CECS 271 Numerical Methods using Matlab

TASK 6.1 Addendum   Due: 11/05/20 (7pm)

1. Write a MATLAB function of the form x=gauss(A,b) to solve an NxN linear system Ax=b by Gaussian elimination in its simplest form and using back substitution to find the solution vector x.
Algorithm: INPUT: Matrix A and constant vector b
OUTPUT: solution vector x

Step 1: Form augmented matrix E=[A,b] and for i=1,2,...N-1, repeat Steps 2 to 4
Step 2: Let p be the smallest integer with $i \le p \le N$ and a_{pi} != 0 and if no integer found, output 'NO UNIQUE SOLUTIONS' and stop.
Step 3: If p!=i, then interchange rows p and i of E
Step 4: For j=i+1, i+2,...N repeat Steps 5 and 6
Step 5: Set $m_{ji} = a_{ji}/a_{ii}$ of E
Step 6: Replace row j of augmented matrix E_j with $E_j - m_{ji}E_i$
Step 7: If a_{NN} of E = 0, output 'NO UNIQUE SOLUTIONS' and stop.
Step 8: Set $x_N = b_N/a_{NN}$ of the now upper triangular augmented matrix E
Step 9: For i = N-1,N-2,..1 set $x_i = \{b_i - (a_{i\,(i+1)}\, x_{i+1} + a_{i\,(i+2)} x_{i+2} + \ldots + a_{i\,N}\, x_N)/a_{ii}\}$
Step 10: Output the solution vector $x=[x_1,x_2,\ldots x_N]$ and stop.

2. Using your program from #1, solve these systems of equations:

(a)
x1 – 2 x2 = -2
3 x1 + 2 x2 = 1

(b)
4 x1 – x2 + x3 = 8
2 x1 + 5 x2 + 2 x3 = 3
x1 + 2 x2 +4 x3 = 11

(c)
x1 + x2 + x4 = 2
2 x1 + x2 - x3 + x4 = 1
4 x1 - x2 - 2 x3 + 2 x4 = 0
3 x1 - x2 - x3 + 2 X4 = -3

3. Given this system of linear equations:

x1 – x2 + S x3 = -2
- x1 + 2 x2 - S x3 = 3
S x1 + x2 + x3 = 2

(a) find a value of S for which the system has no solutions
(b) find a value of S for which the system has an infinite number of solutions
(c) find the unique solution for S=3.

4. Solve this system of linear equations using your Gaussian Elimination Algorithm of #1:

x1 + 1/2 x2 + 1/3 x3 + 1/4 x4 = 1/6

1/2 x1 + 1/3 x2 + 1/4 x3 + 1/5 x4 = 1/7

1/3 x1 + 1/4 x2 + 1/5 x3 + 1/6 x4 = 1/8

1/4 x1 + 1/5 x2 + 1/6 x3 + 1/7 x4 = 1/9

5. Suppose that in a biological system there are n species of animals and m sources of food. Let x_j be the population of the jth species for each j=1,2...n. Let b_i be the available daily supply of the ith food for each i=1,2,...,m and let a_{ij} be the amount of the ith food consumed on the average by a member of the jth species. A linear system represents an equilibrium where there is just enough daily supply of food to meet the daily consumption of each species:

$$\begin{aligned}
a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n &= b_1 \\
a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n &= b_2 \\
\dots \quad \dots \quad \dots \quad &\dots \\
a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n &= b_m
\end{aligned}$$

(a) Suppose $A = \begin{matrix} 1 & 2 & 0 & 3 \\ 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{matrix}$ x =[1000 500 350 400] and b = [3500 2700 900].

Is there sufficient food to meet the average daily consumption by the species?

(b) What is the maximum number of animals of each species that could individually be added to the system with the food supply remaining adequate to meet the consumption?

(c) If species 1 became extinct, how much of an individual increase of each of the remaining species could be supported by the food supply?