Volatility Elasticity

In this section, I empirically test the trading volume elasticity model presented in section 2.7 using SPY data at daily frequency.

To validate my hypothesis H3, which suggests that the expectations channel of ambiguity weakens the elasticity between trading volume and price volatility, I examine whether the series representing the level of mean ambiguity is statistically significant in explaining this elasticity and the sign of their effect on the elasticity. Additionally, I account for the influence of ambiguity volatility.

Methodology

In this analysis, I regress changes in the natural logarithm of SPY trading volume against changes in the natural logarithm of the SPY of (delta) price volatility, denoted as $\sigma_{\Delta p}$, multiplied by the elasticity ξ. I perform the regressions using daily frequency data, striving to closely adhere to the model's approximation for small intervals.

To conduct these linear regressions, I employ an expression that accounts for the non linear normal density and cumulative distribution function (CDF) within the elasticity model. This expression encompasses both linear and quadratic terms of the factors explaining the elasticity. These factors include mean ambiguity, denoted as μ_{δ}^{A} in the model and represented by the regression coefficients $\{\xi_{amb}, \xi_{amb^2}\}$; differences in previous beliefs, expressed as $(\mu_e^A - \mu_e^B)$ and $(\mu_X^B - \mu_X^A)$ in the model and represented by the regression coefficients $\{\xi_{pbel}, \xi_{pbel^2}\}$; and ambiguity volatility, indicated as σ_{δ}^{A} in the model and represented by the regression coefficients $\{\xi_{\sigma_{amb}}, \xi_{\sigma^2_{amb}}\}$. To validate the hypothesis concerning the negative impact of the expectations channel of ambiguity on this elasticity, I examine the statistical significance of the regression coefficients associated with mean ambiguity, specifically $\{\xi_{amb}, \xi_{amb^2}\}$. Additionally, I consider the influence of ambiguity volatility by examining the regression coefficients $\{\xi_{\sigma_{amb}}, \xi_{\sigma^2_{amb}}\}$.

$$\Delta log(V) = \xi * \Delta log(\sigma_{\Delta p}) \tag{6.1}$$
$$\xi = F(\mu_{\delta}^{A}\ ,\ \sigma_{\delta}^{A}\ ,\ \mu_e^A - \mu_e^B\ ,\ \mu_X^B - \mu_X^A,\ \mu_{\delta}^{2\,A}\ , \sigma_{\delta}^{2\,A},\ (\mu_e^A - \mu_e^B)^2\ ,\ (\mu_X^B - \mu_X^A)^2\ )$$

I use the regression below, which also includes a constant term, a lagged series of the dependent variable to account for persistence and yearly time controls.

$$\begin{aligned}\Delta log(V) = c + \big(\, \xi_1 + \xi_{pbel} * PBEL + \xi_{amb} * AMB + \xi_{pbel^2} * PBEL^2 + \xi_{amb^2} * AMB^2 + \\ \xi_{\sigma_{amb}} * \sigma_{amb} + \xi_{\sigma^2_{amb}} * \sigma^2_{amb} \,\big)_{\xi} * \Delta log(\sigma_{\Delta p}) + \varsigma_{t-1} * \Delta log(V)_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p\end{aligned} \tag{6.2}$$

Furthermore, in Appendix E, I introduce two additional explanatory variables to account for variations in how ambiguity volatility influences the elasticity in high ambiguity volatility scenarios. As illustrated in Figure 5, the model suggests that when ambiguity volatility significantly deviates from zero, the direction of its impact on elasticity changes.

I conduct the regression using daily SPY data. The proxy series for differences in previous beliefs (PBEL) are the daily measures of Cookson and Niessner (2020). The ambiguity measure (AMB_{EPUD}) is the daily version of the EPU index extracted from Twitter messages (Baker et al., 2021). To mitigate potential collinearity problems between the explanatory variables and their squared terms, I orthogonalize them by regressing each term against its corresponding squared term and retain the residuals as the explanatory variable. These regressions are carried out for the daily frequency period spanning from 2013 to 2018.

Results Summary

The regression results in Table 4 below demonstrate a statistically significant negative impact of the level of ambiguity (AMB_{EPUD}) on the elasticity across all four daily dataset combinations, validating hypothesis H3. According to these results, the regression coefficient $\{\xi_{amb}\}$ shows that a one-standard-deviation increase in the level of ambiguity level leads to a -9% to -12%-standard-deviation decrease in the average base elasticity between trading volume and price volatility.

H3 Results

In the context of hypothesis H3, Table 4 firstly reveals a consistently positive base elasticity between trading volume and price volatility. This positive elasticity, representing the primary link between price changes and trading volume, ranges from a 0.24%-standard-deviation to a 0.28%-standard-deviation increase in trading volume per one-standard-deviation increase in price volatility. Across all four datasets this relationship is statistically significant with t-values exceeding 5.80. The reduction effect on elasticity caused by the expectations channel of ambiguity, depicted on the right side of the top panel of Figure 5 for positive measures of ambiguity, is reflected in the negative and statistically significant value of the regression coefficient $\{\xi_{amb}\}$ across all four daily datasets. According to the findings, a one-standard-deviation increase in the level ambiguity interacted with the change in log price volatility results in a -9% to -12% standard-deviation reduction in the aforementioned, on average, positive base elasticity between trading volume and price volatility. Additionally, there is a second non-linear effect associated to the regression coefficient $\{\xi_{amb^2}\}$ that is smaller in magnitude and not statistically significant. This aligns with a diminishing marginal effect of the expectations level of ambiguity on elasticity as it grows, as illustrated in Figure 5.

Additional Results

Regarding the proxy measures of differences in prior beliefs, the regression coefficient $\{\xi_{pbel}\}$ suggests a statistically and economically weaker non-stable effect on elasticity across the four datasets. In three out of four datasets, a one-standard-deviation increase in the difference in prior beliefs measures results in a negative -0.02% to -0.03%-standard-deviation decrease on elasticity. Dataset D(4) stands out by exhibiting an effect in the opposite direction, indicating that a one-standard-deviation increase in the difference in prior beliefs measure leads to a 6%-standard-deviation increase in elasticity. These findings underscore a divergence in the impact direction of the difference in prior beliefs measure associated to investors with the same investment approach ($PBEL_{WI,ETF}$) when compared to the one linked to investors employing different investment approaches ($PBEL_{AC,ETF}$). Digging into the differences in construction and nuances between these two prior beliefs measures (Cookson & Niessner, 2020), the observed variation in outcomes indicates diverse elasticity effects contingent on whether differing prior beliefs arise from distinct information sets or distinct methods of interpreting information.

Regarding ambiguity volatility, while not statistically significant, the regression coefficient $\{\xi_{\sigma_{amb}}\}$ suggests a negative linear effect, ranging from a -2% to a -3%-standard-deviation decrease in the base elasticity per one-standard-deviation increase in ambiguity volatility interacted with the change in log price volatility. This linear effect is counteracted by a non-statistically significant positive effect of quadratic ambiguity volatility, represented by the regression coefficient $\{\xi_{\sigma^2_{amb}}\}$, indicating a 2%-standard-deviation to a 3%-standard-deviation increase in elasticity per one-standard-deviation increase in squared ambiguity volatility interacted with the change in log price volatility. As illustrated in the bottom panel of Figure 5, these dynamics align with ambiguity-averse preferences and levels of ambiguity volatility distant from zero. The right side of this figure indicates that for ambiguity-averse investors, as ambiguity volatility increases, the originally negative marginal decreasing impact of ambiguity volatility on elasticity weakens. For detailed results refer to Appendix D.

In the additional regression results detailed in Appendix E, I introduced two further controls, specifically regression coefficients $\{\xi_{h\sigma_{amb}}, \xi_{h\sigma^2_{amb}}\}$, to account for shifts in the sign of the marginal effect of ambiguity volatility on elasticity as ambiguity volatility transitions from low to high levels. This transition is visually represented in the lower panel of Figure 5, where the marginal effect shifts from positive to negative for high levels of ambiguity volatility. Firstly, the regression coefficient $\{\xi_{\sigma_{amb}}\}$ highlights a statistically significant linear negative effect of ambiguity volatility on elasticity, ranging from a -0.12% to a -0.14%-standard-deviation decrease per one-standard-deviation increase in ambiguity volatility interacted with the change in log price volatility. This linear effect maintains its sign beyond the 50% quantile of ambiguity volatility. Regarding the effect of quadratic ambiguity volatility interacted with the change in log price volatility, represented by the regression coefficient $\{\xi_{\sigma^2_{amb}}\}$, the results indicate a statistically significant effect between a 0.30%-standard-deviation to a 0.41%-standard-deviation increase in elasticity. However, as ambiguity volatility surpasses its 50% quantile level, the contribution of quadratic ambiguity volatility undergoes a reversal, resulting in a net -0.32% to -0.38%-standard-deviation decrease in elasticity. This dynamic aligns with the change in sign illustrated in Figure 5.

Overall, the results obtained here from four distinct dataset combinations align with the earlier empirical insights presented by Bollerslev et al. (2018) regarding the role and impact of ambiguity (EPU) on the elasticity between trading volume and price volatility. Both sets of findings underscore the significance of the ambiguity expectations channel in diminishing the connection between trading volume and price volatility. However, there are important distinctions: (1) the results here differentiate between the prior beliefs channel and the ambiguity expectations channel, (2) they show evidence of a change in the sign of the marginal contribution of ambiguity volatility to this elasticity as it transitions from low to high levels, and (3) they show evidence of a varying effect of the prior beliefs measures on the elasticity depending on whether they originate from investors with similar or distinct investment approaches.

Table 4. Daily Elasticity Regression.

This table summarizes the main results of the trading volume to price volatility elasticity regression (6.2). The regressions were performed for the period 2013 to 2018. The row on top indicates the dataset. The dataset D(1): $\text{PBEL}_{\text{WI, IND}}$ & AMB_{EPUD}, D(2): $\text{PBEL}_{\text{AC, IND}}$ & AMB_{EPUD}, D(3): $\text{PBEL}_{\text{WI, ETF}}$ & AMB_{EPUD} and D(4): $\text{PBEL}_{\text{AC, ETF}}$ & AMB_{EPUD}. Where $\text{PBEL}_{\text{WI, IND}}$, $\text{PBEL}_{\text{AC, IND}}$, $\text{PBEL}_{\text{WI, ETF}}$ and $\text{PBEL}_{\text{AC, ETF}}$ are the Cookson and Niessner (2020) previous beliefs measures extracted from Stocktwits and AMB_{EPUD} refers to the daily EPU ambiguity measure extracted from Twitter. γ_p represent time fixed effects. T-values in round brackets are Newey-West autocorrelation robust values. γ_p represent time fixed effects. Detailed regression results in Appendix-D.

$$\Delta log(V) = c \ + \ \Big(\ \xi_1 \ + \ \xi_{pbel} * PBEL \ + \ \xi_{amb} * AMB \ + \ \xi_{pbel^2} * PBEL^2 \ + \ \xi_{amb^2} * AMB^2 \ + \ \xi_{\sigma_{amb}} * \sigma_{amb} \ + \ \xi_{\sigma^2_{amb}} * \sigma^2_{amb} \ \Big)_{\xi} \ * \ \Delta log(\sigma_{\Delta p}) + \varsigma_{t-1} * \Delta log(V)_{t-1} + \ \sum_{p=t+1}^{T} \gamma_p$$

	D(1)	D(2)	D(3)	D(4)
c	-0.02	-0.02	-0.02	-0.02
	(-0.47)	(-0.48)	(-0.45)	(-0.64)
ξ_1	0.24***	0.24***	0.25***	0.28***
	(5.83)	(6.64)	(7.21)	(7.08)
ξ_{pbel}	-0.03	-0.03	-0.02	0.06**
	(-1.10)	(-1.33)	(-0.67)	(2.03)
ξ_{amb}	-0.11***	-0.12***	-0.11***	-0.09***
	(-4.41)	(-4.74)	(-4.17)	(-3.55)
ξ_{pbel^2}	0.01	0.03	-0.02	0.08**
	(0.31)	(0.86)	(-0.44)	(2.17)
ξ_{amb^2}	0.03	0.02	0.03	0.02
	(1.37)	(0.91)	(1.49)	(1.17)
$\xi_{\sigma_{amb}}$	-0.02	-0.03	-0.02	-0.03
	(-0.70)	(-0.82)	(-0.48)	(-0.98)
$\xi_{\sigma^2_{amb}}$	0.03	0.03	0.02	-0.01
	(0.75)	(0.83)	(0.52)	(-0.23)
ς_{t-1}	-0.38***	-0.38***	-0.38***	-0.38***
	(-18.42)	(-18.51)	(-18.45)	(-18.11)
N	1510	1510	1510	1510
R^2_a	0.199	0.2	0.199	0.204

7 Application: Turnover Sorted Portfolios

In this section, I employ the model outlined in Section 2.6 to identify the primary drivers of returns in equity turnover sorted portfolios, which are commonly associated to liquidity in the literature. Furthermore, based on the trading volume components of the trading volume model of Section 2.6, I introduce an improved method for constructing equity turnover sorted portfolios with better risk/return profiles. I begin by introducing the current state of the literature on turnover sorted portfolios and liquidity. Subsequently, I present my key findings, followed by a detailed explanation of the methodology and numerical outcomes.

Several empirical studies have shown that stocks with low turnover tend to exhibit higher future returns (Amihud, 2002, 2018; Chou et al., 2013; Datar et al., 1998; Haugen & Baker, 1996; Lee & Swaminathan, 2000). The asset pricing literature typically regards turnover as a gauge of liquidity and liquidity risk. According to this perspective, theoretically, a greater exposure to low turnover, implying heightened liquidity risk, should be associated to higher returns (Acharya & Pedersen, 2005; Pástor & Stambaugh, 2003). However, second-moment volume based measures of liquidity risk point in the opposite direction (Chordia et al., 2001). This contradiction poses a puzzle within the established asset pricing literature. So, what do turnover sorted portfolios truly measure?.

Utilizing the model outlined in Section 2.6, I provide empirical evidence showing that a significant portion of the returns of a standard U.S. equity turnover sorted portfolio can be attributed to ambiguity. Summary in Figure 6 below. In detail, the results indicate that around 80% of the positive returns of a U.S. long-short turnover sorted portfolio, starting in 1990, can be explained by trading volume driven by ambiguity. These findings align with the Microstructure perspective (Harris & Raviv, 1993), which views turnover not only as an indicator of liquidity risk but also as a manifestation of ambiguity and divergence in beliefs among investors.

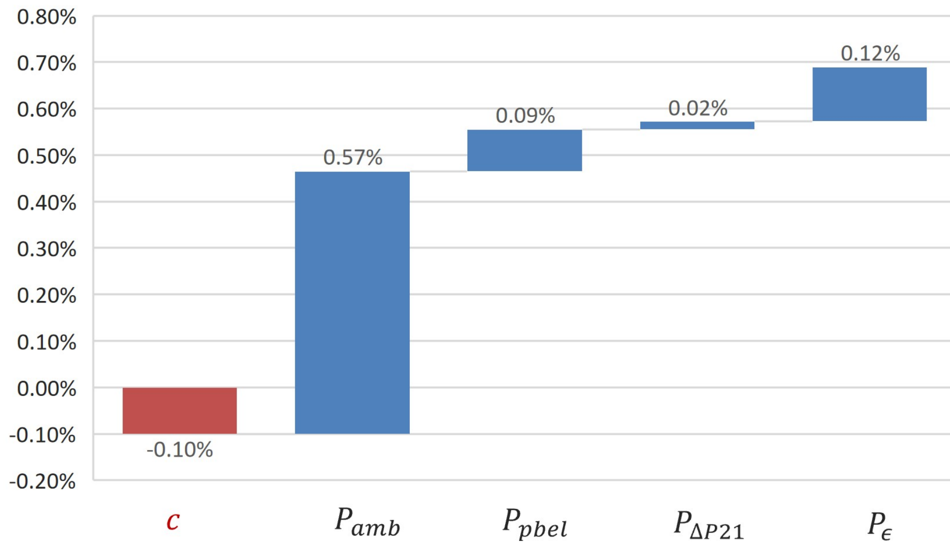

Figure 6. LMH Turnover Portfolio Returns Attribution. Attribution of the monthly returns of the LMH turnover sorted portfolio described in section *Data and Portfolio Construction* from 1990 to 2020. The returns attribution is obtained in regression (7.2) below by regressing the returns on a constant and the LMH portfolios obtained from sorting on the turnover components $\{\alpha_{V_{pbel}}, \alpha_{V_{amb}}, \beta_{V_{\Delta P_{21}}}, \epsilon\}$. The sorting variables of these last portfolios are: turnover driven by difference in prior beliefs ($\alpha_{V_{pbel}}$), turnover driven by ambiguity ($\alpha_{V_{amb}}$), turnover driven by price fluctuations ($\beta_{V_{\Delta P_{21}}}$) and an unexplained turnover portion or error term (ϵ).

Furthermore, I empirically show that creating a bisorted portfolio using the portions of turnover driven by ambiguity and price changes, yields a two to three times higher Sharpe ratio compared to a counterfactual portfolio relying solely on the traditional turnover measure. Figure 7 below.

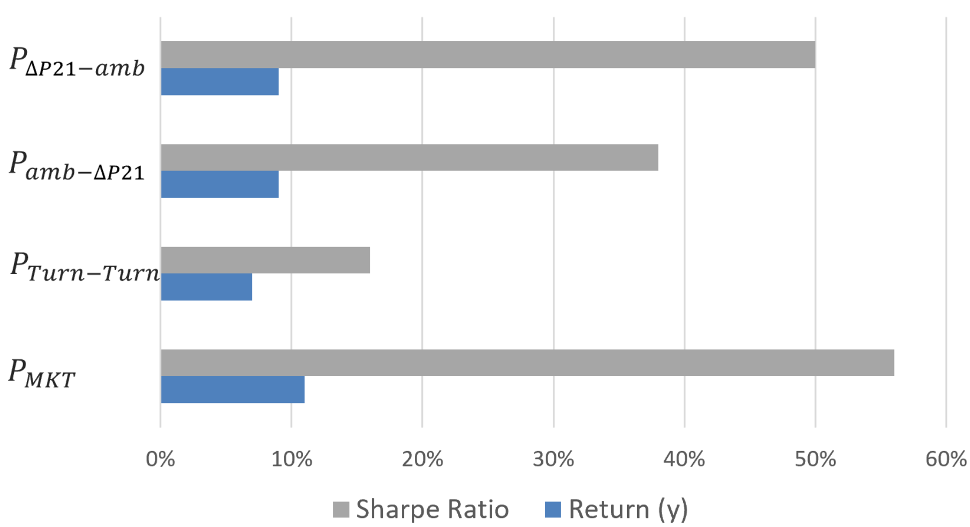

Figure 7. Bisorted Portfolios Returns and Sharpe-Ratios. Yearly returns and Sharpe ratios of bisorted portfolios from 1990 to 2020. P_{MKT} is the Fama-French market factor.

Methodology

I apply the theoretical framework presented in Section 2.6 to empirically deconstruct the turnover measure of each stock analyzed in this chapter. This process results in four distinct turnover components per stock, which in turn serve as the basis for constructing four low-minus-high (LMH) portfolios. Subsequently, I project the original turnover LMH portfolio onto these four distinct LMH portfolios, with the goal of identifying the individual contribution of each turnover component to the overall returns of the original turnover portfolio.

Regarding these four components of turnover, according to the model in Section 2.6 they are: ambiguity driven turnover, turnover stemming from differences in prior beliefs, turnover associated to price fluctuations, and an unexplained residual or error term. Sorting LMH on these measures originates the aforementioned four portfolios.

$$Turnover = \alpha_{V_{pbel}} + \alpha_{V_{amb}} + \beta_{V_{\Delta P_{21}}} + \epsilon \tag{7.1}$$
$$LMH\ Portfolios = \{P_{Turn}, P_{amb}, P_{pbel}, P_{\Delta P_{21}}, P_{\epsilon}\}$$

In a subsequent analysis, taking into account the higher Sharpe ratios of portfolios P_{amb} and $P_{\Delta P_{21}}$ from the prior step, I utilize the respective turnover components associated with ambiguity and changes in price $\{\alpha_{V_{amb}}, \beta_{V_{\Delta P_{21}}}\}$ to form bisorted portfolios. I then compare these portfolios against a similar counterfactual bisorted portfolio based solely on the original turnover metric.

To characterize and compare the above mentioned portfolios, I calculate their average annual excess returns, standard deviations, and Sharpe ratios. Additionally, I conduct regressions (7.2), (7.3) and (7.4) on the monthly returns of the original LMH P_{Turn} portfolio resulting from sorting on turnover in order to analyze, attribute its returns and validate my hypothesis H4 about the contribution of ambiguity to the positive returns of such a turnover sorted portfolio.

$$R_{P_{Turn}} = c + \epsilon \tag{7.2}$$
$$R_{P_{Turn}} = c + R_{free} + \beta_{MKTRF} * R_{MKTRF} + \beta_{HML} * R_{HML} + \beta_{SMB} * R_{SMB} + \epsilon \tag{7.3}$$
$$R_{P_{Turn}} = c + \beta_{P_{amb}} * R_{P_{amb}} + \beta_{P_{pbel}} * R_{P_{pbel}} + \beta_{P_{\Delta P_{21}}} * R_{P_{\Delta P_{21}}} + \beta_{P_{\epsilon}} * R_{P_{\epsilon}} + \epsilon \tag{7.4}$$

In regression (7.2), I analyze the monthly portfolio returns by regressing them against a constant term to determine the average monthly returns and their statistical significance. Regression (7.3) examines the returns using the Fama-French three factor model $\{MKT - R_{Free}, HML, SMB\}$ to gauge the extent to which conventional factors account for the original LMH turnover portfolio's behavior. In regression (7.4), I regress the original LMH turnover portfolio against the LMH portfolios obtained from sorting on $\{\alpha_{V_{pbel}}, \alpha_{V_{amb}}, \beta_{V_{\Delta P_{21}}}, \epsilon\}$, aiming to identify the main turnover components that explain the original turnover portfolio P_{Turn} returns. If my hypothesis H4 is correct, a large portion of the positive returns of the original LMH turnover sorted portfolio P_{Turn} should be associated to the turnover component $\{\alpha_{V_{amb}}\}$. The results of regressions (7.2), (7.3) and (7.4) are displayed in Tables 5 and 6 below. Utilizing the betas obtained from regression (7.4) alongside the mean average returns of each explanatory portfolio, I build the original LMH turnover portfolio P_{Turn} returns attribution displayed in Figure 6 above. In Table 24 of Appendix H, I further validate the regression outcomes from (7.4) by introducing alternative liquidity measures.

Data and Portfolio Construction

I create the portfolios analyzed here by using all CRSP stocks traded on the NYSE and NASDAQ. The monthly portfolio returns span from 1990 to 2020. To account for ambiguity, I utilize the monthly series of the Economic Policy Uncertainty Index (Baker et al., 2016). To account for differences in prior beliefs, I employ an analysts' forecast dispersion measure retrieved from the IBES database. The Fama-French factors for the regression analysis are sourced from the Kenneth French Online Data Library.

I create unisorted portfolios by averaging the turnover measure and its four components: $\{\alpha_{V_{amb}}, \alpha_{V_{pbel}}, \beta_{V_{\Delta P_{21}}}, \epsilon\}$ over a 3-month rolling window for each stock. These five averages represent turnover itself, turnover explained by ambiguity ($\alpha_{V_{amb}}$), turnover explained by differences in prior beliefs ($\alpha_{V_{pbel}}$), and the unexplained portion of turnover (ϵ). On a 3-month basis, using 1-month lagged averages, I segment the stock universe into 10 quantiles (10x), generating ten equally weighted portfolios corresponding to each quantile. This monthly process relies solely on past data. The final unisorted portfolios $\{P_{Turn}, P_{amb}, P_{pbel}, P_{\Delta P_{21}}, P_{\epsilon}\}$ representing one of the five series above, emerge as the difference between the lowest and highest quantile portfolios (LMH).

For the second analysis, I construct bisorted portfolios based on $\{\alpha_{V_{amb}}, \beta_{V_{\Delta P_{21}}}\}$ following the same methodology. Using a lagged 3-month rolling window average of the initially selected measure, I segment the stock universe into 5 quantiles on a 3-month basis. Within these five quantiles, utilizing the lagged 3-month rolling average of the second selected measure, I further segment them into two smaller quantiles. This process solely relies on past data, resulting in a set of 5x2 portfolios. The final portfolio emerges by taking a long position in the portfolio with the lowest $\alpha_{V_{amb}}$ and highest $\beta_{V_{\Delta P_{21}}}$, while shorting the portfolio with the highest $\alpha_{V_{amb}}$ and lowest $\beta_{V_{\Delta P_{21}}}$. I create two versions of this bisorted portfolio, one version employs $\alpha_{V_{amb}}$ as the first sorting dimension (5x) and the other one uses $\beta_{V_{\Delta P_{21}}}$ (2x). I refer to these portfolios as $\{P_{amb-\triangle P_{21}}, P_{\triangle P_{21}-amb}\}$. For comparison and benchmarking purposes I also construct a counterfactal turnover-turnover bisorted portfolio $P_{Turn-Turn}$ following the same methodology. This portfolio employs on both dimensions (5x2) the original turnover measure.

Results

The regression (7.2) results presented in Table 5 below reveal that the LMH turnover portfolio P_{Turn} yielded a positive and statistically significant average monthly return of 0.60% over the period 1990 to 2020. A similar outcome emerged from the LMH portfolio P_{amb} exclusively driven by the turnover component $\alpha_{V_{amb}}$ associated to ambiguity. During this same period, the LMH portfolio $P_{\triangle P_{21}}$ exclusively constructed from the turnover component associated with price changes ($\triangle P_{21}$) showed a statistically significant average negative returns of -0.50% per month. These two portfolios $\{P_{amb}$,$P_{\triangle P_{21}}\}$ linked to the turnover components $\{\alpha_{V_{amb}}, \triangle P_{21}\}$ respectively exhibit the highest Sharpe ratios, albeit in inverse LMH and HML directions.

The results of regressions (7.3) presented in Table 6 highlight that the traditional Fama-French three-factor model fails to account for the positive returns of the LMH Turnover portfolio P_{Turn}. The constant term captures an average monthly return of 1.9% (22% annually) with a robust t-statistic exceeding 4 that can not be explained by the Fama-French factors.

Regression (7.4) results in Table 6, utilizing the LMH Portfolios $\{P_{Turn}, P_{amb}, P_{pbel}, P_{\Delta P_{21}}, P_{\epsilon}\}$ as explanatory variables, reveal that a significant proportion of the LMH Turnover portfolio P_{Turn} positive returns can be attributed to these explanatory portfolios. In this regression, the constant term captures a residual -0.1% monthly return (-1.2% annually) that is not statistically significant. The findings from this Table 6 and the average monthly returns detailed in Table 5 above, indicate that the explanatory portfolio P_{amb} with a beta of 0.942 contributes with an approximate monthly return of 0.57% to the P_{Turn} portfolio. In the second, third, and fourth positions, the explanatory portfolios $\{P_{\epsilon}, P_{pbel}, P_{\Delta P_{21}}\}$ contribute with average monthly returns of {0.12%, 0.09%, 0.02%} respectively to the P_{Turn} portfolio. With a monthly average P_{Turn} portfolio return of 0.69%, the

0.57% contribution from ambiguity driven returns is significant, making up approximately 80% of the total P_{Turn} portfolio returns. These results validate hypothesis H4. The robustness tests in Table 24 of Appendix H show that the regression coefficients, t-stats, and p-values of regression (7.4) remain largely unaffected after the incorporation of alternative liquidity measures.

In relation to the construction of portfolios with better statistical properties using the information contained in the turnover measure, in Table 7 below I show that a bisorted portfolio utilizing solely the components $\{\alpha_{V_{amb}}, \beta_{V_{\Delta P_{21}}}\}$ of turnover for sorting purposes achieves a superior risk-return profile. The bisorted portfolio of dimensions 5x2 that first sorts LMH on $\alpha_{V_{amb}}$ and then HML on $\beta_{V_{\Delta P_{21}}}$ achieves a Sharpe ratio of 0.48. This outperforms the Sharpe ratio of 0.16 achieved by a comparable counterfactual turnover-turnover bisorted portfolio. Similarly, the 5x2 bisorted portfolio that first sorts HML on $\beta_{V_{\Delta P_{21}}}$ and then LMH on $\alpha_{V_{amb}}$ achieves a Sharpe ratio of 0.50 that exceeds the counterfactual turnover-turnover bisorted portfolio sharp ratio and comes close to Fama-French market factor Sharpe ratio.

These findings highlight the important role that the level of ambiguity plays in the dynamics of equity turnover sorted portfolios. Furthermore, they contribute to the longstanding debate on whether Turnover primarily captures liquidity or something else. Additionally, these results offer an alternative explanation on the puzzling relationship between volume based measures of liquidity risk and returns (Chordia et al., 2001).

Table 5. Portfolios Monthly Returns and Sharpe Ratios.

This table summarizes in Panel A the monthly returns of the market factor portfolio, the LMH turnover portfolio (P_{Turn}), the LMH ambiguity driven turnover portfolio (P_{amb}), the LMH prior beliefs disagreement driven turnover portfolio (ΔP_{pbel}), the LMH price changes driven turnover Portfolio (ΔP_{21}) and the LMH non-explained turnover driven portfolio (P_ϵ). Panel B shows the yearly Sharpe ratios for the same portfolios.

	P_{MKT}	P_{Turn}	P_{amb}	P_{pbel}	$P_{\triangle P_{21}}$	P_ϵ
Panel A: Monthly Portfolio Returns (1990 - 2020)						
c	0.009***	0.006*	0.006*	0.002	-0.005***	0.003*
	(3.96)	(1.68)	(1.70)	(0.76)	(-3.27)	(1.75)
N	372	372	372	372	372	372
R_a^2	0	0	0	0	0	0
Panel B: Sharpe Ratios (1990 - 2020, yearly)						
Excess Return	0.09	0.05	0.05	-0.01	-0.03	0.01
Std. Deviation	0.14	0.25	0.21	0.16	0.15	0.11
Sharpe Ratio	0.56	0.20	0.22	-0.05	-0.22	0.04

Table 6. Turnover Portfolio Monthly Returns Regressions.

This table summarizes the attribution of the original LMH turnover sorted portfolio (P_{Turn}) using as explanatory variables the Fama-French factors as well as the ambiguity driven turnover portfolio (P_{amb}), the prior beliefs disagreement driven turnover portfolio (P_{pbel}), price changes driven turnover portfolio ($P_{\Delta P_{21}}$) and the non explained (P_{ϵ}) turnover portfolio.

	P_{Turn}	P_{Turn}	P_{Turn}
c	0.006*	0.019***	-0.001
	(1.68)	(4.81)	(-1.27)
β_{R_f}		-2.380*	
		(-1.77)	
β_{MKT-R_f}		-0.953***	
		(-12.19)	
β_{HML}		0.854***	
		(6.82)	
β_{SMB}		-0.789***	
		(-7.96)	
$\beta_{P_{amb}}$			0.942***
			(41.18)
$\beta_{P_{pbel}}$			0.450***
			(5.93)
$\beta_{P_{\triangle P_{21}}}$			-0.034
			(-0.52)
$\beta_{P_{\epsilon}}$			0.390***
			(7.21)
N	372	372	372
R_a^2	0.000	0.728	0.944

Table 7. Bisorted Portfolios Monthly Returns and Sharpe Ratios.

This tables summarize the statistics of the market portfolio plus the three bisorted portfolios portfolios $P_{Turn-Turn}$, $P_{amb-\Delta P_{21}}$ and $P_{\Delta P_{21}-amb}$. Each bisorted portfolio is obtained as the long-short spread between the extreme quantiles of (bisorted) portfolios sorted on two measures. Each subportfolio that composes the long and short leg of these three portfolios is obtained by double sorting on a 5x2 grid. Panel A shows the monthly returns of the market factor portfolio, the bisorted turnover-turnover portfolio, the bisorted price change - ambiguity driven (turnover) portfolio, and the bisorted ambiguity - price change driven (turnover) portfolio. Panel B shows the yearly Sharpe ratios of these portfolios.

	P_{MKT}	$P_{Turn-Turn}$	$P_{amb-\triangle P_{21}}$	$P_{\triangle P_{21}-amb}$
Panel A: Monthly Portfolio Returnds (1990 - 2020)				
c	0.009***	0.006	0.008***	0.008***
	(3.96)	(1.39)	(2.79)	(3.53)
N	372	372	372	372
R^2_a	0	0	0	0
Panel B: Sharpe Ratios (1990 - 2020, yearly)				
Excess Return	0.09	0.04	0.07	0.07
Std. Deviation	0.14	0.25	0.16	0.13
Sharpe Ratio	0.56	0.16	0.38	0.50

8 Conclusions

In this paper, I present a trading volume model that explains how ambiguity about public information announcements influences trading volume through two channels: expectations and volatility. The ambiguity expectations channel plays a key role in driving trading activity. By biasing investors posterior expectations, this channel induces diverse interpretations of public information among investors, ultimately triggering trading activity through this heterogeneity.

Through the analysis of both daily and monthly equity data, I validate four hypotheses regarding the central role of the expectations channel of ambiguity in the generation of trading volume. The empirical findings confirm a positive relationship between the expectations channel of ambiguity and trading volume. Furthermore, the results validate that ambiguity induced volatility diminishes the impact of this expectations channel. The study also verifies that the expectations channel of ambiguity weakens the generally positive elasticity between trading volume and price volatility. Finally, applying this model to scrutinize the returns of equity turnover sorted portfolios reveals that a substantial portion of the positive returns of a standard U.S. equity turnover sorted portfolio is linked to information ambiguity.

In concrete, the empirical sections of this study reveal that, on average, a one-standard-deviation increase in the level of ambiguity is associated with a roughly 19% to 58%-standard-deviation surge in trading volume at monthly frequencies. At daily frequencies, a one-standard-deviation rise in the level of ambiguity results in an approximately 11%-standard-deviation increase in trading volume. The daily regression results further reveal a statistically significant weakening effect of ambiguity volatility on the expectations channel of ambiguity. This manifests as a -6% to -10%-standard-deviation decrease in trading volume per one-standard-deviation increase in the interaction between the level of ambiguity and its volatility.

Regarding the generally positive relationship between trading volume and price volatility, the empirical findings at daily frequency indicate that a one-standard-deviation increase in the interaction between the level of ambiguity and price volatility leads to a -9% to -12% decrease in trading volume. This weakens the overall positive connection between price volatility and trading volume.

In relation the application of this trading volume model in the analysis of the returns of a standard U.S. equity turnover sorted portfolio between 1990 to 2020, the model's returns attribution breakdown reveals that over 80% of the positive returns of such a standard U.S. turnover sorted portfolio are associated to ambiguity driven turnover.

The model and empirical findings presented here shed light on how ambiguity can help to explain the puzzling large average trading volume observed in markets (Cochrane, 2016; De Bondt & Thaler, 1995; Shleifer, 2000), even in situations marked by minor price shifts and subdued volatility. Furthermore, these results bolster the notion that turnover is more a manifestation of ambiguity and divergent viewpoints rather than mere liquidity, thereby contributing to the understanding of the puzzling relationship between volume based liquidity risk measures and returns (Chordia et al., 2001).

Appendix A Volume and Price Change Relation for Smooth Ambiguity Utility

Ambiguity Neutral Investor (Type-B) At Time 2

The ambiguity-neutral type-B investor takes the following optimal decision at time 2.

$$\max_{\theta_{t2}^B} \ E^B\left[U^B\left(\theta_{t2}^B\right)\middle|S\right] = \max_{\theta_{t2}^B} E^B\left[-e^{-\gamma \, * \, \left(w_{t2}^B \, + \, \theta_{t2}^B * \, \left(\widetilde{X} - P_{t1}\right)\right)} \middle| S\right]$$

$$\max_{\theta_{t2}^B} \ E^B\left[U^B\left(\theta_{t2}^B\right)\middle|S\right] = \max_{\theta_{t2}^B} -e^{-\gamma * \left(w_{t2}^B \, + \, \theta_{t2}^B * \, \left(E^B\left[\widetilde{X}\middle|S\right] - P_{t2}\right)\right) \, + \, \frac{1}{2} * \gamma^2 * \theta_{t2}^{2B} * \mathrm{VAR}^B[\widetilde{X}|S]}$$

Given his prior beliefs and the information received through the signal S, his optimal allocation at time 2 is the following.

$$\theta_{t2}^B = \frac{E^B[\widetilde{X}|S] - P_{t2}}{\gamma * \mathrm{VAR}^B[\widetilde{X}|S]}$$

$$\theta_{t2}^B = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * \ \left(S - \mu_\epsilon^B\right) - P_{t2} * \left(\rho_X^B + \rho_{\mathcal{E}}^B\right)}{\gamma}$$

$$where$$

$$\rho_X^B = \frac{1}{\sigma_X^{2\ B}}$$

$$\rho_{\mathcal{E}}^B = \frac{1}{\sigma_e^{2\ B}} = \frac{1}{\sigma_{\mathcal{E}}^{2\ B}}$$

$$E^B\left[\widetilde{X}\middle|S\right] = \mu_X^B + \frac{\sigma_X^{2\ B}}{\sigma_X^{2\ B} + \sigma_\epsilon^{2\ B}} * \left(S - \mu_X^B - \mu_\epsilon^B\right)$$

$$E^B\left[\widetilde{X}\middle|S\right] = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * \left(S - \mu_\epsilon^B\right)}{\rho_X^B + \rho_{\mathcal{E}}^B}$$

$$\mathrm{VAR}^B\left[\widetilde{X}\middle|S\right] = \sigma_X^{2\ B} - \frac{\sigma_X^{4\ B}}{\sigma_X^{2\ B} + \sigma_e^{2\ B}}$$

$$\mathrm{VAR}^B\left[\widetilde{X}\middle|S\right] = \left(\rho_X^B + \rho_{\mathcal{E}}^B\right)^{-1}$$

The parameter ρ_X^B corresponds to the type-B investor prior belief precision about the payoff X and $\rho_{\mathcal{E}}^B$ corresponds to the agent's belief about the precision of the signal S total error $\mathcal{E}$.

Ambiguity Averse/Loving Investor (Type-A) At Time 2

The ambiguous agent-1 takes interprets the signal in an ambiguous way, according to his Smooth Ambiguity Utility Function (Klibanoff et al., 2005).

The agent's preferences for risk are given by a CARA utility function $U_{1R}(\theta_{t2}^A) = -e^{-\gamma * W(\theta_{t2}^A)}$, where $W(\theta_{t2}^A)$ represents the final wealth of the agent at time 3. The agents preferences for ambiguity are given by the function $U_{1a}(E[U_{1R}]) = -(-E[U_{1R}])^{\gamma_a}$, where γ_a represents the ambiguity attitude of the agent.

The ambiguous agent beliefs that the signal S can be interpreted as $S_M = \widetilde{X} + \widetilde{\delta} + \widetilde{\epsilon}$, where $\widetilde{\delta}$ represents the agent's ambiguity about the correct model M under which he should interpret the signal. The prior belief of the agents is that the ambiguous component $\widetilde{\delta}$ distributes $N(\mu_\delta^A, \sigma_\delta^{2\ A})$.

The type-A investor problem maximizes the following expected utility.

$$\max_{\theta_{t2}^A}\ E^A\Big[-\Big(\ -E^A\Big[-e^{\ -\gamma\ *\ \left(w_{t2}^A\ +\ \theta_{t2}^A *\ \left(\widetilde{X}\ -\ P_{t2}\right)\right)}\ \Big|S, M\Big]\ \ \Big)^{\gamma_a}\Big|S\Big] =$$

$$\mathit{max}_{\theta_{t2}^A}\ E^A\Big[-\Big(e^{\ -\gamma\ *\ \left(w_{t2}^A\ +\ \theta_{t2}^A *\ \left(E^A[\widetilde{X}|S,M] - P_{t2}\right) + \frac{1}{2} * \gamma^2 * \theta_{t2}^{2A} * \mathrm{VAR}^A[\widetilde{X}|S,M]\right)}\ \ \Big)^{\gamma_a}\Big|S\Big] =$$

$$\mathit{max}_{\theta_{t2}^A}\ - e^{\ -\gamma * \gamma_a *\ \left(w_{t2}^A\ +\ \theta_{t2}^A *\ \left(E^A[\widetilde{X}|S] - P_{t2}\right)\right)\ +\ \frac{1}{2} * \gamma^2 * \gamma_a * \theta_{t2}^{2A} * \mathrm{E}^A[\ \mathrm{VAR}^A[\widetilde{X}|S,M]\ |S] +\ \frac{1}{2} * \gamma^2 * \gamma_a^2 * \theta_{t2}^{2A} * \mathrm{VAR}^A[\ \mathrm{E}^A[\widetilde{X}|S,M]\ |S]}$$

Considering that the variance $\mathrm{VAR}^A[\widetilde{X}|S, M]$ is already known before the last expectation operator conditional on the signal S, and that the variance $\mathrm{VAR}^A[\ \mathrm{E}^A[\widetilde{X}|S, M]\ |S]$ can be rewritten using the Law of Total Variance as $(\mathrm{VAR}^A[\widetilde{X}|S] - \mathrm{VAR}^A[\widetilde{X}|S, M])$ (Caskey, 2009), the optimization problem reduces to the maximization of the following certainty equivalence.

$$\mathit{max}_{\theta_{t2}^A}\ w_{t2}^A\ +\ \theta_{t2}^A *\ \left(E^A[\widetilde{X}|S] - P_{t2}\right) - \tfrac{1}{2} * \gamma * \theta_{t2}^{2A} * \mathrm{VAR}^A[\widetilde{X}|S] * \left[1 + (\gamma_a - 1) * \left(\tfrac{\mathrm{VAR}^A[\widetilde{X}|S] - \mathrm{VAR}^A[\widetilde{X}|S,M]}{\mathrm{VAR}^A[\widetilde{X}|S]}\right)\right]$$

Given his prior beliefs, the information received through signal S, and his personal interpretation of information the information contained in S, the optimal allocation of the type-A investor at time 2 is the following bellow. In our parametrization of the signal S, the signal and the payoff $\widetilde{X}$ have a covariance equal to the variance of the payoff $\widetilde{X}$, and we do not consider correlation between the payoff $\widetilde{X}$ and the ambiguity term $\widetilde{\delta}$.

$$\theta_{t2}^A = \frac{E^A[\widetilde{X}|S] - P_{t2}}{\gamma * Var^A[\widetilde{X}|S] * \nu^A}$$

$$\theta_{t2}^A = \frac{\rho_X^A * \mu_X^A + \rho_{\mathcal{E}}^A * (S - \mu_\epsilon^A - \mu_\delta^A) - P_{t2} * (\rho_X^A + \rho_{\mathcal{E}}^A)}{\gamma * \nu^A}$$

$$where$$

$$\rho_X^A = \frac{1}{\sigma_X^{2\ A}}$$

$$\rho_{\mathcal{E}}^A = \frac{1}{\sigma_\delta^{2\ A} + \sigma_\epsilon^{2\ A}}$$

$$\nu^A = \left[1 + (\gamma_a - 1) * \left(\frac{\mathrm{VAR}^A[\widetilde{X}|S] - \mathrm{VAR}^A[\widetilde{X}|S, M]}{\mathrm{VAR}^A[\widetilde{X}|S]}\right)\right]$$

$$E^A\left[\widetilde{X}\middle|S\right] = \mu_X^A + \frac{\sigma_X^{2\ A}}{\sigma_X^{2\ A} + \sigma_\delta^{2\ A} + \sigma_\epsilon^{2\ A}} * \left(S - \mu_X^A - \mu_\delta^A - \mu_\epsilon^A\right)$$

$$= \frac{\rho_X^A * \mu_X^A + \rho_{\mathcal{E}}^A * \left(S - \mu_\delta^A - \mu_\epsilon^A\right)}{\rho_X^A + \rho_{\mathcal{E}}^A}$$

$$Var^A[\widetilde{X}|S] = \left[\rho_X^A + \rho_{\mathcal{E}}^A\right]^{-1}$$

$$E^A\left[\widetilde{X}\middle|S, M\right] = \mu_X^A + \begin{bmatrix} \sigma_{X,S}^A & \sigma_{X,\delta}^A \end{bmatrix} * \begin{bmatrix} \sigma_S^{2\ A} & \sigma_{S,\delta}^A \\ \sigma_{S,\delta}^A & \sigma_\delta^{2\ A} \end{bmatrix}^{-1} * \begin{bmatrix} S - \mu_S^A \\ \delta - \mu_\delta^A \end{bmatrix}$$

$$= \mu_X^A + \begin{bmatrix} \sigma_X^{2\ A} & 0 \end{bmatrix} * \begin{bmatrix} \sigma_S^{2\ A} & \sigma_\delta^{2\ A} \\ \sigma_\delta^{2\ A} & \sigma_\delta^{2\ A} \end{bmatrix}^{-1} * \begin{bmatrix} S - \mu_S^A \\ \delta - \mu_\delta^A \end{bmatrix}$$

$$= \mu_X^A + \frac{\sigma_X^{2\ A} * (S - \delta) - \sigma_X^{2\ A} * \left(\mu_X^A + \mu_\epsilon^A\right)}{\sigma_S^{2\ A} - \sigma_\delta^{2\ A}}$$

$$= \frac{\rho_X^A * \mu_X^A + \rho_\epsilon^A * \left((S - \delta) - \mu_\epsilon^A\right)}{\rho_X^A + \rho_\epsilon^A}$$

$$Var^A[\widetilde{X}|S, M] = \sigma_X^{2\ A} - \begin{bmatrix} \sigma_{X,S}^A & \sigma_{X,\delta}^A \end{bmatrix} * \begin{bmatrix} \sigma_S^{2\ A} & \sigma_{S,\delta}^A \\ \sigma_{S,\delta}^A & \sigma_S^{2\ A} \end{bmatrix}^{-1} * \begin{bmatrix} \sigma_{X,S}^A \\ \sigma_{X,\delta}^A \end{bmatrix}$$

$$= \sigma_X^{2\ A} - \begin{bmatrix} \sigma_X^{2\ A} & 0 \end{bmatrix} * \begin{bmatrix} \sigma_S^{2\ A} & \sigma_\delta^{2\ A} \\ \sigma_\delta^{2\ A} & \sigma_S^{2\ A} \end{bmatrix}^{-1} * \begin{bmatrix} \sigma_X^{2\ A} \\ 0 \end{bmatrix}$$

$$= \sigma_X^{2\ A} - \frac{\sigma_X^{4\ A} * (\sigma_X^{2\ A} + \sigma_\delta^{2\ A} + \sigma_\epsilon^{2\ A})}{(\sigma_X^{2\ A} + \sigma_\delta^{2\ A} + \sigma_\epsilon^{2\ A})^2 - \sigma_\delta^{4\ A}} = \frac{1}{\rho_X^A} - \frac{\rho_S^A * \frac{\rho_\delta^{2\ A}}{\rho_X^{2\ A}}}{\rho_\delta^{2\ A} - \rho_S^{2\ A}}$$

$$\frac{Var^A[\widetilde{X}|S] - Var^A[\widetilde{X}|S, M]}{Var^A[\widetilde{X}|S]} = \frac{\rho_X^A * \rho_\epsilon^{A\ 3}}{(\rho_\delta^A * \rho_\epsilon^A + 2 * \rho_X^A * \rho_\epsilon^A + \rho_X^A * \rho_\delta^A) * (\rho_X^A + \rho_\epsilon^A) * (\rho_\epsilon^A + \rho_\delta^A)}$$

The term ν^A weights by the ambiguity aversion coefficient a the ratio of $\widetilde{X}$'s volatility caused by the ambiguity of the agent's interpretation of the signal. This ratio increases with the ambiguity volatility $\sigma_\delta^{2\ A}$, increasing the volatility divisor $\gamma * Var^A[\widetilde{X}|S] * \nu^A$ that goes inside the type-B investor's optimal allocation.

Equilibrium At Time 2

In equilibrium at time 2 the following equation has to hold.

$$(1-\pi) * \theta_{t2}^B + \pi * \theta_{t2}^A = 0.$$

Replacing the individual agents optimal allocations θ_{t2}^B and θ_{t2}^A in the market clearing condition.

$$\frac{\pi}{\nu^A} * \left[\rho_X^A * \ \mu_X^A \ + \ \rho_\mathcal{E}^A \ (S - \mu_\epsilon^A - \mu_\delta^A) \ - \ P_{t2} * \ (\rho_X^A + \rho_\mathcal{E}^A) \ \right] + (1-\pi) *$$
$$\left[\rho_X^B * \mu_X^B + \rho_\mathcal{E}^B * (S - \mu_\epsilon^B) \ - \ P_2 * (\rho_X^B + \rho_\mathcal{E}^B) \ \right] = 0$$

Grouping together terms associated to the type-A investor previous beliefs about the risky asset expected payment, the signal S, the signal S error and price P_2

$$\left[\frac{\pi}{\nu^A} * \rho_X^A * \mu_X^A + (1-\pi) * \rho_X^B * \mu_X^B\right]_{\bar{\mu}_X} + \left[\frac{\pi}{\nu^A} * \rho_\mathcal{E}^A + (1-\pi) * \rho_\mathcal{E}^B\right]_{\bar{\rho}_\mathcal{E}} * \ \mathrm{S} \ -$$
$$\left[\frac{\pi}{\nu^A} * \rho_\mathcal{E}^A * \mu_\epsilon^A + (1-\pi) * \rho_\mathcal{E}^B * \mu_\epsilon^B\right]_{\bar{\mu}_\epsilon} - \ \left[\frac{\pi}{\nu^A} * \rho_\mathcal{E}^A * \mu_\delta^A\right] - \mathrm{P}_{t2} * \left[\frac{\pi}{\nu^A} * \rho_X^A + (1-\pi) * \rho_X^B\right]_{\bar{\rho}_X} \ -$$
$$\mathrm{P}_{t2} * \left[\frac{\pi}{\nu^A} * \rho_\mathcal{E}^A + (1-\pi) * \rho_\mathcal{E}^B\right]_{\bar{\rho}_\mathcal{E}} = 0$$

Replacing the expression in brackets by the terms $\bar{\mu}_X, \bar{\rho}_\mathcal{E}, \bar{\mu}_\epsilon$ and $\bar{\rho}_X$ we can rewrite the market clearing condition as the expression below.

$$\bar{\mu}_X + \bar{\rho}_\mathcal{E} * \ \mathrm{S} - \bar{\mu}_\epsilon - \frac{\pi}{\nu^A} * \rho_\mathcal{E}^A * \mu_\delta^A - \mathrm{P}_{t2} * (\bar{\rho}_X + \bar{\rho}_\mathcal{E}) = 0$$

From here we can rewrite the price at time 2.

$$\mathrm{P}_{t2} = \frac{\bar{\mu}_X + \bar{\rho}_\mathcal{E} * \ \mathrm{S} - \bar{\mu}_\epsilon - \dfrac{\pi}{\nu^A} * \rho_\mathcal{E}^A * \mu_\delta^A}{\bar{\rho}_X + \bar{\rho}_\mathcal{E}}$$

Signal S At Time 2

We can use this last expression for P_{t2} to rewrite the signal S as a function of the change in price ΔP.

$$\Delta \mathrm{P} = P_{t2} - P_{t1} = \frac{\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * \ \mathrm{S} - \bar{\mu}_\epsilon - \dfrac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A}{\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}} - P_{t1}$$

$$\Delta \mathrm{P} * \Big[\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}\Big] = \Big[\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * S - \bar{\mu}_\epsilon - \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A\Big] - P_1 * \Big[\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}\Big]$$

$$\mathrm{S} = \frac{\Delta \mathrm{P} * [\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] + \bar{\mu}_\epsilon + \dfrac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A - [\bar{\mu}_X - P_{t1} * \bar{\rho}_X]}{\bar{\rho}_{\mathcal{E}}} + P_{t1}$$

In comparison to the Max Min Utility, in this last expression we have the additional term $[\bar{\mu}_X - P_{t1} * \bar{\rho}_X]$ that is different from 0. In the next lines we show that this term is proportional to the investor type-A versus type-B difference in prior beliefs about the mean of the payoff X. From the optimization at time 1 we know the following.

$$\mathrm{P}_{t1} = \frac{\bar{\mu}_{\underset{t1}{X}}}{\bar{\rho}_{\underset{t1}{X}}}$$

$$where$$

$$\bar{\mu}_{\underset{t1}{X}} = \pi * \rho_X^A * \mu_X^A + (1-\pi) * \rho_X^B * \mu_X^B$$

$$\bar{\rho}_{\underset{t1}{X}} = \pi * \rho_X^A + (1-\pi) * \rho_X^B$$

These last terms from time (1) are related to the average terms $\bar{\mu}_X$ and $\bar{\rho}_X$ that appear at time 2 through the following equations.

$$\bar{\mu}_X = \bar{\mu}_{\underset{t1}{X}} + \Big(\bar{\mu}_X - \bar{\mu}_{\underset{t1}{X}}\Big) = \bar{\mu}_{\underset{t1}{X}} + \pi * \left(\frac{1}{\nu^A} - 1\right) * \mu_X^A * \rho_X^A$$

$$\bar{\rho}_X = \bar{\rho}_{\underset{t1}{X}} + \Big(\bar{\rho}_X - \bar{\rho}_{\underset{t1}{X}}\Big) = \bar{\rho}_{\underset{t1}{X}} + \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A$$

Using these last relation, we can rewrite the term $-[\bar{\mu}_X - P_{t1} * \bar{\rho}_X]$ that appears above in the equation for S.

$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = -\left[\bar{\mu}_{\underset{t1}{X}} + \pi * \left(\frac{1}{\nu^A} - 1\right) * \mu_X^A * \rho_X^A\right] + \left[\mathrm{P}_{t1} * \bar{\rho}_{\underset{t1}{X}} + \mathrm{P}_{t1} * \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A\right]$$

$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = -\left[\bar{\mu}_{\underset{t1}{X}} + \pi * \left(\frac{1}{\nu^A} - 1\right) * \mu_X^A * \rho_X^A\right] + \left[\bar{\mu}_{\underset{t1}{X}} + \mathrm{P}_{t1} * \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A\right]$$

$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A * \left(P_{t1} - \mu_X^A\right)$$

$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A * \left(\frac{\pi * \rho_X^A * \mu_X^A + (1-\pi) * \rho_X^B * \mu_X^B}{\pi * \rho_X^A + (1-\pi) * \rho_X^B} - \mu_X^A\right)$$

$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = \pi * \left(\frac{1}{\nu^A} - 1\right) * \rho_X^A * \left(\frac{(1-\pi) * \rho_X^B * (\mu_X^B - \mu_X^A)}{\bar{\rho}_{\underset{t1}{X}}}\right)$$

$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = \pi * (1-\pi) * \left(\frac{1}{\nu^A} - 1\right) * \left(\frac{\rho_X^A * \rho_X^B}{\bar{\rho}_{\underset{t1}{X}}}\right) * \left(\mu_X^B - \mu_X^A\right)$$

$$-\bar{\mu}_X + P_{t1} * \bar{\rho}_X = \eta * \left(\mu_X^B - \mu_X^A\right)$$

Replacing the term $[\bar{\mu}_X - P_{t1} * \bar{\rho}_X]$ by this last expression in the equation for signal S, we obtain the following term.

$$\mathrm{S} = \frac{\Delta\mathrm{P} * [\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] + \bar{\mu}_\epsilon + \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\delta^A + \eta * \left(\mu_X^B - \mu_X^A\right)}{\bar{\rho}_{\mathcal{E}}} + P_{t1}$$

Trading Volume At Time 2

We measure the trading volume from time 1 to 2 as the change in the risky-asset allocation of the type-B agent multiplied by the proportion of this agent type in the economy. By symmetry of this market equilibrium, the volume of risky asset this type-B investors buys/sells is equivalent to the volume the type-A investors sells/buys.

$$V = (1-\pi) * (\theta_{t2}^B - \theta_{t1}^B)$$

where

$$\theta_{t1}^B = \frac{(\mu_X^B - P_{t1}) * \rho_X^B}{\gamma}$$

$$\theta_{t2}^B = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) - P_{t2} * (\rho_X^B + \rho_{\mathcal{E}}^B)}{\gamma}$$

Replacing the allocations of the ambiguity-neutral type-B investor in the expression $(\theta_{t2}^B - \theta_{t1}^B)$ we obtain the following expression.

$$\theta_{t2}^B - \theta_{t1}^B = \left[\frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) - P_{t2} * (\rho_X^B + \rho_{\mathcal{E}}^B)}{\gamma}\right] - \left[\frac{(\mu_X^B - P_{t1}) * \rho_X^B}{\gamma}\right]$$

$$\theta_{t2}^B - \theta_{t1}^B = \frac{\rho_{\mathcal{E}}^B * \left[\ \mathrm{S} - \mu_\epsilon^B\right]}{\gamma} - \frac{P_{t2} * \rho_{\mathcal{E}}^B}{\gamma} - \Delta\mathrm{P} * \frac{\rho_X^B}{\gamma}$$

At this stage we rewrite the signal S expression $(S - U_2)$ as a function of the change in prices between period (1) and (2) using the relation of the previous section.

$$\theta_{t2}^B - \theta_{t1}^B = \rho_{\mathcal{E}}^B * \frac{\left[\left(\frac{\Delta\mathrm{P}*[\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] + \bar{\mu}_\epsilon + \frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^A*\mu_\delta^A + \eta*\left(\mu_X^B - \mu_X^A\right)}{\bar{\rho}_{\mathcal{E}}} + P_{t1}\right)_{\mathrm{S}} - \mu_\epsilon^B\right]}{\gamma} - \frac{P_{t2} * \rho_{\mathcal{E}}^B}{\gamma} - \Delta\mathrm{P} * \frac{\rho_X^B}{\gamma}$$

$$\theta_{t2}^B - \theta_{t1}^B = \frac{\Delta\mathrm{P}}{\gamma} * \left[\rho_{\mathcal{E}}^B * \frac{(\bar{\rho}_X + \bar{\rho}_{\mathcal{E}})}{\bar{\rho}_{\mathcal{E}}} - \rho_{\mathcal{E}}^B - \rho_X^B\right] + \left[\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}}*\gamma} * \eta * \left(\mu_X^B - \mu_X^A\right)\right] + \left[\frac{\rho_{\mathcal{E}}^B*\bar{\mu}_\epsilon}{\gamma*\bar{\rho}_{\mathcal{E}}} - \frac{\rho_{\mathcal{E}}^B*\mu_\epsilon^B}{\gamma} + \frac{\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^B*\rho_{\mathcal{E}}^A*\mu_\delta^A}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]$$

$$\theta_{t2}^B - \theta_{t1}^B = \frac{\Delta\mathrm{P}}{\gamma} * \left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 + \left[\frac{\rho_{\mathcal{E}}^B*\bar{\mu}_\epsilon}{\gamma*\bar{\rho}_{\mathcal{E}}} - \frac{\rho_{\mathcal{E}}^B*\mu_\epsilon^B}{\gamma} + \frac{\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^B*\rho_{\mathcal{E}}^A*\mu_\delta^A}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]_1 + \left[\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}}*\gamma} * \eta * \left(\mu_X^B - \mu_X^A\right)\right]_0$$

We simplify the expressions inside bracket (2)

$$\left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 = \left[\frac{\rho_{\mathcal{E}}^B * \frac{\pi}{\nu^A} * \rho_X^A - \rho_X^B * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A}{\bar{B}}\right]$$

$$\left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 = \frac{\pi}{\nu^A} * \left[\frac{\rho_{\mathcal{E}}^B * \rho_X^A - \rho_X^B * \rho_{\mathcal{E}}^A}{\bar{\rho}_{\mathcal{E}}}\right]$$

We simplify the expression inside bracket (1).

$$[...]_1 = \left[\frac{\left(\rho_{\mathcal{E}}^B*\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^A*\mu_\epsilon^A + \rho_{\mathcal{E}}^B*(1-\pi)*\rho_{\mathcal{E}}^B*\mu_\epsilon^B\right) - \left(\rho_{\mathcal{E}}^B*\mu_\epsilon^B*\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^A + \rho_{\mathcal{E}}^B*\mu_\epsilon^B*(1-\pi)*\rho_{\mathcal{E}}^B\right)}{\gamma*\bar{\rho}_{\mathcal{E}}} + \frac{\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^B*\rho_{\mathcal{E}}^A*\mu_\delta^A}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]$$

$$[...]_1 = \left[\frac{\rho_{\mathcal{E}}^B * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \mu_\epsilon^A - \rho_{\mathcal{E}}^B * \mu_\epsilon^B * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A}{\gamma * \bar{\rho}_{\mathcal{E}}} + \frac{\frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^B * \rho_{\mathcal{E}}^A * \mu_\delta^A}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]$$

$$[...]_1 = \frac{\frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B * \left[\mu_\epsilon^A - \mu_\epsilon^B + \mu_\delta^A\right]}{\gamma * \bar{\rho}_{\mathcal{E}}}$$

Then we can rewrite the expression for the trading volume associated to the type-B investor.

$$\theta_{t2}^B - \theta_{t1}^B = \left[\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}} * \gamma} * \eta * \left(\mu_X^B - \mu_X^A\right)\right]_0 + \left[\frac{\frac{\pi}{\nu^A}*\rho_{\mathcal{E}}^A*\rho_{\mathcal{E}}^B*\left(\mu_\epsilon^A - \mu_\epsilon^B + \mu_\delta^A\right)}{\gamma*\bar{\rho}_{\mathcal{E}}}\right]_1 + \left[\rho_{\mathcal{E}}^B * \frac{\bar{\rho}_X}{\bar{\rho}_{\mathcal{E}}} - \rho_X^B\right]_2 * \frac{\Delta\mathrm{P}}{\gamma}$$

$$\theta_{t2}^B - \theta_{t1}^B = \left[\pi * (1-\pi) * \left(\frac{1}{\nu^A} - 1\right) * \left(\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}} * \gamma}\right) * \left(\frac{\rho_X^A * \rho_X^B}{\bar{\rho}_{\underset{t1}{X}}}\right) * \left(\mu_X^B - \mu_X^A\right)\right]_0 + \left[\frac{\frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B * \left(\mu_\epsilon^A - \mu_\epsilon^B + \mu_\delta^A\right)}{\gamma * \bar{\rho}_{\mathcal{E}}}\right]_1 + \left[\frac{\frac{\pi}{\nu^A} * \left(\rho_{\mathcal{E}}^B * \rho_X^A - \rho_X^B * \rho_{\mathcal{E}}^A\right)}{\bar{\rho}_{\mathcal{E}}}\right]_2 * \frac{\Delta\mathrm{P}}{\gamma}$$

The total trading-volume in the economy is given by the expression below. Here, α_V accounts for the difference in prior beliefs $(\mu_X^B - \mu_X^A)$ between investor types A-B and the expectations channel of ambiguity represented by μ_δ^A. The coefficient β_V is a the term that amplifies or smooths the traditiona trading volume channel associated to changes in prices.

$$V = \left| (1-\pi) * (\theta_{t2}^B - \theta_{t1}^B) \right|$$

$$\begin{aligned} V = \Bigg| \Bigg\{ & \left[\pi * (1-\pi)^2 * \left(\frac{1}{\nu^A} - 1 \right) * \left(\frac{\rho_{\mathcal{E}}^B}{\bar{\rho}_{\mathcal{E}} * \gamma} \right) * \left(\frac{\rho_X^A * \rho_X^B}{\bar{\rho}_{\substack{X \\ t1}}} \right) \right] * \left(\mu_X^B - \mu_X^A \right) \\ & + \left[\frac{(1-\pi) * \frac{\pi}{\nu^A} * \rho_{\mathcal{E}}^A * \rho_{\mathcal{E}}^B}{\gamma * \bar{\rho}_{\mathcal{E}}} \right] * \left(\mu_\epsilon^A - \mu_\epsilon^B + \mu_\delta^A \right) \Bigg\}_\alpha \\ & + \left\{ \left[\frac{\frac{\pi}{\nu^A} * \left(\rho_{\mathcal{E}}^B * \rho_X^A - \rho_X^B * \rho_{\mathcal{E}}^A \right)}{\bar{\rho}_{\mathcal{E}}} \right] \right\}_\beta * \frac{\Delta \mathrm{P}}{\gamma} \Bigg| \end{aligned}$$

$$V = |\alpha_V + \beta_V * \Delta P_{21}|$$

Appendix B Volume and Price Change Relation for Max Min Utility

Market Equilibrium at Time 1

The price P_{t1} represents the equilibrium price at which the risky asset market clears in period 1 and is determined by the following formula.

$$
\begin{aligned}
&P_{t1} = \bar{\mu}_X / \bar{\rho}_X \\
&\textit{where} \\
&\bar{\mu}_X = \pi * \mu_X^A * \rho_X^A + (1-\pi) * \mu_X^B * \rho_X^B \\
&\bar{\rho}_X = \pi * \rho_X^A + (1-\pi) * \rho_X^B
\end{aligned}
$$

The resulting equilibrium allocation at period 1 for both ambiguous and non-ambiguous investors is given by the formulas θ_{t1}^A and θ_{t1}^B below.

$$
\begin{aligned}
\theta_{t1}^B &= \frac{\pi * \rho_X^A * \rho_X^B * \left(\mu_X^B - \mu_X^A\right)}{\gamma * \bar{\rho}_X} \\
\theta_{t1}^A &= \frac{(1-\pi) * \rho_X^A * \rho_X^B * \left(\mu_X^A - \mu_X^B\right)}{\gamma * \bar{\rho}_X}
\end{aligned}
$$

Ambiguity Neutral Investor Type (Type-B) At Time 2

The ambiguity-neutral type-B investor takes the following optimal decision at time (2)

$$
\begin{aligned}
\max_{\theta_{t2}^B} \; E^B\left[U^B\left(\theta_{t2}^B\right)\middle|S\right] &= \max_{\theta_{t2}^B} E^B\left[-e^{-\gamma \,*\, \left(w_{t2}^B \,+\, \theta_{t2}^B * \,\left(\widetilde{X} - P_{t1}\right)\right)} \middle| S\right] \\
\max_{\theta_{t2}^B} \; E^B\left[U^B\left(\theta_{t2}^B\right)\middle|S\right] &= \max_{\theta_{t2}^B} -e^{-\gamma*\left(w_{t2}^B \,+\, \theta_{t2}^B * \,\left(E^B\left[\widetilde{X}\middle|S\right] - P_{t2}\right)\right) \,+\, \frac{1}{2}*\gamma^2*\theta_{t2}^{2B}*\mathrm{VAR}^B[\widetilde{X}|S]}
\end{aligned}
$$

Given his prior beliefs and the information received through the signal S, his optimal allocation at time 2 is the following

$$
\theta_{t2}^B = \frac{E^B[\widetilde{X}|S] - P_{t2}}{\gamma * \mathrm{VAR}^B[\widetilde{X}|S]}
$$

$$\theta_{t2}^{B} = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * \ \left(S - \mu_\epsilon^B\right) - P_{t2} * \left(\rho_X^B + \rho_{\mathcal{E}}^B\right)}{\gamma}$$

$$where$$

$$\rho_X^B = \frac{1}{\sigma_X^{2\ B}}$$

$$\rho_{\mathcal{E}}^B = \frac{1}{\sigma_{\mathcal{E}}^{2\ B}} = \frac{1}{\sigma_\epsilon^{2\ B}}$$

$$E^B\left[\widetilde{X}\middle|S\right] = \mu_X^B + \frac{\sigma_X^{2\ B}}{\sigma_X^{2\ B} + \sigma_e^{2\ B}} * \left(S - \mu_X^B - \mu_e^B\right)$$

$$E^B\left[\widetilde{X}\middle|S\right] = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * \left(S - \mu_e^B\right)}{\rho_X^B + \rho_{\mathcal{E}}^B}$$

$$\mathrm{VAR}^B\left[\widetilde{X}\middle|S\right] = \sigma_X^{2\ B} - \frac{\sigma_X^{4\ B}}{\sigma_X^{2\ B} + \sigma_e^{2\ B}}$$

$$\mathrm{VAR}^B\left[\widetilde{X}\middle|S\right] = \left(\rho_X^B + \rho_{\mathcal{E}}^B\right)^{-1}$$

The parameter ρ_X^B corresponds to the type-B investor prior belief precision about the payoff X and $\rho_{\mathcal{E}}^B$ corresponds to the type-B investor belief about the precision of the signal S total error $\widetilde{\mathcal{E}}$.

Ambiguity Averse Investor (Type-A) At Time 2

The investor type maximizes at time 2 his expected utilitity by selecting the optimal asset mixture in accordance with its Max Min Utility function. In this function, the expectation operator addresses the traditional risk concept associated with the payoff $\widetilde{X}$, while the minimization operator handles the ambiguity surrounding $\widetilde{X}$ by selecting the most pessimistic M model. In this particular setup, each model M represents a specific way of interpreting the public information. Details below.

Regarding this investor type beliefs about the risky-asset payoff $\widetilde{X}$, he initially assumes a normal distribution with parameters $N\left(\mu_X^A,\ \sigma_X^{2\ A}\right)$. Additionally, he believes that the signal S is subject to a measurement bias or error $\widetilde{\epsilon}$ that distributes $N\left(\mu_\epsilon^A,\ \sigma_\epsilon^{2\ A}\right)$.

Despite these prior beliefs, this investor type is not completely certain about the appropriate model for interpreting the signal S. This ambiguity is represented by different models $M \in M^n$, each characterized by a model-dependent signal component $\widetilde{\delta}$ following the normal distribution $N\left(\mu_\delta^A,\ \sigma_\delta^{2\ A}\right)$. The mean and variance of $\widetilde{\delta}$ are specific to each M-model. The mean of $\widetilde{\delta}$ across all models falls within the range $\left[\underline{\mu}_\delta, \overline{\mu}_\delta\right]$ and its variance falls within the range $[\underline{\sigma}_\delta^2, \overline{\sigma}_\delta^2]$.

In summary, the investor type-A believes that the signal S consists of three components: the risky payoff $\widetilde{X}$ information, an ambiguous model-dependent component $\widetilde{\delta}$, and a measurement error $\widetilde{\epsilon}$. The total error term of the signal is denoted as $\widetilde{\mathcal{E}}$.

$$
\begin{aligned}
S &= \widetilde{X} + \widetilde{\delta} + \widetilde{\epsilon} = S + \widetilde{\mathcal{E}} \\
\widetilde{\delta} &\sim N\left(\mu_\delta^A,\ \sigma_\delta^{2\ A}\right) \\
\mu_\delta^A &\in \left[\underline{\mu}_\delta, \overline{\mu}_\delta\right] \\
\sigma_\delta^{2\ A} &\in \left[\underline{\sigma}_\delta^2, \overline{\sigma}_A^2\right]
\end{aligned}
$$

The ambiguity averse type-A investor takes the following optimal decision at time 2.

$$
\begin{aligned}
&\max_{\theta_{t2}^A}\ \min_{M}\ E^A\left[-e^{\,-\gamma\,*\,\left(w_{t2}^A\ +\ \theta_{t2}^{I_A}*\ \left(\widetilde{X}\ -\ P_{t2}\right)\right)}\,\middle|\,S, M\right] \\
&\max_{\theta_{t2}^A}\ \min_{M} -e^{-\gamma*\left(w_{t2}^A\ +\ \theta_{t2}^A*\left(E^A\left[\widetilde{X}\middle|S,M\right]-P_{t2}\right)\right)\ +\ 1/2*\gamma^2*\theta_{t2}^{2\ A}*\mathrm{Var}^A\left[\widetilde{X}\middle|S,M\right]} \\
&\max_{\theta_{t2}^A}\ -e^{-\gamma*\left(w_{t2}^A\ +\ \theta_{t2}^A*\left(E^A\left[\widetilde{X}\middle|S,M^*\right]-P_{t2}\right)\right)\ +\ 1/2*\gamma^2*\theta_{t2}^{2\ A}*\mathrm{Var}^A\left[\widetilde{X}\middle|S,M^*\right]}
\end{aligned}
$$

Given his prior beliefs, the information received through signal S, and his personal interpretation of information (model M) his optimal allocation at time 2 is the following.

$$
\begin{aligned}
\theta_{t2}^A &= \frac{E^A\left[\widetilde{X}\middle|\ S, M^*\right] - P_{t2}}{\gamma * \mathrm{VAR}^A\left[\widetilde{X}\middle|S, M^*\right]} \\
\theta_{t2}^A &= \frac{\rho_X^A * \mu_X^A + \rho_{\mathcal{E}}^{A|M^*} *\ \left(S - \mu_\epsilon^A - \mu_\delta^{A|M^*}\right) - P_{t2} * \left(\rho_X^A + \rho_{\mathcal{E}}^{A|M^*}\right)}{\gamma} \\
&\mathit{where} \\
\rho_X^A &= \frac{1}{\sigma_X^{2\ A}} \\
\rho_{\mathcal{E}}^{A|M^*} &= \frac{1}{\sigma_{\mathcal{E}}^{2\ A|M^*}} = \frac{1}{\sigma_\epsilon^{2\ A} + \sigma_\delta^{2\ A|M^*}}
\end{aligned}
$$

Ambiguity Averse Investor (Type-A) At Time 2: Optimal Paremeters

In this section, I elaborate on the optimal model M parameters, which the Ambiguous agents will employ to interpret the signal S according to their Max Min utility.

To ensure tractability, I adopt a simpler parametrization wherein ambiguity is represented by a range of models with different means u_{AM}, while sharing a fixed ambiguity volatility $\sigma_{A_F}^{2\ A}$.

$\widetilde{A} \sim N\Big(\mu_A^A \in [\ \underline{\mu}_A^A, \overline{\mu}_A^A],\ \sigma_A^{2\ A} \in [\sigma_{A_F}^{2\ A}]\Big)$.

In this parametrization, the Min component of the ambiguous agent's utility gives rise to a function with one kink, located at point $\theta_{t2}^A = 0$. This results in a piecewise function with three regions where the agent maximizes its utility (Condie & Ganguli, 2017). In the first region, the agent selects a model with a mean ambiguity $\underline{u}_A^A$. Moving to the next region, the agent opts for a model with a mean ambiguity $\underline{\bar{\mu}}^A$ inside the range $[\underline{u}_A^A, \overline{u}_A^A]$, ensuring that the expected return $E^A\Big[\widetilde{X}\Big|S, M\Big] - P_{t2}$ equals 0. Lastly, in the final region, the agent goes for the mean ambiguity $\overline{u}_A$. The following equations summarize the piecewise utility of the ambiguous agent across these different regions.

$$\max_{\theta_{t2}^A} E^{I_A}\left[U^A\left(\theta_{t2}^A\right)\middle|S\right] = \begin{cases} \max\limits_{\theta_{t2}^A}\ E^A\Big[-e^{\ -\gamma\ *\ \left(w_{t2}+\theta_{t2}^B*\ \left(\tilde{X}\ -\ P_{t2}\right)\right)}\ \Big|\ S, M = \{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] & \text{if } \theta_{t2}^A < 0 \\ \max\limits_{\theta_{t2}^A}\ E^A\Big[-e^{\ -\gamma\ *\ \left(w_{t2}+\theta_{t2}^B*\ \left(\tilde{X}\ -\ P_{t2}\right)\right)}\ \Big|\ S, M = \{\underline{\bar{\mu}}_A^A, \sigma_{A_F}^{2\ A}\}\Big] & \text{if } \theta_{t2}^A = 0 \\ \max\limits_{\theta_{t2}^A}\ E^A\Big[-e^{\ -\gamma\ *\ \left(w_{t2}+\theta_{t2}^B*\ \left(\tilde{X}\ -\ P_{t2}\right)\right)}\ \Big|\ S, M = \{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\Big] & \text{if } \theta_{t2}^A > 0 \end{cases}$$

One interesting aspect of this utility function is the abrupt changes it exhibits when market conditions and signals shift, causing the utility to transition between different regions. In such instances, adjustments in the optimal mean ambiguity parameter μ_A^A leads to alterations in the utility function's shape.

Another noteworthy aspect of this utility function is the agent's portfolio inertia, wherein $\theta_{t2}^A = 0$. Within this particular region, the concrete allocation quantity remains entirely insensitive to the agent's current ambiguity.

This piecewise function leads to variations in the optimal allocation θ_{t2}^A depending on the region of operation. In one region, the allocation corresponds to the optimal mean variance portfolio resulting from a mean ambiguity $\overline{\mu}_A^A$. In the next region, the agent withdraws completely from the market. Finally, in the last region, the allocation represents the optimal mean variance portfolio resulting from a mean ambiguity $\underline{\mu}_A^A$.

$$\theta_{t2}^{A*} = \begin{cases} \dfrac{E^A\left[\widetilde{X}\middle|S, M = \{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\right] - P_{t2}}{\gamma * \mathrm{VAR}^A\left[\widetilde{X}\middle|S, M = \{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\right]} & \text{if } P_{t2} < E^A\left[\widetilde{X}\middle|S, M = \{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\right] \\ \dfrac{E^A\left[\widetilde{X}\middle|S, M = \{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\right] - P_{t2}}{\gamma * \mathrm{VAR}^A\left[\widetilde{X}\middle|S, M = \{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\right]} & \begin{array}{l}\text{if } E^A\left[\widetilde{X}\middle|S, M = \{\overline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\right] \leq P_{t2} \\ \text{and } P_{t2} \leq E^A\left[\widetilde{X}\middle|S, M = \{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\right]\end{array} \\ \dfrac{E^A\left[\widetilde{X}\middle|S, M = \{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\right] - P_{t2}}{\gamma * \mathrm{VAR}^A\left[\widetilde{X}\middle|S, M = \{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\right]} & \text{if } P_{t2} > E^A\left[\widetilde{X}\middle|S, M = \{\underline{\mu}_A^A, \sigma_{A_F}^{2\ A}\}\right] \end{cases}$$

Equilibrium At Time 2

In equilibrium at time 2 the following equation has to hold.

$$(1 - \pi) * \theta_{t2}^B + \pi * \theta_{t2}^A = 0$$

Replacing the investor types A and B optimal allocations θ_{t2}^B and θ_{t2}^A in the market clearing condition.

$\pi * [\rho_X^A * \ \mu_X^A \ + \rho_{\mathcal{E}}^{A|M^*} * (S - \mu_\epsilon^A - \mu_\delta^{A|M^*}) \ - \ P_{t2} * \ (\rho_X^A + \rho_{\mathcal{E}}^{A|M^*}) \ ] \ + \ (1 - \pi) \ * \ [\rho_X^B * \mu_X^B \ + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) \ - \ P_{t2} * (\rho_X^B + \rho_{\mathcal{E}}^B) \ ] = 0.$

Grouping together terms associated to the investors previous beliefs about the risky asset expected payment, the signal S, the signal S error and price P_{t2}

$[\pi * \rho_X^A * \mu_X^A + (1 - \pi) * \rho_X^B * \mu_X^B]_{\bar{\mu}_X} + [\pi * \rho_{\mathcal{E}}^{A|M^*} + (1 - \pi) * \rho_{\mathcal{E}}^B]_{\bar{\rho}_{\mathcal{E}}} * \mathrm{S} - \left[\pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\epsilon^A + (1 - \pi) * \rho_{\mathcal{E}}^B * \mu_\epsilon^B\right]_{\bar{\mu}_\epsilon} - [\pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}] - P_{t2} * [\pi * \rho_X^A + (1 - \pi) * \rho_X^B]_{\bar{\rho}_X} - P_{t2} * [\pi * \rho_{\mathcal{E}}^{A|M^*} + (1 - \pi) * \rho_{\mathcal{E}}^B]_{\bar{\rho}_{\mathcal{E}}} = 0$

Replacing the expression in brackets by the terms $\bar{\mu}_X, \bar{\rho}_{\mathcal{E}}, \bar{\mu}_\epsilon$ and $\bar{\rho}_X$ we can rewrite the market clearing condition as

$$\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * \ \mathrm{S} - \bar{\mu}_\epsilon - \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*} - P_{t2} * (\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}) = 0.$$

Signal S At Time 2

From here we can rewrite the price at time 2.

$$P_{t2} = \frac{\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * \ \mathrm{S} - \bar{\mu}_\epsilon - \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}}{\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}}.$$

We can use this last expression for P_{t2} to rewrite the signal S as a function of the change in price ΔP

$$\begin{aligned}
\Delta \mathrm{P} = P_{t2} - P_{t1} &= \frac{\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * S - \bar{\mu}_\epsilon - \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}}{\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}} - P_{t1} \\
\Delta \mathrm{P} * [\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] &= \left[\bar{\mu}_X + \bar{\rho}_{\mathcal{E}} * S - \bar{\mu}_\epsilon - \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}\right] - P_1 * [\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] \\
\mathrm{S} &= \frac{\Delta \mathrm{P} * [\bar{\rho}_X + \bar{\rho}_{\mathcal{E}}] + \bar{\mu}_\epsilon + \pi * \rho_{\mathcal{E}}^{A|M^*} * \mu_\delta^{A|M^*}}{\bar{\rho}_{\mathcal{E}}} + P_{t1}
\end{aligned}$$

In this last equation we replaced the price P_{t1} by the formula obtained for it as a result of the market equilibrium at time 1. $P_{t1} = \dfrac{\bar{\mu}_X}{\bar{\rho}_X}$.

Trading Volume from Time 1 to Time 2

We measure the trading volume from time 1 to 2 as the change in the risky-asset allocation of the ambiguity-neutral investor type (B) multiplied by the quantity of this type of investor. By symmetry of this market equilibrium, the volume of risky asset this investor type-B buys/sells is equivalent to the volume the ambiguous investor type-A sells/buys.

$$\begin{aligned}
& V = (1-\pi) * (\theta_{t2}^B - \theta_{t1}^B) \\
where \\
& \theta_{t1}^B = (\mu_X^B - P_{t1}) * \rho_X^B / \gamma \\
& \theta_{t2}^B = \frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) - P_{t2} * (\rho_X^B + \rho_{\mathcal{E}}^B)}{\gamma}
\end{aligned}$$

Replacing the allocations of the ambiguity-neutral investor type-B in the expression $(\theta_{t2}^B - \theta_{t1}^B)$ we obtain

$$\theta_{t2}^B - \theta_{t1}^B = \left[\frac{\rho_X^B * \mu_X^B + \rho_{\mathcal{E}}^B * (S - \mu_\epsilon^B) - P_{t2} * (\rho_X^B + \rho_{\mathcal{E}}^B)}{\gamma}\right] \ - \ \left[\frac{(\mu_X^B - P_{t1}) * \rho_X^B}{\gamma}\right]$$

$$\theta^B_{t2} - \theta^B_{t1} = \frac{\rho^B_{\mathcal{E}} * \left[\, \mathrm{S} - \mu^B_\epsilon \right]}{\gamma} - \frac{P_{t2} * \rho^B_{\mathcal{E}}}{\gamma} - \Delta\mathrm{P} * \frac{\rho^B_X}{\gamma}$$

At this stage we can rewrite the signal S expression $(S - \mu^B_\epsilon)$ as a function of the change in prices between period (1) and (2) using the formula we found in the previous section.

$$\theta^B_{t2} - \theta^B_{t1} = \rho^B_{\mathcal{E}} * \frac{\left[\left(\frac{\Delta\mathrm{P}*[\bar\rho_X + \bar\rho_{\mathcal{E}}] + \bar\mu_\epsilon + \pi*\rho^{A|M^*}_{\mathcal{E}} * \mu^{A|M^*}_\delta}{\bar\rho_{\mathcal{E}}} + P_{t1}\right)_{\mathrm{S}} - \mu^B_\epsilon\right]}{\gamma} - \frac{P_{t2} * \rho^B_{\mathcal{E}}}{\gamma} - \Delta\mathrm{P} * \frac{\rho^B_X}{\gamma}$$

$$\theta^B_{t2} - \theta^B_{t1} = \frac{\Delta\mathrm{P}}{\gamma} * \left[\rho^B_{\mathcal{E}} * \frac{(\bar\rho_X + \bar\rho_{\mathcal{E}})}{\bar\rho_{\mathcal{E}}} - \rho^B_{\mathcal{E}} - \rho^B_X\right] + \left[\frac{\rho^B_{\mathcal{E}} * \bar\mu_\epsilon}{\gamma*\bar\rho_{\mathcal{E}}} - \frac{\rho^B_{\mathcal{E}} * \mu^B_\epsilon}{\gamma} + \frac{\pi*\rho^B_{\mathcal{E}}*\rho^{A|M^*}_{\mathcal{E}} * \mu^{A|M^*}_\delta}{\gamma*\bar\rho_{\mathcal{E}}}\right]$$

$$\theta^B_{t2} - \theta^B_{t1} = \frac{\Delta\mathrm{P}}{\gamma} * \left[\rho^B_{\mathcal{E}} * \frac{\bar\rho_X}{\bar\rho_{\mathcal{E}}} - \rho^B_X\right]_2 + \left[\frac{\rho^B_{\mathcal{E}} * \bar\mu_\epsilon}{\gamma*\bar\rho_{\mathcal{E}}} - \frac{\rho^B_{\mathcal{E}} * \mu^B_\epsilon}{\gamma} + \frac{\pi*\rho^B_{\mathcal{E}}*\rho^{A|M^*}_{\mathcal{E}} * \mu^{A|M^*}_\delta}{\gamma*\bar\rho_{\mathcal{E}}}\right]_1$$

We simplify the expressions inside bracket (2)

$$\left[\rho^B_{\mathcal{E}} * \frac{\bar\rho_X}{\bar\rho_{\mathcal{E}}} - \rho^B_X\right]_2 = \left[\frac{\rho^B_{\mathcal{E}} * \pi * \rho^A_X - \rho^B_X * \pi * \rho^{A|M^*}_{\mathcal{E}}}{\bar B}\right]$$

$$\left[\rho^B_{\mathcal{E}} * \frac{\bar\rho_X}{\bar\rho_{\mathcal{E}}} - \rho^B_X\right]_2 = \pi * \left[\frac{\rho^B_{\mathcal{E}} * \rho^A_X - \rho^B_X * \rho^{A|M^*}_{\mathcal{E}}}{\bar\rho_{\mathcal{E}}}\right]$$

We also rewrite the expression inside bracket (1)

$$[...]_1 = \left[\frac{\left(\rho^B_{\mathcal{E}}*\pi*\rho^{A|M^*}_{\mathcal{E}}*\mu^A_\epsilon + \rho^B_{\mathcal{E}}*(1-\pi)*\rho^B_{\mathcal{E}}*\mu^B_\epsilon\right) - \left(\rho^B_{\mathcal{E}}*\mu^B_\epsilon*\pi*\rho^{A|M^*}_{\mathcal{E}} + \rho^B_{\mathcal{E}}*\mu^B_\epsilon*(1-\pi)*\rho^B_{\mathcal{E}}\right)}{\gamma*\bar\rho_{\mathcal{E}}} + \frac{\pi*\rho^B_{\mathcal{E}}*\rho^{A|M^*}_{\mathcal{E}}*\mu^{A|M^*}_\delta}{\gamma*\bar\rho_{\mathcal{E}}}\right]$$

$$[...]_1 = \left[\frac{\rho^B_{\mathcal{E}} * \pi * \rho^{A|M^*}_{\mathcal{E}} * \mu^A_\epsilon - \rho^B_{\mathcal{E}} * \mu^B_\epsilon * \pi * \rho^{A|M^*}_{\mathcal{E}}}{\gamma * \bar\rho_{\mathcal{E}}} + \frac{\pi * \rho^B_{\mathcal{E}} * \rho^{A|M^*}_{\mathcal{E}} * \mu^{A|M^*}_\delta}{\gamma * \bar\rho_{\mathcal{E}}}\right]$$

$$[...]_1 = \frac{\pi * \rho^{A|M^*}_{\mathcal{E}} * \rho^B_{\mathcal{E}} * \left[\mu^A_\epsilon - \mu^B_\epsilon + \mu^{A|M^*}_\delta\right]}{\gamma * \bar\rho_{\mathcal{E}}}$$

Then we can rewrite the expression for the trade-volume originated by the ambiguity neutral investor type-B as

$$\theta^B_{t2} - \theta^B_{t1} = \left[\frac{\pi*\rho^{A|M^*}_{\mathcal{E}}*\rho^B_{\mathcal{E}}*\left[\mu^A_\epsilon - \mu^B_\epsilon + \mu^{A|M^*}_\delta\right]}{\gamma*\bar\rho_{\mathcal{E}}}\right]_1 + \pi * \left[\frac{\rho^B_{\mathcal{E}}*\rho^A_X - \rho^B_X*\rho^{A|M^*}_{\mathcal{E}}}{\bar\rho_{\mathcal{E}}}\right]_2 * \frac{\Delta\mathrm{P}_{21}}{\gamma}$$

The total trading volume in the economy is given by the expression below. The ambiguity about the signal term δ manifests within this expression through the model dependent mean $\mu^{A|M^*}_\delta$

and the precissions $\rho_{\mathcal{E}}^{A|M^*}$ and $\bar{\rho}_{\mathcal{E}}$, whose specific values depend on the model M^* chosen by the ambiguous investor type-B on the minimization stage of his utility function.

$$V = |(1-\pi) * (\theta_{t2}^B - \theta_{t1}^B)|$$

$$V = \left| \left[\frac{\pi*(1-\pi)*\rho_{\mathcal{E}}^{A|M^*}*\rho_{\mathcal{E}}^{B}*\left[\mu_{\epsilon}^{A}-\mu_{\epsilon}^{B}+\mu_{\delta}^{A|M^*}\right]}{\gamma*\bar{\rho}_{\mathcal{E}}} \right]_1 + \left[\frac{\pi*(1-\pi)*(\rho_{\mathcal{E}}^{B}*\rho_{X}^{A}-\rho_{X}^{B}*\rho_{\mathcal{E}}^{A|M^*})}{\gamma*\bar{\rho}_{\mathcal{E}}} \right]_2 * \Delta \mathrm{P}_{21} \right|$$

$$V_{21} = |\alpha_V + \beta_V * \Delta P_{21}|$$

Appendix C Volume and Price Volatility Elasticity

This section derives the expected market's trading volume one period ahead, as well as the trading volume to price volatility elasticity. These formulas expand the work of Bollerslev et al. (2018) by introducing (1) Ambiguity (Knightian Uncertainty) and (2) a non standard normal distribution.

C.1 Expected Trading Volume

This part shows the derivation of the market's expected trading volume one period ahead, $E[V_{21}]$ in the presence of an Ambiguous Agent. The formula uses the theoretical expression for trading volume V_{21} presented in Section 2.7 and Appendix A. We calculate a closed form expression for $E[V_{21}]$ using the volume formula $V_{21} = |\alpha_V + \beta_V * \Delta P_{21}|$ and assuming a normal distribution $\Delta P_{21} \sim\ N(0, \sigma_{\Delta P})$ for small daily intervals.

Using the standard normal distribution $Z = \frac{\Delta P_{21}}{\sigma_{\Delta P}}$ we rewrite the expression for trading volume as $V_{21} = |\alpha_V + \beta_V * \sigma_{\Delta P} * Z|$ The expected volume calculation can be split in two parts conditional of the sign that the expression inside the absolute value takes

$$E[V_{21}] = \begin{array}{l} E[V_{21} \mid \alpha_V + \beta_V * \sigma_{\Delta P} * Z > 0] * P(\alpha_V + \beta_V * \sigma_{\Delta P} * Z > 0) \\ + E[V_{21} \mid \alpha_V + \beta_V * \sigma_{\Delta P} * Z < 0] * P(\alpha_V + \beta_V * \sigma_{\Delta P} * Z < 0) \end{array}$$

Taking into account the conditional density $f_{z|condition}$ of the standard normal distribution

$$f_{z|\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} = \frac{f_z * I_[[\alpha_V + \beta_V * \sigma_{\Delta P} * Z > 0]}{P(\alpha_V + \beta_V * \sigma_{\Delta P} * Z > 0)}$$

$$f_{z|\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} = \frac{f_Z * I_[[\alpha_V + \beta_V * \sigma_{\Delta P} * Z < 0]}{P(\alpha_V + \beta_V * \sigma_{\Delta P} * Z < 0)}.$$

the expected volume formula simplifies to

$$E[V_{21}] = \int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\alpha_V + \beta_V * \sigma_{\Delta P} * Z) * f_z dZ + \int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\alpha_V + \beta_V * \sigma_{\Delta P} * Z) * f_z dZ$$

Integral of β_V Section

First we calculate the integrals of the expressions $(\beta_V * \sigma_{\Delta P} * Z)$ and $(-\beta_V * \sigma_{\Delta P} * Z)$. For both integrals, there are two sub-cases depending on the sign of the constant β_V, because the inequality that defines the integrals' regions flips direction.

For $\beta_V \geq 0$:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\beta_V * \sigma_{\Delta P} * Z) * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[-\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\frac{\alpha_V}{\beta_V^* \sigma_{\Delta P}}}^{+\infty}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\beta_V * \sigma_{\Delta P} * Z) * f_z dZ = \frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} = \frac{|\beta_V| * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\beta_V * \sigma_{\Delta P} * Z) * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\infty}^{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\beta_V * \sigma_{\Delta P} * Z) * f_z dZ = \frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} = \frac{|\beta_V| * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}$$

For $\beta_V < 0$:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\beta_V * \sigma_{\Delta P} * Z) * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\infty}^{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\beta_V * \sigma_{\Delta P} * Z) * f_z dZ = -\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} = \frac{|\beta_V| * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\beta_V * \sigma_{\Delta P} * Z) * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}}^{+\infty}$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\beta_V * \sigma_{\Delta P} * Z) * f_z dZ = -\frac{\beta_V * \sigma_{\Delta P}}{\sqrt{2*\pi*}} e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} = \frac{|\beta_V| * \sigma_{\Delta P}}{\sqrt{2*\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}$$

We get that independent of the sign of phi1, the sum of both integrals of the expressions $(\beta_V * \sigma_{\Delta P} * Z)$ and $(-\beta_V * \sigma_{\Delta P} * Z)$ is equal to

$$|\beta_V| * \sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}.$$

Integral of α_V Section

Second, we calculate the integrals of the expressions (α_V) and $(-\alpha_V)$. For both integrals, there are four sub-cases depending on the sign of the constants $\{\alpha_V, \beta_V\}$, because the inequality that defines the integrals' regions flips direction with β_V and the sign of both constants affect the standard normal density function.

For α_V^+ and β_V^+:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}}^{+\infty} = \alpha_V * \left(1 - \Phi\left(-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)\right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P*Z>0}} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = |\alpha_V| * \left[1 - \Phi\left(-\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)\right] = |\alpha_V| * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[-\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\infty}^{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}} = -\alpha_V * \Phi\left(-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = -\alpha_V * \left[1 - \Phi\left(\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)\right] = |\alpha_V| * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]$$

For α_V^+ and β_V^-:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P*}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\alpha_V * \frac{1}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\infty}^{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}} = \alpha_V * \Phi\left(-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = |\alpha_V| * \Phi\left(\frac{\alpha_V}{-\beta_V * \sigma_{\Delta P}}\right) = |\alpha_V| * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[-\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\frac{\alpha_V}{\beta_V*\sigma_{\Delta P}}}^{+\infty} = -\alpha_V * \left[1 - \Phi\left(\frac{\alpha_V}{-\beta_V * \sigma_{\Delta P}}\right)\right]$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \alpha_V * \left[\Phi\left(\frac{\alpha_V}{-\beta_V * \sigma_{\Delta P}}\right) - 1\right] = |\alpha_V| * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]$$

For α_V^- and β_V^+:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}}\right]_{-\frac{\alpha_V}{\beta_V*\sigma}}^{+\infty} = \alpha_V * \left(1 - \Phi\left(-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}\right)\right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = (-\alpha_V) * \left[\Phi\left(\frac{-\alpha_V}{\beta_V * \sigma_{\Delta P}}\right) - 1\right] = |\alpha_V| * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[-\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\infty}^{-\frac{\alpha_V}{\beta_V^* \sigma_{\Delta P}}} = -\alpha_V * \Phi\left(\frac{\alpha_V}{-\beta_V * \sigma_{\Delta P}} \right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = |\alpha_V| * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}} \right)$$

For α_V^- and β_V^-:

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\infty}^{-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}} = \alpha_V * \Phi\left(-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}} \right)$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \alpha_V * \left[1 - \Phi\left(\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}} \right) \right] = (-\alpha_V) * \left[\Phi\left(\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}} \right) - 1 \right]$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} \alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \alpha_V * \left[1 - \Phi\left(\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}} \right) \right] = |\alpha_V| * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}} \right) - 1 \right]$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = \left[-\alpha_V * \frac{}{\sqrt{2*\pi}} * e^{\frac{-Z^2}{2}} \right]_{-\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}}}^{+\infty} = -\alpha_V * \left[1 - \Phi\left(\frac{-\alpha_V}{\beta_V * \sigma_{\Delta P}} \right) \right]$$

$$\int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -\alpha_V * \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ = -\alpha_V * \Phi\left(\frac{\alpha_V}{\beta_V * \sigma_{\Delta P}} \right) = |\alpha_V| * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}} \right)$$

We get that independent of the constants $\{\alpha_V, \beta_V\}$ signs, the sum the sum of both integrals of the expressions (α_V) and $(-\alpha_V)$ is equal to

$$|\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}} \right) - 1 \right]$$

Final Result

Finally, we get that independent of the constants $\{\alpha_V, \beta_V\}$ signs, the expression for the one period ahead expected trading volume is

$$E[V_{21}] = \int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z>0} (\alpha_V + \beta_V * \sigma_{\Delta P} * Z) * f_z dZ + \int_{\alpha_V+\beta_V*\sigma_{\Delta P}*Z<0} -(\alpha_V + \beta_V * \sigma_{\Delta P} * Z) * f_z dZ$$

$$E[V_{21}] = |\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}} \right) - 1 \right] + |\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} \right]$$

C.2 Trading Volume to Price Volatility Elasticity

This part makes use of the expected trading volume derived above, to calculate the expected trading volume to price volatility elasticity $\xi = \frac{\partial E[V_{21}]/E[V_{21}]}{\partial\sigma_{\Delta P}/\sigma_{\Delta P}}$ presented in Section-2.7.

The derivation starts by calculating the derivative of the expected trading volume

$$\xi = \frac{\partial E[V_{21}]/E[V_{21}]}{\partial\sigma_{\Delta P}/\sigma_{\Delta P}} = \frac{\partial E[V_{21}]}{\partial\sigma_{\Delta P}} * \frac{\sigma_{\Delta P}}{E[V_{21}]}$$

We can divide the expected trading volume in two parts

$$E[V_{21}] = P1 + P2 = |\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right] + |\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}\right].$$

Then, we proceed to derive the first part of the expected volume P_1 in relation to price volatility $\sigma_{\Delta P}$

$$\frac{\partial \mathrm{P}_1}{\partial\sigma_{\Delta P}} = \frac{\partial\left\{|\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V|*\sigma_{\Delta P}}\right) - 1\right]\right\}}{\partial\sigma_{\Delta P}} = |\alpha_V| * 2 * \frac{\partial\left\{\int_{-\infty}^{\frac{|\alpha_V|}{|\beta_V|*\sigma_{\Delta P}}} \frac{1}{\sqrt{2*\pi}} e^{\frac{-Z^2}{2}} dZ\right\}}{\partial\sigma_{\Delta P}}$$

$$\frac{\partial \mathrm{P}_1}{\partial\sigma_{\Delta P}} = |\alpha_V|^* 2^* \frac{-|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}^2} * \frac{1}{\sqrt{2*\pi}} e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} = \frac{\sqrt{2}}{\sqrt{\pi}} * \frac{-|\alpha_V|^2}{|\beta_V| * \sigma_{\Delta P}^2} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}.$$

Now, we continue by deriving the second part P_2 of the expected trading volume in relation to price volatility $\sigma_{\Delta P}$

$$\frac{\partial \mathrm{P}_2}{\partial\sigma_{\Delta P}} = \frac{\partial\left\{|\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V{}^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}\right]\right\}}{\partial\sigma_{\Delta P}} = |\beta_V|^* \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}} + \frac{\sqrt{2}}{\sqrt{\pi}} * \frac{|\alpha_V|^2}{|\beta_V| * \sigma_{\Delta P}^2} * e^{\frac{-\alpha_V{}^2}{2*\beta_V^2*\sigma_{\Delta P}^2}}$$

$$\frac{\partial E[V_{21}]}{\partial\sigma_{\Delta P}} = \frac{\partial(\mathrm{P}_1 + \mathrm{P}_2)}{\partial\sigma_{\Delta P}} = \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2*\beta_V^2*\sigma_{\Delta P}^2}*} |\beta_V| = \frac{\mathrm{P}_2}{\sigma_{\Delta P}}$$

With the previous results, we can proceed to calculate the elasticity ξ

$$\xi = \frac{\partial E[V_{21}]/E[V_{21}]}{\partial\sigma_{\Delta P}/\sigma_{\Delta P}} = \frac{\partial E[V_{21}]}{\partial\sigma_{\Delta P}} * \frac{\sigma_{\Delta P}}{E[V_{21}]} = \left(\frac{\partial E[V_{21}]}{\partial\sigma_{\Delta P}} * \sigma_{\Delta P}\right) * \frac{1}{E[V_{21}]}$$

$$\xi = \mathrm{P}_2 * \frac{1}{\mathrm{P}_1 + \mathrm{P}_2} = \frac{1}{1 + \mathrm{P}_1/\mathrm{P}_2}.$$

Replacing for the terms P_1 and P_2 we get the final expected trading volume to price volatility elasticity formula ξ used in section Section-2.7. In the formula below ϕ refers to the standard normal density function and Φ refers to the standard normal CDF.

$$\xi = \frac{1}{1 + \frac{|\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]}{|\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2 * \beta_V^2 * \sigma_{\Delta P}^2}}\right]}} = \frac{1}{1 + \frac{|\alpha_V| / (|\beta_V|^* \sigma_{\Delta P}) * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1/2\right]}{\frac{1}{\sqrt{2 * \pi}} * e^{\frac{-\alpha_V^2}{2 * \beta_V^2 * \sigma_{\Delta P}^2}}}}$$

$$\xi = \frac{1}{1 + \frac{|\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]}{|\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2 * \beta_V^2 * \sigma_{\Delta P}^2}}\right]}} = \frac{1}{1 + \frac{|\alpha_V| / (|\beta_V|^* \sigma_{\Delta P}) * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1/2\right]}{\phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)}}$$

$$\xi = \frac{1}{1 + \frac{|\alpha_V| * \left[2 * \Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1\right]}{|\beta_V| * \left[\sigma_{\Delta P} * \frac{\sqrt{2}}{\sqrt{\pi}} * e^{\frac{-\alpha_V^2}{2 * \beta_V^2 * \sigma_{\Delta P}^2}}\right]}} = \frac{1}{1 + \frac{|\alpha_V| / (|\beta_V|^* \sigma_{\Delta P}) * \left[\Phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right) - 1/2\right]}{\phi\left(\frac{|\alpha_V|}{|\beta_V| * \sigma_{\Delta P}}\right)}}$$

Appendix D Model Daily Regressions Results

In this appendix, I present the daily frequency regressions for the trading volume model outlined in Section-2.6, along with the elasticity model detailed in Section-2.7.

Tables 8, 9, and 10 below present the regressions (5.1), (5.2), and (5.3) results of the trading volume model as discussed in Section-5.

$$V = \alpha_V + \beta_V * \Delta_P + \epsilon \tag{5.1}$$

$$\alpha_V = c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P + \varsigma_{t-1} * \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$V = \big(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\big)_{\alpha} \; + \; \big(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\big)_{\beta} * \Delta_P \; + \varsigma_{t-1} * V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \tag{5.2}$$

$$\Delta V = c \; + \; \big(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\big)_{\Delta\alpha} \; + \; \big(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\big)_{\Delta\beta} * \Delta_P \; + \; \big(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\big)_{\beta} * \Delta_P^2 \; + \varsigma_{t-1} * \Delta V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \tag{5.3}$$

The outcomes of regression (6.2) in Section-6 for the elasticity model are illustrated in table-11.

$$\Delta log(V) = c \; + \; \big(\; \xi_1 \; + \xi_{pbel} * PBEL \; + \; \xi_{amb} * AMB \; + \; \xi_{pbel^2} * PBEL^2 \; + \; \xi_{amb^2} * AMB^2 + \xi_{\sigma_{amb}} * \sigma_{amb} + \xi_{\sigma^2_{amb}} * \sigma^2_{amb} \; \big)_{\xi} * \Delta log(\sigma_{\Delta p}) + \varsigma_{t-1} * \Delta log(V)_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \; \epsilon \tag{6.2}$$

The regressions cover the daily period from 2013 to 2018 for the SPY and were conducted on four distinct datasets: D(1), D(2), D(3) and D(4). Each dataset employs the daily EPU measure extracted from Twitter as a proxy for Ambiguity, alongside a distinct proxy for differences in prior beliefs from Stocktwits (Cookson & Niessner, 2020).

Daily Datasets Description	
D(1)	Prior beliefs differences proxied by $\text{PBEL}_{\text{WI, IND}}$ and Ambiguity proxied by AMB_{EPUD}
D(2)	Prior beliefs differences proxied by $\text{PBEL}_{\text{AC, IND}}$ and Ambiguity proxied by AMB_{EPUD}
D(3)	Prior beliefs differences proxied by $\text{PBEL}_{\text{WI, ETF}}$ and Ambiguity proxied by AMB_{EPUD}
D(4)	Prior beliefs differences proxied by $\text{PBEL}_{\text{AC, ETF}}$ and Ambiguity proxied by AMB_{EPUD}

Table 8. Daily Regressions α_V and β_V

This table summarizes the daily regression results for the coefficients α_V and β_V of the trading volume formula, as detailed in regression (5.1) of Section-5. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\alpha_V = c + \alpha_{amb} * AMB + \alpha_{pbel} * PBEL + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_{\sigma_p} * \sigma_P \ + \ \varsigma_{t-1} * \beta_{t-1} + \ \sum_{p=t+1}^{T} \gamma_p + \ \epsilon$$

	D(1)	D(2)	D(3)	D(4)
Panel A: Regression on α_V				
c	0.15	0.16	0.19	0.16
	(3.31) (3.32) (2.80)	(3.46) (3.47) (3.32)	(4.06) (4.07) (4.34)	(3.40) (3.41) (3.43)
	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{pbel}	0.12	0.08	0.10	-0.12
	(6.64) (6.66) (5.39)	(4.21) (4.22) (3.46)	(5.16) (5.18) (6.03)	(-6.62) (-6.64) (-7.24)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.11	0.11	0.11	0.11
	(5.10) (5.11) (3.62)	(5.30) (5.31) (3.80)	(5.13) (5.14) (3.71)	(5.37) (5.38) (3.84)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
ς_{t-1}	0.58	0.58	0.58	0.57
	(28.32) (28.41) (17.50)	(28.32) (28.41) (17.50)	(28.30) (28.38) (16.89)	(27.93) (28.01) (16.75)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1509	1509	1509	1509
R_a^2	0.489	0.481	0.484	0.489
Panel B: Regression on β_V (same for 1, 2, 3, 4)				
c	0.02	0.02	0.02	0.02
	(0.24) (0.24) (0.17)	(0.24) (0.24) (0.17)	(0.24) (0.24) (0.17)	(0.24) (0.24) (0.17)
	[0.81] [0.81] [0.86]	[0.81] [0.81] [0.86]	[0.81] [0.81] [0.86]	[0.81] [0.81] [0.86]
$\beta_{\sigma_{amb}}$	-0.09	-0.09	-0.09	-0.09
	(-1.75) (-1.75) (-1.74)	(-1.75) (-1.75) (-1.74)	(-1.75) (-1.75) (-1.74)	(-1.75) (-1.75) (-1.74)
	[0.08] [0.08] [0.08]	[0.08] [0.08] [0.08]	[0.08] [0.08] [0.08]	[0.08] [0.08] [0.08]
β_{σ_p}	-0.06	-0.06	-0.06	-0.06
	(-1.44) (-1.44) (-1.77)	(-1.44) (-1.44) (-1.77)	(-1.44) (-1.44) (-1.77)	(-1.44) (-1.44) (-1.77)
	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]
ς_{t-1}	0.02	0.02	0.02	0.02
	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)
	[0.48] [0.48] [0.62]	[0.48] [0.48] [0.62]	[0.48] [0.48] [0.62]	[0.48] [0.48] [0.62]
N	1509	1509	1509	1509
R_a^2	0.000	0.000	0.000	0.000

Table 9. Daily Regressions for Trading Volume

This table summarizes the daily regression results of the trading volume formula, as detailed in regression (5.2) of Section-5. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$V = \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\right)_{\alpha} + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\right)_{\beta} * \Delta_P + \varsigma_{t-1} * V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
c	-0.11	-0.09	-0.08	-0.10
	(-2.46) (-2.47) (-2.15)	(-2.16) (-2.17) (-2.06)	(-1.84) (-1.85) (-2.02)	(-2.40) (-2.41) (-2.54)
	[0.01] [0.01] [0.03]	[0.03] [0.03] [0.04]	[0.07] [0.07] [0.04]	[0.02] [0.02] [0.01]
α_{pbel}	0.11	0.08	0.07	-0.13
	(6.21) (6.23) (5.48)	(4.13) (4.15) (3.64)	(3.50) (3.51) (3.85)	(-7.39) (-7.42) (-7.70)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.10	0.10	0.10	0.10
	(5.02) (5.04) (3.78)	(5.17) (5.19) (3.94)	(5.11) (5.13) (3.92)	(5.30) (5.32) (4.10)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
β_{p}	-0.33	-0.33	-0.33	-0.33
	(-14.87) (-14.93) (-8.32)	(-15.04) (-15.10) (-8.48)	(-14.68) (-14.74) (-8.16)	(-15.32) (-15.38) (-8.71)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\sigma_{\text{amb}}}$	-0.05	-0.05	-0.05	-0.05
	(-1.96) (-1.96) (-1.30)	(-2.05) (-2.06) (-1.37)	(-2.11) (-2.12) (-1.40)	(-2.15) (-2.16) (-1.44)
	[0.05] [0.05] [0.20]	[0.04] [0.04] [0.17]	[0.03] [0.03] [0.16]	[0.03] [0.03] [0.15]
$\beta_{\sigma_{\text{p}}}$	0.10	0.10	0.09	0.10
	(4.18) (4.19) (2.37)	(4.08) (4.10) (2.29)	(3.94) (3.96) (2.18)	(4.25) (4.27) (2.41)
	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.03]	[0.00] [0.00] [0.02]
ς_{t-1}	0.61	0.61	0.61	0.60
	(32.09) (32.22) (17.38)	(32.01) (32.14) (17.61)	(32.00) (32.13) (16.97)	(31.68) (31.81) (16.91)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1510	1510	1510	1510
R_a^2	0.538	0.532	0.53	0.543

Table 10. Daily Regressions for ΔTrading Volume

This table summarizes the daily regression results of the first difference trading volume formula, as detailed in regression (5.3) of Section-5. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values. γ_p represent time fixed effects.

$$\Delta V = c \ + \ \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\right)_{\Delta\alpha} \ + \ \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P \ + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\right)_\beta * \Delta_P^2 \ + \varsigma_{t-1} * \Delta log(V)_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
c	-0.01	-0.01	-0.01	-0.01
	(-0.15) (-0.15) (-0.27)	(-0.15) (-0.15) (-0.28)	(-0.16) (-0.16) (-0.29)	(-0.16) (-0.16) (-0.29)
	[0.88] [0.88] [0.78]	[0.88] [0.88] [0.78]	[0.88] [0.88] [0.77]	[0.87] [0.87] [0.77]
α_{pbel}	0.13	0.08	0.08	-0.15
	(5.46) (5.49) (4.59)	(3.53) (3.55) (2.93)	(3.18) (3.20) (3.43)	(-6.27) (-6.29) (-6.65)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.12	0.12	0.12	0.12
	(5.11) (5.14) (3.59)	(5.21) (5.24) (3.62)	(5.10) (5.12) (3.53)	(5.23) (5.25) (3.62)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
β_{p}	-0.23	-0.23	-0.22	-0.23
	(-7.70) (-7.74) (-4.89)	(-7.56) (-7.59) (-4.81)	(-7.28) (-7.32) (-4.64)	(-7.78) (-7.81) (-4.95)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\sigma_{\mathrm{amb}}}$	-0.05	-0.05	-0.05	-0.05
	(-1.52) (-1.53) (-1.17)	(-1.59) (-1.60) (-1.21)	(-1.65) (-1.65) (-1.24)	(-1.65) (-1.65) (-1.24)
	[0.13] [0.13] [0.24]	[0.11] [0.11] [0.23]	[0.10] [0.10] [0.21]	[0.10] [0.10] [0.21]
$\beta_{\triangle\sigma_{\mathrm{amb}}}$	-0.02	-0.02	-0.02	-0.02
	(-0.75) (-0.75) (-0.26)	(-0.73) (-0.74) (-0.25)	(-0.75) (-0.76) (-0.26)	(-0.76) (-0.76) (-0.26)
	[0.45] [0.45] [0.79]	[0.46] [0.46] [0.80]	[0.45] [0.45] [0.79]	[0.45] [0.45] [0.80]
$\beta_{\sigma_{\mathrm{p}}\triangle^2{\mathrm{p}}}$	0.11	0.11	0.11	0.11
	(3.68) (3.69) (2.63)	(3.49) (3.51) (2.47)	(3.40) (3.42) (2.37)	(3.55) (3.56) (2.44)
	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.01]
$\beta_{\triangle\sigma_{\mathrm{p}}}$	-0.04	-0.04	-0.03	-0.04
	(-1.57) (-1.58) (-0.48)	(-1.47) (-1.48) (-0.45)	(-1.40) (-1.40) (-0.42)	(-1.43) (-1.44) (-0.42)
	[0.12] [0.12] [0.63]	[0.14] [0.14] [0.65]	[0.16] [0.16] [0.67]	[0.15] [0.15] [0.67]
ς_{t-1}	-0.29	-0.30	-0.30	-0.30
	(-11.84) (-11.90) (-8.61)	(-12.13) (-12.19) (-8.93)	(-12.07) (-12.12) (-9.17)	(-12.10) (-12.15) (-9.06)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1509	1509	1509	1509
R_a^2	0.176	0.167	0.166	0.181

Table 11. Daily Regressions for Trading Volume Elasticity ξ

This table summarizes the daily regression of the elasticity model, as detailed in regression (6.2) of Section-6. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values. γ_p represent time fixed effects.

$$\Delta log(V) = c + \left(\xi_1 + \xi_{pbel} * PBEL + \xi_{amb} * AMB + \xi_{pbel^2} * PBEL^2 + \xi_{amb^2} * AMB^2 + \xi_{\sigma_{amb}} * \sigma_{amb} + \xi_{\sigma^2_{amb}} * \sigma^2_{amb} \right)_{\xi} * \Delta log(\sigma_{\Delta p}) + \varsigma_{t-1} * \Delta log(V)_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
c	-0.02	-0.02	-0.02	-0.02
	(-0.29) (-0.29) (-0.47)	(-0.30) (-0.30) (-0.48)	(-0.28) (-0.28) (-0.45)	(-0.39) (-0.39) (-0.64)
	[0.77] [0.77] [0.64]	[0.77] [0.77] [0.63]	[0.78] [0.78] [0.65]	[0.70] [0.70] [0.52]
ξ_1	0.24	0.24	0.25	0.28
	(8.81) (8.85) (5.83)	(9.07) (9.11) (6.64)	(8.33) (8.37) (7.21)	(8.70) (8.74) (7.08)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
ξ_{pbel}	-0.03	-0.03	-0.02	0.06
	(-1.16) (-1.16) (-1.10)	(-1.42) (-1.43) (-1.33)	(-0.86) (-0.87) (-0.67)	(2.65) (2.66) (2.03)
	[0.25] [0.25] [0.27]	[0.16] [0.15] [0.18]	[0.39] [0.39] [0.50]	[0.01] [0.01] [0.04]
ξ_{amb}	-0.11	-0.12	-0.11	-0.09
	(-4.71) (-4.73) (-4.41)	(-4.95) (-4.98) (-4.74)	(-4.41) (-4.43) (-4.17)	(-3.63) (-3.65) (-3.55)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
ξ_{pbel^2}	0.01	0.03	-0.02	0.08
	(0.41) (0.41) (0.31)	(1.10) (1.10) (0.86)	(-0.53) (-0.53) (-0.44)	(2.24) (2.26) (2.17)
	[0.68] [0.68] [0.76]	[0.27] [0.27] [0.39]	[0.59] [0.59] [0.66]	[0.02] [0.02] [0.03]
ξ_{amb^2}	0.03	0.02	0.03	0.02
	(1.16) (1.16) (1.37)	(0.75) (0.75) (0.91)	(1.36) (1.37) (1.49)	(1.03) (1.03) (1.17)
	[0.25] [0.24] [0.17]	[0.45] [0.45] [0.36]	[0.17] [0.17] [0.14]	[0.30] [0.30] [0.24]
$\xi_{\sigma_{amb}}$	-0.02	-0.03	-0.02	-0.03
	(-0.94) (-0.95) (-0.70)	(-1.09) (-1.09) (-0.82)	(-0.67) (-0.67) (-0.48)	(-1.25) (-1.26) (-0.98)
	[0.35] [0.34] [0.48]	[0.28] [0.27] [0.41]	[0.50] [0.50] [0.63]	[0.21] [0.21] [0.33]
$\xi_{\sigma^2_{amb}}$	0.03	0.03	0.02	-0.01
	(1.09) (1.09) (0.75)	(1.06) (1.07) (0.83)	(0.72) (0.72) (0.52)	(-0.29) (-0.29) (-0.23)
	[0.28] [0.27] [0.46]	[0.29] [0.29] [0.41]	[0.47] [0.47] [0.60]	[0.77] [0.77] [0.82]
ς_{t-1}	-0.38	-0.38	-0.38	-0.38
	(-16.48) (-16.55) (-18.42)	(-16.47) (-16.55) (-18.51)	(-16.53) (-16.60) (-18.45)	(-16.37) (-16.45) (-18.11)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1510	1510	1510	1510
R^2_a	0.199	0.2	0.199	0.204

Appendix E Model Daily Regressions Results With Ambiguity Volatility Controls

In this appendix, I present the daily frequency regressions for the trading volume model outlined in Section-2.6, along with the elasticity model detailed in Section-2.7 after including additional controls for ambiguity volatility.

Tables 12, 13, and 14 below present the results of the regressions (A.5), (A.5), and (A.5) below.

$$V = \alpha_V + \beta_V * \Delta_P + \epsilon \tag{A.5}$$

$$\alpha_V = c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb} + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P + \varsigma_{t-1} * \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\begin{aligned} V = {} & \big(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb}\big)_{\alpha} + \\ & \big(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P\big)_{\beta} * \Delta_P + \varsigma_{t-1} * V_{t-1} + \\ & \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \end{aligned} \tag{A.5}$$

$$\begin{aligned} \Delta V = {} & c + \big(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB + \alpha_{amb\sigma} * \Delta AMB * \sigma_{amb}\big)_{\Delta\alpha} \\ & + \big(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{h\Delta\sigma_{amb}} * \Delta\sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\big)_{\Delta\beta} * \Delta_P + \\ & \big(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p \Delta_P^2} * \sigma_P\big)_{\beta} * \Delta_P^2 + \varsigma_{t-1} * \\ & \Delta V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \end{aligned} \tag{A.5}$$

The outcomes of regression (4) for the elasticity model are illustrated in table-15 below.

$$\begin{aligned} \Delta log(V) = c + \Big(& \xi_1 + \xi_{pbel} * PBEL + \xi_{amb} * AMB + \xi_{pbel^2} * PBEL^2 + \xi_{amb^2} * \\ & AMB^2 + \xi_{\sigma_{amb}} * \sigma_{amb} + \xi_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \xi_{\sigma^2_{amb}} * \sigma^2_{amb} + \xi_{h\sigma^2_{amb}} * \\ & \sigma^2_{amb} * I_{h\sigma_{amb}} \Big)_{\xi} * \Delta log(\sigma_{\Delta p}) + \varsigma_{t-1} * \Delta log(V)_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \end{aligned} \tag{4}$$

The regressions cover the daily period from 2013 to 2018 for the SPY and were conducted on four distinct datasets: D(1), D(2), D(3) and D(4) used in the previous appendix. The dummy $I_{h\sigma_{amb}}$ marks days with high levels of ambiguity above the 50% quantile of the sample.

Table 12. Daily Regressions α_V and β_V

This table summarizes the daily regression results for the coefficients α_V and β_V of the trading volume formula, as detailed in regression (A.5) of this appendix. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\alpha_V = c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb} + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P + \varsigma_{t-1} * \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
Panel A: Regression on α_V				
c	0.13	0.14	0.17	0.13
	(2.79) (2.79) (2.56)	(2.97) (2.98) (3.07)	(3.53) (3.54) (4.00)	(2.91) (2.92) (3.09)
	[0.01] [0.01] [0.01]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{pbel}	0.13	0.08	0.10	-0.12
	(6.80) (6.82) (5.55)	(4.36) (4.37) (3.63)	(4.96) (4.97) (5.73)	(-6.31) (-6.33) (-6.94)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.16	0.16	0.15	0.15
	(6.77) (6.79) (4.12)	(6.90) (6.93) (4.21)	(6.58) (6.60) (4.03)	(6.69) (6.72) (4.08)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.10	-0.10	-0.09	-0.09
	(-4.81) (-4.83) (-3.83)	(-4.72) (-4.73) (-3.80)	(-4.35) (-4.36) (-3.53)	(-4.13) (-4.14) (-3.44)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
ς_{t-1}	0.56	0.56	0.56	0.56
	(27.18) (27.27) (17.60)	(27.18) (27.27) (17.49)	(27.26) (27.35) (16.75)	(26.97) (27.06) (16.68)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1509	1509	1509	1509
R_a^2	0.497	0.488	0.49	0.495
Panel B: Regression on β_V (same for 1, 2, 3, 4)				
c	0.05	0.05	0.05	0.05
	(0.54) (0.55) (0.41)	(0.54) (0.55) (0.41)	(0.54) (0.55) (0.41)	(0.54) (0.55) (0.41)
	[0.59] [0.59] [0.68]	[0.59] [0.59] [0.68]	[0.59] [0.59] [0.68]	[0.59] [0.59] [0.68]
$\beta_{\sigma_{amb}}$	-0.02	-0.02	-0.02	-0.02
	(-0.16) (-0.16) (-0.18)	(-0.16) (-0.16) (-0.18)	(-0.16) (-0.16) (-0.18)	(-0.16) (-0.16) (-0.18)
	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]
$\beta_{h\sigma_{amb}}$	-0.10	-0.10	-0.10	-0.10
	(-0.75) (-0.75) (-0.75)	(-0.75) (-0.75) (-0.75)	(-0.75) (-0.75) (-0.75)	(-0.75) (-0.75) (-0.75)
	[0.46] [0.45] [0.45]	[0.46] [0.45] [0.45]	[0.46] [0.45] [0.45]	[0.46] [0.45] [0.45]
β_{σ_p}	-0.06	-0.06	-0.06	-0.06
	(-1.44) (-1.44) (-1.78)	(-1.44) (-1.44) (-1.78)	(-1.44) (-1.44) (-1.78)	(-1.44) (-1.44) (-1.78)
	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]	[0.15] [0.15] [0.08]
$\varsigma_{\text{t-1}}$	0.02	0.02	0.02	0.02
	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)	(0.70) (0.70) (0.49)
	[0.49] [0.48] [0.62]	[0.49] [0.48] [0.62]	[0.49] [0.48] [0.62]	[0.49] [0.48] [0.62]
N	1509	1509	1509	1509
R_a^2	0.000	0.000	0.000	0.000

Table 13. Daily Regressions for Trading Volume

This table summarizes the daily regression results of the trading volume formula, as detailed in regression ((A.5)) of this appendix. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$V = \big(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb}\big)_{\alpha} \; + \\ \big(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P\big)_{\beta} * \Delta_P \; + \varsigma_{t-1} * V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
c	-0.13	-0.12	-0.10	-0.12
	(-2.96) (-2.97) (-2.77)	(-2.64) (-2.65) (-2.65)	(-2.31) (-2.32) (-2.62)	(-2.78) (-2.79) (-3.06)
	[0.00] [0.00] [0.01]	[0.01] [0.01] [0.01]	[0.02] [0.02] [0.01]	[0.01] [0.01] [0.00]
α_{pbel}	0.11	0.08	0.06	-0.13
	(6.29) (6.32) (5.58)	(4.22) (4.24) (3.78)	(3.33) (3.34) (3.69)	(-7.13) (-7.17) (-7.54)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.13	0.14	0.13	0.13
	(6.03) (6.06) (4.23)	(6.14) (6.17) (4.31)	(5.95) (5.98) (4.15)	(5.92) (5.95) (4.18)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.07	-0.07	-0.06	-0.06
	(-3.45) (-3.47) (-2.68)	(-3.40) (-3.41) (-2.65)	(-3.11) (-3.12) (-2.44)	(-2.70) (-2.71) (-2.20)
	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.01] [0.01] [0.03]
β_{p}	-0.34	-0.35	-0.34	-0.35
	(-10.42) (-10.47) (-5.95)	(-10.56) (-10.61) (-6.07)	(-10.32) (-10.37) (-5.78)	(-10.70) (-10.75) (-6.12)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\sigma_{\text{amb}}}$	-0.07	-0.07	-0.08	-0.07
	(-1.91) (-1.92) (-1.34)	(-2.00) (-2.01) (-1.41)	(-2.01) (-2.02) (-1.38)	(-1.99) (-2.00) (-1.40)
	[0.06] [0.05] [0.18]	[0.05] [0.04] [0.16]	[0.04] [0.04] [0.17]	[0.05] [0.05] [0.16]
$\beta_{h\sigma_{amb}}$	0.07	0.07	0.07	0.06
	(0.92) (0.92) (0.55)	(0.95) (0.95) (0.58)	(0.91) (0.92) (0.54)	(0.85) (0.85) (0.52)
	[0.36] [0.36] [0.58]	[0.34] [0.34] [0.56]	[0.36] [0.36] [0.59]	[0.40] [0.40] [0.61]
$\beta_{\sigma_{\text{p}}}$	0.10	0.10	0.09	0.10
	(4.16) (4.18) (2.43)	(4.06) (4.08) (2.34)	(3.92) (3.94) (2.22)	(4.20) (4.22) (2.45)
	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.03]	[0.00] [0.00] [0.01]
$\varsigma t-1$	0.60	0.60	0.60	0.59
	(31.13) (31.27) (17.41)	(31.05) (31.20) (17.58)	(31.12) (31.27) (16.86)	(30.91) (31.05) (16.83)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1510	1510	1510	1510
R_a^2	0.542	0.535	0.533	0.545

Table 14. Daily Regressions for ΔTrading Volume

This table summarizes the daily regression results of the first difference trading volume formula, as detailed in regression (A.5) of this appendix. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\Delta V = c + \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB + \alpha_{amb\sigma} * \Delta AMB * \sigma_{amb}\right)_{\Delta\alpha} + \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{h\Delta\sigma_{amb}} * \Delta\sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\right)_{\beta} * \Delta_P^2 + \varsigma_{t-1} * \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
c	0.01	0.01	0.01	0.01
	(0.09) (0.09) (0.17)	(0.10) (0.10) (0.18)	(0.09) (0.09) (0.16)	(0.09) (0.09) (0.16)
	[0.93] [0.92] [0.86]	[0.92] [0.92] [0.86]	[0.93] [0.93] [0.87]	[0.93] [0.93] [0.87]
α_{pbel}	0.12	0.08	0.07	-0.15
	(5.28) (5.31) (4.63)	(3.37) (3.39) (2.89)	(3.06) (3.07) (3.28)	(-6.29) (-6.32) (-6.73)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
α_{amb}	0.13	0.13	0.13	0.13
	(5.17) (5.20) (3.77)	(5.23) (5.26) (3.78)	(5.15) (5.18) (3.74)	(5.28) (5.31) (3.86)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.00	-0.00	-0.00	-0.00
	(-0.16) (-0.16) (-0.10)	(-0.04) (-0.04) (-0.03)	(-0.14) (-0.14) (-0.09)	(-0.14) (-0.14) (-0.09)
	[0.87] [0.87] [0.92]	[0.96] [0.96] [0.98]	[0.89] [0.89] [0.93]	[0.89] [0.89] [0.93]
β_{p}	-0.22	-0.23	-0.22	-0.24
	(-4.96) (-4.99) (-3.32)	(-5.02) (-5.05) (-3.36)	(-4.83) (-4.86) (-3.26)	(-5.28) (-5.31) (-3.54)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\sigma_{\text{amb}}}$	-0.04	-0.05	-0.05	-0.06
	(-0.77) (-0.78) (-0.69)	(-0.97) (-0.97) (-0.85)	(-1.00) (-1.00) (-0.86)	(-1.12) (-1.12) (-0.99)
	[0.44] [0.44] [0.49]	[0.33] [0.33] [0.40]	[0.32] [0.32] [0.39]	[0.26] [0.26] [0.32]
$\beta_{h\sigma_{amb}}$	0.02	0.04	0.04	0.05
	(0.20) (0.20) (0.15)	(0.41) (0.41) (0.31)	(0.40) (0.40) (0.30)	(0.56) (0.56) (0.42)
	[0.84] [0.84] [0.88]	[0.68] [0.68] [0.76]	[0.69] [0.69] [0.76]	[0.58] [0.57] [0.67]
$\beta_{\triangle\sigma_{\text{amb}}}$	0.11	0.11	0.11	0.11
	(3.25) (3.27) (2.72)	(3.29) (3.30) (2.73)	(3.30) (3.32) (2.72)	(3.36) (3.37) (2.64)
	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]
$\beta_{h\triangle\sigma_{amb}}$	-0.27	-0.28	-0.28	-0.28
	(-5.69) (-5.73) (-4.24)	(-5.74) (-5.77) (-4.28)	(-5.78) (-5.81) (-4.21)	(-5.86) (-5.89) (-4.19)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\sigma_{\text{p}}\triangle^2{\text{p}}}$	0.13	0.12	0.12	0.12
	(4.02) (4.04) (2.99)	(3.81) (3.84) (2.81)	(3.73) (3.76) (2.70)	(3.86) (3.89) (2.77)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]
$\beta_{\triangle\sigma_{\text{p}}}$	-0.06	-0.06	-0.06	-0.06
	(-2.41) (-2.43) (-0.98)	(-2.34) (-2.35) (-0.96)	(-2.26) (-2.27) (-0.91)	(-2.30) (-2.32) (-0.91)
	[0.02] [0.02] [0.33]	[0.02] [0.02] [0.34]	[0.02] [0.02] [0.36]	[0.02] [0.02] [0.36]
$\varsigma t-1$	-0.31	-0.31	-0.31	-0.31
	(-12.51) (-12.58) (-9.66)	(-12.79) (-12.87) (-9.94)	(-12.74) (-12.81) (-10.20)	(-12.78) (-12.85) (-10.00)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1509	1509	1509	1509
R_a^2	0.192	0.184	0.182	0.199

Table 15. Daily Regressions for Trading Volume Elasticity ξ

This table summarizes the daily regression of the elasticity model, as detailed in regression (4) of this appendix. The regressions were performed for the SPY ETF from 2013 to 2018 using four different daily data sets {D(1), D(2), D(3), D(4)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\Delta log(V) = c + \Big(\xi_1 + \xi_{pbel} * PBEL + \xi_{amb} * AMB + \xi_{pbel^2} * PBEL^2 + \xi_{amb^2} * AMB^2 + \xi_{\sigma_{amb}} * \sigma_{amb} + \xi_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \xi_{\sigma^2_{amb}} * \sigma^2_{amb} + \xi_{h\sigma^2_{amb}} * \sigma^2_{amb} * I_{h\sigma_{amb}} \Big)_{\xi} * \Delta log(\sigma_{\Delta p}) + \varsigma_{t-1} * \Delta log(V)_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)
c	-0.00	-0.00	0.00	-0.01
	(-0.00) (-0.00) (-0.01)	(-0.02) (-0.02) (-0.04)	(0.02) (0.02) (0.03)	(-0.13) (-0.13) (-0.21)
	[1.00] [1.00] [0.99]	[0.98] [0.98] [0.97]	[0.98] [0.98] [0.97]	[0.90] [0.90] [0.83]
ξ_1	0.58	0.57	0.60	0.54
	(4.80) (4.83) (4.97)	(4.55) (4.57) (4.73)	(5.02) (5.05) (5.04)	(4.48) (4.51) (4.67)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
ξ_{pbel}	-0.02	-0.02	-0.02	0.05
	(-0.77) (-0.77) (-0.77)	(-0.92) (-0.93) (-0.91)	(-0.94) (-0.94) (-0.76)	(2.01) (2.02) (1.65)
	[0.44] [0.44] [0.44]	[0.36] [0.35] [0.36]	[0.35] [0.34] [0.45]	[0.04] [0.04] [0.10]
ξ_{amb}	-0.10	-0.10	-0.10	-0.09
	(-4.23) (-4.25) (-4.47)	(-4.36) (-4.38) (-4.49)	(-4.12) (-4.14) (-4.36)	(-3.49) (-3.51) (-3.57)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
ξ_{pbel^2}	0.00	0.00	-0.00	0.06
	(0.06) (0.06) (0.05)	(0.18) (0.19) (0.14)	(-0.10) (-0.10) (-0.10)	(1.72) (1.73) (1.64)
	[0.95] [0.95] [0.96]	[0.85] [0.85] [0.89]	[0.92] [0.92] [0.92]	[0.09] [0.08] [0.10]
ξ_{amb^2}	0.03	0.03	0.04	0.03
	(1.40) (1.41) (1.62)	(1.18) (1.18) (1.28)	(1.50) (1.51) (1.74)	(1.23) (1.24) (1.33)
	[0.16] [0.16] [0.10]	[0.24] [0.24] [0.20]	[0.13] [0.13] [0.08]	[0.22] [0.22] [0.19]
$\xi_{\sigma_{amb}}$	-0.14	-0.13	-0.14	-0.12
	(-2.86) (-2.87) (-2.33)	(-2.75) (-2.77) (-2.24)	(-2.81) (-2.82) (-2.26)	(-2.44) (-2.45) (-2.01)
	[0.00] [0.00] [0.02]	[0.01] [0.01] [0.03]	[0.01] [0.00] [0.02]	[0.01] [0.01] [0.04]
$\xi_{h\sigma_{amb}}$	-0.10	-0.10	-0.11	-0.09
	(-1.41) (-1.42) (-1.25)	(-1.35) (-1.36) (-1.20)	(-1.47) (-1.47) (-1.29)	(-1.19) (-1.20) (-1.13)
	[0.16] [0.16] [0.21]	[0.18] [0.17] [0.23]	[0.14] [0.14] [0.20]	[0.23] [0.23] [0.26]
$\xi_{\sigma^2_{amb}}$	0.41	0.40	0.42	0.30
	(3.16) (3.17) (3.23)	(2.93) (2.94) (3.12)	(3.19) (3.20) (3.27)	(2.23) (2.24) (2.12)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.03] [0.02] [0.03]
$\xi_{h\sigma^2_{amb}}$	-0.77	-0.74	-0.80	-0.62
	(-2.93) (-2.95) (-2.90)	(-2.72) (-2.73) (-2.74)	(-3.04) (-3.06) (-2.93)	(-2.30) (-2.31) (-2.25)
	[0.00] [0.00] [0.00]	[0.01] [0.01] [0.01]	[0.00] [0.00] [0.00]	[0.02] [0.02] [0.02]
$\varsigma t-1$	-0.38	-0.38	-0.38	-0.38
	(-16.58) (-16.67) (-18.50)	(-16.56) (-16.65) (-18.57)	(-16.63) (-16.72) (-18.57)	(-16.47) (-16.55) (-18.37)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	1510	1510	1510	1510
R^2_a	0.203	0.204	0.204	0.206

Appendix F Model Monthly Regressions Results

In this appendix, I present the monthly frequency regressions for the trading volume model outlined in Section-2.6. These regressions cover the monthly periods 2013-2018 and 2000-2020.

Tables 16, 17, and 18 below present the regressions (5.1), (5.2), and (5.3) results of the trading volume model as discussed in Section-5.

$$V = \alpha_V + \beta_V * \Delta_P + \epsilon \tag{5.1}$$

$$\alpha_V = c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P + \varsigma_{t-1} * \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$V = \big(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\big)_\alpha \ + \ \big(\beta_p + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\big)_\beta * \Delta_P \ + \ \varsigma_{t-1} * V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \tag{5.2}$$

$$\Delta V = c \ + \ \big(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\big)_{\Delta\alpha} \ + \ \big(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\big)_{\Delta\beta} * \Delta_P \ + \ \big(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p \Delta_P^2} * \sigma_P\big)_\beta * \Delta_P^2 \ + \varsigma_{t-1} * \Delta V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \tag{5.3}$$

The regressions using datasets D(1) to D(8) cover the monthly period from 2013 to 2018 for the SPY. The regressions using the datasets D(9) to D(10) cover the monthly period from 2000 to 2020. Datasets (1) ,(2), (3), (4) and (9) employ the monthly EPU measure extracted from newspapers as a proxy for Ambiguity, alongside a distinct proxy for differences in prior beliefs extracted from Stocktwits (Cookson & Niessner, 2020) and the IBES database. Datasets (4), (5), (6), (7) and (10) employ the monthly Ambiguity measure of Izhakian (2020).

Monthly Datasets Description	
D(1)	Prior beliefs differences proxied by $PBEL_{\text{WI, IND}}$ and Ambiguity proxied by AMB_{EPUM}
D(2)	Prior beliefs differences proxied by $PBEL_{\text{AC, IND}}$ and Ambiguity proxied by AMB_{EPUM}
D(3)	Prior beliefs differences proxied by $PBEL_{\text{WI, ETF}}$ and Ambiguity proxied by AMB_{EPUM}
D(4)	Prior beliefs differences proxied by $PBEL_{\text{AC, ETF}}$ and Ambiguity proxied by AMB_{EPUM}
D(5)	Prior beliefs differences proxied by $PBEL_{\text{WI, IND}}$ and Ambiguity proxied by AMB_{IZHM}
D(6)	Prior beliefs differences proxied by $PBEL_{\text{AC, IND}}$ and Ambiguity proxied by AMB_{IZHM}
D(7)	Prior beliefs differences proxied by $PBEL_{\text{WI, ETF}}$ and Ambiguity proxied by AMB_{IZHM}
D(8)	Prior beliefs differences proxied by $PBEL_{\text{AC, ETF}}$ and Ambiguity proxied by AMB_{IZHM}
D(9)	Prior beliefs differences proxied by $PBEL_{\text{IBES}}$ and Ambiguity proxied by AMB_{EPUM}
D(10)	Prior beliefs differences proxied by $PBEL_{\text{IBES}}$ and Ambiguity proxied by AMB_{IZHM}

Table 16. Monthly Regressions α_V and β_V

This table summarizes the monthly regression results for the coefficients α_V and β_V of the trading volume formula, as detailed in regression (5.1) of Section-5. The regressions were performed for the SPY ETF using ten different monthly data sets {D(1), D(2), D(3), D(4), D(6), D(7), D(8), D(9), D(10)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\alpha_V = c + \alpha_{amb} * AMB + \alpha_{pbel} * PBEL + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_{\sigma_p} * \sigma_P \ + \ \varsigma_{t-1} * \beta_{t-1} + \ \sum_{p=t+1}^{T} \gamma_p + \ \epsilon$$

	D(1)	D(2)	D(3)	D(4)	D(5)
Panel A: Regression on α_V					
c	-0.33	-0.38	-0.19	-0.34	-0.10
	(-2.57) (-2.74) (-5.21)	(-2.85) (-3.05) (-5.43)	(-1.29) (-1.38) (-1.68)	(-2.62) (-2.80) (-6.39)	(-1.04) (-1.11) (-1.20)
	[0.01] [0.01] [0.00]	[0.01] [0.00] [0.00]	[0.20] [0.17] [0.10]	[0.01] [0.01] [0.00]	[0.30] [0.27] [0.24]
α_{pbel}	-0.04	-0.08	0.11	-0.04	-0.06
	(-0.64) (-0.68) (-0.67)	(-1.35) (-1.45) (-1.64)	(1.52) (1.63) (1.30)	(-0.81) (-0.87) (-0.89)	(-1.42) (-1.52) (-1.61)
	[0.53] [0.50] [0.50]	[0.18] [0.15] [0.11]	[0.13] [0.11] [0.20]	[0.42] [0.39] [0.38]	[0.16] [0.13] [0.11]
α_{amb}	0.23	0.23	0.20	0.23	0.46
	(2.40) (2.57) (1.99)	(2.38) (2.54) (2.07)	(1.99) (2.13) (1.99)	(2.37) (2.53) (1.95)	(8.82) (9.43) (4.61)
	[0.02] [0.01] [0.05]	[0.02] [0.01] [0.04]	[0.05] [0.04] [0.05]	[0.02] [0.01] [0.06]	[0.00] [0.00] [0.00]
ς_{t-1}	0.16	0.17	0.11	0.16	-0.20
	(1.25) (1.34) (1.31)	(1.31) (1.40) (1.31)	(0.84) (0.90) (0.78)	(1.25) (1.34) (1.28)	(-1.97) (-2.11) (-2.60)
	[0.22] [0.19] [0.19]	[0.20] [0.17] [0.19]	[0.40] [0.37] [0.44]	[0.21] [0.18] [0.21]	[0.05] [0.04] [0.01]
N	72	72	72	72	72
R_a^2	0.255	0.271	0.277	0.258	0.636
Panel B: Regression on β_V(same for 1, 2, 3, 4)					
c	0.05	0.05	0.05	0.05	0.14
	(0.16) (0.17) (0.17)	(0.16) (0.17) (0.17)	(0.16) (0.17) (0.17)	(0.16) (0.17) (0.17)	(0.38) (0.40) (0.45)
	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]	[0.87] [0.87] [0.86]	[0.71] [0.69] [0.65]
$\beta_{\sigma_{amb}}$	-0.15	-0.15	-0.15	-0.15	0.17
	(-0.60) (-0.64) (-0.43)	(-0.60) (-0.64) (-0.43)	(-0.60) (-0.64) (-0.43)	(-0.60) (-0.64) (-0.43)	(0.65) (0.69) (0.93)
	[0.55] [0.53] [0.67]	[0.55] [0.53] [0.67]	[0.55] [0.53] [0.67]	[0.55] [0.53] [0.67]	[0.52] [0.49] [0.35]
β_{σ_p}	0.09	0.09	0.09	0.09	0.17
	(0.30) (0.32) (0.42)	(0.30) (0.32) (0.42)	(0.30) (0.32) (0.42)	(0.30) (0.32) (0.42)	(0.52) (0.55) (0.90)
	[0.77] [0.75] [0.68]	[0.77] [0.75] [0.68]	[0.77] [0.75] [0.68]	[0.77] [0.75] [0.68]	[0.61] [0.58] [0.37]
ς_{t-1}	0.01	0.01	0.01	0.01	0.01
	(0.07) (0.08) (0.10)	(0.07) (0.08) (0.10)	(0.07) (0.08) (0.10)	(0.07) (0.08) (0.10)	(0.09) (0.10) (0.11)
	[0.94] [0.94] [0.92]	[0.94] [0.94] [0.92]	[0.94] [0.94] [0.92]	[0.94] [0.94] [0.92]	[0.93] [0.92] [0.91]
N	72	72	72	72	72
R_a^2	-0.033	-0.033	-0.033	-0.033	-0.032

Table 16. Monthly Regressions α_V and β_V (continuation)

This table summarizes the monthly regression results for the coefficients α_V and β_V of the trading volume formula, as detailed in regression (5.1) of Section-5. The regressions were performed for the SPY ETF using ten different monthly data sets {D(1), D(2), D(3), D(4), D(6), D(7), D(8), D(9), D(10)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\alpha_V = c + \alpha_{amb} * AMB + \alpha_{pbel} * PBEL + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{AMB} + \beta_{\sigma_p} * \sigma_P \; + \; \varsigma_{t-1} * \beta_{t-1} + \; \sum_{p=t+1}^{T} \gamma_p + \; \epsilon$$

	D(6)	D(7)	D(8)	D(9)	D(10)
Panel A: Regression on α_V					
c	-0.13	-0.09	-0.11	-0.10	-1.08
	(-1.39) (-1.48) (-1.43)	(-0.84) (-0.89) (-0.88)	(-1.18) (-1.27) (-1.53)	(-0.57) (-0.69) (-0.79)	(-6.05) (-7.28) (-6.96)
	[0.17] [0.14] [0.16]	[0.41] [0.37] [0.38]	[0.24] [0.21] [0.13]	[0.57] [0.49] [0.43]	[0.00] [0.00] [0.00]
α_{pbel}	-0.08	-0.03	-0.06	-0.14	-0.10
	(-1.91) (-2.04) (-1.99)	(-0.54) (-0.58) (-0.42)	(-1.63) (-1.74) (-2.10)	(-3.38) (-3.30) (-3.34)	(-2.54) (-2.43) (-2.32)
	[0.06] [0.05] [0.05]	[0.59] [0.57] [0.68]	[0.11] [0.09] [0.04]	[0.00] [0.00] [0.00]	[0.01] [0.02] [0.02]
α_{amb}	0.46	0.47	0.46	0.37	0.58
	(8.82) (9.43) (4.68)	(8.16) (8.72) (4.98)	(8.85) (9.46) (4.75)	(6.04) (5.64) (3.03)	(9.33) (10.15) (5.19)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\varsigma t-1$	-0.19	-0.21	-0.20	0.39	0.26
	(-1.95) (-2.09) (-2.40)	(-2.02) (-2.16) (-2.70)	(-1.99) (-2.13) (-2.52)	(7.19) (5.65) (4.97)	(4.93) (3.57) (2.65)
	[0.06] [0.04] [0.02]	[0.05] [0.03] [0.01]	[0.05] [0.04] [0.01]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.01]
N	72	72	72	247	247
R_a^2	0.645	0.626	0.639	0.697	0.746
Panel B: Regression on β_V(same for 1, 2, 3, 4)					
c	0.14	0.14	0.14	0.41	0.68
	(0.38) (0.40) (0.45)	(0.38) (0.40) (0.45)	(0.38) (0.40) (0.45)	(1.30) (1.33) (2.54)	(1.73) (1.78) (2.61)
	[0.71] [0.69] [0.65]	[0.71] [0.69] [0.65]	[0.71] [0.69] [0.65]	[0.20] [0.18] [0.01]	[0.09] [0.08] [0.01]
$\beta_{\sigma_{amb}}$	0.17	0.17	0.17	0.20	0.31
	(0.65) (0.69) (0.93)	(0.65) (0.69) (0.93)	(0.65) (0.69) (0.93)	(1.51) (1.57) (0.99)	(1.63) (1.71) (1.54)
	[0.52] [0.49] [0.35]	[0.52] [0.49] [0.35]	[0.52] [0.49] [0.35]	[0.13] [0.12] [0.32]	[0.10] [0.09] [0.12]
β_{σ_p}	0.17	0.17	0.17	-0.22	-0.16
	(0.52) (0.55) (0.90)	(0.52) (0.55) (0.90)	(0.52) (0.55) (0.90)	(-1.49) (-1.59) (-0.56)	(-1.14) (-1.22) (-0.46)
	[0.61] [0.58] [0.37]	[0.61] [0.58] [0.37]	[0.61] [0.58] [0.37]	[0.14] [0.11] [0.58]	[0.26] [0.22] [0.65]
$\varsigma t-1$	0.01	0.01	0.01	0.00	0.01
	(0.09) (0.10) (0.11)	(0.09) (0.10) (0.11)	(0.09) (0.10) (0.11)	(0.07) (0.07) (0.10)	(0.16) (0.16) (0.22)
	[0.93] [0.92] [0.91]	[0.93] [0.92] [0.91]	[0.93] [0.92] [0.91]	[0.94] [0.94] [0.92]	[0.88] [0.87] [0.83]
N	72	72	72	247	247
R_a^2	-0.032	-0.032	-0.032	0.111	0.113

Table 17. Monthly Regressions for Trading Volume

This table summarizes the monthly regression results of the trading volume formula, as detailed in regression (5.2) of Section-5. The regressions were performed for the SPY ETF using ten different data sets {D(1), D(2), D(3), D(4), D(6), D(7), D(8), D(9), D(10)}.

$$V = \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB\right)_{\alpha} \; + \; \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p} * \sigma_P\right)_{\beta} * \Delta_P \; + \varsigma_{t-1} * V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)	D(5)
c	-0.23	-0.25	-0.16	-0.29	-0.12
	(-2.13) (-2.33) (-3.37)	(-2.20) (-2.41) (-3.37)	(-1.32) (-1.44) (-1.16)	(-2.65) (-2.90) (-4.47)	(-1.27) (-1.39) (-1.33)
	[0.04] [0.02] [0.00]	[0.03] [0.02] [0.00]	[0.19] [0.15] [0.25]	[0.01] [0.01] [0.00]	[0.21] [0.17] [0.19]
α_{pbel}	0.02	-0.00	0.08	-0.05	-0.01
	(0.51) (0.56) (0.55)	(-0.10) (-0.11) (-0.13)	(1.27) (1.39) (0.69)	(-1.13) (-1.24) (-0.86)	(-0.32) (-0.35) (-0.36)
	[0.61] [0.58] [0.58]	[0.92] [0.92] [0.90]	[0.21] [0.17] [0.49]	[0.26] [0.22] [0.40]	[0.75] [0.73] [0.72]
α_{amb}	0.22	0.21	0.19	0.20	0.35
	(2.64) (2.89) (3.87)	(2.57) (2.82) (3.75)	(2.31) (2.53) (5.48)	(2.41) (2.64) (3.14)	(5.71) (6.26) (3.18)
	[0.01] [0.01] [0.00]	[0.01] [0.01] [0.00]	[0.02] [0.01] [0.00]	[0.02] [0.01] [0.00]	[0.00] [0.00] [0.00]
β_{p}	-0.27	-0.26	-0.26	-0.25	-0.09
	(-4.15) (-4.55) (-6.05)	(-3.93) (-4.31) (-5.86)	(-4.16) (-4.56) (-6.54)	(-3.86) (-4.23) (-6.16)	(-1.31) (-1.43) (-1.20)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.20] [0.16] [0.23]
$\beta_{\sigma_{\text{amb}}}$	0.05	0.07	0.05	0.12	-0.03
	(0.24) (0.26) (0.39)	(0.31) (0.34) (0.53)	(0.23) (0.25) (0.38)	(0.58) (0.64) (1.00)	(-0.50) (-0.55) (-0.63)
	[0.81] [0.79] [0.70]	[0.75] [0.73] [0.59]	[0.82] [0.80] [0.71]	[0.56] [0.53] [0.32]	[0.62] [0.59] [0.53]
$\beta_{\sigma_{\text{p}}}$	0.09	0.08	0.11	0.08	-0.01
	(0.62) (0.68) (1.04)	(0.60) (0.65) (1.05)	(0.78) (0.86) (1.31)	(0.57) (0.62) (1.00)	(-0.12) (-0.13) (-0.22)
	[0.54] [0.50] [0.30]	[0.55] [0.52] [0.30]	[0.44] [0.40] [0.19]	[0.57] [0.54] [0.32]	[0.91] [0.90] [0.83]
ς_{t-1}	0.15	0.16	0.13	0.16	-0.16
	(1.43) (1.57) (1.17)	(1.49) (1.63) (1.23)	(1.23) (1.34) (0.96)	(1.55) (1.70) (1.25)	(-1.48) (-1.62) (-1.41)
	[0.16] [0.12] [0.25]	[0.14] [0.11] [0.22]	[0.23] [0.18] [0.34]	[0.13] [0.09] [0.22]	[0.14] [0.11] [0.16]
N	72	72	72	72	72
R_a^2	0.493	0.49	0.504	0.501	0.629
	D(6)	D(7)	D(8)	D(9)	D(10)
c	-0.15	-0.12	-0.18	-0.13	-0.97
	(-1.50) (-1.64) (-1.42)	(-1.18) (-1.29) (-1.05)	(-1.88) (-2.06) (-2.03)	(-0.87) (-1.02) (-1.06)	(-5.53) (-6.55) (-5.56)
	[0.14] [0.11] [0.16]	[0.24] [0.20] [0.30]	[0.07] [0.04] [0.05]	[0.39] [0.31] [0.29]	[0.00] [0.00] [0.00]
α_{pbel}	-0.04	-0.01	-0.07	-0.09	-0.08
	(-0.92) (-1.00) (-0.98)	(-0.21) (-0.23) (-0.17)	(-2.03) (-2.22) (-2.89)	(-2.38) (-2.38) (-3.64)	(-2.07) (-2.03) (-3.24)
	[0.36] [0.32] [0.33]	[0.83] [0.82] [0.86]	[0.05] [0.03] [0.01]	[0.02] [0.02] [0.00]	[0.04] [0.04] [0.00]
α_{amb}	0.36	0.36	0.36	0.36	0.48
	(5.82) (6.37) (3.27)	(5.24) (5.74) (3.65)	(6.04) (6.61) (3.09)	(6.30) (6.07) (3.26)	(7.25) (8.09) (4.15)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
β_{p}	-0.08	-0.09	-0.08	-0.30	-0.19
	(-1.17) (-1.28) (-1.07)	(-1.37) (-1.50) (-1.23)	(-1.16) (-1.27) (-0.99)	(-5.42) (-5.85) (-4.22)	(-3.14) (-3.16) (-2.10)
	[0.25] [0.20] [0.29]	[0.18] [0.14] [0.22]	[0.25] [0.21] [0.33]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.04]
$\beta_{\sigma_{\text{amb}}}$	-0.04	-0.03	-0.05	-0.10	0.03
	(-0.53) (-0.58) (-0.66)	(-0.45) (-0.49) (-0.61)	(-0.71) (-0.78) (-0.88)	(-1.69) (-1.63) (-1.39)	(0.61) (0.54) (0.39)
	[0.60] [0.56] [0.51]	[0.66] [0.63] [0.54]	[0.48] [0.44] [0.38]	[0.09] [0.10] [0.17]	[0.54] [0.59] [0.70]
$\beta_{\sigma_{\text{p}}}$	-0.01	-0.02	-0.00	0.10	-0.04
	(-0.10) (-0.10) (-0.17)	(-0.17) (-0.19) (-0.29)	(-0.02) (-0.02) (-0.03)	(1.37) (1.50) (0.88)	(-0.66) (-0.70) (-0.57)
	[0.92] [0.92] [0.86]	[0.86] [0.85] [0.77]	[0.98] [0.98] [0.97]	[0.17] [0.13] [0.38]	[0.51] [0.48] [0.57]
ς_{t-1}	-0.16	-0.16	-0.17	0.38	0.27
	(-1.52) (-1.66) (-1.45)	(-1.49) (-1.64) (-1.44)	(-1.60) (-1.75) (-1.35)	(7.52) (6.27) (5.31)	(4.90) (3.81) (3.10)
	[0.13] [0.10] [0.15]	[0.14] [0.11] [0.15]	[0.12] [0.09] [0.18]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	72	72	72	248	248
R_a^2	0.634	0.629	0.653	0.732	0.743

Table 18. Monthly Regressions for ΔTrading Volume

This table summarizes the monthly regression results of the first difference trading volume formula, as detailed in regression (5.3) of Section-5.

$$\Delta V = c + \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB\right)_{\Delta\alpha} + \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\right)_{\beta} * \Delta_P^2 + \varsigma_{t-1} * \Delta log(V)_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)	D(5)
c	0.08	0.06	0.05	0.04	0.01
	(0.46) (0.51) (0.82)	(0.36) (0.41) (0.68)	(0.35) (0.39) (0.75)	(0.23) (0.26) (0.50)	(0.09) (0.10) (0.27)
	[0.65] [0.61] [0.42]	[0.72] [0.69] [0.50]	[0.73] [0.70] [0.46]	[0.82] [0.80] [0.62]	[0.93] [0.92] [0.79]
α_{pbel}	0.14	0.10	0.25	-0.02	0.03
	(2.00) (2.24) (1.61)	(1.39) (1.55) (1.09)	(3.89) (4.35) (2.23)	(-0.30) (-0.34) (-0.25)	(0.53) (0.59) (0.53)
	[0.05] [0.03] [0.11]	[0.17] [0.13] [0.28]	[0.00] [0.00] [0.03]	[0.76] [0.74] [0.80]	[0.60] [0.56] [0.60]
α_{amb}	0.24	0.25	0.19	0.23	0.38
	(3.33) (3.72) (4.08)	(3.31) (3.70) (4.03)	(2.86) (3.20) (4.17)	(3.07) (3.43) (3.43)	(7.97) (8.90) (4.05)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.01] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
β_{p}	-0.28	-0.30	-0.26	-0.29	-0.03
	(-2.39) (-2.66) (-2.28)	(-2.47) (-2.75) (-2.52)	(-2.37) (-2.65) (-2.73)	(-2.39) (-2.67) (-3.11)	(-0.25) (-0.28) (-0.31)
	[0.02] [0.01] [0.03]	[0.02] [0.01] [0.01]	[0.02] [0.01] [0.01]	[0.02] [0.01] [0.00]	[0.80] [0.78] [0.76]
$\beta_{\sigma_{amb}}$	0.31	0.27	0.29	0.28	-0.09
	(0.91) (1.01) (1.01)	(0.77) (0.85) (0.92)	(0.94) (1.04) (1.32)	(0.78) (0.88) (1.20)	(-0.81) (-0.90) (-1.83)
	[0.37] [0.32] [0.32]	[0.45] [0.40] [0.36]	[0.35] [0.30] [0.19]	[0.44] [0.38] [0.23]	[0.42] [0.37] [0.07]
$\beta_{\triangle\sigma_{amb}}$	-0.51	-0.45	-0.43	-0.42	0.12
	(-1.53) (-1.71) (-1.13)	(-1.34) (-1.50) (-1.00)	(-1.41) (-1.57) (-1.15)	(-1.25) (-1.39) (-0.99)	(1.67) (1.86) (4.08)
	[0.13] [0.09] [0.26]	[0.18] [0.14] [0.32]	[0.16] [0.12] [0.25]	[0.22] [0.17] [0.32]	[0.10] [0.07] [0.00]
$\beta_{\sigma_p\triangle^2 p}$	0.21	0.17	0.32	0.10	-0.04
	(0.68) (0.76) (0.83)	(0.54) (0.61) (0.64)	(1.10) (1.23) (1.83)	(0.31) (0.34) (0.36)	(-0.14) (-0.15) (-0.19)
	[0.50] [0.45] [0.41]	[0.59] [0.55] [0.53]	[0.28] [0.23] [0.07]	[0.76] [0.73] [0.72]	[0.89] [0.88] [0.85]
$\beta_{\triangle\sigma_p}$	0.19	0.20	0.20	0.23	0.10
	(0.75) (0.84) (0.77)	(0.78) (0.87) (0.76)	(0.87) (0.98) (1.04)	(0.89) (0.99) (0.81)	(0.51) (0.57) (0.77)
	[0.45] [0.40] [0.44]	[0.44] [0.39] [0.45]	[0.39] [0.33] [0.30]	[0.38] [0.33] [0.42]	[0.61] [0.57] [0.45]
ς_{t-1}	-0.33	-0.32	-0.35	-0.30	-0.43
	(-3.25) (-3.63) (-2.00)	(-3.11) (-3.47) (-2.02)	(-3.74) (-4.18) (-2.60)	(-2.88) (-3.21) (-1.90)	(-5.12) (-5.72) (-3.69)
	[0.00] [0.00] [0.05]	[0.00] [0.00] [0.05]	[0.00] [0.00] [0.01]	[0.01] [0.00] [0.06]	[0.00] [0.00] [0.00]
N	71	71	71	71	71
R_a^2	0.404	0.383	0.496	0.363	0.641

	D(6)	D(7)	D(8)	D(9)	D(10)
c	0.01	0.01	0.01	-0.05	-0.05
	(0.07) (0.08) (0.21)	(0.10) (0.11) (0.33)	(0.05) (0.06) (0.17)	(-0.22) (-0.28) (-0.93)	(-0.20) (-0.26) (-1.72)
	[0.94] [0.94] [0.83]	[0.92] [0.91] [0.74]	[0.96] [0.95] [0.87]	[0.83] [0.78] [0.35]	[0.84] [0.80] [0.09]
α_{pbel}	0.02	0.17	0.00	-0.07	-0.04
	(0.28) (0.32) (0.28)	(3.27) (3.65) (3.28)	(0.02) (0.02) (0.02)	(-1.16) (-1.12) (-2.04)	(-0.70) (-0.67) (-0.93)
	[0.78] [0.75] [0.78]	[0.00] [0.00] [0.00]	[0.99] [0.99] [0.98]	[0.25] [0.26] [0.04]	[0.48] [0.51] [0.35]
α_{amb}	0.38	0.33	0.38	0.25	0.53
	(8.08) (9.02) (4.03)	(7.36) (8.21) (4.58)	(8.12) (9.07) (3.94)	(4.42) (4.55) (2.80)	(9.33) (11.13) (5.30)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.00]
β_{p}	-0.03	-0.04	-0.02	-0.29	-0.08
	(-0.23) (-0.26) (-0.29)	(-0.36) (-0.40) (-0.50)	(-0.20) (-0.22) (-0.24)	(-3.51) (-3.97) (-3.50)	(-0.89) (-0.97) (-1.17)
	[0.82] [0.80] [0.77]	[0.72] [0.69] [0.62]	[0.85] [0.83] [0.81]	[0.00] [0.00] [0.00]	[0.38] [0.33] [0.24]
$\beta_{\sigma_{amb}}$	-0.09	-0.10	-0.10	0.19	0.02
	(-0.83) (-0.93) (-1.90)	(-0.98) (-1.10) (-2.05)	(-0.87) (-0.97) (-2.09)	(1.93) (1.84) (2.54)	(0.26) (0.24) (0.35)
	[0.41] [0.36] [0.06]	[0.33] [0.28] [0.05]	[0.39] [0.34] [0.04]	[0.05] [0.07] [0.01]	[0.79] [0.81] [0.73]
$\beta_{\triangle\sigma_{amb}}$	0.12	0.15	0.12	-0.35	-0.09
	(1.70) (1.90) (4.13)	(2.26) (2.53) (5.99)	(1.62) (1.81) (5.55)	(-5.28) (-4.19) (-7.09)	(-1.40) (-1.16) (-1.14)
	[0.09] [0.06] [0.00]	[0.03] [0.01] [0.00]	[0.11] [0.08] [0.00]	[0.00] [0.00] [0.00]	[0.16] [0.25] [0.25]
$\beta_{\sigma_p\triangle^2 p}$	-0.04	0.12	-0.05	-0.04	-0.01
	(-0.16) (-0.18) (-0.23)	(0.47) (0.52) (0.62)	(-0.18) (-0.20) (-0.25)	(-0.32) (-0.37) (-0.34)	(-0.11) (-0.13) (-0.12)
	[0.87] [0.86] [0.82]	[0.64] [0.60] [0.54]	[0.86] [0.84] [0.80]	[0.75] [0.71] [0.73]	[0.91] [0.90] [0.91]
$\beta_{\triangle\sigma_p}$	0.11	0.14	0.11	0.14	-0.03
	(0.57) (0.64) (0.87)	(0.81) (0.91) (1.34)	(0.61) (0.68) (1.01)	(1.87) (2.16) (2.82)	(-0.48) (-0.53) (-0.36)
	[0.57] [0.52] [0.39]	[0.42] [0.37] [0.19]	[0.55] [0.50] [0.31]	[0.06] [0.03] [0.01]	[0.63] [0.60] [0.72]
ς_{t-1}	-0.42	-0.43	-0.42	-0.26	-0.32
	(-5.09) (-5.69) (-3.75)	(-5.61) (-6.26) (-4.36)	(-5.08) (-5.67) (-3.85)	(-4.47) (-3.81) (-2.50)	(-5.88) (-4.77) (-3.92)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.00]
N	71	71	71	247	247
R_a^2	0.639	0.696	0.639	0.317	0.400

Appendix G Model Monthly Regressions Results With Ambiguity Volatility Controls

In this appendix, I present the monthly frequency regressions for the trading volume model outlined in Section-2.6, including additional controls for ambiguity volatility.

Tables 19, 20, and 21 below present the below trading volume model regressions (1), (2), and (3) results.

$$V = \alpha_V + \beta_V * \Delta_P + \epsilon \tag{1}$$

$$\alpha_V = c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb} + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P + \varsigma_{t-1} * \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\begin{aligned} V = &\big(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb}\big)_\alpha \; + \\ &\big(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P\big)_\beta * \Delta_P \; + \varsigma_{t-1} * V_{t-1} + \\ &\textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \end{aligned} \tag{2}$$

$$\begin{aligned} \Delta V = c \; + \; &\big(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB + \alpha_{amb\sigma} * \Delta AMB * \sigma_{amb}\big)_{\Delta\alpha} \\ + \; &\big(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{h\Delta\sigma_{amb}} * \Delta\sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\big)_{\Delta\beta} * \Delta_P \; + \\ &\big(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\big)_\beta * \Delta_P^2 \; + \varsigma_{t-1} * \\ &\Delta V_{t-1} + \textstyle\sum_{p=t+1}^{T} \gamma_p + \epsilon \end{aligned} \tag{3}$$

The regressions using datasets D(1) to D(8) cover the monthly period from 2013 to 2018 for the SPY. The regressions using the datasets D(9) to D(10) cover the monthly period from 2000 to 2020 for the SPY. The ten monthly datasets used here are the same ones presented in the previous appendix. The dummy $I_{h\sigma_{amb}}$ marks days with high levels of ambiguity above the 50% quantile of the sample.

Table 19. Monthly Regressions α_V and β_V

This table summarizes the monthly regression results for the coefficients α_V and β_V of the trading volume formula, as detailed in regression (1) of this appendix. The regressions were performed for the SPY ETF from 2013 to 2018 using four different monthly data sets {D(1), D(2), D(3), D(4), D(5)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\alpha_V = c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb} + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P + \varsigma_{t-1} * \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)	D(5)
Panel A: Regression on α_V					
c	-0.45	-0.47	-0.31	-0.48	-0.07
	(-2.56) (-2.76) (-2.91)	(-2.70) (-2.91) (-3.15)	(-1.65) (-1.78) (-1.86)	(-2.67) (-2.88) (-2.79)	(-0.69) (-0.74) (-0.68)
	[0.01] [0.01] [0.01]	[0.01] [0.00] [0.00]	[0.10] [0.08] [0.07]	[0.01] [0.01] [0.01]	[0.49] [0.46] [0.50]
α_{pbel}	-0.03	-0.07	0.11	-0.05	-0.06
	(-0.46) (-0.50) (-0.44)	(-1.15) (-1.24) (-1.27)	(1.48) (1.60) (1.32)	(-0.84) (-0.90) (-0.89)	(-1.44) (-1.55) (-1.70)
	[0.65] [0.62] [0.66]	[0.25] [0.22] [0.21]	[0.14] [0.12] [0.19]	[0.40] [0.37] [0.38]	[0.15] [0.13] [0.09]
α_{amb}	0.27	0.26	0.24	0.27	0.48
	(2.60) (2.80) (2.05)	(2.50) (2.70) (2.01)	(2.24) (2.41) (2.16)	(2.62) (2.82) (2.21)	(6.94) (7.48) (4.11)
	[0.01] [0.01] [0.04]	[0.01] [0.01] [0.05]	[0.03] [0.02] [0.03]	[0.01] [0.01] [0.03]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.38	-0.32	-0.39	-0.42	-0.02
	(-0.99) (-1.06) (-0.72)	(-0.83) (-0.89) (-0.60)	(-1.04) (-1.12) (-0.84)	(-1.11) (-1.19) (-0.83)	(-0.41) (-0.44) (-0.41)
	[0.33] [0.29] [0.48]	[0.41] [0.38] [0.55]	[0.30] [0.27] [0.41]	[0.27] [0.24] [0.41]	[0.68] [0.66] [0.68]
ς_{t-1}	0.13	0.14	0.08	0.13	-0.20
	(1.00) (1.08) (1.26)	(1.09) (1.17) (1.30)	(0.63) (0.68) (0.70)	(1.01) (1.09) (1.26)	(-1.93) (-2.08) (-2.54)
	[0.32] [0.29] [0.21]	[0.28] [0.25] [0.20]	[0.53] [0.50] [0.49]	[0.32] [0.28] [0.21]	[0.06] [0.04] [0.01]
N	72	72	72	72	72
R_a^2	0.254	0.268	0.277	0.26	0.631
Panel B: Regression on β_V(same for 1, 2, 3, 4)					
c	0.14	0.14	0.14	0.14	-0.01
	(0.45) (0.48) (0.64)	(0.45) (0.48) (0.64)	(0.45) (0.48) (0.64)	(0.45) (0.48) (0.64)	(-0.03) (-0.03) (-0.02)
	[0.66] [0.63] [0.52]	[0.66] [0.63] [0.52]	[0.66] [0.63] [0.52]	[0.66] [0.63] [0.52]	[0.98] [0.98] [0.98]
$\beta_{\sigma_{amb}}$	-1.59	-1.59	-1.59	-1.59	-0.32
	(-3.28) (-3.54) (-4.00)	(-3.28) (-3.54) (-4.00)	(-3.28) (-3.54) (-4.00)	(-3.28) (-3.54) (-4.00)	(-0.27) (-0.29) (-0.20)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.79] [0.77] [0.84]
$\beta_{h\sigma_{amb}}$	3.27	3.27	3.27	3.27	0.52
	(3.40) (3.67) (3.04)	(3.40) (3.67) (3.04)	(3.40) (3.67) (3.04)	(3.40) (3.67) (3.04)	(0.43) (0.47) (0.32)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.67] [0.64] [0.75]
β_{σ_p}	0.77	0.77	0.77	0.77	0.18
	(2.22) (2.39) (2.57)	(2.22) (2.39) (2.57)	(2.22) (2.39) (2.57)	(2.22) (2.39) (2.57)	(0.54) (0.58) (0.93)
	[0.03] [0.02] [0.01]	[0.03] [0.02] [0.01]	[0.03] [0.02] [0.01]	[0.03] [0.02] [0.01]	[0.59] [0.56] [0.35]
ς_{t-1}	-0.13	-0.13	-0.13	-0.13	0.01
	(-1.04) (-1.12) (-1.75)	(-1.04) (-1.12) (-1.75)	(-1.04) (-1.12) (-1.75)	(-1.04) (-1.12) (-1.75)	(0.07) (0.07) (0.09)
	[0.30] [0.27] [0.09]	[0.30] [0.27] [0.09]	[0.30] [0.27] [0.09]	[0.30] [0.27] [0.09]	[0.95] [0.94] [0.93]
N	72	72	72	72	72
R_a^2	0.115	0.115	0.115	0.115	-0.046

Table 19. Monthly Regressions α_V and β_V (continuation)

This table summarizes the monthly regression results for the coefficients α_V and β_V of the trading volume formula, as detailed in regression (1) of this appendix. The regressions were performed for the SPY ETF from 2013 to 2018 using four different monthly data sets {D(6), D(7), D(8), D(9), D(10)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\alpha_V = c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb} + \varsigma_{t-1} * \alpha_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

$$\beta_V = c + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P + \varsigma_{t-1} * \beta_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(6)	D(7)	D(8)	D(9)	D(10)
Panel A: Regression on α_V					
c	-0.11	-0.07	-0.08	-0.05	-1.44
	(-0.98) (-1.05) (-0.91)	(-0.59) (-0.63) (-0.59)	(-0.77) (-0.84) (-0.85)	(-0.28) (-0.34) (-0.32)	(-6.78) (-7.87) (-5.10)
	[0.33] [0.30] [0.37]	[0.56] [0.53] [0.56]	[0.44] [0.41] [0.40]	[0.78] [0.74] [0.75]	[0.00] [0.00] [0.00]
α_{pbel}	-0.08	-0.03	-0.07	-0.12	-0.09
	(-1.94) (-2.09) (-2.09)	(-0.58) (-0.63) (-0.45)	(-1.69) (-1.82) (-2.31)	(-2.96) (-2.91) (-4.06)	(-2.33) (-2.24) (-2.32)
	[0.06] [0.04] [0.04]	[0.56] [0.53] [0.66]	[0.10] [0.07] [0.02]	[0.00] [0.00] [0.00]	[0.02] [0.03] [0.02]
α_{amb}	0.48	0.49	0.49	0.46	0.69
	(7.03) (7.57) (4.18)	(6.42) (6.91) (4.21)	(7.03) (7.57) (4.26)	(6.62) (6.10) (3.19)	(9.59) (10.53) (4.78)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.02	-0.02	-0.02	-0.20	-0.15
	(-0.51) (-0.55) (-0.55)	(-0.35) (-0.38) (-0.34)	(-0.54) (-0.58) (-0.55)	(-2.68) (-3.08) (-2.43)	(-3.00) (-3.14) (-1.92)
	[0.61] [0.58] [0.58]	[0.73] [0.71] [0.73]	[0.59] [0.56] [0.59]	[0.01] [0.00] [0.02]	[0.00] [0.00] [0.06]
ς_{t-1}	-0.19	-0.20	-0.20	0.39	0.25
	(-1.90) (-2.05) (-2.34)	(-1.98) (-2.14) (-2.62)	(-1.94) (-2.10) (-2.44)	(7.20) (5.71) (4.91)	(4.66) (3.43) (2.65)
	[0.06] [0.04] [0.02]	[0.05] [0.04] [0.01]	[0.06] [0.04] [0.02]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.01]
N	72	72	72	247	247
R_a^2	0.64	0.621	0.635	0.705	0.755
Panel B: Regression on β_V(same for 1, 2, 3, 4)					
c	-0.01	-0.01	-0.01	0.06	0.40
	(-0.03) (-0.03) (-0.02)	(-0.03) (-0.03) (-0.02)	(-0.03) (-0.03) (-0.02)	(0.15) (0.15) (0.20)	(0.65) (0.70) (0.61)
	[0.98] [0.98] [0.98]	[0.98] [0.98] [0.98]	[0.98] [0.98] [0.98]	[0.88] [0.88] [0.84]	[0.52] [0.49] [0.54]
$\beta_{\sigma_{amb}}$	-0.32	-0.32	-0.32	-0.15	0.09
	(-0.27) (-0.29) (-0.20)	(-0.27) (-0.29) (-0.20)	(-0.27) (-0.29) (-0.20)	(-0.52) (-0.54) (-0.43)	(0.23) (0.25) (0.18)
	[0.79] [0.77] [0.84]	[0.79] [0.77] [0.84]	[0.79] [0.77] [0.84]	[0.60] [0.59] [0.66]	[0.82] [0.80] [0.86]
$\beta_{h\sigma_{amb}}$	0.52	0.52	0.52	0.51	0.29
	(0.43) (0.47) (0.32)	(0.43) (0.47) (0.32)	(0.43) (0.47) (0.32)	(1.39) (1.44) (1.56)	(0.59) (0.63) (0.53)
	[0.67] [0.64] [0.75]	[0.67] [0.64] [0.75]	[0.67] [0.64] [0.75]	[0.17] [0.15] [0.12]	[0.56] [0.53] [0.60]
β_{σ_p}	0.18	0.18	0.18	-0.27	-0.14
	(0.54) (0.58) (0.93)	(0.54) (0.58) (0.93)	(0.54) (0.58) (0.93)	(-1.80) (-1.92) (-0.71)	(-1.01) (-1.09) (-0.40)
	[0.59] [0.56] [0.35]	[0.59] [0.56] [0.35]	[0.59] [0.56] [0.35]	[0.07] [0.06] [0.48]	[0.31] [0.28] [0.69]
ς_{t-1}	0.01	0.01	0.01	0.00	0.01
	(0.07) (0.07) (0.09)	(0.07) (0.07) (0.09)	(0.07) (0.07) (0.09)	(0.06) (0.06) (0.08)	(0.12) (0.13) (0.18)
	[0.95] [0.94] [0.93]	[0.95] [0.94] [0.93]	[0.95] [0.94] [0.93]	[0.96] [0.95] [0.94]	[0.90] [0.90] [0.86]
N	72	72	72	247	247
R_a^2	-0.046	-0.046	-0.046	0.115	0.11

Table 20. Monthly Regressions for Trading Volume

This table summarizes the monthly regression results of the trading volume formula, as detailed in regression (2) of this appendix. The regressions were performed for the SPY ETF from 2013 to 2018 using four different monthly data sets {D(1), D(2), D(3), D(4), D(5)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$V = \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb}\right)_\alpha + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P\right)_\beta * \Delta_P + \varsigma_{t-1} * V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)	D(5)
c	-0.20	-0.21	-0.11	-0.26	-0.05
	(-1.34) (-1.50) (-2.26)	(-1.40) (-1.56) (-2.39)	(-0.65) (-0.72) (-0.66)	(-1.69) (-1.88) (-2.30)	(-0.39) (-0.44) (-0.40)
	[0.18] [0.14] [0.03]	[0.17] [0.12] [0.02]	[0.52] [0.47] [0.51]	[0.10] [0.06] [0.03]	[0.70] [0.66] [0.69]
α_{pbel}	0.02	-0.00	0.09	-0.05	-0.02
	(0.53) (0.60) (0.60)	(-0.08) (-0.09) (-0.10)	(1.46) (1.63) (0.81)	(-1.03) (-1.15) (-0.75)	(-0.40) (-0.45) (-0.46)
	[0.59] [0.55] [0.55]	[0.94] [0.93] [0.92]	[0.15] [0.11] [0.42]	[0.31] [0.25] [0.45]	[0.69] [0.66] [0.65]
α_{amb}	0.20	0.20	0.17	0.19	0.41
	(2.29) (2.55) (2.70)	(2.20) (2.45) (2.71)	(1.89) (2.11) (2.94)	(2.13) (2.38) (2.78)	(5.43) (6.05) (3.15)
	[0.03] [0.01] [0.01]	[0.03] [0.02] [0.01]	[0.06] [0.04] [0.00]	[0.04] [0.02] [0.01]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.12	-0.09	-0.10	-0.13	-0.06
	(-0.34) (-0.38) (-0.38)	(-0.27) (-0.30) (-0.28)	(-0.29) (-0.32) (-0.35)	(-0.36) (-0.40) (-0.40)	(-1.31) (-1.46) (-1.27)
	[0.73] [0.70] [0.71]	[0.79] [0.77] [0.78]	[0.77] [0.75] [0.73]	[0.72] [0.69] [0.69]	[0.20] [0.15] [0.21]
β_{P}	-0.39	-0.38	-0.40	-0.36	-0.07
	(-3.45) (-3.84) (-4.30)	(-3.34) (-3.72) (-4.04)	(-3.60) (-4.01) (-4.56)	(-3.18) (-3.54) (-3.89)	(-0.93) (-1.03) (-0.76)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.36] [0.31] [0.45]
$\beta_{\sigma_{\text{amb}}}$	-0.29	-0.27	-0.34	-0.20	-0.04
	(-0.87) (-0.97) (-1.19)	(-0.81) (-0.90) (-1.08)	(-1.04) (-1.16) (-1.37)	(-0.58) (-0.65) (-0.71)	(-0.05) (-0.06) (-0.07)
	[0.39] [0.34] [0.24]	[0.42] [0.37] [0.29]	[0.30] [0.25] [0.18]	[0.56] [0.52] [0.48]	[0.96] [0.95] [0.95]
$\beta_{h\sigma_{amb}}$	1.25	1.23	1.41	1.17	-0.05
	(1.29) (1.44) (1.28)	(1.27) (1.42) (1.27)	(1.47) (1.64) (1.37)	(1.22) (1.35) (1.27)	(-0.07) (-0.07) (-0.08)
	[0.20] [0.15] [0.21]	[0.21] [0.16] [0.21]	[0.15] [0.11] [0.18]	[0.23] [0.18] [0.21]	[0.95] [0.94] [0.93]
$\beta_{\sigma_{\text{p}}}$	0.21	0.21	0.26	0.20	-0.01
	(1.25) (1.39) (1.61)	(1.23) (1.37) (1.61)	(1.51) (1.68) (1.73)	(1.16) (1.29) (1.58)	(-0.05) (-0.06) (-0.10)
	[0.22] [0.17] [0.11]	[0.22] [0.18] [0.11]	[0.14] [0.10] [0.09]	[0.25] [0.20] [0.12]	[0.96] [0.95] [0.92]
$\varsigma t-1$	0.14	0.15	0.11	0.15	-0.15
	(1.23) (1.37) (1.17)	(1.32) (1.47) (1.27)	(1.02) (1.14) (0.87)	(1.35) (1.50) (1.31)	(-1.34) (-1.50) (-1.31)
	[0.22] [0.17] [0.25]	[0.19] [0.15] [0.21]	[0.31] [0.26] [0.39]	[0.18] [0.14] [0.20]	[0.18] [0.14] [0.20]
N	72	72	72	72	72
R_a^2	0.49	0.487	0.506	0.497	0.628

Table 20. Monthly Regressions for Trading Volume (continuation)

This table summarizes the monthly regression results of the trading volume formula, as detailed in regression (2) of this appendix. The regressions were performed for the SPY ETF from 2013 to 2018 using four different monthly data sets {D(6), D(7), D(8), D(9), D(10)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$V = \left(c + \alpha_{pbel} * PBEL + \alpha_{amb} * AMB + \alpha_{amb\sigma} * AMB * \sigma_{amb}\right)_{\alpha} + \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p} * \sigma_P\right)_{\beta} * \Delta_P + \varsigma_{t-1} * V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(6)	D(7)	D(8)	D(9)	D(10)
c	-0.08	-0.05	-0.08	-0.11	-1.37
	(-0.64) (-0.71) (-0.62)	(-0.39) (-0.43) (-0.38)	(-0.71) (-0.79) (-0.78)	(-0.68) (-0.80) (-0.83)	(-6.48) (-7.44) (-4.75)
	[0.53] [0.48] [0.54]	[0.70] [0.67] [0.71]	[0.48] [0.43] [0.44]	[0.50] [0.42] [0.41]	[0.00] [0.00] [0.00]
α_{pbel}	-0.04	-0.02	-0.09	-0.09	-0.07
	(-1.04) (-1.16) (-1.13)	(-0.34) (-0.38) (-0.29)	(-2.39) (-2.67) (-3.84)	(-2.25) (-2.25) (-3.58)	(-1.79) (-1.78) (-3.06)
	[0.30] [0.25] [0.26]	[0.74] [0.71] [0.77]	[0.02] [0.01] [0.00]	[0.03] [0.03] [0.00]	[0.07] [0.08] [0.00]
α_{amb}	0.41	0.42	0.44	0.37	0.61
	(5.55) (6.18) (3.26)	(5.07) (5.65) (3.39)	(6.01) (6.70) (3.34)	(5.33) (5.17) (2.92)	(7.99) (8.98) (4.30)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.07	-0.06	-0.09	-0.01	-0.17
	(-1.38) (-1.54) (-1.40)	(-1.31) (-1.46) (-1.29)	(-1.82) (-2.03) (-1.84)	(-0.18) (-0.19) (-0.17)	(-3.24) (-3.38) (-2.38)
	[0.17] [0.13] [0.17]	[0.20] [0.15] [0.20]	[0.07] [0.05] [0.07]	[0.86] [0.85] [0.87]	[0.00] [0.00] [0.02]
β_{P}	-0.06	-0.07	-0.04	-0.21	-0.15
	(-0.82) (-0.92) (-0.68)	(-0.92) (-1.03) (-0.74)	(-0.60) (-0.67) (-0.46)	(-2.75) (-2.89) (-1.85)	(-1.95) (-1.97) (-1.53)
	[0.41] [0.36] [0.50]	[0.36] [0.31] [0.46]	[0.55] [0.50] [0.65]	[0.01] [0.00] [0.07]	[0.05] [0.05] [0.13]
$\beta_{\sigma_{\text{amb}}}$	-0.11	0.06	0.05	0.31	0.08
	(-0.16) (-0.18) (-0.23)	(0.09) (0.10) (0.11)	(0.07) (0.08) (0.11)	(1.20) (1.37) (0.80)	(0.36) (0.34) (0.28)
	[0.87] [0.86] [0.82]	[0.93] [0.92] [0.91]	[0.94] [0.93] [0.91]	[0.23] [0.17] [0.43]	[0.72] [0.74] [0.78]
$\beta_{h\sigma_{amb}}$	0.02	-0.14	-0.16	-0.48	-0.10
	(0.04) (0.04) (0.05)	(-0.20) (-0.22) (-0.24)	(-0.25) (-0.28) (-0.35)	(-1.64) (-1.80) (-1.23)	(-0.44) (-0.40) (-0.35)
	[0.97] [0.97] [0.96]	[0.84] [0.82] [0.81]	[0.80] [0.78] [0.73]	[0.10] [0.07] [0.22]	[0.66] [0.69] [0.73]
$\beta_{\sigma_{\text{p}}}$	-0.00	-0.02	0.01	0.08	-0.03
	(-0.01) (-0.01) (-0.01)	(-0.17) (-0.19) (-0.30)	(0.05) (0.06) (0.09)	(1.10) (1.21) (0.71)	(-0.50) (-0.51) (-0.46)
	[0.99] [0.99] [0.99]	[0.86] [0.85] [0.76]	[0.96] [0.95] [0.93]	[0.27] [0.23] [0.48]	[0.62] [0.61] [0.65]
$\varsigma t-1$	-0.15	-0.15	-0.15	0.38	0.26
	(-1.37) (-1.53) (-1.34)	(-1.37) (-1.53) (-1.34)	(-1.45) (-1.62) (-1.20)	(7.43) (6.17) (5.21)	(4.80) (3.78) (3.17)
	[0.18] [0.13] [0.19]	[0.18] [0.13] [0.19]	[0.15] [0.11] [0.24]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
N	72	72	72	248	248
R_a^2	0.633	0.627	0.66	0.733	0.752

Table 21. Monthly Regressions for ΔTrading Volume

This table summarizes the monthly regression results of the first difference trading volume formula, as detailed in regression (3) of this appendix. The regressions were performed for the SPY ETF from 2013 to 2018 using four different monthly data sets {D(1), D(2), D(3), D(4), D(5)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\Delta V = c \;+\; \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB + \alpha_{amb\sigma} * \Delta AMB * \sigma_{amb}\right)_{\Delta\alpha} + \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{h\Delta\sigma_{amb}} * \Delta\sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P \;+\; \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\right)_{\beta} * \Delta_P^2 \;+ \varsigma_{t-1} * \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(1)	D(2)	D(3)	D(4)	D(5)
c	-0.01	-0.03	-0.02	-0.07	-0.01
	(-0.05) (-0.06) (-0.07)	(-0.16) (-0.18) (-0.21)	(-0.09) (-0.11) (-0.17)	(-0.36) (-0.42) (-0.55)	(-0.09) (-0.10) (-0.20)
	[0.96] [0.96] [0.95]	[0.88] [0.86] [0.83]	[0.93] [0.92] [0.86]	[0.72] [0.68] [0.58]	[0.93] [0.92] [0.84]
α_{pbel}	0.14	0.11	0.23	-0.08	0.03
	(2.06) (2.36) (1.72)	(1.55) (1.78) (1.30)	(3.46) (3.97) (2.37)	(-1.04) (-1.19) (-1.11)	(0.45) (0.52) (0.44)
	[0.04] [0.02] [0.09]	[0.13] [0.08] [0.20]	[0.00] [0.00] [0.02]	[0.30] [0.24] [0.27]	[0.65] [0.61] [0.66]
α_{amb}	0.31	0.31	0.23	0.30	0.39
	(3.80) (4.36) (2.55)	(3.78) (4.33) (2.61)	(2.94) (3.37) (2.41)	(3.57) (4.10) (2.57)	(6.60) (7.57) (3.54)
	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.46	-0.48	-0.24	-0.59	-0.03
	(-1.57) (-1.80) (-0.90)	(-1.59) (-1.82) (-0.90)	(-0.84) (-0.96) (-0.65)	(-1.75) (-2.01) (-1.23)	(-0.41) (-0.47) (-0.50)
	[0.12] [0.08] [0.37]	[0.12] [0.07] [0.37]	[0.40] [0.34] [0.52]	[0.09] [0.05] [0.23]	[0.69] [0.64] [0.62]
β_{P}	-0.09	-0.10	-0.14	-0.12	-0.04
	(-0.44) (-0.50) (-0.41)	(-0.47) (-0.54) (-0.46)	(-0.70) (-0.81) (-0.80)	(-0.53) (-0.61) (-0.54)	(-0.31) (-0.35) (-0.39)
	[0.66] [0.62] [0.68]	[0.64] [0.59] [0.65]	[0.49] [0.42] [0.43]	[0.60] [0.54] [0.59]	[0.76] [0.73] [0.70]
$\beta_{\sigma_{\text{amb}}}$	0.77	0.74	0.61	0.72	-0.36
	(1.28) (1.47) (1.50)	(1.21) (1.39) (1.49)	(1.08) (1.24) (1.57)	(1.17) (1.34) (1.39)	(-0.62) (-0.71) (-0.99)
	[0.21] [0.15] [0.14]	[0.23] [0.17] [0.14]	[0.28] [0.22] [0.12]	[0.25] [0.19] [0.17]	[0.54] [0.48] [0.32]
$\beta_{h\sigma_{amb}}$	-1.49	-1.55	-1.06	-1.23	0.26
	(-0.86) (-0.99) (-0.82)	(-0.88) (-1.01) (-0.88)	(-0.65) (-0.75) (-0.74)	(-0.69) (-0.79) (-0.74)	(0.44) (0.50) (0.74)
	[0.39] [0.33] [0.42]	[0.38] [0.32] [0.38]	[0.52] [0.46] [0.46]	[0.49] [0.43] [0.46]	[0.66] [0.62] [0.46]
$\beta_{\triangle\sigma_{\text{amb}}}$	-0.08	0.01	-0.09	0.12	-0.20
	(-0.17) (-0.20) (-0.24)	(0.03) (0.04) (0.05)	(-0.21) (-0.24) (-0.32)	(0.26) (0.30) (0.51)	(-0.19) (-0.22) (-0.36)
	[0.86] [0.85] [0.81]	[0.98] [0.97] [0.96]	[0.83] [0.81] [0.75]	[0.79] [0.76] [0.61]	[0.85] [0.83] [0.72]
$\beta_{h\triangle\sigma_{amb}}$	0.37	0.41	0.39	0.47	0.32
	(0.86) (0.98) (0.96)	(0.94) (1.08) (1.11)	(0.96) (1.10) (1.22)	(1.06) (1.21) (1.17)	(0.31) (0.35) (0.58)
	[0.39] [0.33] [0.34]	[0.35] [0.28] [0.27]	[0.34] [0.28] [0.23]	[0.29] [0.23] [0.25]	[0.76] [0.73] [0.57]
$\beta_{\sigma_{\text{p}}\triangle^2_{\text{p}}}$	-0.04	-0.08	0.16	-0.19	-0.01
	(-0.11) (-0.12) (-0.10)	(-0.23) (-0.26) (-0.21)	(0.48) (0.55) (0.74)	(-0.54) (-0.62) (-0.49)	(-0.05) (-0.05) (-0.06)
	[0.91] [0.90] [0.92]	[0.82] [0.79] [0.84]	[0.63] [0.58] [0.46]	[0.59] [0.54] [0.63]	[0.96] [0.96] [0.95]
$\beta_{\triangle\sigma_{\text{p}}}$	0.22	0.23	0.25	0.28	0.09
	(0.83) (0.95) (0.80)	(0.87) (0.99) (0.80)	(1.03) (1.19) (1.09)	(1.03) (1.19) (0.84)	(0.46) (0.52) (0.66)
	[0.41] [0.34] [0.43]	[0.39] [0.32] [0.43]	[0.31] [0.24] [0.28]	[0.31] [0.24] [0.40]	[0.65] [0.60] [0.51]
ς_{t-1}	-0.42	-0.41	-0.40	-0.40	-0.43
	(-3.80) (-4.35) (-2.13)	(-3.68) (-4.22) (-2.15)	(-3.86) (-4.43) (-2.64)	(-3.52) (-4.04) (-2.10)	(-4.95) (-5.67) (-3.73)
	[0.00] [0.00] [0.04]	[0.00] [0.00] [0.04]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.04]	[0.00] [0.00] [0.00]
N	71	71	71	71	71
R_a^2	0.418	0.399	0.486	0.384	0.624

Table 21. Monthly Regressions for ΔTrading Volume (continuation)

This table summarizes the monthly regression results of the first difference trading volume formula, as detailed in regression (3) of this appendix. The regressions were performed for the SPY ETF from 2013 to 2018 using four different monthly data sets {D(6), D(7), D(8), D(9), D(10)}. γ_p represent time fixed effects. T-values and P-values from the left side to the right are standard OLS, clustered robust and Newey-West autocorrelation robust values.

$$\Delta V = c \ + \ \left(\alpha_{pbel} * \Delta PBEL + \alpha_{amb} * \Delta AMB + \alpha_{amb\sigma} * \Delta AMB * \sigma_{amb}\right)_{\Delta\alpha} + \ \left(\beta_{\Delta\sigma_{amb}} * \Delta\sigma_{amb} + \beta_{h\Delta\sigma_{amb}} * \Delta\sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\Delta\sigma_p} * \Delta\sigma_P\right)_{\Delta\beta} * \Delta_P \ + \ \left(\beta_P + \beta_{\sigma_{amb}} * \sigma_{amb} + \beta_{h\sigma_{amb}} * \sigma_{amb} * I_{h\sigma_{amb}} + \beta_{\sigma_p\Delta_P^2} * \sigma_P\right)_{\beta} * \Delta_P^2 \ + \varsigma_{t-1} * \Delta V_{t-1} + \sum_{p=t+1}^{T} \gamma_p + \epsilon$$

	D(6)	D(7)	D(8)	D(9)	D(10)
c	-0.02	-0.05	-0.02	-0.06	-0.05
	(-0.13) (-0.15) (-0.29)	(-0.33) (-0.38) (-0.73)	(-0.15) (-0.18) (-0.37)	(-0.23) (-0.29) (-0.84)	(-0.20) (-0.26) (-1.47)
	[0.90] [0.88] [0.78]	[0.74] [0.71] [0.47]	[0.88] [0.86] [0.71]	[0.82] [0.77] [0.40]	[0.84] [0.80] [0.14]
α_{pbel}	0.01	0.18	-0.00	-0.05	-0.03
	(0.23) (0.27) (0.23)	(3.32) (3.80) (3.46)	(-0.00) (-0.00) (-0.01)	(-0.83) (-0.80) (-1.51)	(-0.62) (-0.60) (-0.83)
	[0.82] [0.79] [0.82]	[0.00] [0.00] [0.00]	[1.00] [1.00] [1.00]	[0.41] [0.42] [0.13]	[0.54] [0.55] [0.41]
α_{amb}	0.39	0.35	0.39	0.36	0.54
	(6.64) (7.61) (3.50)	(6.17) (7.07) (4.19)	(6.65) (7.63) (3.43)	(5.24) (5.26) (3.84)	(8.43) (9.65) (4.61)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\alpha_{\text{amb}\sigma}$	-0.02	-0.03	-0.02	-0.18	-0.05
	(-0.38) (-0.44) (-0.48)	(-0.45) (-0.52) (-0.67)	(-0.37) (-0.42) (-0.48)	(-2.74) (-3.21) (-3.47)	(-0.66) (-0.74) (-0.50)
	[0.70] [0.66] [0.64]	[0.65] [0.61] [0.50]	[0.72] [0.68] [0.63]	[0.01] [0.00] [0.00]	[0.51] [0.46] [0.62]
β_{P}	-0.04	-0.05	-0.03	-0.30	-0.12
	(-0.30) (-0.34) (-0.39)	(-0.48) (-0.55) (-0.76)	(-0.28) (-0.32) (-0.35)	(-2.61) (-2.87) (-2.37)	(-0.96) (-1.10) (-1.21)
	[0.76] [0.73] [0.69]	[0.63] [0.58] [0.45]	[0.78] [0.75] [0.73]	[0.01] [0.00] [0.02]	[0.34] [0.27] [0.23]
$\beta_{\sigma_{\text{amb}}}$	-0.38	-0.46	-0.39	0.16	-0.05
	(-0.66) (-0.76) (-1.07)	(-0.88) (-1.01) (-1.83)	(-0.68) (-0.78) (-1.09)	(0.43) (0.51) (0.47)	(-0.30) (-0.30) (-0.54)
	[0.51] [0.45] [0.29]	[0.38] [0.32] [0.07]	[0.50] [0.44] [0.28]	[0.66] [0.61] [0.64]	[0.77] [0.77] [0.59]
$\beta_{h\sigma_{amb}}$	0.28	0.35	0.29	0.05	0.07
	(0.48) (0.55) (0.82)	(0.66) (0.76) (1.47)	(0.49) (0.56) (0.85)	(0.12) (0.14) (0.13)	(0.38) (0.37) (0.60)
	[0.64] [0.59] [0.42]	[0.51] [0.45] [0.15]	[0.63] [0.58] [0.40]	[0.90] [0.89] [0.89]	[0.70] [0.71] [0.55]
$\beta_{\triangle\sigma_{\text{amb}}}$	-0.24	-0.57	-0.26	-0.29	-0.41
	(-0.23) (-0.26) (-0.44)	(-0.60) (-0.69) (-1.32)	(-0.25) (-0.28) (-0.47)	(-2.17) (-2.51) (-2.31)	(-1.37) (-0.98) (-0.73)
	[0.82] [0.80] [0.66]	[0.55] [0.49] [0.19]	[0.81] [0.78] [0.64]	[0.03] [0.01] [0.02]	[0.17] [0.33] [0.47]
$\beta_{h\triangle\sigma_{amb}}$	0.36	0.73	0.38	-0.09	0.31
	(0.34) (0.39) (0.67)	(0.76) (0.87) (1.68)	(0.37) (0.42) (0.70)	(-0.62) (-0.74) (-0.64)	(1.05) (0.74) (0.55)
	[0.73] [0.69] [0.51]	[0.45] [0.39] [0.10]	[0.72] [0.68] [0.49]	[0.54] [0.46] [0.52]	[0.30] [0.46] [0.58]
$\beta_{\sigma_{\text{p}}\triangle^2{\text{p}}}$	-0.02	0.16	-0.02	-0.05	0.02
	(-0.06) (-0.07) (-0.08)	(0.61) (0.70) (0.83)	(-0.08) (-0.09) (-0.10)	(-0.47) (-0.56) (-0.47)	(0.14) (0.16) (0.14)
	[0.95] [0.94] [0.93]	[0.54] [0.49] [0.41]	[0.94] [0.93] [0.92]	[0.64] [0.58] [0.64]	[0.89] [0.87] [0.89]
$\beta_{\triangle\sigma_{\text{p}}}$	0.10	0.13	0.10	0.14	-0.04
	(0.51) (0.58) (0.75)	(0.72) (0.83) (1.15)	(0.53) (0.61) (0.85)	(1.67) (1.97) (2.51)	(-0.51) (-0.58) (-0.35)
	[0.61] [0.56] [0.46]	[0.47] [0.41] [0.26]	[0.60] [0.54] [0.40]	[0.10] [0.05] [0.01]	[0.61] [0.56] [0.73]
ς_{t-1}	-0.43	-0.43	-0.42	-0.26	-0.32
	(-4.93) (-5.65) (-3.80)	(-5.49) (-6.30) (-4.35)	(-4.91) (-5.64) (-3.87)	(-4.56) (-3.91) (-2.45)	(-5.82) (-4.77) (-3.93)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.02]	[0.00] [0.00] [0.00]
N	71	71	71	247	247
R_a^2	0.623	0.686	0.623	0.331	0.397

Appendix H Single Sorted Portfolio Returns

This section shows the regression results for the single sorted portfolios described in Section-7.

Table-24 on this page shows the results of regressing the Fama-French market portfolio, the LMH Turnover Portfolio P_{Turn}, LMH Ambiguity related Turnover sorted portfolio known as P_{amb}, the LMH differences in prior beliefs related Turnover sorted portfolio called P_{pbel}, the LMH price fluctuations related Turnover sorted portfolio called $P_{\Delta P_{21}}$, and finally, the LMH unexplained residual related Turnover sorted portfolio named P_{ϵ}, on a constant as described in regression (4) of Section-7.

$$R_{P_{Turn}} = c + \epsilon \tag{4}$$

$$R_{P_{Turn}} = c + R_{free} + \beta_{MKTRF} * R_{MKTRF} + \beta_{HML} * R_{HML} + \beta_{SMB} * R_{SMB} + \epsilon \tag{5}$$

$$R_{P_{Turn}} = c + \beta_{P_{amb}} * R_{P_{amb}} + \beta_{P_{pbel}} * R_{P_{pbel}} + \beta_{P_{\Delta P_{21}}} * R_{P_{\Delta P_{21}}} + \beta_{P_{\epsilon}} * R_{P_{\epsilon}} + \epsilon \tag{6}$$

Table 22. Monthly Single Sorted Portfolio Returns

Single sorted portfolio returns statistics using monthly data between 1990 to 2020.

	P_{MKT}	P_{Turn}	P_{amb}	P_{pbel}	$P_{\Delta P_{21}}$	P_{ϵ}
Panel A: Monthly Portfolio Returnds (1990 - 2020)						
c	0.009	0.006	0.006	0.002	-0.005	0.003
	(4.10) (4.10) (3.96)	(1.66) (1.67) (1.68)	(1.88) (1.88) (1.70)	(0.65) (0.65) (0.76)	(-2.24) (-2.24) (-3.27)	(1.51) (1.52) (1.75)
	[0.00] [0.00] [0.00]	[0.10] [0.10] [0.09]	[0.06] [0.06] [0.09]	[0.51] [0.51] [0.45]	[0.03] [0.03] [0.00]	[0.13] [0.13] [0.08]
N	372	372	372	372	372	372
R_a^2	0	0	0	0	0	0
Panel B: Sharpe Ratios (1990 - 2020, yearly)						
Excess Return	0.09	0.05	0.05	-0.01	-0.03	0.01
Std. Deviation	0.14	0.25	0.21	0.16	0.15	0.11
Sharpe Ratio	0.56	0.20	0.22	-0.05	-0.22	0.04

Table-23 below shows the results of regressing the LMH Turnover sorted portfolio on a constant, on the Fama-French 3-Factors (MKT, SMB, HML) and on the LMH portfolios $\{P_{amb}, P_{pbel}, P_{\Delta P_{21}}, P_{\epsilon}\}$ derived from the four Turnover components, as described in regressions (4), (5) and (6) of Section-7.

Lastly, you find in Table-24 the outcomes of the regression of the LMH Turnover sorted portfolio P_{Turn} on the LMH portfolios $\{P_{amb}, P_{pbel}, P_{\Delta P_{21}}, P_{\epsilon}\}$ based on the four components of Turnover described in Section-7, plus an explanatory variable controlling for Liquidity. The

Liquidity proxies are the month-end SPY Bid-Ask, the monthly mean SPY Bid-Ask, the Liquidity measure proposed by Hu et al. (2013), and the Intermediary Capital Risk Factor and Ratio introduced by He et al. (2017). These variables are employed both in their level form and as first differences.

Table 23. Monthly Single Sorted Portfolio Returns Attribution

Single sorted portfolio returns statistics using monthly data between 1990 to 2020.

	P_{Turn}	P_{Turn}	P_{Turn}
c	0.006	0.019	-0.001
	(1.66) (1.67) (1.68)	(6.08) (6.07) (4.81)	(-1.53) (-1.54) (-1.27)
	[0.10] [0.10] [0.09]	[0.00] [0.00] [0.00]	[0.13] [0.12] [0.20]
β_{R_f}		-2.380	
		(-2.20) (-2.16) (-1.77)	
		[0.03] [0.03] [0.08]	
β_{MKT-R_f}		-0.953	
		(-19.98) (-17.55) (-12.19)	
		[0.00] [0.00] [0.00]	
β_{HML}		0.854	
		(12.61) (11.43) (6.82)	
		[0.00] [0.00] [0.00]	
β_{SMB}		-0.789	
		(-11.68) (-11.83) (-7.96)	
		[0.00] [0.00] [0.00]	
$\beta_{P_{amb}}$			0.942
			(63.44) (62.43) (41.18)
			[0.00] [0.00] [0.00]
$\beta_{P_{pbel}}$			0.450
			(20.46) (20.70) (5.93)
			[0.00] [0.00] [0.00]
$\beta_{P_{\triangle P_{21}}}$			-0.034
			(-1.26) (-1.27) (-0.52)
			[0.21] [0.21] [0.60]
$\beta_{P_{\epsilon}}$			0.390
			(11.94) (11.94) (7.21)
			[0.00] [0.00] [0.00]
N	372	372	372
R_a^2	0.000	0.728	0.944

Table 24. Monthly Single Sorted Portfolio Returns Attribution with Liquidity Controls

Single sorted portfolio returns statistics using monthly data between 1990 to 2020. Last row indicates the liquidity control used.

	P_{Turn}	P_{Turn}	P_{Turn}	P_{Turn}	P_{Turn}
Panel A: LMH Turnover Portfolio Regressions with Liquidity Controls in Levels					
c	-0.001	-0.001	-0.000	-0.001	-0.002
	(-0.69) (-0.70) (-0.63)	(-0.60) (-0.60) (-0.61)	(-0.16) (-0.16) (-0.17)	(-1.48) (-1.49) (-1.25)	(-0.81) (-0.80) (-0.69)
	[0.49] [0.49] [0.53]	[0.55] [0.55] [0.55]	[0.87] [0.88] [0.87]	[0.14] [0.14] [0.21]	[0.42] [0.42] [0.49]
$\beta_{P_{amb}}$	0.944	0.941	0.942	0.939	0.942
	(61.72) (60.53)	(62.43) (61.13)	(63.43) (62.44)	(57.21) (56.50)	(63.32) (62.41)
	(38.43)	(37.79)	(41.46)	(37.45)	(41.11)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{P_{pbel}}$	0.457	0.457	0.449	0.451	0.450
	(20.31) (20.60) (6.27)	(20.30) (20.60) (6.25)	(20.31) (20.60) (5.84)	(20.44) (20.72) (5.90)	(20.44) (20.71) (5.93)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{P_{\triangle P_{21}}}$	-0.056	-0.056	-0.036	-0.034	-0.033
	(-2.05) (-2.08) (-0.89)	(-2.05) (-2.07) (-0.88)	(-1.32) (-1.34) (-0.54)	(-1.24) (-1.25) (-0.51)	(-1.22) (-1.23) (-0.49)
	[0.04] [0.04] [0.38]	[0.04] [0.04] [0.38]	[0.19] [0.18] [0.59]	[0.21] [0.21] [0.61]	[0.22] [0.22] [0.62]
$\beta_{P_{\epsilon}}$	0.440	0.443	0.392	0.392	0.389
	(12.85) (12.88) (8.85)	(12.89) (12.93) (8.90)	(11.97) (11.97) (7.32)	(11.91) (11.93) (7.37)	(11.85) (11.86) (7.07)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{Control}$	0.001	0.001	-0.000	-0.007	0.013
	(1.06) (1.01) (1.17)	(0.92) (0.89) (0.85)	(-0.92) (-0.85) (-0.86)	(-0.46) (-0.45) (-0.40)	(0.33) (0.33) (0.25)
	[0.29] [0.31] [0.24]	[0.36] [0.38] [0.39]	[0.36] [0.39] [0.39]	[0.65] [0.65] [0.69]	[0.74] [0.75] [0.80]
N	324	324	372	372	372
R_a^2	0.949	0.949	0.944	0.944	0.944
Panel B: LMH Turnover Portfolio Regressions with Liquidity Controls in Differences					
c	-0.001	-0.001	-0.001	-0.001	-0.001
	(-0.92) (-0.93) (-0.80)	(-0.93) (-0.93) (-0.79)	(-1.57) (-1.58) (-1.30)	(-1.55) (-1.56) (-1.28)	(-1.51) (-1.52) (-1.26)
	[0.36] [0.35] [0.43]	[0.35] [0.35] [0.43]	[0.12] [0.12] [0.19]	[0.12] [0.12] [0.20]	[0.13] [0.13] [0.21]
$\beta_{P_{amb}}$	0.942	0.939	0.942	0.946	0.939
	(61.98) (60.71)	(62.36) (61.29)	(63.43) (62.37)	(58.99) (58.44)	(58.02) (57.26)
	(37.18)	(36.47)	(41.23)	(38.07)	(38.48)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{P_{pbel}}$	0.456	0.455	0.450	0.450	0.451
	(20.21) (20.51) (6.23)	(20.27) (20.55) (6.35)	(20.44) (20.73) (5.90)	(20.45) (20.72) (5.97)	(20.44) (20.72) (5.90)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{P_{\triangle P_{21}}}$	-0.055	-0.056	-0.033	-0.033	-0.033
	(-2.02) (-2.04) (-0.86)	(-2.07) (-2.09) (-0.91)	(-1.24) (-1.25) (-0.51)	(-1.22) (-1.23) (-0.51)	(-1.22) (-1.24) (-0.51)
	[0.04] [0.04] [0.39]	[0.04] [0.04] [0.37]	[0.22] [0.21] [0.61]	[0.22] [0.22] [0.61]	[0.22] [0.22] [0.61]
$\beta_{P_{\epsilon}}$	0.440	0.447	0.389	0.389	0.391
	(12.78) (12.81) (8.77)	(13.03) (13.09) (9.55)	(11.90) (11.91) (7.20)	(11.85) (11.86) (7.26)	(11.94) (11.95) (7.29)
	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]	[0.00] [0.00] [0.00]
$\beta_{\triangle Control}$	0.000	-0.004	-0.000	0.007	-0.097
	(0.31) (0.30) (0.44)	(-2.02) (-2.02) (-3.25)	(-0.98) (-0.94) (-1.18)	(0.62) (0.62) (0.68)	(-0.47) (-0.48) (-0.51)
	[0.75] [0.76] [0.66]	[0.04] [0.04] [0.00]	[0.33] [0.35] [0.24]	[0.54] [0.54] [0.50]	[0.64] [0.63] [0.61]
N	323	323	372	372	372
R_a^2	0.949	0.950	0.944	0.944	0.944
Control	BidAsk_{last}	BidAsk_{mean}	Noise	ICapital_{factor}	ICapital_{ratio}

Appendix I Bisorted Portfolio Returns

In this appendix, I provide the monthly frequency return statistics of the bisorted portfolios outlined in Section-7.

Table-25 below displays the mean monthly returns, standard deviations and Sharpe-Ratios of the Fama-French Market Factor, the bisorted LMH Turnover-Turnover portfolio called P_{Turn}, the LMH Turnover related to Ambiguity and Turnover related to price fluctuations bisorted portfolio called $P_{amb-\Delta P_{21}}$ and the LMH Turnover related to price fluctuations and Turnover related to Ambiguity bisorted portfolio called $P_{\Delta P_{21}-amb}$.

Table 26 below presents the annualized average differential returns of the bisorted portfolios. In Panel A, the table illustrates the differential returns achieved by going long on the quantile bisorted portfolio indicated in the top row and simultaneously shorting the quantile bisorted portfolio described in the leftmost column. The first dimension in Panel A, pertaining to Ambiguity driven Turnover, is divided into 5 quantiles, while the second dimension, associated to price fluctuations driven Turnover, is distributed across two quantiles. This arrangement yields a total of 10 bisorted portfolios {00,01,10,11,20,21,30,31,40,41} . In Panel-B of the same table, you can observe the annualized differential returns stemming from bisorted portfolios created by employing Turnover driven by price fluctuations as the first sorting dimension, and Turnover driven by Ambiguity as the second sorting dimension.

Table 25. Bisorted Portfolios Monthly Returns Statistics

This tables summarize the returns of the HML bisorted portfolios of section 5. Each portfolio is obtained by double sorting on a 5x2 grid.

	P_{MKT}	$P_{Turn-Turn}$	$P_{amb-\triangle P_{21}}$	$P_{\triangle P_{21}-amb}$
Panel A: Monthly Portfolio Returns (1990 - 2020)				
c	0.009	0.006	0.008	0.008
	(4.10) (4.10) (3.96)	(1.44) (1.44) (1.39)	(2.96) (2.96) (2.79)	(3.90) (3.91) (3.53)
	[0.00] [0.00] [0.00]	[0.15] [0.15] [0.17]	[0.00] [0.00] [0.01]	[0.00] [0.00] [0.00]
N	372	372	372	372
R_a^2	0	0	0	0
Panel B: Sharpe Ratios (1990 - 2020, yearly)				
Excess Return	0.09	0.04	0.07	0.07
Std. Deviation	0.14	0.25	0.16	0.13
Sharpe Ratio	0.56	0.16	0.38	0.50

Table 26. Bisorted Portfolios Monthly Returns

This tables summarize the annualized differential returns of the bisorted portfolios of Section-7. Each portfolio is obtained by double sorting on a 5x2 grid. The portfolios dimensions are Ambiguity - Price Change driven Turnover in Panel-A, and Price Change - Ambiguity driven Turnover in panel-B. Dimensions are in decimal format, 0.09 means 9% pear year.

Panel A: Ambiguity and $\triangle P_{21}$

	00	01	10	11	20	21	30	31	40	41
00	0.00	-0.03	-0.01	-0.04	0.00	-0.03	0.02	-0.01	0.06	0.03
01	0.03	0.00	0.03	-0.00	0.04	0.01	0.05	0.03	**0.09**	0.07
10	0.01	-0.03	0.00	-0.03	0.01	-0.02	0.03	-0.00	0.07	0.04
11	0.04	0.00	0.03	0.00	0.04	0.01	0.05	0.03	0.10	0.07
20	-0.00	-0.04	-0.01	-0.04	0.00	-0.03	0.01	-0.01	0.06	0.03
21	0.03	-0.01	0.02	-0.01	0.03	0.00	0.05	0.02	0.09	0.06
30	-0.02	-0.05	-0.03	-0.05	-0.01	-0.05	0.00	-0.03	0.04	0.02
31	0.01	-0.03	0.00	-0.03	0.01	-0.02	0.03	0.00	0.07	0.04
40	-0.06	**-0.09**	-0.07	-0.10	-0.06	-0.09	-0.04	-0.07	0.00	-0.03
41	-0.03	-0.07	-0.04	-0.07	-0.03	-0.06	-0.02	-0.04	0.03	0.00

Panel B: $\triangle P_{21}$ and Ambiguity

	00	01	10	11	20	21	30	31	40	41
00	0.00	0.04	-0.02	0.01	-0.02	0.00	-0.05	0.00	-0.05	0.01
01	-0.04	0.00	-0.06	-0.03	-0.06	-0.04	-0.09	-0.04	**-0.09**	-0.03
10	0.02	0.06	0.00	0.03	-0.00	0.02	-0.03	0.02	-0.04	0.03
11	-0.01	0.03	-0.03	0.00	-0.03	-0.01	-0.06	-0.01	-0.07	-0.00
20	0.02	0.06	0.00	0.03	0.00	0.02	-0.03	0.02	-0.04	0.03
21	-0.00	0.04	-0.02	0.01	-0.02	0.00	-0.05	-0.00	-0.06	0.01
30	0.05	0.09	0.03	0.06	0.03	0.05	0.00	0.05	-0.01	0.06
31	-0.00	0.04	-0.02	0.01	-0.02	0.00	-0.05	0.00	-0.06	0.01
40	0.05	**0.09**	0.04	0.07	0.04	0.06	0.01	0.06	0.00	0.07
41	-0.01	0.03	-0.03	0.00	-0.03	-0.01	-0.06	-0.01	-0.07	0.00

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