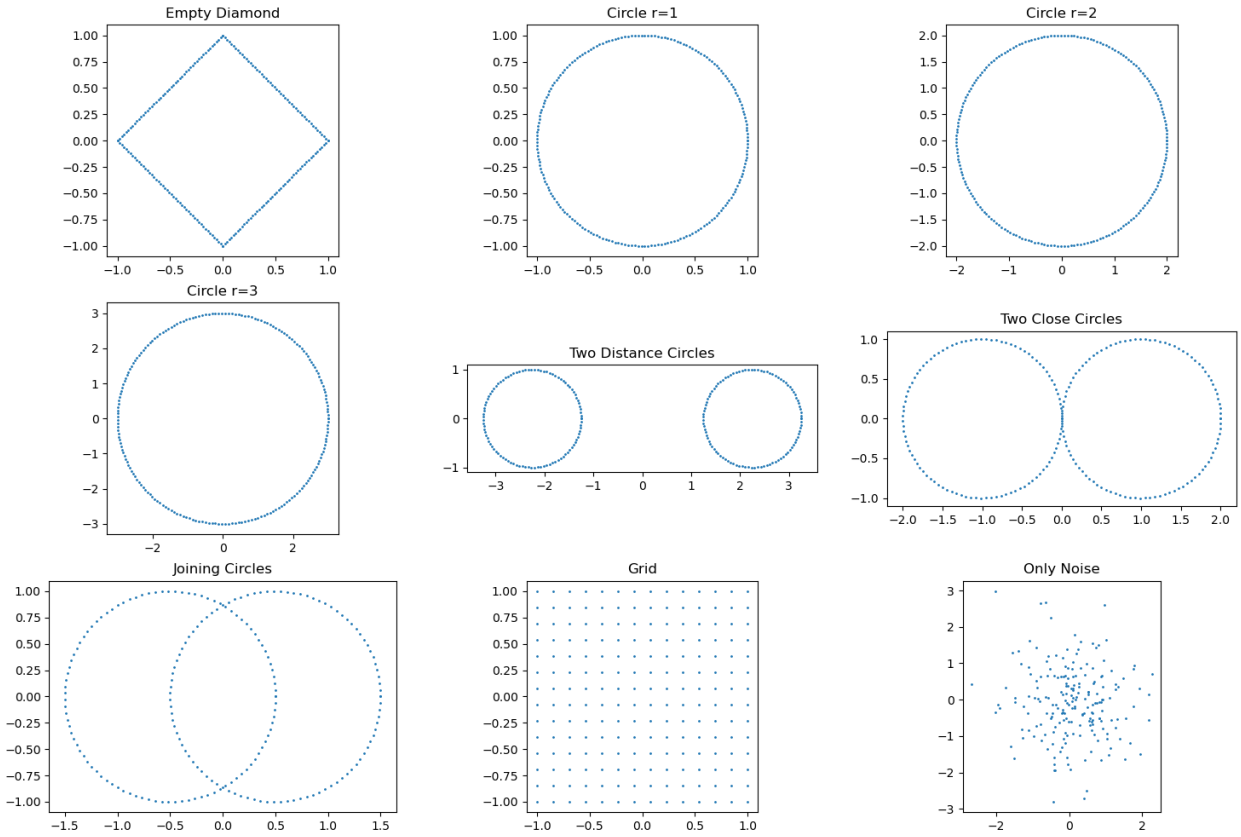
```
import numpy as np
import gudhi as gd
from gudhi.wasserstein import wasserstein_distance
from gudhi.wasserstein.barycenter import lagrangian_barycenter
import matplotlib.pyplot as plt
from scipy.spatial import distance_matrix
import pandas as pd
import seaborn as sns
from scipy.cluster.hierarchy import dendrogram, linkage
from scipy.spatial.distance import squareform
```

## Data Clouds

```
def create_circle(radius, num_points):
    angles = np.linspace(0, 2 * np.pi, num_points)
    x = radius * np.cos(angles)
    y = radius * np.sin(angles)
    return np.vstack((x, y)).T

def create_grid(num_points):
    side_points = int(np.sqrt(num_points))
    x = np.linspace(-1, 1, side_points)
    y = np.linspace(-1, 1, side_points)
    xx, yy = np.meshgrid(x, y)
    return np.vstack((xx.ravel(), yy.ravel())).T

def create_two_circles(radius, num_points, distance):
    angles = np.linspace(0, 2 * np.pi, num_points // 2)
    x1 = radius * np.cos(angles) - distance / 2
    y1 = radius * np.sin(angles)
    x2 = radius * np.cos(angles) + distance / 2
    y2 = radius * np.sin(angles)
    x = np.concatenate((x1, x2))
    y = np.concatenate((y1, y2))
    return np.vstack((x, y)).T

def create_empty_diamond(num_points):
    side_points = num_points // 4
    x1 = np.linspace(-1, 0, side_points)
    y1 = np.linspace(0, 1, side_points)
    x2 = np.linspace(0, 1, side_points)
    y2 = np.linspace(1, 0, side_points)
    x3 = np.linspace(1, 0, side_points)
    y3 = np.linspace(0, -1, side_points)
    x4 = np.linspace(0, -1, side_points)
    y4 = np.linspace(-1, 0, side_points)
    x = np.concatenate((x1, x2, x3, x4))
    y = np.concatenate((y1, y2, y3, y4))
    return np.vstack((x, y)).T
```

```
def create_noise_point(num_points):
    x = np.random.normal(0, 1, num_points)
    y = np.random.normal(0, 1, num_points)
    return np.vstack((x, y)).T

# Parameters
radius = 1
num_points = 200

point_clouds={
    "Empty Diamond":create_empty_diamond(num_points),
    "Circle r=1":create_circle(radius, num_points),
    "Circle r=2":create_circle(2, num_points),
    "Circle r=3":create_circle(3, num_points),
    "Two Distance Circles":create_two_circles(radius, num_points,
4.5),
    "Two Close Circles":create_two_circles(radius, num_points, 2),
    "Joining Circles":create_two_circles(radius, num_points, 1),
    "Grid":create_grid(num_points),
    "Only Noise":create_noise_point(num_points)
}


# Plotting
plt.figure(figsize=(18, 12))
i=1
for name,pc in point_clouds.items():
    plt.subplot(3,3,i)
    i+=1
    plt.scatter(pc[:, 0], pc[:, 1], s=1)
    plt.title(name)
    plt.gca().set_aspect("equal")


plt.show()
```

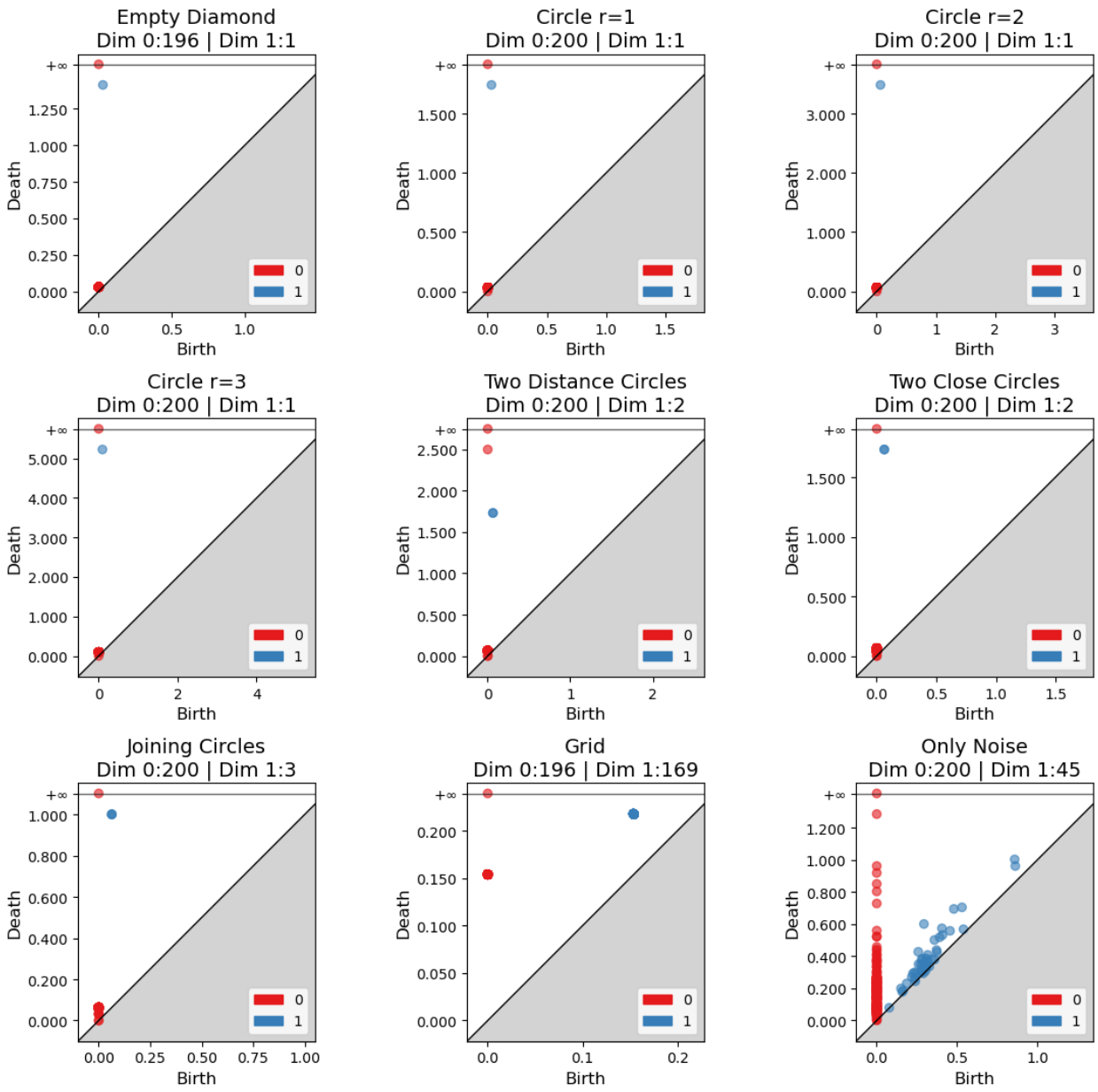



## Persistent Homology

```
### This function is from Giotto Library
def _ideal_thresh(dm, thresh):
    """Compute the enclosing radius of an input distance matrix.

    Under a Vietoris–Rips filtration, all homology groups are trivial
above
    this value because the complex becomes a cone.

    The enclosing radius is only computed if the input matrix is
square."""

    # Check that matrix is square
    if dm.shape[0] != dm.shape[1]:
        return thresh

    # Compute enclosing radius
    enclosing_radius = np.min(np.max(dm, axis=1))

    return min([enclosing_radius, thresh])

plt.figure(figsize=(12, 12))
i=1
barcodesDict={}
```

```
for name,pc in point_clouds.items():
    plt.subplot(3,3,i)
    i+=1

    max_thresh = _ideal_thresh(distance_matrix(pc, pc), np.inf)
    skeleton = gd.RipsComplex(points=pc, max_edge_length=max_thresh)
    simplex_tree = skeleton.create_simplex_tree(max_dimension=2)
    barcode = simplex_tree.persistence()

    diag_0 = simplex_tree.persistence_intervals_in_dimension(0)
    diag_1 = simplex_tree.persistence_intervals_in_dimension(1)
    barcodesDict[name]=(diag_0,diag_1)

    ax=plt.gca()
    gd.plot_persistence_diagram(barcode,axes=ax)
    plt.title(name+"\n"+"Dim 0:"+str(len(diag_0))+" | Dim
1:"+str(len(diag_1)),fontsize=14)
    plt.tight_layout()
    ax.set_aspect("equal")
    plt.xlabel('Birth', fontsize=12)
    plt.ylabel('Death', fontsize=12)

plt.suptitle("Take into account the axis scales \n Dim 0 and Dim 1
show the number of structures.\n",fontsize=22)
plt.tight_layout()
plt.show()
```

```
c:\Users\ragna\anaconda3\Lib\site-packages\gudhi\
persistence_graphical_tools.py:105: UserWarning: This function is not
available.
ModuleNotFoundError: No module named 'matplotlib'.
  warnings.warn(f"This function is not available.\
nModuleNotFoundError: No module named '{import_error.name}'.")
```



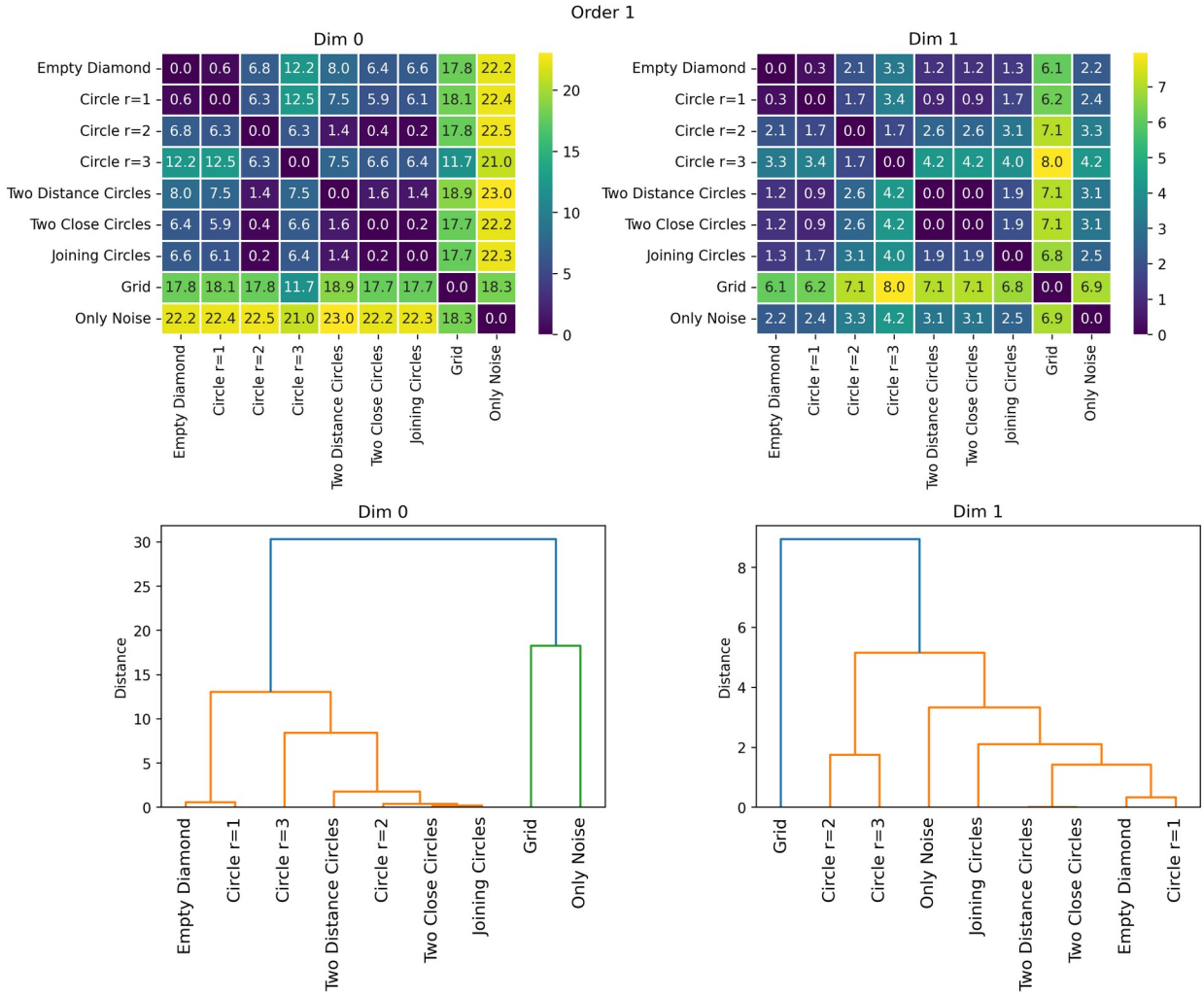

## Wasserstein Distance Matrix

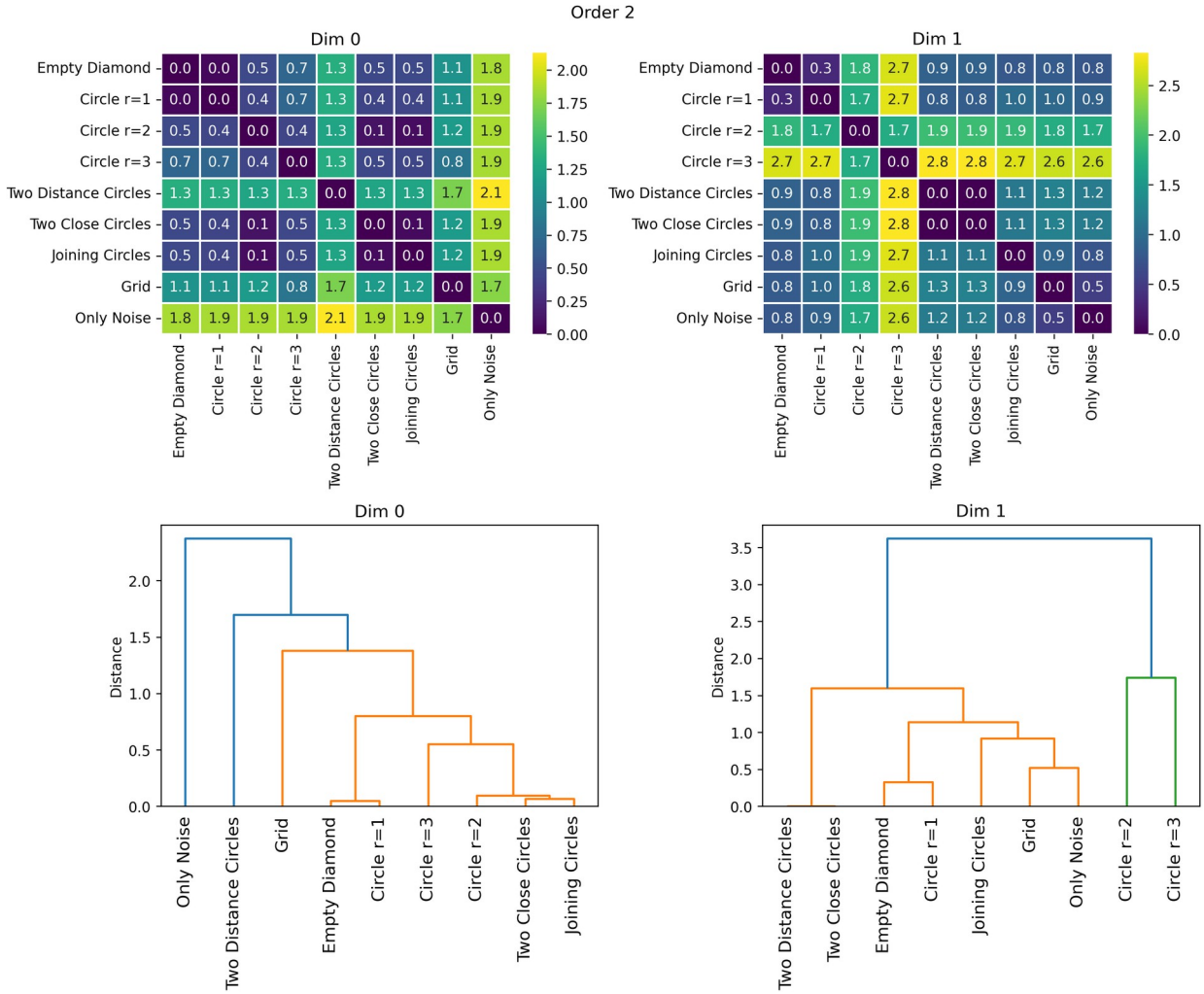
https://gudhi.inria.fr/python/latest/wasserstein_distance_user.html#

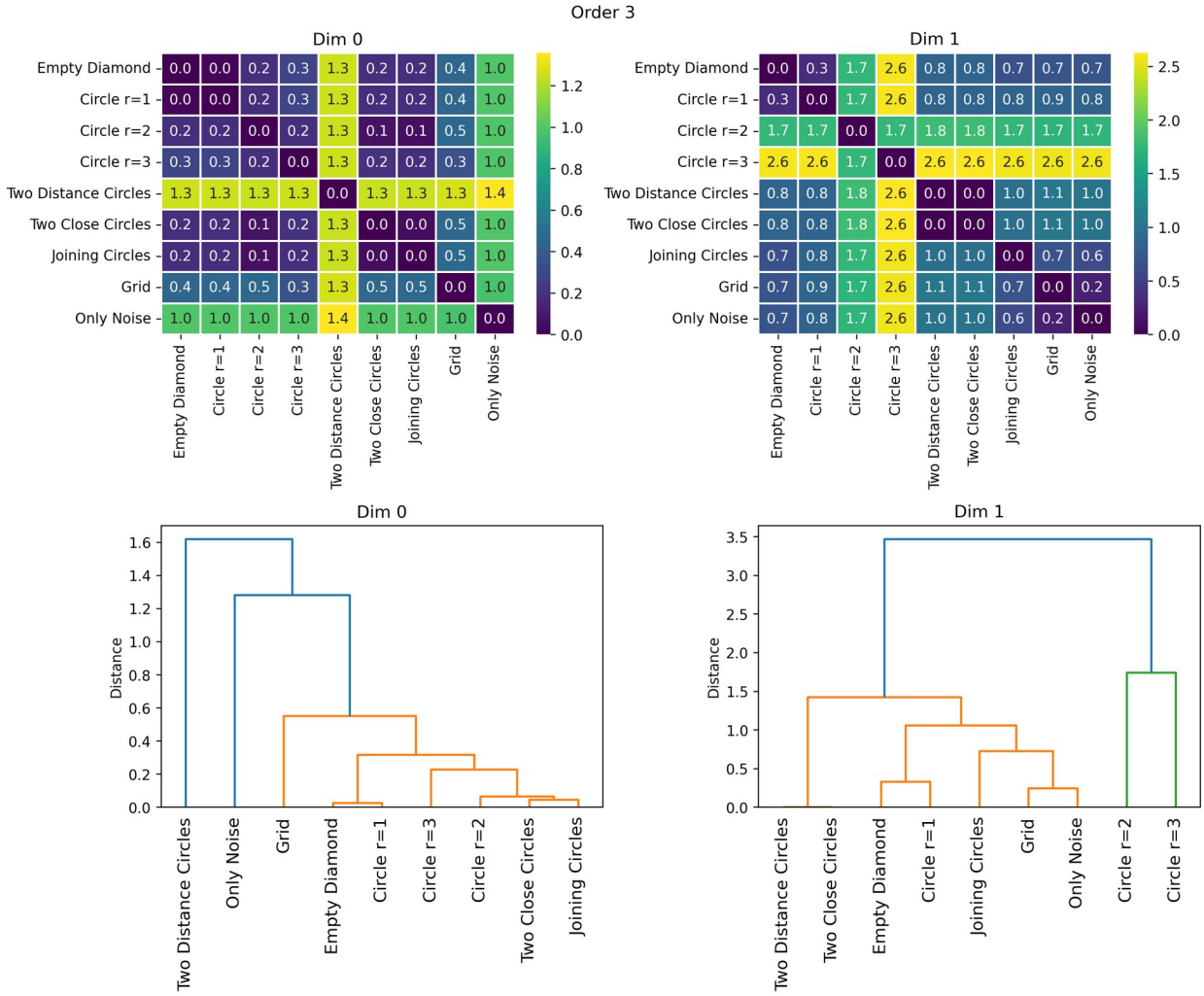
```
def wassersteinDistanceMatrix(barcode,dim,order):
    distanceMatrix={}
    for name1,_ in barcode.items():
        distanceMatrix[name1]={}
        for name2,_ in barcode.items():
```

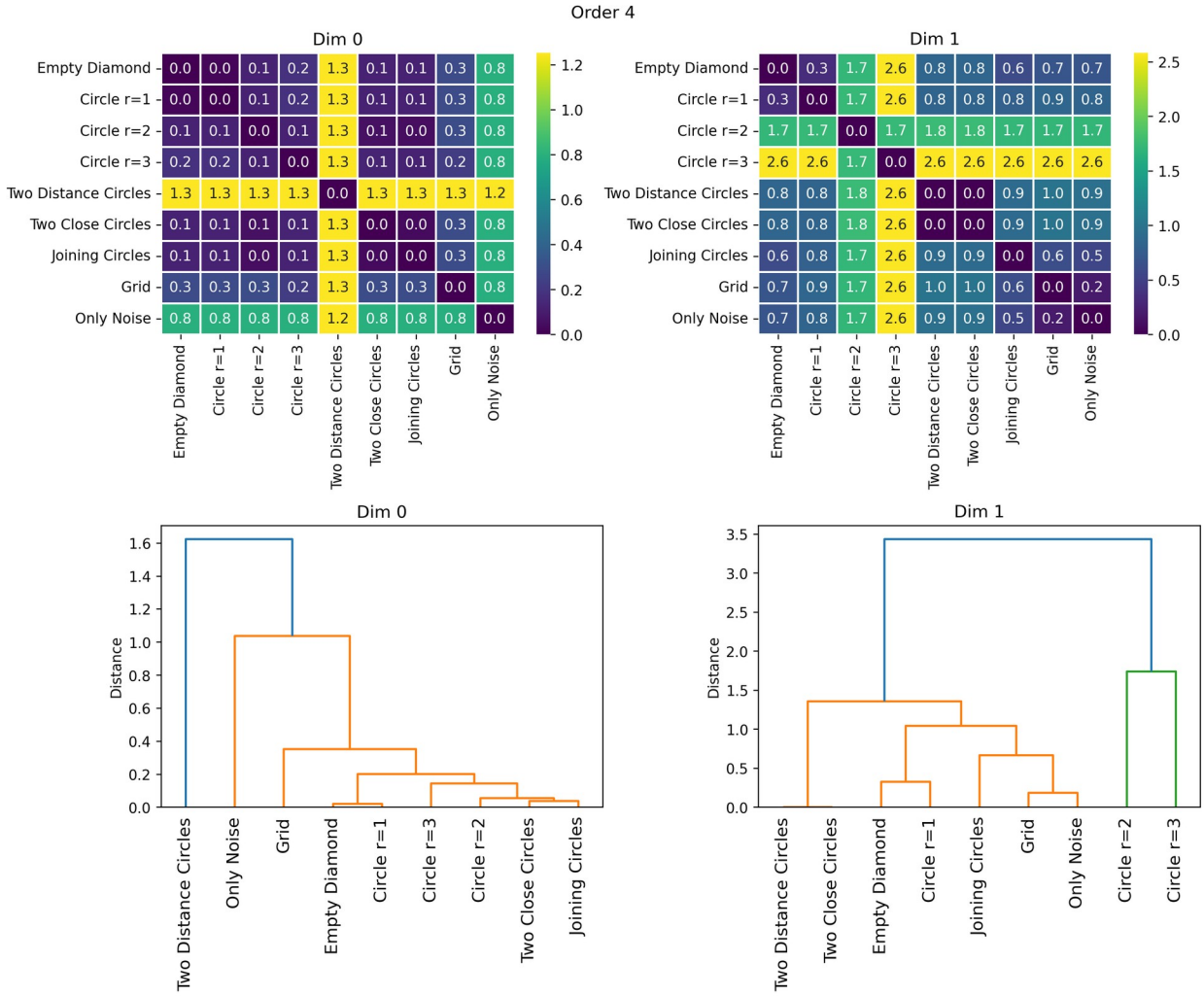
```
            distanceMatrix[name1]
[name2]=round(wasserstein_distance(barcode[name1][dim],barcode[name2]
[dim],order=order),5)
    return pd.DataFrame(distanceMatrix)

def dendrogramFromDM(df):
    # Convert the distance matrix to condensed form
    #https://stackoverflow.com/questions/13079563/how-does-condensed-
distance-matrix-work-pdist
    #https://stackoverflow.com/questions/41416498/dendrogram-or-other-
plot-from-distance-matrix
    condensed_distance_matrix = squareform(df)

    # Compute the linkage matrix
    linkage_matrix = linkage(condensed_distance_matrix,
method='ward',)

    # Dendogram
    dendo = dendrogram(
        linkage_matrix,
        labels=df.columns,
        leaf_rotation=90)

    return dendo

def plotHeatmapsAndDendrogram(df_0,df_1,title):
    plt.figure(figsize=(12,10),dpi=250)

    # Plot the Heatmaps
    plt.subplot(2,2,1)

sns.heatmap(df_0,cmap="viridis",linecolor="white",linewidths=1,fmt='.1
f',annot=True)
    plt.title("Dim 0")

    plt.subplot(2,2,2)

sns.heatmap(df_1,cmap="viridis",linecolor="white",linewidths=1,fmt='.1
f',annot=True)
    plt.title("Dim 1")

    # Plot the Dendrograms
    plt.subplot(2,2,3)
    dendrogramFromDM(df_0)
    plt.title('Dim 0')
    plt.ylabel('Distance')

    plt.subplot(2,2,4)
    dendrogramFromDM(df_1)
    plt.title('Dim 1')
```

```
    plt.ylabel('Distance')

    plt.suptitle(title)
    plt.tight_layout()
    plt.show()

#Testing diferent Orders
distanceMatrixDim0_df=wassersteinDistanceMatrix(barcodesDict,0,1)
distanceMatrixDim1_df=wassersteinDistanceMatrix(barcodesDict,1,1)
plotHeatmapsAndDendrogram(distanceMatrixDim0_df,distanceMatrixDim1_df,
"Order 1")


distanceMatrixDim0_df=wassersteinDistanceMatrix(barcodesDict,0,2)
distanceMatrixDim1_df=wassersteinDistanceMatrix(barcodesDict,1,2)
plotHeatmapsAndDendrogram(distanceMatrixDim0_df,distanceMatrixDim1_df,
"Order 2")

distanceMatrixDim0_df=wassersteinDistanceMatrix(barcodesDict,0,3)
distanceMatrixDim1_df=wassersteinDistanceMatrix(barcodesDict,1,3)
plotHeatmapsAndDendrogram(distanceMatrixDim0_df,distanceMatrixDim1_df,
"Order 3")

distanceMatrixDim0_df=wassersteinDistanceMatrix(barcodesDict,0,4)
distanceMatrixDim1_df=wassersteinDistanceMatrix(barcodesDict,1,4)
plotHeatmapsAndDendrogram(distanceMatrixDim0_df,distanceMatrixDim1_df,
"Order 4")
```


Order 1
Dim 0
Empty Diamond 0.0 0.6 6.8 12.2 8.0 6.4 6.6 17.8 22.2
Circle r=1 0.6 0.0 6.3 12.5 7.5 5.9 6.1 18.1 22.4
Circle r=2 6.8 6.3 0.0 6.3 1.4 0.4 0.2 17.8 22.5
Circle r=3 12.2 12.5 6.3 0.0 7.5 6.6 6.4 11.7 21.0
Two Distance Circles 8.0 7.5 1.4 7.5 0.0 1.6 1.4 18.9 23.0
Two Close Circles 6.4 5.9 0.4 6.6 1.6 0.0 0.2 17.7 22.2
Joining Circles 6.6 6.1 0.2 6.4 1.4 0.2 0.0 17.7 22.3
Grid 17.8 18.1 17.8 11.7 18.9 17.7 17.7 0.0 18.3
Only Noise 22.2 22.4 22.5 21.0 23.0 22.2 22.3 18.3 0.0
Empty Diamond
Circle r=1
Circle r=2
Circle r=3
Two Distance Circles
Two Close Circles
Joining Circles
Grid
Only Noise
0 5 10 15 20
Dim 1
Empty Diamond 0.0 0.3 2.1 3.3 1.2 1.2 1.3 6.1 2.2
Circle r=1 0.3 0.0 1.7 3.4 0.9 0.9 1.7 6.2 2.4
Circle r=2 2.1 1.7 0.0 1.7 2.6 2.6 3.1 7.1 3.3
Circle r=3 3.3 3.4 1.7 0.0 4.2 4.2 4.0 8.0 4.2
Two Distance Circles 1.2 0.9 2.6 4.2 0.0 0.0 1.9 7.1 3.1
Two Close Circles 1.2 0.9 2.6 4.2 0.0 0.0 1.9 7.1 3.1
Joining Circles 1.3 1.7 3.1 4.0 1.9 1.9 0.0 6.8 2.5
Grid 6.1 6.2 7.1 8.0 7.1 7.1 6.8 0.0 6.9
Only Noise 2.2 2.4 3.3 4.2 3.1 3.1 2.5 6.9 0.0
Empty Diamond
Circle r=1
Circle r=2
Circle r=3
Two Distance Circles
Two Close Circles
Joining Circles
Grid
Only Noise
0 1 2 3 4 5 6 7
Dim 0
Distance
0 5 10 15 20 25 30
Empty Diamond
Circle r=1
Circle r=3
Two Distance Circles
Circle r=2
Two Close Circles
Joining Circles
Grid
Only Noise
Dim 1
Distance
0 2 4 6 8
Grid
Circle r=2
Circle r=3
Only Noise
Joining Circles
Two Distance Circles
Two Close Circles
Empty Diamond
Circle r=1


Order 2
Dim 0
Empty Diamond 0.0 0.0 0.5 0.7 1.3 0.5 0.5 1.1 1.8
Circle r=1 0.0 0.0 0.4 0.7 1.3 0.4 0.4 1.1 1.9
Circle r=2 0.5 0.4 0.0 0.4 1.3 0.1 0.1 1.2 1.9
Circle r=3 0.7 0.7 0.4 0.0 1.3 0.5 0.5 0.8 1.9
Two Distance Circles 1.3 1.3 1.3 1.3 0.0 1.3 1.3 1.7 2.1
Two Close Circles 0.5 0.4 0.1 0.5 1.3 0.0 0.1 1.2 1.9
Joining Circles 0.5 0.4 0.1 0.5 1.3 0.1 0.0 1.2 1.9
Grid 1.1 1.1 1.2 0.8 1.7 1.2 1.2 0.0 1.7
Only Noise 1.8 1.9 1.9 1.9 2.1 1.9 1.9 1.7 0.0
2.00 1.75 1.50 1.25 1.00 0.75 0.50 0.25 0.00
Empty Diamond Circle r=1 Circle r=2 Circle r=3 Two Distance Circles Two Close Circles Joining Circles Grid Only Noise
Dim 1
Empty Diamond 0.0 0.3 1.8 2.7 0.9 0.9 0.8 0.8 0.8
Circle r=1 0.3 0.0 1.7 2.7 0.8 0.8 1.0 1.0 0.9
Circle r=2 1.8 1.7 0.0 1.7 1.9 1.9 1.9 1.8 1.7
Circle r=3 2.7 2.7 1.7 0.0 2.8 2.8 2.7 2.6 2.6
Two Distance Circles 0.9 0.8 1.9 2.8 0.0 0.0 1.1 1.3 1.2
Two Close Circles 0.9 0.8 1.9 2.8 0.0 0.0 1.1 1.3 1.2
Joining Circles 0.8 1.0 1.9 2.7 1.1 1.1 0.0 0.9 0.8
Grid 0.8 1.0 1.8 2.6 1.3 1.3 0.9 0.0 0.5
Only Noise 0.8 0.9 1.7 2.6 1.2 1.2 0.8 0.5 0.0
2.5 2.0 1.5 1.0 0.5 0.0
Empty Diamond Circle r=1 Circle r=2 Circle r=3 Two Distance Circles Two Close Circles Joining Circles Grid Only Noise
Dim 0
Distance
2.0 1.5 1.0 0.5 0.0
Only Noise Two Distance Circles Grid Empty Diamond Circle r=1 Circle r=3 Circle r=2 Two Close Circles Joining Circles
Dim 1
Distance
3.5 3.0 2.5 2.0 1.5 1.0 0.5 0.0
Two Distance Circles Two Close Circles Empty Diamond Circle r=1 Joining Circles Grid Only Noise Circle r=2 Circle r=3


Order 3
Dim 0
Empty Diamond 0.0 0.0 0.2 0.3 1.3 0.2 0.2 0.4 1.0
Circle r=1 0.0 0.0 0.2 0.3 1.3 0.2 0.2 0.4 1.0
Circle r=2 0.2 0.2 0.0 0.2 1.3 0.1 0.1 0.5 1.0
Circle r=3 0.3 0.3 0.2 0.0 1.3 0.2 0.2 0.3 1.0
Two Distance Circles 1.3 1.3 1.3 1.3 0.0 1.3 1.3 1.3 1.4
Two Close Circles 0.2 0.2 0.1 0.2 1.3 0.0 0.0 0.5 1.0
Joining Circles 0.2 0.2 0.1 0.2 1.3 0.0 0.0 0.5 1.0
Grid 0.4 0.4 0.5 0.3 1.3 0.5 0.5 0.0 1.0
Only Noise 1.0 1.0 1.0 1.0 1.4 1.0 1.0 1.0 0.0
Empty Diamond
Circle r=1
Circle r=2
Circle r=3
Two Distance Circles
Two Close Circles
Joining Circles
Grid
Only Noise
0.0 0.2 0.4 0.6 0.8 1.0 1.2
Dim 1
Empty Diamond 0.0 0.3 1.7 2.6 0.8 0.8 0.7 0.7 0.7
Circle r=1 0.3 0.0 1.7 2.6 0.8 0.8 0.8 0.9 0.8
Circle r=2 1.7 1.7 0.0 1.7 1.8 1.8 1.7 1.7 1.7
Circle r=3 2.6 2.6 1.7 0.0 2.6 2.6 2.6 2.6 2.6
Two Distance Circles 0.8 0.8 1.8 2.6 0.0 0.0 1.0 1.1 1.0
Two Close Circles 0.8 0.8 1.8 2.6 0.0 0.0 1.0 1.1 1.0
Joining Circles 0.7 0.8 1.7 2.6 1.0 1.0 0.0 0.7 0.6
Grid 0.7 0.9 1.7 2.6 1.1 1.1 0.7 0.0 0.2
Only Noise 0.7 0.8 1.7 2.6 1.0 1.0 0.6 0.2 0.0
Empty Diamond
Circle r=1
Circle r=2
Circle r=3
Two Distance Circles
Two Close Circles
Joining Circles
Grid
Only Noise
0.0 0.5 1.0 1.5 2.0 2.5
Dim 0
Distance
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6
Two Distance Circles
Only Noise
Grid
Empty Diamond
Circle r=1
Circle r=3
Circle r=2
Two Close Circles
Joining Circles
Dim 1
Distance
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5
Two Distance Circles
Two Close Circles
Empty Diamond
Circle r=1
Joining Circles
Grid
Only Noise
Circle r=2
Circle r=3


Order 4
Dim 0
Empty Diamond 0.0 0.0 0.1 0.2 1.3 0.1 0.1 0.3 0.8
Circle r=1 0.0 0.0 0.1 0.2 1.3 0.1 0.1 0.3 0.8
Circle r=2 0.1 0.1 0.0 0.1 1.3 0.1 0.0 0.3 0.8
Circle r=3 0.2 0.2 0.1 0.0 1.3 0.1 0.1 0.2 0.8
Two Distance Circles 1.3 1.3 1.3 1.3 0.0 1.3 1.3 1.3 1.2
Two Close Circles 0.1 0.1 0.1 0.1 1.3 0.0 0.0 0.3 0.8
Joining Circles 0.1 0.1 0.0 0.1 1.3 0.0 0.0 0.3 0.8
Grid 0.3 0.3 0.3 0.2 1.3 0.3 0.3 0.0 0.8
Only Noise 0.8 0.8 0.8 0.8 1.2 0.8 0.8 0.8 0.0
Empty Diamond
Circle r=1
Circle r=2
Circle r=3
Two Distance Circles
Two Close Circles
Joining Circles
Grid
Only Noise
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Dim 1
Empty Diamond 0.0 0.3 1.7 2.6 0.8 0.8 0.6 0.7 0.7
Circle r=1 0.3 0.0 1.7 2.6 0.8 0.8 0.8 0.9 0.8
Circle r=2 1.7 1.7 0.0 1.7 1.8 1.8 1.7 1.7 1.7
Circle r=3 2.6 2.6 1.7 0.0 2.6 2.6 2.6 2.6 2.6
Two Distance Circles 0.8 0.8 1.8 2.6 0.0 0.0 0.9 1.0 0.9
Two Close Circles 0.8 0.8 1.8 2.6 0.0 0.0 0.9 1.0 0.9
Joining Circles 0.6 0.8 1.7 2.6 0.9 0.9 0.0 0.6 0.5
Grid 0.7 0.9 1.7 2.6 1.0 1.0 0.6 0.0 0.2
Only Noise 0.7 0.8 1.7 2.6 0.9 0.9 0.5 0.2 0.0
Empty Diamond
Circle r=1
Circle r=2
Circle r=3
Two Distance Circles
Two Close Circles
Joining Circles
Grid
Only Noise
2.5
2.0
1.5
1.0
0.5
0.0
Dim 0
Distance
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Two Distance Circles
Only Noise
Grid
Empty Diamond
Circle r=1
Circle r=3
Circle r=2
Two Close Circles
Joining Circles
Dim 1
Distance
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Two Distance Circles
Two Close Circles
Empty Diamond
Circle r=1
Joining Circles
Grid
Only Noise
Circle r=2
Circle r=3