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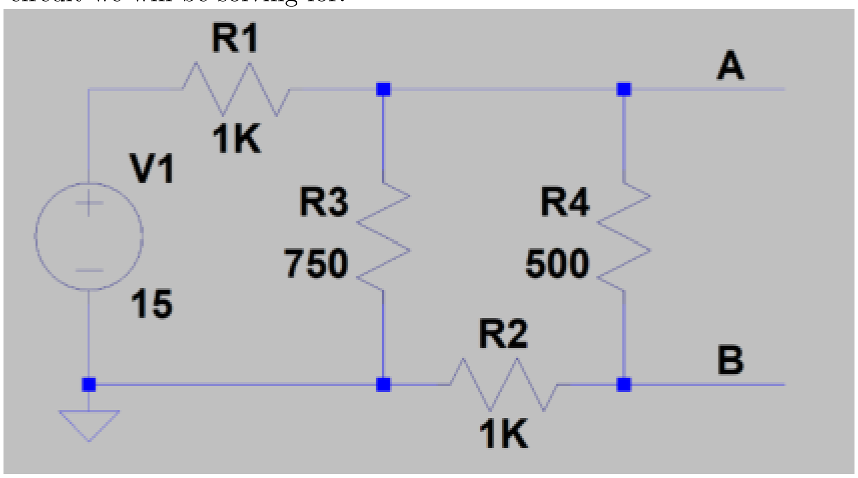
CECS 211 Section 01
Lab 4
2019-10-10–2019-10-22

In this lab, we are tasked with finding the Thevenin and Norton equivalent circuits for a certain given circuit, and verifying if they act the same. Here is the circuit we will be solving for:



1 Finding the Thevenin-Equivalent Circuit

The Thevenin-equivalent circuit has two parts: the Thevenin volatage and the Thevenin resistance. Each must be found separately.

1.1 Thevenin Voltage

The Thevenin voltage is found by figuring out the voltage drop between terminals A and B. To do this, we need to find the total current, and to do this we need to find the equivalent resistance of the circuit.

$$\begin{aligned} R_{\text{net}} &= (R_4 + R_2) \parallel R_3 + R_1 \\ &= (500\,\Omega + 1000\,\Omega) \parallel 750\,\Omega + 1000\,\Omega \\ &= 1500\,\Omega \parallel 750\,\Omega + 1000\,\Omega \\ &= 500\,\Omega + 1000\,\Omega \\ &= 1.5\,\text{k}\Omega \end{aligned}$$

Since Ohm's Law says that $I = V/R$, the total current going through this circuit is

$$\begin{aligned} I_{\text{net}} &= V_1/R_{\text{net}} \\ &= 15\,\text{V}/1.5\,\text{k}\Omega \\ &= 10\,\text{mA} \end{aligned}$$

Knowing the total current going through the circuit, we can calculate the voltage drops across the resistors. The voltage drop across R_1 is $10\,\text{mA} \cdot 1\,\text{k}\Omega = 10\,\text{V}$. That means that the voltage drop across R_3 is $5\,\text{V}$. $V_{R_2} + V_{R_4}$ must equal $5\,\text{V}$ because the two combined are in parallel with R_3.

The current going across R_3 is $5\,\text{V} \cdot 0.75\,\text{k}\Omega = 6.667\,\text{mA}$. By KCL, the current going through both R_4 and R_2 is $I_{\text{net}} - I_{R_3} = 10\,\text{mA} - 6.667\,\text{mA} = 3.333\,\text{mA}$; thus, the voltage going across R_4 and also the voltage from A to B is $0.5\,\text{k}\Omega \cdot 3.333\,\text{mA} = 1.667\,\text{V}$

With this, we can see our Thevenin-equivalent voltage is $V_{\text{Th}} = 1.667\,\text{V}$.

1.2 Thevenin Resistance

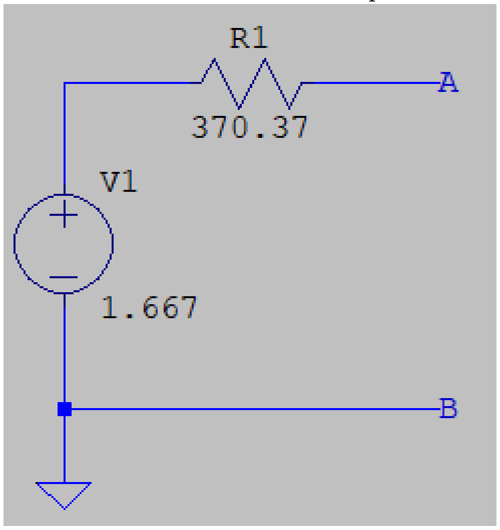
The Thevenin resistance is the resistance as seen from terminals A and B with all voltage sources shorted out. In this example, voltage source V_1 is shorted out and replaced with a wire. This leaves R_1 in parallel with R_3, and that is in series with R_2, and the whole thing is in parallel with R_4:

$$\begin{aligned} R_{\text{Th}} &= (R_1 \parallel R_3 + R_2) \parallel R_4 \\ &= (1000\,\Omega \parallel 750\,\Omega + 1000\,\Omega) \parallel 500\,\Omega \\ &= 10\,000/7\,\Omega \parallel 500\,\Omega \\ &= 10\,000/27\,\Omega \approx 370.37\,\Omega \end{aligned}$$

Our Thevenin-equivalent resistance is $370.37\,\Omega$.

1.3 Schematic

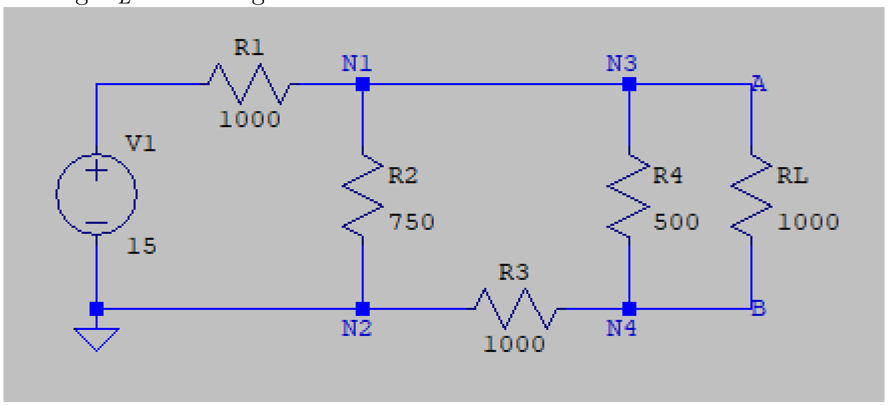
The schematic for this Thevenin-equivalent circuit is as follows:



2 Adding a Load Resistor to the Original Circuit

To verify that the original circuit and the Thevenin-equivalent circuit are indeed equivalent, we will add a resistor of value $1000\,\Omega$ (named R_L) connected to terminals A and B. If the two circuits are indeed equivalent, then both the voltage across R_L and the current going through R_L should be equivalent across both circuits.

Adding R_L to the original circuit looks as follows:



Adding this new load resistor transforms the original circuit into a completely new one. In order to find the voltage across and current going through this new circuit, we will need to find the new equivalent resistance. Here, R_L and R_4 are in parallel, and combined are in series with R_3. All of the previous are in parallel with R_2 which is in series with R_1. After we get the equivalent resistance of the circuit, we can obtain the total current running through it.

$$\begin{aligned}
R_{\text{net}} &= (R_L \parallel R_4 + R_3) \parallel R_2 + R_1 \\
&= (1000\,\Omega \parallel 500\,\Omega + 1000\,\Omega) \parallel 750\,\Omega + 1000\,\Omega \\
&= 4000/3\,\Omega \parallel 750\,\Omega + 1000\,\Omega \\
&= 1480\,\Omega
\end{aligned}$$

$$\begin{aligned}
I_{\text{net}} &= V_1/R_{\text{net}} = 15\,\text{V}/1480\,\Omega \\
&= 3/296\,\text{A} \approx 10.135\,\text{mA}
\end{aligned}$$

The voltage across R_1 is $R_1 \cdot I_{\text{net}} = 1\,\text{k}\Omega \cdot 10.135\,\text{mA} = 10.135\,\text{V}$. This means that the voltage at Node 1 (N_1) is $15\,\text{V} - 10.135\,\text{V} = 4.865\,\text{V}$ (with respect to ground) and the voltage at $N_2 = 0\,\text{V}$; thus, the voltage across R_2 is $4.865\,\text{V}$. The current going through R_2 is given by $V_{R_2}/R_2 = 4.865\,\text{V}/0.750\,\text{k}\Omega \approx 6.487\,\text{mA}$.

Because voltage is the same for every element in parallel, and the combined resistance ($R_L \parallel R_4 + R_3$) is all in parallel with R_2, that means that the voltage of the combined resistance is $4.865\,\text{V}$. The sum of the voltage of $R_4 = R_L$ and R_3 must be equal to $4.865\,\text{V}$ according to Kirchhoff's Voltage Law. The voltage of R_3 can be found using Kirchhoff's Current Law: the sum of currents entering any node must equal the sum of currents exiting the node. In this case, $I_{\text{net}} = 10.135\,\text{mA}$ is being summed at N_2 which is connected directly to the voltage source's negative terminal. This means that the current leaving N_2 is equal to $I_{\text{net}} = 10.135\,\text{mA}$. We know that the current going though R_2 is $6.487\,\text{mA}$. This means that the current going through R_3 must be

$$\begin{aligned}
I_{R_3} &= I_{\text{net}} - I_{R_2} \\
&= 10.135\,\text{mA} - 6.487\,\text{mA} \\
&= 3.648\,\text{mA}
\end{aligned}$$

With this value, we can find the voltage for R_3: $V_{R_3} = 1\,\text{k}\Omega \cdot 3.648\,\text{mA} = 3.648\,\text{V}$. This means that

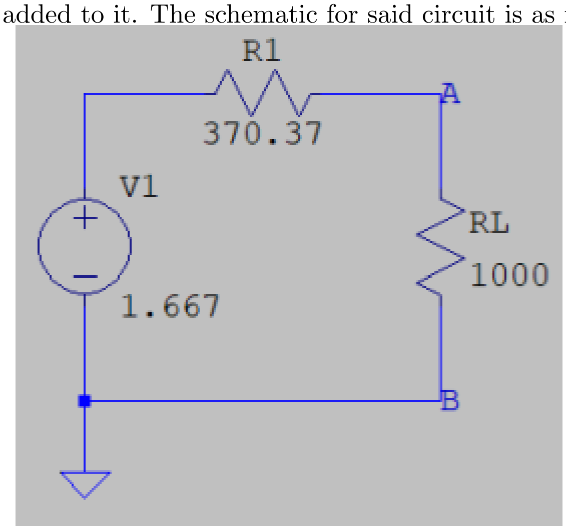
$$\begin{aligned}
V_{R_4} = V_{R_L} &= V_{R_2} - V_{R_3} \\
&= 4.865\,\text{V} - 3.648\,\text{V} \\
&= 1.217\,\text{V}
\end{aligned}$$

The voltage across R_L is $1.217\,\text{V}$.

From this value of the voltage, the current can also be calculated as follows: $I_{R_L} = V_{R_L}/R_L = 1.217\,\text{V}/1000\,\Omega = 1.217\,\text{mA}$.

3 Adding a Load Resistor to the Thevenin-Equivalent Circuit

We will now verify that the values for the I_{R_L} and V_{R_L} match for both circuits by solving the new Thevenin- equivalent circuit when a load resistance of $1000\,\Omega$ is added to it. The schematic for said circuit is as follows:



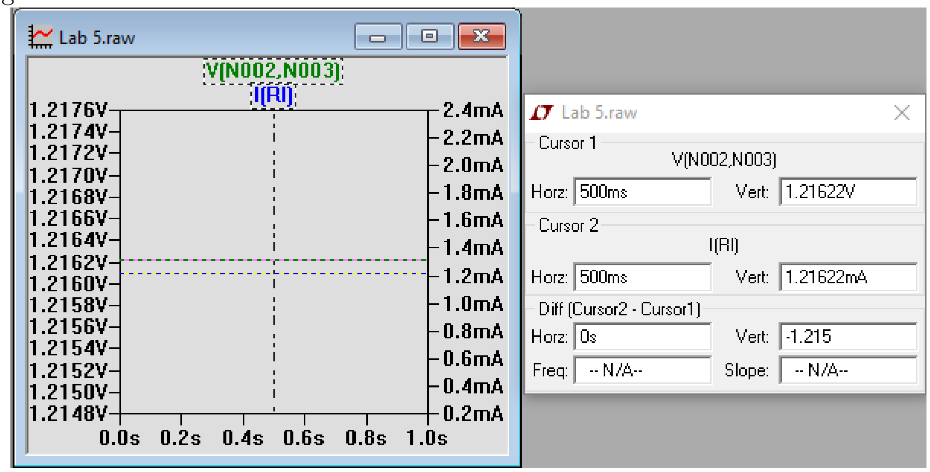
The total resistance of for this circuit is $R_{\text{net}} = R_1 + R_L = 370.37\,\Omega + 1000\,\Omega = 1370.37\,\Omega$. The since this is a series circuit, the total current going though the circuit is the same as the current going through all components, including R_L, and that value is $R_L = R_{\text{net}} = 1.667\,\text{V}/1370.37\,\Omega = 1.216\,\text{mA}$. The voltage across the load resistor is $V_{R_L} = 1.216\,\text{mA} \cdot 1\,\text{k}\Omega = 1.216\,\text{V}$. It clear to see that these values match the values for the original circuit:

	Original	Thevenin
Voltage	1.217 V	1.216 V
Current	1.216 mA	1.216 mA

There was only an error of $\pm 0.001\,\text{V}$ and $\pm 0.001\,\text{mA}$ for the calculations of voltage and current, respectively; this is only due to rounding. These two circuits are indeed equivalent.

4 LTspice Simulation of Original Circuit

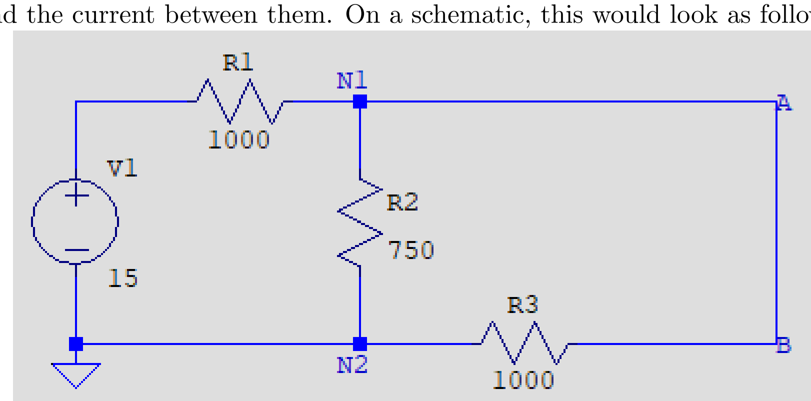
The original circuit can be simulated in LTspice to acquire accurate measured values for our load resistor with uncertainties of ±5 µV and ±5 nA. The following are the waveform and circuit for both voltage and current of R_L of the original circuit:



In the waveform, the top line is the measured voltage, and the bottom line is the measured current.

5 Finding the Norton-Equivalent Circuit

Another form of circuit simplification is Norton simplification. Instead of acquiring an equivalent voltage, Norton simplification finds an equivalent current. The way to find this equivalent current is to short out terminals A and B, and find the current between them. On a schematic, this would look as follows[1]:



[1]Note that when shorting out terminals A and B, the current bypasses resistor R_4, and it effectively disappears from the circuit, as reflected in the above schematic.

5.1 Norton Current

The current running through terminals A and B is the same current running through resistor R_3, so finding the current through R_3 will give us our Norton-equivalent current.

First, as always, it is necessary to find the equivalent resistance of the new circuit. R_3 is connected in parallel with R_2, and the two resistors are connected in series with R_1:

$$\begin{aligned} R_{\text{net}} &= R_3 \parallel R_2 + R_1 \\ &= 1000\,\Omega \parallel 750\,\Omega + 1000\,\Omega \\ &= 0.428\,\text{k}\Omega + 1000\,\Omega \\ &= 1.428\,\text{k}\Omega \end{aligned}$$

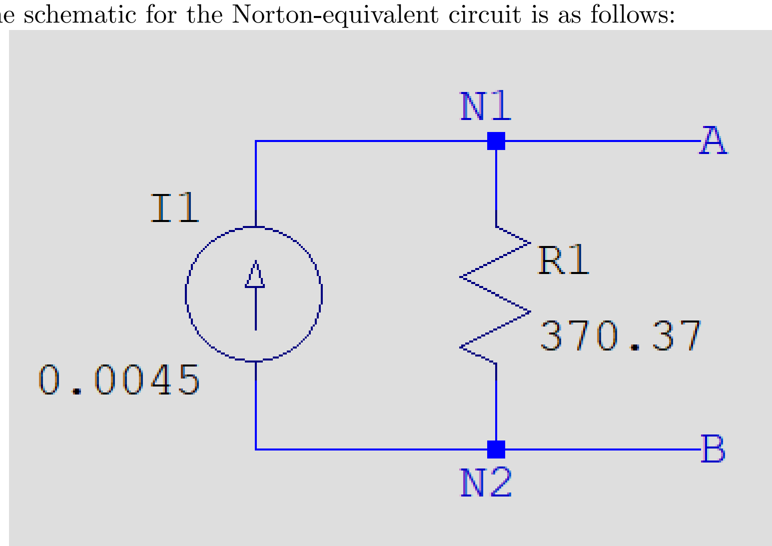
The total current running though the circuit is $15\,\text{V}/1.428\,\text{k}\Omega = 10.50\,\text{mA}$. The voltage across R_1 is $10.50\,\text{mA} \cdot 1\,\text{k}\Omega = 10.50\,\text{V}$. This means that the voltage drop across R_2 and R_3 is $4.5\,\text{V}$. Therefore, the current going through R_3 and between terminals A and B is $4.5\,\text{V}/1\,\text{k}\Omega = 4.5\,\text{mA}$. This is our Norton-equivalent current.

5.2 Norton Resistance

Calculating the Norton-equivalent resistance is the same process as calculating the Thevenin- equivalent resistance, so we can reuse the value calculated in Section 1.2, which is $370.37\,\Omega$.

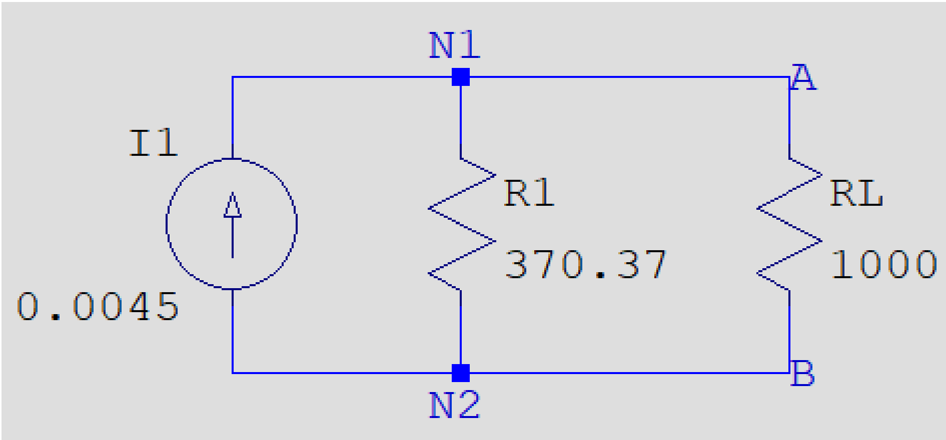
5.3 Schematic

The schematic for the Norton-equivalent circuit is as follows:



6 Adding a Load Resistor to the Norton-Equivalent Circuit

To verify that this circuit is also equivalent to both the original circuit and the Thevenin-equivalent circuit, we will be adding a load resistance of $R_L = 1000\,\Omega$, just like the Thevenin-equivalent circuit. The schematic for this is as follows:



The equivalent resistance of the two resistors is simply $R_1 \parallel R_L$, or $270.27\,\Omega$. The total current going through the circuit is given by I_1, which is 4.5 mA, so the voltage across R_1 and R_L is $4.5\,\text{mA} \cdot 270.27\,\Omega = 1.216\,\text{V}$. The current going though R_l is $1.216\,\text{V}/1\,\text{k}\Omega = 1.216\,\text{mA}$. As it is clear to see, these values corroborate with both the original and Thevenin-equivalent circuits.

7 Table of Values

This lab has clearly shown that both Thevenin- and Norton-equivalent circuits are indeed equivalent to the original circuit. I will end this report with a table compiling the results of all the hand-calculated values, as well as the values obtained by modeling every circuit in LTspice.

	Hand Calculations			LTspice Model		
	Original	Thevenin	Norton	Original	Thevenin	Norton
Voltage	1.217 V	1.216 V	1.216 V	1.216 22 V	1.216 46 V	1.216 22 V
Current	1.217 mA	1.216 mA	1.216 mA	1.216 22 mA	1.216 46 mA	1.216 22 mA