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E E 381 Section 12
Lab 6
12 December 2020

Introduction

The purpose of this lab is to put our knowledge of Markov chains into practice. Markov chains are used to represent sequences.

1 Problem 1

In this problem, a Markov chain was experimentally tested to see if it converges, and it was compared with the expected, calculated values. Figure 1 shows the expected progression of the Markov chain (probability of the given Markov chain vs the iteration) compared to the acquired values obtained by running the Markov chain 10 000 times. Figure 2 shows one single run of a Markov chain, plotted as state number vs iteration.

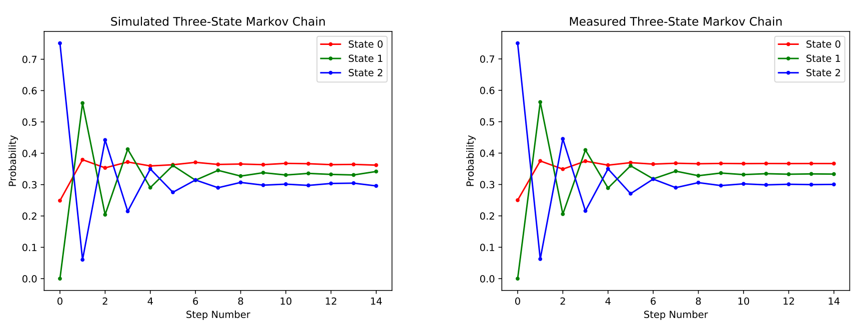

Figure 1: Expected vs actual Markov Chain Progression.

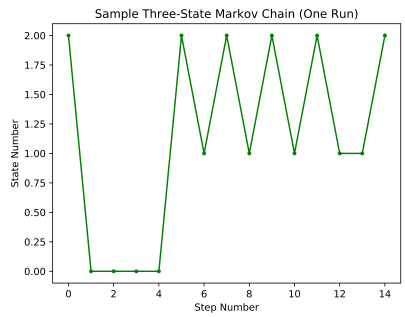

Figure 2: A single Markov chain.

2 Problem 2

In this problem, we were asked to use a simplified version of Google's PageRank algorithm to rank five webpages given a diagram of links. This was done twice: once where each of the five pages have the same probability of being first, and once where Page E was the initial state. In both cases, the pages were ranked as follows:

1. B
2. A
3. C
4. E
5. D

This was regardless of the initial state vector. The following matrix shows the Markov state probability table.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 \end{bmatrix}$$

Below are the plots showing the progression of the Markov chain.

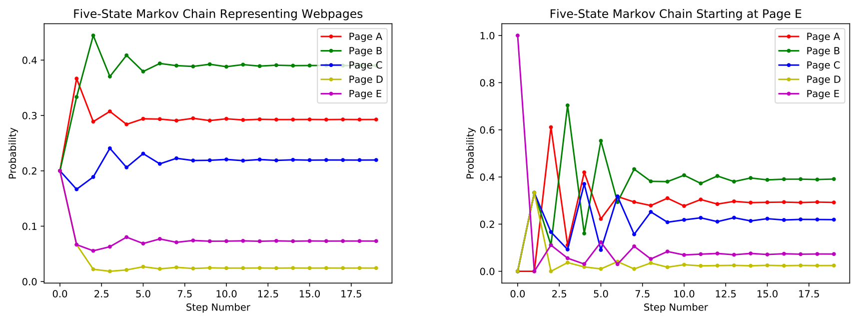

Figure 3: Markov chain progressions.

Initial Probability Vector: v_1		
Rank	Page	Probability Vector
1	B	1/5
2	A	1/5
3	C	1/5
4	E	1/5
5	D	1/5

Initial Probability Vector: v_1		
Rank	Page	Probability Vector
1	B	0
2	A	0
3	C	0
4	E	0
5	D	1

Table 1: Probability vectors and page rankings.

3 Problem 3

The purpose of this problem was to implement a “drunkard’s walk”-type Markov chain where state i only leads to state $i+1$ and $i-1$, and the first and last states are absorbing states, meaning that they are “inescapable.” This was implemented with five states. Two “walks,” one where the end state is 0 and one where the end state is 4, are posted below. Note that the states never skip; i.e. they only progress one state at a time.

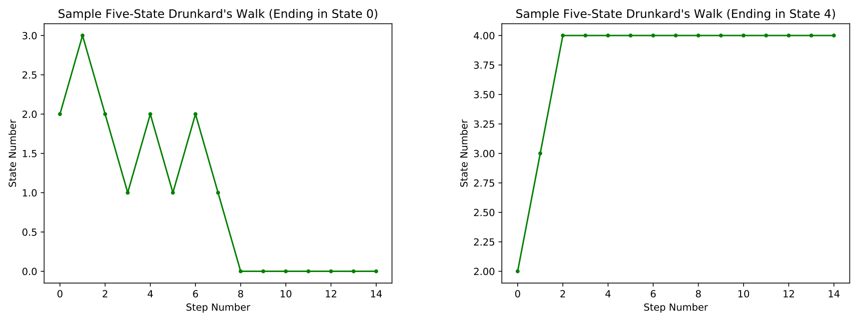

Figure 4: Two walks.

4 Problem 4

Figure 5 is a probability chart consisting of 10 000 walks. Note that State 1 and State 3 are occupying the same space.

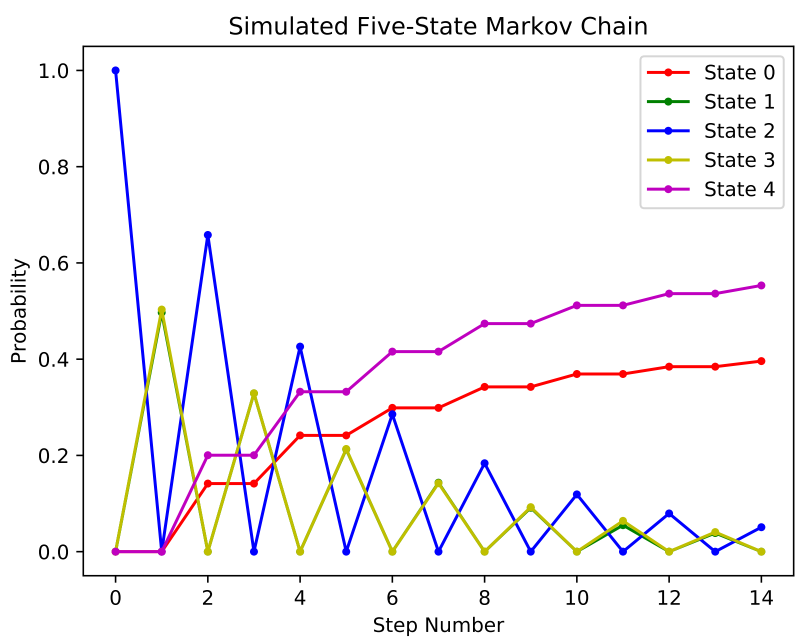

Figure 5: 10 000 combined walks.

Absorbtion Probabilities			
b_{20}	0.4	b_{24}	0.5