Final Project Due: Saturday May 15 before 10 AM

In this project, you will use MATLAB's filterDesigner (aka fdatool) toolbox to obtain a practical understanding of how poles and zeros of the system transfer function (z-Transform) affect the shape of the magnitude of the frequency response, including a single pole or zero, complex conjugate pairs, and multiple poles or zeros. DSP designers use filterDesigner toolbox to design filters to meet target filter specifications.

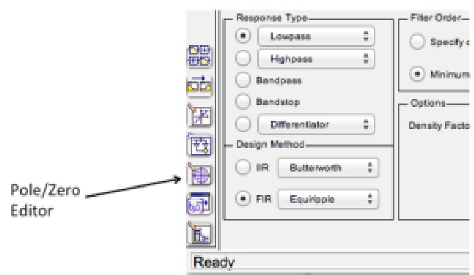
Here is also a review of the needed/useful units of time and frequency terms for Continuous Time (CT) and Discrete Time (DT) signals.

- CT time: t, in units of sec
- CT frequency: f, in units of Hz, or W in rad/sec, and W=2pf
- DT time: n, in units of samples
- DT frequency: w, in units of radians/sample, and w=2pf / fs where fs is in samples/sec (which results from rad/sec ÷ samples/sec)

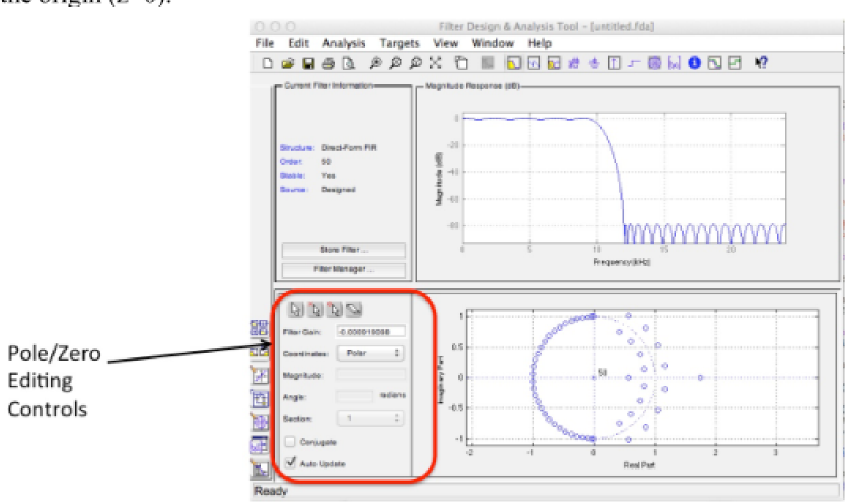
For your Project Report:
Perform all the following steps & **Explain**/provide your **answers to questions #1-11** in the PDF Project Report document (**Project_LastName_FirstName.PDF**) that you upload to the Beachboard.

I. Introduction to the `filterDesigner` GUI

1. Start `filterDesigner` by typing that name on the command line and then hitting the RETURN button. A GUI will pop up, and we will work through the GUI. There are 7 buttons that lie along the lower left edge of the GUI (shown below). One of the buttons (indicated with the arrow) allows you to edit the poles and zeros of a pole/zero plot. When poles/zeros are edited (added, removed or moved), the magnitude response plot will update automatically. This allows you to see in real-time, the effects of pole/zero placement.



2. Click on the *Pole/Zero Editor* button. This will show the pole/zero editor controls, the pole/zero plot for the default LPF (the lower plot in the GUI), and that filter's frequency response (magnitude in dB) (the upper plot in the GUI), as seen in the figure below. This default filter has 50 poles and zeros, and note that all of the poles are at the origin (z=0).



QUESTION #1: Is the default filter an FIR or an IIR filter? Why?

3. Explanation of the pole/zero editing controls:

   a. When the *Auto Update* block is checked, it allows real-time update of the magnitude response plot. Note that the more poles/zeros there are, the slower the update.

   b. The four buttons on the top of this section allow you to: (1) move a pole or zero; (2) add a pole; (3) add a zero; and/or (4) delete a pole or zero.

   c. *Gain* is a value (that you can change) that controls the max magnitude of the filter's frequency response. The value shown by default is the value that forces the max magnitude to be equal to a value of 1 (which is 0 dB). The gain is the constant factor by which the numerator/denominator polynomials are multiplied, as in the following example, where the gain is 10:

$$H(z) = \frac{10(z^2+1)}{z^2+0.25z+4}$$

   Note: to get actual *b*- and *a*-coefficients of *H*(*z*), you need to multiply the numerator polynomial by the gain. The gain factor is used so that the 1st coefficient in the numerator and denominator polynomials is 1.0.

   d. *Coordinates* is used to specify the exact location of a pole or zero, which you can enter in either polar or rectangular coordinates. For polar coordinates, pole/zero location is specified by magnitude and phase (in radians), and for rectangular coordinates by the real and imaginary parts. If you click on a pole or zero, that pole or zero's location is displayed. If you move a pole or zero, its location is updated as it is moved.

   e. When the *Conjugate* block is checked, whenever you add a complex pole or zero, `filterDesigner` will also add the complex conjugate pole/zero. <u>Remember</u>: in order to design a filter that contains real-valued coefficients, any pole or zero that is complex-valued <u>MUST</u> also have its complex conjugate as a pole or zero.

   f. The *Section* portion allows you to investigate poles/zeros of an IIR filter by which *section* the poles/zeros are a part of (we will see in later lessons that IIR filters are implemented in 2nd order sections). When you are looking at the pole/zero plot of an IIR filter and you choose a section number, the poles/zeros associate with that section will be highlighted in some way so that those poles/zeros are the ones that can be edited or evaluated.

II. Using the `filterDesigner` GUI

Now that you're an expert in the pole/zero editing controls, perform the following steps. In doing this, remember the gouge on how single pole/zero placement affects the shape of the frequency response:

-a <u>zero</u> ON the unit circle makes the |H(z)| = 0 at that frequency.
-a <u>zero</u> NEAR the unit circle causes a dip in |H(z)| at that frequency
-a <u>pole</u> ON the unit circle causes |H(z)| –> ∞
-a <u>pole</u> NEAR the unit circle causes a spike in |H(z)| at that frequency

4. You should be looking at the pole/zero plot for the default LPF, with the default sampling frequency of f_s=48 kHz (so the right edge of the magnitude response corresponds to $f_s/2$ = 24 kHz). First, let's switch the magnitude response to showing magnitude instead of magnitude in dB: select the *Analysis* drop down menu, then *Analysis Parameters ...*, and for *Magnitude Display* choose *Magnitude* instead of *Magnitude (dB)*.

5. Select the *Conjugate* block (recall—with this block checked, `filterDesigner` will process poles and zeros in complex conjugate pairs). On the pole/zero plot, click on the zero that is on the unit circle and closest to but not exceeding Ω = π rad/sample. It and its complex conjugate should change to large green circles,

indicated that they are selected. While carefully watching the frequency response, click on the *Delete Pole/Zero* button and then click on one of these green zeros (or you could just push the "delete" key on your keyboard). This deletes one pole and its complex conjugate pole.

QUESTION #2: What happened to the shape of the freq. response when you deleted these zeros? Why?

6. Click on the zero that is now closest to $\Omega = \pi$ rad/sample and delete it (which deletes two poles).

   QUESTION #3: What is the effect on the shape of the frequency response? Note: if the magnitude plot seems to go off the top of the plot's window, click on the plot once and it will update so that the full range of magnitude response can be seen.

7. Right click your mouse somewhere on the pole/zero plot and choose *Select All* to select all poles and zeros. Then right click ON one of the highlighted poles/zeros and choose *Delete current Poles and Zeros* to give you a clean slate to work with (note: you could also just push the "delete" key on your keyboard). Also note that you should be careful when deleting poles or zeros, as there is no "undo" button.

8. Uncheck the *Conjugate* button (so you can add a single pole or zero). Add a single zero at z=1 by pushing the Add Zero button, then clicking where the unit circle intersects with the positive z axis. Since you may not be actually able to click exactly on the z=1 position, you can specify the exact location by setting the *Magnitude* = 1.0 and the *Angle* = 0 radians. The max value of the magnitude response plot looks a little low (around 10^{-4} at $\omega = 0$), so replace the low value of gain by entering *Gain* = 1.0.

   QUESTION #4: What kind of filter is this? Does it make sense with the location of this zero?

9. Let's watch the magnitude response as the zero changes position around the unit circle. Keep the *Magnitude* = 1.0, and set the *Angle* = pi/4.

   QUESTION #5: What changed about the magnitude response when the zero move from z=1 to $z=e^{j\pi/4}$? Why do you think the frequency axes change from first 0 Hz to $f_s/2$ Hz, and now it is $-f_s/2$ to $f_s/2$? Hint: what can you say about the symmetry of the magnitude response about the origin?

10. Now click and hold the zero and move it around the unit circle, trying to keep as close to the unit circle as possible, and answer the following question.

    QUESTION #6: Do the changes you see in the shape of the frequency response match what you expect? When the zero is on the unit circle, does the magnitude response go to 0 at that frequency (realize that the relationship between digital frequency ω and frequency (f) in Hz is $\omega = 2\pi f / f_s$ so $f = \omega f_s/2\pi$)?

11. Delete the zero and now add a pole with *Magnitude* 1 and *Angle* 0.

    QUESTION #7: Does the frequency response go to ∞ at frequency = 0? Is this filter stable? How do you know? Grab the pole and move it around the unit circle. Do you see a spike in the frequency response at the associated frequency as it moves? (again, $\omega = 2\pi f / f_s$ so $f = \omega f_s/2\pi$)

12. Move this pole to *Magnitude* 1.1, *Angle* pi/3.

    QUESTION #8: Is the filter stable? How do you know? By looking at the frequency response only (not at the pole/zero plot), can you tell if the filter is stable?

13. Now move the pole to *Magnitude* 1/1.1, *Angle* pi/3.

    QUESTION #9: Is this filter stable? How can you tell? Can you tell by looking only at the frequency response? Delete this pole.

14. Now let's create a filter by adding poles and zeros to a blank pole/zero plot. Check the *Conjugate* block so that our filter will have real-valued coefficients. Here are the specs to design to:

    Specs:
    - The sampling frequency will be 192 kHz (note: this is DVD quality audio). Select the *Analysis* dropdown menu, then select *Sampling Frequency…* and change it to 192000, then click *Apply*.
    - The filter will be causal.
    - The filter will be stable.
    - The filter will have a spike in its freq. response at 60 kHz, with a peak magnitude of 100.

Let's investigate how these specs relate to pole/zero placement…

- Stable → all poles will be inside unit circle.
- Causal → there are at least as many zeros as poles.
- Spike at 60 kHz → there is a pole close to the unit circle (inside the unit circle) at $\Omega = 2\pi 60{,}000/192{,}000 = 0.6250\ \pi$ rad/sample.
- Depending on where you place the pole, you can either adjust the gain to get a magnitude=100 at 60 kHz or tweak the pole location closer to or farther from the unit circle until you get magnitude=100 at 60 kHz.

15. Add your pole pair (note: you'll be adding 2 poles since they will be complex conjugate pairs), and adjust their location or the gain until you get the peak magnitude of 100. If you adjust the pole position by dragging the pole, be sure to go back and specify the correct angle. Make it a causal filter by adding 2 zeros at the origin. By default, the gain may be negative…you can change it to a positive value if you edit the value in the *Gain* block…whether the gain is positive or negative will have no effect on the shape of the magnitude response.

    QUESTION #10: Based on the poles, zeros and gain you entered on your pole/zero plot, calculate what you think the equation for $H(z)$ should be from your design. Don't forget that since you know the location of the poles and zeros, you can use the MATLAB `conv` function to multiply the numerator terms together to find the numerator and denominator coefficients. Write your answer below in the form given, so the *a*- and *b*-coefficients are evident (note that in the equation below, the gain value is incorporated into the numerator coefficients):

$$H(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2}$$

16. Export your filter to the workspace by choosing *File→Export* from the *File* dropdown menu, and select *Export As*, and choose the type of export to be *Coefficients*. This will give you the numerator and denominator coefficients of the system function as variables *Num* (for numerator coefficients) and *Den* (for denominator coefficients). Or you could change the export names to what we've used: *b* and *a*.

    QUESTION #11: What are your exported coefficients (write the values down)? Do they match what you thought they would be?