Strategic Ad Allocation for Presidential Campaigns

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1 Presidential Super PAC Election

Our objective is to allocate \$100,000,000 optimally and effectively to win the 2024 presidential run for the Democratic Party.

The funds will be used to run social media and television ads across all 50 states and the District of Columbia.

2 Decision Variables

We will have decision variables $x_1, x_2, ..., x_{51}$, one per state. These variables correspond to how much money will be allocated for each state. For example, if Pennsylvania corresponds to variable x_5, having $x_5 = 250000$ means that we will spend \$250000 on ads for the state of Pennsylvania.

3 Objective Function

Since our main objective is to win the presidential race, we want to have as many electoral college votes as possible. Since every state has a different number of electoral votes, we will need to account for this in our objective function. Additionally, we might want to go after swing states more aggressively. Therefore, our objective function's coefficient will be a combination of electoral votes and how close the election is at the moment.

This would look something like the following:

Let e = number of electoral votes and v = current percent poll difference,

$max\ z = e_1(1 - v_1)x_1 + e_2(1 - v_2)x_2 + ... + e_{51}(1 - v_{51})x_{51}$
or
$max\ z = \sum_{i=1}^{51} e(1 - v)x_i$

We have that $(1 - v)$ term in the coefficient because we want to allocate more resources to swing states. A closer poll will have a smaller value of v.

4 The Problem

To set up the problem, we first need to assign states to variable names.

Let:

x_1 : Alabama
x_2 : Alaska
x_3 : Arizona
x_4 : Arkansas
x_5 : California
x_6 : Colorado
x_7 : Connecticut
x_8 : Delaware
x_9 : District of Columbia
x_{10} : Florida
x_{11} : Georgia
x_{12} : Hawaii
x_{13} : Idaho
x_{14} : Illinois
x_{15} : Indiana
x_{16} : Iowa
x_{17} : Kansas
x_{18} : Kentucky
x_{19} : Louisiana
x_{20} : Maine
x_{21} : Maryland
x_{22} : Massachusetts
x_{23} : Michigan
x_{24} : Minnesota
x_{25} : Mississippi
x_{26} : Missouri
x_{27} : Montana
x_{28} : Nebraska
x_{29} : Nevada
x_{30} : New Hampshire
x_{31} : New Jersey
x_{32} : New Mexico
x_{33} : New York
x_{34} : North Carolina
x_{35} : North Dakota
x_{36} : Ohio
x_{37} : Oklahoma
x_{38} : Oregon
x_{39} : Pennsylvania
x_{40} : Rhode Island
x_{41} : South Carolina
x_{42} : South Dakota
x_{43} : Tennessee
x_{44} : Texas
x_{45} : Utah
x_{46} : Vermont
x_{47} : Virginia
x_{48} : Washington
x_{49} : West Virginia
x_{50} : Wisconsin
x_{51} : Wyoming

Now, we need to obtain the number of electoral votes per state. This number is fixed and won't change during the election process. This gives us the first part of our objective function z:

$$
\begin{aligned}
z = {} & 9x_1 + 3x_2 + 11x_3 + 6x_4 + 54x_5 + 10x_6 + 7x_7 + 3x_8 + 3x_9 + 30x_{10} \\
& + 16x_{11} + 4x_{12} + 4x_{13} + 19x_{14} + 11x_{15} + 6x_{16} + 6x_{17} + 8x_{18} \\
& + 8x_{19} + 4x_{20} + 10x_{21} + 11x_{22} + 15x_{23} + 10x_{24} + 6x_{25} + 10x_{26} \\
& + 4x_{27} + 5x_{28} + 6x_{29} + 4x_{30} + 14x_{31} + 5x_{32} + 28x_{33} + 16x_{34} \\
& + 3x_{35} + 17x_{36} + 7x_{37} + 8x_{38} + 19x_{39} + 4x_{40} + 9x_{41} + 3x_{42} \\
& + 11x_{43} + 40x_{44} + 6x_{45} + 3x_{46} + 13x_{47} + 12x_{48} + 4x_{49} + 10x_{50} \\
& + 3x_{51}
\end{aligned}
$$

We need to add the current percent poll difference to our equation. This variable value might change as we approach the election, but we're taking the data available as of October 7th.

This gives us our full objective function z:

$$\begin{aligned}
z = {} & 9 \cdot 0.76x_1 + 3 \cdot 0.91x_2 + 11 \cdot 0.986x_3 + 6 \cdot 0.85x_4 + 54 \cdot 0.751x_5 \\
& + 10 \cdot 0.9x_6 + 7 \cdot 0.84x_7 + 3 \cdot 0.83x_8 + 3 \cdot 0.15x_9 + 30 \cdot 0.975x_{10} \\
& + 16 \cdot 0.988x_{11} + 4 \cdot 0.657x_{12} + 4 \cdot 0.819x_{13} + 19 \cdot 0.84x_{14} + 11 \cdot 0.892x_{15} \\
& + 6 \cdot 0.987x_{16} + 6 \cdot 0.871x_{17} + 8 \cdot 0.843x_{18} + 8 \cdot 0.794x_{19} + 4 \cdot 0.87x_{20} \\
& + 10 \cdot 0.686x_{21} + 11 \cdot 0.642x_{22} + 15 \cdot 0.921x_{23} + 10 \cdot 0.908x_{24} + 6 \cdot 0.841x_{25} \\
& + 10 \cdot 0.92x_{26} + 4 \cdot 0.956x_{27} + 5 \cdot 0.962x_{28} + 6 \cdot 0.947x_{29} + 4 \cdot 0.889x_{30} \\
& + 14 \cdot 0.796x_{31} + 5 \cdot 0.883x_{32} + 28 \cdot 0.706x_{33} + 16 \cdot 0.982x_{34} + 3 \cdot 0.827x_{35} \\
& + 17 \cdot 0.992x_{36} + 7 \cdot 0.77x_{37} + 8 \cdot 0.787x_{38} + 19 \cdot 0.953x_{39} + 4 \cdot 0.694x_{40} \\
& + 9 \cdot 0.929x_{41} + 3 \cdot 0.846x_{42} + 11 \cdot 0.863x_{43} + 40 \cdot 0.989x_{44} + 6 \cdot 0.902x_{45} \\
& + 3 \cdot 0.613x_{46} + 13 \cdot 0.882x_{47} + 12 \cdot 0.77x_{48} + 4 \cdot 0.714x_{49} + 10 \cdot 0.916x_{50} \\
& + 3 \cdot 0.685x_{51}
\end{aligned}$$

5 Constraints

5.1 First Constraint

Since we know we have $100,000,000 to spend, we can have our first constraint:

$\sum_{i=1}^{51} x_i \leq 100,000,000$

5.2 Second Constraint

We also want to make sure we are giving some money to every state, even if we consider it a lost state.

$$\begin{aligned}
x_i \geq 100,000 \quad i = 1, 2, ..., 51 \\
\mathbf{x} \geq \mathbf{100{,}000} \\
I\mathbf{x} \geq \mathbf{100{,}000}
\end{aligned}$$

5.3 Third Constraint

At the same time, we don't want to give all the money to a single state. With our current setup, our linear optimization problem might give all $100,000,000 to a state with a high number of electoral votes and a close poll difference. For this reason, we set an upper bound on every state to ensure that the spending on individual states does not exceed $15,000,000.

$$
\begin{aligned}
x_i \leq 15,000,000 \quad i = 1, 2, ..., 51 \\
\mathbf{x} \leq \mathbf{15{,}000{,}000} \\
I\mathbf{x} \leq \mathbf{15{,}000{,}000}
\end{aligned}
$$

5.4 Fourth Constraint

Around 70% of the total budget is spent on 7 key states and there are 7 swing states that could sway the election this year [1]. The following are those states: Michigan, Wisconsin, Pennsylvania, North Carolina, Georgia, Arizona, Nevada. So we add a constraint where the sum total spending of all of the swing states is equal to $70,000,000 so we make sure that those swing states are getting enough ad money.

$$x_{23} + x_{50} + x_{39} + x_{34} + x_{11} + x_3 + x_{29} \leq 70,000,000$$

6 Results

We used pulp, a Python library to solve our linear programming problem. This method allows us to find the optimal feasible solution, and print out how much money we are spending for each state in the terminal.

The results from our program were:

Alabama : $100,000
Alaska : $100,000
Arizona : $9,800,000
Arkansas : $100,000
California : $15,000,000
Colorado : $100,000
Connecticut : $100,000
Delaware : $100,000
District of Columbia : $100,000
Florida : $100,000
Georgia : $15,000,000
Hawaii : $100,000
Idaho : $100,000
Illinois : $100,000
Indiana : $100,000
Iowa : $100,000
Kansas : $100,000
Kentucky : $100,000
Louisiana : $100,000
Maine : $100,000

[1] `https://www.npr.org/2024/05/24/nx-s1-4980821/ad-spending-presidential-election-biden-trump`

Maryland : \$100,000
Massachusetts : \$100,000
Michigan : \$15,000,000
Minnesota : \$100,000
Mississippi : \$100,000
Missouri : \$100,000
Montana : \$100,000
Nebraska : \$100,000
Nevada : \$100,000
New Hampshire : \$100,000
New Jersey : \$100,000
New Mexico : \$100,000
New York : \$100,000
North Carolina : \$15,000,000
North Dakota : \$100,000
Ohio : \$100,000
Oklahoma : \$100,000
Oregon : \$100,000
Pennsylvania : \$15,000,000
Rhode Island : \$100,000
South Carolina : \$100,000
South Dakota : \$100,000
Tennessee : \$100,000
Texas : \$10,800,000
Utah : \$100,000
Vermont : \$100,000
Virginia : \$100,000
Washington : \$100,000
West Virginia : \$100,000
Wisconsin : \$100,000
Wyoming : \$100,000

Objective Value: 2,124,260,200

These results aligned with our constraints, as swing states and states with high electoral votes received most of the spending. Swing states such as **Georgia** (\$15 million) and **Pennsylvania** (\$15 million) that are close in poll difference are given the maximum budget, and swing states like **Arizona** (\$9.8 million), that are further away in the polls are given less while still satisfying the \$70 million spending constraint.

Strongly leaning states with large electoral influence such as **California** (\$15 million), still receive a large amount of funding since the amount of electoral votes makes states like **California** strategically important.

We are spending the minimum amount of \$100,000 in other states like **Alaska** and **Hawaii**, which have less influence on the election or are less likely to change from increased spending due to a firm lean towards one side.

In our optimal feasible solution, all \$100,000,000 were spent and we calculated the optimal value of our problem to be: 2,124,260,200. We can interpret this optimal value as the total potential impact of the campaign. It factors how close the race is and how many electoral votes are at stake.

A large result indicates that following this campaign strategy could increase the chance of winning critical swing states and potentially winning the election.