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CECS 211 Section 01
Lab 7
2019-11-19 to 2019-11-26

1 Introduction

In this lab, we explored the concept of an L/R circuit using an inductor in series with a resistor. L/R circuits have a property called the L/R time constant, which is simply the value of the inductor (in henries) divided by the value of the resistor (in ohms). This constant has units of seconds, because henries can be written as ohm-seconds:

$$\frac{1\,\mathrm{H}}{1\,\Omega} = \frac{1\,\Omega\,\mathrm{s}}{1\,\Omega} = 1\,\mathrm{s}$$

The amount of current going through the circuit as it is completed is given by the following exponential equation:

$$i_R(t) = \iota \left[1 - \exp\left(-\frac{t}{\tau}\right)\right] \tag{1}$$

where ι is the maximum current allowed through the circuit (given by Ohm's Law), t is time (in seconds), and τ is the L/R time constant given by $\tau = L/R$. As you can see, the time constant must have units of seconds in order for the argument of the exponential function to be unitless.

This equation ensures that after τ seconds, the current will be given by 0.632ι; after 2τ seconds, the current will be 0.865ι; after 3τ, $I = 0.950\iota$; after 4τ, the current will be $I = 0.982\iota$; and after 5τ, the current will be $0.993\iota \approx \iota$. If the inductor were not present, τ would be 0, and the circuit would instantly reach its maximum current ι.

The falling current when the circuit is suddenly broken is given by a similar equation:

$$i_F(t) = \iota \exp\left(-\frac{t}{\tau}\right) \tag{2}$$

This ensures that the current does not immediately fall.

2 Lab

In this particular lab, we are using a $1\,\mathrm{H}$ inductor and a $10\,\Omega$ resistor in series, which means our time constant τ is given by $\tau = L/R = 1\,\mathrm{H}/10\,\Omega = 0.1\,\mathrm{s}$. If the inductor weren't present, the total possible current in this circuit would be given by Ohm's Law: we are using a $10\,\mathrm{V}$ source with a $10\,\Omega$ resistor, which gives us $\iota = V/R = 1\,\mathrm{A}$.

To observe the change of current and how the inductor reacts with the change of current, we used a pulse function for our voltage source: for one second after running the simulation, the voltage source would be off. Then, at $t = 1\,\mathrm{s}$, the source would rise to $10\,\mathrm{V}$ with a negligible rise time ($1\,\mathrm{ns}$). It would be at $10\,\mathrm{V}$ for one second until $t = 2\,\mathrm{s}$, where it would fall back down to $0\,\mathrm{V}$ for $1\,\mathrm{ns}$. It would stay that way until $t = 3\,\mathrm{s}$, where the simulation would end.

2.1 Theoretical Values

When the voltage has its first rise to $10\,\mathrm{V}$ after one second, the current starts to rise as given by Equation 1: since $\tau = 0.1\,\mathrm{s}$, it will reach $0.632\iota = 0.632\,\mathrm{A}$ in $0.1\,\mathrm{s}$: this corresponds to time $t = 1\,\mathrm{s} + \tau = 1.1\,\mathrm{s}$, because the function goes into effect at $t = 1\,\mathrm{s}$, when the voltage rises. At time $t = 1\,\mathrm{s} + 2\tau = 1.2\,\mathrm{s}$, the current is $0.865\iota = 0.865\,\mathrm{A}$. The theoretical results are displayed in the following table. (t_t stands for "theoretical time", $i_R(t)$ is Equation 1, $i_F(t)$ is Equation 2, and ι represents the maximum current of $1\,\mathrm{A}$.) This chart encompasses both rise and fall times.

Rise

t_t	$i_R(-t_t/\tau)/\iota$	$i_R(-t_t/\tau)$
$\tau = 0.1\,\mathrm{s}$	0.632	$0.632\,\mathrm{A}$
$2\tau = 0.2\,\mathrm{s}$	0.865	$0.865\,\mathrm{A}$
$3\tau = 0.3\,\mathrm{s}$	0.950	$0.950\,\mathrm{A}$
$4\tau = 0.4\,\mathrm{s}$	0.982	$0.982\,\mathrm{A}$
$5\tau = 0.5\,\mathrm{s}$	$0.993 \approx 1.000$	$0.993\,\mathrm{A} \approx 1.000\,\mathrm{A}$

Fall

t_t	$i_F(-t_t/\tau)/\iota$	$i_F(-t_t/\tau)/\iota$
$\tau = 0.1\,\mathrm{s}$	0.368	$0.368\,\mathrm{A}$
$2\tau = 0.2\,\mathrm{s}$	0.135	$0.135\,\mathrm{A}$
$3\tau = 0.3\,\mathrm{s}$	0.050	$0.050\,\mathrm{A}$
$4\tau = 0.4\,\mathrm{s}$	0.018	$0.018\,\mathrm{A}$
$5\tau = 0.5\,\mathrm{s}$	$0.007 \approx 0.000$	$0.007\,\mathrm{A} \approx 0.000\,\mathrm{A}$

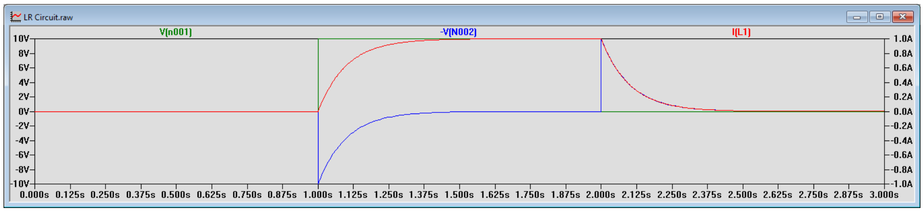
The maximum energy stored by the inductor is given by $E = LI^2$; this is because the units cancel out in such a way where the they give Joules:

$$1\,\mathrm{H} \cdot 1\,\mathrm{A}^2 = 1\,\mathrm{J\,s^2/C^2} \cdot 1\,\mathrm{C^2/s^2} = 1\,\mathrm{J}$$

The maximum energy stored is $1\,\mathrm{H}$ times $1\,\mathrm{A}^2$, which is $1\,\mathrm{J}$, or equivalently $1/3600\,\mathrm{W\,h}$.

2.2 Measured Values

The following graph is the graph of the voltage and current.



V(001) represents the voltage difference output of the voltage source. V(002) represents (but is not equivalent to) the back emf produced by the 1 H inductor. I(L1) represents the current going through the inductor (and therefore the current going through the whole series circuit).

LTSpice lets the user measure current accurately at different points in time. However, it can be difficult to select an exact point in time (for example, exactly 1.1 s), so any measured values have an uncertainty of ± 0.010 s. The following updated table displays measured values along with the theoretical values of the previous table. (t_t stands for "theoretical time", t_m stands for "measured time", $i_R(t)$ is Equation 1 and stands for the instantaneous rising current, $i_F(t)$ is Equation 2 and stands for the instantaneous falling current, I represents the actual measured current with an uncertainty of $\pm 5 \times 10^{-5}$ A, and RE $= \mathrm{abs}(i - I)/i$ represents relative error. All t_m values are adjusted for rise and fall times of voltage.)

Rise

t_t	t_m	$i_R(-t_t/\tau)$	I	RE
$\tau = 0.1$ s	0.104 27 s	0.632 A	0.648 200 A	2.544%
$2\tau = 0.2$ s	0.206 16 s	0.865 A	0.872 719 A	0.9315%
$3\tau = 0.3$ s	0.300 95 s	0.950 A	0.950 670 A	0.048 10%
$4\tau = 0.4$ s	0.400 47 s	0.982 A	0.981 786 A	0.010 35%
$5\tau = 0.5$ s	0.502 37 s	0.993 A	0.993 410 A	0.014 90%

Fall

t_t	t_m	$i_F(-t_t/\tau)$	I	RE
$\tau = 0.1$ s	0.094 79 s	0.368 A	0.387 582 A	5.356%
$2\tau = 0.2$ s	0.201 42 s	0.135 A	0.133 644 A	1.250%
$3\tau = 0.3$ s	0.300 95 s	0.050 A	0.049 213 A	1.153%
$4\tau = 0.4$ s	0.402 84 s	0.018 A	0.017 817 5 A	2.720%
$5\tau = 0.5$ s	0.502 37 s	0.007 A	0.006 566 88 A	2.539%

The values for relative error are decreasing for rise times but stay at about 2.5%; this is likely because the values for decreasing current are small.