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E E 381 Section 12
Lab 3
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# Introduction

The purpose of this lab is to compare an experimental set of results with the predicted binomial distribution and Poisson distribution. A trial is defined as rolling three (unfair) die. A success is defined as rolling a 1 for the first die, a 2 for the second die, and a 3 for the third die. An experiment is defined as 1000 trials; the random variable $X$ is the result of this experiment. The probability vector is given as follows:

$$\rho = [0.2, 0.1, 0.15, 0.3, 0.2, 0.05]$$

where the $n$th element of $\rho$ is the probability of rolling an $n$ on the six-sided die: for example, the probability of rolling a 1 on this die is 0.2.

Our trials are Bernoulli trials, since one trial does not affect the next. The raw output of the code (amount of time the Problems took) is available at the end.

# 1 Problem 1

This problem has us run the experiment (as defined above) 10 000 times. This means that I generated a list of 10 000 $X$s. I then ploted these on a stem plot representing the PMF of $X$ below. Note that this simulation took about 75.7 s.

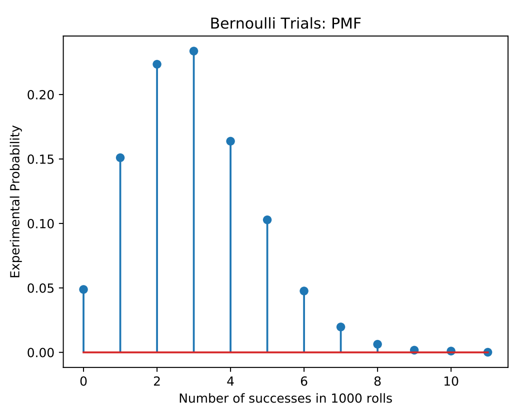



Figure 1: The Probability Mass Function of $X$.

# 2 Problem 2

## 2.1 Question

The purpose of this problem is to find the binomial distribution regarding this problem. The binomial distribution function is defined as follows:

$$P_b(X = x) = \binom{n}{x} p^x q^{n-x}$$

where $X$ is the number of successes in $n$ trials, $p$ is the probability of a success, and $q = 1 - p$. Plotting the results for $x = 0, 1, \ldots, 11$ should give us a plot that is equal (or very close to) our experiment, because our trials were Bernoulli trials.

## 2.2 Results

For our trial, a success is defined as rolling a 1, then a 2, then a 3. With the probability vector $\rho$ as defined above, this gives

$$\begin{aligned} p &= 0.2 \times 0.1 \times 0.15 = 0.003 \\ q &= 1 - p = 0.997 \\ n &= 1000 \end{aligned}$$

Thus, our function becomes

$$P_b(X = x) = \binom{1000}{x} \left(\frac{3}{1000}\right)^x \left(\frac{997}{1000}\right)^{1000-x}$$

Plotting $P_b$ gives the following plot.

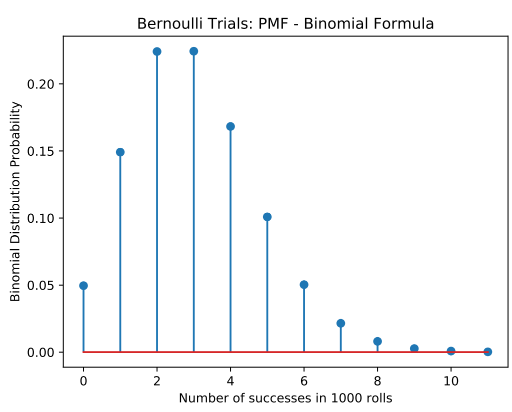



Figure 2: The Binomial Distribution of $X$.

As it is evident, Figures 1 and 2 are very similar; the difference in values is minimal and they are almost the same. The basic shape is the same as well. Note that this simulation took about 1.98 ms.

# 3 Problem 3

## 3.1 Question

The purpose of this problem is to see how well a Poisson distribution can approximate a binomial distribution. Spielgel el al. define the Poisson distribution to be a "good approximation" of the binomial distribution if the number of trials $n \geq 50$ and the probability $p$ is close to zero (or $np \leq 5$). In this example, $n = 1000 \geq 50$ and $np = 3 \leq 5$, so a Poisson distribution is a good approximation.

The function for the Poisson distribution is as follows:

$$P_P(X = x) = \begin{cases} \alpha^x \exp(-\alpha)/x! & \text{if } x \in \mathbb{Z}_0^+ \\ 0 & \text{otherwise} \end{cases}$$

where $X$ denotes the number of success in $n$ trials, $\alpha = np$, and $\mathbb{Z}_0^+ \equiv \mathbb{Z}^+ \cup \{0\}$ is the set of all positive integers and zero.

## 3.2 Results

Plugging in the values for $\alpha$, below is the Poisson distribution function that approximates the binomial distribution.

$$P_P(X = x) = \begin{cases} 3^x / e^3 x! & \text{if } x \in \mathbb{Z}_0^+ \\ 0 & \text{otherwise} \end{cases}$$

Plotting $P_P$ provides the following plot.

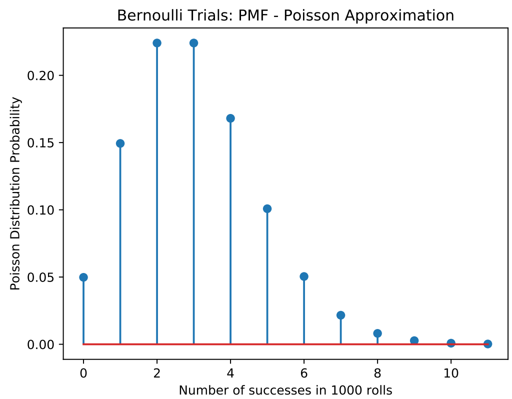
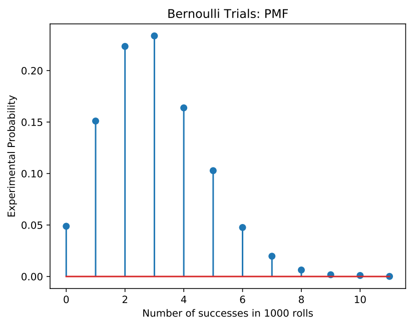


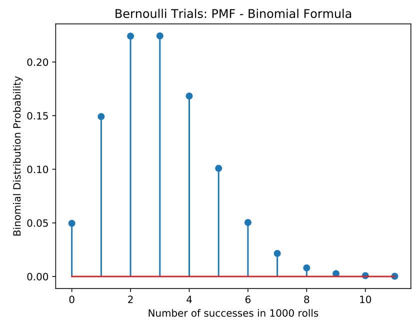

Figure 3: The Poisson Distribution of $X$.

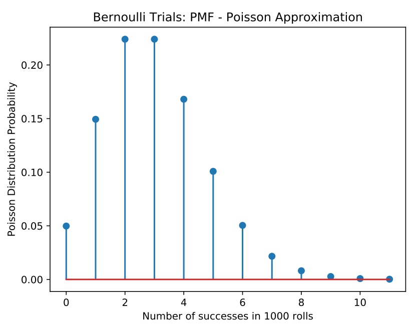
As is evident, the plot for this function is extremely close to that of the binomial distribution plot. Unless one were to compare the two plots side-by-side, it would be hard for him or her to differentiate between the two. This simulation took a fraction of the time as Problem 2, at around 494 µs; that's four times as fast; and it should be noted that on some occasions, the output time was 0 s. Therefore, the Poisson distribution is a good approximation to the binomial distribution for less resources.

# 4 Media

Below is the three plots for quick reference, as well as the raw output of the source file that produced these plots. Note that these graphics are vectors, so feel free to zoom in (if viewing the PDF file).




```
Problem 1 starting...
Problem 1 complete. Time taken: 75.67545175552368 s.

Problem 2 starting...
Problem 2 complete. Time taken: 0.0019783973693847656 s.

Problem 3 starting...
Problem 3 complete. Time taken: 0.0004940032958984375 s.
```

Figure 4: Output of `main.py`.