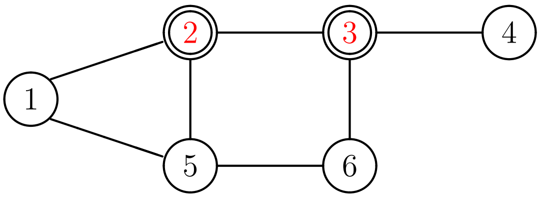
Advanced Algorithms
Exercises 3
DI – NOVA FCT

1. A *dominating set* of an undirected graph $G = (V, E)$ is a subset D of vertices such that every vertex not in D is adjacent to some vertex in D:

$$D \subseteq V \quad \text{and} \quad \forall\, v \in V : \quad v \notin D \Rightarrow (\exists\, w \in D) \;\; (v, w) \in E\,.$$

The **Minimum Dominating Set Problem** can be stated as follows. Given an undirected graph, find a dominating set with minimum size.

For example, $\{2, 3\}$ is an optimal solution of the instance depicted in the figure below.



Apply the **linear programming and rounding** technique for designing an approximation algorithm for this problem. To this end, consider that $G = (V, E)$ is the given undirected graph and perform the following steps:

(a) Formulate the Minimum Dominating Set Problem as an Integer Programming Problem: specify the variables, the constraints and the objective function of the IP instance that corresponds to $G = (V, E)$. Denote that instance by **(IP)**.

(b) Which instance of the Linear Programming Problem would be solved? Denote that instance by **(LP)**.

(c) What is the relation between the value of an optimal solution $\overline{x}^*$ for **(IP)** and the value of an optimal solution $\overline{y}^*$ for **(LP)**?

(d) Prove that **(LP)** is feasible, i.e. there is a solution for **(LP)**.

(e) Prove that **(LP)** is bounded, i.e. there is a number b such that, for every solution $\overline{y}$, $\text{value}(\overline{y}) \geq b$.

(f) Which rounding rule would you define? Prove that, for every optimal solution $\overline{y}^*$ for **(LP)**, the rounding $\overline{x}$ computed from $\overline{y}^*$ with your rule is a solution for **(IP)**.

(g) What is the approximation ratio of the algorithm? Justify your answer.