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E E 381 Section 12
Lab 2
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Introduction

The purpose of this lab is to practice conditional probability using Python. The background situation for all four Problems is the following: we are trying to send a message in binary through a noisy channel, where there is a probability of an individual bit being received as a 0 instead of a 1, or a 1 instead of a 0.

We define the random variables S and R referring to the sent bit and the received bit, respectively, where $P(S = 0) = p_0$, $P(S = 1) = p_1$, and P is the probability function. All Problems respect the following constants:

- The probability that a 0 will be sent $p_0 = 0.6$
- The probability that a 1 will be sent $p1 = 1 - p_0 = 0.4$
- The probability that a sent 0 will be received as a 1 $\varepsilon_0 = 0.05$
- The probability that a sent 1 will be received as a 0 $\varepsilon_1 = 0.03$

An image displaying the raw output of the Python source file can be found at the end of this document.

1 Problem 1

1.1 Question

The purpose of this Problem is to find $P(S \neq R)$, that being the probability of not receiving the same bit that was sent: if the sent bit is 0 and the received bit is 0, or if the sent bit is 1 and the received bit is 1, then the experiment is considered a success; if the two bits do not match, then it is not a success—the task is to find the ratio of non-successes to the total number of bits sent. For this experiment, 100 000 bits were sent.

1.2 Results

Out of the 100 000 bits, 95 703 bits were sent successfully, which means that $100\,000 - 95\,703 = 4297$ bits were received erroneously. This is a probability of 0.04297.

Table 1 displays the results of this experiment. Note that this experiment took about 0.556 s.

Probability of transmission error	
Ans.	$p = 0.04297$

Table 1: Results of Problem 1.

2 Problem 2

2.1 Question

The purpose of this Problem is to find $P(R = 1 \mid S = 1)$, that being the probability of receiving a 1 with the knowledge that the sent bit is also a 1. This is also known as the conditional probability of $R = 1$ given $S = 1$. Again, 100 000 bits were sent in this experiment.

2.2 Results

Out of the 100 000 bits that were sent, only 39 876 of them were 1. Of these 39 876 bits, 38 636 were received as a 1. This makes the probability $P(R = 1 \mid S = 1) = 38\,636/39\,876 = 0.96890$.

Table 2 shows the results of this experiment. Note that this experiment took about 0.545 s.

Conditional probability $P(R = 1 \mid S = 1)$	
Ans.	$p = 0.96890$

Table 2: Results of Problem 2.

3 Problem 3

3.1 Question

The purpose of this Problem is to find $P(S = 1 \mid R = 1)$, that being the probability that, knowing that you received a 1, a 1 was sent. 100 000 bits were sent in this experiment.

3.2 Results

Out of 100 000 bits, 41 642 were received as 1, and 38 616 of *those* were sent as 1. This means that 3026 bits were erroneously received as 1. This means that $P(S = 1 \mid R = 1) = 38\,616/41\,642 = 0.92733$.

Table 3 displays the results of this experiment. Note that this experiment took about 0.365 s.

Conditional probability $P(S = 1 \mid R = 1)$	
Ans.	$p = 0.92733$

Table 3: Results of Problem 3.

4 Problem 4

4.1 Question

The purpose of this Problem is to verify whether a redundancy approach to the problem of sending bits through a noisy channel is effective. In this experiment, the same bit is sent three times (and received three times). The three received bits are then fed into a three-input majority rule circuit, whose truth table is detailed below.

`R3`	`R2`	`R1`	`D`
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Table 4: The truth table for a three-input majority rule circuit.

D becomes the new "received bit." If D is the same as the sent bit S, then the send is successful. 100 000 bits were sent in this experiment. In this Problem, the task is to find the probability that a bit will be sent incorrectly.

4.2 Results

Out of the 100 000 sends, 99 455 were decoded correctly (the decoded value D was the same as the sent bit S), so 545 sends were decoded incorrectly. The probability of receiving an incorrect bit is $p = 545/100\,000 = 0.00545$.

It is worth noting that this probability of failure is significantly smaller than the probability of failure calculated in Problem 1 (0.04297), by one order of magnitude. This means that sending redundant bits and decoding them at the point of arrival is worth the extra effort, because there is a smaller chance that the sent bit will be received incorrectly.

Table 5 displays the results of this experiment. Note that this experiment took about 0.912 s.

Probability of error with enhanced transmission	
Ans.	$p = 0.00545$

Table 5: Results of Problem 4

5 Output of Source File

```
Problem 1 starting...
Problem 1 finished.
95703 successes out of 100 000 (probability of success: 0.95703)
Time taken: 0.5563292503356934 s

Problem 2 starting...
Problem 2 finished.
38636 successes out of 39876 (probability of success: 0.9689036011636072)
Time taken: 0.5448789596557617 s

Problem 3 starting...
Problem 3 finished.
38616 successes out of 41642 (probability of success: 0.9273329811248259)
Time taken: 0.36468935012817383 s

Problem 4 starting...
Problem 4 finished.
99455 successes out of 100 000 (probability of success: 0.99455)
Time taken: 0.9124472141265869 s
```