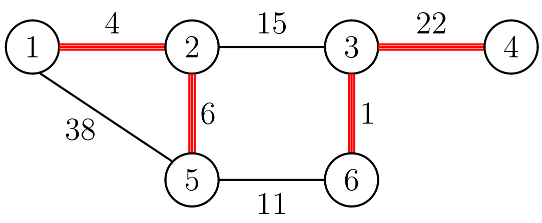
Advanced Algorithms
Exercises 4
DI – NOVA FCT

1. An *edge cover* of an undirected graph $G = (V, E)$ is a subset C of edges such that every vertex is an endpoint of some edge in C:

$$C \subseteq E \quad \text{and} \quad \forall\, v \in V : \ \exists\, (i, j) \in C : \ (\ i = v \ \text{ or } \ j = v\ )\,.$$

The **Minimum Weighted Edge Cover Problem** can be stated as follows. Given an undirected graph $G = (V, E)$ with a weight w_e associated with each edge $e \in E$, find an edge cover with minimum weight. The weight of a set of edges $S \subseteq E$ is $w(S) = \sum_{e \in S} w_e$.

For example, $\{(1, 2), (2, 5), (3, 4), (3, 6)\}$ is an optimal solution of the instance depicted in the figure below (whose weight is 33).



Apply the **primal-dual** method for designing an approximation algorithm for this problem. To this end, perform the following steps:

(a) Formulate the Minimum Weighted Edge Cover Problem as an Integer Programming Problem: specify the variables, the constraints and the objective function of the IP instance that corresponds to ($G = (V, E)$, $\{w_e\}_{e \in E}$). Denote that instance by **(IP)**.

(b) What is the instance of the Linear Programming Problem that corresponds to **(IP)**? Denote that instance by **(P)**.

(c) What is the dual **(D)** of **(P)**?

(d) Implement (in pseudo-code) the algorithm designed by applying the primal-dual method.

(e) What is the time complexity of your algorithm?

(f) Does your algorithm compute an edge cover? Justify your answer.

(g) What is the approximation ratio of your algorithm? Justify your answer.