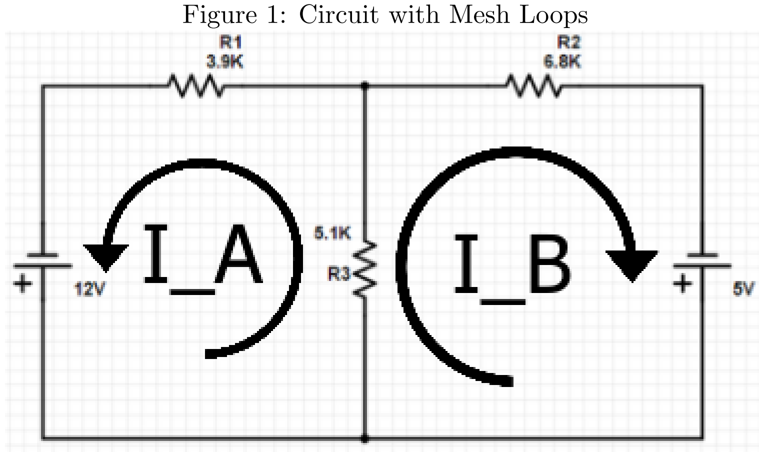
Rodrigo Becerril Ferreyra
CECS 211 Section 01
Lab 6
2019-10-22 to 2019-11-05

For this lab, we are tasked with solving a complex, multi- sourced circuit using mesh analysis techniques. Here is the circuit we will be solving[1]:

Figure 1: Circuit with Mesh Loops


1 Hand Calculations

As with any circuit, is is helpful to calculate all theoretical values by hand before modeling in a computer simulation or in real life, to have an idea of what values one might expect. For mesh analysis, this is done in several steps.

1.1 Assigning Mesh Currents

Mesh currents can be assigned arbitrarily; in this example, I've chosen to assign mesh current I_A in a counter-clockwise fashion and current I_B clockwise, as seen in Figure 1.

1.2 Writing KVL Equations

KVL tells us that the sum of voltage drops in any closed loop must be equal to zero. In mesh analysis, the KVL loops can be written with respect to the mesh loops. Starting from the top-right corner of loop I_A, its respective KVL expression is as follows:

[1] Note that this circuit has the mesh currents added and labeled; the mesh loops do not denote actual current.

$$
\begin{aligned}
0\,\mathrm{V} &= (3.9\,\mathrm{k\Omega} \cdot I_A) - 12\,\mathrm{V} + (5.1\,\mathrm{k\Omega} \cdot I_A) + (5.1\,\mathrm{k\Omega} \cdot I_B) \\
12\,\mathrm{V} &= (9.0\,\mathrm{k\Omega} \cdot I_A) + (5.1\,\mathrm{k\Omega} \cdot I_B)
\end{aligned}
$$

Similarly, the KVL equation for I_B is as follows:

$$
\begin{aligned}
0\,\mathrm{V} &= (5.1\,\mathrm{k\Omega} \cdot I_B) - 5\,\mathrm{V} + (5.1\,\mathrm{k\Omega} \cdot I_B) + (5.1\,\mathrm{k\Omega} \cdot I_A) \\
5\,\mathrm{V} &= (5.1\,\mathrm{k\Omega} \cdot I_A) + (11.9\,\mathrm{k\Omega} \cdot I_B)
\end{aligned}
$$

To solve this system, we will use substitution; more specifically, we will solve for I_B and use that to obtain I_A.

Using the last equation:

$$
\begin{aligned}
5\,\mathrm{V} &= 5.1\,\mathrm{k\Omega} \cdot I_A + 11.9\,\mathrm{k\Omega} \cdot I_B \\
5.1\,\mathrm{k\Omega} \cdot I_A &= 5\,\mathrm{V} - 11.9\,\mathrm{k\Omega} \cdot I_B \\
I_A &= 0.980\,\mathrm{mA} - 2.333 I_B
\end{aligned}
$$

Plugging this value into the other equation, we have:

$$
\begin{aligned}
9.0\,\mathrm{k\Omega} \cdot I_A + 5.1\,\mathrm{k\Omega} \cdot I_B &= 12\,\mathrm{V} \\
9.0\,\mathrm{k\Omega} \cdot (0.980\,\mathrm{mA} - 2.333 I_B) + 5.1\,\mathrm{k\Omega} \cdot I_B &= 12\,\mathrm{V} \\
8.82\,\mathrm{V} - 20.997\,\mathrm{k\Omega} \cdot I_B + 5.1\,\mathrm{k\Omega} \cdot I_B &= 12\,\mathrm{V} \\
-20.997\,\mathrm{k\Omega} \cdot I_B + 5.1\,\mathrm{k\Omega} \cdot I_B &= 3.18\,\mathrm{V} \\
-15.897\,\mathrm{k\Omega} \cdot I_B &= 3.18\,\mathrm{V} \\
I_B &= -0.200\,\mathrm{mA}
\end{aligned}
$$

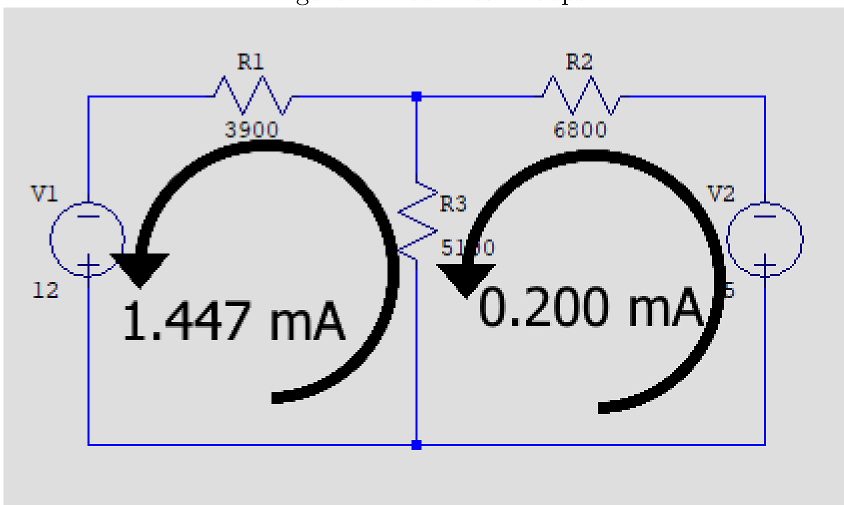
Using this value for I_B, we can substitute back into the equation to find the value for I_A.

$$
\begin{aligned}
I_A &= 0.980\,\mathrm{mA} - 2.333 I_B \\
I_A &= 0.980\,\mathrm{mA} - 2.333(-0.200\,\mathrm{mA}) \\
I_A &= 0.980\,\mathrm{mA} + 0.466\,\mathrm{mA} \\
I_A &= 1.447\,\mathrm{mA}
\end{aligned}
$$

Thus, our values for our mesh currents are $I_A = 1.447\,\mathrm{mA}$ and $I_B = -0.200\,\mathrm{mA}$. Note that I_B is negative, so the actual current goes the other way, in the counter-clockwise direction.

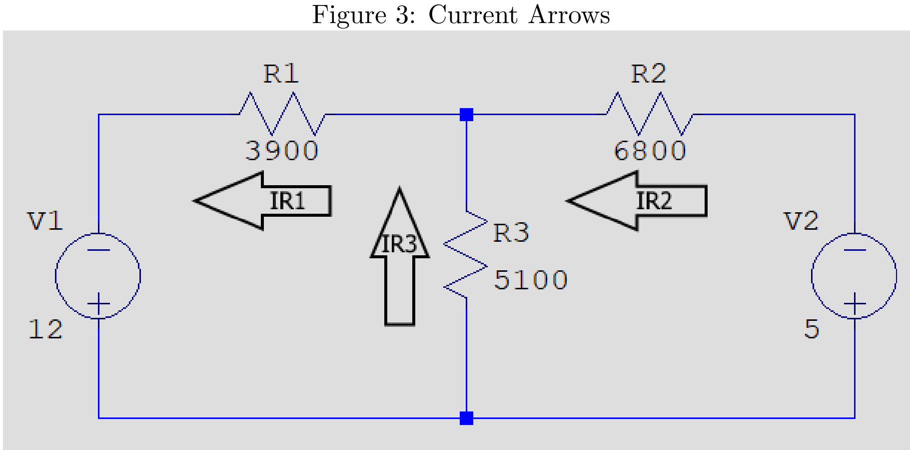
Our new mesh currents look like so:

Figure 2: New Mesh Loops



The current going through R_1 and R_2 are simple to calculate, as there is only one current going through each. However, I_{R_3} has two currents going through it. The current on the left will prevail, however, because its value is larger. The current arrows will look like so:

Figure 3: Current Arrows



The value of I_{R_1} is $1.447\,\text{mA}$ and the the value of I_{R_2} is $0.200\,\text{mA}$. The value of I_{R_3} is $1.447\,\text{mA} - 0.200\,\text{mA} = 1.247\,\text{mA}$.

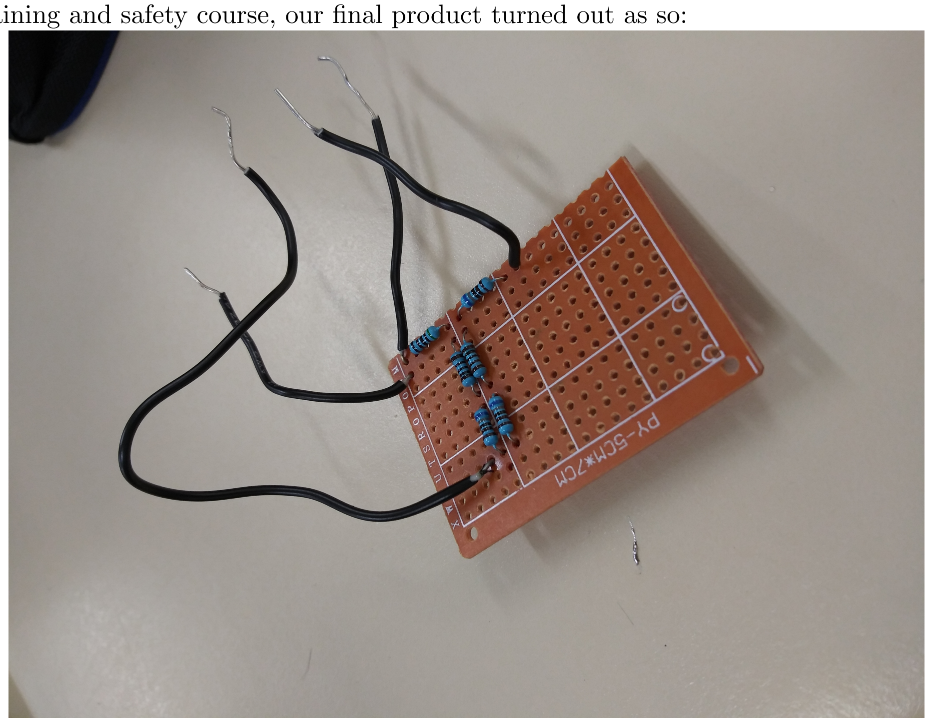
The voltage drop across the three resistors are as follows: $V_{R_1} = 5.64\,\mathrm{V}$, $V_{R_2} = 1.36\,\mathrm{V}$, and $V_{R_3} = 6.36\,\mathrm{V}$.

There are three KVL equations in this circuit. They are as follows (they all begin on the top-left corner and go clockwise):

$$\begin{aligned}
\text{R1 to R3:}\quad & -5.64\,\mathrm{V} - 6.36\,\mathrm{V} + 12\,\mathrm{V} && = 0 \\
\text{R2 to R3:}\quad & -1.36\,{-5}\,\mathrm{V} + 6.36\,\mathrm{V} && = 0 \\
\text{R1 to R2:}\quad & -5.64\,\mathrm{V} - 1.36\,\mathrm{V} - 5\,\mathrm{V} + 12\,\mathrm{V} && = 0
\end{aligned}$$

The next step was to recreate this circuit physically, on a breadboard. We then measured the following values for voltage: $V_{R_1} = 5.58\,\mathrm{V}$, $V_{R_2} = 1.31\,\mathrm{V}$, and $V_{R_3} = 6.35\,\mathrm{V}$. The values for current are as follows: $I_{R_1} = 1.42\,\mathrm{mA}$, $I_{R_2} = 0.19\,\mathrm{mA}$, and $I_{R_3} = 1.22\,\mathrm{mA}$.

The last step was to solder this circuit onto a perfboard. After a brief training and safety course, our final product turned out as so:



The connections (from left to right) are as follows: 12 V positive, 12 V ground, 5 V ground, and 5 V positive.