Name: ____

CECS 271 Introduction to Numerical Methods

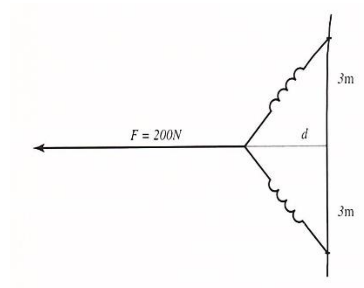
Exam #1 Take Home 9/30/2020 Due: 2 OCT 2020 (5pm)

Using a Matlab script file, develop the code and plots for the following problems. Publish your results as a HTML file, and open it as a DOCX file and then save as a PDF file after editing it for best presentation by starting each problem on a separate page of the pdf file. Drop the file into the Beachboard dropbox by the closing time shown on the dropbox. Title each plot with a figure number the same as the problem and label the axes of all plots.

Note: For each problem use a tolerance of 1e-4 and 25 as the maximum number of iterations.

1. The flow rate in a pipe system connecting two reservoirs (at different surface elevations) depends on the characteristics of the pump, the roughness of the pipe, the length and diameter of the pipe, and the specific gravity of the fluid. For an 800-ft section of 6-inch pipe connecting two reservoirs (with a 5-ft differential in elevation) containing oil of specific gravity of 0.8 with a 6-hp pump, the equation for the flow rate Q is
$$f(Q) = 12Q^3 + 5Q - 40 = 0$$
Using the fixed-point method with starting point x=0, find the real root in the interval $0 \le Q \le 2$. Display the values (Q_k, $g(Q_k)$) for k=1,2,… as you iterate to the root. In Figure 1, as one plot over Q in [0,2], plot the function f(Q) in blue, the function g(Q) in green, the line y=Q in black, and the location of the root identified by a red star (*). Put the value of the root found in the title of the plot.

2. To determine the displacement d of a spring of stiffness 400 N/m and unstretched length of 6 m when a force of 200 N/m is applied, as shown in this figure, two expressions are found for the tension T in each half of the spring. First, T is half the horizontal component of the applied force:
$$T_1 = 100\sqrt{9 + d^2}\,/d$$
Second, T is the product of the spring constant and the amount by which the spring is stretched:
$$T_2 = 400(\sqrt{9 + d^2} - 3)$$
Find d by equating the two equations for T. Use bisection to find the root of T_1-T_2=0 using the initial points a=1 and b=3. Display the values of a, b and c for each iteration as you iterate to a solution. In Figure 2, plot the function f(d) =T_1-T_2 in blue over d in the interval [1,3] and the location of the root identified by a red star (*). Include the value of the root in the plot title. (Note: divide out common factor of 100 to reduce the vertical scale of plot.)



3. The van der Waals equation of state, a simple extension of the ideal gas law discovered in 1873 by the Dutch physicist Johannes Diderik van der Waals is
$$\left(P + \frac{m^2 a}{V^2}\right)(V - mb) = nRT$$
where the constants a and b, characteristic of the gas, are determined experimentally. For P in atmospheres, V in liters, n in moles, T in kelvins. m is the volume in liters occupied by n moles of a gas and R is approximately 0.0820 liter-atm/(deg-mole). The volume of 1 mole of a perfect gas at standard temperature and pressure (STP: 1 atm and 273 °K) is 22.415 liters. At STP, find the volume occupied by n=1 mole of oxygen (O_2) with a=1.36 and b=0.0318. Use the Newton-Raphson method starting at x=0.4. In Figure 3, plot the function in blue and its derivative in green over [0.2,0.6] and the location of the root identified by a red star (*). Include the value of the root in the plot title.

4. The position of a ball thrown vertically upward with a given initial velocity v_0 and initial position x_0, subject to air resistance proportional to its velocity is given by

$$x(t) = \rho^{-1}(v_0 + v_r)(1 - e^{-\rho t}) - v_r t + x_0$$

where ρ is the drag coefficient, g is the gravitational constant, and $v_r = g/\rho$ is the terminal velocity. Using the secant method, find when the ball hits the ground if x_0=5m, v_0=20m/s, ρ=0.35, and g=9.8m/s^2. On Figure 4, plot the trajectory x(t) of the ball in blue over the time interval [0,5] , and the location $x(t_0)$ when the ball hits the ground at $t=t_0$ identified by a red star (*). Include the time t_0 and distance $x(t_0)$ in the title of the plot.

5. According to Archimedes, if a spherical solid that is lighter than water is placed in a tub of water, the solid will sink only to a depth where the weight of the water it displaces equals its own weight. The weight of water displaced by the submerged segment of the sphere is $W_s = \pi x(3r^2 + x^2)/6$. The weight of the cork sphere is $W_c = \frac{4\pi}{3} R^3 \rho$. If the radius of the sphere is R=1 cm and the specific gravity of cork is ρ=0.33, to what depth x does the cork sphere sink? Choose any one of the methods of Chapter 4 to find and display the solution for x.

6.